Market Segmentation, Parallel Imports, and Incomplete Price Discrimination: The Welfare Effects of Regulations*

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Abstract

We consider a regulated monopolist that faces a continuum of markets (e.g., $[0,1]$) although a finite number k of prices must be charged. As a function of the integer k, we provide the optimal profit policy defined as the optimal market segmentation and the discriminatory prices. The social welfare is maximized for three prices, but the result is far below the benchmark, the welfare under the Ramsey price. As this price implies no profit, we study regulatory policies that yield a welfare closer to the benchmark, but compatible with "reasonable" profits. We show that a small amount of price discrimination can substantially enhanced the welfare when the monopolist is subject to an average price constraint.

Keywords : Monopoly, incomplete price discrimination, market segmentation, exhaustion of rights, parallel imports, Ramsey price, average-price regulation, social welfare, stochastic audit.

JEL classification codes: D 42, L 11, L 50.

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1 Introduction

Price discrimination is a very common practice both in domestic and in international markets. For example, the price of a train's ticket generally differs as to whether the purchaser is a senior citizen or not. In the same way, the price of a drug (or a car) depends on the country where the purchase is made. In the first example, the market segmentation for price discrimination is based on an age criteria whereas in the second, it is based on a geographic criteria. In the simplest case of a single firm in a monopoly situation, the present article examines the welfare effects of regulating market segmentation and price discrimination. Its aim is to shed some light on the following questions.

Should a monopolist be allowed to segment its set of customers? If yes, what is the optimal number (if any) of market segments from a social welfare point of view ? Should one only regulate the number of market segments or should one also regulate market segments and/or prices ?

In Economic theory, from the pioneered work of Joan Robinson (1969), the analysis of the welfare effect of (third-degree) price discrimination in monopoly[1] has received a considerable academic attention. The problem has been to identify the conditions under which price discrimination is harmful, or beneficial, for the social welfare. In a constant marginal cost model, Schmalensee (1981) has shown that when the demand function of each market segment is linear, if all the market segments are served under uniform pricing, then, price discrimination reduces the social welfare because the total output does not increase. Varian (1985) has shown that this "output effect" turns out to be true in a much more general model. By deriving the upper and the lower bound to the change in social welfare, Varian (1985) has shown that a necessary condition (but not sufficient) for price discrimination to increase social welfare is that output increases.

However, the analysis of the welfare effect (e.g., Cowan 2007, He and Sun 2006, Kaftal and Pal 2008, Shih et al 1988, Schmalensee 1981, Schwartz 1990, Varian 1985) in the third-degree price discrimination literature[2] always reduces to a pure pricing problem because the underlying market segmentation is always exogenous, i.e., both the number k and the segments are fixed. Thus, for the given market segmentation, the typical problem is to compute the social welfare under two extreme scenarios; no discrimination, and "complete" price discrimination, which is the situation where the monopolist may charge k discriminatory prices when there are k separate segments. Assuming that the market segmentation is given *a priori* is indeed surprising because in practice, market segmentation and discriminatory prices are not chosen independently; they are rather chosen *simultaneously* by the monopolist to maximize profits. In the present framework, market segmentation and prices are endogenous, so that it becomes possible to study the monopolist's optimal profit policy (market segmentation and prices), but also the various ways this optimal profit policy can be regulated to enhance the social welfare.

We consider a patented monopolist that produces an "intellectual property good" (e.g., pharmaceuticals, computer software...) that has required an important investment in terms of research and development, but we focus on the commercialization process, that is, on the segmentation and the pricing problem. A convenient way to study this problem is to consider the case in which the

[1]See e.g., Phlips 1988, Stole 2007, or McAfee 2008 for a review.

[2]The article of Malueg and Schwartz (1994) is a notable exception.

set of customers Ω has the cardinality of the continuum, e.g., $\Omega = [0, 1]$. For a given number k of segments, the *optimal profit policy* is defined by

- the optimal way to segment the set $\Omega = [0, 1]$ in k disjoint groups.
- the optimal discriminatory price to be charged to each of these k groups.

The possibility for the monopolist to freely segment the set of customers critically depends of the *doctrine of right's exhaustion*. Roughly speaking, when there is no exhaustion of rights, once the good of the owner of the intellectual property rights (IPR) has been sold, its rights are not exhausted so that parallel imports (henceforth PI) are not legally possible. On the contrary, under say the "community exhaustion", which is the doctrine adopted by the European Court of Justice[3], the discriminating monopolist can not legally ban PI from a low price country to a high price country within EU. As a consequence, in such an environment, the discriminating monopolist must find strategies to eliminate, or at least to mitigate the parallel import problem (see e.g., Ahmadi and Yang 2000, Cavusgil and Sikora 1988, Danzon and Towse 2003). Two polar scenarios will be considered.

1. The number k of segments is fixed by the regulator, but, due to the perfect enforcement of a no exhaustion doctrine, PI are not possible.
2. The number k of segments can be freely chosen by the monopolist, but under an international exhaustion doctrine so that PI can not be banned.

We solve the optimal profit policy as a function of k when PI are not a threat. It is shown that on each (separate) market segment, the well-known price cost margin (or Lerner index) is satisfied. This optimal profit policy also reveals that if the monopolist is allowed to create $k + 1$ market segments rather than k, this both changes the existing market segmentation and prices. This means that these two quantities (i.e., market segments and prices) can not be chosen independently. We then solve the optimal profit policy with respect to k when the cost of isolating the market segments is an increasing function of k. We show that it is never optimal to create more than few segments even when the cost is low. This best trade-off between the gain and the cost associated to a more complete price discrimination provides a natural foundation of market segmentation.

We then turn to the regulatory aspect of our problem assuming for simplicity no exhaustion of rights. We show that the social welfare under the Ramsey price[4], the benchmark, is approximately 30% higher than the usual welfare measures in the third-degree price discrimination literature (i.e., uniform pricing and "complete" price discrimination). However, under this Ramsey price, the monopolist's profit is equal to zero, so that the rate of return of the underlying R & D investment is negative. Consequently, one must find regulatory policies (that may restrict market segmentation and/or pricing policies) that aim to yield a social welfare closer to the benchmark, but subject to the constraint that profits are "reasonable", e.g., higher than the non discriminatory case. In their well known paper, Malueg and Schwartz (1994) have considered a "mixed system" in which the monopolist has to segment its set of markets. They analyzed the case in which the market segmentation

[3]From the treaty of Rome, the European Union is considered as a single market.

[4]The Ramsey price maximizes the social welfare.

is regulated but not the prices, and show that their mixed system is a Pareto improvement over the uniform pricing case. One of the main result of this paper is to show that, under the optimal market segmentation, when the monopolist's pricing policy must satisfy an average-price constraint, as in Armstrong and Vickers (1991), the social welfare is an aggregate Pareto improvement over the no discrimination case, but also over the Malueg and Schwartz's (1994) "mixed system". With three prices, under the above regulatory policy, we show that, compared to the no discrimination situation, the profit increases by around 6% and the aggregate consumers surplus by 25%. From a dynamic point of view, the patented monopolist may have an incentive to cheat. However, by announcing a stochastic audit, and the shut down of the firm if it is found that the regulatory policy has not been followed, the regulator is able to eradicate this mis-behavior even when the annual probability of being auditing is a very low, typically less than 1%.

The next section of this paper, devoted to the presentation of the model and the results, is organized as follows. In the first subsection, we present the assumptions and the required definitions. In the second and third subsections, we solve the profit maximization problem of the monopolist under two different regimes; no exhaustion and international exhaustion. In the last subsection, two regulatory policies that may yield a social welfare closer to the benchmark are analyzed. Finally, we briefly conclude.

