Property Crime: Targeting versus Private Protection

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Abstract

In this paper, we look at a general equilibrium with private protection and criminal searching activities. In rich and heterogeneous neighbourhoods, criminals are more willing to spend resources to compare potential victims by searching for the most suitable victim. Therefore, private protection is more likely to be observed, and consequently serve as a base of comparison for criminals. If it is costly to observe and compare several victims, criminals will be more inclined to spend resources when the potential gains of comparison are high. Consequently, crime diversion effect induced by this increased observability stimulates private protection in richer and more diverse neighbourhoods. This in turn increases the perceived heterogeneity of the potential victims, which encourages criminals to search even more, which stimulate private protection, and so on. Therefore, there is an amplifying effect present in equilibrium. There implications of our model are tested using data from the Canadian Victimization Survey.

Key Words: Property crime; Private protection; Victimization; Inequality

JEL: K14; K42

1 Introduction

The criminal patterns change rapidly over time and vary significantly across space. According to Glaeser, Sacerdote and Scheinkman (1996) the "high degree of variance of crime rates across space and time is one of the oldest puzzles in the social sciences".

Different determinant of the crime have been analysed: Gleaser and Sacerdott (1999) have looked at the impact of the size of a city on victimization. They have shown that big cities have a higher crime rate. Levitt (1999) showed that inner city is more victimized than the suburb. But, according to Glaeser et al. (1996), the variance of crime rates is way to important and could not be explained by the observed or unobserved geographical attributes. In order to explain these variations, one needs to rely on a multiplier effect of some sort.

An important avenue proposed by Glaeser, Sacerdote and Scheinkman (1996) is based on a social interaction model. In such an environment, a key determinant of an individual behaviour depends on the behaviour of its peers and its neighbours. An individual is more prone to engage in criminal activities when its peers are also engaged in criminal activities. This generates some sort of social multiplier in the criminal activities amplifying the variance of criminal behaviour. Zenou (2003) using the same approach within an urban economics structure, analyses the link between social interaction and distance to job to explain the variance of crime. Those approaches are quite efficient in explaining the geographical repartition of crime, but say little on the way the crime burden is shared among citizens as a function of their wealth.

We propose an original multiplier effect base on the interaction between private protection and criminals' searching and sorting activities. The observability of private protection has been recognised to play a crucial role in the provision of private protection. When the protection is observable, the first objective is to make its belongings less attractive to the criminals; this is known as the crime diversion effect. There are other effects that are taken into account by the potential victims. For example, an increase in private protection can also reduce what is stolen in case of victimization. This is known as the theft reduction effect. Last is the deterrence effect: When private protection increases, the crime is less lucrative and therefore the entry into criminal activity drops. Because of the crime diversion effect, there can easily be an over provision of private protection when private protection is observable. (See Shavell (1992) for an early discussion and Hotte and van Ypersele (2008) for a normative analysis). The literature discussing private protection, assumes an exogenously given observability of the private protection. In the current project, we will develop a general equilibrium framework where the observation of the private protection is costly.

The intuition we develop in this paper is that, in rich and heterogeneous neighbourhoods, criminals are more willing to spend resources to compare potential victims by searching for the most suitable victim. Therefore, private protection is more likely to be observed, and consequently serve as a base of comparison for criminals. If it is costly to observe and compare several victims, criminals will be more inclined to spend resources when the potential gains of comparison are high. If this is true, crime diversion effect induced by this increased observability stimulate private protection in richer and more diverse neighbourhoods. This in turn increases the perceived heterogeneity of the potential victims, which encourages criminals to search even more, which stimulate private protection, and so on. Therefore, there is an amplifying effect present in equilibrium, since a small change in the diversity, or in the wealth of a neighbourhood may have an important impact on the private protection of households.

One of the important consequences associated with the interaction between search and protection is that we should expect to see more private protection in richer and more diverse neighbourhoods. Higher private protection implies lower return to crime in that type of neighbourhoods, and therefore a diversion of the criminals to the poorer and less heterogeneous neighbourhoods. As a consequence, the prediction of our intuition is that 1) within a neighbourhood wealthier households are more victimised than poorer households, 2) rich and diversified neighbourhoods may be less victimised than poor and less diversified ones 3) the level and the heterogeneity in private protection is higher in the wealthy than in the poor neighbourhoods.

The empirical literature on crime showed that there is an important geographical heterogeneity in crime. Different determinant of the victimisation have been analysed: Gleaser and Sacerdott (1999) have analysed the impact of the size of a city on crime. They have shown that big cities are more victimised. Levitt (1999) showed that inner city is more victimized than the suburb. But, according to Glaeser et al. (1996), the variance of crime rates is way to important and could not be explained by the observed or unobserved geographical attributes.

Using three waves of the Canadian Victimisation Survey matched with Census Data we test the three theoretical predictions mentioned above. Most interestingly is the dichotomic effect of intra neighbourghood wealth dispersion versus across neighbourghoods wealth dispersion; rich households could be more victimised than poor households inside of a given neighborhood, but at the same time poor neighbourhoods support higher victimization rates.

In the next section, we present the basic model. All proofs are in the appendix.

2 The Model

Imagine an world with two neighbourhoods denoted by A and B. In each neighbourhoods $j \in \{A, B\}$ resides K_j households. We can also interpret K_j as the amount of capital that can be stolen. A proportion μ_j of those residents are rich, and have belongings worth V_j^R, while a proportion $1 - \mu_j$ are poor (or poorer to be more precise), and have belongings worth V_j^P. Overall, neighbourhoods A is wealthier, so $E[V_A] > E[V_B]$. Denote the difference in income inside neighbourhood j by $\Delta_j = V_j^R - V_j^P$

Every residents can potentially be victim of a robbery, and if "visited" by a criminal, a resident would endures a cost $\gamma_j V_j^i$ per visit. Define by $\alpha_j V_j^i$, for $i \in \{R, P\}$ the gain for a criminal who targets resident i in neighbourhood j. The cost of a robbery for a resident may be lower or larger than what is actually stolen by the criminal. Households maybe insured, and consequently suffer smaller loss, or may suffer larger loss because part of the wealth is destroyed during the robbery, or because the resident suffers some psychological cost associated with the crime. Note that under all those interpretations, total losses can be higher than initial wealth. Each household chooses the level of self-protection θ they desire. A self-protection level θ imposes a cost θV_j^i to a potential burglar. Self protection is costly, and this cost is defined as $V_j^i \theta$ to represent the fact that more wealth is more costly to protect.[1]

A total number of C individuals have the aptitude to commit robbery on the resident of the two neighbourhoods; we refer to those individuals as criminals. Some of the criminals are immobile in the sense that they can only commit a robbery in a given neighbourhood. Denote by N_j the number of immobile criminals in neighbourhood j, where $N_A + N_B = N$. The rest of the criminals are free to commit a crime in either of the two neighbourhoods. Each mobile criminal makes a decision $J \in \{A, B\}$ of whether to plan a robbery in

[1] The marginal cost of protection is normalized to one, but could easily be different than one.

neighbourhoods A, or in neighbourhoods B. Denote by M_j the number of criminals who make choice J. Obviously, $M_A + M_B = M$, and $M + N = C$.

