Mergers and Innovation Portfolios*

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Abstract

This paper studies mergers in markets where firms invest in a portfolio of research projects of different profitability and social value. The portfolio nature of the investment problem brings about novel insights on the external effects of firms' investments. The investment of a firm in one project imposes a negative *business-stealing* externality on the rival firms because it lowers the probability they win the innovation contest for that project; however, the investment of a firm in one project also exerts a positive *business-giving* externality on the rival firms because it increases the likelihood they win the contest for the alternative project.

Merging firms internalize these positive and negative externalities they impose on each other. We show that when the project that is relatively more profitable for the firms is also the more appropriable, then a merger increases consumer welfare by reducing investment in the most profitable project and increasing investment in the alternative (less profitable) project. For the case of linear demand and constant marginal costs, the *portfolio effect* of mergers makes them consumer welfare improving. With constant elasticity of demand and constant marginal costs, a merger increases consumer welfare if the more profitable project corresponds to the market with the higher elasticity of demand. The portfolio effect of mergers may dominate the usual market power effects of mergers.

Keywords: innovation portfolios, R&D contests, mergers

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1 Introduction

Innovation, an essential activity for economic growth and welfare, is often encouraged by public policies such as R&D subsidization and intellectual property policy.[1] Recently, antitrust enforcers are actively taking further action by accounting for the effects of mergers on innnovation. In fact, according to Gilbert and Greene (2015), during the 2004-2014 period, the US Antitrust Agencies invoked innovation-based concerns in about a third of the mergers they challenged. Likewise, the European Commission appealed to an innovation theory of merger harm in the recent Dow/DuPont, GSK Oncology/Novartis and General Electric/Alstom cases.

Starting with Schumpeter (1943) and Arrow (1962), an important and large theoretical literature in economics studies the relationship between competition and innovation. This work, however, does not take into account the specificities of merger activity and cannot readily be used to develop an innovation theory of merger harm. In particular, as demonstrated in the recent papers of Federico, Langus and Valletti (2017, 2018), Motta and Tarantino (2017), Bourreau, Jullien, Lefoulli (2018), Denicolò and Polo (2018) and Gilbert (2019), understanding the impact of merger activity on innovation necessitates a separate analysis because a merger cannot be understood as a mere reduction in the number of competitors in the market, or of the degree of product differentiation, but as a transaction that results in that the partner firms coordinate their strategic decisions. Moreover, the literature on innovation and competition has been somewhat inconclusive about how competition affects investment due to the variety of models analysed, with specific functional forms and modes of competition (see Vives (2008) and Schmutzler (2013)).[2]

The recent literature on mergers and innovation focuses on how a merger impacts R&D investment of merging and non-merging firms. However, it is well known that firms engage themselves into multiple research projects with different chances of success and distinct returns to the innovators and society (see e.g. Mansfield (1981) and Cohen and Klepper (1996)). Typical examples include pharmaceutical firms, which engage in research to develop innovative medicines in distinct therapeutic areas such as ophthalmology, immunology, dermatology, oncology, etc.[3] Because the drugs they produce differ in profits and social value, the question that arises is how does merger activity shape firms' incentives to pursue the socially optimal composition of R&D portfolios.[4] To the best of our knowledge, this question has not been analysed so far. Our paper fills this gap by developing a model of investment portfolios to examine how mergers affect the portfolios chosen by merging and non-merging firms and assess the welfare effects of these choices.

To study how mergers affect equilibrium innovation portfolios, we consider a market where firms invest in two independent research projects. These projects vary in terms of three characteristics, namely, their profitability, difficulty and social value. By investing in a project, an individual firm engages in a contest with the rival firms.[5] In the baseline model, each contest is winner-take-all and the

[1] See e.g. Romer (1990), Aghion and Howitt (1998), and Grossman and Helpman (1994).

[2] Empirically, the relationship between competition and innovation has not reached consensus either, with some authors finding it to be inverted-U shape (Aghion *et al.* (2005)) and others either increasing (Correa and Ornaghi (2014) and Beneito *et al.* (2017)) or even decreasing (see Hashmi (2013)).

[3] One example is *Novartis*; see `https://www.novartis.com/our-company/innovative-medicines`.

[4] In the European GSK Oncology/Novartis case, the Commission had concerns about a deleterious adjustment of investments post-merger. In particular, the Commission's assessed a potential cut in research oriented to treat skin cancer.

[5] In our model firms engage in a conflict involving multiple contests. This type of game was introduced by Borel (1921)

successful firm appropriates the full profits generated by the innovation, while the losing firms obtain zero rewards.[6] The cost of investment function exhibits decreasing returns in aggregate investment across the two projects, which implies that increasing investment in one project raises the marginal cost of investing in the other project. The existence of these negative externalities across the projects of a firm has the novel implication that the investment of a firm in a given project imposes on the rival firms both a *business-stealing* externality for that project and a *business-giving* externality for the alternative project.[7]

We show that firms hold an inefficient portfolio of investments for two reasons. First, because an individual firm ignores the business-stealing and business-giving externalities it imposes on the rest of the firms, it tends to invest excessively in the project with higher expected profitability and insufficiently in the alternative one. Second, because an individual firm fails to fully appropriate the social gains from an innovation, it tends to underinvest in the more socially desirable project and overinvest in the other one. This portfolio inefficiency of the market equilibrium leads us to explore how mergers affect the allocation of investment among these projects by the merging and non-merging firms.[8]

The merging firms internalize the two innovation externalities they exert on one another. Internalization of the business-stealing externality gives the merging firms incentives to lower investment in the more profitable project and raise it in the alternative project, while internalization of the business-giving externality incentivises them to adjust the portfolio of investments in the opposite direction. The tension between these two conflicting externalities determines the impact of a merger on the innovation portfolio of the partner firms. We refer to this new economic effect associated with mergers as the *portfolio effect* of mergers.

When projects differ in expected profitability, a merger results in an increase in the investment of the merging firms in the less profitable project. Whether the non-merging firms increase or decrease their investment in the less profitable project depends on whether investments are strategic substitutes or strategic complements. With Tullock's contest success functions, firms investments are strategic substitutes and the non-merging firms adjust investment in the opposite direction compared to the merging firms. Despite this, we show that aggregate investment in the less profitable project increases and consumer welfare may improve post-merger.[9] We derive a clear policy message: in the absence of any synergies, when the relatively less profitable project is also the relatively less appropriable, a merger raises consumer surplus by increasing investment in it. Otherwise, if the project that is relatively more

and is known as the Colonel Blotto game. For a survey of the literature see Kovenock and Roberson (2012). For a recent application, see Goyal and Vigier (2014).

[6]In Section 4.1 we extend our analysis to the case in which the winning and losing firms compete *à la* Cournot to sell products that are vertically differentiated. We show that the portfolio effects of mergers may have a dominating influence over the usual price effects of mergers.

[7]Business-stealing and business-giving externalities are also at the heart of investment incentives in models with innovation spillovers (see e.g. Katz and Shapiro (1987) and Kamien *et al.* (1992)). These papers feature single-project firms and exogenous inter-firm spillovers. In our model, by contrast, the business-stealing and business-giving externalities originate from the portfolio problem the firms face.

[8]The inefficiency of the market portfolio of investments has also been investigated e.g. in the single-project papers of Klette and de Meza (1986), Bhattacharya and Mookherjee (1986) and Dasgupta and Maskin (1987). Cabral (1994) and section 5 of Bhattacharya and Mookherjee (1986) use a multi-project approach similar to ours. More recently, Hopenhayn and Squintani (2016), Bryan and Lemus (2017) and Chen et al. (2018) study the dynamic efficiency of the direction of innovation and Akcigit et al. (2017) present evidence of a dynamic misallocation in research by which there is overinvestment in applied research and underinvestment in basic research.

[9]This is in contrast with Farrell and Shapiro (1990) and Motta and Tarantino (2017), which show that in the absence of synergies mergers in single product markets cannot increase consumer welfare.

profitable is also significantly less appropriable, then a merger reduces welfare.

We show that for the case of linear demands and constant marginal costs, all mergers are consumer welfare improving. With constant elasticity of demand and constant marginal costs, a merger increases consumer welfare if the more profitable project corresponds to the market with the higher elasticity of demand.

We provide three extensions of our main analysis. First, our results carry over to the case in which the product market is competitive and winning and losing firms engage in quantity competition to sell products of different quality. Firm competition causes an additional *market power effect* of mergers that may operate counter to the portfolio effect. We show that a merger may still be consumer welfare improving despite its price-increasing implications. Second, we show that our results also hold in situations where firms decide on the budget they spend on research and how they allocate it across projects. Allowing firms to choose how much to invest in research does not alter the basic intuition behind the portfolio effects of mergers because investing in one project increases the marginal cost of investing in the other project. Therefore, even if a merger results in a lower aggregate investment as argued by the innovation theory of harm, the portfolio effects of mergers can make a merger socially desirable. Finally, we provide a model with general contest success functions for which firms' investment efforts may even be strategic complements. We provide conditions under which mergers may be welfare improving and develop a graphical approach to show that the main insights of our baseline model hold for general winning probabilities that allow for firms' investments to be strategic substitutes.

Related literature

Our paper adds to a recent literature on the impact of mergers on innovation. As mentioned above, this work has mainly focused on how mergers affect R&D expenditure. This literature has identified two channels through which a merger affects innovation incentives of the partner firms. First, there is an internalization of negative innovation externalities. A firm that invests in R&D increases the likelihood with which it successfully innovates and this lowers the chance other firms appropriate the full gains from their innovation efforts. This channel corresponds to our business-stealing externality and tends to reduce the investment of the partner firms. Second, there is an internalization of negative quantity/pricing externalities through which a merger tends to soften competition in the product market. This weakening of competition may or may not increase the incentives to invest because the pre-merger profits may be more or less responsive than the post-merger profits to innovation effort. Whether a merger ultimately increases investment incentives depends on the relative strength of these two effects.

Motta and Tarantino (2017) study mergers in a deterministic R&D model with product market competition. They analyze a simultaneous innovation and pricing game and show that absent spillovers or synergies, the reduction of output by the merged entity induces a further reduction of cost-reducing investment, which harms consumers. Federico, Langus and Valletti (2018) obtain a similar result in a two-stage model of price competition with stochastic R&D and differentiated products despite the fact that in their model the reduction in the intensity of price competition following a merger favors innovation. Bourreau, Jullien and Lefoulli (2018) further zoom into the market power effect and show that this can be decomposed into two effects working in opposite directions, namely, a negative "margin expansion" effect and a positive "demand expansion" effect. They show that the overall impact of a merger on innovation incentives can be either positive or negative. However, they do not analyze consumer welfare implications. Finally, López and Vives (2018) use an approach based on cross-shareholding agreements between firms to argue that industry-wide mergers can be welfare improving in industries with sufficiently large R&D spillovers.[10]

Our work is closely related to Johnson and Rhodes (2019). They consider mergers in markets where firms choose their product lines and show that, when firms' product lines are asymmetric, mergers may raise consumer surplus due to a product-mix effect through which firms reposition their product lines after a merger. This product-mix effect is different from our portfolio effect of mergers because in the paper of Johnson and Rhodes firms operate in a single market. Nocke and Schutz (2018) also study mergers in a multi-product setting. They show that standard merger results generalize to multi-product settings with nested CES or nested multinomial logit (NMNL) demand systems.

The multi-project setting of our study also brings our paper close to the recent contributions of Letina (2016) and Gilbert (2018), which, instead of looking at the effects of mergers on investment volumes, delve into the effects of mergers on the variety and diversity of R&D. Letina (2016) studies a model where firms can choose the number of projects they wish to activate, knowing that only one of the very many projects will turn out to be successful. Projects differ in the cost of activation. He provides conditions under which the market opens too many research lines and there is too much duplication. A merger decreases the variety of developed projects and decreases the amount of duplication of research, which, depending on parameters, may increase or decrease welfare. Gilbert (2018) extends the model of Federico *et al.* (2017, 2018) by allowing the firms to invest in several research avenues to solve the same problem. He shows that, absent spillovers, mergers generally (but not always) decrease R&D diversity measured by the number of projects undertaken by the industry.

Finally, our paper is also related to studies on the bias of the market equilibrium research portfolios. Part of this literature uses models where firms choose the riskiness of their single research projects and compare the equilibrium with the social optimum (see e.g. Klette and de Meza (1986), Bhattacharya and Mookherjee (1986) and Dasgupta and Maskin (1987)). Klette and de Meza (1986) and Bhattacharya and Mookherjee (1986) find that the firms and the social planner choose the riskiest research program available. Dasgupta and Maskin (1987) introduce convex costs to induce interior solutions in a similar model with differentiated firms and find that the firms choose projects that are too risky from the point of view of social welfare. Bhattacharya and Mookherjee (1986) also study firm choice among two R&D approaches to solve the same hurdle. They show that firms maximally differentiate their R&D approaches, thereby maximising the riskiness and minimizing the correlation of the equilibrium firm portfolios. The market equilibrium is socially optimal. In Cabral (1994) firms also allocate funding across two projects of different riskiness. He shows that market competition results in a bias against the riskier project. Finally, Hopenhayn and Squintani (2016), Bryan and Lemus (2017) and Chen *et al.* (2018) present models of the choice of research direction. They show that firms pursue inefficient research lines, in particular they focus on relatively easy, safe and highly profitable projects. These papers relate to ours in terms of the externalities firms impose on one another and the resulting inefficiency of the investment portfolios held by firms. None of them studies the impact of mergers on social welfare. Their

[10]Markets in which buyers have market power can be quite distinct. Loertscher and Marx (2018) show that in procurement markets a merger increases the incentives to invest of the non-merging suppliers and may also raise the incentives to invest of the merging firms.

focus is on the merits of R&D subsidization and patent policy to improve directional efficiency.

The rest of the paper is organized as follows. We introduce our general model in Section 2. The pre-merger market equilibrium is analyzed in Subsection 2.1. The effect of mergers is discussed in Section 2.2. To deepen into the welfare effects of mergers in Section 3 we analyze the standard case of Tullock's contests. In Section 4 we demonstrate the robustness of our results. In particular, in Subsection 4.3 we study a more general Tullock's contests model where firms not only decide how to allocate funding across projects but also how much money to invest in total. Finally, some concluding remarks are provided in Section 5. All the proofs of the Propositions are relegated to the Appendix.

2 The model and preliminary intuition

We consider a market with $n \geqslant 3$ independent firms, indexed by i. The number of firms is exogenous, reflecting barriers to entry. The market develops over two stages. In the first stage, referred to as the *pre-innovation* stage, firms engage in R&D contests to introduce two types of innovations: A- and B-innovations. We model these contests as "winner-take-all".[11] In the *post-innovation* stage, thus, the "winning" firm obtains the (monopoly) profits from the market created by the innovation; "losing" firms make no profits.[12] We assume that the probability that no firm wins the contest is strictly positive; as we will see later, this assumption makes our welfare analysis meaningful.

The two innovations, indexed by ℓ, differ in three aspects:

(i) the rewards, or profits, they generate for the sole winner of the innovation contest, denoted π_ℓ,

(ii) the intrinsic difficulty of successfully obtaining the innovation, denoted ϵ_ℓ, and

(iii) the social gains the innovation creates, denoted W_ℓ, $\ell = A, B$.

As usual, the social gains equal the sum of firm's profits and consumer surplus, which we denote by S_ℓ, $\ell = A, B$. Summarizing, each innovation j is characterised by a triplet $\{\pi_\ell, \epsilon_\ell, S_\ell\}$, $\ell = A, B$.

We assume that firms have a fixed budget, or a fixed number of scientists, and that their decision is simply how to allocate them across the labs in which research efforts are undertaken to realize the projects. Specifically, assume that firms have a fixed R&D budget that we normalize to 1.[13] Let x_i denote the effort exerted by a firm i in the A-project and, correspondingly, $1 - x_i$ the effort put in by firm i in the alternative B-project.[14]

[11] As discussed in Baye and Hoppe (2003), winner-take-all contests are strategically equivalent to patent races. In that connection, our model can be seen as a model where firms compete to obtain innovations protected by patents.

[12] The winner-take-all assumption serves to focus the discussion on the *portfolio* effects of mergers. Later in Section 4.1 we relax this assumption by allowing the winning and the losing firms to compete *á la* Cournot after they engage in contests to introduce a product of higher quality. In such an extension, in addition to the portfolio effects of mergers, there are price effects of mergers. We show that the portfolio effects of mergers have a dominating influence when the quality gap between the winning and losing firms is sufficiently large.

[13] The fixed budget assumption also serves to place the focus of our paper on the portfolio effects of mergers, rather than on the effects of mergers on investment effort. Later in Section 4.3 we relax this assumption and allow the contestants to choose their research budgets and how they allocate them across the projects.

[14] The cost of allocating the fixed budget to research is constant and will not play any role in the pre-merger market equilibrium. However, when modelling mergers, because the merged entity has double the amount of funds of a non-merging firm, we make the assumption that there are no efficiency gains due to scale economies. This boils down to assuming that the cost function is sufficiently convex. If this is so, after a merger the merged entity will keep the labs of the constituent firms running. Otherwise, with scale economies, the merged entity may prefer to shut down the labs of one

Let $p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)$ denote the probability with which an individual firm i successfully introduces the A-innovation; this probability depends on the effort exerted by firm i, x_i, the efforts put in by the rival firms, denoted $\mathbf{x}_{-i} \equiv (x_1, x_2, ..., x_{i-1}, x_{i+1}, ..., x_n)$, and the intrinsic difficulty of the innovation ϵ_A. Likewise, the probability with which an individual firm i successfully introduces the B-innovation is denoted $q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)$, where $1 - x_i$ is the effort put in by firm i, $\mathbf{1} - \mathbf{x}_{-i}$ the efforts exerted by the rest of the firms and ϵ_B the intrinsic difficulty of successfully completing the B-innovation path.

We now impose certain assumptions on the success functions $p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)$ and $q_i(1-x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)$.

Assumption 1.
For the contest success function $p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)$, and for every firm i, it holds that:
(a) $p_i(\cdot)$ is twice differentiable.
(b) $\sum_{i=1}^{n} p_i(x_i, \mathbf{x}_{-i}, \epsilon_A) < 1$.
(c) $\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i} > 0$, $\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_j} < 0$, $\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial \epsilon_A} < 0$.
(d) $\frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2} < 0$ and $\sum_{j=1}^{n} \frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i \partial x_j} < 0$.
The same assumptions apply to the contest success function $q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)$.

