

THEMA Working Paper n°2018-14
Université de Cergy-Pontoise, France

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Inheritance taxation with agents differing in altruism and productivity. *

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July 2, 2018

Abstract

We analyze a shift from capital income tax towards inheritance tax in a two-period overlapping generation model with rational altruism *à la* Barro, where the population consists of two types of dynasties that differ in altruism and productivity. The tax reform is implemented in a way that leaves the capital-labor ratio constant at steady state. It increases welfare of less altruistic dynasties, but decreases welfare of the most altruistic one. We then extend the model by considering time transfers from the old to the young generation and assuming that the young have elastic labor supply. We discuss the condition for the tax reform to be Pareto-improving in steady state.

Keywords: altruism, bequests, heterogeneity, inheritance tax, redistribution.

JEL Classifications: D64, H22, H24, J22.

*This research has been conducted as part of the project Labex MME-DII (ANR11-LBX-0023-01)

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1 Introduction

Standard models with infinite horizon agents do not make much difference between capital income tax and inheritance tax leading to optimal values equal to zero in the long run (see Chamley (1986, p. 613)). Most of these models consider homogeneous households. However, the distribution of inheritances is highly concentrated in most developed countries and only concerns a part of the population while life cycle savings concern everyone. According to Piketty (2010), in France in 2010, the bottom 50% poorest with respect to inherited wealth received about 5% of aggregate bequests whereas the top 10% richest received about 60%. In addition, one quarter of total bequests is transmitted to the top 1% while a third of deceased people leave no bequests. Hence, these strong disparities in terms of bequests lead to intragenerational inequalities. From this point of view, inheritance tax could play a crucial role in reducing inequality, in a society where dynasties do not have the same accumulation behavior. Another positive argument for stepping inheritance tax against capital income tax, is that it promotes fairness since it concerns unearned resources which does not compensate any effort or work.

With these elements in mind, this paper analyzes whether a shift from capital income tax towards inheritance tax may be welfare enhancing for infinite horizon households (through rational altruism *à la* Barro (1974)), in an economy where bequests are concentrated on a part of population. We consider for this purpose that dynasties have different degrees of altruism, meaning that households within generation care differently about their descendants (see Michel and Pestieau (1998) and Vidal (1996)).

Theoretical literature on rational altruism *à la* Barro (1974) with intragenerational heterogeneity suggests that redistributive incidence of inheritance taxation is likely to worsen welfare of every household even those who behave like life-cyclers. As shown by Michel and Pestieau (2005), if households have homothetic preferences, a uniform lump-sum transfer financed through inheritance tax reduces the steady-state welfare of all dynasties which differ in altruism degree and in productivity. Although inheritance taxation allows to redistribute wealth, the distortive effect of inheritance tax concerning households choice on bequests pushes down the steady-state capital-labor ratio. The fall in capital-labor ratio affects negatively the consumption of all dynasties which results in a negative impact of the tax reform on steady-state households welfare.

Considering a shift from capital income taxation towards inheritance taxation allows to attenuate the fall in the capital-labor ratio. Indeed, this tax reform leads to opposite effects of both taxes on its steady state value: a fall in capital income tax increases the capital-labor ratio, while raising inheritance tax decreases it. In addition, the policy is redistributive since all households pay the capital income tax (in a two-period life cycle model) while inheritance is paid only by dynasties that leave bequests.

We first show that at steady state, a switch from capital income taxation to inheritance taxation leaving constant the capital-labor ratio, is necessarily welfare enhancing for every altruistic dynasty,

except for the most altruistic one. We consider the same kind of framework as Michel and Pestieau (2005): a two-period overlapping generation model with rational altruism *à la* Barro (1974) characterized by two types of dynasties which differ in terms of altruism degree and of human capital level. Both dynasties consume in both periods, work during the first one and then retire in the second period of life. The government finances public spending and a uniform lump-sum transfer using taxes on inheritance, capital income and labor income. We show that the tax reform cannot improve the welfare of agents who do not leave bequests without reducing utility of agents who leave bequests. The main reason is that keeping the capital-labor ratio constant with inelastic labor supply involves constant disposable resources in steady state.

We then propose an extension of the model that allows to modify the household's disposable resources at steady state, even if the capital-labor ratio is constant. We consider a model with elastic labor supply of the young, taking account of family time transfers from old to young adults that may allow the inheritance tax to increase labor supply of the young. Indeed, considering family time transfers, which are substantial,¹ introduces a trade-off between bequests and time transfers since both transfers differently affect the labor supply of the recipient and his life-cycle resources. The model that we are going to consider has many similarities with Cardia and Ng (2003) and Cardia and Michel (2004). In this context, taxing bequests makes time transfers more attractive since inheritance tax affects the trade-off between both family transfers. As shown by Belan and Moussault (2018) in a model with homogeneous agents, the positive effect on time transfers may increase the young's labor supply which attenuates or reverses the potential negative effect of inheritance tax on disposable resources. They show that a shift from capital income tax towards inheritance tax can increase steady-state welfare, depending on the magnitude of the effect of higher time transfers on the labor supply of the young. Consequently, with time transfers, constant capital-labor ratio does not imply constant disposable resources. In the present paper, considering heterogeneous dynasties, we analyze the impact of the tax reform on different dynasties taking into account both types of transfers (in time and money). We identify situations where tax reform can be Pareto-improving. Indeed, the negative effect of the tax reform on the most altruistic dynasty may be attenuated or reversed by the positive impact of the increase of time transfers on the young's labor supply. Thus, depending on the strength of this effect as well as the population distribution, we show that the tax reform can be Pareto improving at steady state.

The paper is organized as follows. Section 2 presents the model. In Section 3, we analyze the steady-state effect of tax reform on the welfare of the two types of dynasties. Then in Section 4, the model is extended to time transfers and we study tax reform impacts on both types of dynasties. Final Section concludes.

¹Numbers of empirical studies indicate that time transfers from parents to their children are on average almost as important as bequests in monetary equivalent in European countries and the United States, such as Schoeni et al. (1997), Cardia and Ng (2003), Attias-Donfut et al. (2005) and Wolff and Attias-Donfut (2007).

2 Equilibrium

2.1 Dynasties and generations

We consider a two-period overlapping generation model. Time is discrete. The population size is constant and normalized to unity. Each parent has only one child. We consider dynastic altruism *à la* Barro (1974) from parents to children. The economy consists of two types of dynasties (types 1 and 2). All agents that belong to the same type of dynasty i ($i \in \{1, 2\}$), whatever the generation, have the same degree of altruism β_i , and the same level of human capital h_i . We assume $0 \leq \beta_1 < \beta_2 < 1$. We define p_i as the proportion of type i 's agents in each generation: $0 < p_i < 1$ and $p_1 + p_2 = 1$.

2.2 Household behavior

An individual born in t that belongs to a type- i dynasty works in period t and retires in period $t + 1$. During its working life, he/she allocates income between market good consumption c_{it} and savings s_{it}

$$c_{it} + s_{it} = (1 - \tau^w) h_i w_t + (1 - \tau^x) x_{it} + a_t$$

where w_t is the real wage, x_{it} is the bequest received from his parent and a_t is a lump-sum transfer. Tax rates on labor income τ^w and bequest τ^x are assumed to be constant.