2 A model of incomplete third-degree price discrimination

2.1 Assumptions, definitions and discussion

Let $q(\omega, P)$ be the demand function of a given customer $\omega \in \Omega$ where $\Omega = [0, \beta]$ is a compact subset of $\mathbb{R}^+$. We consider the case of an *uniform* measure[5] m and of the following set of demand functions.

$$q(\omega, P) = \begin{cases} \omega - P & P \leq \omega \\ & \\ 0 & P > \omega \end{cases} \qquad \forall \omega \in \Omega \tag{1}$$

The parameter ω may be interpreted of the fraction of the income of customer ω devoted to the consumption of the good produced by the monopolist. Equation (1) defines thus the "linear parallel demand", as opposed to the "linear rotating demand" considered in Malueg and Schwartz (1994) (and more recently in Szymanski and Valletti 2006) in which $q(\omega, P) = 1 - \frac{1}{\omega}P$, where ω is uniformly distributed in $\Omega =]1 - x, 1 + x[$ and where the parameter $x \in [0, 1]$ measures the "dispersion" of the continuum. We consider in what follows a monopolist that produces an "intellectual property good" (e.g., pharmaceuticals, computer software...) which is protected by a patent or trademark. While the conception of this good has required an important R & D investment, we focus uniquely on its commercialization process assuming that the research has been successful. As noted by Ganslandt and Maskus (2007) among others, since the essential of the cost of this kind of products is *fixed*, to simplify matters, as e.g. Valletti and Szymanski (2006) or Malueg and Schwartz (1994), we shall

[5]To be precise, $(\Omega, \mathcal{F}, m)$ is our underlying measured space of agents, where $\mathcal{F}$ is the σ-algebra (i.e., the set of subsets that are stable by complementation and by countable union and or intersection) generated by the topology of Ω and where $m : \mathcal{F} \to \mathbb{R}^+$ is the Lebesgue measure, i.e., the σ-additive set function such that for any interval $]a, b[$, $m(]a, b[) = b - a$. Expressed in "differential" terms, $m(d\omega) = d\omega$ so that for any continuous function f, $\int_\Omega f(\omega) m(d\omega) = \int_\Omega f(\omega) d\omega$.

assume that the marginal cost of production C_m is identically equal to zero. For each $\omega \in \Omega$, the profit function $\Pi(\omega, P) = P(\omega - P)$ so that the monopoly price is equal to $P^*(\omega) = \frac{\omega}{2}$.

Definition 1 *We say that the price discrimination problem is complete if the monopolist can charge the monopoly price* $P^*(\omega) = \frac{\omega}{2}$, $\forall\, \omega \in \Omega$. *We say that it is incomplete otherwise.*

Note that the definition of incomplete price discrimination deals with linear pricing and it thus not related to the imperfect price discrimination considered in Chiang and Spatt (1982) as they are concerned with non linear pricing. However, the notion of imperfect price discrimination used in Liu and Serfes (2005) is actually very similar to ours because, in their framework, given k sub-intervals that forms a partition of $[0, \Delta]$ (where $\Delta > 0$), the firm can charge a different price to each of the k groups. In their model, as in ours, $k = 1$ is the no price discrimination situation whereas $k = \infty$ is the perfect (complete for us) price discrimination. Of course, in practice, only a finite number k of prices makes sense. The realistic and interesting cases are thus those in which the price discrimination is incomplete: the patented monopolist must thus divide optimally the set of customers Ω in k disjoint segments and charge the optimal discriminatory price to each segment. The integer k is fixed by the regulator and its choice will be discussed later. This segmentation implicitly assumes that the monopolist can *perfectly isolate* the k segments so that there is no parallel trade problem.

Definition 2 *We say that there is a system of parallel trade if the price differential is arbitraged by an intermediary who sell the product from the low price segment to the high price segment without the authorization of the owner of the intellectual property right.*

When a given consumer "buy low and sell high", this is clearly a parallel trade but not a system of parallel trade because the consumer acts on *her own behalf*, i.e., not to do business. The recent iPhone provides a good example of this. A consumer has typically a strong rebate on the price of the iPhone if she commits to stay with the same carrier for 12 (or 24) months. The price of the new iPhone is around 700 euros with no commitment although it is around 150 euros with 12 (or 24) months contract. Thus, each year, many consumers can have a new iPhone for about 150 euros while they can sell the used one on eBay for a higher price, e.g. 300 euros. Although such parallel resale may not be negligible[6], they should not impact too much the market segmentation policy of Apple as this is not a system of parallel trade. On the contrary, when a system of parallel trade exists (e.g., parallel imports), this may dramatically impacts the pricing strategies of a firm. Valletti and Szymanski (2006) report that 20% of the (branded) pharmaceuticals in U.K was sold via a system of parallel trade. Arfwedson (2004) reports that within the European Union, parallel import of drugs are estimated to represent $3.3 billion in 2001.

From a international monopolist point of view, the legality of parallel import (PI) critically depends on the *exhaustion of right's doctrine*[7]. Roughly speaking, this doctrine means that once the good has been placed on a market segment (nation, region, world), the monopolist that holds

[6] See however Peter Burrow, "Inside the iPhone Gray Market", BusinessWeek, February 12, 2008 in which the author relates that 800.000 to 1 million of iPhones bypass Apples's restriction.

[7] See e.g., Arfwedson (2004), Ganslandt and Maskus (2007), Szymanski and Valleltti (2005), Szymanski and Valleltti (2006). In USA, a similar doctrine, known as "first sale doctrine" is applied.

the IPR (intellectual property right) can not restrict other sales inside the segment. For example, the European Union has adopted the "Community Exhaustion" policy, which means that the holders'exclusive rights are exhausted (or extinct) once the good has been placed in the European Union because the EU is legally a single market. However, parallel imports from *outside* the European Union is prohibited. To borrow the example of Szymanski and Valletti (2006), under the Community Exhaustion, a U.K exporter can not prevent the resale of a given (patented) drug product first sold in France back into the U.K, but can prohibit the re-entry of the drug product designed to be sold outside the European Community (e.g., in Africa). Actually, the Agreement on Trade-Related Aspects of Intellectual Property Rights (TRIPS) gives WTO members the freedom to design their own exhaustion doctrine although the European Court of Justice considers a regional notion of exhaustion[8]. There exists three regimes of doctrine of exhaustion; national, regional, and international. Under national exhaustion, the monopolist can (in principle) charge a high price in a given country and a low price in another one without having to worry about PI. Under regional exhaustion, the IPR owner can only block PI from countries outside the region but not inside. Finally, under the international exhaustion, the right holder of the IPR cannot prevent at all PI. We shall consider two extremes situations; no exhaustion and international exhaustion.

1. Under no exhaustion, PI are not allowed so that the monopolist can freely segment its set of markets.

2. Under international exhaustion, PI can not be banned so that the monopolist must find strategies to counter the PI problem.

Throughout this article, we assume complete (or full) information so that both the monopolist and the regulator knows the set of demand functions given by equation (1) and Ω.

2.2 Profit maximization under no exhaustion when the monopolist is regulated

We consider a monopolist who can charge at most k different prices, and thus forms k different segments. The regulatory constraint comes with the legal enforcement of the patent due to a no exhaustion doctrine. Thus, the monopolist can freely segment the set of markets in k disjoint groups $A_1, ...A_k$ and charge respectively the vector of price $\mathbf{P} = (P_1.....P_k)$, with $P_i < P_{i+1}$ for $i = 1, ...k$; P_i is the price charged for the segment A_i, which is an interval. Thus, a market segmentation is simply a collection of interval A_i that forms a partition of Ω, i.e., such that $\bigcup_{i=0}^{k} A_i = \Omega$, where A_0 is the subset (possibly empty) of customers which are not served under the vector $\mathbf{P}$. Following Wedel and Kamakura (2000), one may define a segmentation basis as a set of variables or characteristics used to assign customers (or potential customers) to *homogeneous groups*. In general, due the multidimensional aspect of the characteristics of each customer (i.e., age, income, sex, country...), market segmentation is a rather complicated problem because it requires the use of clustering methods. In our model, things are much more simple because the market segmentation is based on an unidimensional characteristic, the income $\omega \in [0, 1]$.

[8]See e.g., section 2.2 in Atik and Lidgard 2006.

Lemma 1 *Given equation (1), the set of "interesting" market segmentation reduces to the choice of numbers* $x_1, ...x_k$ *such that* $0 < x_1 < x_2 < ... < x_k < \beta$, *where* x_i *and* x_{i+1} *forms the extremities of the market segment* A_i, *for* $i = 1, 2...k$.

