Information Choice and Diversity: The Role of Strategic Complementarities

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Abstract

We study a class of games where players face restrictions on how much information they can obtain on a common payoff relevant state, but have some leeway in covertly choosing the dependence between their signals, before simultaneously choosing actions. Using a new stochastic dependence ordering between signals, we show that each player chooses information that is more dependent on the information of other players whose actions are either isotonic and complements with his actions or antitonic and substitutes with his actions. Similarly, each player chooses information that is less dependent on the information of other players whose actions are antitonic and complements with his actions or isotonic and substitutes with his actions. We then provide sufficient conditions for information structures such as public or private information to arise in equilibrium. Equilibrium information structures may be inefficient. Making which signals were chosen (but not their realizations) publicly observable may restore efficiency.

Keywords: Dependence ordering; Public information; Stochastic orders; Increasing differences; Complementarities; Information diversity; Endogenous information structure; Value of private and public information; Monotone strategies.

JEL Classifications: C72, D81, D83.

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1 Introduction

Economic agents are often exposed to more information than they can process. They are also often surrounded by more information sources than their limited cognitive abilities enable them to pay attention to (Sims 2003, 2005, 2006). The increase of information flows and the proliferation of information sources has accelerated in the Digital Age. As Google's CEO Eric Schmidt stated it in 2010, *every two days now we create as much information as we did from the dawn of civilization up until 2003*.[1]

This plethora of information sources is perceived as enabling individuals to make better choices by giving them the possibility to learn about various variables relevant to their decision making. In the theory of decision making under uncertainty, various orderings and measures have been developed that allow a decision maker to rank information structures, and the general conclusion of this research is that, for a decision maker, more information is always better.

When many agents interact, information choice is complicated in at least two ways. The first complication is that in a strategic context, information choice becomes a strategic decision, and the value of the various information choices depends on other agents' information choices, as it is the case for any strategic decision. The second complication is that more information is not always better, because in some games, ignorance has commitment value.

In this paper, we study the strategic choice of information, together with the strategic choice of actions. Our focus is not on the quality, quantity or precision of the information acquired –those are the main issues in decision making and remain important in a strategic context–, but on the diversity of the information that agents choose to acquire, which is meaningful only in a model with many agents. Information is diverse if agents choose to acquire dissimilar information and it is homogenous if agents choose to acquire similar information. In our framework, diversity is an endogenous outcome that results from economic fundamentals, such as the payoffs of the agents.

Information asymmetries have played an important role in many economic models in many different fields, including industrial organization, the economics of organizations, political economy, macroeconomics and financial economics. As a first step to understand the influence exerted by information on action choices, the models assume an exogenous information structure. But soon, questions arise

[1]Siegler, MG. (2010, August 4). Eric Schmidt: Every 2 Days We Create As Much Information As We Did Up To 2003. Retrieved from http://techcrunch.com/2010/08/04/schmidt-data/

on where the information structure comes from, who chooses it and how and why they choose it. In an influential paper, Morris and Shin (2002) initiate a literature on the preferences of the central bank (or the planner) over the macroeconomic information structure. In the context of an auction, Bergemann and Pesendorfer (2007) consider the joint design problem of a seller choosing the rules of an auction and the precision of the information of the bidders, in order to maximize his expected profits. Similar questions have been asked in other contexts and a literature on information design has emerged that seeks, more generally, to describe for a given game or for a game to be designed, all possible outcomes a planner could achieve by choosing the players' information structure (Gentzkow and Kamenica, 2011; Bergemman and Morris, 2013; Taneva 2014).

Another approach, the one we adopt, is to model the information structure as the result of the agents' decentralized information choices. Until recently, the literature that followed this path restricted attention to the acquisition of private information and was exclusively concerned with the strategic decision of how much private information (in terms of information quantity, quality or precision) agents choose to acquire when interacting with others. Hellwig and Veldkamp (2009) and Myatt and Wallace (2010) depart from this tradition by allowing agents to acquire potentially public or correlated information in a model of a beauty contest with a continuum of symmetric agents. These authors note that in a strategic context, information about an unknown payoff relevant parameter is at the same time information about what other players know. In these model, actions are either strategic complements or strategic substitutes and agents play actions that are increasing in their signal. The authors make the informal observation that when actions are complements, agents would like to know what the others know and when actions are substitute, they would prefer not to know what the others know, but to have independent information. Thus they make a claim about the *players' preferences over information dependence.* At the same time, in both papers, the authors establish a *complementarity inheritance* result: the sign of the complementarity in actions is passed on to the complementarity in information precisions. If actions are complements, precisions are also complements. If actions are substitutes, precisions are also substitutes. Last, the authors claim that players' preferences over information dependence reflect the players' preferences over action dependence.

Although complementarity inheritance has been noted in several models, all these models are very similar in that they rely on very similar functional forms and distributions. It has been shown that the result is not robust to even slight deviations from the models in which it holds. For example, Jimenez-Martinez (2013) consider the same functional forms as Hellwig and Veldkamp (2009), but assumes two players instead of a continuum, and obtains that complementarity inheritance only holds in a subset of the parameter space. Szkup and Trevino (2014) consider a model that departs from Hellwig and Veldkamp (2009)'s only in that they assume binary actions instead of a continuum. They find that while actions are strategic complements in their model, information precisions need not be complements.

In our view, although complementarity inheritance is related in some way with the players' preferences over information dependence, these are two rather different phenomena. First, complementarity inheritance is a result on preferences over information precision. Preferences over information precision are tricky for two reasons. First, because they mix two different considerations: (i) whether the player wants or not to know more on the uncertain variable; (ii) whether the player wants or not to know what the other players know. Second, because the uncertainty about the other players' knowledge is not held fixed when varying the other players' precisions: it increases as the other players choose a greater precision. Thus, a positive complementarity in precisions, i.e. the fact that following an increase in the information precision by the other players, the remaining player also prefers to increase his precision in reaction, could be due to various reasons. First, it could be that the remaining player is now more willing to know the state more precisely; Second, it could be that he is now more willing to know what the other agents know more precisely; Finally, it could be that he is now more uncertain about what the others know and as a result, is willing to compensate the uncertainty by increasing his own precision. Of course, it could also be some complicated combination of the three reasons we just listed.

This confusion arises even if the players acquire signals that are independent conditional on state realizations. Indeed, when a player acquires private information on the state, he cannot avoid acquiring information on what the other players know, because what they know is correlated with the state. This implies that the issue of precision choice cannot be fully isolated from the players' preferences over information dependence.

It is however possible to study the players' preference over dependence, and the consequences of these preferences on choice and equilibrium, in isolation from the choice of precision. In this paper, we disentangle the two issues and concentrate on the issue of information diversity. We build a model where the only information choice each player has is one between signals that are all equally informative on the state, but are diverse in the sense that they are not perfectly correlated with each other. The amount of information a player can acquire is fixed, perhaps endogenously determined at an earlier stage, but exogenously given from the perspective of the game.[2] Therefore, the only motive driving the players' information choices is whether they want or not to observe the same signal as this or the other player.[3]

We show that there exists a general link between complementarities in payoffs and the players' preferences over information, and that the equilibrium structure can be linked to strategic complementarities in a large class of models, without relying on particular functional forms, distributional assumptions, nor a continuum of agents. Our result is robust in these dimensions, but it requires the amount of information that each player acquires to be held fixed.

We distinguish two components in the choice made by a player. The first one is which signal he chooses to observe. The second one is his action strategy, namely the function that maps the signal he obtains to his actions. One feature of the equilibrium that plays a crucial role are the monotonicity properties of equilibrium action strategies. In general, equilibrium action strategies need not posses any monotonicity property, but in some games they do. Even when this is the case, these properties depend on the information structure.[4]

The problem we study is a complex one: equilibrium action strategies depend on the information structure the players believe in, but the equilibrium information structure depends on the action strategies they expect. We connect player's preferences over information, their equilibrium information choices and the equilibrium information diversity to two types of payoff complementarities: positive or negative complementarities between own action and others' actions, and positive or negative complementarities between own action and the unknown state.

It is useful to decompose our analysis in two parts. First, we need to understand how the information structure determines action strategy monotonicity properties.

[2]This assumption is relatively reasonable in many applications. Firms usually set budgets to information gathering activities, individuals subscribe to newspapers and magazines on a year-term basis, etc. Van Nieuwerburgh & Veldkamp (2009) show how this assumption is not restrictive using a duality argument.

[3]In our model, the players' information choice is akin to a location choice in some abstract information space in which positive dependence can be thought of as an incomplete ranking of distance. By choosing their signals, agents determine the information diversity, in the same way as firms determine the level of brand diversity or geographical dispersion in a market by choosing their brand or their location, as captured in the Hotelling and Salop firm location models.

[4]A large literature studies, for games with exogenous information structures, which fundamentals (in particular, which information structures) guarantee the existence of equilibria where all players' action strategies are increasing in their type (Athey, 2001; McAdams, 2003; Van Zandt and Vives, 2007, Reny, 2010).

Second, we need to understand how the interplay between these monotonicity properties and the payoff complementarities in actions shapes the players' preferences over signals and information structures.

For the first part, we show that for any exogenous information structure, if complementarities between own action and state are strong and complementarities between actions are weak, there exists a Nash Bayesian equilibrium in action strategies where action strategies are monotonic in a way that agrees with the state complementarity between own action and state: if this complementarity is positive, the action strategy is increasing and if it is negative, the action strategy is decreasing. Although action complementarities may work against these monotonicities, if they are weak, they do not prevent the existence of such an equilibrium. When the dominance of state complementarities over action complementarities is sufficiently strong, an equilibrium where action monotonicity agrees with state complementarities exists, regardless of what the information structure is. In some special cases, studied in particular by Van Zandt and Vives (2007), action complementarities work in the same direction as state complementarities. This is the case for example if all complementarities, action and state, are positive. In this case, a monotone equilibrium that agrees with state complementarities can be obtained under weaker assumptions.

For the second part, we study the players' preferences over signals. These preferences are hedonic: a player does not prefer one signal over another per se. Instead the preference depends on what signals the other players choose, on how they react to their signal, and on the dependence properties between the different signals. We show that these hedonic preferences depend on the interplay between action strategy monotonicities and complementarities in actions. More precisely, we show that the preference of a player between two signals depends on which of the two signals is more dependent on (or more similar to) the signals of a certain group of players the agent would like to be informationally close to, and less dependent (or less similar to) the signals of another group of players the agent would like to be informationally far away from. The action strategies of two players are isotonic if they move in the same direction with their signals and antitonic if they move in opposite directions. We show that each player seeks to be informationally close to players whose actions are either isotonic and complement with his own, or antitonic and substitute with his own, and informationally far away from players whose actions are either antitonic and complement with his own, or isotonic and substitute with his own.

We obtain our results for a general class of payoff functions and distributions. With the functional forms and distributions that are usually studied in the literature (e.g. linear and quadratic payoff functions, Gaussian distributions), dependence boils down to the conditional correlation between the signals. To tackle the general case, we define a new notion of statistical dependence between signals. In the case of two players, our dependence ordering between signals coincides with familiar orderings (supermodular, concordance, positive orthant dependence orderings), but in the case of three agents or more, our dependence ordering is novel and of independent interest.

Assembling the two parts of the analysis, we provide sufficient conditions for certain monotonicity patterns and information structures to arise in equilibrium, as well as sufficient conditions for these structures to be the most plausible ones in equilibrium. In particular, we show that if all complementarities (state and action) are positive and public information is feasible, there exists an equilibrium where information is public. In this case, knowing what the other knows allows a player to know a lot on the action of that other player. It is perhaps not surprising that the players choose to obtain the highest level of information by having perfectly correlated information.

However, if all state complementarities are positive, but all action complementarities are negative, and state complementarities sufficiently dominate action complementarities, then there exists an equilibrium where information is as private as possible. It may seem surprising that not knowing what the other players know is better. The reason is that this allows a player to rely more on his signal, without incurring the cost of playing an action that covariates positively with the other players' actions.

Along the way, we show by an example, that equilibrium information structures can be inefficient in an ex ante sense. Interestingly, efficiency can sometimes be restored if the players observe the others' signal choice (but not their realizations). This is because, in this case, the agents internalize some payoff relevant decisions that are ignored when information choice are not observed. In particular, a deviation in information has an effect on actions that is absent when information choice are not observed. These reactions in the action stage may sometimes serve to discipline the players from choosing a suboptimal information structure in the information acquisition stage.

The paper is structured as follows. In Section 2, we present the model. In Section 3, we illustrate all of our results with a simple example. In Sections 4, we introduce definitions that are needed in the analysis of the general case, in particular our new ordering of signal conditional dependence. Section 5 presents the core results of the paper. In Section 6, we show how our model can be applied to different situations. In Section 7, we conclude with a more precise discussion of the literature.

2 The model

In this section, we define a Bayesian game with information choice. Let $I = \{1, ..., N\}$ be a finite set of players. In the game, each player i chooses an action $a_i \in A_i \subseteq \mathbb{R}$. An action profile is denoted $a = (a_1, ..., a_N)$. The players' payoffs depend on a, but also on some unknown state of the world $\theta \in \mathbb{R}$. Each player has a von Neumann-Morgenstern utility function $u_i(a, \theta)$. Actions are chosen simultaneously, but prior to choosing an action, each player chooses a piece of information about θ and observes this information. The information structure is therefore endogenous. We now describe how players choose information.

From the players' point of view, before they acquire information, the state of the world θ is the unknown realization of a random variable Θ, whose support is $T \subseteq \mathbb{R}$. A **signal** is a finite support random variable X_s, which is correlated with the state, and therefore may carry payoff-relevant information, but does not itself directly enter the players' payoffs. Each player i has a set $\mathbb{X}_i$ of signals he can potentially observe, but he can only choose exactly one signal $X_i \in \mathbb{X}_i$, of which he observes the realization $x_i \in \mathbb{R}$. For simplicity, we assume that all the signals that a player can potentially observe have the same finite support $\mathcal{X}_i$ and in addition, we assume without loss of generality that this support is symmetric around zero, i.e. $-\mathcal{X}_i = \mathcal{X}_i$.[5] Let $\mathbb{X} = \bigcup_i \mathbb{X}_i$ be the set of all available signals for all players, $X = (X_1, ..., X_N)$ be a profile of signal choices and $x = (x_1, ..., x_N)$ be a profile of signal realizations. Let F be the joint cdf of the random vector $(\Theta, (X_s : X_s \in \mathbb{X}))$. The tuple $(\mathbb{X}_1, ..., \mathbb{X}_N, F)$ is the **signal structure** of the game. A Bayesian game with information choice is defined by the tuple $\Gamma = (I, (A_1, ..., A_N), (u_1, ..., u_N), (\mathbb{X}_1, ..., \mathbb{X}_N, F))$.

The game unfolds as follows in two stages. Initially all players start with the common prior F, which can be thought of as Nature's mixed strategy. In the first stage, each player i simultaneously chooses a signal $X_i \in \mathbb{X}_i$. In the second

[5]The finite support assumption is made to simplify the exposition, and to avoid uninteresting technical complications. We conjecture that our results extend to the case where $\mathcal{X}$ is infinite. The symmetry assumption is without loss of generality, because the problem is invariant, under any increasing transformation of the set $\mathcal{X}_i$ and any finite set $\mathcal{X}_i$ can be transformed into a symmetric set. The assumption simplifies notations later on.

stage, each player i privately observes his own signal realization x_i, and then simultaneously chooses an action a_i, without having observed the other players' first stage choices, nor the other players' signal realizations.

In the normal form of this game, a pure strategy for player i is a pair (X_i, α_i) in $\mathbb{X}_i \times A_i^{\mathcal{X}_i \times \mathbb{X}_i}$. The first component in the pair is the signal X_i chosen by player i in stage 1. The second component in the pair is the player's **action strategy** α_i, a mapping that determines player i's action choice, given the realization x_i he observed and the source X_i he chose. In most of the paper, we will restrict attention to pure strategies.[6] For simplicity and without loss of generality, given the focus on pure strategies, we restrict attention to action strategies in $A_i^{\mathcal{X}_i}$, such that the action chosen by each player only depends on his signal observation.[7] A strategy profile $(X, \alpha) = (X_i, \alpha_i)_{i \in I}$ is a **full-fledged Nash-Bayesian equilibrium** if for all i and all $(X_i', \alpha_i') \neq (X_i, \alpha_i)$, we have

$$\mathbb{E}_{\Theta, X}\left(u_i\left(\alpha_i\left(X_i\right), \alpha_{-i}\left(X_{-i}\right), \Theta\right)\right) \geq \mathbb{E}_{\Theta, X_i', X_{-i}}\left(u_i\left(\alpha_i'\left(X_i'\right), \alpha_{-i}\left(X_{-i}\right), \Theta\right)\right).$$

Our goal is to understand what type of information structure can arise in a Nash-Bayesian equilibrium of a Bayesian game with information choice Γ, and how the equilibrium information structure relates to the equilibrium action strategies. Of course, both questions are very broad, and could be analyzed from a variety of angles. Important considerations in a player's choice of a signal could be how informative on θ the different available signals are, how costly they are, or which aspects of θ they reveal.[8] We do not consider this type of choice here. Instead, we focus on the choice of signal conditional dependence: does a player want his signal to depend, conditionally on θ, on the other players' signals or not? In order to eliminate the other motives, and to concentrate on the conditional dependence motive, we assume that a player has access to signals that are all equally informative on the state in the sense of Blackwell. Namely, for each i, not only the support of all signals in $\mathbb{X}_i$ is the same, but in addition the joint marginal distribution of Θ and X_s is the same for all signals X_s in $\mathbb{X}_i$.[9]

[6] We consider mixed strategies in Appendix E.

