

Gaining Control by Losing Control*

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ABSTRACT

We study an agency model with two vertical tasks—an upstream (e.g. capital-raising) and a downstream task (e.g. implementation). The effort for the upstream task cannot be verified, and the principal can do it herself (self-performance), or delegate it to the agent (delegated-performance). Only the agent can do the downstream task whose environment is his private information. We show the following. When the cost of effort for the upstream task is small, self-performance is optimal—under delegated-performance, the upstream task is more likely to fail due to the agent’s ‘shirking’ incentive. When the cost of effort is large, by contrast, delegated-performance is optimal—under self-performance, the principal makes an ‘excessive’ effort for the upstream task, and as a result, the distortion in the downstream task’s output schedule due to the agent’s private information becomes more of an issue. Our result also has an implication for the principal’s bias in favor of self- or delegated-performance.

JEL Classification: D82, D86

Key words: Agency, Moral Hazard, Adverse Selection, Excessive Effort, Shirking.

1 Introduction

In a principal-agent relationship, if the principal can perform a task herself, she has to make a decision about whether or not to delegate the task to the agent based on both direct and strategic benefits. This paper provides a new economic rationale for delegating a task to the agent. We show that when the principal performs a task herself, she may exert an “excessive effort” when the project is less profitable—although the task is shirked less frequently, allocation of the organization’s costly effort may be less efficient. When the task is delegated to the agent, although “shirking” takes place more often, our result suggests that the very prospect of shirking sometimes allows the principal to allocate the organization’s effort more efficiently compared to when the task is performed by the principal.

There are two vertical tasks in our model—an upstream task (e.g. capital-raising) and a downstream task (e.g. implementation). The principal can perform the upstream task herself (self-performance), or delegate the task to the agent (delegated-performance). The effort for the upstream task is ‘hidden action’ of the party who is in charge. Only the agent can perform the downstream task, and the task environment (e.g. cost of implementation) is his ‘hidden information.’ An effort for the upstream task increases the likelihood that the task succeeds. If the upstream task succeeds, then the project can generate a revenue from the downstream task. If the upstream task fails, then nothing can be done in the downstream task, and the game ends at that point.

In the absence of moral hazard and/or adverse selection problem, self-performance is always optimal.¹ If an effort for the upstream task is not hidden (an effort can be contracted upon), then it does not matter which party performs the task. That is, it is simply a matter of transferring the cost of effort based on which party performs the task. If information on the downstream task is publicly known, then the principal is strictly better off with self-performance—since the agent reaps no information rent from the downstream task, delegating the upstream task to the agent only brings about a moral hazard problem. In the presence of both the moral hazard and the adverse selection problem, however, the principal faces a time-inconsistency issue when performing the upstream task herself, which leads to our central trade-off between self- and delegated-performance.

¹For expositional purpose, we present the case where the agent’s information on the downstream task is public knowledge, but the upstream effort level cannot be verified (only the moral hazard problem) in Appendix B. The benchmark presented in the main text is the case where the information is privately possessed by the agent, but the upstream effort level is verified (only the adverse selection problem).

The preview of our results is as follows. The principal must provide information rent to the agent for the downstream task, due to the agent's private information about the task environment. For this reason, with self-performance of the upstream task, the principal has less incentive to exert an effort 'ex ante' compared to the case where the agent's information is public. Since the principal's effort level cannot be verified, however, the principal cannot help making more effort 'ex post' to make the upstream task successful, even if the downstream task's environment is not sufficiently favorable. In other words, with self-performance, the principal's effort is excessive for less favorable environments for implementing the project. As a result, the agent obtains excessive information rent from the downstream task, and to reduce his rent, the optimal contract distorts the output schedule further downward (compared to the case where an effort level can be verified).

With delegated-performance, on the other hand, the agent has an incentive to shirk the upstream task unless he reaps enough information rent from the downstream task. We show that the agent's shirking incentive has a positive side. Since preventing shirking is costly to the principal, the agent's effort for the upstream task is induced only when the downstream task's environment is sufficiently favorable. In other words, delegated-performance enables the principal to focus allocation of effort for more favorable environments for implementing the project. In addition, the direction of change in the output distortions under delegated-performance is the opposite of that under self-performance. In order to induce the agent's effort for the upstream task, the principal increases the agent's information rent in the downstream task. As a result, the project's output schedule in the optimal contract gets distorted less (compared to the case where an effort level can be verified).

Comparing self- and delegated performance, when the cost of effort for the upstream task is small, self-performance prevails—for a small effort cost, the principal's excessive effort and a larger information rent for the agent under self-performance is less of a problem when compared to the agent's shirking under delegated-performance. As the cost of effort for the upstream task increases, delegated-performance becomes more attractive for the opposite reason. For a large cost of effort, delegated-performance prevails.

Our result also implies a potential bias in favor of self- or delegated-performance when the principal's and the agent's cost of effort for the upstream task are different. When the costs of upstream effort are small, the principal may perform the task herself, even if her cost is larger than the agent's—a bias in favor of self-performance. When the costs of upstream effort are large, by contrast, the principal may delegate the task to the agent, even if her cost is smaller than the agent's—a bias in favor of delegated-performance.

There are previous contributions, like ours, that provide a rationale for delegation in a principal-agent relationship when the principal’s commitment power is limited.² In the majority of literature, unlike in our paper, the principal is allowed to renegotiate the contracted variables in the beginning, which causes incentive issues such as ratchet problems. Beaudry and Poitevin (1995) point out that delegation of decision making can make it harder to achieve a successful renegotiation. Poitevin (2000) extends their analysis to a multi-agent setting. According to Meyer et al. (1996), assigning the agents joint responsibility for tasks alleviates ratchet effects. Olsen and Torsvik (2000) show that a firm’s ability to learn about the difficulty of the tasks agents engage in will induce the firm to give agents more discretion over tasks and weaker incentives. In our paper, the contracted variables cannot be renegotiated. In addition, unlike in these papers, the principal in our paper is allowed to directly contribute to a task by making an effort—we distinguish “self- vs. delegated-performance” from “centralization vs. delegation” in that sense.³

The following studies demonstrate the optimality of delegation without the possibility of renegotiating the contracted variable. Laffont and Martimort (1998) show that delegation can prevent collusion between the agents. In their model, when the principal cannot discriminate wages, centralized contracting is not collusion proof. They show that delegation of contracting to an agent allows the principal to effectively discriminate wages, thus blocking collusion. Melumad, et al. (1997) also study the optimality of contractual delegation. The authors show that delegation allows decisions to be more sensitive to the agent’s private information, and identify circumstances under which the flexibility gain outweighs the loss of control. Unlike in ours, the principal cannot perform a task herself in these papers.

Lastly, our paper is also linked to the studies that cope with both adverse selection and moral hazard problem. The studies include Lewis and Sappington (1997), Crémer et al. (1998), Gerardi and Maestri (2012), Hoppe and Schmitz (2013), Iossa and Martimort (2015), Ulbricht (2016), and Zambrano (2019). Again, in these studies, there is no costly effort made by the principal to directly contribute to a task of the project.

The rest of the paper is organized as follows. The model is presented in the next section. In Section 3, we present and discuss the optimal outcomes with self- and delegated-performance. We compare the two arrangements for optimality in Section 4. In Section 5, we gather conclusion with remarks. All proofs are relegated to Appendix A.

²See Gibbons et al. (2013) for a survey.

³There are also studies demonstrating that delegation can provide more information. See for example, Aghion and Tirole (1997), Dessein (2002), Aghion et al. (2002, 2004), Harris and Raviv (2005), Alonso and Matouschek (2007, 2008), and Ludema and Olofsgård (2008).

2 Model

A principal hires an agent for a project that involves an upstream and a downstream task. The upstream task can be seen, for example, as capital or fund raising for the project, and the downstream task as implementing the project. At the outset, the principal decides whether to perform the upstream task herself (self-performance: $\varphi = s$), or delegate the task to the agent (delegated-performance: $\varphi = d$). The downstream task must be done by the agent, who privately learns the task environment upon accepting the principal's offer. The principal pays a lump sum transfer t out of the project's revenue to the agent for his service(s).

The Upstream Task The cost of effort for the upstream task is given by $\psi(\alpha) = \beta\alpha$, where $\alpha \in [0, 1]$. Given an effort level α , the upstream task succeeds ($k = Y$: Yes) with probability $\gamma(\alpha)$, and fails ($k = N$: No) with $1 - \gamma(\alpha)$. An effort for the upstream task increases the likelihood of success and we let:

$$\gamma(\alpha) = \alpha\gamma_G + (1 - \alpha)\gamma_B, \quad \text{where } \gamma_G, \gamma_B \in (0, 1) \text{ and } \Delta\gamma \equiv \gamma_G - \gamma_B > 0.$$

Thus, $\gamma(\alpha) = \gamma_G$ with $\alpha = 1$, and $\gamma(\alpha) = \gamma_B$ with $\alpha = 0$.

An effort level for the upstream task (“upstream effort” hereafter), $\alpha \in [0, 1]$, is hidden action and cannot be verified. If the upstream task succeeds ($k = Y$ with $\gamma(\alpha)$), the downstream task can generate an output level $q \in \mathbb{R}_+$. The principal values the project by $v(q)$ that satisfies the Inada condition.⁴ If the upstream task fails ($k = N$ with $1 - \gamma(\alpha)$), the downstream task generates no output: $q_N = 0$. The realized $k \in \{Y, N\}$ is publicly observed.

The Downstream Task The agent's cost of implementing the downstream task for output q is given by $c(q, \theta) = \theta q$, where θ is the marginal cost of production that represents the environment to implement the project. The project environment θ is drawn from a range $\Theta = [\underline{\theta}, \bar{\theta}]$ with the corresponding distribution and density function, $F(\theta)$ and $f(\theta)$ respectively. We make the standard assumption of non-decreasing hazard ratio:

$$\frac{d[F(\theta)/f(\theta)]}{d\theta} \geq 0.$$

⁴The Inada conditions of $v(q)$ assures that the entrepreneur does not want to exclude any $\theta \in \Theta$ provided that the project can generate a strictly positive revenue (i.e., $k = Y$).

The distribution of θ is publicly known, but the agent privately observes the realized θ . Regardless of $\varphi \in \{s, d\}$, once participated, the agent observes θ and makes a report on it, denoted by $\hat{\theta} \in \Theta$.

The Offer and the Payoffs The principal's offer to the agent specifies $\{\varphi, q_k(\hat{\theta}), t_k(\hat{\theta})\}$, where $\varphi \in \{s, d\}$, $k \in \{Y, N\}$ and all contractual variables can be verified. After the agent's report on $\theta \in \Theta$, the party that engages in the upstream task (depending on $\varphi \in \{s, d\}$) chooses her/his effort level α , which determines the likelihood of the project's success $\gamma(\alpha)$. The agent's liability is limited in that he can quit and walk away anytime. Thus, $t_N(\theta) = 0$ in the optimal contract since $q_N(\theta) = 0$ regardless of $\varphi \in \{s, d\}$.⁵ To simplify the notations, we let: $q(\theta) \equiv q_Y(\theta)$ and $t(\theta) \equiv t_Y(\theta)$.

