Competitive information discovery

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Abstract

When agents can make information discoveries, their actions will determine which states can be distinguished and thus which goods are traded. Market equilibria can then be Pareto inefficient even though the classical requirements of competition are satisfied. To restore efficiency, the prices anticipated when agents contemplate an information discovery must be proportional to the probabilities of the events that could be revealed. This rule also eliminates self-confirming equilibria where price expectations lead agents not to undertake the information discoveries that would invalidate those expectations. Efficiency also requires structural assumptions – risk aversion, common priors – that are normally irrelevant for the first welfare theorem.

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1 Introduction

In classical models of competitive markets, the revelation of information descends without human intervention. The archetype is the weather, which is learned regardless of how agents act. But in at least as many cases of economic interest agents must take an action to reveal the state of nature: a firm must experiment with a new technology to find out if it is productive, a consumer must try out a good to gauge its utility. The predominant Schumpeterian analysis of firms' information discoveries is noncompetitive: firms try out new technologies to win a measure of monopoly power and would not conduct experiments without a chance of that reward.[1] Competitive equilibria can nevertheless be defined when agents' actions uncover information. While agents must have expectations of the prices that will rule if they discover information they can otherwise act as price-takers: I will assume that the scale of trades does not affect prices, that there are no arbitrage opportunities, and, to ensure that trades of contingent commodities are verifiable, that no agent enjoys a private information advantage. Despite these conditions, equilibria can be Pareto inefficient. In a characteristic example, it can be socially efficient for a firm to build a new type of capital equipment that will test an innovation but the output price that agents anticipate if the test is conducted and the experiment is successful will not be high enough to justify the investment. In equilibrium the firm consequently does not experiment with the new technology, leaving agents ignorant of the state. This example flips the Schumpeterian story: instead of an innovator using its informational monopoly to manipulate prices in its favor, firms face perfect competition and prices move to an innovator's disadvantage.

To address the inefficiency, I introduce a stronger definition of competition: the prices for a good k that are expected when an information discovery is contemplated must equal the price of k that obtains under ignorance multiplied by the probabilities of the events that would be discovered. This *competitive price rule* extends price-taking to settings where the set of tradable goods is determined endogenously: it ensures that as agents consider information discoveries the price they anticipate paying for an increment in the probability of receiving a good remains constant. The price rule is the main plank that will restore

[1] See Schumpeter (1942). More recent work in this vein includes Aghion and Howitt (1998), Grossman and Helpman (1991), and Romer (1990).

the first welfare theorem: in conjunction with supplementary assumptions, it ensures that competitive equilibria with information discovery are Pareto efficient. As explained in detail in a companion paper, Mandler (2017), firms must take counterintuitive actions to ensure social efficiency: they must select the riskier production sets. The main result of this paper shows that in competitive equilibrium firms will take exactly those actions.

The supplementary assumptions needed for Pareto efficiency – common priors and risk aversion – have a classical pedigree but are alien to the contemporary understanding of the efficiency of markets. A revelation of Arrow (1951) and Debreu (1951) was that the first welfare theorem is nearly assumption-free: the weak Pareto efficiency of equilibria requires no assumptions and strong efficiency requires only that preferences are transitive and locally nonsatiated. The present need for more will make sense in retrospect: without risk aversion and common priors, an agent that fails to make an information discovery can harm other agents by denying them the opportunity to make utility-increasing gambles.

The equilibria that lead to inefficiency are typically 'self-confirming': agents hold unrealistic price expectations that lead them not to make the information discoveries that would disconfirm those expectations, similarly to Fudenberg and Levine (1993) though the analysis here applies to markets rather than games. Ruling out self confirmation is delicate. If we simply impose a subgame perfection requirement that agents accurately perceive the equilibrium that results as they adjust their actions they would no longer behave as price-takers: they would adjust their demands to optimize their effect on prices, a noncompetitive behavior that would lead to inefficiency.

The competitive price rule solves this problem too. First, the prices it mandates coincide with the equilibrium prices that would rule if we could decree or force the state to be revealed and if there were no supply-and-demand effects stemming from the information learned. When information does have a demand impact, the price expectations furnished by the price rule will no longer match the equilibrium prices that obtain when revelation of the state is forced, comparably to the fact that price-taking assumptions do not hold literally when agents contemplate deviations from standard competitive equilibria: knowledge of the state will change demand and thus the prices that clear markets. But if the value of information is small, the price rule lays out price expectations that approximate what would occur in equilibrium when revelation is forced. The small-information assumption is limiting but it provides a clear setting where the price rule has a rationale routed in equilibrium behavior; it also permits a full characterization of the price rule.

The impediments to efficiency considered here stem from the market power that information discoverers potentially wield: since goods are distinguished by state, an agent that causes the state to be revealed can affect which goods are traded and thus indirectly the prices of goods. This leverage resembles the monopoly power that Schumpeterian firms gain when they discover an innovation. The competitive price rule neuters these influences on prices and thus disentangles market power from information discovery. Firm size is pivotal for the former but irrelevant for the latter.

The existence of equilibrium will be a side issue in this paper. Although discrete costs for information discoveries will introduce nonconvexities that can interfere with existence, those difficulties have classical fixes and, when firms are the discoverers, the convexity and continuity assumptions that guarantee existence can be applied successfully (see section 6). Though work remains to confirm that there are no intractable existence problems, the first task is to define what a competitive equilibrium with information discovery actually is.

Boldrin and Levine (2002, 2017a, 2017b) pursue a compatible agenda where innovative goods are competitively produced under constant returns and optimality obtains. Boldrin and Levine do not allow for uncertainty, however, a prominent feature of technological development; one goal of the competitive price rule is to fill this gap. My aim though is not to argue that efficiency is the norm. For competitive equilibria to be efficient, one must assume that information externalities are absent. There are cases where that assumption makes sense (consumers figuring out their own tastes) and other Schumpeterian cases where it does not (expensive technological research that free riders can copy).

2 Equilibrium with information discovery

Consumers, firms, states, and goods.

The sets of consumers $\mathcal{I}$, firms $\mathcal{J}$, and states Ω are all finite with I, J, and S elements respectively. There are L_1 goods in the first period and L_2 goods at each state in the second period. The total number of goods is therefore $L = L_1 + SL_2$.

A consumption for $i \in \mathcal{I}$ and a production for $j \in \mathcal{J}$ are given by

$$x^i = \left(x_1^i(1), ..., x_{L_1}^i(1), \left(x_1^i(\omega), ..., x_{L_2}^i(\omega)\right)_{\omega \in \Omega}\right) \in \mathbb{R}_+^L,$$
$$y^j = \left(y_1^j(1), ..., y_{L_1}^j(1), \left(y_1^j(\omega), ..., y_{L_2}^j(\omega)\right)_{\omega \in \Omega}\right) \in \mathbb{R}^L.$$

Define also

$$x^i(1) = \left(x_1^i(1), ..., x_{L_1}^i(1)\right), \; x^i(\omega) = \left(x_1^i(\omega), ..., x_{L_2}^i(\omega)\right), \; x = \left(x^i\right)_{i \in \mathcal{I}},$$
$$y^j(1) = \left(y_1^j(1), ..., y_{L_1}^j(1)\right), \; y^j(\omega) = \left(y_1^j(\omega), ..., y_{L_2}^j(\omega)\right), \; y = \left(y^j\right)_{j \in \mathcal{J}},$$

and let x^{-i} and y^{-j} denote $(x^{i'})_{i' \in \mathcal{I} \setminus \{i\}}$ and $(y^{j'})_{j' \in \mathcal{J} \setminus \{j\}}$.

The probabilities of states and events in Ω are given by $\pi(\cdot)$ and we fix, for any event $E \subset \Omega$, conditional probabilities $\pi(\cdot|E)$ that satisfy Bayes rule when applicable.

Each consumer i at each state ω has a concave and locally nonsatiated utility $u_\omega^i : \mathbb{R}_+^{L_1+L_2} \to \mathbb{R}$, henceforth called a *vNM utility*, which defines an expected utility function $U^i : \mathbb{R}_+^L \to \mathbb{R}$ by $U^i(x^i) = \sum_{\omega \in \Omega} \pi(\omega) u_\omega^i(x^i(1), x^i(\omega))$.

Consumer i's endowments are given by $\left(e^i(1), (e^i(\omega))_{\omega \in \Omega}\right) \in \mathbb{R}_+^L$.

Each firm $j \in \mathcal{J}$ has a production set $Y^j \subset \mathbb{R}^L$. Consumer i's ownership share of j is $\theta^{ij} \geq 0$, where $\sum_{i \in \mathcal{I}} \theta^{ij} = 1$. For each state ω, Y^j defines a *single-state production set* Y_ω^j for first-period goods and the second-period goods that appear at ω equal to the projection of Y^j onto its first L_1 coordinates and its L_2 ω-coordinates:

$$Y_\omega^j = \Big\{ (y^j(1), y^j(\omega)) \in \mathbb{R}^{L_1+L_2} : \\ (y^j(1), \left(y^j(\omega')\right)_{\omega' \in \Omega}) \in Y^j \text{ for some } \left(y^j(\omega')\right)_{\omega' \in \Omega \setminus \{\omega\}} \in \mathbb{R}^{L_2(S-1)} \Big\}.$$

A simple and plausible model of the relationship between Y^j and the Y_ω^j would be to assume that firm j's production set at state ω equals Y_ω^j regardless of what j chooses at other states: the firm chooses $y^j(1)$ in the first period, nature then selects the state ω, and the firm then chooses any vector in $\{y^j(\omega) \in \mathbb{R}^{L_2} : (y^j(1), y^j(\omega)) \in Y^j_\omega\}$ in the second period.[2] But there is no need to rule out all inter-dependencies of production across states; it is sufficient that if the Y^j_ω coincide at some set of states then the same productions can be chosen simultaneously at these states, fixing what j does elsewhere. Formally, we assume that if $Y^j_{\omega'} = Y^j_{\omega''}$ and $y^j \in Y^j$ then the $\widehat{y}^j$ defined by $\widehat{y}^j(\sigma) = y^j(\sigma)$ for $\sigma = 1$ or $\sigma \in \Omega \backslash \{\omega''\}$ and $\widehat{y}^j(\omega'') = y^j(\omega')$ is an element of Y^j: j can choose at ω'' whatever it chooses at ω', all else remaining fixed.

Information.

The actions of the agents in the first period uncover information. For each consumer i, the choice $x^i \geq 0$ informs all agents of a cell of the partition $\mathcal{P}^i(x^i)$ of Ω, where, since it is i's first-period consumption that reveals the information, $\mathcal{P}^i(x^i) = \mathcal{P}^i(x^{i\prime})$ if $x^i(1) = x^{i\prime}(1)$. Similarly, for each firm j, the production $y^j \in Y^j$ informs all agents of a cell of the partition $\mathcal{P}^j(y^j)$, where $\mathcal{P}^j(y^j) = \mathcal{P}^j(y^{j\prime})$ if $y^j(1) = y^{j\prime}(1)$. Since goods can be useless and have a 0 price, the model can let agents costlessly select a partition from an arbitrary menu of partitions without affecting their useful consumption (see section 5). The following examples consider the information that one agent can discover; typically many agents will make discoveries simultaneously.

Example 1 Suppose consumer i can uncover information by buying $c \in \mathbb{R}^{L_1}_+$ or more in the first period: for some nontrivial partition $\widehat{\mathcal{P}}$ of Ω, $\mathcal{P}^i(x^i) = \widehat{\mathcal{P}}$ if $x^i(1) \geq c$ and $\mathcal{P}^i(x^i) = \{\Omega\}$ otherwise. The partition $\widehat{\mathcal{P}}$ could indicate whether i likes good 1 with the cells of $\widehat{\mathcal{P}}$ given by

$$\begin{aligned} P_I &= \{\omega \in \Omega : u^i_\omega \text{ is increasing in } x^i_1(\omega)\}, \\ P_D &= \{\omega \in \Omega : u^i_\omega \text{ is decreasing in } x^i_1(\omega)\}, \\ P_N &= \Omega \backslash (P_I \cup P_D). \end{aligned}$$

The cost c might consist of a minimum-size sampling of the good in the first period: $c_1 > 0$ and $c_k = 0$ for $2 \leq k \leq L_1$. ■

[2] Firm j's production set would then be given by

$$Y^j = \left\{ \left(y^j(1), \left(y^j(\omega)\right)_{\omega \in \Omega} \right) \in \mathbb{R}^L : \left(y^j(1), y^j(\omega)\right) \in Y^j_\omega \text{ for all } \omega \in \Omega \right\}.$$

Example 2 A firm j tries out a production process by building a prototype of a good in the first period. The prototype requires inputs of $c \in \mathbb{R}_+^{L_1}$ or greater, which reveals if the experiment succeeds, the event P_s, or fails, $P_f = \Omega \backslash P_s$. So $\mathcal{P}^j(y^j) = \{P_s, P_f\}$ if $y^j(1) \leq -c$ and $\mathcal{P}^j(y^j) = \{\Omega\}$ otherwise. If the firm builds the prototype and $\omega \in P_s$ then it has second-period access to a production function $g : \mathbb{R}_+^n \to \mathbb{R}_+^m$ such that $g(0) = 0$; otherwise j cannot engage in second-period production. Letting the second-period inputs have the initial indices among second-period goods, the production set is given by

$$y^j \in Y^j \Leftrightarrow \begin{cases} y^j(1) \leq -c,\ y^j(\omega) \leq (-l, g(l)) \text{ for some } l \geq 0 \text{ if } \omega \in P_s,\ y^j(\omega) \leq 0 \text{ if } \omega \in P_f, \\ \text{or} \\ y^j(1) \nleq -c,\ y^j(1) \leq 0,\ y^j(\omega) \leq 0 \text{ for all } \omega \in \Omega. \quad \blacksquare \end{cases}$$

Examples 1 and 2 above share the nonconvexity that information is discovered only if a threshold payment c is made. This feature is by no means necessary. Information discovery can readily fit standard convexity assumptions, particularly in production where the harnessing of inputs can by itself reveal information. In the following example, mentioned in the Introduction, a firm can build a capital good in the first period to produce output in the second period. In the course of constructing the capital, the firm finds out if its experimental technology 'works', whether it has high or low productivity.