[7] With pure strategies, it is without loss of generality to restrict attention to action strategies α_i that do not depend on X_i. Indeed, holding a strategy profile for the other players (X_{-i}, α_{-i}) fixed, any joint distribution over $T \times A^N$ induced by some profile (X, α) such that α_i depends on X_i, can also be induced by some other profile $(X_i, \alpha_i', \alpha_{-i})$ such that α_i' does not depend on X_i.

[8] For example, one signal could reveal θ's sign, whereas another could reveal θ's absolute value.

[9] In particular, if all players have access to the same signals, i.e. all the sets $\mathbb{X}_i$ are equal to $\mathbb{X}$, then our assumption is that all signals in $\mathbb{X}$ are equally informative on θ in the sense of Blackwell.

Given this restriction, the only remaining degree of freedom the players have when choosing their information is the conditional dependence of their signals with each other. The goal of the paper is to study which dependence patterns between players' signals can arise in a Nash Equilibrium. We interpret these dependence patterns in terms of informational diversity. We also relate the dependence patterns with the monotonicity properties of action strategies and the payoff complementarities in actions. For some information structures, we provide sufficient conditions on the primitives of the model, which ensure that this information structure is chosen by the players in some equilibrium of the game.

In the rest of the paper, we refer to Γ as the game with endogenous information structure, that is the game where the players choose which signal to observe. We also refer to Γ_X as the game with an exogenous information structure such that the profile of signal observed by the players is X.

In Section 3, we first examine the questions in a simple example and provide complete answers in this context. We then show in Section 5 that several of the insights gained from studying the example can be generalized to a large class of games, and do not rely on specific payoffs nor on a particular information structure.

3 An illustrative example

To fix ideas, we start with a simple example.[10] Suppose that $N = 2$, $\mathbb{X}_1 = \mathbb{X}_2 = \{X_I, X_{II}\}$ and player i chooses an action $a_i \in \mathbb{R}$. The payoffs of the game are

$$u_i(a, \theta) = -a_i^2 + 2b_{ia}a_i a_j + 2b_{i\theta}a_i\theta + K(a_j, \theta) \tag{1}$$

where b_{ia} and $b_{i\theta}$ are real numbers for $i \in \{1, 2\}$ and $K(\cdot, \cdot)$ is a function that does not affect the set of Nash-Bayesian equilibria, but may have an effect on welfare. The parameter b_{ia} captures the level of strategic interaction between player i's and player j's actions and the parameter $b_{i\theta}$, the strategic interaction between player i's action and the state. Positivity implies action complementarity and negativity, action substitutability.

The information structure is as follows. The random vector (θ, X_I, X_{II}) is distributed in $\{-1, 1\}^3$ according to a probability distribution function such that

[10] One could also consider the normal quadratic payoff setting to illustrate our results. Such an example, however, is not strictly speaking a special case of our model, because the support of the signals is infinite. Note, that the finiteness assumption is made to keep the exposition simple, not for more fundamental reasons.

the vectors (Θ, X_I) and (Θ, X_{II}) have the same joint marginal distribution given by

	$X_\ell = -1$	$X_\ell = 1$
$\Theta = -1$	$\frac{1-\varepsilon}{2}$	$\frac{\varepsilon}{2}$
$\Theta = 1$	$\frac{\varepsilon}{2}$	$\frac{1-\varepsilon}{2}$

,

for $\ell \in \{I, II\}$ and where $\varepsilon \in (0, 1/2)$. Moreover, we assume that $\mathbb{P}(\theta = -1) = \mathbb{P}(\theta = 1) = 1/2$, and that the joint distribution of two signals, conditional on $\Theta = \theta \in \{-1, 1\}$ is given by the following matrix:

	$X_{II} = \theta$	$X_{II} \neq \theta$
$X_I = \theta$	$(1-\varepsilon)^2$	$\varepsilon(1-\varepsilon)$
$X_I \neq \theta$	$\varepsilon(1-\varepsilon)$	ε^2

.

Fixing signal choices $X_i \in \{X_I, X_{II}\}$ for $i = 1, 2$, the ex ante expected payoff of player i given the profile of signal choice X is

$$\begin{aligned}
&\mathbb{E}_{\Theta,X}(u_i(\alpha(X), \Theta)) = \\
&\mathbb{P}(X_i = -1) \cdot \Big(\mathbb{P}(X_j = 1 | X_i = -1)\mathbb{E}(u_i(\Theta, \alpha_i(-1), \alpha_j(1)) | X_i = -1) \\
&\qquad + \mathbb{P}(X_j = -1 | X_i = -1)\mathbb{E}(u_i(\Theta, \alpha_i(-1), \alpha_j(-1))) | X_i = -1)\Big) \qquad (2) \\
&+ \mathbb{P}(X_i = 1) \cdot \Big(\mathbb{P}(X_j = -1 | X_i = 1)\mathbb{E}(u_i(\Theta, \alpha_i(1), \alpha_j(-1))) | X_i = 1) \\
&\qquad + \mathbb{P}(X_j = 1 | X_i = 1)\mathbb{E}(u_i(\Theta, \alpha_i(1), \alpha_j(1))) | X_i = 1)\Big),
\end{aligned}$$

where $\mathbb{P}(X_j = x | X_i = x) = 1$ if $X_i = X_j$ and $\mathbb{P}(X_j = x | X_i = x) = 1 - 2\varepsilon(1-\varepsilon)$ if $X_i \neq X_j$. By taking the first-order condition to (2) with respect to $\alpha_i(1)$ and $\alpha_i(-1)$ for $i = 1, 2$ and then solving for $(\alpha_1(-1), \alpha_1(1), \alpha_2(-1), \alpha_2(1))$, we can compute the equilibrium in the second-stage, that is, once the information structure is fixed. In particular, we obtain $\alpha_i(-1) = -\alpha_i(1)$ and

$$\alpha_i(1) = \frac{(b_{i\theta} + b_{ia} b_{j\theta} [2\mathbb{P}(X_j = x | X_i = x) - 1])}{1 - b_{ia} b_{ja} [2\mathbb{P}(X_j = x | X_i = x) - 1]^2} (1 - 2\varepsilon). \qquad (3)$$

Because $\alpha_i(-1) = -\alpha_i(1)$, the number $\alpha_i(1)$ equals the slope of the action strategy of player i, and its sign indicates whether this strategy is increasing or decreasing in his signal.

To avoid non generic trivial cases, we will assume that for all $i, j \in \{1, 2\}$, such that $i \neq j$, we have $b_{i\theta} + b_{ia} b_{j\theta} \neq 0$, $b_{ia} b_{ja} \neq 1$, $b_{i\theta} + b_{ia} b_{j\theta} (1 - 2\varepsilon)^2 \neq 0$ and $b_{ia} b_{ja} (1 - 2\varepsilon)^4 \neq 1$. These conditions ensure that (a) for any profile of pure signal strategies (X_1, X_2), a unique pure Nash-Bayesian equilibrium exists in the action game with exogenous information structure (X_1, X_2), and (b) that in this Nash-Equilibrium, each player's action strategy is strictly monotonic: either it is strictly increasing, or it is strictly decreasing.

A look at Equation (3) shows that whether player i's action strategy is strictly increasing or strictly decreasing in his signal depends on b_{ia}, the level of strategic complementarity, $b_{i\theta}$, the level of state complementarity and $\mathbb{P}(X_j = x|X_i = x)$, the information structure.

The profile of action strategies (α_1, α_2) is said to be strictly isotonic if both players's actions are either strictly increasing $(\alpha_i(1) > 0)$ or strictly decreasing $(\alpha_i(1) < 0)$. This occurs when $b_{1\theta} + b_{1a}b_{2\theta}(2\mathbb{P}(X_2 = x|X_1 = x) - 1)$ and $b_{2\theta} + b_{2a}b_{1\theta}(2\mathbb{P}(X_1 = x|X_2 = x) - 1)$ have the same sign.

3.1 Information choices

We turn now to the information choice stage of the game. The main question that motivates our work is to understand which assumptions about the payoffs are necessary for information diversity to emerge as a result of the players' individual choice. The binary example allows us to illustrate very clearly the main contribution of the paper.

Fixing the action strategies to (α_1, α_2), where the α_i are the odd functions given by (3), the expected payoff for player i can be written as

$$\mathbb{E}_{\Theta,X}(u_i(\alpha(X), \Theta)) = 2b_{ia}(2\mathbb{P}(X_j = x|X_i = x) - 1)\,\alpha_i(1)\,\alpha_j(1) + \text{Constant}.$$

Player i's information choice determines $\mathbb{P}(X_j = x|X_i = x)$ and which one is optimal depends on the monotonicity of actions strategies (the sign of $\alpha_i(1)\,\alpha_j(1)$) and on the strategic motive in actions (the sign of b_{ia}). In our example, since the players have access to exactly the same signal, the information structure is either public, if both players observe the same signal, or private, if the players observe different signals.

3.1.1 Conflict on the information structure

A first observation is that whenever the players have conflicting preferences over the information structure, which is this context means that one of the two players would prefer the information to be public, while the other would prefer it to be private, there cannot be an equilibrium of the endogenous information game that is in pure strategies. This is because the players are playing a game akin to Matching Pennies in the first stage. The players have conflicting preferences over the information structure when $b_{1a}b_{2a} < 0$.

Proposition 1. *In the binary quadratic example, if $b_{1a}b_{2a} < 0$, players have conflicting preferences over the information structure. In this case, the game with endogenous information acquisition does not have an equilibrium in pure strategies.*

However, it can be shown in a more general context, that this game always has an equilibrium in mixed strategies.[11] In the rest of the analysis of the binary quadratic example, we will restrict attention to the case where players agree on the information structure they like best, that is, we assume that $b_{1a}b_{2a} > 0$.

3.1.2 Agreement on the information structure

Proposition 2 characterizes the equilibrium information structure. With isotonic action strategies, a low value for $\mathbb{P}(X_j = x|X_i = x)$ is desirable for player i only if $b_{ia} < 0$, i.e., when actions are substitutes. With antitonic action strategies, a low value for $\mathbb{P}(X_j = x|X_i = x)$ is desirable for player i only if $b_{ia} > 0$, i.e., when actions are complements.

Proposition 2. *In the binary quadratic example, let $b_{1a}b_{2a} > 0$ and let $(X, (\alpha_1, \alpha_2))$ be a pure Nash-Bayesian equilibrium of the game. Then,*

1. *$X_1 = X_2$ only if $\alpha_1(1)\,\alpha_2(1)\,b_{ia} > 0$ for $i = 1, 2$.*
2. *$X_1 \neq X_2$ only if $\alpha_1(1)\,\alpha_2(1)\,b_{ia} < 0$ for $i = 1, 2$.*

The result in Proposition 2 is an instance of a more general phenomenon, which we will analyze in greater generality in Theorem 4.

A general feature of games with endogenous information structure is that multiple equilibria can exist. For instance, it can be the case that the players choose to acquire the same signal, so that they hold public information, but that the actual signal they observe can be either one contained in the set $\mathbb{X}$. This type of multiplicity is trivial since the dependence pattern among the players' signals is the same for all equilibria. More interesting is the fact that non-trivial multiplicity can also occur with endogenous information choice. One such example would be a game where two types of equilibria can be sustained, an equilibrium where the players choose the same signal and another one where the players choose different signals.

[11] See Appendix E.

Theorem 1. *In a game with endogenous information choice, non-trivial multiple equilibria can exist.*

Basically, Theorem 1 establishes that the dependence pattern in information choice is not always uniquely pin down by action complementarities. We prove Theorem 1 using our binary quadratic example. More specifically, we construct an example with strategic substitutability in actions and complementarities in a player's action and the state and show that both public and private information can be sustained in some equilibrium of the game.

3.2 Ex ante constrained inefficiency of the equilibrium information structure

Next, we use the binary quadratic example to show that, under certain conditions, the players' equilibrium signal choices do not result in the information structure a planner would design. We compare the equilibrium of the endogenous information game (in cases covered by Proposition 2) with an auxiliary game in which the planner chooses the information structure. In particular, we assume the planner chooses between either (X_I, X_I) or (X_I, X_{II}), then this information structure becomes common knowledge, and the players simultaneously choose actions in a noncooperative manner. Of course, it is not necessarily obvious what the preferences of the planner should be. In order to avoid this difficulty, we focus on the case where the two players have symmetric payoffs, given by

$$u_i(\theta, a) = -a_i^2 + 2b_a a_i a_j + 2b_\theta a_i \theta + 2b_{\theta a} a_j \theta + 2b_{aa} a_j^2. \tag{4}$$

This is a special case of Equation (1) considered before, when the players have symmetric payoffs and with $K(a_j, \theta) = 2b_{\theta a} a_j \theta + 2b_{aa} a_j^2$. The terms in $K(a_j, \theta)$ capture an externality that does not affect the Nash-Bayesian equilibrium in the game with endogenous information choice, but contributes to determine which information structures are constrained efficient.

Since the players' payoffs are symmetric, the unique equilibrium is also symmetric. We may then safely assume that the planner maximizes the expected payoff of player 1. Given a profile of signal choices (X_1, X_2) and action strategies α as in (3) , the ex ante expected utility of player 1 is written as

$$\begin{aligned}\mathbb{E}_{\Theta,X}(u_1(\alpha(X),\Theta)) &= 2b_a\left(2\mathbb{P}(X_2=x|X_1=x)-1\right)\alpha_1(1)\,\alpha_2(1)-(\alpha_1(1))^2 \\ &\quad +2b_{aa}(\alpha_2(1))^2+2b_\theta\alpha_1(1)(1-2\varepsilon) \\ &\quad +4b_{\theta a}\alpha_2(1)(1-2\varepsilon) \qquad (5)\end{aligned}$$

Player 1 would optimize his signal choice by considering that he has a direct impact on the first term through a change in $\mathbb{P}(X_2 = x|X_1 = x)$. On the other hand, the planner, when pondering over which information structure to impose, considers that player 1's utility, also depends indirectly on $\mathbb{P}(X_2 = x|X_1 = x)$ as this term enters $\alpha_1(1)$ and $\alpha_2(1)$.

Therefore, the social planner, since he knows the signal choices, uses a different expected payoff function when maximizing welfare, and thus, would not necessarily choose the Nash-Bayesian equilibrium for the signal choice structure.

Theorem 2. *A pure Nash-Bayesian equilibrium* (X, α) *of the game with endogenous information acquisition need not be constrained ex-ante Pareto efficient.*

Note that Theorem 2 applies to every game that fits the description of our model in Section 2 and not just the particular binary quadratic example. Essentially, the reason for the inefficiency is that the planner will take into consideration the impact of the signal choices on the actions when making a choice on the information structure, an effect that the players do not individually consider. This result suggests that policy intervention is sometimes beneficial in markets for information. In a decentralized system, players may choose either too similar or too dissimilar information, and policy intervention can help to mitigate this type of inefficiency.

In the main model, we make the assumption that signal choices of the first stage are not observed by the players. This is important, since it implies that a deviation from equilibrium play does not affect the other player's action choices in the second stage: the choice of signal and actions are strategically simultaneous. One can imagine situations where signal choices are observable. For example, a company may sign a contract with a market research firms and this may be observable by all other companies.

Interestingly, this difference can have important effects. To see this, consider again the case of two symmetric players. In this case, both players in stage 1 face the problem of the planner, which we analyzed earlier. As we showed, the planner's solution may be disjoint from the set of Nash equilibria of the game where signal choices are unobservable. We can thus deduce the following result.

Theorem 3. *Suppose the players publicly observe the profile of signal choices. Then, any pure Nash-Bayesian equilibrium* (X, α) *of the game is constrained ex ante Pareto efficient.*

In this alternative model where the players publicly observe the profile of signal choices, the actions in the second stage are functions of the profile of signal choices. Therefore, a shift of signal by a player has an impact on the other players' actions, which is in turn acknowledged by the deviating player. So it turns out that allowing for the public observation of information choices induces the players to internalize the impact of their signal choice and to behave as the planner would want them to.

Theorem 3 suggests that an intervention that mandates players to publicly disclose their sources of information may sometimes be desirable, in that it could help to mitigate excessive information similarity or dissimilarity that may result from a decentralized market for information.

4 General case: preliminary definitions

In this Section, we introduce the concepts that are needed in order to generalize some of the insights obtained in the example studied in Section 3. We first introduce monotonicity properties, then strategic complementarities in actions. Last, we introduce a new partial order on a set of information structures, the "dependence ordering," which compares the positive dependence between a single player's signal and all the other players' signals across information structures.

4.1 Monotonicity properties of action strategies

For any action strategy $\alpha_i : \mathcal{X} \to \mathbb{R}$, we say that α_i is **increasing** if for all $x_i, x_i' \in \mathcal{X}$, we have $x_i \leq x_i' \Longrightarrow \alpha_i(x_i) \leq \alpha_i(x_i')$, and that α_i is **strictly increasing** if for all $x_i, x_i' \in \mathcal{X}$, we have $x_i < x_i' \Longrightarrow \alpha_i(x_i) < \alpha_i(x_i')$. We say that α_i is **(strictly) decreasing** if $-\alpha_i$ is (strictly) increasing.

A profile of action strategies α is **(strictly) monotonic** if for all i, the action strategy α_i is either (strictly) increasing or (strictly) decreasing. It is (strictly) **isotonic** if either, for all i, the action strategy α_i is (strictly) increasing, or for all i, α_i is (strictly) decreasing.

We also want to encompass the cases where the action profiles are (strictly) monotonic, but not necessarily (strictly) isotonic. For any vector $m \in \{1, -1\}^I$, we say that the profile of action strategies α is **(strictly) m-monotonic** if for all i, the function $m_i \alpha_i$ is (strictly) increasing. In particular, for any vector m, a (strictly) m-monotonic profile of action strategies α is (strictly) isotonic if, for all i, the m_i have the same sign. Fixing a player i, a profile of action strategies α is (**strictly) antitonic for** i if it is (strictly) m-monotonic, and m satisfies $m_i = -m_j$ for all $j \neq i$.