The total number of criminals operating in neighbourhood j is define as $C_j = N_j + M_j$. Since criminals commit only one crime each, we can define the crime rate in neighbourhood j as $c_j = C_j/K_j$.[2] A criminal who plans a robbery in neighbourhoods j is presented with up to two opportunities at random. When presented with an opportunity, a criminal is able to observe the value of the potential victim belongings V_j^i, and the protection level θ. Being presented with a choice of opportunities is not automatic. Criminals must search for those suitable opportunities. For simplicity, but without lost of generality, criminals are presented with one or two options. Every criminals operating in a neighbourhood is presented with at least one option. Denote by q the probability a criminal finds a second suitable option. We will refer to q as search effort for the rest of the paper. Search effort comes at a cost of $\lambda s(q)$. The cost of effort $s(q)$ is increasing, at an increasing rate. We assume that $s(0) = 0$, $s'(0) = 0$ and $s'(1) \to \infty$ to guarantee that effort is always positive, but strictly lower than one. We also assume that $s'''(q) > 0$, so that the cost function is sufficiently convex to guarantee the uniqueness of equilibrium. The parameter λ represent the importance of the search cost. Denote by C_{j1} the expected number of criminals in neighbourhood j who have one option, and C_{j2} the expected number of criminals who have two options. The none capital letters c_{j1} and c_{j2} are reserved for the relative expected number of criminals to households in a neighbourhood.

The timing is as follow. Mobile criminals make their location decision J, and all criminals make their search decision q. At the same time, residents in each neighbourhoods j invest in effort θ. All criminals matched with only one household commit a robbery against that particular one, and all criminals matched with two households select the most suitable one to commit a robbery at. We assume that there is only time for one robbery.

Criminals with two opportunities select the one delivering the highest payoff $(\alpha_j - \theta_j^i)V_j^i$. Denote by $\Omega^j(q)$ the expected payoff for a criminal operating in neighbourhood j, where:

$$\Omega^j(q) = (1-q)E[(\alpha_j - \theta)V_j^i] + qE\max[(\alpha_j - \theta)V_j^i, (\alpha_j - \theta')V_j^{i'}] - \lambda s(q). \qquad (1)$$

[2]We can also define $n_j = N_j/K_j$ and $m_j = M_j/K_j$ the same way.

We now define the payoff obtain by a households of wealth i in neighbourhood j. Denote by $P\left[(\alpha_j - \theta_j^i)V_j^i > (\alpha_j - \theta_j^{i'})V_j^{i'}\right]$ as the probability that a household i in neighbourhood j is strictly preferred when compared against another household in the same neighbourhood. We assume when a criminal is indifferent, each household is selected with equal probability. We must now compute the expected number of robberies for a given household when there are c_j criminals, among which there are c_{j1} with one option, and c_{j2} with two options. The expected number of visits by criminals with only one options is given by:

$$\eta_{j1} = \sum_{t=0}^{C_{j1}} t \frac{C_{j1}!}{t!(C_{j1}-t)!}\left(\frac{1}{K_j}\right)^t \left(1 - \frac{1}{K_j}\right)^{C_{j1}-t} = \frac{C_{j1}}{K_j} = c_{j1}. \tag{2}$$

The expected number of visits by criminals with two options for an household of type $i \in \{R, P\}$ are respectively given by:[3]

$$\eta_{j2}^R(\theta) = 2c_{j2}\left[1 - \mu P[\theta < \theta'] - (1-\mu)P\left[(\alpha_j - \theta)V_j^R > (\alpha_j - \theta')V_j^P\right]\right]; \tag{3}$$

$$\eta_{j2}^P(\theta) = 2c_{j2}\left[1 - \mu P\left[(\alpha_j - \theta)V_j^P > (\alpha_j - \theta')V_j^R\right] - (1-\mu)P[\theta < \theta']\right]. \tag{4}$$

When compared to the same type of potential victim, only the level of private protection matters. When compared to a potential victim of a different type, the difference in private protection must be important enough so that the difference in robbery costs $\theta_j^R V_j^R - \theta_j^P V_j^P$ is high enough to compensate for the difference in bounty $\alpha_j[V_j^R - V_j^P]$. The expected payoff for a household of type i in neighbourhood j is given by:

$$W_j^i(\theta) = V_j^i - [\eta_{j1} + \eta_{j2}^i(\theta_j^i)]\gamma_j V_j^i - V_j^i\theta. \tag{5}$$

Note that every time an individual of type i is paired with an individual of the same type, investing just a little bit more in private protection than the other household guaranties not to be selected. Because of this discontinuity, we show in Lemma 1 that there exists no pure symmetric strategy equilibrium.

[3] The computation of these numbers are provided in Appendix B.

Lemma 1: *For any $c_{j2} > 0$ there exist no private protection pure symmetric strategy Nash equilibrium.*

2.1 Investment in Private Protection

We now solve for the equilibrium investment in private protection θ_j^i by residents of type $i \in \{R, P\}$ in neighbourhoods $j \in \{A, B\}$. First note that in the absence search effort ($c_{j2} = 0$), all residents would simply not invest in private protection. In our model, the sole reason to invest in private protection is to push crime away onto another household.[4] However, search effort will be positive since $n_j q$ is always positive.

We already know that a pure strategy equilibrium does not exist in this game. However, there exist mixed strategy equilibria. To simplify the exposition, will abstain from using the neighbourhood index $j \in \{A, B\}$ when describing these equilibria. Denote by $H^i(\theta)$ the cumulative distribution function used by each resident of type $i \in \{R, P\}$ in a mixed strategy equilibrium. Also denote by $\bar{\theta}^i$ the maximal level of self protection played in equilibrium a household of type i. Since the function $H^i(\theta)$ is a cumulative distribution function, it must be non-decreasing, and it must be the case that $H^i(\bar{\theta}^i) = 1$. Given all other individuals in the neighbourhood play θ according to $H^i(\theta)$, an individual choosing θ would earn an expected payoff $W^i(\theta)$. Given those distribution functions, rich and poor households expect to be visited by η_2^i criminals who have two options, where:

$$\eta_2^R(\theta^R) = 2c_2 \left[1 - \mu H^R(\theta) - (1-\mu) H^P\left(\frac{\theta V^R - \alpha\Delta}{V^P}\right)\right]; \tag{6}$$

$$\eta_2^P(\theta^P) = 2c_j \left[1 - \mu H^R\left(\frac{\theta V^P + \alpha\Delta}{V^R}\right) - (1-\mu) H^P(\theta)\right]. \tag{7}$$

Depending on parameters values different types of equilibria are possible, and those equilibria may or may not be unique. We concentrate on parameters values such that

[4] It is standard to assume that each individual household does not consider the impact of its own investment on the overall crime rate in the neighbourhood, because each household has a "small[redacted] impact on such crime rate. If private protection had an impact on the size of a robbery loss, investment in private protection would be strictly positive even when $c_{j2} = 0$. Any positive amount of search effort by criminals would increase private protection above this minimal level. Assuming the loss minimization incentives away simply normalized the minimum investment in private protection to zero, but keep all the qualitative results the same.

a simple, and unique equilibrium exist. Such equilibrium features the fact that inside a neighbourhood, all rich households are preferred by criminals compare to any poor households. The existence of such type of equilibrium requires a sufficient differences in income between rich and poor households, defined as $\Delta = V^R - V^P$. More precisely, it is sufficient to assume that $\Delta / V^R \geq 2\mu\frac{\gamma}{\alpha}(n+m)$. With less income variation, equilibria are no longer unique, and are characterized by the fact that some poor households are preferred to some rich households. We concentrate on the first type of equilibrium for tractability reasons, but we show in appendix C that the two important features of the model are also true under the second type of equilibria. First, private protection is linearly increasing in the number of criminals with two option c_2. Second, inside a neighbourhood, rich households are always more victimized than poor households on average.