When retired, the individual divides return on savings between market good consumption d_{it+1} and bequest to his child x_{it+1} ,

$$d_{it+1} + x_{it+1} = (1 - \tau^R) R_{t+1} s_{it}$$

where R_{t+1} is the gross interest rate. The tax rate on capital income τ^R is also assumed to be constant.

Utility U_{it} of a type- i individual born in t is

$$U_{it} = u(c_{it}, d_{it+1}) + \beta_i U_{it+1}$$

where the lifetime utility function u is increasing in both arguments, strictly quasi-concave and satisfies Inada conditions. We also assume that both consumptions are normal goods.

It is assumed that parents cannot leave negative bequest to their children:

$$x_{it+1} \geq 0$$

If $\beta_i > 0$, optimality conditions with respect to s_{it} and x_{it+1} write

$$-u'_c(c_{it}, d_{it+1}) + (1 - \tau^R) R_{t+1} u'_d(c_{it}, d_{it+1}) = 0 \quad (1)$$

$$-u'_d(c_{it}, d_{it+1}) + \beta_i (1 - \tau^x) u'_c(c_{it+1}, d_{it+2}) \leq 0 \quad (= 0 \text{ if } x_{it+1} > 0) \quad (2)$$

If $\beta_1 = 0$, then type-1 dynasties consist of life-cyclers. Their bequest is zero and their saving satisfies (1).

2.3 Firms and production

The production sector consists in a representative firm that behaves competitively and combines capital K_t and efficient labor L_t to produce output. Technology $F(K_t, L_t)$ is linear homogenous. Profit maximization of the representative firm leads to equality between marginal products and real input prices

$$R_t = F_K(K_t, L_t) \text{ and } w_t = F_L(K_t, L_t) \quad (3)$$

assuming total depreciation of the capital stock in one period. F_K and F_L stand for the partial derivative of F with respect to capital and efficient labor.

2.4 Government

The government has to finance public spendings defined as a fraction of the production of the private sector, $\Gamma F(K_t, L_t)$, with $0 \leq \Gamma < 1$. Let Δ_t denote the public debt at the beginning of period t . The government budget constraint writes

$$\Delta_{t+1} + \tau^R \sum_i p_i R_t s_{it-1} + \tau^w L_t w_t + \tau^x \sum_i p_i x_{it} = R_t \Delta_t + a_t + \Gamma F(K_t, L_t) \quad (4)$$

Notice that production of the private sector creates an externality on public spendings. Capital and efficient labor, by increasing production, also increase public spendings and reduce product disposable for consumption and investment. Therefore, capital and labor demands have a social marginal product lower than the real input prices R_t and w_t .

2.5 Market equilibrium

In period t , the labor market equilibrium is given by

$$L_t = \sum_i p_i h_i = \bar{h}$$

where \bar{h} is the average productivity. The resource constraint in period t writes

$$(1 - \Gamma) F(K_t, \bar{h}) = \sum_i p_i c_{it} + \sum_i p_i d_{it} + K_{t+1} \quad (5)$$

The Walras' law implies equilibrium on the capital market is also satisfied:

$$K_{t+1} + \Delta_{t+1} = \sum_i p_i s_{it} \quad (6)$$

Indeed, it is obtained from the household budget constraints, the government budget constraint and linear homogeneity of the production function F , which implies $F(K_t, \bar{h}) = R_t K_t + w_t \bar{h}$.

The instruments considered allow the social planner to reach a Pareto optimum. Indeed, tax rates on capital income and labor income can be set in order to equalize social and private marginal products of capital and labor. It will be done with $\tau^R = \tau^w = \Gamma$.

Then, with a zero tax rate on bequest ($\tau^x = 0$), the government budget constraint for all $t \geq 0$ reduces to $\Delta_{t+1} = (1 - \Gamma) R_t \Delta_t + a_t$. This means that an initial public debt would be shared among generations and dynasties through uniform lump-sum tax a_t ($t \geq 0$). Inheritance tax would create inefficiency by distorting the household choice on bequests. Nevertheless, inheritance tax may help to reduce wealth inequalities in an economy where dynasties do not have the same accumulation behavior.

2.6 Steady state

As stressed by Becker (1980), Altig and Davis (1992), Vidal (1996), Michel and Pestieau (1998, 2005) and Nourry and Venditti (2001), the dynasties that leave positive bequests, at steady-state equilibrium, can only be those with the highest degree of altruism. Other dynasties behave as life-cyclers and accumulate no wealth. The same result applies in our model. At steady state, optimality conditions (1) and (2) imply

$$\beta_i (1 - \tau^x) (1 - \tau^R) R \leq 1 \quad (= 1 \text{ if } x_i > 0)$$

Since $\beta_1 < \beta_2$, the preceding condition implies that type-1 dynasties leave no bequest at steady state: $x_1 = 0$. As shown by Nourry and Venditti (2001), bequests of type-2 dynasties are positive iff the capital stock K_M consistent with the modified Golden-rule – *i.e.* the capital stock that satisfies $\beta_2 (1 - \tau^x) (1 - \tau^R) F_K(K_M, \bar{h}) = 1$ – is higher than savings that would be obtained if all agents were life-cyclers, with $R = F_K(K_M, \bar{h})$ and $w = F_L(K_M, \bar{h})$. We need to extend this result to take account of fiscal instruments.

In the following, we assume that the government chooses the tax instruments $(\tau^R, \tau^w, \tau^x, a)$. Then, considering situations with positive bequests of type-2 dynasties ($x_2 > 0$), a steady-state equilibrium

is a vector $(c_1, d_1, c_2, d_2, K_M, x_2, R_M, w_M)$ such that

$$\beta_2 (1 - \tau^x) (1 - \tau^R) R_M = 1 \quad (7)$$

$$MRS_i^{d/c} = \beta_2 (1 - \tau^x) \quad (8)$$

$$c_1 + \beta_2 (1 - \tau^x) d_1 = (1 - \tau^w) h_1 w_M + a \quad (9)$$

$$c_2 + \beta_2 (1 - \tau^x) d_2 = (1 - \tau^w) h_2 w_M + a + (1 - \tau^x) x_2 \quad (10)$$

$$w_M = F_L(K_M, \bar{h}), \quad \text{and} \quad R_M = F_K(K_M, \bar{h}) \quad (11)$$

$$\sum_i p_i (c_i + d_i) = (1 - \Gamma) F(K_M, \bar{h}) - K_M \quad (12)$$

where $MRS_i^{d/c} = u'_d(c_i, d_i)/u'_c(c_i, d_i)$ is the marginal rate of substitution of type- i between d and c , for $i = 1, 2$. The public debt Δ then results from the budget constraint of the government (4) at steady state:

$$[1 - (1 - \tau^R) R_M] \Delta = \Gamma F(K_M, \bar{h}) + a - \tau^R R_M K_M - \tau^w w_M \bar{h} - \tau^x p_2 x_2 \quad (13)$$

We get the following Lemma.

