Currency Union with or without Banking Union*

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Abstract

This paper builds a symmetric two-country monetary model with credit to study the interplay between currency arrangements and credit market integration. We show that the currency regime impacts on credit availability through changes in default incentives. We capture differences in credit market integration by variations in the cost for agents to obtain credit for cross-border transactions. With the euro area context in mind, we label as banking union a situation of low enough cross-border cost. For high enough levels of this cost, currency integration may magnify default incentives, leading to more stringent credit rationing and lower welfare than in a regime of two currencies. The integration of credit markets restores the optimality of the currency union.

Keywords: currency union, monetary union, cross-border credit, default.

J.E.L. codes: E42, E50, F3, G21

1 Introduction

This paper constructs a model to analyze whether the desirability of a currency union depends on the degree of credit market integration across state borders.

The unification of banking markets is an overlooked issue of academic discussions on the design of monetary unions. This stands in contrast with historical experience. In the two prominent examples of monetary unions—the U.S. dollar and the euro—the initial design defined common rules governing the legal tender and endowed a single organization with the right to issue currency. Bank regulation and supervision originally remained in the state domain. Both unions ended up fostering greater credit market integration across states and devolved most of the banking regulation and supervision to the federal authorities.[1] Within the European Union today, this policy agenda is being implemented under the label "banking union".[2] This paper

[1] In 19th century U.S. periodic systemic banking crises triggered political discussions questioning the organization of monetary issuance (Rousseau, 2013). Differences in regulatory frameworks during the National Banking Era of 1865–1913 caused credit market distortions that "stimulated the public to press for currency and banking reform" (White, 1982).

[2] James (2012) documents that integration of banking markets was considered as an important dimension in the discussions of monetary integration in Europe as early as the 1970s. Political

suggests that these policy initiatives did not occur by chance, but instead that in a low inflation environment, credit market integration across states is a requisite to reap the gains of a unique currency.

The issue is topical today in the euro area where credit markets remain imperfectly integrated along states borders. The cross-border financing of firms and households represents a small fraction of total financing to non-bank entities. For example, the share of cross-border bank lending to non-bank entities across member states has varied between 3% and 6% since the creation of the euro. Recent policy discussions on the sustainability of the European monetary union have revealed that there is no consensus on whether more integration of credit and financial markets occurring through an increase of cross-border lending would be beneficial to the performance of euro area economies. Federal institutions—the ECB and the European commission—have supported policies fostering integration of those markets, including retail finance, in order to complete monetary unification. In the words of the ECB President M. Draghi, the insufficient credit market integration in the euro area is related to *"hidden barriers to cross-border activity linked to national preferences"* (Draghi, 2014b).[3] A contrasting standpoint in the policy debate defends instead greater credit market segmentation across member states with the view that currency arrangements and financial market structures are to a large extent two independent matters (Cerutti, Ilyina, Makarova, and Schmieder, 2010).

To analyze the interplay between currency and credit market integration, we develop a symmetric two-country model with currency and bank credit. Agents are entrepreneurs who alternate between buying inputs and selling goods. They use currency to purchase inputs. Bank credit provides insurance against individual productivity shocks that cannot be efficiently insured by cash holdings. Entrepreneurs can borrow from banks to relax their cash constraint. By lending out all the cash

support eventually came up following the European sovereign debt crisis.

[3]See also Constâncio (2014). The trend towards the ring-fencing of banking activities at the state level has been reversed by the devolution of the supervision of banks to the ECB in November 2014. In this respect, a stated objective of the ECB is that *"a Spanish firm should be able to borrow from a Spanish bank at the same price at which it would borrow from a Dutch bank"* (Draghi, 2013).

received in deposits, banks effectively redistribute cash according to agents' current transaction needs. Lending is potentially limited by the fact that agents cannot commit to repay their debt. Banks impose borrowing limits and use the threat of exclusion from future access to the banking system to sustain debt repayment.

An entrepreneur produces with either foreign or domestic input. Her production function is such that she is sometimes more productive using the foreign rather than the domestic input. But the cost of credit may be more expensive for cross-border purchases. In choosing which input to use, she thus faces a trade-off between the efficiency gain of purchasing abroad versus the higher cost of debt to finance foreign purchases. Throughout the paper we refer to this extra cost as the cross-border credit premium. The introduction of this premium is intended to capture in a stylized manner various institutional frictions that may plague the efficiency of the use of cross-border credit—such as the cost of cross-jurisdiction collateral seizure, the inter-operability cost of using multi-platform payment systems or instruments, or the cost of sharing information on borrowers' creditworthiness.[4] The combination of those costs jointly determine the degree of integration of inter-state credit markets. In turn, the lack of integration of cross-border credit limits cross-border trade and ultimately triggers a home bias in spending decisions.

To evaluate the interplay between the lack of credit market integration and the monetary regime, we compare two monetary arrangements: a single currency regime, and a 'one country-one currency' regime. The only difference between the two regimes lies in the costs of converting currencies, and we ask whether the case with strictly positive conversion costs is dominated in terms of welfare by a currency union. We embed those features into the framework developed by Lagos and Wright (2005), Rocheteau and Wright (2005) and Berentsen, Camera, and Waller (2007), but we believe that the result would go through in other settings. Given our emphasis on monetary versus credit integration, our modelling approach has

[4]The level of cross-border credit to non-financial agents may also depend on other factors, such as the knowledge of specifics of local markets, the role of relationship-based information, the degree of harmonization across state bankruptcy legal procedures, and the automaticity of enforcement of cross-border foreclosure procedures.

the advantage of allowing for a precise distinction between money and credit.

Our analysis delivers two sets of results.

The first set of results defines the conditions for the optimality of a currency union. We show that with sufficiently integrated credit markets, a unique currency is always optimal. When the cross-border credit markets are imperfectly integrated, a unique currency is optimal if the borrowing constraint is not binding, which occurs when inflation is high enough. A regime of separate currencies may be preferred for a low level of credit market integration if the borrowing constraint is binding, which occurs if inflation is low enough.

The intuition for potential welfare gains associated with a breakup of a monetary union is as follows. When credit market integration is imperfect, the reduction in conversion costs associated with a single currency may *worsen* default incentives on bank loans. Given the cross-border credit premium, the wedge between the cost of financing foreign versus domestic purchases induces borrowers—agents with no record of default—to be biased towards domestic goods. Instead agents who have defaulted and lost access to credit—something that does not happen on the equilibrium path—are not impacted by the cross-border credit premium. Unlike agents with access to credit, agents who have defaulted are not home biased since they make their purchase decisions solely based on inputs' relative productivities. Therefore, positive conversion costs can make default less attractive, as they affect defaulters more severely than non-defaulters, thereby relaxing borrowing constraints and allowing for a higher amount of credit in equilibrium. By contrast, when financing conditions are the same for domestic and cross-border purchases, there is no home bias, and a conversion cost between currencies does not attenuate default incentives.

The second set of results characterizes how credit varies with the cross-border premium when there is credit rationing. We first show that for both monetary regimes, the volume of credit is monotonically decreasing in the cross-border credit premium. The logic is that a higher cross-border credit premium reduces the value of maintaining future access to bank credit. This negatively impacts repayment incentives, and results in a lower volume of credit. We then investigate how the

premium has a differential impact across monetary regimes. We show that credit crunches—defined as a reduction in the quantity of credit caused by a substantial increase in the cross-border credit premium—are sharper in a currency union than in a regime of separate currencies. The intuition is that by inducing a sufficiently strong increase in home bias, an increase in the cross-border credit premium can trigger the positive effect of conversion costs on repayment incentives, which ultimately outweighs the negative impact of conversion costs on trade.

These results have implications for the current policy debate on the architecture of the euro area. Our focus on stationary equilibrium highlights the long-term (structural) ingredients needed for a sustainable currency union, and independently of the design of the tools tailored to cope with financial crises. The policy agenda of the European Commission aims at deepening credit market integration, and is negotiated under the headings "banking union" for banking matters and "capital market union" for direct finance matters, see Valiante (2016). The model suggests that in a low inflation economy deeper banking and capital market integration across member states improves the efficiency of the currency union by reducing the incentives to default on credit and thereby supporting both a higher level of credit and welfare.

1.1 Credit market integration in the euro area

Our paper is partly motivated by the situation of the euro area characterized by a level of cross-border credit integration that has varied substantially over the last 30 years. European credit markets were segmented along state borders before the creation of the euro. With the prospects of the creation of the monetary union in 1999, various policy initiatives were taken during the 1990s to promote a single European financial and credit market (James, 2012, ECB, 2007, 2012). As a result markets became more integrated and credit activity increased (Allen, Beck, Carletti, Lane, Schoenmaker, and Wagner, 2011). The money market and the government bonds market became fully integrated, and the degree of integration of corporate bonds and equity markets across states also increased (De Haan, Oosterloo, and Schoenmaker, 2009). European banks opened or purchased subsidiaries and branches in

other European states (Claessens and van Horen, 2012). A dramatic reversal in cross-border banking activity in the aftermath of the subprime crisis has been documented (Milesi-Ferreti and Tille, 2011, Manna, 2011).

By contrast, the financing of households and small and medium enterprises, highly reliant on bank credit, has always remained segmented across member states since the creation of the euro (Kleimeier and Sander, 2007, ECB, 2008, Gropp and Kashyap, 2009). As the European Central Bank asserts, "*cross-border banking through branches or subsidiaries has remained limited*"(ECB, 2012, p.90-91). ECB President M. Draghi stated that "*integration [in the euro area] was largely based on short term interbank debt rather than on equity or direct cross-border lending to firms and households*" (Draghi, 2014a).

Several institutional features make it ultimately more difficult for borrowers to obtain cross-border credit in the euro area. Differences in debt recovery and foreclosure procedures with no automatic judicial cooperation among states hinder cross-border credit. The diversity in standards for property valuation, tax systems and even languages across member states also limits the provision of credit across the borders (Allen et al., 2011, European Commission, 2014b,a). Finally, the extension of cross-border credit has been constrained by supervisory and regulatory policies at the state level.[5] Although the creation of the euro was accompanied by EU initiatives to reduce barriers to the inter-state exchange of financial services, the timing of the transposition of the EU directives reflected each state's preference towards cross-border financial integration (Kalemli-Ozcan, Papaioannou, and Peydró, 2010). While between 1999 and 2014 banks' supervision remained a state-level prerogative in the euro area, during the financial crisis some policies of supervisors could have encouraged the fragmentation of local credit markets (Gros, 2012). The existence of country-specific financial safety nets is also found to act as a barrier to cross-border banking (Bertay, Demirguç-Kunt, and Huizinga, 2017).

[5]See Aglietta and Scialom (2003) for a discussion related to the euro area supervisory authorities and Houston, Lin, and Ma (2012) for an empirical investigation showing that banks' activity is influenced by the regulatory environment.

Limits to cross-border information circulation also contribute to the low level of cross-border credit to non-financial entities within the euro area. Creditors' access to information on non-resident borrowers remains limited despite regulatory initiatives taken by the European Commission to ensure non-discriminatory access to credit data.[6] While data on debtors is reported at the state level to credit registers operated by central banks and to private credit bureaus, cross-border information sharing occurs only among a subset of public credit registers and mainly on legal persons. In addition, the lack of harmonization among states both on the type of information reported and on the standards for processing it hampers the use of credit information by foreign creditors (Jentzsch, 2007). As a result, for borrowers it is difficult to obtain credit in a member state in which they have no credit history. The informational disadvantage of foreign creditors within the euro area has also negatively affected their entry through branches into other member states (Giannetti, Jentzsch, and Spagnolo, 2009).

1.2 Related literature

Our paper contributes to the macroeconomic literature on the costs and benefits of monetary unions. This literature has surprisingly overlooked the issue of credit markets integration.[7] To underscore our contribution, our framework deliberately sets aside several dimensions previously emphasized. First, we abstract from any source of heterogeneity or asymmetric shocks across countries to make the point that sustainable currency unions require integrated cross-border credit markets, independently of cross-country risk-sharing considerations (Mundell, 1961, Kenen, 1969, Cooper and Kempf, 2004, Gali and Monacelli, 2008). From that perspective, our analysis suggests that fiscal integration within a currency union does not obviate

[6] See for instance European Commission (2014b). Jentzsch and San José Rientra (2003) report that EU banks have less access to cross-border information on their EU customers than U.S. banks have on their customers across U.S. states. Jentzsch (2007) discusses discriminatory rules on cross-border data exchange adopted by EU countries to limit competition within their jurisdictions.

[7] For instance, in the survey by Beetsma and Giuliodori (2010), financial integration is only discussed with respect to sovereign debt markets integration.

the need for financial integration.[8]

Second, our results are unrelated to interactions between fiscal and monetary policies—another common theme in the literature. Several papers discuss the need for fiscal constraints or coordination in a currency union to contain the risk of monetary financing of the fiscal deficits of (sub)national governments when public authorities lack commitment (Chari and Kehoe, 2007, Cooper, Kempf, and Peled, 2010).[9] Other papers argue that a currency union may be unsustainable by preventing over-indebted governments from reducing their real debt-burden through inflation and currency devaluation (De Grauwe, 2013, Sims, 2013). We have no role for fiscal policy and consider monetary authorities fully committed to a given inflation rate, and instead focus on how default incentives of private borrowers threaten the sustainability of a currency union.

We also contribute to monetary theory by suggesting a new rationale for the optimality of multiple currencies *vis-à-vis* a unique currency. While a unique currency reduces transaction costs, multiple currencies may lead to welfare gains by inducing a greater division of labour (Kiyotaki and Moore, 2003), or by serving as a mechanism to signal agents' preferred basket of consumption goods (Kocherlakota and Krueger, 1999) or money holdings (Kocherlakota, 2002). We show that with limited credit integration, higher transaction costs in cross-border trades can turn into a benefit—rather than a cost—of multiple currencies by mitigating default incentives.

The rest of the paper is organized as follows. The environment is laid out in Section 2. The conditions for the existence of equilibria are presented in Section 3. Section 4 presents the results pertaining to the welfare implications of a regime of unique versus multiple currencies for different degrees of credit market integration. Section 5 concludes.

[8]By contrast, financial integration and fiscal integration substitute one another from a risk-sharing perspective. Gros and Belke (2015) argue that in the U.S. the fraction of regional financial shocks that is absorbed through the fiscal union is small compared to that absorbed through the banking union dimension.

[9]Relatedly, it has been argued that countries lacking internal discipline can attain monetary stability by joining a currency with a low inflation anchor country (Alesina and Barro, 2002).

2 Environment

Time is discrete and continues forever. The economy is composed of two symmetric countries labeled H and F, each populated by a unit-mass continuum of infinitely-lived agents who discount utility across periods with factor β. For the sake of interpretation, we refer to these agents as entrepreneurs. In every period, two competitive markets open sequentially in each country. Most action—in particular cross-border trades—happens in the first market. The second market allows agents to rebalance their money holdings before the next period.

In the first market of country H (F), agents from both countries trade good h (f) against currency. Goods h and f are perfectly divisible and non-storable. Entrepreneurs randomly alternate between being suppliers (with probability s) or purchasers (with probability $1-s$) across periods. Suppliers can produce goods and do not derive utility from consumption in the first market. Purchasers can use the goods produced by suppliers as inputs to produce output for their own consumption. For the ease of exposition, in what follows we describe what happens to entrepreneurs from country H.[10] When they are suppliers, they stay in country H and produce a quantity q_s^H of good h at cost $c\left(q_s^H\right) = q_s^H$. When they are purchasers, they can use either good h or f as input to produce for their own consumption. If they decide to purchase (a quantity q_h^H of) good h, they stay in country H. If instead they decide to purchase (a quantity q_f^H of) good f, they travel to country F. The relative productivity of using either h or f as input varies according to an idiosyncratic shock η that is realised at the beginning of each period (there is no aggregate uncertainty). Specifically the quantity of output is given by

$$\max\left[y\left(q_f^H\right) + \eta q_f^H, y\left(q_h^H\right)\right] \tag{1}$$

where the productivity shock η takes values $\eta \in \{0, \eta_1, \eta_2\}$ with probabilities π_0, π_1 and π_2, respectively, and $0 < \eta_1 < \eta_2$. The function y satisfies $y'(q), -y''(q) > 0$, $y'(0) = \infty$ and $y'(\infty) = 0$. In addition we assume that $-y''(q)\, q \leq y'(q)$. The

[10] For entrepreneurs from country F, the description and notations are symmetric and obtained by interchanging f and h, and F and H.

utility derived by purchasers from consuming their output is linear.[11] With this specification, the choice of inputs depends on the realisation of the productivity shock. One can see from expression (1) that a higher realization of η favors the purchase of the foreign input.[12]

In the second market, entrepreneurs produce and consume a quantity of a generic good x in their country. Utility from consuming ($x > 0$) or disutility from producing ($x < 0$) is linear.