In the type of equilibrium we explore, criminals always prefer visiting a rich household compare to any poor household, and this for any levels of self protection, i.e. $\bar{\theta}^R < \alpha\Delta/V^R$. Rich households provide protection effort θ^R according to the overall cumulative distribution function $H^R(\theta)$ on the support $[0, \bar{\theta}^R]$. Similarly, poor households provide protection effort θ^P according to the overall cumulative distribution function $H^P(\theta)$ on the support $[0, \bar{\theta}^P]$. Note that zero protection is always in the interval played by both types of households. If a household (rich or poor) was to play a positive lower bound level on private protection, then reducing protection would not increase the probability of being visited by a criminal, but would be strictly less costly. We first solve for expected payoff for a rich households playing any level of private protection on the support $[0, \bar{\theta}^R]$. Since the expected number of robberies targeting a rich households performed by criminals with two options is given by

$$\eta_2^R(\theta^R) = 2c_2\left[1 - \mu H^R(\theta)\right], \tag{8}$$

when any rich households are preferred to any poor household, then the expected payoff for a rich household is given by:

$$W^R(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2\left[1 - \mu H^R(\theta)\right] - \theta\right] V^R. \tag{9}$$

Similarly, the expected number of robberies by criminals with two options against poor households is given by:

$$\eta_2^P(\theta^P) = 2c_2(1-\mu)\left[1 - H^P(\theta)\right], \tag{10}$$

and so the expected payoff for poor households is:

$$W^P(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2(1-\mu)\left[1 - H^P(\theta)\right] - \theta\right] V^P. \tag{11}$$

Using those expected payoffs, we solve the following mixed strategies equilibrium.

Proposition 1: *There exist a unique mixed strategies equilibrium, where rich households invest in private protection on the support* $[0, \bar{\theta}^R]$*, according to the cumulative distribution function :*

$$H^R(\theta) = \frac{\theta}{2\mu\gamma c_2}, \quad \text{where} \quad \bar{\theta}^R = 2\mu\gamma c_2,$$

and where poor households invest in private protection on the support $[0, \bar{\theta}^P]$*, according to the cumulative distribution function :*

$$H^P(\theta) = \frac{\theta}{2(1-\mu)\gamma c_2}, \quad \text{where} \quad \bar{\theta}^R = 2(1-\mu)\gamma c_2.$$

Under such equilibrium, criminals prefer a rich household with protection level $\bar{\theta}^R$ compare to a poor household with no private protection. This implies that $\Delta/V^R \geq 2\mu\frac{\gamma}{\alpha}c_2$. Given that $c_2 = cq$, if this condition is satisfied when $c = n + m$, then it is always satisfied.

Note that private protection increases with the number of criminals with two option (c_2) operating in a given neighbourhood (this is also true in all other equilibria as shown in appendix C). Naturally, households invest less in private protection when the cost of protection is large. Finally, each individual types of households, invest more in private protection when the proportion of comparable houses (same type) increases.

2.2 Criminals Searching Decisions

We now look at search effort decisions q by criminals in a given neighbourhood. Again, we omit to use the neighbourhood index j. An equilibrium search effort q must be a best response to all residents' private protection investments strategies, and residents' strategies must also be a best response to the equilibrium search effort. For given distributions of private protection investments, a criminal maximizes the following objective function:

$$\Omega(q) = [1-q]E[\alpha V - \theta] + qE\max[\alpha V - \theta, \alpha V' - \theta'] - \lambda s(q). \tag{12}$$

More search effort increases the chance of having two options, and grants a criminal with the possibility of taking the best one. In Lemma 2 bellow, we compute the objective function of a criminal given the distributions of private protection efforts described in the last section.

Lemma 2: *The expected payoff for a criminal is given by:*

$$\Omega(q) = \alpha E[V] - E[\theta V] + q\alpha MAD[V] + q\gamma R(V^R, V^P)c_2 - \lambda s(q), \tag{13}$$

The first two terms in the equation above represent the net benefit from targeting a random household, where $E[\theta V] = [\mu^2 V^R + (1-\mu)^2 V^P]\gamma c_2$ is the average cost of a robbery. The third term represent the fact that having two options increases the chance of selecting a rich household. We define $MAD[V] = \mu(1-\mu)\Delta$ as the Mean Absolute Deviation of Wealth.[5] The fourth term represent the fact that criminals with two options are able to pick the best opportunity, where $R(V^R, V^P) = \frac{\mu^2(3-2\mu)V^R + (1-\mu)^2(3-2(1-\mu))V^P - 6\mu^2(1-\mu)V^R}{3}$ represent the reduction in robbery costs generated by having two options to choose from. Given the equilibrium we selected, this term is positive because criminals always pick the lowest stealing cost.[6] Together, the third and fourth terms represent the benefit of search, while the last term of all represent the cost of search.

Proposition 2: *There exist a unique search effort* $q(c_2) \in]0,1[$ *that maximizes the expected return for criminals, where* $q(c_2)$ *is given by*

[5]Note that the Mean Absolute Deviation is a measure of dispersion like the Standard Deviation, but where the weighted absolute value of all deviations are taken instead of the weighted square value of all deviations.

[6]In the equilibrium described in appendix C, however, this term could be negative. Criminals may select a rich household who invested more in private protection instead of a poor household with lower private protection investment.

$$\alpha MAD[V] + \gamma R(V^R, V^P)c_2 = \lambda s'\Bigg(q(c_2)\Bigg). \tag{14}$$

Proposition 2 describes a criminal's search effort levels, when taking other criminals behaviour as given. At the aggregate level, one criminal■s search effort level influences other criminals search effort via c_2, which is a combination of the number of criminals operating in the neighbourhood, and their search effort level. There exist a negative externality associated with search effort; if one individual increases his or her search effort, it increases the expected number of criminals with two options. This in turn, increases protection effort by households. In the proposition bellow, we solve for the aggregate search effort.