The principal's and the agent's *ex post* payoffs are respectively:

$$\pi = \begin{cases} \gamma(\alpha(\hat{\theta})) [v(q(\hat{\theta})) - t(\hat{\theta})] - \beta\alpha(\hat{\theta}) & \text{with } \varphi = s, \\ \gamma(\alpha(\theta)) [v(q(\hat{\theta})) - t(\hat{\theta})] & \text{with } \varphi = d. \end{cases}$$

$$u = \begin{cases} \gamma(\alpha(\hat{\theta})) [t(\hat{\theta}) - \theta q(\hat{\theta})] & \text{with } \varphi = s, \\ \gamma(\alpha(\theta)) [t(\hat{\theta}) - \theta q(\hat{\theta})] - \beta\alpha(\theta) & \text{with } \varphi = d. \end{cases}$$

With $\varphi = s$, the principal incurs the cost of upstream effort, whereas with $\varphi = d$, the agent incurs the cost of effort. The agent's reservation payoff is normalized to zero.

The Timing We summarize the timing of the game as follows:

1. The principal offers $\{\varphi, q(\hat{\theta}), t(\hat{\theta})\}$ to the agent.
2. If the offer is accepted, the agent observes $\theta \in \Theta$ and sends a report on it.
3. The principal/the agent (according to φ) chooses α , and $k \in \{Y, N\}$ is realized.
4. If $k = Y$, then the agent produces $q(\hat{\theta})$ and the principal pays $t(\hat{\theta})$ to the agent.

The First-Best Outcome When neither adverse selection nor moral hazard is an issue, the optimal outcome is the first-best. The first-best outcome maximizes the total surplus, and is characterized as:

$$v'(q^*(\theta)) = \theta,$$

⁵We will discuss more on this point in the concluding section.

and the optimal effort levels are:

$$\alpha^* = 1 \text{ for } \theta \leq \theta^* \text{ and } \alpha^* = 0 \text{ for } \theta > \theta^*,$$

where θ^* is characterized by:

$$v(q^*(\theta^*)) - \theta^* q^*(\theta^*) = \beta / \Delta\gamma \text{ and } \Delta\gamma \equiv \gamma_G - \gamma_B > 0.$$

In our model, the principal's choice of θ^* is equivalent to her choice of effort. The cutoff θ^* is interior ($\theta^* < \bar{\theta}$) for $\beta > \beta^* \equiv \Delta\gamma [v(q^*(\bar{\theta})) - \bar{\theta}q^*(\bar{\theta})]$. For $\beta \leq \beta^*$, the effort choice is $\alpha^* = 1$ for $\forall \theta \in \Theta$, i.e., $\theta^* = \bar{\theta}$. Without adverse selection and moral hazard problem, the principal is indifferent to $\varphi \in \{s, d\}$.

3 Analyses and Results

For later convenience, we first establish the following expressions.

Definition 1 Define the following expressions for the principal's surplus for any given output level $q(\theta)$:

$$\begin{aligned} \pi^*(q(\theta), \theta) &\equiv v(q(\theta)) - \theta q(\theta), \\ \pi^b(q(\theta), \theta) &\equiv v(q(\theta)) - \left(\theta + \frac{F(\theta)}{f(\theta)} \right) q(\theta). \end{aligned}$$

In the literature, $\pi^b(q(\theta), \theta)$ is referred to as “virtual surplus,” which has taken account of the rent given to the agent. For convenience, we will refer to $\pi^*(q(\theta), \theta)$ as “real surplus.” Notice that, for any given $q(\theta)$, $\pi^*(q(\theta), \theta) > \pi^b(q(\theta), \theta)$. The maximizers of $\pi^*(q(\theta), \theta)$ and $\pi^b(q(\theta), \theta)$ are denoted by $q^*(\theta)$ and $q^b(\theta)$ respectively, and:

$$\pi_q^*(q^*(\theta), \theta) = 0 \quad \text{and} \quad \pi_q^b(q^b(\theta), \theta) = 0,$$

where $q^*(\theta)$ is the first-best output schedule presented in the previous section. The following lemma establish the properties of these surpluses and output schedules.

Lemma 1 $\frac{d\pi^b(q^b(\theta), \theta)}{d\theta} < 0$ where $\frac{dq^b(\theta)}{d\theta} < 0$, and $\pi_q^*(q^b(\theta), \theta) \equiv \frac{d\pi^*(q(\theta), \theta)}{dq} \Big|_{q=q^b} > 0$.

We first discuss our benchmark case below—the case where upstream effort $\alpha \in [0, 1]$ is publicly verifiable. For expositional purpose, the case in which the agent's information on the downstream task is public knowledge, but the upstream effort level cannot be verified (the case where only the moral hazard is an issue) is relegated to Appendix B.⁶

⁶As shown in Appendix B, self-performance ($\varphi = s$) dominates delegated-performance ($\varphi = d$) in that case.

When Upstream Effort Can be Verified (Benchmark)

When the upstream effort can be verified, it does not matter which party performs the upstream task. It is simply a matter of transferring the effort cost $\beta\alpha$ based on which party bears upstream effort—as in the first-best case, the principal is indifferent to $\varphi \in \{s, d\}$. Without loss of generality, therefore, we allocate the upstream task to the principal here ($\varphi = s$).

In setting out the principal’s problem, we list the constraints that must be satisfied. To induce the agent’s truth-telling on θ , the following incentive compatibility constraint must be satisfied:

$$\theta \in \arg \max_{\hat{\theta}} u(\hat{\theta}|\theta), \text{ where } u(\hat{\theta}|\theta) \equiv \gamma(\alpha(\hat{\theta})) \left[t(\hat{\theta}) - \theta q(\hat{\theta}) \right]. \quad (IC)$$

In addition, to induce the agent’s participation, the optimal contract must satisfy:

$$u(\theta) \geq 0 \text{ for } \forall \theta \in \Theta, \text{ where } u(\theta) \equiv u(\theta|\theta). \quad (PC)$$

The principal’s problem, when upstream effort level $\alpha \in [0, 1]$ is publicly verified, is:

$$\text{Max}_{\alpha, q, t} \int_{\Theta} [\gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] - \beta\alpha(\theta)] f(\theta) d\theta,$$

subject to (IC) and (PC).

As in standard models of adverse selection, the participation constraint (PC) is binding for $\theta = \bar{\theta}$, and we substitute the incentive constraint into the principal’s objective function. Then, recalling that $\gamma(\alpha) = \alpha\gamma_G + (1 - \alpha)\gamma_B$, the principal’s problem can be written as in the following lemma.

Lemma 2 *Suppose upstream effort is publicly verified. Then, there exists θ^b such that $\alpha = 1$ for $\theta \leq \theta^b$, and $\alpha = 0$ for $\theta > \theta^b$. The principal’s problem becomes:*

$$\mathcal{P}^b : \text{Max}_{\theta^b} \int_{\underline{\theta}}^{\theta^b} \{ \gamma_G [\pi^b(q(\theta), \theta)] - \beta \} f(\theta) d\theta + \int_{\theta^b}^{\bar{\theta}} \gamma_B [\pi^b(q(\theta), \theta)] f(\theta) d\theta.$$

Again, the principal’s choice of θ^b is equivalent to her choice of effort. Thus, the first term in the principal’s objective function in Lemma 1 is her surplus less the effort cost with $\alpha = 1$ for $\theta \leq \theta^b$, and the second term is her surplus with $\alpha = 0$ for $\theta > \theta^b$.

Definition 2 *Let $\beta^b \equiv \Delta\gamma\pi^b(q^b(\bar{\theta}), \bar{\theta})$.*

The following proposition presents the optimal outcome when upstream effort is publicly verified.

Proposition 1 *The optimal outcome in \mathcal{P}^b entails that:*

- (i) $q(\theta) = q^b(\theta)$,
- (ii) θ^b is characterized by $\pi^b(q^b(\theta^b), \theta^b) = \beta/\Delta\gamma$, where $\theta^b < \bar{\theta}$ for $\beta > \beta^b$,
- (iii) θ^b is decreasing in β .

The optimal output schedule, $q^b(\theta)$, that maximizes $\pi^b(q(\theta), \theta)$ in Definition 1 is characterized by $\pi_q^b(q^b(\theta), \theta) = 0$, and is the well-known second-best output schedule (Baron and Myerson, 1982): $q^b(\theta) < q^*(\theta)$. The intuition behind the downward distortion is standard and well known—the agent can reap information rent by exaggerating θ , and the principal reduces the agent’s rent by distorting the project’s optimal output schedule downward.

For β small enough ($\beta \leq \beta^b$), $\alpha = 1$ for $\forall \theta \in \Theta$, i.e., $\theta^b = \bar{\theta}$. In such a case, the marginal benefit of upstream effort is always greater than the marginal cost of effort *ex ante*. For $\beta > \beta^b$, however, as a result of balancing the marginal benefit and the marginal cost, the upstream task is shirked when the project environment is not attractive enough. The interior cutoff θ^b is determined by the virtual surplus with the second best-output, $\pi^b(q^b(\theta), \theta)$.

We now proceed to the main part of this paper—the cases where upstream effort cannot be verified.

When Upstream Effort Cannot be Verified

Without verifiability, upstream effort is chosen based on the ex post incentive of the party in charge. Therefore, the contract offered by the principal must take the effort making party’s incentive at the point of making an effort for the upstream task. We will first discuss the case with self-performance, followed by the case with delegated-performance.

Self-Performance ($\varphi = s$)

Recall from the principal’s problem in \mathcal{P}^b (the benchmark case) that, when upstream effort is publicly verifiable thus can be contracted upon, the principal is indifferent to $\alpha \in [0, 1]$ *ex ante* at $\theta = \theta^b$, with $\alpha = 1$ for $\theta < \theta^b$ and $\alpha = 0$ for $\theta > \theta^b$. When upstream effort cannot be verified (thus not contractible upon), however, the principal can change her effort level due to her interest *ex post*. Thus, we need to see that if the principal would still be indifferent to $\alpha \in [0, 1]$ at $\theta = \theta^b$ at the point of making an effort (*not* at the point of contracting).

With the optimal outcome in \mathcal{P}^b in Proposition 2, let us evaluate the principal's *ex post* incentive regarding her own effort with self-performance. The principal's payoffs are:

$$\begin{aligned} & \gamma_G [v(q^b(\theta^b)) - t(\theta^b)] - \beta \quad \text{if } \alpha(\theta^b) = 1, \\ & \gamma_B [v(q^b(\theta^b)) - t(\theta^b)] \quad \text{if } \alpha(\theta^b) = 0, \end{aligned}$$

Again, we stress that the principal is indifferent to $\alpha \in [0, 1]$ *ex ante*, according to the principal's problem in \mathcal{P}^b , where she can fully commit to her effort. This, however, does not mean that she is indifferent to her effort *ex post*. Comparing the principal's payoffs at $\theta = \theta^b$, the following must be true regarding her *ex post* incentive for α with the optimal outcome in \mathcal{P}^b :

$$\alpha(\theta^b) = \begin{cases} 1 & \text{if } \gamma_G [v(q^b(\theta^b)) - t(\theta^b)] - \beta \\ & > \gamma_B [v(q^b(\theta^b)) - t(\theta^b)], \\ 0 & \text{if } \gamma_G [v(q^b(\theta^b)) - t(\theta^b)] - \beta \\ & < \gamma_B [v(q^b(\theta^b)) - t(\theta^b)]. \end{cases}$$

The next claim shows the principal's *ex post* incentive.

Claim 1 *In \mathcal{P}^b , the principal strictly prefers making an effort at $\theta = \theta^b$ *ex post*.*

The claim above implies that, unless θ is not sufficiently larger than θ^b , the principal will choose $\alpha = 1$ with the optimal outcome in \mathcal{P}^b —again, when upstream effort is contractible, $\alpha = 0$ for any $\theta > \theta^b$, which is her optimal choice *ex ante*. Thus, the optimal contract under self-performance must respect the principal's *ex post* incentive for her upstream effort.