We now describe the currency and credit arrangements. Entrepreneurs are anonymous on the market for goods, implying that suppliers require immediate compensation to produce. Currencies serve as media of exchange. A currency is a storable, perfectly divisible and intrinsically useless object. We first describe the 'one country-one currency' case. In country H (F), the quantity of currency at the beginning of period t is denoted as M^H (M^F), and grows at the gross rate γ^H (γ^F). Monetary injections are made by the central bank in each country using lump-sum transfers in the second market. Since we consider symmetric countries, we set $\gamma^H = \gamma^F \equiv \gamma$ and $M^H = M^F \equiv M$. We assume that agents can only hold the currency of their country of residence across periods. This implies in particular that currencies circulate only in their issuing country. Purchasers from country H (F) who wish to buy input f (h) access a foreign exchange market where they exchange their local currency H (F) against currency F (H), before the first market opens. Exchanging currencies entails a conversion cost which for tractability we model as a disutility cost. We denote by $\varepsilon \geq 0$ the conversion cost per real unit of money converted.[13]

[11] The assumption that purchasers consume their own output is a simplification. Our results would be unaffected if we assumed instead that output matures at the end of the period and is sold in the second market.

[12] Assuming that η takes three values rather than two allows for non-trivial trading patterns: we could make our point with a binomial distribution, but with no trade between countries in equilibrium. Taking $\eta = 0$ as the lowest value is a simplification and captures all cases where the productivity of input h is greater than that of input f (for agents from country H).

[13] Modelling explicitly the exchange of currencies is beyond the scope of our paper. One way to provide micro-foundations for the FOREX market in this type of environment is developed by

We now describe the monetary union case. There is one common currency with quantity $2M$ distributed evenly across the two countries. Since our objective is to analyse the impact of conversion costs *per se*, we abstract from differences in monetary policy and assume the same money growth rate (γ) as in the separate currencies case. Consistently with the current euro area situation, we assume that the central bank has no power to tax agents, so that $\gamma \geq 1$.[14] Given these assumptions, the monetary union case is formally equivalent to the separate currencies case with $\varepsilon = 0$.

In each country, there is a competitive banking system. Suppliers can deposit their idle currency balances and purchasers can borrow from banks to purchase a greater quantity of goods in the first market. Following Berentsen, Camera, and Waller (2007), loans and deposits are intra-period: they are contracted before the opening of the first market and reimbursed during the second market. Timing is such that purchasers make their travel decision before banks open. Consider purchasers from country H. Those who stay can contact a bank in country H to obtain credit. Those who travel can contact a bank in country F which acts as an intermediary between a lending bank in country H and a purchaser; i.e., the intermediary bank receives money from a bank in country H and transfers it to a purchaser. We refer to the former case as domestic credit, and to the latter as cross-border credit. In both cases, purchasers take out loans in the currency of their country of residence.

Debt is unsecured and agents cannot commit to repay. To ensure that agents do not default on their loans, banks use borrowing limits and the threat of exclusion from the banking system.[15] This requires that banks recognize agents and have

Geromichalos and Jung (2017). In a real world setting, the cost of converting currencies comprises conversion fees paid to FOREX dealers, taxes paid to governments as well as exchange rate risks. The parameter ε is meant to capture these costs in a parsimonious way.

[14]This restriction implies that the Friedman rule is not a feasible policy, so that it is optimal for agents to insure against idiosyncratic shocks using both (costly) cash holdings and banks.

[15]We assume that defaulters are also excluded from monetary transfers. This is useful to prove the existence of an unconstrained equilibrium but is not necessary for our main result on the effect of conversion costs when agents are credit constrained. Assuming instead that defaulters receive monetary transfers would make default more attractive and reinforce the role of conversion costs

information on borrowers' past financial history. To capture the degree of credit integration across countries, we assume that access to this type of information is costless for domestic loans, but is potentially costly for cross-border loans. For simplicity, we model this cost as being incurred directly by a borrower from country H (F) to disclose her identity and financial history to the bank in country F (H) which serves as the intermediary for her lending bank. This disclosure cost is $c \geq 0$ per real unit of money borrowed. We discuss the interpretation in section 2.1.

Throughout the paper we refer to the parameter c as the cross-border credit premium. When $c = 0$, using credit to finance purchases in country H or F is equivalent; i.e. credit market integration is perfect. When $c > 0$, financing purchases abroad is more costly than financing purchases in the local country; i.e., there is imperfect credit market integration.

To summarize, the sequence of events within a period is as follows (see figure 1). At the beginning of the period, the productivity shocks are realized. Then, purchasers stay in their country of residence or travel to the other country. Next, banking activities (loans and deposits) and foreign exchange take place. Then the first market opens; people trade and come back to their country of residence to rebalance their money holdings on the second market.

2.1 Discussion of modelling choices

In this section, we briefly discuss some of our assumptions on money and credit.

The agents in our model are aimed to represent small and medium firms, self-employed individuals and households; i.e., agents who most often participate in local markets but do business or part of their production activities in foreign markets from time to time. In the real world, these agents primarily rely on local banks to fund themselves and their revenues are mainly in local currency. Thus, restricting agents to hold the local currency across periods and to contract debt in their local currency allows us to keep the analysis tractable while being consistent with the type of agents

in mitigating default incentives.

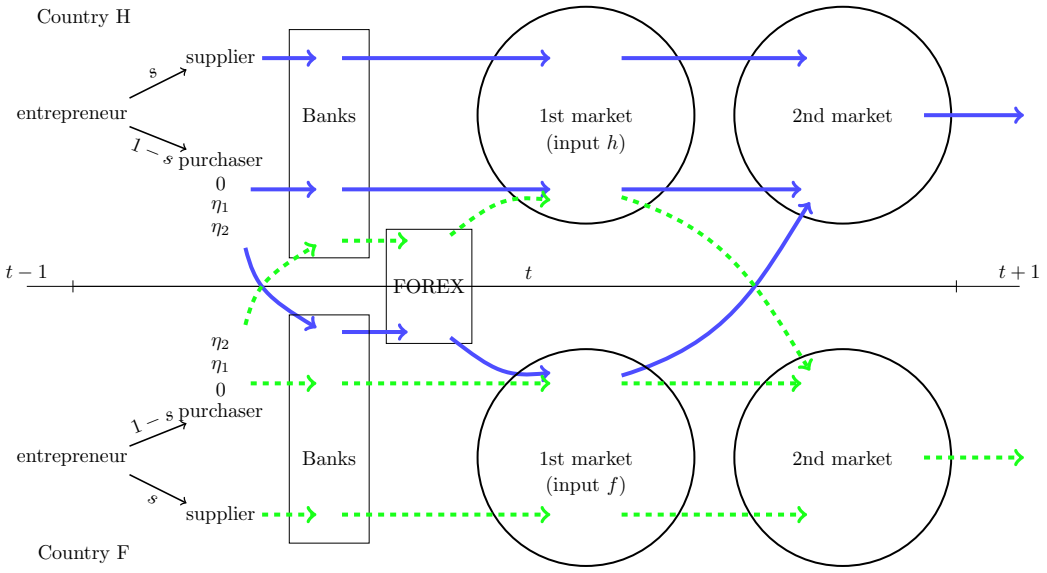

Figure 1: Sequence within a period

and countries we have in mind.[16]

We assume that the loan used to finance the purchase of foreign input is denominated in the local currency of the borrower. This specification captures the empirically relevant observation that due to exchange risk considerations, foreign currency borrowing is limited for retail clients with limited revenues in foreign currency (see e.g. Brown, Ongena, and Yeşin (2011)). Our main result, however, would go through with foreign currency borrowing but with added analytical complexity due to the fact that the conversion cost would be borne at the repayment—rather than at the borrowing—stage.

In our model, cross-border credit is more expensive because agents incur a

[16]By definition, the question of currency integration—or lack thereof—has relevance only in a world where agents face costs or legal restrictions in the use of foreign currencies. Since our aim is not to explain which asset circulates as medium of exchange in each country, but why different countries would choose a common currency, we take for granted that only the local currency circulates in the 'one country-one currency' case. See King, Wallace, and Weber (1992) for a paper with different types of agents, some forced by law to hold the currency of their country of residence, and some free to hold any currency. See Engineer (2000), Camera, Craig, and Waller (2004), Zhang (2014), Geromichalos and Simonovska (2014) for models of currency portfolio choice.

(higher) disclosure cost when contacting the foreign bank acting as a correspondent bank for their home bank. However, this disclosure cost should not be interpreted too literally. It is meant to capture various costs associated with information sharing and communication that banks face when arranging credit with a cross-border correspondent. What matters for our result is not the precise friction that makes cross-border credit more expensive, but the existence of a cross-border credit premium, which as discussed in section 1.1 is supported by a large empirical literature. Our specification with a cost borne directly by the agent at the borrowing stage brings analytical tractability. Alternatively, we could model the cross-border credit premium as arising from information and operating costs incurred by banks when extending cross-border credit. For instance, we could assume that cross-border credit is granted by foreign banks and have these banks pay a cost to access credit information on non-resident borrowers. Banks would ultimately pass these costs onto customers (through higher interest rates for cross-border loans), but such a model would be considerably more complicated without adding insight for our purpose.

3 Symmetric equilibria

This section describes stationary and symmetric monetary equilibria under both monetary arrangements. Recall that the monetary union case is formally equivalent to the separate currencies case with $\varepsilon = 0$; hence for our purpose it suffices to consider the separate currencies arrangement with $\varepsilon \geq 0$.

In a stationary and symmetric monetary equilibrium, end-of-period real money balances are constant, implying that

$$\gamma = M^H/M^H_{-1} = \phi^H_{-1}/\phi^H \qquad \text{and} \qquad \gamma = M^F/M^F_{-1} = \phi^F_{-1}/\phi^F \tag{2}$$

where ϕ^H (ϕ^F) is the price of currency H (F) in real terms in the second market.[17]

Since countries are perfectly symmetric, we only present the optimal choices by agents from country H, and drop the country index when no confusion should result. Let $V\left(m^H\right)$ denote the value function of an agent who holds an amount m^H of

[17] Throughout the paper, the subscript '-1' ('$+1$') indicates the previous (next) period.

money—currency H in the 'one country-one currency' case or the common currency in the monetary union—at the beginning of a period, before the realization of her idiosyncratic shock. Let $W_k\left(m^H, \ell_k^H\right)$ denote the value from entering the second market with m^H units of money and an amount ℓ_k^H of loans (> 0) or deposits (< 0), where $k = h, f, s$ indicates whether the agent has taken out a loan to buy good h, a loan to buy good f, or deposited in the current period.

3.1 The second market

In the second market, entrepreneurs consume or produce, reimburse loans or redeem deposits, and adjust money balances. Let i_h^H (i_f^H) denote the interest rate on loans for purchases of good h (f) and i_s^H the interest rate on deposits. The representative agent chooses next period monetary holdings, m_{+1}^H, and consumption (production) of the generic good, x, to solve

$$\begin{aligned} &\max_{x, m_{+1}^H} W_k\left(m^H, \ell_k^H\right) = x + \beta V\left(m_{+1}^H\right) \\ &\text{s.t. } x + \phi^H \ell_k^H\left(1 + i_k^H\right) + \phi^H m_{+1}^H = \phi^H m^H + \phi^H T^H \end{aligned} \tag{3}$$

where T^H is the lump-sum transfer received from the central bank. The budget constraint (3) states that the sum of consumption (production), loan repayment ($k = h, f$) or deposit's redemption ($k = s$) and future money holdings equals the sum of current money holdings and monetary transfer. This problem can be simplified to

$$\max_{m_{+1}^H}\left[-\phi^H m_{+1}^H + \phi^H m^H - \phi^H \ell_k^H\left(1 + i_k^H\right) + \phi^H T^H + \beta V\left(m_{+1}^H\right)\right]. \tag{4}$$

The solution satisfies the first order condition

$$\beta \frac{\partial V\left(m_{+1}^H\right)}{\partial m_{+1}^H} = \phi^H, \tag{5}$$

which states that the marginal value of bringing an additional unit of money into the next period equals the real price of money. One can see from (5) that all agents choose to carry the same money holdings, m_{+1}^H, into the next period. The envelope

conditions are

$$\frac{\partial W_k\left(m^H,\ell_k^H\right)}{\partial m^H}=\phi^H \tag{6}$$

$$\frac{\partial W_k\left(m^H,\ell_k^H\right)}{\partial \ell_k^H}=-\phi^H\left(1+i_k^H\right). \tag{7}$$

3.2 The first market

Suppliers. Suppliers do not derive utility from consumption for the current period, so they deposit their currency holdings from the previous period. More precisely, they strictly prefer to deposit than to keep idle balances when $i_s^H>0$ and are indifferent when $i_s^H=0$ (in the latter case, we assume w.l.o.g. that they deposit their entire balance). Formally,

$$-\ell_s^H=m_{-1}^H. \tag{8}$$

Given (8), the supplier chooses her production q_s^H to solve

$$\max_{q_s^H}\left[-q_s^H+W_s\left(m_{-1}^H+\ell_s^H+p^Hq_s^H,\ell_s^H\right)\right]\equiv\max_{q_s^H}\left[-q_s^H+W_s\left(p^Hq_s^H,-m_{-1}^H\right)\right],$$

with p^H the price of good h. Using (6), the first order condition on q_s^H writes

$$p^H\phi^H=1. \tag{9}$$

Condition (9) states that prices ϕ^H and p^H are such that suppliers must be indifferent between producing in the first market and producing in the second market.

Purchasers. Purchasers decide to use good h or good f as input depending on their productivity shock η. Given specification (1), their purchasing and travel decision can be described by a simple cutoff rule: they buy good h when $\eta\leq\eta^{H*}$ and buy good f when $\eta>\eta^{H*}$, where η^{H*} is some endogenously determined threshold (see section 3.4).

From (1), observe that if a purchaser decides to use good h as input, the quantity purchased does not depend directly on η; by contrast, η affects the quantity of input for a purchaser who uses good f. To stress this, we denote by q_h^H the choice of the

former, and by $q^H_{f,\eta}$ that of the latter. We denote by ℓ^H_h and $\ell^H_{f,\eta}$, respectively, the loans taken to finance these purchases. In principle, banks can set different borrowing limits, $\bar{\ell}^H_h$ and $\bar{\ell}^H_f$, for domestic or cross-border loans.

Consider first an agent purchasing good h (with $\eta \leq \eta^{H*}$). She solves

$$\max_{q^H_h, \ell^H_h} y\left(q^H_h\right) + W_h\left(m^H_{-1} + \ell^H_h - p^H q^H_h, \ell^H_h\right)$$

$$\text{s.t. } p^H q^H_h = m^H_{-1} + \ell^H_h \tag{10}$$

$$\ell^H_h \leq \bar{\ell}^H_h, \tag{11}$$

where (10) states that purchases are financed by initial money holdings and borrowed money,[18] and (11) is the borrowing constraint on domestic loans.

Using (10) to eliminate ℓ^H_h, the above program simplifies to

$$\max_{q^H_h} y\left(q^H_h\right) + W_h\left(0, p^H q^H_h - m^H_{-1}\right)$$

$$\text{s.t. } p^H q^H_h - m^H_{-1} = \ell^H_h \leq \bar{\ell}^H_h, \tag{12}$$

Using (7) and (9), the first order condition on q^H_h yields

$$y'\left(q^H_h\right) = 1 + i^H_h + \lambda^H_h / \phi^H, \tag{13}$$

where λ^H_h is the multiplier associated with the borrowing constraint.

Consider next an agent purchasing good f (with $\eta > \eta^{H*}$). This agent bears two extra costs compared to an agent purchasing good h. First, she incurs the cross-border credit premium (c) on her real money borrowed ($\ell^H_{f,\eta}/p^H$). Second, she incurs conversion costs (ε) on the real amount of currency H converted to obtain the amount $p^F q^H_{f,\eta}$ of currency F. Formally, she solves

$$\max_{q^H_{f,\eta}, \ell^H_{f,\eta}} -c\frac{\ell^H_{f,\eta}}{p^H} - \varepsilon\frac{p^F q^H_{f,\eta}}{e^{H/F}}\frac{1}{p^H} + y\left(q^H_{f,\eta}\right) + \eta q^H_{f,\eta} + W_f\left(m^H_{-1} + \ell^H_{f,\eta} - \frac{p^F q^H_{f,\eta}}{e^{H/F}}, \ell^H_{f,\eta}\right)$$

$$\text{s.t. } p^F q^H_{f,\eta} = e^{H/F}\left(m^H_{-1} + \ell^H_{f,\eta}\right), \tag{14}$$

$$\ell^H_{f,\eta} \leq \bar{\ell}^H_f, \tag{15}$$

[18] Note that (10) holds as an equality because purchasers can always deposit idle money balances.

where $e^{H/F}$ is the nominal exchange rate between currency H and currency F.[19] Using similar steps as with the previous case, the first-order condition on $q^H_{f,\eta}$ is

$$\left[y'\left(q^H_{f,\eta}\right)+\eta\right]\frac{e^{H/F}}{p^F}=\left(1+\varepsilon+c+i^H_f\right)\phi^H+\lambda^H_{f,\eta} \tag{16}$$

with $\lambda^H_{f,\eta}$ the multiplier associated with the borrowing constraint (15).