Lemma 1. *Assume there exists a capital stock K_M that corresponds to the modified Golden-rule, i.e. that satisfies equality (7). Consider an inheritance tax rate τ^x close to zero. The steady-state bequest of type-2 agents x_2 is positive iff*

$$K_M + \Delta > \sum_{i=1}^2 p_i s((1 - \tau^w) h_i w_M + a, 0, (1 - \tau^R) R_M) \quad (14)$$

where the public debt Δ satisfies equation (13).

Proof. Let us first define the following saving function

$$s(\omega_1, \omega_2, R) = \arg \max_z u(\omega_1 - z, \omega_2 + Rz)$$

Under the normal good assumption, s increases with ω_1 and decreases with ω_2 . The capital market equilibrium (6) can be rewritten as

$$\begin{aligned} K_M + \Delta &= p_1 s((1 - \tau^w) h_1 w_M + a, 0, (1 - \tau^R) R_M) \\ &+ p_2 s((1 - \tau^w) h_2 w_M + a + (1 - \tau^x) x_2, -x_2, (1 - \tau^R) R_M) \end{aligned} \quad (15)$$

where Δ depends on x_2 . Differentiating equation (13) with respect to Δ and x_2 leads to:

$$d\Delta = \frac{\tau^x p_2}{(\beta_2 (1 - \tau^x))^{-1} - 1} dx_2$$

Then, assuming τ^x close to zero leads to a small effect of x_2 on Δ .

Moreover, the saving function s increases with its first argument and decreases with the second one. Consequently, the right-hand side of (15) increases with x_2 . Then, under condition (14), equality (15) is satisfied iff $x_2 > 0$. \square

For high inheritance tax, the condition in Lemma 1 may be not sufficient. Indeed, as τ^x increases, the public debt Δ may become an increasing function of x_2 . This positive effect on the left-hand side of (14) may then dominate the positive effect of x_2 on the right-hand side. Consequently, condition (14) may become irrelevant for guaranteeing positive bequests x_2 .

The effect of τ^w and a on condition (14) can also be analyzed. Indeed, the public debt increases with τ^w and decreases with a , while the saving function on the right-hand side of (14) varies in the opposite direction. Then condition (14) is likely to be satisfied with low value of a and high value of τ^w . As the young receive low after-tax labor income or low public transfer, inheritance is likely to be positive.

3 Fiscal reform at steady state

With coexistence of dynasties that leaves bequest or behaves like life-cyclers, the inception of an inheritance tax allows to redistribute wealth. Nevertheless, it also reduces the capital-labor ratio. Indeed, equation (7) leads to a negative relation between the capital-labor ratio and the inheritance tax rate. Considering homothetic preferences, Michel and Pestieau (2005) have shown that a uniform lump-sum transfer financed through inheritance tax reduces the steady-state lifetime utility of all dynasties. One may explain the result in the following way. Two forces affect welfare of the life-cyclers. First, they receive a lump-sum public transfer. Second, the fall in the capital-labor ratio increases the real interest rate and pushes down the real wage rate. The latter effect on the wage rate overcompensates the other forces leading to a fall in the well-being of the life-cyclers. The main driving force here is the fact that, at a steady state with underaccumulation, any fall in the capital-labor ratio reduces the product disposable for consumption $((1 - \Gamma) F(K_M, \bar{h}) - K_M)$. Dynasties that leave bequests also experience a fall in their welfare, for two additional reasons: (i) the inheritance tax creates a distortion in their bequest decision and (ii) the lump-sum transfer they receive is lower than their contribution.

The point we want to stress in this paper is that there exist fiscal reforms that include an increase in the inheritance tax combined with changes in the other tax rates that attenuate or eliminate the fall in the capital-labor ratio. In the following, we explore the consequence of a tax reform that consists in a switch from capital income taxation towards inheritance taxation. These changes have opposite effects on the capital-labor ratio. A fall in the capital income tax rate τ^R increases the capital-labor ratio while raising the inheritance tax rate τ^x decreases it. Moreover, such a policy still allows to redistribute wealth since the capital income tax is paid by all agents while the inheritance tax is paid only by the dynasties that leave bequests.

It is possible for instance to set both tax rates in order to leave the capital-labor ratio constant. If, additionally, we assume constant labor income tax rate and constant lump-sum transfer, the reduction of τ^R financed through an increase in τ^x is necessarily welfare enhancing for life-cyclers. They do not pay inheritance tax and pay less capital income taxes, while wage rate and interest rate remain unchanged. We can then state the following Proposition, only using the intertemporal budget constraint (9) of type-1 agents.

Proposition 1. *At steady state, any increase in the inheritance tax τ^x that leaves the first-period income of type-1 agents (i.e. $(1 - \tau^w) h_1 w_M + a$) constant increases steady-state life-cycle utility of type-1 agents.*

First-period income of type-1 agents is constant if, for instance, the capital-labor ratio is left unchanged (constant w_M) as well as the instruments τ^w and a . Such a situation can be obtained by setting τ^R in order to keep the product $(1 - \tau^x) (1 - \tau^R)$ constant. In this case, the capital stock K_M , characterized by equation (7), is unchanged, as well as the wage rate. Nevertheless, keeping $(1 - \tau^x) (1 - \tau^R)$ constant also modifies fiscal receipts. Indeed, the fiscal base of the inheritance tax is $p_2 x_2$ while the fiscal base of the capital income tax is

$$R_M \sum_i p_i s_i = \frac{p_1 d_1 + p_2 (d_2 + x_2)}{1 - \tau^R}$$

The latter is higher than $p_2 x_2$ at least if the capital income tax rate is positive. Type-2 individuals save not only in order to leave bequests to their offsprings, but also to consume during old-age. To keep $(1 - \tau^x) (1 - \tau^R)$ constant, the fall in fiscal receipts from capital income tax will be larger than the increase in fiscal receipts from the inheritance tax. This results in a decrease in the steady-state public debt² that involves intergenerational redistribution from the first generations towards the ones living at a time where the economy is closed to the steady state.

We now complete the preceding Proposition by analyzing how life-cycle utility of type-2 agents is affected.

Proposition 2. *At steady state, for given τ^w and a , a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant increases the steady-state life-cycle utility of type-1 agents and reduces the one of type-2 agents.*

²Differentiating equation (13) and assuming $d\tau^w = 0$, $da = 0$ and initially $\tau^x = 0$, one gets

$$\left(1 - \frac{1}{\beta_2}\right) d\Delta = -R_M (K_M + \Delta) d\tau^R - p_2 x_2 d\tau^x$$

where, to keep the capital-labor ratio constant: $d[(1 - \tau^R) (1 - \tau^x)] = 0$. This implies

$$\left(1 - \frac{1}{\beta_2}\right) d\Delta = \left[(1 - \tau^R) R_M (K_M + \Delta) d\tau^R - p_2 x_2\right] d\tau^x$$

Then, $d\Delta$ and $d\tau^x$ have opposite signs.