4.2 Strategic complementarities in actions

Let $a_{-i,j} \in \mathbb{R}^{I \setminus \{i,j\}}$ be the action strategies of players $I \setminus \{i, j\}$. We say that **player i has (strict) positive complementarities in actions with player** $j \neq i$, if for all $a'_i < a''_i$, and all $a_{-i,j} \in \mathbb{R}^{I \setminus \{i,j\}}$ the difference $u_i (a''_i, a_j, a_{-i,j}) - u_i (a'_i, a_j, a_{-i,j})$ is (strictly) increasing in a_j. We say that player i has **(strict) negative complementarities in actions with player** $j \neq i$, if for all $a'_i < a''_i$, and all $a_{-i,j} \in \mathbb{R}^{I \setminus \{i,j\}}$, the difference $u_i (a''_i, a_j, a_{-i,j}) - u_i (a'_i, a_j, a_{-i,j})$ is (strictly) decreasing in a_j. We say that player i has (strict) positive complementarities in actions if he has (strict) positive complementarities with all the other players. We say that he has (strict) negative complementarities in actions if he has strict (negative) complementarities with all the other players.[12]

Although these definitions can be used to describe many situations, we are interested in a richer class of payoff functions where each player may have a (strict) positive complementarity in actions with some players and a (strict) negative complementarity with some other players. The complementarity properties of a payoff function u_i are encoded by a complementarity vector $c^i = \left(c^i_j\right)_{j \in I} \in \{-1, 1\}^I$ with $c^i_i = 1$. We say that player i has **(strict) c^i-complementarities in actions** if he has (strict) positive complementarities in actions with all players $j \neq i$ such that $c^i_j = 1$ and (strict) negative complementarities in actions with all players $j \neq i$ such that $c^i_j = -1$. In particular, the case where $c^i = (1, ..., 1)$ corresponds to the case of a player that has a (strict) positive complementarity in actions. Similarly, the case where $c^i_{-i} = (-1, ..., -1)$ corresponds to the case of a player that has a (strict) negative complementarity in actions.[13]

[12] In a complete information game in which the best response function of player i is well defined, if u_i has positive (negative) complementarities in actions with player j, his best response function is increasing (decreasing) in a_j (Topkis, 1998; Milgrom and Roberts, 1994).

[13] If the payoff u_i is twice continuously differentiable, then player i has c^i-complementarities in actions if and only if $c^i_j \frac{\partial^2 u_i}{\partial a_i \partial a_j} (a) \geq 0$ for all $j \neq i$ and all a, and he has strict complementarities

4.3 Conditional dependence orderings

We now introduce a family of weak partial ordering on a set $\mathbb{X}_i$ of signals accessible to player i, the "conditional dependence orderings." Each such ordering is indexed by some fixed profile of the other player's signals X_{-i} and compares across the signals X_i accessible to player i the positive dependence between X_i and X_{-i}. These orderings play a central role in all the results in Section 5.

This notion requires the following definition. For any $k \geq 1$, a subset $L \subseteq \mathcal{X}^k$ is an **increasing subset** of $\mathcal{X}^k$ if for all $x, x' \in \mathcal{X}^k$, such that $x \leq x'$, $x \in L \Rightarrow x' \in L$. Equivalently, L is an increasing subset if its indicator function $\mathbf{1}_L(x)$ is increasing.

Definition 1 (weakly greater conditional dependence)**.** *Let $i \in I$ and let X_{-i} be a profile of signals for all players different from i. For all X_i' and X_i'' in $\mathbb{X}_i$, we say that X_i' **depends at least as much as** X_i'' **on** X_{-i} **conditionally on** Θ, if for all (θ, x), and all increasing set $L \subseteq \mathcal{X}^{I\setminus\{i\}}$, we have*

$$\mathbb{P}(X_i' \geq x \mid X_{-i} \in L,\ \Theta = \theta) \geq \mathbb{P}(X_i'' \geq x \mid X_{-i} \in L,\ \Theta = \theta).$$[14]

For each profile X_{-i} of signals chosen by the other players, this defines a weak partial order over the signals accessible to player i.

Similarly, we define a larger class of weak partial orders over $\mathbb{X}_i$. It enables us to compare, for two signals X_i' and X_i'', whether one signal depends more on some other player signal, but less on another player's signal than the other.

Definition 2 (weakly greater conditional d^i-dependence)**.** *Let $i \in I$ and $d^i \in \{-1, 1\}^I$. For all X_i' and X_i'' in $\mathbb{X}_i$, we say that X_i' d^i**-depends at least as much as** X_i'' **on** X_{-i} **conditionally on** Θ, if for all (θ, x), $d_i^i X_i'$ depends at least as much as $d_i^i X_i''$ on $\left(d_j^i X_j\right)_{j \neq i}$.*

For each profile X_{-i} of signals chosen by the other players, and for each dependence vector d^i, this defines a weak partial order over the signals accessible to player i.[15]

Our interpretation of weakly greater conditional d^i-dependence is that player i's signal X_i' depends at least as much on the signals of the players j such that $d_i^i = d_j^i$

in actions if this inequalities hold strictly, almost everywhere. But we do not assume that payoffs have this regularity property.

[14] We provide an equivalent definition, based on the notion of multivariate first order stochastic dominance in Appendix A.

[15] Similarly to Definition 1 (respectively to Definition 2), one can define a weakly greater unconditional dependence (respectively d^i-dependence) partial ordering, which is weaker than the orderings in this definition. Only the conditional versions play a role in the paper, because we are interested in information structures, not in random variables.

and at most as little on the signals of the players j such that $d_i^i = -d_j^i$, as player i's signal X_i'', conditional on θ. Note that weakly greater conditional dependence is precisely the special case of weakly greater conditional d^i-dependence, when $d_1^i = ... = d_N^i \in \{-1, 1\}$.

For the two weak partial orders defined in this subsection, a strict partial order is defined as follows: we say that X_i' **depends more than** X_i'' **on** X_{-i} if X_i' depends as much as X_i'' on X_{-i}, and X_i'' does not depend as much as X_i' on X_{-i}.

5 Equilibrium information structures

In this section, which is the core of the paper, we determine which information structures can be part of a full-fledged Nash-Bayesian equilibrium of the game with information choice.

The analysis is in two steps, each subdivided in two sub-steps. In the first step (Sections 5.1 and 5.2), we assume that action strategies are m-monotonic, for some exogenously fixed vector m. The action strategies themselves may not be fixed, but their monotonicity is. We show in Section 5.1 that together with m, the complementarities in actions determine preferences over conditional dependence between own and others' signals. For each pair (i, j) of players, we determine whether player i wants his signal to be as conditionally dependent as possible on player j's signal, whichever this signal is, or as independent as possible of this signal, whichever this signal is. The answer to this question depends on the action complementarities between i and j for player i and on the monotonicities m_i and m_j. More precisely, it only depends on the sign of the product $m_i m_j c_j^i$.

We then characterize in Section 5.2 the set of signal profiles which are "compatible" with the maximization of the preferences over conditional dependence described in Section 5.1, while still holding the monotonicity vector m of the action strategies fixed. Throughout this first step, the problem we study is akin to the study of a "location game," where a finite number of players choose a *location* from a set of possible locations, and have preferences over locating *close t*o of *far from* each of the other players, except that the "locations" are in fact the signals and the distance is replaced by our notion of conditional dependence.

In the second step (Sections 5.3 and 5.4), we proceed to endogenize the monotonicity vector m, so as to obtain a full-fledged Nash-Bayesian equilibrium of the Bayesian game with information choice, where both the information structure and the action strategies are jointly determined. In Section 5.3, we provide sufficient conditions for a signal profile X to be part of an equilibrium. The way this works is that if the Bayesian game Γ_X with exogenous information equilibrium X admits an equilibrium α in m-monotonic strategies, such that in addition, X is compatible with α in the sense of the characterization of Section 5.2, then an equilibrium (X, α) turns out to be a full-fledged equilibrium, and then, it follows from this that X is the signal profile of some equilibrium. In Section 5.4, we provide conditions under which the equilibrium information structure is essentially unique, in the sense that all (possibly multiple) equilibria have the same information structure.

5.1 Preferences for conditional (in)dependence

We now show how the monotonicity m of action strategies and the complementarities in actions for a given player jointly determine this player's preferences over the conditional dependence between his own and other players' signals. Throughout Section 5.1, we suppose that a monotonicity vector $m \in \{-1, 1\}^I$ is fixed and that players are restricted to play second stage action strategies that are m-monotonic. The restriction to m-monotonic action strategies for some given m is a step in the analysis, but in some cases, the restriction may follow from the primitives. For example, the restriction could result from an external constraint, or from iterative elimination of never best-response action strategies.

5.1.1 Preferences for conditional dependence

We are now ready to present our characterization of the preferences for conditional dependence.

Theorem 4. *Let $i \in I$ and let c^i be a complementarity vector for i. Fix a monotonicity profile $m \in \{-1, 1\}^I$ and a profile of signals X_{-i}.*

(i) Suppose that u_i has c^i-complementarity in actions. Suppose that X_i' and X_i'' are two signals in $\mathbb{X}_i$ such that X_i' d^i-depends at least as much on X_{-i} as X_i'' does, where d^i is the conditional dependence vector such that

$$d_j^i = m_i m_j c_j^i \tag{6}$$

for all $j \in I \setminus \{i\}$ and $d_i^i = 1$. Then for any profile of pure m-monotonic action strategies α, player i finds signal X_i' at least as good as signal X_i'' :

$$\mathbb{E}_{\Theta, X_i', X_{-i}} \left(u_i \left(\alpha_i \left(X_i' \right), \alpha_{-i} \left(X_{-i} \right), \Theta \right) \right) \geq \mathbb{E}_{\Theta, X_i'', X_{-i}} \left(u_i \left(\alpha_i \left(X_i'' \right), \alpha_{-i} \left(X_{-i} \right), \Theta \right) \right). \tag{7}$$

(ii) If, in addition, u_i has strictly c^i-complementarity in actions, and X_i' d^i-depends more on X_{-i} than X_i'' does, then for any profile of pure strictly m-monotonic action strategies α, player i strictly prefers signal X_i' to signal X_i'' :

$$\mathbb{E}_{\Theta,X_i',X_{-i}}\left(u_i\left(\alpha_i\left(X_i'\right),\alpha_{-i}\left(X_{-i}\right),\Theta\right)\right) > \mathbb{E}_{\Theta,X_i'',X_{-i}}\left(u_i\left(\alpha_i\left(X_i''\right),\alpha_{-i}\left(X_{-i}\right),\Theta\right)\right). \tag{8}$$

Part (i) in Theorem 4 is tight in the sense that if X_i' and X_i'' do *not* satisfy X_i' d^i-depends at least as much on X_{-i} as X_i'', where d^i is player i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarity in actions, then a payoff function u_i with c^i-complementarity in actions and an m-monotonic profile of action strategies can be found, such that the inequality (7) does not hold. Part *(ii)* is also tight, in a similar sense. In other words, the d^i-dependence ordering over signals is the weakest ordering for which the inequalities (7) and (8) hold.

In a nutshell, Theorem 4 states that player i prefers a signal that is

- as conditionally <u>dependent</u> as possible on the signals of players who belong to one of two groups: first, the players whose actions are complement to his own and whose monotonic strategy varies in the same direction as his own; and second, the players whose actions are substitute to his own and whose monotonic strategy varies in the direction opposite to his own;

- as conditionally <u>independent</u> as possible of the signals of players who belong to one of two groups: first, the players whose actions are complement to his own and whose monotonic strategy varies in a direction opposite to his own; and second, the players whose actions are substitute to his own and whose monotonic strategy varies in the same direction as his own;

Moreover, part *(i)* of Theorem 4 simplifies in the four following cases:

a. If player i has a positive complementarity in actions, i.e. $c^i = (1,...,1)$, and α is *isotonic,* i.e. $m = (1,...,1)$ or $m = (-1,...,-1)$, then player i prefers a signal that is as conditionally dependent as possible on X_{-i}.

b. If player i has a negative complementarity in actions, i.e. $c^i_{-i} = (-1,...,-1)$, and α is *isotonic,* i.e. $m = (1,...,1)$ or $m = (-1,...,-1)$, then player i prefers a signal that is as conditionally independent as possible of X_{-i}.

c. If player i has a positive complementarity in actions, and α is *antitonic for* i, i.e. $m_j = -m_i$ for all $j \neq i$, then player i prefers a signal that is as conditionally independent as possible of X_{-i}.

d. If player i has a negative complementarity in actions, and α is *strictly antitonic for* i, then player i prefers a signal that is as conditionally dependent as possible on X_{-i}

Part *(ii)* of Theorem 4 also simplifies is a similar way in the analogous four cases. Theorem 4 motivates and justifies the following definition.

Definition 3. *For any $i \in I$, any monotonicity vector m and any complementarity vector c^i, let **player i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarity in actions** be the vector d^i such that for all $j \in I$, the equation (6) holds.*

5.1.2 Link with the literature on dependence orderings

Before we proceed with the rest of the analysis, we shall now pause and discuss how Theorem 4 relates to the literature in applied probability, which studies dependence orderings and the logical relations between them.

Part (i) in Theorem 4 can be viewed as a generalization of a classic result in this literature, due to Tchen (1980). This scholar compares, for N-variate random vectors with fixed marginals, two dependence orderings: the *Positive Quadrant Dependence ordering* (PQD) and the *Supermodular Dependence ordering* (SPM). While it is well known that SPM dependence implies PQD dependence, Tchen shows that in the case $N = 2$, PQD dependence also implies SPM dependence, i.e. the two are equivalent.[16] In contrast, for $N \geq 3$, PQD dependence no longer implies SPM dependence. Müller and Scarsini (2003) provide a counterexample in the case $N = 3$.[17]

One difference between the two cases, which in our view is crucial for the difference in results, is that while in two dimensions, dependence only involves a single pair of components $(1, 2)$, in three dimensions, it involves three pairs of components: $(1, 2)$, $(2, 3)$ and $(1, 3)$. In other words, it becomes a measure of multilateral dependence (or interdependence).

[16] For definitions of SPM and PQD, see the Appendix D. See Müller and Stoyan (2002, Theorem 3.8.2) for related results.

[17] One way implications between various interdependence orderings and some equivalences have been obtained for the case $N \geq 3$. They are reviewed by Strulovici and Meyer (2012), who also establish new implications (see also Christofides and Vaggelatou, 2004; Müller and Stoyan, 2002; and Hu, Müller and Scarsini, 2004).

The paradigm in the applied probability literature is to conceive dependence orderings as multilateral dependence between multiple univariate components. Our result departs from this paradigm by defining dependence between two components of a multivariate random vector, one of which is itself multivariate.

In applied probability literature, different concepts of dependence relate to our definition of dependence between an univariate and a multivariate components.[18] These are called concepts of *setwise dependence* (e.g. Chhetry et al., 1989) and can be seen as generalizations of the positive upper (lower) orthant dependence concept.[19]

The various concepts of setwise dependence describe the dependence between random vectors, while disregarding the dependence between the univariate components within each of these vectors. In our particular problem, we only need to study the dependence between the signal choice of a given player and the choices of other players, not the dependence patterns among these other players' signals. Although such concepts of setwise dependence between vectors have been studied, we are not aware of any work studying setwise dependence *orderings,* such as the one we define.

Our Theorem 4 can be viewed as an extension of Tchen's result to the more general setwise case. Indeed, our concept of dependence is an appropriate generalization of PQD dependence in a setwise setting, and the inequality (7) is also an appropriate generalization of SPM dependence.[20] More specifically, in the special case where the state Θ is deterministic, $N = 2$, $i = 1$, α_1 and α_2 are the identity functions (so that $m_1 = m_2 = 1$) and $c^1 = (1, 1)$, we obtain the following result.

Corollary 1 (Tchen, 1980)**.** *Suppose that u_1 has $(1,1)$-complementarity in actions. Fix a signal $X_2 \in \mathbb{X}_2$. Suppose that X_1 and X_1' are two signals in $\mathbb{X}_1$ such that X_1' depends at least as much as X_1 on X_2. Then player 1 finds signal X_1' at least as good as signal X_1*

$$\mathbb{E}_{X_1',X_2}\left(u_1\left(X_1',X_2\right)\right) \geq \mathbb{E}_{X_1,X_2}\left(u_1\left(X_1,X_2\right)\right).$$

[18] We thank Marco Scarsini for pointing us to this literature.

[19] One such concept in Chhetry et al. (1989) is setwise positive upper (lower) orthant dependence, SPUOD (SPLOD). The set $(X_1, ..., X_k)$ with X_t a $p_t \times 1$ vector in $\mathbb{R}_t^p$ is said to be setwise positively upper (lower) orthant dependent if for all $x_t \in \mathbb{R}_t^p,\ t = 1, .., k$,

$$\mathbb{P}\left[\cap_{t=1}^k\{X_t > (\leq) x_t\}\right] \geq \Pi_{t=1}^k \mathbb{P}\left[X_t > (\leq) x_t\right].$$

[20] An appropriate name for this generalization of the SPM dependence ordering would be *Increasing Differences dependence ordering.*

This Corollary is a reformulation of Tchen's result, because $(1, 1)$-complementarity in actions coincides with supermodularity for a function of two variables, and the assumption that X_1' depends at least as much on X_2 as X_1 is equivalent to the assumption that (X_1', X_2) is at least as PQD dependent as (X_1, X_2) in the bivariate case.

Another way in which Theorem 4 extends Tchen's result is not mathematical, but purely conceptual. Our comparison dependence orderings are conditional on the state Θ. While this does not raise any mathematical difficulty, it allows us to interpret our dependence ordering as similarity between information sets, rather than between the components of a random vector. Similarly, inequality (7) indicates a preference for an information set over another. This generalization enables us to interpret Theorem 4 as telling us, between two pieces of information, which one a player prefers to have, depending on his preferences over actions, when one piece is more (or less) similar than the other to the other players' information.