It has been assumed that the monopolist both chooses the market segments $A_1, ...A_k$ and the prices $\mathbf{P} = (P_1.....P_k)$. To prove lemma 1, assume for a moment that decisions are *decentralized* in the following sense. The monopolist is now the headquarters whose job is to choose the price $\mathbf{P} = (P_1.....P_k)$, with $P_i < P_{i+1}$ for $i = 1, ...k$, and Ω is the set of subsidiaries; each subsidiary $\omega \in \Omega$ is in a monopoly situation and faces a linear demand function $q(\omega, P) = \omega - P$. Now, for a given vector $\mathbf{P}$, each subsidiary ω is *free* to choose one price P_i of $\mathbf{P} = (P_1.....P_k)$. Each subsidiary $\omega \in \Omega$ chooses indeed the price P_i that maximizes its own profit function $\Pi(\omega, P) = P(\omega - P)$. Let A_i be the subset of subsidiaries that choose the price P_i. Note that A_i may be empty for some i but not for all as each subsidiary must choose a price. It is rather natural to assume that if, for a given ω, $\Pi(\omega, P_i) = 0$ for all $i = 1...k$, then, ω chooses the lowest price, i.e., P_1. The set of subsidiaries that chooses the price P_1 but for which $q(\omega, P_1) = 0$ forms the group A_0. As a tie breaking rule, if ω is indifferent between two prices, we assume that she chooses the lowest one. We are now ready to prove lemma 1

Proof. It suffices now to show that if two subsidiaries $\underline{\omega}$ and $\bar{\omega}$, with $\bar{\omega} > \underline{\omega}$ choose the price P_i, then, for all $\omega \in]\underline{\omega}; \bar{\omega}[$, the subsidiary ω also chooses the price P_i. Assume that subsidiaries $\underline{\omega}$ and $\bar{\omega}$ choose the price P_i. We thus have to show that, for all $\alpha \in]0, 1[$, the subsidiary $\alpha\bar{\omega} + (1 - \alpha)\underline{\omega}$ also chooses the price P_i. The choices of $\bar{\omega}$ and $\underline{\omega}$ imply that $\Pi(\underline{\omega}, P_i) > \Pi(\underline{\omega}, P_{i-1})$ and that $\Pi(\bar{\omega}, P_i) > \Pi(\bar{\omega}, P_{i+1})$, which is equivalent to the two inequalities; $\underline{\omega} - P_i > P_{i-1}$ and $\bar{\omega} - P_i < P_{i+1}$. Consequently, it is easy to see that for all $\alpha \in]0, 1[$, $\alpha\bar{\omega} + (1-\alpha)\underline{\omega} - P_i > P_{i-1}$ and $\alpha\bar{\omega} + (1-\alpha)\underline{\omega} - P_i < P_{i+1}$ for all $i = 1...k$. It is thus never interesting, from a total profit point of view, to consider a market segmentation in which say $\underline{\omega}$ and $\bar{\omega}$ would charge the price P_i although a subset of $]\underline{\omega}; \bar{\omega}[$ would charge a different price. The set of interesting market segmentation, from a total profit point of view, reduces thus to the numbers $x_1, x_2...x_k$, the extremities of the market segments $\square$

Lemma 1 shows that in our framework, one can restrict the analysis to the case in which the market segments are "consecutive". It is important to point out that this property depends on the fact that demand functions have the same functional form. Consider the case in which $\Omega = \{1, 2...n\}$, where the demand function of each market segment is given by $q(\omega_j, P) = (\omega_j - P)^{\alpha_j}$, with $\omega_{j+1} > \omega_j$ and $\alpha_j > 0$ for all $j \in \Omega$. It can be shown that if $\alpha_j = \alpha$ for all j, then, market segments[9] are consecutive. However, if α_j may be higher or lower than one, i.e., if demand functions may be concave or convex, then, the market segments need not be consecutive.

Pricing with a fixed segmentation

Before we solve the full problem, let us consider the simpler case in which the various segments $A_i =]x_i, x_{i+1}]$ for $i = 1, ...k$ are *fixed.* Let P_i be the price charged to the segment A_i. There are thus two cases; either $P_i \leq x_i$ or $P_i \in]x_i; x_{i+1}]$ because it is never optimal to charge a price $P_i > x_{i+1}$. The aggregate demand of the fixed segment A_i denoted $Q(x_i, x_{i+1}, P_i) \equiv Q_i(P_i)$ is equal to

[9]See Braouezec (2009) for a proof and definitions in a model with a finite set of markets where decisions are decentralized.

$$Q_i(P_i) = \begin{cases} \int_{x_i}^{x_{i+1}} (\omega - P_i) d\omega = (x_{i+1} - x_i) \left(\frac{x_{i+1} + x_i}{2} - P_i \right) & \text{if } P_i \le x_i \\ \int_{P_i}^{x_{i+1}} (\omega - P_i) d\omega = \frac{1}{2}(x_{i+1} - P_i)^2 & \text{if } P_i \in]x_i; x_{i+1}] \end{cases} \tag{2}$$

Note that when $P_i \le x_i$, the demand function $Q_i(P_i)$ is a *linear* function of the price P_i while when $P_i \in]x_i; x_{i+1}]$, it is a *convex* function of the price. Because there are no production costs, the total profit of the segment A_i is equal to

$$\Pi_i(P_i) = \begin{cases} P_i(x_{i+1} - x_i) \left(\frac{x_{i+1} + x_i}{2} - P_i \right) & \text{if } P_i \le x_i \\ \frac{P_i}{2}(x_{i+1} - P_i)^2 & \text{if } P_i \in]x_i; x_{i+1}] \end{cases} \tag{3}$$

Let P_i^* be the solution of $\frac{d\Pi_i(P_i^*)}{dP_i} = 0$. It is easy to show that $P_i^* = \frac{x_{i+1}}{3}$ if $P_i^* \in]x_i; x_{i+1}]$ and that $P_i^* = \frac{x_{i+1}+x_i}{4}$ if $P_i^* \le x_i$. It thus follows that

$$P_i^* = \begin{cases} \frac{x_{i+1}}{3} & \text{if } \; x_{i+1} > 3x_i \\ \frac{x_{i+1} + x_i}{4} & \text{if } \; x_{i+1} \le 3x_i \end{cases} \tag{4}$$

In both cases, the second order conditions are satisfied. Thus, when the segmentation of the set of markets Ω is fixed, the problem reduces to k independent pricing problems.

Optimal profit policy: definition and characterization

As the market segmentation is not fixed but rather part of the optimization problem, the monopolist must find *simultaneously* the optimal segmentation of the set of markets in k groups, and the optimal price to be charged to each group. Let

$$A_0 = [0, x_1] \tag{5}$$
$$A_i =]x_i, x_{i+1}] \quad \text{for } i = 1, 2...k \tag{6}$$
$$\text{with} \quad x_{k+1} = \beta \tag{7}$$

be a segmentation of the set of markets Ω in k (disjoint) segments that forms a partition of Ω, and let P_i be the price charged to the segment A_i. From the previous paragraph, the case in which $P_i > x_i$ is unlikely to be optimal as it implies that $x_{i+1} > 3x_i$ for $i = 1, 2....k$, so that $x_k = 3^{k-1}x_1$, with $x_k < \beta$. We shall thus directly focus on the case in which $P_i \le x_i$.

Definition 3 *Let*

$$\mathcal{X} = \{\mathbf{x} \in \Omega^k : x_1 \le x_2 \le ... \le x_k \le \beta\} \tag{8}$$
$$\mathcal{P}_{\mathbf{x}} = \{\mathbf{P} \in \mathbb{R}^{k+} : P_i \le x_i, \forall i = 1, 2...k\} \tag{9}$$
$$\mathcal{U} = \mathcal{X} \times \mathcal{P}_{\mathbf{x}} \tag{10}$$

where $\mathcal{X}$ is the set of segmentation policies, $\mathcal{P}_{\mathbf{x}}$ the set of pricing policies given $\mathbf{x}$ and $\mathcal{U}$ the set of profit policies.

