Fatter attraction: anthropometric and socioeconomic matching on the marriage market*

Pierre-André Chiappori
Columbia University

Sonia Oreffice
Universitat d'Alacant and IZA

Climent Quintana-Domeque
Universitat d'Alacant and IZA

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Abstract

We construct a marriage market model of matching along multiple dimensions, some of which unobservable, where individual preferences can be summarized by a one-dimensional index combining the various characteristics. We show that, under reasonable assumptions, these indices are ordinally identified, and that the male and female trade-offs between their partners' characteristics are overidentified. Using PSID data on spouses' physical and socioeconomic attributes, we recover the marginal rates of substitutions between body mass index (BMI) and wages or education: Men may compensate a 10%-increase in BMI with a 2%-increase in wages. For women, an additional year of education may compensate up to two BMI units.

Keywords: marriage market, multidimensional matching, trade-offs, body mass index, wage/education.

JEL Codes: D1, J1.

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1 Introduction

The analysis of matching patterns in the population has recently attracted considerable attention, from both a theoretical and an empirical perspective. Most models focus on exactly one characteristic on which the matching process is assumed to be exclusively based. Various studies have thus investigated the features of assortative matching on income, wages or education (e.g., Becker, 1991; Choo and Siow, 2006; Pencavel, 1998), but also on such preference-based notions as risk aversion (e.g., Chiappori and Reny, 2004; Legros and Newman, 2007) or desire to have a child (Chiappori and Oreffice, 2008).

One-dimensional matching models offer several advantages. Their formal properties are by now well established. In a transferable utility context, they provide a simple and elegant way to explain the type of assortative matching patterns that are currently observed; namely, the stable match is positive (negative) assortative if the surplus function is super (sub) modular. Moreover, it is possible, from the shape of the surplus function, to recover the equilibrium allocation of resources within each match, a feature that proves especially useful in many theoretical approaches. Arguments of this type have been applied, for instance, to explain why female demand for university education may outpace that of men (Chiappori, Iyigun and Weiss, 2009), or how women unwilling to resort to abortion still benefited from its legalization (Chiappori and Oreffice, 2008).

These advantages, however, come at a cost. The transferable utility assumption generates strong restrictions. For instance, the efficient decision at the group level does not depend on the distribution of Pareto weights within the group. This implies not only that the group behaves as a single individual – a somewhat counterfactual statement, as illustrated by numerous empirical studies – but also that a redistribution of powers, say to the wife, cannot by assumption alter the group's aggregate behavior. Secondly, matching models with supermodular surplus can only predict *perfectly* assortative matching – while reality is obviously much more complex, if only because of the role played by chance (or unobservable factors) in the assignments. Thirdly, and more importantly, empirical evidence strongly suggests that, in real life, matching processes are actually multidimensional; spouses tend to be similar in a variety of characteristics, including age, education, race, religion, and anthropometric characteristics such as weight or height (e.g., Becker, 1991; Hitsch, Hortaçsu, and Ariely, 2010; Oreffice and

Quintana-Domeque, 2010; Qian, 1998; Silventoinen et al., 2003; Weiss and Willis, 1997). Sexual selection studies in biology and evolutionary psychology analyze trade-offs between mate attributes, and point to the relative importance of several fitness indicators (e.g., Miller, 2000; Rodriguez-Muñoz et al., 2010).

Each of these criticisms has, in turn, generated further research aimed at addressing the corresponding concerns. Models of frictionless matching without transferable utility have been developed by Chiappori and Reny (2004) and Legros and Newman (2007). Following the seminal theoretical contribution by Shimer and Smith (2000), several empirical studies (e.g., Choo and Siow, 2006) introduce randomness into the matching process, to account for the deviations from perfectly assortative matching that characterize actual data. Hitsch et al. (2010), working on online dating, introduce several dimensions by modeling individual utility as a linear valuation of the mates' attributes within a Gale-Shapley framework (in which transfers between mates are ruled out). However, they lack the relevant information on the matches actually formed. Furthermore, Galichon and Salanié (2009) explicitly model multidimensional matching in a frictionless framework under transferable utility.

The goal of the present paper is to *simultaneously* address the concerns described above. We investigate the relative importance of *multiple* characteristics on the marriage market, and the way men and women assess them. In addition, we assume that some of the relevant characteristics are not observable to the econometrician; as a consequence, the matching process is partly random, at least from an exterior perspective, and does not result in a perfectly assortative outcome. Finally, we do not focus on a specific setting or matching game. Our approach is compatible with a large variety of matching mechanisms, including frictionless models with and without transferable utility, random matching à la Shimer and Smith, search models and others.

We consider a model in which individual 'attractiveness' on the marriage market is fully determined by a set of (observable and non-observable) characteristics. Our framework relies on two crucial assumptions. One is that attractiveness is separable in the observable variables, in the sense that it depends on these variables only through some (unknown) index. Secondly, *conditional on the same indices*, the distributions of observables and unobservables are independent. We show that, under these assumptions, it is possible to non-parametrically identify the form of the relevant indices up to some increasing transform. Therefore, one can non-parametrically recover the trade-offs between the various observable dimensions that characterize each individual. Technically, the index we postulate allows to define 'iso-attractiveness profiles' and marginal rates of substitution (MRSs) between the various individual characteristics. We show that these profiles are ordinally identified and the MRSs are exactly identified from the matching patterns. In addition, we derive a host of overidentifying restrictions on the MRSs. These restrictions can be tested regardless of the non-linearity or non-monotonicity of the index; in particular, our overidentifying restrictions apply even when the MRSs vary with individual characteristics in arbitrary ways. Deriving how men and women trade-off their partners' characteristics, and showing how to estimate these trade-offs is the main contribution of this paper.

We apply our approach to marital trade-offs in the United States, using data from the Panel Study of Income Dynamics (PSID) from 1999 to 2007, which contain anthropometric and socioeconomic characteristics of married men and women. We proxy a man's socio-economic status by his wage; for women, since participation is a serious issue (a significant fraction of females in our sample do not work), we use education as our main socioeconomic variable. Regarding anthropometric characteristics, the PSID provides data on individual weight and height, which we use to construct the individual body mass index (BMI)[1], our main proxy for physical attractiveness. We identify the trade-offs between economic and physical dimensions: For women, an additional year of education may compensate up to two BMI units, and men may compensate a 10%-increase in BMI with a 2%-increase in wages. Interestingly, male physical attractiveness matters as well.

Our work is also linked to a large economic research agenda on the effects of anthropometric measures. Many economists have been working on assessing the effects of BMI, height, and weight on labor-market outcomes. The consensus is that BMI in the overweight or obese range has negative effects on the probability of employment and on hourly wages, particularly for women (e.g., Cawley, 2004; Garcia and Quintana-Domeque, 2007; Rooth, 2009), while height has a positive effect on hourly wages (e.g., Case and Paxson, 2008; Lundborg, Nystedt, Rooth, 2009).

A related body of literature using National Longitudinal Survey of Youth data links

[1]BMI is defined as individual's body weight (in kilograms) divided by the square of his or her height (in meters).

women's weight to lower spousal earnings or lower likelihood of being in a relationship (Averett and Korenman, 1996; Averett, Sikora and Argys, 2008; Mukhopadhyay, 2008). However, these data provide anthropometric measures of the respondent only, so that the weight-income trade-off across spouses is estimated without controlling for the men's physical attributes. The same can be said about the influential work by Hamermesh and Biddle (1994), which shows that physically unattractive women are matched with less educated husbands. Indeed, assortative mating in body weights has been established in the medical and psychological literatures, which also document the importance of examining the effect of both spouses' characteristics on their marriage (e.g., Jeffrey and Rick, 2002; McNulty and Neff, 2008). More recently, Oreffice and Quintana-Domeque (2011), in a collective labor supply framework, find evidence that men and women who are heavier than their spouses tend to work more hours.

The paper is organized as follows. Section 2 presents the general setting on which our approach is based, and the intuitions for the main results. Section 3 contains a formal analysis. Section 4 specifies the econometric model. Section 5 discusses how to measure the attractiveness dimensions that mates care about. Section 6 describes the data used in the empirical analysis, and documents preliminary evidence on the observed matching patterns. Section 7 provides the main empirical results. Section 8 considers some extensions. Finally, Section 9 concludes.

2 The model: general setting and main intuitions

2.1 Matching and search

We consider a finite population of men and a finite population of women, of respective sizes N_m and N_w. Each potential husband, say $i \in B$, is characterized by a vector $Y_i = \left(Y_i^1, ..., Y_i^K\right) \in R^K$ of observable characteristics, and by some vector of unobservable characteristics $\eta_i \in R^N$; similarly, woman $j \in G$ is defined by a vector of observable variables $X_j = \left(X_j^1, ..., X_j^L\right) \in R^L$ and some unobservable characteristics $\varepsilon_j \in R^N$, where the random components η and ε are drawn from continuous and atomless distributions.[2] Let X (resp. Y) denote the space of female (male) characteristics; i.e., a typical element of $\mathcal{X}$ (resp. $\mathcal{Y}$) is a vector (X, ε) $((Y, \eta))$.

[2] Ths assumption can however be slightly relaxed; we only need that *one* compnent (at least) of each vector is drawn from an atomless distribution.

Similarly, we define $\mathcal{X}_C$ (resp. $\mathcal{Y}_C$) as the space of observable female (male) characteristics; i.e., a typical element of $\mathcal{X}$ (resp. $\mathcal{Y}$) is a vector (X) $((Y))$. Finally, to allow for the possibility that some agents choose not to marry, we define the augmented spaces $\mathcal{X}^A := \mathcal{X} \cup \{\emptyset_X\}$ and $\mathcal{Y}^A := \mathcal{Y} \cup \{\emptyset_Y\}$ by including an isolated point in each - a partner $\emptyset_X$ for any unmatched man and a partner $\emptyset_Y$ for any unmatched woman. We can similarly augment the spaces of observable characteristics to $\mathcal{X}_C^A$ and $\mathcal{Y}_C^A$.

Men and women match on the marriage market, according to some mechanism. An interesting property of our approach is that we do not need to specify the particular matching process at stake; our technology applies to a number of different frameworks. These include:

1. Frictionless matching without transferable utility (NTU). If Ms. j, characterized by the vector (X_j, ε_j), is matched with Mr. i, characterized by the vector (Y_i, η_i), she (resp. he) derives a gain equal to:

   $$W_{ij} = \Psi\left(Y_i, \eta_i, X_j, \varepsilon_j\right) \quad \left(\text{resp. } M_{ij} = \Phi\left(Y_i, \eta_i, X_j, \varepsilon_j\right)\right)$$

   As always, a matching is stable if (i) no married person would rather remain single, and (ii) one cannot find two individuals who would *both* rather be married together than remain in their current situation.