When upstream effort is hidden action, the principal's problem with $\varphi = e$ is as follows:

$$\text{Max}_{\alpha, q, t} \int_{\Theta} \{\gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] - \beta\alpha(\theta)\} f(\theta) d\theta,$$

subject to (IC), (PC) and

$$\alpha(\theta) \in \arg \max_{\hat{\alpha}} \{\gamma(\hat{\alpha}(\theta)) [v(q(\theta)) - t(\theta)] - \beta\hat{\alpha}(\theta)\}.^7 \quad (EC^s)$$

⁷We rely on the revelation principle in Myerson (1986) since our setup features moral hazard and adverse selection. According to the revelation principle, there is no loss in focusing on mechanisms under which the agent truthfully reports his type and follows the mechanism's recommendations. If the principal does not delegate the upstream decision, the principal is also a player (i.e., an agent). The principal has no private information but makes a uncontractible decision. The mechanism according to Myerson (1986) asks the agent to report his type and then sends the principal either the recommendation to invest or not to invest. The recommendation is informative about the agent's type, but the principal does not perfectly know theta when making the investment decision.

Constraint (EC^s) reflects that the principal's upstream effort is hidden action. The principal's inability to commit to her effort level is the central issue with self-performance—her effort level is chosen at the point of making an effort, which in turn may affect the outcome in the optimal contract.

The principal's problem described above can be rewritten as in the following lemma.

Lemma 3 *Suppose the upstream effort is hidden action. With $\varphi = s$, there exists θ^s such that $\alpha = 1$ for $\theta \leq \theta^s$, and $\alpha = 0$ for $\theta > \theta^s$. The principal's problem becomes:*

$$\begin{aligned} \mathcal{P}^s : \quad & \underset{q, \theta^s}{\text{Max}} \int_{\underline{\theta}}^{\theta^s} \left\{ \gamma_G \left[\pi^b(q(\theta), \theta) \right] - \beta \right\} f(\theta) d\theta + \int_{\theta^s}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta, \\ \text{s.t.} \quad & \pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta) d\theta = \beta / \Delta\gamma. \end{aligned}$$

The difference from the benchmark problem \mathcal{P}^b is readily apparent from the constraint in \mathcal{P}^s (without the constraint, \mathcal{P}^s becomes equivalent to \mathcal{P}^b). Again, with self-performance, the principal exerts an effort when her expected payoff is high enough to cover the cost of effort as well as the agent's *ex post* rent. We make the following definition to present our first main result.

Definition 3 *Let $\beta^s \equiv \Delta\gamma\pi^*(q^b(\bar{\theta}), \bar{\theta})$.*

The next proposition presents the optimal outcome with self-performance when upstream effort is hidden action.

Proposition 2 *The optimal outcome in \mathcal{P}^s entails that:*

$$(i) \quad q^s(\theta) \begin{cases} = q^b(\theta) & \text{for } \theta \leq \theta^s, \\ < q^b(\theta) & \text{for } \theta > \theta^s. \end{cases}$$

$$(ii) \quad \theta^s \text{ is characterized by } \pi^*(q^b(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q^s(\theta) d\theta = \beta / \Delta\gamma, \text{ where } \theta^s < \bar{\theta} \text{ for } \beta > \beta^s,$$

(iii) θ^s is decreasing in β .

The key result in Proposition 3 is that $q^s(\theta) < q^b(\theta)$ for $\theta > \theta^s$. When upstream effort is hidden action, downward output distortion in the optimal contract becomes larger under self-performance. Since the principal has an incentive to make an excessive effort, as pointed out in Claim 1, the agent can potentially obtain more information rent in the downstream task, even for not so favorable environment (compared to when upstream effort

can be verified). To reduce the agent's rent, therefore, the principal distorts the output schedule further down in the optimal contract.

Comparing θ^b in Propositions 1 and θ^s in Proposition 2, we have the following corollary.

Corollary 1 *Suppose $\beta > \beta^b$. Under self-performance ($\varphi = s$), the upstream task is carried out more frequently, i.e., $\theta^s > \theta^b$, when upstream effort cannot be verified.*

When upstream effort cannot be verified, the cutoff level θ^s is determined by the principal's incentive *ex post*. Although this *ex post* incentive is expected, the optimal contract does not make the principal to reduce her effort all the way to the *ex ante* optimal level—she still makes an excessive effort on the upstream task in equilibrium from the *ex ante* view point. This result indicates that, under self-performance, the organization's resources may be excessively expended even when the project is not attractive enough *ex ante*.

Delegated-Performance ($\varphi = d$)

We revisit our benchmark case where upstream task is publicly verified—recall that in \mathcal{P}^b , self- and delegated-performance yield the same optimal outcome. At $\theta = \theta^b$, the following must be true regarding the agent's incentive for α with the optimal outcome in \mathcal{P}^b :

$$\alpha(\theta^b) = \begin{cases} 1 & \text{if } \begin{aligned} &\gamma_G [t(\theta^b) - \theta^b q(\theta^b)] - \beta \\ &> \gamma_B [t(\theta^b) - \theta^b q(\theta^b)], \end{aligned} \\ 0 & \text{if } \begin{aligned} &\gamma_G [t(\theta^b) - \theta^b q(\theta^b)] - \beta \\ &< \gamma_B [t(\theta^b) - \theta^b q(\theta^b)]. \end{aligned} \end{cases}$$

The next claim shows the agent's incentive with the optimal outcome in \mathcal{P}^b when upstream effort cannot be verified.

Claim 2 *In \mathcal{P}^b , the agent strictly prefers shirking at $\theta = \theta^b$.*

Whereas inefficiency under self-performance comes from the principal's excessive effort, under delegated-performance, inefficiency comes from the agent's shirking incentive. At $\theta = \theta^b$, the effort cost is larger than his expected rent in \mathcal{P}^b , and hence the agent will shirk on the upstream task if his effort cannot be verified.

We now lay out the constraints the principal faces under delegated-performance. The agent's truth-telling on θ with delegated-performance requires the following condition:

$$\theta \in \arg \max_{\hat{\theta}} u(\hat{\theta}|\theta), \text{ where } u(\hat{\theta}|\theta) \equiv \gamma(\alpha(\theta)) [t(\hat{\theta}) - \theta q(\hat{\theta})] - \beta \alpha(\theta). \quad (IC^d)$$

To induce the agent's participation, the optimal contract must satisfy:

$$u(\theta) \geq 0 \text{ for } \forall \theta \in \Theta, \quad \text{where } u(\theta) \equiv u(\theta|\theta). \quad (PC^d)$$

Lastly, the optimal contract under delegated-performance must respect the agent's incentive for his upstream effort, and therefore must satisfy:

$$\alpha(\theta) \in \arg \max_{\tilde{\alpha}} \gamma(\tilde{\alpha}(\theta)) [t(\theta) - \theta q(\theta)] - \beta \tilde{\alpha}(\theta). \quad (EC^d)$$

When upstream effort is hidden action, the principal's problem with $\varphi = d$ is:

$$\text{Max}_{\alpha, q, t} \int_{\Theta} \gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] f(\theta) d\theta,$$

subject to (IC^d) , (PC^d) and (EC^d) . The principal's problem under delegated-performance is rewritten as follows.

Lemma 4 *Suppose upstream effort is hidden action. With $\varphi = d$, there exists θ^d such that $\alpha = 1$ for $\theta \leq \theta^d$, and $\alpha = 0$ for $\theta > \theta^d$. In addition, there exists $\underline{\theta}^d (> \underline{\theta})$ such that, if $\theta^d < \underline{\theta}^d$, the principal does not induce the agent's effort for the entire range $\Theta = [\underline{\theta}, \bar{\theta}]$.*

The principal's problem becomes:

$$\begin{aligned} \mathcal{P}^d : \text{Max}_{q, \theta^d} & \int_{\underline{\theta}}^{\theta^d} \{ \gamma_G [\pi^b(q(\theta), \theta)] - \beta \} f(\theta) d\theta + \int_{\theta^d}^{\bar{\theta}} \gamma_B [\pi^b(q(\theta), \theta)] f(\theta) d\theta, \\ \text{s.t.} & \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta = \frac{\beta}{\Delta\gamma} \text{ for } \theta^d \in [\underline{\theta}^d, \bar{\theta}], \text{ and } \mathcal{P}^d = \mathcal{P}^b(\theta^b = \underline{\theta}) \text{ for } \theta^d \in [\underline{\theta}, \underline{\theta}^d]. \end{aligned}$$

The LHS of the constraint, $\int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta$, is the agent's *ex post* rent for $\theta = \theta^d$. The constraint reflects the fact that the agent's incentive to make an effort depends on his rent. For $\theta < \theta^d$ ($\theta > \theta^d$), an increase in the agent's rent from making upstream effort is larger (smaller) than the cost of effort. As mentioned in the lemma, if θ^d , the cutoff level of θ , is too small (i.e., the range of θ with $\alpha = 0$ is too large), then the principal simply chooses the optimal outcome in \mathcal{P}^b with no effort for the entire range of θ . Since inducing the agent's upstream effort requires an extra constraint in the principal's problem, if the range of θ with $\alpha = 0$ is too large, then it is not worth inducing the agent's effort for a small range of θ . As a result, the principal chooses to implement the optimal outcome in \mathcal{P}^b with no effort for the entire range Θ .

The following proposition presents the optimal outcome under delegated-performance when upstream effort cannot be verified.

Proposition 3 When $\theta^d \in [\underline{\theta}^d, \bar{\theta}]$, the optimal outcome in \mathcal{P}^d entails that:

$$(i) \quad q^d(\theta) \begin{cases} = q^b(\theta) & \text{for } \theta \leq \theta^d, \\ > q^b(\theta) & \text{for } \theta > \theta^d. \end{cases}$$

(ii) θ^d is characterized by $\int_{\theta^d}^{\bar{\theta}} q^b(\theta) d\theta = \beta / \Delta\gamma$, where $\theta^d < \bar{\theta}$ for $\beta > 0$.

(iii) θ^d is decreasing in β .

When $\theta^d \in [\underline{\theta}, \underline{\theta}^d)$, $q^d(\theta) = q^b(\theta)$ and $\alpha(\theta) = 0$ for $\theta \in \Theta$.

The key result in Proposition 3 is that $q^d(\theta) > q^b(\theta)$ for $\theta > \theta^d$. Since θ^d is decreasing in β , this is the case when β is not too large that $\theta^d \in [\underline{\theta}^d, \bar{\theta}]$. When upstream effort is hidden action, downward output distortion in the optimal contract becomes smaller under delegated-performance. As pointed out in Claim 2, the agent has an incentive to shirk the upstream task, and inducing his effort for the task requires more rent provision (compared to when upstream effort can be verified). Since distorting the output schedule downward is used as the device to extract the agent's information rent, by increasing the output schedule in the optimal contract, the principal provides more rent to the agent, thereby incentivizing him to exert an effort for the upstream task.

For $\theta^d \in [\underline{\theta}, \underline{\theta}^d)$, the optimal outcome in \mathcal{P}^d is the same as that in \mathcal{P}^b with $\theta^b = \underline{\theta}$, which follows directly from Lemma 4. When $\theta^d < \underline{\theta}^d$ due to a sufficiently large β , the principal is providing too much rent to the agent to induce his effort for a small favorable range of the project project environment. Consequently, the principal completely abandons inducing the agent's effort in such a case.

In what follows, we will assume that following condition holds.

