Sequencing bilateral negotiations with externalities*

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Preliminary Version

Abstract

We study the optimal sequence of bilateral negotiations between one principal and two agents, whereby the agents have different bargaining power. The principal chooses whether to negotiate first with the stronger or the weaker agent. We show that the joint surplus is highest when the principal negotiates with the stronger agent first, independent of externalities between agents being positive or negative. The sequence chosen by the principal maximizes the joint surplus if there are negative externalities. Instead, if externalities are positive, the principal often prefers to negotiate with the weaker agent first. We also demonstrate that the sequence can be non-monotonic in the externalities and provide conditions for simultaneous timing to be optimal.

Keywords: bargaining, sequential negotiations, externalities, bilateral contracting, endogenous timing

JEL-codes: C72, C78, D62, L14

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1 Introduction

In many situations, a principal needs to negotiate with several agents, and the outcome of the negotiation between the principal and one agent imposes externalities on the other agents. Examples include the following situations:

1. Vertical relations between a supplier and retailers who compete in the consumer market. Externalities between the retailers are negative, if they sell substitutes, but positive if they sell complements.

2. A seller of a product contracts with R&D firms (e.g., research labs) to improve the product's quality. Again, externalities between R&D firms can be negative (e.g., because research labs provide similar quality improvements) or positive (e.g., because one improvement makes the other more effective).

3. A big country negotiates bilateral trade agreements with several smaller countries. Externalities are negative if the smaller countries export substitute goods but positive if the goods are in complementary nature.

A salient feature in these settings is that the principal often bargains with each agent bilaterally (e.g., because it is too costly to bring all agents together). An important strategic choice of the principal is then the sequence in which these negotiations are conducted. A key variable driving this choice is the bargaining power of an agent. Specifically, the question arises if the principal prefers to bargain first with a strong agent and later with a weak agent or if the reverse order is optimal. In this paper, we study this choice and analyze if the sequence chosen by the principal corresponds to the efficient sequence. We determine if and how the choice and its efficiency consequences depend on the externalities between agents.

We consider a stylized model where a principal bargains with two agents who differ in their bargaining power. Bargaining is modeled as random proposer take-it-or-leave-it bargaining.[1] The principal chooses with which agent to bargain first. We focus on the case where negotiations are over binding contracts that fix a vector of quantities and a transfer, and do not condition on any actions taken later in the game. While there is, in general, an incentive to renegotiate a contract signed in the first negotiation (or to reopen failed negotiations) after the principal has come to an agreement with the second agent, in practice, requirements of time or significant legal costs often make renegotiation difficult. We focus on the case where no renegotiation is possible.

[1] Our results are equivalent if negotiations were modeled according to the Nash Bargaining Solution. Therefore, our model can also be interpreted in this way.

We study which sequence of negotiations maximizes the payoff of the principal, and which maximizes welfare (defined as the joint surplus of all three players). To trace out the effect of unequal bargaining power, we derive our main results under the assumption that agents are symmetric except for bargaining power. To keep the model as simple as possible, an agent's bargaining power is modeled as the probability of making the offer.

We demonstrate that two effects are at work, which drive the privately and socially optimal negotiation sequence. The first effect arises because the principal only obtains a fraction of the joint surplus in the second-stage negotiation. The utility of the agent who bargains in the second stage is therefore only partially considered in the negotiation in the first stage. The decision about quantities in the first stage will then be distorted due to externality that these quantities have on the negotiation in the second stage. This distortion is the larger, the smaller is the share the principal obtains in the second stage. We call this effect the *forward effect*. The second effect occurs because negotiated quantities in the second stage affect the payoff of the agent with whom the principal bargained in the first stage. However, this is not taken into account in the negotiation in stage 2, which only maximizes the bilateral surplus of those bargaining in the stage 2. We call this effect the *backward effect*.

We first show that welfare is maximized if the principal bargains first with the agent who has higher bargaining power. This result holds under very general assumptions on the payoff functions and is independent of externalities between agents being positive or negative. The intuition is easiest to grasp in the extreme case in which one agent has no bargaining power. When bargaining with this agent in the second stage, the principal obtains the full surplus. Therefore, he will take the externalities that arise from the negotiation in the first stage fully into account. As a consequence, there is no distortion in the first stage. In other words, with the negotiation sequence of bargaining first with the stronger agent, there is no distortion through the forward effect. The distortion implied by the backward effect is the same in both timings because in the second stage players always maximize their bilateral surplus. As a consequence, joint surplus is higher when the principal negotiates first with the agent who has some bargaining power. We show that this insight carries over to the case in which both agents have positive bargaining power but one of them is the stronger bargainer, as long as both agents are symmetric but for bargaining power.

We then look at the sequence chosen by the principal. We find that the principal chooses the welfare maximizing sequence if externalities are negative. With negative externalities, both the forward effect and the backward effect favor the sequence of negotiating with the stronger agent first. First, the principal obtains a larger surplus in the negotiation in the second stage, which implies that the distortion implies by the forward effect is smaller when bargaining with the stronger bargainer first. Second, because externalities are negative, the surplus in the first-stage negotiation is lower if the bargainers in the second-stage agree on a positive quantities (i.e., the backward effect). The principal suffers less from the backward effect if he negotiates with the stronger bargainer first because he then obtains a relatively low share of the surplus. Due to the fact that he gets a comparably large share in the second negotiation, the principal secures a high outside option in the first negotiation allowing him to demand a higher piece of the cake in this negotiation.

With positive externalities, however, the principal may prefer to bargain with the weaker agent first. This results in an inefficient timing. With positive externalities, the backward effect favors the sequence of negotiating with the weaker bargainer first. In particular, the joint surplus of the negotiation in the first stage is now increased through the positive externality. The principal benefits more from this increase if he bargains first with the weaker agent because she obtains a larger share in this negotiation. The principal is therefore willing to sacrifice efficiency to obtain a larger piece of a smaller pie.

We show that this inefficient timing occurs if the positive externality is relatively small. By contrast, if the positive externality is large, the efficiency effect becomes more important, inducing the principal to bargain first with the stronger agent. Therefore, the sequence chosen by the principal changes non-monotonically in the externalities between agents: when externalities are negative, the principal prefers to bargain with the stronger agent first, if externalities are moderately positive, he prefers to bargain with the weaker agent first, but when externalities are sufficiently positive, he prefers again to bargain with the stronger agent first.

Even without externalities the principal prefers to bargain with the stronger agent first if the principal's payoff function is not additive separable in the quantities of the agents. This holds, for example, if agents are retailers but monopolists in their respective product market (i.e., exerting no externalities on each other) and the principal is the supplier with a cost function that is convex in quantities (i.e., not additive separable). This scenario implies that the quantities to the two agents are interdependent in the principal's payoff function. In this case (without externalities) the backward effect is immaterial for the principal. However, the forward effect is still present because the principal cares more about the second-stage surplus in the first stage negotiation, when receiving larger fraction of it. This forward effect ultimately favors negotiating first with the stronger agent.

Finally, we consider simultaneous negotiations. We show that for negative externalities, the sequential timing in which the principal bargains with the stronger agent first, dominates the simultaneous timing. The same holds true for large positive externalities due to the efficiency considerations described above. However, with positive externalities, the simultaneous timing becomes optimal for the principal. The intuition is rooted in the fact that with simultaneous bargaining agents cannot observe the outcome in the other negotiation. Each agent suppose that an agreement will be reached there (as is true on the equilibrium path). With sequential negotiations, the agent bargaining at the second stage can observe if the bargainers in the first stage failed to reach an agreement. With positive externalities this implies that the principal when being selected as the porposer, can extract more surplus from the agent in the simultaneous timing. Although disagreement dos not happen on the equilibrium path, this effect increases the outside option of the principal.

Related literature. Our paper relates to a growing literature on one-to-many negotiations. Stole and Zwiebel (1996), Cai (2000), and Bagwell and Staiger (2010) study one-to-many negotiations in different situations, such as bargaining between a firm and several workers, an buyer and multiple sellers, or between countries, respectively. These papers focus on an exogenously given bargaining sequence.

Several recent papers analyze the sequencing of negotiations. Noe and Wang (2004) consider a situation in which the principal can keep the order of negotiations confidential, and determine conditions for efficient equilibria to exist.[2] Agents are symmetric in their model. Marx and Shaffer (2007, 2010) study a buyer who bargains with two sellers, and allow for contracts conditioning on the quantity supplied by both sellers. The cost function of a seller depends only on own quantity, implying that there are no direct externalities.[3] They show that in this situation, the payoff of a seller can be decreasing in own bargaining power. Krasteva and Yildirim (2012b) analyze a model in which a buyer negotiates with two sellers supplying complementary products and the buyer's valuation for the stand-alone products are uncertain. They show that the optimal sequence depends on the extent of complementarity and the difference in bargaining power. Xiao (2015) endogenizes the bargaining order in the model of Cai (2000), in which a buyer negotiates with several sellers who own perfectly complementary goods. He shows that the buyer wants to negotiate with small sellers first.[4]

Another strand of the literature analyzes simultaneous versus sequential negotiations. Horn and Wolinsky (1988) study the situation of a union bargaining over wages with two competing firms, and find that sequential bargaining is always preferred for the union.[5] Marshall and Merlo (2004) consider pattern bargaining (i.e., the first agreement sets the

[2] Krasteva and Yildirim (2012a) provide a complementary analysis and e.g., distinguish between exploding and non-exploding offers.

[3] Raskovich (2007) also considers the case wthout direct externalities and focuses on private contracts between buyer-seller pairs.

[4] Sequencing has also been studied in the literature on agenda formation (e.g. Winter 1997, Inderst 2000). However, sequencing here refers to the order of different issues.

[5] See Banerji (2002) for a related analysis.

pattern for all subsequent negotiations) and demonstrate how it affects the optimal structure of negotiations. Guo and Iyer (2013) analyze a supplier selling through two competing retailers and allow for renegotiation. They demonstrate that the optimal sequencing choice of the supplier depends on the size difference between buyers.