Proof. Since τ^w and a are not modified, the first-period income of type-1 agents is unchanged. Then applying Proposition 1 allows to state the result for type-1 agents.

For type-2 agents, differentiation of lifetime utility $u_2 = u(c_2, d_2)$ leads to

$$du_2 = u'_{c_2} [dc_2 + \beta_2 (1 - \tau^x) dd_2]$$

From the resource constraint (5) ,

$$p_1 (c_1 + d_1) + p_2 (c_2 + d_2) = (1 - \Gamma) F (K_M, \bar{h}) - K_M$$

one gets

$$dc_2 + dd_2 = -\frac{p_1}{p_2} (dc_1 + dd_1)$$

Moreover, equality (8) implies

$$\frac{\partial MRS_2^{d/c}}{\partial c_2} dc_2 + \frac{\partial MRS_2^{d/c}}{\partial d_2} dd_2 = -\beta_2 d\tau^x$$

The last two equalities lead to

$$\begin{aligned} & \left(\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2} \right) [dc_2 + \beta_2 (1 - \tau^x) dd_2] \\ = & - (1 - \beta_2 (1 - \tau^x)) \beta_2 d\tau^x - \left[\beta_2 (1 - \tau^x) \frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2} \right] \frac{p_1}{p_2} (dc_1 + dd_1) \end{aligned}$$

As shown in Appendix 6.1, strict concavity of u implies $\beta_2 (1 - \tau^x) \frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2} > 0$ and normal good assumption implies $\frac{\partial MRS_2^{d/c}}{\partial c_2} > 0 > \frac{\partial MRS_2^{d/c}}{\partial d_2}$. To conclude, one needs to sign $dc_1 + dd_1$. Differentiation of the intertemporal budget constraint of type-1 agents (9) and of the marginal condition $MRS_1^{d/c} = \beta_2 (1 - \tau^x)$ leads to

$$dc_1 + dd_1 = \frac{1 - \beta_2 (1 - \tau^x) + \left(\frac{\partial MRS_1^{d/c}}{\partial c_1} - \frac{\partial MRS_1^{d/c}}{\partial d_1} \right) d_1}{\frac{\partial MRS_1^{d/c}}{\partial c_1} \beta_2 (1 - \tau^x) - \frac{\partial MRS_1^{d/c}}{\partial d_1}} \beta_2 d\tau^x > 0$$

since the strict concavity of u implies that the denominator is positive and the normal good assumption implies that $\frac{\partial MRS_1^{d/c}}{\partial c_1} > 0 > \frac{\partial MRS_1^{d/c}}{\partial d_1}$. Consequently $dc_2 + \beta_2 (1 - \tau^x) dd_2 < 0$. This concludes the proof. \square

For type-2 agents, the introduction of the inheritance tax reduces the relative price of old-age consumption. This positive effect on their utility is overcompensated by a fall in after-tax bequest $(1 - \tau^x) x_2$, leading to a reduction in utility.

Leaving the capital-labor ratio constant with the tax reform involves that aggregate resources available for consumption are constant. Thus, any consumption gain for one type of agent is offset by a loss of consumption for the other. For both types of dynasties, the fall in the relative price of old-age consumption leads the agents to shift part of their resources from the youth period to old-age. In addition, the marginal rate of transformation between d and c ($MRT_i^{d/c}$) is equal to one whereas the marginal rate of substitution ($MRS_i^{d/c} = \beta_2(1 - \tau^x)$) is lower than one. As a result, any shift of consumption from c to d creates an inefficiency in the resource allocation for consumption.

As the utility of type-1 agents increases with the tax reform considered in Proposition 2, the utility of type-2 agents decreases both because of the transfer of resources to type 1-agents and also because of a greater inefficiency in the resource allocation between consumption when young and consumption when old.

The following Proposition states that a tax reform leaving the steady-state capital-labor ratio constant cannot increase lifetime utility of type-2 agents. The tax reform considered allow for changes in the labor income tax rate τ^w or the lump-sum transfer a , that we have kept constant until now.

Proposition 3. *Consider an initial steady-state equilibrium where $\beta_2(1 - \tau^x) < 1$. Assume that government implements a tax reform that consists in a marginal increase in inheritance tax rate ($d\tau^x > 0$) and marginal changes in other tax instruments ($d\tau^R, d\tau^w, da$) such that the capital-labor ratio remains constant. If the reform does not reduce the lifetime utility of type-1 agents at steady state, then lifetime utility of type-2 agents necessarily decreases.*

Proof. The fiscal reform ($d\tau^x, d\tau^R, d\tau^w, da$) is such that

- $d\tau^x > 0$.
- The capital-labor ratio remains unchanged, that is $d[(1 - \tau^x)(1 - \tau^R)] = 0$, or equivalently (since initially $\tau^x = 0$):

$$\frac{d\tau^R}{1 - \tau^R} = -\frac{d\tau^x}{1 - \tau^x}.$$

We consider the extreme case where the reform does not change lifetime utility of type-1 agents ($du_1 = 0$). We then check whether lifetime utility of type-2 agents can increase ($du_2 > 0$). Recall that du_i has the same sign as

$$dc_i + \beta_2(1 - \tau^x) dd_i$$

Then, type-1 utility does not change iff $dc_1 + \beta_2(1 - \tau^x) dd_1 = 0$. To compute dc_2 and dd_2 , we combine the last equation with

- differentiation of the resource constraint (12) under the assumption of constant capital-labor ratio:

$$p_1(dc_1 + dd_1) + p_2(dc_2 + dd_2) = 0$$

- and differentiation of the marginal conditions $MRS_i^{d/c} = \beta_2 (1 - \tau^x)$, for $i = 1, 2$, that is

$$\frac{\partial MRS_i^{d/c}}{\partial c_i} dc_i + \frac{\partial MRS_i^{d/c}}{\partial d_i} dd_i = -\beta_2 d\tau^x, \text{ for } i = 1, 2$$

Straightforward calculations lead to

$$dc_1 + dd_1 = \frac{[1 - \beta_2 (1 - \tau^x)] \beta_2 d\tau^x}{\beta_2 (1 - \tau^x) \frac{\partial MRS_1^{d/c}}{\partial c_1} - \frac{\partial MRS_1^{d/c}}{\partial d_1}} > 0$$

and

$$\begin{aligned} dc_2 + \beta_2 (1 - \tau^x) dd_2 \\ = \frac{-[1 - \beta_2 (1 - \tau^x)] \beta_2 d\tau^x}{\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}} - \frac{\beta_2 (1 - \tau^x) \frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}}{\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}} \frac{p_1}{p_2} (dc_1 + dd_1) \end{aligned}$$

which implies $du_2 < 0$. This concludes the proof. \square

Therefore, the tax reform cannot increase the utility of type-2 agents whereas, as we have seen in Propositions 1 and 2, it is possible to design that reform in a way that increases lifetime utility of type-1 agents.

The crucial point in the preceding result is that disposable resources for consumption cannot vary since we have assumed a constant capital-labor ratio.