These assumptions are standard in innovation contests (see e.g. Tullock, 1980; Dixit, 1987 and Skaperdas, 1996). Assumption (a) allows us to use calculus to address our research questions. Assumption (b) avoids that the welfare criterion is constant in the vector of firms' investments and thus makes our welfare analysis interesting (more details later). Assumption (c) signifies that if a firm exerts more effort in a research project, the probability that it succeeds increases and the probability that other firms succeed decreases. Moreover, the assumption implies that an increase in the parameter ϵ_A lowers the probability that any of the firms innovates and therefore measures the industry difficulty of successfully completing the innovation path. The first part of Assumption (d) means that the probability of success exhibits decreasing returns with respect to own investment. The second part is a standard dominant diagonal condition, which serves to establish uniqueness of equilibrium. At this stage we do not impose any assumption on the sign of the second cross-partial derivative of the success probabilities with respect to own and rival firms efforts; this implies that the best-replies of the firms may be decreasing (in which case the game is one of strategic substitutes) or increasing (in which case the game is one of strategic complements).

We assume that firms pick their investments in the A- and B-innovation projects simultaneously. The focus of the paper will be on symmetric Nash equilibria (SNE), that is, equilibria in which similar firms make equal investments. Specifically, in the pre-merger market, a SNE satisfies $x_i = x^*$, $i = 1, 2, .., n$. In the post-merger market, the symmetry holds for the merging firms on the one hand and for the non-merging ones on the other hand.

of the constituent firms and allocate the entire budget to the labs of the other firm. Later in Section 4.3 when we relax the fixed budget assumption, we demonstrate that maintaining the assumption that the costs of hiring additional scientists is convex enough, the same insights arise. The reason is that when the cost is convex, allocating one more scientist to one project increases the marginal cost of allocating one more scientist to the alternative project. Our fixed budget assumption here is nothing else than an extreme version of this idea.

2.1 Pre-merger market equilibrium

The payoff to a firm i putting efforts x_i and $1 - x_i$ in the A- and B-innovation contests is:

$$u_i(x_i; \mathbf{1} - \mathbf{x}_{-i}) = p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B \tag{1}$$

Note that $u_i(x_i, \mathbf{x}_{-i})$ is strictly concave in own effort x_i. Therefore, assuming an interior equilibrium exists, the first order condition (FOC) for profits-maximization is:

$$\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B = 0. \tag{2}$$

Equation (2) simply means that the marginal gains from investing in a project should be equal across projects. Note that $\partial q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)/\partial x_i < 0$ because the probability with which a firm i wins the contest for the B-innovation decreases with its effort in the A-innovation.

The solution to (2) in x_i gives the best-reply function of a firm i to the vector of rivals' investments $\mathbf{x}_{-i}$. As it is known from the work of Salant *et al.* (1983) and Deneckere and Davidson (1985), the strategic nature of the variables will be important later in our analysis of mergers. Applying the implicit function theorem to the FOC (2), we obtain:

$$\frac{\partial x_i}{\partial x_j} = -\frac{\frac{\partial^2 p_i}{\partial x_i \partial x_j}\pi_A + \frac{\partial^2 q_i}{\partial x_i \partial x_j}\pi_B}{\frac{\partial^2 p_i}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i}{\partial x_i^2}\pi_B}.$$

When this derivative is negative (positive), we have a game of strategic substitutes (complements).

If an interior pre-merger market equilibrium exists, we can derive it by applying symmetry in (2), i.e. $x_i = x^*$, $i = 1, 2, ..., n$, and solving for x^*. It is straightforward to provide conditions under which an interior equilibrium in this general model exists. We postpose these details to Section 4.2 and focus here on developing the main intuition behind the portfolio effects of mergers.

2.2 Mergers

Consider now that firms i and j merge. Because there are no efficiency gains due to scale economies, it is optimal for the merged entity to continue to allocate scientists to each of the labs of the constituent firms. In such a case, the merged entity chooses investments x_i and x_j in the A-project (and by implication $1 - x_i$ and $1 - x_j$ in the B-project) to maximise the (joint) payoff:

$$\begin{aligned} u_m(x_i, x_j; \cdot) = {} & p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B + p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\pi_A \\ & + q_j(1 - x_j, \mathbf{1} - \mathbf{x}_{-j}, \epsilon_B)\pi_B. \end{aligned} \tag{3}$$

Non-merging firms continue to maximise the payoff in (1).

Assuming that an interior equilibrium exists, the FOC for the maximisation of the profits of the merged entity with respect to x_i is given by:

$$\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(1-x_i, \mathbf{1}-\mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B + \underbrace{\frac{\partial p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)}{\partial x_i}\pi_A}_{\text{business-stealing externality on } j}$$
$$+ \underbrace{\frac{\partial q_j(1-x_j, \mathbf{1}-\mathbf{x}_{-j}, \epsilon_B)}{\partial x_i}\pi_B}_{\text{business-giving externality on } j} = 0. \tag{4}$$

The FOC for profits maximisation with respect to x_j is similar. The FOC corresponding to the payoff of a non-merging firm is the same as that given in (2).

If an interior post-merger equilibrium exists, we can derive it by imposing symmetry among the merging and non-merging firms, i.e. $x_i = x_j = x^m$ and $x_k = x^{nm}$, $k \neq i, j$, and solving for x^m and x^{nm}. Later in Section 4.2 we provide existence conditions for an interior post-merger equilibrium. Now, let us focus on understanding how merging and non-merging firms adjust their portfolio of investments after a merger.

Inspection of the FOC (4) reveals that each of the merging units internalises *two* externalities. On the one hand, a merging firm that engages in a contest for a project imposes a *business-stealing* externality on the partner firm for the same project. This is because when a merging firm, say i, increases its investment x_i in the A-innovation, it lowers the probability the partner entity j gets that innovation. On the other hand, a merging firm that engages in a contest for a project imposes a *business-giving* externality on the partner firm for the alternative project. This is because when firm i increases its investment x_i in the A-innovation, it consequently lowers its investment in the B-innovation and this therefore increases the probability the partner entity j wins the contest for the B-innovation.

Internalization of these two externalities leads to a new economic effect of mergers, which we call the portfolio effect of mergers. The first, business-stealing, externality is negative, pushing the merged entity to reduce x_i relative to the pre-merger market.[15] The second, business-giving, externality is positive and works counter to the negative business-stealing externality. The net effect is thus ambiguous and, depending on parameters, may be towards more investment in the A-innovation and less in the B-innovation or viceversa.

Whether the portfolio adjustment of the merging firms tends to benefit consumers or not depends on which of the two projects generates more surplus for them. If, for example, the internalization of the business-stealing and business-giving externalities results in the merging firms moving resources from the A-project to the B-project and consumers gains from the B-project are higher than those from the A-project, then the merger will tend to increase consumer welfare.

Of course, even if a merger causes the merging firms to adjust their investment in a way that favours consumers, it is not a priori clear whether the post-merger equilibrium consumer welfare will be higher than pre-merger. For this, we also need to take into consideration the reaction of the non-merging firms, which depends on whether the game is one of strategic substitutes or one of strategic complements. If, compared to the pre-merger market equilibrium, the merged entity increases investment in the A-

[15] The literature refers to this negative externality as the "innovation externality" and identifies it as an additional source of detrimental merger effects for consumers (cf. Federico *et al.* (2018)).

innovation (and so decreases investment in the B-innovation) and the game is of strategic substitutes, the non-merging firms will instead reduce investment in the A-project (and so increase it in the B-project). With strategic complements, the non-merging firms will adjust their investments in the same way as the merged entity.

To better understand the likely effects of a merger on welfare, let us now present the social optimum.

2.3 Welfare

We will evaluate mergers from a consumer welfare perspective but the analysis that follows can straightforwardly be generalized to the adoption of a social welfare standard.[16] The social planner picks a portfolio of investments $\mathbf{x}$ to maximize:

$$W(\mathbf{x}, \mathbf{1}-\mathbf{x}) = \left(\sum_{j=1}^{n} p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\right) S_A + \left(\sum_{j=1}^{n} q_j(1-x_j, \mathbf{1}-\mathbf{x}_{-j}, \epsilon_B)\right) S_B. \tag{5}$$

This welfare expression is equal to the sum across projects of the market probability with which an innovation is introduced times the consumer surplus generated by the innovation. (If the planner maximizes social welfare, we simply replace S_ℓ by $W_\ell = \pi_\ell + S_\ell$ in (5), $\ell = A, B$.) Given our assumptions on the success probabilities $p_j(\cdot)$ and $q_j(\cdot)$, the social welfare function is strictly concave and hence there is a unique global maximizer.

Assuming the social optimum is interior, the FOC for social welfare maximization with respect to x_i is:

$$\frac{\partial W(\cdot)}{\partial x_i} = \left(\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i} + \underbrace{\sum_{j\neq i} \frac{\partial p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)}{\partial x_i}}_{\text{negative externality}}\right) S_A$$

$$+ \left(\frac{\partial q_i(1-x_i, \mathbf{1}-\mathbf{x}_{-i}, \epsilon_B)}{\partial x_i} + \underbrace{\sum_{j\neq i} \frac{\partial q_j(1-x_j, \mathbf{1}-\mathbf{x}_{-j}, \epsilon_B)}{\partial x_i}}_{\text{positive externality}}\right) S_B = 0. \tag{6}$$

It is interesting to compare the FOC (6) with the equilibrium conditions in the pre- and post-merger markets to understand the difference between the social optimum and the market equilibria. Consider the case of the pre-merger market equilibrium first. The FOCs (2) and (6) differ in two important regards. First, the social planner cares about the market probability that the innovations are introduced, and not just about the individual firms' success probabilities. This implies that the social planner takes into account *all* the externalities that the investments of an individual firm impose on all other firms in the market. The fact that an individual firm ignores these externalities, which are negative and positive as indicated in equation (6), creates a source of inefficiency of the market equilibrium (unless they accidentally happen to cancel out). Second, the fact that the social planner cares about the consumer

[16] EU and US competition authorities usually employ a consumer welfare standard to assess mergers.

value of an innovation S_ℓ (or more generally social value W_ℓ) and the firm only about the private reward π_ℓ, $\ell = A, B$ constitutes a second source of inefficiency of the market equilibrium (unless they coincidentally happen to be exactly equal).[17]

No matter whether we adopt a consumer or a social welfare criterion, at this level of generality, it is unclear how the post-merger market equilibrium will fare relative to the pre-merger equilibrium in terms of welfare. It is possible that all the merging and non-merging firms adapt their portfolio of investments in the right direction, in which case a merger would be desirable. But even if the merging firms adapt their portfolio of investments in the right direction, it may be the case that the non-merging firms respond in an offsetting way, thereby possibly rendering a merger undesirable. In order to better understand when a merger is appealing, we thus need to introduce more structure into our model.

In the next section, we use the well-known Tullock's contest formulation to characterize welfare improving/harming mergers and derive a clear-cut policy recommendation. We return to the general model in Section 4.2 and show that the insights obtained apply more generally.

3 Tullock's (1980) R&D contests

Following Tullock (1980), let us assume that the probabilities with which a firm i succeeds to introduce A- and B-innovations are given by:[18]

$$p_i(x_i, \mathbf{x}_{-i}, \epsilon_A) = \frac{x_i}{\sum_{k=1}^{n} x_k + \epsilon_A}, \tag{7}$$

$$q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B) = \frac{1 - x_i}{\sum_{k=1}^{n} (1 - x_k) + \epsilon_B}. \tag{8}$$

To ensure that our game of portfolio choice is one of strategic substitutes, we assume that $\epsilon_\ell \geqslant 1$, $\ell = A, B$. (For a proof, see the Appendix.)

3.1 Pre-merger market equilibrium

In the pre-merger market equilibrium, a firm i maximises the expression:

$$u_i(x_i, \mathbf{x}_{-i}) = \frac{x_i}{\sum_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{1 - x_i}{\sum_{k=1}^{n} (1 - x_k) + \epsilon_B}\pi_B. \tag{9}$$

[17]Interestingly, with only one firm in the market there are no external effects and the first source of inefficiency disappears. Moreover, the second source of inefficiency disappears when the winning firm can perfectly price discriminate in the product market; or, alternatively, when the product market demand is rectangular. For these special cases, the winning firm extracts the whole surplus in the product market, i.e. $\pi_\ell = W_\ell$, $\ell = A, B$. Therefore, in our model the investment portfolio of a perfectly price discriminating monopolist is socially optimal. For a similar result in a related model of R&D portfolios see Cabral (1994).

[18]To focus solely on the portfolio effects of mergers, we purposely adopt a constant returns to scale version of the Tullock contest success functions (see Tullock, 1980). Because a merger alters the "size" of the merged entity, with increasing or decreasing returns mergers would have scale effects that would need to be separated from the portfolio effects of mergers.

The FOC for an interior equilibrium is given by:

$$\frac{\sum_{k\neq i} x_k + \epsilon_A}{\left(\sum_{k=1}^{n} x_k + \epsilon_A\right)^2}\pi_A - \frac{n-1+\epsilon_B - \sum_{k\neq i} x_k}{\left(n+\epsilon_B - \sum_{k=1}^{n} x_k\right)^2}\pi_B = 0. \tag{10}$$

It is straightforward to verify that the second order condition for profits maximization holds.

The following result characterises the SNE, and provides conditions on the parameters under which the equilibrium is interior.

Proposition 1. *If a unique interior SNE of the portfolio choice game exists, firms invest an amount* $x^* \in (0,1)$ *in the A-innovation project, where* x^* *is given by the unique solution to:*

$$\frac{(n-1)x^* + \epsilon_A}{(nx^* + \epsilon_A)^2}\pi_A - \frac{n+\epsilon_B - 1 - (n-1)x^*}{(n+\epsilon_B - nx^*)^2}\pi_B = 0. \tag{11}$$

The rest of the budget, $1 - x^*$*, is invested in the B- innovation project. The equilibrium investment* x^* *increases in* π_A *and* ϵ_B*, and decreases in* ϵ_A *and* π_B*.*

The equilibrium exists if the parameters of the model satisfy the inequality:

$$\frac{\pi_A\epsilon_B - 2n\pi_B + \sqrt{\pi_A\epsilon_B\left(\pi_A\epsilon_B - 4\pi_B\right)}}{2\pi_B} < \epsilon_A < \frac{\pi_A(n+\epsilon_B)^2}{\pi_B(n-1+\epsilon_B)}. \tag{12}$$

Proof. See the Appendix. ■

The condition in the proposition ensures that corner solutions cannot be equilibria: if all firms invest solely in the B-project, the condition ensures that the payoff of a firm would strictly increase if the firm deviated by investing in the A-project. In other words, the payoff (9) is strictly increasing in a neighbourhood of zero. Likewise, suppose all firms solely invested in the A-project, then the condition ensures that an individual firm would gain by cutting its investment in the A-project and raising it in the B-project. In what follows we assume that the parameters of the model satisfy the condition in (12) and focus on the interior equilibrium.

Because we analyze mergers in an n-player game, it is illustrative to follow the graphical tool devised by Deneckere and Davidson (1985) and build the "pseudo" reaction functions corresponding to our setting. Deneckere and Davidson plot the joint reaction function of the potentially merging firms against the non-merging firms on one hand, and the joint reaction of the non-merging firms against the potentially merging firms on the other hand. The crossing point between these two reaction functions depicts the pre-merger symmetric equilibrium.

Let firms i and j be the firms involved in a tentative merger. Let us define $x_m = x_i + x_j$ as the joint effort put by these firms in the A-innovation path. Likewise, let $x_{nm} = \sum_{k\neq i,j} x_k$ be the corresponding joint effort of the non-merging firms.[19] Using this notation, we can compute the joint best-response function of the potentially merging firms against the non-merging firms as follows. First write the FOCs

[19] To be sure, we defined above x^m and x^{nm} to refer to the *individual* effort of merging and non-merging firms, respectively. Now, we use the notation x_m and x_{nm} to refer to the joint effort of merging and non-merging firms.

for firms i and j in this way:

$$x_i \quad : \qquad \frac{x_j + x_{nm} + \epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_j - x_{nm}}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0$$

$$x_j \quad : \qquad \frac{x_i + x_{nm} + \epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_i - x_{nm}}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0$$

and sum them to obtain:

$$\frac{x_m + 2x_{nm} + 2\epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{2(n - 1 + \epsilon_B) - x_m - 2x_{nm}}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0. \tag{13}$$

This expression defines implicitly the joint best-response function of firms i and j against the rest of the firms. By the implicit function theorem, the slope of the best response is easily shown to be negative. Therefore, the joint best-response of the potentially merging firms is downward sloping.

We can construct the joint best-response of the non-merging firms in a similar way, that is, writing the FOC of a typical non-merging firm as:

$$x_\ell : \qquad \frac{x_m + \sum_{k \neq i,j,\ell} x_k + \epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_m - \sum_{k \neq i,j,\ell} x_k}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0,$$

and summing across $\ell \neq i, j$ to obtain:

$$\frac{x_m + \frac{n-3}{n-2}x_{nm} + \epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 1 + \epsilon_B - x_m - \frac{n-3}{n-2}x_{nm}}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0. \tag{14}$$

This expression defines implicitly the joint best-response function of the non-merging firms against the potentially merging firms i and j. Using the implicit function theorem, it is readily seen that the slope of this pseudo best-response function is also negative for all $n \geqslant 4$. Therefore, for $n \geqslant 4$, the two pseudo best-response functions are decreasing and cross only once. The crossing point gives the pre-merger symmetric Nash equilibrium.

We represent these pseudo best-response functions in Figure 1. In this figure we have the joint effort of the potentially merging firms, denoted x_m, in the vertical axes, while in the horizontal axes we have the joint effort of the non-merging firms, denoted x_{nm}. The best response functions are plotted for an oligopoly market with 4 firms, and for parameters $\epsilon_A = 4$, $\pi_A = 2$, $\epsilon_B = 1$ and $\pi_B = 1$. The crossing point is $x_m = x_{nm} = 1.084$, which signifies that each firm invests $x^* = 0.541$ in project A and $1 - x^* = 0.459$ in project B.