The literature that is connected closest to our paper is the one on contracting with externalities. In most of this literature, one side has all the bargaining power. For example, the seminal papers by Segal (1999, 2003) analyze the *offer game* where the principal has all the bargaining power. In this context, Möller (2007) studies the principal's choice of simultaneous versus sequential offers. He focuses on the impact of early negotiations on the outside option of the agents who bargain later and shows that if externalities are declining in the amount of trade, simultaneous contracting is optimal for the principal. Genicot and Rey (2006) also analyze contracting over time and demonstrate how the principal extract most surplus from agents by combining simultaneous and sequential offers. Instead, Bernheim and Whinston (1986) study the *bidding game* where the agents make the offers. Contrary to these papers, we consider a situation with intermediate bargaining power and demonstrate how the bargaining power affect the optimal negotiation sequence.

Galasso (2008) combines the offer and the bidding game in a sequential bargaining model along the lines of Rubinstein (1982), thereby allowing both sides to have bargaining power. He focuses on negative externalities between agents and shows that the principal's payoff can be decreasing in his bargaining power. In contrast our paper, he does not analyze sequencing of negotiations.

2 The Model

Assumptions. There are three players: a principal (A, "she") and two agents (B and C). A and B negotiate over a decision $b \in \mathcal{B} \subset \mathbb{R}_+^{n_b}$, with $0 \in \mathcal{B}$, and a monetary transfer $t_B \in \mathbb{R}$ from B to A. Similarly, A and C negotiate over a decision $c \in \mathcal{C} \subset \mathbb{R}_+^{n_c}$, $0 \in \mathcal{C}$, and a transfer $t_C \in \mathbb{R}$. The payoff of the principal is $u_A(b,c) + t_B + t_C$, the payoffs of the agents are $u_B(b,c) - t_B$ and $u_C(b,c) - t_C$, respectively.

Negotiations are bilateral, and the order is chosen by A. Within each stage, there is random proposer take-it-or-leave-it bargaining.[6] Bargaining power is modelled as the probability of making the offer: B proposes with probability $\beta \in [0,1]$, C proposes with $\gamma \in [0,1]$. Without loss of generality, assume that $\beta \geq \gamma$; that is, B is the stronger bargainer among the agents. As it is the objective of the paper to analyze which agent the principal will approach

[6] Alternatively, one can think of the outcome of each negotiation as given by the asymmetric Nash bargaining solution (see, for example, Muthoo 1999). All our results then continue to hold.

first, we follow the literature on sequencing decisions and rule out renegotiation.[7]

Moreover, we assume that the contract negotiated in stage 1 cannot condition on any actions chosen later in the game, because of exogenous legal constraints, or other reasons for incomplete contracting. For example, if A is an upstream firm serving two retailers B and C, a contract between A and B that conditions on c might be in conflict with competition law. As noted by Möller (2007), in practice, contingent contracts are rare, and hard to enforce.

The timing of the game is as follows. In stage 0, A chooses timing BC or timing CB. In *timing* BC, in stage 1, A bargains with B. With probability β, B proposes a contract $(b, t_B) \in \mathcal{B} \times \mathbb{R}$, and A either accepts or rejects. With probability $1-\beta$, A proposes, and B then accepts or rejects. If A and B reach an agreement on a contract (b, t_B), the decision b is implemented and the transfer t_B is made. In case of rejection, $b = t_B = 0$. In $t = 2$, C observers the outcome of stage 1. Then A and C bargain. With probability γ, C proposes a contract $(c, t_C) \in \mathcal{C} \times \mathbb{R}$; with probability $1-\gamma$, A proposes. If they reach an agreement on a contract (c, t_C), the decision c is implemented and the transfer t_C is paid. Otherwise, $c = t_C = 0$.[8] *Timing* CB is similar, except that A bargains with C in stage 1 and with B in stage 2.

Our bargaining game implies that the principal negotiates with one agent at a time. This is a very relevant situation in reality because negotiations often require physical presence of the principal and it is too costly to communicate to all agents at the same time.[9] However, there can be circumstances in which the principal can delegate the negotiations, which gives rise to the possibility of simultaneous negotiations. We will consider this case in Section 5.

We assume that there are no externalities on the nontraders: $u_B(0, c)$ is constant in c, and $u_C(b, 0)$ is constant in b. Moreover, we normalize the utility functions such that $u_A(0,0) = u_B(0, c) = u_C(b, 0) = 0$.

We say that b has negative (no, positive) externalities on C if $u_C(b, c)$ is decreasing (constant, increasing) in b. In other words, b has negative externalities on C if $u_C(b', c) \leq u_C(b'', c)$ for $b' > b''$. As b can be a vector, $b' > b''$ means that $b'_i \geq b''_i$ for all $i = 1, ..., n$ and $b'_i > b''_i$ for at least one $i = 1, ..., n$. Similarly, c has negative (no, positive) externalities on B if $u_B(b, c)$ is decreasing (constant, increasing) in c. Finally, there are negative (no, positive) externalities if b has negative (no, positive) externalities on C, and c has negative (no, positive) externalities on B.

Moreover, we say that b has strictly negative (strictly positive) externalities on C if

[7] See Möller (2007) or Montez (2014), among others, for reasons why renegotiation is often not possible.

[8] After stage 2, a game between $i = A, B, C$ might ensue, provided it has unique expected equilibrium payoffs $u_i(b, c)$ for all $(b, c) \in \mathcal{B} \times \mathcal{C}$, and the contracts cannot condition on any actions taken in the game.

[9] Due to this reason, many recent studies on bargaining such as Cai (2000), Noe and Wang (2004), and Krasteva and Yildirim (2012a,b) analyze sequential negotiations.

$u_C(b,c)$ is strictly decreasing (strictly increasing) in b whenever $c \neq 0$,[10] and (ii) c has strictly negative (strictly positive) externalities on B if $u_B(b,c)$ is strictly decreasing (strictly increasing) in c whenever $b \neq 0$.

To isolate the impact of differences in bargaining power, our main results assume some degree of symmetry between players B and C. We say that *agents are symmetric except for bargaining power* if $\mathcal{B} = \mathcal{C}$ and for all $(b,c) \in \mathcal{B}^2$, (i) u_A is a symmetric function, i.e. $u_A(b,c) = u_A(c,b)$, and (ii) $u_C(c,b) = u_B(b,c)$. Note that under symmetry, b (c) has negative externalities on C (B) if and only if there are negative externalities, and similarly for positive externalities.

Define welfare as the joint surplus of all three players, $W(b,c) := \sum_{i \in \{A,B,C\}} u_i(b,c)$. We impose the tie-breaking rule that, if A is indifferent, but welfare is strictly higher in one of the timings, A selects the welfare maximizing timing.

Preliminaries. Since within each stage there is take-it-or-leave-it-bargaining, the decisions reached in the stage maximize the joint expected surplus of the two bargaining players. Moreover, whoever proposes chooses the transfer such that the other player is just willing to accept.

Consider timing BC (timing CB can be analyzed similarly). In stage 2, the decision b and transfer t_B are already fixed. The decision reached in stage 2 maximizes the joint surplus of A and C, given b. We assume that, for any b, there exists a unique solution

$$c^*(b) := \arg\max_{c \in \mathcal{C}} \{u_A(b,c) + u_C(b,c)\}.$$

Existence is ensured when (i) the sets $\mathcal{B}$ and $\mathcal{C}$ are finite, or (ii) the payoff functions u_i $(i = A, B, C)$ are continuous on $\mathcal{B} \times \mathcal{C}$ and the sets $\mathcal{B}$ and $\mathcal{C}$ are compact. A sufficient condition for uniqueness of decisions in case (ii) is that $u_A(b,c) + u_B(b,c)$ is strictly quasiconcave in b, and $u_A(b,c) + u_C(b,c)$ is strictly quasiconcave in c.

The expected payoff of A in stage 2 of timing BC is

$$(1-\gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b,0) + t_B.$$

When $b = t_B = 0$, the expected payoff of A in stage 2 is

$$O_A^{BC} = (1-\gamma) \max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\}.$$

[10]Again, we use the vector inequality notation where $c \neq 0$ means that there is at least one element $i = 1, .., n$ such that $c_i \neq 0$.

This is the expected utility of A when the first stage negotiation with B fails; it therefore is the outside option of A in the first stage.

In the first stage of timing BC, the joint surplus of A and B consists of player B 's payoff, and the expected payoff of A in stage 2:

$$S_{AB}^{BC}(b) := u_B(b, c^*(b)) + (1-\gamma)(u_A(b, c^*(b)) + u_C(b, c^*(b))) + \gamma u_A(b, 0). \tag{1}$$

In any equilibrium of timing BC, A and B reach a decision $b^{BC} \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$,[11] and the expected payoff of A is

$$U_A^{BC} = (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}.$$

In case that there exists several $b \in \arg\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$, note that they all lead to the same payoffs for A and B. In case they lead to different welfare, we asssume that a decision that maximizes $W(b, c^*(b))$ is selected. Therefore, the welfare in any equilibrium of timing BC is unique, even if the first stage decisions are not unique. We impose the corresponding assumptions on timing CB.

3 Welfare maximizing sequence

Welfare is defined as

$$\max_{b\in\mathcal{B},\, c\in\mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\}.$$

There are two reasons why, in general, the equilibrium decisions are not welfare maximizing. The first is that the negotiation in the second stage maximizes the surplus of the two players involved. Therefore, it does not take into account the effect that the decision in this negotiation has on the agent with whom A has already signed a contract. We call this the *backward effect.* The backward effect works through the externality of c on B in timing BC, and through the externality of b on C in timing CB. As an example, suppose A is a supplier and B and C are competitors in a downstream market. Then, agreeing on a larger quantity in the second-stage negotiation has a negative effect on the agent with whom A bargained first.

The second reason why equilibrium decisions are not welfare maximizing is because A only receives a fraction of the surplus in the second-stage negotiation. This implies that,

[11] Existence of a maximum of $S_{AB}^{BC}(b)$ is ensured under the conditions discussed above (in case (ii), $b^*(c)$ is continuous by the Maximum Theorem, thus $S_{AB}^{BC}(b)$ is continuous, and a solution to $\max_{b\in\mathcal{B}} S_{AB}^{BC}(b)$ exists by the Weierstrass Theorem).

in the first stage, A does only partially consider the second-stage surplus. Therefore, first-stage decisions may be distorted away from the welfare maximizing outcome. We call this the *forward effect*. It works through two channels. First, through the externality of b on C in timing BC, and through the externality of c on B in timing CB. In the example above, if A signs a contract with a large quantity in the first stage, the surplus A and his negotiation partner can achieve in the second stage is lower due to the negative externalities of the decisions. Second, through interaction of b and c in A's utility function. This occurs because the agent with whom A bargains in the second stage, extracts A utility with some probability. As an example, suppose that A is a supplier with a convex cost function (e.g., $u_A(b,c) = -(b+c)^2$) and the negotiation sequence is BC. Then, the first-stage decision b might be chosen too high from a welfare point of view, because with some probability A will not be the proposer in the second stage, implying that C has to bear this higher cost.