Proposition 3: *There exist a unique equilibrium search effort* $q(c) \in]0,1[$ *given by*

$$\alpha MAD[V] + \gamma R(V^R, V^P)cq(c) = \lambda s'\Bigg(q(c)\Bigg).$$

To describe the geographical allocation of criminals, we first need to understand how aggregate search effort varies with income level, and income dispersion. However, with this level of generality, such exercise can be difficult. For example, the equation above may suggest search effort is increasing with the $MAD[V]$. However, $MAD[V]$ depends on μ, and so does $R(V^R, V^P)$. The same applies for the average income. For this reason, we make some assumptions relative to the income distribution to render comparative static more meaningful. More precisely, we assume that $V^P = V$ and that $V^R = V + \Delta$. We can then interpret V as a measure of income level, and Δ as a measure of income dispersion. We take μ as given, since it not only influences both income level and income dispersion, but it also influence directly protection via the mix strategy equilibrium in private protection. With these assumptions, the aggregate search effort level is given by:

$$\alpha\mu(1-\mu)\Delta + \gamma cR(V, \Delta)q(c) = \lambda s'\Bigg(q(c)\Bigg),$$

where $R(V, \Delta) = \Big[\frac{1-6\mu^2(1-\mu)}{3}V + \mu^2\frac{4\mu-3}{3}\Delta\Big]$.

Lemma 3: *Aggregate level of effort* $q(c)$ *is increasing with wealth level* V*, and is also increasing with income dispersion unless* γ *is excessively larger than* α*. Search effort elasticities* ϵ_{qV} *and* $\epsilon_{q\Delta}$ *with respect to a change in* V *and* Δ *are respectively given by:*

$$\epsilon_{\{q,V\}} = -\frac{\gamma V R_V c}{\gamma R(V,\Delta)c - \lambda s''(q)} \in [0,\infty]; \qquad \epsilon_{\{q,\Delta\}} = -\frac{\alpha\mu(1-\mu)\Delta + \gamma\Delta R_\Delta c}{\gamma R(V,\Delta)c - \lambda s''(q)}.$$

When wealth V increases, the expected robbery cost imposed to criminals due to private protection not only increases, it also becomes more heterogeneous. Sampling two houses becomes more attractive. When income dispersion increases, two effects operate in opposite directions. With more income dispersion, having a second chance to draw a rich household is more valuable. Moreover, this benefit is proportional to the proportion of wealth stolen, α. On the other hand, more income dispersion induces criminals to accept a higher robbery cost in exchange of targeting a rich household instead of a poor one. Since average private protection increases with the damage caused by a robbery, this effect is stronger for high value of γ. Conceptually, search effort could decrease with Δ, but this would require that criminals■ gain are much smaller than the damage imposed to victims.[7] In the next section, we look at the location decisions by criminals.

2.3 Criminals Location Decisions

The final step is to determine criminals' allocation decisions $J \in \{A, B\}$. In a given neighbourhood, the expected return to crime is: (we still omit the index j for the moment):

$$\begin{aligned}\Omega(c) = {} & \alpha[V+\mu\Delta] - \gamma q(c)\Big[[\mu^2+(1-\mu)^2]V + \mu^2\Delta\Big]c \\ & + q(c)\alpha\mu(1-\mu)\Delta + \gamma R(V^R,V^P)q(c)^2 c - \lambda s\Big(q(c)\Big),\end{aligned} \tag{15}$$

where $R(V,\Delta) = \left[\frac{1-6\mu^2(1-\mu)}{3}V + \mu^2\frac{4\mu-3}{3}\Delta\right]$. The expected payoff for a criminal is not necessarily increasing in neither wealth level, nor in wealth dispersion. Wealthier neighbourhoods are more attractive for an obvious reason. However, higher wealth stimulates the incentive for criminals to search, generating a higher q(c). Facing more chance to be compared, households have more incentives to invest in private protection. Consequently, operating in such neighbourhood may be less attractive. A similar ambiguity applies to wealth dispersion. In more heterogeneous neighbourhood, the benefit of having two

[7] With a 100% crime rate, the damage would need to be four time higher than the benefit to criminals, and for a 10% crime rate it would require a factor of fourthy to one.

options is grater. This directly makes such neighbourhood more attractive, but it also increases the incentive to search. Again, search stimulate private protection, and reduce the benefit to operate in such neighbourhood.

It is also very important to notice that criminals inflict a negative externality on each other. When households anticipate that there will be more criminals in their neighbourhood, they invest more in private protection. When criminals anticipate more protection, they search more. Since more search also generate more private protection, small increases in the number of criminal can lead to sharp decrease in expected payoff. The fact that the expected payoff is decreasing with c is particularly interesting, since in our model we don't have any matching frictions, nor congestion effects. We will now use the index $j \in \{A, B\}$ do designate the neighbourhood, as introduced earlier.

Proposition 4: *Provided that $\Omega^B(n_B) > \Omega^A(n_a + m)$ there exist a unique equilibrium allocation of criminals c_A and c_B as displayed in Figure 1*

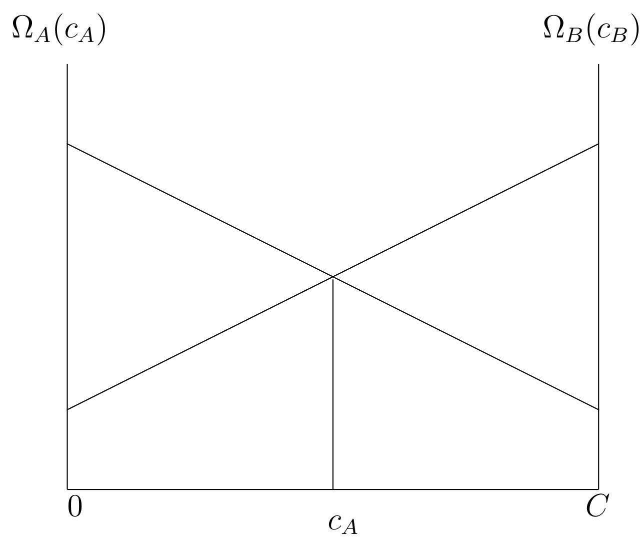

Figure 1: Allocation of criminals

Given the equilibrium described in Proposition 4, any reduction in the expected payoff from operating in a given neighbourhood reduces the crime rate in such neighbourhood, because of the migration of criminals it induces. Proposition 5 bellow shows that an increase in V, can lead to a reduction in the net expected benefit from operating in the neighbourhood, leading to a reduction in crime rate. This implies that even if neighbourhood A is wealthier, more criminal may locate in region B. Imagine that we start in an environment where $E[V^A] = E[V^B]$, an increase in V^A may be equivalent to a downward shift of the Ω^A curve in Figure 1, leading to more criminal locating in region B.

Proposition 5: *An increase in V reduces the net expected benefit $\Omega(c)$ if and only if:*

$$\epsilon_{\{q,c\}} > -\frac{\epsilon_{\{\Omega,V\}}\big|_{\text{at } q(e)c \text{ constant}}}{\epsilon_{\{\Omega,c\}}}.$$

The expected return to crime will be decreasing in average wealth only if search effort is sufficiently responsive to changes in V; enough responsive to overcome the direct benefit of facing wealthier potential victims.

Note that when V increase, private protection also increases because households have a higher marginal benefit of protecting their own wealth. In our model, this increase in private protection is not sufficient by itself to generate a reduction in the return to crime. We need the addition of search effort. Without search effort, in a given neighbourhood criminals would always prefer wealthier target, and would also prefer operating in wealthier neighbourhoods. Altering the model in such a way that wealthier individuals invest so much in private protection that poorer target are preferable would be easy, but then less wealthy individuals in a neigbourhood would be targeted. The addition of search effort generates the possibility that the intra-neighbourhood versus across neigbourhoods incentives be different, an implication that we will test in the next section.