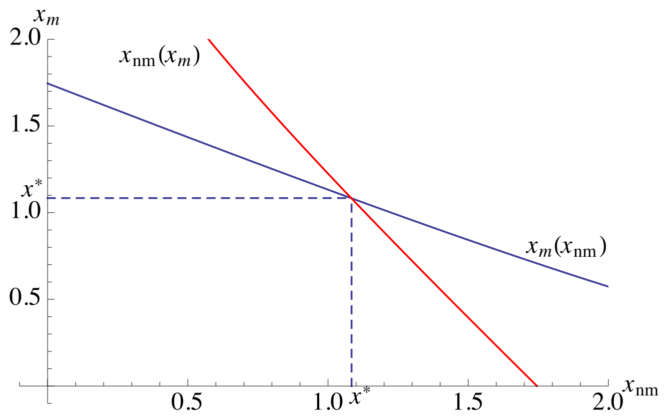


Figure 1: Pre-merger market equilibrium

3.2 Post-merger market equilibrium

We now study the effects of a merger on the equilibrium allocation of investment efforts. Suppose firms i and j merge. After simplification, the payoff to the merged entity is:

$$u_m(x_i, x_j; \cdot) = \frac{x_i + x_j}{\sum_{k=1}^{n} x_k + \epsilon_A}\pi_A + \frac{2 - x_i - x_j}{\sum_{k=1}^{n}(1 - x_k) + \epsilon_B}\pi_B. \tag{15}$$

Inspection of the payoff in Equation (15) reveals that what matters for the merged entity is the joint investment in the projects. Correspondingly, let us define $x_m \equiv x_i + x_j$ as the total investment of the merged entity into the A-innovation path. The total investment of the merged entity into the B-innovation path is then $2 - x_m$. Non-merging firms continue to maximize the payoff in (9).

The FOC necessary for an interior equilibrium for the merged entity is:

$$\frac{\sum_{k\neq m} x_k + \epsilon_A}{\left(x_m + \sum_{k\neq m}^{n} x_k + \epsilon_A\right)^2}\pi_A - \frac{n - 2 + \epsilon_B - \sum_{k\neq m} x_k}{\left(n + \epsilon_B - x_m - \sum_{k\neq m}^{n} x_k\right)^2}\pi_B = 0, \tag{16}$$

The FOC for the non-merging firms continues to be (10). As before, the second order conditions hold so the existence of a Nash equilibrium is established by the same arguments as in Proposition 1.

Applying symmetry for the non-merging firms and setting $x_{nm} = \sum_{k\neq i,j} x_k$, the FOC for the merging firms can be written as:

$$\frac{x_{nm} + \epsilon_A}{(x_m + x_{nm} + \epsilon_A)^2}\pi_A - \frac{n - 2 + \epsilon_B - x_{nm}}{(n + \epsilon_B - x_m - x_{nm})^2}\pi_B = 0. \tag{17}$$

This equation defines implicitly the new pseudo best-response of the merging firms after a merger. Let us denote such a best-response function as $\tilde{x}_m(x_{nm})$. The pseudo best-response of the non-merging firms against the merging firms continues to be implicitly defined by equation (14). The crossing point between these pseudo best-response functions gives the post-merger market equilibrium. Let (x^*_m, x^*_{nm}) denote the post-merger equilibrium aggregate investments of merging and non-merging firms.

Comparing the FOCs of the potentially merging firms before and after the merger we can prove that:

Proposition 2. *If a merger occurs, the merged entity increases investment in the A-project (and so cuts it in the B-project) if and only if*

$$\frac{\pi_B}{\epsilon_B} > \frac{\pi_A}{\epsilon_A}. \quad (18)$$

The non-merging firms, by strategic substitutability, reduce investment in the A-project (and therefore raise it in the B-project).

Proof. See the Appendix. ■

As advanced above, when choosing how to adjust the funding allocated to the A- and B-projects, the merged entity internalizes the business-stealing and the business-giving externality it imposes on the partner firm. The condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$ refers to the relative profitability of the two projects and thus governs the relative strength of these two externalities. When π_A is small relative to π_B and/or ϵ_A is large relative to ϵ_B, investing in the A-project is relatively less attractive for the firms. This also implies that the negative externalitity a firm imposes on another firm when investing is the A-project is weaker than the positive externality. The merged entity, internalizing these externalities, raises investment in the A-project and cuts investment in the B-project.

Using the notion of pseudo best-response functions, we illustrate Proposition 2 in Figure 2. This Figure builds on Figure 1 by adding the pseudo best-response function of the merging firms after the merger, which is denoted by $\tilde{x}_m(x_{nm})$. Because the parameters of the model ($\epsilon_A = 4$, $\pi_A = 2$, $\epsilon_B = 1$ and $\pi_B = 1$) satisfy the condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$, the best-response function of the merging firms after the merger lies above the best-response function of the merging firms before the merger in a neighbourhood of the pre-merger equilibrium x^*. As a result, the merged entity puts in a greater effort in the A-project and a lower in the B-project. Because the best-replies are decreasing, the non-merging firms do exactly the opposite.

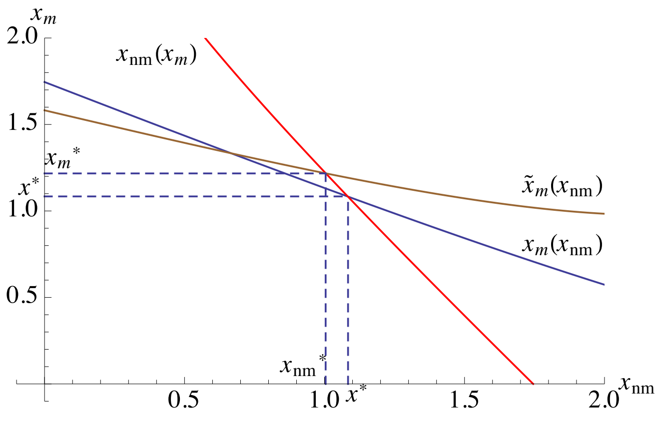
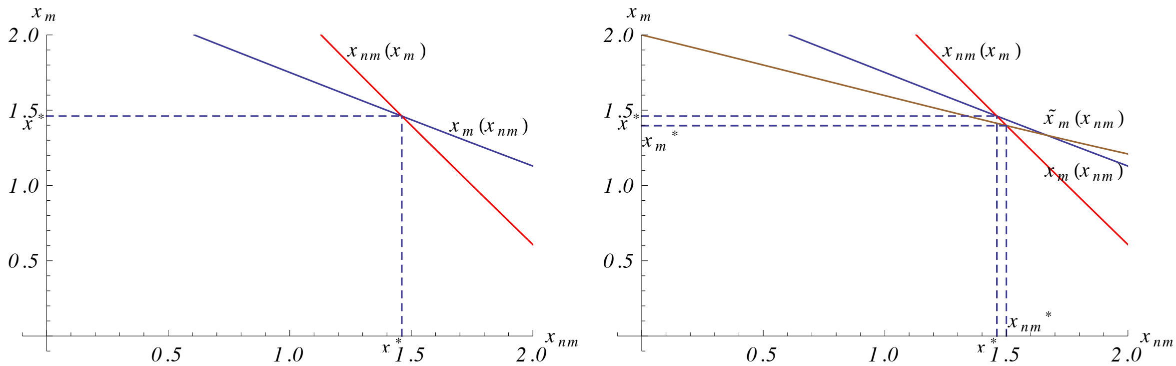


Figure 2: Post-merger market equilibrium

When the parameters of the model violate the condition in the proposition so that $\pi_B/\epsilon_B < \pi_A/\epsilon_A$, the merger results in the opposite result. That is, the merging firms put less effort in the A-project (and more in the B-project) and the non-merging firms do otherwise. This situation is illustrated in Figure 3. In this Figure we increase the parameter ϵ_B to 2.5; the rest of the parameters remain the same as in Figures 1 and 2. The graph on the left shows the pseudo best-response functions of the merging and non-merging firms in the pre-merger symmetric equilibrium. In the graph on the right we add the pseudo best-response function of the merging firms after the merger. It can be seen that the pseudo best-response function of the merging firms after the merger falls below the pseudo best-response function of the merging firms before the merger in a neighbourhood of the pre-merger market equilibrium. As a result, post-merger, the merging firms cut investment in the A-innovation (and increase it in the B-innovation) while the non-merging firms do the opposite.



(a) Pre-merger (b) Post-merger

Figure 3: Pre- and post-merger market equilibrium

As we will see later, irrespective of whether we use the consumer surplus or the social welfare standard, our welfare criterion depends on the aggregate investment level. Because investments of merging and non-merging firms move in opposite directions, the question that arises is what happens on aggregate. Suppose for example that $\pi_B/\epsilon_B > \pi_A/\epsilon_A$. As stated in Proposition 2, after a merger the merging firms increase their investment in the A-innovation path and decrease it in the B-innovation path; the non-merging firms do exactly the opposite. What happens on aggregate? Our next result addresses this issue.

Proposition 3. *The industry-wide investment in the A-project increases after a merger (and so industry-wide investment in the B-project decreases) if and only if condition (18) holds.*

Proof. See the Appendix. ■

Proposition 3 demonstrates that, on aggregate, after a merger the merging firms have a dominating influence over the non-merging firms. That is, despite the fact that after a merger the non-merging firms adjust their portfolio of investments in a direction opposite to that of the merging firms, aggregate investment changes in the same direction as the investment of the merging firms. Therefore, the market as a whole will allocate more funding to the A-project (and hence less to the B project) if and only if the parameters of the model satisfy the condition in Proposition 2.

Now that we know how the market will adjust investments after a merger, we move to examine the welfare effects of a merger. In doing so, we adopt a consumer welfare standard.

3.3 Consumer welfare effects of a merger

We define the social welfare criterion as the expected consumer surplus.[20] Using the Tullock's winning probabilities in the expression for expected consumer surplus (5) gives:

$$W = \frac{X}{X+\epsilon_A} S_A + \frac{n-X}{n+\epsilon_B - X} S_B, \tag{19}$$

where $X \equiv x_m + x_{nm}$ is the aggregate industry investment in the A-innovation path and $n - X$ the corresponding aggregate investment in the B-innovation one. Pre-merger, we denote this aggregate investment as $X^{pre} = nx^*$; post-merger, we denote it as $X^{post} = x_m^* + x_{nm}^*$. From Proposition 3 we know that $X^{post} > X^{pre}$ if and only if the market parameters satisfy $\pi_B/\epsilon_B > \pi_A/\epsilon_A$.

Note that the consumer welfare expression in (19) is strictly concave in X. Therefore, the aggregate industry investment in the A-innovation path that maximises the welfare criterion is given by the solution to the first order condition:

$$\frac{\epsilon_A}{(X+\epsilon_A)^2} S_A - \frac{\epsilon_B}{(n-X+\epsilon_B)^2} S_B = 0. \tag{20}$$

Let X_W^* be the unique solution to (20). We say that the market over- (under)-invests in the A-project when the aggregate investment in A exceeds (resp. falls short of) the socially optimal investment. In the pre-merger market equilibrium, over- (under)-investment thus occurs when $nx^* > (<) X_W^*$. Likewise, in the post-merger market equilibrium over- (under)-investment occurs when $x_m^* + x_{nm}^* > (<) X_W^*$.

Our next result provides conditions under which the pre-merger market equilibrium is distorted relative to the consumer surplus maximizing portfolio. For this purpose it is convenient to define the function:

$$f\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big) \equiv \frac{S_A}{S_B}\left(\frac{(n-1)(n+\epsilon_A)\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}} + \epsilon_A\Big(1 + n\sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}{(n+\epsilon_B)(n-1) + \epsilon_A\Big(n + \sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}\right)$$

Proposition 4. *Suppose the social planner maximizes the expected consumer surplus. The pre-merger market equilibrium exhibits under-investment in the A-project and correspondingly over-investment in the B-project if and only if:*

$$\frac{\pi_A}{\pi_B} < f\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big). \tag{21}$$

The pre-merger market over-invests in the A-innovation and under-invests in the B-innovation if and only if the above inequality is reversed.

Proof. See the Appendix. ■

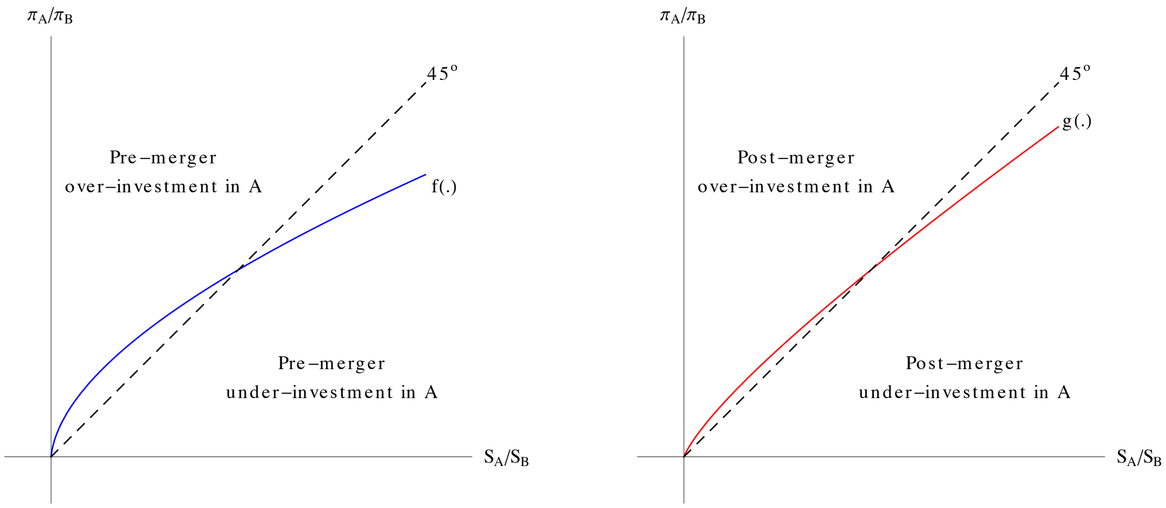
Proposition 4 characterizes the nature of the bias of the market investment portfolio in the pre-merger market equilibrium. As we discussed above, there are two sources of inefficiency in the portfolio of investments chosen by the firms. On the one hand, there is an inefficiency associated to competition because firms ignore the business-stealing and business-giving externalities associated to their portfolio

[20] Our results can easily be extended to the usual notion of social welfare, which includes firms' profits and consumer surplus.

choice. On the other hand, firms focus on their private profits rather than on the consumer surplus the various projects generate.

These two sources of inefficiency are nicely captured in condition (21) of Proposition 4. To see this, notice that if we set $n = 1$ in the condition we shut down the source of inefficiency related to competition and then condition (21) only reflects the inefficiency associated to the sole business focus of the firms. In fact, when $n = 1$ we have $f(S_A/S_B; n, \epsilon_A, \epsilon_B) = S_A/S_B$, which is the 45^o line in Figure 4(a), and the condition in the Proposition simplifies to $\pi_A/\pi_B < S_A/S_B$. Written as $\pi_A/S_A < \pi_B/S_B$, the Proposition then implies that a monopolist under-invests in the A-project so long as the (total) surplus appropriability in the market created by the A-innovation is lower than that in the market created by the B-innovation.[21]

The insight that the market tends to allocate too little funding to the A-innovation path and too much to the B-innovation one when $\pi_A/S_A < \pi_B/S_B$ also applies to the case of oligopoly. In fact, we note that the function f is increasing and concave in S_A/S_B and takes on value zero when S_A/S_B is zero. Therefore, given the other parameters of the model, the market invests too much in the A-project whenever π_A/π_B is sufficiently large compared to S_A/S_B. To illustrate, Figure 4(a) represents the regions of parameters for which there is over- and under-investment in the A-project. (Parameters are set to $n = 3$, $\epsilon_A = 3/2$ and $\epsilon_B = 3/4$.)



(a) Pre-merger

(b) Post-merger

Figure 4: Inefficiency of the market equilibrium

The effect of competition, n, on the market bias towards the more appropriable project is hard to disentangle analytically but numerical analysis of the RHS of condition (21) reveals that it depends on the ratio of consumer surpluses S_A/S_B. When S_A/S_B is small, then an increase in the number of firms tends to make the overinvestment problem in the more profitable project more likely. When S_A/S_B is large, the result is opposite.

Following the same approach as above, we can derive conditions under which the industry's innovation portfolio is biased in the post-merger market. For that purpose, consider the system of the FOCs for the

[21] Note that the condition $\pi_A/\pi_B > S_A/S_B$ is exactly equivalent to the condition $\pi_A/\pi_B > W_A/W_B$.

merged and non-merged firms. The FOC for the merged entity is given by equation (17). That for the non-merging firms is given by (14). Recall that the total investment in the A-innovation by the merged entity is $x_m = x_i + x_j$, while the aggregate investment by all non-merging firms is $x_{nm} = \sum_{k \neq i,j} x_k$. Summing up the FOC (17) and $(n-2)$ times the FOC (14) we get:

$$\frac{\frac{n-2}{n-1}X_m + \epsilon_A}{(X_m + \epsilon_A)^2}\pi_A - \frac{\frac{n-2}{n-1}(n - X_m) + \epsilon_B}{(n - X_m + \epsilon_B)^2}\pi_B = 0 \tag{22}$$

The above equation is expressed in terms of the post-merger industry's aggregate investment in the A-innovation, $X_m = x_m + x_{nm}$. Comparing this equation with the FOC (20) that gives the socially optimal investment, our next result provides conditions under which the post-merger market equilibrium is distorted relative to the consumer surplus maximizing portfolio. For that purpose, we define the function:

$$g\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big) \equiv \frac{S_A}{S_B}\left(\frac{(n-2)(n+\epsilon_A)\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}} + \epsilon_A\Big(1 + (n-1)\sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}{(n+\epsilon_B)(n-2) + \epsilon_A\Big(n - 1 + \sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}\Big)}\right).$$

Proposition 5. *Suppose the social planner maximizes the expected consumer surplus. The post-merger market equilibrium exhibits under-investment in the A-innovation and correspondingly over-investment in the B-innovation if and only if:*

$$\frac{\pi_A}{\pi_B} < g\big(S_A/S_B; n, \epsilon_A, \epsilon_B\big). \tag{23}$$

The market over-invests in the A-innovation and under-invests in the B-innovation if and only if the above inequality is reversed.

Proof. See the Appendix. ■

Proposition 5 shows that the post-merger market equilibrium portfolio will also in general deviate from the expected consumer surplus maximizing one due to competition and the sole business focus of the firms. Similarly to the pre-merger market equilibrium, if we set $n = 2$ in condition (23), we shut down the competition effect and we get exactly the same condition as above: firms overinvest in the A-project (and hence underinvest in the B-project) so long as $\pi_A/S_A < \pi_B/S_B$. Note that for $n = 2$, $g(S_A/S_B; n, \epsilon_A, \epsilon_B) = S_A/S_B$, which is given by the 45^o line in Figure 4(b). For arbitrary n, the condition in (23) is similar to that in (21): the RHS of (23) is an increasing and concave function of S_A/S_B, dividing the space $\pi_A/\pi_B - S_A/S_B$ into a region of parameters for which there is over-investment in the A-project and a region for which there is under-investment. We illustrate this in Figure 4(b).