Note that although the set of pricing policies $\mathcal{P}_{\mathbf{x}}$ takes as given a segmentation policy, from a mathematical point of view, we shall determine *simultaneously* the optimal segmentation policy and the optimal pricing policy. Let $\mathbf{u} = (\mathbf{x}, \mathbf{P}) \equiv (x_1, ...x_k; P_1, ...P_k)$ be a given profit policy. Without further information, $\mathbf{x}$ and $\mathbf{P}$ are of dimension k. For a given profit policy $\mathbf{u} \in \mathcal{U}$, the aggregate profit function is equal to

$$\Pi(\mathbf{u}) = \sum_{i=1}^{k} \Pi_i(x_i, x_{i+1}, P_i) \tag{11}$$

where the profit function of the segment A_i (see equation (3)) is equal to

$$\Pi_i(x_i, x_{i+1}, P_i) \equiv \Pi_i(P_i) = P_i(x_{i+1} - x_i)\left(\frac{x_{i+1} + x_i}{2} - P_i\right) \tag{12}$$

From a mathematical point of view, this optimization problem turns out to be related to the article of Moorthy (1984) (especially section 8) and Oren et al. (1984). However, from an economic point of view, we depart from the two mentioned articles in that we focus on a linear pricing/homogeneous good model. Let $\mathbf{u}^* = \arg\max_{\mathbf{u} \in \mathcal{U}} \Pi(\mathbf{u})$ be the optimal profit policy.

Proposition 1 *Let $\beta = 1$. The optimal profit policy $\mathbf{u}^* \in \mathcal{U}$ is given by*

1. $x_i^* = \dfrac{2i-1}{2k+1}$ *for $i = 1...k$*
2. $P_i^* = \dfrac{i}{2k+1}$ *for $i = 1...k$, where P_i^* is the price charged to the segment $A_i^* =]x_i^*; x_{i+1}^*]$.*

Proof. See the appendix

Unless otherwise specified, we now assume that $\beta = 1$. Proposition 1 says that it is optimal for the monopolist to charge the price $P_i^* = \frac{i}{2k+1}$ to the segment $A_i^* = \left]\frac{2i-1}{2k+1}; \frac{2i+1}{2k+1}\right]$ for $i = 1, ...k$. In the particular case in which $k = 1$, $x^* = P^* = \frac{1}{3}$; it is thus optimal to serve the segment $]\frac{1}{3}; 1]$ so that only 66% of the markets are served. When $k \geq 2$, it is optimal for the monopolist to slice the set $[0, 1]$ in k segments of *identical length* equal to $\frac{2}{2k+1}$. As one can expect, when k increases, the length of each segment decreases, which means that the market segmentation becomes finer, and the difference between the highest and the lowest price tends to $\frac{1}{2}$. Note importantly that when we move from k to $k+1$, this *both changes* the optimal segmentation and the optimal pricing policy. As an example, when $k = 2$, the two optimal segments of markets are $]\frac{1}{5}; \frac{3}{5}]$ and $]\frac{3}{5}; 1]$ and their associated optimal prices are respectively $\frac{1}{5}$ and $\frac{2}{5}$. As the intuition suggests, one price is higher than the uniform monopoly price and one price is lower. When $k = 3$, the three optimal segments are now given by $]\frac{1}{7}; \frac{3}{7}]$, $]\frac{3}{7}; \frac{5}{7}]$, $]\frac{5}{7}; 1]$ and their three associated optimal prices are $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$. Note that only the highest price is higher than the uniform monopoly price P^*. To understand proposition 1 with standard economic theory, recall that from basic monopoly theory, we know that the optimal price P^* of a given market must be such that

$$\frac{P^* - C_m}{P^*} = \frac{1}{|\mathcal{E}(P^*)|} \tag{13}$$

where $\mathcal{E}(P^*)$ is the elasticity of demand evaluated at P^*, and C_m is the constant marginal cost. The left hand side of equation (13) is generally called the *price-cost margin* (or Lerner index) and is implied by the first order condition of the profit maximization problem. When $C_m = 0$, the price-cost margin reduces to one so that $|\mathcal{E}(P^*)| = 1$. Returning to our model in which the marginal cost is zero, it is easy to show that the price cost margin condition is satisfied because

$$|\mathcal{E}(P_i^*)| = \left|\frac{dQ_i(P_i^*)}{dP_i}\right| \frac{P_i^*}{Q_i(P_i^*)} = 1 \qquad i = 1, 2...k \tag{14}$$

so that one can safely conclude that $(\mathbf{x}^*, \mathbf{P}^*) \in \mathcal{U}$ is the optimal profit policy. From a mathematical point of view, the difficult part is to show that the Hessian matrix of the profit function, whose dimension is $(2k \times 2k)$, is negative definite for *any* value of $k \in \mathbb{N}^*$. The total profit under the optimal profit policy, denoted $\Pi(\mathbf{x}^*, \mathbf{P}^*) \equiv \Pi_k^*$, is equal to

$$\Pi_k^* = \sum_{i=1}^{k} \Pi_i(P_i^*) = \sum_{i=1}^{k} \frac{2i^2}{(2k+1)^3} = \frac{k(1+k)}{3(2k+1)^2} \tag{15}$$

When $k = 1$, we know that $P^* = x^* = \frac{1}{3}$, so that the profit is equal to $\Pi_1^* = \frac{2}{27}$. Consider now the case in which the monopolist can completely price discriminate. As $P^*(\omega) = \frac{\omega}{2}$, $\Pi(P^*(\omega)) = \frac{\omega^2}{4}$ for each $\omega \in \Omega$, the total profit under the complete third-degree price discrimination is equal to

$$\Pi^*_{\text{cplte}} = \int_0^1 \Pi(P^*(\omega))d\omega = \frac{1}{12} \tag{16}$$

From equation (15), it is easy to see that when $k \to \infty$, $\Pi_k^* \to \Pi^*_{\text{cplte}}$, as one can expect. By moving from a uniform pricing to a complete third-degree price discrimination situation, the monopolist increases its profit by 12.5%. Note interestingly that $\frac{\Pi_3^*}{\Pi^*_{\text{cplte}}} = 0.9796$, which means that the monopolist is able to capture 98% of the maximum total profit with only 3 prices! In a recent article, Chu Leslie and Sorensen (2009) get a similar result in a bundle pricing problem. They find that with a rather simple pricing rule, the monopolist is able to generate 99% of the maximum total profit. Using part 2 of proposition 1, let

$$f(k) = 1 - \frac{1}{2k+1} \tag{17}$$

be the proportion of markets that are served under the optimal profit policy $(\mathbf{x}^*, \mathbf{P}^*)$.

Corollary 1 *Assuming that $k \in \mathbb{R}^+$, the functions $f(k)$ and Π_k^* are increasing and concave functions of k.*

Proof. It is easy to show that $f'(k) > 0$ and $f''(k) < 0$ and that $\Pi_k'^* > 0$ and $\Pi_k''^* < 0$ for all $k \in \mathbb{R}^+$ $\square$

It is only when the number of prices k goes to the infinity that all the markets will be served.

Remark 1. Consider once again the "decentralized decisions" version of our model (see the proof of lemma 1) in which the monopolist chooses the price $\mathbf{P}$ and each subsidiary ω chooses one price P_i of $\mathbf{P}$. It is not difficult to show that if the headquarters proposes the vector of optimal prices $\mathbf{P}^*$ given in proposition 1, then, the resulting endogenous segments $A_1, A_2 ... A_k$ are exactly those given in proposition 1. It is also very easy to show that under the optimal profit policy, each subsidiary x_i^*, for $i = 1, 2...k$ is actually *indifferent* between the prices P_i^* and P_{i-1}^*. This "indifference property" turns out to be used directly by Oren et al (1984) as an optimality condition to analyze the solution of their optimization problem. As we shall now see, the so called weak-strong partition, frequently used in the third-degree price discrimination literature (e.g., Robinson 1969, Schmalensee 1981, Shih et al 1988), does not satisfy this indifference property.