We believe that both our new bilateral (conditional) dependence ordering for multivariate distributions and the extension of Tchen's result in Theorem 4 are of independent interest, and that they are likely to have applications in economics, in addition to the particular one we study in this paper. We now return to the analysis of Bayesian games with information choice.

5.2 Equilibria of the information choice game

Every Bayesian game with information choice Γ and every fixed profile of action strategies α induce an information choice game Γ_α, which is the normal form in which each players $i \in I$ chooses a signal $X_i \in \mathbb{X}_i$ and receives the payoffs $\mathbb{E}_{\Theta,X}\left(u_i\left(\alpha_i\left(X_i\right), \alpha_{-i}\left(X_{-i}\right), \Theta\right)\right)$, where α is the fixed profile of action strategies.

We now use part (ii) in Theorem 4 to obtain a characterization of the equilibria of the information choice game Γ_α when α is a strictly m-monotonic action strategy profile, for some monotonicity vector m.

Corollary 2. *Let $\left(c^1, ..., c^N\right)$ be a profile of complementarity vectors. Suppose that for each i, player i has strict c^i-complementarities in actions. Suppose that (X, α) is a full-fledged Nash-Bayesian equilibrium profile in pure strategies. Suppose that α is strictly m-monotonic, for some monotonicity vector m. Then for all i, there exists no signal X_i' that d^i-depends more on X_{-i} than X_i, where d^i is player i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarity in actions.*

In some cases, by this result, Nash-Bayesian equilibrium conditions imply that m-monotonicity of action strategies together with c^i-complementarity in actions pin down an essentially unique information structure. Whether or not this is the case depends on the geometric structure of the signal structure. We now study this question in more detail.

5.2.1 Most d^i-dependent signals

The first set of situations where monotonicity pins down the information structure is when the dependence preference of each player are to some extent independent on the information choices of the other players. For example, if there are two players and both have access to two different signals X_I and X_{II}, none of these signals is intrinsically more public than the other. For each player, the more public signal is the signal the other player chooses. Yet, in many contexts, the different available signals do not possess that kind of symmetry. Some signals are unambiguously more public than others, some are unambiguously more private than others. We push this idea even further and introduce the concept of a most d^i-dependent signal, independently of the signals chosen by others.

For any signal structure $(\mathbb{X}_1, ..., \mathbb{X}_N)$, any distribution F and any dependence vector d^i, and for all X_i in $\mathbb{X}_i$, we say that X_i^* is the **most d^i-dependent signal in** $\mathbb{X}_i$ if X_i^* has the property that, for all signal profiles X_{-i}, the signal X_i^* is a greatest element of the "as d^i-dependent on X_{-i} as" weak partial order on $\mathbb{X}_i$. In plain words, the signal X_i^* provides player i with information that is more d^i-dependent on the other player's signals, than any other signal player i could choose to observe, regardless of what signals the other players choose to observe. In particular, for $d^i = (1, ..., 1)$, we call this signal player i**'s most public signal** and for $d^i_{-i} = (-1, ..., -1)$, for $d^i_{-i} = (d^i_j)_{j \neq i}$ we call this signal player i**'s most private signal.**[21]

We obtain the following direct implication of Corollary 2 (a direct implication of Theorem 4).

Corollary 3. *Let $\left(c^1, ..., c^N\right)$ be a profile of complementarity vectors. Suppose that for each i, player i has strict c^i-complementarities in actions. Suppose that (X, α) is a full-fledged Nash-Bayesian equilibrium profile in pure strategies. Suppose*

[21]Most d^i-dependent signals in $\mathbb{X}_i$ need not be unique, but they are payoff-equivalent under m-monotonic action strategies. See the last paragraph of Section 5.4. If two different players i and j both have a most public signal, then it must be that the intersection $\mathbb{X}_i \cap \mathbb{X}_j$ has at most (essentially) one element. Moreover, if this intersection is indeed nonempty, its (essentially) unique element is both i's and j's most public signal.

that α is strictly m-monotonic, for some monotonicity vector m. Suppose that for all $i \in I$, player i has a most d^i-dependent signal in $\mathbb{X}_i$, where d^i is player i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarity in actions. Then X_i must be a most d^i-dependent signal in $\mathbb{X}_i$.

To illustrate the usefulness of this result, it is helpful to consider the following three special cases:

1. If every player has a most public signal, and $m_i m_j c^i_j = 1$ for all i, j such that $i \neq j$, then in any Nash-Bayesian equilibrium in which actions are strictly m-monotonic, every player must be choosing his most public signal.

2. More specifically, if every player has a most public signal and strict positive complementarities in actions, then in any Nash-Bayesian equilibrium in which actions are strictly isotonic, every player must be choosing his most public signal.

3. If every player has a most private signal, and $m_i m_j c^i_j = -1$ for all i, j such that $i \neq j$, then in any Nash-Bayesian equilibrium in which actions are strictly m-monotonic, every player must be choosing his most private signal.

Of course, whether or not Corollary 3 has bite hinges upon whether there exists or not a most d^i-dependent signal for each player i. The answer to this question depends on the geometry of the signal structure $\mathbb{X}$.

To gain intuition, it is helpful to visualize the information choice game (where action monotonicities are held fixed) as a location game in a spatial setting. For each player i, the set $\mathbb{X}_i$ is the analog of a set of admissible locations where this player can locate. Each player's preferences over locations only depends on where the other players locate. He wants to be close to some of them and far away from some of them. The preferences of player i are summarized by the vector d^i. For each $j \neq i$, if $d^i_j = 1$, call j a friend of i, and if $d^i_j = -1$, call j an enemy of i. Note that j may be a friend of i, while i is an enemy of j. Then each player i wants to locate as close as possible to all his friends and as far as possible from all his enemies. Corollary 3 says that if each player has a location X_i that minimizes distance with all his friends and maximizes distance with all his enemies, regardless of where all of them locate, then player i must choose this location in any Nash equilibrium in pure strategies.

In Appendix B we show how to construct examples of signal structures $\mathbb{X}$ that admit a most public signal, a most private signal and a most d^i-dependent signal.

One case of interest where Corollary 3 does not apply is when the signal structure is symmetric, i.e. when $\mathbb{X}_1 = ... = \mathbb{X}_N$. In this case, except in degenerate cases, players do not have most d^i-dependent signals. But, in the symmetric case, we can still obtain sharp predictions in the case where $N = 2$.

If $N = 2$ and the two players have strict positive (negative) complementarities in actions, in any pure Nash-Bayesian equilibrium, whose actions are strictly isotonic (antitonic for both players) they choose to acquire essentially the same information. We say that (X_1, X_2) is **public information** if the event $X_1 = X_2$ has probability one.

Corollary 4. *Suppose that $N = 2$, that $\mathbb{X}_1 = \mathbb{X}_2$ and suppose that the payoff functions u_i have strict positive (negative) complementarities in actions, for $i = 1, 2$. Let $(X_1, X_2, \alpha_1, \alpha_2)$ be a pure full-fledged Nash-Bayesian equilibrium of the game. If α is strictly isotonic (antitonic for both players), the equilibrium information structure must be public information.*[22]

Note that, under the assumptions of Corollary 4, the equilibrium is never unique: if $(X_I, X_I, \alpha_1, \alpha_2)$ is a full-fledged Nash-Bayesian equilibrium, then for any $X_{II} \in \mathbb{X}$, the profile $(X_{II}, X_{II}, \alpha_1, \alpha_2)$ is also a Nash-Bayesian equilibrium. But all of these equilibria are payoff equivalent. The players higher order beliefs are also the same across all the equilibria. In the example of Section 3 we referred to this as trivial multiplicity.

Corollary 4 describes a case where the two players "agree" on the information structure they want. In the case of a symmetric signal structure with two players, this agreement is necessary. Indeed, in the case where they do not agree, it is easy to see that no equilibrium in pure strategies can exist, even when, for any information structure X, the game with exogenous information structure X admits a Nash-Bayesian equilibrium in pure strategies. This was illustrated in the example studied in Section 3.1.1, but it is a more general phenomenon.

Corollary 5. *Suppose that $N = 2$, that $\mathbb{X}_1 = \mathbb{X}_2$ and contain at least two signals whose realizations are not equal with probability one, and suppose that both players have strict complementarities in actions, but of opposing signs. There is no pure full-fledged Nash Bayesian equilibrium $(X_1, X_2, \alpha_1, \alpha_2)$ such that α is strictly monotonic.*

To see why Corollary 5 is true, suppose for example that player 1 has a strict positive complementarity in actions, while player 2 has a strict negative

[22] It is worth noting that Corollary 4 does not generalize to the case of three players or more.

complementarity in actions. Suppose further that action strategies are set to be strictly isotonic in the second stage. Then, there cannot be an equilibrium in pure strategies in the first stage. This is because player 1 wants to observe the same signal as player 2, in order to increase dependence, whereas player 2 wants to observe a signal different from player 1, in order to decrease dependence. Similarly, if action strategies are set to be strictly antitonic in the second stage, there cannot be an equilibrium in pure strategies in the first stage either. This is because player 2 now wants to observe the same signal as player 1, in order to increase dependence, whereas player 1 wants to observe a signal different from player 2, in order to decrease dependence. In both cases, the signal choice in the first stage of the game has a structure à la matching pennies. No equilibrium in pure strategies exists, although (as shown in Appendix E) a mixed equilibrium can always be constructed.

Note, however, that Corollary 5 need not hold when the signal structure is not symmetric. For example, as shown in Corollary 3, and with the payoff configuration of the previous paragraph, if player 1 has access to a most-dependent signal and player 2 has access to a less dependent signal, then the information structure defined by these two signals is a (unique) candidate for an equilibrium where the strategies are strictly isotonic. Similarly, if player 1 has access to a least-dependent signal and player 2 has access to a most dependent signal, then the information structure defined by these two signals is a (unique) candidate for an equilibrium where the strategies are strictly isotonic.

5.3 Full-fledged equilibrium information structures

In general, the information structure and the actions strategies (and their monotonicity properties) are jointly determined in equilibrium. The conditional dependence properties between the signals chosen by the different players contribute to determine incentives to choose actions strategies that are either increasing or decreasing in signal realizations. Conversely, in Sections 5.1 and 5.2, we showed how the monotonicity properties of the action strategies chosen by the players contribute to determine their incentives to choose more or less conditionally dependent signals. In this section, we propose conditions that guarantee that an equilibrium exists, with certain pre-specified monotonicity characteristics and with a certain pre-specified information structure.

The key condition that guarantees existence of an equilibrium is a form of compatibility between the monotonicity properties of the candidate action strategies and the candidate information structure. But other conditions are required as well. In the equilibrium we construct, the m-monotonicity of the equilibrium action strategies agrees with the state complementarity in the players' payoffs. For example, if for player i, the state and his action are positive complements, his equilibrium action strategy will be increasing in the signal. If they are negative complements instead, his equilibrium strategy will be decreasing in the signal. Existence of such an equilibrium is established in two cases. First, when the action complementarities are aligned with the state complementarities, and in effect reinforce them, by giving players additional incentives to play m-monotonic action strategies. Second, when the sign of the monotonicity of equilibrium action strategies is predictable.

5.3.1 When action complementarities reinforce state complementarities

We first provide sufficient conditions that ensure that an equilibrium exists where players have public information (or most public information). We show that a very simple condition ensuring this is that all players have positive complementarities, both in actions and in state. But we also show that it is the case for a weaker condition. It only requires that the state complementarities be *aligned* with the action complementarities in the sense that if the sign of the players' state complementarities are given by the vector m and their action complementarities are given by the vectors $(c^i)_{i \in I}$, then for all $i \neq j$, we have $c^i_j = m_i m_j$. Clearly, the situation where all (action and state) complementarities are positive is the special case where $m = (1, ..., 1)$ and $c^i = (1, ..., 1)$ for all i.

First, when state complementarities are aligned with action complementarities, we establish, using a result due to Van Zandt and Vives (2007), that fixing the information structure X, the game Γ_X admits an m-monotonic Nash-Bayesian equilibrium α^X. A partial intuition for why this is true is that the monotonicity of each player i's equilibrium strategy m_i is dictated by his state complementarity, also m_i, but is further reinforced by the action complementarities.

For example, consider the situation where all players have positive action and state complementarity. Suppose further that each signal depends positively on the state so that a high realization is evidence that the state is likely to be high, and that conditionally on the state, all signals are positively dependent among each other. Thus, when a player observes a high (low) realization, he believes that the state is high and that other players' realizations are also high. A first-order effect is that he wants to play a high (low) action, so that his action will be aligned with the state, which he believes to be high (low). But there is a second-order effect, which is that he also believes that, conditional on the state, the other players' realizations are high (low), so that they are likely to be playing high (low) actions. Because the action complementarities are positive, this gives an additional reason to play a high (low) action. Since all players can realize this, there is then a third order effect which further increases the incentive to play a high (low) action. The process then goes on ad infinitum.

Second, we show that when state complementarities are aligned with action complementarities, and players choose either a public or a most public information structure X, and play an m-monotonic Nash-Bayesian equilibrium α^X of the game Γ_X, no player can strictly gain by deviating to another strategy $(X_i', \alpha_i') \neq \left(X_i, \alpha_i^X\right)$. To establish this, we first argue that assuming by contradiction that the deviation (X_i', α_i') is a profitable deviation, there exists another deviation (X_i', α_i'') that is even better, where α_i'' is m_i-monotonic. But in this case, we can show that the deviation (X_i, α_i'') is even better, so it must be profitable, which contradicts the fact that α^X was an equilibrium of the game Γ_X in the first place.

The precise conditions that guarantee the existence of an equilibrium where dependence among signals is maximized (most public signals are chosen) are the following.

Theorem 5. *Let $N \geq 2$. Let m be a monotonicity vector and let $(c^i)_{i\in I}$ be the profile of complementarity vectors such that for all i, j, $c^i_j = m_i m_j$. Suppose that*

i. For each $i \in I$, u_i has c^i-complementarities in actions.

ii. For each $i \in I$, u_i has m_i-complementarities in a_i and θ.

iii. For all $x_i < x_i'$, the distribution of θ conditional on $X_i = x_i'$ first order stochastically dominates the distribution of θ conditional on $X_i = x_i$.

iv. For every profile X, all i and all $x_i < x_i'$, the distribution of X_{-i} conditional on $X_i = x_i'$ first order stochastically dominates the distribution of X_{-i} conditional on $X_i = x_i$.

Then for any profile X of signal choices such that for each i, the signal X_i is most dependent on X_{-i} in $\mathbb{X}_i$, there exists an m-monotonic action strategy profile α such that (X, α) is a full-fledged Nash-Bayesian equilibrium for the game.

The following result is a direct implication of Theorem 5.

Corollary 6. *Let $N \geq 2$. Suppose that conditions (i) to (iv) of Theorem 5 hold. In addition, suppose that a public (most public) signal profile exists. Then for any public (most public) information signal profile X, there exists an m-monotonic action strategy profile α such that (X, α) is a full-fledged Nash-Bayesian equilibrium for the game Γ.*

5.3.2 When the monotonicity of action strategies is predictable

We now move away from the case where action complementarities reinforce state complementarities and consider cases where action complementarities may create incentives for the players to play actions that vary in the direction opposite to the one which agrees with the state complementarity. In that case, it is not possible to predict in general whether the action strategies of any full-fledged Nash-Bayesian equilibrium will be m-monotonic for any particular monotonicity vector m.

Nevertheless, in some context, we may have enough information to know that in equilibrium, action strategies are m-monotonic for some pre-specified vector m. For example, it could be that all profiles of action strategies α that are not m-monotonic are strictly dominated, or do not survive iterated elimination of strictly dominated strategies.

One natural reason why m may be predictable is that each player i could have an m_i-complementarity in state which is strong enough that it dominates any potential higher order effect and that it single-handedly determines the monotonicity of equilibrium action strategies. For example, in an symmetric Cournot duopoly, where firms 1 and 2 should produce a larger quantity when the state is high and a lower quantity when the state is low, the negative action complementarity creates a contrarian incentive. But if this second order effect and all other higher order effects are negligible compared to the first order effect, it could be predictable that equilibrium action strategies are increasing in signal realizations.

Alternatively, the monotonicity of action strategies could be predictable for other reasons. For example, in the same setting, it could be that firm 1's complementarity in state dominates firm 1's (negative) action complementarities, so that this firm always plays an increasing action strategy in any equilibrium. In contrast, firm 2 could have a (negative) action complementarity that is much stronger than its positive complementarity in state. This and the fact that firm 1 plays an increasing action strategy in any equilibrium could imply that firm 2 plays a decreasing action strategy in any equilibrium. As a result, in any equilibrium, the action strategy profile is $(1, -1)$-monotonic.

When the monotonicity of action strategies is predictable in this sense, and under the assumption that each player i has c^i-complementarities in actions, in light of Theorem 4, a natural candidate X may emerge for an equilibrium information structure: one that has the property that for each i, X_i is most d^i-dependent on X_{-i} than any other signal in $\mathbb{X}_i$, where d^i is agent i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarities in actions (defined in Equation (6)). Our main result in this section provides a simple sufficient condition for this candidate to form a full-fledged Nash-Bayesian equilibrium, together with some m-monotonic action strategy α. The condition imposes that for any player i, and for any signal deviation X_i', in the game $\Gamma_{X_i', X_{-i}}$, player i has at least one best response to α_{-i} that is m_i-monotonic.

The logic at play here is the same as in Theorem 5. There, because action complementarities reinforce state complementarities, monotonic equilibrium action strategies are known to exist. We thus obtain $d_j^i = m_i m_j c_i^j = 1 \cdot 1 \cdot 1$ for all i and j and the natural candidate that emerges is any information structure where positive dependence is maximized for all players. The same condition on deviations to other signals X_i' as the one stated in the previous paragraph can then be obtained from primitives and is sufficient to establish that this candidate is indeed an equilibrium.

