The pure advantage of risk in production

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This version: October 2017

Abstract

An increase in the riskiness of technologies brings a benefit along with the cost of raising the volatility of consumption. If the productivity realization of a technology is large the technology will be used intensively but if it is small its use can be curtailed. This asymmetry implies that increases in risk raise total expected output and can be Pareto-improving (even for risk-averse agents) in contrast to the effect of riskier endowments. The *observed* expected output of risky technologies, however, will typically be less than that of safer technologies: empirical estimates of expected output are therefore a poor measure of efficiency. Risky production sets can be placed into a classical general equilibrium model in which the gains to greater risk are realized in equilibrium and thus do not have to spread as an externality.

JEL codes: D20, D81, D51

Keywords: risk, risk aversion, production

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1 Introduction

Since Schumpeter (1942), economists have predominantly taken the view that firms will bear the risks of experimenting with new technologies only if they can earn a monopoly reward when their experiments succeed.[1] This monopoly power subsequently dissipates as discoveries spread as an externality to other firms. The riskiness of new technologies is a disadvantage in this account; a certain technical improvement would be superior to a risky improvement. This paper takes issue with this story. First, riskiness is an inherent advantage of new technologies: increases in risk will raise expected output. Second, markets can guide firms and investors to undertake the riskiest and hence most productive investments; externalities need not play any role in spreading the benefits of risk.

A risky technology brings the obvious drawback of increasing the variability of consumption. To see the benefit, suppose as a matter of comparative statics that we replace a technology t with a riskier technology $\widetilde{t}$ that has productivity realizations that form a mean-preserving spread of t's realizations. High and low productivity realizations are then more likely. If a high outcome occurs $\widetilde{t}$ will be used intensively but if a low outcome occurs then $\widetilde{t}$ can be curtailed in favor of alternative methods of production. This asymmetry argument will show that any increase in risk, as defined by second-order stochastic dominance, will expand expected output in the economy as a whole (Theorems 1 and 2).

Empirical estimates of the expected output of efficient, risky technologies will make them appear to be failures. Since the use of risky technologies will fall when they suffer a poor realization, their observed level of expected output will, ceteris paribus, be small compared to safer alternatives (Theorem 3): greater risk for a firm should positively correlate with smaller observed expected outputs. This simple conclusion bears on the risk-return trade-off, the theoretical positive correlation between risk and return that has proved difficult to detect empirically.[2] While the theory of that trade-off is sound, I will show that the trade-off need not appear as a positive correlation between the variance and mean of output (as opposed to returns), which casts doubt on the practice, common since Mehra and Prescott (1985), of

[1] See also Schumpeter (1934) and, more recently, Aghion and Howitt (1998), Grossman and Helpman (1991), and Romer (1990).

[2] A classic treatment of the risk-return trade-off is Merton (1973b). Lettau and Ludvigson (2010) consider and survey the evidence.

equating consumption and output with the flow of dividends.

The productivity gain from risky production also sheds light on the private equity premium puzzle – the empirical finding that entrepreneurs are willing to accept a low rate of return on their own risky, nondiversified investments.[3] I argue that the more efficient riskier technologies *should* have smaller observed expected outputs, which fits with the private equity premium puzzle fact that the expected output levels of new technologies are surprisingly small.[4]

After examining the comparative statics of increases in risk, we move one period back in time to see whether profit-making firms will adopt the riskier and more productive technologies. It might seem that the advantages of risky technologies cannot be realized via competitive markets. A risky technology $\widetilde{t}$ can have a smaller realized expected output due to the option of resorting to an alternative technology t when $\widetilde{t}$ turns out to be a failure and that gain goes to the owner of t rather than $\widetilde{t}$. The benefits of risk nevertheless do not have to spread as an externality with imitators free riding on the experiments of innovators; the efficient use of uncertain technologies can be embedded in a traditional, convex general equilibrium model. If to use a technology a producer must invest in technology-specific capital goods built while testing the technology, then the gains from experimenting with riskier technologies will be realized in equilibrium. Profit-seeking firms will initiate an efficient risky technology because they understand that the quasi-rents they earn on their capital when the technology is successful will be sufficient to make up for the losses they suffer when the technology fails. Even an economy of risk-averse agents can benefit from an increase in production risk and achieve these benefits in equilibrium.

These conclusions may seem to be at odds with the classical view that risk is welfare-worsening in a convex world.[5] Indeed an increase in endowment risk in a general equilibrium model with risk-averse agents will always harm some agent. But production risk differs from

[3] See Hamilton (2000) and Moskowitz and Vissing-Jorgensen (2002).

[4] There are alternative behavioral explanations of the puzzle, for example, Vereshchagina and Hopenhayn (2009) who argue that entrepreneurs are drawn to riskier projects in order to avoid intermediate rates of return on investment that, due to the minimum capital requirements of businesses, would force them into the labor force. For another behavioral explanation of the innovation-risk linkage, see Cogley and Jovanovic (2016) who argue that increases in the riskiness of output raise precautionary savings and thus growth.

[5] See Arrow (1965) and Stiglitz (1982) for examples of the classical view, with Stiglitz emphasizing an exception.

endowment risk: the use of a stochastic technology can be endogenously increased or decreased depending on its productivity realization, while the harm of a stochastic endowment is unavoidable.

In the Schumpeterian tradition, efficient technological experimentation is incompatible with competition: if the results of a successful production experiment are public then the non-experimenters will free ride while if the results are private then either the non-experimenters produce inefficiently or the experimenter gains market power. In this paper, no agent has private information about technology but the need for technology-specific capital goods prevents free-riding; experimentation is therefore conducted optimally. This competitive approach has much in common with the Boldrin and Levine (2002) account of innovation, where the further production of an innovative good proceeds by copying earlier versions.[6] One aim of the present paper is to show that a Boldrin-Levine style competitive theory can incorporate experiments with uncertain technologies. But my goal is not to argue that risky technologies usually disseminate via competitive markets. The goal is to show that the advantages of production risk can appear and propagate regardless of how innovations spread; the presence or absence of externalities is immaterial.

This paper's initial model will conclude that risky production methods are always superior, but the following general equilibrium model will introduce two channels that can mitigate the 'crowding out' of safer technologies. The first is the effect of risk aversion which counts against risky technologies. To excavate this channel, I show that with smooth technologies a risky alternative to a technology can always be exploited to deliver a Pareto improvement, though the risky alternative might be adopted only on a small scale (Theorem 7). With linear activities, there is a revealing exception: if agents are risk-averse then investment in a technology with outcomes that are only *slightly* riskier than a safe technology will always diminish welfare, regardless of the scale of investment (Theorem 8). Risk is thus not only potentially beneficial but substantial risks are better than minor risks.

The second channel stems from the scalability of investment in uncertain technologies, which will allow many experiments to be conducted at once. Equilibrium investment in a riskier technology $\widetilde{t}$ can then sometimes fail to fully crowd out a safer technology t even

[6] See also Boldrin and Levine (2017a, 2017b).

when agents are risk neutral: the safer t can complement the riskier $\widetilde{t}$ by itself serving as the alternative to be used when $\widetilde{t}$ turns out to be a failure (Theorem 6). The routine operation of competitive markets will determine the allocation of resources between the technologies as a function of their productivity realizations.

The paper's general equilibrium model is distinctive in that firms through their testing decisions determine what information is revealed. This feature makes 'self-confirming' equilibria possible where agents can have beliefs that persist only because the testing decisions that would invalidate those beliefs are not conducted in equilibrium (somewhat like Fudenberg and Levine (1993)). These equilibria are noncompetitive in some respects; and so to avoid suspicion I do not use them to analyze production risk.[7]

Roberts and Weitzman (1981), in a continuous time model of normally distributed and evolving investment returns, find that an increase in the variance of returns can make an investment project a more attractive prospect for preliminary funding and worth a sacrifice in mean return. Their explanation is that a greater variance of benefits implies that there is more opportunity to reduce uncertainty and they argue that their conclusions hinge on the sequential character of investment – the plug on a project can be pulled at any point. In a similar vein, Bar-Ilan and Strange (1996, 1998) use continuous time models of price uncertainty to show that it is possible for greater price uncertainty to hasten investment, with the ability to abandon an investment again appearing to be crucial.[8]

These papers consider the timing and the implicit value of a discrete 0-1 investment choice. For our focus on expected output and welfare, the passage of time and the discreteness of investment choices are less relevant. The gain in expected output due to production risk appears in static one-period models and the adjustment of output need not be discrete. Moreover the response of a firm to its random productivity realizations is nothing more than standard competitive supply behavior (see section 5).

Roberts and Weitzman (1981) and Bar-Ilan and Strange (1996) also take a partial equi-

[7] A companion paper, Mandler (2017), develops a theory of competitive equilibria when agents can uncover information, presents a first welfare theorem for this environment, and lays out alternative equilibria that are self-confirming.

[8] Bar-Ilan and Strange (1996) point out the link between their paper and results that show that greater price uncertainty will increase investment (Hartman (1972), Abel (1983), Caballero (1991)). This latter work in turn relies on the convexity of the profit function and thus was anticipated by Oi's (1961) argument that greater price dispersion increases expected profits. It notable that time plays no role in Oi's paper.

librium perspective: they either consider increases in the risk of an endogenous variable (such as price) or the profitability of an investment decision rather than equilibrium outcomes.[9] By embedding production risk in a general equilibrium model, we can examine the effect on equilibrium output of exogenous changes in riskiness and see which technologies will be adopted in competitive equilibrium. Our finding is that the advantage of risk does not turn on the sequential nature of investment: it holds as a comparative statics conclusion.

Green and Scotchmer (1995) provide a modern treatment of the Schumpeterian linkage between innovation and market power that pays particular attention to the problem of imitators who free ride on innovators. Chamley and Gale (1994) also emphasize the free-riding problem when information about technology spreads as an externality; Jovanovic and Nyarko (1996) can be understood similarly. Technology in this paper resembles a multi-armed bandit (Berry and Fristedt (1985), Bolton and Harris (1999), Weber (1992)) though agents discover the relevant information about the productivity of a new technology from a single data sample.

2 The superiority of risky technologies

Suppose production can use one of two methods, a certain method a^c or an uncertain method a^u. Each method a produces a quantity of value $v(a) \geq 0$ which can be thought of as output or output per unit of input. The value $v(a)$ is a random variable: $v(a)(\omega)$ is the value produced by a at state ω in the state space Ω. Probabilities and expectations are denoted by $\mathbb{P}$ and $\mathbb{E}$ respectively. Since a^c is certain, $v(a^c)(\omega)$ equals the same constant, v^c, for all ω. For each a^u we consider, assume that $\mathbb{E}v(a^u)$ is well-defined.

Before production takes place, a 'test' of a^u reveals the true state and thus the value $v(a^u)$. What is important is that some information about $v(a^u)$ is acquired: it would be sufficient for the conditional expectation of $v(a^u)$ given the test sometimes to differ from $\mathbb{E}v(a^u)$. A test could simply be a trial round of production that generates some fraction of $v(a')$.[10]

[9] See Pindyck (1993) on the drawbacks of treating the distribution of prices as an exogenous variable.

[10] Tests of this form are compatible with Theorem 1 since if ϕ if the fraction of $v(a^u)$ generated in the trial run then for any a^u the sum of the expected output generated by a^u in the trial run and production proper

After learning the state, the decision-maker will choose a^u if $v(a^u)(\omega) > v^c$. The economy thus resorts to a^c when a^u has a poor outcome and we label v^c the 'reserve value'. Call $\mathbb{E} \max [v(a^u), v^c]$ the **expected value achieved** by a^u. Since a^u needs to be chosen only if its revealed value exceeds v^c, the gain delivered by the uncertain method a^u (relative to being able to use a^c only) can be seen as the payoff of an option that delivers $v(a^u)$ – the return of the underlying asset a^u – if one pays the strike price v^c. If each production method has a separate owner, however, then neither owner's return would align with society's total expected value $\mathbb{E} \max [v(a^u), v^c]$. The option interpretation is helpful only in this section since later in the paper the reserve value delivered by alternative technologies will be determined in equilibrium and not given exogenously.[11]

Now consider two uncertain production methods a^u and $\widetilde{a}^u$ that have the same expected value, $\mathbb{E}[v(\widetilde{a}^u)] = \mathbb{E}[v(a^u)]$, but where $\widetilde{a}^u$ has a riskier distribution of value than a^u. As a matter of comparative statics, which uncertain method will lead economy-wide expected value to be greater?

To answer this question, view the determination of $v(\widetilde{a}^u)$ as a two-stage process. In the first stage, $v(\widetilde{a}^u)$ assumes a provisional value v governed by the same distribution as $v(a^u)$. In the second stage, the actual $v(\widetilde{a}^u)$ is set by applying a mean-preserving spread (MPS) to v. One reason that $\widetilde{a}^u$ is a more productive method than a^u is that when its provisional value v is greater than the reserve value v^c the dispersion of the MPS can allow $v(\widetilde{a}^u)$ to fall below v^c. One of these poor realizations for the MPS does only mild harm since producers can resort to a^c while the good realizations deliver an undiluted gain. In terms of the production that actually takes place, the bad outcomes for the MPS are therefore not as harmful as the good outcomes are beneficial. Similarly, suppose the provisional value v of $\widetilde{a}^u$ has a poor realization less than v^c. Without the MPS producers would resort to a^c but with the MPS producers can continue to turn to a^c when the MPS has a poor realization and use $\widetilde{a}^u$ when $v(\widetilde{a}^u) > v^c$. So now the good outcomes for the MPS are more beneficial than the bad outcomes are harmful, again in terms of the production that actually takes

would equal $(1 + \phi)v(a^u)(\omega)$ for all $\omega \in \Omega$.

[11] Theorem 1 is consistent with letting $v(a^c)$ be uncertain: what would then be important is that $v(a^u)$ does not correlate with $v(a^c)$.

place.[12]

If a^u and $\widetilde{a}^u$ have the same expected value, we consider $\widetilde{a}^u$ to be riskier than a^u if the distribution of $v(a^u)$ second-order stochastically dominates the distribution of $v(\widetilde{a}^u)$. Second-order stochastic dominance can be defined in several equivalent ways (Rothschild and Stiglitz (1970)) but mean-preserving spreads are the most convenient. Given the two production methods a and $\widetilde{a}$, define $\widetilde{a}$ **to be riskier than** a if $v(\widetilde{a})$ has the same distribution as the sum of $v(a)$ and a mean-preserving spread: there is a random variable Z such that (1) $\mathbb{E}(Z|v(a) = v) = 0$ for all $v \in \mathbb{R}$, and (2) $v(\widetilde{a}) \stackrel{d}{=} v(a) + Z$. Production method $\widetilde{a}$ is **strictly riskier than** method a if $\widetilde{a}$ is riskier than a but a is not riskier than $\widetilde{a}$.