We are now ready to address the question whether a merger increases consumer surplus or not. Suppose, for example, that the parameters of the model are such that both pre-merger and post-merger there is under-investment in the A-project and hence over-investment in the B-project. If after a merger firms collectively increase their aggregate investment in the A-project, which, as shown in Proposition 3, happens when $\pi_A/\epsilon_A < \pi_B/\epsilon_B$, then a merger is welfare improving.

Putting together Figures 4(a), 4(b) and the condition $\pi_A/\epsilon_A < \pi_B/\epsilon_B$, we can easily evaluate the welfare effects of a merger. We do this in Figure 5. Pairs of innovations above both curves f and g lead to over-investment in the A project and hence under-investment in the B-project, both in the pre- and the post-merger market equilibrium. Likewise, pairs of innovations below the curves f and g exhibit under-investment in the A-project and over-investment in the alternative project. For pairs of innovations that are above one curve, but below the other curve, a merger changes the market distortion. For pairs of innovations in between the two curves f and g and to the left of the crossing point there is under-investment in the A-project pre-merger but over-investment in the same project post-merger. The opposite happens to the right of the crossing point. The horizontal line with intercept ϵ_A/ϵ_B divides the space into two regions. Above the line, the A-innovation is relatively more profitable than the B-innovation and the opposite holds below the line.

As a result, we can split the space of parameters into four main regions. These regions are depicted in Figure 5 and it is apparent that they can be classified in terms of the relative profitability of the projects and the relative consumer surplus appropriability of the projects.

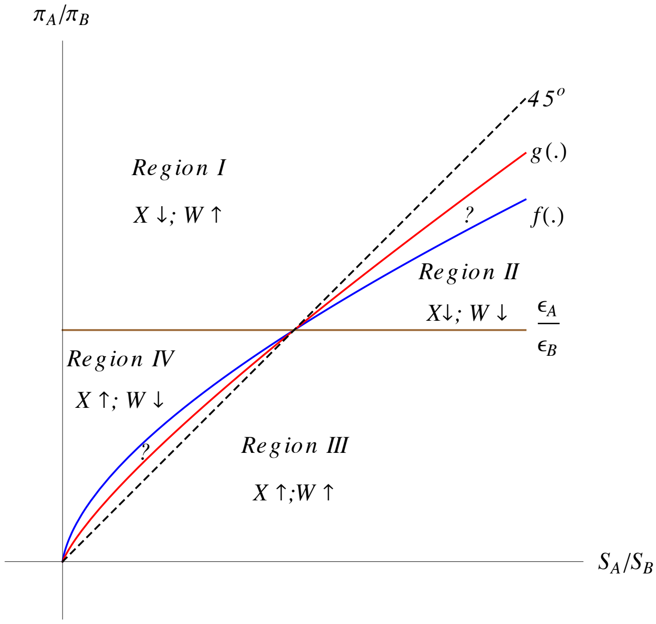


Figure 5: Consumer welfare effects of mergers

In Region I, the parameters satisfy the inequalities:

$$\text{Region I:} \qquad \frac{\pi_A}{\pi_B} > \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} > g(\cdot).$$

As explained above, for this region of parameters, both pre-merger and post-merger, there is over-investment in the A-project. Because the A-project is relatively more profitable than the B-project, by Proposition 3 we know that aggregate investment in the A project post-merger is lower than pre-merger. This implies that in Region I consumer welfare post-merger is higher than pre-merger, hence a merger is beneficial for consumers.

To better see the intuition behind this result, one can consider the subset of parameters in Region I for which

$$\frac{\pi_A}{\pi_B} > \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{S_A}{\epsilon_A} \ll \frac{S_B}{\epsilon_B}.$$

As these inequalities indicate, the relatively more profitable project, the A-project, is the one with the relatively lower consumer surplus. Because the merger reduces investment in the more profitable project, it increases investment in the more socially desirable project, the B-project, which renders the merger consumer welfare improving.

In Region II, the parameters satisfy the inequalities:

$$\text{Region II:} \qquad \frac{\pi_A}{\pi_B} > \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} < f(\cdot).$$

In contrast to the previous set of parameters, in Region II investment in the A-project is insufficient while it is excessive in the B-project. By Proposition 3, because the A-project is relatively more profitable than the B-project, investment in the A-project post-merger is lower than pre-merger. This aggravates the situation and welfare decreases.

In Region III of the figure, the parameters satisfy the inequalities:

$$\text{Region III:} \qquad \frac{\pi_A}{\pi_B} < \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} < g(\cdot),$$

As in the previous parameter set, in this region the market allocates too little funding to the A-project and too much to the B-project, both before and after a merger. However, because the A-project is relatively less profitable than the B-project, by Proposition 3, investment in the A-project after a merger is higher than pre-merger. As a result, a merger increases consumer welfare.

Finally, the parameters in Region IV satisfy the inequalities:

$$\text{Region IV:} \qquad \frac{\pi_A}{\pi_B} < \frac{\epsilon_A}{\epsilon_B} \text{ and } \frac{\pi_A}{\pi_B} > f(\cdot),$$

For these parameters, investment in the A-project is excessive and hence investment in the B-project insufficient. Because the A-project is relatively less profitable than the B-project, investment in the A-project after a merger is higher than pre-merger. This aggravates the misallocation problem and so consumer welfare decreases.

The following result summarises our findings:

Proposition 6. *Suppose the social planner maximizes the expected consumer surplus. For any fixed n, ϵ_A and ϵ_B, if the parameters of the model fall in:*
(i) Region I (resp. III): a merger increases social welfare by reducing (resp. increasing) investment in project A and increasing (resp. decreasing) it in project B.
(ii) Region II (resp. IV): a merger decreases social welfare by further reducing (resp. increasing) investment in project A and increasing (resp. reducing) it in project B.

We have not yet described the effects of a merger when the parameters of the model satisfy the inequalities:

$$\text{Ambiguity regions:} \qquad f(\cdot) > \frac{\pi_A}{\pi_B} > g(\cdot) \text{ and } f(\cdot) < \frac{\pi_A}{\pi_B} < g(\cdot).$$

When the parameters satisfy the first inequality and a merger occurs, the market moves from an under-investment problem to an over-investment problem in the A-project. When the parameters satisfy the second inequality, it is the other way around. Without specifying further the values of the parameters, for these ambiguity regions it is not possible to say whether welfare post-merger is higher or lower than pre-merger.

The analysis above yields a clear policy message:

Corollary 1. *When the project that is relatively more profitable for the firms is also the more appropriable, then a merger increases consumer welfare by reducing investment in the more profitable project and increasing investment in the alternative (less profitable) project. Otherwise, if the project that is relatively more profitable is also significantly less appropriable, then a merger reduces welfare.*

It is useful to provide a couple of examples to illustrate the power of Proposition 6.

Example 1: Linear demands and constant marginal costs. Suppose the winning firms face linear demands in markets A and B, and constant marginal costs. It is easy to verify that with linear demands and constant marginal costs $\pi_A/\pi_B = S_A/S_B$.[22] Hence, all the possible parameters lie in the 45 degrees line of Figure 5. From the analysis above, this implies that with linear demands, mergers always increase consumer surplus. In fact, when $\pi_A/\pi_B = S_A/S_B < \epsilon_A/\epsilon_B$, both the pre- and post-merger markets put too little funding on project A. Because in the post-merger equilibrium there is more investment in project A than in the pre-merger equilibrium, welfare is higher. When $\pi_A/\pi_B = S_A/S_B > \epsilon_A/\epsilon_B$, the situation is the opposite. There is too much investment in A both pre-merger and post-merger but a merger results in a cut of investment in A. As a result, welfare increases post-merger.

Example 2: Non-linear demand and constant marginal costs. Suppose the winning firms face demands $Q_\ell = K_\ell p^{-\eta_\ell}$, where $K_\ell > 0$, and $\eta_\ell > 1$, $\ell = A, B$. The parameter η_ℓ is the elasticity of demand. It is relatively easy to check that for each market ℓ it holds that $\pi_\ell/S_\ell = (\eta_\ell - 1)/\eta_\ell$. Therefore, we have:

$$\frac{\pi_A}{\pi_B} = \frac{\frac{\eta_A - 1}{\eta_A}}{\frac{\eta_B - 1}{\eta_B}} \frac{S_A}{S_B}$$

This is an increasing function of S_A/S_B, with a slope greater than 1 if and only if $\eta_A \geqslant \eta_B$. When the two markets have similar elasticities, the situation is alike to that with linear demands so a merger is welfare improving. However, when the elasticities are quite different, a merger may reduce welfare. For example, suppose that $\eta_A \gg \eta_B$; in this case, if $\pi_A/\epsilon_A > (<) \pi_B/\epsilon_B$ then a merger will tend to be favourable (unfavourable) for consumers.

4 Extensions

4.1 Price effects of mergers: a model of quality innovation and quantity competition

So far, we have examined the portfolio effects of mergers in a two-period model with winner-take-all contests. In the first period, the *pre-innovation* market, firms chose investment efforts for the A- and B-projects. In the second period, the *post-innovation market*, after the outcome of the firms' research efforts became known, the winning firm in each market put its product on sale and obtained monopoly profits. The simplicity of the model has served to isolate the novel issues that arise when examining firms' investment portfolios at the expense of ignoring other relevant aspects of innovation and mergers.

[22] With linear demand $D_\ell = a_\ell - b_\ell q_\ell$ in market $\ell = A, B$, profits are $\pi_\ell = (a_\ell - c_\ell)^2/4b_\ell$ and consumer surplus is $S_\ell = \pi_\ell/2$.

In particular, and most importantly, the winner-take-all contest formulation has *de facto* ruled out any price effect of mergers (Salant *et al.*, 1983; Davidson and Deneckere, 1985).

In order to demonstrate that the portfolio effects of mergers can have a dominating influence over the usual and well understood price effects of mergers, we now extend our model to a richer situation in which firms compete in quantities to sell differentiated products. To be precise, we adopt the well-known model of vertical and horizontal product differentiation of Sutton (1997, 1998), further studied e.g. by Symeonidys (2003). To serve our purpose, it will be sufficient to assume that there is competition in one of the markets.

Specifically, we examine the following two-stage game:

- In stage 1, firms engage in the research contests for the A- and B-innovations. In market A, the winning firm produces a good of high quality; the losing firms produce goods of low quality. If no firm succeeds, all firms produce goods of low quality. The market for the B-innovation continues to be a winner-take-all market as in Section 2.

- In stage 2, in the market generated by the A-innovation winning and losing firms engage in Cournot competition and choose quantities to maximize their profits. In the market created by the B-innovation the winning firm produces the monopoly quantitiy and gets a profit of π_B; the losing firms obtain zero profits.

In market A, thus, there are n firms selling differentiated products. Let us denote the quantity-quality combination produced by firm i by $\{q_i, s_i\}$, and the price received p_i. The representative consumer's utility in market A is given by

$$U^A = \sum_{i=1}^{n} \left[q_i - \left(\frac{q_i}{s_i} \right)^2 \right] - \sigma \sum_{i<j} \frac{q_i}{s_i} \frac{q_j}{s_j} - \sum_{i=1}^{n} p_i q_i, \tag{24}$$

where

$$s_i = \begin{cases} \overline{s} \text{ if } i \text{ wins contest for } A\text{-innovation} \\ \underline{s} \text{ otherwise.} \end{cases}$$

The corresponding system of inverse demands for the products of the firms in market A is:

$$p_i = 1 - \frac{2q_i}{s_i^2} - \frac{\sigma}{s_i} \sum_{j \neq i} \frac{q_j}{s_j}, \ i = 1, 2, ..., n. \tag{25}$$

The parameter $\sigma \in [0, 2]$ is an inverse measure of the degree of horizontal product differentiation. When $\sigma \to 2$, unless they are differentiated in terms of quality, products are homogeneous. When $\sigma \to 0$, products become independent. In order to present a model as close as possible to our baseline model of Section 3, we set $\sigma = 2$ in what follows. In such a case, we are back to our baseline model in the limiting case when $\underline{s} = 0$. When $\underline{s} = 0$ only the winner of the contest obtains positive profits and when all firms lose the contest they all make zero profits. This is the case that isolates the portfolio effects of mergers. As we increase $\underline{s}$ (relative to $\overline{s}$), the product of the losing firms becomes more attractive for the consumers. Yet, if $\underline{s}$ is positive but sufficiently low, the winner of a contest can monopolize the market and therefore losing firms only obtain positive profits when no firm wins the contest and therefore they all produce low quality. Here mergers have price effects. When $\underline{s}$ becomes sufficiently large, losing firms make positive profits even if there is a winner of the contest. We also normalize the marginal cost of production to zero. In this situation, mergers have even stronger price effects.[23]

Pre-merger market

We first consider the portfolio problem in the pre-merger market corresponding to the A-project. To characterize the equilibrium of the two-stage game, we proceed by backwards induction. That is, we first solve the continuation game firms play in the post-innovation market. Then, folding the game backwards, we examine firms' decisions in the pre-innovation market.

Stage 2, post-contests markets

In market A there are two types of subgames. In the first type of subgame, one of the firms has succeeded at innovating and produces the good of high quality $\overline{s}$; the rest of the firms all produce a good of low quality $\underline{s}$. In the second subgame, all firms fail at obtaining the innovation and produce a good of low quality $\underline{s}$. In market B there are also two such subgames but, because of the winner-take-all feature of the B-innovation, the continuation payoffs are the same as in the main model of Section 2.

Therefore, we now focus on market A. Consider first the subgame in which one of the firms innovates and sells a high-quality product, $\overline{s}$, while the rest of the firms sell a low-quality product, $\underline{s}$. Standard derivations yield the Cournot equilibrium:

$$\overline{q}_A = \begin{cases} \frac{\overline{s}^2}{4} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[n(\overline{s}-\underline{s})+\underline{s}]}{2(n+1)} & \text{otherwise} \end{cases} \tag{26}$$

$$\underline{q}_A = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2(n+1)} & \text{otherwise} \end{cases} \tag{27}$$

where $\overline{q}_A$ is the quantity put in the market by the high-quality seller and $\underline{q}_A$ the quantity sold by each of the low-quality sellers.

From expression (26)-(27), note that the firms that lose the contest and sell quality $\underline{s}$ put a positive quantity in the market only if the quality disadvantage vis-á-vis the winning firm is not excessive. If this condition is violated, the winning firm monopolizes the market.

The winner's and losers' profits, denoted $\overline{\pi}$ and $\underline{\pi}$ respectively, are equal to:

$$\overline{\pi}_A = \left(\frac{\overline{q}_A}{\overline{s}}\right)^2, \underline{\pi}_A = \left(\frac{\underline{q}_A}{\underline{s}}\right)^2, \tag{28}$$

while consumer surplus is given by

$$\overline{S}_A = \left(\frac{(n-1)\overline{s}\underline{q}_A + \underline{s}\overline{q}_A}{\overline{s}\underline{s}}\right)^2 \tag{29}$$

[23] We have performed the analysis for an arbitrary degree of product differentiation and the insights remain the same. Later we shall report some results for cases in which the firms' products are horizontally differentiated.

The other type of subgame is one in which no firm succeeds at innovating in the pre-innovation stage, in which case all firms sell a product of quality $\underline{s}$. Standard derivations yield the symmetric Cournot equilibrium:

$$q_A^* = \frac{\underline{s}^2}{2(n+1)}. \tag{30}$$

In this situation, all firms earn profits

$$\pi_A^* = \left(\frac{q_A^*}{\underline{s}}\right)^2,$$

and consumer surplus is given by

$$S_A^* = \left(n\frac{q_A^*}{\underline{s}}\right)^2 = \left(\frac{n\underline{s}}{2(n+1)}\right)^2.$$

It is obvious that for $\overline{s} > \underline{s}$, the inequalities $\overline{\pi}_A > \pi_A^* > \underline{\pi}_A$ and $\overline{S}_A > S_A^*$ hold.

Stage 1

We are now ready to fold the game backwards and look at the pre-innovation stage of the pre-merger market. In this stage, anticipating the payoffs from the continuation subgames, firms choose their portfolio of investments to maximize expected profits. Using the profits notation introduced above, the expected profits of a firm i investing an amount x_i in project A and an amount $1 - x_i$ in project B are:

$$\begin{aligned} u_i(x_i, \mathbf{x}_{-i}) &= p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\overline{\pi}_A + \sum_{j\neq i} p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\underline{\pi}_A + \Big(1 - \sum_{k=1}^{n} p_k(x_k, \mathbf{x}_{-k}, \epsilon_A)\Big)\pi_A^* \\ &+ q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B, \end{aligned} \tag{31}$$

where the success probabilities are the same as the Tullock probabilities given in (7)-(8).

The first line of this expression is the expected payoff from investing in the A-project. With probability $p_i(\cdot)$, firm i wins the contest and obtains the profits corresponding to a high quality seller in the Cournot market; with probability $\sum_{j\neq i} p_j(\cdot)$, some other firm wins the contest so firm i obtains the profits corresponding to a low-quality seller; finally, with probability $1 - \sum_{k=1}^{n} p_k(\cdot)$, no firm wins the contest and firm i gets the symmetric equilibrium profits π_A^*. The second line of the payoff in (31) is the expected profit from investing in the B-project. Firm i only obtains rents when winning the contest, which happens with probability $q_i(\cdot)$ and the rents are equal to π_B.

In terms of the incentives of the firms to invest in the A- and B-projects, the new firm's payoff (31) represents a similar trade-off as that in the basic model of Section 2. It is then straightforward to extend the existence result in Proposition 1 to this setting, which we omit to save on space. To derive the pre-merger equilibrium investment, we take the FOC, apply symmetry $x_i = x^*$, $i = 1, 2, ..., n$ and solve for x^*.