Example 3 There are two states ω_H and ω_L indicating high and low productivity and one good at each date and state. A firm j can use a linear activity with coefficients $(-2, 3, 1)$ for the first-period, ω_H, and ω_L goods respectively and therefore has the production set

$$Y^j = \{(y(1), y(\omega_H), y(\omega_L)) : (y(1), y(\omega_H), y(\omega_L)) \leq \lambda(-2, 3, 1) \text{ for some } \lambda \geq 0\}.$$

The state is revealed if and only if the firm chooses $y(1) < 0$: $\mathcal{P}(y) = \{\{\omega_H\}, \{\omega_L\}\}$ if $y(1) < 0$ and $\mathcal{P}(y) = \{\Omega\}$ otherwise. Notably, the production set Y^j is convex. $\blacksquare$

Examples 2 and 3 baked into a firm j's production set the measurability restriction that it cannot vary its actions more finely than its information reveals. When other agents besides j make decisions that leave information undiscovered then measurability restrictions on j's actions must be imposed directly.

All information is shared: the agents face a common information partition, from which a cell is revealed at the end of the first period. When agents take the actions $(x, y) \in \mathbb{R}_+^{IL} \times \prod_{j \in \mathcal{J}} Y^j$, this partition will be the coarsest common refinement of the partitions in $\{\mathcal{P}^i(x^i) : i \in \mathcal{I}\}$ and $\{\mathcal{P}^j(y^j) : j \in \mathcal{J}\}$ which we denote by $\mathcal{P}_{x,y}$. The absence of asymmetric information is important for efficiency: both sides of any trade can confirm its execution and agents can condition their consumption or production on the finest information possible.

For $i \in \mathcal{I}$ and $j \in \mathcal{J}$, let $\bigwedge \mathcal{P}^i$ and $\bigwedge \mathcal{P}^j$ be finest common coarsening of the partitions in $\{\mathcal{P}^i(x^i) : x^i \geq 0\}$ and $\{\mathcal{P}^j(y^j) : y^j \in Y^j\}$ respectively: $\bigwedge \mathcal{P}^l$ represents the information agent l will necessarily uncover regardless of what action l takes. We assume that, for each consumer i, the random variable e^i is measurable with respect to the coarsest common refinement of the partitions in $\left\{\bigwedge \mathcal{P}^l : l \in \mathcal{I} \cup \mathcal{J}\right\}$. So each consumer i's endowment will be known to i (and any other agent) in the second period whatever actions the agents take.[3] Consumer i therefore always has the option of delivering any portion of his endowment to the market. The assumption also ensures that if the market for delivery of a good at a state in $P \in \mathcal{P}_{x,y}$ clears then so will the markets for the same good at other states in P.

Markets and equilibrium.

Let $p = \left(p_1(1), ..., p_{L_1}(1), (p_1(\omega), ..., p_{L_2}(\omega))_{\omega \in \Omega}\right) \in \mathbb{R}_+^L$ denote a price vector. Prices for second-period goods have operational meaning only in relation to the partition $\mathcal{P}_{x,y}$ that agents face: the price of good k in the event $P \in \mathcal{P}_{x,y}$ is $\sum_{\omega \in P} p_k(\omega)$. The $p_k(\omega)$ need *not* indicate the prices of k at the states in Ω since those states need not form cells of a partition that agents might face. The $p_k(\omega)$ nevertheless play an important role: they determine the prices agents face as they alter their actions and thereby change $\mathcal{P}_{x,y}$.

Given prices p, productions y, and consumptions x^{-i} for all consumers besides i, the budget set for consumer i is

$$B^i(p, x^{-i}, y) = \left\{ x^i \in \mathbb{R}_+^L : x^i \text{ is } \mathcal{P}_{x^i, x^{-i}, y}\text{-measurable and } p \cdot x^i \leq p \cdot e^i + \sum_j \theta^{ij} p \cdot y^j \right\}.$$

Given the consumptions x and productions y^{-j} for all firms besides j, the action set for firm

[3] Conversely, if second-period endowments are publicly observed in the second period then a violation of this measurability assumption would mean that, for some set of first-period actions the agents might take, the agents would be failing to learn all that can be inferred from their observations.

j is given by

$$A^j(x, y^{-j}) = \left\{ y^j \in Y^j : y^j \text{ is } \mathcal{P}_{x,y^j,y^{-j}}\text{-measurable} \right\}.$$

These measurability requirements ensure that agents cannot take actions that are a function of information they are not privy to. But, since the measurability requirements adjust as agents vary their actions, agents do take account of how their information changes with their actions. Price-taking has no clear-cut meaning in this setting: since actions can change the set of purchasable goods, agents enjoy a measure of market power.

Definition 1 *An **equilibrium** is a* (p, x, y) *such that, for each* $i \in \mathcal{I}$ *and* $j \in \mathcal{J}$*,*

- $x^i \in B^i(p, x^{-i}, y)$ *and* $U^i(x^i) \geq U^i(x^{i\prime})$ *for each* $x^{i\prime} \in B^i(p, x^{-i}, y)$*,*
- $y^j \in A^j(x, y^{-j})$ *and* $p \cdot y^j \geq p \cdot y^{j\prime}$ *for each* $y^{j\prime} \in A^j(x, y^{-j})$*,*
- $\sum_{i \in \mathcal{I}} x^i \leq \sum_{j \in \mathcal{J}} y^j + \sum_{i \in \mathcal{I}} e^i$*, with* $p_k(\sigma) = 0$ *if strict inequality obtains for good* k *and period* $\sigma = 1$ *or state* $\sigma = \omega$*.*

3 Inefficiency and its cure

We begin with two examples of inefficiency. The first returns to the firm in Example 3 that tested a convex technology in the course of building its capital equipment. Although it is socially efficient for the firm to perform this test, if agents expect a low price for output when the test is undertaken and the technology is successful the firm will not do so. Outside of information discovery being an active choice, the example is entirely orthodox.

Example 3 Continued There is one firm, an arbitrary number of consumers, two states ω_H and ω_L, and one good at each date and state: $J = 1$, $S = 2$, with probabilities given by $\pi(\omega_H) = \frac{2}{3}$ and $\pi(\omega_L) = \frac{1}{3}$. Endowments satisfy $\sum_{i \in \mathcal{I}} e^i(1) > 0$ and $e^i(\omega_L) = e^i(\omega_H) > 0$ for each i, with the equality ensuring that agents cannot deduce the state from the endowment profile. Since there is one good at every date and state, we omit the subscripts denoting goods.

As described the original example, the firm can use a linear activity with coefficients $(-2, 3, 1)$ for the first-period, ω_H, and ω_L goods respectively and the state is revealed if and only if the firm chooses $y(1) < 0$. Each consumer i is risk-neutral and does not discount the future and therefore has the utility $U^i(x^i(1), x^i(\omega_H), x^i(\omega_L)) = x^i(1) + \frac{2}{3}x^i(\omega_H) + \frac{1}{3}x^i(\omega_L)$.

With the prices $p(1) = 3$, $p(\omega_H) = 1$, $p(\omega_L) = 2$, the firm will choose $y = (0, 0, 0)$ rather than produce: an investment of two units of the first-period good would cost 6 and earn a return of 5. When the state is not revealed, the consumers are happy to consume their endowments since units of first-period and second-period consumption (the latter delivered at both states) each cost 3.

This equilibrium is inefficient: a sacrifice of 2 units of the first-period consumption yields $\left(\frac{2}{3} \times 3\right) + \left(\frac{1}{3} \times 1\right) = 2\frac{1}{3}$ units of expected second-period consumption. ■

The equilibrium in Example 3 Continued has the self-confirming property that agents' price expectations lead the firm not to make the information discovery that could disconfirm those expectations. The price expectations consistent with a no-discovery equilibrium can therefore vary widely. If the state is unknown, the price that consumers pay for second-period consumption is the sum $p(\omega_H) + p(\omega_L)$ and as long as $p(1) = p(\omega_H) + p(\omega_L)$ the decomposition of that sum into state-by-state prices is irrelevant: any decomposition that leads the firm to refrain from information discovery will be consistent with equilibrium. We examine self-confirmation in more detail in section 4.

In the next Example, a consumer rather than a firm makes the information discovery and consequently the prices that lead to inefficiency have to be chosen more carefully in some cases. The convexity that Example 3 Continued enjoyed is less plausible with a consumer as the discoverer and we therefore use a variant of Example 1: the consumer reveals information by paying a discrete cost. Since that information is valuable only to the same consumer, no externality is present.

Example 4 There are two consumers a and b, no firms, two states ω_H and ω_L, and one good at each date and state: $I = 2$, $J = 0$, $S = 2$, with probabilities given by $\pi(\omega_H) = \frac{1}{2}$ and $\pi(\omega_L) = \frac{1}{2}$. Both agents are risk-neutral. Consumer a has vNM utilities $2x^a(1) + 3x^a(\omega_H)$ at ω_H and $2x^a(1) + x^a(\omega_L)$ at ω_L and b has the vNM utility $x^b(1) + x^b(\omega)$ at each ω. Thus ω_H and ω_L indicate whether a's marginal utility of consumption is high or low. We again omit subscripts denoting goods.

Consumer a can discover the state by spending $c \geq 0$ of the first-period good on an information-discovery technology and is thus the only agent with a state-dependent utility and the only agent that can discover information.[4] Let endowments satisfy $e^a(1) > c$ and $e^i(\omega_L) = e^i(\omega_H) > 0$ for each i.

With the prices $p(1) = 4$, $p(\omega_H) = 3$, $p(\omega_L) = 1$, consumer a has no incentive to discover the state. Whether or not the state is known, a dollar buys $\frac{1}{4}$ units of $x^a(1)$ and hence a utility gain of $\frac{1}{2} = 2 \times \frac{1}{4}$. If the state is not known a dollar buys a bundle of $\frac{1}{4}$ units each of $x^a(\omega_H)$ and $x^a(\omega_L)$ and thus a utility gain of $\frac{1}{2} = \left(\frac{1}{2} \times 3 \times \frac{1}{4}\right) + \left(\frac{1}{2} \times 1 \times \frac{1}{4}\right)$, while if the state is known a dollar buys $\frac{1}{3}$ units of $x^a(\omega_H)$ for a utility gain of $\frac{1}{2} = \left(\frac{1}{2} \times 3 \times \frac{1}{3}\right)$ or 1 unit of $x^a(\omega_L)$ for a utility gain of $\frac{1}{2} = \left(\frac{1}{2} \times 1 \times 1\right)$. Thus, putting aside the discovery cost, consumer a experiences neither a gain nor a loss if the state is revealed. Consequently, if $c > 0$ then the only equilibrium with the above prices is for consumer a not to discover.

For c sufficiently small, this equilibrium is inefficient: if the state were known, both a and b would be strictly better off if b transferred a unit of second-period consumption to a at state ω_H and received a unit from a at ω_L. ■

When the discovery cost c in Example 4 is 0, the price expectations that will lead consumer a not to discover the state must assume the precise values that equalize the marginal utility of income across states; otherwise a would discover the state in order to concentrate consumption in the state with the higher marginal utility. If $c > 0$ though the price expectations consistent with nondiscovery enjoy greater latitude.

3.1 The competitive price rule

When agents can discover information, competition by itself will not lead to Pareto efficiency: the information that one agent can uncover might be valuable to others and, as with standard externalities, potential discoverers will ignore those consequences. But as the Examples above have shown an absence of information externalities is not enough.

[4] When $c > 0$, this discovery can be modeled as the purchase of c units of an additional first-period good produced by a firm that uses c of the original first-period good as its only input. So $\mathcal{P}^a(x^a) = \{\{\omega_L\}, \{\omega_H\}\}$ if $x_2^a(1) \geq c$ and $\mathcal{P}^a(x^a) = \{\Omega\}$ otherwise. We can leave out further mention of these details by having the firm buy and sell at the same price, thus leaving no profits to distribute to its owners. When $c = 0$, discovery can be modeled as the purchase of a free good: see section 5.