On the Robinson-Schmalensee weak-strong markets partition

In the third-degree price discrimination literature, after Joan Robinson (1969) and Schmalensee (1981) it is usual to call weak-strong partition, denoted $W - S$, the segmentation of set of markets in two groups defined as follows.

$$W = \{\omega \in \Omega : P^*(\omega) \leq P^*\} \qquad S = \{\omega \in \Omega : P^*(\omega) > P^*\} \tag{18}$$

The group W (S) is the set of markets for which the optimal price of the market ω is lower (higher) than the uniform monopoly price. Historically, Joan Robinson (1969) analyzed the case in which there are only two markets while Schmalensee (1981) extended it to the finite case. After Schmalensee (1981), it became common in the third-degree price discrimination literature to use the weak-strong partition for the social welfare analysis (see e.g., Phlips 1988 for a survey). It is indeed assumed that it is the *optimal segmentation* of the set of markets in two groups. To understand its origin, let us follow the reasoning of Joan Robinson (1969).

> *"The profitability of the monopoly will depend upon the manner in which the market is broken up. (...) It is therefore necessary to inquire in what way a monopolist would divide his market if he were perfectly free to do so in the manner most profitable to himself. Let us suppose that the monopolist is in possession of some device which enables him to separate buyers from each other at will, and let us suppose that he is at first charging a single monopoly price throughout the market, and then proceeds to divide it up by successive stages. (...) if the elasticities of demand are different, he will first divide all individual buyers into two classes such that the highest elasticity of demand in the one class is less than the least elasticity of demand in the other class. To the first class, he will raise the price, and to the second, he will lower it"* p 185-186.

Assume now that the monopolist is allowed to charge two different prices. When there are two markets, the weak strong partition is clearly the optimal one because it is the unique one. However, when there are more than two markets, the optimality of the weak strong partition is not so clear... Since $P^* = \frac{1}{3}$ and $P^*(\omega) = \frac{\omega}{2}$, the Robinson-Schmalensee weak-strong partition (and its associated optimal prices) is given by

$$W = \left[\frac{2}{9}; \frac{2}{3}\right], \; P_W^* = \frac{2}{9} \qquad S = \left]\frac{2}{3}; 1\right], \; P_S^* = \frac{5}{12} \tag{19}$$

From remark 1, it should be immediate that the profit policy given by $[(W, S); (P_W^*; P_S^*)]$ can not be the optimal one because the subsidiary $\omega = \frac{2}{3}$ is *not indifferent* between the prices $\frac{2}{9}$ and $\frac{5}{12}$. Actually, if she had the choice, she would choose the price $\frac{5}{12}$ and not $\frac{2}{9}$. Thus, if we leave the choice of the price to each subsidiary $\omega \in [0, 1]$, given the prices $\frac{2}{9}$ and $\frac{5}{12}$, a fraction of the segment W will choose the price $\frac{5}{12}$. But then, the market segments change and the profit increases, which contradicts the optimality of the profit policy[10] $[(W, S); (P_W^*; P_S^*)]$. By proposition 1, the optimal segmentation policy is given by $A_1^* = \left]\frac{1}{5}; \frac{3}{5}\right]; A_2^* = \left]\frac{3}{5}; 1\right]$ and the optimal pricing policy is given by $P_1^* = \frac{1}{5}; P_2^* = \frac{2}{5}$. Under the optimal profit policy when $k = 2$, the total profit is equal to 0.08 while under the weak-strong partition (and its associated optimal prices), the total profit is equal to 0.0798. The loss of profit due to the suboptimal profit policy can thus be considered as negligible.

2.3 Profit maximization under international exhaustion when the monopolist is not regulated

Firms frequently employ simple pricing strategy (Chu, Leslie and Sorensen 2009) so that they typically discriminate *less* than what they could do. For example, Apple, through iTunes store, charges a uniform price of \$0.99 for each song although it could price discriminate (Shiller and Waldfogel 2009). Why ? If we assume that firms' decisions are rational, this means that the marginal cost associated to a finer price discrimination may be higher than its marginal gain. Ideally, unregulated firms would like to achieve a first-degree price discrimination in which each consumer would pay a price equal to her willingness to pay so that all the surplus would be absorbed by the monopolist. However, the cost of such a segmentation would be prohibitively costly because the efficiency of the price discrimination critically depends on the monopolist's ability to preserve each market segment as a distinct market, or, to put it differently, to remove PI. We argue that the cost of maintaining each segment as a *separate market* may explain why firms choose a few number of prices. We consider the case of an unregulated monopolist who is free to choose the number k of prices (or segments) but this no-regulation situation comes under a legal environment in which an international doctrine of exhaustion prevails, i.e., PI cannot be legally banned. Consequently, the monopolist that wishes to price discriminate must develop its own strategy to count the PI problem, as long as it is optimal to do so.

Non pricing strategies to count parallel imports

As noted by Cavusgil and Sikora (1998) and more recently by Ahmadi and Yang (2000) and Danzon and Towse (2003), manufacturers can use in practice various *non pricing strategies* to mitigate parallel imports. Following Ahmadi and Yang (2000), they fall into three categories

[10]Note that in a model with a finite number of markets, the weak strong partition need not be the optimal one. In Braouezec (2009), we show that this partition may indeed fail to satisfy a necessary condition to be the optimal partition. We also give conditions under which this weak strong partition is the unique optimal partition.

- Monitoring
- Differentiating products
- Differentiating services

Monitoring is the possibility for a manufacturer to control its dealers' sales through product registration. For say the automobiles industry, a serial number is associated to each car that informs about e.g., its destination. Product differentiation is another possibility that may be used and concerns the differentiation of the product, which may be minor (e.g., only the packaging) or major (product itself). Cavusgil and Sikora (1998) provide the example of a manufacturer of tractors that alters the functional characteristics of the tractor as a function of its country's destination; tractors designed to be sold in South America for a low price may not perform very well in extremely cold weather so that a Canadian contractor may prefer to use the authorized distribution channel instead of the gray market. As noted by the two authors, this non pricing strategy is not difficult to implement but it may clearly be quite costly. Eventually, a firm may leave invariant the product's characteristics but add services such as technical support and maintenance, but only to the authorized product. Although the following example is not a gray market example, it shows that support and maintenance may make a big difference. Consider the case of Matlab and Scilab[11], which are two softwares of scientific computing that are rather very similar. Matlab is costly while Scilab is free. However, many institutions (e.g., university, banks...) still prefer Matlab[12] not only because it offers specialized toolboxes (e.g., wavelet, statistics, finance...) but also, and perhaps essentially, because of technical support and maintenance. If a Matlab user has a problem, he or she can call immediately the technical support to solve it. Such a service is of course not available with Scilab. For the specific case of drugs, Danzon (1997), and Danzon and Towse (2003) have suggested that price discrimination could be implemented through confidential rebates contracts.

On the optimal number of segments

As suggested by Cavusgil and Sikora (1998), the cost of mitigation of parallel imports (PI) should be part of the overall optimization problem. In Ahmadi and Yang (2000), they explicitly consider the reduction of PI as a function of a stylized decision variable $q \in [0, 1]$, where $q = 0$ means no impact on PI while $q = 1$ means complete elimination of PI. In their model, Ahmadi and Yang (2000) assume that the cost function is quadratic in q.

We shall here consider a different model in which, when the monopolist supports a cost C, the market segment is supposed to be perfectly isolated. Thus, when the monopolist decides to create k segments, the total cost associated to the complete elimination of PI is just $C\,k$. Let

$$TC(k) = k\,C\,\mathbf{1}_{k\geq 2} \tag{20}$$

be this total cost function where $\mathbf{1}_{k\geq 2}$ is an indicator function. We consider here a "reduced form" model in which C is simply a percentage x of $\Pi_1^* = \frac{2}{27}$, the total profit under uniform pricing. Thus,

[11] See their web site www.scilab.org/

[12] Actually, the figures included in this article are realized with Matlab...

for a given value of $k \in \mathbb{N}^*$, the total cost is equal to $\frac{2}{27}\, x\, k$. The full profit function to be maximized with respect to k is thus given by

$$\Pi(\mathbf{x}^*, \mathbf{P}^*, C) = \frac{k(1+k)}{3(2k+1)^2} - x\,\frac{2}{27}\,k\,\mathbf{1}_{k\geq 2} \tag{21}$$

The maximization of equation (21), for a given x, gives thus the optimal trade-off between the gain associated to a more complete price discrimination and the cost associated to the perfect isolation (or separation) of each market segment. The following table provides the optimal value k, denoted k^*_π as a function of the percentage x.