Remark 1 illustrates that the forward effect and the backward effect are indeed the *only* reasons for inefficiencies. It shows that the equilibrium decisions maximize welfare in timing BC if $\gamma = 0$ (which shuts down the forward effect because A receives the full surplus in the negotiation with C) and c has no externality on B (which shuts down the backward effect). Denote the welfare in timing BC by W^{BC}, and welfare in timing CB by W^{CB}.

Remark 1 *Suppose that* $1 \geq \beta > \gamma = 0$*, and* c *has no externalities on* B*. Then* $W^{BC} = W^{FB} \geq W^{CB}$.

Proof. Consider timing BC. In the second stage, the decision reached is

$$\begin{aligned} c^*(b) &= \arg\max_{c \in \mathcal{C}} \{u_A(b,c) + u_C(b,c)\} \\ &= \arg\max_{c \in \mathcal{C}} \{u_A(b,c) + u_B(b,c) + u_C(b,c)\} \\ &= \arg\max_{c \in \mathcal{C}} W(b,c). \end{aligned}$$

Since $u_B(b,c)$ is independent of c, b is predetermined from the first stage, and adding a constant does not change the location of the maximum. In the first stage, the decision maximizes the joint surplus $S_{AB}^{BC}(b)$ of A and B. Since $\gamma = 0$, $S_{AB}^{BC}(b) = W(b, c^*(b))$. Therefore, $W^{BC} = \max_{b \in \mathcal{B}} W(b, c^*(b)) = W^{FB} \geq W^{CB}$. ■

The next proposition shows that the insight derived in the remark also applies if C has some bargaining power (i.e., $\gamma > 0$) and agents are symmetric but for bargaining power.

Proposition 1 *(i)* W^{BC} *is decreasing in* γ *and constant in* β*. Similarly,* W^{CB} *is decreasing in* β *and constant in* γ*. (ii) Suppose that agents are symmetric except for bargaining power, and* $1 \geq \beta > \gamma \geq 0$*. Then* $W^{BC} \geq W^{CB}$.

The proof of part (i) of Proposition 1 uses the following Lemma:

Lemma 1 *Suppose that $w : \mathcal{B} \to \mathbb{R}$ and $v : \mathcal{B} \to \mathbb{R}$ are functions and suppose that*

$$b^*(\gamma) := \arg\max_{b \in \mathcal{B}} (1-\gamma)\, w(b) + \gamma v(b)$$

exists for all $\gamma \in [0,1]$. Then for all $\gamma_1 \in [0,1]$ and $\gamma_0 \in [0,1]$, $\gamma_1 > \gamma_0$ implies $w(b^*(\gamma_1)) \leq w(b^*(\gamma_0))$.

Proof. See Appendix 7.1. ■

Proof of Proposition 1. Part (i). Consider timing BC (the result concerning timing CB can be established similarly). It is evident from (1) that the equilibrium decisions $\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ do not depend on β. Therefore, $W^{BC} = W\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is constant in β. Moreover, $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$, where

$$\begin{aligned} S_{AB}^{BC}(b) &= u_B(b, c^*(b)) + (1-\gamma)\left(u_A(b, c^*(b)) + u_C(b, c^*(b))\right) + \gamma u_A(b, 0) \\ &= (1-\gamma)\, W(b, c^*(b)) + \gamma\left[u_A(b, 0) + u_B(b, c^*(b))\right] \end{aligned}$$

Applying Lemma 1 with $w(b) = W(b, c^*(b))$ and $v(b) = u_A(b,0) + u_B(b, c^*(b))$ shows that $W\left(b^{BC}, c^*\left(b^{BC}\right)\right)$ is decreasing in γ.

Part (ii). Suppose agents are symmetric. If $\beta = \gamma$, timings BC and CB differ only in the names of the agents. Since equilibrium welfare is unique, $W^{BC} = W^{CB}$. Part (i) therefore implies that, if $\beta > \gamma$, $W^{BC} \geq W^{CB}$. ■

The Proposition shows that, under symmetry, welfare is higher when the principal bargains with the stronger agent first, irrespective of whether externalities are negative or positive.[12] The intuition is rooted in the forward effect: with symmetry, the backward effect plays out similarly in the two timings. This is because the players in the second-stage negotiation always maximize their joint profits.

However, the forward effect is different in both timings. If the principal negotiates with the weaker agent in the second stage, she receives a larger share of the surplus in this stage. Therefore, the utility of the agent with whom the principal bargains in the second stage is taken into account to a larger extent in the first stage negotiation. The forward effect leads to a larger distortion when the bargaining power of the agent with whom the principal negotiates in stage 2 increases. As a consequence, welfare is higher in case the principal

[12] Interestingly, it also does not matter whether the principal has more or less bargaining power than the agents, or one of them. Whenever $\beta \geq \gamma$, $W^{BC} \geq W^{CB}$, no matter whether the principal's bargaining power is low compared with the agents' bargaining power.

bargains with the weaker player in the second stage. This explains our main insight that the welfare optimal bargaining sequence is BC independent of the externalities. As we proceed to show, the sequence preferred by the principal depends on the nature of externalities.

We finally note that while part (i) of Proposition 1 does not need symmetry, part (ii) does. In fact, if agents were asymmetric, welfare can be higher in timing CB than in timing BC.[13]

4 The sequence preferred by the principal

We start this section by considering the special case in which $\beta = 1$, that is, B has all bargaining power. This case shows in a particularly transparent way how the externalities affect the principal's preference over the bargaining sequences.

Let U_A^{BC} $\left(U_A^{CB}\right)$ denote the expected payoff of A in timing BC (CB).

Remark 2 *Suppose that $\beta = 1$, $\gamma \in [0,1)$. If b has negative (no, positive) externalities on C, then $U_A^{BC} \geq U_A^{CB}$ $\left(U_A^{BC} = U_A^{CB},\ U_A^{BC} \leq U_A^{CB}\right)$. Moreover, when externalities are strictly negative (strictly positive) and equilibrium decisions in timing CB are not zero, then $U_A^{BC} > U_A^{CB}$ $\left(U_A^{BC} < U_A^{CB}\right)$.*

Proof. Since $\beta = 1$, $U_A^{BC} = O_A^{BC} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(0,c)\}$. In contrast, in timing CB, $U_A^{CB} = (1-\gamma)\max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}$ where

$$b^*(c) = \arg\max_{b\in\mathcal{B}}\{u_A(b,c) + u_B(b,c)\}.$$

Therefore,

$$U_A^{BC} - U_A^{CB} = (1-\gamma)\left(\max_{c\in\mathcal{C}}\{(u_A(0,c) + u_C(0,c))\} - \max_{c\in\mathcal{C}}\{u_A(0,c) + u_C(b^*(c),c)\}\right)$$

When there are negative externalities of b on C, then $u_C(0,c) \geq u_C(b,c)$ for all b, c. Hence $U_A^{BC} \geq U_A^{CB}$. Moreover, when externalities are strictly negative and $c \neq 0 \neq b^*(c)$, then $U_A^{BC} > U_A^{CB}$. The results on positive and no externalities can be established similarly. ■

The remark shows that for $\beta = 1$, the principal's preference is solely driven by the externality of b on C. The principal prefers timing BC if externalities are negative and

[13] The following example illustrates the point. Assume that $\gamma = 0 < \beta < 1$. Suppose that $b \in [0, 1/2]$ and $c \in \{0,1\}$. Let $u_A(b,c) = 0$; $u_B(b,1) = -b$ and $u_B(b,0) = k\left(b - b^2\right)$ with $k > 4/(1-\beta)$; $u_C(b,0) = 0$ and $u_C(b,1) = 1-b$. In timing BC, in the second stage $c^*(b) = 1$; therefore, $b^{BC} = 0$ and $U_A^{BC} = u_C(0,1) = 1$. Welfare is $W^{BC} = 1$. In timing CB, if $c = 0$, then in the second stage $b^*(c) = 1/2$ and the principal achieves a payoff of $(1-\beta)k/4$; if $c = 1$, the principal achieves 1 as before. Since by assumption $k > 4/(1-\beta)$, $c^{CB} = 0$, and $U_A^{CB} = (1-\beta)k/4$. Welfare is $W^{CB} = k/4 > W^{BC}$.

CB if externalities are positive. The intuition behind the result is again rooted in the interplay between the backward and the forward effect. If B has full bargaining power, the principal only receives a profit from the negotiation with C. When bargaining with B first, the backward effect plays no role for the principal because she receives no surplus in the negotiation with B. Only the forward effect is important. The principal's threat in the first stage is to reject B's offer, which implies $b = 0$. Hence, A can always assure herself a payoff that gives her $1 - \gamma$ of the joint surplus of A and C, where the decision c maximizes this surplus, given $b = 0$.

By contrast, when bargaining with C first, the forward effect is immaterial for A because A receives no surplus in the second stage. However, the backward effect is important because the decision A and B agree upon in the second stage affects the surplus made in the first stage. In fact, C will foresee the decision that A and B will make in the second stage. Therefore, A and C will maximize the joint surplus, taking into account that b is decided upon in the second stage. As a consequence, A can assure herself a payoff that gives her $1 - \gamma$ of the joint surplus of A and C, given that b will be positive.

The optimal sequence for the principal follows from this consideration. If externalities are negative, C's profit is higher if b equals zero than if b is positive. Since the principal obtains a share of C's profit, she prefers the sequence BC, where $b = 0$. By contrast, if externalities are positive, the joint surplus of those who bargain in stage 1 is increased through the externality. The principal then prefers the sequence CB where b is positive.

Finally if there are no externalities, the principal is indifferent. As we will show later, this last result only holds for $\beta = 1$. If the principal has strictly positive bargaining power against both agents, perhaps surprisingly, she prefers the sequence BC even without externalities.