Condition 1 $\pi(q^d(\theta), \theta) > r(q^d(\theta), \theta)$ for $\forall \theta \in \Theta$, where $r(q^d(\theta), \theta) \equiv \int_{\theta}^{\bar{\theta}} q^d(\tau) d\tau$.

By imposing the condition above, we are considering parameter values such that, under delegated-performance, the principal's *ex post* surplus is greater than the agent's *ex post* rent. Recall that, when upstream effort is verifiable (in \mathcal{P}^b), it does not matter which party performs the upstream task. Comparing the cutoff levels give the next corollary.

Corollary 2 Suppose $\beta > \beta^b$. Under delegated-performance ($\varphi = w$), the upstream task is shirked more frequently, i.e., $\theta^d < \theta^b$ when upstream effort cannot be verified.

Without verification, the upstream task under delegated-performance fails more frequently due to the agent’s shirking incentive. The cutoff level θ^d under delegated-performance is determined by the agent’s incentive regarding the upstream task. Although the agent’s shirking incentive is well expected, the principal does not induce the agent’s effort all the way to the benchmark level in the optimal contract—the agent still shirk the upstream task in equilibrium from the *ex ante* view point. The result suggests that, under delegated-performance, the organization’s resources may not be fully utilized even when the project is attractive enough *ex ante*.

4 Self- vs. Delegated-Performance

We now compare self- and delegated-performance. Under self-performance ($\varphi = s$), the principal faces a time-inconsistency problem. Without ability to commit to her effort, the principal cannot help but make an effort for the upstream task *ex post*, unless the cost of effort is prohibitively large. As a result, upstream effort is exerted more frequently compared to the benchmark case where there is no hidden action ($\theta^s > \theta^b$). Since the project will be carried out even for less favorable environments, the agent can obtain more information rent, and to reduce the agent’s rent, the output schedule is distorted more compared to the benchmark case. As aforementioned, under self-performance, the organization’s resources may be excessively expended even when the project is not attractive enough *ex ante*.

Under delegated-performance ($\varphi = d$), the principal faces a shirking problem—upstream effort is exerted less frequently compared to the benchmark case ($\theta^d < \theta^b$). To induce the agent’s effort for the upstream task, the principal increases the agent’s rent in the downstream task by increasing the optimal output levels—as a result, the output schedule is distorted less compared to the benchmark case. Contrast to the case under self-performance, under delegated performance, the organization’s resources may not be fully utilized even when the project is not attractive enough *ex ante*.

As will be shown below, the optimality of self- or delegated-performance depends on the cost of upstream effort β . We make the following definition to compare self- and delegated-performance.

Definition 4 Let $\bar{\beta}^s \equiv \Delta\gamma[\pi^*(q^*(\underline{\theta}), \underline{\theta}) - \int_{\underline{\theta}}^{\bar{\theta}} q^s(\theta)d\theta]$.

For $\beta \geq \bar{\beta}^s$, the principal makes $\theta^s = \underline{\theta}$, i.e., she makes $\alpha^s = 0$ for the entire range of θ in \mathcal{P}^s . Our final result is presented in the following proposition.

Proposition 4 *There exist $\underline{\beta}$ and $\overline{\beta}$ such that:*

- *The principal's payoff is strictly higher with self-performance ($\varphi = s$) for $\beta \in (0, \underline{\beta}]$.*
- *The principal's payoff is strictly higher with delegated-performance ($\varphi = d$) for $\beta \in [\overline{\beta}, \overline{\beta}^s)$.*

When the effort cost β is small, an excessive frequency of upstream effort is less of a problem from the principal's *ex ante* view point. By contrast, shirking is relatively costlier to the principal since the upstream task is not carried out even when it is easy to be done. When β is large, the trade-off shifts to the other direction. Too much effort is made with self-performance even for less favorable project environments when the upstream task is very costly. In addition, compared to the benchmark, the agent can reap more rent under either self- or delegated-performance (due to the principal's excessive effort with self-performance, and due to the agent's shirking incentive with delegated-performance), but under self-performance the downward distortion in the optimal output schedule becomes exacerbated, whereas the distortion becomes smaller under delegated-performance—this effect also becomes significant for large β .

The optimal output schedules and upstream efforts under self- and delegated-performance with interior cutoff levels of θ are illustrated in Figure 1.

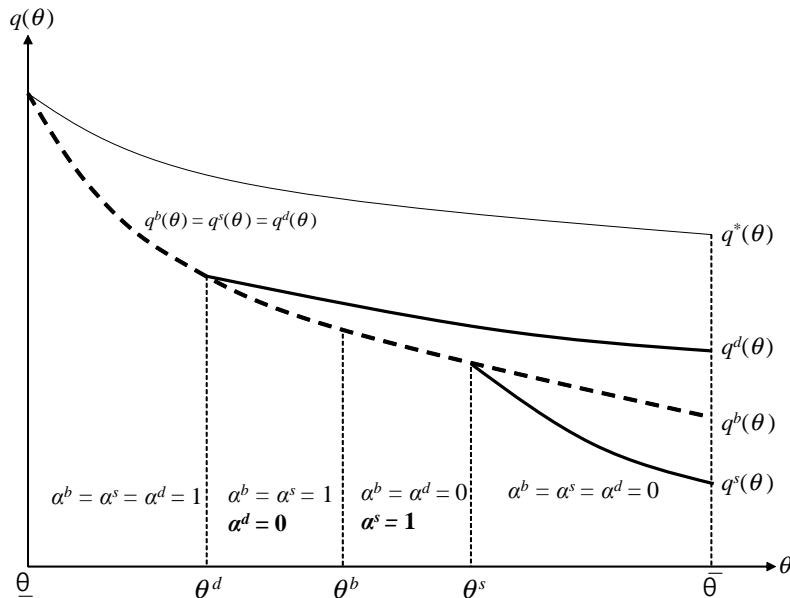


Fig 1. Optimal Upstream Effort and Output Schedule

Bias in Favor of Self- or Delegated Performance

Up to this point, we have assumed that the cost of upstream effort is the same for the principal and the agent. We now relax this assumption. When the principal's and the agent's cost of upstream effort are different, Proposition 5 has the following implication.

Corollary 3 *Let $\Delta\beta \equiv \beta_P - \beta_A$, where β_P and β_A denote the principal's and the agent's cost of upstream effort respectively. There exist $\overline{\Delta\beta} > 0$ and $\underline{\Delta\beta} < 0$ such that:*

- *For $\beta_P, \beta_A \in (0, \underline{\beta}]$ and $\Delta\beta < \overline{\Delta\beta}$, the principal prefers $\varphi = s$ even if $\Delta\beta > 0$.*
- *For $\beta_P, \beta_A \in [\overline{\beta}, \overline{\beta}^s)$ and $\Delta\beta > \underline{\Delta\beta}$, the principal prefers $\varphi = d$ even if $\Delta\beta < 0$.*

The corollary above states that when the upstream task is easy (β_P and β_A are small), the principal may assign the upstream task to herself, even if her cost is larger than the agent's (a bias in favor of self-performance). When the upstream task is hard (β_P and β_A are large), by contrast, the principal may assign the task to the agent, even if her opportunity cost is smaller than the agent's (a bias in favor of delegated-performance). Our previous results, together with Corollary 3, also imply that when the principal is biased in favor of self-performance, distortions in the optimal output levels can only be larger, whereas when she is biased in favor of delegated-management, such distortions can be smaller.

5 Conclusion with Remarks

This paper has provided a novel rationale for delegated-performance even when the principal can directly contribute to a task. We have argued that “loss of control” under delegated-performance may enable the principal to have more effective allocation of effort in the organization. In our model, there are two vertical tasks. The upstream task can be performed by the principal or delegated to the agent, while only the agent can perform the downstream task. We have shown that, while self-performance of the upstream task increases the likelihood of the project's success, the principal exerts an effort even when the project is not profitable enough.

Under delegated-performance, the agent's effort is induced only when the downstream task's environment is sufficiently favorable—delegated-performance enables the principal to focus allocation of effort for more favorable environments. We have also shown that directions of additional distortions in the output schedule under the two management styles

are opposite—output schedule is distorted more under self-performance, and less under delegated-performance.

Our result suggests that, for the cost of upstream effort small (large) enough, the principal prefers self-performance (delegated-performance). This result has an implication for a potential bias of the principal toward self- or delegated-performance.

We assumed that when the upstream task fails (e.g. no fund raised), no output can be produced in the downstream task ($q_N(\theta) = 0$). As aforementioned, in this setting (recall that $q(\theta) \equiv q_Y(\theta)$ and $t(\theta) \equiv t_Y(\theta)$), we have $t_N(\theta) = 0$ in the optimal contract. To illustrate, the principal's problem in the benchmark case (\mathcal{P}^b) is:

$$\text{Max}_{\alpha, q, t} \int_{\Theta} \{\gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] - [1 - \gamma(\alpha(\theta))] t_N(\theta) - \beta\alpha(\theta)\} f(\theta) d\theta.$$

Since $q_N(\theta) = 0$, the agent's rent is linked only to $q(\theta)$ and thus it is clear from the objective function that $t_N(\theta) = 0$. The same argument holds for the case of self-performance under moral hazard (\mathcal{P}^s). For the case of delegated-performance under moral hazard (\mathcal{P}^d), the effort condition for the agent enters as:

$$\Delta\gamma [t(\theta) - \theta q(\theta)] - \beta = \Delta\gamma t_N(\theta).$$

It is clear from the above equation that, if $t_N(\theta) > 0$ then the agent's shirking incentive increases compared to when $t_N(\theta) = 0$. Thus, together with the principal's objective function above, it is implied that $t_N(\theta) = 0$ at the optimum.

Finally, our model can be extended to a hierarchical agency, in which the principal hires, for example, two agents: one for the upstream task and the other for the downstream task.⁸ Under self-performance, the principal directly deals with both agents. Under delegated-performance, the principal only contracts with one agent, who in turn, contracts with the other agent. If the contract for one agent can be linked to the contract for the other, then self-performance can always implement the optimal outcome under delegated-performance. If that arrangement is not possible, however, then moral hazard and adverse selection do not interplay with each other under self-performance. Delegated-performance under such environments then can be optimal because it links the upstream agent's effort to the downstream agent's information rent.

⁸For studies on hierarchical structures in multi-agent setting, see Baron and Besanko (1992), Gilbert and Riordan (1995), Melumad et al. (1995), Mookherjee and Reichelstein (2001) and Mookherjee (2006) among others. Gromb and Martimort (2007) analyze the optimality of using multiple agents when there are both vertical and horizontal collusion possibilities.

Appendix A: Proofs

Proof of Lemma 1.