3.3 Market clearing conditions

Three market equilibrium conditions must be satisfied in each country. In country H, the demand for good h by purchasers from both countries must be equal to the supply by sellers from country H

$$(1-s)\sum_{\eta\leq\eta^{H*}}\pi_\eta q^H_h+(1-s)\sum_{\eta>\eta^{F*}}\pi_\eta q^F_{h,\eta}=sq^H_s, \tag{17}$$

money supply is equal to money demand

$$M^H_{-1}=m^H_{-1}, \tag{18}$$

while the sum of domestic and cross-border loans is equal to deposits

$$s\ell^H_s+(1-s)\sum_{\eta\leq\eta^{H*}}\pi_\eta\ell^H_h+(1-s)\sum_{\eta>\eta^{H*}}\pi_\eta\ell^H_{f,\eta}=0. \tag{19}$$

Symmetric conditions hold for country F.

Finally, in the FOREX market the quantity of currency H converted by agents from country H must be equal, given the prevailing nominal exchange rate, to the quantity of currency F converted by agents from country F

$$(1-s)\sum_{\eta>\eta^{H*}}\pi_\eta\left(\ell^H_{f,\eta}+m^H_{-1}\right)=e^{H/F}(1-s)\sum_{\eta>\eta^{F*}}\pi_\eta\left(\ell^F_{h,\eta}+m^F_{-1}\right). \tag{20}$$

Since we focus on symmetric equilibria, all nominal and real variables are the same for both countries. In particular, $p^H=p^F\equiv p$, $i^H_h=i^F_f$, $i^H_f=i^F_h$, $i^H_s=i^F_s$, $\phi^H=\phi^F\equiv\phi$, and $\eta^{H*}=\eta^{F*}\equiv\eta^*$, and (20) simply reduces to $e^{H/F}=1$.

[19] Formally, 1 unit of currency H=$e^{H/F}$ units of currency F. Symmetrically, 1 unit of currency F=$e^{F/H}$ units of currency H, with $e^{F/H}=1/e^{H/F}$.

3.4 Travel decision

In making their travel decisions, purchasers compare the values of using either good h or good f as input. Given optimal choices $(q_h^H, q_{f,\eta}^H)$, continuation value (4), and the fact that we focus on symmetric equilibria, one can show that the travel threshold η^* is defined by

$$y\left(q_h^H\right) - \phi\ell_h^H\left(1+i_h^H\right) = -c\frac{\ell_{f,\eta^*}^H}{p} - \varepsilon q_{f,\eta^*}^H + y\left(q_{f,\eta^*}^H\right) + \eta^* q_{f,\eta^*}^H - \phi\ell_{f,\eta^*}^H\left(1+i_f^H\right).$$

Using (9) and rearranging, this reduces to

$$y\left(q_h^H\right) - \phi\ell_h^H\left(1+i_h^H\right) = y\left(q_{f,\eta^*}^H\right) + \left(\eta^*-\varepsilon\right)q_{f,\eta^*}^H - \phi\ell_{f,\eta^*}^H\left(1+i_f^H+c\right). \tag{21}$$

The left side of (21) shows the value of purchasing q_h^H, viz the utility from consuming the output produced net of the cost of reimbursing the loan. On the right side, the value of purchasing $q_{f,\eta}^H$ is equal to the utility from consumption net of conversion costs, the cost of reimbursing the loan and the cross-border credit premium.

3.5 Marginal value of money

The expected utility for an agent who starts a period with m^H units of money H is

$$\begin{aligned} V\left(m^H\right) = {} & (1-s)\sum_{\eta\le\eta^*}\pi_\eta\left[y\left(q_h^H\right) + W_h\left(0, pq_h^H - m^H\right)\right] \\ & + (1-s)\sum_{\eta>\eta^*}\pi_\eta\left[y\left(q_{f,\eta}^H\right) + \left(\eta-\varepsilon\right)q_{f,\eta}^H - \phi\ell_{f,\eta}^H c + W_f\left(0, pq_{f,\eta}^H - m^H\right)\right] \\ & + s\left[-q_s + W_s\left(pq_s^H, -m^H\right)\right]. \end{aligned}$$

Differentiating with respect to m^H, and using (9), cash constraints (10) and (14), and the envelope condition (7), one gets the marginal value of money

$$\partial V/\partial m^H = (1-s)\,\phi\sum_{\eta\le\eta^*}\pi_\eta y'\left(q_h^H\right) + (1-s)\,\phi\sum_{\eta>\eta^*}\pi_\eta\left[y'\left(q_{f,\eta}^H\right) + \eta - \varepsilon\right] + s\phi\left(1+i_s^H\right).$$

Using (2) and (5), this condition becomes

$$\gamma/\beta = (1-s)\sum_{\eta\le\eta^*}\pi_\eta y'\left(q_h^H\right) + (1-s)\sum_{\eta>\eta^*}\pi_\eta\left[y'\left(q_{f,\eta}^H\right) + \eta - \varepsilon\right] + s\left(1+i_s^H\right). \tag{22}$$

The left side of (22) represents the marginal cost of acquiring an additional unit of money while the right side represents its marginal benefit: With probability $(1-s)$ the agent purchases good h (for $\eta \leq \eta^*$) or good f (for $\eta > \eta^*$), and with probability s the agent is a supplier and earns interest on her deposits.

3.6 Borrowing constraint

Banks set the borrowing limits $\bar{\ell}_h^H$ and $\bar{\ell}_f^H$ to ensure repayment, which requires that the continuation value from repayment be higher than the outside option of default. Denoting with a hat ($\hat{}$) the values and choices for an entrepreneur who has defaulted in the past, we have

Lemma 1 *An agent who borrows ℓ_k^H at a rate i_k^H repays her loan if and only if*

$$-\phi\ell_k^H\left(1+i_k^H\right)-\phi m_{+1}^H+\phi T^H+\beta V\left(m_{+1}^H\right)\geq-\phi\hat{m}_{+1}^H+\beta\hat{V}\left(\hat{m}_{+1}^H\right). \tag{23}$$

In equilibrium, interest rates satisfy $i_h^H=i_f^H=i_s^H\equiv i^H$, and (23) implies that banks set identical limits $\bar{\ell}_h^H=\bar{\ell}_f^H\equiv\bar{\ell}^H$ for domestic and cross-border loans.

The left side of (23) is the pay-off to an agent who does not default. In period t, she works to pay her loan and to replenish her money holdings, and receives lump-sum transfers. her expected lifetime utility from $t+1$ onwards is $V\left(m_{+1}^H\right)$. The right side is the value of the outside option of default. In period t, a defaulter saves the cost of loan repayment and works to obtain money holdings $\hat{m}_{+1}^H$. Her expected lifetime utility from $t+1$ onwards is $\hat{V}\left(\hat{m}_{+1}^H\right)$.

Lemma 1 also shows that interest rates on domestic and cross-border loans are equalized in equilibrium and that banks set identical borrowing limits for both types of loans. The reason is that both costs c and ε are incurred at the borrowing stage, and thus do not affect the continuation value at the repayment stage. In addition, the zero profit condition implies that the deposit rate is equal to the loan rate.

To compute the right side of (23), note that the main difference between a defaulter and a non-defaulter is exclusion from versus access to the banking system. Hence the optimal choices of a defaulter can be derived from the analysis of those of a non-defaulter, by setting $\hat{\ell}_h^H=\hat{\ell}_{f,\eta}^H=\hat{\ell}_s^H=0$.

In particular, from (21), the travel threshold for a defaulter, $\hat{\eta}^*$, is defined by

$$y\left(\hat{q}_h^H\right) = y\left(\hat{q}_{f,\hat{\eta}^*}^H\right) + \left(\hat{\eta}^* - \varepsilon\right)\hat{q}_{f,\hat{\eta}^*}^H. \tag{24}$$

We show in the appendix that for all relevant parameter values, defaulters are cash-constrained for all realizations of η, so that $\hat{q}_h^H = \hat{q}_{f,\hat{\eta}^*}^H \equiv \hat{q}^H$ and $\hat{m}_{-1}^H = p\hat{q}^H \quad \forall \eta$. Equation (24) thus reduces to

$$\hat{\eta}^* = \varepsilon. \tag{25}$$

Comparing with (21) one can see that contrary to the travel decision of a non-defaulter, the travel decision of a defaulter depends on the conversion cost ε, but not on the cross-border credit premium c. This comes from the fact that the latter does not access the banking market.

Using a reasoning similar to that of section 3.5, the value for a defaulter of starting the period with $\hat{m}^H$ units of money H can be computed as

$$\begin{aligned}\hat{V}\left(\hat{m}^H\right) = {} & (1-s)\sum_{\eta\leq\hat{\eta}^*}\pi_\eta\left[y\left(\hat{q}^H\right) + \hat{W}\left(0\right)\right] + (1-s)\sum_{\eta>\hat{\eta}^*}\left[y\left(\hat{q}^H\right) + \left(\eta-\varepsilon\right)\hat{q}^H + \hat{W}\left(0\right)\right] \\ & + s\left(-\hat{q}_s^H + \hat{W}\left(\hat{m}^H + p\hat{q}_s^H\right)\right)\end{aligned}$$

where $\hat{q}^H$ is given by the optimal condition on the money holdings of the defaulter:

$$\gamma/\beta = (1-s)\,y'\left(\hat{q}^H\right) + (1-s)\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right) + s. \tag{26}$$

3.7 Unconstrained and fully-constrained equilibria

This section provides conditions for the existence of symmetric and stationary equilibria. We consider unconstrained equilibria, in which purchasers are not credit constrained regardless of the value of their productivity shock η, and fully constrained equilibria in which all purchasers are credit constrained.

Definition 1 *An equilibrium is a vector of quantities* $\left(q_h^H, q_{f,\eta}^H, \hat{q}^H\right)\left(=\left(q_f^F, q_{h,\eta}^F, \hat{q}^F\right)\right)$, *thresholds* $\left(\eta^*, \hat{\eta}^*\right)$, *loans* $\left(\ell_h^H, \ell_{f,\eta}^H\right)\left(=\left(\ell_f^F, \ell_{h,\eta}^F\right)\right)$, *money holdings* $m_{-1}^H\left(=m_{-1}^F\right)$, *prices* $\left(i, p, \phi\right)$, *borrowing limits* $\bar{\ell}^H\left(=\bar{\ell}^F\right)$ *and multipliers* $\left(\lambda_h^H, \lambda_{f,\eta}^H\right)\left(=\left(\lambda_f^F, \lambda_{h,\eta}^F\right)\right)$

which satisfy (9)-(10),(13)-(14), (16), (18), and (21)-(26). An equilibrium is unconstrained if the borrowing constraint (23) is slack for all values of η. An equilibrium is fully constrained if (23) binds for all values of η.

Proposition 1 refers to the existence of the unconstrained equilibrium and Proposition 2 to the fully constrained equilibrium.

Proposition 1 *If β is sufficiently high there is $\tilde{\gamma}$ such that if $\gamma \geq \tilde{\gamma} \geq 1$, a unique unconstrained equilibrium exists.*

Proposition 1 states that if the rate of money growth γ is high enough, agents are unconstrained and borrow as much as they desire in equilibrium. This result extends Proposition 4 in Berentsen, Camera, and Waller (2007) to our two-country framework with potentially imperfect credit market integration, and echoes usual results in monetary models with limited commitment.[20]

This result comes from the impact of inflation on consumption and thus on lifetime utility. Agents choose the quantity of input to purchase in the first market by equating their marginal productivity to the marginal cost of carrying money from the second market in t to the first market in $t+1$. Carrying money throughout periods is costly because the rate of money growth γ is higher than the discount factor β. The higher γ, the higher the cost of carrying money, therefore the lower the amount of output agents produce—and the lower the utility derived from its consumption. Now, the cost of carrying money is mitigated for non-defaulters by the interest earned on idle cash balances when they turn out to be suppliers, and by monetary transfers. Therefore, the existence of banks allows agents with access to the banking system to enjoy a higher level of consumption. On the contrary, defaulters cannot deposit their cash balances and hence bear a higher cost of carrying money across periods—and enjoy a lower level of consumption. When inflation reaches a certain point, and if agents are patient enough, the resulting difference in lifetime utilities is such that agents are unwilling to default, and hence can borrow their desired amount of money at the equilibrium interest rates.

[20] See for instance Aiyagari and Williamson (2000).

Proposition 2 *If β, η_1 and η_2 are sufficiently low, there is $\{\gamma^1, \gamma^2\}$ with $1 \leq \gamma^1 < \gamma^2 < \tilde{\gamma}$ such that if $\gamma \in [\gamma^1, \gamma^2]$ a fully constrained equilibrium exists. In this fully constrained equilibrium the threshold η^* satisfies*

$$\eta^* = \varepsilon + sc. \tag{27}$$

If $\eta_1 > \eta^*$, purchasers in country H (F) buy input h (f) with probability π_0. If $\eta^* \geq \eta_1 > \varepsilon$ purchasers in country H (F) buy input h (f) with probability $(\pi_0 + \pi_1)$.

In a fully constrained equilibrium, all purchasers would like to borrow more money than the banks are willing to lend at the equilibrium interest rate. Proposition 2 states that this arises when the inflation rate is positive and low enough, provided that the discount factor β and the values of the productivity shocks η_1 and η_2 are low enough.[21] When inflation is low, the marginal cost of carrying money is low, and defaulters obtain a relatively high level of consumption. Incentives to default are high and the borrowing constraint is binding: only a limited amount of credit can be sustained because the threat of being excluded from the banking system imposes too mild a cost of default.

Next, we discuss the travel decision by an agent from country H in this equilibrium (the decision is symmetric for an agent from country F). Purchasers are credit constrained for all realizations of η, thus they borrow the same amount of credit and consume the same quantity of output regardless of which input, h or f, they purchase. Therefore equation (21) reduces to (27). In (27), the threshold η^* depends on the extra cost of purchasing input f which consists of the cross-border credit premium and the conversion cost. The conversion cost is paid on the total amount purchased whereas the cross-border credit premium is paid on the share of the purchase that is financed with a loan, equal to s.[22]

Given the realized productivity shock, there is a level of the financing cost above which an agent from country H switches from input f to input h even for a positive

[21] The case where c is so high that purchasers never buy input abroad is covered in the appendix.

[22] In this equilibrium, $\ell^H = spq^H$ and $m^H = (1-s)\,pq^H$ (see appendix, eq. (43)). Intuitively, cash holdings increase with the probability $1-s$ of being a purchaser since agents are more inclined to accumulate costly money holdings when they have a greater opportunity to spend them.

value of η. As stated in Proposition 2, if $\eta_1 > \eta_1^* = \varepsilon + sc$, the cross-border credit premium is low, so purchasers buy the input h only when η is zero—with probability π_0—and there is no home-bias. If $\eta_2 > \eta_1^* = \varepsilon + sc \geq \eta_1 > \varepsilon$ the cross-border credit premium is high and purchasers buy good h when η is equal to zero or to η_1—i.e., with probability $(\pi_0 + \pi_1)$. This defines a home bias in the choice of inputs which is triggered by a sufficiently high cross-border credit premium and (or) conversion cost. When the cost of converting one currency into the other is negligible, an agent's bias towards the local good is due to imperfect credit market integration.

4 Currency conversion costs, credit and welfare

This section presents our main results. We analyze the effect of making currency exchange costly on both credit and welfare; i.e., the expected lifetime utility of the representative agent. By symmetry, we focus on the welfare of agents in country H, which using (1), (9) and (17), is given by the following expression:

$$\mathcal{W} = \frac{1-s}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[y\left(q_h^H\right)-q_h^H\right]+\sum_{\eta>\eta^*}\pi_\eta\left[y\left(q_{f,\eta}^H\right)+(\eta-1-\varepsilon)\,q_{f,\eta}^H-\phi\ell_{f,\eta}^H c\right]\right\}.$$

We ask when a monetary union is optimal; i.e., for which parameter values welfare is maximal when $\varepsilon = 0$. Formally, we evaluate the impact of introducing positive conversion costs and we derive conditions on c and γ such that agents prefer a regime of separate currencies ($\varepsilon > 0$) instead of a common currency ($\varepsilon = 0$).[23] We also provide a comparative statics result on how credit and welfare depend on c.

4.1 When is a monetary union optimal?

In this section, we first show that in economies with money and credit agents prefer a monetary union if the inflation rate γ is high enough or if the credit market integration between countries is deep enough. The next proposition assesses the effect of implementing conversion costs when agents are not credit constrained.

[23] We focus on the comparison of steady state welfare levels and abstract from any cost of entry or exit from a currency union. This comparison can be extended to a setup in which the cost of exit is fixed, as suggested by the empirical discussion in Eichengreen (2007).

Proposition 3 *In an unconstrained equilibrium, the imposition of (any) conversion costs $\varepsilon > 0$ does not impact the use of input h (q_h^H), decreases the use of input f ($q_{f,\eta}^H$) ($\forall \eta > \eta^\star$), and reduces welfare.*

Proposition 3 states that imposing positive conversion costs is detrimental to welfare if agents are not credit constrained. On the one hand, a positive conversion cost increases the marginal cost of purchasing good f, implying that purchasers reduce the quantity of input f so that marginal productivity matches marginal cost. On the other hand, conversion costs do not affect the equilibrium quantity of input h. Since the output produced with input f decreases while that produced with input h is unaffected, the overall effect on expected utility is negative.