The difference now is that, both existence and the condition on deviation to signals X_i' are assumed rather than derived from primitives, and therefore need to be verified directly. But the result shows that this approach can be adapted beyond the case where complementarities reinforce each other.

For any strategy profile (X, α), we say that the action strategy α_i is a best response for player i in game Γ_X, if for all x_i,

$$\alpha_i (x_i) \in \arg\max_{a_i \in A_i} \mathbb{E}_{\Theta, X} \ \left(u_i \left(a_i, \alpha_{-i} \left(X_{-i}\right), \Theta\right) \mid X_i = x_i\right).$$

Theorem 6. *Let $N \geq 2$. Let m be a monotonicity vector. For each i, let c^i be a complementarity vector, and let d^i be agent i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarities in actions. Let (X, α) be a strategy profile such that:*

i. For each i, the payoff of player i has c^i-complementarities in actions.

ii. For each i, X_i is most d^i-dependent on X_{-i} in $\mathbb{X}_i$.

iii. The profile α is an m-monotonic Nash-Bayesian equilibrium of the game Γ_X.

iv. For each i and each X_i', in the game $\Gamma_{X_i',X_{-i}}$, player i has a best response α_i' to the action profile α_{-i} that is m_i-monotonic.

Then, the profile (X, α) is a full-fledged Nash-Bayesian equilibrium of the game Γ.

Note that assumptions (*iii*) and (*iv*) are not on primitives, but they could be derived from assumptions on primitives: for example, one could assume that only m-monotonic profiles survive iterated elimination of strictly dominated strategies (and existence of a Nash-Bayesian equilibrium of game Γ_X could be established using Kakutani's fixed point theorem). However, we feel that in practice, both assumptions (*iii*) and (*iv*) are much less restrictive than those assumptions on primitives, and can often be easily checked in most applications. The scope of our result is thus larger than if we imposed those assumptions.

5.4 Conditions for a unique full-fledged equilibrium information structure

We would now like to provide conditions under which the information structure is essentially the same in all full-fledged Nash-Bayesian equilibria. In fact, this question is already answered in Corollary 3, which only needs to be reformulated and reinterpreted.

In the reformulation, the key assumptions we make are (*i*) that for all information structures X, all Nash-Bayesian equilibria are strictly m-monotonic for some monotonicity vector m such that there is a profile of dependence vectors $\left(d^1, ..., d^N\right)$ where each d^i is agent i's most preferred dependence vector under m-monotonic action strategies and his actual c^i-complementarities in actions and (*ii*) that each agent has a most d^i-dependent signal in $\mathbb{X}_i$. When this holds, then all full-fledged Nash-Bayesian equilibria of the game must have an information structure where each agent chooses a most d^i-dependent signal in $\mathbb{X}_i$.

Again, as in Section 5.3.2, the predetermined monotonicity m in assumption (*i*) could be the resulting balance of a number of contrarian forces.

Corollary 7. *Let $N \geq 2$. Suppose that for each i, player i has c^i-complementarities in actions. Suppose for any signal profile X, all Nash-Bayesian equilibria of the game Γ_X are strictly m-monotonic, for some m such that there exist vectors $\left(d^1, ..., d^N\right)$ such that for all i, d^i is player i's most preferred dependence vector under m-monotonic action strategies and c^i-complementarities in actions. Suppose that each player i has a most d^i-dependent signal in $\mathbb{X}_i$. Then in any full-fledged*

Nash-Bayesian equilibrium (X, α) *of the game* Γ, *for each* i, *the signal* X_i *is a most* d^i*-dependent signal in* $\mathbb{X}_i$.

Although the formulation and the interpretation is different, this result is formally equivalent to Corollary 3. Strictly speaking, Corollary 7 does not pin down a unique information structure, because each player i may have multiple most d^i-dependent signals. But most d^i-dependent signals are interchangeable when actions are played according to a strictly m-monotonic profile α, in the sense that if X and X' are two signal profiles, each made up of (possibly distinct) most d^i-dependent signals X_i and X_i', where for each i, d^i is player i's most preferred dependence vector under m-monotonic action strategies and his c^i-complementarities in actions, then (X', α) is also a full-fledged Nash Bayesian equilibrium of the game and all players obtain the same expected payoff in both equilibria.

6 Applications

In this section, we now illustrate how the model can be applied to different contexts.

6.1 Currency speculation

This example is adapted from the model of currency speculation of Morris and Shin (1998), to which we add information choice. The game is played between N agents. Each agent i decides whether ($a_i = 1$) or not ($a_i = 0$) she speculates against a currency. The bank then observes the realization of the number n of agents who speculate and the realization $\theta \in \{\theta_1, ..., \theta_p\}$, with $\theta_1 < ... < \theta_p$, of an uncertain but relevant fundamental state Θ. It defends the currency if and only if $n \leq \theta$. If this condition holds, the attack is "unsuccessful" and it is "successful" otherwise. The payoff of an agent who chooses to speculate is $\pi - b\theta - K$, with $\pi > 0$, $b > 0$ and $K > 0$ if the attack is successful and $-K$ if it is unsuccessful. The payoff of not speculating is 0.

Before deciding whether or not to speculate, each of the agents i chooses a single signal X_i from a common set of accessible signals $\mathbb{X}_i = \mathbb{X}$. Suppose that assumptions (*iii*) and (*iv*) of Theorem 5 hold, so that high realizations of any of the accessible signals are associated with high realizations both of the state and of the other accessible signals.

In this game, all agents have negative complementarities in state and positive complementarities in the other agents' actions: if other agents are more likely to attack, the agent is more willing to attack.

All assumptions of Theorem 5 are satisfied. Therefore we know that at least one full-fledged Nash-Bayesian equilibrium exists, where all agents choose to observe the same (i.e. public) signal. Doing so enables them to perfectly coordinate: whenever the realization of the public signal is lower than some threshold, all agents attack.[23]

An implication is that unanimous attacks followed by devaluation sometimes occur even for high realizations of the fundamental.

These unanimous attacks are a poor signal of the fundamental, since they only reflect part of the information of one signal, and ignore the information contained in all the other signals that the agents choose not to observe.

In this equilibrium, the movements of the currency are essentially driven by random realizations of a signal used by speculators for coordination purposes.

While such an equilibrium may be detrimental to society, it may be good for speculators. But in a version of the model, this "herding on the same signal" equilibrium may be bad for the speculators themselves.

Following Goldstein, Ozdenoren and Yuan (2011), consider instead a setting where the central bank does not observe the realization of Θ at all, nor does it observe a signal of it, and where n does not directly enter its decision on whether or not to defend the currency. Suppose instead that the central bank learns about the state from the occurrence and potentially also from the size of an attack, and that it defends the currency peg if and only if $\mathbb{E}(\Theta \mid n) > 0$. Taking again the bank's decision rule as given, the agents play a coordination game with information choice, the payoffs of which are endogenous, since they depend on which signal action strategies the bank expects them to play. For the same reasons as in the previous model, there exists an equilibrium where all agents choose to observe the same signal. This implies that a unanimous attack is a weak signal that the realization of the fundamental is low. Consider the case where $\mathbb{E}(\Theta) > 0$ and $\mathbb{E}(\Theta \mid X_i \leq x^*) > 0$, where x^* is the threshold realization of the common signal below which the agents choose to attack. Then, when observing an attack, the bank does not find evidence in favor of abandoning the peg convincing enough, because the unanimous attack only reflects the information of one signal.

[23] Under our assumption that the set of accessible signals $\mathbb{X}_i$ is the same for all speculators, we cannot rule out other information structures in equilibria. Under the alternative assumption that each speculator i has a unique most public signal in $\mathbb{X}_i$, an argument in the spirit of Corollary 7 can be used to establish that for an open set of parameters, a unique information structure arise in all full-fledged Nash-Bayesian equilibria, such that all agents choose their most public signal (possibly the same one).

Because of the excessive similarity in speculators' information, the bank chooses to always defend the currency. Consequently, attacks are never successful and therefore they never occur in equilibrium.

What happens in this case is that, while individually, each speculator has an incentive to observe the same signal as the others, their collective interest is that the bank expects them to acquire diversified information. The equilibrium is however determined by their individual interest. From the speculators' point of view, informational diversity is a public good that they under-provide in equilibrium.

The above analysis can also be applied to technology adoption in the presence of positive network externalities, or the problem of collective action in a revolutionary movement. In both cases, complementarities imply that players might observe the same signal so that the aggregate action is not a good aggregator of all available information.

6.2 Other Applications

In each of the following examples, θ is an uncertain parameter with support $\{-1,1\}$, with $\mathbb{P}(\Theta=-1)=\mathbb{P}(\Theta=1)=1/2$. Moreover, available signals are $\mathbb{X}=(X_1,...,X_L)$ with $L\geq N$, each with support in $\{-1,1\}$ such that the random vector (θ,X_ℓ) is distributed in $\{-1,1\}^2$ according to a joint marginal distribution given by

$$\begin{array}{c|c|c} & X_\ell=-1 & X_\ell=1 \\ \hline \Theta=-1 & \frac{1-\varepsilon}{2} & \frac{\varepsilon}{2} \\ \hline \Theta=1 & \frac{\varepsilon}{2} & \frac{1-\varepsilon}{2} \end{array} \tag{9}$$

for all X_ℓ and $\varepsilon\in(0,1/2)$. The signals X_ℓ are independent, conditional on any realization of θ. Conditional on $\theta\in\{-1,1\}$, two signals (X_s,X_ℓ) have the joint distribution

$$\begin{array}{c|c|c} & X_\ell=\theta & X_\ell\neq\theta \\ \hline X_s=\theta & (1-\varepsilon)^2 & (1-\varepsilon)\varepsilon \\ \hline X_s\neq\theta & (1-\varepsilon)\varepsilon & \varepsilon^2 \end{array} \tag{10}$$

Given the information structure, when two players observe the same signal, their signal's realization is the same with probability one. When two players do not observe the same signal, their signal's realization is the same with probability $1-\varepsilon$.

6.2.1 Supply chain

Suppose there is a supply chain with 1 manufacturer (denoted M) and 2 retailers (denoted R_1 and R_2). The manufacturer chooses the wholesale price w, and retailer R_i, the markup p_i over the wholesale price. Final prices are $w + p_1$ and $w + p_2$. The demand for retailer R_i is

$$Q_i(p_i, p_j, w; \theta) = A + b_i\theta + \lambda_i(w + p_j) - (w + p_i), \tag{11}$$

where $b_i > 0$, $A > b_i$ and $0 < \lambda_i < 1$ for $i = 1, 2$. All three players are uncertain on the intercept of the retailers' demand function θ and have access to the signals $(X_1, ..., X_L)$. Retailer R_i's profits function is given by

$$\Pi_{R_i}(p_i, p_j, w; \theta) = p_i \cdot (A + b_i\theta + \lambda_i(w + p_j) - (w + p_i)), \tag{12}$$

for $i = 1, 2$, and the manufacturer's profits function is given by

$$\Pi_M(w, p_i, p_j; \theta) = w\cdot(2A+(b_1+b_2)\theta-(1-\lambda_1)p_2-(1-\lambda_2)p_1+w(\lambda_1+\lambda_2-2)). \tag{13}$$

The vector encoding action complementarities is $c^{R_i} \equiv (c^{R_i}_{R_j}, c^{R_i}_M) = (1, -1)$ for retailer R_i, and $c^M \equiv (c^M_{R_1}, c^M_{R_2}) = (-1, -1)$ for the manufacturer.

It is natural to look for an equilibrium where (p_1, p_2, w) are strictly increasing in θ, so that the action strategies are strictly m-monotonic with $m \equiv (m_{R_1}, m_{R_2}, m_M) = (1, 1, 1)$. Then, assuming that m describes the monotonicity of equilibrium action strategies, Theorem 4 implies that the players' dependence preferences are obtained by combining the monotonicity and the complementarity vectors, such that $d^{R_i} \equiv (d^{R_i}_{R_j}, d^{R_i}_M) = (1, -1)$ for retailer R_i and $d^M \equiv (d^M_{R_1}, d^M_{R_2}) = (-1, -1)$ for the manufacturer. Hence, the retailers prefer to observe the same signal and the manufacturer a signal different from the one observed by the retailers.

6.2.2 Beauty contests

Suppose a set I of players interacts in a beauty contest game. Two versions of the beauty contest model can be found in the literature. In both versions, each player $i \in I$ chooses an action and his payoff depends on the others' average action $\bar{a} = \frac{1}{N}\left(\sum_{j\in I\setminus\{i\}} a_j\right)$. Version 1 (as in Myatt and Wallace (2011)) assumes the payoff function

$$u_i(a_i, \bar{a}; \theta) = -(1 - r)(a_i - \theta)^2 - r(a_i - \bar{a})^2,$$

while version 2 (as in Hellwig and Veldkamp (2009)) assumes the payoff function

$$u_i(a_i, \bar{a}; \theta) = -(a_i - (1-r)\theta - r\bar{a})^2,$$

where $r \in (-1, 1)$. In both versions, all players have positive complementarities in actions if $r > 0$ and negative complementarities in actions if $r < 0$, and they all have positive complementarities in state.

For both versions, given the information structure is fixed, a player's best response in action is $\alpha_i(x_i) = (1-r)\mathbb{E}(\Theta|x_i, X)) + r\mathbb{E}(\overline{a}|x_i, X)$.

If r is sufficiently small (such that $1 - r$ is big enough), then the best response will be increasing in the signal's realization for all players. Then, by Theorem 6, the situations where all players play action strategies that are strictly increasing in state and i) all players observe the same signal if $r > 0$, and ii) all players observe different signals if $r < 0$ are equilibria of the game with an endogenous information structure.

6.2.3 Technological Spillovers

Suppose a set I of players have the possibility to exert effort in developing a new technology. A player chooses his level of effort a_i to the development of this technology. The cost of effort a_i is ca_i^2, with $c > 0$, while the benefit for each individual is $\left((Ka_i + b\theta)\sum_{j\in I} a_j\right)$, with $b > 0$ so that the payoff of player i is

$$u_i(a;\theta) = \left((Ka_i + b\theta)\sum_{j\in I} a_j\right) - ca_i^2.$$

The effort exerted by the other players has a positive impact on player i's payoff. Hence, all players have positive complementarities in actions and positive complementarities in state. When all players observe the same signals, there exists an equilibrium where all players choose an effort level that is increasing in their signal if b is large enough. Then, Theorem 6 ensures that the situations where all players choose a strictly increasing action strategies and all choose to observe the same signals is still an equilibrium of the game with an endogenous information structure. Therefore, information acquisition on the state is suboptimal from a social viewpoint since acquired information will be homogenous.

6.2.4 Policy choice in federations

This example is adapted from Loeper (2011). Assume there is a set I of jurisdictions and that each of them needs to decide on a policy. A jurisdiction's policy choice is a_i, and its payoff is

$$u_i(a;\theta) = -(a_i - \beta_{i\theta}\theta)^2 - \sum_{j\neq i} \beta_{ij}(a_i - a_j)^2,$$

where $\beta_{i\theta}$ is jurisdiction i's alignment preference with the state θ and $\beta_{ij} \in (-1,1)$ the coordination externality that jurisdiction j imposes on i. Jurisdiction i has positive (negative) complementarities in state when $\beta_{i\theta} > (<)\ 0$ and positive (negative) complementarity in actions with jurisdiction j when $\beta_{ij} > (<)\ 0$.

A player's action best response is

$$\alpha_i(x_i) = \frac{\beta_{i\theta}\mathbb{E}[\theta|x_i] + \sum_{j\neq i}\beta_{ij}\mathbb{E}[a_j|x_i]}{(1+\sum_{j\neq i}\beta_{ij})}.$$

When all jurisdictions have positive (negative) complementarity in state and positive complementarity in actions, such that the action complementarities are aligned with the state complementarities, Theorem 5 ensures that there exists an equilibrium where policy choices are increasing (decreasing) in the signal and all jurisdictions observe the same signal. In this type of equilibrium, the vector of policy choices would not be very informative on the fundamental θ, since all policies will be based on the same signal.

Consider next the case where all jurisdictions have negative (positive) complementarity in state and negative complementarity in actions. Suppose further that when it is exogenously determined that all jurisdictions observe different signals then, in this case, there exists an equilibrium where policy choices are decreasing (increasing) in the signal. Then, Theorem 6 implies that observing different signals is still an equilibrium in Γ, the game with endogenous information choice, when all jurisdictions have sufficiently negative (positive) complementarity in state.

6.2.5 Imperfect competition

These examples are adapted from Jiménez-Martinez (2013).

Cournot: Consider a set I of firms interacting in an oligopoly and competing in Cournot. Let the aggregate demand function be given by $P(q_1, ..., q_I; \theta) = A + b\theta - \delta(q_1 + ... + q_I)$, with $b > 0$, $\delta > 0$.

Each of the firms sets a quantity q_i, $i = 1, ..., I$ assuming the marginal cost is constant and equals to $c_i > 0$ for each firm. Firm i's payoff is

$$u_i(q_i, q_{-i}; \theta) = \big(A + b\theta - \delta(q_1 + ... + q_I) - c_i\big)q_i.$$

In this context, the firms have negative complementarities in actions and positive complementarities in state.

Thus Theorem 6 implies that when condition *iv*) holds, which is the case if b is large enough (and ε is small enough relative to the other parameters), there exists an equilibrium where all firms choose different signals and play a quantity strategy increasing in their signal. In particular, the price P is very informative on the state since it aggregates the information of N independent signals.