From $\pi_q^b(q^b(\theta), \theta) = 0$, i.e., $v'(q^b(\theta)) = \theta + F(\theta)/f(\theta)$, where $d[F(\theta)/f(\theta)]/d\theta \geq 0$, we have $v''(q^b(\theta))dq^b(\theta)/d\theta = 1 + d[F(\theta)/f(\theta)]/d\theta$, implying that:

$$\frac{dq^b(\theta)}{d\theta} = \frac{1 + d[F(\theta)/f(\theta)]/d\theta}{v''(\cdot)} < 0.$$

Differentiating $\pi^b(q^b(\theta), \theta) \equiv v(q^b(\theta)) - (\theta + [F(\theta)/f(\theta)])q^b(\theta)$ gives:

$$\begin{aligned} \frac{d\pi^b(q^b(\theta), \theta)}{d\theta} &= v'(q^b(\theta))\frac{dq^b(\theta)}{d\theta} - \left(1 + \frac{d[F(\theta)/f(\theta)]}{d\theta}\right)q^b(\theta) - \left(\theta + \frac{F(\theta)}{f(\theta)}\right)\frac{dq^b(\theta)}{d\theta} \\ &= \underbrace{\left[v'(q^b(\theta)) - \left(\theta + \frac{F(\theta)}{f(\theta)}\right)\right]}_{=0}\frac{dq^b(\theta)}{d\theta} - \left(1 + \frac{d[F(\theta)/f(\theta)]}{d\theta}\right)q^b(\theta) < 0. \end{aligned}$$

Finally, since $\pi_q^b(q^b(\theta), \theta) = 0$ and $\pi_q^*(q^*(\theta), \theta) = 0$, we have $q^b(\theta) < q^*(\theta)$. This implies that:

$$\left.\frac{d\pi^*(q(\theta), \theta)}{dq}\right|_{q=q^b} > 0.$$

■

Proof of Lemma 2.

By the envelope theorem, the necessary condition of (IC) is: $u'(\theta) = -\gamma(\alpha(\theta))q(\theta) < 0$. This is valid “almost everywhere”—as will be shown later, the effort level drops from $\alpha = 1$ (thus $\gamma(\alpha(\theta)) = \gamma_G$) to $\alpha = 0$ (thus $\gamma(\alpha(\theta)) = \gamma_B$) at the cutoff level of θ . However, it is innocuous to use this expression in our model since what is needed is that the agent’s expected utility is decreasing in θ . Integration gives:

$$u(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \gamma(\alpha(\tau))q(\tau)d\tau. \quad (A1)$$

The second order condition of (IC) is:

$$\Delta\gamma\alpha'(\theta)q(\theta) + \gamma(\alpha(\theta))q'(\theta) \leq 0,$$

and as usual in the model of this type, this condition is automatically satisfied by the solution without it. Again, as will be shown later, $\alpha'(\theta) = 0$ except the cutoff level of θ (at the cutoff point, $\alpha'(\theta) = -\infty$), and $q(\theta) = q^b(\theta)$ in the optimal contract in all cases, where $dq^b(\theta)/d\theta < 0$ from Lemma 1.

Using integration by part in (A1), the agent's expected rent, $E[u(\theta)] = \int_{\underline{\theta}}^{\bar{\theta}} u(\theta) f(\theta) d\theta$, is expressed as follows:

$$E[u(\theta)] = u(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \gamma(\alpha(\theta)) q(\theta) F(\theta) d\theta, \quad (A2)$$

where $u(\bar{\theta}) = 0$ in the optimum. Since $u(\theta) = \gamma(\alpha(\theta))[t(\theta) - \theta q(\theta)]$, the transfer is: $t(\theta) = \theta q(\theta) + [u(\theta)/\gamma(\alpha(\theta))]$. Substituting for $t(\theta)$ with the expression in (A2), the principal's problem becomes:

$$\text{Max}_{q, \alpha} \int_{\underline{\theta}}^{\bar{\theta}} \{\gamma(\alpha(\theta)) [\pi^b(q(\theta), \theta)] - \beta \alpha(\theta)\} f(\theta) d\theta. \quad (A3)$$

The objective function is linear in $\alpha(\theta)$ since $\gamma(\alpha(\theta)) = \alpha(\theta)\gamma_G + (1 - \alpha(\theta))\gamma_B$, and $\pi^b(q^b(\theta), \theta)$ is decreasing in θ in (A3) by Lemma 1, implying that optimization with respect to $\alpha(\theta)$ gives a cutoff θ such that:

$$\alpha = \begin{cases} 1 & \text{for } \theta \leq \theta^b \\ 0 & \text{for } \theta > \theta^b \end{cases} \quad (A4)$$

Using (A4), the objective function in (A3) is rewritten as the one in Lemma 2. ■

Proof of Proposition 1.

Point-wise maximizing the objective function in Lemma 2 with respect to $q(\theta)$ gives $\pi_q^b(q(\theta), \theta) = 0$ implying that $q(\theta) = q^b(\theta)$. The cutoff θ^b is characterized by differentiating the objective function with respect to θ^b :

$$\Delta \gamma \pi^b(q^b(\theta^b), \theta^b) - \beta = 0. \quad (A5)$$

The second-order condition for maximization is satisfied by Lemma 1. The equation in (A5) implies that, for $\beta > \Delta \gamma \pi^b(q^b(\bar{\theta}), \bar{\theta})$, the cutoff $\theta^b < \bar{\theta}$. To see the sign of $d\theta^b/d\beta$, we differentiate (D1) to have:

$$\Delta \gamma [\pi_q^b(q^b(\theta^b), \theta^b) \frac{dq^b}{d\theta^b} + \pi_\theta^b(q^b(\theta^b), \theta^b)] d\theta^b - d\beta = 0. \quad (A6)$$

Since $\pi_q^b(q^b(\theta), \theta) = 0$, (A6) gives: $d\theta^b/d\beta = 1/[\Delta \gamma \pi_\theta^b(q^b(\theta^b), \theta^b)] < 0$. ■

Proof of Claim 1.

We will show that in \mathcal{P}^b the principal strictly prefers choosing $\alpha(\theta^b) = 1$ ex post. The principal (weakly) prefers choosing $\alpha(\theta^b) = 1$ if:

$$\gamma_G [v(q^b(\theta^b)) - t(\theta^b)] - \beta \geq \gamma_B [v(q^b(\theta^b)) - t(\theta^b)]. \quad (A7)$$

Since $u(\theta) = \gamma(\alpha(\theta)) [t(\theta) - \theta q(\theta)]$, we have $t(\theta) = \theta q(\theta) + [u(\theta)/\gamma(\alpha(\theta))]$. Substituting for $t(\theta)$, the inequality in (A7) becomes:

$$\begin{aligned} & \gamma_G \left[v(q^b(\theta^b)) - \theta^b q^b(\theta^b) - \frac{[u(\bar{\theta}) + \int_{\theta^b}^{\bar{\theta}} \gamma_G q^b(\theta) d\theta]}{\gamma_G} \right] - \beta \\ & \geq \gamma_B \left[v(q^b(\theta^b)) - \theta^b q^b(\theta^b) - \frac{[u(\bar{\theta}) + \int_{\theta^b}^{\bar{\theta}} \gamma_B q^b(\theta) d\theta]}{\gamma_B} \right]. \end{aligned}$$

Since $u(\bar{\theta})$ cancels out as it enters both LHS and the RHS of the above inequality, it can be rewritten as:

$$\Delta\gamma [v(q^b(\theta^b)) - \theta^b q^b(\theta^b) - \int_{\theta^b}^{\bar{\theta}} q^b(\theta) d\theta] \geq \beta. \quad (A8)$$

Since $\pi^*(q(\theta), \theta) \equiv v(q(\theta)) - \theta q(\theta)$, (A8) is rewritten as:

$$\pi^*(q^b(\theta^b), \theta^b) \geq \frac{\beta}{\Delta\gamma} + \int_{\theta^b}^{\bar{\theta}} q^b(\theta) d\theta. \quad (A9)$$

We prove next that constraint (A9) must be slack, i.e., satisfied as a strict inequality in \mathcal{P}^b .

Consider the principal's problem \mathcal{P}^b complemented by the constraint (A9):

$$\mathcal{P}^b : \quad \underset{\theta^b}{\text{Max}} \int_{\underline{\theta}}^{\theta^b} \left\{ \gamma_G [\pi^b(q^b(\theta), \theta)] - \beta \right\} f(\theta) d\theta + \int_{\theta^b}^{\bar{\theta}} \gamma_B [\pi^b(q^b(\theta), \theta)] f(\theta) d\theta$$

subject to:

$$\pi^*(q^b(\theta^b), \theta^b) \geq \frac{\beta}{\Delta\gamma} + \int_{\theta^b}^{\bar{\theta}} q^b(\theta) d\theta.$$

The Lagrangian function for the optimization problem is:

$$\begin{aligned} \mathcal{L} = & \int_{\underline{\theta}}^{\theta^b} \left\{ \gamma_G [\pi^b(q^b(\theta), \theta)] - \beta \right\} f(\theta) d\theta + \int_{\theta^b}^{\bar{\theta}} \gamma_B [\pi^b(q^b(\theta), \theta)] f(\theta) d\theta \\ & + \lambda [\pi^*(q^b(\theta^b), \theta^b) - \frac{\beta}{\Delta\gamma} - \int_{\theta^b}^{\bar{\theta}} q^b(\theta) d\theta], \end{aligned}$$

where the multiplier $\lambda \in R$ is a constant. Differentiating \mathcal{L} with respect to θ^b we obtain:

$$\Delta\gamma [\pi^b(q(\theta^b), \theta^b)] - \beta + \lambda \left[\frac{d\pi^*(q(\theta^b), \theta^b)}{d\theta^b} + q(\theta^b) \right] = 0. \quad (A10)$$

Recall that in \mathcal{P}^b the optimal θ^b is given by $\pi^b(q(\theta^b), \theta^b) = \beta/\Delta\gamma$, which together with (A10) implies that:

$$\lambda \left[\frac{d\pi^*(q(\theta^b), \theta^b)}{d\theta^b} + q(\theta^b) \right] = 0. \quad (A11)$$

We show next that $[d\pi^*(q(\theta^b), \theta^b)/d\theta^b] + q(\theta^b) < 0$, which will imply that $\lambda = 0$, i.e., the constraint (A9) is slack. We rewrite the expression for $[d\pi^*(q(\theta^b), \theta^b)/d\theta^b] + q(\theta^b)$ as follows:

$$\begin{aligned} & [d\pi^*(q(\theta^b), \theta^b)/d\theta^b] + q(\theta^b) \\ &= v'(q(\theta^b)) \frac{dq(\theta^b)}{d\theta^b} - q(\theta^b) - \theta^b \frac{dq(\theta^b)}{d\theta^b} + q(\theta^b) \\ &= [v'(q(\theta^b)) - \theta^b] \frac{dq(\theta^b)}{d\theta^b} < 0, \end{aligned}$$

where $v'(q(\theta^b)) - \theta^b > 0$ and $dq(\theta^b)/d\theta^b < 0$. Therefore, $\lambda = 0$ and the constraint (A9) is satisfied a strict inequality. ■

Proof of Lemma 3.