Definition 2 *An equilibrium* (p, x, y) *satisfies the* ***competitive price rule*** *if, for each* $P \in \mathcal{P}_{x,y}$*,* $\omega \in P$*, and good* k*,*

$$p_k(\omega) = \pi(\omega|P) \sum_{\omega' \in P} p_k(\omega').$$

To understand the sense in which this rule is competitive, suppose at some equilibrium (p, x, y) that consumer i, by deviating to $x^{i\prime}$, reveals some event E contained in an event that is observable at (p, x, y). That is, suppose $E \subset P \in \mathcal{P}_{x,y}$ and $E \in \mathcal{P}_{x^{i\prime},x^{-i},,y}$. The competitive price rule then leads to a price of good k at E given by

$$\sum_{\omega \in E} p_k(\omega) = \sum_{\omega \in E} \pi(\omega|P) \sum_{\omega' \in P} p_k(\omega') = \pi(E|P) \sum_{\omega' \in P} p_k(\omega').$$

The derivative of the price of k at E with respect to the conditional probability of E given P is thus a constant, $\sum_{\omega' \in P} p_k(\omega')$. So, although agents can change the set of purchasable goods and thus cannot be traditional price-takers, under the competitive price rule they face a constant price for good k per increment of likelihood. Agents can adjust these likelihoods by varying the information discoveries they choose. When E is an arbitrary event in Ω the price of k at E according to the price rule will equal

$$\sum_{\omega \in E} p_k(\omega) = \sum_{P \in \mathcal{P}_{x,y}} \sum_{\omega \in E} \pi(\omega|P) \sum_{\omega' \in P} p_k(\omega') = \sum_{P \in \mathcal{P}_{x,y}} \pi(E|P) \sum_{\omega' \in P} p_k(\omega').$$

The interpretation is similar. At any $P \in \mathcal{P}_{x,y}$ agents face a constant price for good k per increment of likelihood: the marginal impact on the price of good k of increasing the conditional probability of E given P by the same amount at every $P \in \mathcal{P}_{x,y}$ is again a constant, $\sum_{\omega \in \Omega} p_k(\omega)$, that does not vary with the conditional probabilities.

Consider a weaker version of the price rule that requires the price of good k at a subevent E of $P \in \mathcal{P}_{x,y}$ plus the price of k at the complementary event $P \backslash E$ to equal the price of k at P: using obvious notation, $p_k(E) + p_k(P \backslash E) = p_k(P)$. This condition will hold in equilibrium if agents can costlessly undertake a discovery that refines P into the subevents E and $P \backslash E$: if it were violated then either buyers or sellers of k would gain by making the discovery. See Theorem 4 in section 5 for a result in this vein. The competitive price rule implies this weaker version but says more: the prices of k at subevents must be proportional to their conditional likelihoods.

The following Example illustrates the price rule and makes clear that it can be applied even when informational externalities make efficiency impossible.

Example 5 There are three firms, one consumer, two states, with labor and output present at each date and state: $I = 1$, $J = 3$, $S = 2$, with probabilities $\pi(\omega_g) = \pi(\omega_b) = \frac{1}{2}$. Each firm at each period and state can use a linear production function f that takes labor l as its input, subject to a capacity constraint: $f(l) = \min[l, 1]$ for $l \geq 0$. In the first period, any of the firms can use a unit of output to test a production function h equal to $h(l) = \min[2l, 2]$ in state ω_g and $h(l) = 0$ in state ω_b.[5] If some firm tests the technology then agents face the partition $\{\{\omega_g\}, \{\omega_b\}\}$ while if no firm tests they face $\{\{\omega_g, \omega_b\}\}$.

The consumer is risk-neutral and elastically supplies labor at a price equal to half a unit of output: letting the subscripts o and l indicate output and labor/leisure and omitting the superscript, the consumer's utility is

$$U(x) = x_o(1) + \frac{1}{2}x_l(1) + \sum_{\omega \in \{\omega_g, \omega_b\}} \frac{1}{2}\left(x_o(\omega) + \frac{1}{2}x_l(\omega)\right).$$

The consumer has the same endowment of labor $e_l > 3$ at each date and state.

In the only equilibrium that satisfies the competitive price rule (up to a normalization), no firm tests the uncertain technology, prices are $p_o(1) = 1$, $p_l(1) = \frac{1}{2}$, $p_o(\omega_g) = p_o(\omega_b) = \frac{1}{2}$, $p_l(\omega_g) = p_l(\omega_b) = \frac{1}{4}$, each firm j chooses $y^j(1) = y^j(\omega_g) = y^j(\omega_b) = (1, -1)$ (output has the first index). At each date and state, consumption is 3 plus any output endowment. Since $p_o(\omega_g) = p_o(\omega_b) = \frac{1}{2}(p_o(\omega_g) + p_o(\omega_b))$ and $p_l(\omega_g) = p_l(\omega_b) = \frac{1}{2}(p_l(\omega_g) + p_l(\omega_b))$, the price rule is satisfied. If a firm tests the technology it gains $p_o(\omega_g) \times 1 = \frac{1}{2}$ in revenue but loses the cost of 1: the decision not to test is profit-maximizing. Due to the externality, that decision is socially inefficient: expected output would increase by $\frac{1}{2} \times 3$ while the output cost of testing is 1. ■

[5] Formally, this discovery can be modeled as the production by some firm j of an additional first-period good d with a technology that produces positive output if and only if at least one unit of labor is applied. So $\mathcal{P}^j(y^j) = \{\{\omega_g\}, \{\omega_b\}\}$ if $y^j_d(1) \geq 1$ and $\mathcal{P}^j(y^j) = \{\Omega\}$ otherwise. Implicitly good d has price 0 below.

3.2 The first welfare theorem

To achieve efficiency we need to assume that information about an agent's utility function or production set is conditionally independent of the information that the remaining agents can discover, given one of the cells that could be revealed in equilibrium. Otherwise, as in Example 5, there would be an externality: one agent's discovery would be valuable to another agent. Conditional independence, unlike unconditional independence, lets agents receive common information, e.g., they all read the same weather report or learn some fact that some agent always uncovers, while requiring that more refined information is independent across agents.

Let $\mathcal{Q}^{-l}$ for $l \in \mathcal{I} \cup \mathcal{J}$ equal the coarsest common refinement of the partitions in

$$\left\{\mathcal{P}^i(x^i) : i \in \mathcal{I} \backslash \{l\} \text{ and } x^i \geq 0\right\} \text{ and } \left\{\mathcal{P}^j(y^j) : j \in \mathcal{J} \backslash \{l\} \text{ and } y^j \in Y^j\right\}.$$

This partition represents the information that all agents besides l can obtain.

For $i \in \mathcal{I}$, let $\mathcal{V}^i$ be the partition of Ω that demarcates the utilities consumer i might have: $W \in \mathcal{V}^i$ if and only if there exists a vNM utility v such that $W = \{\omega \in \Omega : u^i_\omega = v\}$. Similarly, for $j \in \mathcal{J}$, let $\mathcal{Y}^j$ be the partition of Ω that demarcates the single-state production sets firm j might have: $W \in \mathcal{Y}^j$ if and only there exists a $Y \subset \mathbb{R}^{L_1+L_2}$ such that $W = \{\omega \in \Omega : Y^j_\omega = Y\}$.

Definition 3 ***No externalities*** *is satisfied if the realization of an agent's utility function or production set is conditionally independent of the information other agents can obtain, given the information all agents do obtain: for all $x \geq 0$, $y \in \prod_{j\in\mathcal{J}} Y^j$, $P \in \mathcal{P}_{x,y}$, $l \in \mathcal{I} \cup \mathcal{J}$, $W \in \mathcal{V}^l$ for $l \in \mathcal{I}$, $W \in \mathcal{Y}^l$ for $l \in \mathcal{J}$, and $Q \in \mathcal{Q}^{-l}$,*

$$\pi\left(W \cap Q | P\right) = \pi(W|P)\pi(Q|P),$$

and $W \in \mathcal{Y}^l$ and $W \cap Q \cap P \neq \varnothing$ imply $W' \cap Q \cap P \neq \varnothing$ for any $W' \in \mathcal{Y}^l$ such that $W' \cap P \neq \varnothing$.

Under no externalities, what all other agents besides l can learn (beyond $P \in \mathcal{P}_{x,y}$) provides no information about l's utility if l is a consumer or about l's production possibilities if l is a firm. The last part of Definition 3 extends conditional independence to 0-probability events: even if $\pi(W|P) = 0$ or $\pi(Q|P) = 0$, for $W \in \mathcal{Y}^l$ and $Q \in \mathcal{Q}^{-l}$ that intersect P, a switch of the single-state production set from W to W' still defines a non-null event in P (though with probability 0). This requirement needs to be applied only to firms, not consumers, and thus is irrelevant in an exchange economy.

Examples 3 Continued and 4 satisfy no externalities, in both cases because the only agent with a state-dependent utility function or production set is also the only agent that can discover information. That efficiency requires an absence of information externalities is no surprise; the notable fact is that ruling them out is not enough.

An allocation $x \in \mathbb{R}_+^{IL}$ is *feasible* if x^i is $\mathcal{P}_{x,y}$ measurable for each $i \in \mathcal{I}$, there exists a $y \in \mathbb{R}^{JL}$ such that $y^j \in A^j(x, y^{-j})$ for each $j \in \mathcal{J}$ and $\sum_{i \in \mathcal{I}} x^i \leq \sum_{j \in \mathcal{J}} y^j + \sum_{i \in \mathcal{I}} e^i$. An equilibrium (p, x, y) is *Pareto efficient* if there does not exist a feasible allocation x' such $U^i(x^{i\prime}) \geq U(x^i)$ for all $i \in \mathcal{I}$ and with strict inequality for some $i \in \mathcal{I}$.

Theorem 1 *If an equilibrium satisfies no externalities and the competitive price rule then it is Pareto efficient.*

It is easy to confirm that Examples 3 Continued and 4 both have equilibria that satisfy the competitive price rule and which are therefore efficient: the market can induce agents to undertake the optimal information discoveries. In Example 3 Continued, for instance, the firm can earn enough when the technology it is testing has high productivity to make up for its losses when productivity is low.

The proof constructs an artificial economy where agents have to satisfy only those measurability requirements implied by their own information discoveries. We then show that an equilibrium of the true economy is an equilibrium of the artificial economy. If there were a gain in the artificial economy to a consumer i from letting x^i vary with respect to the information uncovered by other agents then by no externalities and the concavity of i's vNM utilities an 'averaged' version of that action that would also be a gain for i in the true economy, and the competitive price rule implies that the averaged action would be affordable in the true economy. Similarly, if there were a gain in the artificial economy for a firm j from letting y^j vary with respect to information uncovered by other agents then due to no externalities j could, in the true economy, choose whichever action from among these new possibilities yields the greatest profits, and the competitive price rule implies that this choice would also lead to an increase in profits in the true economy. The presence of an advantageous deviation in the artificial economy for either a consumer or a firm thus implies that the true economy could not have been in equilibrium. Since, by the first welfare theorem, the equilibrium allocation is Pareto efficient in the artificial economy, it must be Pareto efficient in the true economy as well.[6] Proofs can be found in Appendix B.

3.3 Why are the assumptions so strong?

The assumptions needed for the first welfare theorem are famously weak. The Pareto efficiency of competitive equilibria requires only that preferences are transitive and locally nonsatiated. Weak Pareto efficiency – the absence of an allocation that gives a strict improvement to every consumer – requires no assumptions at all.

Several assumptions in contrast underlie Theorem 1. No externalities and the competitive price rule address points that do not arise in the standard general equilibrium model, namely actions that reveal information and the prices of goods that are not marketed in equilibrium. The need for no externalities is plain: if a firm can discover the productivity of its technology or if a consumer can discover his tastes only when another agent pays the costs then inefficiency will result. The competitive price rule is part of the definition of equilibrium rather than being an assumption on primitives: it specifies the prices of goods at events that are not observed in equilibrium. The price rule, considered in sections 4 and 5, will be our main focus.

Theorem 1 also imposes additional structural assumptions, the concavity of the vNM utilities and implicitly that the agents share a common probability π. Since these conditions are unnecessary for the standard first welfare theorem, their presence requires explanation.

A consumer with a nonconcave vNM utility can prefer a lottery of consumption bundles over receiving the lottery's expected value with certainty. But if the lottery is conducted

[6] An equilibrium of the true economy will not generally be an equilibrium of a different artificial economy that omits all measurability requirements on agents: a consumer i that is not measurability restricted could want to vary his consumption to take advantage of information about his own utility but not be willing to pay to reveal that information.

only if a distinct agent undertakes an information discovery then the first consumer can fail to reap this benefit (or his part of one of the Pareto improvements the lottery can make possible). Example 7 in Appendix A illustrates the inefficiency that can result.

If consumers hold different prior probabilities they will typically be able to achieve a Pareto improvement by betting against each other. But they will not be able to make these bets if the lottery that allows them to do so is conducted only when some other agent undertakes an information discovery. See Example 8 in Appendix A.

Since consumers can have state-dependent utilities, any model where consumers have diverse probabilities is behaviorally identical to a model with common priors and utilities that are rescaled separately by state. This modification would however convert a model that satisfies no externalities – such as Example 8 – to one where it would be violated. While the inefficiency that can accompany diverse probabilities can therefore be diagnosed as a type of externality, the story becomes awkward: one agent's discovery might provide information to others only with respect to a hypothetical probability distribution that none of the agents holds.