4 Time transfers and elastic labor supply

We now consider a case where keeping the capital-labor ratio constant does not imply that resources for consumption are fixed. The tax reform that consists in introducing an inheritance tax is reconsidered in a framework that combines elastic labor supply of the young and intergenerational time transfers from the old to the young as an alternative to leaving money.

Indeed, considering time transfers introduces a substitution effect of the inheritance tax on the trade-off between both types of intergenerational transfers. Inheritance tax makes time transfers more attractive and may increase time transfers and thus the young's labor supply.

4.1 A framework with time transfers

Households of generation t that belongs to type- i dynasties ($i \in \{1, 2\}$) consume a composite good that aggregates market good c_{it} when young (resp. when old d_{it+1}) and time spent in home production when young T_{it}^y (resp. T_{it+1}^o when old). Labor supply is elastic and the agent's labor

supply decision depends on the trade-off between formal work and home production. The lifetime utility function becomes:

$$u(f^y(c_{it}, T_{it}^y), f^o(d_{it+1}, T_{it+1}^o)) \quad (16)$$

where u is increasing and strictly quasi-concave and f^y and f^o are linear homogeneous functions, with positive and decreasing first-order derivatives. f^y (resp. f^o) is the quantity of composite good when young (resp. when old).

The household's budget constraint during his working life is rewritten as follows:

$$c_{it} + s_{it} = (1 - \tau^w) h_i w_t \ell_{it} + (1 - \tau^x) x_{it} + a_t \quad (17)$$

where ℓ_{it} denotes type- i agent's labor supply in the formal sector, and satisfies:

$$\ell_{it} = 1 - T_{it}^y + \mu_i (1 - T_{it}^o) \quad (18)$$

where μ_i represents the productivity parameter of time transfer in home production of the young. We assume μ_i is the same for all dynasties of the same type.

When retired, type- i agent's budget constraint corresponds to the one of the preceding Section:

$$d_{it+1} + x_{it+1} = (1 - \tau^R) R_{t+1} s_{it} \quad (19)$$

The first-order conditions of type- i agent are given in the Appendix section 6.2.

On the production side, the factor prices w_t and R_t of the representative firm are equal to their marginal products (see equation (3)). The budget constraint of the government is the same as equation (4). The labor market equilibrium becomes:

$$L_t = \sum_i p_i h_i \ell_{it}$$

and the resource constraint is the same as equation (5) where \bar{h} has been replaced with $\sum_i p_i h_i \ell_{it}$. The capital market equilibrium (6) is then satisfied as a consequence of the Walras' law.

4.2 Steady state

As shown in Appendix 6.2, only the more altruistic dynasties can leave bequests in the long-run. In the following, we assume that type-2 agents make positive bequests. Thus, $x_1 = 0$ and $x_2 > 0$, and the gross interest rate satisfies the modified Golden rule such that:

$$\beta_2 (1 - \tau^x) (1 - \tau^R) R_M = 1 \quad (20)$$

The steady-state capital-labor ratio z_M is then characterized by the equality between marginal product of capital $F_K(z_M, 1)$ and the gross interest rate R_M . The resource constraint at steady state then rewrites as

$$\sum_{i=1}^2 p_i (c_i + d_i) = C_M \sum_{i=1}^2 p_i h_i (1 + \mu_i) \ell_i \quad (21)$$

where

$$C_M \equiv (1 - \Gamma) F(z_M, 1) - z_M \quad (22)$$

We also assume that the productivity parameters of time transfers μ_i are high enough for all dynasties to leave time transfers: $T_i^o < 1$, for $i = 1, 2$.

Under non-negativity of time transfers and bequests at steady state, the first-order conditions of type- i agents can be rewritten as follows, for $i = 1, 2$,

$$MRS_i^{d/c} = \beta_2 (1 - \tau^x) \equiv P^R \quad (23)$$

$$MRS_i^{T^y/c} = (1 - \tau^w) h_i w \equiv P_i^y \quad (24)$$

$$MRS_i^{T^o/d} = \frac{\beta_i \mu_i (1 - \tau^w) h_i w}{\beta_2 (1 - \tau^x)} \equiv P_i^o \quad (25)$$

where P^R , P_i^y and P_i^o denote the relative prices respectively between d and c , between T^y and c , and between T^o and d .

4.3 Tax reform with time transfers

The tax reform is the same as in Section 3. To present its consequences, it is useful to distinguish interperiod and intraperiod effects:

1. *Interperiod effects.* The introduction of an inheritance tax decreases the relative price of the second-period market-good consumption P^R for both types agents. The fall in P^R is an *interperiod effect* which involves a negative effect on the consumption in composite good when young (negative effect on c_i and T_i^y) whereas the effect is positive for the composite good consumed when old (positive effect on d_i and T_i^o). The effect on young's labor supply is ambiguous since the fall of T_i^y increases the labor supply while the increase of T_i^o leads to the opposite effect. The magnitude of these effects depends on the intertemporal elasticity of substitution σ_i^u between the composite goods when young f^y and old f^o :

$$\sigma_i^u = \frac{d \ln (f^y / f^o)}{d \ln (u'_{f^o} / u'_{f^y})} \quad (26)$$

where u'_{f^o} and u'_{f^y} stands for the marginal utilities of both composite goods.

2. *Intraperiod effects.* The tax reform leads also to increase the relative prices P_i^o ($i = 1, 2$)

between market good and time used in home production when old. This *intraproduct effect* has a positive impact on d_i and a negative effect on T_i^o . The negative effect on T_i^o affects positively time transfers and, therefore, the young's labor supply. Both dynasties have incentives to increase the young's labor supply through higher time transfers. The magnitude of this effect depends on the elasticity of substitution σ_i^o between d_i and T_i^o :

$$\sigma_i^o = \frac{d \ln (d_i/T_i^o)}{d \ln P_i^o} \quad (27)$$

The interperiod effect (through the fall in P^R) has some similarity with the one obtained in the model of the preceding Section with inelastic labor supply and no time transfers, but adds in the present framework new consequences on labor supply that depend on the relative changes in T_i^y and T_i^o , since $\ell_i = 1 - T_i^y + \mu_i (1 - T_i^o)$. Indeed, interperiod effects involves lower T^y and higher T^o . Moreover, labor supply of the young is also modified by intraperiod effects through the fall in P_i^o , that also affect both times devoted to home production: higher T_i^y and lower T_i^o .

In the standard Barro (1974) model with inelastic labor supply, implemented a tax reform leaving the capital-labor ratio constant involves that the resources available for consumption are fixed. Indeed, as stated in Section 3, any consumption gain for one type of agent involves a loss of consumption for the other.

With time transfers and elastic labor supply, the tax reform affects the aggregate labor supply and thus may increase the aggregate resources for consumption of market goods. We then ask whether a Pareto-improving tax reform is possible. The most favorable situations for higher steady-state resources are those where the aggregate labor supply increases significantly.

In the following Proposition, we first consider the consequence of the tax reform on lifetime utility of type-1 agents.