We now analyze the effect of conversion costs when agents are credit constrained. The next proposition refers to the case where credit market integration is sufficiently deep.

Proposition 4 *Let $c < \eta_1/s$. In a fully constrained equilibrium, the imposition of (small) conversion costs $\varepsilon > 0$ triggers a reduction in the use of both goods ($q_h^H, q_{f,\eta}^H$) and in the real quantity of credit ($\phi \ell_h^H$, $\phi \ell_{f,\eta}^H$) and worsens welfare.*

Proposition 4 states that imposing positive conversion costs is welfare worsening when agents are credit constrained and the credit markets of the two countries are relatively well integrated; i.e., $c < \eta_1/s$. In the fully constrained equilibrium, agents are constrained for all realizations of η. Thus, they all borrow the same amount, equal to the borrowing limit. In addition agents reduce their money holdings when conversion costs increase, since the marginal value of money decreases with conversion costs (see eq. 22). As a result, an increase in conversion costs entails a reduction in the output produced with both inputs h and f. As for the case in which agents are not constrained, when agents are credit constrained and credit market integration is deep enough, the imposition of conversion costs makes agents reduce their consumption and so unambiguously worsens welfare.

From Propositions 3 and 4, we can conclude that a monetary union is always optimal when no agent is credit constrained and when all agents are credit con-

strained and the cross-border credit premium is low. In our symmetric setup, this corresponds to the standard positive effect of lowering transaction costs.

We now show how the previous result on the optimality of a monetary union can be reversed. This reversal stems from explicitly considering in our analysis imperfect credit integration across countries and how it interacts with default incentives. We start from a situation of a monetary union between countries—agents do not pay any currency conversion cost ε—and imperfect credit market integration, i.e. high premium on cross-border credit c. We ask whether agents' welfare may be improved by imposing a positive conversion cost between currencies.

Proposition 5 *Let $c > \eta_1/s$. There are $\hat{\pi}_2 > 0$ and $\hat{\gamma}^2$ with $\gamma^1 < \hat{\gamma}^2 \leq \gamma^2$, such that for $\pi_2 \leq \hat{\pi}_2$ and $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$ in a fully constrained equilibrium the imposition of (small) conversion costs $\varepsilon > 0$ increases the use of both goods ($q_h^H, q_{f,\eta}^H$) and the quantity of credit ($\phi\ell_h^H$, $\phi\ell_{f,\eta}^H$), and improves welfare.*

Proposition 5 states that imposing positive conversion costs is welfare improving if agents are credit constrained, the cross-border credit premium c is sufficiently high, and the probability π_2 of a large productivity advantage in using the foreign input is sufficiently low.[24] A positive conversion cost has a differential impact on the lifetime utility of a defaulter on loan repayment, compared to a non-defaulter. The reason is that defaulters use input f more often than non-defaulters and hence pay the conversion cost more frequently. A positive conversion cost therefore reduces the *ex ante* incentives to default, which relaxes the borrowing constraint. To understand why defaulters are not home-biased while non-defaulters are, let us compare their respective travel and input choices. A high level of c reduces the willingness of a non-defaulter to use input f. When the cost of using credit to finance purchases abroad $s \cdot c$ is greater than η_1, purchasers choose to use input f only when the realized value of η is η_2, and choose to use input h when $\eta = 0, \eta_1$. Input f is used with probability π_2. By contrast, a defaulter cannot borrow and hence her decision

[24] Proposition takes the inflation rate γ as exogenous. We provide in the appendix an example in which a regime of separate currencies is optimal, even if inflation is optimally chosen.

$\hat{\eta}^*$ is independent of c (see equation 25). When $\varepsilon = 0$, he uses input f for any η higher than 0 (for $\eta = \eta_1, \eta_2$); i.e., with probability $(\pi_1 + \pi_2)$.

Since defaulters pay the conversion cost more often than home-biased non-defaulters, a positive conversion cost makes default less attractive. In equilibrium a higher level of credit can be sustained, thereby allowing higher consumption. However, conversion costs increase the marginal cost of purchasing goods for non-defaulters as well. Therefore, for conversion costs to be welfare improving, it must be that the probability π_2 is sufficiently small so that the negative effect of conversion costs on the use of good f is more than compensated by the effect of conversion costs on the incentives to default. The condition that π_2 is lower than the threshold value $\hat{\pi}_2$ in Proposition 5 states that the probability π_2 that non-defaulters pay the conversion cost must be relatively low.[25]

By contrast, the positive effect of conversion costs does not arise for the case of low cross-border credit premium covered in Proposition 4. The reason is that the purchasing pattern is the same for defaulters and non-defaulters. For $c < \eta_1 s$ and small ε, non-defaulters travel if their productivity shock η is η_1 or η_2, since $\eta^* < \eta_1$ by (27). Similarly, $\hat{\eta}^* < \eta_1$ by (25). Hence non-defaulters use input f with probability $\pi_1 + \pi_2$ and pay the conversion costs as often as defaulters.

4.2 Monetary and non-monetary causes for monetary disunion

In this section, we use our model to discuss two potential causes for monetary disunion: first a monetary cause—a variation of the level γ of monetary injections—and then a non-monetary cause—an increase in the cross-border credit premium c.

Monetary cause for currency disunion. We now ask whether a currency disunion may be optimal following a variation in the growth rate of the money supply

[25] If c is high enough to lead purchasers to use good h for all realizations of the productivity shock η, conversion costs are only borne by defaulters and hence their unique effect is to relax the borrowing constraint. Therefore an increase in conversion costs unambiguously improves welfare regardless of the probabilities associated with the different values of the productivity shock. See the appendix for the proof of this result.

and hence in the rate of inflation. Proposition 1 states that agents are unconstrained for sufficiently high values of γ, in which case they always prefer trading in a monetary union according to Proposition 3, regardless of the level of the cross-border credit premium. Proposition 2 states that agents may be credit constrained for values of γ below a certain threshold γ^2. Propositions 4 and 5 refer to the case in which agents are credit constrained. They state that if the cross-border credit premium c is low enough, welfare is higher in a regime with no conversion costs between currencies than in a regime with positive conversion costs (Proposition 4), whereas the opposite is true when the cross-border credit premium is sufficiently high if the probability π_2 is sufficiently low (Proposition 5). Therefore comparison of Propositions 1 and 3 with Propositions 2 and 5 suggests the following interpretation: for any sufficiently high level of the cross-border credit premium and a sufficiently low level of π_2, a reduction in the level of monetary injections below γ^2 makes agents switch from a preference for the monetary union to a preference for separate monies. The following corollary sums up this discussion.

Corollary 6 *Using propositions 1 to 5, (i) if $c < \eta_1/s$, the currency union is optimal regardless of the level of inflation; (ii) if $c \geq \eta_1/s$ (and π_2 is small), the level of inflation matters for the optimality of the currency union. In particular, a decrease from some $\gamma > \widetilde{\gamma}$ to $\gamma < \gamma^2$ can lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

Non-monetary cause for currency disunion. We now look at a potential non-monetary cause for the sub-optimality of a monetary union. We follow a traditional interpretation of financial crises that sees their origin in an increase in the real cost associated with the extension of bank credit, see for example Bernanke (1983), Gertler and Kiyotaki (2007). In our model, the non-monetary factor is a variation of the real cost c for agents to obtain cross-border loans. This interpretation is consistent with recent empirical evidence which has shown that the Japanese and the subprime crises had an asymmetric impact on bank lending to the economy: Credit granted by foreign banks decreased more than credit granted by domestic

banks, something that may be interpreted as a differential cost of getting different types of credit.[26] In the euro area, the fall of cross-border credit activity that occurred before 2012 has been blamed on supervisory pressures aimed at favoring domestic over cross-border credit (Gros, 2012). In the model this type of policy corresponds to an increase of c.

Following this view, our model suggests that the sustainability of a monetary union is directly impacted by the non-monetary factor given by an increase in the cost c when the inflation rate is low enough. The next corollary summarizes the effect of an increase in c in a situation of low inflation.

Corollary 7 *Comparison of Propositions 4 and 5 shows that for low levels of inflation, an increase in the cross-border credit premium from a low level ($c < \eta_1/s$) to a high level ($c > \eta_1/s$) may lead to a shift from a situation in which a currency union is optimal to one in which separate currencies are preferred.*

Credit crunch across monetary regimes. We define a credit crunch as a decrease in the real amount of credit triggered by an exogenous increase in c that is sufficiently high to induce a home bias in the choice of input. Proposition 8 establishes that any increase in the cross-border credit premium c reduces the quantity of credit when agents are credit constrained, and compares the size of the credit crunch across currency arrangements.

Proposition 8 *Let $0 < c_0 < \eta_1/s < c_1$ be such that a fully constrained equilibrium exists for all $c \in [c_0, c_1]$. An increase in c from c_0 to $c \leq c_1$ leads to a reduction in the real amount of credit $\left(\phi \ell_h^H, \phi \ell_{f,\eta}^H\right)$ and welfare, for any $\varepsilon \leq 0$. In addition, for π_2 sufficiently low there is a range of values of γ such that the decrease in credit is greater if $\varepsilon = 0$ than if $\varepsilon > 0$.*

Proposition 8 shows that an in a fully constrained equilibrium an increase in c reduces the amount of credit both when it impacts the travel decision and when it does not. The reason is that a greater value of c reduces agents' expected lifetime

[26] See Peek and Rosengren (1997), De Haas and van Lelyveld (2010), Popov and Udell (2012).

utility and hence their repayment incentives. The dashed curve in Figure 2 plots the volume of credit as a function of c in a fully constrained equilibrium under a regime of currency union.[27] For low levels of c, credit is continuously decreasing in c. When c reaches the threshold value η_1/s, credit shrinks sharply—the credit crunch—because the agents' decision on how often to purchase good f gets distorted, with a resulting fall in their lifetime utility. Agents who previously bought good f with probability $(\pi_1 + \pi_2)$ now buy it with probability π_2. For values of c greater than η_1/s, the effect of c on credit is negative.

The second part of Proposition 8 states that when the increase in c is sufficiently high to generate a home bias in the use of input, the decrease in the quantity of credit is sharper in a regime of currency union—when $\varepsilon = 0$—than in a regime of separate currencies—i.e. when $\varepsilon > 0$. The solid line in Figure 2 represents the volume of credit in a regime of separate currencies. Comparison with the dashed line shows that a currency union is the regime that provides the highest volume of credit and consumption when $c < \eta_1/s$. However the credit crunch triggered by an increase in c above the threshold η_1/s is less acute in a regime of separate currencies than in a currency union.

5 Conclusion

This paper provides a stylized model in which there is perfect integration with respect to the currency dimension but potentially imperfect integration of credit markets across different jurisdictions. The model shows that the welfare gains from currency union depend on the degree of credit market integration. We capture a high level of credit market integration by a low premium on cross-border credit costs. We show that when this premium is low enough agents always prefer using a unique currency. If countries are unable or unwilling to sufficiently reduce the cross-border

[27] Figure 2 is drawn assuming that $y(q) = (q^{\alpha})/\alpha$ and parameter values $\alpha = 0.2$, $\beta = 0.9$, $s = 0.7$, $\eta_1 = 0.02$, $\eta_2 = 0.05$, $\pi_1 = 0.2$, $\pi_2 = 0.02$, $\gamma = 1.01$ and $\varepsilon = 0.001$ for the regime of separate currencies. The software program Mathematica was used to check that the conditions for the existence of the fully constrained equilibrium are satisfied.

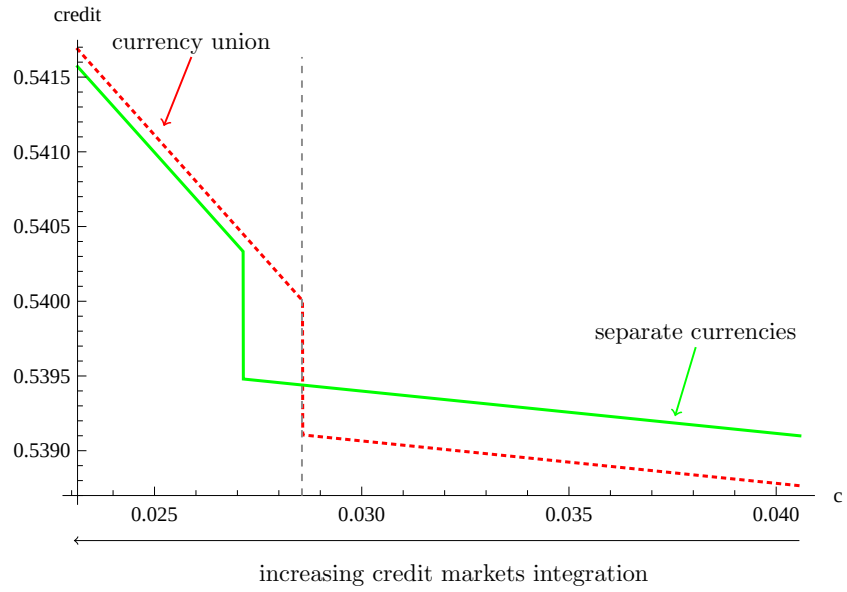

Figure 2: Quantity of credit as a function of the cross-border credit premium c in a currency union and in a regime of separate currencies

credit premium, welfare may be impaired by the adoption of a unique currency. The reason is that a currency union may be a cause of credit rationing when the supply of bank credit is reduced to cope with borrowers' default incentives. This issue may be especially acute in times of crisis when impediments to cross-border credit increase. Our analysis remains silent on the specific obstacles to credit market integration such as the limited capacity of banks to seize collateral or revenue across jurisdictions, and the absence of automatic inter-state judicial cooperation. Those elements are pinned down by public authorities, whose policy objective may be endogenized in future research.

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Appendix A - Conversion cost and optimal inflation

Proposition 5 shows that under appropriate conditions a strictly positive conversion cost—separate currencies—may relax the borrowing constraint and improve welfare compared to the benchmark case of a currency union. This result is obtained taking the inflation rate (γ) as given. However, previous studies of economies with credit and limited commitment show that inflation can be used to curb default incentives. In this type of environment default is a cash-intensive activity. A positive inflation rate thus acts as a tax that discourages default. In the setup we consider, default is a conversion-intensive activity. In this appendix, we present a parametrization in which using positive conversion costs in combination with the inflation rate is necessary to maximize welfare.

Figure 3 is drawn assuming that $y(q) = (q^{\alpha})/\alpha$ and parameter values $\alpha = 0.2$, $\beta = 0.9$, $s = 0.7$, $c = 0.1$, $\eta_1 = 0.02$, $\eta_2 = 0.05$, $\pi_1 = 0.7$, $\pi_2 = 0.02$ and $\varepsilon = 0.015$ for the regime of separate currencies. Notice that in our example η_1 is lower than sc and the value of β is such that defaulters are cash contrained for all realizations of η (see Lemma 2 in Appendix B). The maximum level of welfare is 1.19688 with no conversion costs and 19.691 with positive conversion costs. The software program Mathematica was used to check that the conditions for the existence of the different equilibria are satisfied. Given parameter values, in a regime of currency union a fully constrained equilibrium exists up to the threshold value of γ equal to $\gamma^2 = 1.021$. The unconstrained equilibrium exists for values of γ higher than $\widetilde{\gamma} = 1.026$. For intermediate values the equilibrium is partially constrained since agents with productivity shocks η_0 and η_1 are credit constrained whereas agents with productivity shock η_2 are not.

Appendix B - Proofs

Before presenting the proofs of the Lemma and Propositions stated in the paper, it is useful to present Lemma 2 that states that defaulters are cash-constrained for all realizations of the shock η if agents are sufficiently impatient. In order to alleviate

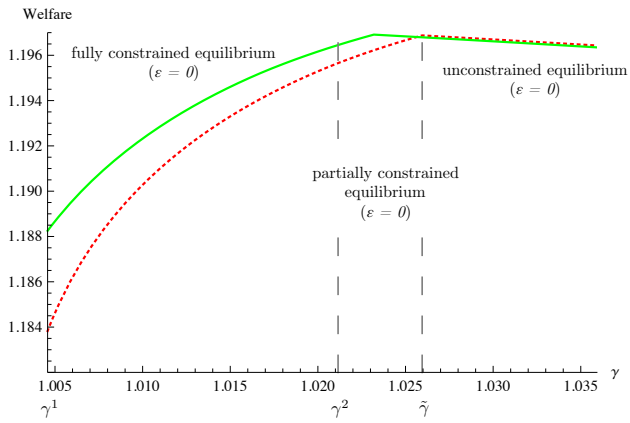

Figure 3: Welfare as a function of inflation in a currency union (dashed line) and in a regime of separate currencies (solid line)

the notation for the proofs, without loss in generality we focus on country H and drop the index H unless necessary to avoid confusion.

Lemma 2 *Let* $\tilde{\beta} = [1 + (1-s)(\pi_1\eta_1 + \pi_2\eta_2)]^{-1}$. *Defaulters are cash-constrained for all realizations of* η *for* $\beta < \tilde{\beta}/\gamma$. *In particular, if* $\beta < \tilde{\beta}$ *defaulters are cash-constrained for all realizations of* η *for all* $\gamma \geq 1$.