   Technically, the matching problem is defined in this context by the distributions of characteristics in the male and female populations and the two functions Φ and Ψ. A matching is a measure γ on the product space $\mathcal{X}^A \times \mathcal{Y}^A$, the marginals of which coincide with the initial distributions on each set; intuitively, $\gamma\left(Y_i, \eta_i, X_j, \varepsilon_j\right)$ denotes the probability that a man with characteristics (Y_i, η_i) is matched to a woman with characteristics (X_j, ε_j). Note that the measure can be degenerate, in the sense that the matching can be deterministic: for (almost) all (Y_i, η_i) there exists exactly one (X_j, ε_j) to which (Y_i, η_i) is matched with probability 1, and conversely. For instance, with finite populations, there is always a deterministic stable match.

2. Frictionless matching with transferable utility (TU). Now, a match of Mr. i and Ms. j

generates a *total* surplus of the form:

$$S_{ij} = \Gamma\left(Y_i, \eta_i, X_j, \varepsilon_j\right)$$

that has to be shared between the spouses. The matching problem is again defined by the distributions of characteristics in the male and female populations and the surplus function Γ. A matching consists of a measure γ on the product space $\mathcal{X}^A \times \mathcal{Y}^A$ *and* of two functions $u\left(Y_i, \eta_i\right)$ and $v\left(X_j, \varepsilon_j\right)$ such that:

$$u\left(Y_i, \eta_i\right) + v\left(X_j, \varepsilon_j\right) = \Gamma\left(Y_i, \eta_i, X_j, \varepsilon_j\right) \quad \text{for all } \left(Y_i, \eta_i, X_j, \varepsilon_j\right) \text{ in the support of } \gamma$$

Here, $u\left(Y_i, \eta_i\right)$ (resp. $v\left(X_j, \varepsilon_j\right)$) is the utility received by Mr. $\left(Y_i, \eta_i\right)$ (resp. Ms. $\left(X_j, \varepsilon_j\right)$) at the stable match; they are endogenously determined at the equilibrium.

Stability is defined in the usual way; under TU, however, a matching is stable if and only if it maximizes the total aggregate surplus

$$S = \int_{\mathcal{X}^A \times \mathcal{Y}^A} \Gamma\left(Y_i, \eta_i, X_j, \varepsilon_j\right) d\gamma\left(Y_i, \eta_i, X_j, \varepsilon_j\right)$$

This property guarantees existence under mild conditions (see for instance Chiappori, McCann, and Nesheim, 2010).

3. Frictionless matching with imperfectly transferable utility. Unlike the previous case, the surplus is shared in a non-linear way; i.e., there exists a function Θ such that:

$$u\left(Y_i, \eta_i\right) = \Theta\left(Y_i, \eta_i, X_j, \varepsilon_j, v\left(X_j, \varepsilon_j\right)\right) \quad \text{for all } \left(Y_i, \eta_i, X_j, \varepsilon_j\right)$$

but Θ needs not be additively separable in $v\left(X_j, \varepsilon_j\right)$.

4. Search models. Finally, frictions can be introduced in the matching technology. Specifically, in the matching under a transferable utility framework, one can introduce the search component that agents meet randomly, and that at any meeting each agent must decide whether to accept the current match, or decline and resume searching – at the cost of a (random) waiting time. The matching problem is still defined by the distribu-

tions of characteristics in the male and female populations, and the surplus function, but now also by the meeting technology. One may, for instance, follow Shimer and Smith (2000) and assume that the meeting rate is proportional to the mass of those unmatched, and that any existing match is destroyed with some (exogenous) probability, although none of these assumptions is crucial. At any rate, the outcome of such a model is now typically random: any (Y_i, η_i) is matched with positive probability to several (possibly a continuum of) (X_j, ε_j) – and conversely – due to the randomness introduced by the meeting (and separation) technology. Again, a search equilibrium results in a measure γ on the product space $\mathcal{X}^A \times \mathcal{Y}^A$.

These various settings each lead to specific equilibrium concepts. Our approach applies to all of these, which highlights its robustness, although it comes at the cost of not empirically distinguishing between these various models.

Two remarks are in order. First, in all these contexts, it is implicitly assumed that the probability that a man i and a woman j match (including, in the search version, the probability that they meet) depends only on their characteristics; in other words, the vectors (Y_i, η_i) and (X_j, ε_j) provide an exhaustive definition of the matching-relevant characteristics. Second, remember that the η_i and ε_j are not observable. From an econometrician's perspective, therefore, the observed matching patterns will always look random, even though the actual match may be deterministic. That is, Mr. (Y_i, η_i) may actually be matched with probability one to Ms. (X_j, ε_j); but the econometrician only observes that several individuals, all characterized by the same vector Y_i of observables (although probably by different unobservable vectors η_i), end up being matched with women with different vectors X_j. This remark will be crucial in the empirical work that follows.

2.2 Two crucial assumptions

We now introduce our key assumptions. The first concerns observable characteristics.

Assumption S (Separability) *The observable characteristics* $Y = \left(Y^1, ..., Y^K\right)$ *(resp.* $X = \left(X^1, ..., X^L\right)$*) only matter through a one-dimensional index* $I = I\left(Y^1, ..., Y^K\right)$ *(resp.* $J = J\left(X^1, ..., X^L\right)$*).*

In the next Section, we shall specialize this assumption for different, specific contexts;

essentially, we shall assume that the various functions introduced above (Φ and Ψ, Γ, or Θ, depending on the theoretical context) are weakly separable in Y and in X. More intuitively, the assumption implies the existence of two 'attractiveness indices' – one for men, one for women – so that the impact of a spouse's observable characteristics on the couple's welfare is fully summarized by their corresponding index. This is a strong assumption; it suggests that all individuals have similar 'tastes' regarding the opposite sex – technically, they trade-off the various observable components at the same rate. Note, however, that we do *not* assume monotonicity; the index may well be non-monotonic in the attributes. Also, the index needs not be linear; in particular, the 'marginal rate of substitution' (MRS) between the k-th and l-th characteristics in the male index, defined as $-\left(\partial I/\partial Y^k\right)/\left(\partial I/\partial Y^l\right)$, may take different values for different profiles of characteristics.

The separability assumption has an immediate application, which can be intuitively described as follows. Assume that two males, i and i', have different vectors of observable characteristics ($Y_i \neq Y_{i'}$), but the same index ($I(Y_i) = I(Y_{i'})$). If they are endowed with the same vectors of unobservables ($\eta_i = \eta_{i'}$), they are perfect substitutes on the marriage market: any woman will be indifferent between marrying one or the other.

We now shift our attention to unobservables characteristics. This is a crucial issue, because the econometrician will never be able to know whether two agents are endowed with the same vectors of unobservables. Therefore, the conditional distribution of unobservables given the observables will play a key role in any empirical assessment. We therefore introduce our second assumption:

Assumption CI (Conditional Independence) *Conditional on the index $I = I\left(Y^1, ..., Y^K\right)$ (resp. $J = J\left(X^1, ..., X^L\right)$) the distribution of η (resp. ε) is atomless and independent of $\left(Y^1, ..., Y^K\right)$ (resp. $\left(X^1, ..., X^L\right)$).*

In words, Assumption CI states that the conditional distribution of η and ε given the observables only depends on the respective indices. If two males i and i' have the same index, then their respective unobservable characteristics are drawn from the same distribution. Conditional independence is weaker than independence, which is often assumed in the empirical work on matching and search: the distribution of unobservables may depend on the vector of observables, but only through the index.

Coming back to our two males with different characteristics but the same index, Assumption CI introduces an additional requirement – namely, that they are equally likely to draw any specific vector of unobservables. In that case, we may expect that they are 'equally likely' to marry any given woman – i.e., that they have the same probability distribution of potential spouses. Of course, a more precise statement requires a formal description of the stochastic structure implicit in the notion of 'equally likely'. This is provided in the next Section, in two formal versions of the model dealing respectively with transferable and non-transferable utility.

2.3 The additively separable model

At this point, it is useful to check that the assumptions just introduced are compatible. Is there a model that would satisfy them? The question is especially relevant because the list of observable variables may vary across data sets; a given characteristic may belong to the observable vector Y (resp. X) in some cases, and to the unobservable vector η (resp. ε) in others. Is this setting compatible with separability and conditional independence? The answer is clearly positive. The simplest model that satisfies our assumptions is probably the additively separable one. In this case, the relevant functions (Φ and Ψ, Γ, or Θ) depend on two sums, respectively characterizing the male and the female partners. For a man with characteristics $Y_i = \left(Y_i^1, ..., Y_i^K\right)$ and $\eta = \left(\eta_i^1, ..., \eta_i^N\right)$, the sum has the form:

$$M_i = \sum_k m_k \left(Y_i^k\right) + \sum_n \mu_n \left(\eta_i^n\right)$$

and similarly for women:

$$W_j = \sum_l w_l \left(X_j^l\right) + \sum_n \omega_n \left(\varepsilon_j^n\right)$$

for some functions m, μ, w and ω; again, these functions need not to be linear or even monotonic. Note that these forms can be seen as first order approximations of more complex expressions; in this regard, the main issue is now the empirical relevance of this approximation, a question that will be addressed in the next sections. Of course, any index may, without loss of generality, be replaced by an increasing function of itself. For instance, one could equivalently refer to multiplicatively separable versions:

$$M_i' = \prod_k m_k \left(Y_i^k\right) \prod_n \mu_n \left(\eta_i^n\right), W_j' = \prod_l w_l \left(X_j^l\right) \prod_n \omega_n \left(\varepsilon_j^n\right)$$

It is well known that additive separability implies weak separability with respect to all subsets of variables, so Assumption S is satisfied irrespective of the particular division between observables and non-observables. Regarding Assumption CI, independence between Y and η (resp. X and ε) is sufficient. Also, remember that the surplus can be any function of the indices. If this function is strictly supermodular, for instance, only matches that are strictly assortative with respect to the indices can be stable.

2.4 Property of the equilibrium: an intuitive presentation

We have previously discussed that our setting is compatible with several theoretical frameworks. A common feature is that the corresponding equilibrium is characterized, among other things, by a (possibly degenerate) distribution γ on the product space $\mathcal{X}^A \times \mathcal{Y}^A$, the marginals of which coincide with the initial distributions on each set. Integrating over the unobservables generates a new distribution μ over the product space $\mathcal{X}_C^A \times \mathcal{Y}_C^A$, where $\mathcal{X}_C^A$ (resp. $\mathcal{Y}_C^A$) is the augmented space of female (male) *observable* characteristics.