Optimal number of segments $k^*_\pi(x)$ as a function of x

x	0.25%	0.5%	1%	2%	3%	4%
$k^*_\pi(x)$	5, 6 or 7	4	3	3	3 or 2	1

For example, when $x = 1\%$, $k^*_\pi = 3$, and the total profit is equal to 0.07941. Since the cost of separation $TC(k)$ is linear in k while Π^*_k is concave in k, it is optimal for the monopolist to create only few segments even when the marginal cost C is rather low. When $x > 4\%$, it is not anymore interesting for the monopolist to price discriminate.

2.4 Social welfare

In the rest of this article, we consider the case of a benevolent-fully informed regulator that maximizes the social welfare defined as the sum of consumers surplus and profits. As before, because the monopolist is subject to some regulation, the no exhaustion doctrine prevails so that there is no system of parallel trade. Let

$$W_\mu(P) = \Pi(P) + \mu CS(P) \tag{22}$$

where CS is the aggregate consumers surplus, and μ is a fixed value between 0 and 1 that reflects the weight assigns to consumers'surplus by the regulator. Following Laffont, Rey, Tirole (1998), let us first consider the *Ramsey price*, denoted P^*_R, which is defined as follows

$$P^*_R = \arg\max_{P\in[0,1]} W_\mu(P) \tag{23}$$

It is shown (see e.g., Phlips 1988) that P^*_R is such that

$$P^*_R - C_m = (1-\mu)\frac{1}{|\mathcal{E}(P^*_R)|}P^*_R \tag{24}$$

where, as before, $|\mathcal{E}(P)|$ denote the demand elasticity at price P. Using equation (2) and the definition of the elasticity, it is easy to see that in our model, equation (24) reduces to

$$P^*_R - C_m = (1-\mu)\left(\frac{1-P^*_R}{2}\right) \tag{25}$$

When $\mu = 0$, the maximization of the social welfare is equivalent to the maximization of total profits. Equation (25) reduces in that case to

$$P_R^* = \frac{1 + 2C_m}{3} \tag{26}$$

so that, when $C_m = 0$, the uniform monopoly price is equal to 1/3, as expected.

When $\mu = 1$, the regulator maximizes the sum of profits and consumers surplus. From equation (24), we immediately get that $P_R^* = C_m$, i.e., the Ramsey price is equal to the marginal cost. As $C_m = 0$, it thus follows that[13]

$$P_R^* = 0 \tag{27}$$

so that the social welfare is equal to the total consumers' surplus

$$W_{\text{Rsey}} \equiv W(P_R^*) = CS(P_R^*) = \frac{1}{6} \tag{28}$$

Let us now compute the social welfare as a function of k under the optimal profit policy.

Lemma 2 *Under the optimal profit policy $\mathbf{u}^* \in \mathcal{U}$, the social welfare $W(\mathbf{u}^*) \equiv W_k^*$ as a function of $k \geq 1$ is equal to*

$$W_k^* = \frac{1}{3}\left[\frac{3}{2}\frac{k(1+k)}{(2k+1)^2} + \frac{2k}{2(2k+1)^3}\right] \tag{29}$$

Proof. See the appendix

By considering $k = 1$, we immediately get $W_1^* = \frac{10}{81}$, the social welfare under uniform pricing. Let W_{cplte}^* be the social welfare under complete price discrimination. By taking the limit when $k \to \infty$, it is easy to show that $W_k^* \to W_{\text{cplte}}^* = \frac{1}{8}$, as one may expect. Consequently, $W_{\text{cplte}}^* > W_1^*$. However, we are still very far from the benchmark. Equipped with lemma 2, the next interesting question is whether or not one can find a *finite value of* k, denoted k_W^* when it exists, that maximizes W_k^*.

Proposition 2 *Under the optimal profit policy $\mathbf{u}^* \in \mathcal{U}$, $k_W^* = 3$, i.e., the social welfare is maximized for three prices and the resulting social welfare is equal to $W_3^* = \frac{129}{1029} \approx 0.12536$.*

Proof. Assume that $k \in \mathbb{R}^+$. It is easy to show that the sign of $\frac{dW_k^*}{dk}$ is given by the function $g(k) = \frac{1}{2} - \frac{1}{(2k+1)^2}\left(2k(1+k) - \frac{1}{3} + \frac{2k}{2k+1}\right)$. It is easy to show that $g(1) >$, $g(2) > 0$, $g(3) < 0$ and $g(k) < 0$ for $k > 3$. By lemma 1, one get immediately the social welfare with three prices $\square$

Figure (1) shows the evolution of the social welfare as a function of k. Although $W_3^* > W_{\text{cplte}}^*$, the difference remains negligible because there is only a 0.3% increase in the social welfare. Actually, the essential economic meaning of proposition 2 may be as follows: *to achieve the welfare effects of price discrimination, a few number of prices is enough.* Since $k_W^* = 3$, it is optimal to open 86% of the set of markets.

[13] A simple way to get this result is as follows. Consider a given market ω. The social welfare is equal $W(\omega, P) = \frac{(\omega-P)^2}{2} + P(\omega - P)$ so that $\frac{dW(\omega,P)}{dP} = -P < 0$. Thus, the price that maximizes the social welfare is equal to zero.

It is important to note that proposition 2 is not in contradiction with the classical results of Varian (1985) on the welfare effect of third-degree price discrimination. In a fairly general model, with a given (finite) number k of market segments, Varian (1985) has shown that the change in social welfare ΔW is such that $\Delta W \leq (P^* - C_m)\Delta Q$, where ΔQ is the change in total output that results from the complete price discrimination (i.e., k prices) with uniform pricing as the initial situation. What the Varian's (1985) result says is that for ΔW to be positive, ΔQ must be positive. However, the converse is not true; $\Delta Q > 0$ does not imply that $\Delta W > 0$. Thus, proposition 2 does not contradict the Varian's (1985) result. It is actually an example in which, when $k \geq 3$, ΔQ increases with k whereas ΔW decreases with k.

In the third-degree price discrimination literature, it is standard to compute only W_1^* and W_{cplte}^*, that is, the social welfare under uniform pricing and under complete discrimination. We have shown here than an intermediate value of k turns out to be optimal from a social welfare point although the resulting social welfare is far below the benchmark. However, this benchmark implies zero profits. Since we assume that the conception of the "intellectual good" has required an important R & D investment, no profit is thus equivalent to a (highly) negative rate of return on the underlying investment. Thus, from a regulatory point of view, at least to maintain the incentive to further invest in R & D, profits should be positive, and indeed higher than the uniform price profit. Consequently, the next challenge is to exhibit regulatory policies that should yield a social welfare closer to the benchmark, but compatible with "reasonable" profits.

2.5 Regulations of market segmentation and price discrimination

Although there is a rather huge literature on monopoly regulation, (see e.g., Armstrong and Sappington 2007, Laffont and Tirole 1993, Spulber 1989), only few articles (Armstrong and Vickers 1991, Hausman and Mason 1988, Ireland 1992, Malueg and Schwartz 1994) have been devoted to the analysis of the regulatory welfare effects of third-degree price discrimination. Within our framework, various regulatory policies can be considered since one may both restrict the set of market segmentation and/or the set of prices. For example, Malueg and Schwartz (1994) analyze the impact of regulating the market segmentation, but not the prices, on social welfare. Armstrong and Vickers (1991) consider various average-revenue constraints to regulate a multiproduct monopolist while Ireland (1992) study a specific pricing policy using Varian's (1985) result. However, as far as we know, there are no theoretical results on the joint regulation of market segmentation and prices. This is the subject of this paragraph.

