Pricing effects of ambiguous private information*

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Abstract

Ambiguous private information need not be revealed by market prices in a rational expectations equilibrium. This partial revelation property is due to inertia with respect to information on the part of the ambiguity averse recipient. We show how and when such informational inefficiency arises endogenously in an otherwise standard model of asset markets with private information. Partial revelation takes the form of intermediate or moderate information not being revealed, while extreme information is revealed. This has the following asset-pricing implications: (1) informationally inefficient prices may be less volatile than informationally efficient ones, (2) informational inefficiency in prices may lead to discontinuous changes in asset prices and in price volatility even when the volatility of asset fundamentals does not change, and (3) the price impact of a given trade may be larger when prices are informative than when price are uninformative and price impact can change discontinuously. Public information can affect the informational efficiency of price. Trade volume may be higher and prices lower under partial revelation than under full revelation.

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1 Introduction

Asset markets are continually beset by new information. In addition to allocating ownership rights to assets, markets also aggregate and convey information through the observation of relative prices for assets.

However, the quality and reliability of information can vary widely across sources, asset classes, and time, which means information is not homogeneous in its usefulness. For instance, suppose a trader learns that a firm has filed a patent application on a potentially profitable technology. While informative about the firm's future prospects and its stock market value, this news is not as useful as the knowledge of the outcome of the application in assessing the likelihood of increases in the firm's value. Information about a patent application has been submitted might properly be modeled as being ambiguous. That is, the trader does not exactly know the probability of success or failure in the patent process and may instead consider the information as being useful about the range of possible probabilities of success.

Another instance is provided by non-traded assets, like labor income, whose payoff is uncertain and correlated with that of traded financial assets. If private information about labor income arrives and is perceived to be ambiguous, due to uncertainty about its source or its systemic or idiosyncratic implications, then this will translate into ambiguous private information about the traded assets.

This paper investigates whether uninformed investors are able to glean such ambiguous information from the traded financial assets' price. If not, under what conditions would such information be revealed and what effect might the nature of this information have on other properties of market prices? Do prices that reveal information behave differently from prices that do not reveal information?

We present a model to analyse these issues. We show that ambiguous information need not be revealed in equilibrium and that when it is revealed this occurs because the signal is sufficiently extreme, the ambiguity about the signal is sufficiently small, or those who receive the signal have sufficient wealth.

This has the following implications for asset prices. (1) Informative prices may be more volatile than uninformative prices. (2) Price and price volatility may exhibit jumps. (3) The price impact of a trade may be larger when prices are informative than when prices are not informative and may also exhibit jumps.

Market prices aggregate and communicate information as formalized in the rational expectations equilibrium (REE) concept developed in Radner (1979) and Grossman (1976) among others. Traders in the market may be averse toward any ambiguity that they may perceive in information they observe and we use well-known Gilboa and Schmeidler (1989) representation with multiple priors to model their decision-making.

Most existing analyses of REE only include Savage (1954) subjective expected utility (SEU) investors who are not sensitive to ambiguity. Ellsberg (1961) emphasized that the exclusion of any role for ambiguity by the SEU framework has important behavioral implications.[1]

Incorporating concern for ambiguity in models of financial markets has provided a number of insights. Epstein and Schneider (2010) is a recent survey of the growing literature on the effects of ambiguity and ambiguity aversion in financial markets, particularly non-smooth ambiguity aversion. As noted there, much of this work has been conducted in the context of representative agent or homogeneous information models.[2]

In this paper, market participants are allowed to differ in their ambiguity attitude and there is differential information since the ambiguity averse participants receive private information. This information need not be revealed due to the recipient's ambiguity aversion, as modeled by Gilboa and Schmeidler (1989). This non-smooth ambiguity aversion leads to portfolio inertia with respect to information on the part of the recipient and as a consequence, the information may not be revealed. Condie and Ganguli (2011a) showed that the nature of partial revelation considered here has the desirable property of being robust in the context of general financial market economies. The mechanism is distinct from the portfolio inertia in prices property identified by Dow and da Costa Werlang (1992b), but related since both relate to the non-differentiability of the representation.

The partial revelation property studied here does not rely on the presence of noise or taste shocks, which are commonly used methods for generating partial revelation in financial market models with unambiguous information. Dow and Gorton (2008)

[1]Ellsberg (1961), p.657, describes ambiguity as an informational phenomenon –"a quality depending on the amount, type, reliability and "unanimity" of information, and giving rise to one's degree of "confidence" in an estimate of relative likelihoods." Keynes (1921) and Knight (1921) earlier emphasized the distinction between risky situations with known probabilities and uncertain situations with unknown probabilities.

[2]Chapman and Polkovnichenko (2009) present evidence of how a representative investor framework can give significantly different estimates for the equity premium and risk free rate if heterogeneity present in the underlying economy is ignored.

provide a very nice recent discussion of this mechanism and also of mechanisms used in strategic interaction models to generate informationally inefficient prices.[3]

As we show, partial revelation of ambiguous information leads to properties of equilibrium price that are different from those found in models of partial revelation with noise traders. In the present model, partial revelation takes the form of a subset of signal values not being revealed with the rest being revealed. The revealed signals are those with relatively extreme values while the non-revealed signals are those with intermediate values. This form of partial revelation implies the possibility of discontinuous price changes and discontinuous variations in price volatility and price impact of trade.

This is in contrast to noise-based partial revelation, where all signal values are obscured by the noise shock in equilibrium and as such the partial revelation does not have implications for price volatility beyond what noise traders add. That is, noise traders contribute to the properties of equilibrium prices in these models but in a way that is qualitatively similar to how noise traders would alter prices in a setting without private information. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these may forms of partial revelation may be useful in different ways, possibly even complementary, in studying financial markets.

There is a small but growing literature examining the informational efficiency of financial market prices in the presence of ambiguity. This includes Tallon (1998), Caskey (2008), Ozsoylev and Werner (2011), Mele and Sangiorgi (2011), Easley, O'Hara, and Yang (2011), Condie and Ganguli (2011a), and Condie and Ganguli (2011b). However, in these papers except the last three, any partial revelation property is driven by noise traders. [4]

Smooth representations of ambiguity averse preferences such as in Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007) will not generate the partial revelation we study here since these will not exhibit inertia with respect to information. The experimental work of Ahn, Choi, Kariv, and Gale (2011) and Bossaerts, Ghirardato, Guarneschelli, and Zame (2010) also provide persuasive evidence in support of non-smooth models of ambiguity

[3] We do not discuss the strategic interaction model mechanisms here since we analyse a general equilibrium financial market model with price-taking investors.

[4] de Castro, Pesce, and Yannelis (2010) define a new concept called maximin rational expectations equilibria and prove universal existence, incentive compatibility, Pareto efficiency of these equilibria.

aversion in financial market settings.

The paper proceeds as follows. We first develop the financial market model in section 2. Section 3 describes the nature of partial revelation in this framework and the conditions needed for partial revelation to be possible. Section 5 discusses some comparative statics which illustrate the properties of this form of partial revelation further. Section 4.1 discusses the implications for price volatility and swings. Section 6.1 discusses how the model of section 2 can be used to think about ambiguous information to non-tradeable labor income and section 7 concludes.

2 A model of ambiguous private information

There are two types of investors indexed by $n \in \mathcal{N} = \{I, U\}$, where I denotes informed and U denotes uninformed, who live for 3 periods and trade assets in the market. Time is indexed by $t = 0, 1, 2$. Investors observe information and trade at $t = 1$. All uncertainty is resolved and consumption occurs at $t = 2$.

There is one asset whose payoff is certain and denoted by V_f, called the risk-free asset or bond.[5] This asset is in zero net supply. There is another asset whose payoff or terminal value denoted by V is uncertain and it is assumed to have unit net supply. Each investor n is endowed with a fraction $x_0^n > 0$ of the uncertain asset at time 0. Trade occurs in period 1 with the resolution of uncertainty occurring in period 2.

We assume that $\ln V$, denoted v henceforth, is normally distributed with mean μ and variance σ^2. In period 0 all investors have identical information about the expected value of the uncertain assets. However, the two types of traders differ in their perception of information as we describe next.

Prior beliefs of I- and U-investors. Both types of investors believe that v is normally distributed with variance σ^2. Both types are uncertain about the mean of v and their beliefs over μ are given by a normal distribution that has mean μ_0 and precision ρ_0.

Private information. At $t = 1$ I-investors receive *a private signal* that conveys information about the mean μ of the log payoff. The signal takes the form $s = \mu + \epsilon$, where ϵ is a stochastic error term.[6] The signal is interpreted differently by the

[5] It would perhaps be more appropriate to use the term 'uncertainty-free' to describe this asset in our setting, but we stay with the usual terminology.

[6] A similar signal structure without ambiguity appears in Peress (2009) relating to the analysis of Peress (2004).

informed I-investors and the uninformed U-investors, if the latter observe it. This differential interpretation is related to the signal error term ϵ.

Both types of investor agree that the signal error ϵ is distributed normally with precision ρ_ϵ. However, they have different assessments of the mean μ_ϵ of the error term. I- investors perceive *ambiguity in the signal* in the sense that they know only that $\mu_\epsilon \in [-\delta, \delta]$ where $\delta > 0$. We denote I-investors' assessment of the mean by μ_ϵ^I. This structure means that I-investors use a set of likelihoods, indexed by $\mu_\epsilon^I \in [-\delta, \delta]$, in updating their beliefs, which we discuss below in section 2.1.1.

This ambiguity in the signal reflects the possibility that the signal provides biased information about the payoff of the asset. I-investors may doubt the unbiasedness of a signal because of concerns about the signal source, because the information is intangible in the sense of Daniel and Titman (2006), or because the relationship between the signal and the asset is ambiguous, among other possibilities (see for example, the discussion in Epstein and Schneider (2008) and Illeditsch (2011)).[7] As we discuss in section 6.1, ambiguous private information about a non-traded asset like labor income, whose payoff is correlated with that of the stock will also lead to a signal structure like that above.

On the other hand, U-investors believe the signal is unbiased, i.e. their assessment of the mean $\mu_\epsilon^U = 0$. Imposing this structure means that the informational inefficiency derives from the ambiguity in information perceived by the ambiguity-averse recipients and not the uninformed investors. That is, it not the uninformed investors' inability to interpret information which drives informational inefficiency. This assumption that U-investors are sure about the (non-)bias in the signal and hence do not perceive any ambiguity in it could be relaxed at the cost of some additional complexity in notation, but without much additional insight into the cause and nature of partial revelation. We discuss how in section 6.3.

2.1 Decision making

The investors maximize the expected utility of terminal wealth W_2. Their von Neumann-Morgenstern utility, denoted by u^n, is in the constant relative risk aver-

[7] In Epstein and Schneider (2008) and Illeditsch (2011), ambiguity in the signals is captured through an interval of signal variances, rather than through an interval of signal means as done here.

sion class (CRRA) with common CRRA coefficient γ, i.e.

$$u^n(W_2) = \frac{W_2^{1-\gamma}}{1-\gamma}. \tag{1}$$

Since I-investors receive signals which they perceive as ambiguous, their updated beliefs will not be represented by a single probability distribution and will instead be represented by a set of distributions as we discuss in section 2.1.1.

If investors perceive ambiguity after incorporating all information from private and public signals and from prices, their decision-making is modeled using the Gilboa and Schmeidler (1989) representation. Denoting by M^n the set of distributions representing investor n's beliefs given his information, the utility from a portfolio θ^n is

$$\mathbf{U}^n(\theta^n) = \min_{m \in M^n} \mathbb{E}_m[u^n(W_2)] = \min_{m \in M^n} \mathbb{E}_m\left(\frac{W_2^{1-\gamma}}{1-\gamma}\right) \tag{2}$$

This representation includes the case of U-investors who do not perceive any ambiguity. In this case, M^U is a single probability distribution and the utility $\mathbf{U}^U(\cdot)$ corresponds to the Savage (1954) and Anscombe and Aumann (1963) subjective expected utility representation.

This is a non-smooth representation of decision-making under ambiguity which has been fruitfully used to study a wide array of financial market phenomena (see for example, the discussion in Epstein and Schneider (2010)). Here, building on the result of Condie and Ganguli (2011a), we use the non-smoothness to construct partially revealing rational expectations equilibria different from those obtained via the assumption of noise trading, endowment shocks, or taste shocks or the assumption of higher-dimensional private information. Dow and Gorton (2008) provide a recent discussion of the noise and endowment and taste shocks mechanisms, while Ausubel (1990) presents an application of partial revelation due to higher-dimensional private information.[8]

In the present set up, the utility $\mathbf{U}^n$ is everywhere differentiable except when the terminal wealth from portfolio holdings is not uncertain, i.e. when the investor trades away his holdings of the stock and holds only the risk-free asset. These positions will be key to our analysis since the utility is non-differentiable at this value, which in

[8] Allen and Jordan (1998) provide an extensive discussion of the results on existence of rational expectations equilibria, fully or partially revealing, covering the higher, equal, and lower dimensional cases for smooth models of preferences.

turn is key for the partial revelation equilibria in the present model as we discuss in section 3.[9]

There has also been work on smooth representations of ambiguity averse preferences, inter alia, by Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Hansen and Sargent (2007). These representations will not generate the partial revelation we study here. The reason for this is very similar to that for standard (smooth) Savage (1954) and Anscombe and Aumann (1963) expected utility preferences which do not exhibit sensitivity to ambiguity. The smoothness of these preferences imply that market-clearing prices in markets populated by only traders with such preferences will always respond to changes in private information, which rules out the possibility of the partial revelation we study here almost surely as we further clarify in section 3. See also Radner (1979), Grossman (1981), Allen and Jordan (1998), and Condie and Ganguli (2011a) for closely related analyses and discussion.

2.1.1 Information and updating

In order to understand the nature of partial revelation in this model, we must first specify how information is incorporated into the beliefs of investors. In the present framework, information is processed and incorporated using an updating rule developed in Epstein and Schneider (2007) and Epstein and Schneider (2008).[10] This updating rule includes standard Bayesian updating with unambiguous beliefs as a special case, which we discuss first.

In Bayesian updating with unambiguous beliefs, suppose that the probability of an uncertain event depends on a parameter B over which the decision maker has a prior denoted $Pr(B)$ and that given a parameter value B_0, the likelihood of receiving a signal s is given by $L(s|B_0)$. Then Bayes' rule indicates that the updated distribution of B conditional on having observed the signal s is

$$Pr(B|s) = \frac{Pr(B)L(s|B)}{\int L(s|B)dB}. \tag{3}$$

[9] Though we will not explore this here, other portfolio positions where utility is non-differentiable could be used for studying the kind of partial revelation we present here.

[10] All investors make decisions only once after receiving information, so issues of dynamic inconsistency do not arise here. Nevertheless, this updating rule and our assumptions ensure that all investors' make dynamically consistent decisions if and when any intertemporal comparisons are made.