Note that Remark 2 does not assume any symmetry. In particular, the externality of c on B does not influence the principal's choice of the bargaining sequence. To understand why, note that in timing BC, the backward effect on the joint stage-one-surplus of A and B is fully borne by B when $\beta = 1$. Likewise, in timing CB, the forward effect in the second-stage surplus is fully borne by B.

Remarks 1 and 2 have a straightforward implication for the efficiency of equilibrium timing in the case where B has all the bargaining power and C has no bargaining power.

Remark 3 *Suppose that $\beta = 1$, $\gamma = 0$, and c has no externalities on B. The equilibrium timing is efficient if b has negative externalities or no externalities on C. If b has positive externalities on C, the equilibrium timing is inefficient, unless the principal is indifferent between the two timings.*

Proof. By Remark 1, $W^{BC} \geq W^{CB}$. Suppose that b has negative externalities, or no externalities, on C. By Remark 2, $U_A^{BC} \geq U_A^{CB}$. Moreover, we assumed that if $U_A^{BC} = U_A^{CB}$ but $W^{BC} > W^{CB}$, A selects the timing BC. It follows that the equilibrium timing is welfare maximizing. Now suppose that b has positive externalities on C. By Remark 2, $U_A^{BC} \leq U_A^{CB}$. Thus, if the principal is not indifferent between the timings, $U_A^{BC} < U_A^{CB}$. ■

We now turn to the analysis of the case in which the bargaining power of both agents is strictly below 1. In particular, we are interested how the conclusions of Remark 2 need to be modified if $\beta < 1$. To isolate the effect of differing bargaining power, we focus our analysis on the symmetric case, that is, agents are symmetric but for bargaining power. We start with the case of negative externalities.

Proposition 2 *Assume that the agents are symmetric except for bargaining power, and* $1 > \beta > \gamma$. *If externalities are negative, then* $U_A^{BC} \geq U_A^{CB}$, *with strict inequality if externalities are strictly negative and equilibrium decisions are not zero.*

Proof. The symmetry of the agents has two implications that will be used in the proof. First,

$$\arg\max_{c \in \mathcal{C}} \{u_A(x,c) + u_C(x,c)\} = \arg\max_{b \in \mathcal{B}} \{u_A(b,x) + u_C(b,x)\} =: f(x) \tag{2}$$

for all $x \in \mathcal{B} = \mathcal{C}$. The function f defined in (2) gives the second stage decision that ensues after a first stage decision x; under symmetry, it is the same function in both timings. Second, symmetry implies that

$$\max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\} = \max_{b \in \mathcal{B}} \{u_A(b,0) + u_B(b,0)\}.$$

Since the outside options of A in stage one are

$$\begin{aligned} O_A^{BC} &= (1-\gamma) \max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\}, \\ O_A^{CB} &= (1-\beta) \max_{b \in \mathcal{B}} \{u_A(b,0) + u_B(b,0)\}, \end{aligned}$$

it follows that symmetry implies that

$$\beta O_A^{BC} - \gamma O_A^{CB} = (\beta - \gamma) \max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\}. \tag{3}$$

The surplus of A and B in timing BC as a function of b is

$$S_{AB}^{BC}(b) = (1-\gamma)(u_A(b,f(b)) + u_C(b,f(b))) + \gamma u_A(b,0) + u_B(b,f(b)).$$

In equilibrium of timing BC, $b = b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$. Similarly, the surplus of A and C in timing CB as a function of c is

$$S_{AC}^{CB}(c) = (1-\beta)(u_A(f(c), c) + u_B(f(c), c)) + \beta u_A(0, c) + u_C(f(c), c).$$

In equilibrium of timing CB, $c = c^{CB} \in \arg\max_{c \in \mathcal{C}} S_{AC}^{CB}(c)$. The expected payoffs of A in timing BC and CB are, respectively,

$$\begin{aligned} U_A^{BC} &= (1-\beta) S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC} \\ U_A^{CB} &= (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) + \gamma O_A^{CB}. \end{aligned}$$

Since $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$,

$$S_{AB}^{BC}\left(b^{BC}\right) \geq S_{AB}^{BC}\left(c^{CB}\right). \tag{4}$$

Moreover, by symmetry,

$$S_{AB}^{BC}\left(c^{CB}\right) = \left((1-\gamma)\left(u_A\left(f\left(c^{CB}\right), c^{CB}\right) + u_B\left(f\left(c^{CB}\right), c^{CB}\right)\right) + \gamma u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)$$

and therefore

$$(1-\beta) S_{AB}^{BC}\left(c^{CB}\right) - (1-\gamma) S_{AC}^{CB}\left(c^{CB}\right) = (\gamma - \beta)\left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right). \tag{5}$$

From (3), (4), and (5),

$$\begin{aligned} U_A^{BC} - U_A^{CB} &\geq (\beta-\gamma)\left(\max_{c \in \mathcal{C}}\{u_A(0,c) + u_C(0,c)\} - \left(u_A\left(0, c^{CB}\right) + u_C\left(f\left(c^{CB}\right), c^{CB}\right)\right)\right) \\ &\geq (\beta-\gamma)\left(\max_{c \in \mathcal{C}}\{u_A(0,c) + u_C(0,c)\} - \max_{c \in \mathcal{C}}\{(u_A(0,c) + u_C(f(c), c))\}\right). \end{aligned}$$

Negative externalities imply $u_C(0,c) \geq u_C(b,c)$ for all $b \geq 0$, and therefore $U_A^{BC} \geq U_A^{CB}$. Moreover, whenever externalities are strictly negative and $b > 0$, $u_C(0,c) > u_C(b,c)$ for all $c > 0$, therefore $U_A^{BC} > U_A^{CB}$. ■

The proposition shows that the main insight obtained in Remark 2 carry through if externalities are negative. If $\beta < 1$, in both timings the forward and the backward effect are present. When externalities are negative, both effects point in the same direction of favoring the timing BC over CB. First, in the timing BC the distortion implied by the forward effect is smaller than in timing CB because the utility of the agent A bargains with in the second stage is taken into account in the first stage to larger extent. Because A obtains a fraction of the full surplus, she favors BC over CB. Second, because externalities are negative, the agent in the first stage negotiation knows that she will receive a lower utility if the decision in the second stage is large. This distorts first-stage decisions (i.e., the backward effect). The principal suffers less from this effect if she negotiates with B first because she then obtains a lower share of the surplus in the first stage than when bargaining with C first.

It is interesting that the principal strictly prefers timing BC even if the decisions are the same in both timings, as long as externalities are strict. This in in contrast to the timing that maximizes welfare: if decisions are the same in both timings, both timings are welfare-equivalent. The intuition for why the principal strictly prefers BC is that she obtains a different share of the surplus in the two timings, even if decisions are the same. His outside option in timing BC is strictly larger because receives a larger share in the negotiation with C. The is the decisive effect for the principal if b^* and c^* are the same, leading to a strict timing preference.

We now turn to the case in which there are no externalities between agents. As demonstrated in Remark 2, if $\beta = 1$, then the principal is indifferent between the two timings. However, this is no longer true if $\beta < 1$. The reason is that even without externalities, the two bargaining problems are not independent of each other because the decisions b and c interact through the principal's payoff function. The timing of negotiations then still plays a role. As the next proposition shows the principal still prefers timing BC over CB in this case.

Proposition 3 *Assume agents are symmetric except for bargaining power, there are no externalities, and* $1 > \beta > \gamma$. *Then* $U_A^{BC} \geq U_A^{CB}$. *Moreover, the inequality is strict if first-stage decisions in the two timing differ from each other; a sufficient condition is that (i) equilibrium first-stage and second-stage decisions are interior, (ii)* u_A, u_B, u_C *and* $c^*(b)$ *are differentiable, and (iii) whenever* $c \neq c'$, *then for any* $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$ *there exists some* $i = 1, ..., n_B$, *such that*

$$\frac{\partial}{\partial b_i} u_A\left(b^{BC}, c\right) \neq \frac{\partial}{\partial b_i} u_A\left(b^{BC}, c'\right). \tag{6}$$

Proof. See Appendix 7.2 ■

Conditions (i)-(iii) are used to ensure that the first-stage decisions in the two timings problems differ from each other.[14]

[14]More generally, the proof of Proposition 3 shows that, if there are no externalities, and $b^{BC} = c^{CB}$, and

We point out that (iii) will be satisfied in many economic applications. A sufficient condition for (iii) is that the marginal returns to some b_i are strictly monotone (increasing or decreasing) in c.[15] It is satisfied, for example, when A is a supplier, sells a single homogeneous good to B and C, and has strictly increasing marginal costs. Assumption (iii) rules out the case of an additively separable u_A where there is no interaction between the bargaining problems. Assumption (iii) alone is not sufficient to rule out the possibility that first-stage decisions might be identical in the two timings, be it because they occur at a boundary of the feasible set, or because the payoff functions are not differentiable; assumptions (i) and (ii) serve to rule these possibilities out.[16]

The intuition behind result of the last Proposition lies in the forward effect, which favors timing BC. Without externalities, the backward effect is immaterial because the second-stage decision has no effect on the utility of the agent negotiating in the first stage. However, the forward effect is still important. Since b and c interact only through A's payoff function, A would like to decide about both variables at one stage. In the timing BC, she takes the second-stage maximization into account to a greater extent than in the timing CB. Therefore, the distortion in the two decisions is smaller in the timing BC. It is worth mentioning that the result holds independent of the concrete way b and c interact in u_A.

We now turn to the case of positive externalities. As shown above, with $\beta = 1$, the principal unambiguously prefers timing CB. However, in what follows we demonstrate that this is no longer true if $\beta < 1$. In fact, both timings BC and timing CB can emerge in equilibrium, even with additive separability of b and c in the principal's utility function. In order to focus on the pure effect of positive externalities, we give more structure to the utility function by considering the case of "parametric externalities". This allows us to show that given u_A is additive separable and some differentiability assumptions, A strictly prefers CB when externalities are small.

Case of parametric externalities. *The utility functions of B and C are parametrized by $k \in \mathbb{R}$ and written $u_B(b,c,k)$ and $u_C(b,c,k)$. k parametrizes the importance of externalities in the following sense: (1) u_A is constant in k; (2) if $k = 0$ there are no externalities, thus $u_B(b,c;0)$ is constant in c; (3) k has no effect on u_B when $c = 0$;*[17] *(4)*

c^{CB} maximizes $(u_A(0,c) + u_C(0,c))$, then $U_A^{BC} = U_A^{CB}$. This is the case in Krasteva and Yildirim (2012a) in the benchmark case with commonly known valuations.