Notice that, in this model, without productivity shocks that determine the productivity of good h and f ($\pi_0 = 1, \pi_1, \pi_2 = 0$) and given that $\gamma \geq 1$, the condition in Lemma 2 is simply $\beta \leq 1$.

Proof of Lemma 2. If defaulters are cash-constrained for all realizations of η, it must be that $y'(\hat{q}), y'(\hat{q}) + \eta_1 - \varepsilon, y'(\hat{q}) + \eta_2 - \varepsilon > 1$. Since we only consider parameter values such that $\eta_1, \eta_2 > \varepsilon$, it is sufficient to show that in the conjectured equilibria $y'(\hat{q}) > 1$ holds to ensure that defaulters are cash-constrained for all realizations of η. From (25) and (26) we get

$$y'(\hat{q}) - 1 = (\gamma/\beta - 1)/(1-s) - [\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)]. \tag{28}$$

Thus if $\beta > \{[1 + (1-s)(\pi_1\eta_1 + \pi_2\eta_2)]\gamma\}^{-1}$ it follows that $y'(\hat{q}) > 1$ always

holds for $\varepsilon \geq 0$ whereas if $\beta \leq \tilde{\beta}$ it follows that $y'(\hat{q}) > 1$ always holds for $\gamma \geq 1$ and $\varepsilon \geq 0$.

Proof of Lemma 1. We first show that any agent repays iff (23) holds. Conjecture that $i_h = i_f = i$. First, observe that (23) corresponds to the incentive constraint for any agent at the repayment date (in the second market), and as such must hold for any η. To show that condition (23) is also sufficient, it suffices to show that no type η has an incentive to deviate at the borrowing stage. Let $\Gamma = \beta\left(V(m_{+1}) - \hat{V}(\hat{m}_{+1})\right) - \phi(m_{+1} - T - \hat{m}_{+1})$, and rewrite (23) as

$$\phi\ell(1+i) \leq \Gamma. \tag{29}$$

This defines a first debt limit $\bar{\ell}^1 \equiv \frac{\Gamma}{\phi(1+i)}$ for all η. Now, consider an agent with productivity shock η and debt $\ell^\eta \leq \bar{\ell}$ (with $\bar{\ell} \geq 0$ arbitrary). With no loss of generality, consider the of domestic debt. For this agent not to deviate at the borrowing stage, it must be the case that

$$y\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) - \phi m_{+1} + \phi T + \beta V(m_{+1}) \geq y\left(\frac{m+\bar{\ell}}{p}\right) - \phi\hat{m}_{+1} + \beta\hat{V}(\hat{m}_{+1}), \tag{30}$$

since an agent that will default borrows up to the limit $\bar{\ell}$. Notice that because the right-hand side is increasing in $\bar{\ell}$, (30) defines a second debt limit $\bar{\ell}^2$ to be imposed on type η. To show that (30) is redundant, we show that $\bar{\ell}^1 \leq \bar{\ell}^2$. Assume the contrary, that is $\bar{\ell}^1 > \bar{\ell}^2$. Using (30),

$$y\left(\frac{m+\bar{\ell}^2}{p}\right) = y\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) + \Gamma, \tag{31}$$

where ℓ^η is the equilibrium borrowing for type η. Since ℓ^η is chosen optimally (and $\bar{\ell}^2$ can be chosen) we have

$$y\left(\frac{m+\ell^\eta}{p}\right) - \phi\ell^\eta(1+i) \geq y\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2(1+i). \tag{32}$$

From (31) and (32),

$$y\left(\frac{m+\bar{\ell}^2}{p}\right) \geq y\left(\frac{m+\bar{\ell}^2}{p}\right) - \phi\bar{\ell}^2(1+i) + \Gamma,$$

which gives $\bar{\ell}^2 \geq \frac{\Gamma}{\phi(1+i)} = \bar{\ell}^1$, a contradiction. Hence, $\bar{\ell}^1 \leq \bar{\ell}^2$ and (23) is both sufficient and necessary for repayment incentives.

Since $\bar{\ell}^1$ does not depend on η, it also follows that $\bar{\ell}_h = \bar{\ell}_f = \bar{\ell}$. Furthermore, since agents' continuation value at the settlement stage is equal for all purchasers regardless of the good that they have acquired in the first market, the interest rate on domestic loans and the interest rate on cross-border loans must also be equal in equilibrium; i.e. $i_h = i_f$. In addition, since banks make no profits it must be that $i_h = i_f = i_s = i$. Otherwise a bank could attract all borrowers by offering a lower interest rate and/or all depositors by offering a higher interest rate.

To conclude, we rewrite (23) as

$$\begin{aligned}
&-\phi\left[\ell\left(1+i\right)+m_{+1}-T\right] \\
&+\frac{\beta\left(1-s\right)}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[y\left(q_h\right)-q_h\right]+\sum_{\eta>\eta^*}\pi_\eta\left[y\left(q_{f,\eta}\right)+\left(\eta-1-\varepsilon\right)q_{f,\eta}-\phi\ell_{f,\eta}c\right]\right\} \\
&\geq\frac{\beta\left(1-s\right)}{1-\beta}\left[y\left(\hat{q}\right)-\hat{q}+\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\hat{q}\right]-\frac{\left(\gamma-\beta\right)\hat{q}}{1-\beta}. \qquad (33)
\end{aligned}$$

To verify that (33) is equivalent to (23), denote as $x_{j,\eta}$ and x_s the amount of consumption by the purchaser with productivity shock η who purchases good $j=(h,f)$ and the amount of consumption by the supplier, respectively, in the second market. When the settlement stage arrives, the pay-off to a purchaser with productivity given by η who repays her debt is:

$$\begin{aligned}
&x_{j,\eta}+\frac{\beta\left(1-s\right)}{1-\beta}\left\{\sum_{\eta\leq\eta^*}\pi_\eta\left[y\left(q_{h,\eta}\right)+x_{h,\eta}\right]+\sum_{\eta>\eta^*}\pi_\eta\left[y\left(q_{f,\eta}\right)+\left(\eta-\varepsilon\right)q_{f,\eta}-\phi\ell_{f,\eta}c+x_{f,\eta}\right]\right\} \\
&-\frac{\beta s}{1-\beta}\left(q_s-x_s\right).
\end{aligned}$$

The pay-off to a defaulter with productivity shock η who purchases good $j=(h,f)$ is

$$\bar{x}_j^\eta+\frac{\beta\left(1-s\right)}{1-\beta}\left\{y\left(\hat{q}\right)+\sum_{\eta\leq\hat{\eta}^*}\pi_\eta\hat{x}_{h,\eta}+\sum_{\eta>\hat{\eta}^*}\pi_\eta\left[\left(\eta-\varepsilon\right)\hat{q}+\hat{x}_{f,\eta}\right]\right\}-\frac{\beta s}{1-\beta}\left(q_s-\hat{x}_s\right)$$

where $\bar{x}_{j,\eta}$ is consumption by the agent in the period in which she defaults and $\hat{x}_{h,\eta}$, $\hat{x}_{f,\eta}$ and $\hat{x}_s$ are net consumption by the defaulter in subsequent periods in case she

is a purchaser with a productivity shock $\eta \leq \hat{\eta}^*$, a purchaser with a productivity shock $\eta > \hat{\eta}^*$, or a supplier.

Consumption quantities $x_{j,\eta}$ and x_s are

$$\begin{aligned} x_{j,\eta} &= -\phi \ell_{j,\eta} (1+i) - \phi m_{+1} + \phi T \\ x_s &= -\phi \ell_s (1+i) + \phi p q_s - \phi m_{+1} + \phi T. \end{aligned} \tag{34}$$

where $T = (\gamma - 1) M_{-1}$. In a symmetric equilibrium, $m_{-1} = M_{-1}$. In addition, $m_{-1} = -\ell_s$. Using (8)-(10), (14), (17)-(19) and (34), we verify the market clearing condition in the second market:

$$(1-s) \sum_{\eta \leq \eta^*} \pi_\eta x_{h,\eta} + (1-s) \sum_{\eta > \eta^*} \pi_\eta x_{f,\eta} + s x_s = 0.$$

Consumption quantities by the defaulter $\bar{x}_{j,\eta}$, $\hat{x}_{h,\eta}$, $\hat{x}_{f,\eta}$ and $\hat{x}_s$ are

$$\begin{aligned} \bar{x}_j^\eta &= \hat{x}_{h,\eta} = \hat{x}_{f,\eta} = -\phi \hat{m}_{+1} = -\gamma \hat{q} \\ \hat{x}_s &= \hat{x}_j^\eta + \phi \hat{m}_{-1} + q_s = -(\gamma - 1) \hat{q} + q_s \end{aligned} \tag{35}$$

since $\phi \hat{m}_{-1} = \hat{q}$ and $\hat{m}_{+1} / \hat{m}_{-1} = \gamma$. Using (34) and (35), the borrowing constraint can be rewritten as in (33).

Proof of Proposition 1. We first rewrite the equilibrium equations that correspond to an unconstrained equilibrium and then show that the borrowing constraint is effectively slack for γ sufficiently high.

Conjecture an unconstrained equilibrium by setting $\lambda_h = 0$ and $\lambda_{f,\eta} = 0$ for all η. Then (13) and (16) become

$$\begin{aligned} y'(q_h) &= 1 + i \\ y'(q_{f,\eta}) + \eta - \varepsilon - c &= 1 + i. \end{aligned} \tag{36}$$

Hence (22) can be rewritten as

$$\gamma / \beta - (1-s) \sum_{\eta > \eta^*} \pi_\eta c = 1 + i. \tag{37}$$

Thus, q_h and $q_{f,\eta}$ are immediately pinned down for a given value of γ, and the value of $\bar{\ell}$ is not part of the solution of the unconstrained equilibrium.

From (8), (10), (14), and (19), we get

$$\phi \ell_h = s q_h - (1-s) \sum_{\eta > \eta^*} \pi_\eta \left(q_{f,\eta} - q_h \right) \tag{38}$$

and

$$q_h - \phi \ell_h = q_{f,\eta} - \phi \ell_{f,\eta} \tag{39}$$

for all η.

From (28), a defaulter effectively purchases $\hat{q}$ for γ sufficiently high regardless of the value of the productivity shock. From (26), (36), and (37), it follows that $\hat{q} < q_h, q_{f,\eta}$ for γ sufficiently high. Hence, by the mean value theorem $y\left(q_h\right) - y\left(\hat{q}\right) > y'\left(q_h\right)\left(q_h - \hat{q}\right)$. Similarly, $y\left(q_{f,\eta}\right) - y\left(\hat{q}\right) > y'\left(q_f^\eta\right)\left(q_{f,\eta} - \hat{q}\right)$. Therefore, a sufficient condition for the borrowing constraint (33) to be non-binding is

$$\begin{aligned}
&- \phi \left[\ell \left(1+i\right) + m_{-1} \right] + \frac{\beta \left(1-s\right)}{1-\beta} \left\{ \sum_{\eta > \eta^*} \pi_\eta \left[\left(\eta - \varepsilon\right) q_{f,\eta} - \phi \ell_{f,\eta} c \right] - \sum_{\eta > \hat{\eta}^*} \pi_\eta \left(\eta - \varepsilon\right) \hat{q} \right\} \\
&+ \frac{\beta \left(1-s\right)}{1-\beta} \left\{ \sum_{\eta \leq \eta^*} \pi_\eta \left[y'\left(q_h\right) - 1 \right] \left(q_h - \hat{q}\right) + \sum_{\eta > \eta^*} \pi_\eta \left[y'\left(q_{f,\eta}\right) - 1 \right] \left(q_{f,\eta} - \hat{q}\right) \right\} \\
&\geq - \frac{\left(\gamma - \beta\right) \hat{q}}{1-\beta}.
\end{aligned}$$

Given (36), (38) and (39), this condition can be rewritten as

$$\begin{aligned}
&- \phi \left[\ell \left(1+i\right) + m_{-1} \right] + \frac{\beta \left(1-s\right)^2 c}{1-\beta} \sum_{\eta > \eta^*} \pi_\eta \left[q_h + \sum_{\eta > \eta^*} \pi_\eta \left(q_{f,\eta} - q_h \right) \right] \\
&+ \frac{\beta \left(1-s\right) i}{1-\beta} \left(\sum_{\eta \leq \eta^*} \pi_\eta q_h + \sum_{\eta > \eta^*} \pi_\eta q_{f,\eta} \right) + \frac{\beta \left(1-s\right)}{1-\beta} \left[\sum_{\eta > \eta^*} \pi_\eta \left(\eta - \varepsilon\right) - \sum_{\eta > \hat{\eta}^*} \pi_\eta \left(\eta - \varepsilon\right) \right] \hat{q} \\
&\geq - \frac{\left(\gamma - \beta\right) \hat{q}}{1-\beta} + \frac{\beta \left(1-s\right)}{1-\beta} \left(i + c \sum_{\eta > \eta^*} \pi_\eta \right) \hat{q}.
\end{aligned} \tag{40}$$

We consider two different cases. First, consider the case of an agent who has bought

good h in the current period. Using (10), (14), (37) and (38), (40) becomes

$$
\begin{aligned}
&-sq_h i+(1-s)\sum_{\eta>\eta^*}\pi_\eta\left(q_{f,\eta}-q_h\right)i-q_h+\frac{\beta\left(1-s\right)^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left[q_h+\sum_{\eta>\eta^*}\pi_\eta\left(q_{f,\eta}-q_h\right)\right]\\
&+\frac{\beta\left(1-s\right)i}{1-\beta}\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h+\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\right)\\
&\geq-\beta i\frac{s}{1-\beta}\hat{q}-\frac{\beta\left(1-s\right)}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\right]\hat{q}.
\end{aligned}
$$

Since all terms with $q_{f,\eta}$ in the above inequality are positive, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$
\begin{aligned}
&-q_h i-q_h+\frac{\beta\left(1-s\right)^2 cq_h}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\sum_{\eta\leq\eta^*}\pi_\eta+q_h i\frac{1-s}{1-\beta}\sum_{\eta\leq\eta^*}\pi_\eta\\
&\geq-\frac{\beta is}{1-\beta}\hat{q}-\frac{\beta\left(1-s\right)}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\right]\hat{q}.
\end{aligned}
$$

Since $\eta^*\geq\hat{\eta}^*$, note that if γ is high enough the right-hand side in the above inequality is unambiguously negative given (37). Therefore, the right-hand side can be dismissed and it is sufficient for this inequality to hold that

$$
\left(-1+\frac{(1-s)}{1-\beta}\sum_{\eta\leq\eta^*}\pi_\eta\right)i\geq1-\frac{\beta\left(1-s\right)^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\sum_{\eta\leq\eta^*}\pi_\eta. \tag{41}
$$

From (37), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high (it is sufficient that $\beta>1-\left(1-s\right)\pi_0$ since $\sum_{\eta\leq\eta^*}\pi_\eta\geq\pi_0$).

Second, consider the case of an agent who has purchased good f in the current period. Using (10), (14), (37), (38) and (39), (40) can be written as

$$
\begin{aligned}
&\left[-q_{f,\eta}+(1-s)\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}+(1-s)\sum_{\eta\leq\eta^*}\pi_\eta q_h\right]i-q_{f,\eta}\\
&+\frac{\beta\left(1-s\right)^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h+\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\right)+\frac{\beta\left(1-s\right)i}{1-\beta}\left(\sum_{\eta\leq\eta^*}\pi_\eta q_h+\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\right)\\
&\geq-\beta i\frac{s}{1-\beta}\hat{q}-\frac{\beta\left(1-s\right)}{1-\beta}\left[\sum_{\eta>\eta^*}\pi_\eta\left(\eta-\varepsilon\right)-\sum_{\eta>\hat{\eta}^*}\pi_\eta\left(\eta-\varepsilon\right)\right]\hat{q}.
\end{aligned}
$$

In the above inequality, all terms with q_h are positive and the right-hand side is negative if γ is high enough (as stated in the case of the borrowing constraint for domestic loans). Thus, one way to show that this inequality holds for γ sufficiently high is to consider the following sufficient condition

$$\left(-q_{f,\eta}+\frac{(1-s)}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\right)i-q_{f,\eta}+\frac{\beta\left(1-s\right)^2 c}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\geq 0.$$

Since in an equilibrium with exchange among agents of the two countries $q_{f,\eta}\leq q_f^{\eta_2}$ and $\sum_{\eta>\eta^*}\pi_\eta q_{f,\eta}\geq \pi_2 q_f^{\eta_2}$, it is sufficient that

$$\left(-q_f^{\eta_2}+\frac{(1-s)}{1-\beta}\pi_2 q_f^{\eta_2}\right)i-q_f^{\eta_2}+\frac{\beta\left(1-s\right)^2 c\pi_2 q_f^{\eta_2}}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta\geq 0.$$

If $\beta>1-(1-s)\,\pi_2$, then a sufficient condition is

$$\left(-1+\frac{(1-s)\,\pi_2}{1-\beta}\right)i\geq 1-\frac{\beta\left(1-s\right)^2 c\pi_2}{1-\beta}\sum_{\eta>\eta^*}\pi_\eta. \tag{42}$$

From (37), the left-hand side in the above inequality is increasing in γ, provided that β is sufficiently high.