For the case of ambiguous beliefs, Epstein and Schneider (2007) consider the set of possible Bayes updates that arise from a set of possible likelihoods and a prior. If the prior is $Pr(B)$ and the set of likelihoods is $\{L(s|B)\}_{L\in\mathcal{L}}$ for some index set $\mathcal{L}$, then the set of updated beliefs is given by

$$\{Pr(B|s)\} = \left\{ \frac{Pr(B)L(s|B)}{\int L(s|B)dB} \middle| L \in \mathcal{L} \right\} \tag{4}$$

For the model with ambiguous beliefs presented here, the prior over the mean of v is a normal distribution with mean μ_0. The set of likelihoods is the set of normal distributions with mean $\mu_0 + \mu_\epsilon$ with μ_ϵ in the set $[-\delta, \delta]$.

Standard results on Bayesian updating with normal distributions imply that given μ_0 and $\mu_\epsilon \in [-\delta, \delta]$, the mean of v, conditional on having observed the signal s is normally distributed with mean

$$\mu|s = \frac{\rho_0\mu_0 + \rho_\epsilon (s + \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \tag{5}$$

and precision

$$\rho|s = \rho_0 + \rho_\epsilon. \tag{6}$$

Therefore, the set of updated priors representing the ambiguity of an investor is the set of normal distributions with precision $\rho_0 + \rho_\epsilon$ and means

$$\{\mu|s\} = \left\{ \frac{\rho_0\mu_0 + \rho_\epsilon (s + \mu_\epsilon)}{\rho_0 + \rho_\epsilon} \middle| \mu_\epsilon \in [-\delta, \delta] \right\} \tag{7}$$

In what follows, we will denote the set of distributions after observing the private signal s by $M^n(s)$ for investor n. Given the assumptions about prior beliefs and signals, notice that this set can be indexed by the interval $[\underline{\mu}|s, \overline{\mu}|s]$, which is the interval of means of beliefs about μ as defined in equation (7).

2.2 Market prices and rational expectations equilibria

Trade in the assets occurs in period 1 and equilibrium requires that markets for all assets clear. Market prices play the role of information aggregators and communicators through a price function.

A price function $\mathbf{P}$ maps signal values s to prices, i.e. $\mathbf{P}(s) = (P(s), R_f(s))$, where

$P(\cdot)$ denotes the price of the uncertain asset while $R_f(\cdot)$ denotes the gross return on the risk-free asset. Information is revealed through prices when the prevailing market prices under two signals that convey different information are different, i.e. the function $\mathbf{P}$ is invertible. When this occurs for all signals, market participants can correctly infer the signal by observing the prices in the market and the price function $\mathbf{P}$ is said to be *fully-revealing.*

The market price may not reveal all privately held information if the function from signal information into equilibrium prices is not invertible. In this case, the function is said to be *partially revealing.* When prices are partially revealing, more than one signal may be consistent with the observed price. Upon observing the market prices (P, R_f), each investor knows that the signal s is in the set $\mathbf{P}^{-1}(P, R_f)$.

The holdings of investor n in the uncertain and risk-free assets are x_t^n and $b_t^n, t = 0, 1$, respectively with $b_0^n = 0$ and $0 < x_0^n < 1$. Hence, initial wealth for investor n at price P is $W_0^n = x_0^n P$, whereas the terminal or period 2 wealth of investor n at time 2 given choices in period 1 is $W_2^n = x_1^n V + b_1^n V_f$. The fraction of wealth put into the risky asset at time 1 is labeled θ^n. By definition

$$x_1^n = \frac{\theta^n W_0^n}{P} \tag{8}$$

The market clearing conditions for the assets are

$$\begin{aligned} \sum_n \frac{\theta^n}{P} W_0^n &= 1 \\ \sum_n (1-\theta^n) R_f W_0^n &= 0. \end{aligned} \tag{9}$$

We now provide a definition of rational expectations equilibrium (REE) for this setting.

Definition 1. A *rational expectations equilibrium* is a set of portfolio weights $\{\theta^n(s)\}_{n\in\mathcal{N}}$ and a price function $\mathbf{P}$, which specifies prices $P(s)$ and $R_f(s)$ for each signal s, such that the following hold almost surely.

1. Each I-investor has information s and $\mathbf{P}^{-1}(P(s), R_f(s))$ and chooses a portfolio $\theta^I(s)$ that satisfies

$$\theta^I(s) \in \arg\max \mathbf{U}^I(\theta | s, \mathbf{P}^{-1}(P(s), R_f(s))) \tag{10}$$

2. Each U-investor has information $\mathbf{P}^{-1}(P(s), R_f(s))$ and chooses a portfolio $\theta^U(s)$ that satisfies

$$\theta^U(s) \in \arg\max \mathbf{U}^U(\theta|\mathbf{P}^{-1}(P(s), R_f(s))) \tag{11}$$

3. The market clearing equations given in (9) are satisfied.

Given this definition, an REE is said to be *fully revealing* when the equilibrium price function is fully revealing and it is said to be *partially revealing* otherwise. In the above definition, we specify I-investors information as the private signal s and the price information $\mathbf{P}^{-1}(P(s), R_f(s))$ for completeness. In the present structure, price does not convey more information than their private information s to I-investors.

2.3 Investor demand and inertia

To solve this model, we will first solve for investor demand by adapting the standard method for approximating asset returns given the lognormality assumption on the payoff distribution (see for example Campbell and Viciera (2002)). This solution method becomes exact as the discrete time interval shrinks to zero. We briefly discuss here the approximation as applicable to I-investors, since this covers the case of U-investors also. Details are provided in section 8.1.1.

Let M^n denote the set of distributions that represent the beliefs of investor n conditional on any information that she may have received. Let σ^2 denote the conditional variance of the investor's log portfolio payoff.

We approximate the return on initial wealth W_0^n as a function of the returns to the individual assets. Throughout, lowercase letters represent the natural log of model variables. Given n's portfolio $(\theta^n, 1-\theta^n)$ and using $R = V/P$ to denote the return on the uncertain asset, terminal wealth is given by

$$W_2^n = W_0^n(\theta^n R + (1-\theta^n)R_f). \tag{12}$$

If terminal wealth W_2^n is lognormally distributed then the solution to the individual's optimization problem is equivalent to the solution to

$$\max_{\theta} \min_{m \in M^n} \ln \mathbb{E}_m \left[\frac{(W_2^n)^{1-\gamma}}{1-\gamma} \right]. \tag{13}$$

Using the approximation of returns (given in (94) in section 8.1.1), we can rewrite the investor's optimization problem as

$$\max_{\theta}\min_{m\in M^n}\mathbb{E}_m\theta\left(r-r_f\right)+\frac{\theta(1-\theta)}{2}\sigma^2+\frac{(1-\gamma)\theta^2}{2}\sigma^2. \tag{14}$$

Using $[\underline{\mu}^n,\overline{\mu}^n]$ to denote the interval of means for $v(\equiv lnV)$ given by the set of distributions M^n, investor n demand is as expressed in the following result.

Lemma 2.1. *The optimal portfolio weight for the stock under beliefs M^n is given by*

$$\theta^n(M^n)=\begin{cases}\frac{1}{\gamma\sigma^2}\left(\underline{\mu}-r_f+\frac{1}{2}\sigma^2-p\right) & \underline{\mu}-r_f+\frac{1}{2}\sigma^2-p>0\\ 0 & \underline{\mu}-r_f\le p-\frac{1}{2}\sigma^2\le\overline{\mu}-r_f\\ \frac{1}{\gamma\sigma^2}\left(\overline{\mu}-r_f+\frac{1}{2}\sigma^2-p\right) & \overline{\mu}-r_f+\frac{1}{2}\sigma^2-p<0\end{cases} \tag{15}$$

In the above expression, note that the case of $\underline{\mu}-r_f\le p-\frac{1}{2}\sigma^2\le\overline{\mu}-r_f$ corresponds to a situation where the investor trades from his non-zero initial stock position to a zero position in the stock. Thus, this demand does involve trading and is not a no-trade position.

Since we can work with relative prices, we normalize $R_f=1$, i.e. $r_f=0$ hereafter and work with the stock price P, with $p\equiv\ln P$ in analysing rational expectations equilibrium prices.

We use $[\underline{\mu}|s,\overline{\mu}|s]$ to denote the updated interval of means using the rule given in (7), where

$$\underline{\mu}|s=\frac{\rho_0\mu_0+\rho_\epsilon\left(s-\delta\right)}{\rho_0+\rho_\epsilon}\text{ and }\overline{\mu}|s=\frac{\rho_0\mu_0+\rho_\epsilon\left(s+\delta\right)}{\rho_0+\rho_\epsilon}. \tag{16}$$

Using the above and the optimal portfolio expression provided in (15), the demand for the uncertain asset from I-investors is described in the following result.

Proposition 1. *The optimal portfolio weight on the stock for an I-investor who observes signal s about the mean stock payoff is given by*

$$\theta^I(s)=\begin{cases}\frac{1}{\gamma\sigma^2}\left(\underline{\mu}|s+\frac{1}{2}\sigma^2-p\right) & \underline{\mu}|s+\frac{1}{2}\sigma^2-p>0\\ 0 & \underline{\mu}|s\le p-\frac{1}{2}\sigma^2\le\overline{\mu}|s\\ \frac{1}{\gamma\sigma^2}\left(\overline{\mu}|s+\frac{1}{2}\sigma^2-p\right) & \overline{\mu}|s+\frac{1}{2}\sigma^2-p<0\end{cases} \tag{17}$$

Given the expressions for $\underline{\mu}|s$ and $\overline{\mu}|s$ in (16), the demand expression shows that I-investors require an uncertainty premium whenever they do not trade away their stock holding to a zero position in addition to the usual premium demanded by any investor with the same information who does not perceive any ambiguity. When they are long in the stock, i.e. $\theta^I(s) > 0$, I-investors require a reduction in price of $\delta\rho_\epsilon/(\rho_0 + \rho_\epsilon)$ relative to the case of no ambiguity given their effective belief $\underline{\mu}|s$. Similarly, when they short the stock, i.e. $\theta^I(s) < 0$, they require that an increase in price of $\delta\rho_\epsilon/(\rho_0 + \rho_\epsilon)$ given their effective belief $\overline{\mu}|s$. Whenever the price does not incorporate this additional uncertainty premium, they trade away their stock holding to a zero position.

The demand expression also exhibits two interesting and complementary facts about the demand θ^I of A investors. The first is that for any given signal value s, there exists a range of prices for which it is optimal for A investors to trade away their stock holdings to a zero position ($\theta^I = 0$). This fact was first noted by Dow and da Costa Werlang (1992b) as portfolio inertia with respect to prices.

The second fact is that for a fixed price p, the A investors will still find it optimal to trade to a zero position even if they observe a different signal $s' \neq s$ instead of s. That is, at $\theta^I_1 = 0$, there is *portfolio inertia with respect to information.* We will show below that this inertia can lead to the existence of a partially revealing rational expectations equilibrium price.[11] Given the above discussion, whether or not the price incorporates the uncertainty premium of $\delta\rho_\epsilon/(\rho_0 + \rho_\epsilon)$ plays an important role since it determines whether the inertia position is possible. Finally, note also that this inertia does not appear in smooth models of ambiguity averse preferences and so these will not lead to the partial revelation property we study here.

3 Partial revelation and inertia

Recall that non-revelation of signals s and s' requires that the price is the same for both, i.e. $P(s) = P(s') = P$. Given the optimal portfolio expression provided in (15), at price P (with $p = \ln P$), investor I will trade to a zero-position in the uncertain asset under both signal realisations s and s' if $[\underline{\mu}|s, \overline{\mu}|s] \cap [\underline{\mu}|s', \overline{\mu}|s']$ is non-empty and

$$\max\{\underline{\mu}|s, \underline{\mu}|s'\} \leq p - \frac{1}{2}\sigma^2 \leq \min\{\overline{\mu}|s, \overline{\mu}|s'\}. \tag{18}$$

[11] Condie and Ganguli (2011a) use this property in the context of general financial market exchange economies to establish robust existence of partially revealing rational expectations equilibria.

This observation is key in the existence of partially-revealing equilibria with ambiguous information and is where the property of inertia with respect to information under non-smooth preferences will be utilised.

3.1 A benchmark: unambiguous information and revelation

To begin, it is useful to consider the case of unambiguous information first and note that in this case, private information will (almost surely) be revealed by the market price. This is the message of Grossman (1976), Grossman (1981), and Radner (1979) among others. We present the analysis here to clarify the role of ambiguous information in partial revelation and it will be further useful in discussing the conditions under which ambiguous information is not revealed.

Suppose that all information is unambiguous and so each investor's beliefs can be represented by a single probability distribution.[12] That is, suppose I-investors also believe the signal is unbiased, $\mu^I_\epsilon = 0$.[13]

In this case, there is no ambiguity and their updated belief about the mean of v is given by a normal distribution with precision $\rho_0 + \rho_\epsilon$ and mean

$$\underline{\mu}|s = \overline{\mu}|s = \mu^I|s = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon}. \tag{19}$$

At stock price P, with $p \equiv \ln P$, the demand of A investors is then

$$\theta^I(s) = \frac{1}{\gamma\sigma^2}\left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - p\right). \tag{20}$$

Since $\theta^I(s)$ varies linearly with s, there is no inertia with respect to information. This in turn will imply that the rational expectations equilibrium market-clearing price will reveal s since it will vary with s monotonically. To see this, suppose first that U-investors do not use any information from price to update their beliefs, i.e. their demand is given by $\theta^U = \mu_0 + 0.5\sigma^2 - p$.

Using the market clearing condition (9) for the stock, the price $p^1(s)$, is

$$\left(x^A_0\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + x^U_0\mu_0\right) + \frac{1-2\gamma}{2}\sigma^2 = p^1(s), \tag{21}$$

[12]Equivalently, consider a market that begins with all investors having homogeneous information and participating in the market.

[13]Similar reasoning applies for any fixed value of $\mu^I_\epsilon \neq 0$.

which is linear in s.

Thus observing this price reveals the signal s to the uninformed investors E. As envisaged in the rational expectations equilibrium framework U-investors then use this information to update their beliefs, which in turn means that their demand is now given by $\theta^U(s) = \frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon} + 0.5\sigma^2 - p$.

Using the market clearing condition (9) for the stock again, the price $p^0(s)$ is

$$\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1-2\gamma}{2}\sigma^2 = p^0(s), \tag{22}$$

which is linear in s and hence reveals s to U-investors and in fact is a fully-revealing rational expectations equilibrium price function.

Finally, notice also in the above analysis that a change in information s will change demand almost surely. Thus, there is no inertia in demand with respect to information and (18) will (almost surely) not be satisfied for distinct signals s and s' at any price p, thus ruling out non-revelation of signals. This observation is also helpful in the discussion below where information is ambiguous.