[15]This sufficient condition, however, rules out some economically interesting cases covered by (iii). For example, (iii) is also satisfied when $u_A(b,c) = -\sum_{i=1}^{n}(b_i + c_i)^2$. Here, there is no single good i such that the marginal returns to b_i are strictly monotone in c. Moreover, (iii) assumes that marginal returns are unequal, not that they are monotone.

[16]Similarly, Edlin and Shannon (1998) rely on interiority and differentiability assumptions for strictly monotone comparative statics.

[17]This assumption is motivated from the idea that k should parametrize externalities *and nothing else.*

for all $b > 0$*, all* c *and* $c' > c$*,* $u_B(b, c'; k) - u_B(b, c; k)$ *is strictly increasing in* k*. Since* $u_B(b, c'; 0) = u_B(b, c; 0)$ *, it follows that, for all* $k > 0$*,* $u_B(b, c'; k) > u_B(b, c'; 0)$ *.*
W*e employ the slightly stronger assumption*[18] *that* u_B *is differentiable in* k *and*

$$\frac{\partial u_B(b, c; k)}{\partial k} > 0, \tag{7}$$

whenever $b > 0$ *and* $c > 0$*.*[19]

In the following Proposition, let $c^*(b, k) := \arg\max_{c \in \mathcal{C}} (u_A(b, c) + u_C(b, c, k))$ denote the second stage decision in timing BC, and define $b^*(c, k)$ similarly.

Proposition 4 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power,* $1 > \beta > \gamma$*,* u_A *is additively separable, and (i)* u_i *(*$i = A, B, C$*) is* C^1 *in* (b, c, k) *, (ii)* $c^*(b, k)$ *is interior and* C^1 *in* (b, k) *, and (iii)* $\mathcal{B} = \mathcal{C}$ *is compact. Then, there exists exists a* $\hat{k} > 0$ *such that* $U_A^{BC} < U_A^{CB}$ *for all* $k \in \left(0, \hat{k}\right)$ *.*

Proof. See Appendix 7.3. ■

If externalities are small, the principal prefers the inefficient timing CB. For the intuition behind the result, first note that the principal is more interested in the joint surplus of her and agent C because she receives a larger share in this negotiation than in the negotiation with B. With positive externalities, the surplus realized by A and C is larger if A and B decide on a positive b in the second stage. Therefore, the backward effect favors timing CB with positive externalities. In particular, as outlined after Remark 2, when $\beta = 1$, in the timing BC the principal receives a surplus in the negotiation with C given $b = 0$, whereas b is positive in the timing CB. This is less extreme for $\beta < 1$. However, by continuity, b is still higher in timing CB than in BC. Hence, with positive externalities the surplus in the negotiation with C is higher in the timing CB. If u_A is additive-separable and externalities are small, this effect is decisive in the preferred timing of the principal. With positive externalities, the principal is therefore willing to sacrifice overall surplus to obtain a larger piece of a smaller cake.

Proposition 4 focused on the small externalities. The question remains if the principal also prefers the timing CB if externalities are positive but large. Remark 2 showed that this

[18]The issue is that $u_B(b, c', k) - u(b, c, k)$ could have a zero derivative with respect to k on sets of measure zero.

[19]We note that if $u_B(b, c; k)$ is differentiable in c, then condition (4) together with $\partial u_B(b, c; k) / \partial k > 0$ implies $\partial^2 u_B(b, c; k) / (\partial c \partial k) > 0$.

is true for $\beta = 1$. However, as the next example demonstrates, this is not true in general if $\beta < 1$. In particular, the chosen sequence may switch from CB to BC when externalities grow.[20]

Example 1 *Let* $u_A(b,c) = -b-c$, $u_B(b,c) = \left(\sqrt{b} + k\sqrt{c}\right)$ *if* $b > 0$, $u_C(b,c) = \left(\sqrt{c} + k\sqrt{b}\right)$ *if* $c > 0$, *and* $u_B(0,c) = u_C(b,0) = 0$. *Moreover, let* $\beta > \gamma$. *Then* $U_A^{CB} > U_A^{BC}$ *whenever* $0 < k < \hat{k} := 2/\left((1-\beta)(1-\gamma)\right)$, *and* $U_A^{CB} < U_A^{BC}$ *whenever* $k > \hat{k}$.

The intuition behind the result of Example 1 is that the joint surplus increases as k gets bigger. From Section 3, we know that the joint surplus is higher with timing BC. This difference becomes more important as k increases. The principal then wants to maximize the cake an is willing to obtain a smaller share from it. He does so by choosing the welfare-optimal sequence.

Example 1 shows that the principal's chosen sequence changes non-monotonically in the externalities. If externalities are negative, the principal prefers BC, if externalities are positive but small she prefers CB whereas if externalities are positive and large, she prefers BC again. The intuition is that if externalities become large, efficiency considerations become more important. In particular, the difference in efficiency between the sequences BC and CB increases with k; this leads to $U_A^{BC} > U_A^{CB}$ when k becomes sufficiently large.[21]

We also note that the equilibrium sequence need not be non-monotonic in the externalities. For instance, replace k in Example 1 by $1 - 1/(1+k)$. The example then still fulfills all requirements for parametric externalities. However, it is then easy to show that $U_A^{CB} > U_A^{BC}$ for all $k > 0$ (i.e., even if $k \to \infty$). The reason why the equilibrium sequence does not change with k here is that when k grows large, the effect of b on u_C (and by symmetry the effect of c on u_B) stays bounded. Therefore, although the importance of externalities increases with k, they will not become dominant. From the principal's perspective, efficiency considerations are then dominated by the effect that she obtains a higher surplus in the negotiation with C.

Finally, our results allow us to compare the equilibrium sequence with the efficient one. The last proposition summarizes these insights.

Proposition 5 *Suppose agents are symmetric except for bargaining power. The equilibrium timing is efficient when there are negative or no externalities. The equilibrium timing is inefficient if externalities are positive and small and can either be inefficient or efficient if externalities are positive and large.*

[20] See Appendix 7.4 for details on the derivation of the equilibrium timing in Example 1.

[21] The threshold value $\hat{k}$ at which the principal's preferred sequence changes from CB to BC is in fact larger than 1 in Example 1. This implies that the decision variable b (c) must have a stronger effect on C (B) to render timing BC optimal for the principal.

5 Simultaneous Negotiations

We so far focused on the optimal timing of sequential negotiations. In some scenarios, simultaneous negotiations are also possible. Since the principal bargains bilaterally with each agent and cannot divide himself, a way to think about this case is that the principal delegates the negotiations to two delegates who act on his behalf. Each delegate maximizes the bilateral profit in the negotiation he is involved in, taking into account that the other delegate bargains at the same time.

We assume that no information exchange between the two delegates is possible. In particular, we rule out the case in which the delegates before the first offer is made can exchange the information on who is the proposer in each bargaining game. In this case, an offer by a delegate also has a signaling role to an agent. The mechanisms at work are then very different to the ones identified in our analysis of sequential bargaining, which makes the comparison between the to scenarios difficult. In addition, the negotiations then do no longer correspond to the Nash Bargaining Solution. This is an undesirable feature as it is natural to consider the outcome of simultaneous negotiations as the outcome of two Nash bargaining procedures. In fact, most of the literature (e.g., Horn and Wolinsky, 1988, and Marshall and Merlo, 2004) focuses on this case.

Therefore, we analyze the situation in which the two pairs of bargainers are negotiating at the same time and do not observe what is happening in the other negotiation. Every player needs to form a belief about the outcome in the other negotiation. In line with the literature, we assume that players have passive belief, that is, if a player receives an out-of-equilibrium offer, he or she does not revise his or her belief about the outcome in the other negotiation. This belief formation is very reasonable in our situation in which the principal's delegates do not exchange information. We note that if $\beta = \gamma = 0$, that is, each delegate has bargaining power, our game is then equivalent to the vertical contracting game with secret offers considered by e.g., Hart and Tirole (1990), McAfee and Schwartz (1994), or Rey and Tirole (2007), which has become a workhorse model in literature on contracting in supplier-retailer relationships.

Given passive beliefs, in the negotiation between A and B, the solution $b^{\star}$ is given by

$$b^{*}(c) := \arg\max_{b \in \mathcal{B}} \{u_A(b, c) + u_B(b, c)\},$$

where c is the belief about the outcome in the other negotiation. Similarly, in the negotiation between A and C, $c^{\star}$ is given by

$$c^{*}(b) := \arg\max_{b \in \mathcal{B}} \{u_A(b, c) + u_C(b, c)\},$$

where b is the belief about the outcome in the other negotiation. In equilibrium, beliefs are correct. In what follows, we assume that there is unique solution (b^*, c^*).

Turning to the transfers, if A is drawn as the proposer in the negotiation with B, he sets $t_B = u_B(b^\star, c^\star)$. Similarly, in the negotiation with C, he sets $t_C = u_C(b^\star, c^\star)$. By contrast, if B is selected as the proposer in the negotiation with A, she offers $t_B = -u_A(b^\star, c^\star) + u_A(0, c^\star)$. This occurs because the principal or his delegate obtains as an outside option $u_A(0, c^\star)$ when rejecting B's contract. By the same argument, if C is selected as the proposer in the negotiation with A, he sets $t_C = -u_A(b^\star, c^\star) + u_A(b^\star, 0)$.

The payoff of the principal can then be written as

$$(1-\beta)(1-\gamma)\{u_A(b^*, c^*) + u_B(b^*, c^*) + u_C(b^*, c^*)\} + (1-\beta)\gamma\{u_A(b^*, 0) + u_B(b^*, c^*)\} \quad (8)$$

$$\beta(1-\gamma)\{u_A(0, c^*) + u_C(b^*, c^*)\} + \beta\gamma\{u_A(b^*, 0) + u_A(0, c^*) - u_A(b^*, c^*)\}.$$

We can now compare the principal's payoff in the simultaneous timing with the one in the sequential timing. As above, we start with the case of negative externalities. We focus on the timing BC because we know from Proposition 2 that this timing dominates timing CB in case of negative externalities.