While the exact implications of our two assumptions obviously depend on the context (and will be discussed in the next Section), one can give a general intuition of their common content. The key idea is that, in all cases, there exists an equilibrium (or a stable matching) for which the measure $\mu\left(Y^1, ..., Y^K, X^1, ..., X^L\right)$ has the form:

$$\mu\left(Y^1, ..., Y^K, X^1, ..., X^L\right) = \nu\left[I\left(Y^1, ..., Y^K\right), J\left(X^1, ..., X^L\right)\right] \tag{1}$$

for some measure ν on $\mathbb{R}^2$. The crucial property, here, is that the conditional distribution of $\left(X^1, ..., X^L\right)$ given $\left(Y^1, ..., Y^K\right)$ only depends on the value $I\left(Y^1, ..., Y^K\right)$; similarly, the conditional distribution of $\left(Y^1, ..., Y^K\right)$ given $\left(X^1, ..., X^L\right)$ only depends on the value $J\left(X^1, ..., X^L\right)$. In other words, the index I, which only depends on observables, is a sufficient statistic for the distribution of characteristics of a man's spouse; and the same holds with index J for women. This property simply reflects the fact that, from a male's viewpoint, two women j and j' with different profiles $\left(X_j^1, ..., X_j^L\right)$ and $\left(X_{j'}^1, ..., X_{j'}^L\right)$ but identical indices $J\left(X_j^1, ..., X_j^L\right) = J\left(X_{j'}^1, ..., X_{j'}^L\right)$ offer equivalent marital prospects. Any difference between the distributions of their mates' respective profiles must therefore be driven by the unobservable characteristics. Since all agents with the same index have the same distribution of unobservables by Assumption CI, the two distributions are identical.

Formal statements will be provided in specific contexts in the next Section. Let us explore, for the time being, their intuitive implications. Essentially, it is in general possible, from data on matching patterns, to (ordinally) identify the underlying attractiveness indices. Indeed, consider the distribution of a wife's characteristics, *conditional on the vector of characteristics of the husband*. This distribution only depends on the index $I\left(Y_i^1, ..., Y_i^K\right)$. It follows, in particular, that *any of its moments* only depends on the index. For instance, the expected value of the sth characteristic of the wife, conditional on the vector of characteristics of the husband, is of the form:

$$E\left[X^s \mid Y_i^1, ..., Y_i^K\right] = \phi_s\left[I\left(Y_i^1, ..., Y_i^K\right)\right] \tag{2}$$

for some function ϕ_s. The same is true for the variance, the median, any covariance, etc.

This remark, in turn, has two consequences. One is that the function I is identified up to some transform (ϕ_s in equation (2)). It follows that the trade-off between various characteristics can easily be recovered. Since attractiveness is fully summarized by the indices I and J, we can define 'iso-attractiveness' profiles, i.e., profiles of observable characteristics that generate the same (distribution of) attractiveness. These are defined, for men, by $I\left(Y_i^1, ..., Y_i^K\right) = C$, where C is a constant, and similarly for women by $J\left(X_j^1, ..., X_j^L\right) = C'$. Assuming I and J to be differentiable, the marginal rate of substitution between characteristics r and t can be defined (for male i) by:

$$MRS_i^{r,t} = \frac{\partial I/\partial Y^t}{\partial I/\partial Y^r}$$

where the partials are taken at $\left(Y_i^1, ..., Y_i^K\right)$ (and a similar definition can be given for women). From (2), these MRSs are also equal to:

$$\frac{\partial I/\partial Y^t}{\partial I/\partial Y^r} = \frac{\partial E\left[X^s \mid Y_i^1, ..., Y_i^K\right]/\partial Y^t}{\partial E\left[X^s \mid Y_i^1, ..., Y_i^K\right]/\partial Y^r}, \tag{3}$$

and the right-hand side of this equation can be recovered from the data; therefore the MRSs are exactly identified. In addition, this property generates a host of overidentifying restrictions. Indeed, the left-hand side of the expression above does not depend on s, so neither should the right-hand side. Moreover, the sth conditional mean could be replaced with any moment of the (joint) distribution; again, the ratio should remain unchanged when varying s.

2.5 Uniqueness of the equilibrium

Finally, we discuss uniqueness issues. Here, the conclusion depends on the specific model under consideration. Take, for instance, a search model. There, uniqueness cannot be expected to hold, even with a finite number of agents: because of frictions, for any male there exists in general a set of females with whom he can be matched at equilibrium (and conversely), and the final outcome depends on the (random) meeting technology.[3] In the case of frictionless matching without transferable utility, we know that the stable match needs not be unique, even with a finite number of agents; and the same conclusion holds with imperfectly transferable utility, since the existence proof in that case relies on a generalization of the Gale-Shapley algorithm (e.g., Kelso and Crawford, 1982; Chiappori and Reny, 2004).

The case of frictionless matching with transferable utility is different. To see why, assume that the surplus function $\Gamma(Y_i, \eta_i, X_j, \varepsilon_j)$ is such that, for any i, k, j, l, X and Y, the partials $\partial\Gamma/\partial\eta_i^k$ and $\partial\Gamma/\partial\varepsilon_j^l$ are non zero outside of a set of measure zero - an assumption that we maintain in what follows.[4] This implies that the probability (over the draw of η and ε) that two males i and i', when matched with the same female j, generate the same surplus is zero. In that case, for almost all realizations of the draw, the measure associated to a stable matching (which defines who marries whom) is unique. Indeed, it is well known that for any stable matching, the corresponding measure maximizes aggregate surplus over the set of measures on the product space $\mathcal{X}^A \times \mathcal{Y}^A$ with the same marginals. For a finite set of agents, the set of such measures is itself finite, and for each of them the value of the aggregate surplus is a continuous random variable; the probability that two such variables take exactly the same

[3] Specifically, an agent's optimal strategy typically involves a threshold; they will marry any person they meet whose 'quality' exceeds the threshold. Therefore, the matching actually realized at equilibrium depends on the realization of the random meetings.

[4] Again, this property can be relaxed: it suffices that the partial with respect to one component of η_i that has an atomless distribution and the partial with respect to one component of ε_j that has an atomless distribution be non zero outside a set of measure zero.

value is zero. Note that the marital patterns – who marries whom – are (generically) exactly pinned down by the equilibrium conditions, while the way the surplus is shared in each couple is not – a spouse's share is simply bounded above and below by the equilibrium conditions. Clearly, the uniqueness of the measure is the relevant concept here, since we only observe marital patterns.

3 A formal analysis

We now provide a formal translation of the intuitions described above. This can be done only on specific models. We will consider two frameworks, involving respectively non-transferable and transferable utility.

3.1 Non-transferable utility

The notations are as above: if Ms. j, characterized by the vector (X_j, ε_j), is matched with Mr. i, characterized by the vector (Y_i, η_i), she (resp. he) derives a gain equal to:

$$W_{ij} = \Psi\left(Y_i, \eta_i, X_j, \varepsilon_j\right) \quad \text{(resp. } M_{ij} = \Phi\left(Y_i, \eta_i, X_j, \varepsilon_j\right)\text{)}$$

If Ms. j (resp. Mr. i) remains single then her (his) utility is $W_{0j} = \Psi_0\left(X_j, \varepsilon_j\right)$ $(M_{i0} = \Phi_0\left(Y_i, \eta_i\right))$.

In our finite setting, we may without loss of generality define a matching as a mapping $\mathcal{F}$ from $\mathcal{X}^A$ to $\mathcal{Y}^A$. We first specialize our separability assumption for that case:

Assumption S' *The functions Φ, Φ_0, Ψ and Ψ_0 are weakly separable in $X = \left(X^1, ..., X^L\right)$ and $Y = \left(Y^1, ..., Y^K\right)$; i.e., there exist two functions $I = I\left(Y^1, ..., Y^K\right)$ and $J = J\left(X^1, ..., X^L\right)$ such that:*

$$\begin{aligned}
\Phi\left(Y_i, \eta_i, X_j, \varepsilon_j\right) &= \tilde{\Phi}\left(I\left(Y_i^1, ..., Y_i^K\right), \eta_i, J\left(X_j^1, ..., X_j^L\right), \varepsilon_j\right) \\
\Psi\left(Y_i, \eta_i, X_j, \varepsilon_j\right) &= \tilde{\Psi}\left(I\left(Y_i^1, ..., Y_i^K\right), \eta_i, J\left(X_j^1, ..., X_j^L\right), \varepsilon_j\right) \\
\Psi_0\left(X_j, \varepsilon_j\right) &= \tilde{\Psi}_0\left(J\left(X_j^1, ..., X_j^L\right), \varepsilon_j\right) \\
\Phi_0\left(Y_i, \eta_i\right) &= \tilde{\Phi}_0\left(I\left(Y_i^1, ..., Y_i^K\right), \eta_i\right)
\end{aligned} \qquad (4)$$

for some $\tilde{\Phi}, \tilde{\Psi}, \tilde{\Phi}_0$ *and* $\tilde{\Psi}_0$.

Clearly, the observable marital patterns at a stable matching depend on the draw of the unobservable components $\varepsilon = \{\varepsilon_j, j = 1, ..., N_w\}$ and $\eta = \{\eta_i, i = 1, ..., N_m\}$. For any draw, (4) defines a NTU matching problem, for which one stable matching (at least) exists. If the problem has several stable matches, then we select one of them, say the one who is preferred by the female population.[5] For any such stable matching $\mathcal{F}$, we can consider the projection $\mathcal{G}$ of $\mathcal{F}$ over the augmented spaces of observable characteristics, defined as follows. Take any mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$ and any draw $(\varepsilon, \eta) = \left(\varepsilon_1, ..., \varepsilon_{N_w}, \eta_1, ..., \eta_{N_m}\right)$. We say that the mapping $\mathcal{G}$ is stable-compatible for the draw (ε, η) if the female preferred stable matching $\mathcal{F}$ of the matching problem thus defined is such that $(Y_i = \mathcal{G}(X_j), \eta_i) = \mathcal{F}(X_j, \varepsilon_j)$ for all i, j. By extension, we say that the mapping $\mathcal{G}$ is stable-compatible if there exists at least one draw (ε, η) for which $\mathcal{G}$ is stable-compatible. In words, $\mathcal{G}$ is stable-compatible if one can find a draw such that, in the matching problem thus defined, (X_j, ε_j) is matched with $(Y_i = \mathcal{G}(X_j), \eta_i)$ for all i, j at the female-preferred stable matching.

This defines a probability measure μ over the (finite) set of possible mappings of observable characteristics; i.e., we define the probability of a mapping $\mathcal{G}$ being stable-compatible by the measure of the set of draws for which $\mathcal{G}$ is stable-compatible. Finally, we define the probability that a particular vector X_j of observable female characteristics is matched with a particular vector Y_i of observable male characteristics at a stable matching by the measure of the set of stable-compatible mappings $\mathcal{G}$ such that $Y_i = \mathcal{G}(X_j)$.