3.2 Ambiguous information and partial revelation

Now consider the receipt of an ambiguous private signal by I-investors. Their updated beliefs are represented by the set $[\underline{\mu}|s, \overline{\mu}|s]$, where the endpoints are given in equation (16). While, U-investors do not receive the signal, they may infer it if it is revealed by the market price P. We use $\mu^U_{PR}|\cdot$ to generically denote the updated beliefs of U-investors about the mean of v. We will be explicit about the construction of these beliefs shortly.

For now, using similar reasoning as in section 3.1 note that if the price distinguishes two distinct signal values s and s', i.e. $P(s) \neq P(s')$, then these signals are revealed by the price to U-investors and hence $\mu^U_{PR}|s \neq \mu^U_{PR}|s'$.[14] On the other hand, if price does not reveal the signals, i.e. $P(s) = P(s')$, then $\mu^U_{PR}|s = \mu^U_{PR}|s'$. As noted above, this non-revelation requires inertia with respect to information, in particular for (18) to hold.

[14]More precisely, $\mu^U_{PR}(s) \neq \mu^U_{PR}|s'$ almost surely given the operation of standard Bayesian updating with unambiguous beliefs.

3.2.1 Reduced stockholding by I-investors

Since (18) is needed for partial revelation to be possible, we now show that it can be satisfied when I-investors exhibit inertia with respect to information. In turn, given the preference representation and structure of information in this model, I-investors will exhibit inertia when they trade away their stock holdings to a zero position.

Partial revelation further requires that markets clear, so we first investigate when I-investors trading away their stock holdings to a zero position is consistent with market-clearing. First, recall from the discussion following equation (17) that I-investors will hold a positive position in equilibrium only if market-clearing price include an uncertainty premium of $\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$. On the other hand, if I-investors do not hold a positive position in equilibrium then U-investors have to hold all of the stock.

If the total number of stockholders is smaller, those that are holding the stock are holding more risk and must be compensated for it. That is, market clearing prices must include a premium to compensate the U-investors for holding all of the stock. This reduced stockholding premium is given by $\frac{\gamma\sigma^2}{x_0^U}$. Since $0 < x_0^U < 1$, this premium is larger than the usual risk premium $\gamma\sigma^2$ which would be required if only U-investors populated the market. Thus comparing the two premia will clarify whether or not markets clear with I-investors at their inertia position of no stockholding. This intuition is summarized in the following result, whose proof is in the appendix.

Proposition 2. *If* $\frac{\gamma\sigma^2}{x_0^U} > \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$ *then markets clear with* $\theta^I(s) > 0$ *almost surely.*

Thus, if the uncertainty premium required by I-investors for a positive position is less than that the reduced stockholding premium required by U-investors for holding all of the stock, then in equilibrium, markets can clear with prices reflecting the lower uncertainty premium required by I-investors and with $\theta^I(s) > 0$ almost surely.[15] If $\theta^I(s) > 0$, then analogous reasoning as in section 3.1 shows that there is no inertia with respect to information and hence the partial revelation condition (18) can not be satisfied. Moreover, using the same reasoning shows that if $\theta^I(s) > 0$ when markets clear for some signal value s, then $\theta^I(s') > 0$ for all $s' \neq s$ given the relation between the updated beliefs of U-investors and I-investors noted earlier, i.e. $\mu^U_{PR}|s \in [\underline{\mu}^A|s, \overline{\mu}^A|s]$ for all s.

[15] This reasoning is related to that in Easley and O'Hara (2009), but not the same since there is no updating of beliefs in Easley and O'Hara (2009).

As the inertia property requires $\theta^I = 0$ to be consistent with market clearing, the above discussion yields the following result about informational efficiency of prices.

Corollary 1. *If* $\frac{\gamma\sigma^2}{x_0^U} > \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$ *then any rational expectations equilibrium price function is monotonic in signals and hence fully revealing.*

In light of this result, we next consider market-clearing prices where $\theta^I(s) = 0$, i.e. I-investors trade away their stock holdings to a zero position, to study prices which are not fully revealing. That is, we consider markets where the uncertainty premium required by I-investors to hold the stock is too high relative to the premium required by U-investors to hold the all of the stock.

Suppose markets clear at price P with the U-investors holding the entire stock supply and the I-investors trading away their stock holding to hold a zero position in the stock. Given signal s, the market-clearing price $p(s)$ satisfies

$$p(s) - 0.5\sigma^2 = \mu_{PR}^U|s - \frac{\gamma\sigma^2}{x_0^U}. \tag{23}$$

As noted above, the term $\frac{\gamma\sigma^2}{x_0^U}$ reflects the reduced stockholding premium required by U-investors over and above the usual risk premium $\gamma\sigma^2$ by a factor of $\frac{1}{x_0^U}$. We next consider when price given by (23) reveals and does not reveal private signals received by I-investors.

3.2.2 Revelation of extreme information

We now turn to whether the market price may reveal the signal when the I-investors trade to a zero position in the stock. In the present setup, as we will show below, partial revelation takes the form of market price P not revealing intermediate information, in particular an interval of signal values, while revealing extreme information, i.e. signal values outside of the interval.

The reason that market prices may reveal relatively very good news or very bad news in a rational expectations equilibrium is as follows. The price that I-investors demand for their stock holdings will depend on their information. When I-investors receive very good news in the form of a high signal value, they will demand a higher price for selling their stock holdings given their updated belief that mean stock payoff is high. In turn, U-investors are willing to pay a higher price for the stock holdings if they believe a high enough signal has been received since this would mean a higher (estimate of) mean stock payoff. Moreover, they will also be more sensitive to price and through the price to I-investor information when they believe it is extreme.

For high enough signal values, this means the market clearing price will be responsive to signal values, i.e. will change as the signal value changes. Given the self-fulfilling nature of rational expectations equilibrium, this means that market price will reveal high enough signals in equilibrium. Similar reasoning applies to the case of very bad news in the form of very low signal values. I-investors would be willing to accept a lower price for selling stock holding since their updated belief about the mean stock payoff is low and U-investors would be willing to only pay lower prices if they believe a low enough signal has been received, meaning a lower (estimate of) mean stock payoff. Again, this will mean prices are responsive signals and hence revealing. To summarize, with extreme information, the signal extraction problem may actually be easier for the uninformed investors.

3.2.3 Non-revelation of intermediate information

For moderate information in the form of intermediate signal values, price may not be responsive to information as A- and U-investors may trade at the same price for a range of signal values. This in turn means that price will not reveal changes in signals when information is moderate as we show next.

As noted above, for partial revelation to be possible, market-clearing requires U-investors to hold the entire stock supply and I-investors to trade away their holdings of the stock. Hence, the equilibrium price must satisfy

$$p = \mu^U_{PR} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x^U_0}\sigma^2. \tag{24}$$

Combining this with the condition given in equation (18) for stock price to not reveal, i.e. not distinguish two distinct signal values s and s' yields

$$\max\{\underline{\mu}|s, \underline{\mu}|s'\} \leq \mu^U_{PR} - \frac{\gamma\sigma^2}{x^U_0} \leq \min\{\overline{\mu}|s, \overline{\mu}|s'\}. \tag{25}$$

These inequalities and the expressions for updated beliefs $\underline{\mu}|s$ and $\overline{\mu}|s$ given in equation (16) can be used to characterise the beliefs μ^U_{PR} of the uninformed E investors for unrevealed signals as we describe next. The form of μ^U_{PR} can then be used to characterise the set of unrevealed signals.

3.3 The price function and uninformed investor beliefs

Rearranging the inequalities in (25) suggests that for the uninformed investors to hold the entire stock supply in equilibrium with the stock price not changing in response to changes in signal values, the signals must not be too extreme and must lie in some intermediate range or interval with finite bounds. We proceed to solve for uninformed investors beliefs and the partially revealing price function with this structure. Using the explicit expressions for $\underline{\mu}|s$ and $\overline{\mu}|s$ given in equation (16) and rearranging terms, this range for s is given by

$$\mu^U_{PR}+\delta+\frac{\rho_0}{\rho_\epsilon}(\mu^U_{PR}-\mu_0)-\frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x^U_0} \geq s \geq \mu^U_{PR}-\delta+\frac{\rho_0}{\rho_\epsilon}(\mu^U_{PR}-\mu_0)-\frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x^U_0} \quad (26)$$

Let a denote the lower bound on this range, as given by the second inequality and b denote the upper bound on this range as given by the first inequality. Thus, for the beliefs of U-investors to be consistent with informed I-investors selling off their stockholding at the same price for a range of signal values, the U-investors will only know that the signal observed by I-investors lies an interval $[a, b]$ of potential signals. This means that U-investor belief $\mu^U_{PR}|s$ will be constant over the interval $[a, b]$ and we denote by $\mu^U_{PR}|[a, b]$ this constant belief.

Before proceeding to solve for uninformed belief μ^U_{PR}, we note that the above analysis and equation (26) together suggest the following form for the price function p_{PR} under partial revelation.

$$p_{PR}(s) = \begin{cases} \mu^U_{PR}|[a, b] + \frac{1}{2}\sigma^2 - \frac{\gamma}{x^U_0}\sigma^2 & \text{if } s \in [a, b] \\ \mu^U_{PR}|s + \frac{1}{2}\sigma^2 - \frac{\gamma}{x^U_0}\sigma^2 & \text{if } s < a \text{ or } s > b \end{cases} \quad (27)$$

Beliefs $\mu^U_{PR}|s$ are monotone and linear in s for $s \notin [a, b]$ while they are constant at $\mu^U_{PR}|[a, b]$ for $s \in [a, b]$. This suggests that the price function is non-linear in signal values. We explore this and further properties of the price function after investigating the nature of uninformed beliefs μ^U_{PR} next.

3.3.1 Uninformed investor beliefs

The boundaries of the interval $[a, b]$ are determined endogenously according to equation (26). First, note that if the uninformed U-investors infer the signal s from price, their updated beliefs about the mean of v would be

$$\mu_{PR}^{U}|s = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} \tag{28}$$

When the U-investors are not able to infer the signal, their belief about the mean of v is obtained by using the updated beliefs conditional on the knowledge that the signal is in $[a, b]$. Let $f(s|a \leq s \leq b)$ denote the marginal probability density function over signals conditional on the signal being between a and b. Then this expected value is

$$\begin{aligned} \mathbb{E}\left[\mu|a \leq s \leq b\right] &= \int_a^b \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} f(s|a \leq s \leq b)ds \\ &= \frac{1}{\rho_0 + \rho_\epsilon}\left(\rho_0\mu_0 + \rho_\epsilon \int_a^b sf(s|a \leq s \leq b)ds\right) \\ &= \frac{1}{\rho_0 + \rho_\epsilon}\left(\rho_0\mu_0 + \rho_\epsilon \mathbb{E}[s|a \leq s \leq b]\right) \end{aligned} \tag{29}$$

Recall that $s = \mu + \epsilon$ where μ and ϵ are independent, normally distributed random variables. Since the uninformed U-investors believe that the signal is unbiased, and thus that s is normally distributed with mean μ_0 and precision $\rho_0 + \rho_\epsilon$, the expected value of s conditional on s being in the interval $[a, b]$ is therefore

$$\mathbb{E}\left[s|s \in [a, b]\right] = \mu_0 + \Delta(a, b) \tag{30}$$

where

$$\Delta(a,b) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b-\mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a-\mu_0)\right)}\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}, \tag{31}$$

with ϕ and Φ denoting the standard normal density and distribution functions respectively. The above is derived from the properties of the truncated normal distribution (see e.g. Johnson and Kotz (1970)).

Simplifying expression (29) gives

$$\mu_{PR}^{U}|[a, b] = \mathbb{E}\left[\mu|a \leq s \leq b\right] = \mu_0 + \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\Delta(a, b). \tag{32}$$

The term

$$\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\Delta(a,b) \tag{33}$$

represents the change in beliefs U-investors when they know only that the I-investors received a signal that is not revealed by price, i.e. $s \in [a,b]$. Note that since $\mu^U_{PR}|s$ is continuous in s, there exists a signal value $\hat{s} \in [a,b]$ such that $\mu^U_{PR}|\hat{s} = \mu^U_{PR}|[a,b]$.

3.3.2 The price function

Using the above results, we can solve for the partially revealing price function. First, plugging these expression for $\mu^U_{PR}|[a,b]$ into equation (26) gives

$$\frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\left(\mu_0 - \frac{\gamma\sigma^2}{x^U_0}\right) - \frac{\rho_0}{\rho_\epsilon}\mu_0 + \delta + \Delta(a,b) \geq s \geq \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\left(\mu_0 - \frac{\gamma\sigma^2}{x^U_0}\right) - \frac{\rho_0}{\rho_\epsilon}\mu_0 - \delta + \Delta(a,b). \tag{34}$$

The left-hand side inequality relates to b and the right-hand side relates to a. So, the signal bounds that are consistent with the behavior of the informed agents are found by solving the following system of equations.

$$\begin{aligned} \mu_0 - \delta + \Delta(a,b) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x^U_0} &= a \\ \mu_0 + \delta + \Delta(a,b) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x^U_0} &= b \end{aligned} \tag{35}$$

Subtracting the second equation from the first and rearranging yields

$$b - a = 2\delta. \tag{36}$$

So, the system can be reduced to the following equation in one unknown.

$$\mu_0 - \delta + \Delta(a, a+2\delta) - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x^U_0} - a = 0. \tag{37}$$

This implicit equation in a can be solved numerically in general. Since the length of the interval of unrevealed signals is fixed at 2δ, the existence of the interval $[a,b]$ reduces to the question of when equation (37) has a finite solution for a. While we know that a solution does not exist when the condition identified in Proposition 2 holds, i.e. $\frac{\gamma\sigma^2}{x^U_0} > \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$, we do not have a proof of the existence of a solution when it does not. However, we compute an explicit solution for a special case next and all numerical configurations we attempted for other cases yielded a solution.

When all investors are risk neutral ($\gamma = 0$), this equation can be solved for a explicitly. The fixed point problem for the solution of beliefs when the signal is not revealed becomes

$$\mu_0 - \delta + \Delta(a, a + 2\delta) - a = 0. \tag{38}$$

Given the symmetry properties of the standard normal density ϕ and the expression for $\Delta(a, b)$ in equation (31), it follows that $\Delta(a, a + 2\delta) = 0$ when a and $a + 2\delta$ are symmetric around μ_0. Since the interval distance must be 2δ, we note that value $a = \mu_0 - \delta$ solves equation (38). Thus for the risk-neutral case, $b = \mu_0 + \delta$.[16]

We summarize this discussion in the following result. This confirms that the partial revelation takes the form of moderate information, given by an interval of signals, not being revealed, while extreme information, given by signals which lie outside the interval, is revealed.