Example 1 Suppose $\Omega = \{g, b\}$ and $\mathbb{P}(\{g\}) = \mathbb{P}(\{b\}) = \frac{1}{2}$ and define the coefficients $0 < \gamma \leq \beta$. If $v(a^u)(g) = v(a^u)(b) = \beta$ while $v(\widetilde{a}^u)(g) = \beta + \gamma$ and $v(\widetilde{a}^u)(b) = \beta - \gamma$ then $\widetilde{a}^u$ is strictly riskier than a^u. ■

If $\widetilde{a}$ is riskier than a then define its greater risk to be v^c-**nontrivial** if there is a positive chance that adding Z to $v(a)$ can change whether $v(a)$ is greater than or less than v^c, that is, if

$$\mathbb{P}\left[(v(a) - v^c)(v(a) + Z - v^c) < 0\right] > 0.$$

Theorem 1 *If the uncertain production method $\widetilde{a}^u$ is riskier than the uncertain method a^u then the expected value achieved by $\widetilde{a}^u$ will be greater than or equal to the expected value achieved by a^u. If $\widetilde{a}^u$'s greater risk is v^c-nontrivial then the expected value achieved will be strictly greater with $\widetilde{a}^u$.*

We have posed Theorem 1 as a comparative statics result, that expected output will be greater with the riskier method. It can be given a choice interpretation as well. Suppose we have the option of testing one uncertain method from a pool of possibilities A^u. If we wish to make the testing decision to maximize expected output, $\widetilde{a}^u$ is nontrivially riskier than a^u, and both a^u and $\widetilde{a}^u$ are in A^u then a^u cannot solve the problem.

[12] If $v(a^u)$ and hence the provisional value v of $\widetilde{a}^u$ are certain or nearly certain then the second effect does not come into play: if $v(a^u)$ is certain and less than v^c it would not be productive to use a^u under any circumstances and thus $\widetilde{a}^u$ could not displace a^u. In this sense, the first effect is more fundamental than the second effect, a point we return to in section 3.2. Another reason for the primacy of the first effect appears in footnote 25.

One corollary of Theorem 1 is that a production method $\widetilde{a}^u$ that has smaller expected value than a^u but that displays greater risk than a^u can increase expected value achieved.[13] That the social value of an uncertain $\widetilde{a}^u$ is an increasing function of its risk fits with the option interpretation: the price of an option to buy a stock is increasing in the risk of the underlying stock.[14]

Proof of Theorem 1. Let $F(\cdot)$ denote the distribution function of $v(a^u)$ and let $H_v(\cdot)$ denote the conditional distribution function of Z given $v(a^u) = v$. If $v(\widetilde{a}^u) \leq v^c$ then the value achieved with $\widetilde{a}^u$ is v^c. So the expected value achieved with $\widetilde{a}^u$ equals

$$\int \left(\int_{Z>v^c-v} (v+Z) dH_v(Z) + \int_{Z\leq v^c-v} v^c dH_v(Z) \right) dF(v).$$

Observe that for any $v \in \mathbb{R}$,

$$\int_{Z>v^c-v} (v+Z) dH_v(Z) + \int_{Z\leq v^c-v} v^c dH_v(Z) \geq \int (v+Z) dH_v(Z) = v, \tag{2.1}$$

where the equality follows from first part of the definition of $\widetilde{a}^u$ being riskier than a^u. Still fixing $v \in \mathbb{R}$, we also have

$$\int_{Z>v^c-v} (v+Z) dH_v(Z) + \int_{Z\leq v^c-v} v^c dH_v(Z) \geq v^c. \tag{2.2}$$

Hence

$$\int_{Z>v^c-v} (v+Z) dH_v(Z) + \int_{Z\leq v^c-v} v^c dH_v(Z) \geq \max[v, v^c].$$

and

$$\int \left(\int_{Z>v^c-v} (v+Z) dH_v(Z) + \int_{Z\leq v^c-v} v^c dH_v(Z) \right) dF(v) \geq \int \max[v, v^c] dF(v). \tag{2.3}$$

[13] The framework of Theorem 1 bears some similarity to the Weitzman (1979) Pandora Box problem (I am grateful to Richard Weber for pointing this out to me). When applied to the Pandora problem, Theorem 1 identifies a comparative statics advantage to replacing a safer box with a riskier box. If the uncertainty in Karlan et al. (2012) is reinterpreted as a feature of technology rather than entrepreneurial type then that paper identifies a Theorem 1-like advantage of low-expected-value but higher-risk technologies.

[14] See Merton (1973a) where risk is also measured by second-order stochastic dominance. Merton's proof is not helpful for our purposes since it considers only weak increases in risk and weak increases in price.

With a^u the expected value achieved equals $\int \max[v, v^c]dF(v)$, the expected value achieved is at least as great with $\widetilde{a}^u$ rather than with a^u.

Since $\widetilde{a}^u$'s greater risk is v^c-nontrivial at least one of the following cases must obtain: (i) $\mathbb{P}\left(v(a^u) > v^c \text{ and } Z + v(a^u) < v^c\right) > 0$ or (ii) $\mathbb{P}\left(v(a^u) < v^c \text{ and } Z + v(a^u) > v^c\right) > 0$. In case (i), the event where the realization v of $v(a^u)$ satisfies

$$v > v^c \text{ and } \int_{Z \leq v^c - v} v^c dH_v(Z) > \int_{Z \leq v^c - v} (v + Z) dH_v(Z)$$

has positive probability. Hence the event where v is such that (2.1) holds with strict inequality has positive probability and (2.3) holds with strict inequality. Similarly, in case (ii), the event where v satisfies

$$v < v^c \text{ and } \int_{Z > v^c - v} (v + Z) dH_v(Z) > \int_{Z > v^c - v} v^c dH_v(Z)$$

has positive probability. Hence (2.2) is strict with positive probability and (2.3) is again strict.[15] ■

3 Risky technologies in a classical economy

We now place the comparison of riskier and safer technologies into a classical economic setting. While Theorem 1's endorsement of production risk suggests that the replacement of a safer production set with a riskier one will raise expected output and welfare, the desirability of risk seems to run against the principle that risk is welfare-diminishing in a classical general equilibrium model. To show that Theorem 1 generalizes and that the putative principle is false, we eliminate from our earlier model the features that violate the rules of general equilibrium theory; production decisions will be scalable and, when we move to the complete intertemporal model, it will be possible to test multiple technologies simultaneously. The conclusion that an increase in production risk raises economy-wide

[15] We could weaken v^c-nontriviality (without threatening Theorem 1) to the assumption that either $\mathbb{P}\left[v(a) \geq v^c \text{ and } v(a) - v^c + Z < 0\right] > 0$ or $\mathbb{P}\left[v(a) \leq v^c \text{ and } v(a) - v^c + Z > 0\right] > 0$ (or both). The first terms of the last two displayed conditions in the proof would then become $v(a) \geq v^c$ and $v(a) \leq v^c$ with the remainder of the argument unaffected.

expected output will remain standing. Unlike increases in endowment risk, which always diminish welfare, increases in production risk can be beneficial.

Though the replacement of a certain technology by a risky technology raises expected output, the *observed* expected output of the risky technology will normally be less than that of the certain technology. Statistical estimates of a technology's expected output therefore provide a misleading measure of efficiency, a conclusion that helps clarify the private equity premium puzzle and the risk-return trade-off.

In the presence of competitive markets, the gains to experimenting with risky technologies will be realized in equilibrium. Since a version of the first welfare theorem will hold, firms will reject safer technologies in favor of riskier technologies when the latter are more efficient. Experimentation with new technologies will be efficient even though part of the social benefit is reaped by other firms, the ones that take over production when an experiment turns out to be a failure.

The scalability of production will show that the advantage of risk is unrelated to the discreteness of investment decisions as for instance in the Roberts-Weitzman (1981) and Bar-Ilan-Strange (1996) models. The allocation of resources that occur as firms learn their productivity realizations will in fact be indistinguishable from the standard supply adjustments that occur in a competitive equilibrium.

This section lays out the economic model and shows that riskier technologies are both advantageous and can display smaller observed expected outputs. Showing that markets will realize the advantages of risk follows in section 4.

3.1 The static economy

Though the economy will ultimately operate at two dates, 0 and 1, we begin with the latter date. At date 1, agents produce and consume by applying a primary input, labor, either to sets of certain technologies T^c or uncertain technologies T^u. Let $T = T^c \cup T^u$.

A state ω specifies an output level for each technology and input vector, and Ω is again the set of states, henceforth assumed to be finite. For a technology $t \in T^u$ we will state explicitly the amount of capital accumulated in the past that is used in t. The output of technology $t \in T^u$ with capital and labor inputs (k, l) at state ω is given by $v^t(k, l)(\omega)$, with $v^t(k,l)$ denoting the random variable. The capital used by technology t is specific to t and is called *t-capital*. For $t \in T^c$ there can also be accumulated capital but it will not be necessary to record it explicitly. Thus the output of $t \in T^c$ with input l at state ω is given by $v^t(l)(\omega)$ though, given l, $v^t(l)(\omega)$ does not vary with ω. For each $t \in T$ and state ω, we assume that (i) $v^t(\cdot)(\omega)$ is continuous, concave and weakly increasing, (ii) $v^t(k,l)(\omega) > 0$ implies $(k,l) \gg 0$ when $t \in T^u$ and $v^t(l) > 0$ implies $l > 0$ when $t \in T^c$, and (iii) $v^t(\cdot)(\omega)$ satisfies constant returns to scale when $t \in T^u$.[16] We assume that there is a $t \in T^c$ such that $v^t(\cdot)(\omega)$ is strictly increasing.

Although the decisions to accumulate capital have occurred in the past and capital stocks are fixed at date 1, those decisions determine the information that agents receive. For an uncertain $t \in T^u$, the installation of capital prior to date 1 discloses or 'tests' how the abstract description of the technology works in practice. All agents at the beginning of date 1 therefore learn $v^t(k,l)$ for each $(k,l) \geq 0$ if t-capital has been accumulated. At no point, before or after the test, is there any asymmetric information. We could let the exact state be revealed regardless of what investment decisions are made but it would be awkward to assume that a technology's productivity is revealed even when it remains completely untouched.[17]

Each agent i in the finite set $\mathcal{I}$ has endowments of labor given by $e^i_{L_1} > 0$ and of t-capital given by $e^i_t \geq 0$ for each $t \in T^u$, and consumes $x^i_1(\omega)$ when state ω obtains. Defining $x^i_1 = (x^i_1(\omega)_{\omega\in\Omega}) \in \mathbb{R}^{|\Omega|}_+$, agent i receives utility $U^i(x^i_1) \equiv \sum_{\omega\in\Omega} \mathbb{P}(\omega) u^i\left(x^i_1(\omega)\right)$ when consuming x^i_1, where $u^i : \mathbb{R}_+ \to \mathbb{R}$ is concave, differentiable, and strictly increasing.[18] An **allocation** x_1 is the vector $(x^i_1)_{i\in\mathcal{I}}$.

The economy's inputs of labor are labeled $L_1 = (L^t_1)_{t\in T} = \left(\left((L^t_1(\omega))_{\omega\in\Omega}\right)_{t\in T}\right)$.

Let the aggregate endowments of labor, t-capital, and all types of capital be given re-

[16]The uncertain technologies satisfy constant returns since all of their inputs are specified explicitly. If we specified the capital used by the certain technologies explicitly, it would be reasonable to require those technologies to satisfy constant returns as well.

[17]The measurability issues discussed in section 4 would then disappear and in addition small investments would no longer reveal discrete quanta of information. As in section 2, it is not important that agents learn the value of $v^t(k,l)$ at every state: date-0 decisions could be based on the conditional expectation of $v^t(k,l)$ given the test results. What matters is that testing or simply the passage of time uncovers some information.

[18]Differentiability is used only for Theorem 7.

spectively by $e_{L_1} = \sum_{i \in \mathcal{I}} e^i_{L_1}$, $e_t = \sum_{i \in \mathcal{I}} e^i_t$, and $e_T = (e_t)_{t \in T^u}$.

Since typically some uncertain technologies will have no capital accumulated from the past for these t – the true state will remain unknown. We therefore require each agent to consume a bundle that is measurable with respect to the information that is revealed: each agent's consumption must be constant across states that cannot be distinguished given the capital that has been accumulated.[19] To see which states are indistinguishable, define for each $t \in T^u$ the partition $\mathcal{P}^t$ of Ω that indicates the information revealed when t is tested: $P \in \mathcal{P}^t$ if and only if P is a maximal subset of Ω with the property that if $\omega, \omega' \in P$ then

$$v^t(k,l)(\omega) = v^t(k,l)(\omega') \text{ for all } (k,l) \geq 0.$$[20]

Let the partition $\mathcal{P}$ represent the information about the v^t that the earlier capital investment e_T reveals: $\mathcal{P}$ is the coarsest common refinement of the $\mathcal{P}^t$ such that $t \in T^u$ and $e_t > 0$. A random variable Y (or a vector that has Y as a coordinate) is $\mathcal{P}$-**measurable** if Y is constant on the cells of $\mathcal{P}$: for any $P \in \mathcal{P}$, if $\omega, \omega' \in P$ then $Y(\omega) = Y(\omega')$.

An allocation x_1 is **feasible** if there exists a L such that x_1 and L are $\mathcal{P}$-measurable and, for each $\omega \in \Omega$,

$$\begin{aligned} &\textstyle\sum_{i \in \mathcal{I}} x^i_1(\omega) \leq \sum_{t \in T^c} v^t(L^t_1(\omega)) + \sum_{t \in T^u} v^t(e_t, L^t_1(\omega))(\omega) \\ &\textstyle\sum_{t \in T} L^t_1(\omega) \leq e_{L_1}. \end{aligned}$$

The definition of greater production risk needs to be adjusted to cover technologies. Let $\mathcal{P}^{-\tilde{t}}$ represent the productivity information regarding all the uncertain technologies except $\tilde{t}$: the partition $\mathcal{P}^{-\tilde{t}}$ is the coarsest common refinement of the $\mathcal{P}^{t'}$ with $t' \in T^u \backslash \{\tilde{t}\}$. Also let $X|P$ will denote the restriction of the random variable X to $P \subset \Omega$, endowed with its conditional distribution given P.

Definition 1 *Technology $\tilde{t}$ is **partitionally riskier** than t if for each $(k,l) \geq 0$ there is a random variable $Z_{k,l}$ such that, for each $P \in \mathcal{P}^{-\tilde{t}}$, (1) $\mathbb{E}(Z_{k,l}|P) = 0$ and (2) $v^{\tilde{t}}(k,l)|P \overset{d}{=}$*

[19] As mentioned, we could alternatively assume that the true state is revealed automatically at the beginning of date 1: the measurability requirement can then be omitted.

[20] A set $P \subset \Omega$ with a property is *maximal* if there does not exist a $P' \subset \Omega$ that satisfies the property such that $P' \neq P$ and $P \subset P'$.

$v^t(k,l)|P + Z_{k,l}|P$. *Technology* $\widetilde{t}$ *is* ***strictly partitionally riskier*** *than* t *if* $\widetilde{t}$ *is partitionally riskier than* t *but* t *is not partitionally riskier than* $\widetilde{t}$.

Greater partitional risk is somewhat stronger than the greater risk definition of section 2. In the mean-preserving spreads of productivity outcomes, which can now vary with the input vector (k,l), it is the *conditional* expected value of each $Z_{k,l}$, given the productivity outcomes for all technologies besides $\widetilde{t}$, that must equal 0. Greater partitional risk taken as a whole is not unnatural: $\widetilde{t}$ might be a risky attempt to improve on t.