Outside of no externalities, Theorem 1 imposes no extra assumptions on firms: the convexity of the Y^j is not needed and, as in the standard general equilibrium model, probabilities do not enter into firm decision-making.

4 Self-confirming equilibria

The striking deviation in Theorem 1 from the classical first welfare theorem is the competitive price rule. One rationale for the price rule is that violations of the rule can lead to equilibria that are self-confirming: the hypothetical prices that agents assign to events that they do not learn in equilibrium would not be borne out if agents could trade all state-contingent goods (they are released from their measurability requirements). These misguided price expectations can in turn validate the equilibrium decision not to discover information. We first return to our earlier examples of inefficiency to underscore the mismatch between the prices agents assign to events and the prices that would rule if all markets were open.

Examples 3 Continued and 4 (revisited) In the equilibrium in Example 3 Continued, the price ratio $\frac{p(\omega_H)}{p(\omega_L)} = \frac{1}{2}$ that agents assign to the goods that appear at the high and low states does not coincide with the ratio given by the competitive price rule, namely $\frac{\pi(\omega_H)}{\pi(\omega_L)} = \frac{2}{1}$. If we could force the state to be revealed – say by requiring the firm to use ε units of the first-period good as an input – the market-determined price ratio for the high and low state goods will equal the ratio of the consumer's marginal utilities, $\frac{2}{1}$, in accord with the price rule. Although the prices that agents assign to states thus do not match the prices that would rule if revelation of the state were forced, the fact that the state is not discovered in equilibrium allows these beliefs to be sustained.

In Example 4, suppose we require consumer 1 to pay the ε revelation cost thus revealing the state. If consumer a is small relative to b (specifically if $2e^a(1) + e^a(\omega_L) \le e^b(\omega_H)$) then the ratio of equilibrium prices $\frac{p(\omega_H)}{p(\omega_L)}$ will be determined by consumer b's ratio of high-state to low-state marginal utility, $\frac{1}{1}$, which is also the ratio given by the competitive price rule.[7] In the equilibrium without forced revelation, in contrast, agents assign a price ratio of $\frac{3}{1}$ to the high-state and low-state goods. ■

The revisited examples are unusual in that the prices enforced by the market if the revelation of the state is forced will *exactly* equal the prices prescribed by the competitive price rule. This precision is exceptional since in most cases the additional information provided by the state will change demand and thus have an impact on prices. In Example 4, for instance, the variation by state of b's demands will affect equilibrium prices when the revelation of the state is mandated and b fails to be small relative to a.

We cannot eliminate the combination of self-confirmation and inefficiency simply by requiring, in the spirit of subgame perfection, that agents' price expectations must coincide with the market equilibrium prices that would obtain as they adjust their information discoveries and demands. That requirement would give agents a strategic, noncompetitive

[7] To find the equilibrium when the state is revealed, set $p(1) = 1$. Since for both a and b the ratio of the marginal utility of first-period consumption to state-invariant second-period consumption equals 1, $p(\omega_H) + p(\omega_L) = p(1) = 1$ must hold in equilibrium. If $p(\omega_H) \le \frac{1}{2}$ then a consumes only $x^a(\omega_H)$ and thus has an excess demand $z^a(\omega_H)$ determined by $p(\omega_H)z^a(\omega_H) = e^a(1) + p(\omega_L)e^a(\omega_L)$. If $p(\omega_H) < \frac{1}{2}$ then $x^b(\omega_L) = 0$ but $x^a(\omega_L) + x^b(\omega_L) = 0$ cannot occur in equilibrium. If $p(\omega_H) > \frac{1}{2}$ then $x^b(\omega_H) = 0$ and $z^a(\omega_H) \le \frac{1}{p(\omega_H)}e^a(1) + \frac{p(\omega_L)}{p(\omega_H)}e^a(\omega_L) < 2e^a(1) + e^a(\omega_L)$. The assumption that $2e^a(1) + e^a(\omega_L) \le e^b(\omega_H)$ thus implies $z^a(\omega_H) < e^b(\omega_H)$ and hence $z^a(\omega_H) + z^b(\omega_H) < 0$, which also cannot occur in equilibrium. Hence $p(\omega_H) = \frac{1}{2}$.

power to manipulate prices and would allow inefficiency to return.

The standard response to the manipulation problem is to argue that the price effect of any deviation from equilibrium actions is small; so even though price-taking assumptions cannot hold literally, they can hold approximately. We follow a similar path by showing that the competitive price rule will be a good predictor of the prices that obtain when revelation of the state is forced if and only if the demand-and-supply effects of revelation are small. The analogy is imperfect – the demand effect of information on prices will normally be substantial while an individual's demand effect in a conventional general equilibrium model will typically be small – and hence in many cases the competitive price rule will not approximate the market prices that would rule if revelation were forced. But the assumption that the demand effect of information is small also serves a theoretical goal: it will allow a full characterization of the price rule.

We first show that if the true state does not convey valuable information beyond what is learned in equilibrium then the predictions of the competitive price rule hold exactly if and only if there is no difference between an equilibrium as previously defined and the equilibrium that would occur if revelation of the state were forced. We then show that if the information content of further information and the costs of discovery are small then the price rule holds approximately if and only if the difference between an equilibrium and a full-revelation equilibrium is small. Costs as well as information content need to be small since no agent will spend resources on information discovery if the state is going to be revealed anyway; large costs would therefore lead to a discrepancy between equilibrium demand and forced-revelation demand.

To concentrate on essentials, we consider exchange economies ($J = 0$) and therefore omit all y's from our notation. Recall that $\mathcal{V}^i$ is the partition of Ω that defines the utilities consumer i might have: $V \in \mathcal{V}^i$ if and only if there exists a vNM utility v such that $V = \{\omega \in \Omega : u^i_\omega = v\}$.

Definition 4 *Information discoveries are* ***conclusive*** *if, for all $x \geq 0$, $P \in \mathcal{P}_x$, $i \in \mathcal{I}$, and $V \in \mathcal{V}^i$, $\pi(V|P)$ equals 0 or 1.*

When information discoveries are conclusive, a $P \in \mathcal{P}_x$ for a potential equilibrium (p, x)

fully reveals the utilities agents have. If information discoveries were not conclusive then revelation of the exact state P would provide some consumer i with utility information and change i's demand, which would typically cause the competitive price rule to fail to hold.

Forcing the revelation of the state translates formally into releasing agents from their measurability requirements. Any result seeking a correspondence between equilibrium outcomes and the competitive price rule has to drop those requirements: if delivery of a good k at ω must be bundled with delivery at ω', no market mechanism could patrol the relative price of k at these states.

Definition 5 *A **full-revelation equilibrium** satisfies all of the requirements of an equilibrium except that, for each $i \in \mathcal{I}$ and $j \in \mathcal{J}$, the measurability conditions in B^i and A^j are omitted.*[8]

When, for each $i \in \mathcal{I}$ and $\omega \in \Omega$, u^i_ω is differentiable and strictly increasing in each good, the economy is *differentiable*. An equilibrium (p, x) (in the sense of Definition 1) is *interior* if $x^i \gg 0$ for each $i \in \mathcal{I}$.

Theorem 2 *For a differentiable exchange economy in which information discoveries are conclusive, any interior equilibrium is a full-revelation equilibrium if and only if the competitive price rule holds.*

So, when information discoveries are conclusive, a violation of the competitive price rule implies that an equilibrium cannot persist undisturbed if agents gain the right to trade for delivery at any state: the equilibrium would have to jump in response. The proof in this direction ('only if') argues that if a full-revelation equilibrium does not satisfy the price rule then there must be a mismatch between the ratio of probabilities for two subevents of some $P \in \mathcal{P}_x$ and the ratio of the prices of goods delivered at those subevents; a shift of consumption to the underpriced subevent would then increase a consumer i's utility at P (i has only one utility function at P due to conclusiveness), violating the assumption of equilibrium. For 'if', a standard equilibrium that satisfies the price rule must also be a full-revelation equilibrium: if a utility improvement were available when we drop the

[8] We will later use full-revelation equilibria when firms are present.

measurability requirements on agents then an 'averaged' version of that improvement would also be an improvement and would satisfy the measurability requirements, a contradiction similar to the one used in the proof of Theorem 1. The equivalence of equilibria and full-revelation equilibria (given conclusiveness) thus provides a complete characterization of the price rule.

To extend Theorem 2 to information discoveries that are less than conclusive, we model the cost of information discovery as a distinct expenditure of goods, $c^i(\mathcal{P}^i) \in \mathbb{R}^L_+$, for any discovery $\mathcal{P}^i$ (a partition of Ω) that i can undertake from a menu of partitions $\mathcal{M}^i$.[9] For equilibria that nearly satisfy the competitive price rule to approximate the full-revelation equilibria, discovery costs must be small: in a full-revelation equilibrium no one will agree to pay any discovery costs.

Given the partitions $\mathcal{P}^{-i}$ and letting $\mathcal{R}_{\mathcal{P}^i,\mathcal{P}^{-i}}$ denote the coarsest common refinement of $\mathcal{P}^i$ and the $\mathcal{P}^{-i}$, consumer i's budget set will now be

$$B^i(p,\mathcal{P}^{-i}) = \left\{ \left(x^i,\mathcal{P}^i\right) \in \mathbb{R}^L_+ \times \mathcal{M}^i : x^i \text{ is } \mathcal{R}_{\mathcal{P}^i,\mathcal{P}^{-i}}\text{-measurable and } p\cdot x^i + p\cdot c^i(\mathcal{P}^i) \leq p\cdot e^i \right\}.$$

Accordingly an equilibrium is now a (p,x) such that there exists $\left(\mathcal{P}^1,...,\mathcal{P}^I\right) \in \prod_{i\in\mathcal{I}}\mathcal{M}^i$ where, for every consumer i, $(x^i,\mathcal{P}^i) \in B^i(p,\mathcal{P}^{-i})$ and $U^i(x^i) \geq U^i(x^{i\prime})$ for each $(x^{i\prime},\mathcal{P}^{i\prime}) \in B^i(p,\mathcal{P}^{-i})$ and where the market-clearing inequality in Definition 1 is replaced by $\sum_{i\in\mathcal{I}}(x^i + c^i(\mathcal{P}^i)) \leq \sum_{i\in\mathcal{I}} e^i$.

If, for each $i \in \mathcal{I}$, u^i differentiably strictly concave and increasing then the economy is *smooth*. Fixing u^i, e^i, and $\mathcal{M}^i$ for each agent $i \in \mathcal{I}$, a *model* $\mathcal{E}$ consists of a cost $c^i(\mathcal{P})$ for each $i \in \mathcal{I}$ and $\mathcal{P} \in \mathcal{M}^i$ and a probability π. For a sequence of models $\mathcal{E}_n$, the *costs of information converge to* 0 if $c^i_n(\mathcal{P}) \to 0$ for each $i \in \mathcal{I}$ and $\mathcal{P} \in \mathcal{M}^i$ and the *inconclusiveness of information converges to* 0 if there is a π, called the *probability identified* by $\mathcal{E}_n$, that satisfies Definition 4 and $\pi_n \to \pi$.

If $\mathcal{E}_n$ is a sequence for a smooth economy such that the inconclusiveness of information converges to 0 then each consumer i has well-defined full-revelation demands $x^i(p)$ when

[9] As with endowments, we should assume, for each $i \in \mathcal{I}$ and $\mathcal{P}^i \in \mathcal{M}^i$, that $c^i(\mathcal{P}^i)$ is measurable with respect to the coarsest common refinement of the partitions in $\left\{\bigwedge \mathcal{P}^i : i \in \mathcal{I}\right\}$.

facing $p \gg 0$ and the probability identified by $\mathcal{E}_n$.[10] A (p,x) is *regular* if $D\sum_{i\in\mathcal{I}} x^i(p)$ has rank $L-1$ and $(p,x)\gg 0$. The rank condition is generically satisfied at equilibria.[11]

Given $\mathcal{E}_n$, the sequence (p_n,x_n) *satisfies the competitive price rule in the limit* if

$$p_{k,n}(\omega) - \pi_n(\omega|P_n(\omega)) \sum_{\omega'\in P_n(\omega)} p_{k,n}(\omega') \to 0$$

for each state ω and good k, where $P_n(\omega)$ indicates the cell of $\mathcal{P}_{x_n}$ that contains ω.

Theorem 3 *If for a sequence of smooth exchange economies the costs and inconclusiveness of information converge to* 0 *and the equilibria* (p_n,x_n) *converge to a regular point then there exist full-revelation equilibria* (p_n^*,x_n^*) *such that the distance between* (p_n^*,x_n^*) *and* (p_n,x_n) *converges to* 0 *if and only if* (p_n,x_n) *satisfies the competitive price rule in the limit.*

So if the costs and inconclusiveness of information are small the equilibria that satisfy the competitive price rule would not be disturbed by much if markets for all state-contingent goods were to open. The competitive-price-rule equilibria thus do not display the suspicious self-confirmation pattern where the hypothetical prices of goods at events that will not be observed lie far from the values that would obtain if markets for goods at those events were operating.