We shall compare the welfare generated pre-merger to that after a merger. In the pre-merger market, using the consumer welfare standard, social welfare is given by the expression:

$$W(x_i, x_{-i}) = \sum_{i=1}^{n} p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\overline{S}_A + \Big(1 - \sum_{i=1}^{n} p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\Big)S_A^* + \sum_{i=1}^{n} q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)S_B.$$

In the above formula, the first two terms represent the expected consumer benefits from firm participation in the contest for introducing the A-innovation. With probability $\sum_{i=1}^{n} p_i(\cdot)$, the A-innovation is obtained by one of the firms, in which case consumer surplus is $\overline{S}_A$; with the remaining probability, no firm wins the contest for the A-innovation and the surplus generated is S_A^*. The last term is the value to consumers arising from the B-innovation.

Plugging the symmetric equilibrium x^* into this consumer surplus expression gives the pre-merger market social welfare level $W(x^*, \mathbf{x}^*)$.

Post-merger Market

We now present the analysis for the post-merger market. Suppose firms i and j merge. As in the basic model of Section 2, firms will choose their portfolio of investments to maximize the sum of the partners' pre-innovation profits. What is new in this richer model is that a merger brings detrimental price effects in market A. Compared to the situation before the merger, the equilibrium quantities of the merging firms in market A will reflect the internalization of the (negative) quantity externalities they impose on one another (cf. Salant *et al.*, 1983).

Stage 2

Post-merger, in the market created by the A-project, there are three types of subgames. We label these subgames as I, II and III. In the first type of subgame, subgame I, one of the merging firms wins the contest for the innovation. In such a case, the two partner firms produce a product of high quality $\overline{s}$, while the rest of the firms produce a good of low quality $\underline{s}$. In the second type of subgame, subgame II, one firm other than the partners to the merger obtains the innovation and produces a good of high quality $\overline{s}$; the rest of the firms, including the merging firms, produce a good of low quality $\underline{s}$. In the third type of subgame, subgame III, all firms, merging and non-merging, fail to innovate and all produce a good of low quality $\underline{s}$. In all these subgames, the merging firms coordinate their quantities to maximize the joint post-innovation profits and this coordination generates detrimental price effects of mergers.

Consider the first type of subgame. The merged firms produce high quality and coordinate their production; the rest of the firms sell low quality and operate independently. Standard derivations yield the Cournot equilibrium:

$$\overline{q}_A^{m,I} = \begin{cases} \frac{\overline{s}^2}{8} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[(n-1)\overline{s}-(n-2)\underline{s}]}{4n} & \text{otherwise} \end{cases} \tag{32}$$

$$\underline{q}_A^{nm,I} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2n} & \text{otherwise} \end{cases} \tag{33}$$

where $\overline{q}^{m,I}$ denotes the quantity produced by each of the merging firms and $\overline{q}^{nm,I}$ the quantity produced by each of the non-merging firms. Note that the assumption $\overline{s} < 4\underline{s}$ suffices for the low-quality non-merging firms to supply a positive quantity in the market.

The equilibrium profits of the merging and non-merging firms in this subgame are

$$\overline{\pi}_A^{m,I} = 8\left(\frac{\overline{q}^{m,I}}{\overline{s}}\right)^2, \ \underline{\pi}_A^{nm,I} = 2\left(\frac{\underline{q}^{nm,I}}{\underline{s}}\right)^2.$$

Consumer surplus is given by

$$S_A^I = \left(\frac{(n-2)\overline{s}\underline{q}_A^{nm,I} + 2\underline{s}\overline{q}_A^{m,I}}{\overline{s}\underline{s}}\right)^2.$$

In the second type of subgame, denoted II, a non-merging firm, say k, successfully introduces the A-innovation, in which case it produces high quality and commercializes it independently. The rest of non-merging firms sell low quality and also operate independently. Finally, the merging firms also sell low-quality but coordinate their quantities to maximize profits. Standard derivations yield the Cournot equilibrium:

$$\underline{q}_A^{m,II} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{4n} & \text{otherwise} \end{cases} \tag{34}$$

$$\overline{q}_A^{nm,II} = \begin{cases} \frac{\overline{s}^2}{4} & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\overline{s}[(n-1)\overline{s}-(n-2)\underline{s}]}{2n} & \text{otherwise} \end{cases} \tag{35}$$

$$\underline{q}_A^{nm,II} = \begin{cases} 0 & \text{if } \underline{s} < \overline{s}/2 \\ \frac{\underline{s}(2\underline{s}-\overline{s})}{2n} & \text{otherwise} \end{cases} \tag{36}$$

where $\underline{q}_A^{m,II}$ is the equilibrium quantity of the low-quality merging firms, $\overline{q}_A^{nm,II}$ is the quantity of the high-quality non-merging firm and $\underline{q}_A^{nm,II}$ is the quantity of the rest of the non-merging firms, which also sell low quality.

The corresponding profits are:

$$\underline{\pi}_A^{m,II} = 8\left(\frac{\underline{q}_A^{m,II}}{\underline{s}}\right)^2, \ \overline{\pi}_A^{nm,II} = 2\left(\frac{\overline{q}_A^{nm,II}}{\overline{s}}\right)^2, \ \underline{\pi}_A^{nm,II} = 2\left(\frac{\underline{q}_A^{nm,II}}{\underline{s}}\right)^2.$$

In this second type of subgame, consumer surplus is given by

$$S_A^{II} = \left(\frac{(n-3)\overline{s}\underline{q}_A^{nm,II} + 2\overline{s}\underline{q}_A^{m,II} + \underline{s}\overline{q}_A^{nm,II}}{\overline{s}\underline{s}}\right)^2.$$

Finally, in the last type of subgame, no firm introduces the A-innovation and all the firms produce a low-quality product. Non-merging firms operate independently while merging firms coordinate their quantities. Standard derivations yield the Cournot equilibrium quantities:

$$\underline{q}_A^{m,III} = \frac{\underline{s}^2}{4n}, \ \underline{q}_A^{nm,III} = \frac{\underline{s}^2}{2n}.$$

The profits firms obtain are given by:

$$\pi_A^{m,III} = 8\left(\frac{q_A^{m,III}}{\underline{s}}\right)^2, \ \pi_A^{nm,III} = 2\left(\frac{q_A^{nm,III}}{\underline{s}}\right)^2.$$

Consumer surplus is given by

$$S_A^{III} = \left(\frac{2\underline{q}_A^{m,III} + (n-2)\underline{q}_A^{nm,III}}{\underline{s}}\right)^2 = \frac{(n-1)^2\underline{s}^2}{4n^2}.$$

Stage 1

In the pre-innovation stage of the post-merger market, anticipating the payoffs in the continuation subgames I, II, and III, merging and non-merging firms choose their portfolio of investments to maximize their expected payoffs.

The expected payoff of the merged entity is given by:

$$\begin{aligned} u_m(x_i, \mathbf{x}_{-i}) &= \Big(p_i(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)\overline{\pi}_A^{m,I} + \sum_{k \neq i,j} p_k(x_k, \mathbf{x}_{-k}, \epsilon_A)\underline{\pi}_A^{m,II} \\ &+ \Big(1 - \sum_{k=1}^{n} p_k(x_k, \mathbf{x}_{-k}, \epsilon_A)\Big)\underline{\pi}_A^{m,III} + q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)\pi_B, \end{aligned}$$

where the success probabilities are again the Tullock's ones as in (7)-(8).

Likewise, the expected payoff of a non-merging firm is given by

$$\begin{aligned} u_{nm}(x_k, x_{-k}) &= p_k(x_k, \mathbf{x}_{-k}, \epsilon_A)\overline{\pi}_A^{nm,II} + \Big(p_i(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)\underline{\pi}_A^{nm,I} \\ &+ \sum_{l \neq i,j,k} p_l(x_l, \mathbf{x}_{-l}, \epsilon_A)\underline{\pi}_A^{nm,II} + \Big(1 - \sum_{m=1}^{n} p_m(x_m, \mathbf{x}_{-m}, \epsilon_A)\Big)\underline{\pi}_A^{nm,III} \\ &+ q_k(1 - x_k, \mathbf{1} - \mathbf{x}_{-k}, \epsilon_B)\pi_B \end{aligned}$$

To derive the equilibrium of the investment game, we take the FOCs, apply symmetry for the merging and non-merging firms, i.e. $x_i = x_j = x^m$ and $x_k = x^{nm}$, $k \neq i, j$, and solve for x^m and x^{nm}. Let us denote the equilibrium vector of investments post-merger by $\mathbf{x}^m$.

We evaluate the merger in terms of consumer welfare, which again depends on whether the innovations are obtained or not, as well as on whether the successful firms are merging or non-merging firms. Taking into account the probability with which the different events occur, consumer welfare is given by:

$$\begin{aligned} W_m(x_i, x_{-i}) &= \Big(p_i(x_i, \mathbf{x}_{-i}, \epsilon_A) + p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\Big)S_A^I + \sum_{k=i,j}^{n} p_k(x_k, \mathbf{x}_{-k}, \epsilon_A)S_A^{II} \\ &+ \Big(1 - \sum_{l=1}^{n} p_l(x_l, \mathbf{x}_{-l}, \epsilon_A)\Big)S_A^{III} + \sum_{j=1}^{n} q_j(1 - x_j, \mathbf{1} - \mathbf{x}_{-j}, \epsilon_B)S_B \end{aligned}$$

In the above expression, the first two terms represent the expected benefit to consumers from introducing the A-innovation depending on who innovates: the merging firm or one of the non-merging firms. The third term is the expected benefit when all firms fail to introduce the A-innovation in the market. The last term corresponds to the market generated by the B-innovation.

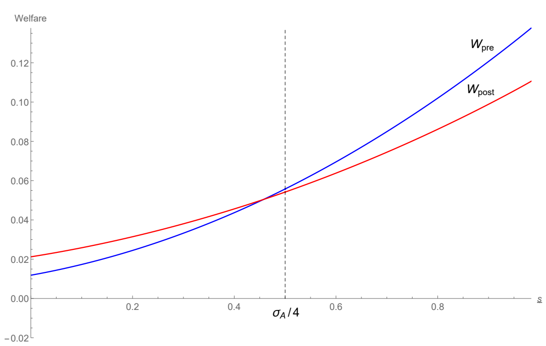
Results

In the basic model of this paper, we have analyzed the impact of mergers on equilibrium research portfolios and concluded that mergers align the incentives of the merging firms to those of the planner when the relatively more profitable projects is also the relatively more appropriable. In that analysis, standard market power effects of mergers (price increases) did not play a role because of the winner-take-all feature of the contests. The present model incorporates the detrimental price effects of mergers because merging firms are allowed to coordinate their prices after merger. The question that arises is whether the positive portfolio effects of mergers can have a dominating influence over the negative price effects of mergers. We address this question in the remaining of this section.

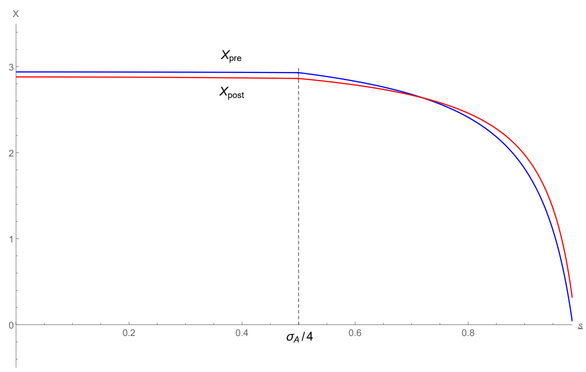
An analytical approach to the above question is more difficult than in the baseline model of Section 3 because of the richness of subgames in stage 2, specially after a merger. We thus proceed to a numerical analysis that shows that mergers may improve consumer welfare, despite the detrimental price effects of mergers.

In Figure 6 we represent pre- and post-merger welfare levels and aggregate investment in the A project as a function of $\underline{s}$. In these graphs we set the parameters of the model as follows. For the A project, we set $\epsilon_A = 1$, $\overline{s} = 1$ and let $\underline{s}$ vary from 0 to 1. For the B-project, which is winner-take-all, we just set $\epsilon_B = 1/2$, $\pi_B = 1/72$ and $S_B = 1/144$.

We emphasize that when $\underline{s} = 0$, we are back to our model of Section 3 in which mergers have no price effects at all. As shown earlier in the paper, if the relatively more profitable project is also the relatively more appropriable, then a merger increases consumer welfare by reducing investment in the more profitable project. This is exactly what we see in Figure 6. It is straightforward to verify that, in a neighbourhood of $\underline{s} = 0$, the inequality $\overline{\pi}_A/\epsilon_A > \pi_B/\epsilon_B$ holds. Therefore, a merger increases welfare (see Figure 6(a)) by reducing investment (see Figure 6(b)) in the more profitable project. Note further that when $\underline{s} = 0$ demands are linear (and marginal costs are constant) so all mergers are profitable.



(a) Consumer welfare



(b) Aggregate investment in A

Figure 6: Pre- and post-merger welfare and aggregate investment in the A-project

As we increase $\underline{s}$ away from zero, in addition to the portfolio effects of mergers, the price effects of mergers start playing a role. In the graphs we depict the threshold value corresponding to the condition

$\underline{s} = \overline{s}/2$ (see e.g. the equilibrium quantities (26)-(27)), which is $\sigma_A/4 = 1/2$. When $\underline{s} < 1/2$, the winner of the contest for the A-project monopolizes the post-innovation market. However, if no firm obtains the innovation, then all the firms, merging and non-merging, produce low quality and compete. This competition is the source of detrimental price effects when firms i and j merge in this region of parameters. As $\underline{s}$ increases towards 1/2, the price effects remain similar (they do not depend on $\underline{s}$) while the profitability of the A-project decreases relative to that of the B-project. (Investment in A pre-and post-merger correspondingly decreases, though this is hardly perceptible in the graph.) This weakens the portfolio effect of mergers while keeping the price effects similar. Eventually, as Figure 6(a) shows, as the degree of quality differentiation vanishes ($\underline{s} \to \overline{s}$), the merger becomes detrimental for consumers.

Beyond the threshold value $\underline{s} > 1/2$, much of the same occurs but note that the price effects of mergers in this region are much larger than when $\underline{s} < 1/2$ because they also arise when one of the firms wins the contest. Observe also that when $\underline{s}$ approaches $\underline{s} = 1$, the market associated to the A-project turns extremely competitive because the products are virtually identical (neither vertical nor horizontal differentiation whatsoever). In this situation price effects are strongest (like in a Cournot market for homogeneous products) and moreover the profitability of the A-project ends up being below that of project B so that a merger results in an increase in investment in the A-project rather than in a decrease.

While constructing Figure 6 we have set $\sigma_A = 2$, which implies that products are not horizontally differentiated at all. In the Appendix we report Figure 9, which corresponds to the case in which $\sigma_A = 1$. As explained in the Appendix, the insights we obtain are similar: for low $\underline{s}$, a merger increases welfare by reducing investment in the relatively more profitable project, while for high $\underline{s}$ a merger reduces welfare because the price effects are quite strong.

We close this section with a comment on the incentive-compatibility of mergers. In the baseline model of Section 3, mergers tend to be not incentive-compatible. This result is reminiscent of mergers in Cournot settings. As shown above, investments are strategic substitutes so that when the merged entity, say, reduces investment in the relatively more profitable project, rival firms increase it. This reaction of the rival firms lowers the probability the merged entity wins the contest for the relatively more profitable project, which makes merger profitability more difficult. When project A is not winner-take-all and firms sell horizontally differentiated products, mergers are incentive-compatible. In the Appendix we report the profits of the merging and non-merging firms for the case in which $\sigma_A = 1$ in Figure 10. It can be seen that mergers increase the profits of the merging and non-merging firms; moreover, the profits of the latter increase to a lower extent when $\underline{s}$ is relatively low, while the opposite holds when $\underline{s}$ is relatively high. Note that with $\sigma_A = 1$ mergers are profitable even if $\underline{s} \to 0$. The reason is that products are horizontally differentiated and in such a case the quantity effects of a merger increase the continuation game payoff of the merged entity. Even though the investments in the A-project are strategic substitutes and merger profitability is compromised, the former effect dominates and merging turns out to be incentive-compatible. However, when $\underline{s} \to \overline{s}$, firms' products are not anymore vertically differentiated and, like in the merger paradox, the internalization of the price effects of mergers tend to lower the profitability of the insiders and favor the outsiders.

4.2 General success probabilities and strategic complements

In the baseline model of Section 3, we have assumed a specific functional form for the success probabilities. The reason for adopting a particular functional form was the difficulty to derive clear-cut parameter conditions under which mergers were welfare improving/reducing. In this section we return to the general formulation of Section 2 and show that the main insights from the Tullock model continue to hold in general. Two important differences should however be highlighted. First, with general success probabilities, the game may be one of strategic complements. Second, with general success probabilities, the welfare function depends on the vector of investments, rather than on the aggregate level of investment in the A-project. Despite these two differences, the mechanism by which mergers may increase welfare is the same and we develop below a graphical approach to demonstrate it.

Pre-merger market equilibrium

We start by stating a more general existence result. Using the general payoff formulation in (1) we can show that:

Proposition 7. *In the general game of portfolio choice, there exists a unique pre-merger SNE denoted* $x_i^* = x^* \in [0,1]$, $i = 1, 2, ..., n$. *The SNE is interior provided that*

$$\frac{\partial p_i(0, \mathbf{0}_{-i}, \epsilon_A)}{\partial x_i}\pi_A > \left|\frac{\partial q_i(1, \mathbf{1}_{-i}, \epsilon_B)}{\partial x_i}\right|\pi_B \text{ and } \frac{\partial p_i(1, \mathbf{1}_{-i}, \epsilon_A)}{\partial x_i}\pi_A < \left|\frac{\partial q_i(0, \mathbf{0}_{-i}, \epsilon_B)}{\partial x_i}\right|\pi_B,$$

in which case it is given by the x^* *that solves:*

$$\frac{\partial p_i(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(1 - x^*, \mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B = 0. \tag{37}$$

Proof. See the Appendix.

As we are interested only in interior solutions, we will assume from now on that the conditions for an interior equilibrium hold. Like in the model of Section 3, a higher reward from the A-innovation shifts up the marginal gains from investing in it and correspondingly x^* increases. A higher reward from the B-innovation raises the marginal gains from investing in it and correspondingly $1 - x^*$ increases, so x^* decreases. It is also easy to conclude that x^* decreases in the difficulty ϵ_A with which the A-innovation materialises, and decreases in ϵ_B.