Proposition 3.

1. *The length of the interval* $[a, b]$ *of unrevealed signals, if it exists, is* 2δ.
2. *The existence of an interval* $[a, b]$ *of unrevealed signals and hence the existence of partially revealing rational expectations equilibrium price function follows from the existence of a solution to equation* (37).
3. *If investors are risk-neutral, the interval of unrevealed signals is* $[\mu_0 - \delta, \mu_0 + \delta]$.

The first result means that the length of the interval of unrevealed signals is directly related to the amount of ambiguity in the signal. More ambiguity in the signals increases the set of signals which are not revealed. Moreover, even when we allow for ambiguity-averse U-investors, the size of the interval of unrevealed signals is 2δ, i.e. given by the ambiguity perceived by I-investors only (see Proposition 12 in section 6.3).

The third result with risk-neutral investors shows that knowledge of the non-revelation region $[a, b]$ may provide no meaningful information by which U-investors update beliefs, i.e. the update to belief $\Delta(a, a + 2\delta) = 0$.

[16]This is exactly the solution we obtain by starting with risk-neutral investors and repeating the above process directly.

Moreover, by the continuity of the functions involved in the fixed point problem being considered since a solution to (37) exists for $\gamma = 0$, a solution will also exist for values of $\gamma > 0$ that are small enough. Further, for $\gamma > 0$, the update $\Delta(a, a + 2\delta)$ will not usually be zero since a and $b = a + 2\delta$ will not usually be symmetric around μ_0.

Using the expressions for beliefs μ^U_{PR} obtained above, we obtain the partial revelation price function, which is unique as there is no indeterminacy in equilibrium prices. Finally, note that since the I-investors are endowed with a positive amount of the uncertain asset, i.e. $x^I_0 > 0$, partial revelation is not based on a no-trade outcome. Indeed, in the present model, trade volume under signal s is given by $|x^I_1(s) - x^I_0|$ which equals x^I_0 when I-investors trade away their stockholding. These results are summarized in the following.

Proposition 4.

1. *The price function under partial revelation is given by*

$$p_{PR}(s) = \begin{cases} \frac{\rho_0\mu_0 + \rho_\epsilon(\mu_0 + \Delta(a,b))}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x^U_0}\sigma^2 & \text{if } s \in [a, b] \\ \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x^U_0}\sigma^2 & \text{if } s < a \text{ or } s > b \end{cases} \tag{39}$$

   where $b = a + 2\delta$ and the value of a is obtained by solving (37).

2. *Trade volume under partial revelation is*

$$|x^I_1(s) - x^I_0| = x^I_0 > 0 \tag{40}$$

   for all s.

The price function is non-linear in signals and exhibits discontinuities at signal values a and b. When the signal is not revealed, U-traders' updated beliefs are based only on the knowledge that the signal could be any one of those in $[a, b]$. The updated belief based on this information $\mu^U_{PR}|[a, b]$ lies strictly between the updated belief based on the signal value a, $\mu^U_{PR}|a$, and the updated belief based on the signal value b, $\mu^U_{PR}|b$.[17] This in turn implies the discontinuity and non-linearity of the price

[17] As we discuss in section 7, it is possible to relax the assumption that U-investors perceive no ambiguity with qualitatively similar results, but with additional notational complexity.

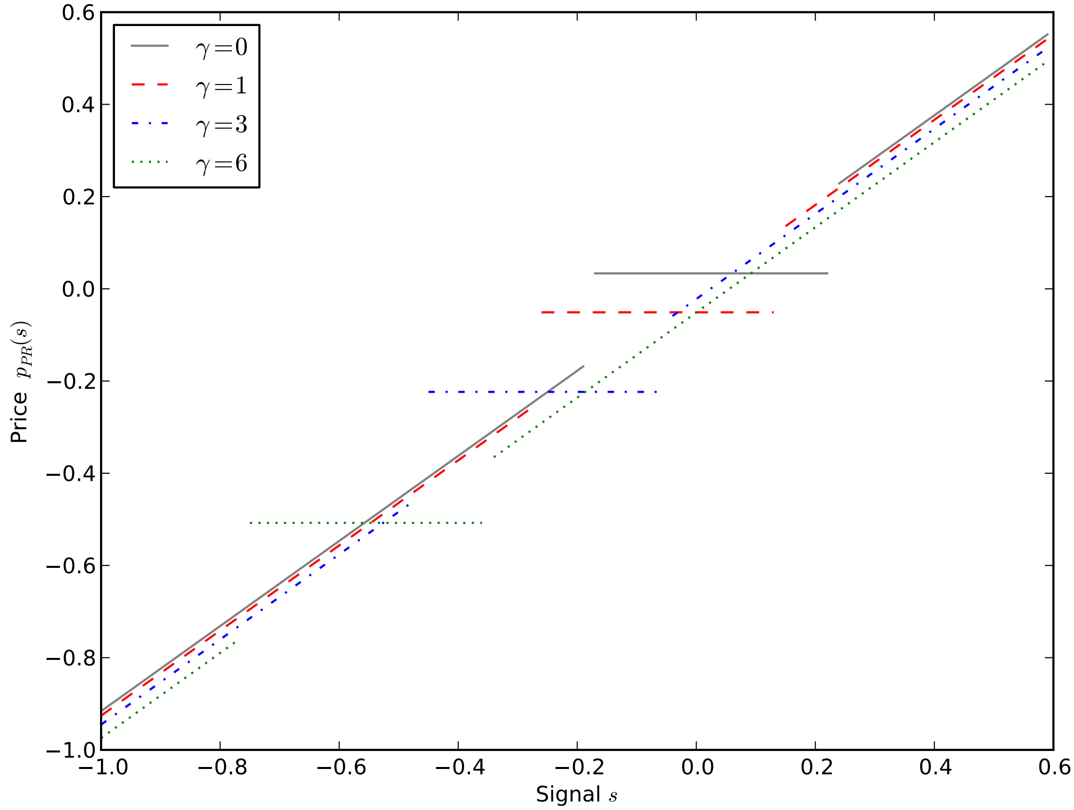

Figure 1: Partial revelation price function p_{PR}

function as shown in Figure 1, which depicts the price function for different values of γ.

Figure 1 also illustrates two other features of the price function. First, the set $[a, b]$ of unrevealed signal values moves to the left as the risk aversion γ increases. That is, the more risk-averse the U-investors are, the worse the signal values that are not revealed. Second, the update to U-investor beliefs given by $\Delta(a, b)$ is positive for risk averse U-investors ($\gamma > 0$).

It is possible to draw implications for stock price even without explicit solutions to the above equation, and we proceed to these in the next section 4. Before doing so, we compare the the nature of partial revelation obtained here, including the form of the price function p_{PR}, with that obtained from noise-based partial revelation.

REE models with noise traders and ambiguity-averse traders such as Ozsoylev and Werner (2011) and Mele and Sangiorgi (2011) and without noise traders such as Easley, O'Hara, and Yang (2011) feature continuous price functions.[18] The price function in the commonly-used Grossman and Stiglitz (1980) framework is linear, which is driven by the assumption of normal distributions, CARA utility with no wealth constraints, and unambiguous beliefs. Other models of noise-based partial revelation impose different distributional or utility assumptions such as Mailath and Sandroni (2003) and Barlevy and Veronesi (2003) or wealth constraints such as Yuan (2005) to get non-linear price functions under noise-based partial revelation. A discontinuous price function appears in Gennotte and Leland (1990) due to the presence of exogenous portfolio insurance or hedging demand.[19]

In the present model, partial revelation involves an intermediate range of signal values not being revealed and relatively extreme values being revealed. As we show in section 4.1, this form of partial revelation implies the possibility of discontinuous price changes and large discontinuous variations in price volatility.

Under noise-based partial revelation without ambiguity, all signal values are obscured by the noise shock in equilibrium and as such the partial revelation does not have implications for price volatility beyond what noise adds. That is, noise contributes to the properties of equilibrium prices in these models but in a way that is qualitatively similar to how noise traders would alter prices in a setting without private information.

Another feature which distinguishes partial revelation discussed here from that in some common noise-based models is that information on volume does not change the informational properties of the prices here. In CARA-normal models of noise-based partial revelation adding trading volume information makes partially revealing prices fully revealing. See Blume, Easley, and O'Hara (1994) and Schneider (2009) for a discussion of this issue. Overall, the differences in partial revelation due to ambiguous information and noise-based partial revelation suggest that in principle, these differing forms of partial revelation may be differentially useful, possibly even

[18] In Ozsoylev and Werner (2011), ambiguity-averse traders do not receive any private signals, while in Mele and Sangiorgi (2011) private information eliminates ex-ante ambiguity. In Easley, O'Hara, and Yang (2011), ambiguity-averse uninformed 'simple' traders are ambiguous about the trading strategy of 'opaque' traders and this yields a price function which is not fully informative for the 'simple' traders.

[19] The price function in Barlevy and Veronesi (2003) is discontinuous at a point in the noise trade variable. They analyse a set up with risk-neutral traders and binomially distributed values for the stock payoff.

complementary, in studying financial markets.

Noise-based partial revelation is also used to provide a resolution to the Grossman and Stiglitz (1980) paradox of costly information acquisition. The present model does not address this issue. It is not clear that all information used in financial markets involves a direct cost. One example would be information from a non-traded asset like labor income, whose payoff is correlated with, and hence informative about, the stock payoff. We discuss this in the present model's context in section 6.1. Also, work by Bernardo and Judd (2000), Muendler (2007), and Krebs (2007) suggests that the co-existence of informationally efficient prices and costly information is not paradoxical outside of the widely-used CARA-normal models, where wealth effects are absent and normality assumptions yields linearity of the equilibrium price function.

3.3.3 Self-fulfilling full revelation

We have thus far focused on the possibility of partial revelation rational expectations equilibrium when markets clear with I-investors trading away their stockholding to U-investors. However, the self-fulfilling nature of rational expectations equilibrium (Definition 1) means that it is also possible for a full revelation rational expectations equilibrium to exist with this trading behavior.

This full revelation equilibrium is one where U-investors correctly conjecture a price function that is monotonic and revealing in s irrespective of the trading behavior of I-investors. Their demand then reflects their updated beliefs. In equilibrium, for each signal, markets clear with the price given by this conjectured fully-revealing price function, with I-investors selling all their stockholding to U-investors at that price. Thus, given the self-fulfilling nature of rational expectations equilibrium, this price function will be a fully-revealing equilibrium price function. The details of this equilibrium are provided in section 8.2.[20]

However, this full revelation equilibrium depends on U-investors on correctly conjecturing the right revealing price for each signal observed by I-investors. While this equilibrium can not be formally ruled out or refined away, it seems less natural than the partial revelation equilibrium given the preceding discussion in sections 3.3.1 and 3.3.2 and so, we focus on the partial revelation equilibria.

[20]The analyses of Radner (1979), Grossman (1981), and Condie and Ganguli (2011b) also indicate that such a full revelation equilibrium will exist.

4 Pricing implications of partial revelation

4.1 Price volatility and jumps

The revelation and non-revelation of privately held signals has implications for the volatility of equilibrium stock prices. To see this, we consider now the variance of p_{PR} conditional on the signal being revealed or not revealed. When the signal is not revealed, i.e. $s \in [a, b]$, then the price is constant at

$$p_{PR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon(\mu_0 + \Delta(a,b))}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2. \tag{41}$$

Hence, price volatility conditional on signals not being revealed is zero, i.e.

$$Var\left[p_{PR}|s \in [a,b]\right] = 0. \tag{42}$$

On the other hand, if the signal is revealed, i.e. $s < a$ or $s > b$ then the equilibrium price is

$$p_{PR}(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2. \tag{43}$$

Thus, conditional on the signal being outside the non-revelation interval the variance of price is

$$Var(p_{PR}|s < a \text{ or } s > b) = \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right)^2 Var(s|s < a \text{ or } s > b) \tag{44}$$

We can use the properties of the truncated normal distribution, to obtain an expression for $Var(s|s < a \text{ or } s > b)$. First, note that $Var(s|s < a)$ is

$$Var(s|s<a) = \frac{1}{\rho_\epsilon}\left[1 - \frac{\phi\left(\sqrt{\rho_\epsilon}\,(\mu_0 - a)\right)}{1 - \Phi\left(\sqrt{\rho_\epsilon}\,(\mu_0 - a)\right)}\left(\frac{\phi\left(\sqrt{\rho_\epsilon}\,(\mu_0 - a)\right)}{1 - \Phi\left(\sqrt{\rho_\epsilon}\,(\mu_0 - a)\right)} - \sqrt{\rho_\epsilon}\,(\mu_0 - a)\right)\right] \tag{45}$$

and $Var(s|s > b)$ are

$$Var(s|s>b) = \frac{1}{\rho_\epsilon}\left[1 - \frac{\phi\left(\sqrt{\rho_\epsilon}\,(b - \mu_0)\right)}{1 - \Phi\left(\sqrt{\rho_\epsilon}\,(b - \mu_0)\right)}\left(\frac{\phi\left(\sqrt{\rho_\epsilon}\,(b - \mu_0)\right)}{1 - \Phi\left(\sqrt{\rho_\epsilon}\,(b - \mu_0)\right)} - \sqrt{\rho_\epsilon}\,(b - \mu_0)\right)\right] \tag{46}$$

To calculate the variance of s conditional on s not being an element of $[a, b]$, denote the probability density function of s by $f(s)$ and note that the conditional density of s is

$$f(s|s < a \text{ or } s > b) = \frac{f(s)}{F(a) + 1 - F(b)} \tag{47}$$

Using $\hat{F}(a) = \frac{F(a)}{F(a)+1-F(b)}$ and $\hat{F}(b) = \frac{1-F(b)}{F(a)+1-F(b)}$, the conditional variance is then defined as

$$\begin{aligned} Var(s|s < a \text{ or } s > b) &= \int_{-\infty}^{a} (s-\mu_0)^2 \frac{f(s)}{F(a)+1-F(b)} ds + \int_{b}^{\infty} (s-\mu_0)^2 \frac{f(s)}{F(a)+1-F(b)} ds \\ &= \hat{F}(a) \int_{-\infty}^{a} (s-\mu_0)^2 \frac{f(s)}{F(a)} ds + \hat{F}(b) \int_{b}^{\infty} (s-\mu_0)^2 \frac{f(s)}{F(b)} ds \\ &= \hat{F}(a) Var(s|s < a) + \hat{F}(b) Var(s|s > b) \end{aligned} \tag{48}$$

Together, this implies that

$$Var(p_{PR}|s < a \text{ or } s > b) = \left(\frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right)^2 \left(\hat{F}(a) Var(s|s < a) + \hat{F}(b) Var(s|s > b)\right) \tag{49}$$

We can summarize the above analysis as follows.