5 The price rule as a positive feature of equilibrium

Theorems 2 and 3 raise the question of whether the competitive price rule will be a necessary property of (standard, not full-revelation) equilibrium when information discoveries are conclusive and costless. A violation of the price rule means that the prices that agents assign to goods at some unobserved event E will be disproportionately low relative to the probability of E, which gives agents an incentive to undertake an information discovery that can distinguish E from its complement. As we will now see, this argument is correct but it does not imply the price rule, which typically requires the coordinated discoveries of many agents.

[10] That is, $x^i(p)$ is the solution to $\max\sum_{\omega\in\Omega}\pi(\omega)u^i_\omega(x^i_\omega)$ s.t. $p\cdot x^i \le p\cdot e^i$, $x^i\ge 0$.
[11] See, e.g., Mas-Colell (1985), chapter 8.

The positive result that a partial price rule must hold in equilibrium requires only a weaker form of conclusiveness. Since the information discoveries we consider might not reveal the exact state, we do not need to assume that agents know their utilities once they learn $P \in \mathcal{P}_{x,y}$. It is enough that any further information beyond P is independent of the information agents receive about their utilities from P.

The information discovery (partition) $\mathcal{D}$ is *conditionally conclusive for consumer i at the equilibrium* (p, x, y) if i's vNM utility and the information i can discover are conditionally independent given i's information in equilibrium: if for all $D \in \mathcal{D}$, $P \in \mathcal{P}_{x,y}$, and $V \in \mathcal{V}^i$,

$$\pi(D \cap V|P) = \pi(D|P)\pi(V|P).$$

The information discovery $\mathcal{D}$ is *costless* for consumer i at equilibrium (p, x, y) if there is a $x^{i\prime} \geq 0$ such that (i) $\mathcal{D} = \mathcal{P}_{x^{i\prime},x^{-i},y}$, (ii) $\mathcal{D}$ refines $\mathcal{P}_{x,y}$, (iii) $p \cdot x^{i\prime} = p \cdot x^i$, and (iv) $U^i(x^{i\prime}) = U^i(x^i)$. So $\mathcal{D}$ is costless to a consumer if paying for the discovery does not lead to a utility loss and the consumer gains information relative to what he knows in equilibrium. As mentioned earlier, the general model of section 2 can accommodate costless information discoveries if we introduce goods that provide no utility with a price of 0. Define consumer i's *utility for k to be differentiable at* $(\overline{p}, \overline{x}, \overline{y})$ if, for each $\omega \in \Omega$, u^i_ω is differentiable and strictly increasing with respect to $x^i_k(\omega)$ and $\overline{x}^i_k(\omega) > 0$.

Theorem 4 *Assume consumer's i utility for k is differentiable at the equilibrium (p, x, y). If $\mathcal{D}$ is conditionally conclusive and costless for consumer i at (p, x, y) then the competitive price rule obtains with respect to $\mathcal{D}$ and good k: for each $P \in \mathcal{P}_{x,y}$ and $D \in \mathcal{D}$ with $D \subset P$,*

$$\sum_{\omega \in D} p_k(\omega) = \pi(D|P) \sum_{\omega \in P} p_k(\omega).$$

The reasoning behind Theorem 4 is similar to the proof of Theorem 2: a violation of the price rule at some event D would induce i to reveal D and buy a little bit more of good k either at D or its complement in P at a disproportionately low price.

Theorem 4 is as far as we can go and even its strong assumptions do not imply the competitive price rule, as the following Example shows. To avoid the inefficiency that accompanies information discovery, we have to impose the price rule: it will not impose itself.

Example 6 Set $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ with probabilities given by:

$\pi(\omega_1) = .1$	$\pi(\omega_2) = .4$
$\pi(\omega_3) = .4$	$\pi(\omega_4) = .1$

.

There are two consumers and no firms, $I = 2$ and $J = 0$. Consumer 1 can costlessly discover the rows above, the cells $\{\omega_1, \omega_2\}$ and $\{\omega_3, \omega_4\}$ while 2 can costlessly discover the columns, $\{\omega_1, \omega_3\}$ and $\{\omega_2, \omega_4\}$.

Assume there is one first-period good (in addition to a useless good that triggers information discoveries) and one second-period good per state. If each consumer i has the vNM utility $u^i_\omega(x^i(1), x^i(\omega)) = x^i(1) + x^i(\omega)$ at state ω then it is an equilibrium action for each agent to consume his endowment at the prices $p(1) = 1$ and $p(\omega) = .25$ for all $\omega \in \Omega$ and to not uncover the information he could discover: the sum of the prices at any event D an agent can discover, $\sum_{\omega \in D} p(\omega)$, equals $\frac{1}{2}$ as does $\pi(D) \sum_{\omega \in \Omega} p(\omega)$. The conclusion of Theorem 4 is therefore satisfied. The competitive price rule in contrast requires that $(p(\omega))_{\omega \in \Omega}$ be proportional to $(\pi(\omega))_{\omega \in \Omega}$. Notice that if at least one of the two agents does undertake discovery then the competitive price rule must hold. ■

The limited enforcement of the price rule given in Theorem 4 applies only to the discoveries of consumers, not firms. Firms do not hold probability judgements and do not maximize an expected value of any random variable: they therefore are not in a position to exploit prices that are not proportional to probabilities. See Example 3 Continued.

6 Conclusion

When agents can discover information, competitive equilibria can be inefficient. Ruling out information externalities will not by itself solve the problem; we need to impose a competitive price rule that the prices agents assign to goods at undiscovered events are proportional to the probabilities of those events. Additional classical assumptions that are usually irrelevant for efficiency – agreement on probabilities and risk aversion – are also required. From the glass-is-half-full perspective, the price rule extends the concept of price-taking to settings where agents can discover information: the rule holds constant the price of increases in the likelihood that a good is received. When the value of information is small, the price rule also has the virtue of avoiding the self-confirmation phenomenon where prices jump from their equilibrium values if markets for goods at unobserved events were to open.

Though several examples have illustrated the existence of equilibrium, this paper has focused on efficiency. For purposes of countering the Schumpeterian view that optimal experimentation with new technologies leads inevitably to market power, it is the compatibility of efficiency and competition that is most relevant. Moreover the existence issues that do arise can be addressed by known tools. For instance, one natural case of information acquisition occurs when agents must pay a discrete cost to make a discovery (Examples 1 and 2). This nonconvexity can block existence since it introduces a discontinuity of demand: agents can respond to a small price change by discretely deciding to start or stop paying the discovery cost. The Starr (1969) approach to existence can tackle this problem: a continuum of agents can bridge the discontinuities by letting a fraction of agents (a continuous variable) pay the discrete discovery cost.

The existence of equilibria is easier to establish when information discoveries are made by firms. As Example 3 indicated, discovery can occur as a byproduct of trying out a technology in the first period or building capital equipment. The conditions that guarantee existence of an Arrow-Debreu equilibrium – convexity, continuity, and positive endowments – will then imply that a full-revelation equilibrium exists.[12] As long as second-period production requires first-period inputs, one can then build a Definition 1 equilibrium from a full-revelation equilibrium. We close with a brief sketch.

Recall that $\mathcal{Y}^j$ is the partition that indicates firm j's single-state production sets.

Definition 6 *If there exists a partition $\mathcal{P}$ of Ω and a $Y_1^j \subset \mathbb{R}^{L_1}$ for each $j \in \mathcal{J}$ such that*

1. *for all $j \in \mathcal{J}$, $y^j(1) \in Y_1^j$ implies $\mathcal{P}^j(y^j)$ is the coarsest common refinement of $\mathcal{Y}^j$ and $\mathcal{P}$ while $y^j(1) \notin Y_1^j$ implies $\mathcal{P}^j(y^j) = \mathcal{P}$ and $y^j(\omega) = 0$ for all $\omega \in \Omega$,*

[12] A full-revelation equilibrium amounts to an Arrow-Debreu equilibrium in which the state is revealed independently of agent actions at the end of the first period.

2. *for all* $i \in \mathcal{I}$ *and* $x^i \geq 0$, $\mathcal{P}^i(x^i) = \mathcal{P}$,

3. *the coarsest common refinement of* $\mathcal{P}$ *and the* $\mathcal{Y}^j$, $j \in \mathcal{J}$, *equals the partition of singletons* $\{\{\omega\} : \omega \in \Omega\}$,

*then **first-period production is fully revealing**.*

The partition $\mathcal{P}$ above indicates the information revealed by nature independently of agent actions. Condition 1 says that each firm j can discover its second-period production possibilities and go on to produce in the second period if and only if it tries out production in the first period by taking an action in Y_1^j. Conditions 2 and 3 say that the information revealed by firms and nature form the whole of the economy's uncertainty.

Proposition 1 *If a full-revelation equilibrium exists and production sets are convex, no externalities is satisfied, and first-period production is fully revealing then an equilibrium exists.*

The proof, which we omit, builds an equilibrium from a full-revelation equilibrium by having each agent, at each cell P of the partition $\mathcal{P}_{x,y}$ that arises in a full-revelation equilibrium (p, x, y), instead take the average of the actions the agent takes at P. As in the proof of Theorem 1, the new actions are feasible given the convexity assumption and deliver the same utility or profits.[13] It may be that some information is not revealed in equilibrium since some firm j may decide not to take an action in Y_1^j. But the assumption that first-period production is fully revealing ensures that this information loss does not hamper firm j: by Definition 6-1, when j fails to take an action in Y_1^j the missing information is useless.

A Appendix: risk aversion and common priors

Example 7 Let there be two consumers a and b and two states in an exchange economy with one consumption good at each date and state: $I = 2$, $J = 0$, $S = 2$ with probabilities $\pi(\omega_1) = \pi(\omega_2) = \frac{1}{2}$. Endowments for both consumers are constant across dates and states,

[13] In effect, this is a full-revelation equilibrium that is constant across sunspots. See Cass and Polemarchakis (1990) for a similar argument.

$e^a = e^b = (1,1,1)$. Consumer a is risk-loving with vNM utility $x^a(1) + (x^a(\omega))^2$ at each state ω while consumer b is risk-neutral with vNM utility $x^b(1) + 2x^b(\omega)$ at each ω. (We omit subscripts on consumptions and prices.) Since the vNM utilities do not vary by state, no externalities is vacuously satisfied.

Consumer b can reveal the state by spending $\varepsilon \geq 0$ of the first-period good on an information-discovery technology.[14] In the following equilibrium, b does not reveal the state – with a strict disincentive if $\varepsilon > 0$ – but it would be socially efficient to do so. The prices are $p = (p(1), p(\omega_1), p(\omega_2)) = (1,1,1)$, which satisfies the competitive price rule, and each agent i consumes $(x^i(1), x^i(\omega_1), x^i(\omega_2)) = (1,1,1)$. But if $\varepsilon \geq 0$ and $\delta > 0$ are sufficiently small, the consumption profile

$$\begin{aligned}(x^a(1), x^a(\omega_1), x^a(\omega_2)) &= (1-\varepsilon-\delta, 2, 0),\\ (x^b(1), x^b(\omega_1), x^b(\omega_2)) &= (1+\delta, 0, 2),\end{aligned}$$

strictly Pareto dominates equilibrium consumption. ■

Example 8 Suppose there are three consumers a, b, and c, and two states in an exchange economy with one good at each date and state: $I = 3$, $J = 0$, $S = 2$, with probabilities now varying by individual and defined by $\pi^a(\omega_1) = \frac{1}{2}$, $\pi^b(\omega_1) = \frac{1}{4}$, $\pi^c(\omega_1) = \frac{3}{4}$. Each consumer i is endowed with one unit of the good at each date and state, $e^i = (1,1,1)$.

Consumer a by devoting $\varepsilon \geq 0$ of the first-period good to a production technology can reveal the state. We build an equilibrium where the consumer does not reveal the state but where it is socially efficient to do so. The price vector will be $p = (p(1), p(\omega_1), p(\omega_2)) = (1, \frac{1}{2}, \frac{1}{2})$ which satisfies the price rule when $\pi(\omega_1) = \frac{1}{2}$. (We again omit the subscripts on prices and consumptions.)

For consumers b and c, set the utility functions so that at p: if the state is revealed, the agent consumes $\frac{3}{2}$ at the state to which the agent assigns the probability $\frac{3}{4}$ and consumes $\frac{1}{2}$ at the state to which the agent assigns probability $\frac{1}{4}$ and if the state is not revealed, the agent consumes 1 unit of each good. That is, $(x^b(1), x^b(\omega_1), x^b(\omega_2)) =$

[14] As in Example 4, discovery must be modeled formally as the purchase of an additional first-period good produced by a firm that uses the original first-period good as its input. See footnote 4.