Post-merger market equilibrium

When firms i and j merge, the merged entity chooses investments x_i and x_j in the A-project (and by implication $1-x_i$ and $1-x_j$ in the B-project) to maximise the payoff in (3). The existence of an interior equilibrium can easily be established by providing additional conditions under which the payoff of the merged entity is strictly concave in x_i and x_j on the one hand, and corresponding conditions for an internal solution to the system of FOCs on the other hand. [24]

Let $(x_i^m, x_j^m, \mathbf{x}_{-i,j}^m)$ denote the post-merger equilibrium vector of investments in the A-project. Assuming that the equilibrium is interior, we can state the following result comparing the pre- and post-merger equilibria.

Proposition 8. *Assume that a merger between firms i and j occurs in the general game of portfolio choice. Then, compared to the pre-merger equilibrium, the merged firms will invest more in the A-innovation and less in the B-innovation, that is, $x_i^m > x^*$ and $x_j^m > x^*$, if and only if for firms i and j at the pre-merger equilibrium of Proposition 7 it holds that:*

$$\frac{\pi_A}{\pi_B} < -\frac{\frac{\partial q_j(\mathbf{1}-\mathbf{x}^*,\epsilon_B)}{\partial x_i}}{\frac{\partial p_j(\mathbf{x}^*,\epsilon_A)}{\partial x_i}}. \tag{38}$$

The non-merging firms will invest less (more) in the A-innovation, that is $\mathbf{x}_{-i,j}^m < \mathbf{x}_{-i,j}^*$ ($\mathbf{x}_{-i,j}^m > \mathbf{x}_{-i,j}^*$) and more (less) in the B-innovation if firms' investments are strategic substitutes (complements).

Proof. See the Appendix.

Proposition 8 states that, compared to the situation before a merger, the merging firms will put more effort in the A-innovation and less in the B-innovation if and only if condition (38) holds. This condition governs the nature of the total externality exerted by the investment of a firm in the A-innovation path and it tends to be satisfied when the rewards from the A-innovation relative to the B-innovation are low enough.[25]

Social welfare

We evaluate the performance of mergers using the welfare function in (5). Our next result characterizes the inefficiency of the pre-merger market equilibrium. To present it, it is useful to denote the gradient

[24] For this, the Hessian matrix has to be definite negative. It is then sufficient that

(a) $\left|\frac{\partial^2 p_i}{\partial x_i^2}\right| > \left|\frac{\partial^2 p_j}{\partial x_i^2}\right|$ and $\left|\frac{\partial^2 q_i}{\partial x_i^2}\right| > \left|\frac{\partial^2 q_j}{\partial x_i^2}\right|$

(b) $\left|\frac{\partial^2 p_j}{\partial x_j^2}\right| > \left|\frac{\partial^2 p_i}{\partial x_j^2}\right|$ and $\left|\frac{\partial^2 q_j}{\partial x_j^2}\right| > \left|\frac{\partial^2 q_i}{\partial x_j^2}\right|$

(c) $\left(\frac{\partial^2 p_i}{\partial x_i^2}\pi_A + \frac{\partial^2 p_j}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i}{\partial x_i^2}\pi_B + \frac{\partial^2 q_i}{\partial x_i^2}\pi_B\right)^2 - \left(\frac{\partial^2 p_i}{\partial x_i \partial x_j}\pi_A + \frac{\partial^2 p_j}{\partial x_i \partial x_j}\pi_A + \frac{\partial^2 q_i}{\partial x_i \partial x_j}\pi_B + \frac{\partial^2 q_j}{\partial x_i \partial x_j}\pi_B\right)^2 > 0.$

[25] Formally, this follows from the following observations. An increase in π_A (or a decrease in π_B) raises the LHS of (38). At the same time, as shown in Proposition 7, the pre-merger equilibrium investment increases. Because of Assumption 1d, this decreases the numerator and increases the denominator of the RHS of (38), making the inequality more difficult to hold.

of the social welfare function evaluated at the pre-merger equilibrium as:[26]

$$\Delta W(\mathbf{x}^*) = \left(\frac{\partial W(\mathbf{x}^*)}{\partial x_1}, \frac{\partial W(\mathbf{x}^*)}{\partial x_2}, ..., \frac{\partial W(\mathbf{x}^*)}{\partial x_n} \right).$$

Proposition 9. *In the general game of portfolio choice, the pre-merger market equilibrium investment in the A-innovation is insufficient (and, correspondingly, excessive in the B-innovation) from the point of view of social welfare, that is,* $\mathbf{x}^* < \mathbf{x}^o$*, if and only if*

$$\Delta W(\mathbf{x}^*) > 0. \tag{39}$$

When the inequality (39) is reversed, the market invests too much in the A-innovation and too little in the B-innovation.

Proof. See the Appendix.

Proposition 9 characterizes the nature of the bias of the pre-merger market equilibrium investment portfolio. Under the condition in this Proposition, firms invest too little in the A-innovation path. As shown in Proposition 8, a merger satisfying condition (38) will give incentives to the merging firms to invest more in the A innovation path. Will then a merger increase social welfare? As mentioned earlier, addressing this question requires us to pay attention to two aspects. First, even though the merged firms will invest more in the A-innovation, the non-merging firms may or may not invest more in the A innovation. This depends on whether the game is of strategic complements or of strategic substitutes. The second aspect is that even if all the firms invest more in the A-innovation after a merger this does not guarantee that welfare increases because it is possible that they invest too much. That is, a merger may cause the market to move from under-investment to over-investment in project A. If the over-investment problem is not too large, welfare will increase after the merger but otherwise welfare will decrease.

Our next result provides conditions under which a merger is a corrective device. Before presenting it,

[26]Each component of the gradient evaluated at the pre-merger equilibrium is given by the expression:

$$(S_A - \pi_A)\frac{\partial p_i(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i} + S_A \sum_{j \neq i}^{n} \frac{\partial p_j(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}$$
$$+ (S_B - \pi_B)\frac{\partial q_i(1 - x^*, \mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i} + S_B \sum_{j \neq i}^{n} \frac{\partial q_j(1 - x^*, \mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i} > 0, \; i = 1, 2, ..., n,$$

where we have used the fact that x^* solves the FOC (37).

let us denote the gradient of the social welfare function evaluated at the post-merger equilibrium by:[27]

$$\Delta W(x_i^m, x_j^m, \mathbf{x}_{-i,j}^m) = \left(\frac{\partial W(x_i^m, x_j^m, \mathbf{x}_{-i,j}^m)}{\partial x_i}, \frac{\partial W(x_i^m, x_j^m, \mathbf{x}_{-i,j}^m)}{\partial x_j}, \frac{\partial W(x_i^m, x_j^m, \mathbf{x}_{-i,j}^m)}{\partial \mathbf{x}_{-ij}} \right) \tag{40}$$

Proposition 10. *In the general game of portfolio choice, suppose the parameters of the model are such that firms' investments are strategic complements. Then:*

a. *If conditions (38), (39) hold and the gradient (40) is strictly positive, then a merger will increase welfare by raising innovation in the A-project.*

b. *If neither condition (38) nor (39) hold and the gradient (40) is strictly negative, then a merger will increase welfare by raising innovation in the B-project.*

c. *If condition (38) does not hold, condition (39) holds, and the gradient (40) is strictly positive, a merger will reduce welfare by decreasing innovation in the A-project.*

d. *If condition (38) holds, condition (39) does not hold and the gradient (40) is strictly negative, a merger will reduce welfare by decreasing innovation in the B-project.*

Proof. See the Appendix.

The result in Proposition 10 is intuitive. If there is too little funding allocated to project A in the pre-merger equilibrium and all the firms invest more in such a project in the post-merger equilibrium, then it is likely that welfare post-merger is higher than welfare pre-merger.[28] To ensure that welfare increases after a merger, it suffices to require condition (40) on the gradient evaluated at the post-merger equilibrium in Proposition 10. This condition ensures that post-merger there is still under-investment and therefore the unique global maximum of the welfare function has not been reached yet.

It is useful to illustrate the result in Proposition 10 graphically. In Figure 7 we have represented an arbitrary family of iso-welfare curves in the $x_{-ij}^m - x_i^m$ space. In the vertical axes we have the investment in the A-project of a representative merging firm, x_i^m, while in the horizontal axis we place

[27]To be sure, the n components of the gradient evaluated at the post-merger equilibrium are:

$$(S_A - \pi_A)\frac{\partial p_i(x_i^m, x_j^m, \mathbf{x}_{-ij}^m, \epsilon_A)}{\partial x_i} + (S_A - \pi_A)\frac{\partial p_j(x_i^m, x_j^m, \mathbf{x}_{-ij}^m, \epsilon_A)}{\partial x_i} + S_A \sum_{k \neq i,j}^{n} \frac{\partial p_k(x_i^m, x_j^m, \mathbf{x}_{-ij}^m, \epsilon_A)}{\partial x_i}$$
$$+ (S_B - \pi_B)\frac{\partial q_i(1 - x_i^m, 1 - x_j^m, \mathbf{1} - \mathbf{x}_k^m, \epsilon_B)}{\partial x_i} + (S_B - \pi_B)\frac{\partial q_j(1 - x_i^m, 1 - x_j^m, \mathbf{1} - \mathbf{x}_{-ij}^m, \epsilon_B)}{\partial x_i}$$
$$+ S_B \sum_{k \neq i,j}^{n} \frac{\partial q_k(1 - x_i^m, 1 - x_j^m, \mathbf{1} - \mathbf{x}_{-ij}^m, \epsilon_B)}{\partial x_i} > 0 \text{ for } i, j.$$

and

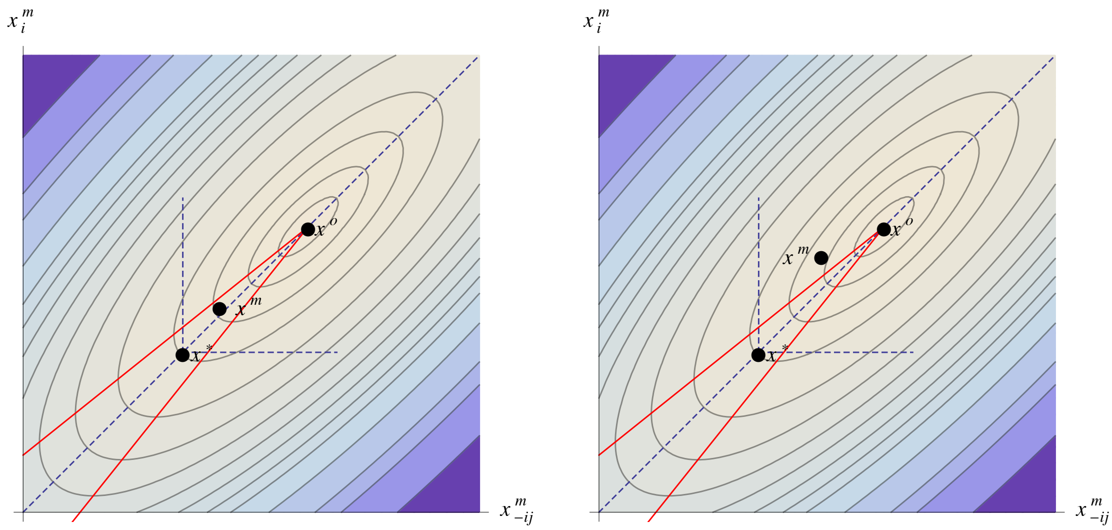
$$(S_A - \pi_A)\frac{\partial p_k(x_i^m, x_j^m, \mathbf{x}_{-ij}^m, \epsilon_A)}{\partial x_k} + S_A \sum_{h \neq k}^{n} \frac{\partial p_h(x_i^m, x_j^m, \mathbf{x}_{-ij}^m, \epsilon_A)}{\partial x_k}$$
$$+ (S_B - \pi_B)\frac{\partial q_k(1 - x_i^m, 1 - x_j^m, \mathbf{1} - \mathbf{x}_{-ij}^m, \epsilon_B)}{\partial x_k} + S_B \sum_{j \neq i}^{n} \frac{\partial q_h(1 - x_i^m, 1 - x_j^m, \mathbf{1} - \mathbf{x}_{-ij}^m, \epsilon_B)}{\partial x_k} > 0, \text{ for } k \neq i, j,$$

where we have used the fact that the post-merger equilibrium vector of investments satisfies the FOCs (37) and (4).

[28]More investment in project A post-merger does not guarantee that welfare increases even if pre-merger there is under-investment in such a project. One reason for this is that the market equilibrium may move from an under-investment situation to an over-investment one.

the corresponding investment of a representative non-merging firm. Assume the rest of the firms are in the background playing the same strategies as the representative firms. Lighter colours indicate higher welfare levels. The social optimum is represented by the point $\mathbf{x^o}$, while the pre-merger equilibrium by $\mathbf{x}^*$. The space in between the red lines going through the social optimum determines the area for which the social welfare function has a positive gradient.

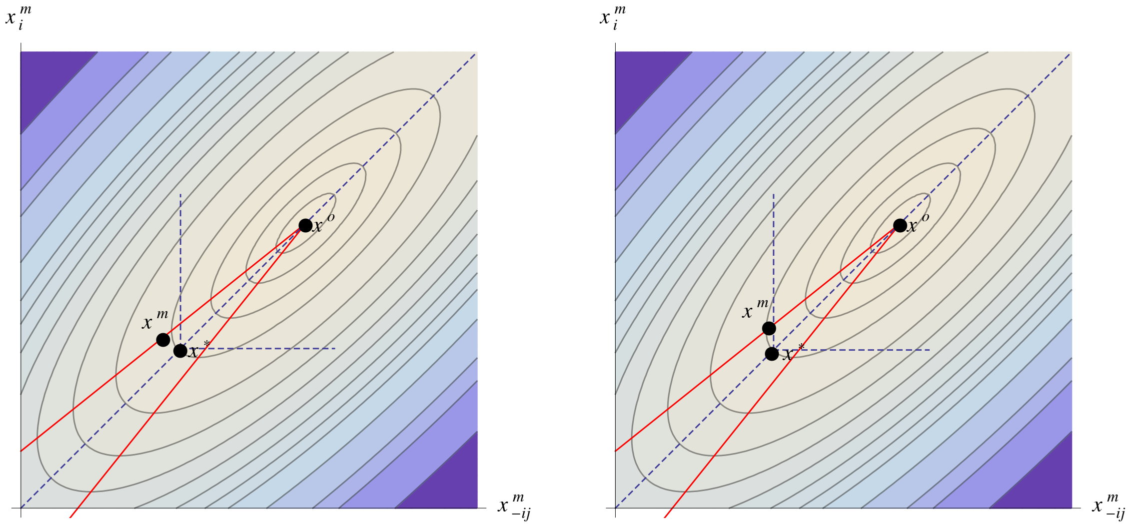
The left graph of Figure 7 shows the situation depicted by the Proposition 10a. At the pre-merger equilibrium, firms invest insufficiently in the A-project. Post-merger equilibrium, all the firms invest more in the A-project than pre-merger and because the gradient at the post-merger equilibrium is positive, welfare surely increases. That the gradient is positive is a sufficient condition but not necessary for welfare to increase after all firms raise their investments in the A-project. This can be seen in the right graph, where we have plotted a situation where welfare increases but the component of the gradient corresponding to x_i^m is negative.



(a) Merger raises welfare (gradient positive) (b) Merger raises welfare (gradient non-positive)

Figure 7: Pre- and post-merger equilibrium and welfare (strategic complements)

When firms' investments are strategic substitutes, the non-merging firms change their investment in opposite direction to the merging firms. Because of this, the result in Proposition 10 is invalid. Even if the gradient is positive at the post-merger equilibrium, the fact that the non-merging firms cut their investment may result in a decrease in social welfare. Nevertheless, with strategic substitutes it is possible that welfare increases. We illustrate these two points in Figure 8. On the left graph, we show a case in which welfare falls, even if the gradient is positive. On the right graph welfare increases.



(a) Merger lowers welfare

(b) Merger raises welfare

Figure 8: Pre- and post-merger equilibrium and welfare (strategic substitutes)

4.3 Endogenous research budget

So far we have restricted the analysis to a situation where firms are research-budget constrained. This restriction has served to focus our discussion solely on the portfolio effects of mergers. In this extension, we allow the contestants to choose their research effort.

Let y_i denote the total research effort of a firm i. As in the model of Section 2, let x_i be the investment of a firm i in the A-project, and $y_i - x_i$ the corresponding investment in the B-project. Let the cost of investment be $c(y_i)$, with $c(\cdot)$ increasing and convex. The rest of the model assumptions remain the same. Thus, we are back to the model of Section 2 when the total research effort is exogenously set to $y_i = 1$, $i = 1, 2, ..., n$.

The payoff to a firm i that invests $x_i \geqslant 0$ in the A-innovation path and $y_i - x_i \geqslant 0$ in the B-innovation path is:

$$u_i(x_i, y_i; \mathbf{x}_{-i}, \mathbf{y}_{-i}) = p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)\pi_B - c(y_i) \tag{41}$$

In the pre-merger market, a firm i chooses y_i and x_i to maximize the payoff in (41). The FOCs for profits maximization are:

$$\frac{\partial q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i}\pi_B - c'(y_i) = 0. \tag{42}$$

$$\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B = 0 \tag{43}$$

Because $\partial q_i(\cdot)/\partial y_i = -\partial q_i(\cdot)/\partial x_i$, equations (42) and (43) can be combined to get:

$$\frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A = \frac{\partial q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i}\pi_B = c'(y_i), \tag{44}$$

which means that the marginal gains from investing in a project must be equal across projects and equal to the marginal cost of total investment.[29]

Pre-merger equilibrium

The bi-dimensional nature of the problem makes it difficult to analyze the existence of equilibrium. We follow an approach recently developed by Hefti (2017) that imposes regularity conditions on the payoff function that guarantee the existence and uniqueness of a symmetric equilibrium.

Proposition 11. *In the model with endogenous research budget, there exists a unique SNE denoted $y_i^* = y^* \geqslant 0$ and $x_i^* = x^* \geqslant 0$, $i \in N$, and is given by the solution to (44). The SNE is interior provided that $\frac{\partial p_i(0,\mathbf{0},\epsilon_A)}{\partial x_i}\pi_A > \frac{\partial q_i(0,\mathbf{0},\epsilon_B)}{\partial y_i}\pi_B > c'(y^*)$.*

Proof. See the Appendix. ■

As we are interested only in the interior solutions, we assume henceforth that the conditions for an interior equilibrium hold.