Proposition 5. *The volatility of stock price conditional on revelation of signals is* $Var\left(p_{PR}|s < a \text{ or } s > b\right) > 0$ *where* $Var(p_{PR}|s < a \text{ or } s > b)$ *is given by equation* (49)*. The volatility of stock price conditional on non-revelation of signals is* $Var\left(p_{PR}|s \in [a, b]\right) = 0$.

The above result leads to the following observations about large movements in price and in price volatility.

Observation 1. *Periods of non-revelation due to ambiguous signals will have strictly less volatility, ceteris paribus, than market periods when either ambiguous signals are revealed in the market or signals are not perceived to be ambiguous.*

This observation describes two aspects of information transmission through markets as studied here. The first is the role price movements play in information revelation of any kind. Periods of higher price volatility coincide with the revelation of information since the arrival of information changes traders' beliefs about the payoffs of the assets. Under partial revelation this information may not be revealed through market prices depending on the informational regime. These price movements under the revelation regime are the result of the market incorporating private information and means by which market prices convey this information to other traders.

This suggests caution in the face of policy options that might unduly limit market volatility, whether this is the goal of the policy or not. Periods of high price volatility are not necessarily bad if prices are successfully incorporating new information and transmitting that information to market participants. Likewise, for these same reasons, periods of low market volatility are not necessarily desirable.

The second point is that partial revelation of ambiguous signals differs in both the cause and the empirical implications from traditional, noise-based partial revelation. In noise-based partial revelation, every signal value is obscured, i.e. there is only one informational regime, and moreover excess volatility arises directly from the random noise. As such price volatility does not differ due to different informational regimes unlike in the current model. See also the discussion in section 5.

The second observation is that periods of low market volatility may portend large price swings followed by periods of high volatility.

Observation 2. *If partial revelation occurs because of ambiguous information as described in this model, changes in the information content of market prices will coincide with jumps in asset prices.*

As discussed above (equation (29)), when information is not revealed, U-investors will formulate beliefs by averaging over the set of signals that are not revealed by price. Very good or very bad news will be revealed, while intermediate information will not.

Since traders had previously been using only the average of the unrevealed signals, when a signal becomes available to them there is a discontinuous change in their beliefs about market fundamentals which implies a discontinuous change in market prices. This happens even if the revealed signal is very close in value to the unrevealed signal. This jump in asset prices will be positive if the previously unrevealed signal was better news than the average of the unrevealed signals. Likewise, this price swing will be negative if the unrevealed signal was worse than the average unrevealed signal.

A similar phenomenon occurs when the economy moves from revelation of signals to non revelation of signals. Although all previously revealed signals are known by all market participants, when the market moves into periods of non-revelation, unrevealed signals must be averaged over by U-investors and this can lead to a discontinuous movement in market prices, even if the change in signal value is small.[21] In general, the discontinuous nature of the price function is suggestive of crashes and jumps.

The above two observations mean that the transition out of periods of low market volatility can be hectic. In this model, periods of low market volatility occur when information is not being revealed. When the market changes in such a way that information is revealed, this happens concurrently with a large price swing, followed by a period of relatively high market volatility.

In the model of Illeditsch (2011) with a CARA representative investor and normally distributed payoffs, ambiguous information leads to a discontinuity in the price as a function of signals at the signal value which confirms the prior mean. Hence, there is also a jump in price volatility at the point of discontinuity.[22]

Dow and da Costa Werlang (1992a) provide an example of excess volatility when the standard standard variance decomposition formula used in Bayesian updating with unambiguous beliefs is violated due to ambiguity aversion.[23] Mele and Sangiorgi (2011) find that non-smooth ambiguity aversion can lead to price swings in a model with costly information acquisition under noise-based partial revelation.

4.2 Price impact

The revelation and non-revelation of signals also has implications for the price impact of a given trade, which can be used as a measure of market liquidity in the presence of adverse selection due to differentially informed investors (see for example, Brennan and Subrahmanyam (1996) and O'Hara (2003)).[24] Under partial revelation, trade volume is $x_0^I > 0$ for all signal values since the I-investors trade to a zero position in the stock. Comparing the partial revelation price for signal s with the partial

[21] This applies even when the fundamental v remains the same, but the signal value changes due to a change in the error term. This discontinuity due to a switch in informational regime can be traced back to the inertia with respect to information property exhibited by I-investors.

[22] Illeditsch (2011) models ambiguous information through a range of signal precisions, rather than through a range of means as done in the present paper. The representative investor uses the highest signal precision when the signal is below the prior mean to update his beliefs and the lowest signal precision when it is above the mean.

[23] Investors update ambiguous prior beliefs via the Dempster-Shafer rule, which does not guarantee dynamic consistency of decisions. Excess price variance follows since the variance bounds implied by Bayesian updating are violated.

[24] The notion of illiquidity due to adverse selection can be traced back at least to Bagehot (1971).

revelation price for different signal s' provides a measure of the price impact due to private information.

The price impact of this trade differs depending on whether the price p_{PR} reveals the signals or not. For unrevealed signals $s, s' \in [a, b]$, the price impact is zero since the same price $p_{PR}(s) = p_{PR}(s')$ prevails. On the other hand, for revealed signals $s \notin [a, b]$ or $s' \notin [a, b]$, the price impact is non-zero since the price $p_{PR}(\cdot)$ changes with the signal value.

Moreover, given the discontinuous price function, there can be a large change in price impact if the information content of price changes, i.e. either of the signal is not revealed while the other is revealed. Let $\lambda(s, s') \equiv |p_{PR}(s) - p_{PR}(s')|$ denote the price impact of trade by I-investors due a change in the private signal from s to s'. Then,

$$\lambda(s, s') = \begin{cases} 0 & \text{if } s \in [a, b] \text{ and } s' \in [a, b] \\ \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} |s - s'| & \text{if } s \notin [a, b] \text{ and } s' \notin [a, b] \\ \frac{\rho_\epsilon}{\rho_0 + \rho_\epsilon} |\mu_0 + \Delta(a, b) - s'| & \text{if } s \in [a, b] \text{ and } s' \notin [a, b]. \end{cases} \tag{50}$$

Clearly, if $s \notin [a, b]$ and $s' \notin [a, b]$, then there positive price impact, but it is continuous in the change $|s - s'|$. There is no price impact if $s, s' \in [a, b]$, i.e. neither signal is revealed, i.e. it is constant at zero and so it is continuous in the change $|s - s'|$. However, if one signal is revealed and the other is not, then the price impact is discontinuously large relative to the change $|s - s'|$. This discussion can be summarized as follows.

Proposition 6. *The price impact of trade due to a change in information is (i)* 0 *if neither signal is revealed, (ii) positive if either of the signals is revealed, and (iii) discontinuously large if one signal is not revealed while the other is revealed.*

Figure 2 depicts the price impact $\lambda(s, s')$ for the case where $s \in [a, b]$ and the case that $s \notin [a, b]$ as a function of s'. There are discontinuities at $s' = a$ and $s' = b$ as indicated by the discussion above. The price impact is positive, except for $s, s' \in [a, b]$ and for $s' = s$, and continuous otherwise.

The above discussion suggests that markets with low price impact may be performing poorly in aggregating and communicating information, with the consequence that uninformed investors do not obtain compensation for the adverse selection risk they face. On the other hand, a positive price impact may just reflect the mar-

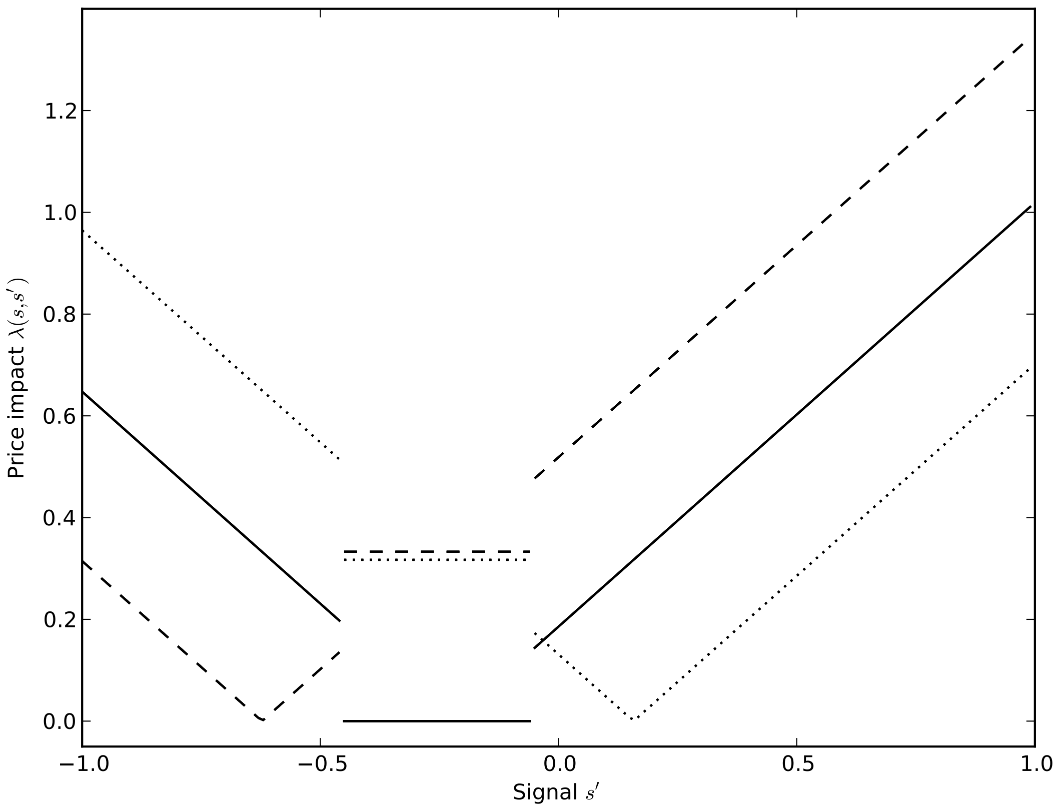

Figure 2: Price impact $\lambda(s, s')$ as a function of s' for a given s: (i) solid lines for $s \in [a, b]$, (ii) dashed lines for $s < a$, and (iii) dotted lines for $s > b$. Price impact $\lambda(s, s') = 0$ when $s = s'$.

ket's informational efficiency. In particular, a jump in price impact may in fact be a consequence of the market moving into a regime of informational efficiency from inefficiency.

4.3 Public information

The discussion so far has excluded any public information about the mean (log) stock payoff. Public information affects both I- and U-investors by reducing the disparity in their beliefs and this will have an impact on asset prices as we now show.

Suppose investors observe *a public signal* $\zeta = \mu + \epsilon_\zeta$, where ϵ_ζ is normally distributed with mean 0 and precision ρ_ζ. Since public information is observed by all investors, for simplicity, we assume that there is no ambiguity in the public signal.

I-investors' beliefs about the mean of v after the receipt of a public signal ζ with precision ρ_ζ are given by the set

$$\left\{\mu^I|(s,\zeta)\right\} = \left\{\frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon\,(s+\mu_\epsilon)}{\rho_0+\rho_\zeta+\rho_\epsilon} : \mu_\epsilon \in [-\delta,\delta]\right\}. \tag{51}$$

and the end points of the interval $\left[\underline{\mu}^I|(s,\zeta), \overline{\mu}^I|(s,\zeta)\right]$ are

$$\underline{\mu}|(s,\zeta) = \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon\,(s-\delta)}{\rho_0+\rho_\zeta+\rho_\epsilon} \text{ and } \overline{\mu}|(s,\zeta) = \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon\,(s+\delta)}{\rho_0+\rho_\zeta+\rho_\epsilon}. \tag{52}$$

Let $\hat{\mu}_0 = \rho_0\mu_0 + \rho_\zeta\zeta$ and $\hat{\rho}_0 = \rho_0 + \rho_\zeta$. Reasoning similar to that for equation (26) indicates that with public information, partial revelation will again involve a range of moderate information not being revealed. Moreover, the public information can affect the range of unrevealed private information. We denote the range of unrevealed private information by $[a_\zeta, b_\zeta]$ to make explicit the dependence on public signal ζ.

Then proceeding along the same lines as in section 3.3.1 shows that U-investor beliefs are given by

$$\mu^U_{PR}|([a_\zeta,b_\zeta],\zeta) = \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon(\mu_0 + \Delta(a_\zeta,b_\zeta))}{\rho_0+\rho_\zeta+\rho_\epsilon} = \frac{\hat{\mu}_0 + \rho_\epsilon(\mu_0+\Delta(a_\zeta,b_\zeta))}{\hat{\rho}_0+\rho_\epsilon} \tag{53}$$

if private information $s \in [a_\zeta, b_\zeta]$, i.e. is not revealed, where

$$\Delta(a_\zeta, a_\zeta+2\delta) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a_\zeta-\mu_0)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b_\zeta-\mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b_\zeta-\mu_0)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a_\zeta-\mu_0)\right)}\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}} \tag{54}$$

while U-investor beliefs are

$$\mu^U_{PR}|(s,\zeta) = \frac{\rho_0\mu_0 + \rho_\zeta\zeta + \rho_\epsilon s}{\rho_0+\rho_\zeta+\rho_\epsilon} = \frac{\hat{\mu}_0 + \rho_\epsilon s}{\hat{\rho}_0+\rho_\epsilon} \tag{55}$$

if private information $s \notin [a_\zeta, b_\zeta]$, i.e. is revealed.

Reasoning as before shows that the length of the interval is 2δ, i.e. $b_\zeta = a_\zeta + 2\delta$ and the existence of the interval follows from the existence of a solution to the following equation,

$$\mu_0 - \delta + \Delta(a_\zeta, a_\zeta + 2\delta) - \frac{\gamma\sigma^2}{x_0^U}\frac{\rho_0 + \rho_\zeta + \rho_\epsilon}{\rho_\epsilon} - a_\zeta = 0. \tag{56}$$

The preceding analysis suggests that public information can affect whether a given private signal s is revealed or not under partial revelation, which in turn has pricing implications. These are summarized in the following result.

Proposition 7. *Public information has the following effects under partial revelation.*

1. *The price function under partial revelation with public information $p_{PR,\zeta}$ is given by*

$$p_{PR,\zeta}(s) = \begin{cases} \frac{\hat{\mu}_0 + \rho_\epsilon(\mu_0 + \Delta(a_\zeta, b_\zeta))}{\hat{\rho}_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } s \in [a_\zeta, b_\zeta] \\ \frac{\hat{\mu}_0 + \rho_\epsilon s}{\hat{\rho}_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } s < a_\zeta \text{ or } s > b_\zeta. \end{cases} \tag{57}$$

2. *Stock price changes discontinuously if private information $s \in [a, b]$ and $s \notin [a_\zeta, b_\zeta]$ or $s \notin [a, b]$ and $s \in [a_\zeta, b_\zeta]$.*

3. *Price volatility changes discontinuously if private information not revealed without public information is revealed or vice versa.*

4. *Price impact $\lambda(s, s')$ increases discontinuously if $s' \in [a, b]$ and $s' \notin [a_\zeta, b_\zeta]$ given $s \in [a, b]$.*

This possibility for transition from non revelation to revelation due to a public signal means that otherwise anomalous price behavior can occur. The receipt of a public signal that is bad news will usually lead to a decline in the price of the stock. However, if that bad news implies that price reveals I-investors' private information, then this revelation may influence price to increase or decrease relative to where they would be were the economy to remain in partial revelation.