The expression for the agent's rent and expected rent are the same as in \mathcal{P}^b and they are:

$$\begin{aligned} u(\theta) &= u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \gamma(\alpha(\tau))q(\tau)d\tau \quad \text{and} \\ \alpha[u(\theta)] &= u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} \gamma(\alpha(\theta))q(\theta)F(\theta)d\theta. \end{aligned}$$

Using these expressions with $t(\theta) = \theta q(\theta) + [u(\theta)/\gamma(\alpha(\theta))]$, the principal's objective function is rewritten as:

$$\int_{\underline{\theta}}^{\bar{\theta}} \{ \gamma(\alpha(\theta))[\pi^b(q(\theta), \theta)] - u(\bar{\theta}) - \beta\alpha(\theta) \} f(\theta)d\theta. \quad (\text{A12})$$

The principal's *ex post* effort rule from (EC^s) gives characterization of the cutoff θ^s :

$$\begin{aligned} & \gamma_G \left[v(q(\theta^s)) - \theta^s q(\theta^s) - \frac{[u(\bar{\theta}) + \int_{\theta^s}^{\bar{\theta}} \gamma_G q(\theta)d\theta]}{\gamma_G} \right] - \beta \\ &= \gamma_B \left[v(q(\theta^s)) - \theta^s q(\theta^s) - \frac{[u(\bar{\theta}) + \int_{\theta^s}^{\bar{\theta}} \gamma_B q(\theta)d\theta]}{\gamma_B} \right], \end{aligned}$$

which simplifies to:

$$\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta)d\theta = \frac{\beta}{\Delta\gamma}. \quad (\text{A13})$$

We next show that the LHS of (A13), $\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta)d\theta$, is a monotonically decreasing function of θ^s , which will imply that the effort level $\alpha(\theta)$ will be $\alpha(\theta) = 1$ if $\theta \leq \theta^s$

and $\alpha(\theta) = 0$ if $\theta > \theta^s$. Consider the first-order derivative of $\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta) d\theta$ with respect to θ^s :

$$\begin{aligned} & \frac{d[\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta) d\theta]}{d\theta^s} \\ &= \frac{d[v(q(\theta^s)) - \theta^s q(\theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta) d\theta]}{d\theta^s} \\ &= v'(q(\theta^s)) \frac{dq(\theta^s)}{d\theta^s} - q(\theta^s) - \theta^s \frac{dq(\theta^s)}{d\theta^s} + q(\theta^s) \\ &= \frac{dq(\theta^s)}{d\theta^s} [v'(q(\theta^s)) - \theta^s]. \end{aligned}$$

As will be shown in Proof of Proposition 2, $dq(\theta^s)/d\theta^s < 0$ and the output schedule is distorted downward in the optimal contract, i.e., $v'(q(\theta^s)) > \theta^s$. Therefore, $\frac{dq(\theta^s)}{d\theta^s} [v'(q(\theta^s)) - \theta^s] < 0$ and, as a result:

$$\alpha = \begin{cases} 1 & \text{for } \theta \leq \theta^s \\ 0 & \text{for } \theta > \theta^s \end{cases}. \quad (A14)$$

Using (A14), the principal's objective function in (A12) is rewritten as:

$$\int_{\underline{\theta}}^{\theta^s} \{\gamma_G [\pi^b(q(\theta), \theta)] - u(\bar{\theta}) - \beta\} f(\theta) d\theta + \int_{\theta^s}^{\bar{\theta}} \{\gamma_B [\pi^b(q(\theta), \theta)] - u(\bar{\theta})\} f(\theta) d\theta. \quad (A15)$$

The principal's problem is maximizing (A15) subject to (A13). Since $u(\bar{\theta})$ enters only the objective function, $u(\bar{\theta}) = 0$ in the optimum and the principal's problem is expressed as the one in the lemma. ■

Proof of Proposition 2.

The principal's problem in \mathcal{P}^s from Lemma 3 is written as:

$$\mathcal{P}^s : \quad \underset{q(\cdot), \theta^s}{Max} \int_{\underline{\theta}}^{\theta^s} \{\gamma_G [\pi^b(q(\theta), \theta)] - \beta\} f(\theta) d\theta + \int_{\theta^s}^{\bar{\theta}} \gamma_B [\pi^b(q(\theta), \theta)] f(\theta) d\theta, \quad (A16)$$

subject to:

$$\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\tau) d\tau = \frac{\beta}{\Delta\gamma}. \quad (A17)$$

We first show that the output will be at the benchmark level, $q^s(\theta) = q^b(\theta)$, for $\theta \leq \theta^s$, and will be distorted down more, $q^s(\theta) < q^b(\theta)$, for $\theta > \theta^s$. Replacing θ^s with $\theta^s(\cdot) \equiv \theta^s(q(\theta^s), \{q(\theta)\}_{\theta \geq \theta^s})$ in (A16), the principal's maximization problem in $q(\theta)$ is written as:

$$\underset{q}{Max} \int_{\underline{\theta}}^{\theta^s(\cdot)} \{\gamma_G [\pi^b(q(\theta), \theta)] - \beta\} f(\theta) d\theta + \int_{\theta^s(\cdot)}^{\bar{\theta}} \gamma_B [\pi^b(q(\theta), \theta)] f(\theta) d\theta.$$

Since $q(\theta)$ depends on $\theta^s(\cdot)$ only for $\theta \geq \theta^s$ it is clear from the problem that regardless of the value of θ^s , the point-wise maximization for $\theta \leq \theta^s$ gives that:

$$\pi_q^b(q(\theta), \theta) = 0 \text{ for } \theta \leq \theta^s,$$

implying that $q^s(\theta) = q^b(\theta)$ for $\theta \leq \theta^s$. Next, consider $\theta > \theta^s$. Using the Leibniz integral rule, the point-wise maximization with respect to q implies:

$$\left[\gamma_G \pi^b(q, \theta^s(\cdot)) - \beta \right] f(\theta^s(\cdot)) \frac{d\theta^s}{dq} - \gamma_B \pi^b(q, \theta^s(\cdot)) f(\theta^s(\cdot)) \frac{d\theta^s}{dq} + \int_{\theta^s(\cdot)}^{\bar{\theta}} \gamma_B \pi_q^b(q, \theta) f(\theta) d\theta = 0,$$

which simplifies to:

$$[\Delta \gamma \pi^b(q, \theta^s(\cdot)) - \beta] f(\theta^s(\cdot)) \frac{d\theta^s}{dq} + \int_{\theta^s(\cdot)}^{\bar{\theta}} \gamma_B \pi_q^b(q, \theta) f(\theta) d\theta = 0.$$

From (A16) it follows that $d\theta^s/dq < 0$ and $\Delta \gamma \pi^b(q, \theta^s(\cdot)) - \beta > 0$, and hence we have $q^s(\theta) < q^b(\theta)$ for $\theta > \theta^s$. Thus we have shown that:

$$q^s(\theta) \begin{cases} = q^b(\theta) & \text{for } \theta \leq \theta^s \\ < q^b(\theta) & \text{for } \theta > \theta^s \end{cases}.$$

The cutoff θ^s is then obtained from (A17) with $q^s(\theta)$ described above:

$$\pi^*(q^b(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q^s(\theta) d\theta = \beta / \Delta \gamma, \quad (\text{A18})$$

For $\theta^s = \bar{\theta}$, (A18) gives $\beta = \beta^s \equiv \Delta \gamma \pi^*(q^b(\bar{\theta}), \bar{\theta})$.

To show that θ^s is decreasing in β , we differentiate (A18) to have:

$$\left[\underbrace{\pi_q^*(q^b(\theta^s), \theta^s) \frac{dq^b(\theta^s)}{d\theta^s}}_{(-)} + \underbrace{\pi_\theta^*(q^b(\theta^s), \theta^s)}_{(-)} \right] d\theta^s = \frac{1}{\Delta \gamma} d\beta. \quad (\text{A19})$$

In (A19), $\pi_q^*(q^b(\theta^s), \theta^s) > 0$ and $dq^b(\theta^s)/d\theta^s < 0$ by Lemma 1. That $\pi_\theta^* < 0$ follows from $\pi^*(q(\theta), \theta) = v(q(\theta)) - \theta q(\theta)$. Together, it is implied from (A19) that $d\theta^s/d\beta < 0$. ■

Proof of Corollary 1.

The Lagrangian for the optimization problem in \mathcal{P}^s is:

$$\begin{aligned} \mathcal{L} &= \int_{\underline{\theta}}^{\theta^s} \left\{ \gamma_G \left[\pi^b(q(\theta), \theta) \right] - \beta \right\} f(\theta) d\theta + \int_{\theta^s}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta \\ &+ \lambda \left[\pi^*(q(\theta^s), \theta^s) - \int_{\theta^s}^{\bar{\theta}} q(\theta) d\theta - \frac{\beta}{\Delta \gamma} \right], \end{aligned}$$

where $\lambda \in R$ is a constant. Differentiating \mathcal{L} with respect to $q(\theta)$ for $\theta > \theta^s$ we obtain:

$$\frac{d\mathcal{L}}{dq(\theta)} = \int_{\theta^s}^{\bar{\theta}} \gamma_B \left[\pi_q^b(q(\theta), \theta) \right] f(\theta) d\theta - \lambda \int_{\theta^s}^{\bar{\theta}} \frac{dq(\theta)}{dq(\theta)} d\theta = 0. \quad (A20)$$

Since $\pi_q^b(q(\theta), \theta) < 0$, it follows from (A20) that $\lambda < 0$. Differentiating \mathcal{L} with respect to θ^s we obtain:

$$\Delta\gamma \left[\pi^b(q(\theta^s), \theta^s) \right] - \beta + \lambda \left[\frac{d\pi^*(q(\theta^s), \theta^s)}{d\theta^s} + q(\theta^s) \right] = 0. \quad (A21)$$

To simplify further (A21), we rewrite the expression for $[d\pi^*(q(\theta^s), \theta^s)/d\theta^s] + q(\theta^s)$ as follows:

$$\begin{aligned} & \frac{d\pi^*(q(\theta^s), \theta^s)}{d\theta^s} + q(\theta^s) \\ &= v'(q(\theta^s)) \frac{dq(\theta^s)}{d\theta^s} - q(\theta^s) - \theta^s \frac{dq(\theta^s)}{d\theta^s} + q(\theta^s) \\ &= (v'(q(\theta^s)) - \theta^s) \frac{dq(\theta^s)}{d\theta^s}, \end{aligned}$$

where $v'(q(\theta^s)) - \theta^s > 0$ and $dq(\theta^s)/d\theta^s < 0$ at the optimum (from Proof of Proposition 2). Therefore cutoff θ^s is determined by:

$$\left[\pi^b(q^b(\theta^s), \theta^s) \right] = \frac{\beta}{\Delta\gamma} - \underbrace{\lambda \left[\frac{d\pi^*(q(\theta^s), \theta^s)}{d\theta^s} + q(\theta^s) \right]}_{(+)}. \quad (A22)$$

Recall that in \mathcal{P}^b , the cutoff θ^b is determined by:

$$\pi^b(q^b(\theta^b), \theta^b) = \beta/\Delta\gamma. \quad (A23)$$

Since $\pi^b(q^b(\theta), \theta)$ is decreasing in θ , by comparing (A22) and (A23) it follows that $\theta^b < \theta^s$.

■

Proof of Claim 2.