To sum up, (41) and (42) hold if γ is sufficiently high. In addition from (37) a high value of γ ensures $i\geq 0$. Hence an unconstrained equilibrium exists. Since (37) pins down a unique value of i and (36) pins down unique values of q_h and $q_{f,\eta}$ for all η this equilibrium is unique.

Proof of Proposition 2. First, we derive the threshold η^* in a conjectured fully constrained equilibrium. In this equilibrium all purchasers are credit constrained. Since the multiplier associated to the borrowing constraint is positive for all realizations of η, it follows from (13) and (16) that $y'\left(q_h\right)>1+i$ and $y'\left(q_{f,\eta}\right)+\eta q_{f,\eta}>1+i+\varepsilon+c$, meaning that all purchasers are cash-constrained. From Lemma 1, $\bar{\ell}_h=\bar{\ell}_f$. Thus we can write $q_h=q_{f,\eta}=q$ and $\ell_h=\ell_{f,\eta}=\ell$ for all η. Combining (8), (10), and (19) yields

$$\begin{aligned}\phi\ell&=sq\\ \phi m_{-1}&=(1-s)\,q.\end{aligned} \tag{43}$$

Therefore, from (21) the threshold η^* is equal to $\varepsilon + sc$.

Second, we prove the existence of a fully constrained equilibrium. We distinguish three cases depending on the value of c: $\eta_1 > \varepsilon + sc$, $\varepsilon < \eta_1 \leq \varepsilon + sc < \eta_2$ and $\varepsilon + sc > \eta_2$. We show for the three cases that a fully constrained equilibrium exists for $\gamma \in \left[\gamma^1, \gamma^2\right]$ where γ^1 and γ^2 depend on the value of c. However, we are unable to ensure the unicity of the credit-constrained equilibrium.

The proof proceeds as follows. First, we rewrite equilibrium equations by conjecturing a fully constrained equilibrium and show that $i \geq 0$ for $\gamma \geq \gamma^1$. Then we show that there is an interval $\left[\gamma^1, \gamma^2\right]$ such that the borrowing constraint binds for all purchasers; i.e., for any value of η.

For the cases $\eta_1 \leq \varepsilon + sc < \eta_2$ and $\eta_2 \leq \varepsilon + sc$, we show that sufficiently low values of η_1 and η_2 ensure that an agent with productivity shock η_1 or η_2 always prefers borrowing in order to purchase good h instead of purchasing good f by using only her money holdings.

Case $\eta_1 > \varepsilon + sc$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (25) and (27), $\eta^*, \hat{\eta}^* < \eta_1$. (22) and (26) can be rewritten as

$$\gamma/\beta - 1 = (1-s)\left[y'(q) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right] + si \tag{44}$$

and

$$\gamma/\beta - 1 = (1-s)\left[y'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right]. \tag{45}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$, which requires $q \geq \hat{q}$ given (44) and (45). Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (44) and (45), $\gamma^1 = \gamma^{1\prime}$. Rewrite the borrowing constraint (33) by conjecturing a fully constrained equilibrium for the case $\eta^*, \hat{\eta}^* < \eta_1$. Using (43) equation (33) becomes

$$\begin{aligned} &- isq - q \\ &+ \frac{\beta(1-s)}{1-\beta}\left[y(q) - q + \pi_1(\eta_1 - \varepsilon)q + \pi_2(\eta_2 - \varepsilon)q - (\pi_1 + \pi_2)scq\right] \\ &= \frac{\beta(1-s)}{1-\beta}\left[y(\hat{q}) - \hat{q} + \pi_1(\eta_1 - \varepsilon)\hat{q} + \pi_2(\eta_2 - \varepsilon)\hat{q}\right] - \frac{(\gamma - \beta)\hat{q}}{1-\beta}. \end{aligned} \tag{46}$$

From (46) it follows that

$$\gamma^{1\prime} = 1 + \beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc.$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime} = \gamma^1$. Differentiate (46) with respect to γ to get

$$\begin{aligned} &-\frac{\partial i}{\partial\gamma}sq - \left(is+1\right)\frac{\partial q}{\partial\gamma} \\ &+\frac{\beta\left(1-s\right)}{1-\beta}\left\{y'\left(q\right)-1+\pi_1\left(\eta_1-\varepsilon\right)+\pi_2\left(\eta_2-\varepsilon\right)-\left(\pi_1+\pi_2\right)sc\right\}\frac{\partial q}{\partial\gamma} \\ &=\frac{\beta\left(1-s\right)}{1-\beta}\left\{y'\left(\hat{q}\right)-1+\pi_1\left(\eta_1-\varepsilon\right)+\pi_2\left(\eta_2-\varepsilon\right)\right\}\frac{\partial\hat{q}}{\partial\gamma}-\frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\gamma}-\frac{\hat{q}}{1-\beta}. \end{aligned} \tag{47}$$

From (44),

$$s\frac{\partial i}{\partial\gamma} = 1/\beta - \left(1-s\right)y''\left(q\right)\frac{\partial q}{\partial\gamma}. \tag{48}$$

Use (44), (45), (47) and (48) to get

$$\frac{\partial q}{\partial\gamma} = \frac{\left(1-\beta\right)q/\beta-\hat{q}}{\left(1-\beta\right)\left(1-s\right)y''\left(q\right)q+\gamma-1-is-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc}$$

and

$$s\beta\frac{\partial i}{\partial\gamma} = \frac{\gamma-1-is-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)\left(1-b\right)c+\beta\left(1-s\right)y''\left(q\right)\hat{q}}{\left(1-\beta\right)\left(1-s\right)y''\left(q\right)q+\gamma-1-is-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc}. \tag{49}$$

From (46), we get

$$\begin{aligned} &\gamma-is-1-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc \\ &=\gamma-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc+\frac{\beta\left(1-s\right)}{1-\beta}\frac{y\left(\hat{q}\right)+\left[-1+\pi_1\left(\eta_1-\varepsilon\right)+\pi_2\left(\eta_2-\varepsilon\right)\right]\hat{q}}{q} \\ &-\frac{\left(\gamma-\beta\right)\hat{q}/q}{1-\beta}-\frac{\beta\left(1-s\right)}{1-\beta}\frac{y\left(q\right)-q+\pi_1\left(\eta_1-\varepsilon\right)q+\pi_2\left(\eta_2-\varepsilon\right)q-\left(\pi_1+\pi_2\right)scq}{q}. \end{aligned} \tag{50}$$

By the mean value theorem, $y\left(q\right)-y\left(\hat{q}\right) > y'\left(q\right)\left(q-\hat{q}\right)$ for $q > \hat{q}$. Therefore, for $q > \hat{q}$ (or $i > 0$) we verify from (50) that

$$\gamma-is-1-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc < -\beta si\frac{\hat{q}}{q}$$

so $\gamma-is-1-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc$ is unambiguously negative for $i > 0$ and given γ^1 it is equal to zero for $i = 0$. Therefore from (49) it follows that $\partial i/\partial\gamma > 0$ for

$i \geq 0$ provided that the borrowing constraint binds. Since $\partial i/\partial\gamma > 0$ at $\gamma = \gamma^1$, $i > 0$ at γ slightly higher than γ^1. In turn, this implies that $\partial i/\partial\gamma > 0$ for γ slightly higher than γ^1. Therefore, $i > 0$ for a higher value of γ. Thus there is an interval of values of $\gamma \geq \gamma^1$ for which $i \geq 0$.

To conclude, we must ensure that the representative agent is credit constrained for all values of η as we conjectured at the beginning of the proof. We show that she is credit constrained for a range of values of γ. Since $\eta_1 > sc + \varepsilon$, two subcases may exist: $\eta_1 - c - \varepsilon > 0$ and $\eta_1 - c - \varepsilon \leq 0$.

Subcase $\eta_1 - c - \varepsilon > 0$: For the agent who purchases good h, given (13) the multiplier of the borrowing constraint (33) is positive at $\gamma = \gamma^1$ if $y'(q) - 1 > 0$. From (44), this is the case if $\gamma/\beta - 1 - (1-s)\pi_1(\eta_1 - \varepsilon) - (1-s)\pi_2(\eta_2 - \varepsilon) > 0$ at $\gamma = \gamma^1 = \gamma^{1\prime}$. Since $\gamma \geq 1$ and $\varepsilon \geq 0$, this inequality always holds if $1/\beta - 1 - (1-s)\pi_1\eta_1 - (1-s)\pi_2\eta_2 > 0$. Since $\eta_1 - c - \varepsilon > 0$ and $\eta_2 > \eta_1$, given (16) this condition implies that the multiplier of the borrowing constraint is also positive for the agent who purchases good f. It follows that if β is sufficiently low agents are credit constrained for all realizations of the productivity shock for an interval of values of $\gamma \geq \gamma^1$.

Subcase $\eta_1 - c - \varepsilon \leq 0$: For an agent with productivity shock η_1, given (16) the multiplier of the borrowing constraint (33) is positive at $\gamma = \gamma^1$ if $y'(q) + \eta_1 - 1 - c - \varepsilon > 0$. From (44), this is the case if $(\gamma/\beta - 1)/(1-s) - \pi_1(\eta_1 - \varepsilon) - \pi_2(\eta_2 - \varepsilon) + \eta_1 - c - \varepsilon > 0$ at $\gamma = \gamma^1 = \gamma^{1\prime}$. Since $\gamma \geq 1$, $\varepsilon \geq 0$ and $\eta_1 > \varepsilon + sc$, this inequality always holds if $(1/\beta - 1)/(1-s) - \pi_1\eta_1 - \pi_2\eta_2 - (1-s)\eta_1/s > 0$. Since $\eta_2 > \eta_1$ and $\eta_1 - c - \varepsilon \leq 0$, this condition implies that the multiplier of the borrowing constraint is also positive for the agent whose productivity shock is η_2 and for the agent who purchases good h given (13). It follows that if β is sufficiently low agents are credit constrained for all realizations of the productivity shock for an interval of values of $\gamma \geq \gamma^1$.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Case $\varepsilon < \eta_1 \leq \varepsilon + sc < \eta_2$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (25) and (27), $\eta^* > \eta_1$ and $\hat{\eta}^* < \eta_1$. (22) and (26) can be rewritten as

$$\gamma/\beta = (1-s)\, y'(q) + (1-s)\, \pi_2 (\eta_2 - \varepsilon) + s(1+i) \tag{51}$$

and

$$\gamma/\beta - 1 = (1-s)\left[y'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right]. \tag{52}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$. Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (51) and (52) it follows that $si = (1-s)\,\pi_1(\eta_1 - \varepsilon)$ at $\gamma = \gamma^{1\prime}$ so $i > 0$. Rewrite the borrowing constraint (33) by conjecturing a fully constrained equilibrium for the case $\eta_1 \leq \eta^* < \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (43) equation (33) becomes

$$\begin{aligned} &-isq - q + \frac{\beta(1-s)}{1-\beta}\left\{y(q) - q + \pi_2(\eta_2 - \varepsilon - sc)\, q\right\} \\ &= \frac{\beta(1-s)}{1-\beta}\left\{y(\hat{q}) + \left[-1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right]\hat{q}\right\} - \frac{(\gamma-\beta)\,\hat{q}}{1-\beta}. \end{aligned} \tag{53}$$

From (53) it follows that

$$\gamma^{1\prime} = 1 + \beta(1-s)\,\pi_2 sc + (1-s)\,\pi_1(\eta_1 - \varepsilon). \tag{54}$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (53) with respect to γ to get

$$\begin{aligned} &-\frac{\partial i}{\partial\gamma} sq - (is+1)\frac{\partial q}{\partial\gamma} + \frac{\beta(1-s)}{1-\beta}\left\{y'(q) - 1 + \pi_2[\eta_2 - \varepsilon - sc]\right\}\frac{\partial q}{\partial\gamma} \\ &= \frac{\beta(1-s)}{1-\beta}\left\{y'(\hat{q}) - 1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right\}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\hat{q}}{1-\beta}. \end{aligned} \tag{55}$$

From (51),

$$s\frac{\partial i}{\partial\gamma} = 1/\beta - (1-s)\, y''(q)\frac{\partial q}{\partial\gamma}. \tag{56}$$

Use (51), (52), (55) and (56) to get

$$\frac{\partial q}{\partial\gamma} = \frac{(1-\beta)\, q/\beta - \hat{q}}{(1-\beta)(1-s)\, y''(q)\, q + \gamma - 1 - si - \beta(1-s)\,\pi_2 sc}$$

and

$$s\beta\frac{\partial i}{\partial\gamma}=\frac{\beta\left(1-s\right)y''\left(q\right)\hat{q}+\gamma-1-si-\beta b\pi_2 sc}{\left(1-\beta\right)\left(1-s\right)y''\left(q\right)q+\gamma-1-si-\beta\left(1-s\right)\pi_2 sc}. \tag{57}$$

From (53), we get

$$\begin{aligned}&\gamma-is-1-\beta\left(1-s\right)\pi_2 sc \\ &=\gamma-\beta\left(1-s\right)\pi_2 sc+\frac{\beta\left(1-s\right)}{1-\beta}\frac{y\left(\hat{q}\right)+\left[-1+\pi_1\left(\eta_1-\varepsilon\right)+\pi_2\left(\eta_2-\varepsilon\right)\right]\hat{q}}{q}-\frac{\left(\gamma-\beta\right)\hat{q}/q}{1-\beta} \\ &-\frac{\beta\left(1-s\right)}{1-\beta}\frac{y\left(q\right)-q+\pi_2\left(\eta_2-\varepsilon-sc\right)q}{q}.\end{aligned} \tag{58}$$

By the mean value theorem, $y\left(q\right)-y\left(\hat{q}\right)>y'\left(q\right)\left(q-\hat{q}\right)$ for $q>\hat{q}$. Therefore, for $q>\hat{q}$ (or $si>\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)$) we verify from (58) that

$$\gamma-is-1-\beta\left(1-s\right)\pi_2 sc<\beta\left[\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)-si\right]\hat{q}/q$$

so $\gamma-is-1-\beta\left(1-s\right)\pi_2 sc$ is unambiguously negative for $si>\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)$ and given $\gamma^{1\prime}$ it is equal to zero at $si=\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)$. Therefore from (57) it follows that $\partial i/\partial\gamma>0$ for $i\geq\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)/s>0$ provided that the borrowing constraint binds. Since $\partial i/\partial\gamma>0$ at $\gamma=\gamma^{1\prime}$, si is slightly higher than $\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)$ for γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\partial i/\partial\gamma>0$ for γ slightly higher than $\gamma^{1\prime}$. Therefore $i>\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)/s>0$ for a higher value of γ. Since $i>0$ at $\gamma=\gamma^{1\prime}$, $\gamma^1<\gamma^{1\prime}$ and there is an interval of values of $\gamma\geq\gamma^1$ for which $i\geq0$.

To conclude, we must ensure that the representative agent is credit constrained for all values of η as we conjectured at the beginning of the proof. We show that she is credit constrained for a range of values of γ. Since $\varepsilon<\eta_1\leq\varepsilon+sc<\eta_2$, two subcases may exist: $\eta_2-c-\varepsilon>0$ and $\eta_2-c-\varepsilon\leq0$.

Subcase $\eta_2-c-\varepsilon>0$: For the agent who purchases good h, given (13) the multiplier of the borrowing constraint (33) is positive if $y'\left(q\right)-1-i>0$. From (51), at $\gamma=\gamma^{1\prime}$ this is the case if $\gamma/\beta-1-\left(1-s\right)\pi_2\left(\eta_2-\varepsilon\right)-\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)/s>0$. Since $\gamma\geq1$ and $\varepsilon\geq0$, this always holds if $1/\beta-1-\left(1-s\right)\pi_2\eta_2-\left(1-s\right)\pi_1\eta_1/s>0$. It is straightforward that this condition also implies that $y'\left(q\right)-1-i>0$ for $0\leq si\leq\left(1-s\right)\pi_1\left(\eta_1-\varepsilon\right)$ and $\gamma\in\left[\gamma^1,\gamma^{1\prime}\right]$. Since $\eta_2-c-\varepsilon>0$, given (16) this

condition also implies that the multiplier of the borrowing constraint is also positive for the agent who purchases good f. It follows that if β is sufficiently low agents are credit constrained for all realizations of the productivity shock for an interval of values of $\gamma \geq \gamma^1$.

Subcase $\eta_2 - c - \varepsilon \leq 0$: For an agent with productivity shock η_2, given (16) the multiplier of the borrowing constraint (33) is positive if $y'(q) - 1 - i + \eta_2 - c - \varepsilon > 0$. From (51), at $\gamma = \gamma^{1\prime}$ this is the case if $(\gamma/\beta - 1)/(1-s) - \pi_2(\eta_2 - \varepsilon) - \pi_1(\eta_1 - \varepsilon)/s + \eta_2 - c - \varepsilon > 0$. Since $\gamma \geq 1$, $\varepsilon \geq 0$ and $\eta_2 > sc + \varepsilon$, this always holds if $(1/\beta - 1)/(1-s) - \pi_2\eta_2 - \pi_1\eta_1/s + (1-s)\eta_2/s > 0$. It is straightforward that this condition also implies that $y'(q) - 1 - i + \eta_2 - c - \varepsilon > 0$ for $0 \leq si \leq (1-s)\pi_1(\eta_1 - \varepsilon)$ and $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Since $\eta_2 - c - \varepsilon < 0$, given (13) this condition also implies that the multiplier of the borrowing constraint is also positive for the agent who purchases good f. It follows that if β is sufficiently low agents are credit constrained for all realizations of the productivity shock for an interval of values of $\gamma \geq \gamma^1$.