Example 2 The simplest case of greater partitional risk occurs with linear technologies and mimics Example 1. Let $\Omega = \{g, b\}$ and $\mathbb{P}(g) = \mathbb{P}(b) = \frac{1}{2}$, assume the productivity of t is unaffected by the state, $\mathcal{P}^t = \{\Omega\}$, and set

$$v^t(k,l)(g) = v^t(k,l)(b) = \beta \min[k,l],$$

where $\beta > 0$.[21] For $\widetilde{t}$, set $\mathcal{P}^{\widetilde{t}} = \{\{g\},\{b\}\}$ and

$$v^{\widetilde{t}}(k,l)(g) = (\beta + \gamma)\min[k,l],\ v^{\widetilde{t}}(k,l)(b) = (\beta - \gamma)\min[k,l],$$

where $0 < \gamma \leq \beta$. Then $\widetilde{t}$ is strictly partitionally riskier than t. ■

3.2 The comparative-statics advantage of risk

We begin with a stark comparative statics experiment that replaces a safer with a riskier technology and show that expected output increases, comparably to Theorem 1. If utilities are linear, welfare increases as well. After we introduce the initial period of the model, we will be able to conduct the less stark experiment of introducing of a variable quantity of a risky technology. Risk will again be welfare improving (and even if agents are risk-averse). Risky and safe technologies will then efficiently operate side by side with output levels in equilibrium that adjust to productivity realizations. See section 5.

[21] Setting $\mathcal{P}^t = \{\Omega\}$ is consistent with t being an uncertain technology: in fact, since t uses capital explicitly, t must be in T^u.

Suppose a model has a set of technologies T that contains the uncertain technologies t and $\widetilde{t}$ and where $e_t = e_{\widetilde{t}} > 0$. Letting the state space Ω, probabilities $\mathbb{P}$, utilities $(u^i)_{i\in\mathcal{I}}$, and the aggregate resources e_{L_1} and e_T remain fixed, define two further models, one with the set of technologies $T\backslash\{t\}$ and the other with $T\backslash\{\widetilde{t}\}$. We then say that the model with $T\backslash\{t\}$ then **replaces** t **with** $\widetilde{t}$. Since the endowments of t-capital and $\widetilde{t}$-capital are identical, the only potential difference between the models is the production functions v^t and $v^{\widetilde{t}}$. Given the model with t but not $\widetilde{t}$ and the inputs L_1, define **labor is used in** t to mean that there is positive probability that labor is applied to t at date 1, $\mathbb{P}(L_1^t > 0) > 0$.

The counterpart in the present setting of the reserve value of section 2 is the marginal product of labor of the certain technologies.[22] Let the aggregate production function for the certain technologies, $V^c : \mathbb{R}_+ \to \mathbb{R}_+$, be defined by

$$V^c(l) = \max_{(l^t)_{t\in T^c}} \sum_{t\in T^c} v^t(l^t) \text{ s.t. } \sum_{t\in T^c} l^t \leq l \text{ and } \left(l^t\right)_{t\in T^c} \geq 0.$$

The recast version of reserve value, d^r, is defined to equal the left derivative of V^c evaluated at e_{L_1}.[23] For $\widetilde{t} \in T^u$ (and an arbitrary $k > 0$), let $d^{\widetilde{t}}$ equal the random variable that at ω equals the right derivative of $v^{\widetilde{t}}(k,\cdot)(\omega)$ evaluated at $l = 0$.[24]

Recalling that $\mathcal{P}^{-\widetilde{t}}$ is the coarsest common refinement of the $\mathcal{P}^{t'}$ with $t' \in T^u\backslash\{\widetilde{t}\}$, define technology $\widetilde{t}$'s greater risk to be **nontrivial** if, for each $P \in \mathcal{P}^{-\widetilde{t}}$,

$$\mathbb{P}\left(d^{\widetilde{t}} < d^r \middle| P\right) > 0.$$

That is, there is a positive chance, given the outcomes for the non-$\widetilde{t}$ technologies, that the output of the first increment of labor used in the $\widetilde{t}$ technology falls below d^r, analogously to the chance in section 2 that the value of a risky production method would fall below the

[22] If $v(a)$ in section 2 is interpreted as the average product of a linear single-factor production function $f(l)$ then $v(a)$ would equal $f'(l)$.

[23] If $\langle l_n \rangle$ is an increasing sequence of real numbers where each $DV^c(l_n)$ (the derivative of V^c evaluated at l_n) is well-defined and $l_n \to e_{L_1}$ then $d^r = \lim_{n\to\infty} DV^c(l_n)$. Since V^c is concave, this limit exists and is independent of the $\langle l_n \rangle$ chosen.

[24] Since $v^{\widetilde{t}}$ satisfies constant returns, the value of each $d^{\widetilde{t}}(\omega)$ is not affected by the choice of $k > 0$.

reserve value.[25] Define the (economy-wide) **expected output** of L_1 to equal

$$\sum_{t\in T^c} \mathbb{E}v^t(L_1^t) + \sum_{t\in T^u} \mathbb{E}v^t(e_t, L_1^t).$$

Theorem 2 *Suppose labor is used in the uncertain technology t when t but not $\widetilde{t}$ is available and that $\widetilde{t}$ is nontrivially partitionally riskier than t. Then an increase in economy-wide expected output is feasible if $\widetilde{t}$ replaces t. If in addition utilities are linear a Pareto improvement over the initial allocation is feasible if $\widetilde{t}$ replaces t.*

As in the proof of Theorem 1, the increased riskiness of $\widetilde{t}$ does no harm and will deliver a benefit if the realization of the marginal product of labor in $\widetilde{t}$ falls below the marginal product of labor for the certain technologies: a transfer of labor from $\widetilde{t}$ to the certain technologies will then increase total output. An explicit proof is in Appendix B with other proofs omitted from the text. An example illustrates.

Example 3 Let there be one certain 'reserve' technology with production function $2l$. The uncertain technologies t and $\widetilde{t}$ follow Example 2: we again set $\Omega = \{g, b\}$ and $\mathbb{P}(g) = \mathbb{P}(b) = \frac{1}{2}$, let the technology t be constant across states, and let $\widetilde{t}$ be strictly partitionally riskier than t. For concreteness, let $v^t(k,l)(\omega) = 5\min[k,l]$ for $\omega \in \{g, b\}$, while $v^{\widetilde{t}}(k,l)(g) = 9\min[k,l]$ and $v^{\widetilde{t}}(k,l)(b) = \min[k,l]$. Then $d^r = 2$ while $d^{\widetilde{t}}(g) = 9$ and $d^{\widetilde{t}}(b) = 1$. Since $\mathbb{P}\left(d^{\widetilde{t}} < 2 \middle| \Omega\right) = \frac{1}{2}$, $\widetilde{t}$'s greater risk is nontrivial. If e_t units of t-capital are available and employ e_t units of date-1 labor then output will equal $5e_t$. If instead e_t units of $\widetilde{t}$-capital are available for technology $\widetilde{t}$ and are used in state g but abandoned in state b, then e_t units of date-1 labor will produce $9e_t$ units of output in state g (using $\widetilde{t}$) and $2e_t$ units of output in state b (using the certain technology). Since expected output increases from $5e_t$ to $\frac{11}{2}e_t$ a Pareto improvement is achievable when $\widetilde{t}$ replaces t and utilities are linear. ■

[25] Details aside, there is a substantive difference between nontriviality and v^c-nontriviality. In section 2, v^c-nontriviality can be satisfied when v^c is greater than the realized value of the safer production method but less than the realized value of the riskier method. Nothing comparable appears in the present definition of nontriviality. The reason is that if $D_l v^t(1,0) < d^r < D_l v^{\widetilde{t}}(1,0)$ occurs (where as usual t is the safer and $\widetilde{t}$ is the riskier technology) then it still might not be efficient to use $\widetilde{t}$: there may be capital available for several uncertain technologies and it could be more productive to use the non-$\widetilde{t}$ uncertain technologies when t suffers a poor realization. In this sense, it is the potential for riskier technologies to fail that provides the main route by which greater production risk delivers its benefits.

3.3 Efficient risky technologies with smaller expected output

When a riskier technology $\widetilde{t}$ substitutes for a safer technology t the expected output observed when $\widetilde{t}$ is available will typically be less than when t is available: in Example 3, expected output is $\frac{9}{2}e_t$ for $\widetilde{t}$ versus $5e_t$ for t. Of course $\widetilde{t}$ still delivers an overall benefit due to the output gained from resorting to the reserve technology when $\widetilde{t}$ is abandoned.

This example generalizes: the use of a riskier technology can bring an economy-wide benefit even though its observed expected output falls short of the observed output of its safer counterpart. Since greater risk and social benefit can negatively correlate with observations of expected output, it is misleading to use those observations to measure the value of a technology.

Call $\mathbb{E}v^t(e_t, L_1^t)$ the **observed expected output of technology** t when inputs L_1 are applied.

Theorem 3 *Suppose labor is used in the uncertain technology t when t but not $\widetilde{t}$ is available and that $\widetilde{t}$ is nontrivially partitionally riskier than t. If t is replaced with $\widetilde{t}$ then economy-wide expected output can be increased even though the observed expected output of $\widetilde{t}$ falls relative to the initial observed expected output of t.*

Since riskier technologies can have lower expected output, it is important for phenomena like the risk-return trade-off and the private equity premium puzzle not to measure the return of a firm or its efficiency by its expected output. The tradition of identifying dividend flows with consumption and output will miss the greater productivity of risky technologies.[26] In line with the premium puzzle, we should in fact expect risky entrepreneurial investments to have poor output performance. In the next section, we will see that these risky investments will be undertaken in competitive equilibrium.

4 Technology tests in the market

To endogenize the construction of capital, we add an initial period and compIete markets to the section 3 model and show that the first welfare theorem holds. Firms therefore make

[26] See Mehra and Prescott (1985), Whitelaw (2000), and Lettau and Ludvigson (2001).

efficient production decisions and will test the risky technologies that should be tested; the quasi-rents earned when a technology is successful ensure that the efficient capital investments are profitable. The efficiency of competition holds despite the fact that when an experiment with a risky technology fails, it is the reserve producers that reap the gains. Our conclusions argue against the view that investment in technical change either brings an externality that allows other producers to free ride or leads to monopoly.

So far we have considered only the comparative-statics benefit of risk: one uncertain production method or technology has replaced another. The initial capital-construction period will let investments in uncertain technologies be scalable and let many risky technologies be tested at once. These features will allow us to see when riskier technologies are so dominant that safer technologies remain entirely untouched.

4.1 The intertemporal economy

In the initial period, date 0, labor is used to produce consumption and to build capital to be used in the uncertain technologies at date 1.

We keep the assumptions and terminology already laid out except where noted. Let each agent $i \in \mathcal{I}$ at date 0 now additionally be endowed with a quantity of labor $e^i_{L_0} > 0$ and consume x^i_0. Redefining $x^i = (x^i_0, x^i_1) \in \mathbb{R}^{1+|\Omega|}_+$, agent i now has utility $U^i(x^i) = \sum_{\omega \in \Omega} \mathbb{P}(\omega) u^i\left(x^i_0, x^i_1(\omega)\right)$ when consuming x^i, where u^i remains concave and is strictly increasing in both arguments.

Let L^t_0 indicate the labor used in a certain technology when $t \in T^c$ and the labor used to build t-capital when $t \in T^u$. Define $L_0 = (L^t_0)_{t \in T}$, $L = (L_0, L_1)$, and $L^t = (L^t_0, L^t_1)$.

Since we are free to choose a measurement unit for capital, we assume without loss of generality that if l units of date-0 labor are invested in technology $t \in T^u$ then l units of t-capital are produced. As capital is now produced, agents have no initial capital endowment and the aggregate produced quantity of t-capital is L^t_0. The usage of t-capital at date 1 and state ω is denoted K^t. Define $K = (K^t)_{t \in T^u} = (K^t(\omega)_{\omega \in \Omega})_{t \in T^u}$.

Since the quantities of capital and hence the testing of uncertain technologies will be determined in equilibrium rather than exogenously, the information agents receive will now be endogenous. As before, agents know the v^t functions only for the uncertain technologies that have been tested via positive investment. Given L, the information partition $\mathcal{P}_L$ that represents this information partition is the coarsest common refinement of the $\mathcal{P}^t$ such that $t \in T^u$ and $L_1^t > 0$.[27] A random variable Y is L-**measurable** if Y is constant on the cells of $\mathcal{P}_L$.

Definition 2 *A (x, L, K) is **feasible** (in an intertemporal economy) if x, L, and K are L-measurable and*

$$\begin{array}{ll}
\sum_{i\in\mathcal{I}} x_0^i \le \sum_{t\in T^c} v^t(L_0^t), & \\
\sum_{i\in\mathcal{I}} x_1^i(\omega) \le \sum_{t\in T^c} v^t(L_1^t(\omega)) + \sum_{t\in T^u} v^t(K^t(\omega), L_1^t(\omega))(\omega) & \text{for each } \omega \in \Omega, \\
K^t(\omega) \le L_0^t & \text{for each } t \in T^u \text{ and } \omega \in \Omega, \\
\sum_{t\in T} L_0^t \le \sum_{i\in\mathcal{I}} e_{L_0}^i, & \\
\sum_{t\in T} L_1^t(\omega) \le \sum_{i\in\mathcal{I}} e_{L_1}^i & \text{for each } \omega \in \Omega.
\end{array}$$

If (x, L, K) is feasible then we also say that x is **feasible using the inputs** (L, K).

Finally, (1) **utilities are linear** if, for each i, x^i, and ω, $u^i(x_0^i, x_1^i(\omega)) = x_0^i + x_1^i(\omega)$, and (2) the inputs $(L, K) \ge 0$ **test** (or **invest in**) **the uncertain technology** $t \in T^u$ if $L_0^t > 0$.

4.2 Competitive equilibria

The definition of equilibrium is routine except on one point. While agents must make choices that are measurable with respect to their information, they need to form expectations of the prices that would rule if they were to uncover more information, even if that discovery is not conducted in equilibrium: if $t \in T^u$ is not tested in equilibrium ($L_0^t = 0$), producers still have the option to choose $L_0^t > 0$ and then follow a date-1 plan that varies with the information about v^t that would be revealed. Producers thus need to anticipate the prices that would prevail at each event they could observe.[28] To be reasonable, anticipated prices must be able to clear markets if the exact state were known and must be competitive in the sense that agents should not be able to buy at disproportionately low prices (or sell at

[27] Previously this partition was $\mathcal{P}$ given a capital endowment of $(L_0^t)_{t\in T^u}$.

[28] There would be no issue of price expectations under the alternative assumption that the true state is always revealed at the beginning of date 1 regardless of the date 0 investment decision. Each state-contingent good would then be traded and the measurability requirement would be omitted.

disproportionately high prices) at some of the states they cause to be revealed. Although at first glance price-taking would seem to be a meaningless idea when agents can uncover information, since the set of available goods changes with agent actions, the concept can be adapted suitably.

The competitive rules for state-by-state prices that we impose will ensure that the first welfare theorem holds. Without these rules, inefficiency could result as we show in Appendix A. The inefficient equilibria have a self-confirming character where agents' beliefs persist only because their own choices stop any disconfirming information from appearing, as in Fudenberg and Levine (1993).[29] The presence of inefficient equilibria is surprising since, outside of the activity discovery of information, every classical assumption of a competitive equilibrium is satisfied.