Proposition 6 *Suppose externalities are negative and that u_A is additive separable. The principal prefers timing BC to the simultaneous timing; moreover, the preference is strict if externalities are strictly negative.*

Proof. See Appendix 7.5. ■

The intuition behind this result is driven by two effects. The first one is similar to the intuition explaining the result in the comparison between the two sequential timings. In the sequential timing BC, the two bargainers take the utility of agent C partially into account because the principal receives a share of it. By contrast, in the simultaneous timing, the delegate of the principal and agent B do not consider the utility of agent C. As a consequence, in the simultaneous timing, the decision made by A and B is further away from the welfare-optimal decision, implying that the overall cake is lower with simultaneous timing.

The second effect, which is inherent in the simultaneous timing, is rooted in the fact that the bargainers in each negotiation cannot observe the outcome of the other negotiation (because negotiations take place simultaneously). In particular, agent C cannot observe if an agreement was reached between A and B. She supposes (correctly so on the equilibrium path) that the decision in the other negotiation was $b^* > 0$. In the sequential timing BC, agent C instead observes if there was an agreement in the negotiation between A and B. This difference per se does not affect the decisions on the equilibrium path but it affects the expected transfer that A obtains. Specifically, in the simultaneous timing, he can demand a transfer from C that equals C's utility given that $b = b^*$, whereas in the sequential timing he can demand a transfer from C that equals C's utility given that $b = 0$, in case A and B failed to reach an agreement. With negative externalities, that latter is higher than the former, thereby favoring the sequential timing.

Let us now turn to the case without externalities. We obtain the following proposition:

Proposition 7 *Assume agents are symmetric except for bargaining power and that that there are no externalities.*
(i) If u_A is super-modular in b and c, timing BC is preferred over the simultaneous timing.
(ii) If u_A is sub-modular in b and c, then timing BC is preferred over the simultaneous timing for γ close to 0, whereas the simultaneous timing if preferred over BC for γ close to 1.

Proof. See Appendix 7.6. ■

Without externalities but interaction of b and c in the principal's utility function, the first effect described after Proposition 6 is still present. This works in favor of timing BC. However, in contrast to the comparison of the sequential timings, it now matters how b and c interact in u_A (i.e., if u_A is super-modular or sub-modular). If u_A is sub-modular, A receives a positive payoff even if both agents are selected as the proposers in the respective negotiation. The reason is that in the negotiation with, say, agent B, the outside option of the principal's delegate is to reject B's offer and keep the outside option of $u_A(0, c^*)$. Therefore, the agent can only demand a transfer equal to $u_A(b^*, c^*) - u_A(0, c^*)$, which leaves a positive rent to A.[22] This effect works in favor of the simultaneous timing if u_A is sub-modular because the principal obtains no rent in the sequential timing.

The result of Proposition 7 therefore depends on the bargaining power of the agents. If the principal has a lot of bargaining power (i.e., β and γ or relatively small), the effect just described has only little bite. The principal then prefers the sequential timing. By contrast, if the agents have a high bargaining power (i.e., β and γ or relatively large), the effect is particularly strong, and the principal favors the simultaneous timing. Finally, if u_A is super-modular, the effect works in the opposite direction and favors the sequential timing over the simultaneous one.

[22]This can be seen in the last term of (8), which is positive if u_A is sub-modular, that is, is $u_A(b, 0) + u_A(0, c) > u_A(b, c)$).

The result is case of sub-modularity can be illustrated with the help of a simple example. Let $u_A(b,c) = -(b+c)^2$, $u_B(b,c) = kb - b^2$, and $u_C(b,c) = kc - c^2$. There are no externalities and u_A is sub-modular. In the sequential timing, $b^{BC} = k(1+\gamma)/(2(3+\gamma))$ and $c^\star(b^{BC}) = k/(2(3+\gamma))$. The utility of the principal is

$$\frac{k^2(4-\beta-2\beta\gamma-\beta\gamma^2)}{8(3+\gamma)}. \tag{9}$$

In the symmetric equilbrium of the simultaneous timing, $b^\star = c^\star = 1/6k$ and the utility of the principal is

$$\frac{k^2}{18}(3-\beta-\gamma). \tag{10}$$

It is evident, that (9) is larger than (10) for γ close to 0, as (9) then equals $k^2(1/6-\beta/24)$, whereas (10) is $k^2(1/6-\beta/18)$. By contrast, for γ close to 1, (9) is smaller than (10), because the former goes to zero, whereas the latter becomes $k^2/6 > 0$. (Remember that γ close to 1 also implies β close to 1.) Overall, the threshold is

$$\gamma = \frac{4\sqrt{\beta(19\beta-3)}-14\beta}{2(9\beta-4)}$$

and the simultaneous timing is preferred for γ above this threshold.

Finally, we turn to the case of positive externalities and again derive a result for small positive externalities.

Proposition 8 *Consider the case of parametric externalities. Suppose agents are symmetric except for bargaining power, $1 > \beta > \gamma$, u_A is additively separable, and (i) u_i $(i = A, B, C)$ is C^1 in (b,c,k), (ii) $c^*(b,k)$ is interior and C^1 in (b,k), and (iii) $\mathcal{B} = \mathcal{C}$ is compact. Then, there exists exists a $\hat{k} > 0$ such that $U_A^{sim} > U_A^{CB}$ for all $k \in \left(0, \hat{k}\right)$.*

Proof. See Appendix 7.7. ■

The result demonstrates that the simultaneous timing can dominate the sequential timing even if u_A is additive-separable. The result is perhaps not obvious at first glance because the simultaneous timing has the disadvantage that it does not allow the principal to commit to one of the decision variables. However, there is a clear intuition why the simultaneous timing can be optimal if externalities are positive, which rests on the non-observability of outcomes.

In any sequential timing, the bargainers in the second stage know the outcome of the first stage. If there was no agreement reached in the first stage, the principal, when being selected as the proposer, can extract from the agent in the second stage an amount that equals her payoff, given that the decision variable of the first stage is 0. By contrast, in the simultaneous timing, an agent does not observe the outcome of the other bargaining game and supposes that an agreement was reached there. If externalities are positive, this implies that the principal can demand more from the agent. Although rejections do not happen on the equilibrium path, the effect just described increases the outside option of the principal.

Finally, we demonstrate that when allowing for simultaneous and sequential timing, all three timing an indeed occur when externalities are positive. We do so again with the help of Example 1.

Example 1 (continued) *Comparing the timing BC with the simultaneous timing, we obtain*

$$U_A^{BC} - U_A^{sim} = \frac{k(1-\gamma)\left(k(1-\gamma)(1-\beta) - 2\beta\right)}{4}.$$

As obtained in the proof, for all $k < 0$*, the expression is strictly positive. However, for* $k > 0$*, the expression is negative if* $k < 2\beta / \left((1-\beta)(1-\gamma)\right)$*. Proceeding in the same in the comparison between* U_A^{CB} *and* U_A^{sim}*, we obtain that* U_A^{CB} *is larger than* U_A^{sim} *if* $k > 2\gamma / \left((1-\beta)(1-\gamma)\right)$*.*

Because $\gamma < \beta$*, we obtain the following result in the ranking of the timings:*
(i) For $k < 0$*, timing BC is optimal.*
(ii) For $0 < k < 2\gamma / \left((1-\beta)(1-\gamma)\right)$*, the simultaneous timing is optimal.*
(iii) For $2\gamma / \left((1-\beta)(1-\gamma)\right) < k < 2\beta / \left((1-\beta)(1-\gamma)\right)$*, the timing CB is optimal.*
(iv) For $k > 2 / \left((1-\beta)(1-\gamma)\right)$*, the timing BC is optimal.*

6 Conclusion

This paper has studied the optimal sequence of negotiations between one principal and two agents. We have shown that welfare is higher when the principal bargains with the stronger agent first, independent of externalities between agents are positive or negative. By contrast, the sequence chosen by the principal depends on the externalities. If externalities are negative, the principal chooses the welfare maximizing sequence. By contrast, with positive externalities, the equilibrium timing is to bargain with the weaker agent first, as long as as externalities are relatively small. If externalities are large, the principal prefers to bargain first with the stronger agent, leading to a non-monotonicity. As a consequence, the equilibrium timing can be inefficient only if externalities are positive. In addition, we also contribute to the debate if the principal prefers simultaneous or sequential bargaining. We show that simultaneous negotiations are optimal is externalities are positive but only slightly so.

In our study, we focused on the role of bargaining power and derived our main results under the assumption that agents are symmetric except for bargaining power.[23] Our analysis can therefore be extended in many dimensions. For example, agents may differ in their contribution to the total surplus instead of the bargaining power. Also, agents may be asymmetric in the externalities they exert on each other. It is interesting to analyze how these differences drive the welfare-optimal sequence and the sequence chosen by the principal. In particular, asymmetries in those other dimensions may bring in new effects that could qualify are strengthen the effects shown in the paper. We leave this for future research.

[23]Without the assumption of symmetry, we have derived some results for limiting cases of bargaining power. In particular, if one agent has all the bargaining power, the principal will negotiate with this agent first if externalities are negative but with the weaker agent first if externalities are positive.

7 Appendix

7.1 Proof of Lemma 1

Proof. Suppose to the contrary that $w(b^*(\gamma_1)) > w(b^*(\gamma_0))$. From the definition of $b^*(\gamma)$,

$$(1-\gamma_0)\, w(b^*(\gamma_0)) + \gamma_0 v(b^*(\gamma_0)) \geq (1-\gamma_0)\, w(b^*(\gamma_1)) + \gamma_0 v(b^*(\gamma_1)),$$

or equivalently,

$$(1-\gamma_0)\left(w(b^*(\gamma_0)) - w(b^*(\gamma_1))\right) \geq \gamma_0 \left(v(b^*(\gamma_1)) - v(b^*(\gamma_0))\right) \tag{11}$$

Since $w(b^*(\gamma_1)) > w(b^*(\gamma_0))$ and $1 \geq \gamma_1 > \gamma_0$, the left side of inequality (11) is strictly negative. Therefore, $v(b^*(\gamma_1)) < v(b^*(\gamma_0))$.