We can now state the main result of this subsection:

Proposition 1 *Assume that Assumptions CI and S' are satisfied. Take any two vectors* $X_j = \left(X_j^1, ..., X_j^L\right)$ *and* $X_{j'} = \left(X_{j'}^1, ..., X_{j'}^L\right)$ *of female observable characteristics, such that* $J(X_j) = J\left(X_{j'}\right)$. *Then for any vector* Y_i *of male observable characteristics, the probability that* X_j *is matched with* Y_i *at a stable matching is equal to the probability that* $X_{j'}$ *is matched with* Y_i *at a stable matching.*

Similarly, for any two vectors $Y_i = \left(Y_i^1, ..., Y_i^L\right)$ *and* $Y_{i'} = \left(Y_{i'}^1, ..., Y_{i'}^L\right)$ *of male observable characteristics, such that* $I(Y_i) = I(Y_{i'})$ *and for any vector* X_j *of female observable character-*

[5] The existence and generic uniqueness of such a match is well known (see Gale and Shapley, 1962). Alternative choices are of course possible; for instance, one may select the matching preferred by males, or randomize over the (finite) set of stable matches. All of the conclusions below would remain valid.

istics, the probability that Y_i is matched with X_j at a stable matching is equal to the probability that $Y_{i'}$ is matched with X_j at a stable matching.

Proof. *For obvious symmetry reasons, it is sufficient to prove the first statement. The proof relies on the following Lemma:*

Lemma 1 *Take any two vectors $X_j = \left(X_j^1, ..., X_j^L\right)$ and $X_{j'} = \left(X_{j'}^1, ..., X_{j'}^L\right)$ such that $J(X_j) = J\left(X_{j'}\right)$. For any stable-compatible mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$, such that $Y_i = \mathcal{G}(X_j)$ and $Y_{i'} = \mathcal{G}\left(X_{j'}\right)$, there exists an equally probable stable-compatible mapping $\mathcal{G}'$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$, such that $Y_i = \mathcal{G}'\left(X_{j'}\right)$ and $Y_{i'} = \mathcal{G}'(X_j)$.*

Proof. *For any stable-compatible mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$ such that $Y_i = \mathcal{G}(X_j)$ and $Y_{i'} = \mathcal{G}\left(X_{j'}\right)$, consider a draw $(\varepsilon, \eta) = \left(\varepsilon_1, ..., \varepsilon_{N_w}, \eta_1, ..., \eta_{N_m}\right)$ for which $\mathcal{G}$ is stable-compatible. Define the draw $(\varepsilon', \eta) = \left(\varepsilon'_1, ..., \varepsilon'_{N_w}, \eta_1, ..., \eta_{N_m}\right)$ by:*

$$\varepsilon'_j = \varepsilon_{j'}, \varepsilon'_{j'} = \varepsilon_j, \varepsilon'_k = \varepsilon_k \text{ for all } k \neq j, j'$$

and the matching $\mathcal{G}'$ by:

$$\begin{aligned}
\mathcal{G}'(X_j) &= \mathcal{G}\left(X_{j'}\right) \\
\mathcal{G}'\left(X_{j'}\right) &= \mathcal{G}(X_j) \\
\mathcal{G}'(X_k) &= \mathcal{G}(X_k) \text{ for all } k \neq j, j'
\end{aligned}$$

Now, take any (Y_i, η_i). From Assumption S', we have that:

$$\begin{aligned}
\Phi(Y_i, \eta_i, X_j, \varepsilon_j) &= \tilde{\Phi}(I(Y_i), \eta_i, J(X_j), \varepsilon_j) \\
&= \tilde{\Phi}\left(I(Y_i), \eta_i, J\left(X_{j'}\right), \varepsilon'_{j'}\right) \\
&= \Phi\left(Y_i, \eta_i, X_{j'}, \varepsilon'_{j'}\right)
\end{aligned}$$

and by the same token:

$$\begin{aligned}
\Psi(Y_i, \eta_i, X_j, \varepsilon_j) &= \Psi\left(Y_i, \eta_i, X_{j'}, \varepsilon'_{j'}\right) \\
\Psi_0(X_j, \varepsilon_j) &= \Psi_0\left(X_{j'}, \varepsilon'_{j'}\right)
\end{aligned}$$

This implies that the inequalities that are satisfied by stable-compatibility of $\mathcal{G}$ for the draw (ε,η) also prove stable-compatibility of $\mathcal{G}'$ for the draw (ε',η). Finally, these two draws are equally likely by Assumption CI, which proves the Lemma. ■

To conclude the proof, remember that the probability that X_j is matched with Y_i at a stable matching is the (finite) sum of probabilities of all stable-compatible mappings $\mathcal{G}$ such that $Y_i = \mathcal{G}(X_j)$. The Lemma directly implies the conclusion. ■

Lastly, it is important to note that, while the result has been derived under a specific equilibrium selection device (female's preferred stable matching), it would hold under any alternative mechanism (male's preferred stable matching, randomization between all stable matches, etc.); the only constraint being that the selection device treats individuals with the same index identically.

3.2 Transferable utility

Now, the matching of Ms. j, characterized by the vector (X_j,ε_j), with Mr. i, characterized by the vector (Y_i,η_i), generates a total surplus equal to:

$$S_{ij} = \Gamma\left(Y_i,\eta_i,X_j,\varepsilon_j\right)$$

Moreover, we assume, as above, that for any i,k,j,l,X and Y, the partials $\partial\Gamma/\partial\eta_i^k$ and $\partial\Gamma/\partial\varepsilon_j^l$ are non zero almost everywhere.

Again, in a finite context, a matching can equivalently be defined as a mapping $\mathcal{F}$ from $\mathcal{X}^A$ to $\mathcal{Y}^A$, together with two functions $u\left(Y_i,\eta_i\right)$ and $v\left(X_j,\varepsilon_j\right)$ such that:

$$u\left(Y_i,\eta_i\right)+v\left(X_j,\varepsilon_j\right)=\Gamma\left(Y_i,\eta_i,X_j,\varepsilon_j\right) \quad \text{for all } \left(Y_i,\eta_i,X_j,\varepsilon_j\right) \text{ with } \left(Y_i,\eta_i\right)=\mathcal{F}\left(X_j,\varepsilon_j\right)$$

Also, if Ms. j (resp. Mr. i) remains single then her (his) surplus is normalized to zero.

Our separability assumption becomes:

Assumption S" *The function Γ is weakly separable in $X=\left(X^1,...,X^L\right)$ and $Y=\left(Y^1,...,Y^K\right)$; i.e., there exist two functions $I=I\left(Y^1,...,Y^K\right)$ and $J=J\left(X^1,...,X^L\right)$ such that:*

$$\Gamma\left(Y_i,\eta_i,X_j,\varepsilon_j\right)=\tilde{\Gamma}\left(I\left(Y_i^1,...,Y_i^K\right),\eta_i,J\left(X_j^1,...,X_j^L\right),\varepsilon_j\right) \tag{5}$$

for some $\tilde{\Gamma}$.

Note that since $\partial\Gamma/\partial\eta_i^k = \partial\tilde{\Gamma}/\partial\eta_i^k$ and $\partial\Gamma/\partial\varepsilon_j^l = \partial\tilde{\Gamma}/\partial\varepsilon_j^l$, the partials $\partial\tilde{\Gamma}/\partial\eta_i^k$ and $\partial\tilde{\Gamma}/\partial\varepsilon_j^l$ are also non zero almost everywhere.

As before, for any draw of the (ε, η) vector, (5) defines a TU matching problem, for which there exists a (generically unique) stable mapping $\mathcal{F}$. As before, for any mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$ and any draw $(\varepsilon, \eta) = \left(\varepsilon_1, ..., \varepsilon_{N_w}, \eta_1, ..., \eta_{N_m}\right)$, we say that the mapping $\mathcal{G}$ is stable-compatible for the draw (ε, η) if there exist two functions $u\left(Y_i, \eta_i\right)$ and $v\left(X_j, \varepsilon_j\right)$ such that the mapping $\mathcal{F}$ from $\mathcal{X}^A$ to $\mathcal{Y}^A$ defined by $\mathcal{F}\left(X_j, \varepsilon_j\right) = \left(Y_i = \mathcal{G}\left(X_j\right), \eta_i\right)$, together with the functions u and v, is stable. The mapping $\mathcal{G}$ is stable-compatible if there exists at least one draw (ε, η) for which it is stable-compatible; and we define the probability of a mapping $\mathcal{G}$ being stable-compatible by the measure of the set of draws for which it is stable-compatible. Finally, we define the probability that a particular female observable vector X_j is matched with a particular male observable vector Y_i at a stable matching by the measure of the set of stable-compatible mappings $\mathcal{G}$ such that $Y_i = \mathcal{G}\left(X_j\right)$.

A key feature of the TU framework is that stability is equivalent to surplus maximization. That is, for any given draw (ε, η), a mapping $\mathcal{F}$ is associated to a stable-compatible matching if and only if it solves:

$$\Sigma\left(\varepsilon, \eta\right) = \max_{\sigma} \sum_j \Gamma\left(Y_{\sigma(j)}, \eta_{\sigma(j)}, X_j, \varepsilon_j\right)$$

where $\left(Y_{\sigma(j)}, \eta_{\sigma(j)}\right) = \mathcal{G}\left(X_j, \varepsilon_j\right)$ for all j.

Our second result is then:

Proposition 2 *Assume that Assumptions CI and S" are satisfied. Take any two vectors* $X_j = \left(X_j^1, ..., X_j^L\right)$ *and* $X_{j'} = \left(X_{j'}^1, ..., X_{j'}^L\right)$ *of female observable characteristics, such that* $J\left(X_j\right) = J\left(X_{j'}\right)$. *Then for any vector* Y_i *of male observable characteristics, the probability that* X_j *is matched with* Y_i *at a stable matching is equal to the probability that* $X_{j'}$ *is matched with* Y_i *at a stable matching.*

Similarly, for any two vectors $Y_i = \left(Y_i^1, ..., Y_i^L\right)$ *and* $Y_{i'} = \left(Y_{i'}^1, ..., Y_{i'}^L\right)$ *of male observable characteristics, such that* $I\left(Y_i\right) = I\left(Y_{i'}\right)$, *and for any vector* X_j *of female observable characteristics, the probability that* Y_i *is matched with* X_j *at a stable matching is equal to the* *probability that $Y_{i'}$ is matched with X_j at a stable matching.*

Proof. *Again, we need to prove the first statement only. The proof relies on the following Lemma, which is the exact equivalent (in the TU context) of the previous one:*

Lemma 2 *Take any two vectors $X_j = \left(X_j^1, ..., X_j^L\right)$ and $X_{j'} = \left(X_{j'}^1, ..., X_{j'}^L\right)$ such that $J(X_j) = J\left(X_{j'}\right)$. For any stable-compatible mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$, such that $Y_i = \mathcal{G}(X_j)$ and $Y_{i'} = \mathcal{G}\left(X_{j'}\right)$, there exists an equally probable stable-compatible mapping $\mathcal{G}'$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$, such that $Y_i = \mathcal{G}'\left(X_{j'}\right)$ and $Y_{i'} = \mathcal{G}'(X_j)$. Moreover, the aggregate surplus is the same in both cases.*