5 Price and volume across informational regimes

The preceding discussion shows the pricing effects from partial revelation of ambiguous information when different signals such as an intermediate one and an extreme one, are compared. We now discuss pricing and trading volume implications when the information of I-investors is held constant across informational regimes of partial revelation and full revelation, which in turn correspond to differences in the market composition of investors in terms of their wealth shares.

5.1 Full revelation informational regime

A change in the wealth share of U-investors as captured by a change in x_0^U can lead to a switch in informational regimes. If I-investors are poor enough, their effect on the risk premium from selling their stock endowment is relatively small. However, the reduced stockholding premium required by U-investors gets larger as the wealth share of I-investors increases, i.e. as x_0^U decreases. This means eventually I-investors will too rich, i.e. the reduced participation premium will be too high for them to trade away all their stockholding with markets clearing. Thus I-investors will trade to a positive position in the stock and this in turn implies that any signal will be revealed as the economy switches to a full revelation equilibrium as noted in Corollary 1.

We denote by p_{FR} the natural log of price P_{FR}, by $x_{0,FR}^I$ (respectively $x_{0,FR}^U$) the wealth share of informed (respectively uninformed) investors, and by $x_{1,FR}^I$ the stock demand from the I-investors under full revelation. Note from Corollary 1 that

$$1 - x_{0,FR}^I = x_{0,FR}^U < \frac{\rho_0 + \rho_\epsilon}{\delta \rho_\epsilon} \gamma \sigma^2. \tag{58}$$

The price function and trade volume under full revelation are characterised in the next result.

Proposition 8.

1. *The price function under full revelation is given by*

$$p_{FR}(s) = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \gamma\sigma^2 - x_{0,FR}^I \frac{\delta \rho_\epsilon}{\rho_0 + \rho_\epsilon}. \tag{59}$$

   for all s.

2. *Trade volume under full revelation is*

$$|x_{1,FR}^I(s) - x_{0,FR}^I| = \frac{x_{0,FR}^U}{\gamma\sigma^2} \frac{\delta \rho_\epsilon}{\rho_0 + \rho_\epsilon} x_{0,FR}^I. \tag{60}$$

   for all s.

The price under full revelation incorporates the uncertainty premium $\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$ required by each I-investor to be long in the stock. This premium is weighted by the wealth share $x^I_{0,FR}$ of I-investors reflecting their market presence. The price also incorporates the usual risk premium $\gamma\sigma^2$ when all investors are long in the stock.

Trade volume is the same across all signals under full revelation and depends on both the uncertainty premium required by each I-investor and the reduced stockholding premium required by U-investors. Further, it increases as the share of I-investors in the market $x^I_{0,FR}$ increases. In particular, given that $\frac{x^U_{0,FR}}{\gamma\sigma^2}\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon} < 1$, I-investors final stock position involves only partial sale of their stockholding to U-investors.

The preceding results and discussion suggest that for a given signal s price and will differ across full and partial revelation. To clarify the difference, we denote by $x^I_{0,PR}$ (respectively $x^U_{0,PR}$) the wealth share of I-investors (respectively U-investors) in the partial revelation regime. Again, from Corollary 1 and the discussion on partial revelation previously, we have

$$1 - x^I_{0,PR} = x^U_{0,PR} \geq \frac{\rho_0+\rho_\epsilon}{\delta\rho_\epsilon}\gamma\sigma^2. \tag{61}$$

The next result summarizes the comparison of price and trade volume across full and partial revelation regimes using the preceding discussion and results.

Corollary 2.

1. *For signals revealed under partial revelation, i.e. $s \notin [a,b]$, the stock price is lower under partial revelation than under full revelation when*

$$\frac{x^I_{0,PR}}{x^I_{0,FR}} > \left(\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)\left(\frac{x^U_{0,PR}}{\gamma\sigma^2}\right) \tag{62}$$

2. *For signals not revealed under partial revelation, i.e. $s \in [a,b]$, the stock price is lower under partial revelation than under full revelation when*

$$s > \hat{s} \text{ and } \frac{x^I_{0,PR}}{x^I_{0,FR}} > \left(\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}\right)\left(\frac{x^U_{0,PR}}{\gamma\sigma^2}\right) \tag{63}$$

where $\hat{s} \in [a,b]$ satisfies $\mu^U_{PR}|\hat{s} = \mu^U_{PR}|[a,b]$.

3. Trade volume is higher under partial revelation than full revelation when

$$\frac{x^I_{0,PR}}{x^I_{0,FR}} > \left(\frac{\delta\rho_\epsilon}{\rho_0 + \rho_\epsilon}\right)\left(\frac{x^U_{0,FR}}{\gamma\sigma^2}\right) \tag{64}$$

and lower otherwise.

For signals that are revealed under partial revelation, lower partial revelation stock price means higher trade volume under partial revelation compared to full revelation. For signals that are not revealed the comparison is depends on whether the private signal is above or below the signal corresponding to the averaged belief of uniformed investors $\mu^U_{PR}|[a,b]$.

Figure 3 shows the market-clearing asset price for a given signal as a function of I-investor wealth. For a given signal, if the wealth share x^I_0 is high enough then as indicated in corollary 1, the reduced participation premium is too high relative to the uncertainty premium required by I-investors, and in equilibrium, I- and U-investors have positive holdings of the stock. The price is decreasing as function of x^I_0 throughout since both the reduced stockholding premium $\gamma\sigma^2/(1-x^I_0)$ demanded by U-investors under partial revelation and the total uncertainty premium demanded by I-investors under full revelation $x^I_0\delta\rho_\epsilon/(\rho_0+\rho_\epsilon)$ increase with x^I_0.

However, as x^I_0 decreases, the reduced participation premium $\gamma\sigma^2/x^U_0 = \gamma\sigma^2/(1-x^I_0)$ eventually becomes small enough relative to the uncertainty premium and the I-investors can trade off their stock endowment to U-investors and hold no position in the stock with markets clearing. This yields a discontinuous jump downward in the price to the partially-revealing price value, as shown in figure 3. As noted in proposition 4, the partially-revealing price is an increasing function of x^U_0 and so the price increases as x^I_0 decreases after the discontinuous jump downward.

Finally, figure 3 also depicts the set of partially-revealing price values corresponding to non-revealed signals for each wealth level x^I_0 for which partial revelation is possible. For each value of x^I_0, there is a set of signals that are not revealed. This set of signals that are not revealed shrinks as x^I_0 increases since the reduced participation premium increases and so the requirement for full participation indicated in corollary 1 is satisfied by an increasing set of signal values, which implies these signals are revealed by price.

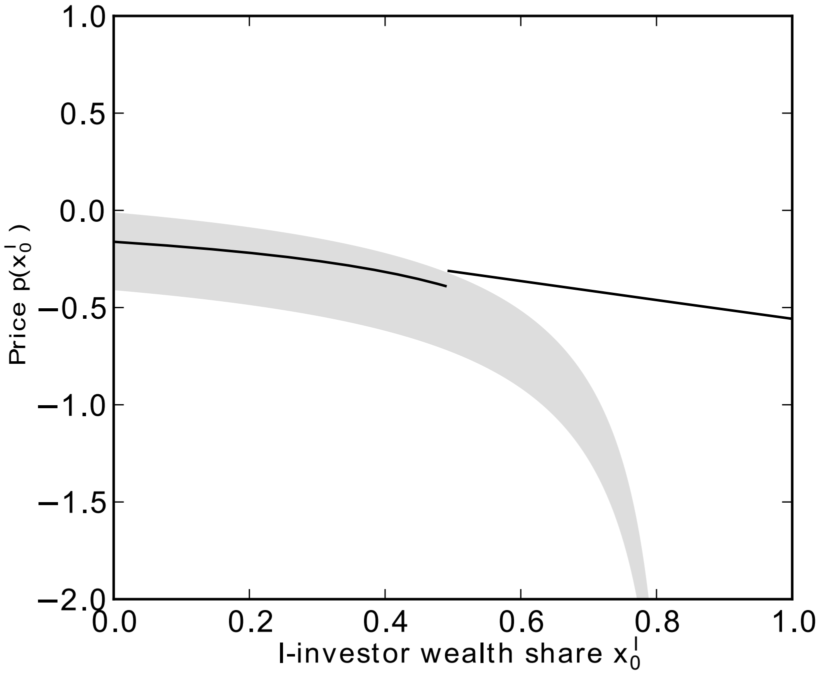

Figure 3: Market-clearing price p as a function of I-investor wealth x_0^I. The gray region depicts the set of partially-revealing price values p_{PR} corresponding to the set of non-revealed signals at each value of x_0^I.

6 Labor income, multiple signals, ambiguity-averse uninformed investors

6.1 Non-tradeable labor income

In this section we provide conditions under which the signal structure described previously can be reinterpreted as one in which investors receive signals about a non-tradeable asset such as labor income whose payoff is correlated with the stock parameter about which investors learn. The investors can use the stock to hedge against their labor income fluctuations and in turn use information about labor income to update their information about the stock payoff.

Such formulations of hedging motives are commonly used in the rational expectations equilibrium literature, see for example Biais, Bossaerts, and Spatt (2010), Schneider (2009), Goldstein and Guembel (2008), Watanabe (2008), and the references therein.[25]

Suppose that each investor has non-tradeable labor income that provides a return on initial wealth R_l. The return from labor income that the investor receives is correlated with the mean stock payoff. In particular, $r_l = \ln R_l$ and μ are jointly normally distributed with means (μ_l, μ_0) and covariance η. The precision of r_l is ρ_l.

Instead of receiving private signal directly about μ, I-investors receive a signal $s = r_l + \epsilon$ that provides information about the value of labor income. For clarity, first assume that there is no ambiguity present in this signal. Assume that ϵ is normally distributed with mean 0 and precision ρ_ϵ and that ϵ and r_l are independent. Let $\rho_s = \rho_l \rho_\epsilon / (\rho_l + \rho_\epsilon)$ be the precision of the signal s. Since r_l and μ are correlated, this signal also provides information about the mean payoff to the asset. We assume the joint normality of s, r_l and μ_0 which implies that the covariance of s and μ is η. As such, the updated distribution of μ given the observation of the private signal s is normal with mean

$$\mu|s = \mu_0 + \frac{\eta}{\sigma_l^2 + \sigma_\epsilon^2}(s - \mu_l) \tag{65}$$

As the previous updated mean can be written as

$$\mu|s = \mu_0 + \frac{\rho_s}{\rho_0 + \rho_s}(s - \mu_0) \tag{66}$$

we see that the form of this equation and that of equation (5) are similar. The difference is that the change in the updated mean of the asset payoff when a signal has been received now depends on the covariance of μ with the signal, which is the covariance of μ with labor incomes. In the previous structure the update depended on the covariance of s with μ which was just the variance of μ (or one over the precision). Both of these updated means can be written as $\mu_0 + \beta(s - \mathbb{E}s)$ although β and $\mathbb{E}s$ differ depending on whether the signal is about the mean payoff directly or about some other variable that is correlated with the mean payoff.

If the mean of the signal (or equivalently, of ϵ) is ambiguous, then the set of updates becomes

$$\{\mu|s\} = \left\{\mu_0 + \frac{\eta}{\sigma_l^2 + \sigma_\epsilon^2}(s + \mu_\epsilon - \mu_l) : \mu_\epsilon \in [-\delta, \delta]\right\}. \tag{67}$$

[25] In these papers, the hedging motive is closely tied to the noise which prevents prices from revealing information.

6.1.1 Investor demand with non-tradeable labor income

We model the wealth shock as being a random return on initial wealth. Hence, terminal wealth is given by

$$W_2 = W_0(\theta R + (1-\theta)R_f + R_l) \tag{68}$$

Using an approximation of payoffs similar to that in section 2.3, we have the following result regarding investor demand.

Lemma 6.1. *The optimal portfolio weight on the stock for investor n who observes information about non-tradable labor income that is correlated with the mean stock payoff is given by*

$$\theta^n(s) = \begin{cases} \frac{\underline{\mu}|s - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \underline{\mu}|s - r_f + \frac{1}{2}\sigma^2 - p > 0 \\ 0 & \text{if } \underline{\mu}|s - r_f \le p - \frac{1}{2}\sigma^2 \le \overline{\mu}|s - r_f \\ \frac{\overline{\mu}|s - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \overline{\mu}|s - r_f + \frac{1}{2}\sigma^2 - p < 0. \end{cases} \tag{69}$$

This demand is similar to that in the case when signals are directly related to the asset. The difference here, is that now the range of values for $\mathbb{E}(r)$ given the ambiguity in the labor income signal will be slightly different. Despite this difference, given the above, this model with non-tradeable labor income can be mapped directly into the model presented previously by changing a single parameter—the relative importance of the signal (modeled there by $\rho_\epsilon/(\rho_0 + \rho_\epsilon)$) in the updating of beliefs about the mean of the (log) asset payoff. The rest of the analysis conducted previously will then follow immediately.

6.2 Multiple signals observed by I-investors

We now consider the scenario where I-investors may observe a sequence of identically and independently drawn signals before trading occurs. We show that partial revelation does not disappear with such repeated observation of information by I-investors. For simplicity, we assume that I-investors perceive the same ambiguity in all of the signals. Wealth shares and all other parameters are fixed as in section 2.

Suppose I-investors observe a sequence denoted by $(s_1, s_2, \ldots, s_K)$ of $K \geq 1$ such signals. Their updated beliefs are represented by set of normal distributions with precision $\rho_0 + K\rho_\epsilon$ and means

$$\{\mu|(s_1, \ldots, s_K)\} = \left\{ \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K (s_k + \mu_{\epsilon k})}{\rho_0 + K\rho_\epsilon} : \mu_{\epsilon k} \in [-\delta, \delta] \text{ for all } k = 1, \ldots, K \right\}. \tag{70}$$

We denote the interval of means of v by $[\underline{\mu}|(s_1, \ldots, s_K), \overline{\mu}|(s_1, \ldots, s_K)]$, where

$$\underline{\mu}|(s_1, \ldots, s_K) = \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K s_k - \rho_\epsilon K\delta}{\rho_0 + K\rho_\epsilon} \tag{71}$$

and

$$\overline{\mu}|(s_1, \ldots, s_K) = \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K s_k + \rho_\epsilon K\delta}{\rho_0 + K\rho_\epsilon}. \tag{72}$$

Note that the length of the interval is given by $2K\delta/(\rho_0 + K\rho_\epsilon)$, i.e. repeated observation of ambiguous information increases the ambiguity perceived by I-investors.