Proposition 4. *For given τ^w and a , a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant increases the steady-state welfare of type-1 agents when the intraperiod effects dominate the interperiods effects, i.e. $\sigma_1^u \leq \sigma_1^o$.*

Proof. Differentiating steady-state life-cycle utility V_i and using marginal conditions (23)-(25), dV_i , for $i = 1, 2$, has the same sign as:

$$dV_i = dc_i + P_i^y dT_i^y + P^R dd_i + \beta_i \mu_i P_i^y dT_i^o \quad (28)$$

Since τ^w and the capital-labor ratio are kept constant with the tax reform, the relative prices P_i^y remains unchanged: $dP_i^y = 0$. Then, linear homogeneity of f^y implies that the ratio c_i/T_i^y is also unchanged, that is,

$$\frac{dc_i}{c_i} = \frac{dT_i^y}{T_i^y}, \text{ for } i = 1, 2. \quad (29)$$

Moreover, since $d \ln P_i^o = d \ln P_i^y - d \ln P^R$ and $dP_i^y = 0$, the definition of σ_i^o (see equation (27)) implies

$$\frac{dd_i}{d_i} - \frac{dT_i^o}{T_i^o} = -\sigma_i^o \frac{dP^R}{P^R}. \quad (30)$$

Finally, from the marginal condition (23), we get

$$\frac{u_{f_i^o}}{u_{f_i^y}} = \frac{f_{c_i}^y P^R}{f_{d_i}^o}.$$

which implies, using the definition of σ_i^u (see equation (26)),

$$\frac{df_i^o}{f_i^o} - \frac{df_i^y}{f_i^y} = -\sigma_i^u \left(\frac{df_{c_i}^y}{f_{c_i}^y} - \frac{df_{d_i}^o}{f_{d_i}^o} + \frac{dP^R}{P^R} \right).$$

Then, we obtain

$$\frac{dT_i^y}{T_i^y} - \frac{dT_i^o}{T_i^o} = -\alpha_i^o (\sigma_i^o - \sigma_i^u) \frac{dP^R}{P^R} \quad (31)$$

using the following relations deduced from linear homogeneity of f^y and f^o :

$$\frac{df_{c_i}^y}{f_{c_i}^y} = 0, \quad \frac{df_{d_i}^o}{f_{d_i}^o} = \frac{f_{d_i d_i}^o d_i}{f_{d_i}^o} \left(\frac{dd_i}{d_i} - \frac{dT_i^o}{T_i^o} \right) \quad \text{and} \quad \frac{-d_i f_{d_i d_i}^o}{f_{d_i}^o} = \frac{1 - \alpha_i^o}{\sigma_i^o}$$

where $\alpha_i^o \equiv d_i f_{d_i}^o / f_i^o$.

Replacing equations (29)-(31) in (28) yields

$$dV_i = \Omega_i \left[\frac{dT_i^o}{T_i^o} + S_i \alpha_i^o \left(\frac{-dP^R}{P^R} \right) \right] \quad (32)$$

where

$$S_i \equiv \gamma_i (\sigma_i^o - \sigma_i^u) + (1 - \gamma_i) \sigma_i^o, \quad \gamma_i \equiv \frac{c_i + P_i^y T_i^y}{\Omega_i} \quad \text{and} \quad \Omega_i \equiv c_i + P_i^y T_i^y + P^R (d_i + P_i^o T_i^o)$$

Furthermore, differentiating the intertemporal budget constraint of type-1 agents

$$c_1 + P_1^y T_1^y + P^R d_1 + \mu_1 P_1^y T_1^o = P_1^y (1 + \mu_1) + a$$

leads to (using equations (29)-(31)):

$$\frac{dT_1^o}{T_1^o} = \frac{\alpha_1^o (1 - \gamma_1 - S_1)}{1 + (1 - \gamma_1) \frac{(1 - \beta_1)(1 - \alpha_1^o)}{\beta_1}} \left(\frac{-dP^R}{P^R} \right) \quad (33)$$

Thus, replacing in dV_1 (see equation (32) for $i = 1$), and given that the tax reform implies $dP^R < 0$,

life-cycle utility of type-1 agents increases iff

$$1 + S_1 \frac{(1 - \beta_1)(1 - \alpha_1^o)}{\beta_1} > 0$$

which is true if $\sigma_1^u \leq \sigma_1^o$ since this implies $S_1 > 0$. This concludes the proof. \square

From equation (33), notice that an increase in time transfer of type-1 dynasties arises iff $S_1 > 1 - \gamma_1$, or equivalently

$$\gamma_1(\sigma_1^o - \sigma_1^u) + (1 - \gamma_1)(\sigma_1^o - 1) > 0$$

Therefore if the elasticity σ_1^o is larger than σ_1^u and 1, time resources of the young type-1 agents increase.

This suggests that, for high σ_1^o , intraperiod effect leads type-1 agents to increase the ratio d_1/T_1^o through the rise of time transfers. This may involve higher resources for the young type-1 agents. Additionally, if σ_1^u is high (for instance $\sigma_1^u = \sigma_1^o > 1$), the interperiod effect (fall in c_1 and T_1^y due to the fall in P^R) may dominate the intraperiod effect (increase in c_1 and T_1^y due to higher time transfers). In fact, the higher σ_1^u , the lower T_1^y , reinforcing the positive effect on labor supply of type-1 young agents ($\ell_1 = 1 - T_1^y + \mu_1(1 - T_1^o)$).

Compared to the previous Section, the resources available for consumption of private goods may increase with the tax reform. Thus, the consumption gain for type-1 agents is not necessarily offset by a loss of consumption for type-2 agents.

Proposition 5. *For given τ^w and a , a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant is Pareto-improving iff the two following conditions are satisfied (with a strict inequality for at least one of the two conditions)*

- $dV_1 \geq 0$, or equivalently

$$\theta_1 \equiv 1 + \left(\frac{1}{\beta_1} - 1\right)(1 - \alpha_1^o)S_1 \geq 0 \quad (34)$$

- $dV_2 \geq 0$, or equivalently

$$\sum_{i=1}^2 p_i \Omega_i^M \alpha_i^o (S_i - S_i^M) \geq \frac{p_1 \Omega_1^M \alpha_1^o (1 - \gamma_1)}{1 + \left(\frac{1}{\beta_1} - 1\right)(1 - \alpha_1^o)(1 - \gamma_1)} \theta_1 \quad (35)$$

where $S_i^M \equiv \gamma_i^M (\sigma_i^o - \sigma_i^u) + (1 - \gamma_i^M) \sigma_i^o \left[1 - (1 - \alpha_i^{oM}) \left(1 - \frac{P_i^o}{C_M h_i \mu_i}\right)\right]$, $\gamma_i^M \equiv \frac{c_i + C_M h_i T_i^y}{\Omega_i^M}$, $\alpha_i^{oM} \equiv \frac{d_i}{d_i + C_M h_i \mu_i T_i^o}$, $\Omega_i^M \equiv c_i + d_i + C_M h_i (T_i^y + \mu_i T_i^o)$, and C_M defined by (22).