Post-merger equilibrium

Consider now that firms i and j merge. In such a case, the merged entity chooses total investments y_i and y_j as well as an allocation across the projects of the partner firms to maximise the payoff:

$$\begin{aligned} u_m(x_i, y_i; \mathbf{x}_{-i}, \mathbf{y}_{-i}) = {} & p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)\pi_A + q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)\pi_B - c(y_i) \\ & + p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)\pi_A + q_j(y_j - x_j, \mathbf{y}_{-j} - \mathbf{x}_{-j}, \epsilon_B)\pi_B - c(y_j) \end{aligned}$$

The FOCs for profits maximisation with respect to y_i and x_i are given by:

$$\frac{\partial q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i}\pi_B + \frac{\partial q_j(y_j - x_j, \mathbf{y}_{-j} - \mathbf{x}_{-j}, \epsilon_B)}{\partial y_i}\pi_B - c'(y_i) = 0 \tag{45}$$

$$\begin{aligned} & \frac{\partial p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i}\pi_B + \frac{\partial p_j(x_j, \mathbf{x}_{-j}, \epsilon_A)}{\partial x_i}\pi_A \\ & \quad \frac{\partial q_j(y_j - x_j, \mathbf{y}_{-j} - \mathbf{x}_{-j}, \epsilon_B)}{\partial x_i}\pi_B = 0. \end{aligned} \tag{46}$$

The FOC for profits maximisation with respect to x_j and y_j are similar. Non-merging firms continue to maximise the payoff in (41) and the corresponding FOCs for an interior equilibrium are the same as those given in (42) and (43).

As in the pre-merger market, the two FOCs in (45)-(46) can be combined into:

$$\left(\frac{\partial p_i(\cdot)}{\partial x_i} + \frac{\partial p_j(\cdot)}{\partial x_i}\right)\pi_A = \left(\frac{\partial q_i(\cdot)}{\partial y_i} + \frac{\partial q_j(\cdot)}{\partial y_i}\right)\pi_B = c'(y_i), \tag{47}$$

where we have removed the arguments of the probability functions to shorten the expression.

This expression says that the marginal gains from investing in a project must be equal across projects and equal to the marginal cost of total investment. Compared to the pre-merger market, the marginal

[29] Recall that budget-constrained firms chose the optimal portfolio to equalize the marginal profits across innovations. When the firms are not budget-constrained, in addition to this requirement, the marginal profits from either project must equal the marginal cost.

gains now include the business-stealing and business-giving externalities a firm i's portfolio choice imposes on the partner firm. If the merging firms did not change their total investment after merging, the analysis would be similar to that in the main body of the paper. However, because total investment may change after merging, we need to adapt the result on the equilibrium portfolio choice.

The net effect of the internalisation of the negative and positive externalities due to a merger is ambiguous, and can therefore be towards either more or less investment in the more profitable and/or socially desirable type of innovation. Our next result explores this net effect. Before presenting it, a few comments on the existence of an interior post-merger Nash equilibrium are in order. Notice that the payoff of the merged entity is the sum of the payoffs of firms i and j. The additive structure retains the properties of the payoff function, and hence the existence of an equilibrium can easily be established. To ensure that the equilibrium is interior, we need further regularity conditions. In what follows we assume that these conditions hold and we focus on comparing the pre- and post-merger equilibria of our model.

Proposition 12. *Assume that firms i and j merge in the model with endogenous research budget. Then, compared to the pre-merger equilibrium, the merged firms will invest more in the A-innovation and less in the B-innovation if and only if:*

$$\frac{\pi_A}{\pi_B} < -\frac{\frac{\partial q_j(\hat{\mathbf{y}}-\mathbf{x}^*,\epsilon_B)}{\partial x_i} + \frac{\partial q_i(\hat{\mathbf{y}}-\mathbf{x}^*,\epsilon_B)}{\partial x_i} - \frac{\partial q_i(\mathbf{y}^*-\mathbf{x}^*,\epsilon_B)}{\partial x_i}}{\frac{\partial p_j(\mathbf{x}^*,\epsilon_A)}{\partial x_i}}. \tag{48}$$

where y^* and $\hat{y}$ are the unique solutions to the FOCs (42) and (45) evaluated at $\mathbf{x}^*$.

The non-merging firms, by contrast, will invest less in the A-innovation and more in B-innovation.

Proof. See Appendix. ■

Proposition 12 suggests that, compared to the situation before a merger, the merging firms will put more effort in the A-project and less in the B one provided that condition (48) holds. This condition governs the nature of the total externality exerted by the investment of a firm in the A-innovation path and it tends to be satisfied when the rewards from the A-innovation relative to the B-innovation are low enough. This condition is similar to that obtained when the budget is constrained in Section 4.2. In fact, condition (48) is identical to (38) if $y^* = \hat{y} = 1$.

Next, we study the effect of a merger on the total investment of the partner firms.

Proposition 13. *Assume that firms i and j merge in the model with endogenous research budget and condition (48) is satisfied. Then, the merged entity reduces total investment in the research projects. That is: $y^*_m < y^*$.*

Proposition 13 implies that the increase in the effort put into the A-innovation is less than the decrease in the effort exerted into the B-innovation one if condition (48) holds. Hence, the total innovation effort of the merging firms decreases.

From the point of view of welfare maximization, this result introduces a novel tradeoff for mergers: in the post-merger equilibrium the merged firms reduce investment but the allocation of funding across projects is more sound. The question is how these two effects fare against one another. To understand this, consider the limiting case in which the slope of the cost function $c(.)$ is close to zero up to the point where total investment is 1, and thereafter the slope becomes very large approaching infinity. This situation is virtually equivalent to the one in the main body of the paper where the budget is constrained to be equal to 1. For such a case, the impact of the merger on total investment ought to be negligible and therefore most of its effect would be related to the portfolio adjustment we have conducted in section 3.

As we depart from this limiting case, the reduction of total investment becomes more and more significant from a welfare viewpoint. By continuity, we expect that there will be a sufficiently steep cost function for which the investment effect dominated the portfolio effect and mergers turn welfare reducing. A detailed analysis of the tradeoff between the investment effect and the portfolio effect in more general situations is left for future research.

5 Conclusions

There is a growing consensus that innovation should be protected by economic policy and a literature has recently emerged on the effects of mergers on innovation. We have contributed to this literature by presenting a model of mergers in which firms pursue an array of projects. While the focus of the existing literature has been on how mergers affect investment effort, our paper has paid attention to the question how mergers affect the portfolio of projects firms invest in.

We have studied a market in which firms can invest in two independent research projects. The projects vary in terms of profitability and surplus appropriation. By investing in a project a firm engages in a contest with the rival firms. The portfolio nature of the firms' problem leads to new insights about firm interaction in the market. In settings in which investing in a project raises the marginal cost of investing in another project, putting effort in one project creates both a negative business-stealing externality and a positive business-giving externality on the rival firms. The negative externality relates to the project in which a firm raises investment and is the usual innovation externality by which the effort of a firm decreases the chance rival firms innovate. The positive externality is novel and goes via the alternative project. Because raising investment in one project increases the marginal cost of investing in the alternative project, rival firms are conferred a positive externality in the competition for that project.

We have shown that, compared to the optimal portfolio, the market portfolio is typically biased due to two reasons. First, ignoring the business-stealing and business-giving externalities, firms tend to allocate too much funding to the more profitable project and too little to the less profitable one. Second, because firms only care about the profits generated by the projects and not about consumer surplus, they typically tend to put too little effort on projects that generate a large social surplus.

We have characterized the impact of a merger on the portfolio of the merging and non-merging firms. Merging firms adjust their portfolio of investments to internalize the business-stealing and business-giving externalities they impose on one another. The strength of these externalities depend on the relative profitability of the projects and after merging they cut investment in the more profitable project and raises it in the less profitable one. In the absence of synergies, even if the non-merging firms adjust their portfolio in the opposite direction, we have seen that a merger may increase consumer welfare. This occurs when the relatively more profitable project is also the relatively more appropriable. In such a case, a merger increases welfare by reducing investment in the more profitable and more appropriable project and increasing it in the less profitable project but also less appropriable project. When profitability and surplus appropriability are negatively correlated, a merger reduces welfare by further misaligning the market and the social incentives.

We have argued that our results carry over to cases in which, in addition to portfolio effects, there are price effects of mergers. Moreover, we have seen that the results hold if firms not only choose how to allocate funding across projects but also how much money they spend on research.

Appendix

Proof of strategic substitutes.

To show that $\epsilon_\ell \geqslant 1$, $\ell = A, B$, suffices for strategic substitutability, we need to prove that the marginal benefit a firm obtains from its investment in the A-project decreases as the rival firm investment in the same project raises. We thus compute the second cross partial derivative of the payoff of a firm i with respect to own and rival investment:

$$\begin{aligned}\frac{\partial u(x_i, x_j)}{\partial x_i \partial x_j} &= -\frac{(X + \epsilon_A - 2x_i)\,\pi_A}{(X + \epsilon_A)^3} - \frac{(n - X - 2 + 2x_i + \epsilon_B)\,\pi_B}{(n - X + \epsilon_B)^3} \\ &= -\frac{(X_{-i} + \epsilon_A - x_i)\,\pi_A}{(X + \epsilon_A)^3} - \frac{(n - 1 - X_{-i} - (1 - x_i) + \epsilon_B)\,\pi_B}{(n - X + \epsilon_B)^3},\end{aligned} \tag{49}$$

where, in the first line we have used the notation $X = \sum_{i=1}^n x_i$, and in the second line $X_{-i} = \sum_{j \neq i}^n x_j$.

It is now straightforward to see that $\epsilon_A \geqslant 1$ guarantees that the first summand of this expression is negative and that $\epsilon_B \geqslant 1$ ensures that the second summand is also negative. Hence, the investments of the firms in the A-project are strategic substitutes. ■

Proof of Proposition 1.

Because the payoff function (9) is strictly concave in firm i's own investment and continuous in x_{-i}, the existence of equilibrium follows from the Debreu-Glicksberg-Fan theorem. Because the game is symmetric, there exists a SNE $x_i = x^*$ (see Hefti, 2017).

Equation (11) follows from applying symmetry, i.e. $x_i = x^*$ for all i, in the FOC (10). Let us denote the LHS of (11) as $h(x^*)$ and first notice that h is continuous and monotone decreasing in x^*. In fact:

$$\frac{dh}{dx^*} = -\frac{n((n-1)x^* + \epsilon_A) + \epsilon_A}{(nx^* + \epsilon_A)^3}\pi_A - \frac{n(n-1)(1 - x^*) + (n+1)\epsilon_B}{(n + \epsilon_B - nx^*)^3}\pi_B < 0.$$

Therefore, the symmetric equilibrium is unique.

Moreover, note that

$$h(0) = \frac{\pi_A}{\epsilon_A} - \frac{n + \epsilon_B - 1}{(n + \epsilon_B)^2}\pi_B > 0 \text{ if and only if } \epsilon_A < \frac{\pi_A(n + \epsilon_B)^2}{\pi_B(n + \epsilon_B - 1)}$$

and that

$$h(1) = \frac{\pi_A(n - 1 + \epsilon_A)}{(n + \epsilon_A)^2} - \frac{\pi_B}{\epsilon_B} < 0 \text{ if and only if } \epsilon_A > \frac{\pi_A\epsilon_B - 2n\pi_B + \sqrt{\pi_A\epsilon_B\,(\pi_A\epsilon_B - 4\pi_B)}}{2\pi_B}$$

Therefore, the unique equilibrium is interior provided that (12) holds.

To show that equilibrium investment x^* increases in π_A and ϵ_B and decreases in ϵ_A and π_B, we apply implicit differentiation to Equation (11). We then obtain:

$$\begin{aligned}\frac{\partial x^*}{\partial \pi_A} &= -\frac{\partial h/\partial \pi_A}{\partial h/\partial x^*} > 0, \ \frac{\partial x^*}{\partial \epsilon_A} = -\frac{\partial h/\partial \epsilon_A}{\partial h/\partial x^*} < 0 \\ \frac{\partial x^*}{\partial \pi_B} &= -\frac{\partial h/\partial \pi_B}{\partial h/\partial x^*} < 0, \ \frac{\partial x^*}{\partial \epsilon_B} = -\frac{\partial h/\partial \epsilon_B}{\partial h/\partial x^*} > 0,\end{aligned}$$

where we have used the derivatives

$$\frac{\partial h}{\partial x^*} = -\frac{n((n-1)x^* + \epsilon_A) + \epsilon_A}{(nx^* + \epsilon_A)^3}\pi_A - \frac{n(n-1)(1-x^*) + (n+1)\epsilon_B}{n^2(1-x^*)^2}\pi_B < 0$$

$$\frac{\partial h}{\partial \pi_A} = \frac{(n-1)x^* + \epsilon_A}{(nx^* + \epsilon_A)^2} > 0, \ \frac{\partial h}{\partial \epsilon_A} = -\frac{(n-2)x^* + \epsilon_A}{(nx^* + \epsilon_A)^3}\pi_A < 0,$$

$$\frac{\partial h}{\partial \pi_B} = -\frac{(n-1)(1-x^*) + \epsilon_B}{(n + \epsilon_B - nx^*)^2} < 0, \text{ and } \frac{\partial h}{\partial \epsilon_B} = \frac{(n-2)(1-x^*) + \epsilon_B}{(n + \epsilon_B - nx^*)^3}\pi_A < 0.$$

■

Proof of Proposition 2.

To prove this result, we compute the difference between the FOC of the merged entity after the merger with that of a potentially merging firm before the merger and evaluate it at the pre-merger equilibrium x^*. Because the FOCs of the non-merging firms remain the same, if this difference is positive we conclude that the merged entity invests more in A-innovation after the merger.

$$\Delta(x^*) = \frac{\partial u_m}{\partial x_i}(x^*) - \frac{\partial u}{\partial x_i}(x^*) = \frac{-x^*}{(nx^* + \epsilon_A)^2}\pi_A + \frac{(1-x^*)}{(n + \epsilon_B - nx^*)^2}\pi_B$$

Using the first order condition evaluated at the pre-merger equilibrium (11), this difference can be rewritten as

$$\Delta(x^*) = \frac{\pi_B((1-x^*)\epsilon_A - x^*\epsilon_B)}{((n-1)x^* + \epsilon_A)(n(1-x^*) + \epsilon_B))^2}$$

This difference is positive whenever $(1-x^*)\epsilon_A - x^*\epsilon_B > 0$, which requires that

$$x^* < \frac{\epsilon_A}{\epsilon_A + \epsilon_B}. \tag{50}$$

In order to find the parameters under which this condition holds, we evaluate the FOC (11) at the value $\frac{\epsilon_A}{\epsilon_A + \epsilon_B}$, which gives:

$$-\frac{(\epsilon_A + \epsilon_B)(n - 1 + \epsilon_A + \epsilon_B)(\epsilon_A\pi_B - \epsilon_B\pi_A)}{\epsilon_A\epsilon_B(n + \epsilon_A + \epsilon_A\pi_B)^2}$$

This expression is negative whenever $\pi_B/\epsilon_B > \pi_A/\epsilon_A$. Because the FOC is a decreasing function of x^*, this implies that condition (50) holds. Therefore, the merged entity increases investment in the A-innovation project. This concludes the proof. ■

Proof of Proposition 3.

To prove this result, we make use of the pseudo best-response functions analysed above. In the pre-merger symmetric equilibrium, this total investment is equal to nx^*, where x^* solves (11). In the post-merger equilibrium, we have denoted the total investment in the A-innovation path by $x_m + x_{nm}$. The change from nx^* to $x_m + x_{nm}$ after the merger is determined by the slope of the joint best-response function of the non-merging firms, given by the expression (14).

Under the condition $\pi_B/\epsilon_B > \pi_A/\epsilon_A$, we know that x_m increases compared to $2x^*$ after the merger while x_{nm} decreases compared to $(n-2)x^*$. Consequently, if the slope of the pseudo best-response of the non-merging firm is smaller than -1 then the increase in x_m is larger than the reduction in x_{nm} and therefore aggregate investment in the A-innovation path increases after the merger.

The slope of the pseudo best-response of the non-merging firms is given by:

$$\frac{\partial x_{nm}}{\partial x_m} = -\frac{\frac{\partial \beta}{\partial x_m}}{\frac{\partial \beta}{\partial x_{nm}}} = -\frac{-\frac{(n-2)x_m+(n-4)x_{nm}+(n-2)\epsilon_A}{(x_m+x_{nm}+\epsilon_A)^3}\pi_A - \frac{(n-2)(n-2+\epsilon_B)-(n-2)x_m-(n-4)x_{nm}}{(n+\epsilon_B-x_m-x_{nm})^3}\pi_B}{-\frac{(n-1)x_m+(n-3)x_{nm}+(n-1)\epsilon_A}{(x_m+x_{nm}+\epsilon_A)^3}\pi_A - \frac{n(n-3)+(n-1)\epsilon_B+4-(n-1)x_m-(n-3)x_{nm}}{(n+\epsilon_B-x_m-x_{nm})^3}\pi_B}. \tag{51}$$

This slope if smaller than -1 provided that

$$\frac{(n-1)x_m+(n-3)x_{nm}+(n-1)\epsilon_A}{(x_m+x_{nm}+\epsilon_A)^3}\pi_A + \frac{n(n-3)+(n-1)\epsilon_B+4-(n-1)x_m-(n-3)x_{nm}}{(n+\epsilon_B-x_m-x_{nm})^3} >$$
$$\frac{(n-2)x_m+(n-4)x_{nm}+(n-2)\epsilon_A}{(x_m+x_{nm}+\epsilon_A)^3}\pi_A + \frac{(n-2)(n-2+\epsilon_B)-(n-2)x_m-(n-4)x_{nm}}{(n+\epsilon_B-x_m-x_{nm})^3}\pi_B.$$

After rearranging, this condition is equivalent to

$$\frac{1}{(x_m+x_{nm}+\epsilon_A)^2}\pi_A > -\frac{1}{(n+\epsilon_B-x_m-x_{nm})^2}\pi_B,$$

which is always satisfied because the RHS is negative. As a result, total investment $x_m + x_{nm}$ increases after a merger. ■

Proof of Proposition 4.