In \mathcal{P}^b , the agent prefers shirking at θ^b if:

$$\gamma_G \left[t(\theta^b) - \theta^b q^b(\theta^b) \right] - \beta \leq \gamma_B \left[t(\theta^b) - \theta^b q^b(\theta^b) \right]. \quad (A24)$$

Since $u(\theta) = \gamma(\alpha(\theta)) [t(\theta) - \theta q(\theta)]$, we have $t(\theta) = \theta q(\theta) + [u(\theta)/\gamma(\alpha(\theta))]$. Substituting for $t(\theta)$, the inequality in (A24) becomes:

$$\begin{aligned} & \gamma_G \left[\theta^b q^b(\theta^b) \frac{\left[u(\bar{\theta}) + \int_{\theta^b}^{\bar{\theta}} \gamma_G q^b(\theta) d\theta \right]}{\gamma_G} - \theta^b q^b(\theta^b) \right] - \beta \\ & \leq \gamma_B \left[\theta^b q^b(\theta^b) + \frac{\left[u(\bar{\theta}) + \int_{\theta^b}^{\bar{\theta}} \gamma_B q^b(\theta) d\theta \right]}{\gamma_B} - \theta^b q^b(\theta^b) \right]. \end{aligned}$$

Since $u(\bar{\theta})$ cancels out as it enters both LHS and the RHS of the above inequality, it can be rewritten as:

$$\frac{\beta}{\Delta\gamma} - \int_{\theta^b}^{\bar{\theta}} q^b(\theta)d\theta \geq 0. \quad (A25)$$

Consider the principal's problem \mathcal{P}^b complemented by the constraint (A25):

$$\mathcal{P}^b : \quad \text{Max}_{\theta^b} \int_{\underline{\theta}}^{\theta^b} \left\{ \gamma_G \left[\pi^b(q^b(\theta), \theta) \right] - \beta \right\} f(\theta)d\theta + \int_{\theta^b}^{\bar{\theta}} \gamma_B \left[\pi^b(q^b(\theta), \theta) \right] f(\theta)d\theta$$

subject to:

$$\frac{\beta}{\Delta\gamma} - \int_{\theta^b}^{\bar{\theta}} q^b(\theta)d\theta \geq 0.$$

The Lagrangian function for the optimization problem is:

$$\begin{aligned} \mathcal{L} = & \int_{\underline{\theta}}^{\theta^b} \left\{ \gamma_G \left[\pi^b(q^b(\theta), \theta) \right] - \beta \right\} f(\theta)d\theta + \int_{\theta^b}^{\bar{\theta}} \gamma_B \left[\pi^b(q^b(\theta), \theta) \right] f(\theta)d\theta \\ & + \mu \left[\frac{\beta}{\Delta\gamma} - \int_{\theta^b}^{\bar{\theta}} q^b(\theta)d\theta \right], \end{aligned}$$

where $\mu \in R$ is a constant.

Differentiating \mathcal{L} with respect to θ^b we obtain:

$$\Delta\gamma \left[\pi^b(q(\theta^b), \theta^b) \right] - \beta + \mu[q^b(\theta^b)] = 0. \quad (A26)$$

Recall that in \mathcal{P}^b the optimal θ^b is given by $\pi^b(q(\theta^b), \theta^b) = \beta/\Delta\gamma$, which together with (A26) imply that:

$$\mu q^b(\theta^b) = 0. \quad (A27)$$

Since $q^b(\theta^b) > 0$, (A27) implies that $\mu = 0$ and the constraint (A24) is satisfied a strict inequality. ■

Proof of Lemma 4.

By the envelope theorem, the necessary condition of (IC) is: $u'(\theta) = -\gamma(\alpha(\theta))q(\theta)$. Since the agent's rent, $u(\theta)$, is decreasing in θ , it is implied from (EC^s) that there exists the cutoff θ^d such that:

$$\alpha = \begin{cases} 1 & \text{for } \theta \leq \theta^d \\ 0 & \text{for } \theta > \theta^d \end{cases}$$

Therefore, the agent's rent can be expressed as:

$$u(\hat{\theta}|\theta) = \begin{cases} \gamma_G[t(\hat{\theta}) - \theta q(\hat{\theta})] - \beta & \text{for } \theta \leq \theta^d, \\ \gamma_B[t(\hat{\theta}) - \theta q(\hat{\theta})] & \text{for } \theta > \theta^d, \end{cases} \quad (A28)$$

which in turn gives:

$$u'(\theta) = -\gamma_G q(\theta) \quad \text{and} \quad u(\theta) = \gamma_G \int_{\theta}^{\theta^d} q(\tau) d\tau + u(\theta^d) \quad \text{for } \theta \leq \theta^d, \quad (\text{A29})$$

$$u'(\theta) = -\gamma_B q(\theta) \quad \text{and} \quad u(\theta) = \gamma_B \int_{\theta}^{\bar{\theta}} q(\tau) d\tau + u(\bar{\theta}) \quad \text{for } \theta > \theta^d. \quad (\text{A30})$$

From (A30), we have:

$$u(\theta^d) = \gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + u(\bar{\theta}). \quad (\text{A31})$$

Therefore, from (A29):

$$u(\theta) = \gamma_G \int_{\theta}^{\theta^d} q(\tau) d\tau + \gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + u(\bar{\theta}) \quad \text{for } \theta \leq \theta^d. \quad (\text{A32})$$

Applying integration by part to the expected values of (A30) for $\theta > \theta^d$ and (A32) for $\theta \leq \theta^d$, the agent's expected rents are:

$$\begin{aligned} E_{\theta \leq \theta^d} [u(\theta)] &= \gamma_G \int_{\underline{\theta}}^{\theta^d} q(\theta) F(\theta) d\theta + u(\theta^d) F(\theta^d) + \int_{\underline{\theta}}^{\theta^d} u(\bar{\theta}) f(\theta) d\theta \quad \text{for } \theta \leq \theta^d, \\ E_{\theta > \theta^d} [u(\theta)] &= \gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) F(\theta) d\theta - u(\theta^d) F(\theta^d) + \int_{\theta^d}^{\bar{\theta}} u(\bar{\theta}) f(\theta) d\theta \quad \text{for } \theta > \theta^d. \end{aligned}$$

Since $\gamma_G t(\theta) = \gamma_G \theta q(\theta) + u(\theta) + \beta$ for $\theta \leq \theta^d$ and $\gamma_B t(\theta) = \gamma_B \theta q(\theta) + u(\theta)$ for $\theta > \theta^d$ from (A28), using the the expressions for the agent's expected rents above, we can replace for $E_{\theta \leq \theta^d} [\gamma_G t(\theta)]$ and $E_{\theta > \theta^d} [\gamma_B t(\theta)]$ in the objective function to have:

$$\begin{aligned} &\int_{\underline{\theta}}^{\theta^d} \{ \gamma_G [\pi^b(q(\theta), \theta)] - u(\bar{\theta}) - \beta \} f(\theta) d\theta + \int_{\theta^d}^{\bar{\theta}} \{ \gamma_B [\pi^b(q(\theta), \theta)] - u(\bar{\theta}) \} f(\theta) d\theta \\ &= \int_{\underline{\theta}}^{\theta^d} \{ \gamma_G [\pi^b(q(\theta), \theta)] - \beta \} f(\theta) d\theta + \int_{\theta^d}^{\bar{\theta}} \gamma_B [\pi^b(q(\theta), \theta)] f(\theta) d\theta - u(\bar{\theta}). \quad (\text{A33}) \end{aligned}$$

The first order condition for (EC^d) gives:

$$\begin{aligned} &\Delta \gamma [t(\theta^d) - \theta^d q(\theta^d)] - \beta = 0 \\ &\iff \gamma_G [t(\theta^d) - \theta^d q(\theta^d)] - \beta = \gamma_B [t(\theta^d) - \theta^d q(\theta^d)] \\ &\iff \frac{\gamma_G}{\gamma_B} \gamma_B [t(\theta^d) - \theta^d q(\theta^d)] - \beta = \gamma_B [t(\theta^d) - \theta^d q(\theta^d)] \\ &\iff \frac{\gamma_G}{\gamma_B} \left[\gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + u(\bar{\theta}) \right] - \beta = \gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + u(\bar{\theta}), \quad (\text{A34}) \end{aligned}$$

where the last step in (A34) follows from the fact that $u(\theta^d) = \gamma_B [t(\theta^d) - \theta^d q(\theta^d)]$ from (A28) with $u(\theta) \equiv u(\theta|\theta)$, and $u(\theta^d) = \gamma_B \int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + u(\bar{\theta})$ from (A31). A simple rearrangement of (A34) gives:

$$\int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta + \frac{u(\bar{\theta})}{\gamma_B} = \frac{\beta}{\Delta \gamma}. \quad (\text{A35})$$

The principal's problem in \mathcal{P}^d is maximizing (A33) subject to (A35) and $u(\bar{\theta}) \geq 0$. The principal can optimally choose $u(\bar{\theta}) = 0$, and hence the problem becomes the one presented in Lemma 4 for $\theta^d \in [\underline{\theta}^d, \bar{\theta}]$. Lastly, we show that $\mathcal{P}^d = \mathcal{P}^b(\theta^b = \underline{\theta})$ for $\theta^d \in [\underline{\theta}, \underline{\theta}^d)$, i.e., $q^d(\theta) = q^b(\theta)$ with $\alpha(\theta) = 0$ for the entire range $\Theta = [\underline{\theta}, \bar{\theta}]$. Suppose $\theta^d = \underline{\theta}$. Then the principal's problem is written as:

$$\begin{aligned} & \underset{q}{Max} \int_{\underline{\theta}}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta, \\ & \text{s.t.} \quad \int_{\underline{\theta}}^{\bar{\theta}} q(\theta) d\theta = \frac{\beta}{\Delta\gamma}. \end{aligned} \tag{A36}$$

The optimal outcome of this problem, however, is strictly dominated by the optimal outcome of following problem:

$$\underset{q}{Max} \int_{\underline{\theta}}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta, \tag{A40}$$

which is $\mathcal{P}^b(\theta^b = \underline{\theta})$. That is, at $\theta^d = \underline{\theta}$, the principal's expected payoff in (A40) is strictly higher than her expected payoff in (A39). Since $\mathcal{P}^b(\theta^b = \underline{\theta})$ is implementable under delegated-performance, this shows that there exists $\underline{\theta}^d (> \underline{\theta})$ such that $\mathcal{P}^d = \mathcal{P}^b(\theta^b = \underline{\theta})$ for $\theta^d \in [\underline{\theta}, \underline{\theta}^d)$. ■

Proof of Proposition 3.

Again from Lemma 4, the principal's problem in \mathcal{P}^d when $\theta^d \in [\underline{\theta}^d, \bar{\theta}]$ is:

$$\underset{q, \theta^d}{Max} \int_{\underline{\theta}}^{\theta^d} \{ \gamma_G \left[\pi^b(q(\theta), \theta) \right] - \beta \} f(\theta) d\theta + \int_{\theta^d}^{\bar{\theta}} \gamma_B \left[\pi^d(q(\theta), \theta) \right] f(\theta) d\theta, \tag{A41}$$

subject to:

$$\int_{\theta^d}^{\bar{\theta}} q(\theta) d\theta = \frac{\beta}{\Delta\gamma}. \tag{A42}$$

From (A42), we can express θ^d as a function of $q(\theta)$ for the relevant interval, i.e., $\theta^d = \theta^d(q(\theta))$, $\theta \in [\theta^d, \bar{\theta}]$. Replacing θ^d with $\theta^d(q(\theta))$ in (A41), the principal's maximization problem in $q(\theta)$ is written as:

$$\underset{q, u(\bar{\theta})}{Max} \int_{\underline{\theta}}^{\theta^d(q(\theta))} \{ \gamma_G \left[\pi^b(q(\theta), \theta) \right] - \beta \} f(\theta) d\theta + \int_{\theta^d(q(\theta))}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta.$$

We next show that the output will be at the benchmark level, $q^d(\theta) = q^b(\theta)$, for $\theta \leq \theta^d$, and $q^d(\theta) > q^b(\theta)$, for $\theta > \theta^d$. Replacing θ^d with $\theta^d(\cdot) \equiv \theta^d(q(\theta^d))$, $\{q(\theta)\}_{\theta \geq \theta^d}$ in (A42),

the principal's maximization problem in $q(\theta)$ is written as:

$$Max_q \int_{\underline{\theta}}^{\theta^d(\cdot)} \left\{ \gamma_G \left[\pi^b(q(\theta), \theta) \right] - \beta \right\} f(\theta) d\theta + \int_{\theta^d(\cdot)}^{\bar{\theta}} \gamma_B \left[\pi^b(q(\theta), \theta) \right] f(\theta) d\theta.$$