Bertrand: Consider next an oligopoly where two firms produce a differentiated product and compete in Bertrand. Each of the two firms $i = 1, 2$ sets a price p_i for its product and faces the linear demand $Q_i = b_0 + b_1\theta - \gamma(p_i - p_j)$ with $b_0 > 0$, $b_1 > 0$ and $\gamma > 0$. Each firm i has a constant marginal cost c_i.

Firm i's profits function is then

$$u_i(p_i, p_j; \theta) = \big(b_0 + b_1\theta - \gamma(p_i - p_j)\big)\big(p_i - c_i\big).$$

The firms have positive complementarities in actions and positive complementarities in states. Thus, since state and action complementarities reinforce each other, Theorem 5 implies that there exists an equilibrium where both firms choose the same (public) signal and choose a price strategy that is increasing in the signal. In this case, the quantity $Q_1 + Q_2$ only reflects the information contained in one signal and does not aggregate all the information potentially available.

7 Related literature

We contribute to the applied probability literature on dependence orderings. We explain this contribution in section 5.1.2.

Regarding decentralized information acquisition, the most commonly studied framework is a two stage game where players start with a common prior on some unknown common value state that affects all players' payoffs.[24] In the first stage,

[24]See Li, McKelvey and Page (1987), Vives (1988), Hellwig and Veldkamp (2009), Myatt and Wallace (2011), Szkup and Trevino (2014), Yang (2014), and many others. Veldkamp's monograph (2011) and Hellwig, Kohls and Veldkamp (2013) provide excellent surveys on the widely studied special case of the beauty contest games with a continuum of actions and

each player makes an information choice (for example, the precision of the signal he receives) that determines the information on the state that he has when entering the second stage. In the second stage, players simultaneously choose an action. Two different extensive forms have been considered, depending on whether the choices made at the first stage are observed or not. In some models, the acquisition is publicly observed. The game is then an extensive form game where each profile of information acquisition choices defines a subgame, and in each subgame, the information structure is common knowledge: in these games, acquisition is *overt.* In other models, the choices of the players in the first period are not observed before actions are taken. Acquisition is then *covert.* A game where information acquisition is covert is essentially static, as it is equivalent to one where all players simultaneously choose both their information and a commitment to an action strategy that maps the signal they will observe to the action they choose. The difference between overt and covert information acquisition is in the way a deviation in the first stage is treated: Under overt acquisition, a deviation on information choice is commonly observed, and the information structure is common knowledge in the second stage subgame following the deviation; Under covert acquisition, players form a belief of what the information structure is in the second stage, and this belief is correct in equilibrium. But whenever a player deviates, all other players' belief on the information structure is incorrect. It should be noted that in games with a continuum of players (Hellwig and Veldkamp, 2009; Myatt and Wallace, 2011; Szkup and Trevino, 2014), where players' payoffs only depend on the statistical distribution of the other players' actions, the two forms of acquisition are equivalent. Thus, there is no need to make a distinction in this case. The distinction matters only for games with finitely many players. In this paper, we derive results that apply to games with covert acquisition and finitely many players, and to games with covert or overt acquisition and a continuum of players. The case where acquisition is overt and the number of players is finite is also considered but only in section 7.3.

7.1 The motive inheritance result

The main focus in the literature has been on the player's choice of amount of information (their signal's precision for signals that are independent conditional on the state), and on the acquisition of private information. A central question in this

players, quadratic payoffs and a Gaussian information structure, and their applications to macroeconomics and finance. Our paper covers a larger class of models, since we do not rely on specific functional forms and allow for a finite number of players.

context is whether the players' amount of information acquisition are complements, substitutes, or neither complements nor substitutes. With finitely many players, the question is meaningful only when acquisition is overt. With a continuum of players, the question is meaningful for both overt and covert acquisition, since the two are in this case equivalent. Li, McKelvey and Page (1987) study a Cournot market with finitely many firms and overt acquisition. The unknown common value state is the demand intercept and the information structure satisfies certain conditions. Actions are substitutes and they find that the precision levels of the private information acquired in the first stage are substitutes as well.[25] Vives (1988) obtains a similar result in the case of a continuum of players. Assuming as well a continuum of players, Hellwig and Veldkamp (2009) obtain a similar result in the context of a beauty contest game, where actions can be either substitutes or complements. They find that when actions are substitutes, acquisition levels are substitutes and when actions are complements, acquisition levels are complements: the strategic motive in actions is *inherited* by the acquisition game. All of these papers assume an unbounded continuum of actions (the real line), quadratic payoffs, and a Gaussian information structure.

In spite of the large number of contexts where the inheritance result is confirmed, it does not generalize to the larger class of all games with strategic complementarities or substitutabilities. In particular, the unbounded continuum of actions, the continuum of players and a Gaussian information structure seem to be crucial for the result. Even with unbounded actions, quadratic payoffs and a Gaussian information structure, but only two players (as in the case of a differentiated Bertrand game, which the author uses as an example), Jimenez-Martinez (2013) shows that it only holds for some parameters: when the complementarity in actions is strong, levels of acquired precision may be substitutes. And even with a continuum of players, quadratic payoffs, a Gaussian information structure, but binary actions, i.e. a global game, Szkup and Trevino (2014) present a model where the actions are complements but the acquired precision levels are not.

In contrast with this literature, we do not allow players to choose how much information they acquire. We hold the amount of information fixed. Instead, we let them choose whether the information they acquire is private or public. More

[25]Hwang (1993) exploits this result in a duopoly to derive various comparative statics results. Hwang (1995) studies a similar model but focuses on payoff comparisons between different market structures and different ways in which the levels of information precision of the firms is set. Bergemann, Shi and Välimäki (2009) obtain conditions under which information acquisition levels are substitutes or complements, in a VCG auction with interdependent valuations. Their setting differs from the common value models listed here in several ways.

generally, we allow players to choose the level of conditional dependence between their signals. We show that another type of inheritance result holds: complementarity in actions implies a preference for positive informational dependence, and substitutability in actions implies a preference for informational independence. But unlike the motive inheritance result on precision, our dependence inheritance results hold for all games where actions are strategic complements or substitutes and do not rely on specific functional forms, provided that the second stage strategies are monotonic (which is a possibility in some games, and an implication of Nash-Bayesian equilibrium in a subset of these games).

7.2 Public and private information

The issue of the role of public and private information is a central one in the entire literature on endogenous information structures. Morris and Shin (2002) for example, show that in a beauty contest game with a continuum of players, when the planner (the central bank) increases the precision of public information, it can be detrimental to welfare, because players rely less on their private information.

In the context of information acquisition, Hellwig and Veldkamp (2009) and Myatt and Wallace (2011) let players choose whether the information is private and potentially public. They provide models where public information is an equilibrium outcome of players choosing to observe the same potentially public signals. Thus in their models, unlike Morris and Shin (2002), public information is not provided by an external third party, but is the result of the market forces themselves. We follow up on this idea, and go one step further. While their model has private and potentially public signals (public signals are the potentially public ones that all players chose to observe), in a version of our model, all signals are potentially public: a private signal is one that only one player chose to observe, while a public signal is one that all players chose to observe.

Like Morris and Shin (2002), Hellwig and Veldkamp (2009) are interested in the marginal value of additional public information compared to an initial situation. But because no information is intrinsically public or private, what they really look at is the marginally value of additional potentially public information. They make the important observation that marginal value of acquiring *more* potentially public information is kinked at some profiles that they call symmetric. At a symmetric profile, defined as one where all players observe the same potentially public signals, if a player deviates and observes one more potentially public signal, he obtains additional information that in effect is private, since nobody else observes it. If he instead drops one of his potential signals, he decreases his own access to public information. This asymmetry and discontinuity causes multiple equilibria that differ in the level of public information. In Myatt and Wallace (2011), public information obtains when all players pay a substantial amount of attention to the same signal. An implication of this assumption is that players who hold public information are necessarily well informed players. In both of these two papers, the problem of the division of information between private and public (and everything in between) is intrinsically intertwined with the more widely studied issue of the amount of information that the players acquire. In Hellwig and Veldkamp (2009), it is because of the question they choose to ask, and in Myatt and Wallace (2011), it is in the way they define public information.

In contrast, we choose to completely disentangle the two issues. At the risk of making the model seem less realistic (because in practice, economic agents often face the choice of how much information to acquire), we hold the amount of information fixed by assuming that all signals an agent can choose to observe are equally informative of the unknown state: they all have the same joint marginal distribution with the state. By doing so, we isolate the issue of the partition of the information structure between public, private and neither private nor public information, from the issue of the amount of information. Doing so enables us to identify a robust force and to obtain general results that hold for a large class of games, not only the Gaussian-quadratic model with a continuum of actions and players. As we argued earlier, no such result holds when the issue of the amount of information is not excluded, even when only private information can be acquired.

Our assumption that players are restricted in the amount of information they acquire (formally, the joint marginal distribution between their own signal and the state is fixed, no matter what signal they choose) can be thought of as a form of rational inattention. Players are limited in how much information they can acquire, (Sims, 2003, 2005, 2006), and thus face a choice of what to observe.

7.3 Inefficiency of equilibrium under hidden information acquisition

A number of papers are dedicated to the analysis of inefficiencies in the collection of information and in the use of that information when the information structure is exogenously given. Angeletos and Pavan (2007) for instance, study a model with a continuum of players, quadratic payoff and a Gaussian information structure, where each player observe a private and a public signals. By comparing the equilibrium use of information to an efficiency benchmark (the best society could achieve keeping information decentralized), they show that information use can be inefficient when the incentives to coordinate actions and the social value for coordination are different. The welfare impact depends on the degree of strategic interaction and on its nature (complementarity or substitutability).

Angeletos and Pavan (2007)'s finding is recurrent in the literature. Morris and Shin (2002) among others also show that an increase in the amount of public information can impair welfare. This, however, does not hold necessarily if information is a choice for the players. Chahrour (2012) proposes a model of endogenous information acquisition where public information can still have a detrimental effect. In the model, a central authority chooses both how many signals to divulge and their precisions. He finds that the authority always chooses the highest possible precision and releases a positive but finite number of signals. An important result is that too many signals can cause the players to decrease the amount of information they acquire which in turn decreases welfare.

Colombo and Femminis (2008, 2011), on the other hand, are examples where endogenizing the information structure makes additional public information beneficial for welfare. By allowing the players to choose the precision of their private signals once the central authority has announced the precision of the public signal, they show that the precisions of private and public signals are strategic substitutes. Moreover, if the cost of public information is lower than the cost of private information, then increasing the precision of the public information increases welfare. While Colombo and Femminis (2008, 2011) investigate the welfare implications of public information provision on incentives to acquire private information, Llosa Gonzalo and Venkateswaran (2012), by considering models different from the beauty-contest type, study how different links and externalities among players affect the acquisition process of private information.

Existing work allows the players to choose the level of information precision. In this paper, our approach was different. Indeed, we take the analysis in an other direction by keeping the amount of information fixed and focusing instead on information dependence. We show that covert information acquisition sometimes leads to inefficiencies when there are payoff externalities that are not reflected in the players' equilibrium choice. Interestingly, we show that these inefficiencies can sometimes be eliminated when information acquisition is overt.

Appendix

A Multivariate first-order stochastic dominance and dependence orderings

In this section, we provide equivalent definitions of our dependence ordering, based on multivariate first order stochastic dominance, which we define next. Note that the following definition specializes to the usual first-order stochastic dominance in the univariate case.

Definition 4 (Multivariate first-order stochastic dominance)**.**

*i. Let f and g be two multivariate probability distribution functions (pdf) on the support $\mathcal{X}^k$. We say that g **first-order stochastically dominates (FOSD)** f if or all increasing L, we have*

$$\sum_{x \in L} f(x) \leq \sum_{x \in L} g(x).$$

*Moreover, we say that g **strictly FOSD** f if g FOSD f, but f does not FOSD g.*

*ii. Let $X = (X_1, ..., X_k)$ and $X' = (X'_1, ..., X'_k)$ be two random vectors on the support $\mathcal{X}^k$. We say that X **FOSD** X' if the pdf of X FOSD the pdf of X'. Moreover we say that X **strictly FOSD** X' if X FOSDX', but X' does not FOSD X.*

The following result due to Lehman (1955), Levhari, Paroush and Peleg (1975) and Østerdal (2010) provides four alternative and equivalent definitions of multivariate stochastic dominance.

Theorem 7. *Let X and Y be random vectors with respective pdfs f and g on the support $\mathcal{X}^k$. The following conditions are equivalent:*

i. Y FOSD X.

ii. For all decreasing L, we have

$$\sum_{x \in L} f(x) \geq \sum_{x \in L} g(x).$$

iii. *For all nondecreasing mapping* $W : \mathcal{X}^k \to \mathbb{R}$, $\mathbb{E}(W(Y)) \geq \mathbb{E}(W(X))$.

iv. *There exist two random vectors* X' *and* Y' *with respective pdfs* f *and* g *such that* Y' *FOSD* X'.

v. *There exist a finite list of vector pairs* $(x_t, y_t)_{t=1,...,T}$ *with* $x_t \leq y_t$ *and a list of reals* $(\Delta_t)_{t=1,...,m}$, *with* $\Delta_t \in [0,1]$ *such that*

$$g(x) - f(x) = \sum_t \Delta_t \left(1_{\{y_t\}}(x) - 1_{\{x_t\}}(x)\right).$$

This enables us to provide the following definition equivalent to Definition (1) which is in turn used in Definition (2), using the notion of multivariate first-order stochastic dominance.

Definition 5 (weakly greater conditional dependence)**.** *Let* $i \in I$ *and let* X_{-i} *be a profile of signals for all players different from* i*. For all* X_i *and* X_i' *in* $\mathbb{X}_i$*, we say that* X_i' ***depends at least as much as*** X_i ***on*** X_{-i} ***conditionally on*** Θ*, if for all* (θ, x) *the conditional pdf* $\mathbb{P}(X_{-i} \mid X_i' \geq x,\ \Theta = \theta)$ *FOSD the conditional pdf* $\mathbb{P}(X_{-i} \mid X_i \geq x,\ \Theta = \theta)$.

B Most public signal, most private signal and most d-dependent signal

In this section, we show how to construct two examples of signal structures $(\mathbb{X}_1, ..., \mathbb{X}_N)$ that admit a most public signal, a most private signal and most d-dependent signal.

Example 1. *Let* $I = \{1, 2\}$. *Let* $\left(\Theta, X_1^*, X_2^*, X_P^*, Y_1^I, Y_1^{II}, Y_2^I, Y_2^{II}\right)$ *be a random vector distributed on* $\{-1,1\}^4 \times \{0,1\}^4$ *so that, the three vectors* (Θ, X_1^*), (Θ, X_2^*) *and* (Θ, X_P^*) *are distributed as* (Θ, X_I) *in the binary information structure presented in Section 3. Moreover, let the random vector* $\left(X_1^*, X_2^*, X_P^*, Y_1^I, Y_1^{II}, Y_2^I, Y_2^{II}\right)$ *be independent conditionally on* $\Theta = \theta$*, for all* $\theta \in \{-1,1\}$ *and let the vector* $\left(\Theta, Y_1^I, Y_1^{II}, Y_2^I, Y_2^{II}\right)$ *be independent. For each* $i \in \{1,2\}$, *we assume that* $\mathbb{P}\left(Y_i^I = 1\right) < \mathbb{P}\left(Y_i^{II} = 1\right)$ *holds. Last. for each* i*, let the set* $\mathbb{X}_i$ *consist of two signals*

$$\begin{cases} X_i^I &= X_P^* Y_i^I + X_i^*\left(1 - Y_i^I\right) \\ X_i^{II} &= X_P^* Y_i^{II} + X_i^*\left(1 - Y_i^{II}\right). \end{cases}$$

In the signal structure constructed in Example 1, it is easily verified that for each i, the signals $\left(X_i^I, X_i^{II}\right)$ are such that $X_i^I \prec X_i^{II}$, regardless of what signals the other players choose. The signal X_i^I is player i's most private signal and the signal X_i^{II} is player i's most public signal. It is also clear that one can generalize this construction to more than two players, where each player i has a set of signals $\mathbb{X}_i = \{X_i^1, ..., X_i^{m_i}\}$, so that $X_i^1 \prec_i ... \prec_i X_i^{m_i}$. For each i, the signal is player i's most private signal and the signal $X_i^{m_i}$ is player i's most public signal. When generalizing this construction to more than two players and more than two signals per player, each player i still has a most private signal, namely X_i^1, and a most public signal, namely $X_i^{m_i}$, but for an arbitrary dependence vector d^i, it is not the case that player i has a most d^i-dependent signal.

We now provide an example of a signal structure $(\mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3)$ for three players, such that each of the three players i has a most d^i-dependent signal, for each dependence vector d^i.

Example 2. *Let $I = \{1, 2, 3\}$. Let*

$$(\Theta, X_{12}^*, X_{23}^*, X_{13}^*, X_{11}^*, X_{22}^*, X_{33}^*, Y_1, Y_2, Y_3)$$

be random vector such that Θ and the X_{ij}^* have support $\{-1, 1\}$ and the random variable Y_i has full support $\{\{i\}, \{i, j\}, \{i, k\}, I\}$. For all i, j, let the vector $\left(\Theta, X_{ij}^*\right)$ be distributed as (Θ, X_I) in the binary signal structure presented in Section 3. Moreover, let the random vector

$$(X_{12}^*, X_{23}^*, X_{13}^*, X_{11}^*, X_{22}^*, X_{33}^*, Y_1, , Y_2, Y_3)$$

be independent conditionally on $\Theta = \theta$, for all $\theta \in \{-1, 1\}$ and let the vector (Θ, Y_1, Y_2, Y_3) be independent. Last. for each i, let the set $\mathbb{X}_i$ consist of four signals defined as follows:

$$\begin{cases} X_i^{\{i,j,k\}} & = & \mathbf{1}_{\{Y_i = ij\}} X_{ij}^* + \mathbf{1}_{\{Y_i = ik\}} X_{ik}^* + \mathbf{1}_{\{Y_i = i\}} X_{ii}^* \\ X_i^{ij} & = & \mathbf{1}_{\{Y_i = ij\}} X_{ij}^* + \mathbf{1}_{\{Y_i = ik\}} X_{ii}^* + \mathbf{1}_{\{Y_i = i\}} X_{ii}^*. \\ X_i^{\{i,k\}} & = & \mathbf{1}_{\{Y_i = ik\}} X_{ik}^* + \mathbf{1}_{\{Y_i = ij\}} X_{ii}^* + \mathbf{1}_{\{Y_i = i\}} X_{ii}^* \\ X_i^{\{i\}} & = & X_{ii}^* \end{cases}$$

In the signal structure constructed in Example 2, for each player i and each dependence vector i, the signal X_i^S, with $S = \left\{j : d_j^i = 1\right\}$ is player i's most d^i-dependent signal. One can generalize this construction to more than two players and to more signals.