Proof. From equation (32), the marginal effect of the tax reform on the steady-state life-cycle utility of type-2 agents is positive iff:

$$\frac{dT_2^o}{T_2^o} \geq -S_2 \alpha_2^o \left(\frac{-dP^R}{P^R}\right)$$

To compute dT_2^o/T_2^o , we use the resource constraint (21):

$$\sum_{i=1}^2 p_i \Omega_i^M = C_M \sum_{i=1}^2 p_i h_i (1 + \mu_i)$$

where Ω_i^M is defined in the statement of Proposition 5 and C_M corresponds to production of market goods disposable for consumption per efficient labor unit (see equation (22)).

Differentiation of the resource constraint yields

$$\sum_{i=1}^2 p_i \Omega_i^M \frac{d\Omega_i^M}{\Omega_i^M} = 0$$

where

$$\frac{d\Omega_i^M}{\Omega_i^M} = \frac{dT_i^o}{T_i^o} + S_i^M \alpha_i^o \left(\frac{-dP^R}{P^R} \right)$$

This implies

$$\frac{dT_2^o}{T_2^o} = -\frac{p_1 \Omega_1^M}{p_2 \Omega_2^M} \frac{dT_1^o}{T_1^o} - \left(\frac{p_1 \Omega_1^M}{p_2 \Omega_2^M} S_1^M \alpha_1^o + S_2^M \alpha_2^o \right) \left(\frac{-dP^R}{P^R} \right)$$

Then $dV_2 \geq 0$ iff

$$\frac{p_1 \Omega_1^M}{p_2 \Omega_2^M} \frac{dT_1^o}{T_1^o} + \left(\frac{p_1 \Omega_1^M}{p_2 \Omega_2^M} S_1^M \alpha_1^o + S_2^M \alpha_2^o \right) \left(\frac{-dP^R}{P^R} \right) \leq S_2 \alpha_2^o \left(\frac{-dP^R}{P^R} \right)$$

where $\frac{dT_1^o}{T_1^o}$ is given by equation (33). One gets that $dV_2 \geq 0$ iff

$$p_1 \Omega_1^M \alpha_1^o \left(\frac{1 - \gamma_1 - S_1}{1 + (1 - \gamma_1) \frac{(1 - \beta_1)(1 - \alpha_1^o)}{\beta_1}} + S_1^M \right) + p_2 \Omega_2^M \alpha_2^o (S_2^M - S_2) \leq 0$$

which is equivalent to condition (35). This concludes the proof. \square

Let us consider, as a benchmark, the case where there is no type-1 dynasties, i.e. $p_1 = 0$. Condition (35) rewrites as $S_2 > S_2^M$. We show in Appendix 6.3 that, under the condition $\mu_2 C_M h_2 > P_2^o$,³ this inequality is satisfied in at least one of the two following cases: (i) σ_2^o relatively high with respect to

³From the definitions of C_M and P_2^o (equations (22) and (25)), the inequality $\mu_2 h_2 C_M > P_2^o$ is equivalent to

$$(1 - \Gamma) F(z_M, 1) - z_M > \frac{(1 - \tau^w) F_L(z_M, 1)}{1 - \tau^x}.$$

It is satisfied for instance if $\tau^w = \Gamma$, $\tau^x = 0$ and $\tau^R \geq 0$ at the initial steady state. Indeed, in this case, it rewrites as

$$[(1 - \Gamma) F_K(z_M, 1) - 1] z_M > 0$$

or equivalently (from equation (20))

$$\beta (1 - \tau^R) < 1.$$

σ_2^u ; (ii) $\gamma_2 < \gamma_2^M$. These results are the same as the ones obtained in Belan and Moussault (2018) in an economy with homogeneous agents.

Notice that the assumption $\mu_2 C_M h_2 > P_2^o$ means that the marginal rate of transformation ($MRT_2^{T^o/d} = \mu_2 C_M h_2$) is higher than the marginal rate of substitution ($MRS_2^{T^o/d} = P_2^o$), that is, for a given (c_2, T_2^y) any reduction in T_2^o leads to higher time transfers which increase the labor supply and then leaves enough additional resources for second-period consumption to maintain the type-2 agents with the same utility.

Let us give some interpretations of conditions (i) and (ii):

- $\sigma_2^o \gg \sigma_2^y$ implies that the intraperiod effects (through higher P_2^o) dominate the interperiod effects (through the fall in P^R). In this context, the tax reform introduces a strong substitution effect between consumption of market good d_2 and time devoted to home production T_2^o , that increases time transfers and labor supply of the young, enhancing type-2 agents' welfare.
- The condition $\gamma_2 < \gamma_2^M$ is equivalent to

$$\frac{\frac{c_2}{T_2^y} + P_2^y}{\frac{c_2}{T_2^y} + C_M h_2} < P^R \frac{\frac{d_2}{T_2^o} + P_2^o}{\frac{d_2}{T_2^o} + \mu_2 C_M h_2}$$

A necessary condition for the last inequality is $C_M h_2 > \frac{P_2^y}{P^R}$ which implies that $C_M h_2 > P_2^y$ and hence $\mu_2 C_M h_2 > P_2^o$. Assuming $\tau^x = 0$ at the initial steady state, the inequality $\mu_2 C_M h_2 > P_2^o$ is equivalent to $C_M h_2 > P_2^y$. Then the condition $\gamma_2 < \gamma_2^M$ is likely to be satisfied if $\frac{c_2}{T_2^y}$ is low and $\frac{d_2}{T_2^o}$ is high, that is, domestic production is more intensive in time when young than when old.

By continuity, with small values of p_1 , the tax reform creates a Pareto improvement, since it increases type-2 life-cycle utility without reducing type-1 utility (assuming $\sigma_1^o \geq \sigma_1^u$). However, for higher values of p_1 , the increase in type-2 life-cycle utility is no longer guaranteed.

Nevertheless, differences between Ω_i^M can also make the tax reform Pareto improving. Indeed

$$\Omega_i^M = c_i + d_i + C_M h_i (T_i^y + \mu_i T_i^o)$$

Consumptions in time and market goods depend on the distribution of resources among both types of dynasties.

Using the intertemporal budget constraint of type-1 agents

$$c_1 + P_1^y T_1^y + P^R (d_1 + P_1^o T_1^o) = \Omega_1 = (1 - \tau^w) h_1 w (1 - (1 - \beta_1) \mu_1 T_1^o) + a$$

and marginal conditions (23)-(25), consumptions of the bundle (c_1, d_1, T_1^y, T_1^o) increase with Ω_1

under the normal good assumption. Then the higher Ω_1 , the higher Ω_1^M .

Similarly, type-2 agents consumptions (c_2, d_2, T_2^y, T_2^o) that satisfy the intertemporal budget constraint

$$c_2 + P_2^y T_2^y + P^R (d_2 + P_2^o T_2^o) = \Omega_2 = (1 - \tau^w) h_2 w (1 - (1 - \beta_2) \mu_2 T_2^o) + a + (1 - \beta_2) (1 - \tau^x) x_2$$

and marginal conditions (23)-(25), increase with Ω_2 . Consequently differences between Ω_i 's involve differences between Ω_i^M 's. Then high bequests for type-2 agents can make the reform Pareto improving (as well as higher productivity for type-2 agents).