$(1, \frac{1}{2}, \frac{3}{2})$ and $(x_1^c, x^c(\omega_1), x^c(\omega_2)) = (1, \frac{3}{2}, \frac{1}{2})$ if the state is revealed and $(x_1^b, x^b(\omega_1), x^b(\omega_2)) = (x_1^c, x^c(\omega_1), x^c(\omega_2)) = (1, 1, 1)$ if the state is not revealed. To satisfy these features, let $\widehat{u}$ be differentiable and strictly concave such that $\frac{1}{4}\widehat{u}'(\frac{1}{2}) = \frac{1}{2}\widehat{u}'(1) = \frac{3}{4}\widehat{u}'(\frac{3}{2})$ and assume that each consumer i (including $i = a$) has the expected utility

$$U^i\left(x^i\right) = \widehat{u}(x^i(1)) + \pi^i(\omega_1)\widehat{u}(x^i(\omega_1)) + \pi^i(\omega_2)\widehat{u}(x^i(\omega_2)).$$

As the vNM utilities do not vary by state, no externalities is satisfied regardless of which distribution π^i is used in Definition 3.

If there were no discovery cost, consumer a would consume $(1, 1, 1)$ at p and thus enjoy the same expected utility level (2) whether or not the state is revealed. Thus a has no incentive to reveal the state and has a strict disincentive if $\varepsilon > 0$. But when b and c both consume $(1, 1, 1)$, the ratio of b's marginal utilities for consumption at ω_1 and ω_2, $\frac{\pi^b(\omega_1)}{\pi^b(\omega_2)} = \frac{1}{3}$, does not equal the same ratio for c, $\frac{\pi^c(\omega_1)}{\pi^c(\omega_2)} = 3$. Consequently if the state were revealed a reallocation between b and c of their consumption at ω_1 and ω_2 can increase both agents' expected utility. Hence, when ε is sufficiently small, consumer a could be compensated enough to reveal the state, achieving a Pareto improvement.

Observe that the price rule is satisfied both with respect to π^a, the belief of the consumer who makes the decision to reveal the state, and the average beliefs of the consumers, $\frac{1}{3}\left(\pi^a + \pi^b + \pi^c\right)$. ∎

B Appendix: proofs

Throughout Appendix B, we use the notation $x^i_\omega = (x^i(1), x^i(\omega))$ and $y^j_\omega = (y^j(1), y^j(\omega))$.

Proof of Theorem 1. Let $(\overline{p}, \overline{x}, \overline{y})$ be an equilibrium that satisfies the price rule. We show that $(\overline{p}, \overline{x}, \overline{y})$ is a competitive equilibrium of the following model $\widehat{\mathcal{E}}$. Let each $i \in \mathcal{I}$ have the consumption set

$$\widehat{X}^i = \left\{x^i \in \mathbb{R}^L_+ : x^i \text{ is measurable w.r.t. the coarsest common refinement of } \mathcal{P}^i(x^i) \text{ and } \mathcal{Q}^{-i}\right\},$$

and, given $p \in \mathbb{R}^L_+$ and $y \in \mathbb{R}^{LJ}_+$, the budget set $\widehat{B}^i(p, y) = \{x^i \in \widehat{X}^i : p{\cdot}x^i \leq p{\cdot}e^i + \sum_j \theta^{ij} p{\cdot}y^j\}$. Let each $j \in \mathcal{J}$ have the production set

$$\widehat{Y}^j = \left\{y^j \in Y^j : y^j \text{ is measurable w.r.t. the coarsest common refinement of } \mathcal{P}^j(y^j) \text{ and } \mathcal{Q}^{-j}\right\}.$$

Each $i \in \mathcal{I}$ must choose a $x^i \in \widehat{B}^i(p, y)$ but can violate the further measurability requirements in $B^i(p, x^{-i}, y)$ and each $j \in \mathcal{J}$ must choose a $y^j \in \widehat{Y}^j$ but can violate the further measurability requirements in $A^j(x, y^{-j})$. Otherwise the definition of equilibrium remains unchanged. Given that preferences are utility-representable and hence transitive and our local nonsatiation assumption, the first welfare theorem applies to $\widehat{\mathcal{E}}$ and $\overline{x}$ is therefore Pareto efficient among allocations in

$$\widehat{F} = \left\{ x \in \prod_{i \in \mathcal{I}} \widehat{X}^i : \text{ there exists } y \in \prod_{j \in \mathcal{J}} \widehat{Y}^j \text{ such that } \sum_{i \in \mathcal{I}} \widehat{x}^i \leq \sum_{j \in \mathcal{J}} y^j + \sum_{i \in \mathcal{I}} e^i \right\}.$$

Since $\widehat{F}$ contains the set of feasible allocations for the original model, Pareto efficiency in $\widehat{F}$ implies that $\overline{x}$ is Pareto efficient in the original model.

To conclude that $(\overline{p}, \overline{x}, \overline{y})$ is a competitive equilibrium of $\widehat{\mathcal{E}}$, suppose to the contrary that there is either (1) a $i \in \mathcal{I}$ and $\widehat{x}^i \in \widehat{X}^i$ such that $\overline{p} \cdot \widehat{x}^i \leq \overline{p} \cdot e^i + \sum_{j \in \mathcal{J}} \theta^{ij} \overline{p} \cdot \overline{y}^j$ and $U^i\left(\widehat{x}^i\right) > U^i\left(\overline{x}^i\right)$ (an affordable but possibly non-$\mathcal{P}_{\overline{x},\overline{y}}$-measurable $\widehat{x}^i$ that increases i's utility relative to $\overline{x}^i$), or (2) a $j \in \mathcal{J}$ and $\widehat{y}^j \in \widehat{Y}^j$ such that $\overline{p} \cdot \widehat{y}^j > \overline{p} \cdot \overline{y}^j$ (a feasible but possibly non-$\mathcal{P}_{\overline{x},\overline{y}}$-measurable $\widehat{y}^j$ that increases j's profits relative to $\overline{y}^j$).

For (1), define the $\mathcal{P}_{\widehat{x}^i, \overline{x}^{-i}, \overline{y}}$-measurable $\widetilde{x}^i$ by setting $\widetilde{x}^i(1) = \widehat{x}^i(1)$ and, for each $P \in \mathcal{P}_{\widehat{x}^i, \overline{x}^{-i}, \overline{y}}$ and $\omega \in P$, $\widetilde{x}^i(\omega) = \sum_{\omega' \in P} \pi(\omega'|P) \widehat{x}^i(\omega')$. Since $\widetilde{x}^i$ is $\mathcal{P}_{\widehat{x}^i, \overline{x}^{-i}, \overline{y}}$-measurable, $\widetilde{x}^i \in \widehat{X}^i$. We show that $U^i\left(\widetilde{x}^i\right) \geq U^i\left(\widehat{x}^i\right)$ and $\widetilde{x}^i \in B^i(p, \overline{x}^{-i}, \overline{y})$, thus contradicting the fact that $(\overline{p}, \overline{x}, \overline{y})$ is a competitive equilibrium.

Fix $P \in \mathcal{P}_{\widehat{x}^i, \overline{x}^{-i}, \overline{y}}$ and $V^* \in \mathcal{V}^i$ such that $\pi(V^* \cap P) > 0$. For any $Q \in \mathcal{Q}^{-i}$, the assumption that $\widehat{x}^i \in \widehat{X}^i$ implies there is a $\widehat{x}^i_Q \in \mathbb{R}^{L_1 + L_2}$ such that $\widehat{x}^i_\omega = \widehat{x}^i_Q$ for all $\omega \in Q \cap P$. Hence, for any $\omega \in P$,

$$\widetilde{x}^i(\omega) = \sum_{V \in \mathcal{V}^i} \pi(V|P) \sum_{Q \in \mathcal{Q}^{-i}} \pi(Q|V \cap P) \widehat{x}^i_Q.$$

For all $V \in \mathcal{V}^i$ with $\pi(V \cap P) > 0$ and all $Q \in \mathcal{Q}^{-i}$, no externalities implies $\pi(Q|V \cap P) = \pi(Q|V^* \cap P)$ and thus $\sum_{V \in \mathcal{V}^i} \pi(V|P) \sum_{Q \in \mathcal{Q}^{-i}} \pi(Q|V \cap P) \widehat{x}^i_Q = \sum_{Q \in \mathcal{Q}^{-i}} \pi(Q|V^* \cap P) \widehat{x}^i_Q$. For $x^i \in \mathbb{R}^L_+$, let $v^i(x^i)$ denote the random variable equal to $u^i_\omega(x^i_\omega)$ at $\omega \in \Omega$ and let $u^i_{V^*}$ denote the vNM utility u^i_ω where ω is any state in V^*. Then

$$E[v^i(\widetilde{x}^i)|V^* \cap P] = u^i_{V^*}\left(\sum_{Q \in \mathcal{Q}^{-i}} \pi(Q|V^* \cap P) \widehat{x}^i_Q\right).$$

Since

$$E[v^i(\widehat{x}^i)|V^* \cap P] = \sum_{Q \in \mathcal{Q}^{-i}} \pi(Q|V^* \cap P) u^i_{V^*}(\widehat{x}^i_Q),$$

the concavity of the u^i_ω and Jensen's inequality imply $E[v^i(\widetilde{x}^i)|V^* \cap P] \geq E[v^i(\widehat{x}^i)|V^* \cap P]$. Consequently $U^i\left(\widetilde{x}^i\right) \geq U^i\left(\widehat{x}^i\right)$.

To confirm that $\widetilde{x}^i \in B^i(p, \overline{x}^{-i}, \overline{y})$, fix $P \in \mathcal{P}_{\widehat{x}^i, \overline{x}^{-i}, \overline{y}}$ and a good k. The competitive price rule then implies the third equality below:

$$\sum_{\omega \in P} \overline{p}_k(\omega) \widetilde{x}^i_k(\omega) = \sum_{\omega \in P} \overline{p}_k(\omega) \sum_{\omega' \in P} \pi(\omega'|P) \widehat{x}^i_k(\omega') = \sum_{\omega' \in P} \pi(\omega'|P) \sum_{\omega \in P} \overline{p}_k(\omega) \widehat{x}^i_k(\omega') = \sum_{\omega' \in P} \overline{p}_k(\omega') \widehat{x}^i_k(\omega').$$

Hence $p \cdot \widetilde{x}^i = p \cdot \widehat{x}^i$. Since $\overline{p} \cdot \widehat{x}^i \leq \overline{p} \cdot e^i + \sum_{j \in \mathcal{J}} \theta^{ij} \overline{p} \cdot \overline{y}^j$, $\widetilde{x}^i \in B^i(p, \overline{x}^{-i}, \overline{y})$. Combined with

$U^i(\widetilde{x}^i) \geq U^i(\widehat{x}^i) > U^i(\overline{x}^i)$, this contradicts $\overline{x}^i$ being an equilibrium choice for i.

For (2), the assumption that $\widehat{y}^j \in \widehat{Y}^j$ implies that for each $P \in \mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$ and $Q \in \mathcal{Q}^{-j}$ there is a $\widehat{y}^j_{P\cap Q} \in \mathbb{R}^{L_2}$ such that $\widehat{y}^j(\omega) = \widehat{y}^j_{P\cap Q}$ for $\omega \in P\cap Q$. For each $P \in \mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$, set Q_P to be an element of $\arg\max_Q \sum_{\omega\in P} \overline{p}(\omega)\cdot\widehat{y}^j_{P\cap Q}$ s.t. $Q\cap P \neq \varnothing$, where $\overline{p}(\omega) = (\overline{p}_1(\omega), ..., \overline{p}_{L_2}(\omega))$, and define $\widetilde{y}^j \in \mathbb{R}^L$ by $\widetilde{y}^j(1) = \widehat{y}^j(1)$ and $\widetilde{y}^j(\omega) = \widehat{y}^j_{P(\omega)\cap Q_P}$ for each $\omega \in P$, where $P(\omega)$ denotes the $P \in \mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$ such that $\omega \in P$.

We first show that $\widetilde{y}^j \in A^j(x, y^{-j})$. Since $\widetilde{y}^j$ is $\mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$-measurable, we need to show that $\widetilde{y}^j \in Y^j$. To that end, enumerate the cells in the coarsest common refinement of $\mathcal{Y}^j$ and $\mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$ as $\mathcal{R} = \{R_1, ..., R_n\}$ and let $P(R_m) \in \mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$ satisfy $P(R_m) \supset R_m$. Beginning with $\widehat{y}^j \in Y^j$, change $\widehat{y}^j(\omega)$ to $\widetilde{y}^j(\omega)$ for ω in the sequence of cells $R_1, ..., R_n$ until arriving at $\widetilde{y}^j$. Formally, for $m \in \{0, ..., n\}$, define $y^j(m) \in \mathbb{R}^L$ by

$$y^j_\omega(m) = \begin{cases} \widetilde{y}^j_\omega & \text{if } \omega \in R_l \text{ and } 1 \leq l \leq m, \\ \widehat{y}^j_\omega & \text{otherwise.} \end{cases}$$

To argue by induction that $y^j(n) = \widetilde{y}^j \in Y^j$, note that $y^j(0) \in Y^j$ and suppose $y^j(m-1) \in Y^j$ for some $m \in \{1, ..., n\}$. Recall our assumption that if $Y^j_{\omega'} = Y^j_{\omega''}$ and $y^j \in Y^j$ then $y^{j\prime} \in Y^j$ for the $y^{j\prime}$ defined by $y^{j\prime}_\omega = y^j_\omega$ for $\omega \in \Omega\backslash\{\omega''\}$ and $y^{j\prime}_{\omega''} = y^j_{\omega'}$. Since (i) $Y^j_{\omega'} = Y^j_{\omega''}$ for all $\omega', \omega'' \in R_m$, (ii) $y^j(m-1) \in Y^j$, and (iii) $Q_{P(R_m)} \cap P(R_m) \neq \varnothing$ and no externalities imply that $Q_{P(R_m)} \cap R_m \neq \varnothing$ and hence there is a $\omega \in R_m$ such that $y^j_\omega(m-1) = (\widehat{y}^j(1), \widehat{y}^j_{P(R_m)\cap Q_{P(R_m)}})$, we conclude that $y^j(m) \in Y^j$.