C Proofs

Abusing the terminology defined in section (4.1), for all $k \geq 0$, and all $d \in \{-1,1\}^k$, we say that a mapping Φ from $\mathcal{X}^k$ to $\mathbb{R}$ is (strictly) **d-monotonic** if it is (strictly) increasing in each x_j such that $d_j = 1$ and (strictly) decreasing in each x_j such that $d_j = -1$.

The following Lemma is useful in the Proof of Theorem 4.

Lemma 1. *Let X_{-i} be a profile of signals, and let X_i and X_i' be signals. Let $d \in \{-1,1\}^I$ be a dependence vector, with $d_i = 1$ and let Φ be a mapping from $\mathcal{X}^{I \setminus \{i\}}$ to $\mathbb{R}$.*

i. Suppose that X_i' conditionally d-depends as least as much on X_{-i} as X_i does and that Φ is d_{-i}-monotonic. Then for all $z \in \mathcal{X}$,

$$\mathbb{E}_{X_i,X_{-i}}\left[\Phi\left(X_{-i}\right) \mid X_i > z\right] \leq \mathbb{E}_{X_i',X_{-i}}\left[\Phi\left(X_{-i}\right) \mid X_i' > z\right]. \tag{14}$$

ii. Suppose that X_i' conditionally d-depends strictly more on X_{-i} than X_i does and that Φ is strictly d_{-i}-monotonic. Then for all $z \in \mathcal{X}$ such that $z \neq \max \mathcal{X}$,

$$\mathbb{E}_{X_i,X_{-i}}\left[\Phi\left(X_{-i}\right) \mid X_i > z\right] < \mathbb{E}_{X_i',X_{-i}}\left[\Phi\left(X_{-i}\right) \mid X_i' > z\right] \tag{15}$$

Proof of Lemma 1: (i) By definition, since X_i' d-depends as least as much on X_{-i} as X_i does, the pdf of $\left(d_j X_j\right)_{j \neq i}$, conditional on $X_i' > z$, first-order stochastically dominates the pdf of $\left(d_j X_j\right)_{j \neq i}$, conditional on $X_i > z$. Then consider the function $\Gamma\left(x_{-i}\right) = \Phi\left(\left(d_j X_j\right)_{j \neq i}\right)$. Because Φ is d_{-i}-monotonic, the function Γ is increasing. Together, and by the equivalence between (i) and (iii) in Theorem 7, these last two claims imply that

$$\mathbb{E}_{X_i,X_{-i}}\left[\Gamma\left(\left(d_j X_j\right)_{j \neq i}\right) \mid X_i > z\right] \leq \mathbb{E}_{X_i',X_{-i}}\left[\Gamma\left(\left(d_j X_j\right)_{j \neq i}\right) \mid X_i' > z\right],$$

which is equivalent to inequality (14).

(ii) Under assumptions of point (ii), the inequalities hold strictly. ■

Proof of Theorem 1:

Let $A_{i,X} : \alpha_{-i} \longrightarrow \alpha_i$ be the action best-response of player i under information structure X. Then, the following lemma is useful for the proof of Theorem 1.

Lemma 2. *Let (α_1, α_2) be a Nash Bayesian equilibrium of the game Γ_X with an exogenous information structure X, such that for all $i = 1,2$ the following holds*

1) $X_1 = X_2$ *if and only if* $\alpha_1(1)\alpha_2(1)b_{ia} > 0$
$X_1 \neq X_2$ *if and only if* $\alpha_1(1)\alpha_2(1)b_{ia} < 0$

2) $A_{i,(X_i,X_{-i})}(\alpha_{-i})$ *has a sign that does not depend on* X_i.

Then, $(X, (\alpha_1, \alpha_2))$ *is a Nash Bayesian equilibrium of the game* Γ *with an endogenous information structure.*

Proof of Lemma 2: First, for each $i = 1, 2$, because (α_1, α_2) is a Nash Bayesian equilibrium of Γ_X, player i does not have a profitable deviation of the form (X_i, α_i') with $\alpha_i' \neq \alpha_i$.

Suppose by contradiction that $(X, (\alpha_1, \alpha_2))$ is not a Nash Bayesian equilibrium of the game Γ with an endogenous information structure. Then this means that a player i has a profitable deviation (X_i', α_i') with $X_i' \neq X_i$. Then, $\left(X_i', A_{i,(X_i',X_{-i})}(\alpha_{-i})\right)$ is an even better deviation, thus it is also profitable. But because $A_{i,(X_i',X_{-i})}(\alpha_{-i})$ is of the same sign as α_i, and by 1), the deviation $\left(X_i, A_{i,(X_i',X_{-i})}(\alpha_{-i})\right)$ is even better and thus must be profitable. But then, this contradicts the assumption that (α_1, α_2) is a Nash Bayesian equilibrium of the game Γ_X. ■

We now proceed to prove Theorem 1. Consider the game given in Section 3 and assume that $b_{1\theta} = 4.8, b_{2\theta} = 5$, $b_{1a} = -0.8$, $b_{2a} = -1.2$ and $\varepsilon = 0.26$.

Fix the information structure X, then by taking the first-order condition to (2) with respect to $\alpha_i(1)$ and $\alpha_i(-1)$ for $i = 1, 2$, we can observe that $-\alpha_i(1) = \alpha_i(-1)$ and that the best response functions are

$$\begin{cases} \alpha_1(1) = \frac{288}{125} - \frac{4}{5}\alpha_2(1) \text{ and } \alpha_2(1) = \frac{12}{5} - \frac{6}{5}\alpha_1(1) & \text{if } X_1 = X_2 \\ \alpha_1(1) = \frac{288}{125} - \frac{576}{3125}\alpha_2(1) \text{ and } \alpha_2(1) = \frac{12}{5} - \frac{864}{3125}\alpha_1(1) & \text{if } X_1 \neq X_2 \end{cases}. \tag{16}$$

Given the information structure is $X_1 \neq X_2$, the unique equilibrium in action strategies is $((1.9616, -1.9616), (1.85766, -1.85766))$. Whereas, when the information structure is $X_1 = X_2$, the unique equilibrium in action strategies is $\left(\left(\frac{48}{5}, -\frac{48}{5}\right), \left(-\frac{228}{25}, \frac{228}{25}\right)\right)$.

Then, by checking that conditions 1) and 2) of Lemma 2 hold using (16), we can show that the profile $(X, (1.9616, -1.9616), (1.85766, -1.85766))$ with $X_1 \neq X_2$ and the profile $\left(X, \left(\frac{48}{5}, -\frac{48}{5}\right), \left(-\frac{228}{25}, \frac{228}{25}\right)\right)$ with $X_1 = X_2$ are both Nash Bayesian equilibria of the game Γ.

Figure 1 depicts the players' action best-responses and shows the equilibrium when $X_1 = X_2$ (in blue) and when $X_1 \neq X_2$ (in red). Figure 2 zooms in on the equilibrium when $X_1 \neq X_2$ and shows that Condition 2) of Lemma 2 holds. Then, Figure 3 zooms in on the equilibrium when $X_1 = X_2$ and shows that Condition 2) of Lemma 2 holds also. ■

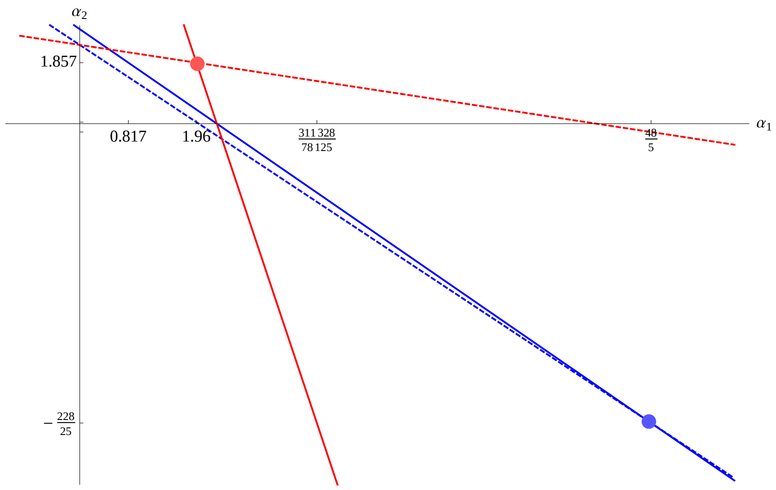

Figure 1: Players 1's action best-response (full line), Players 2's action best-response (dashed line) given information structure X

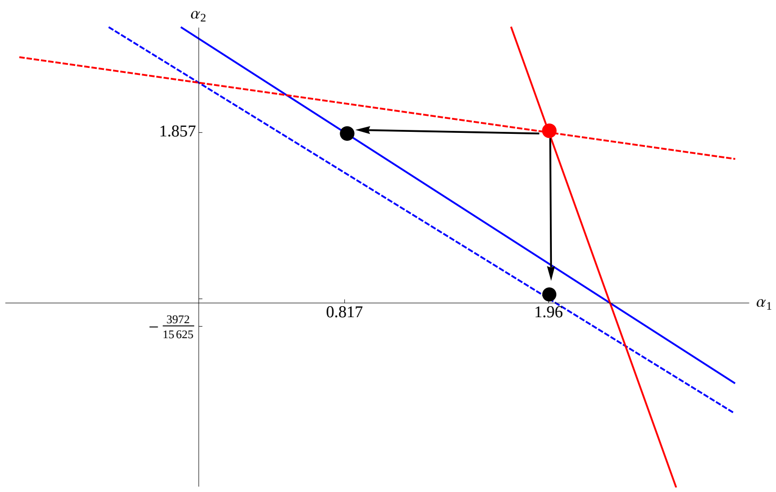

Figure 2: Deviation from $X_1 \neq X_2$: Players 1's action best-response (full line), Players 2's action best-response (dashed line)

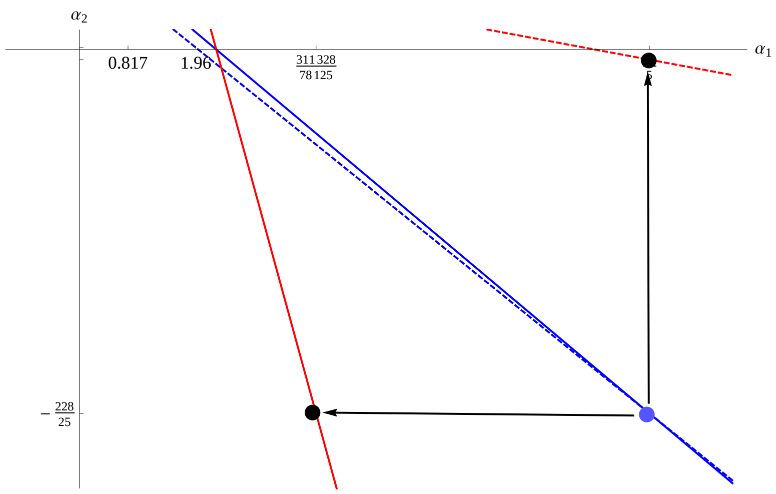

Figure 3: Deviation from $X_1 = X_2$: Players 1's action best-response (full line), Players 2's action best-response (dashed line)

Proof of Theorem 2: It suffices to give an example where the planner chooses an information structure that differs from what the players choose in the decentralized game. Suppose that player i's payoff is given by Equation (5) and that $b_a = -3, b_\theta = 1, b_{\theta a} = -2, b_{aa} = 0.75$ and $\varepsilon = 0.25$. In this case, an equilibrium exists and according to Proposition 2, the players will choose to obtain private information. The social planner, however, will prefer to impose public information.■

Proof of Theorem 4: *Proof of* (i) . Let $i \in I$ and let c^i be a complementarity vector for i. Suppose that u_i has c^i-increasing differences in own and others' actions. Let $m \in \{-1, 1\}^I$ be a monotonicity profile and let X_{-i} be a profile of signals such that $X_j \in \mathbb{X}_j$ for all j. Suppose that X_i and X_i' are two signals in $\mathbb{X}_i$ such that X_i' d^i-depends as least as much on X_{-i} as X_i does, where d^i is the dependence vector such that the relation (6) holds for all j. Let α be profile of pure m-monotonic action strategies. We will show that

$$\mathbb{E}_{\Theta,X_i,X_{-i}}\left(u_i\left(\alpha\left(X_i,X_{-i}\right),\Theta\right)\right) \leq \mathbb{E}_{\Theta,X_i',X_{-i}}\left(u_i\left(\alpha\left(X_i',X_{-i}\right),\Theta\right)\right) \tag{17}$$

holds.

Let $z^1 < ... < z^m$ be the elements of $\mathcal{X}$. Also, for each $k \in \{1,..,m-1\}$, and each $\theta \in T$, let $\Phi_{k,\theta}$ be the function from $\mathcal{X}^{I\backslash\{i\}}$ to $\mathbb{R}$ such that

$$\Phi_{k,\theta}\left(x_{-i}\right) = u_i\left(\alpha\left(z^{k+1},x_{-i}\right),\theta\right) - u_i\left(\alpha\left(z^k,x_{-i}\right),\theta\right).$$

The proof is in four steps.

Step 1: The function $\Phi_{k,\theta}$ is d_{-i}^i-monotonic.

Proof of Step 1: Since for each θ, the function u_i has c^i-monotonic differences in x, and the function α is m-monotonic, it follows that $\Phi_{k,\theta}$ is d^i_{-i}-monotonic. $\square$

Step 2: For all $k \in \{1, .., m-1\}$, and all $\theta \in T$, we have

$$\mathbb{E}_{X_i, X_{-i}} \left[\Phi_{k,\theta} \left(X_{-i} \right) \mid X_i > z^k, \; \Theta = \theta \right] \le \mathbb{E}_{X_i', X_{-i}} \left[\Phi_{k,\theta} \left(X_{-i} \right) \mid X_i' > z^k, \; \Theta = \theta \right]. \tag{18}$$

Proof of Step 2: By Step 1, the function $\Phi_{k,\theta}$ is d^i_{-i}-monotonic. By assumption, X_i' d-depends more on X_{-i} than X_i does, thus by Lemma 1, inequality (18) holds. $\square$

Step 3: For all $k \in \{1, .., m-1\}$, and all $\theta \in T$, we have

$$u_i \left(\alpha \left(X_i, X_{-i} \right), \theta \right) = \sum_{k=0}^{m-1} \mathbb{1}_{\{X_i > k\}} \Phi_{k,\theta} \left(X_{-i} \right) \tag{19}$$

and

$$u_i \left(\alpha \left(X_i', X_{-i} \right), \theta \right) = \sum_{k=0}^{m-1} \mathbb{1}_{\{X_i' > k\}} \Phi_{k,\theta} \left(X_{-i} \right). \tag{20}$$

Proof of Step 3: These identities are easily verified. We leave them to the reader. $\square$

Step 4:

$$\mathbb{E}_{\Theta, X_i, X_{-i}} \left[u_i \left(\alpha \left(X_i, X_{-i} \right), \Theta \right) \mid \Theta = \theta \right] \le \mathbb{E}_{\Theta, X_i', X_{-i}} \left[u_i \left(\alpha \left(X_i', X_{-i} \right), \Theta \right) \mid \Theta = \theta \right]. \tag{21}$$