If U-investors observe this sequence of signals through price, then their updated beliefs about the mean of v are given by a normal distribution with precision $\rho_0 + K\rho_\epsilon$ and mean

$$\mu^U_{PR}|(s_1, \ldots, s_K) = \frac{\rho_0\mu_0 + \rho_\epsilon \sum_{k=1}^K s_k}{\rho_0 + K\rho_\epsilon}. \tag{73}$$

Using expressions (71) and (72) for I-investor beliefs and (73) for U-investor beliefs, we obtain the following result, parallel to Proposition 2 and Corollary 1, regarding full-revelation due to I-investors holding a positive position $\theta^I(s_1, \ldots, s_K) > 0$ in the market in equilibrium.

Proposition 9. *If $\frac{\gamma\sigma^2}{x_0^U} > \frac{K\delta\rho_\epsilon}{\rho_0 + K\rho_\epsilon}$, then markets clear with $\theta^I(s_1, \ldots, s_K) > 0$. Hence, any rational expectations equilibrium price function is fully-revealing.*

$\frac{K\delta\rho_\epsilon}{\rho_0 + K\rho_\epsilon}$ denotes the uncertainty premium required by I-investors to hold a positive position in the stock with markets clearing. Partial revelation requires $\frac{\gamma\sigma^2}{x_0^U} \leq \frac{K\delta\rho_\epsilon}{\rho_0 + K\rho_\epsilon}$, i.e. the uncertainty premium is too high relative to the reduced stockholding premium $\frac{\gamma\sigma^2}{x_0^U}$ required by U-investors to hold all the stock. Note that $\frac{K\delta\rho_\epsilon}{\rho_0 + K\rho_\epsilon}$ is increasing in K. Thus, as I-investors observe more ambiguous information, conditions for partial revelation are easier to satisfy.

When U-investors hold all the stock, their demand satisfies $\theta^U(s_1, \ldots, s_K) = \frac{1}{\gamma\sigma^2}\mu^U_{PR}(s_1, \ldots, s_K) + \frac{1}{2}\sigma^2 - p$ and the market-clearing price satisfies $p = \mu^U_{PR}(s_1, \ldots, s_K) - \frac{\gamma\sigma^2}{x_0^U} + \frac{1}{2}\sigma^2$. This price includes the reduced stockholding premium required by U-investors.

When I-investors observe K signals before trading, information is revealed or not revealed in the form of $\sum_{k=1}^{K} s_k$. Reasoning similar to section 3.3 shows that a range of intermediate values of $\sum_{k=1}^{K} s_k$, while extreme values of $\sum_{k=1}^{K} s_k$ are revealed. This range of unrevealed values is given by the following inequalities, similar to (26).

$$K(\mu_{PR}^U+\delta)+\frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^U-\mu_0)-\frac{\rho_0+K\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} \geq \sum_{k=1}^{K} s_k \geq K(\mu_{PR}^U-\delta)+\frac{\rho_0}{\rho_\epsilon}(\mu_{PR}^U-\mu_0)-\frac{\rho_0+K\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} \tag{74}$$

Let a_K and b_K denote the lower and upper bounds of this range of unrevealed information. The beliefs μ_{PR}^U of the U-investors are constant over this range and denoted by $\mu_{PR}^U|[a_K, b_K]$. Reasoning similarly to section 3.3.1 shows that

$$\mu_{PR}^U|[a_K, b_K] = \mu_0 + \Delta_K(a_K, b_K) \tag{75}$$

where

$$\Delta_K(a_K, b_K) = \frac{\phi\left(\sqrt{\frac{\rho_0 K\rho_\epsilon}{\rho_0+K\rho_\epsilon}}(a-K\mu_0)\right)-\phi\left(\sqrt{\frac{\rho_0 K\rho_\epsilon}{\rho_0+K\rho_\epsilon}}(b-K\mu_0)\right)}{\Phi\left(\sqrt{\frac{\rho_0 K\rho_\epsilon}{\rho_0+K\rho_\epsilon}}(b-K\mu_0)\right)-\Phi\left(\sqrt{\frac{\rho_0 K\rho_\epsilon}{\rho_0+K\rho_\epsilon}}(a-K\mu_0)\right)}\sqrt{\frac{\rho_0 K\rho_\epsilon}{\rho_0+K\rho_\epsilon}}. \tag{76}$$

Using (74) and (75), we get that

$$b_K - a_K = 2\delta K, \tag{77}$$

and so the existence of partial revelation is given by the existence of a solution to the following equation.

$$K(\mu_0-\delta)+\Delta_K(a_K, a_K+2\delta K)-\frac{\rho_0+K\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U}-a_K=0. \tag{78}$$

This leads to the following result, which is analogue of Proposition 3 and Proposition 4.

Proposition 10.

1. *The length of the interval $[a_K, b_K]$ of unrevealed signals, if it exists, is $2\delta K$.*

2. *The existence of an interval $[a_K, b_K]$ of unrevealed signals and hence the exis-*

tence of partially revealing rational expectations equilibrium price function follows from the existence of a solution to equation (78)*.*

3. *The price function p_{PR} under partial revelation is given by*

$$p_{PR}(s_1,\ldots,s_K) = \begin{cases} \frac{\rho_0\mu_0+\rho_\epsilon(K\mu_0+\Delta_K(a_K,b_K))}{\rho_0+K\rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \textit{if } \sum_{k=1}^K s_k \in [a_K, b_K] \\ \frac{\rho_0\mu_0+\rho_\epsilon\sum_{k=1}^K s_k}{\rho_0+K\rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \textit{if } \sum_{k=1}^K s_k < a_K \textit{ or } \sum_{k=1}^K s_k > b_K. \end{cases} \tag{79}$$

6.3 Ambiguity averse U-investors

In this section, we outline the model when U-investors also perceive ambiguity in the signals. We retain the structure outlined in section 2 except that U-investors also perceive ambiguity in the signal if they observe it through price. Like I-investors, the U-investors also consider a range $[-\delta^U, \delta^U]$ of possible values for the mean μ_ϵ^U of the error term ϵ, where $\delta^U > 0$.

We assume that $\delta^U < \delta$, so that $[-\delta^U, \delta^U]$ is a strict subset of $[-\delta, \delta]$. This assumption is needed for market clearing to be consistent with I-investors trading away all their stockholding, i.e. $\theta^I = 0$, as we show below. This portfolio position is where I-investors exhibit inertia with respect to information, which is required for partial revelation to be possible. Partial revelation will still take the form of an intermediate range of unrevealed signals with relatively extreme signals being revealed.

For any signal s, if U-investors observe it through the price, their updated beliefs about the mean of v are represented by the set of normal distributions with precision $\rho_0 + \rho_\epsilon$ and means

$$\{\mu_{PR}^U|s\} = \left\{ \frac{\rho_0\mu_0 + \rho_\epsilon(s + \mu_\epsilon^U)}{\rho_0 + \rho_\epsilon} \middle| \mu_\epsilon^U \in [-\delta^U, \delta^U] \right\}. \tag{80}$$

Thus, when U-investors infer the signal s, the interval of means is given by $[\underline{\mu}_{PR}^U|s, \overline{\mu}_{PR}^U|s]$, where

$$\underline{\mu}_{PR}^U|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s - \delta^U)}{\rho_0 + \rho_\epsilon} \text{ and } \overline{\mu}_{PR}^U|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s + \delta^U)}{\rho_0 + \rho_\epsilon}. \tag{81}$$

We use $\underline{\mu}^U_{PR}$ (respectively, $\overline{\mu}^U_{PR}$) to generically denote the lower bound (respectively, upper bound) of the interval of means for U-investors. Then the demand for U-investors is expressed similarly to that in (15).

Recall that partial revelation requires that I-investors trade away all their stockholding and U-investors holding all of the stock. This leads to the following result, which is the analogue of Proposition 2 for the current setting.

Proposition 11. *If* $\frac{\gamma\sigma^2}{x_0^U} > \frac{\rho_\epsilon(\delta-\delta^U)}{\rho_0+\rho_\epsilon}$*, then markets clear with* $\theta^I(s) > 0$*. Hence, any rational expectations equilibrium is fully revealing.*

The term $\frac{\rho_\epsilon(\delta-\delta^U)}{\rho_0+\rho_\epsilon}$ reflects the additional uncertainty premium required by the I-investors relative to U-investors given their relative perception of ambiguity in the signal. Thus, for markets to clear with I-investors selling all their stockholding requires that $\frac{\gamma\sigma^2}{x_0^U} \leq \frac{\rho_\epsilon(\delta-\delta^U)}{\rho_0+\rho_\epsilon}$, i.e. the uncertainty premium required by I-investors is too high relative to the reduced stockholding premium $\frac{\gamma\sigma^2}{x_0^U}$ required by U-investors to hold all the stock.

When U-investors hold all of the stock, their demand satisfies $\theta^U(s) = \frac{1}{\gamma\sigma^2}\underline{\mu}^U_{PR} + \frac{1}{2}\sigma^2 - p$. Moreover, the market-clearing price then satisfies $p = \underline{\mu}^U_{PR} - \frac{\gamma\sigma^2}{x_0^U} + \frac{1}{2}\sigma^2$. This market price includes an uncertainty premium for U-investors given their perception of ambiguity in the signal captured in $\underline{\mu}^U_{PR}$.

Reasoning similar to that in section 3.3 shows that there is an intermediate range of signals which aren't revealed while extreme signals are revealed. This range of unrevealed signals satisfies the following set of inequalities, similar to those in (26),

$$\underline{\mu}^U_{PR}+\delta+\frac{\rho_0}{\rho_\epsilon}(\underline{\mu}^U_{PR}-\mu_0)-\frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} \geq s \geq \underline{\mu}^U_{PR}-\delta+\frac{\rho_0}{\rho_\epsilon}(\underline{\mu}^U_{PR}-\mu_0)-\frac{\rho_0+\rho_\epsilon}{\rho_\epsilon}\frac{\gamma\sigma^2}{x_0^U} \quad (82)$$

Let a and b the lower and upper bounds of this interval. As before, the beliefs of U-investors are constant over this interval and we denote these by $[\underline{\mu}^U_{PR}|[a,b], \overline{\mu}^U_{PR}|[a,b]]$. The lower bound $\underline{\mu}^U_{PR}$ is obtained as follows.

$$\underline{\mu}^U_{PR}|[a,b] = \min_{\mu^U_\epsilon \in [-\delta^U,\delta^U]} \mathbb{E}_{\mu^U_\epsilon}\left[\mu | s \in [a,b]\right]. \quad (83)$$

where $\mathbb{E}_{\mu^U_\epsilon}[\cdot]$ denotes the expectation with respect to the distribution of s corresponding to the mean μ^U_ϵ of the error term ϵ.

As before, evaluating this expectation involves use of truncated normal distribution.

$$\min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \mathbb{E}_{\mu_\epsilon^U}\left[\mu | s \in [a,b]\right] = \min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\int_a^b \frac{\rho_0\mu_0 + \rho_\epsilon(s + \mu_\epsilon^U)}{\rho_0 + \rho_\epsilon} f_{\mu_\epsilon^U}(s | s \in [a,b]) ds \right]. \tag{84}$$

This simplifies to

$$\min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\int_a^b \frac{\rho_0\mu_0 + \rho_\epsilon(s + \mu_\epsilon^U)}{\rho_0 + \rho_\epsilon} f_{\mu_\epsilon^U}(s | s \in [a,b]) ds \right] = \mu_0 + \min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\Delta_{\mu_\epsilon^U}(a,b) + 2\mu_\epsilon^U \right]. \tag{85}$$

where $f_{\mu_\epsilon^U}$ is the density function and

$$\Delta_{\mu_\epsilon^U}(a,b) = \frac{\phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a - \mu_0 - \mu_\epsilon^U)\right) - \phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b - \mu_0 - \mu_\epsilon^U)\right)}{\Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(b - \mu_0 - \mu_\epsilon^U)\right) - \Phi\left(\sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0+\rho_\epsilon}}(a - \mu_0 - \mu_\epsilon^U)\right)} \sqrt{\frac{\rho_0\rho_\epsilon}{\rho_0 + \rho_\epsilon}}. \tag{86}$$

Thus, the inequalities for the range of signals in (82) simplify to

$$\mu_0 + \delta + \min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\Delta_{\mu_\epsilon^U}(a,b) + 2\mu_\epsilon^U \right] - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^U} \geq s \tag{87}$$

and

$$s \geq \mu_0 - \delta + \min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\Delta_{\mu_\epsilon^U}(a,b) + 2\mu_\epsilon^U \right] - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^U}. \tag{88}$$

From these inequalities it follows again that the length of the interval $[a,b]$ is 2δ and hence the existence of the interval boils down to the existence of a finite value of a that solves

$$\mu_0 - \delta + \min_{\mu_\epsilon^U \in [-\delta^U, \delta^U]} \left[\Delta_{\mu_\epsilon^U}(a, a + 2\delta) + 2\mu_\epsilon^U \right] - \frac{\rho_0 + \rho_\epsilon}{\rho_\epsilon} \frac{\gamma\sigma^2}{x_0^U} = a. \tag{89}$$

This then leads to the following result, which is the analogue of Proposition 3 and Proposition 4.

Proposition 12.

1. *The length of the interval $[a,b]$ of unrevealed signals, if it exists, is 2δ.*

2. *The existence of an interval $[a,b]$ of unrevealed signals and hence the existence of partially revealing rational expectations equilibrium price function follows from the existence of a solution to equation* (89)*.*

3. The price function p_{PR} under partial revelation is given by

$$p_{PR}(s) = \begin{cases} \mu_0 + \frac{\rho_\epsilon\left(\min_{\mu_\epsilon^U \in[-\delta^U,\delta^U]}\left[\Delta_{\mu_\epsilon^U}(a,a+2\delta)+2\mu_\epsilon^U\right]\right)}{\rho_0+\rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } s \in [a,b] \\ \frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma}{x_0^U}\sigma^2 & \text{if } s < a \text{ or } s > b \end{cases} \tag{90}$$

The partial revelation price function is non-linear and discontinuous in signals since $\underline{\mu}_{PR}^U|s$ is constant at $\underline{\mu}_{PR}^U|[a,b]$ for $s \in [a,b]$, while it is monotone and linear for $s \notin [a,b]$. Using these results, we can then obtain qualitatively similar results to those in the main text.