To finish, we show that $\overline{p}\cdot\widetilde{y}^j \geq \overline{p}\cdot\widehat{y}^j$ and therefore $\overline{p}\cdot\widetilde{y}^j > \overline{p}\cdot\overline{y}^j$ which contradicts $\overline{y}^j$ being an equilibrium choice for j. Fix $P \in \mathcal{P}_{\overline{x},\widehat{y}^j,\overline{y}^{-j}}$. For $y^j \in \mathbb{R}^L$, define $\Pi(y^j) = \sum_{\omega\in P} \overline{p}(\omega)\cdot y^j(\omega)$ (the profits earned by y^j at P) and let $Q(\omega)$ denotes the $Q \in \mathcal{Q}^{-j}$ such that $\omega \in Q$. Then $\Pi(\widetilde{y}^j) = \sum_{\omega\in P} \overline{p}(\omega) \cdot \widehat{y}^j_{P\cap Q_P}$ while the competitive price rule implies

$$\Pi(\widehat{y}^j) = \sum_{\omega\in P} \overline{p}(\omega)\cdot\widehat{y}^j_{P\cap Q(\omega)} = \sum_{\omega\in P} \left(\pi(\omega|P) \sum_{\omega'\in P} \overline{p}(\omega')\right)\cdot\widehat{y}^j_{P\cap Q(\omega)} = \sum_{\omega\in P} \pi(\omega|P) \left(\sum_{\omega'\in P} \overline{p}(\omega') \cdot \widehat{y}^j_{P\cap Q(\omega)}\right).$$

Since the definition of Q_P implies $\sum_{\omega\in P} \overline{p}(\omega)\cdot\widehat{y}^j_{P\cap Q_P} \geq \sum_{\omega\in P} \overline{p}(\omega)\cdot\widehat{y}^j_{P\cap Q(\omega)}$, we have $\Pi(\widetilde{y}^j) \geq \Pi(\widehat{y}^j)$ and hence $\overline{p}\cdot\widetilde{y}^j \geq \overline{p}\cdot\widehat{y}^j > \overline{p}\cdot\overline{y}^j$. ■

Proof of Theorem 2. Suppose (p, x) is a full-revelation equilibrium and the competitive price rule fails: there exist $P \in \mathcal{P}_x$, $\omega' \in P$, and a good k such that $p_k(\omega') \neq \pi(\omega'|P) \sum_{\omega\in P} p_k(\omega)$.

Observation 1: for any $\widehat{P} \subset \Omega$, $\pi(\widehat{P}) = 0$ iff $\sum_{\omega\in\widehat{P}} p_{k'}(\omega) = 0$ for each good k'. Proof: if $\pi(\widehat{P}) = 0$ then $x^i \gg 0$ implies $p_{k'}(\omega) = 0$ for each $\omega \in \widehat{P}$, while if $\sum_{\omega\in\widehat{P}} p_{k'}(\omega) = 0$ then the increasingness of the utilities implies $\pi(\widehat{P}) = 0$.

Observation 2: $\pi(P) > 0$. Proof: if $\pi(P) = 0$ then, by Observation 1, $\sum_{\omega\in P} p_k(\omega) = 0$ and $p_k(\omega') = 0$ which imply $p_k(\omega') = \pi(\omega'|P) \sum_{\omega\in P} p_k(\omega)$, a contradiction.

To conclude that there is a $P' \subset P$ such that $\sum_{\omega\in P'} p_k(\omega) < \pi(P'|P) \sum_{\omega\in P} p_k(\omega)$, note that $\{\omega'\}$ can serve as P' if $p_k(\omega') < \pi(\omega'|P) \sum_{\omega\in P} p_k(\omega)$. If $p_k(\omega') > \pi(\omega'|P) \sum_{\omega\in P} p_k(\omega)$ then $\sum_{\omega\in P\backslash\{\omega'\}} p_k(\omega) < \pi(P\backslash\{\omega'\}|P) \sum_{\omega\in P} p_k(\omega)$ and so $P\backslash\{\omega'\}$ can serve as P'.

Next we show that $\sum_{\omega \in P \backslash P'} p_k(\omega) > 0$. If instead $\sum_{\omega \in P \backslash P'} p_k(\omega) = 0$ then, by Observation 1, $\pi(P'|P) = 1$ and hence $\sum_{\omega \in P'} p_k(\omega) = \pi(P'|P) \sum_{\omega \in P} p_k(\omega)$, a contradiction. This fact permits the following definitions.

For $\varepsilon > 0$ and $\omega \in \Omega$, define $\widetilde{x}^i(\varepsilon, \omega) \in \mathbb{R}^{L_2}$ by setting, for each good k',

$$\widetilde{x}^i_{k'}(\varepsilon, \omega) = \begin{cases} x^i_k(\omega) + \varepsilon & \text{if } k' = k \text{ and } \omega \in P', \\ x^i_k(\omega) - \frac{\sum_{\widehat{\omega} \in P'} p_k(\widehat{\omega})}{\sum_{\widehat{\omega} \in P \backslash P'} p_k(\widehat{\omega})} \varepsilon & \text{if } k' = k \text{ and } \omega \in P \backslash P', \\ x^i_{k'}(\omega) & \text{otherwise,} \end{cases}$$

and also $\widetilde{x}^i_\omega(\varepsilon) = (x^i(1), \widetilde{x}^i(\varepsilon, \omega))$ and $\widetilde{x}^i(\varepsilon) = \left(x^i(1), (\widetilde{x}^i(\varepsilon, \omega))_{\omega \in \Omega}\right)$.

Since $\sum_{\omega \in P'} p_k(\omega) < \pi(P'|P) \sum_{\omega \in P} p_k(\omega)$ implies $\sum_{\omega \in P \backslash P'} p_k(\omega) > \pi(P \backslash P'|P) \sum_{\omega \in P} p_k(\omega)$ and therefore $\pi(P') > \pi(P \backslash P') \frac{\sum_{\omega \in P'} p_k(\omega)}{\sum_{\omega \in P \backslash P'} p_k(\omega)}$, we have

$$E\left[\widetilde{x}^i_k(\varepsilon, \cdot)\right] - E\left[x^i_k(\cdot)\right] = \left(\pi(P') - \pi(P \backslash P') \frac{\sum_{\omega \in P'} p_k(\omega)}{\sum_{\omega \in P \backslash P'} p_k(\omega)}\right) \varepsilon > 0.$$

Given our differentiability assumption, Arrow (1965) implies that, for any vNM utility u and all $\varepsilon > 0$ sufficiently small, $\sum_{\omega \in P} \pi(\omega|P) u(\widetilde{x}^i_\omega(\varepsilon)) > \sum_{\omega \in P} \pi(\omega|P) u(x^i_\omega)$.

For $x^i \in \mathbb{R}^L_+$, let $v^i(x^i)$ denote the random variable equal to $u^i_\omega(x^i_\omega)$ at ω; for $V \in \mathcal{V}^i$, let u^i_V denote the vNM utility u^i_ω where ω is any state in V. Since $\pi(V|P)$ equals either 0 or 1 for each $V \in \mathcal{V}^i$ there is one $V \in \mathcal{V}^i$ such that $\pi(V|P) = 1$, which we label V^*. Hence $E[v^i(\widetilde{x}^i(\varepsilon))|P] = \sum_{\omega \in P} \pi(\omega|P) u^i_{V^*}(\widetilde{x}^i_\omega(\varepsilon))$. Since $\sum_{\omega \in P} \pi(\omega|P) u^i_{V^*}(\widetilde{x}^i_\omega(\varepsilon)) > \sum_{\omega \in P} \pi(\omega|P) u^i_{V^*}(x^i_\omega)$, we conclude that

$$E[v^i(\widetilde{x}^i(\varepsilon))|P] > u^i_{V^*}\left(x^i_P\right) = E[v^i(x^i)|P].$$

Since, by Observation 2, $\pi(P) > 0$, $U^i\left(\widetilde{x}^i(\varepsilon)\right) > U^i(x^i)$ for all $\varepsilon > 0$ sufficiently small, and since

$$\sum_{\omega \in P} p_k(\omega) \widetilde{x}^i_k(\varepsilon, \omega) = \sum_{\omega \in P'} p_k(\omega) \left(x^i_k(\omega) + \varepsilon\right) + \sum_{\omega \in P \backslash P'} p_k(\omega) \left(x^i_k(\omega) - \frac{\sum_{\omega \in P'} p_k(\omega)}{\sum_{\omega \in P \backslash P'} p_k(\omega)} \varepsilon\right)$$
$$= \sum_{\omega \in P} p_k(\omega) x^i_k(\omega),$$

$\widetilde{x}^i(\varepsilon) \in B^i(p, x^{-i})$.

Since $\widetilde{x}^i(\varepsilon)$ for any ε sufficiently small is therefore a utility-increasing deviation, (p, x) could not be an equilibrium.

Conversely suppose the competitive price rule holds at the equilibrium (p, x) but (p, x) is not a full-revelation equilibrium: there is a $i \in \mathcal{I}$ and $\widehat{x}^i \geq 0$ such that $U^i(\widehat{x}^i) > U^i(x^i)$ and $p \cdot \widehat{x}^i \leq p \cdot e^i$.

Fix $P \in \mathcal{P}_x$ such that $\pi(P) > 0$, and let V^* be the sole element of $\mathcal{V}^i$ such that $\pi(V^* \cap P) > 0$ and let $u^i_{V^*}$ denote the vNM utility u^i_ω where ω is any state in V^*. Following the proof of Theorem 1, define the $\mathcal{P}_x$-measurable $\widetilde{x}^i$ by setting $\widetilde{x}^i(1) = \widehat{x}^i(1)$ and, for each $P \in \mathcal{P}_x$ and $\omega \in P$, $\widetilde{x}^i(\omega) = \sum_{\omega' \in P} \pi(\omega'|P)\widehat{x}^i(\omega')$. Let $v^i(x^i)$ denote the random variable defined by $v^i(x^i)(\omega) = u^i_\omega(x^i_\omega)$. Since $E[v^i(\widetilde{x}^i)|P] = u^i_{V^*}\left(\sum_{\omega' \in P} \pi(\omega'|P)\widehat{x}^i(\omega')\right)$ and $E[v^i(\widehat{x}^i)|P] = \sum_{\omega' \in P} \pi(\omega'|P) u^i_{V^*}(\widehat{x}^i(\omega'))$, the concavity of the u^i_ω and Jensen's inequality imply $E[v^i(\widetilde{x}^i)|P] \geq E[v^i(\widehat{x}^i)|P]$ and consequently $U^i(\widetilde{x}^i) \geq U^i(\widehat{x}^i)$. Mention budget constraint. As in the proof of Theorem 1, $\sum_{\omega \in P} p_k(\omega)\widetilde{x}^i_k(\omega) \leq \sum_{\omega \in P} p_k(\omega)\widehat{x}^i_k(\omega)$ for each good k and hence $p \cdot \widetilde{x}^i \leq p \cdot e^i$. Thus (p, x) could not be an equilibrium. ■

Proof of Theorem 3. Let $(\overline{p}, \overline{x})$ denote the regular point to which the equilibria (p_n, x_n) converge, let $\overline{\pi}$ denote the probabilities to which π_n converges, and let $\left(\mathcal{P}^1_n, ..., \mathcal{P}^I_n\right) \in \prod_{j \in \mathcal{I}} \mathcal{M}^j$ be the partitions chosen at equilibrium (p_n, x_n). Given the finiteness of Ω, there must be a $\left(\mathcal{P}^1, ..., \mathcal{P}^I\right)$ and a subsequence of positive integers $\langle n' \rangle$ such that $\left(\mathcal{P}^1, ..., \mathcal{P}^I\right) = \left(\mathcal{P}^1_n, ..., \mathcal{P}^I_n\right)$ for all n' in the subsequence. Let $\mathcal{R}$ denote the coarsest common refinement of the $\mathcal{P}^j$.

Note that since $(\overline{p}, \overline{x}) \gg 0$ our smoothness assumption implies $\overline{\pi} \gg 0$.