The prices of output and labor are p_0 and w_0 at date 0 and $p_1 = (p_1(\omega))_{\omega\in\Omega}$ and $w_1 = (w_1(\omega))_{\omega\in\Omega}$ at date 1. Define $w = (w_0, w_1)$ and $p = (p_0, p_1)$ and let the prices of capital goods at date 1 be $p_K = (p_K^t(\omega))_{t\in T^u,\omega\in\Omega}$. As the exact state will not usually be revealed, the prices of goods at the events relevant to an agent will equal the sum of the prices for the states in those events. If for example $\overline{L}$ obtains and agents therefore face the partition $\mathcal{P}_{\overline{L}}$ then a consumer buying a unit of date-1 output at the event $P \in \mathcal{P}_{\overline{L}}$ would pay $\sum_{\omega\in P} p_1(\omega)$.[30]

Given $\overline{L}$, the prices p_1, w_1, and p_K^t are $\overline{L}$-**compatible** if there exists a conditional probability $\mathbb{P}(\cdot|\cdot)$ that satisfies Bayes' rule when applicable such that, for all $P \in \mathcal{P}_{\overline{L}}$ and all $\omega \in P$,

$$\begin{aligned} p_1(\omega) &= \mathbb{P}(\omega|P)\sum\nolimits_{\omega'\in P} p_1(\omega'), \\ w_1(\omega) &= \mathbb{P}(\omega|P)\sum\nolimits_{\omega'\in P} w_1(\omega'), \\ p_K^t(\omega) &= \mathbb{P}(\omega|P)\sum\nolimits_{\omega'\in P} p_K^t(\omega'). \end{aligned}$$

We also call p, w, and p_K $\overline{L}$-compatible when their pertinent components are $\overline{L}$-compatible. To see the competition rationale for these proportionality rules, consider a purchase of a

[29] These issues are studied in detail in Mandler (2017).

[30] A firm that contemplates testing a technology t that is not in fact tested in equilibrium would consider the events in the coarsest common refinement of $\mathcal{P}_{\overline{L}}$ and $\mathcal{P}^t$. If Q is one of these events, the firm when buying a unit of date-1 labor at Q would pay $\sum_{\omega\in Q} w_1(\omega)$.

unit of date-1 output at $P \in \mathcal{P}_{\overline{L}}$ for a price of $\sum_{\omega' \in P} p_1(\omega')$. If the anticipated price of date-1 output at $P' \subset P$ were less than $\mathbb{P}(P'|P) \sum_{\omega' \in P} p_1(\omega')$ then an agent that can reveal P' or $P \backslash P'$ could do so and then buy output only at P', thus paying a smaller fraction of the price than the likelihood of P' given P: there would be a non-competitive price reward to undertaking investments that reveal information. With the above proportionality rules, agents are price-takers in the sense that when they uncover information the price per unit of probability mass remains unchanged.

The profits earned by technology $t \in T^c$ when $L^t \in \mathbb{R}_+^{1+|\Omega|}$ is chosen are given by

$$\pi^t(L^t) = p_0 v^t(L_0^t) - w_0 L_0^t + \sum_{\omega \in \Omega} \left(p_1(\omega) v^t(L_1^t(\omega)) - w_1(\omega) L_1^t(\omega)\right)$$

and the profits earned by $t \in T^u$ when $(L^t, K^t) \in \mathbb{R}_+^{1+2|\Omega|}$ is chosen are given by

$$\begin{aligned}\pi^t(L^t, K^t) = &\left(\sum_{\omega \in \Omega} p_K^t(\omega) - w_0\right) L_0^t \\ &+ \sum_{\omega \in \Omega} \left(p_1(\omega) v^t(K^t(\omega), L_1^t(\omega))(\omega) - w_1(\omega) L_1^t(\omega) - p_K^t(\omega) K^t(\omega)\right).\end{aligned}$$

The first term above equals the portion of profits earned from the construction of t-capital and thus incorporates the quasi-rents earned on successful investments. The random variable p_K^t is thus the return of a date-0 investment in t.

Consumer i has an ownership share $0 \leq \theta^{it} \leq 1$ for each $t \in T^c$ where $\sum_{i \in \mathcal{I}} \theta^{it} = 1$. Due to the constant returns assumption on v^t for $t \in T^u$, these technologies generate no profits in equilibrium. Given (p, w) and $\overline{L}$, $\overline{x}^i$ is **utility-maximizing for** i if $\overline{x}^i$ is $\overline{L}$-measurable and in budget set

$$\begin{aligned}B^i(p, w, \overline{L}) = \Big\{ x^i \in \mathbb{R}_+^{1+|\Omega|} : p_0 x_0^i &+ \sum_{\omega \in \Omega} p_1(\omega) x_1^i(\omega) \\ &\leq w_0 e_{L_0}^i + \sum_{\omega \in \Omega} w_1(\omega) e_{L_1}^i + \sum_{t \in T^c} \theta^{it} \pi^t(\overline{L}^t) \Big\},\end{aligned}$$

and if $U^i\left(\overline{x}^i\right) \geq U^i\left(x^i\right)$ for all $x^i \in B^i(p, w, \overline{L})$ that are $\overline{L}$-measurable.

Given (p, w, p_K) and $\overline{L}$, $(L^{t\prime}, K^{t\prime}) \geq 0$ is profit-maximizing for technology $t \in T^u$ if $(L^{t\prime}, K^{t\prime})$ is measurable with respect to the coarsest common refinement of $\mathcal{P}_{\overline{L}}$ and $\mathcal{P}^t$ and

$\pi^t(L^{t\prime}, K^{t\prime}) \geq \pi^t(L^t, K^t)$ for each $(L^t, K^t) \geq 0$ that is measurable with respect to the same partition. Similarly, and again given (p, w, p_K) and $\overline{L}$, $L^{t\prime} \geq 0$ is **profit-maximizing for technology** $t \in T^c$ if $L^{t\prime}$ is $\overline{L}$-measurable and $\pi^t(L^{t\prime}) \geq \pi^t(L^t)$ for each $L^t \geq 0$ that is $\overline{L}$-measurable.

Definition 3 *A* ***limited-information competitive equilibrium*** *is a* $\left(\overline{x}, \overline{L}, \overline{K}, p, w, p_K\right)$ *such that the prices* (p, w, p_K) *are* $\overline{L}$*-compatible and*

1. *for each agent* $i \in \mathcal{I}$, $\overline{x}^i$ *is utility-maximizing for* i,
2. *for each* $t \in T^u$ *and* $t' \in T^c$, $\left(\overline{L}^t, \overline{K}^t\right)$ *is profit-maximizing for* t *and* $\overline{L}^{t'}$ *is profit-maximizing for* t',
3. $\left(\overline{x}, \overline{L}, \overline{K}\right)$ *is feasible and, for each feasibility inequality in Definition 2, equality holds if the corresponding price is positive.*[31]

Theorem 4 *A limited-information competitive equilibrium exists.*[32]

It turns out that any limited-information competitive equilibrium qualifies as a standard competitive equilibrium (where the measurability requirement is omitted) for a fully orthodox model where the state is always revealed at the beginning of date 1 regardless of what investments are undertaken. Since the standard first welfare theorem implies that an equilibrium allocation x cannot be Pareto dominated by any allocation that is feasible in the orthodox model, x also cannot be Pareto dominated by any allocation that happens to satisfy the measurability requirement of Definition 2. The first welfare theorem therefore holds. As this argument indicates, we could drop the measurability requirement for feasible allocations and the Theorem below would still hold.

[31] In the order of the feasibility inequalities, the prices are p_0, $p_1(\omega)$, $p_K^t(\omega)$, w_0, $w_1(\omega)$.

[32] Given our positive endowment assumptions, the only nonstandard part of the proof is to establish measurability and compatibility. If $\left(\overline{x}, \overline{L}, \overline{K}, p, w, p_K\right)$ satisfies every other equilibrium condition then, fixing some $P \in \mathcal{P}_{\overline{L}}$, aggregate output remains constant across the positive-probability states in P: for any technology t that has been invested in, the realizations of v^t coincide at these states. We can therefore let all equilibrium quantities remain constant. Each agent i can consume $\mathbb{E}(\overline{x}_1^i|P)$ at the states in P and input levels can equal the values that hold at some positive-probability state in P. The weak risk aversion of the u^i implies that these replacement bundles weakly increase each U^i and it is easy to confirm that the replacements are affordable. As this argument applies to each $P \in \mathcal{P}_{\overline{L}}$, there is an equilibrium that satisfies measurability (in effect, an equilibrium that is constant across sunspots – see Cass and Polemarchakis (1990)).

Theorem 5 *The allocation of a limited-information competitive equilibrium is Pareto efficient.*

4.3 Crowding out safe technologies in the market

We now consider the extent to which riskier technologies will optimally displace safer technologies in competitive equilibrium when both are available. One point is straightforward: due to Theorem 5, the competitive equilibria of a risk-neutral economy must maximize expected output. While Theorems 1 and 2 might therefore seem to imply that only the riskiest technologies will be tested, those results did not allow many technologies to be tested at once. When two uncertain technologies, t and a riskier $\widetilde{t}$, are both available, they might be natural complements, with technology t *itself* providing the reserve technology to use when $\widetilde{t}$ suffers a poor realization. Although this twist means that full crowding out does not always obtain in equilibrium, it remains a robust outcome.

Call $t \in T^c$ **linear** if there is a $\lambda^t > 0$ such that $v^t(l) = \lambda^t l$ for all $l \geq 0$ and, elaborating Example 2, call $t \in T^u$ **linear** if for each $P \in \mathcal{P}^t$ there is a $\left(\kappa^{t,P}, \lambda^{t,P}\right) \geq 0$ such that $v^t(k,l)(\omega) = \min[\kappa^{t,P}k, \lambda^{t,P}l]$ for all $(k,l) \geq 0$ and $\omega \in P$. Finally an economy is **linear** if each u^i is linear and if each $t \in T$ is linear. We view sets of linear technologies or economies as points in the Euclidean space that has one dimension for each production coefficient and each independently-specifiable probability (a total of $|T^c| + \sum_{t \in T^u} 2\,|\mathcal{P}^t| + |\Omega| - 1$).

Example 4 Suppose the economy is linear, there is one certain technology t^r (r for reserve) with $v^r \equiv \lambda^{t^r} > 0$, $\Omega = \{g, b\}$, $\mathbb{P}(g) = \mathbb{P}(b) = \frac{1}{2}$, and there are two uncertain technologies. The first is t where productivity is unaffected by the state and $\left(\kappa^{t,\Omega}, \lambda^{t,\Omega}\right) = (\overline{v}, \overline{v})$ and the second is $\widetilde{t}$ where $\left(\kappa^{\widetilde{t},\{b\}}, \lambda^{\widetilde{t},\{b\}}\right) = (0,0)$ and $\left(\kappa^{\widetilde{t},\{g\}}, \lambda^{\widetilde{t},\{g\}}\right) = (2\overline{v}, 2\overline{v})$.[33] The parameter $\overline{v}$ increases the productivity of investment in both t and $\widetilde{t}$. With these coefficients, $\widetilde{t}$ is strictly partitionally riskier than t. Assume also that $(v^r, \overline{v}) \gg 0$ and $e_{L_0} > e_{L_1}$.

Let the *initial* (L, K) have $K = 0$ and use all labor in technology t^r: $L_0^{t^r} = e_{L_0}$ and $L_1^{t^r}(g) = L_1^{t^r}(b) = e_{L_1}$. If an $\varepsilon > 0$ investment in $\widetilde{t}$ is undertaken, $L_0^{\widetilde{t}} = \varepsilon$, a date-1

[33] Keep in mind that the defining feature of a t in T^u is that it requires explicit capital and is available only date 1; its output need not vary with the state.

consumption gain occurs only in state g and the expected gain is $\varepsilon\frac{1}{2}(2\overline{v} - v^r)$ while the loss in date-0 consumption is εv^r. Expected gains therefore exceed losses if

$$2\overline{v} \geq 3v^r. \tag{4.2.1}$$

If 4.2.1 is satisfied the linearity of the model implies that any investment level up to e_{L_1} units of date-0 labor in $\widetilde{t}$ will increase expected consumption relative to what the initial (L, K) achieves.

Consider therefore a (L, K) where $L_0^{\widetilde{t}}$ equals e_{L_1}, thus producing just the amount of $\widetilde{t}$-capital to fully employ date-1 labor, and let the remaining date-0 labor be used in t^r. If we withdraw $\varepsilon > 0$ labor from t^r-production at date 0 and invest that labor instead in t then a gain is achieved only in state b: the expected gain is therefore $\varepsilon\frac{1}{2}(\overline{v} - v^r)$. As the loss is again εv^r, expected gains now exceed losses if

$$\overline{v} \geq 3v^r. \tag{4.2.2}$$

When 4.2.2 is satisfied, technology t is sufficiently productive that it is worth investing in it only to provide a fallback technology when $\widetilde{t}$ fails.

For small values of $\overline{v}$ neither 4.2.1 nor 4.2.2 is satisfied and then expected consumption maximization requires that there is no investment (it follows from 4.2.1 that an investment in t will also diminish expected output). For large values of $\overline{v}$ both inequalities are satisfied and hence expected consumption maximization requires there to be investment in both t and $\widetilde{t}$. Efficient investment therefore need not imply full crowding out; it depends on $\overline{v}$.

For intermediate values of $\overline{v}$ in $\left[\frac{3v^r}{2}, 3v^r\right]$, investing in $\widetilde{t}$ will increase expected output but investing in t to provide a fallback is not worth the cost. Since in this case a shift of investment from $\widetilde{t}$ to t will also reduce expected consumption, it is efficient for all labor invested at date 0 to go to $\widetilde{t}$ and for none to go to t: technology $\widetilde{t}$ fully crowds out t.[34]

[34]Suppose first that $\left(L_0^t + L_0^{\widetilde{t}}\right) < \sum_{i\in\mathcal{I}} e_{L_1}^i$. Then (4.2.1) implies that an increase in $L_0^{\widetilde{t}}$ will increase expected output. If $\left(L_0^t + L_0^{\widetilde{t}}\right) > \sum_{i\in\mathcal{I}} e_{L_1}^i$ then (4.2.2) implies that a reduction in L_0^t will increase expected output. Finally if $\left(L_0^t + L_0^{\widetilde{t}}\right) = \sum_{i\in\mathcal{I}} e_{L_1}^i$ and $L_0^t > 0$ then the expected output effect of a $\varepsilon > 0$ reduction in L_0^t and a ε increase in $L_0^{\widetilde{t}}$ equals $\varepsilon\left(\frac{1}{2}(\beta\overline{v}) - \frac{1}{2}(\beta\overline{v} - v^r)\right) > 0$.

Our assumption that $e_{L_0} > e_{L_1}$ has meant that the economy has ample investment resources, thus making it worthwhile in some cases to invest in both $\widetilde{t}$ and t. With the complementary assumption $e_{L_0} \leq e_{L_1}$, full crowding out always obtains when investment is efficient: if some labor is invested at date 0, all of it goes to $\widetilde{t}$. ■

Since the competitive equilibria of linear economies must maximize aggregate expected consumption, Example 4 shows that both full and partial crowding out are robust equilibrium events. The following Theorem sums up. If we were to introduce appropriate spaces of utilities and technologies, a similar result would hold for the general, nonlinear model. **Robust** means nonempty and open.