7 Concluding remarks

In this paper, we show how partially revealing rational expectations prices may arise when with ambiguous private information is received by ambiguity averse investors in a financial market. This informational inefficiency can arise due to inertia with respect to information, which in turn arises under non-smooth ambiguity averse preferences such as the Gilboa and Schmeidler (1989) representation.

The partial revelation property is different from that generated though noise and can lead to large variation in price and price volatility without a large change in fundamentals or any change in volatility of fundamentals. Moreover, informationally inefficient prices can coincide with lower price impact of trade than informationally efficient prices and there can be large changes in price impact without a large change in fundamentals. The arrival of public information and changes in wealth shares may directly affect the informational efficiency of prices. This inefficiency need not disappear with the repeated observation of information by informed traders or by the uninformed traders also being ambiguity-averse.

This paper has focused on a single type of informed investor and a single asset. Future work would allow for for multiple types of informed I-investors who receive different information or multiple assets being traded. The requirement for information to not be revealed would then require that informed be able to trade to their respective portfolio positions which exhibit inertia with respect to information. This would result in conditions similar to those in equation 37 that would need to be analysed.

8 Appendix

8.1 Proofs and derivations not in main text.

8.1.1 Proofs for Section 2

Proof of Lemma 2.1. Terminal wealth is given by

$$W_2^n = W_0^n(\theta^n R + (1-\theta^n)R_f). \tag{91}$$

Expressing the returns in natural logs implies

$$\begin{aligned}\ln \frac{\theta^n R + (1-\theta^n)R_f}{R_f} &= \ln\left(\theta^n \frac{R}{R_f} + 1 - \theta^n\right) \\ &= \ln(1+\theta^n\left(e^{r-r_f}-1\right))\end{aligned} \tag{92}$$

We use a second order Taylor series approximation around $r - r_f = 0$ to get

$$\theta^n\left(r - r_f\right) + \frac{1}{2}\theta^n(r-r_f)^2. \tag{93}$$

We use the unconditional expectation of the second-order term and obtain

$$\theta^n\left(r - r_f\right) + \frac{1}{2}\theta^n(1-\theta^n)\sigma^2 \tag{94}$$

as our approximation of returns. This function is the lognormally distributed function to the actual market return.

If terminal wealth W_2^n is lognormally distributed then the solution to the individual's optimization problem is equivalent to the solution to

$$\max_{\theta} \min_{m\in M^n} \ln \mathbb{E}_m\left[\frac{(W_2^n)^{1-\gamma}}{1-\gamma}\right]. \tag{95}$$

The term $\ln \mathbb{E}_m\left[(W_2^n)^{1-\gamma}\right]$ by the lognormality of W_2^n can be rewritten as

$$\begin{aligned}\mathbb{E}_m \ln(W_2^n)^{1-\gamma} + \frac{1}{2}Var\ln(W_2^n)^{1-\gamma} = \\ (1-\gamma)\mathbb{E}_m w_2^n + \frac{1}{2}(1-\gamma)^2 Var \ln W_2^n = \\ (1-\gamma)\mathbb{E}_m w_0^n + \ln(\theta^n R + (1-\theta^n)R_f) + \frac{1}{2}(1-\gamma)^2\sigma^2.\end{aligned} \tag{96}$$

Since w_0^n and r_f are both non-stochastic and $1-\gamma$ is a scale factor that won't affect the solution to the problem, solving the optimization problem is equivalent to solving

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \ln(\theta R + (1-\theta)R_f) - r_f + \frac{1-\gamma}{2}\sigma^2. \tag{97}$$

Using the approximation given in (94), we can rewrite the optimization problem as

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \theta\,(r - r_f) + \frac{\theta(1-\theta)}{2}\sigma^2 + \frac{(1-\gamma)\theta^2}{2}\sigma^2. \tag{98}$$

The first order conditions for investor n are given by

$$\begin{array}{ll} 0 = \min_{m \in M^n} \mathbb{E}_m r - r_f + \frac{1}{2}\sigma^2 - \gamma\theta\sigma^2 & \text{if } \theta > 0 \\ 0 \in \left\{\mathbb{E}_m r - r_f + \frac{1}{2}\sigma^2 : m \in M^n\right\} & \text{if } \theta = 0 \\ 0 = \max_{m \in M^n} \mathbb{E}_m r - r_f + \frac{1}{2}\sigma^2 - \gamma\theta\sigma^2 & \text{if } \theta < 0. \end{array} \tag{99}$$

Therefore, investor n will not hold a position in the risky asset if and only if

$$\min_{m \in M^n} \mathbb{E}_m v - r_f + \frac{1}{2}\sigma^2 - p \leq 0 \leq \max_{m \in M^n} \mathbb{E}_m v - r_f + \frac{1}{2}\sigma^2 - p. \tag{100}$$

With $[\underline{\mu}^n, \overline{\mu}^n]$ denoting the interval of means under the set M^n, investor n will not hold the uncertain asset if

$$\underline{\mu}^n - r_f + \frac{1}{2}\sigma^2 \leq p \leq \overline{\mu}^n - r_f + \frac{1}{2}\sigma^2. \tag{101}$$

Using the above provides the optimal portfolio weight as expressed in (15). □

8.1.2 Proofs for Section 3

Proof of Proposition 2. Recall that after observing signal s, I-investors belief about the mean of v is given by the interval $[\underline{\mu}|s, \overline{\mu}|s]$, where $\underline{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta)}{\rho_0+\rho_\epsilon}$ and $\overline{\mu}|s = \frac{\rho_0\mu_0 + \rho_\epsilon(s+\delta)}{\rho_0+\rho_\epsilon}$. When the signal value s is revealed by the price, the updated belief $\mu_{PR}^U|s$ of U-investors about the mean of v is $\frac{\rho_0\mu_0+\rho_\epsilon s}{\rho_0+\rho_\epsilon}$, since they believe the signal is unbiased, i.e. $\mu_\epsilon^U = 0$. Hence, $\mu_{PR}^U|s \in [\underline{\mu}|s, \overline{\mu}|s]$.

Shorting the asset ($\theta^I(s) < 0$) is not consistent with market clearing. For any price P, A investors will short only if $\overline{\mu}|s < p - \frac{1}{2}\sigma^2$. However, in this case, U-investors will also short since $\mu_{PR}^U|s < \overline{\mu}|s$ and so the stock market will not clear. The market will also not clear with only U-investors shorting, i.e. $\mu_{PR}^U|s < p - \frac{1}{2}\sigma^2 \leq \overline{\mu}|s$ since the A investors will want to trade to a zero position at this price.

Hence, markets will only clear with both types of investors trading to non-negative positions in the stock. Suppose both types of investors hold long positions, $\theta^I(s) > 0$ and $\theta^U(s) > 0$ at price P. Then for the stock market to clear, using (9), we get $p - 0.5\sigma^2 = x_0^U \mu_{PR}^U|s + x_0^I \underline{\mu}|s - \gamma\sigma^2$. Moreover, $\theta^I(s) > 0$ requires $p - 0.5\sigma^2 < \underline{\mu}|s$ which, using the above, in turn requires $\mu_{PR}^U|s - \frac{\gamma\sigma^2}{x_0^U} < \underline{\mu}|s$. Using the expressions for $\mu_{PR}^U|s$ and $\underline{\mu}|s$, this simplifies to $\frac{\gamma\sigma^2}{x_0^U} > \frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}$. □

8.1.3 Proofs for Section 5

Proof of Proposition 8.

1. When I-investors trade to a positive position in the stock, their demand is given by
$$\theta^I(s) = \frac{1}{\gamma\sigma^2}\left(\frac{\rho_0\mu_0 + \rho_\epsilon(s-\delta)}{\rho_0+\rho_\epsilon} + \frac{1}{2}\sigma^2 - p(s)\right) \tag{102}$$
at price $p(s)$. With this demand expression, reasoning similar to that used in section 3.1 shows that the market clearing price will reveal the signal s. Thus, U-investors will infer s through the price and their demand is given by
$$\theta^U(s) = \frac{1}{\gamma\sigma^2}\left(\frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0+\rho_\epsilon} + \frac{1}{2}\sigma^2 - p(s)\right). \tag{103}$$
Using these demands in the market-clearing condition (9) yields the market clearing price
$$p(s) = \frac{\rho_0\mu_0 + \rho_\epsilon s}{\rho_0+\rho_\epsilon} + \frac{1}{2}\sigma^2 - \gamma\sigma^2 - x_{0,FR}^I\frac{\delta\rho_\epsilon}{\rho_0+\rho_\epsilon}. \tag{104}$$
As conjectured, this price is linear and monotone in s, thus reveals s. This yields the fully revealing price function in equation (59).

2. Using the demand expression for $\theta^I(s)$ given above and $x_{1,FR}^I(s) = \frac{\theta^I(s)W_{0,FR}^I(s)}{P_{FR}(s)}$, where $W_{0,FR}^I(s) = x_{0,FR}^I P_{FR}(s)$ and $p_{FR}(s) = \ln P_{FR}(s)$ yields the trading volume under full revelation given in equation (60).

□

8.1.4 Proofs for Section 6.

Proof of Lemma 6.1. Terminal wealth is given by

$$W_2 = W_0(\theta R + (1-\theta)R_f + R_l) \tag{105}$$

and, as before, we approximate the return on initial wealth by a lognormal random variable based on a second order Taylor series approximation of the portfolio return.

We find first the second-order Taylor series approximation of $\ln(\theta R + (1-\theta)R_f + R_l)/R_f$. This can be rewritten as

$$f(r - r_f, R_l/R_f) = \ln\left[1 + \frac{R_l}{R_f} + \theta(e^{r-r_f} - 1)\right]. \tag{106}$$

After substituting in the appropriate derivatives evaluated at the point $(r - r_f, R_l/R_f) = (0, 0)$, the approximation becomes

$$f(r-r_f, R_l/R_f) \approx \theta(r-r_f) + e^{r_l-r_f} + \frac{\theta-\theta^2}{2}(r-r_f)^2 + \frac{1}{2}e^{2(r_l-r_f)} - \frac{\theta}{2}(r_l-r_f)e^{r_l-r_f} \tag{107}$$

We assume that the signal provides information about the asset payoff only through its covariance with the asset's mean payoff. As such, conditional on having observed the signal and updated her prior about the mean of the asset payoff, the covariance of the asset with the return on labor income is zero.

As do Campbell and Viciera (2002), we replace the second order terms with their expectations to get the final form of the approximation

$$f(r - r_f, R_l/R_f) \approx \theta(r - r_f) + \frac{R_l}{R_f} + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 \tag{108}$$

Given the assumptions above,

$$Var(r_w) = Var(\theta(r - r_f) + r_l - r_f) = \theta^2\sigma^2 + \sigma_l^2. \tag{109}$$

Using M^n to denote the set of distributions and $[\underline{\mu}^n, \overline{\mu}^n]$ to denote the corresponding interval of means, we can use the above in the investor's objective function to get

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r - r_f) + \frac{R_l}{R_f} + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1}{2}\sigma_l^2 - r_f + \frac{1-\gamma}{2}\left[\theta^2\sigma^2 + \sigma_l^2\right]\right] \tag{110}$$

The terms r_f, $\frac{1}{2}\sigma_l^2$, $\frac{R_l}{R_f}$, and $\frac{1-\gamma}{2}\sigma_l^2$ are monotonic increases which will not affect the optimal choice of θ and hence can be ignored. This leaves the objective function

$$\max_{\theta} \min_{m \in M^n} \mathbb{E}_m \left[\theta(r - r_f) + \frac{\theta - \theta^2}{2}\sigma^2 + \frac{1-\gamma}{2}\theta^2\sigma^2 \right] \tag{111}$$

The first order condition of this objective with respect to θ is

$$\begin{array}{ll} 0 = \min_{m \in M^n} \mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2 \right] & \text{if } \theta > 0 \\ 0 \in \left\{ \mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2 \right] : m \in M^n \right\} & \text{if } \theta = 0 \\ 0 = \max_{m \in M^n} \mathbb{E}_m \left[(r - r_f) + \frac{1}{2}\sigma^2 - \theta\sigma^2 + (1-\gamma)\,\theta\sigma^2 \right] & \text{if } \theta < 0 \end{array} \tag{112}$$

which implies a demand of

$$\theta^n(M^n) = \begin{cases} \frac{\underline{\mu} - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \underline{\mu} - r_f + \frac{1}{2}\sigma^2 - p > 0 \\ 0 & \text{if } \underline{\mu} - r_f \le p - \frac{1}{2}\sigma^2 \le \overline{\mu} - r_f \\ \frac{\overline{\mu} - r_f + \frac{1}{2}\sigma^2 - p}{\gamma\sigma^2} & \text{if } \overline{\mu} - r_f + \frac{1}{2}\sigma^2 - p < 0 \end{cases} \tag{113}$$

Using the above and denoting the lower bound of the updated beliefs of the investor about the mean log stock payoff by $\underline{\mu}|s$ and upper bound by $\overline{\mu}|s$. □

Proof of Proposition 11. The proof is similar to that for Proposition 2. The market-clearing price when I- and U-investors are long in the stock satisfies $p - 0.5\sigma^2 = x_0^I \underline{\mu}|s + x_0^U \underline{\mu}^U|s - \gamma\sigma^2$. Moreover, the price must satisfy $p - 0.5\sigma^2 < \underline{\mu}|s$ and $p - 0.5\sigma^2 < \underline{\mu}^U|s$. Since $\delta^U < \delta$, the second inequality is satisfied whenever the first one is. Simplifying the first inequality and using the expressions for $\underline{\mu}|s$ and $\underline{\mu}^U|s$ provides the result. □

8.2 Fully-revealing equilibrium with reduced stockholding

The self-fulfilling nature of rational expectations equilibrium as defined in Definition 1 means that there can exist a fully-revealing equilibrium even when I-investors sell off their stockholding to U-investors. This full-revelation equilibrium exists due to the possibility that U-investors could conjecture exactly the right price function that is needed for all signals to be revealed even with reduced participation. The self-fulfilling nature of rational expectations equilibrium means that such a price function will be an equilibrium function.

To construct this equilibrium, suppose U-investors conjecture the price function $\hat{p}$ as

$$\hat{p}(s) = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon} + \frac{1}{2}\sigma^2 - \frac{\gamma \sigma^2}{x_0^U} \text{ for all } s. \tag{114}$$

This price function clearly reveals every signal s since it is linear and monotone in s. This in turn means that with this price function, U-investors updated beliefs are given by $\mu^U | s = \frac{\rho_0 \mu_0 + \rho_\epsilon s}{\rho_0 + \rho_\epsilon}$. Then checking the demand expression (17) of I-investors and the market clearing condition shows that this price indeed clears markets with I-investors selling off all their stockholding to U-investors. This in turn shows that there is a fully-revealing rational expectations equilibrium with the equilibrium price function given by $\hat{p}$.

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