Proof. *For any stable-compatible mapping $\mathcal{G}$ from $\mathcal{X}_C^A$ to $\mathcal{Y}_C^A$ such that $Y_i = \mathcal{G}(X_j)$ and $Y_{i'} = \mathcal{G}\left(X_{j'}\right)$, consider a draw $(\varepsilon, \eta) = \left(\varepsilon_1, ..., \varepsilon_{N_w}, \eta_1, ..., \eta_{N_m}\right)$ for which $\mathcal{G}$ is stable-compatible. Define the draw $(\varepsilon', \eta) = \left(\varepsilon'_1, ..., \varepsilon'_{N_w}, \eta_1, ..., \eta_{N_m}\right)$ by:*

$$\varepsilon'_j = \varepsilon_{j'}, \varepsilon'_{j'} = \varepsilon_j, \varepsilon'_k = \varepsilon_k \text{ for all } k \neq j, j'$$

and the matching $\mathcal{G}'$ by:

$$\begin{aligned} \mathcal{G}'(X_j) &= \mathcal{G}\left(X_{j'}\right) \\ \mathcal{G}'\left(X_{j'}\right) &= \mathcal{G}(X_j) \\ \mathcal{G}'(X_k) &= \mathcal{G}(X_k) \text{ for all } k \neq j, j' \end{aligned}$$

Now, take any (Y_i, η_i). From Assumption S", we have that:

$$\begin{aligned} \Gamma(Y_i, \eta_i, X_j, \varepsilon_j) &= \tilde{\Gamma}(I(Y_i), \eta_i, J(X_j), \varepsilon_j) \\ &= \tilde{\Gamma}\left(I(Y_i), \eta_i, J\left(X_{j'}\right), \varepsilon'_{j'}\right) \\ &= \Gamma\left(Y_i, \eta_i, X_{j'}, \varepsilon'_{j'}\right) \end{aligned}$$

Let $\mathcal{V}(\mathcal{G}, \varepsilon, \eta)$ denote the aggregate surplus generated by $\mathcal{G}$ for the draw (ε, η):

$$\mathcal{V}(\mathcal{G}, \varepsilon, \eta) = \sum_j \Gamma(Y_i = \mathcal{G}(X_j), \eta_i, X_j, \varepsilon_j)$$

then

$$\Sigma(\varepsilon,\eta)=\mathcal{V}(\mathcal{G},\varepsilon,\eta)=\mathcal{V}\left(\mathcal{G}',\varepsilon',\eta\right)\leq\Sigma\left(\varepsilon',\eta\right)$$

But the construct is symmetric in $\mathcal{G}$ *and* $\mathcal{G}'$*; therefore* $\Sigma(\varepsilon,\eta)\geq\Sigma(\varepsilon',\eta)$ *and finally:*

$$\Sigma(\varepsilon,\eta)=\mathcal{V}(\mathcal{G},\varepsilon,\eta)=\mathcal{V}\left(\mathcal{G}',\varepsilon',\eta\right)=\Sigma\left(\varepsilon',\eta\right)$$

We conclude that $\mathcal{G}'$ *maximizes total surplus for the draw* (ε',η)*, which proves the Lemma* ■

To conclude the proof, remember that the probability that X_j *is matched with* Y_i *at a stable matching is the sum of probabilities of all stable-compatible mappings* $\mathcal{G}$ *such that* $Y_i=\mathcal{G}(X_j)$*. The Lemma directly implies the conclusion.* ■

3.3 Additional remarks

Measure on the product space The previous subsections derive formal results in two specific frameworks, both involving a finite set of agents and a frictionless model. Similar results could easily be derived for either an imperfectly transferable utility or a search framework. In the first case, the proof is similar to the NTU case – not surprisingly, since the main existence result in that case relies on a generalization of the Gale-Shapley algorithm. Regarding search, the only additional condition is that the meeting technology treats identically agents with the same index. The proofs are available upon request.

Also, both Propositions have a common Corollary, which simply translates the properties of the stable mappings in terms of measures on the product space:

Corollary 1 *The probability measure* μ *over the set of observable characteristics only depends on the indices* I *and* J*; i.e., there exists a measure* ν *on* $\mathbb{R}^2$ *such that*

$$\mu\left(Y^1,...,Y^K,X^1,...,X^L\right)=\nu\left[I\left(Y^1,...,Y^K\right),J\left(X^1,...,X^L\right)\right]$$

This is exactly the property described in the previous Section by equation (1).

Practical implementation Finally, how can this results be used in practice? One answer is to compare matching patterns from a collection of finite-size markets on which the surplus function is the same, and the realizations of male and female characteristics are i.i.d draws from the same distribution, as in Fox (2010). The markets can be defined geographically (by counties, states, countries, etc.), temporally (as in Chiappori, Salanié and Weiss 2011), or by any alternative indicator (language, religion, ethnicity, etc.), although the identical distribution assumption may be more acceptable in some interpretations than in others. Such 'local' markets need not be directly observable by the econometrician. It is possible, for instance, that we only observe outcomes at the level of the global market; we may know that these outcomes stem from the aggregation of several, local submarkets without being able to independently identify these submarkets. In that case, our results directly apply: although on each particular submarket the matching patterns are exactly determined by the submarket's specific draw, on aggregate the distribution of matching patterns will reflect the distribution of the independent draws on the various submarkets. In particular, if two individuals have the same index, they should have the same distribution of spouses, a property that is easy to test. Our empirical section will exploit this insight.

4 Econometric specification

Consider the conditional characteristics of the wife, $X = \left(X^1, ..., X^L\right)$, given those of the husband, $Y = \left(Y^1, ..., Y^K\right)$ – the opposite case is similar. We typically observe a finite sample drawn from a joint conditional distribution. This distribution may be quite complex; it reflects both the randomness inherent to the matching process (for instance, in a search model) *and* the distribution of unobserved characteristics of both spouses; remember, moreover, that the latter is typically multidimensional. Still, under the null, the distribution (therefore all its moments) depends on the husband's observable characteristics $Y = \left(Y^1, ..., Y^K\right)$ only through a single (and unknown) index $I\left(Y^1, ..., Y^K\right)$. Testing for this property is in principle feasible non-parametrically. A two-stage procedure could (i) non-parametrically estimate each conditional mean and possibly other moments (variance, median, etc.), and (ii) check the non-linear restrictions implied by (3).[6] Alternatively, one could, in a more parametric spirit, simultaneously estimate the various moments with and without the restrictions, and base the test on a comparison of these estimates.

[6] For instance, we may define α_i^s by

$$\alpha_i^s = Y_i^s - E\left[Y^s \mid X\right]$$

In practice, we start with the benchmark case in which the functions I and J are linear, similarly to Hitsch et al. (2010):

$$\begin{aligned} I\left(Y_i^1, ..., Y_i^K\right) &= \sum_k f_k Y_i^k \\ J\left(X_j^1, ..., X_j^L\right) &= \sum_l g_l X_j^l \end{aligned}$$

We have concluded above that the distribution of any female characteristic conditional on the husband's vector $\left(Y_i^1, ..., Y_i^K\right)$ only depends on $I\left(Y_i^1, ..., Y_i^K\right)$. It follows from (3) that, for any female characteristic s:

$$\frac{\partial E\left[X_j^s \mid Y_i^1, ..., Y_i^K\right] / \partial Y^t}{\partial E\left[X_j^s \mid Y_i^1, ..., Y_i^K\right] / \partial Y^r} = \frac{f_t}{f_r}$$

and by the same token:

$$\frac{\partial E\left[Y_i^s \mid X_j^1, ..., X_j^L\right] / \partial X^t}{\partial E\left[Y_i^s \mid X_j^1, ..., X_j^L\right] / \partial X^r} = \frac{g_t}{g_r}$$

Assume, moreover, that the conditional expectations at stake (the ϕ_s functions in (2)) are also linear in the index:

$$\begin{aligned} E\left[X_j^s \mid Y_i^1, ..., Y_i^K\right] &= b^s I\left(Y_i^1, ..., Y_i^K\right) \\ &= b^s \left(\sum_k f_k Y_i^k\right) \text{ and} \\ E\left[Y_i^s \mid X_j^1, ..., X_j^L\right] &= a^s J\left(X_j^1, ..., X_j^L\right) \\ &= a^s \left(\sum_l g_l X_j^l\right) \end{aligned}$$

Then, one can simply regress the various characteristics of male i over the characteristics of i's wife, say j, on the sample of married couples; the resulting coefficients should be proportional across the various regressions. The regression of the kth male attribute on the wife's

Intuitively, α_i^s is the projection, over the corresponding direction, of the (multidimensional) randomness just mentioned. By construction, $E\left[\alpha^s \mid X\right] = 0$. Rewriting the relationship as:

$$Y_i^s = E\left[Y^s \mid X\right] + \alpha_i^s$$

suggests to use a non-linear regression of the Y_i^s on the X_i.

characteristics takes the form:

$$Y_i^k = \sum_l \gamma_l^k X_j^l + \alpha_i^k \tag{6}$$

where the random term $\alpha_i^k = Y_i^k - E\left[Y^k \mid X_j^1, ..., X_j^L\right]$ captures the impact of the unobserved heterogeneity, as well as other shocks affecting the process. Note that, as remarked above, the α_i^k also contains the projection of the (multidimensional) set of unobservable characteristics over the corresponding axis; we must therefore allow for the α_i^k to be correlated across k. The theory then predicts that there exist some $\phi_1, ..., \phi_K$ such that:

$$\gamma_l^k = \phi_k \ f_l \text{ for all } (k, l) \tag{7}$$

Equivalently, the γs must be such that:

$$\frac{\gamma_t^k}{\gamma_r^k} = \frac{\gamma_t^s}{\gamma_r^s} = \frac{f_t}{f_r} \text{ for all } (k, r, t) \tag{8}$$

Hence, we can estimate (6) simultaneously for all characteristics k using Seemingly-Unrelated-Regression (SUR), and test for (8). If we cannot reject the equality of the ratios of the coefficients[7], then we are confident to obtain the marginal rate of substitution between characteristics t and r:

$$MRS_i^{r,t} = \frac{f_t}{f_r}$$

Alternatively, we can estimate (6) simultaneously for all characteristics k subject to (7), and then test this constrained model against the unconstrained one. If the constrained model is not rejected to be nested in the unconstrained, then we are confident to obtain the marginal rate of substitution.

The same strategy can be used for female characteristics. The $MRS_i^{r,t}$ is constant in this linear specification of the index. However, the linearity assumption, which is used only for empirical convenience, is *independently testable*. Indeed, one can nest it into a more general formulation involving non-linear terms, and test whether these terms are significant. We perform several tests of this kind in Section 8.

[7] Note that we can also test for the equality of the corresponding products of the coefficients.

5 Measuring attractiveness

5.1 Physical attractiveness

There exists a considerable literature on measuring physical attractiveness in which weight scaled by height (BMI) is widely used as a proxy for socially defined physical attractiveness (e.g., Gregory and Rhum, 2011). Indeed, BMI is shown to be negatively related to physical attractiveness. For instance, Rooth (2009) found that photos that were manipulated to make a person of normal weight appear to be obese caused a change in the viewer's perception, from attractive to unattractive.