Theorem 6 *There are robust sets of linear economies, where $t, \widetilde{t} \in T^u$ and $\widetilde{t}$ is partitionally riskier than t, such that any limited-information competitive equilibrium invests in $\widetilde{t}$ but not in t. There are also robust sets, again with $t, \widetilde{t} \in T^u$ and $\widetilde{t}$ partitionally riskier than t, such that any equilibrium invests in both t and $\widetilde{t}$.*

Example 4 continued To illustrate the competitive equilibria of Theorem 6, consider the prices that rule when there is full investment in $\widetilde{t}$ and where all other labor is devoted to t^r. The following 0-profit conditions for processes in use and nonpositive profit conditions for the other processes must then hold:

t^r	$w_0 = p_0 v^r,\ w_1(b) = p_1(b) v^r,\ w_1(g) \geq p_1(g) v^r,$
t- and $\widetilde{t}$-investment	$w_0 \geq p_K^t(b) + p_K^t(g),\ w_0 = p_K^{\widetilde{t}}(b) + p_K^{\widetilde{t}}(g),$
$\widetilde{t}$	$p_K^{\widetilde{t}}(g) + w_1(g) = 2\overline{v} p_1(g),\ p_K^{\widetilde{t}}(b) + w_1(b) \geq 0,$
t	$p_K^t(b) + w_1(b) \geq p_1(b)\overline{v},\ p_K^t(g) + w_1(g) \geq p_1(g)\overline{v}.$

The linearity of the utilities implies $p_1(g) = p_1(b) = \frac{1}{2} p_0$ and the fact that $\widetilde{t}$-capital will be in excess supply in state b implies $p_K^{\widetilde{t}}(b) = 0$. Given these constraints, it is easy to confirm that the conditions above have a solution $(p, w, p_K) \geq 0$ if and only $\overline{v} \in \left[\frac{3v^r}{2}, 3v^r\right]$, which confirms the first welfare theorem fact that all equilibrium investment can be devoted to $\widetilde{t}$ only if it is efficient to do so.

It is easy to confirm that for the equilibria under discussion,

$$w_1(g) - w_1(b) = \overline{v} - \frac{3v^r}{2} \geq 0,$$

where we have set the numéraire $p_0 = 1$. The higher productivity of $\widetilde{t}$ in state g drives up the wage and makes t^r unprofitable in that state. Investors, foreseeing the high value of $w_1(g)$, do not invest in t. ■

Inefficient self-confirming equilibria, where prices fail to be compatible, can be embedded in Example 4. See Appendix A.

5 Risk aversion and partial crowding out

With risk aversion it will be impossible to conclude in any generality that a riskier technology will fully crowd out a safer technology when both are available: there is no reason why the productivity advantage of risk should fully outweigh the penalty of more variable consumption. We can however apply the principle that a sufficiently small quantity of a positive-expected-value asset or gamble increases expected utility when ex ante consumption is constant across states.

Partial crowding out of a safer technology t by a riskier $\widetilde{t}$ will bring the helpful side effect that we can do without separate reserve technologies. By retaining some investment in t, that technology can itself provide the reserve technology to be used when $\widetilde{t}$ suffers a low productivity realization. The only assumption needed is that there is a positive probability that t and $\widetilde{t}$ have different marginal products of labor: there will then be an adjustment of labor between t and $\widetilde{t}$ that increases aggregate output.

Suppose in this section that T^u contains the technologies t and $\widetilde{t}$ and let $\mathcal{P}^{-\widetilde{t}}$ again denote the coarsest common refinement of the $\mathcal{P}^{t'}$ with $t' \in T^u \backslash \{\widetilde{t}\}$ (the information revealed when all technologies except $\widetilde{t}$ are tested). Define technology $\widetilde{t}$ to have **different derivatives** than t if, for each $\omega \in \Omega$, $v^t(\cdot,\cdot)(\omega)$ and $v^{\widetilde{t}}(\cdot,\cdot)(\omega)$ are differentiable functions and, for all $(k,l) \gg 0$ and $P \in \mathcal{P}^{-\widetilde{t}}$,

$$\mathbb{P}\left(D_l v^{\widetilde{t}}(k,l) \neq D_l v^t(k,l) \middle| P \right) > 0.$$

This condition is very mild; there only needs to be a positive chance that a knife-edge equality is violated.

Although consumption is *not* constant across states in the present model, we can nevertheless invoke the principle that small amounts of positive-expected-value gambles are utility increasing. Consumption is constant on the cells of $\mathcal{P}_L$, the partition that indicates the information revealed by the investments the economy undertakes, and we can show that a small investment in $\widetilde{t}$ is utility-improving on each cell taken individually.

Define inputs (L, K) to **invest productively** in technology t if $\mathbb{E}\left(v^t(L_0^t, L_1^t)\right) > 0$.

Theorem 7 *Suppose the uncertain technology $\widetilde{t}$ is partitionally riskier and has different derivatives than t and that initially inputs invest productively in t and do not invest in $\widetilde{t}$. Then a Pareto improvement is feasible using inputs that invest in $\widetilde{t}$.*

If t and $\widetilde{t}$ are the only technologies in T then Theorem 7 implies that to achieve Pareto efficiency there must be investment in *both* t and $\widetilde{t}$: operating $\widetilde{t}$ alone would introduce consumption risk with no compensating increase in output.

The following Corollary translates Theorem 7 into equilibrium behavior and follows from Theorem 5.

Corollary 1 *If $\widetilde{t}$ is partitionally riskier and has different derivatives than t then a limited-information competitive equilibrium that invests in t also invests in $\widetilde{t}$.*

When there is investment in both t and $\widetilde{t}$ the standard functioning of competitive markets at date 1 will efficiently reallocate labor between the technologies as different productivity realizations obtain.

6 Conclusion: a small investment in a big risk is good, any investment in a small risk is bad

To conclude, we show for linear technologies that 'small' risks as measured by their range of productivity outcomes are harmful.

Recall from Theorem 2 that a risky technology $\widetilde{t}$ is beneficial when it is possible for it to have a realization that is poor enough that its marginal of product of labor falls below that of the certain technologies. Risks that are substantial in this sense deliver a benefit. Even if consumers were risk averse, at least a small-scale investment in one of these substantial risks can deliver a Pareto improvement.

For a converse that small risks are harmful at any scale, we assume that ex ante every technology is certain and show that the intrusion of a small risk into such a world must lower welfare. Though technologies are certain ex ante, we continue to divide technologies into those that require new capital investment (and are therefore in T^u) and those that do not. We assume in this section that all technologies are linear, perhaps because of doubts that the gains to factor substitution between riskier and safer technologies, exploited in Theorem 7, could be substantial. Consequently, when a riskier $\widetilde{t}$ replaces a safer t, the only source of a potential gain occurs when the productivities of t and $\widetilde{t}$ fall on different sides of the productivity of some alternative reserve technology.

As before, a technology t is linear if at each state the production function takes the form $\lambda^t l$ or $\min[\kappa^t k, \lambda^t l]$, where λ^t and κ^t vary as a function of the state. Accordingly we view λ^t and κ^t as random variables.[35] In line with section 3.3, a linear technology t is **constant** if λ^t and κ^t are constant functions of $\omega \in \Omega$. Let the **risk magnitude** of a technology $\widetilde{t}$ equal $\left(\max \lambda^{\widetilde{t}} - \min \lambda^{\widetilde{t}}\right) + \left(\max \kappa^{\widetilde{t}} - \min \kappa^{\widetilde{t}}\right)$.

We consider a set of models by fixing the utilities, endowments, and the number of technologies in T^c and T^u. When technologies are linear, define a set of models to be **generic** if the technologies in the model form an open, full-measure set (see section 4.3).

Theorem 8 *Suppose utilities are strictly concave. For a generic set of models where technologies are linear and constant, if an allocation is Pareto efficient and feasible using inputs that invest in technology t and t is replaced by a technology $\widetilde{t}$ that (i) is strictly partitionally riskier than t and (ii) has sufficiently small risk magnitude, then any feasible allocation reduces some agent's utility.*

The argument behind the omitted proof is that, since the ex ante allocation is efficient,

[35] Formally, if $P(\omega)$ denotes the $P \in \mathcal{P}^t$ such that $\omega \in P$ then κ^t for example is the random variable $\kappa^{t,P(\cdot)}$.

generically every other technology $t' \neq t$ has $\lambda^{t'} < \lambda^t$. Hence the realization of $\lambda^{\widetilde{t}}$ must lie above $\max_{t' \in T^c} \lambda^{t'}$ when the risk magnitude of $\widetilde{t}$ is sufficiently small. Consequently there is no expected output gain from $\widetilde{t}$'s greater risk; the only effect is to increase the dispersion of consumption, which lowers welfare due to the strict concavity of the utilities.

A Appendix: Inefficient, self-confirming equilibria

If in Example 4 prices are allowed to not be compatible then the resulting equilibria, which we call 'weak', can be inefficient. Let $\overline{v} \in \left[\frac{3v^r}{2}, 3v^r\right]$ in which case efficiency requires that all investment is devoted to $\widetilde{t}$. In the following weak equilibrium, all investment is devoted to t. If date-0 output is the numéraire, $p_0 = 1$, then with investment only in t profit maximization requires:

t^r	$w_0 = v^r,\ w_1(b) + w_1(g) \geq p_1(b)v^r + p_1(g)v^r,$
t- *and* $\widetilde{t}$-*investment*	$w_0 = p^t_K(b) + p^t_K(g),\ w_0 \geq p^{\widetilde{t}}_K(b) + p^{\widetilde{t}}_K(g),$
$\widetilde{t}$	$p^{\widetilde{t}}_K(g) + w_1(g) \geq 2\overline{v}p_1(g),\ p^{\widetilde{t}}_K(b) + w_1(b) \geq 0,$
t	$p^t_K(b) + w_1(b) + p^t_K(g) + w_1(g) = p_1(b)\overline{v} + p_1(g)\overline{v}.$

Only the $\widetilde{t}$ producer must satisfy a nonnegative profit condition at each of the states since only that producer can cause ω to be revealed; the other producers take the coarse partition $\mathcal{P}_L = \{\Omega\}$ as given.

The following (deliberately extreme) price vector satisfies these conditions when $\overline{v} \geq 2v^r$: $p_1(b) = 1$, $p_1(g) = 0$, $w_0 = v^r$, $w_1(b) = w_1(g) = \frac{\overline{v}-v^r}{2}$, $p^{\widetilde{t}}_K(b) = 0$, $p^{\widetilde{t}}_K(g) = v^r$, $p^t_K(b) = p^t_K(g) = \frac{v^r}{2}$. It is easily confirmed that if $L^{t^r}_0 = e_{L_0} - e_{L_1}$, $L^t_0 = e_{L_1}$, $K^t(b) = K^t(g) = L^t_0$, and $L^t_1(b) = L^t_1(g) = e_{L_1}$ then all markets clear. In weak equilibrium, consumers must pay the same amount, $p_0 = p_1(b) + p_1(g)$, for one unit of date-0 consumption and one unit of expected date-1 consumption. But $p_1(b) = p_1(g) = \frac{1}{2}$ need not obtain, which is precisely the suspicious feature of the above equilibrium: the prices that producers expect are not the equilibrium prices that would rule if the true state were revealed. Although revelation is nearly costless – it requires only a ε investment in $\widetilde{t}$ – it does not occur and so agents' beliefs in the prices that would obtain at b and g are not disconfirmed.

B Appendix: Remaining proofs

Proof of Theorem 2. Recalling that in our notation, T is assumed to contain both t and $\widetilde{t}$, define $(\widehat{L}^{t'}_1)_{t' \in (T \backslash \{t\})}$ by $\widehat{L}^{\widetilde{t}}_1 = L^t_1$ and $\widehat{L}^{t'}_1 = L^{t'}_1$ for $t' \in T \backslash \{t, \widetilde{t}\}$.

Fix some $P \in \mathcal{P}^{-\tilde{t}}$ and let $l = L_1^t(P)$. Then

$$\begin{aligned}\sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) v^{\tilde{t}} \left(e_{\tilde{t}}, \widehat{L}_1^{\tilde{t}}(\omega)\right)(\omega) &= \sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) v^{\tilde{t}} \left(e_t, L_1^t(\omega)\right)(\omega) \\ &= \sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) \left(v^t \left(e_t, L_1^t(\omega)\right)(\omega) + Z_{e_t,l}(\omega)\right) \\ &= \sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) v^t \left(e_t, L_1^t(\omega)\right)(\omega).\end{aligned}$$

The second and third equalities are due to Definition 1. Hence

$$\begin{aligned}&\sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \widehat{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\widehat{L}_1^{t'}(\omega)\right)(\omega)\right) \\ &\qquad = \sum\nolimits_{\omega \in P} \mathbb{P}(\omega|P) \left(\sum\nolimits_{t' \in T^u \backslash \{\tilde{t}\}} v^{t'} \left(e_{t'}, L_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(L_1^{t'}(\omega)\right)(\omega)\right).\end{aligned}$$

Since this inequality applies to any $P \in \mathcal{P}^{-\tilde{t}}$,

$$\begin{aligned}&\sum\nolimits_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \widehat{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\widehat{L}_1^{t'}(\omega)\right)(\omega)\right) \\ &\qquad = \sum\nolimits_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum\nolimits_{t' \in T^u \backslash \{\tilde{t}\}} v^{t'} \left(e_{t'}, L_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(L_1^{t'}(\omega)\right)(\omega)\right).\end{aligned}$$

That is, expected output with $(\widehat{L}_1^{t'})_{t' \in T \backslash \{t\}}$ equals expected output with $(L_1^{t'})_{t' \in T \backslash \{\tilde{t}\}}$.

By assumption there is a $P' \in \mathcal{P}$ such that $\mathbb{P}(P') > 0$ and $L_1^t(\omega) > 0$ for $\omega \in P'$. Due to the nontriviality assumption, $\mathbb{P}\left(d^{\tilde{t}} < d^r \middle| P'\right) > 0$. Letting $\mathcal{P}^*$ denote the coarsest common refinement of $\mathcal{P}$ and $\mathcal{P}^{\tilde{t}}$, there is consequently a $P^* \in \mathcal{P}^*$ such that $P^* \subset P'$, $\mathbb{P}(P^*) > 0$, and $d^{\tilde{t}}(\omega) < d^r$ for all $\omega \in P^*$. Define $(\overline{L}_1^{t'})_{t' \in T \backslash \{t\}}$ by $\overline{L}_1^{t'} = \widehat{L}_1^{t'}$ if $t' \in T^u \backslash \{t, \tilde{t}\}$, $\overline{L}_1^{\tilde{t}}(\omega) = \widehat{L}_1^{\tilde{t}}(\omega)$ if $\omega \in \Omega \backslash P^*$, $\overline{L}_1^{t'}(\omega) = \widehat{L}_1^{t'}(\omega)$ if $t' \in T^c$ and $\omega \in \Omega \backslash P^*$, $\overline{L}_1^{\tilde{t}}(\omega) = 0$ if $\omega \in P^*$, and finally set the $\overline{L}_1^{t'}(\omega)$ for $t' \in T^c$ and $\omega \in P^*$ so that $\sum_{t' \in T^c} \overline{L}_1^{t'}(\omega) + \sum_{t' \in T \backslash \{t, \tilde{t}\}} \overline{L}_1^{t'}(\omega) = e_{L_1}$ and $\sum_{t' \in T^c} v^{t'}(\overline{L}_1^{t'}(\omega)) = V^c \left(\sum_{t' \in T^c} \overline{L}_1^{t'}(\omega)\right)$. Since $d^{\tilde{t}}(\omega) < d^r$ for $\omega \in P^*$,

$$\begin{aligned}&\sum\nolimits_{\omega \in P^*} \mathbb{P}(\omega|P^*) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \overline{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\overline{L}_1^{t'}(\omega)\right)(\omega)\right) \\ &\qquad > \sum\nolimits_{\omega \in P^*} \mathbb{P}(\omega|P^*) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \widehat{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\widehat{L}_1^{t'}(\omega)\right)(\omega)\right).\end{aligned}$$

Since $\mathbb{P}(P^*) > 0$,

$$\begin{aligned}&\sum\nolimits_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \overline{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\overline{L}_1^{t'}(\omega)\right)(\omega)\right) \\ &\qquad > \sum\nolimits_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum\nolimits_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \widehat{L}_1^{t'}(\omega)\right)(\omega) + \sum\nolimits_{t' \in T^c} v^{t'} \left(\widehat{L}_1^{t'}(\omega)\right)(\omega)\right).\end{aligned}$$

If utilities are linear, any increase in expected output allows a Pareto domination.