Finally note that an agent with $\eta = \eta_1$ could prefer to buy good f by using only her money holdings instead of borrowing and buying good h, but this possibility can be dismissed. That is, the following condition is satisfied

$$y(q) - \phi\ell(1+i) \geq y(m_{-1}) + (\eta_1 - \varepsilon) m_{-1}.$$

From (43), this expression can be written as

$$y(q) - sq(1+i) \geq y((1-s)q) + (\eta_1 - \varepsilon)(1-s)q.$$

Since $y(q) - y((1-s)q) > y'(q)sq$ and in a fully constrained equilibrium $i \leq y'(q) - 1$, it follows that it is always possible to define a value $\bar{\eta}_1$ such that if $\eta_1 \leq \bar{\eta}_1$ the above inequality holds.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Case $\eta_2 < \varepsilon + sc$.

Using the solutions for η^* and $\hat{\eta}^*$ stated in (25) and (27), $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$.

(22) and (26) can be rewritten as

$$\gamma/\beta - 1 = (1-s)\left[y'(q) - 1\right] + si \tag{59}$$

and

$$\gamma/\beta - 1 = (1-s)\left[y'(\hat{q}) + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon) - 1\right]. \tag{60}$$

For a constrained equilibrium to exist, it must be that $i \geq 0$. Denote as $\gamma^{1\prime}$ the value of γ such that $\hat{q} = q$ and as γ^1 the value of γ such that $i = 0$ in a fully constrained equilibrium. From (59) and (60), $si = (1-s)\left[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right]$ at $\gamma = \gamma^{1\prime}$. Rewrite the borrowing constraint (33) by conjecturing a fully constrained equilibrium for the case $\eta^* > \eta_2$ and $\hat{\eta}^* < \eta_1$. Using (43) it becomes

$$\begin{aligned} &- isq - q + \frac{\beta(1-s)}{1-\beta}\left[y(q) - q\right] \\ &= \frac{\beta(1-s)}{1-\beta}\left[y(\hat{q}) - \hat{q} + \pi_1(\eta_1 - \varepsilon)\hat{q} + \pi_2(\eta_2 - \varepsilon)\hat{q}\right] - \frac{(\gamma-\beta)\hat{q}}{1-\beta}. \end{aligned} \tag{61}$$

From (61) it follows that

$$\gamma^{1\prime} = 1 + (1-s)\left[\pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right].$$

Next we must ensure that $\partial i/\partial\gamma \geq 0$ for $\gamma \geq \gamma^{1\prime}$. Differentiate (61) with respect to γ to get

$$\begin{aligned} &- \frac{\partial i}{\partial\gamma}sq - (is+1)\frac{\partial q}{\partial\gamma} + \frac{\beta(1-s)}{1-\beta}\left[y'(q) - 1\right]\frac{\partial q}{\partial\gamma} \\ &= \frac{\beta(1-s)}{1-\beta}\left\{y'(\hat{q}) - 1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right\}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\gamma} - \frac{\hat{q}}{1-\beta}. \end{aligned} \tag{62}$$

From (59),

$$s\frac{\partial i}{\partial\gamma} = 1/\beta - (1-s)\,y''(q)\frac{\partial q}{\partial\gamma}. \tag{63}$$

Use (59), (60), (62) and (63) to get

$$\frac{\partial q}{\partial\gamma} = \frac{(1-\beta)\,q/\beta - \hat{q}}{\gamma - 1 - is + (1-\beta)(1-s)\,y''(q)\,q}$$

and

$$s\beta\frac{\partial i}{\partial\gamma} = \frac{\gamma - 1 - is + \beta(1-s)\,y''(q)\,\hat{q}}{\gamma - 1 - is + (1-\beta)(1-s)\,y''(q)\,q}. \tag{64}$$

From (61), we get

$$\gamma - is - 1 + \frac{\beta(1-s)}{1-\beta}\frac{y(q)-q}{q}$$
$$= \gamma + \frac{\beta(1-s)}{1-\beta}\frac{y(\hat{q})-\hat{q}+\pi_1(\eta_1-\varepsilon)\hat{q}+\pi_2(\eta_2-\varepsilon)\hat{q}}{q} - \frac{(\gamma-\beta)\hat{q}/q}{1-\beta}. \qquad (65)$$

By the mean value theorem, $y(q) - y(\hat{q}) > y'(q)(q-\hat{q})$ for $q > \hat{q}$. Therefore, for $q > \hat{q}$ (or $i > (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]/s$) we verify from (65) that

$$\gamma - is - 1 < \beta\{(1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)] - si\}\hat{q}/q$$

so $\gamma - is - 1$ is unambiguously negative for $si > (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]$ and given $\gamma^{1\prime}$ it is equal to zero at $si = (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]$. Therefore from (64) it follows that $\partial i/\partial\gamma > 0$ for $i \geq (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)] > 0$ provided that the borrowing constraint binds. Since $\partial i/\partial\gamma > 0$ at $\gamma = \gamma^{1\prime}$, si is slightly higher than $(1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]$ for γ slightly higher than $\gamma^{1\prime}$. In turn, this implies that $\partial i/\partial\gamma > 0$ for γ slightly higher than $\gamma^{1\prime}$. Therefore $i > (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]/s > 0$ for a higher value of γ. Since $i > 0$ at $\gamma = \gamma^{1\prime}$, $\gamma^1 < \gamma^{1\prime}$ and there is an interval of values of $\gamma \geq 0$ for which $i \geq 0$.

To conclude, we must ensure that the representative agent is credit constrained for all values of η as we conjectured at the beginning of the proof. When $\eta_2 < \varepsilon + sc$, the agent purchases good h for all realizations of the productivity shock. Given (13), the multiplier of the borrowing constraint (33) is positive if $y'(q) - 1 - i > 0$. From (59), at $\gamma = \gamma^{1\prime}$ this is the case if $\gamma/\beta - 1 - (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]/s > 0$. Since $\gamma \geq 1$ and $\varepsilon \geq 0$, this always holds if $1/\beta - 1 - (1-s)(\pi_1\eta_1+\pi_2\eta_2)/s > 0$. It is straightforward that this condition also implies that $y'(q) - 1 - i > 0$ for $0 \leq si \leq (1-s)[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)]$ and $\gamma \in [\gamma^1, \gamma^{1\prime}]$. It follows that if β is sufficiently low agents are credit constrained for all realizations of the productivity shock for an interval of values of $\gamma \geq \gamma^1$.

Finally note that an agent with $\eta = \eta_1, \eta_2$ could prefer to buy good f by using only her money holdings instead of borrowing and buying good h, but this possibility can be dismissed. That is, the following condition is satisfied

$$y(q) - \phi\ell(1+i) \geq y(m_{-1}) + (\eta_2 - \varepsilon)m_{-1}.$$

From (43), this expression can be written as

$$y(q) - sq(1+i) \geq y((1-s)q) + (\eta_2 - \varepsilon)(1-s)q.$$

Since $y(q) - y((1-s)q) > y'(q)sq$ and in a fully constrained equilibrium $i \leq y'(q) - 1$, it follows that it is always possible to define a value $\bar{\eta}_2$ such that if $\eta_2 \leq \bar{\eta}_2$ the above inequality holds. Further, the above inequality implies that $y(q) - sq(1+i) \geq y(bq) + (\eta_1 - \varepsilon)(1-s)q$ since $\eta_2 > \eta_1$.

Therefore there is an interval $\left[\gamma^1, \gamma^2\right]$ such that if $\gamma \in \left[\gamma^1, \gamma^2\right]$ then a fully constrained equilibrium in which $i \geq 0$ exists.

Proof of Proposition 3. Given (36), (22) can be written as

$$\gamma/\beta = y'(q_{f,\eta}) + \eta - \varepsilon - \left(1 - (1-s)\sum_{\eta > \eta^*} \pi_\eta\right) c.$$

Hence

$$\frac{\partial q_{f,\eta}}{\partial \varepsilon} = \frac{1}{y''(q_{f,\eta})} \tag{66}$$

so that $\partial q_{f,\eta}/\partial \varepsilon < 0$. From (36) and (66), $\partial q_h/\partial \varepsilon = 0$. From (38) and (39), we get

$$\frac{\partial(\phi \ell_h)}{\partial \varepsilon} = -(1-s)\sum_{\eta > \eta^*} \pi_\eta \frac{\partial q_{f,\eta}}{\partial \varepsilon}$$

and

$$\frac{\partial(\phi \ell_{f,\eta})}{\partial \varepsilon} = \left(1 - (1-s)\sum_{\eta > \eta^*} \pi_\eta\right) \frac{\partial q_{f,\eta}}{\partial \varepsilon}.$$

Given (66), we get $\partial(\phi \ell_h)/\partial \varepsilon > 0$ and $\partial(\phi \ell_{f,\eta})/\partial \varepsilon < 0$.

Let us replicate here the equation for welfare as stated in the main body of the paper

$$\mathcal{W} = \frac{1-s}{1-\beta}\left\{\sum_{\eta \leq \eta^*} \pi_\eta \left[y(q_h) - q_h\right] + \sum_{\eta > \eta^*} \pi_\eta \left[y(q_{f,\eta}) + (\eta - 1 - \varepsilon) q_{f,\eta} - \phi \ell_{f,\eta} c\right]\right\}. \tag{67}$$

Differentiating (67) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon} = \frac{1-s}{1-\beta}\sum_{\eta > \eta^*} \pi_\eta \left\{\left[y'(q_{f,\eta}) - 1 + \eta - \varepsilon - c + (1-s)\sum_{\eta > \eta^*} \pi_\eta c\right] \frac{\partial q_{f,\eta}}{\partial \varepsilon} - q_{f,\eta}\right\}.$$

Since $y'(q_{f,\eta}) - 1 + \eta - \varepsilon - c > 0$ for all $\eta > \eta^*$ from (36) and $\partial q_{f,\eta}/\partial \varepsilon < 0$ from (66), it follows that $\partial \mathcal{W}/\partial \varepsilon < 0$.

Proof of Proposition 4. Let $c < (\eta_1 - \varepsilon)/s$ and consider a fully constrained equilibrium in which $\lambda_{h,\eta}, \lambda_{f,\eta} > 0$ and the borrowing constraint (33) holds with equality. As in the proof of Proposition 2, we can set $\phi \ell_h = \phi \ell_{f,\eta} = \phi \ell$ and $q_h = q_{f,\eta} = q$ for all η. Given (27) and (43), welfare defined in (67) becomes

$$\mathcal{W}\left(\frac{1-s}{1-\beta}\right)^{-1} = y(q) + (-1 + \pi_1\eta_1 + \pi_2\eta_2)\, q - (\pi_1 + \pi_2)(\varepsilon + sc)\, q. \tag{68}$$

Differentiate the borrowing constraint for the case $c < (\eta_1 - \varepsilon)/s$ stated in (46) with respect to ε to get

$$\begin{aligned} &-(1+si)\frac{\partial q}{\partial \varepsilon} - s\frac{\partial i}{\partial \varepsilon} q - \frac{\beta(1-s)(\pi_1+\pi_2)}{1-\beta} q \qquad (69)\\ &+ \frac{\beta(1-s)}{1-\beta}\left\{y'(q) - 1 + \pi_1\eta_1 + \pi_2\eta_2 - (\pi_1+\pi_2)(\varepsilon + sc)\right\}\frac{\partial q}{\partial \varepsilon}\\ &= \frac{\beta(1-s)}{1-\beta}\left[y'(\hat{q}) - 1 + \pi_1(\eta_1 - \varepsilon) + \pi_2(\eta_2 - \varepsilon)\right]\frac{\partial \hat{q}}{\partial \varepsilon} \qquad (70)\\ &- \frac{\beta(1-s)(\pi_1+\pi_2)}{1-\beta}\hat{q} - \frac{\gamma - \beta}{1-\beta}\frac{\partial \hat{q}}{\partial \varepsilon}. \end{aligned}$$

Differentiating (44) with respect to ε yields

$$s\frac{\partial i}{\partial \varepsilon} = -(1-s)\, y''(q)\frac{\partial q}{\partial \varepsilon} + (1-s)(\pi_1 + \pi_2). \tag{71}$$

Rewrite (69) using (44), (45) and (71) to get

$$\frac{\partial q}{\partial \varepsilon} = \frac{(1-s)(\pi_1+\pi_2)\left[(1-\beta)q + \beta(q - \hat{q})\right]}{\gamma - 1 - si - \beta(1-s)(\pi_1+\pi_2)sc + (1-\beta)(1-s)qy''(q)}. \tag{72}$$

From (44) and (45), $q = \hat{q}$ when $i = 0$ and $q > \hat{q}$ when $i > 0$. From the proof of Proposition 2, when $c < (\eta_1 - \varepsilon)/s$ the value of γ such that $i = 0$ and $q = \hat{q}$ is $\gamma^1 = 1 + \beta(1-s)(\pi_1 + \pi_2)sc$. Further, $q \geq \hat{q}$ for $\gamma \in \left[\gamma^1, \gamma^2\right]$. Therefore, the numerator at the right-hand side in (72) is positive for $\gamma \in \left[\gamma^1, \gamma^2\right]$. From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side in (72) is negative for $\gamma \in \left[\gamma^1, \gamma^2\right]$. It follows that in a fully constrained equilibrium $\partial q/\partial \varepsilon < 0$ for $\gamma \in \left[\gamma^1, \gamma^2\right]$. Since $\phi \ell_h = \phi \ell_{f,\eta} = \phi \ell = sq$ for all η from (43), $\partial(\phi \ell)/\partial \varepsilon < 0$.

Differentiating (68) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{1-s}{1-\beta}\right)^{-1}=\left\{y'(q)-1+\pi_1\eta_1+\pi_2\eta_2-(\pi_1+\pi_2)(\varepsilon+sc)\right\}\frac{\partial q}{\partial \varepsilon}-(\pi_1+\pi_2)q.$$

Since $\eta_1,\eta_2>\varepsilon+sc$ and $\partial q/\partial\varepsilon<0$ from (72), it follows that $\partial\mathcal{W}/\partial\varepsilon<0$.

Proof of Proposition 5. For this proof we distinguish two cases, $\varepsilon+sc>\eta_2$ and $\varepsilon<\eta_1\leq\varepsilon+sc<\eta_2$. In the first case showing that $\partial\mathcal{W}/\partial\varepsilon>0$ is straightforward since the non-defaulter purchases only good h and hence does not incur conversion costs. For the second case it is shown that $\partial\mathcal{W}/\partial\varepsilon>0$ holds for π_2 sufficiently low.

Consider a fully constrained equilibrium in which $\lambda_h,\lambda_{f,\eta}>0$ and the borrowing constraint (33) holds with equality. As in the proof of Proposition 2, we can set $\phi\ell_h=\phi\ell_{f,\eta}=\phi\ell$ and $q_h=q_{f,\eta}=q$ for all η.

Case $\varepsilon+sc>\eta_2$.

Given (27), welfare defined in (67) becomes

$$\mathcal{W}\left(\frac{1-s}{1-\beta}\right)^{-1}=y(q)-q. \tag{73}$$

Differentiate the borrowing constraint for the case $\varepsilon+sc>\eta_2$ stated in (61) with respect to ε to get

$$\begin{aligned}&-s\frac{\partial i}{\partial\varepsilon}q+\frac{\beta(1-s)(\pi_1+\pi_2)}{1-\beta}\hat{q}-(1+si)\frac{\partial q}{\partial\varepsilon}\\&+\frac{\beta(1-s)}{1-\beta}\left[y'(q)-1\right]\frac{\partial q}{\partial\varepsilon}\\&=\frac{\beta(1-s)}{1-\beta}\left\{y'(\hat{q})-1+\left[\pi_1(\eta_1-\varepsilon)+\pi_2(\eta_2-\varepsilon)\right]\right\}\frac{\partial\hat{q}}{\partial\varepsilon}-\frac{\gamma-\beta}{1-\beta}\frac{\partial\hat{q}}{\partial\varepsilon}.\end{aligned} \tag{74}$$

Differentiating (59) with respect to ε yields

$$s\frac{\partial i}{\partial\varepsilon}=-(1-s)y''(q)\frac{\partial q}{\partial\varepsilon}. \tag{75}$$

Rewrite (74) using (59), (60) and (75)

$$\frac{\partial q}{\partial\varepsilon}=\frac{-\beta(1-s)(\pi_1+\pi_2)\hat{q}/(1-\beta)}{(1-s)y''(q)q+(\gamma-1-si)/(1-\beta)}. \tag{76}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (76) is negative. Since the numerator at the right-hand side of

(76) is also negative, it follows that in a fully constrained equilibrium $\partial q/\partial \varepsilon > 0$. Since $\phi \ell_h = \phi \ell_{f,\eta} = \phi \ell = sq$ for all η from (43), it follows that $\partial (\phi \ell) / \partial \varepsilon > 0$.