Market segmentation and prices are regulated: the ζ-average-price constraint

Following Armstrong and Vickers (1991), the average-price constraint is defined as follows

$$\sum_{i=1}^{k} w_i(\mathbf{P})\, P_i \leq P^0 \tag{30}$$

where P^0 is intended to be the monopoly price, $\mathbf{P} = (P_1...P_k)$ is the chosen k-vector of prices, and $w_i(\mathbf{P})$ is the ratio defined as the produced quantity of segment i over the total produced quantity. Let us consider this constraint applied to our model when $P^0 = P^*$ and $P_i = P_i^*$, $i = 1, 2...k$, for a fixed value of k. Using equation (2), it is easy to obtain that $w_i^*(\mathbf{P}^*) = \frac{2i}{k(k+1)}$, so that for all $k \geq 2$, $\sum_{i=1}^{k} w_i^* P_i^* = P^* = \frac{1}{3}$. The constraint is indeed binding for any value of $k \in \mathbb{N}^*$. Instead of $P^0 = P^*$, one may consider the case in which $P^0 = \zeta P^*$ for some $\zeta < 1$. Let $\mathcal{P}_{\mathbf{x}^*}$ be the set of prices under $\mathbf{x}^* \in \mathcal{X}$, the optimal market segmentation given in proposition 1. Consider a ζ-average-price regulation defined as follows.

Definition 4 *The vector of prices $\boldsymbol{P} \in \mathcal{P}_{\mathbf{x}^*}$ is said to satisfy the ζ-average-price constraint if, for a given $\zeta \in]0,1[$,*

$$\sum_{i=1}^{k} w_i^*(\boldsymbol{P})\, P_i \leq \zeta P^* \tag{31}$$

where $w_i^*(\boldsymbol{P}) = \dfrac{Q_i(x_i^*, x_{i+1}^*, P_i)}{\sum_{i=1}^{k} Q_i(x_i^*, x_{i+1}^*, P_i)}$.

Equipped with this definition, one may define the set of prices $\mathcal{P}_{\mathbf{x}^*}(\zeta) \subseteq \mathcal{P}_{\mathbf{x}^*}$ as follows

$$\mathcal{P}_{\mathbf{x}^*}(\zeta) = \{\mathbf{P} \in \mathcal{P}_{\mathbf{x}^*} : \ \mathbf{P} \ \text{ satisfies definition 4}\} \tag{32}$$

where ζ is a parameter chosen by the regulator. Of course, the monopolist is free to choose the price $\mathbf{P} \in \mathcal{P}_{\mathbf{x}^*}(\zeta)$ that maximizes its profits. Instead of deriving the optimal prices, we shall consider a simple suboptimal pricing policy which turns out to be a particular case of a pricing scheme proposed by Ireland (1992). Recall that the optimal profit policy (see proposition 1) is denoted by $\mathbf{u}^* = (\mathbf{x}^*, \mathbf{P}^*) \in \mathcal{U}$. We examine the simple pricing policy given by $\mathbf{P}^*(\zeta) = (\zeta\, P_1^*, ...\zeta\, P_k^*)$. As we shall see, under the optimal market segmentation $\mathbf{x}^*$, when $\zeta \geq 0.8$, our simple suboptimal pricing policy is indeed nearly optimal.

Proposition 3 *Assume that the optimal market segmentation $\mathbf{x}^* \in \mathcal{X}$ given in proposition 1 is fixed by the regulator and that the monopolist's pricing policy must satisfy a ζ-average-price constraint. Under the (nearly optimal) pricing policy defined as*

$$\mathbf{P}^*(\zeta) = (\zeta\, P_1^*, ...\zeta\, P_k^*) \tag{33}$$

the social welfare is equal to

$$W_k^*(\zeta) = \frac{1}{3}\left[\frac{\zeta(2-\zeta)k(1+k)}{(2k+1)^2} + \frac{2k + (2-\zeta)^2 k(k+1)(2k+1)}{2(2k+1)^3}\right] \qquad k \in \mathbb{N} \tag{34}$$

and the price given in equation (33) is such that

$$\mathbf{P}^*(\zeta) \in \mathcal{P}_{\mathbf{x}^*}(\zeta) \qquad \forall \zeta \in]0,1[\tag{35}$$

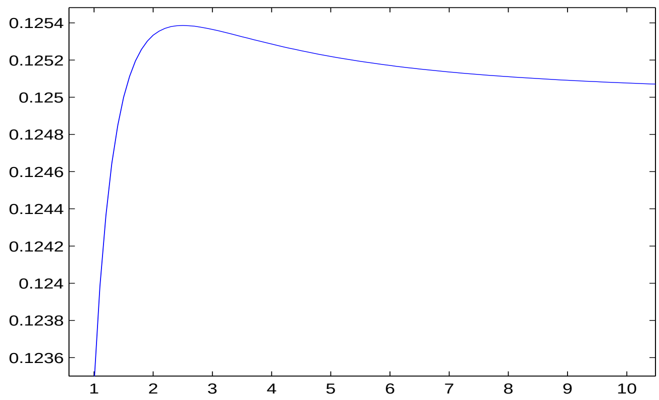

Figure 1: Total welfare as a function of k when $\zeta = 1$

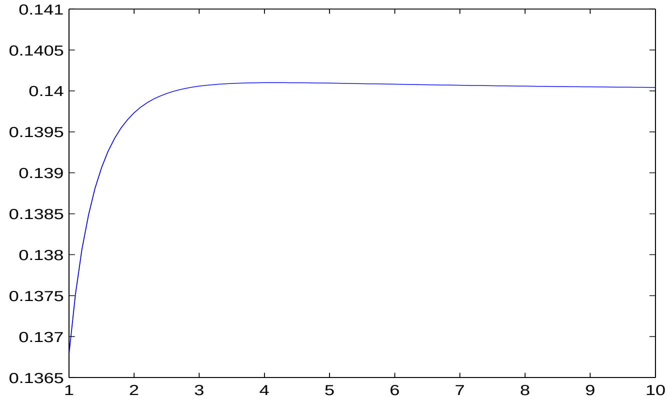

Figure 2: Total welfare as a function of k when $\zeta = 0.8$

Proof. See the appendix.

In a article devoted to the regulation of price discrimination, Ireland (1992) proposed the following pricing formula

$$\mathbf{P} = \Gamma\beta + C_m \tag{36}$$

where C_m is a vector (the marginal cost), Γ is a matrix and β is a vector. It is easy to see that the pricing policy defined in equation (33) can be written as $\mathbf{P}^*(\zeta) = \zeta I\,\mathbf{P}^*$. By taking $C_m = 0$, $\beta = \mathbf{P}^*$, $\Gamma = \zeta I$, where I is the $k \times k$ identity matrix, we obtain a special case of equation (36). Before we discuss proposition 4, let us analyze the pricing error on the aggregate profit. Without loss of generality, we consider the case in which $k = 2$ and $\zeta \in [0.8; 1]$ because the profits must be higher than the uniform pricing profits, i.e., ζ can not be too low. Consider the extreme case in which $\zeta = 0.8$ and let $\mathbf{P}^\diamond(\zeta) = (P_1^\diamond(\zeta); P_2^\diamond(\zeta)) \in \mathcal{P}_{\mathbf{x}^*}(\zeta)$ be the optimal pricing policy. We show in appendix that $P_1^\diamond(0.8) \approx 0.139$ and $P_2^\diamond(0.8) \approx 0.339$ whereas the suboptimal pricing policy gives $P_1^*(0.8) = 0.16$ and $P_2^*(0.8) = 0.32$. Actually, the loss of profit that results from the suboptimal profit policy is around 0.25%, and is thus negligible. It is interesting to note that for a given value of $\zeta < 1$, the total profit under the nearly optimal pricing policy is equal to

$$\Pi(\mathbf{x}^*, \zeta\mathbf{P}^*) = \zeta(2-\zeta)\Pi_k^* \tag{37}$$

and is thus a *non linear* function of ζ. For example, when $\zeta = 0.9$, the monopolist realizes 99% of the total (unregulated) profit. We give in the following table the surplus, the profit and the social welfare under the nearly optimal profit policy for few values of ζ when $k = 3$. As can be seen from figures (1) and (2), more than three prices is not very interesting.