Proof of Step 4: We know that

$$
\begin{aligned}
& \mathbb{E}_{X_i,X_{-i}}\left[u_i\left(\alpha\left(X_i,X_{-i}\right),\Theta\right)\mid\Theta=\theta\right] \\
= \; & \mathbb{E}_{X_i,X_{-i}}\left[\sum_{k=0}^{m-1}\mathbb{1}_{\{X_i>k\}}\left(\Phi_{k,\theta}\left(X_{-i}\right)\right)\mid\Theta=\theta\right] \\
= \; & \sum_{k=0}^{m-1}\mathbb{E}_{X_i,X_{-i}}\left[\mathbb{1}_{\{X_i>k\}}\Phi_{k,\theta}\left(X_{-i}\right)\mid\Theta=\theta\right] \\
= \; & \sum_{k=0}^{m-1}\mathbb{E}_{X_{-i}}\left[\Phi_{k,\theta}\left(X_{-i}\right)\mid X_i>k\text{ and }\Theta=\theta\right]\mathbb{P}\left(X_i>k\mid\Theta=\theta\right) \\
\leq \; & \sum_{k=0}^{m-1}\mathbb{E}_{X_{-i}}\left[\Phi_{k,\theta}\left(X_{-i}\right)\mid X_i'>k\text{ and }\Theta=\theta\right]\mathbb{P}\left(X_i'>k\mid\Theta=\theta\right) \qquad (22)\\
= \; & \sum_{k=0}^{m-1}\mathbb{E}_{X_i',X_{-i}}\left[\mathbb{1}_{\{X_i'>k\}}\left(\Phi_{k,\theta}\left(X_{-i}\right)\right)\mid\Theta=\theta\right] \\
= \; & \mathbb{E}_{X_i',X_{-i}}\left[\sum_{k=0}^{m-1}\mathbb{1}_{\{X_i'>k\}}\left(\Phi_{k,\theta}\left(X_{-i}\right)\right)\mid\Theta=\theta\right] \\
= \; & \mathbb{E}_{X_i',X_{-i}}\left[u_i\left(\alpha\left(X_i',X_{-i}\right),\Theta\right)\mid\Theta=\theta\right]
\end{aligned}
$$

where the first and sixth equalities follow from Step 3, and the inequality follows from Step 2 and from the assumption that (X_i,Θ) and (X_i',Θ) have the same joint marginal distribution. □
Final Step: Since for all θ,

$$\mathbb{E}_{X_i,X_{-i}}\left[u_i\left(\alpha\left(X_i,X_{-i}\right),\Theta\right)\mid\Theta=\theta\right]\leq\mathbb{E}_{X_i',X_{-i}}\left[u_i\left(\alpha\left(X_i',X_{-i}\right),\Theta\right)\mid\Theta=\theta\right], \quad (23)$$

taking expectations of both sides on Θ, we obtain

$$\mathbb{E}_{\Theta,X_i,X_{-i}}\left[u_i\left(\alpha\left(X_i,X_{-i}\right),\Theta\right)\right]\leq\mathbb{E}_{\Theta,X_i',X_{-i}}\left[u_i\left(\alpha\left(X_i',X_{-i}\right),\Theta\right)\right], \quad (24)$$

the desired conclusion. □■
Proof of (ii). Under the additional assumptions we will prove that the inequality (22) is strict at least for some realization θ. First, Step 1 can be modified as follows. since for each θ, the function u_i has <u>strict</u> c^i-complementarities in actions, and the function α is <u>strictly</u> m-monotonic, the function $\Phi_{k,\theta}$ is <u>strictly</u> d_{-i}^i-monotonic for all θ and k. Second, because X_i' d-depends <u>more</u> than X_i on X_{-i}, there exists a realization θ° and some integer k such that the pdf $\mathbb{P}\left(\left(d_j^iX_j\right)_{j\neq i}\mid d_i^iX_i'>k\text{ and }\Theta=\theta\right)$ <u>strictly</u> stochastically dominates the pdf $\mathbb{P}\left(\left(d_j^iX_j\right)_{j\neq i}\mid d_i^iX_i'>k\text{ and }\Theta=\theta\right)$. For

this realization θ° and this integer k, the inequality (18) holds strictly. As a result, the inequality (23) holds strictly as well. Finally, since all realizations θ of Θ have positive probability, the inequality (24) holds strictly as well. ■

Proof of Corollary 4: Let $(X_1, X_2, \alpha_1, \alpha_2)$ be a pure Nash-Bayesian equilibrium of the game such that α is strictly isotonic (if the payoff complementarities in actions are strictly positive) or antitonic (if they are strictly negative) and suppose by contradiction that $X_1 \neq X_2$ with positive probability. Then by Theorem 4, the deviation (X_1', α_1) with $X_1' = X_2$ is strictly profitable for player 1, since X_1' depends more on X_2 than X_1. Therefore X must be public information. ■

Proof of Theorem 5. By changing variables $a_i' = m_i a_i$, the game is equivalent to one where $m = (1, ..., 1)$ and all u_i have increasing differences in actions and in θ. In the continuation we will thus restrict attention to the case where $m = (1, ..., 1)$ and $c^i = (1, ..., 1)$.

For any signal profile X, let Γ_X denote the game with exogenous information structure X, and Γ the game with endogenous information. The main result in Van Zandt and Vives (2007) implies that there exists an increasing action strategy profile α such that in the game Γ_X, the profile α is a Nash-Bayesian equilibrium of Γ_X. Let α be such a profile. We will now show that the profile (X, α) is a Nash-Bayesian equilibrium of the game with endogenous information Γ.

Suppose by contradiction that (X_i', α_i') is a profitable deviation for player i in this game. Let α_i'' be a player i's best response to α_{-i} in the game $\Gamma_{X_i', X_{-i}}$. By Proposition 11 in Van Zandt and Vives (2007), the action strategy α_i'' is increasing. Since (X_i', α_i') is a profitable deviation for player i from profile (X, α) in Γ, it follows that (X_i', α_i'') is also a profitable deviation for player i from profile (X, α) in Γ. But, because X_i depends more on X_{-i} than X_i', the same argument used in Theorem 4 implies that (X_i, α_i'') is an at least as good profitable deviation for player i from profile (X, α) in Γ. But this implies that α_i'' is a profitable deviation for player i from profile α in Γ_X, which contradicts the statement that α is Nash-Bayesian equilibrium of Γ_X. Therefore no player has any profitable deviation from (X, α) in Γ, the desired conclusion. ■

Proof of Theorem 6. Suppose by contradiction that (X_i', α_i') is a profitable deviation from (X, α) for player i in this game Γ. Let α_i'' be a pure m_i-monotonic action strategy that is a best response for player i's to α_{-i} in the game $\Gamma_{X_i', X_{-i}}$. Such an action strategy exists by assumption (iv) and because α is m-monotonic by assumption (iii). Since (X_i', α_i') is a profitable deviation for player i from profile

(X, α) in Γ, it follows that (X_i', α_i'') is an even better deviation in Γ, and is therefore also a profitable deviation for player i from (X, α) in Γ. But, because X_i d^i-depends more on X_i than X_i', and because $(\alpha_i'', \alpha_{-i})$ is m-monotonic, the same argument used in Theorem 4 implies that (X_i, α_i'') is at least as good as (X_i', α_i''), and therefore at least as good as (X_i', α_i'). Therefore (X_i, α_i'') is a profitable deviation for player i from profile (X, α) in Γ. But this implies that α_i'' is a profitable deviation for player i from profile α in Γ_X, which contradicts the statement that α is Nash-Bayesian equilibrium of Γ_X. Therefore no player has any profitable deviation from (X, α) in Γ, the desired conclusion.■

D PQD and SPM dependence

For any two random vectors $X = (X_1, ..., X_N)$ and $Y = (Y_1, ..., Y_N)$ with identical marginals, and respective cdfs F and G, we define the following dependence orderings.

We say that X is **at least as Positive Quadrant Dependent (PQD) as** Y if for all $x \in \mathbb{R}^N$, we have

$$F(x) \leq G(x).$$

A function $u : \mathbb{R}^N \to \mathbb{R}$ is said to be supermodular if for any $x, y \in \mathbb{R}^N$ it satisfies

$$u(x) + u(y) \leq u(x \wedge y) + u(x \vee y),$$

where the operators $\wedge$ and $\vee$ denote coordinate-wise minimum and maximum respectively.

We say that X is **at least as Supermodular Dependent (SPM) as** Y if

$$\mathbb{E}_X(u(X)) \geq \mathbb{E}_Y(u(Y))$$

for all supermodular functions $u : \mathbb{R}^N \to \mathbb{R}$.

E Mixed strategies

The results obtained in Theorem 4 generalize to mixed strategies, but they imply very few restrictions for Nash-Bayesian equilibria where players play non degenerate mixed strategies. For example, consider a game with two players and two signals, with a fixed information structure such that both players observe each of the two signals with equal probabilities (independent draws). Suppose that this game admits a pure Nash-Bayesian equilibrium in action strategies (they could be pure or not).

Then it is easy to see that the game with an endogenous information structure admits a Nash-Bayesian equilibrium, where both players randomize with equal probabilities between the two signals. To see this, suppose that player 2 uses this strategy. From the point of view of the player 1, the two signals are then equally informative in a Blackwell sense on the vector (θ, α_2), which is all he cares about. It is then a best response for him to play this half half mixed strategy and the same argument holds for player 2. This phenomenon is more general. A symmetric fully mixed equilibrium exists, for any number of players, if and only if the Bayesian game where this structure is fixed admits a Nash-Bayesian equilibrium. What is important for the result is that there are only two signals in $\mathbb{X}$. A more general result can be obtained for a larger number of signals in $\mathbb{X}$, provided that some symmetry condition, which automatically holds in the case of two signals, is imposed on the signal structure.

Theorem 8. *Let $N \geq 2$ and $\mathbb{X}_i = \{X_I, X_{II}\}$ for all $i \in I$. Consider the game with an exogenous information structure, where each player observes X_I or X_{II} with probability $1/2$ (independent draws across players). Suppose that this game admits a pure Nash-Bayesian equilibrium in action strategies (pure or not). Then this action profile and this information structure form a Nash-Bayesian equilibrium of the game Γ where the information structure is endogenous.*

References

Amador, M. 2010. Learning from Prices: Public Communication and Welfare. *Journal of Political Economy*, **118**, 866–907.

Amador, M. 2012. Learning from Private and Public Observation of Others' Actions. *Journal of Economic Theory*, **147**, 910–940.

Amir, R., & Lazzati, N. 2014. *Endogenous Information Acquisition in Bayesian Games with Strategic Complementarities.* Working Paper.

Angeletos, G.M., & Pavan, A. 2007. Efficient Use of Information and Social Value of Information. *Econometrica*, **75**(4), 1103–1142.

Athey, S. 2001. Single Crossing Properties and the Existence of Pure Strategy Equilibria in Games of Incomplete Information. *Econometrica*, **69**(4), 861–889.

Athey, S., & Levin, J. 2001. *The Value of Information in Monotone Decision Problems*. MIT Working Paper 98-24.

Bergemann, D., & Morris, S. 2013. Robust Predictions in Games with Incomplete Information. *Econometrica*, **81**(4), 1251–1308.

Bergemann, D., & Pesendorfer, W. 2007. Information Structure in Optimal Auctions. *Journal of Economic Theory*, **137**, 580–609.

Bergemann, D., X. Shi, & Välimäki, J. 2009. Information Acquisition in Interdependent Value Auctions. *Journal of the European Economic Association*, **7**(1), 61–89.

Calvo-Armengol, A., & de Marti Beltran, J. 2007. Communication Networks: Knowledge and Decisions. *The American Economic Review*, **97**(2), 86–91.

Calvo-Armengol, A., & de Marti Beltran, J. 2009. Information Gathering In Organizations: Equilibrium, Welfare, And Optimal Network Structure. *Journal of the European Economic Association*, **7**(1), 116–161.

Calvo-Armengol, A., de Marti Beltran, J., & Prat, A. 2009. *Endogenous Communication in Complex Organizations*. Working Paper.

Chahrour, Ryan. 2014. Public Communication and Information Acquisition. *American Economic Journal: Macroeconomics*, **6**(3), 73–101.

Chhetry, D., Sampson, A.R., & Kimeldorf, G. 1989. Concepts of Setwise Dependence. *Probability in the Engineering and Informational Sciences*, **3**, 367–380.

Christofides, T.C., & Vaggelatou, E. 2004. A Connection between Supermodular Ordering and Positive/Negative Association. *Journal of Multivariate Analysis*, **88**(1), 138–151.

Colombo, L., & Femminis, G. 2008. The Social Value of Public Information with Costly Information Acquisition. *Economic Letters*, **100**, 196–199.

Colombo, L., Femminis, G., & Pavan, A. 2014. Information Acquisition and Welfare. *Forthcoming in Review of Economic Studies.*

Cornand, C., & Heinemann, F. 2008. Optimal Degree of Public Information Dissemination. *The Economic Journal*, **118**, 718–742.

Dewan, T., & Myatt, D.P. 2008. The Qualities of Leadership: Direction, Communication and Obfuscation. *American Political Science Review*, **102**(3), 351–368.

Goldstein, I., Ozdenoren, E., & Yuan, K. 2011. Learning and Complementarities in Speculative Attacks. *The Review of Economic Studies*, **78**(1), 263–292.

Hauk, E., & Hurkens, S. 2001. Secret Information Acquisition in Cournot Markets. *Economic Theory*, **18**(3), 661–681.

Hellwig, C., & Veldkamp, L. 2009. Knowing what Others Know: Coordination Motives in Information Acquisition. *Review of Economic Studies*, **76**, 223–251.

Hellwig, C., S. Kohls, & Veldkamp, L. 2012. Information Choice Technologies. *American Economic Review: Papers and Proceedings*, **102**(3), 35–40.

Hu, T., A. Müeller, & Scarsini, M. 2004. Some Counterexamples in Positive Dependence. *Journal of Statistical Planning and Inference*, **1**(153-158).

Hwang, H.S. 1993. Optimal Information Acquisition for Heterogenous Duopoly Firms. *Journal of Economic Theory*, **59**(2), 385–402.

Hwang, H.S. 1995. Information Acquisition and Relative Efficiency of Competitive, Oligopoly and Monopoly Markets. *International Economic Review*, **36**(2), 325–340.

James, J.G., & Lawler, P. 2011. *Optimal Policy Intervention and the Social Value of Public Information.*

Jewitt, I. 2007. *Information Order in Decision and Agency Problems.* Unpublished manuscript.

Jiménez-Martinez, A. 2011 (July). *Strategic Interactions in Information Decisions with a Finite Set of PLayers.* Working Paper.

Jiménez-Martinez, A. 2013. Information Acquisition Interactions in Two-Player Quadratic Games. *International Journal of Game Theory*, August.

Lehmann, E.L. 1955. Ordered Families of Distributions. *The Annals of Mathematical Statistics*, **26**, 399–419.

Levhari, D., Paroush, J., & Peleg, B. 1975. Efficiency Analysis for Multivariate Distributions. *Review of Economic Studies*, **42**, 87–91.

Li, Lode, McKelvey, Richard D, & Page, Talbot. 1987. Optimal research for cournot oligopolists. *Journal of Economic Theory*, **42**(1), 140 – 166.

Llosa Gonzalo, L., & Venkateswaran, V. 2012. *Efficiency with Endogenous Information Choice*. Working Paper.

Loeper, Antoine. 2011. Coordination in Heterogeneous Federal Systems. *Journal of Public Economics*, **95**, 900–912.

Meyer, M., & Strulovici, B. 2012. Increasing Interdependence of Multivariate Distributions. *Journal of Economic Theory*, **147**(4), 1460–1489.

Meyer, M., & Strulovici, B. 2013. *The Supermodular Stochastic Ordering*. Unpublished manuscript.

Morris, S., & Shin, H.S. 1998. Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks. *The American Economic Review*, **88**(3), pp. 587–597.

Morris, S., & Shin, H.S. 2002. Social Value of Public Information. *American Economic Review*, **92**(5), 1521–1534.

Morris, S., & Shin, H.S. 2007. Optimal Communication. *Journal of the European Economic Association*, **5**(2-3), 594–602.

Morris, S., Shin, H.S., & Tong, H. 2006. Social Value of Public Information: Morris and Shin (2002) Is Actuallt Pro-Transparency, not Con.: Reply. *American Economic Review*, **96**(1), 453–455.

Myatt, D.P., & Wallace, C. 2011. Endogenous Information Acquisition in Coordination Games. *Review of Economic Studies*, **78**(4), 1–38.

Østerdal, LP. 2010. The Mass Transfer Approach to Multivariate Discrete First Order Stochastic Dominance: Direct Proof and Implications. *Journal of Mathematical Economics*, **46**, 1222–1228.

Radner, R. 1962. Team Decision Problems. *The Annals of Mathematical Statistics*, **33**(3), 857–881.

Reny, P. 2011. On the Existence of Monotone Pure Strategy Equilibria in Bayesian Games. *Econometrica*, **79**(2), 499–553.

Russo, G., & Senatore, L. 2012. A Note on Contribution Game with Loss Function. *Economic Letters*, **115**, 211–214.

Schofield, N., & Sened, I. 2002. Local Nash Equilibrium in Multiparty Politics. *Annals of Operation Research*, **109**, 193–211.

Shaked, M., & Shanthikumar, G.J. 2007. *Stochastic Orders*. Springer Series in Statistics. Springer.

Sims, C. 1998. Stickiness. *Carnegie-Rochester Conference Series on Public Policy*, **49**(1), 317–356.

Sims, C. 2003. Implications of Rational Inattention. *Journal of Monetary Economics*, **50**(3), 665–690.

Sims, C. 2005. *Rational Inattention: A Research Agenda*. Discussion Paper Series 1: Economic Studies 2005, 34. Deutsche Bundesbank, Research Centre.

Sims, C. 2006. Rational Inattention: Beyond the Linear-Quadratic Case. *The American Economic Review*, **96**(2), 158–163.

Sonderegger, S. 2012. Centralized or Decentralized Information: Which Is Better For Providing Incentives? *Economic Inquiry*, **48**(2), 290–305.

Svensson, L.E.O. 2006. Social Value of Public Information: Comment: Morris and Shin (2002) Is Actuallt Pri-Transparency, not Con. *American Economic Review*, **96**(1), 448–452.

Szkup, M., & Trevino, I. 2014. *Information Acquisition and Transparency in Global Games*. Unpublished manuscript.

Taneva, I. 2014. *Information Design*. Unpublished manuscript.

Tchen, A.H. 1980. Inequalities for Distributions with Given Marginals. *The Annals of Probability*, **8**(4), 814–827.

Ui, T., & Yoshizawa, Y. 2013 (October). *Characterizing Social Value of Information*. Working Paper.

Van Nieuwerburgh, S., & Veldkamp, L. 2009. *Information Acquisition and Under-Diversification*. Working Paper.

Van Zandt, T., & Vives, X. 2007. Monotone Equilibria in Bayesian Games of Strategic Complementarities. *Journal of Economic Theory*, **134**(1), 339–360.

Veldkamp, L. 2011. *Information Choice in Macroeconomics and Finance*. Princeton University Press.

Vives, X. 1988. Aggregation of Information in Large Cournot Markets. *Econometrica*, **56**(4), 851–876.

Vives, X. 2011. *Endogenous Public Information and Welfare*. Working Paper.

Yang, M. 2014. *Coordination with Flexible Information Acquisition*. Unpublished manuscript.