The last question is when the condition stated in Proposition 5 can be satisfied. To answer this question, this we will at the following example.

Example: Imagine that μ is equal to zero or one, and that $s(q) = \frac{s^{2+1/a}}{2+1/a}$. With homogeneity inside the neighbourhood, we can explicitly solve for $q(c)$, and with the assumption on the $s(q)$, we get that $q(c)$ is iso-elastic in V with a elasticity of a. REST TO COME

3 Empirical Analysis

By looking at the intra-neighbourhood versus across neighbourhoods incentives our model can predict patterns that may be difficult to generate with competing models. Our model can predict that wealthier individuals inside of a neighbourhood may face higher victimization rates, but that wealthier neighbourhoods support lower crime rate. At the same time, individuals in wealthier neighbourhoods invest more in private protection. Moreover, our model predicts that more heterogeneous neighbourhood may support lower crime rate, and higher private protection.

Before even looking at the data, we should reflect on the types of patterns competing models would generate. The simplest model where only the direct elasticity of private protection matter, would have a hard time generating different intra-neighbourhood versus across neighbourhoods differences. If the elasticity of private protection with respect to wealth were to be lower than one, wealthier individuals and wealthier neighbourhood would be more attractive. The opposite would be true with an elasticity higher than one. A political economy model with public protection could easily generate lower crime rates in wealthier neighbourhood, but the same rational for heterogonous neighbourhood would require many additional assumptions. More importantly, in such model, it could be difficult to explain why private protection is higher in wealthy neighbourhoods since they would benefit from much more public protection. Another competing argument that can be made is that the supply of criminals is higher in less wealthy neighbourhood. This poses the question, why criminals don■t move. Never the less, if it were to be the case the incentives to invest in private protection in wealthier neighbourhood would be much smaller.

To test if inside a neighbourhood wealthier households are more victimized than poorer households, we use the respondent placement into the neighbourhood income distribution (2, 4 and 10 Quantiles). To assess whether wealthy and heterogeneous neighbourhoods may be less victimized than poorer and less diversified ones, we will use average income and standard deviation of income inside a neighbourhood. The same exercise will be done for investment in private protection.

In our model two variables - the victimization probability and the protection effort simultaneously ■ are simultaneously determined. A higher victimization probability leads to more private protection. More private protection leads to lower victimization probability. None of them are directly observed. What is observed are dummies that take a value of 1 when the agent is victimized or when the agent undertakes more than a certain level of protection and zero otherwise. Our estimation strategy is based on Maddala (1983). More precisely the process we describe corresponds to the model 6 presented page 246–247 of that book. We will follow the estimation procedure and the correction techniques. To do this, we need at least one exclusion variable that affects only one of the endogenous variables and not the other. We propose that the number of young children in the household would affect the probability to exert a protection effort and not the probability of victimization. Take V_{ij} the victimization dummy of agent i in location j and the P_{ij} the protection dummy with respectively $\hat{V}_{ij}$ and $\hat{P}_{ij}$ their latent variables that are the victimization probability and the protection effort. We want to identify the direct impact of I_{ij} the income of individual i, $\bar{I}_j$ the average income of location j, σ_j the standard deviation of the income distribution in location j on both the protection effort and on the victimization probability. Our model predicts that victimization and protection depend on each other:

$$\hat{V}_{ij} = a_0 + a_1 I_{ij} + a_2 \bar{I}_j + a_3 X_j + a_4 \hat{P}_{ij}; \tag{16}$$

$$\hat{P}_{ij} = b_0 + b_1 I_{ij} + b_2 \bar{I}_j + b_3 \sigma_j + b_4 n_{ij} + b_5 \hat{V}_{ij}. \tag{17}$$

Mallar (1977) and Maddala (1983) showed that in terms of parameter restrictions, criteria for identification are identical to those for linear simultaneous equations systems.

This specification enables us to identify the direct effect of income on the victimization and the indirect effect that transmits via the protection effort. Our model predicts that a_1 , b_1, b_2, b_3, b_4, and b_5 are all positive, while a_2, a_3, and a_4 should be negative.

We will also estimate an equation including some interaction variables like $I_{ij}(\bar{I} - \bar{I}_j)$ that would indicate whether being rich in a richer than average localization has a positive impact on victimization and on protection.

3.1 Data

The first step will be to construct our measure of a neighbourhood. We will concentrate only on urban area. Among the geographic units as defined by Statistic Canada the one that corresponds most to our model is the Census Tract. Other geographic units will be used as a measure of comparison, such as, the dissemination Area for a smaller definition of a neighbourhood, and the Federal Electoral District for a larger definition of a neighbourhood. We are using the three cycles of the GSS-Victimization (23, 18 and 13) to maximize the number of observations. Each of these cycles needs to be matched with the appropriate years of the Census. Since victimization happens before the GSS 1999, 2004 and 2009, we will use the Census from 1996, 2001, and 2006 respectively.

The main variable of interest is about victimization incidents, more specifically about attempt to break in or break and enter. To test our first prediction pertaining to whether within a neighbourhood the wealthier are more victimized than those who are poorer, we plan to use household incomes in the GSS, and the income distribution in the neighbourhood using the corresponding Census.

To test our second prediction about whether rich and heterogeneous neighbourhoods may be less victimized than poor and less diversified ones we will need to match the geographic units of a B&E in the GSS with the average and standard deviation of income from the corresponding Census

To test our last prediction about whether the heterogeneity in private protection is higher in the wealthy and more heterogeneous neighbourhoods, we will need information about private protection. The GSS-Victimization asks the some questions about whether the respondent has ever done (and in the last 12 months) any of the following things to protect oneself or one■s property from crime. We are interested in variables about locks and bars, about alarms and about dogs.

4 Conclusion

5 References

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6 Appendix A: Proofs

Proof to Lemma 1: We proceed by contradiction. Take a *pure symmetric strategy Nash equilibrium,* $\{\theta_j^{R*}, \theta_j^{P*}\}$. This allocation is an equilibrium iff

$$W_j^i(\theta) \leq W_j^i(\theta^{i*}) \text{ for all } \theta \geq 0.$$

Using (5), we rewrite the payoff of a rich victim as:

$$\begin{aligned} W_j^R(\theta) = {} & V_j^R - \theta V_j^R - \gamma_j \eta_{j1} V_j^R \\ & -2\gamma_j V_j^R c_{j2} \left[1 - \mu P(\theta < \theta_j^{R*}) - (1-\mu) P([\alpha_j - \theta] V_j^R > [\alpha_j - \theta_j^{P*}] V_j^P)\right]. \end{aligned} \tag{18}$$

Announcing $\theta = \theta_j^{R*} + \varepsilon$ generates a discrete gain $\mu \gamma_j V_j^R c_{j2}$ at an infinitely small cost εV_j^R. Similarly for a poor victim:

$$\begin{aligned} W_j^P(\theta) = {} & V_j^P - \theta V_j^P - \gamma_j \eta_{j1} V_j^P \\ & -2\gamma_j V_j^P c_{j2} \left[1 - \mu P([\alpha_j - \theta] V_j^P > [\alpha_j - \theta_j^{R*}] V_j^R) - (1-\mu) P(\theta < \theta_j^{P*})\right]. \end{aligned} \tag{19}$$