To conclude, we adjust $\overline{L}$ so that it is $\widetilde{\mathcal{P}}$-measurable, where $\widetilde{\mathcal{P}}$ is the coarsest common refinement of the $\mathcal{P}^{t'}$ such that $t' \in T^u \backslash \{t\}$ and $e_{t'} > 0$. Fix $P \in \widetilde{\mathcal{P}}$. For $\omega, \omega' \in P$ and t' such that $e_{t'} > 0$, $v^{t'}(k,l)(\omega) = v^{t'}(k,l)(\omega')$ for all $(k,l) \geq 0$. Therefore, setting $\omega \in P$ arbitrarily, let $\overline{\overline{L}}_1(\omega')$ for each $\omega' \in P$ equal the $\left(l^{t'}\right)_{t' \in T\backslash\{t\}}$ that solves

$$\begin{aligned} \max \quad & \textstyle\sum_{t' \in T^c} v^{t'}(l^{t'}) + \sum_{t' \in T^u \backslash \{t\}} v^t(e_{t'}, l^{t'})(\omega) \\ \text{s.t.} \quad & \textstyle\sum_{t' \in T\backslash\{t\}} l^{t'} \leq e_{L_1}, \ \left(l^{t'}\right)_{t' \in T} \geq 0. \end{aligned}$$

Repeating for each $P \in \widetilde{\mathcal{P}}$, the $\overline{\overline{L}}_1$ that results is $\widetilde{\mathcal{P}}$-measurable and

$$\begin{aligned} &\sum_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \overline{\overline{L}}_1^{t'}(\omega) \right)(\omega) + \sum_{t' \in T^c} v^{t'} \left(\overline{\overline{L}}_1^{t'}(\omega) \right)(\omega) \right) \\ &\qquad \geq \sum_{\omega \in \Omega} \mathbb{P}(\omega) \left(\sum_{t' \in T^u \backslash \{t\}} v^{t'} \left(e_{t'}, \overline{L}_1^{t'}(\omega) \right)(\omega) + \sum_{t' \in T^c} v^{t'} \left(\overline{L}_1^{t'}(\omega) \right)(\omega) \right), \end{aligned}$$

again allowing a Pareto domination. ■

Proof of Proposition 3. Let L_1 be the input levels of the original model where t is available. By assumption there is a $P' \in \mathcal{P}$ such that $\mathbb{P}(P') > 0$ and $L_1^t(\omega) > 0$ for $\omega \in P'$. Let $(\widehat{L}_1^{t'})_{t' \in T\backslash\{t\}}$, $\mathcal{P}^*$, $P^* \in \mathcal{P}^*$, and $(\overline{L}_1^{t'})_{t' \in T\backslash\{t\}}$ be defined as in the proof of Theorem 2.

As in the proof of Theorem 2, the expected outputs of $\widehat{L}_1$ and L_1 are equal and the expected output of $\overline{L}_1$ is strictly greater than the expected output of $\widehat{L}_1$. For each $P \in \mathcal{P}^*$ besides P^*,

$$\sum_{\omega \in P} \mathbb{P}(\omega|P) v^{\tilde{t}} \left(e_{\tilde{t}}, \overline{L}_1^{\tilde{t}}(\omega) \right)(\omega) = \sum_{\omega \in P} \mathbb{P}(\omega|P) v^{\tilde{t}} \left(e_{\tilde{t}}, \widehat{L}_1^{\tilde{t}}(\omega) \right)(\omega),$$

while for P^*,

$$\sum_{\omega \in P^*} \mathbb{P}(\omega|P^*) v^{\tilde{t}} \left(e_{\tilde{t}}, \overline{L}_1^{\tilde{t}}(\omega) \right)(\omega) < \sum_{\omega \in P^*} \mathbb{P}(\omega|P^*) v^{\tilde{t}} \left(e_{\tilde{t}}, \widehat{L}_1^{\tilde{t}}(\omega) \right)(\omega).$$

Since $\mathbb{P}(P^*) > 0$, the expected output of $\tilde{t}$ in the model that replaces t with $\tilde{t}$ is strictly smaller than the expected output of t in the original model.

It remains to adjust $\overline{L}_1$ so that it is $\widetilde{\mathcal{P}}$-measurable. If X is a random variable constant on $E \subset \Omega$, let $X(E)$ denote $X(\omega)$ for any $\omega \in E$. For each $t' \in T\backslash\{t\}$, $P \in \widetilde{\mathcal{P}}$, and $\omega \in P$, let $\mathcal{P}_P^*$ denote the cells of $\mathcal{P}^*$ contained in P and set $\overline{\overline{L}}_1^{t'}(\omega) = \sum_{P''' \in \mathcal{P}_P^*} \mathbb{P}(P'''|P) \overline{L}_1^{t'}(P''')$. By the concavity of $v^{t'}$, $\mathbb{E}\left[v^{t'}\left(e_{t'}, \overline{\overline{L}}_1^{t'} \right) \middle| P \right] = v^{t'}\left(e_{t'}, \overline{\overline{L}}_1^{t'}(P) \right)(P) \geq \mathbb{E}\left[v^{t'}\left(e_{t'}, \overline{L}_1^{t'} \right) \middle| P \right]$ for $t' \in T^u \backslash \{\tilde{t}\}$ and $\mathbb{E}\left[v^{t'}\left(\overline{\overline{L}}_1^{t'} \right) \middle| P \right] = v^{t'}\left(\overline{\overline{L}}_1^{t'}(P) \right)(P) \geq \mathbb{E}\left[v^{t'}\left(\overline{L}_1^{t'} \right) \middle| P \right]$ for $t' \in T^c$, thus achieving $\mathcal{P}$-measurability while preserving feasibility and the increase in expected output economy-wide determined in the previous paragraph. Further reset inputs into $v^{\tilde{t}}$ to equal $\lambda \overline{\overline{L}}_1^{\tilde{t}}$ where $\lambda \in (0,1]$ is set so that $\mathbb{E}\left[v^{\tilde{t}}\left(e_{\tilde{t}}, \lambda \overline{\overline{L}}_1^{\tilde{t}} \right) \right] = \mathbb{E}\left[v^{\tilde{t}}\left(e_{\tilde{t}}, \overline{L}_1^{\tilde{t}} \right) \right]$, thus preserving the decline in the expected output of $\widetilde{t}$ determined in the previous paragraph. Since, for any ω, $v^{\widetilde{t}}(\cdot)(\omega)$ is continuous and $v^{\widetilde{t}}\left(e_{\widetilde{t}},0\right)(\omega)=0$, such a λ exists. ■

Proof of Theorem 4. Standard existence arguments ensure there exists a $\left(\overline{x},\overline{L},\overline{K},p,w,p_K\right)$ that satisfies every equilibrium condition except possibly $\overline{L}$-measurability and $\overline{L}$-compatibility, which we call a *base equilibrium.* Fix some $P\in\mathcal{P}_{\overline{L}}$ and call $\left(\overline{x},\overline{L},\overline{K},p,w,p_K\right)$ the *initiating* base equilibrium for P. We build a new equilibrium where the quantities $\widetilde{x}$, $\widetilde{L}$, and $\widetilde{K}$ are constant across $\omega\in P$. To that end, set $\widetilde{x}_0=\overline{x}_0$, $\widetilde{L}_0=\overline{L}_0$, and, for $\omega\notin P$, $\widetilde{x}_1(\omega)=\overline{x}_1(\omega)$, $\widetilde{L}_1(\omega)=\overline{L}_1(\omega)$, $\widetilde{K}^t(\omega)=\overline{K}^t(\omega)$ for $t\in T^u$.

(1) For any $\omega\in\Omega$, $\mathbb{P}(\omega)=0$ if and only if $p_1(\omega)=0$. Proof. If $p_1(\omega)=0$ then the fact that utilities are increasing implies $\mathbb{P}(\omega)=0$. If $\mathbb{P}(\omega)=0$ and, to the contrary, $p_1(\omega)>0$ then utility maximization implies $\sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega)=0$ and profit maximization implies $w_1(\omega)>0$. But then market-clearing (Definition 3.3) implies $\overline{L}_1^t(\omega)=e_{L_1}(\omega)$ which, in combination with $p_1(\omega)\sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega)=0$, violates profit-maximization.

(2) If, for some $\omega\in\Omega$, $p_1(\omega)=0$ then $w_1(\omega)=0$ and $p_K^t(\omega)=0$ for each $t\in T^u$ with $\overline{L}_0^t>0$. Proof. If $w_1(\omega)>0$ (resp. $p_K^t(\omega)>0$ for some $t\in T^u$ with $\overline{L}_0^t>0$) then, as in (1), market-clearing would imply $\overline{L}_1^t(\omega)=e_{L_1}(\omega)$ (resp. $\overline{K}^t(\omega)=\overline{L}_0^t$) in violation of profit-maximization.

(3) Suppose that $\sum_{\omega\in P}p_1(\omega)=0$. Then, for $\omega\in P$, set $\widetilde{x}_1(\omega)=\widetilde{L}_1(\omega)=0$ and $\widetilde{K}^t(\omega)=0$ for each $t\in T^u$. Given (1), $\mathbb{P}(\omega)=0$ for each $\omega\in P$ and hence, for each $i\in\mathcal{I}$, $\widetilde{x}^i$ is utility-maximizing. Given (2), $w_1(\omega)=0$ and $p_K^t(\omega)=0$ for each $\omega\in P$ and $t\in T^u$ with $\overline{L}_0^t>0$. Hence profit-maximization and market-clearing are satisfied.

(4) Given (3), assume through (7) that $\sum_{\omega\in P}p_1(\omega)>0$. For each $i\in\mathcal{I}$ and $\omega\in P$, set $\widetilde{x}_1^i(\omega)=\frac{\sum_{\omega'\in P}p_1(\omega')\overline{x}_1^i(\omega')}{\sum_{\omega'\in P}p_1(\omega')}$ which, given $\widetilde{x}_0=\overline{x}_0$ and $\widetilde{x}_1(\omega)=\overline{x}_1(\omega)$ for $\omega\notin P$, defines a $\widetilde{x}^i$ for each $i\in\mathcal{I}$.

(5) Given the $\widetilde{x}^i$ defined in (4), aggregate date-1 consumption at any $\omega\in P$ equals

$$\sum_{i\in\mathcal{I}}\widetilde{x}_1^i(\omega)=\frac{\sum_{i\in\mathcal{I}}\sum_{\omega'\in P}p_1(\omega')\overline{x}_1^i(\omega')}{\sum_{\omega'\in P}p_1(\omega')}=\frac{\sum_{\omega'\in P}\left(p_1(\omega')\sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega')\right)}{\sum_{\omega'\in P}p_1(\omega')}. \tag{A.1}$$

Since for fixed $(k^t,l^t)\in\mathbb{R}_+^2$, $v^t(k^t,l^t)(\omega)$ is constant across $\omega\in P$, the first welfare theorem implies that, for any $\omega'\in P$ with $\mathbb{P}(\omega')>0$, $\sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega')$ must equal

$$\begin{aligned}
\max\quad & \textstyle\sum_{t\in T^c}v^t(l^t)+\sum_{t\in T^u}v^t(k^t,l^t)(\omega)\\
\text{s.t.}\quad & k^t\le\overline{L}_0^t \text{ for } t\in T^u \text{ and } \textstyle\sum_{t\in T}l^t\le e_{L_1},\\
& (k^t)_{t\in T^u}\ge 0,(l^t)_{t\in T}\ge 0.
\end{aligned}$$

Since $\mathbb{P}(\omega')=0$ implies $p_1(\omega')=0$ (see (1)), there exists a $\widehat{\omega}\in P$ with $\mathbb{P}(\widehat{\omega})>0$. So, substituting $\sum_{i\in\mathcal{I}}\overline{x}_1^i(\widehat{\omega})$ for each $\sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega')$ in A.1 and again using the fact that $\mathbb{P}(\omega')=0$ implies $p_1(\omega')=0$, we have $\sum_{i\in\mathcal{I}}\widetilde{x}_1^i(\omega)=\sum_{i\in\mathcal{I}}\overline{x}_1^i(\widehat{\omega})$ for all $\omega\in P$.

(6) Due to the concavity of u^i, $U^i(\widetilde{x}^i)\ge U^i(\overline{x}^i)$. If $\sum_{\omega\in P}p_1(\omega)\widetilde{x}_1^i(\omega)<\sum_{\omega\in P}p_1(\omega)\overline{x}_1^i(\omega)$ for any i then $\overline{x}_1^i$ could not be an equilibrium choice i. So $\sum_{\omega\in P}p_1(\omega)\widetilde{x}_1^i(\omega)\ge\sum_{\omega\in P}p_1(\omega)\overline{x}_1^i(\omega)$ for all i. Hence if $\sum_{\omega\in P}p_1(\omega)\widetilde{x}_1^{\widehat{i}}(\omega)>\sum_{\omega\in P}p_1(\omega)\overline{x}_1^{\widehat{i}}(\omega)$ holds for some $\widehat{i}\in\mathcal{I}$ then

$\sum_{i\in\mathcal{I}}\sum_{\omega\in P} p_1(\omega)\widetilde{x}_1^i(\omega) > \sum_{i\in\mathcal{I}}\sum_{\omega\in P} p_1(\omega)\overline{x}_1^i(\omega)$. But, by (5) and (1), $\sum_{i\in\mathcal{I}}\widetilde{x}_1^i(\omega) = \sum_{i\in\mathcal{I}}\overline{x}_1^i(\omega)$ for all $\omega \in P$ with $p_1(\omega) > 0$. Hence $\sum_{\omega\in P} p_1(\omega)\widetilde{x}_1^i(\omega) = \sum_{\omega\in P} p_1(\omega)\overline{x}_1^i(\omega)$ for all i. Since, for each i, $\widetilde{x}^i$ is therefore affordable at (p, w), $\widetilde{x}^i$ maximizes i's utility.