Similarly,

$$-(1-\gamma_1)\left(w(b^*(\gamma_0)) - w(b^*(\gamma_1))\right) \geq -\gamma_1 \left(v(b^*(\gamma_1)) - v(b^*(\gamma_0))\right) \tag{12}$$

Adding (12) to (11) shows that

$$(\gamma_1 - \gamma_0)\, w(b^*(\gamma_0)) - w(b^*(\gamma_1)) \geq (\gamma_0 - \gamma_1)\left(v(b^*(\gamma_1)) - v(b^*(\gamma_0))\right)$$

This is a contradiction because the left hand side is strictly smaller than zero, and the right hand is strictly greater than zero. ■

7.2 Proof of Proposition 3

Proof. The proof of Proposition (2) also establishes that with no externalities, $U_A^{BC} \geq U_A^{CB}$. Moreover, when $b^{BC} \neq c^{CB}$, then inequality (4) is strict. Since $\beta < 1$, it follows that $U_A^{BC} > U_A^{CB}$ when $b^{BC} \neq c^{CB}$ for any $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$ and $c^{CB} \in \arg\max_{c \in \mathcal{C}} S_{AC}^{CB}(c)$. We show that (i)-(iii) imply this is the case.

By (ii), $S_{AB}^{BC}(b)$ and $S_{AC}^{CB}(c)$ are differentiable. Since any $b^{BC} \in \arg\max_{b \in \mathcal{B}} S_{AB}^{BC}(b)$ is interior by (i), it satisfies the first order condition

$$\begin{aligned}\frac{\partial S_{AB}^{BC}\left(b^{BC}\right)}{\partial b_i} &= \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i} + (1-\gamma)\frac{\partial}{\partial b_i} u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + \gamma \frac{\partial}{\partial b_i} u_A\left(b^{BC}, 0\right) \\ &\quad + (1-\gamma)\left(\sum_k \frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right) + u_C\left(f\left(b^{BC}\right)\right)\right) \frac{df_k\left(b^{BC}\right)}{db_i}\right) = 0.\end{aligned}$$

Since $f\left(b^{BC}\right)$ is interior by (i), and $u_A(b,c)+u_C(c)$ is differentiable by (ii), the first order condition

$$\frac{\partial}{\partial c_k}\left(u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+u_C\left(f\left(b^{BC}\right)\right)\right)=0$$

holds, thus

$$\frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)=0.$$

Since $f\left(b^{BC}\right)$ is interior by (i), $f\left(b^{BC}\right)>0$. Thus (6) implies

$$\begin{aligned}
& \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\gamma)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\gamma\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)\\
\neq\ & \frac{\partial u_B\left(b^{BC}\right)}{\partial b_i}+(1-\beta)\frac{\partial}{\partial b_i}u_A\left(b^{BC}, f\left(b^{BC}\right)\right)+\beta\frac{\partial}{\partial b_i}u_A\left(b^{BC},0\right)\\
=\ & \frac{\partial u_C\left(b^{BC}\right)}{\partial c_i}+(1-\beta)\frac{\partial}{\partial c_i}u_A\left(f\left(b^{BC}\right), b^{BC}\right)+\beta\frac{\partial}{\partial c_i}u_A\left(0,b^{BC}\right)\\
=\ & \frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i}
\end{aligned}$$

where the first equality is from symmetry. We have shown that

$$\frac{\partial S_{AC}^{CB}\left(b^{BC}\right)}{\partial c_i}\neq 0.$$

Since any $c^{CB}\in\arg\max_{c\in\mathcal{C}} S_{AC}^{CB}(c)$ is interior by (i), it satisfies the first order condition

$$\frac{\partial S_{AC}^{CB}\left(c^{CB}\right)}{\partial c_i}=0,$$

thus $b^{BC}\neq c^{CB}$. ■

7.3 Proof of Proposition 4

Proof. In what follows, we denote the first stage decision in timing BC, which depends on k, by $b^{BC}(k)$.

To show the result in the most concise way, we first determine in the next lemma how the social surpluses in the two timings change with k. We note that the proof of the lemma uses a version of the envelope theorem applied to the joint first-stage surplus. We cannot directly apply to standard versions of the envelope theorem (e.g., Simon and Blume 1994, Theorem 19.4) for two reasons. First, we do not assume $b^{BC}(k)$ to be differentiable in k.

We solve this issue by using an envelope theorem from Milgrom and Segal (2002) that does not presuppose differentiability of the maximizer. Second, the choices in the second-stage do in general not maximize the joint surplus of those who bargain in the first stage. As in the envelope theorem for Stackelberg games (Caputo 1998), we need to take into account the effect of k on the second-stage reaction function. Under the assumptions of Proposition 4, however, at $k = 0$ the second-stage decision also maximizes the surplus of the negotiation in the first stage, therefore the corresponding terms disappear.

Lemma 2 *Under the assumptions of Proposition 4,* $S_{AB}^{BC}(k) = \max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ *and* $S_{AC}^{CB}(k) = \max_{c\in\mathcal{C}} S_{AC}^{CB}(b^*(c,k), c, k)$ *are differentiable in* k *at* $k = 0$*, and*

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta-\gamma)\frac{\partial}{\partial k}u_B(b,c;k)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} > 0.$$

Proof. If $k = 0$, there is no interaction between the two bargaining problems, and

$$\arg\max_b S_{AB}^{BC}(b,0) = \arg\max_b u_A(b,0) + u_B(b,0,0)$$

Our assumption that second stage decision are unique ensures that $\arg\max_b u_A(b,0) + u_B(b,0,0)$ is unique. Therefore, when $k = 0$, the first stage decision in timing BC is unique. Since $c^*(b,0)$ is interior by assumption (ii), symmetry implies that if $k = 0$, $c^*(b,0) = b^{BC}(0)$. Thus $b^{BC}(0)$ is interior. Moreover, the function $S_{AB}^{BC}(b, c^*(b,k), k)$ is continous in b and continuously differentiable in k.

Therefore, Corollary 4 from Milgrom and Segal (2002) applies (here we use assumption (iii)), and $\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)$ is differentiable in k at $k = 0$, with

$$\begin{aligned}\frac{d}{dk}\max_{b\in\mathcal{B}} S_{AB}^{BC}(b, c^*(b,k), k)|_{k=0} &= \frac{\partial}{\partial k}\left(u_B(b, c^*(b;k);k) + (1-\gamma)u_C(b, c^*(b;k);k)\right)\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} \\ &\quad + \sum_{i=1}^{n}\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c_i}\frac{\partial c_i^*(b;k)}{\partial k}\Bigg|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}}.\end{aligned}$$

The first term of the right-hand side is the direct effect of k, keeping b and c constant, whereas the second term captures that the second-stage reaction function depends on k.

We next show that

$$\left.\frac{\partial S_{AB}^{BC}(b,c,k)}{\partial c_i}\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0$$

for all $i = 1, ..., n$. We have

$$\frac{\partial S_{AB}^{BC}(b,k)}{\partial c_i} = \frac{\partial}{\partial c_i} u_B(b, c^*(b); k) + (1-\gamma)\frac{\partial}{\partial c_i}(u_A(b, c^*(b;k)) + u_C(b, c^*(b;k); k)).$$

Since $c^*(b,0)$ maximizes $u_A(b,c) + u_C(b,c;0)$ and is interior,

$$\left.\frac{\partial}{\partial c_i}(u_A(b,c) + u_C(b,c,k))\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0.$$

Moreover, at $k = 0$ there are no externalities, thus

$$\left.\frac{\partial}{\partial c_i} u_B(b,c,k)\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC},0\right)}} = 0.$$

It follows that

$$\frac{d}{dk} S_{AB}^{BC}(k) = \left.\frac{\partial}{\partial k}(u_B(b,c;k) + (1-\gamma)u_C(b,c;k))\right|_{\substack{k=0\\ b=b^{BC}(0)\\ c=c^*\left(b^{BC}(0),0\right)}}.$$

Similarly,

$$\frac{d}{dk} S_{AC}^{CB}(k) = \left.\frac{\partial}{\partial k}(u_C(b,c;k) + (1-\beta)u_B(b,c;k))\right|_{\substack{k=0\\ b=b^*\left(c^{CB}(0),0\right)\\ c=c^{CB}(0)}}.$$

By symmetry, for all x, y, and k, $u_B(x,y;k) = u_C(y,x;k)$ and thus

$$\frac{\partial}{\partial k} u_B(x,y;k) = \frac{\partial}{\partial k} u_C(y,x;k) \tag{13}$$

Moreover, symmetry implies $b^{BC}(0) = c^{CB}(0)$ and $b^*\left(c^{CB}(0);0\right) = c^*\left(b^{BC}(0),0\right)$. Evaluating (13) at $k = 0$, $x = b^{BC}(0) = c^{CB}(0)$, and $y = b^*\left(c^{CB}(0);0\right) = c^*\left(b^{BC}(0),0\right)$

gives

$$\frac{\partial}{\partial k} u_B(b, c; k) \Big|_{\substack{k=0 \\ b=b^{BC}(0) \\ c=c^*\left(b^{BC}(0),0\right)}} = \frac{\partial}{\partial k} u_C(b, c; k) \Big|_{\substack{k=0 \\ b=b^*\left(c^{CB}(0),0\right) \\ c=c^{CB}(0)}}.$$

Similarly, evaluating (13) at $k = 0$, $x = b^*\left(c^{CB}(0); 0\right) = c^*\left(b^{BC}(0), 0\right)$, and $y = b^{BC}(0) = c^{CB}(0)$ gives

$$\frac{\partial}{\partial k} u_B(b, c; 0) \Big|_{\substack{k=0 \\ b=b^*\left(c^{CB}(0),0\right) \\ c=c^{CB}(0)}} = \frac{\partial}{\partial k} u_C\left(b^{BC}, c^*\left(b^{BC}, 0\right); 0\right) \Big|_{\substack{k=0 \\ b=b^{BC}(0) \\ c=c^*\left(b^{BC}(0),0\right)}}.$$

Therefore,

$$\frac{d}{dk}\left((1-\gamma) S_{AC}^{CB}(k) - (1-\beta) S_{AB}^{BC}(k)\right)|_{k=0} = (\beta - \gamma) \frac{\partial}{\partial k} u_B(b, c; k) \Big|_{\substack{k=0 \\ b=b^{BC}(0) \\ c=c^*\left(b^{BC}(0),0\right)}},$$

which is strictly positive since by assumption $c^*(b, k) > 0$ and, as shown above, $b^{BC}(0) > 0$. ■