Announcing $\theta = \theta_j^{P*} + \varepsilon$ generates a discrete gain $\mu \gamma_j V_j^R c_{j2}$ at an infinitely small cost εV_j^R. Therefore, when $c_{j2} > 0$, then$\{\theta_j^{R*}, \theta_j^{P*}\}$ can not be an equilibrium. QED

Proof of Proposition 1: In a mixed strategy equilibrium, players must be indifferent between all strategies, we can then solve for $H^R(\theta)$ by solving for $W^R(\theta) = W^R(0)$:

$$H^R(\theta) = \frac{\theta}{2\mu\gamma c_2}. \tag{20}$$

For $H^R(\theta)$ to be an suitable c.d.f., it must be none-decreasing in θ, and take value smaller or equal to unit. Moreover, it must take the unit value on the upper bound of the strategic support, so $H^R(\bar{\theta}^R) = 1$. This implies that the upper bond on protection effort for a rich households is given by:

$$\bar{\theta}^R = 2\mu\gamma c_2. \tag{21}$$

Similarly, we can solve for the mixed strategy $H^P(\theta)$ for a poor resident by solving for $W^P(\theta) = W^P(0)$, where:

$$W^P(0) = [1 - \gamma c_1 - 2(1-\mu)\gamma c_2]\, V^P. \tag{22}$$

Consequently,

$$H^P(\theta) = \frac{\theta}{2(1-\mu)\gamma c_2}. \tag{23}$$

Again, $H^P(\bar{\theta}^P)$ must equal to one, so Iit implies that the upper bond on protection effort by the poor households is given by:

$$\bar{\theta}^P = 2(1-\mu)\gamma c_2. \tag{24}$$

QED

Proof of Lemma 2: For any positive search effort level q, a criminal is match with two households with probability q. In such a case, a criminal may be matched with two poor households with probability $(1-\mu)^2$, with two rich households with probability μ^2 and with one rich and one poor households with probability $2\mu(1-\mu)$. If matched with two rich households, the expected payoff is given by:

$$\begin{aligned} E\max[\alpha V^R - \theta, \alpha V^{R'} - \theta'] &= \alpha V^R - 2\textstyle\int_0^{\bar{\theta}^R} \theta[1 - H^R(\theta)]h^R d\theta \\ &= \alpha V^R - \tfrac{2}{3}\mu\gamma c_2 V^R. \end{aligned} \tag{25}$$

If matched with two poor households, the expected payoff is given by:

$$\begin{aligned} E\max[\alpha V^P - \theta, \alpha V^{P'} - \theta'] &= \alpha V^P - 2\textstyle\int_0^{\bar{\theta}^P} \theta[1 - H^P(\theta)]h^P d\theta \\ &= \alpha V^P - \tfrac{2}{3}(1-\mu)\gamma c_2 V^P. \end{aligned} \tag{26}$$

If matched with one rich and one poor household, the criminal opts for the rich household, and the payoff is given by:

$$\begin{aligned} E\max[\alpha V^R - \theta, \alpha V^P - \theta] &= \alpha V^R - \textstyle\int_0^{\bar{\theta}^R} \theta h^R d\theta \\ &= \alpha V^R - \mu\gamma c_2 V^R. \end{aligned} \tag{27}$$

Given that average wealth is define by $E[V] = \mu V^R + (1-\mu)V^P$ and that the mean average deviation is define as $MAD[V] = \mu[V^R - E(V)] + (1-\mu)[E(V) - V^P]$ we can rewrite the expected payoff of a criminal as shown by equation (13). QED

Proof of Proposition 2: Search effort q chosen by criminals in a given neighbourhood is defined by the following first order condition:

$$\alpha MAD[V] + \gamma R(V^R, V^P)c_2 = \lambda s'\Big(q(c_2)\Big). \tag{28}$$

Note that even if $R(V^R, V^P)$ is negative, the LHS is always positive since the return to crime is always positive. The second order condition is given by:

$$-\lambda s''(q)/2 < 0 \tag{29}$$

Since $s'(0) = 0$ and $s'(1) \to \infty$ there exist a unique solution to this first order condition. QED

Proof of Proposition 3: Aggregate search effort $q(c)$ is given by:

$$\alpha MAD[V] + \gamma R(V^R, V^P)q(c)c = \lambda s'\Big(q(c)\Big). \tag{30}$$

The LHS is linearly increasing in $q(c)$, while the RHS is increasing and convex in $q(c)$ since we assumed that $s'''(q) > 0$. Assuming that $s'(0) = 0$ and $s'(1) \to \infty$, guarantee a unique solution. QED

Proof of Lemma 3: Comparative static on $q(c)$ and V, reveals that

$$\frac{\partial q(c)}{\partial V} = -\frac{\gamma R_V qc}{\gamma R(V, \Delta)c - \lambda s''(q)} \geq 0. \tag{31}$$

The numerator is positive since $1 - 6\mu^2(1-\mu)$ is positive, and the denominator is negative since our unique equilibrium is stable. Similarly, comparative static on $q(c)$ and Δ, reveals that

$$\frac{\partial q(c)}{\partial \Delta} = -\frac{\alpha\mu(1-\mu) + \gamma R_\Delta qc}{\gamma R(V, \Delta)c - \lambda s''(q)}. \tag{32}$$

The numerator is definitively positive when $\gamma < 4\alpha$ for all value of c, q and μ including one. QED

Proof of Proposition 4: First, we will define the search effort elasticity ϵ_{qc} with respect to the number of criminal in the neighbourhood by:

$$\epsilon_{\{q,c\}} = -\frac{\gamma R(V, \Delta)c}{\gamma R(V, \Delta)c - \lambda s''(q)} \in [-1, \infty].$$

We now show that $\Omega(c)$ is always decreasing in c. Using the first order condition on $q(c_2)$, we can see:

$$\frac{\partial \Omega(c)}{\partial c} = -q(c)\gamma\left[[\mu^2 + (1-\mu)^2]V + \mu^2\Delta - q(c)R(V,\Delta)\right]\left[1 + \frac{c}{q(c)}\frac{\partial q(c)}{\partial c}\right] < 0 \quad (33)$$

The term in bracket is always positive, and since $\frac{c}{q(c)}\frac{\partial q(c)}{\partial c} > -1$, the expected benefit is always decreasing with c. As long as $\Omega^B(n_B) > \Omega^A(n_a + m)$ the two benefit (in A and B) will be equalized when criminals locate in both regions. QED

Proof of Proposition 4: The effect of V on Ω using the first order condition on $q(c_2)$ is given by:

$$\frac{\partial \Omega(c)}{\partial V} = \left.\frac{\partial \Omega(c)}{\partial V}\right|_{\text{at } q(e)c \text{ constant}} + \frac{\partial \Omega(c)}{\partial c}\frac{\partial q(c)}{\partial V} \quad (34)$$

We can see that Ω is decreasing with V if only if:

$$\frac{\partial q(c)}{\partial V} > -\frac{\left.\frac{\partial \Omega(c)}{\partial V}\right|_{\text{at } q(e)c \text{ constant}}}{\frac{\partial \Omega(c)}{\partial c}} \quad (35)$$