Both body shape and body size are important determinants of physical attractiveness; in practice, BMI provides information on body size, while the waist-to-hip ratio (WHR) and the waist-to-chest ratio (WCR) provide information on body shape. The available empirical evidence, e.g., the literature review on body shape, body size and physical attractiveness by Swami (2008), seems to point to BMI being the dominant cue for female physical attractiveness, with WHR (the ratio of the width of the waist to the width of the hips) playing a more minor role. Regarding male physical attractiveness, WCR (waist-to-chest) plays a more important role than either the WHR or BMI, but it must be emphasized that BMI and WCR are strongly positively correlated. Not surprisingly, BMI is correlated with the male attractiveness rating by women, though this correlation is lower than the one with WCR.[8] We are not aware of any study with detailed measures of body shape *and* socioeconomic characteristics which simultaneously provides these data for *both* spouses. Since BMI has been shown to constitute a good proxy for both male and female physical attractiveness, we will use this measure in our analysis.[9]

We conclude with two remarks. First, our notion of attractiveness postulates that individuals of one gender rank the relevant characteristics of the opposite sex in the same way – say, all men prefer thinner women. Such a 'vertical' evaluation may not hold for other

[8] Wells, Treleaven and Cole (2007), using a large survey of adults in the UK (more than 4,000 men and more than 5,000 women) and a sophisticated technique to assess body shape (three-dimensional body scanning), investigate the relationship of shape and BMI. They find that BMI conveys different information about men and women: the two main factors associated with weight in men after adjustment for height are chest and waist, whereas in women they are hip and bust. They suggest that chest in men but hips in women reflect physique (i.e., physical appearance), whereas waist in men and bust in women reflects fatness.

[9] Notice also that our analysis refers to the Western culture, as in some developing countries the relationship between female attractiveness and BMI may be different.

characteristics. Age is a typical example: while a female teenager is likely to prefer a male adolescent over a middle-age man, a mature woman would probably have the opposite ranking. In this regard, we follow most of the applied literature on matching in assuming that different age classes constitute different matching populations. Since, however, preferences on other characteristics (like BMI) may vary across these populations, we control for age in all our regressions. Second, another possible indicator of physical attractiveness is height. Again, whether the height criterion is valued in a unanimous way (all men prefer taller women) or in an individual-specific one (say, tall men prefer tall women, but short males prefers petites) is not clear and it seems to be a measure of male, rather than female, physical attractiveness (Herpin, 2005).

5.2 Socioeconomic attractiveness

In our model, men and women observe potential mates' ability in the labor market and in the household, such as ability to generate income, earnings capacity, and household productivity. Since most of these are not directly observed by the econometrician, we need to define an acceptable proxy for both genders. The most natural indicator of socioeconomic attractiveness is probably wage; not only does wage directly measure a person's ability to generate income from a given amount of input (labor supply), but it is also strongly correlated with other indicators of socioeconomic attractiveness, such as prestige or social status. The main problem with wage, however, is that it is only observed for people who actually work. This is a relatively minor issue for men, since their participation rate, at least in the age category we shall consider, is close to one; but it may be a serious problem for women. One solution could be to estimate a potential wage for non-working women, the drawback of this strategy being to introduce an additional layer of measurement error in some of the key variables. In practice, however, potential wages are predicted from a small number of variables: age, education, number of children and various interactions of these (plus typically time and geographical dummy variables). Here, we are interested in the matching patterns at first marriage; we therefore consider a female population that is both relatively homogeneous in age and typically without children. We may therefore assume that education is an acceptable proxy for female socioeconomic attractiveness. Additionally, female education may also capture ability to produce quality household goods, which is likely to be valued by men. We can now proceed to the empirical analysis of matching patterns along these two dimensions – i.e., physical and socioeconomic attractiveness.

6 Data description

Our empirical work uses data from the Panel Study of Income Dynamics (PSID). The PSID is a longitudinal household survey collecting a wide range of individual and household demographic, income, and labor-market variables. In addition, in all the most recent waves since 1999 (1999, 2001, 2003, 2005, and 2007), the PSID provides the weights (in pounds) and heights (in feet and inches) of both household heads and wives, which we use to calculate the BMI of each spouse, defined as an individual's body weight (in kilograms) divided by the square of his or her height (in meters).[10]

In each of the survey years under consideration, the PSID comprises about 4,500 married households. We select households with a household head and a wife where both are actually present. In our sample years, all the married heads with spouse present are males, so we refer to each couple as husband and wife, respectively. We confine our study to those couples whose wife is between 20 and 50 years old, given that the median age at first marriage of women in the US was 25.1 in 2000 and 26.2 in 2008 (U.S. Census Bureau, Current Population Survey, 2005; American Community Survey, 2008). The upper bound 50 is chosen to focus on prime-age couples. Our main analysis comprises white spouses with working husbands, so that we include couples with both working and non-working wives. We focus on white couples for two reasons. First, because the sample size for black couples in the PSID is much smaller. Second, and more importantly, because perceptions of attractiveness regarding BMI can be very different between blacks and whites. Indeed, several researchers argue that standards and experiences of beauty vary by gender and race (e.g., Craig, 2006; Conley and McCabe, 2011). Moreover, following Conley and Glauber (2007), we discard those couples whose height and weight values include any extreme ones: a weight of more than 400 or less than 70 pounds,

[10] Weight and height are originally reported in pounds and inches in the PSID. The pounds/inches BMI formula is: Weight (in pounds) $\times$ 704.5 divided by Height (in inches) $\times$ Height (in inches). Oreffice and Quintana-Domeque (2010) have shown that non-response to body size questions appears to be very small in the PSID data. Specifically, item non-response for husband's height is below 1.4% in each year, for wife's height is below 1.4% in each year, and for husband's weight is below 2.2% in each year. Regarding wife's weight, item non-response is below 5.5% in each year.

a height above 84 or below 45 inches. In our main analysis we consider individuals who are in the normal- and over- weight range (18.5≤BMI<30), that is, the medically underweight or obese individuals are excluded (WHO, 2003).

Because the PSID main files do not contain any direct question concerning the duration of the marriages, we rely on the "Marital History File: 1985-2007" Supplement of the PSID to obtain the year of marriage and number of marriages, to account for the duration of the couples' current marriage. We merge this information to our main sample using the unique household and person identifiers provided by the PSID. We establish a threshold of less than or equal to *three* years of marriage, as a proxy for how recently a couple formed. From a theoretical perspective this demographic group is particularly adequate for studying matching patterns, because the marriage market penalties for BMI should arise through sorting *at the time of the match*. Clearly, the price to pay is a serious reduction in the sample size.

In the PSID, all the variables, including the information on the wife, are reported by the head of the household. Reed and Price (1998) found that family proxy-respondents tend to overestimate heights and underestimate weights of their family members, so that family proxy-respondent estimates follow the same patterns as self-reported estimates. The authors suggest that the best proxy-respondents are those who are in frequent contact with the target. Since we are considering married couples, the best proxy-respondents are likely to be the spouses.[11]

The main characteristics we use in our empirical analysis are age, log hourly wage, and education. Education is defined as the number of completed years of schooling and is top-coded at 17 for some completed graduate work. We establish a minimum threshold of 9 years of schooling. State dummy variables are included to capture constant differences in labor and marriage markets across geographical areas in the US. To account for omitted variables bias, we also use additional spousal characteristics and household variables. Specifically, the following variables are considered: health status (1 if excellent, very good, or good; 0 if fair or poor); an individual dummy variable for being a smoker; number of children in the household under 18 years; a dummy variable for the presence of children aged two years or less (to control

[11] Cawley (2004) used the National Health and Nutrition Examination Survey III (NHANES III) to estimate the relationship between measured height and weight and their self-reported counterparts. First, he estimated regressions of the corresponding measured variable to its self-reported counterpart by age and race. Then, assuming transportability, he used the NHANES III estimated coefficients to adjust the self-reported variables from the NLSY. The results for the effect of BMI on wages were very similar, whether corrected for measurement error or not. Recent papers confirm that the BMI adjustment makes no difference (Kelly et al., 2011).

for a recent pregnancy); and the ratio of the expenditures on food at home versus total food ones (food ratio).

As the original sample consists of several PSID waves, to decrease measurement error concerns we take the means of our variables of interest by household head id over the wave years. From a total of 871 observations concerning recently married couples satisfying the criteria indicated above, we reach a sample of 667 couples, with one observation per couple. After removing observations with missing values in some variables of interest, the final sample consists of 659 observations.

The main characteristics of our sample are described in the top panel of Table 1. The average number of years of schooling slightly exceeds 14, and the wives are on average more educated than their husbands. The average age difference within couples is about 2 years, which is the standard age gap estimated for couples in the US. As to weight, a salient feature is that male BMI is on average much larger than female; the average man is actually overweight (BMI above 25), whereas female average BMI is inferior to 23.

[**Insert Table 1 about here**]

Regarding the correlation of individual characteristics within couples, the bottom panel of Table 1 summarizes some clear patterns. We first note, as expected, a significant level of assortative matching on economic characteristics. The wife's education is strongly correlated with both the husband's education ($> .53$) and log wage ($> .23$); these correlations are statistically significant at the 1% level, and consistent with previous studies (e.g., Qian, 1998). A second conclusion is the existence of a negative correlation between education and BMI, at least for women ($-.14$). An interesting remark, however, is that the correlation between male log wage and BMI is actually positive (.10) and statistically significant at the 5% level. Finally, since the wife's education is both positively correlated with her husband's log wage and negatively correlated with her BMI, one might expect a negative relationship between male log wage and female BMI. Table 1 indeed confirms this prediction, the correlation being $-.11$ (p-value < 0.01). However, although wealthier husbands tend both to be fatter and to have thinner wives, and husband's BMI is negatively correlated with female education ($-.07$, p-value < 0.1), male and female BMIs are actually *positively* correlated (.09, p-value < 0.05). This result, which is consistent with previous studies in the medical (e.g., Jeffrey and Rick,

2002) and economic (Hitsch et al., 2010; Oreffice and Quintana-Domeque, 2010) literatures, suggests that, as argued in the introduction, physical appearance is another element of the assortative matching pattern. Not only these correlations show that assortative matching takes place along the two dimensions of physical and socioeconomic attractiveness, but a trade-off seems to exist, whereby a lower level of physical attractiveness can be compensated by better socioeconomic characteristics, and conversely. However, these findings do not constitute clean tests of our theory, which are presented in the next Section.

7 Estimating matching patterns and trade-offs

Table 2 presents the regressions of wife's BMI and education on husband's characteristics. Two specifications are presented for each regression: a standard one, with controls for own age and state fixed effects, and an augmented one, where we also control for the number of children, recent pregnancy, ratio of food at home relative to total food expenditure, spousal health status, and spousal smoking status, in an attempt to capture omitted variables related to (socioeconomic and physical) attractiveness, such as health aspects.