(7) Turning to the date-1 inputs, for each $t \in T^u$ set $\widetilde{K}^t(\omega) = \overline{L}_0^t$ for $\omega \in P$. Letting $\widehat{\omega}$ continue to denote the state identified in (5), set $\widetilde{L}_1^t(\omega) = \overline{L}_1^t(\widehat{\omega})$ for each $\omega \in P$ and $t \in T$. Letting $v^t(L_1^t(\omega))$ denote $v^t(\overline{L}_0^t, L_1^t(\omega))$ for $t \in T^u$, we have

$$\sum_{t\in T} v^t(\widetilde{L}_1^t(\omega)) = \sum_{t\in T} v^t(\overline{L}_1^t(\widehat{\omega})) \tag{A.2}$$

for $\omega \in P$. By the first welfare theorem $\sum_{i\in\mathcal{I}}\overline{x}_1^i(\widehat{\omega}) = \sum_{t\in T} v^t(\overline{L}_1^t(\widehat{\omega}))$. Combining this fact, the conclusion of (5), and A.2, we conclude that the date-1 output market clears when consumption equals $\widetilde{x}$. Since $\sum_{t\in T}\widetilde{L}_1^t(\omega) = \sum_{t\in T}\overline{L}_1^t(\widehat{\omega})$ and $\widetilde{K}^t(\omega) = \overline{L}_0^t$ for $\omega \in P$ and $t \in T^u$, the labor and capital markets clear. Finally, to see that $\widetilde{K}$ and $\widetilde{L}_1$ are profit-maximizing, observe first that $\sum_{t\in T}\overline{L}_1^t(\widehat{\omega}) = e_{L_1}$ since otherwise aggregate output could be increased at $\widehat{\omega}$ without decreasing aggregate output in any other date or state. Thus, since $w_1(\omega) > 0$ implies $\sum_{t\in T}\overline{L}_1^t(\omega) = e_{L_1}$ (by Definition 3.3), $w_1(\omega) > 0$ also implies $\sum_{t\in T}\widetilde{L}_1^t(\omega) = \sum_{t\in T}\overline{L}_1^t(\omega)$. So, for all values of $w_1(\omega)$, $w_1(\omega)\sum_{t\in T}\widetilde{L}_1^t(\omega) = w_1(\omega)\sum_{t\in T}\overline{L}_1^t(\omega)$. Similarly, for each $t \in T^u$, if $p_K^t(\omega) > 0$ then $\overline{K}^t(\omega) = \overline{L}_0^t$ and hence $\widetilde{K}^t(\omega) = \overline{K}^t(\omega)$. So, for all values of $p_K^t(\omega)$, $p_K^t(\omega)\widetilde{K}^t(\omega) = p_K^t(\omega)\overline{K}^t(\omega)$. Given A.2, we therefore have

$$\sum_{t\in T^c}\pi^t(\widetilde{L}^t) + \sum_{t\in T^u}\pi^t(\widetilde{L}^t, \widetilde{K}^t) \geq \sum_{t\in T^c}\pi^t(\overline{L}^t) + \sum_{t\in T^u}\pi^t(\overline{L}^t, \overline{K}^t).$$

But given profit maximization, $\pi^t(\overline{L}^t) \geq \pi^t(\widetilde{L}^t)$ for $t \in T^c$ and $\pi^t(\overline{L}^t, \overline{K}^t) \geq \pi^t(\widetilde{L}^t, \widetilde{K}^t)$ for $t \in T^u$. Hence $\pi^t(\widetilde{L}^t) = \pi^t(\overline{L}^t)$ for $t \in T^c$ and $\pi^t(\widetilde{L}^t, \widetilde{K}^t) = \pi^t(\overline{L}^t, \overline{K}^t)$ for $t \in T^u$.

We conclude that $\left(\widetilde{x}, \widetilde{L}, \widetilde{K}, p, w, p_K\right)$ is a base equilibrium and we say it is *generated by* P. Endow $\mathcal{P}_{\overline{L}}$ with an arbitrary order and proceed in sequence through each $P \in \mathcal{P}_{\overline{L}}$: if P'' is the immediate successor of P' in the sequence let the $\left(\widetilde{x}, \widetilde{L}, \widetilde{K}, p, w, p_K\right)$ generated by P' be the initiating base equilibrium for P''. The base equilibrium generated by the final P has a $\widetilde{x}$, $\widetilde{L}$, and $\widetilde{K}$ that are $\overline{L}$-measurable.

For $\overline{L}$-compatibility, let $\left(\widetilde{x}, \widetilde{L}, \widetilde{K}, p, w, p_K\right)$ be the base equilibrium generated by the final P. For each $P \in \mathcal{P}_{\overline{L}}$ and $\omega \in P$, define $\overline{L}$-compatible prices by $\widetilde{p}_1(\omega) = \mathbb{P}(\omega|P)\sum_{\omega'\in P} p_1(\omega')$, $\widetilde{w}_1(\omega) = \mathbb{P}(\omega|P)\sum_{\omega'\in P} w_1(\omega')$, and $\widetilde{p}_K^t(\omega) = \mathbb{P}(\omega|P)\sum_{\omega'\in P} p_K^t(\omega')$. To see that $\left(\widetilde{x}, \widetilde{L}, \widetilde{K}, \widetilde{p}, \widetilde{w}, \widetilde{p}_K\right)$ is a limited-information competitive equilibrium, suppose that $x^{i\prime}$ is $\overline{L}$-measurable and that $P \in \mathcal{P}_L$. Letting $x_1^{i\prime}$ denote $x_1^{i\prime}(\omega)$ for $\omega \in P$,

$$\sum_{\omega\in P}\widetilde{p}_1(\omega)x_1^{i\prime}(\omega) = x_1^{i\prime}\sum_{\omega\in P}\left[\mathbb{P}(\omega|P)\sum_{\omega'\in P} p_1(\omega')\right] = x_1^{i\prime}\sum_{\omega'\in P} p_1(\omega')\sum_{\omega\in P}\mathbb{P}(\omega|P) = \sum_{\omega'\in P} p_1(\omega')x_1^{i\prime}(\omega),$$

where the first and last equalities follow from the $\overline{L}$-measurability of $x_1^{i\prime}$. Hence, for each

$i \in \mathcal{I}$, $\widetilde{x}^i$ is utility-maximizing relative to all affordable $\overline{L}$-measurable alternative when prices equal $(\widetilde{p}, \widetilde{w}, \widetilde{p}_K)$.

Similarly, suppose that $t \in T^u$, (L^t, K^t) is measurable with respect to the coarsest common refinement of $\mathcal{P}_{\overline{L}}$ and $\mathcal{P}^t$, and P^* is an element of that refinement. Then, letting K^t and L_1^t denote $K^t(\omega)$ and $L_1^t(\omega)$ for $\omega \in P^*$ and letting P denote the element of $\mathcal{P}_{\overline{L}}$ that contains P^*,

$$\begin{aligned}
&\sum_{\omega \in P^*} \left(\widetilde{p}_1(\omega) v^t(K^t, L_1^t) - \widetilde{p}_K^t(\omega) K^t - \widetilde{w}_1(\omega) L_1^t\right) \\
&= \sum_{\omega \in P^*} \left(\mathbb{P}(\omega|P) \sum_{\omega' \in P} p_1(\omega') v^t(K^t, L_1^t) - \mathbb{P}(\omega|P) \sum_{\omega' \in P} p_K^t(\omega') K^t - \mathbb{P}(\omega|P) \sum_{\omega' \in P} w_1(\omega') L_1^t\right) \\
&= \sum_{\omega \in P^*} \left(v^t(K^t, L_1^t) \mathbb{P}(\omega|P) \sum_{\omega' \in P} p_1(\omega') - K^t \mathbb{P}(\omega|P) \sum_{\omega' \in P} p_K^t(\omega') - L_1^t \mathbb{P}(\omega|P) \sum_{\omega' \in P} w_1(\omega')\right) \\
&= \sum_{\omega \in P^*} \left(p_1(\omega) v^t(K^t, L_1^t) - p_K^t(\omega) K^t - w_1(\omega) L_1^t\right).
\end{aligned}$$

Summing over in P^* in P,

$$\begin{aligned}
&\sum_{\omega \in P} \left(\widetilde{p}_1(\omega) v^t(K^t(\omega), L_1^t(\omega)) - \widetilde{p}_K^t(\omega) K^t(\omega) - \widetilde{w}_1(\omega) L_1^t(\omega)\right) \\
&\qquad = \sum_{\omega \in P} \left(p_1(\omega) v^t(K^t(\omega), L_1^t(\omega)) - p_K^t(\omega) K^t(\omega) - w_1(\omega) L_1^t(\omega)\right).
\end{aligned}$$

So, for each $t \in T^u$, $\left(\widetilde{L}^t, \widetilde{K}^t\right)$ is profit-maximizing relative to all (L^t, K^t) that are measurable with respect to the coarsest common refinement of $\mathcal{P}_{\overline{L}}$ and $\mathcal{P}^t$. By a similar calculation, for each $t \in T^c$, $\widetilde{L}^t$ is profit-maximizing relative to all $\overline{L}$-measurable L^t. ∎

Proof of Theorem 5. Let $\left(\overline{x}, \overline{L}, \overline{K}, p, w, p_K\right)$ be a limited-information competitive equilibrium. Let $\mathcal{P}^*$ be the coarsest common refinement of $\mathcal{P}_{\overline{L}}$ and $\mathcal{P}^t$. To show that $\left(\overline{x}, \overline{L}, \overline{K}, p, w, p_K\right)$ is a standard competitive equilibrium, to which the standard first welfare theorem applies, when the state is revealed at the beginning of date 1, it is sufficient to show that (1) for each $i \in \mathcal{I}$, if $U^i(x^i) > U^i(\overline{x}^i)$ and x^i is not $\overline{L}$-measurable then $p_0 x_0^i + \sum_{\omega \in \Omega} p_1(\omega) x_1^i(\omega) > w_0 e_{L_0}^i + \sum_{\omega \in \Omega} w_1(\omega) e_{L_1}^i + \sum_{t \in T^c} \theta^{it} \pi^t(\overline{L}^t)$, (2) for each $t \in T^u$, if $(L^t, K^t) \geq 0$ is not measurable with respect to $\mathcal{P}^*$ then $\pi^t(L^t, K^t) \leq \pi^t(\overline{L}^t, \overline{K}^t)$, and (3) for each $t \in T^c$, if $L^t \geq 0$ is not $\overline{L}$-measurable then $\pi^t(L^t) \leq \pi^t(\overline{L}^t)$. Let $\mathbb{P}(\cdot|\cdot)$ throughout be the conditional probability given by compatibility.

For (1), suppose that $U^i(x^i) > U^i(\overline{x}^i)$ and $p_0 x_0^i + \sum_{\omega \in \Omega} p_1(\omega) x_1^i(\omega) \leq p_0 \overline{x}_0^i + \sum_{\omega \in \Omega} p_1(\omega) \overline{x}_1^i(\omega)$. Define the $\overline{L}$-measurable $\widetilde{x}_1^i$ by setting, for each $P \in \mathcal{P}_{\overline{L}}$ and each $\omega \in P$, $\widetilde{x}_1^i(\omega) = \sum_{\omega' \in P} \mathbb{P}(\omega'|P) x_1^i(\omega')$. By the concavity of U^i, $U^i\left(x_0^i, \widetilde{x}_1^i(\omega)_{\omega \in \Omega}\right) \geq U^i\left(x^i\right)$. Since

$$\sum_{\omega \in P} p_1(\omega) \widetilde{x}_1^i(\omega) = \sum_{\omega \in P} p_1(\omega) \sum_{\omega' \in P} \mathbb{P}(\omega'|P) x_1^i(\omega') = \sum_{\omega' \in P} \mathbb{P}(\omega'|P) \sum_{\omega \in P} p_1(\omega) x_1^i(\omega') = \sum_{\omega' \in P} p_1(\omega') x_1^i(\omega'),$$

we have

$$p_0 x_0^i + \sum\nolimits_{\omega\in\Omega} p_1(\omega)\widetilde{x}_1^i(\omega) = p_0 x_0^i + \sum\nolimits_{\omega\in\Omega} p_1(\omega) x_1^i(\omega) \le$$
$$p_0 \overline{x}_0^i + \sum\nolimits_{\omega\in\Omega} p_1(\omega)\overline{x}_1^i(\omega) \le w_0 e_{L_0}^i + \sum\nolimits_{\omega\in\Omega} w_1(\omega) e_{L_1}^i + \sum\nolimits_{t\in T^c} \theta^{it}\pi^t(\overline{L}^t)$$

which, combined with $U^i\left(x_0^i, (\widetilde{x}_1^i(\omega))_{\omega\in\Omega}\right) > U^i\left(\overline{x}^i\right)$, contradicts the maximality of $\overline{x}^i$.

For (2), suppose that $(L^t, K^t) \ge 0$ and $\pi^t(L^t, K^t) > \pi^t(\overline{L}^t, \overline{K}^t)$. Define the $\mathcal{P}^*$-measurable $\left(\widetilde{L}^t, \widetilde{K}^t\right)$ by setting $\widetilde{L}_0^t = \overline{L}_0^t$ and, for each $P \in \mathcal{P}^*$ and $\omega \in P$,

$$\widetilde{K}^t(\omega) = \sum_{\omega'\in P} \mathbb{P}(\omega'|P) K^t(\omega') \text{ and } \widetilde{L}_1^t(\omega) = \sum_{\omega'\in P} \mathbb{P}(\omega'|P) L_1^t(\omega').$$

Fix some $P \in \mathcal{P}^*$ and let $\widetilde{K}^t$ and $\widetilde{L}_1^t$ denote $\widetilde{K}^t(\omega)$ and $\widetilde{L}_1^t(\omega)$ for $\omega \in P$. Let Q denote the cell of $\mathcal{P}_{\overline{L}}$ that contains P (and so $Q = P$ if $\overline{L}_0^t > 0$). The t-capital and labor prices given by $\overline{L}$-compatibility are thus $\mathbb{P}(\omega''|Q)\sum_{\omega\in Q} w_1(\omega)$ and $\mathbb{P}(\omega''|Q)\sum_{\omega\in Q} p_K^t(\omega)$ for $\omega'' \in P$. Then, using the concavity of v^t for the inequality and the definition of compatibility for the first and final equalities,

$$\begin{aligned}
&\sum_{\omega\in P} p_1(\omega) v^t(\widetilde{K}^t, \widetilde{L}_1^t) - \sum_{\omega'\in P} p_K^t(\omega')\widetilde{K}^t - \sum_{\omega'\in P} w_1(\omega')\widetilde{L}_1^t \\
&\quad = \sum_{\omega\in P} p_1(\omega) v^t(\widetilde{K}^t, \widetilde{L}_1^t) - \sum_{\omega''\in P} \mathbb{P}(\omega''|Q) \sum_{\omega\in Q} p_K^t(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) K^t(\omega') \\
&\qquad\qquad - \sum_{\omega''\in P} \mathbb{P}(\omega''|Q) \sum_{\omega\in Q} w_1(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) L_1^t(\omega') \\
&\quad \ge \sum_{\omega\in P} p_1(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) v^t(K^t(\omega'), L_1^t(\omega')) - \mathbb{P}(P|Q) \sum_{\omega\in Q} p_K^t(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) K^t(\omega') \\
&\qquad\qquad - \mathbb{P}(P|Q) \sum_{\omega\in Q} w_1(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) L_1^t(\omega') \\
&\quad = \sum_{\omega\in P} p_1(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|P) v^t(K^t(\omega'), L_1^t(\omega')) - \sum_{\omega\in Q} p_K^t(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|Q) K^t(\omega') \\
&\qquad\qquad - \sum_{\omega\in Q} w_1(\omega) \sum_{\omega'\in P} \mathbb{P}(\omega'|Q) L_1^t(\omega') \\
&\quad = \sum_{\omega'\in P} p_1(\omega') v^t(K^t(\omega'), L_1^t(\omega')) - \sum_{\omega'\in P} p_K^t(\omega') K^t(\omega') - \sum_{\omega'\in P} w_1(\omega') L_1^t(\omega'),
\end{aligned}$$

where, given the restriction to P, I have omitted the ω argument of v^t. Since the profits from constructing t-capital are maximized at $\widetilde{L}_0^t = \overline{L}_0^t$, $\pi^t\left(\widetilde{L}^t, \widetilde{K}^t\right) \ge \pi^t\left(L^t, K^t\right)$. Hence $\pi^t\left(\widetilde{L}^t, \widetilde{K}^t\right) > \pi^t\left(\overline{L}^t, \overline{K}^t\right)$, contradicting the maximality of $\left(\overline{L}^t, \overline{K}^t\right)$.