Differentiating (73) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon} \left(\frac{1-s}{1-\beta} \right)^{-1} = \left[y'(q) - 1 \right] \frac{\partial q}{\partial \varepsilon}. \tag{77}$$

Since $\partial q/\partial \varepsilon > 0$ from (76), (77) implies that $\partial \mathcal{W}/\partial \varepsilon > 0$.

Case $\varepsilon < \eta_1 \leq \varepsilon + sc < \eta_2$.

Given (27) and (43), welfare defined in (67) becomes

$$\mathcal{W} \left(\frac{1-s}{1-\beta} \right)^{-1} = y(q) - q + \pi_2 (\eta_2 - \varepsilon - sc) q. \tag{78}$$

Differentiate the borrowing constraint for the case $\varepsilon < \eta_1 \leq \varepsilon + sc < \eta_2$ stated in (53) with respect to ε to get

$$\begin{aligned} & -s \frac{\partial i}{\partial \varepsilon} q - \frac{\beta (1-s)}{1-\beta} \left[\pi_2 q - (\pi_1 + \pi_2) \hat{q} \right] \\ & - (1+si) \frac{\partial q}{\partial \varepsilon} + \frac{\beta (1-s)}{1-\beta} \left\{ y'(q) - 1 + \pi_2 \left[\eta_2 - \varepsilon - sc \right] \right\} \frac{\partial q}{\partial \varepsilon} \\ & = \frac{\beta (1-s)}{1-\beta} \left\{ y'(\hat{q}) - 1 + \left[\pi_1 (\eta_1 - \varepsilon) + \pi_2 (\eta_2 - \varepsilon) \right] \right\} \frac{\partial \hat{q}}{\partial \varepsilon} - \frac{\gamma - \beta}{1-\beta} \frac{\partial \hat{q}}{\partial \varepsilon}. \end{aligned} \tag{79}$$

Differentiating (51) with respect to ε yields

$$s \frac{\partial i}{\partial \varepsilon} = -(1-s) y''(q) \frac{\partial q}{\partial \varepsilon} + (1-s) \pi_2. \tag{80}$$

Rewrite (79) using (51), (52) and (80)

$$\frac{\partial q}{\partial \varepsilon} = \frac{(1-s) \pi_2 q + \beta (1-s) \left[\pi_2 q - (\pi_1 + \pi_2) \hat{q} \right] / (1-\beta)}{(1-s) y''(q) q + \left[\gamma - 1 - si - \beta (1-s) s \pi_2 c \right] / (1-\beta)}. \tag{81}$$

From the proof of Proposition 2, it can be deduced that the denominator at the right-hand side of (81) is negative. At $\gamma = \gamma^{1\prime}$, $q = \hat{q}$ and hence the numerator at the right-hand side of (81) is negative if $\pi_2 - \beta \pi_1 / (1-\beta) < 0$. Therefore, $\partial q/\partial \varepsilon > 0$ at $\gamma = \gamma^{1\prime}$ if π_2 is sufficiently low. Since q is increasing in ε as long as the numerator at the right-hand side of (81) is negative and $\hat{q}$ is decreasing in ε given (52), the numerator at the right-hand side of (81) is increasing in ε. Define $\gamma^{2\prime}$ the value of γ such that the numerator at the right-hand side of (81) is zero given $\{q, \hat{q}, i\}$

that solve (51), (52) and (53). In addition, let $\bar{\gamma}^2 = \min\left(\gamma^2, \gamma^{2\prime}\right)$. Then in a fully constrained equilibrium $\partial q/\partial \varepsilon > 0$ for $\gamma \in \left[\gamma^1, \bar{\gamma}^2\right]$. Since $\phi \ell_h = \phi \ell_{f,\eta} = \phi \ell = sq$ for all η from (43), $\partial\left(\phi \ell\right)/\partial \varepsilon > 0$ for $\gamma \in \left[\gamma^1, \bar{\gamma}^2\right]$.

Differentiating (78) with respect to ε yields

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{1-s}{1-\beta}\right)^{-1} = \left\{y'\left(q\right) - 1 + \pi_2\left(\eta_2 - \varepsilon - sc\right)\right\}\frac{\partial q}{\partial \varepsilon} - \pi_2 q. \tag{82}$$

Using (81) for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$ with $\gamma^{1\prime}$ stated in (54) it follows that

$$\frac{\partial \mathcal{W}}{\partial \varepsilon}\left(\frac{1-s}{1-\beta}\right)^{-1} > \frac{y'\left(q\right) - 1 + \pi_2\left(\eta_2 - \varepsilon - sc\right)}{-y''\left(q\right)}\left(\frac{\beta \pi_1}{1-\beta} - \pi_2\right) - \pi_2 q.$$

Since assumed productivities satisfy $-y''\left(q\right)q \leq y'\left(q\right)$ and $\eta_2 - \varepsilon - sc > 0$, a sufficient condition for $\partial \mathcal{W}/\partial \varepsilon > 0$ for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$ is

$$\frac{y'\left(q\right) - 1}{y'\left(q\right)}\left(\frac{\beta \pi_1}{1-\beta} - \pi_2\right) - \pi_2 > 0. \tag{83}$$

The left-hand side at (83) is positive at $\pi_2 = 0$ and given (51) is decreasing in π_2 for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Therefore there is a value $\bar{\pi}_2 > 0$ such that if $\pi_2 \leq \bar{\pi}_2$ the left-hand side in (83) is positive for $\gamma \in \left[\gamma^1, \gamma^{1\prime}\right]$. Since condition (83) is sufficient (but not necessary) there is $\hat{\pi}_2 > \bar{\pi}_2 > 0$ and $\hat{\gamma}^2 > \gamma^{1\prime}$ such that if $\pi_2 \leq \hat{\pi}_2$ then $\partial \mathcal{W}/\partial \varepsilon > 0$ for $\gamma \in \left[\gamma^1, \hat{\gamma}^2\right]$.

Proof of Proposition 8. We proceed in three steps to show that the amount of credit is decreasing in c. Consider two cases: $\varepsilon = 0$ and $\varepsilon > 0$. First, we show that q is decreasing in c from some value c_0 up to some value $c < \eta_1/s$ in the case $\varepsilon = 0$ and up to some value $c < \left(\eta_1 - \varepsilon\right)/s$ in the case $\varepsilon > 0$. To prove that an increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon + sc \leq \eta_1$ with $\varepsilon \geq 0$, differentiate the borrowing constraint stated in (46) with respect to c:

$$\begin{aligned} &-\left(si+1\right)\frac{\partial q}{\partial c} - sq\frac{\partial i}{\partial c} - \frac{\beta\left(1-s\right)}{1-\beta}\left(\pi_1 + \pi_2\right)sq \\ &+ \frac{\beta\left(1-s\right)}{1-\beta}\left\{y'\left(q\right) - 1 + \pi_1\eta_1 + \pi_2\eta_2 - \left(\pi_1 + \pi_2\right)\left(\varepsilon + sc\right)\right\}\frac{\partial q}{\partial c} = 0. \end{aligned} \tag{84}$$

From (44) we get

$$s\frac{\partial i}{\partial c} = -\left(1-s\right)y''\left(q\right)\frac{\partial q}{\partial c}. \tag{85}$$

Use (44) and (85) to rewrite (84) as follows

$$\frac{\partial q}{\partial c}=\frac{\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sq/\left(1-\beta\right)}{\left(1-s\right)y''\left(q\right)q+\left[\gamma-1-si-\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc\right]/\left(1-\beta\right)}. \tag{86}$$

From the proof of Proposition 2, in the case $\varepsilon+sc<\eta_1$ of a fully constrained equilibrium the denominator at the right-hand side in (86) is negative. Since in the fully constrained equilibrium $\ell_h=\ell_f^\eta=\ell$ for all η and $\phi\ell=sq$ from (43), it follows that $\partial\left(\phi\ell\right)/\partial c<0$ for $c<\left(\eta_1-\varepsilon\right)/s$. Since $\partial q/\partial c<0$ for all $c<\left(\eta_1-\varepsilon\right)/s$ it follows that, in the case $\varepsilon=0$, q and $(\phi\ell)$ are decreasing in c up to $c=\eta_1/s$ and, in the case $\varepsilon>0$, q and $\phi\ell$ are decreasing in c up to $c=\left(\eta_1-\varepsilon\right)/s$.

Second, we show that the function $q=q\left(c\right)$ is not continuous. For this, we evaluate the function $q=q\left(c\right)$ at a particular value of γ and infer that its properties hold for a range of values of γ. Consider the case $\varepsilon=0$. The function $q=q\left(c\right)$ jumps below at $c=\eta_1/s$; i.e., $q\left(c^-\right)>q\left(c^+\right)$ with $c^-=\eta_1/s-dc$, $c^+=\eta_1/s+dc$ and $dc\to0$. From (44) and (51), it follows that

$$\left(1-s\right)\left[y'\left(q\left(c^-\right)\right)+\pi_1\eta_1\right]+si\left(c^-\right)=\left(1-s\right)y'\left(q\left(c^+\right)\right)+si\left(c^+\right) \tag{87}$$

where $q\left(c^-\right)$ and $i\left(c^-\right)$ solve (46) and (44) (with $\hat{q}$ being determined by (45)), whereas $q\left(c^+\right)$ and $i\left(c^+\right)$ solve (53) and (51) (with $\hat{q}$ being determined by (52)). At $\gamma=\gamma^1\left(c^+,\varepsilon=0\right)=1+\beta\left(1-s\right)\pi_2sc^+ +\left(1-s\right)\pi_1\eta_1$, $i\left(c^+\right)=\left(1-s\right)\pi_1\eta_1/s$. Hence, at $\gamma=\gamma^1\left(c^+,\varepsilon=0\right)$ (87) becomes

$$\left(1-s\right)y'\left(q\left(c^-\right)\right)+si\left(c^-\right)=\left(1-s\right)y'\left(q\left(c^+\right)\right). \tag{88}$$

Note that $\gamma^1\left(c^+,\varepsilon=0\right)>\gamma^1\left(c^-,\varepsilon=0\right)=1+\beta\left(1-s\right)\left(\pi_1+\pi_2\right)sc^-$ provided that $\beta<1$. Thus, at $\gamma=\gamma^1\left(c^+,\varepsilon=0\right)$, $i\left(c^-\right)>0$ since $i\left(c^-\right)=0$ at $\gamma^1\left(c^-,\varepsilon=0\right)$ and $\partial i/\partial\gamma>0$ in a fully constrained equilibrium with $c<\eta_1/s$ from Proposition 2. Hence, from (88) $q\left(c^+\right)<q\left(c^-\right)$. It follows that the function is discontinuous at $c=\eta_1/s$ with $q\left(c^+\right)<q\left(c^-\right)$. Since all functions in (87) ($y'\left(q\left(c^-\right)\right)$ and $i\left(c^-\right)$ which solve (46) and (44), and $y'\left(q\left(c^+\right)\right)$ and $i\left(c^+\right)$ which solve (53) and (51)) are continuous, we can infer that there is a range of values of γ for which the function $q=q\left(c\right)$ is not continuous at $c=\eta_1/s$ with $q\left(c^+\right)<q\left(c^-\right)$. From (43), it follows that at $c=\eta_1/s$ the function $\phi\ell$ also jumps below.

Similarly, in the case $\varepsilon > 0$, the function $q = q(c)$ jumps below at $c = (\eta_1 - \varepsilon)/s$; i.e., $q(c^-) > q(c^+)$ with $c^- = (\eta_1 - \varepsilon)/s - dc$, $c^+ = (\eta_1 - \varepsilon)/s + dc$ and $dc \to 0$. From (44) and (51), it follows that

$$(1-s)\left[y\left(q\left(c^-\right)\right) + \pi_1(\eta_1 - \varepsilon)\right] + si\left(c^-\right) = (1-s)\, y\left(q\left(c^+\right)\right) + si\left(c^+\right) \tag{89}$$

where $q(c^-)$ and $i(c^-)$ solve (46) and (44) (with $\hat{q}$ being determined by (45)), whereas $q(c^+)$ and $i(c^+)$ solve (53) and (51) (with $\hat{q}$ being determined by (52)). At $\gamma = \gamma^1(c^+, \varepsilon > 0) = 1 + \beta(1-s)\pi_2 s c^+ + (1-s)\pi_1(\eta_1 - \varepsilon)$, $i(c^+) = (1-s)\pi_1(\eta_1 - \varepsilon)/s$. Hence, at $\gamma = \gamma^1(c^+, \varepsilon > 0)$ (89) becomes

$$(1-s)\, y\left(q\left(c^-\right)\right) + si\left(c^-\right) = (1-s)\, y\left(q\left(c^+\right)\right). \tag{90}$$

At $\gamma = \gamma^1(c^+, \varepsilon > 0)$, $i(c^-) > 0$ since $i(c^-) = 0$ at $\gamma^1(c^-, \varepsilon > 0)$, $\partial i/\partial\gamma > 0$ in a fully constrained equilibrium with $c < (\eta_1 - \varepsilon)/s$ from Proposition 2, and $\gamma^1(c^+, \varepsilon > 0) > \gamma^1(c^-, \varepsilon > 0)$. Thus, from (90) $q(c^+) < q(c^-)$. It follows that the function is discontinuous at $c = (\eta_1 - \varepsilon)/s$ with $q(c^+) < q(c^-)$. Since all functions in (89) are continuous, we can infer that there is a range of values of γ for which the function $q = q(c)$ is not continuous at $c = (\eta_1 - \varepsilon)/s$ with $q(c^+) < q(c^-)$. From (43), it follows that the function $\phi\ell$ also jumps below at $c = (\eta_1 - \varepsilon)/s$.

Third, we show that q is decreasing in c for $c > \eta_1/s$ in the case $\varepsilon = 0$ and for $c > (\eta_1 - \varepsilon)/s$ in the case $\varepsilon > 0$. To prove that this increase in c entails a decrease in credit in a fully constrained equilibrium in which $\varepsilon + sc > \eta_1$ with $\varepsilon \geq 0$, differentiate the borrowing constraint stated in (53) with respect to c:

$$\begin{aligned} &-(si+1)\frac{\partial q}{\partial c} - sq\frac{\partial i}{\partial c} - \frac{\beta(1-s)}{1-\beta}\pi_2 sq \\ &+ \frac{\beta(1-s)}{1-\beta}\left\{y'(q) - 1 + \pi_2(\eta_2 - \varepsilon - sc)\right\}\frac{\partial q}{\partial c} = 0. \end{aligned} \tag{91}$$

From (51) we get

$$s\frac{\partial i}{\partial c} = -(1-s)\, y''(q)\frac{\partial q}{\partial c}. \tag{92}$$

Use (51) and (92) to rewrite (91) as follows

$$\frac{\partial q}{\partial c} = \frac{\beta(1-s)\pi_2 sq/(1-\beta)}{(1-s)\, y''(q)\, q + [\gamma - 1 - si - \beta(1-s)\pi_2 sc]/(1-\beta)}. \tag{93}$$

As shown in the proof of Proposition 2, in a fully constrained equilibrium in the case $\varepsilon < \eta_1 \leq sc + \varepsilon < \eta_2$ the denominator at the right-hand side in (93) is negative, so $\partial q/\partial c < 0$. Since in the fully constrained equilibrium $\ell_h = \ell_{f,\eta} = \ell$ for all η and $\phi\ell = sq$ from (43), it follows that $\partial(\phi\ell)/\partial c < 0$.

Finally, from Proposition 4 for $\varepsilon + sc < \eta_1$, in a fully constrained equilibrium q and $\phi\ell$ are decreasing in ε. In addition, from Proposition 5 for $sc > \eta_1$, there is a range of values of γ for which q and $\phi\ell$ are increasing in ε. Then it is straightforward to verify that if c increases from c_0 to c_1 and a fully constrained equilibrium exists for c_0 and c_1 for this range of values of γ, the decrease in q and $\phi\ell$ is stronger in the case $\varepsilon = 0$ than in the case $\varepsilon > 0$.

Differentiating (68) with respect to c yields

$$\frac{\partial \mathcal{W}}{\partial c}\left(\frac{1-s}{1-\beta}\right)^{-1} = \left\{y'(q) - 1 + \pi_1\eta_1 + \pi_2\eta_2 - (\pi_1 + \pi_2)(\varepsilon + sc)\right\}\frac{\partial q}{\partial c} - (\pi_1 + \pi_2)\, sq.$$

Thus $\partial \mathcal{W}/\partial c > 0$ for all $sc < \eta_1 - \varepsilon$ since $\partial q/\partial c < 0$ for $sc < \eta_1 - \varepsilon$. Similarly, after differentiating (78) it is straightforward to verify that $\partial \mathcal{W}/\partial c < 0$ for all $\varepsilon < \eta_1 \leq sc + \varepsilon < \eta_2$ since $\partial q/\partial c < 0$ in this case as well. Further, since $q(c^+) < q(c^-)$ for $c^- = (\eta_1 - \varepsilon)/s - dc$, $c^+ = (\eta_1 - \varepsilon)/s + dc$ and $dc \to 0$, comparison of (68) and (78) demonstrates that welfare at c^+ is lower than welfare at c^-.