We can easily re-arrange the expression in term of elasticity. QED

7 Appendix B: Number of visits

We now compute the expected number of robberies against individuals of type i from the c_{j2} criminals with two options. With probability

$$\Pi(t) = \binom{C_{j2}}{t}\left(\frac{2}{K_j}\right)^t\left(1 - \frac{2}{K_j}\right)^{C_{j2}-t}, \quad (36)$$

the victim is matched with t other criminals. With probability $\Pi(t^R;t)$ out of those t criminals t^R are also matched with another rich households, and $t^P = t - t^R$ are matched with a poor one. Consequently:

$$\Pi(t^R;t) = \binom{t}{t^R}\mu^{t^R}(1-\mu)^{t-t^R}. \quad (37)$$

Out of those potential matches, only the cases where the household is matched with a less profitable victim will trigger a robbery against agent i. A victim of type i is less profitable to a criminal if than another agent i if $\theta < \theta^i$, while a type $-i$ is less profitable if $\theta < \alpha(V_i - V_{-i})/V_i + \theta^{-i}V_{-i}/V_i \Leftrightarrow \theta^{-i} > \theta V_i/V_{-i} - \alpha(V_i - V_{-i})/V_{-i} = \tilde{\theta}^{-i}(\theta)$.

Let write $(\theta^i_1, ...\theta^i_{t^i})$ the t^i order of that sample i.e. $\theta^i_1 < \theta^i_2 ... < \theta^i_{t^i}$. A particular household will be visited by $t^i - g$ of the criminals matched with another type i household if $\theta \in [\theta^i_g, \theta^i_{g+1}]$. For any cumulative distribution $F(\cdot)$, the joint density distribution (x,y) of $(g, g+1)$ order statistics is given by

$$\frac{t^i!}{(g-1)!(t^i-g-1)!} F^i(x)^{g-1} \left(1 - F^i(y)\right)^{t-g-1} f^i(x) f^i(y). \tag{38}$$

Therefore the probability that $\theta \in [\theta^i_g, \theta^i_{g+1}]$ is given by

$$\begin{aligned} p^i(g) &= \int_{\theta}^{\bar{\theta}^i} \int_0^{\theta} \frac{t^i!}{(g-1)!(t^i-g-1)!} F^i(x)^{g-1} \left(1 - F^i(y)\right)^{t-g-1} f^i(x) f^i(y) \; dx \; dy; \\ &= \frac{t^i!}{(g-1)!(t^i-g-1)!} \frac{F^i(\theta)^g}{g} \frac{\left(1 - F^i(\theta)\right)^{t^i-g}}{t^i-g}; \\ &= \frac{t^i!}{g!(t^i-g)!} F^i(\theta)^g (1 - F^i(\theta))^{t^i-g}. \end{aligned} \tag{39}$$

as $\int F(X)^{g-1} f(x) dx = \frac{F(X)^g}{g}$ and $\int (1 - F(y))^{t-g-1} f(y) dy = \frac{(1-F(y))^{t-g}}{g-t}$.

We also write $(\theta^{-i}_1, ...\theta^{-i}_{t^{-i}})$ the t^{-i} order of that sample $i.e. \theta^{-i}_1 < \theta^{-i}_2 ... < \theta^{-i}_{t^i}$. My victim will be visited by $t^i - l$ of the other criminals matched with type i victim if $\theta - \alpha(V_i - V_{-i}) \in [\theta^{-i}_l, \theta^{-i}_{l+1}]$.

The probability that $\theta - \alpha(V_i - V_{-i}) \in [\theta^{-i}_l, \theta^{-i}_{l+1}]$. is given by

$$\begin{aligned} p^{-i}(l) &= \int_{\tilde{\theta}^{-i}(\theta)}^{\bar{\theta}^{-i}} \int_0^{\tilde{\theta}^{-i}(\theta)} \frac{t^{-i}!}{(l-1)!(t^{-i}-l-1)!} F^{-i}(x)^{l-1} \left(1 - F^{-i}(y)\right)^{t-l-1} f^{-i}(x) f^{-i}(y) \; dx \; dy; \\ &= \frac{t^{-i}!}{(l-1)!(t^{-i}-l-1)!} \frac{F^{-i}(\tilde{\theta}^{-i}(\theta))^l}{l} \frac{\left(1 - F^{-i}(\tilde{\theta}^{-i}(\theta))\right)^{t^i-l}}{t^i-l}; \\ &= \frac{t^{-i}!}{l!(t^{-i}-l)!} F^{-i}(\tilde{\theta}^{-i}(\theta))^l \left(1 - F^{-i}(\tilde{\theta}^{-i}(\theta))\right)^{t^{-i}-l}. \end{aligned} \tag{40}$$

We can therefore compute the joint density of the distribution of the (g, l) for an agent of type i that is $p^i(g)p^{-i}(l)$. The expected number of visits is given by

$$\begin{aligned} n_1^i &= \sum_{t=0}^{c_2} \Pi(t) \sum_{t^i=0}^{t} \Pi(t^i;t) \sum_{l=0}^{t-t^i} \sum_{k=0}^{t^i} (t-g-l) p^i(g) p^{-i}(l); \\ &= 2c_2 \left[1 - \mu_i F^i(\theta) - \mu_{-i} F^{-i}(\tilde{\theta}^{-i}(\theta))\right]. \end{aligned} \tag{41}$$

8 Appendix C: Some poor households are preferred

In any equilibria of that type, all poor households provide protection effort according to an overall cumulative distribution function $H^P(\theta)$ on the full support $\left[0, \bar{\theta}^P\right]$. As in the equilibrium described earlier, some rich households with low level of private protection will be selected when compared to any poor households; a rich household with zero effort for example. More precisely, any rich households with protection on the support $\left[0, \frac{\alpha\Delta}{V^R}\right]$, will always be selected against any poor households. Rich households who play above $\frac{\alpha\Delta}{V^R}$, on the other hand, will not necessarily be selected. Consequently, there exists two different cumulative distribution functions for rich individuals. Private protection will be selected according to a cumulative distribution $H_\ell^R(\theta)$ on the lower par of the support $\left[0, \frac{\alpha\Delta}{V^R}\right]$, and according to $H_u^R(\theta)$ on the upper par of the support $\left[\frac{\alpha\Delta}{V^R}, \bar{\theta}^R\right]$. The expected payoff for a rich household over the lower and upper supports are given by:

$$W_\ell^R(\theta) = \left[1 - \gamma c_1 - 2\gamma c_2 \left[1 - \mu H_\ell^R(\theta)\right] - \theta\right] V^R; \tag{42}$$

$$W_u^R(\theta) = \left[1 - c_1\gamma - 2\gamma c_2 \left[1 - \mu H_u^R(\theta) - (1-\mu) H^P\left(\frac{\theta V^R - \alpha\Delta}{V^P}\right)\right] - \theta\right] V^R. \tag{43}$$

Similarly, we can solve for the expected payoff for poor households over the unique range. Denote by $W^P(\theta)$ the expected payoff of a poor household where:

$$W^P(\theta) = \left[1 - c_1\gamma - 2\gamma c_2 \left[1 - \mu H_u^R\left(\frac{\theta V^P + \alpha\Delta}{V^R}\right) - (1-\mu) H^P(\theta)\right] - \theta\right] V^P. \tag{44}$$