The top panel in the table shows that, as expected, the wife's BMI is negatively related to the husband's log wage and positively to his BMI, while her education exhibits the opposite patterns. This finding is consistent with the view that wage positively contributes to a man's attractiveness, while excess weight has a negative impact. It is reassuring that the estimates are very similar in the standard and the augmented specifications, indicating that our results are unlikely to be driven by omitted variables bias.

We then report the ratios and the products of the coefficients of interest within or across columns. The corresponding Wald tests on the proportionality of these factors are not rejected (p-values $> .32$ and $> .50$, standard and augmented regressions) indicating that the marginal rates of substitution are identified. In addition, we perform constrained estimations, corresponding to the regressions presented above but imposing the proportionality constraint. These allow us to use likelihood-ratio (LR) tests, which are invariant to non-linear transformations of the parameters (Gregory and Veall, 1985), thus yielding stronger support to our results.[12] As shown in the bottom panel of Table 2, these estimates are consistent with

[12] We thank one anonymous referee for suggesting the use of likelihood-ratio tests.

the previous unconstrained ones. Most of all, our evidence shows that the LR test of the constrained versus the unconstrained model does not reject our predicted proportionality constraint. Specifically, the MRS between BMI and log wage is estimated to be between -7.97 (standard regression) and -5.25 (augmented regression), both of them strongly significant.

[Insert Table 2 about here]

Table 3 exhibits identical features for a woman's attractiveness, with the husband's BMI being negatively related to wife's education and positively to her BMI, while husband's log wage exhibits correlations of opposite signs. As before, one can see that the estimates are very similar in the standard and the augmented specifications. Again, the corresponding Wald tests on the proportionality of the ratios and the products of the coefficients of interest within or across columns are not rejected (p-values $> .33$ and $> .52$, standard and augmented regressions), meaning that we can identify the marginal rates of substitution. Finally, the estimation of the constrained model at the bottom of the table confirms and reinforces the results from the unconstrained one. Specifically, the estimated MRS between BMI and education is -2.27 (standard regression) and -1.84 (augmented regression), both of them strongly significant.

[Insert Table 3 about here]

Numerically, the above point estimates from the augmented regressions suggest, for the ratio of the coefficient of husband's log wage to his BMI, a value of -5.3 (or -0.21 if BMI is substituted with its logarithm); in other words, a 10%-increase in male BMI can be compensated by a 2%-increase in his wage. Similarly, the ratio between the wife's education and BMI coefficients is close to -2; i.e., for women, an additional year of education compensates about 2 BMI units, which is almost the gap between the average female BMI in our sample (22.7) and the threshold for being overweight (25).

8 Extensions

An obvious weakness of the linear specification adopted so far is that it assumes the MRSs to be constant – i.e., that the trade-offs between physical and socioeconomic attractiveness are the same for all agents. Remember, however, that linearity is not required to identify the MRSs. We now relax this assumption in different ways, namely, analyzing whether the MRSs differ across spousal height-groups, and exploring potential non-monotonicities in the socioeconomic and physical attributes of our index, as well as potential interactions between them.

First, we allow for different MRSs across different *spousal* height classes, enriching the form adopted for the respective indices by introducing an indicator for being tall ($T_j = 1$ if spouse j's height is above the median, 0 otherwise) and the interaction of this indicator with the physical and the socioeconomic characteristics. This new unrestricted model is written as:

$$\begin{aligned} BMI_{-j} &= \beta_1 SES_j \ + \pi_1 BMI_j \ + \ \rho_1 T_j \ + \ \theta_1 SES_j \times T_j \ + \ \delta_1 BMI_j \times T_j \ + \ P\mathbf{\Lambda}_1 \ + \ u_{-j,1} \\ SES_{-j} &= \beta_2 SES_j \ + \pi_2 BMI_j \ + \ \rho_2 T_j \ + \ \theta_2 SES_j \times T_j \ + \ \delta_2 BMI_j \times T_j \ + \ P\mathbf{\Lambda}_2 \ + \ u_{-j,2} \end{aligned}$$

where the subindices $-j$ and j are defined as $j = \{wife, husband\}$ and $-j = \{husband, wife\}$, SES_{wife} =wife's education, $SES_{husband}$ =husband's log wage, and P is a vector of standard controls (age of $-j$ and state dummy variables). This unrestricted model is tested against the following restricted model:

$$\begin{aligned} BMI_{-j} &= \kappa_j \varphi_1 SES_j \ + \varphi_1 BMI_j \ + \ \lambda_1 T_j \ + \ \kappa_j \varsigma_1 SES_j \times T_j \ + \ \varsigma_1 BMI_j \times T_j \ + \ P\Xi_1 \ \ + \ e_{-j,1} \\ SES_{-j} &= \kappa_j \varphi_2 SES_j \ + \varphi_2 BMI_j \ + \ \lambda_2 T_j \ + \ \kappa_j \varsigma_2 SES_j \times T_j \ + \ \varsigma_2 BMI_j \times T_j \ + \ P\Xi_2 \ \ + \ e_{-j,2} \end{aligned}$$

which imposes the proportionality constraint, implying not only that the MRSs are identified, but also that they are constant and equal to κ_j. Table 4 reports the estimates corresponding to these models, and the corresponding LR tests. The top panel in the table presents the estimates corresponding to the unconstrained model: only two out of eight coefficients on the interaction terms are statistically different from zero. In the bottom panel, the estimates corresponding to the new constrained model are presented: the estimated MRS between wife's BMI and education is -7.37 (se = 2.20), the estimated MRS between husband's BMI and log wage is -2.60 (se = 0.976), and the ςs are close to zero and cannot be rejected to be statistically different from zero. The LR tests do not reject the proportionality-restricted model against the unrestricted one (in which it is nested), both for men and women, so that the

MRSs are the same irrespective of spousal height, which allows us to interpret this evidence also as a test of linearity of the indices I and J.

[**Insert Table 4 about here**]

We also test the sensitivity of our previous results to other possible deviations from linearity. First, we include an interaction between the physical and the socioeconomic characteristics, hence allowing for the importance of the physical component of attractiveness to vary with the socioeconomic level, and vice versa. Second, we add quadratic terms in both the physical and the socioeconomic characteristics. We test each of these models against our linear model with the proportionality constraint. Table 5 summarizes the results of these tests: our likelihood-ratio tests cannot reject our linear model with the proportionality constraint against these non-linear models.[13]

[**Insert Table 5 about here**]

Overall these findings consistently show that not only the proportionality cannot be rejected, so that the MRSs are identified, but also that they are constant, (at least) across spousal heights. Moreover, when testing our restricted linear model with the proportionality constraint against more flexible models that allow for non-monotonicities or interactions, we cannot reject our restricted model, again suggesting that the linear version is an acceptable approximation – although one cannot exclude the possibility that we are not able to find non-monotonicities or interactions on account of our small sample size. Still, if larger data sets are available in the future, and the existence of non-monotonicities or interactions is not rejected, these flexible models could be tested against their proportional-restricted counterparts. If proportionality were not to be rejected, the MRSs would still be identified, but they would differ across individuals.

Finally, we address the fact that the presentation given above is asymmetric across genders, since the socioeconomic indicator is log wage for men and education for women. To investigate whether this asymmetry may affect our results, in Table 6 we run the regressions using the education of the husband (instead of his log wage) to proxy for his socioeconomic

[13] Regression estimates available upon request.

attractiveness. The qualitative results are similar, as well as the LR tests, which represent additional support for our framework.

[**Insert Table 6 about here**]

9 Conclusions

Our paper relies on a few simple ideas. One is that the nature of the matching process taking place on marriage markets is multidimensional, and involves both physical and socioeconomic ingredients. Secondly, we explore the claim that this matching process may admit a one-dimensional representation. In other words, the various characteristics only matter through some one-dimensional index. We present a formal model in which this assumption can be taken to data. Under the assumptions of separability and conditional independence, we show that our framework generates testable predictions. Moreover, should these predictions be satisfied, then the indices are identified in the ordinal sense; therefore, the marginal rates of substitution between characteristics, which summarize the trade-offs between the various attributes involved, can be exactly identified. In addition, we derive a host of overidentifying restrictions on the MRSs, which can be tested regardless of the non-linearity or non-monotonicity of the index.

Using data from the PSID, we find that our predictions are not rejected. An estimation of the trade-offs suggests that among men, a 10%-increase in BMI can be compensated by a higher wage, the supplement being estimated to be around 2%. Similarly, for women, an additional year of education may compensate up to two BMI units.

Our approach clearly relies on specific and strong assumptions. One-dimensionality is a serious restriction, if only because it assumes that a woman's attractiveness involves the same arguments with identical weighting for all men (and conversely). Still, it can be seen as a first and parsimonious step in a promising direction – i.e., including several dimensions in the empirical analysis of matching. Although we are interested here in marriage markets, other applications (to labor markets in particular) could also be considered. Perhaps the main contribution of this paper is to show that models of this type, once correctly specified, can generate strong testable restrictions that allow to identify and estimate the marginal rates of substitution among partners' characteristics, and that these restrictions do not seem to be obviously counterfactual.

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Table 1: Summary statistics.

A. Sample descriptive statistics

I. Main variables	N	Mean	SD	Min	Max
Wife's Age	659	29.31	6.89	20	50
Husband's Age	659	31.29	7.37	19	68
Wife's BMI	659	22.67	2.62	18.56	29.95
Husband's BMI	659	25.47	2.46	18.56	29.98
Wife's Education	659	14.31	2.04	9	17
Husband's Log Wage	659	2.89	0.593	1.22	5.07
Husband's Education	640	14.07	2.04	9	17
II. Additional variables	N	Mean	SD	Min	Max
Wife's Good Health	659	0.966	0.176	0	1
Husband's Good Health	658	0.973	0.160	0	1
Wife's Smoking	659	0.192	0.387	0	1
Husband's Smoking	659	0.227	0.412	0	1
Number of children	659	0.653	0.898	0	5
Recent pregnancy	656	0.224	0.387	0	1
Food ratio	641	0.688	0.172	0.071	1

B. Sample correlations

	Wife's BMI	Husband's BMI	Wife's Education	Husband's Log Wage	Husband's Education
Wife's BMI	1.000				
Husband's BMI	0.0939** (0.0159)	1.000			
Wife's Education	−0.1408*** (0.0003)	−0.0675* (0.0831)	1.000		
Husband's Log Wage	−0.1117*** (0.0041)	0.0980** (0.0118)	0.2394*** (0.0000)	1.000	
Husband's Education	−0.1806*** (0.0000)	−0.0125 (0.7517)	0.5370*** (0.0000)	0.2717*** (0.0000)	1.000

Sample descriptive statistics use sampling weights. p-values in parentheses. *** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1

Table 2: SUR Regressions of Wife's Characteristics on Husband's Characteristics.