If. Suppose (p_n, x_n) satisfies the competitive price rule in the limit. We begin by showing that $(\overline{p}, \overline{x})$ is a full-revelation equilibrium when probabilities equal $\overline{\pi}$. Suppose to the contrary that there is a $i \in \mathcal{I}$ and $\widehat{x}^i \geq 0$ such that $\overline{p} \cdot \widehat{x}^i \leq \overline{p} \cdot e^i$ and $\sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\widehat{x}^i_\omega) > \sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\overline{x}^i_\omega)$. Define the $\mathcal{R}$-measurable $\widetilde{x}^i$ by setting $\widetilde{x}^i(1) = \widehat{x}^i(1)$ and, for each $P \in \mathcal{R}$ and $\omega \in P$, $\widetilde{x}^i(\omega) = \sum_{\omega' \in P} \overline{\pi}(\omega'|P)\widehat{x}^i(\omega')$. Since $p_{k,n'}(\omega) - \pi_{n'}(\omega|P) \sum_{\omega' \in P} p_{k,n'}(\omega') \to 0$ as $n' \to \infty$ for each good k and $\overline{\pi} \gg 0$, $P \in \mathcal{R}$, and $\omega \in P$, we have $\overline{p}_k(\omega) = \overline{\pi}(\omega|P) \sum_{\omega' \in P} \overline{p}_k(\omega')$. As in the proofs of Theorem 1 and 2, $\overline{p} \cdot \widetilde{x}^i = \overline{p} \cdot \widehat{x}^i$ and

$$\sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\widetilde{x}^i_\omega) \geq \sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\widehat{x}^i_\omega) > \sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\overline{x}^i_\omega).$$

Next define, for each n' in the subsequence, the $\mathcal{P}$-measurable $\widetilde{x}^i_{n'}$ by setting $\widetilde{x}^i_{n'}(1) = \widetilde{x}^i(1) - c^i_{n'}(\mathcal{P}^i)$ and $\widetilde{x}^i_{n'}(\omega) = \widetilde{x}^i(\omega)$ for each $\omega \in \Omega$. Since $\pi_{n'} \to \overline{\pi}$, $c^i_{n'}(\mathcal{P}^i) \to 0$, $(p_n, x_n) \to (\overline{p}, \overline{x})$, and the u^i_ω are continuous,

$$\begin{aligned}
\sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(\widetilde{x}^i_{n'}(1), \widetilde{x}^i_{n'}(\omega)) &\to \sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\widetilde{x}^i(1), \widetilde{x}^i(\omega)), \\
\sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(x^i_{n'}(1), x^i_{n'}(\omega)) &\to \sum_{\omega \in \Omega} \overline{\pi}(\omega) u^i_\omega(\overline{x}^i(1), \overline{x}^i_\omega), \\
p_{n'} \cdot \widetilde{x}^i_{n'} &\to \overline{p} \cdot \widetilde{x}^i.
\end{aligned}$$

Thus $p_{n'} \cdot \widetilde{x}^i_{n'} \to \overline{p} \cdot \widehat{x}^i$ and hence $p_{n'} \cdot \widetilde{x}^i_{n'} + p_{n'} \cdot c^i_{n'}(\mathcal{P}^i) - p_{n'} \cdot e^i \to 0$ and there exists a $\varepsilon > 0$ such that, for all n' sufficiently large,

$$\sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(\widetilde{x}^i_{n'}(1), \widetilde{x}^i_{n'}(\omega)) \geq \sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(x^i_{n'}(1), x^i_{n'}(\omega)) + \varepsilon.$$

There is consequently a $\delta > 0$ such that

$$\sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(\widetilde{x}^i_{n'}(1) - (\delta, ..., \delta), \widetilde{x}^i_{n'}(\omega)) > \sum_{\omega \in \Omega} \pi_{n'}(\omega) u^i_\omega(x^i_{n'}(1), x^i_{n'}(\omega))$$

and $p_{n'} \cdot (\widetilde{x}^i_{n'}(1) - (\delta, ..., \delta), \widetilde{x}^i_{n'}(\omega)) \leq p_{n'} \cdot e^i$ for all n' sufficiently large, which contradicts the assumption that each (p_n, x_n) is an equilibrium. Thus $(\overline{p}, \overline{x})$ is a full-revelation equilibrium when probabilities are $\overline{\pi}$.

Let $x^i(p, \pi)$ denote agent i's full-revelation demand as a function of p and π. Given that u^i is differentiably strictly concave and weakly increasing, u^i is strictly increasing which combined with concavity implies $DU^i(x^i) \gg 0$ for any $x^i \geq 0$. Using this fact and the differentiably strict concavity of u^i, a standard application of the implicit function theorem to i's optimization problem implies that $x^i(\cdot)$ is continuously differentiable. Since $(\overline{p}, \overline{x})$ is a full-revelation equilibrium, $p = \overline{p}$ is a solution of $x^i(p, \overline{\pi}) = \sum_{i \in \mathcal{I}} e^i$. Since $(\overline{p}, \overline{x})$ is regular, the implicit function theorem implies that if $\pi_n \to \overline{\pi}$ then for all n sufficiently large $x^i(p, \pi_n) = \sum_{i \in \mathcal{I}} e^i$ has a solution $p = p^*_n$ such that $p^*_n \to \overline{p}$, and so $\left(p^*_n, (x^i(p^*_n, \pi_n))_{i \in \mathcal{I}}\right)$ provides the desired sequence of full-revelation equilibria.

Only if. Suppose there is a full-revelation equilibrium (p^*_n, x^*_n) for each $\mathcal{E}_n$ such that $(p^*_n, x^*_n) - (p_n, x_n) \to 0$ and that, for some good k and state ω',

$$p_{k,n}(\omega') - \pi_n(\omega' | P_n(\omega')) \sum_{\omega \in P_n(\omega)} p_{k,n}(\omega)$$

fails to converge to 0. Taking a further subsequence of $\langle n' \rangle$ if necessary, there must be a $P \in \mathcal{R}$ and $a \neq 0$ such that $P_{n'}(\omega') = P$ for all n' and $p_{k,n'}(\omega') - \pi_{n'}(\omega' | P)) \sum_{\omega \in P} p_{k,n'}(\omega) \to a$. Therefore $\overline{p}_k(\omega') \neq \overline{\pi}(\omega' | P) \sum_{\omega \in P} \overline{p}_k(\omega)$.

We now follow the proof of Theorem 2 and its notation except that $\overline{x}^i_{k'}(\omega)$ replaces $x^i_{k'}(\omega)$ in the definition of $\widetilde{x}^i(\varepsilon, \omega)$ and E_n (resp. U^i_n) and $E_{\overline{\pi}}$ (resp. $U^i_{\overline{\pi}}$) indicate expectations (resp. expected utilities) calculated using π_n and $\overline{\pi}$ respectively. Since there exists a $P' \subset P$ such that $\sum_{\omega \in P'} \overline{p}_k(\omega) < \overline{\pi}(P' | P) \sum_{\omega \in P} \overline{p}_k(\omega)$, $E_{\overline{\pi}}\left[\widetilde{x}^i_k(\varepsilon, \cdot)\right] - E_{\overline{\pi}}\left[\overline{x}^i_k(\cdot)\right] > 0$. Hence for any vNM utility u and all $\varepsilon > 0$ sufficiently small, $\sum_{\omega \in P} \overline{\pi}(\omega | P) u(\widetilde{x}^i_\omega(\varepsilon)) > \sum_{\omega \in P} \overline{\pi}(\omega | P) u(\overline{x}^i_\omega)$. Since the inconclusiveness of information converges to 0, there is a $V^* \in \mathcal{V}^i$ such that $\overline{\pi}(V^* | P) = 1$. Hence $E_{\overline{\pi}}[v^i(\widetilde{x}^i(\varepsilon)) | P] = \sum_{\omega \in P} \overline{\pi}(\omega | P) u^i_{V^*}(\widetilde{x}^i_\omega(\varepsilon))$ and therefore

$$E_{\overline{\pi}}[v^i(\widetilde{x}^i(\varepsilon)) | P] > u^i_{V^*}\left(x^i_P\right) = E_{\overline{\pi}}[v^i(x^i) | P].$$

Since $E_n[v^i(\widetilde{x}^i(\varepsilon)) | P] \to E_{\overline{\pi}}[v^i(\widetilde{x}^i(\varepsilon)) | P]$ and $E_n[v^i(\overline{x}^i) | P] \to E_{\overline{\pi}}[v^i(\overline{x}^i) | P]$, we conclude that $E_n[v^i(\widetilde{x}^i(\varepsilon)) | P] > E_{\overline{\pi}}[v^i(\overline{x}^i) | P]$ for all large n. Hence $U^i_n(\widetilde{x}^i(\varepsilon)) > U^i_n(\overline{x}^i)$ for large n. Since $\widetilde{x}^i(\varepsilon) \in B^i(p_n, x^{-i}_n)$, (p_n, x_n) could not be an equilibrium for large n. ■

Proof of Theorem 4. Suppose that the partition $\mathcal{D}$ is conditionally conclusive and costless for consumer i at an equilibrium (p, x, y), $P \in \mathcal{P}_{x,y}$, $D \in \mathcal{D}$ with $D \subset P$, and there is a good k such that $\sum_{\omega \in D} p_k(\omega) \neq \pi(D | P) \sum_{\omega \in P} p_k(\omega)$ and $x^i_k(\omega) > 0$ for $\omega \in P$.

Following the proof of Theorem 2, $\pi(D) > 0$ and there is no loss in generality in assuming $\sum_{\omega \in D} p_k(\omega) < \pi(D | P) \sum_{\omega \in P} p_k(\omega)$. With $D = P'$, let $\widetilde{x}^i(\varepsilon, \omega)$, $\widetilde{x}^i_\omega(\varepsilon)$, $\widetilde{x}^i(\varepsilon)$, the random variable $v^i(x^i)$, and the vNM utility u^i_V assume their earlier definitions. Then

$$E\left[\widetilde{x}^i_k(\varepsilon, \cdot)\right] - E\left[x^i_k(\cdot)\right] = \left(\pi(D) - \pi(P \backslash D) \frac{\sum_{\omega \in D} p_k(\omega)}{\sum_{\omega \in P \backslash D} p_k(\omega)}\right) \varepsilon > 0$$

and hence, for any vNM utility u and all $\varepsilon > 0$ sufficiently small, $\sum_{\omega \in P} \pi(\omega|P)u(\widetilde{x}^i_\omega(\varepsilon)) > \sum_{\omega \in P} \pi(\omega|P)u(x^i_\omega)$. Letting $\widetilde{x}^i_D(\varepsilon)$ (resp. $\widetilde{x}^i_{P\backslash D}$) denote $\widetilde{x}^i_\omega(\varepsilon)$ for $\omega \in D$ (resp. $\omega \in P\backslash D$), we therefore have $\pi(D|P)u\left(\widetilde{x}^i_D(\varepsilon)\right)+\pi(P\backslash D|P)u\left(\widetilde{x}^i_{P\backslash D}(\varepsilon)\right) > u\left(x^i_P\right)$ for small $\varepsilon > 0$. Using this fact for the inequality, the fact that $\mathcal{D}$ refines $\mathcal{P}$ for the first equality, and the conditional conclusiveness of $\mathcal{D}$ for the third equality, we have, for all $\varepsilon > 0$ sufficiently small,

$$\begin{aligned}
E[v^i(\widetilde{x}^i(\varepsilon))|P] &= \pi(D|P)\sum_{V\in\mathcal{V}^i}\pi(V|D)u^i_V\left(\widetilde{x}^i_D(\varepsilon)\right) + \pi(P\backslash D|P)\sum_{V\in\mathcal{V}^i}\pi(V|P\backslash D)u^i_V\left(\widetilde{x}^i_{P\backslash D}(\varepsilon)\right) \\
&= \sum_{V\in\mathcal{V}^i}\left(\pi(V|D)\pi(D|P)u^i_V\left(\widetilde{x}^i_D(\varepsilon)\right) + \pi(V|P\backslash D)\pi(P\backslash D|P)u^i_V\left(\widetilde{x}^i_{P\backslash D}(\varepsilon)\right)\right) \\
&= \sum_{V\in\mathcal{V}^i}\pi(V|P)\left(\pi(D|P)u^i_V\left(\widetilde{x}^i_D(\varepsilon)\right) + \pi(P\backslash D|P)u^i_V\left(\widetilde{x}^i_{P\backslash D}(\varepsilon)\right)\right) \\
&> \sum_{V\in\mathcal{V}^i}\pi(V|P)u^i_V\left(x^i_P\right) = E[v^i(x^i)|P].
\end{aligned}$$

Since $\pi(D) > 0$ and hence $\pi(P) > 0$, $U^i\left(\widetilde{x}^i(\varepsilon)\right) > U^i(x^i)$ for all $\varepsilon > 0$ sufficiently small, and since

$$\begin{aligned}
\sum_{\omega\in P} p_k(\omega)\widetilde{x}^i_k(\varepsilon,\omega) &= \sum_{\omega\in D} p_k(\omega)\left(x^i_k(\omega)+\varepsilon\right) + \sum_{\omega\in P\backslash D} p_k(\omega)\left(x^i_k(\omega) - \frac{\sum_{\omega\in D} p_k(\omega)}{\sum_{\omega\in P\backslash D} p_k(\omega)}\varepsilon\right) \\
&= \sum_{\omega\in P} p_k(\omega)x^i_k(\omega),
\end{aligned}$$

and $\mathcal{D}$ is costless, $\widetilde{x}^i(\varepsilon) \in B^i(p, x^{-i}, y)$.

Since $\widetilde{x}^i(\varepsilon)$ for any ε sufficiently small is therefore a utility-increasing deviation, (p, x, y) could not be an equilibrium. ■

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