Therefore, there exist situations where inequality (35) is satisfied which involves that the tax reform may be Pareto-improving.

5 Conclusion

By considering the inheritance taxation in a fiscal reform that keeps the capital-labor ratio constant, we have shown that inheritance taxation is welfare enhancing for every altruistic dynasty except for the more altruistic one. However, keeping the capital-labor ratio constant with inelastic labor supply involves that disposable resources are constant. Thus, the tax reform consists in a transfer of resources across dynasties (through inheritance taxation) which can not be Pareto-improving. When we extend the model by introducing time transfers in a economy with elastic labor supply, the aggregate resources can increase with the tax reform (keeping the capital-labor ratio constant). Indeed, the implementation of inheritance tax makes time transfers more attractive which may increase the young's labor supply through the positive effect on time transfers. Finally, we have shown that the Pareto improvement of the tax reform strongly depends on the population distribution, as well as wealth distribution, between both types of agents and on the strength of the positive effect on the labor supply of every agent.

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6 Appendix

6.1 Properties of the utility function

Let us consider a strictly concave utility function that satisfies normal goods assumption. Let us define the marginal rate of substitution as

$$MRS^{d/c} = \frac{u'_d(c, d)}{u'_c(c, d)}$$

Then, a bundle (c, d) that satisfies

$$MRS^{d/c} = P$$

is such that

$$\frac{\partial MRS^{d/c}}{\partial c} P - \frac{\partial MRS^{d/c}}{\partial d} > 0$$

and

$$\frac{\partial MRS^{d/c}}{\partial c} > 0 \text{ and } \frac{\partial MRS^{d/c}}{\partial d} < 0$$

The former inequality results from concavity, while the latter comes from the normal good assumption.

Indeed,

$$\begin{aligned} \frac{\partial MRS^{d/c}}{\partial c} P - \frac{\partial MRS^{d/c}}{\partial d} &= \frac{u'_d}{u'_c} \left(\frac{u''_{cd}}{u'_c} - \frac{u'_d u''_{cc}}{u'_c u'_c} \right) - \left(\frac{u''_{dd}}{u'_c} - \frac{u'_d u''_{cd}}{u'_c u'_c} \right) \\ &= \frac{1}{u'_c} \left[- \left(\frac{u'_d}{u'_c} \right)^2 u''_{cc} + 2 \frac{u'_d}{u'_c} u''_{cd} - u''_{dd} \right] > 0 \end{aligned}$$

where the last inequality results from strict concavity of u . To establish the condition on u that implies c is a normal good, let us add a budget constraint $c + Pd = I$, where I would be the life-cycle income. Then

$$\frac{dc}{dI} = 1 - P \frac{dd}{dI}$$

where, from $MRS^{d/c} = P$,

$$\frac{\partial MRS^{d/c}}{\partial c} \frac{dc}{dI} + \frac{\partial MRS^{d/c}}{\partial d} \frac{dd}{dI} = 0$$

Consequently

$$\begin{aligned} \frac{dc}{dI} &= \left(\frac{\partial MRS^{d/c}}{\partial c} P - \frac{\partial MRS^{d/c}}{\partial d} \right)^{-1} \left(- \frac{\partial MRS^{d/c}}{\partial d} \right) \\ \frac{dd}{dI} &= \left(\frac{\partial MRS^{d/c}}{\partial c} P - \frac{\partial MRS^{d/c}}{\partial d} \right)^{-1} \frac{\partial MRS^{d/c}}{\partial c} \end{aligned}$$

Thus, both derivatives are positive iff

$$\frac{\partial MRS^{d/c}}{\partial c} > 0 \text{ and } \frac{\partial MRS^{d/c}}{\partial d} < 0$$

6.2 First-order conditions of the dynastic problem with time transfers

Plugging the consumptions c_{it} , d_{it+1} and c_{it+1} from the budget constraints (17)-(19) into the utility function (16), we get the following marginal conditions, for $i = 1, 2$:

- with respect to s_{it}

$$-u_{f_{it}^y} f_{c_{it}}^y + (1 - \tau^R) R_{t+1} u_{f_{it+1}^o} f_{d_{it+1}}^o = 0$$

- with respect to T_{it}^y (assuming interior solution $\mu_i (1 - T_{it}^o) < T_{it}^y < 1 + \mu_i (1 - T_{it}^o)$)

$$-(1 - \tau^w) h_i w_t f_{c_{it}}^y + f_{T_{it}^y}^y = 0$$

- with respect to x_{it+1}

$$-u_{f_{it+1}^o} f_{d_{it+1}}^o + \beta_i (1 - \tau^x) u_{f_{it+1}^y} f_{c_{it+1}}^y \leq 0, = 0 \text{ if } x_{it+1} > 0$$

- with respect to T_{it+1}^o (assuming $T_{it+1}^o > 0$)

$$u_{f_{it+1}^o} f_{T_{it+1}^o}^o - \beta_i \mu_i (1 - \tau^w) h_i w_{t+1} u_{f_{it+1}^y} f_{c_{it+1}}^y \geq 0, = 0 \text{ if } T_{it+1}^o < 1$$

At steady state, marginal conditions with respect to x_{it+1} imply

$$\frac{u_{f_i^o} f_{d_i}^o}{u_{f_i^y} f_{c_i}^y} \geq \beta_2 (1 - \tau^x) > \beta_1 (1 - \tau^x)$$

Therefore, type-1 agents cannot leave positive bequests. In the text, we will assume that

- type-2 agents leave positive bequests;
- all time transfers are positive.

6.3 Tax reform with homogeneous agents

Consider the case with $p_1 = 0$. Steady-state lifetime utility of type-2 agents increases if and only if

$$S_2 > S_2^M$$

where

$$\begin{aligned}
S_2 &= \gamma_2 (\sigma_2^o - \sigma_2^u) + (1 - \gamma_2) \sigma_2^o \\
S_2^M &= \gamma_2^M (\sigma_2^o - \sigma_2^u) + (1 - \gamma_2^M) \sigma_2^o \left[1 - (1 - \alpha_2^{oM}) \left(1 - \frac{P_2^o}{C_M h_2 \mu_2} \right) \right]
\end{aligned}$$

The inequality then rewrites

$$\frac{\sigma_2^o}{\sigma_2^u} (1 - \gamma_2^M) (1 - \alpha_2^{oM}) \left(1 - \frac{P_2^o}{C_M h_2 \mu_2} \right) > \gamma_2 - \gamma_2^M$$

Under the condition $\mu_2 C_M h_2 > P_2^o$, the inequality is true if $\gamma_2 < \gamma_2^M$. By contrary, if $\gamma_2 > \gamma_2^M$, then the inequality is satisfied if σ_2^o is high with respect to σ_2^u , that is, if intraperiod effects dominate interperiod effects.