First, since $q(\theta)$ depends on $\theta^d(\cdot)$ only for $\theta \geq \theta^d$ by (A42), it is clear from the problem that regardless of the value of θ^d , the point-wise maximization for $\theta \leq \theta^d$ gives that:

$$\pi_q^b(q(\theta), \theta) = 0 \text{ for } \theta \leq \theta^d,$$

implying that $q^d(\theta) = q^b(\theta)$ for $\theta \leq \theta^d$. Second, consider $\theta > \theta^d$. Using the Leibniz integral rule, the point-wise maximization with respect to $q(\theta)$ implies:

$$\left[\gamma_G \pi^b(q, \theta^d(\cdot)) - \beta \right] f(\theta^d(\cdot)) \frac{d\theta^d}{dq} - \gamma_B \pi^b(q, \theta^d(\cdot)) f(\theta^d(\cdot)) \frac{d\theta^d}{dq} + \int_{\theta^d(\cdot)}^{\bar{\theta}} \gamma_B \pi_q^b(q, \theta) f(\theta) d\theta = 0,$$

which simplifies to:

$$[\Delta \gamma \pi^b(q, \theta^d(\cdot)) - \beta] f(\theta^d(\cdot)) \frac{d\theta^d}{dq} + \int_{\theta^d(\cdot)}^{\bar{\theta}} \gamma_B \pi_q^b(q, \theta) f(\theta) d\theta = 0.$$

From (A42) it follows that $d\theta^d/dq > 0$ and $\Delta \gamma \pi^b(q, \theta^d(\cdot)) - \beta > 0$, and thus we have $q^d(\theta) > q^b(\theta)$ for $\theta < \theta^d$. Therefore:

$$q^d(\theta) \begin{cases} = q^b(\theta) & \text{for } \theta \leq \theta^d \\ > q^b(\theta) & \text{for } \theta > \theta^d \end{cases} \quad (A43)$$

The cutoff θ^d is obtained from (A42) with $q^b(\theta)$:

$$\int_{\theta^d}^{\bar{\theta}} q^b(\theta) d\theta = \frac{\beta}{\Delta \gamma}. \quad (A44)$$

For $\theta^d = \bar{\theta}$, (A44) gives $\beta = 0$. It follows directly from (A44) that, as β increases, θ^d must decrease. Lemma 4, together with that θ^d is decreasing in β , implies that, when $\theta^d \in [\underline{\theta}, \bar{\theta}^d)$, $q^d(\theta) = q^b(\theta)$ and $\alpha(\theta) = 0$ for $\theta \in \Theta$. ■

Proof of Corollary 2.

From Proposition 4, for a given β , the cutoff θ^d in \mathcal{P}^d is determined by:

$$\int_{\theta^d}^{\bar{\theta}} q^d(\theta) d\theta = \beta / \Delta \gamma,$$

where $q^d(\theta^d) = q^b(\theta^d)$. Since $\pi(q^d(\theta^d), \theta^d) = \pi^b(q^b(\theta^d), \theta^d)$, from Condition 1, we thus have:

$$\pi^b(q^b(\theta^d), \theta^d) > \int_{\theta^d}^{\bar{\theta}} q^d(\theta) d\theta = \beta/\Delta\gamma. \quad (A45)$$

Recall that, from Proposition 2, the cutoff θ^b in \mathcal{P}^b is determined by:

$$\pi^b(q^b(\theta^b), \theta^b) = \beta/\Delta\gamma \quad (A46)$$

Comparing (A45) and (A46), $\pi^b(q^b(\theta^d), \theta^d) > \pi^b(q^b(\theta^b), \theta^b)$, and $d\pi^b/d\theta < 0$ implies that $\theta^d < \theta^b$. ■

Proof of Proposition 4.

We have $\theta^s = \bar{\theta}$ at $\beta = \beta^s$, and $\theta^b = \bar{\theta}$ at $\beta = \beta^b$. Corollary 1 and 2 imply that $\beta^s > \beta^b > 0$. Therefore at $\beta = \beta^b$, the optimal outcome in \mathcal{P}^s gives the same optimal outcome in \mathcal{P}^b . This implies that there exists $\underline{\beta}$ such that, for $\beta \in (0, \underline{\beta}]$, the principal's expected payoff with $\varphi = s$ is strictly higher than her expected payoff with $\varphi = d$. Now, let $\bar{\beta}^b \equiv \Delta\gamma\pi^b(q^b(\underline{\theta}), \underline{\theta})$. For $\beta \geq \bar{\beta}^b$, the principal makes $\theta^b = \underline{\theta}$, i.e., $\alpha^b = 0$ for the entire range of θ in \mathcal{P}^b , and Corollary 1 implies that $\bar{\beta}^b < \bar{\beta}^s$. At $\beta = \bar{\beta}^b$, the optimal outcome in \mathcal{P}^d gives the same optimal outcome in \mathcal{P}^b by Proposition 3. This implies that there exists $\bar{\beta}$ such that, for $\beta \in [\bar{\beta}, \bar{\beta}^s)$, the principal's expected payoff with $\varphi = d$ is strictly higher than her expected payoff with $\varphi = s$. ■

Proof of Corollary 3.

Corollary 3 is implied by Proposition 4. ■

Appendix B. When θ is Public Information

In this section, we present the case where θ is publicly observed—i.e., only hidden action is an issue. As will be demonstrated below, the principal strictly prefers self-performance to delegated-performance under such an environment.

Under self-performance ($\varphi = s$), the principal's problem is:

$$\text{Max}_{\alpha, q, t} \int_{\Theta} [\gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] - \beta\alpha(\theta)] f(\theta) d\theta, \text{ subject to}$$

$$\gamma(\alpha(\theta)) [t(\theta) - \theta q(\theta)] \geq 0, \text{ and}$$

$$\alpha(\theta) \in \arg \max_{\hat{\alpha}} \{\gamma(\hat{\alpha}(\theta)) [v(q(\theta)) - t(\theta)] - \beta\hat{\alpha}(\theta)\}.$$

The first constraint induces the agent's participation. The second constraint is for the principal's upstream effort—since her effort is hidden, and thus non-contractible upon, the principal must respect her ex post incentive when she makes an offer.

Under delegated-performance ($\varphi = d$), the principal's problem is:

$$\begin{aligned} & \underset{\alpha, q, t}{Max} \int_{\Theta} \gamma(\alpha(\theta)) [v(q(\theta)) - t(\theta)] f(\theta) d\theta, \text{ subject to} \\ & \gamma(\alpha(\theta)) [t(\theta) - \theta q(\theta)] - \beta \alpha \geq 0, \text{ and} \\ & \alpha(\theta) \in \arg \max_{\tilde{\alpha}} \gamma(\tilde{\alpha}(\theta)) [t(\theta) - \theta q(\theta)] - \beta \tilde{\alpha}(\theta). \end{aligned}$$

The first constraint is for the agent's participation, and the second constraint is for the agent's upstream effort.

The following lemma presents the principal's choice between self- and delegated-performance when the agent's information is public.

Lemma 5 *If θ is public information, the principal's expected payoff with $\varphi = s$ is strictly higher than her expected payoff with $\varphi = d$.*

Proof. With $\varphi = s$, the participation constraint for the agent is binding for $\forall \theta \in \Theta$, implying that the principal's problem can be rewritten as:

$$\begin{aligned} & \underset{\alpha, q}{Max} \int_{\Theta} [\gamma(\alpha(\theta)) [v(q(\theta)) - \theta q(\theta)] - \beta \alpha(\theta)] f(\theta) d\theta, \text{ subject to} \\ & \alpha(\theta) \in \arg \max_{\hat{\alpha}} \{\gamma(\hat{\alpha}(\theta)) [v(q(\theta)) - \theta q(\theta)] - \beta \hat{\alpha}(\theta)\}, \end{aligned}$$

where the constraint for the principal's ex post incentive for upstream effort becomes: $\Delta \gamma [v(q(\theta)) - \theta q(\theta)] = \beta$. This constraint is automatically satisfied with the first-best outcome. Suppose there is no effort constraint. Then, the optimal output schedule is the first-best: $q(\theta) = q^*(\theta)$. Also, the objective function is linear in $\alpha(\theta)$ since $\gamma(\alpha(\theta)) = \alpha(\theta)\gamma_G + (1 - \alpha(\theta))\gamma_B$, and $v(q^*(\theta)) - \theta q^*(\theta)$ is decreasing in θ , implying that optimization with respect to $\alpha(\theta)$ gives a cutoff θ such that:

$$\alpha = \begin{cases} 1 & \text{for } \theta \leq \theta^{s*} \\ 0 & \text{for } \theta > \theta^{s*} \end{cases}.$$

The principal's problem without the effort constraint becomes:

$$\underset{\theta^{s*}}{Max} \int_{\underline{\theta}}^{\theta^{s*}} \{\gamma_G [v(q^*(\theta)) - \theta q^*(\theta)] - \beta\} f(\theta) d\theta + \int_{\theta^{s*}}^{\bar{\theta}} \gamma_B [v(q^*(\theta)) - \theta q^*(\theta)] f(\theta) d\theta.$$

Optimization with respect to θ^{s^*} gives: $\Delta\gamma[v(q^*(\theta)) - \theta q^*(\theta)] = \beta$, which coincide with the principal's ex post incentive from the effort constraint. Also, θ^{s^*} is characterized by $\Delta\gamma[v(q^*(\theta^{s^*})) - \theta^{s^*} q^*(\theta^{s^*})] = \beta$, implying that $\theta^{s^*} = \theta^*$. Thus, when θ is public information, the principal can achieve the first-best outcome with $\varphi = s$.

With $\varphi = d$, the effort constraint for the agent gives:

$$t(\theta) = \theta q(\theta) + \frac{\beta}{\Delta\gamma},$$

implying that the participation constraint for the agent is automatically satisfied. As in the case with $\varphi = s$, the transfer schedule above implies that $q(\theta) = q^*(\theta)$, and again the principal induces the agent's effort for θ less than a cutoff point. The principal's problem is rewritten as:

$$\underset{\theta^{d^*}}{\text{Max}} \int_{\underline{\theta}}^{\theta^{d^*}} \gamma_G \left[v(q^*(\theta)) - \theta q^*(\theta) - \frac{\beta}{\Delta\gamma} \right] f(\theta) d\theta + \int_{\theta^{d^*}}^{\bar{\theta}} \gamma_B [v(q^*(\theta)) - \theta q^*(\theta)] f(\theta) d\theta.$$

Optimization with respect to θ^{d^*} gives: $v(q^*(\theta)) - \theta q^*(\theta) = \gamma_G \beta$, and θ^{d^*} is characterized by $v(q^*(\theta^{d^*})) - \theta^{d^*} q^*(\theta^{d^*}) = \gamma_G \beta$. This implies that $\theta^{d^*} < \theta^* = \theta^{s^*}$, which in turn implies that the principal's expected payoff is strictly higher with $\varphi = s$. ■

When the project environment is public information, the agent commands no rent under self-performance, and the principal's ex ante and ex post incentives are the same. As a result, under self-performance, the optimal outcome is the first-best when the project environment is public information.

Under delegated-performance, although the project environment is public information, inducing the agent's upstream effort requires rent provision. To be specific, the transfer that induces the agent's upstream effort is:

$$t(\theta) = \theta q(\theta) + \beta/\Delta\gamma,$$

where the second term, $\beta/\Delta\gamma$, represents the extra cost to the principal to induce the agent's upstream effort. Thus, when the agent has no private information, there is no trade-off between self- and delegated-performance—while the former implements the first-best outcome, the latter invites a moral hazard problem. The principal's optimal payoff strictly lower with delegated-performance when θ is public information.

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