Since in a mixed strategy equilibrium, players must be indifferent between all their strategies, we can find $H_\ell^R(\theta)$ for $\theta < \frac{\alpha\Delta}{V^R}$, by solving for $W_\ell^R(\theta) = W^R(0)$:

$$H_\ell^R(\theta) = \frac{\theta}{2\mu\gamma c_2}. \tag{45}$$

To guarantee that $H_\ell^R(\theta) < 1$ for all $\theta \in \left[0, \frac{\alpha\Delta}{V^R}\right]$, it requires that $\Delta/V^R < 2\mu\frac{\gamma}{\alpha}c_2$. Similarly, we can find $H_u^R(\theta)$ for $\frac{\alpha\Delta}{V^R} < \theta < \bar{\theta}^R$, by solving for $W_u^R(\theta) = W^R(0)$:

$$H_u^R(\theta) = \frac{\theta}{2\mu\gamma c_2} - \frac{(1-\mu)}{\mu} H^P \left(\frac{\theta V^R - \alpha\Delta}{V^P} \right) \tag{46}$$

There are many distribution functions that may satisfy equation (46) above.

Lemma C1: *In any mixed strategy equilibrium, it must be the case that* $\bar{\theta}_R^{II} = 2\gamma c_2$ *and* $\bar{\theta}^P = \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$.

Proof of Lemma C1: We prove this statement by contradiction. First assume that $\bar{\theta}^P < \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, this implies that a rich household by announcing the maximal level of protection is never chosen when compared to a poor household. Therefore,

$$\begin{aligned} W_u^R(\bar{\theta}^R) &= W_\ell^R(0); \\ V^R\left[1 - c_1\gamma - \bar{\theta}^R\right] &= V^R\left[1 - c_1\gamma - 2\gamma c_2\right]; \\ \bar{\theta}^R &= 2\gamma c_2. \end{aligned} \tag{47}$$

Since $\bar{\theta}^P \leq \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, a poor household may be chosen in some cases, so:

$$\begin{aligned} W^P(\bar{\theta}^P) &= W^P(0); \\ 2c_2\gamma\left[1 - \mu H_u^R\left(\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right) - (1-\mu)\right] + \bar{\theta}^P &= 2c_2\gamma. \end{aligned} \tag{48}$$

Using equation (47), we get that:

$$\bar{\theta}^P = (1-\mu)\bar{\theta}^R + \mu\bar{\theta}^R H_u^R\left(\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right). \tag{49}$$

This implicitly defines $\bar{\theta}^P$.The LHS and RHS are linearly increasing in $\bar{\theta}^P$. To see the second note that by (46), the RHS has a slope of $\frac{1}{2\mu\gamma c_2}$. This is enough to show that when $\bar{\theta}^P < \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$, the only possible solution is that $\bar{\theta}^P = \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. Consequently. $\bar{\theta}^P$ can not be strictly smaller than $\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$

Second, assume that $\bar{\theta}^P > \frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. A poor households by investing the maximal level of private protection is never chosen when compared to a rich household. Therefore,

$$\begin{aligned} W^P(\bar{\theta}^P) &= W^P(0); \\ V^P\left[1 - c_1\gamma - \bar{\theta}^P\right] &= V^P\left[1 - c_1\gamma - 2c_2\gamma\right]. \end{aligned} \tag{50}$$

Consequently,

$$\bar{\theta}^P = 2\gamma c_2. \tag{51}$$

On the other hand, a rich household may be chosen, so

$$\begin{aligned} W_\ell^R(0) &= W_u^R(\bar{\theta}^R); \\ \gamma 2c_2 &= \gamma 2c_2(1-\mu)\left[1 - H^P\left(\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}\right)\right] - \bar{\theta}^R. \end{aligned} \tag{52}$$

This is to say,

$$\bar{\theta}^R = \left[\frac{\bar{\theta}^P V^P + \alpha\Delta}{V^R}\right]\left[1 - (1-\mu)\left(1 - H^P\left(\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}\right)\right)\right]. \tag{53}$$

As above, the RHS and LHS are linearly increasing in θ, and generate a unique solution when $\bar{\theta}^P = \frac{\bar{\theta} = ^R V^R - \alpha\Delta}{V^P}$. This implies that $\bar{\theta}^P$ can not be strictly larger than $\frac{\bar{\theta}^R V^R - \alpha\Delta}{V^P}$. QED

Among the interesting properties of such equilibria, both $\bar{\theta}^P$ and $\bar{\theta}^R$ are increasing with c_2; more criminal who are searching implies that all households invest more in private protection. Moreover, since all rich households with protection under $\frac{\alpha\Delta}{V^R}$ are always selected, rich households face higher probability of being victimized.

In the proposition bellow, we restrict our attention to mixed strategy equilibria with uniform distribution.

Proposition C1: *There exist a mixed strategies equilibrium where rich households invest in private protection on the supports* $[0, \frac{\alpha\Delta}{V^R}]$ *and* $[\frac{\alpha\Delta}{V^R}, \bar{\theta}^R]$*, according to the cumulative distribution functions:*

$$H_\ell^R(\theta) = \frac{\theta}{2\mu\gamma c_2}, \quad \text{and}$$

$$H_u^R(\theta) = H_\ell^R\left(\frac{\alpha\Delta}{V^R}\right) + \frac{\mu\bar{\theta}^R V^R - \alpha\Delta}{\bar{\theta}^R V^R - \alpha\Delta}\frac{1}{\mu\bar{\theta}^R V^R}[\theta V^R - \alpha\Delta],$$

$$\text{where} \quad \bar{\theta}^R = 2\gamma c_2.$$

Similarly, poor households invest in private protection on the support $[0, \bar{\theta}^P]$*, according to the cumulative distribution function:*

$$H^P(\theta) = \frac{\theta}{2\gamma c_2 - \alpha\Delta}, \quad \text{where} \quad \bar{\theta}^P = 2\gamma c_2 - \alpha\Delta.$$

Proof of Proposition C1: We will propose two uniform distributions for $H_u^R(\theta)$ and $H^P(\theta)$. Note that $W^P(0)$ is given by:

$$W^P(0) = [1 - c_1\gamma - 2c_2\gamma]\, V^P, \tag{54}$$

We can now derive the following uniform distribution for poor households:

$$H^P(\theta) = \frac{\theta}{\bar{\theta}^P}, \tag{55}$$

From equation (46), we get that:

$$H_u^R(\theta) = H_\ell^R\left(\frac{\alpha\Delta}{V^R}\right) + \frac{\mu\bar{\theta}^R V^R - \alpha\Delta}{\bar{\theta}^R V^R - \alpha\Delta}\frac{1}{\mu\bar{\theta}^R}[\theta V^R - \alpha\Delta]$$

Finally, we can see that when $c_2 > \frac{1}{2\mu}\frac{\alpha}{\gamma}\Delta V^R$, then $H_\ell^R(\theta) < 1$ for all $\theta \in [0, \alpha\frac{\Delta}{V^R}]$. QED