We can now show the result of Proposition 4. Since u_A does not depend on k, and $u_C(0, c; k)$ is independent of k,

$$O_A^{BC} = (1-\gamma) \max_{c \in \mathcal{C}} \{u_A(0, c) + u_C(0, c; k)\}$$

does not depend on k. Similarly, O_A^{CB} is independent of k. They payoff of A in timings BC and CB is

$$\begin{aligned} U_A^{BC}(k) &:= (1-\beta) S_{AB}^{BC}(k) + \beta O_A^{BC}, \\ U_A^{CB}(k) &:= (1-\gamma) S_{AC}^{CB}(k) + \gamma O_A^{CB}. \end{aligned}$$

Therefore, Lemma 2 implies that

$$\frac{\partial}{\partial k}\left(U_A^{CB}(k) - U_A^{BC}(k)\right)|_{k=0} > 0.$$

If $k = 0$, the bargaining problems do not interact, and $U_A^{BC}(0) = U_A^{CB}(0)$. By continuity, it follows that for sufficiently small $k > 0$, $U_A^{CB}(k) > U_A^{BC}(k)$. ■

7.4 Details on Example 1

In timing BC, the second stage decision is $c^*(b) = 1/4$, the joint surplus of A and B on stage 1 is

$$S_{AB}^{BC}(b) = \left(\sqrt{b} + \frac{k}{2}\right) + (1-\gamma)\left(-b + \frac{1}{4} + k\sqrt{b}\right) - \gamma b,$$

which is maximized by

$$b^{BC} = \frac{(k(1-\gamma)+1)^2}{4}.$$

Thus

$$S_{AB}^{BC}\left(b^{BC}\right) = \frac{1}{4}\left[k^2(1-\gamma)^2 + (2-\gamma)(2k+1)\right].$$

The outside option of A in the first stage of timing BC is $O_A^{BC} = (1-\gamma)/4$. Therefore,

$$U_A^{BC} = (1-\beta)\left(\frac{1}{4}\left[k^2(1-\gamma)^2 + (2-\gamma)(2k+1)\right]\right) + \beta\left(\frac{1-\gamma}{4}\right)$$

A similar argument shows that in timing CB, $b^*(c) = 1/4$,

$$\begin{aligned} c^{CB} &= \frac{(k(1-\beta)+1)^2}{4}, \\ U_A^{CB} &= (1-\gamma)\left(\frac{1}{4}\left[k^2(1-\beta)^2 + (2-\beta)(2k+1)\right]\right) + \gamma\left(\frac{1-\beta}{4}\right). \end{aligned}$$

Moreover,

$$U_A^{BC} - U_A^{CB} = \frac{1}{4}k(\beta-\gamma)(k(1-\beta)(1-\gamma)-2).$$

Thus $U_A^{BC} > U_A^{CB}$ if, and only if,

$$k > \hat{k} := \frac{2}{(1-\beta)(1-\gamma)}.$$

7.5 Proof of Proposition 6

Proof. A's payoff in timing BC is given by

$$U_A^{BC} = (1-\beta)\, S_{AB}^{BC}\left(b^{BC}\right) + \beta O_A^{BC}, \tag{14}$$

where $S_{AB}^{BC}\left(b^{BC}\right)$ is given by

$$S_{AB}^{BC}\left(b^{BC}\right) := u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + (1-\gamma)\left(u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right) + \gamma u_A\left(b^{BC}, 0\right)$$

and

$$O_A^{BC} = (1-\gamma) \max_{c \in \mathcal{C}} \{u_A(0,c) + u_C(0,c)\}.$$

Inserting the last two expressions into (14) and rearranging yields

$$(1-\beta) \left\{u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + (1-\gamma)\left(u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right) + \gamma u_A\left(b^{BC}, 0\right)\right\}$$

$$+\beta(1-\gamma)\{u_A(0, c^*(0)) + u_C(0, c^*(0))\}.$$

This can be written as

$$(1-\beta)(1-\gamma)\left\{u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) + u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right\} \quad (15)$$

$$+(1-\beta)\gamma\left\{u_A(b^{BC}, 0) + u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right)\right\} + \beta(1-\gamma)\{u_A(0, c^*(0)) + u_C(0, c^*(0))\}.$$

We now compare (15) with (8). If u_A is additive separable in b and c, then $u_A(b,c) = u_A(b,0) + u_A(0,c)$. This implies that the last term of (8) equals zero.

Looking at the first and the second term of (15), it is easy to see that the structure is the same as the one of the first two terms of (8). However, the arguments are different. In (8), they are b^* and c^* or b^* and 0, whereas in (15) they are b^{BC} and $c^*\left(b^{BC}\right)$ or b^{BC} and 0. If b^{BC} were equal to b^*, then $c^*\left(b^{BC}\right)$ will also be equal to c^* because the maximization problem with respect to c is then the same in the simultaneous and the sequential timing. However, b^{BC} is chosen to maximize the first two terms of (15) (i.e., taken into account the reaction of c in the second stage). Therefore, by a revealed preference argument, if b^{BC} differs from b^*, the first two terms of (15) must be larger than the one of (8). In fact, $b^{BC} \neq b^*$, if c^* depends on b. This holds if there are either externalities in the agents' payoff functions or if u_A is not additive separable in b and c (or both).

Finally, we need to compare the last term of (15) (i.e., $\beta(1-\gamma)\{u_A(0, c^*(0)) + u_C(0, c^*(0))\}$), with the third term of (8) (i.e., $\beta(1-\gamma)\{u_A(0, c^*) + u_C(b^*, c^*)\}$). Since $b^* \geq 0$ and $c^*(0)$ maximizes $u_A(0,c) + u_C(0,c)$, it is evident that the latter term is lower then the former if externalities are negative. It follows that all terms in (8) are weakly lower than those in (15) if externalities are negative and u_A is additive separable. In addition, (8) is strictly lower than (15) if externalities are strictly negative. ■

7.6 Proof of Proposition 7

Proof. We know from Proposition 3 of the paper that timing BC is preferred over timing CB in case of no externalties. Therefore, can we focus on timing BC in our comparison with the simultaneous timing.

We start with u_A being super-modular in b and c. As in the proof of the previous proposition, we need to compare (8) with (15). Let us first look at the last term of (8). It is easy to see that this term is negative if u_A is super-modular in b and c, that is, $u_A(b,c) > u_A(b,0) + u_A(0,c)$.

Comparing the third term of (8) with the last term of (15), it is also easy to see that they are the same with no externalities. This is because $u_C(b^*,c^*) = u_C(0,c^*)$ without externalities. The difference in the remaining terms between (8) and (15) can be written as

$$(1-\beta)\Big\{u_A(b^*,c^*) + u_B(b^*,c^*) + u_C(b^*,c^*) \tag{16}$$

$$-u_A\left(b^{BC}, c^*\left(b^{BC}\right)\right) - u_B\left(b^{BC}, c^*\left(b^{BC}\right)\right) - u_C\left(b^{BC}, c^*\left(b^{BC}\right)\right)\Big\}.$$

By setting b^{BC} equal to $b^\star$, the difference equals zero because then $c^*\left(b^{BC}\right) = c^*$. However, b^{BC} is chosen to maximize the second line of (16). Therefore, the difference must be weakly negative. It follows that timing BC is preferred by the principal if u_A is super-modular.

We now turn to the case in which u_A is sub-modular in b and c. Suppose first that $\gamma = 0$. Then, the last term in (8) drops out. However, the arguments just given for the case of u_A being super-modular continue to hold. In particular, the difference in (16) is still weakly negative. Hence, timing BC is still preferred by the principal if u_A is sub-modular and $\gamma = 0$. By continuity, the result also holds in the vicinity of $\gamma = 0$.

Finally, suppose $\gamma = 1$, which implies that $\beta = 1$ (since $\gamma \leq \beta$). It is evident that (15) is equal to zero, whereas (8) equals $u_A(b^*,0)+u_A(0,c^*)-u_A(b^*,c^*)$. But u_A being sub-modular implies $u_A(b^*,0) + u_A(0,c^*) > u_A(b^*,c^*)$; hence, (8) is positive. Again, by continuity, the result also holds n the vicinity of $\gamma = 1$. ■

7.7 Proof of Proposition 8

Proof. From the proof of Proposition 4 of the paper we know that

$$\frac{d}{dk}u_A^{CB}(k) = \frac{\partial}{\partial k}\left((1-\gamma)u_C(b,c;k) + (1-\gamma)(1-\beta)u_B(b,c;k)\right)\Bigg|_{\substack{k=0 \\ b=b^*\left(c^{CB}(0),0\right) \\ c=c^{CB}(0)}}$$

Applying the same logic to (8) , we obtain

$$\frac{d}{dk}u_A^{sim}(k) = \frac{\partial}{\partial k}\Big((1-\beta)(1-\gamma)\left(u_B(b,c;k) + u_C(b,c;k)\right)$$

$$+ (1-\beta)\,\gamma u_B\,(b,c;k) + \beta(1-\gamma)u_C\,(b,c;k)\Big)\Bigg|_{\substack{k=0\\ b=b^*\\ c=c^*}}$$

$$= \frac{d}{dk}u_A^{sim}\,(k) = \frac{\partial}{\partial k}\left((1-\gamma)u_C\,(b,c;k) + (1-\beta)\,u_B\,(b,c;k)\right)\Bigg|_{\substack{k=0\\ b=b^*\\ c=c^*}}$$

Symmetry of agents, no externalities and u_A being additive separable implies $b = b^*\left(c^{CB}\,(0)\,,0\right) = b^*$ and $c^{CB}\,(0) = c^*$. As a consequence,

$$\frac{d}{dk}\left\{u_A^{sim}\,(k) - u_A^{CB}\,(k)\right\} = \frac{\partial}{\partial k}\left(\gamma(1-\beta)u_B\,(b,c;k)\right)\Bigg|_{\substack{k=0\\ b=b^*\\ c=c^*}} > 0$$

If $k=0$, the bargaining problems do not interact, and $U_A^{sim}\,(0) = U_A^{CB}\,(0)$. By continuity, it follows that for sufficiently small $k>0$, $U_A^{sim}\,(k) > U_A^{CB}\,(k)$. ■

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