We first calculate the socially optimal investment in the A-innovation, X_W^*, by solving equation (20):

$$X_W^* = \frac{n+\epsilon_B-\epsilon_A\sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}}{1+\sqrt{\frac{S_B\epsilon_B}{S_A\epsilon_A}}}. \tag{52}$$

We now rewrite the FOC (11) in terms of aggregate investment X in the A-innovation path as follows:

$$\frac{\partial u_i(\cdot)}{\partial x_i} = \frac{\frac{n-1}{n}X+\epsilon_A}{(X+\epsilon_A)^2}\pi_A - \frac{\frac{n-1}{n}(n-X)+\epsilon_B}{(n-X+\epsilon_B)^2}\pi_B. \tag{53}$$

At the pre-merger market equilibrium $X^* = nx^*$, this expression is equal to zero. We now evaluate it at the socially optimal aggregate investment, X_W^*:

$$\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X=X_W^*} = n\Big(1+\frac{\epsilon_A}{\epsilon_B}+\frac{n-1}{\epsilon_B}\Big)\Big(1-\frac{\frac{\pi_B}{S_B}}{\frac{\pi_A}{S_A}}\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}}\Big) - \Big(1+\frac{\epsilon_A}{\epsilon_B}\frac{\frac{\pi_B}{S_B}}{\frac{\pi_A}{S_A}}\Big)\Big(1-\sqrt{\frac{S_B\epsilon_A}{S_A\epsilon_B}}\Big). \tag{54}$$

If the above expression is negative, it implies that $X^* < X_W^*$; in different words, the market under-invests in the A-innovation path and correspondingly over-invests in the B-innovation path. Solving the inequality $\left.\frac{\partial u_i(\cdot)}{\partial x_i}\right|_{X=X_W^*} < 0$ in $\frac{\pi_A}{\pi_B}$ gives condition (21) in the proposition. If (54) is instead positive, then the market over-invests in the A-innovation path and under-invests in the B-innovation path. ■

Proof of Proposition 5 The socially optimal investment in the A-innovation continues to be

$$X_W^* = \frac{n + \epsilon_B - \epsilon_A \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}{1 + \sqrt{\frac{S_B \epsilon_B}{S_A \epsilon_A}}}. \tag{55}$$

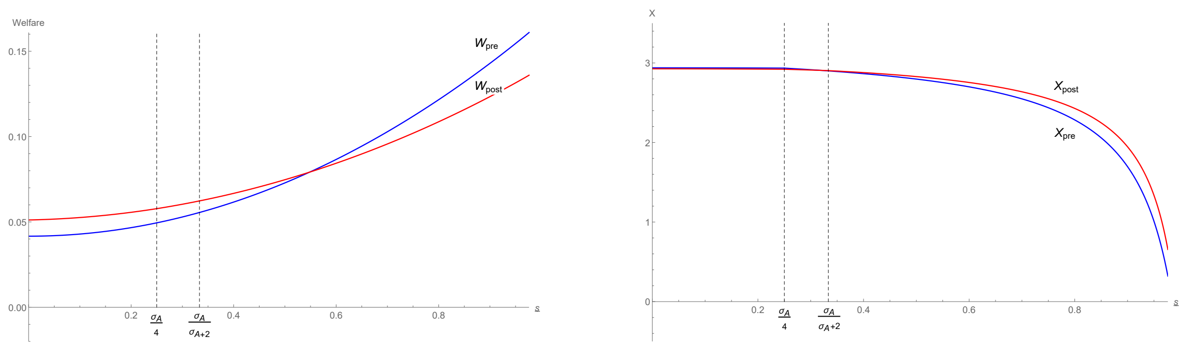
From equation (22) we have the following FOC:

$$\frac{\frac{n-2}{n-1} X + \epsilon_A}{(X + \epsilon_A)^2} \pi_A - \frac{\frac{n-2}{n-1}(n - X) + \epsilon_B}{(n - X + \epsilon_B)^2} \pi_B = 0. \tag{56}$$

Following the approach in Proposition 4, we evaluate the above expression at the socially optimal aggregate investment, X_W^* and analyze the case in which the FOC (56) is negative. This gives condition (23). ■

Price effects of mergers when the products of the winning and losing firms are horizontally differentiated.

In Figure 6 we report results for the case in which $\sigma_A = 1$. The rest of the parameters are exactly the same as in Section 4.1. In the graphs of Figure 6, we depict two threshold values. The first, $\sigma_A/4$, corresponds to the pre-merger equilibrium. When $\underline{s} < 1/4$, the winning firm monopolizes the market and therefore price effects only arise when no firm wins the contest for the A-innovation. The second threshold, $\sigma_A/(\sigma_A + 2)$, corresponds to the post-merger. When $\underline{s} < 1/3$, the winning firm monopolizes the market.



(a) Consumer welfare (b) Aggregate investment in A

Figure 9: Pre- and post-merger welfare and aggregate investment in the A-project

The figure shows that introducing product differentiation does not change the main insights. For low $\underline{s}$ a merger increases consumer welfare by (slightly) reducing investment in the A-project. As we increase $\underline{s}$ the price effects of a merger become stronger, eventually turning a merger undesirable for consumers.

We finally report the profits of merging and non-merging firms. It can be seen that mergers are incentive compatible. For low levels of the quality parameter $\underline{s}$, the non-merging firms benefit less than the merging firms. However, when $\underline{s}$ is quite large and the price effects are quite strong, the non-merging firms benefit more than the merging firms.

Proof of Proposition 7.

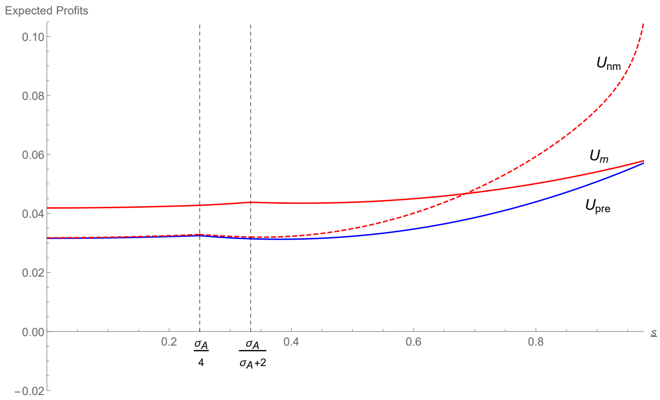


Figure 10: Pre- and post-merger profits of merging and non-merging firms.

Note that the payoff function (1) is strictly concave in firm i's own investment because the second derivatives $\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)/\partial x_i^2$ and $\partial^2 q_i(1 - x_i, \mathbf{1} - \mathbf{x}_{-i}, \epsilon_B)/\partial x_i^2$ are negative. The existence of equilibrium then follows from the Debreu-Glicksberg-Fan theorem because the strategy spaces are compact and convex sets, and the payoff functions are strictly concave in x_i and continuous in x_{-i}. Because our game is symmetric, there exists a SNE $x_i = x^*$ (see Hefti, 2017).

Uniqueness follows from Assumption 1d. Rewrite equation (37) as follows:

$$\frac{\partial p_i(x^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A = -\frac{\partial q_i(1 - x^*, \mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B. \tag{57}$$

Note now that Assumption 1d implies that the LHS of (57) is strictly decreasing in x^*, while the RHS of (57) is strictly increasing in x^*. Hence, given the conditions in the Proposition, the LHS and RHS surely intersect once and only once at x^*.

Finally, the comparative statics of x^* with respect to the parameters of the model follow from a straightforward application of the implicit function theorem to equation (37). For the signs, we employ again Assumption 1d. ■

Proof of Proposition 8

Evaluating the LHS of the FOC (4) at the pre-merger symmetric equilibrium gives the expression:

$$\frac{\partial p_j(\mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_j(\mathbf{1} - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B,$$

where we have used the fact that x^* satisfies (37). This expression is positive if and only if condition (38) holds, in which case the payoff of the merged entity increases at $x_i = x^*$. This implies that, relative to the pre-merger situation, the best-reply of partner i of the merged entity shifts out. The same applies to partner j of the merged entity. Regarding the non-merging firms, when the game is of strategic substitutes (decreasing best-replies), they cut investment in the A-project and correspondingly increase it in the B-project. When the game is of strategic complements (increasing best-replies), the non-merging firms also raise investment in the A-project and thus cut it in the B-project. ■

Proof of Proposition 9 Evaluating the LHS of the social planner FOC (6) at the pre-merger symmetric equilibrium $\mathbf{x}^*$ gives the LHS of (39), where we have used the fact that x^* solves (37) and

$W_\ell = \pi_\ell + S_\ell$, $\ell = A, B$. When this expression is postive, the payoff of the social planner increases at $x = x^*$. This implies that, relative to the planner's choice, the market puts too little funding in the A-project and, correspondingly, too much funding is allocated to the B-project. ■

Proof of Proposition 10.

The following arguments prove result *a*. Suppose conditions (38) and (39) hold. Note first that condition (38) means that the merging firms allocate more funding to the A-project in the post-merger equilibrium than in the pre-merger one. Because of strategic complementarity, so do the non-merging firms. Hence, all the firms invest more in the A-project post-merger than pre-merger. Observe now that condition (39) means that, from the point of view of social welfare maximization, in the pre-merger equilibrium firms under-invests in the A-project. Therefore, if the gradient of the social welfare function at the post-merger equilibrium is strictly positive, which is ensured by condition (40), then we are sure that the welfare level increases after a merger. Results *b, c* and *d* follow from similar arguments and we omit the details to save on space. ■

Proof of proposition 11.

To determine the concavity of firm i's payoff, we calculate the Hessian for the payoff function.

$$\text{Hess}\left(u_i(x_i, y_i; \mathbf{x}_{-i}, \mathbf{y}_{-i})\right) = \begin{bmatrix} \frac{\partial^2 u_i}{\partial x_i^2} & \frac{\partial^2 u_i}{\partial x_i \partial y_i} \\ \frac{\partial^2 u_i}{\partial x_i \partial y_i} & \frac{\partial^2 u_i}{\partial y_i^2} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i^2}\pi_B & \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i \partial y_i}\pi_B \\ \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i \partial x_i}\pi_B & \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i^2}\pi_B - c''(y_i) \end{bmatrix}$$

The determinant of the leading principal minors are

$$|L_1| = \frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i^2}\pi_B$$
$$|L_2| = \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i^2}\frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2}\pi_A\pi_B$$
$$- c''(y_i)\left(\frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i^2}\pi_B\right)$$

where we have used the fact that:

$$-\frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i \partial y_i} = \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i^2} = \frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i^2}$$

to simplify $|L_2|$.

As $\frac{\partial^2 p_i(x_i, \mathbf{x}_{-i}, \epsilon_A)}{\partial x_i^2} < 0$, $\frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial x_i^2} < 0$ and $\frac{\partial^2 q_i(y_i - x_i, \mathbf{y}_{-i} - \mathbf{x}_{-i}, \epsilon_B)}{\partial y_i^2} < 0$, by assumption 1d we conclude $|L_1| < 0$ and $|L_2| > 0$. Hence, the Hessian is negative definite and the payoff of firms is strictly concave in their own strategies

To find symmetric equilibria, a simplified approach, called the symmetric opponents form approach (SOFA) hereafter, is useful. The SOFA takes an arbitrary indicative player (i) and restricts all opponents to play the same strategies, i.e., $\bar{\mathbf{x}}_{-i} = (\bar{x}, \bar{x}..., \bar{x})$ and $\bar{\mathbf{y}}_{-i} = (\bar{y}, \bar{y}..., \bar{y})$.

Let

$$\hat{p}_i(x_i, \bar{x}, \epsilon_A) \equiv p_i(x_i, \bar{\mathbf{x}}_{-i}, \epsilon_A)$$
$$\hat{q}_i(y_i - x_i, \bar{y} - \bar{x}, \epsilon_B) \equiv q_i(y_i - x_i, \bar{\mathbf{y}}_{-i} - \bar{\mathbf{x}}_{-i}, \epsilon_B)$$

A symmetric payoff function u_i and identical strategy set across players implies the existence of at least one symmetric equilibrium: $x_i = x_j = x^*$ and $y_i = y_j = y^*$ for all firms $i, j \in N$.

To check for multiple equilibria, we follow the method proposed by Hefti (2017). We use index theory approach based on Poincare-Hopf index. However, applying the index theory to our bi-dimensional n-player game would involve a Jacobian matrix of dimensionality $2n \times 2n$. The SOFA condition allows us to reduce the dimensionality of the game to a two-player game that requires us to solve a 2×2 Jacobian matrix.

In order to do that, we first apply SOFA to the gradient of u_i:

$$\nabla u_i(x_i, y_i; \mathbf{x}_{-i}, \mathbf{y}_{-i}) \equiv \nabla \hat{u}_i(x_i, y_i, \bar{x}, \bar{y}) = \begin{bmatrix} \frac{\partial p_i(x_i, \bar{x}, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(y_i - x_i, \bar{y} - \bar{x}, \epsilon_B)}{\partial x_i}\pi_B \\ \frac{\partial q_i(y_i - x_i, \bar{y} - \bar{x}, \epsilon_B)}{\partial y_i}\pi_B - c'(\bar{y}) \end{bmatrix} \tag{58}$$

Next, evaluate $\nabla \hat{u}_i(x_i, y_i, \bar{x}, \bar{y})$ at $\bar{x} = x_i, \bar{y} = y_i$ and calculate the Jacobian of $\nabla \hat{u}_i$

$$J(x_i, y_i) = \begin{bmatrix} \frac{\partial^2 p_i(x_i, x_i, \epsilon_A)}{\partial x_i^2}\pi_A + \frac{\partial^2 q_i(y_i - x_i, y_i - x_i, \epsilon_B)}{\partial x_i^2}\pi_B & \frac{\partial^2 q_i(y_i - x_i, y_i - x_i, \epsilon_B)}{\partial x_i \partial y_i}\pi_B \\ \frac{\partial^2 q_i(y_i - x_i, y_i - x_i, \epsilon_B)}{\partial y_i \partial x_i}\pi_B & \frac{\partial^2 q_i(y_i - x_i, y_i - x_i, \epsilon_B)}{\partial y_i^2}\pi_B - c''(y_i) \end{bmatrix} \tag{59}$$

If the $D\Big(- J(x^*, y^*)\Big) > 0$, where $\{(x^*, y^*) \ |\nabla \hat{u}_i(x^*, y^*) = 0 \ \}$, then a unique solution exists.

$$\begin{aligned} D\Big(- J(x^*, y^*)\Big) &= \Big(\frac{\partial^2 \hat{p}_i(x^*, x^*, \epsilon_A)}{\partial x_i^2}\frac{\partial^2 \hat{q}_i(y^* - x^*, y^* - x^*, \epsilon_B)}{\partial y_i^2}\Big)\pi_A\pi_B \\ &- \Big(\frac{\partial^2 \hat{p}_i(x^*, x^*, \epsilon_A)}{\partial x_i^2}\pi_A + \frac{\partial^2 \hat{q}_i(y^* - x^*, y^* - x^*, \epsilon_B)}{\partial x_i^2}\pi_B\Big)c''(y^*) \end{aligned}$$

From assumption 1(d) we have $\frac{\partial^2 \hat{p}_i(x^*, x^*, \epsilon_A)}{\partial x_i^2}, \frac{\partial^2 \hat{q}_i(y^* - x^*, y^* - x^*, \epsilon_B)}{\partial y_i^2}, \frac{\partial^2 \hat{q}_i(y^* - x^*, y^* - x^*, \epsilon_B)}{\partial x_i^2} < 0$, while $c''(y^*) > 0$. Hence, $D\Big(- J(x^*, y^*)\Big) > 0$. This implies the existence of a unique equilibrium (see Hefti (2017)). ■

Proof of proposition 12.

As the function $c'(.)$ is invertible, the FOCs (42) and (45) define implicitly the functions

$$y^* = r(\mathbf{x}, \pi_A, \epsilon_A)$$
$$\hat{y} = \hat{r}(\mathbf{x}, \pi_A, \epsilon_A)$$

which map a given vector of investments $\mathbf{x}$ into a research budget.

Next, we evaluate the FOCs (43) and (46) at the pre-merger symmetric equilibrium, $\mathbf{x}^*$. This gives:

$$FOC(x^*) = \frac{\partial p_i(\mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(\mathbf{y}^*(\mathbf{x}^*) - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B = 0.$$

$$FOC_m(x^*) = \frac{\partial p_i(\mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_i(\hat{\mathbf{y}}(\mathbf{x}^*) - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B + \frac{\partial p_j(\mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial q_j(\hat{\mathbf{y}}(\mathbf{x}^*) - \mathbf{x}^*, \epsilon_B)}{\partial x_i}\pi_B.$$

The post-merger investment in the A-project is greater than pre-merger if and only if $FOC_m(x^*) - FOC(x^*) > 0$. After simplification, this condition can be written as equation (48). ■

Proof of proposition 13.

As usual, let (x^*, y^*) and (x_m^*, y_m^*) denote the pre- and post-merger equilibrium investments, respectively. These equilibrium investments satisfy the FOCs (42) and (45). Therefore:

$$c'(y^*) = \frac{\partial p_i(\mathbf{x}^*, \mathbf{x}^*, \epsilon_A)}{\partial x_i}\pi_A$$

$$c'(y_m^*) = \frac{\partial p_i(\mathbf{x}_m^*, \mathbf{x}_{nm}^*, \epsilon_A)}{\partial x_i}\pi_A + \frac{\partial p_j(\mathbf{x}_m^*, \mathbf{x}_{nm}^*, \epsilon_A)}{\partial x_i}\pi_A$$

Subtracting the two equations above gives:

$$c'(y_m^*) - c'(y^*) = \frac{\partial p_j(\mathbf{x}_m^*, \mathbf{x}_{nm}^*, \epsilon_A)}{\partial x_i}\pi_A + \Big(\frac{\partial p_i(\mathbf{x}_m^*, \mathbf{x}_{nm}^*, \epsilon_A)}{\partial x_i} - \frac{\partial p_i(x^*, \mathbf{x}_{-i}^*, \epsilon_A)}{\partial x_i}\Big)\pi_A.$$

When condition (48) of Proposition 12 holds, we have $x_m^* > x^*$ and $x_{nm}^* < x^*$. In such a case, Assumption 1(d) implies $\frac{\partial p_i(\mathbf{x}_m^*,\mathbf{x}_{nm}^*,\epsilon_A)}{\partial x_i} < \frac{\partial p_i(x^*,\mathbf{x}_{-i}^*,\epsilon_A)}{\partial x_i}$ and Assumption 1(c) means that $\frac{\partial p_j(\mathbf{x}_m^*,\mathbf{x}_{nm}^*,\epsilon_A)}{\partial x_i} < 0$. Together, these two inequalities imply that the marginal cost of the merged firm decreases post merger:

$$c'(y_m^*) - c'(y^*) < 0.$$

As the cost function is strictly convex, $c'' > 0$, this means that the total investment decreases post merger, $y_m^* < y^*$. ■

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