I. Unconstrained model	Wife's BMI	Wife's Education	Wife's BMI	Wife's Education
Husband's Log Wage	−0.585***	0.762***	−0.568***	0.541***
	(0.187)	(0.136)	(0.188)	(0.127)
Husband's BMI	0.111***	−0.080***	0.135***	−0.088***
	(0.041)	(0.030)	(0.042)	(0.028)
Standard Controls	YES	YES	YES	YES
Additional Controls	NO	NO	YES	YES
N	659		638	
Corr(residuals)	−0.1013***		−0.0667*	
Breusch-Pagan Test	$\chi^2(1) = 6.760$		$\chi^2(1) = 2.840$	
	p-value = 0.0093		p-value = 0.0919	
Wald Tests				
Within columns:				
Husband's Log Wage / Husband's BMI	−5.27**	−9.58**	−4.20**	−6.13***
	(2.47)	(3.88)	(1.84)	(2.35)
	$\chi^2(1) = 0.96$		$\chi^2(1) = 0.44$	
	p-value = 0.3263		p-value = 0.5053	
Across columns:				
Husband's Log Wage × Husband's BMI	0.047**	0.085**	0.050**	0.073**
	(0.023)	(0.035)	(0.023)	(0.028)
	$\chi^2(1) = 0.98$		$\chi^2(1) = 0.45$	
	p-value = 0.3224		p-value = 0.5021	
II. Constrained model				
Ratio of coefficients	−7.97***		−5.25***	
	(2.53)		(1.55)	
Husband's BMI	0.082***	−0.092***	0.120***	−0.097***
	(0.029)	(0.027)	(0.034)	(0.025)
LR Test				
H_o: Constrained nested in unconstrained	$\chi^2(1) = 1.00$		$\chi^2(1) = 0.46$	
	p-value = 0.3175		p-value = 0.4999	

We consider individuals who are in the normal-overweight range, BMI [18.5, 30). Wife's age is in the range [20, 50]. Standard errors in parenthesis. Standard controls: own age, and state dummy variables. Additional controls: number of children, recent pregnancy indicator, food ratio, spousal good health, and spousal smoking.

*** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1

Table 3: SUR Regressions of Husband's Characteristics on Wife's Characteristics.

I. Unconstrained model	Husband's BMI	Husband's Log Wage	Husband's BMI	Husband's Log Wage
Wife's Education	−0.091*	0.054***	−0.126**	0.047***
	(0.049)	(0.011)	(0.055)	(0.012)
Wife's BMI	0.076**	−0.020**	0.095**	−0.021***
	(0.036)	(0.008)	(0.037)	(0.008)
Standard Controls	YES	YES	YES	YES
Additional Controls	NO	NO	YES	YES
N	659		638	
Corr(residuals)	0.0995**		0.0781**	
Breusch-Pagan Test	$\chi^2(1) = 6.527$		$\chi^2(1) = 3.896$	
	p-value = 0.0106		p-value = 0.0484	
Wald Tests				
Within columns:				
Wife's Education / Wife's BMI	−1.19	−2.76**	−1.33	−2.21**
	(0.914)	(1.28)	(0.813)	(1.04)
	$\chi^2(1) = 0.92$		$\chi^2(1) = 0.41$	
	p-value = 0.3381		p-value = 0.5217	
Across columns:				
Wife's Education × Wife's BMI	0.0018	0.0042*	0.0027*	0.0044**
	(0.0012)	(0.0022)	(0.0015)	(0.0021)
	$\chi^2(1) = 0.78$		$\chi^2(1) = 0.40$	
	p-value = 0.3781		p-value = 0.5257	
II. Constrained model				
Ratio of coefficients	−2.27***		−1.84***	
	(0.829)		(0.648)	
Wife's BMI	0.050**	−0.023***	0.079***	−0.024***
	(0.022)	(0.007)	(0.028)	(0.007)
LR Test				
H_o: Constrained nested in unconstrained	$\chi^2(1) = 0.79$		$\chi^2(1) = 0.41$	
	p-value = 0.3745		p-value = 0.5237	

We consider individuals who are in the normal-overweight range, BMI [18.5, 30). Wife's age is in the range [20, 50]. Standard errors in parenthesis. Standard controls: own age, and state dummy variables. Additional controls: number of children, recent pregnancy indicator, food ratio, spousal good health, and spousal smoking.
*** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1

Table 4: SUR Regressions of Individual Characteristics on Spousal Characteristics allowing for *spousal* height (above the median) interactions.

$BMI_{-j} = \beta_1 SES_j + \pi_1 BMI_j + \rho_1 T_j + \theta_1 SES_j \times T_j + \delta_1 BMI_j \times T_j + P\Lambda_1 + u_{-j,1}$
$SES_{-j} = \beta_2 SES_j + \pi_2 BMI_j + \rho_2 T_j + \theta_2 SES_j \times T_j + \delta_2 BMI_j \times T_j + P\Lambda_2 + u_{-j,2}$

I. Unconstrained model	Wife's BMI	Wife's Education	Husband's BMI	Husband's Log Wage
β	−0.655**	0.865***	−0.035	0.074***
	(0.256)	(0.186)	(0.064)	(0.014)
π	0.175***	−0.108***	0.101**	−0.015
	(0.056)	(0.041)	(0.051)	(0.011)
ρ	3.03	−0.593	3.23	0.853
	(2.21)	(1.61)	(2.30)	(0.491)
θ	0.147	−0.233	−0.135	−0.043**
	(0.351)	(0.255)	(0.096)	(0.021)
δ	−0.138*	0.062	−0.055	−0.012
	(0.081)	(0.059)	(0.072)	(0.015)
II. Constrained model				

$BMI_{-j} = \kappa_j \times \varphi_1 SES_j + \varphi_1 BMI_j + \lambda_1 T_j + \kappa_j \times \varsigma_1 SES_j \times T_j + \varsigma_1 BMI_j \times T_j + P\Xi_1 + e_{-j,1}$
$SES_{-j} = \kappa_j \times \varphi_2 SES_j + \varphi_2 BMI_j + \lambda_2 T_j + \kappa_j \times \varsigma_2 SES_j \times T_j + \varsigma_2 BMI_j \times T_j + P\Xi_2 + e_{-j,2}$

	Wife's BMI	Wife's Education	Husband's BMI	Husband's Log Wage
κ	−7.37***		−2.60***	
	(2.20)		(0.976)	
φ	0.111***	−0.114***	0.032	−0.026***
	(0.039)	(0.033)	(0.023)	(0.009)
λ	0.153	0.161	0.484	0.115
	(0.447)	(0.339)	(0.566)	(0.137)
ς	−0.049	0.038	0.029	0.010
	(0.044)	(0.032)	(0.032)	(0.007)
N	659		659	
LR Test				
H_o: Constrained nested in unconstrained	$\chi^2(3) = 2.88$		$\chi^2(3) = 4.35$	
	p-value = 0.4105		p-value = 0.2258	

We consider individuals who are in the normal-overweight range, BMI [18.5, 30). Wife's age is in the range [20, 50]. Standard errors in parenthesis. P is a vector of standard controls: own age and state dummy variables.
*** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1

Table 5: Nonlinearities. Quadratic or Interactions terms.

I. Ho: Linear model with proportionality constraint nested in quadratic model

Constrained model: Linear model with proportionality constraint

$BMI_{-j} = \kappa_j \times \beta_1 SES_j + \beta_1 BMI_j + P\Xi_1 + e_{-j,1}$

$SES_{-j} = \kappa_j \times \beta_2 SES_j + \beta_2 BMI_j + P\Xi_2 + e_{-j,2}$

Unconstrained model: Quadratic model in SES and BMI variables

$BMI_{-j} = \beta_1 SES_j + \pi_1 BMI_j + \rho_1 SES_j^2 + \delta_1 BMI_j^2 + P\Lambda_1 + u_{-j,1}$

$SES_{-j} = \beta_2 SES_j + \pi_2 BMI_j + \rho_2 SES_j^2 + \delta_2 BMI_j^2 + P\Lambda_2 + u_{-j,2}$

LR Test	**Wife's Equations**	**Husband's Equations**
H_o: Constrained nested in unconstrained	$\chi^2(5) = 2.41$ p-value = 0.7893	$\chi^2(5) = 2.81$ p-value = 0.7288

II. Ho: Linear model with proportionality constraint nested in model with interaction

Constrained model: Linear model with proportionality constraint

$BMI_{-j} = \kappa_j \times \beta_1 SES_j + \beta_1 BMI_j + P\Xi_1 + e_{-j,1}$

$SES_{-j} = \kappa_j \times \beta_2 SES_j + \beta_2 BMI_j + P\Xi_1 + e_{-j,2}$

Unconstrained model: Model with an interaction term SES×BMI

$BMI_{-j} = \beta_1 SES_j + \pi_1 BMI_j + \rho_1 SES_j \times BMI_j + P\Lambda_1 + u_{-j,1}$

$SES_{-j} = \beta_2 SES_j + \pi_2 BMI_j + \rho_2 SES_j \times BMI_j + P\Lambda_2 + u_{-j,2}$

LR Test	**Wife's Equations**	**Husband's Equations**
H_o: Constrained nested in unconstrained	$\chi^2(5) = 1.39$ p-value = 0.7074	$\chi^2(5) = 3.10$ p-value = 0.3765

We consider individuals who are in the normal-overweight range, BMI [18.5, 30). Wife's age is in the range [20, 50]. Standard errors in parenthesis. P is a vector of standard controls: own age and state dummy variables.

*** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1

Table 6: SUR Regressions of Individual Characteristics on Spousal Characteristics.

I. Unconstrained model	Wife's BMI	Wife's Education	Husband's BMI	Husband's Education
Spousal Education	−0.254***	0.495***	−0.093*	0.504***
	(0.051)	(0.033)	(0.050)	(0.035)
Spousal BMI	0.091**	−0.052*	0.075**	−0.100***
	(0.041)	(0.027)	(0.037)	(0.026)
N	640		640	
Corr(residuals)	−0.0310		0.0137	
Breusch-Pagan Test	$\chi^2(1) = 0.616$ p-value = 0.4325		$\chi^2(1) = 0.121$ p-value = 0.7281	
II. Constrained model				
Ratio of coefficients	−7.73**		−4.73***	
	(3.16)		(1.22)	
Spousal BMI	0.035**	−0.064**	0.025**	−0.106***
	(0.015)	(0.026)	(0.011)	(0.025)
LR Test				
H_o: Constrained nested in unconstrained	$\chi^2(1) = 2.15$ p-value = 0.1425		$\chi^2(1) = 1.97$ p-value = 0.1602	

We consider individuals who are in the normal-overweight range, BMI [18.5, 30). Wife's age is in the range [20, 50]. Standard errors in parenthesis. All regressions include standard controls (own age and state dummy variables).
*** p-value < 0.01, ** p-value < 0.05, * p-value < 0.1