Condition (3) involves only minor notational changes from the argument for (2). ■

Proof of Theorem 7. Let the assumed inputs be (L, K) and let x be the allocation they produce. We use P_L^* to denote any cell in $\mathcal{P}_L$ such that $\mathbb{P}(P_L) > 0$ and $v^t(L_0^t, L_1^t(\omega))(\omega) > 0$ for $\omega \in P_L$. Since (L, K) invests productively in t, a P_L^* exists. Since $v^t(L_0^t, L_1^t(\omega))(\omega) > 0$ for $\omega \in P_L$, $L_1^t(\omega) > 0$ for $\omega \in P_L$.

If for each P_L^* we have $x_1^i(\omega) = 0$ for all $i \in \mathcal{I}$ and $\omega \in P_L^*$, the result is trivial: reset L_0^t and L_1^t to equal 0, use the released labor for $L_0^{\tilde{t}}$ and $L_1^{\tilde{t}}$, and assign the additional output to some agent, thereby achieving a Pareto improvement. So assume there exists a P_L^* and a $i \in \mathcal{I}$ such that $x_1^i(\omega) > 0$ for $\omega \in P_L^*$. Call such a pair (P_L^*, i) *fruitful*.

Fix $\varepsilon > 0$ and set $\widetilde{x}_0^j = x_0^j$ for all $j \in \mathcal{I}$, $\widetilde{L}_0^t = (1-\varepsilon)L_0^t$, $\widetilde{L}_0^{\tilde{t}} = \varepsilon L_0^t$, and $\widetilde{L}_0^{t'} = L_0^{t'}$ if $t' \neq t, \tilde{t}$. For $\omega \in \Omega$, set $\widetilde{L}_1^t(\omega) = (1-\varepsilon)L_1^t(\omega)$, $\widetilde{L}_1^{\tilde{t}}(\omega) = \varepsilon L_1^t(\omega)$, $\widetilde{L}_1^{t'}(\omega) = L_1^{t'}(\omega)$ for $t' \in T \backslash \{t, \tilde{t}\}$, and $\widetilde{K}^{t'}(\omega) = \widetilde{L}_0^{t'}$ for $t' \in T$.

For any fruitful (P_L^*, i) and $\omega \in P_L^*$, set $\widetilde{x}_1^i(\omega) = x_1^i(\omega) + v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega)) - v^t(\varepsilon L_0^t, \varepsilon L_1^t(\omega))$ and $\widetilde{x}_1^j(\omega) = x_1^j(\omega)$ for $j \neq i$. If P_L^* is not part of a fruitful pair, set $\widetilde{x}_1^j(\omega) = x_1^j(\omega)$ for all $j \in \mathcal{I}$ and $\omega \in P_L$. Finally if $P_L \in \mathcal{P}_L$ does not qualify as a P_L^*, set each $\widetilde{x}_1^i(\omega) = 0$ when $\mathbb{P}(P_L) = 0$ and set each $\widetilde{x}_1^i(\omega) = x_1^i(\omega)$ when $\mathbb{P}(P_L) > 0$ and $v^t(L_0^t, L_1^t(\omega))(\omega) = 0$ for $\omega \in P_L$.

Since, for any fruitful (P_L^*, i), $x_1^i(\omega) > 0$ and $v^t(L_0^t, L_1^t(\omega)) > 0$ for $\omega \in P_L^*$ and since Ω is finite, $\widetilde{x}_1^i(\omega) > 0$ for $\omega \in P_L^*$ and for all $\varepsilon > 0$ sufficiently small. Hence $\widetilde{x} \geq 0$ for all $\varepsilon > 0$ sufficiently small.

Implicitly, $\widetilde{L}_0^t$, $\widetilde{L}_0^{\tilde{t}}$, and $\widetilde{L}_1^t(\omega)$, $\widetilde{L}_1^{\tilde{t}}(\omega)$, and $\widetilde{x}$ are functions of ε and we insert a ε argument, e.g., $\widetilde{x}_1^i(\omega, \varepsilon)$, when we need to indicate this fact.

1. We show that $\mathbb{E}(\widetilde{x}_1^i) = \mathbb{E}(x_1^i)$ for all $i \in \mathcal{I}$. Since this holds trivially if i is never part of a fruitful pair, assume for i that there is a P_L^* such that (P_L^*, i) is fruitful.

Recall from the proof of Theorem 2 that, when $P \in \mathcal{P}^{-\tilde{t}}$, and $\mathbb{P}(P) > 0$, we have $\mathbb{E}\left[v^{\tilde{t}}(L_0^t, L_1^t(\omega)) \middle| P \right] = \mathbb{E}\left[v^t(L_0^t, L_1^t(\omega)) \middle| P \right]$. Hence, using constant returns,

$$\begin{aligned} &\mathbb{E}\left[v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega)) \middle| P \right] + \mathbb{E}\left[v^t((1-\varepsilon)L_0^t, (1-\varepsilon)L_1^t(\omega)) | P \right] \\ &\qquad = \varepsilon \mathbb{E}\left[v^{\tilde{t}}(L_0^t, L_1^t(\omega)) \middle| P \right] + (1-\varepsilon)\mathbb{E}\left[v^t(L_0^t, L_1^t(\omega)) \middle| P \right] = \mathbb{E}\left[v^t(L_0^t, L_1^t(\omega)) | P \right]. \end{aligned}$$

(To simplify notation, we omit the final (ω) from $v^{t'}(L_0^{t'}, L_1^{t'}(\omega))(\omega)$ for each t'.)

Since this equality obtains when P is set to any $P^* \in \mathcal{P}^{-\tilde{t}}$ such that $P^* \subset P_L^*$ and $\mathbb{P}(P^*) > 0$, we have $\mathbb{E}(\widetilde{x}_1^i | P_L^*) = \mathbb{E}(x_1^i | P_L^*)$. Hence $\mathbb{E}(\widetilde{x}_1^i) = \mathbb{E}(x_1^i)$.

2. Let $\mathcal{P}^T$ denote the coarsest common refinement of the $\mathcal{P}^{t'}$ with $t' \in T^u$. Fix a fruitful (P_L^*, i). Since

$$\mathbb{P}\left(D_l v^{\tilde{t}}(L_0^t, L_1^t(\omega)) \neq D_l v^t(L_0^t, L_1^t(\omega)) \middle| P^* \right) > 0$$

for each $P^* \in \mathcal{P}^{-\tilde{t}}$ such that $P^* \subset P_L^*$ and $\mathbb{P}(P^*) > 0$, there is a $\widetilde{P} \subset P_L^*$ with $\mathbb{P}(\widetilde{P}) > 0$ such that $\widetilde{P}$ equals *either* the union of all $P^T \in \mathcal{P}^T$ such that $P^T \subset P_L^*$ and

$$D_l v^{\tilde{t}}(L_0^t, L_1^t(\omega)) > D_l v^t(L_0^t, L_1^t(\omega)) \tag{A}$$

for $\omega \in P^T$ *or* the union of all $P^T \in \mathcal{P}^T$ such that $P^T \subset P_L^*$ and

$$D_l v^{\tilde{t}}(L_0^t, L_1^t(\omega)) < D_l v^t(L_0^t, L_1^t(\omega)) \tag{B}$$

for $\omega \in P^T$. Due to constant returns, $D_l v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega)) > D_l v^t(\varepsilon L_0^t, \varepsilon L_1^t(\omega))$ in (A) and $D_l v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega)) < D_l v^t(\varepsilon L_0^t, \varepsilon L_1^t(\omega))$ in (B). Hence fixing some $\overline{\varepsilon} > 0$ there is a $\overline{\delta} \in \mathbb{R}$ such that

$$v^{\tilde{t}}(\overline{\varepsilon} L_0^t, \overline{\varepsilon} L_1^t(\omega) + \overline{\delta}) + v^t(\overline{\varepsilon} L_0^t, \overline{\varepsilon} L_1^t(\omega) - \overline{\delta}) - v^{\tilde{t}}(\overline{\varepsilon} L_0^t, \overline{\varepsilon} L_1^t(\omega)) - v^t(\overline{\varepsilon} L_0^t, \overline{\varepsilon} L_1^t(\omega)) > 0$$

and $\left(\overline{\varepsilon} L_1^t(\omega) + \overline{\delta}, \overline{\varepsilon} L_1^t(\omega) - \overline{\delta}\right) \gg 0$ for $\omega \in \widetilde{P}$. Hence, defining

$$\Delta(\omega, \varepsilon) = v^{\tilde{t}}\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) + v^t\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) - \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) - v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega)) - v^t(\varepsilon L_0^t, \varepsilon L_1^t(\omega))$$

for $\omega \in \widetilde{P}$, constant returns implies that, for $\varepsilon > 0$, $\Delta(\omega, \varepsilon) > 0$. We also have

$$\left(\varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}, \varepsilon L_1^t(\omega) - \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) \gg 0$$

for $\omega \in \widetilde{P}$.

Given $\varepsilon > 0$, define $\widehat{x}$ by $\widehat{x}^j = \widetilde{x}^j$ for $j \neq i$, $\widehat{x}_0^i = \widetilde{x}_0^i$, $\widehat{x}_1^i(\omega, \varepsilon) = \widetilde{x}_1^i(\omega, \varepsilon)$ for $\omega \notin \widetilde{P}$, and

$$\begin{aligned}
\widehat{x}_1^i(\omega, \varepsilon) &= \widetilde{x}_1^i(\varepsilon, \omega) + \left(v^{\tilde{t}}\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) - v^{\tilde{t}}(\varepsilon L_0^t, \varepsilon L_1^t(\omega))\right) - \\
&\qquad \left(v^t\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) - v^t(\varepsilon L_0^t, \varepsilon L_1^t(\omega))\right) \\
&= x_1^i(\omega) + v^{\tilde{t}}\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right) - v^t\left(\varepsilon L_0^t, \varepsilon L_1^t(\omega) + \frac{\varepsilon \overline{\delta}}{\overline{\varepsilon}}\right)
\end{aligned}$$

for $\omega \in \widetilde{P}$ and accordingly set $\widehat{L}_1^t(\omega, \varepsilon) = (1 - \varepsilon) L_1^t(\omega) - \varepsilon \frac{\overline{\delta}}{\overline{\varepsilon}}$ and $\widehat{L}_1^{\tilde{t}}(\omega, \varepsilon) = \varepsilon L_1^t(\omega) + \varepsilon \frac{\overline{\delta}}{\overline{\varepsilon}}$ for $\omega \in \widetilde{P}$ and leave the remaining input levels unchanged: $\widehat{L}_0^{t'} = \widetilde{L}_0^{t'}$ for all $t' \in T$, $\widehat{L}_1^{t'}(\omega) = \widetilde{L}_1^{t'}(\omega)$ for $t' \in T \backslash \{t, \tilde{t}\}$ and all $\omega \in \Omega$, and $\widehat{L}_1^t(\omega, \varepsilon) = \widetilde{L}_1^t(\omega, \varepsilon)$ and $\widehat{L}_1^{\tilde{t}}(\omega, \varepsilon) = \widetilde{L}_1^{\tilde{t}}(\omega, \varepsilon)$ for $\omega \notin \widetilde{P}$.

Since $\Delta(\omega, \varepsilon) > 0$ for $\omega \in \widetilde{P}$ and $\varepsilon > 0$, $\widehat{x}_1^i(\omega, \varepsilon) > \widetilde{x}_1^i(\omega, \varepsilon)$ for $\omega \in \widetilde{P}$ and $\varepsilon > 0$, and $\widehat{x}_1^i(\omega, \varepsilon) > 0$ for all $\omega \in \Omega$ and all $\varepsilon > 0$ sufficiently small. To conclude that $\widehat{x}_1^i$ and $\widehat{L}_1$ are $\widehat{L}$-measurable, it is sufficient to show that $\widetilde{P}$ equals a union of cells in $\mathcal{P}_{\widehat{L}}$. To show that $P_{\widehat{L}} \subset \widetilde{P}$ when $P_{\widehat{L}} \in \mathcal{P}_{\widehat{L}}$ and $P_{\widehat{L}} \cap \widetilde{P} \neq \varnothing$, suppose $P_{\widehat{L}} \in \mathcal{P}_{\widehat{L}}$, $\omega \in P_{\widehat{L}} \cap \widetilde{P}$, and $\omega' \in P_{\widehat{L}}$. Since $\widetilde{P} \subset P_L^*$ and $P_{\widehat{L}} \cap \widetilde{P} \neq \varnothing$, $P_{\widehat{L}} \cap P_L^* \neq \varnothing$. Since $\mathcal{P}_{\widehat{L}}$ is the coarsest common refinement of P_L^* and $\mathcal{P}^{\tilde{t}}$, we have $P_{\widehat{L}} \subset P_L^*$ and therefore $\omega, \omega' \in P_L^*$. Hence $v^t(\cdot, \cdot)(\omega) = v^t(\cdot, \cdot)(\omega')$ and, since $\omega, \omega' \in P_{\widehat{L}}$, $v^{\tilde{t}}(\cdot, \cdot)(\omega) = v^{\tilde{t}}(\cdot, \cdot)(\omega')$. Therefore (A) (resp. (B)) holds for ω if and only if (A) (resp. (B)) holds for ω' and so $\omega' \in \widetilde{P}$. Hence $P_{\widehat{L}} \subset \widetilde{P}$.

Since $\widehat{x}_1^i(\omega) > \widetilde{x}_1^i(\omega)$ for $\omega \in \widetilde{P}$ and $\mathbb{E}(\widetilde{x}_1^i | P_L^*) = \mathbb{E}(x_1^i | P_L^*)$, $\mathbb{E}\left(\widehat{x}_1^i \middle| P_L^*\right) > \mathbb{E}\left(x_1^i \middle| P_L^*\right)$ for any $\varepsilon > 0$. Given the classical result that a sufficiently small quantity of positive-expected-value gamble if added to a constant ex ante consumption vector will increase a differentiable concave expected utility (see, e.g., Mas-Colell et al. (1995)), for any $\varepsilon > 0$ sufficiently small, $\mathbb{E}\left[u^i\left(\widehat{x}^i\right) \middle| P_L^*\right] > \mathbb{E}\left[u^i(x^i) \middle| P_L^*\right]$. Since this inequality holds for each (P_L^*, i),

$\mathbb{E}\left[u^j\left(\widehat{x}^j\right)\right] > \mathbb{E}\left[u^j(x^j)\right]$ for each j that is part of fruitful pair (and $\mathbb{E}\left[u^j\left(\widehat{x}^j\right)\right] \geq \mathbb{E}\left[u^j(x^j)\right]$ for the remaining j). ■

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