Social welfare under the nearly optimal pricing policy

$k = 3$	Values of ζ	0.75	0.8	0.85	0.9	1
Surplus		0.0667	0.0617	0.0569	0.0523	0.04376
Profit		0.0765	0.0784	0.0798	0.0808	0.08163
Social welfare		0.1432	0.014	0.01367	0.0133	0.012536

From the above table, we suggest to take $\zeta = 0.8$ so that the social welfare is equal to $W_3^*(0.8) = 0.014$. The profit is (approximately) equal to 0.0784 whereas the consumers surplus is (approximately) equal to 0.0617. Compared to the social welfare under uniform pricing, there is a 6% increase in the profit and a 25% increase in the surplus! Our regulatory policy is thus a substantial aggregate Pareto improvement.

On Pareto improvement of price discrimination over uniform pricing

In Hausman and Mason (1988) and Malueg and Schwartz (1994), they explicitly show that price discrimination may yield a Pareto improvement over uniform pricing.

In Hausman and Mason (1988), they show that price discrimination may yield a Pareto improvement when the discriminatory price in each market segment is lower than the uniform price. Let us show that we can get a similar result within our model. When $\zeta = 0.75$, the three discriminatory prices are equal to $\frac{3}{28}$; $\frac{6}{28}$; $\frac{9}{28}$ and are lower than $P^* = \frac{1}{3}$. The profit is equal to 0.0765, which is higher than $\frac{2}{27}$, and the consumers's surplus is equal to 0.0667, which is much higher than $\frac{4}{81} \approx 0.0494$. This is thus a Pareto improvement over uniform pricing.

In Malueg and Schwartz (1994), they consider a form of regulation, called "mixed suystem", in which the market segmentation is regulated but not the discriminatory prices. Let us now study the Malueg and Schwartz's (1994) mixed system within our model. When $k = 1$, we know that $x^* = P^* = \frac{1}{3}$ so that only the segment $[\frac{1}{3}, 1]$ is served. Suppose now that the monopolist is given the opportunity to charge another price to the residual segment $[0, \frac{1}{3}]$. The optimal price to this segment is thus equal to $\frac{1}{9}$. Consquently, if the monopolist is asked to choose two prices to the regulated (i.e., fixed) segments $[\frac{1}{9}, \frac{1}{3}]$ and $[\frac{1}{3}, 1]$, she will charge the price $\frac{1}{3}$ and $\frac{1}{9}$. Let $\mathbf{x}_{ms} = (x_1, x_2, ...x_k)$ but now with $x_1 > x_2 > ...x_k$, where "ms" means "mixed system". From the above discussion, the Malueg and Schwartz's (1994) segmentation is given by

$$\mathbf{x}_{ms} = \left(\frac{1}{3^k}, \frac{1}{3^{k-1}}, ..., \frac{1}{3^2}, \frac{1}{3} \right) \quad (38)$$

where, as before, a given segment is of the form $A_i =]\frac{1}{3^i}; \frac{1}{3^{i-1}}]$ for $i = 1, 2...k$. Let $\mathbf{P}^\star \in \mathcal{P}_{\mathbf{x}_{ms}}$ be the optimal pricing policy. It is clear that

$$\mathbf{P}^\star = \left(P_k^\star = \frac{1}{3^k}, P_{k-1}^\star = \frac{1}{3^{k-1}}, ...\ P_1^\star = \frac{1}{3} \right) \quad (39)$$

where $P_1^\star$ is now the highest price and $P_k^\star$ the lowest. As noted by Malueg and Schwartz (1994), this "k-segmentation" given by equation (38) and its associated optimal prices given by equation (39) is a *Pareto improvement* not only over the uniform pricing, but also over the "$(k-1)$-segmentation" so that profits, consumers surplus and thus the social welfare are increasing functions of k.

Lemma 3 *Assume that the market segmentation $\mathbf{x}_{ms} \in \mathcal{X}$ given by equation (38) is fixed by the regulator. Then, under the optimal pricing policy $\boldsymbol{P}^\star \in \mathcal{P}_{\mathbf{x}_{ms}}$ given by equation (39), the social welfare is equal to $W(\mathbf{x}_{ms}, \boldsymbol{P}^\star) \equiv W_k^\star = \frac{10}{81}\frac{27}{26}(1 - \upsilon_k)$, where $\upsilon_k = \frac{1}{27^{k+1}}$.*

Proof. When $k = 1$, for a given $\beta > 0$, the uniform monopoly price is equal to $\frac{\beta}{3}$ and the total welfare is equal to $\frac{10}{81}\beta^3$. Under the profit policy $(\mathbf{x}_{ms}, \mathbf{P}^\star) \in \mathcal{U}$, when $\beta = 1$, the social welfare is equal to $W_k^\star = \sum_{i=0}^{k} \frac{10}{81} \left(\frac{1}{3} \right)^{3i}$ so that the result follows $\square$

As the social welfare is an increasing function of k, the maximum is reached for $k = \infty$, and $W_\infty^\star = \frac{5}{39}$, where $\Pi_\infty^\star = \frac{1}{13}$ and $CS_\infty^\star = \frac{2}{39}$. However, as before, with a few number of prices, the social welfare will be almost maximized. We find that $W_3^\star \approx 0.128198$ so that with only 3 prices or segments, 99.99% of the maximum social welfare is already achieved[14].

Corollary 2 *There exists a value of $\zeta \in]0, 1[$ and $k \in \mathbb{N}^*$ for which the regulatory policy given in proposition 3 is an aggregate Pareto improvement over the Malueg and Schwartz's (1994) mixed system.*

When $\zeta = 0.8$ and $k = 3$, as shown in the previous table, the profit is equal to 0.0784 and the consumers' surplus is equal to 0.0617. Compared to the Malueg and Schwartz's (1994) mixed system, our regulatory policy yields a 2% increase in the profit and a 20% increase in the consumers' surplus.

Possible cheating and stochastic audit: a simple approach to dynamic aspects

Up to now, we focused on a static model in which time plays no role. However, in practice, the exclusivity of the patent is given for a number of years, e.g., 20 years. Let T be the maturity of the

[14] In their rotating case, Malueg and Schwartz (1994) obtain the same result with three prices as $\frac{8.76}{8.82} \approx 99.3\%$ of W_∞ is achieved.

patent and I be the investment's cost in the R & D process[15]. We assume that I is invested at time $t = 0$, and that the production and the commercialization processes start at this time $t = 0$. In a dynamic model, the parameter β, which may be thought of as the *size* of the set of markets, need not be constant over time. It may for example depend of the introduction of other (substitutable) goods and/or of the underlying change in the consumers's preferences. Thus, β may be time dependent, i.e., deterministic or stochastic. Consider the regulatory policy considered in proposition 3. For a given known value of β_t, under the mentioned regulatory policy, the profit is equal to

$$\Pi(\mathbf{x}^*, \zeta \mathbf{P}^*, \beta_t) = \zeta(2-\zeta)\,\Pi_k^*\,\beta_t^3 \tag{40}$$

It is easy to see that when $\beta_t = 1$ and $\zeta = 1$, equation (40) reduces to equation (15). Assuming that $\zeta_t = 0.8$ for all $t \in [0, T]$, $k = 3$, and that $\beta^3 = (\beta_t^3)_{t \in \mathbb{R}^+}$ follows some stochastic process in continuous time (such that $\beta_t \geq 0$ for all $t > 0$) the expected present value of the rate of return, when positive[16], is equal to

$$Y = \frac{\mathbb{E}\left(\int_0^T e^{-rt}\,\Pi_3^*(0.8)\,\beta_t^3 dt\right)}{I} - 1 \tag{41}$$

where r is the risk-free rate. Let us suppose that $\beta_t^3 = e^{((\gamma - 0.5\sigma^2)t + \sigma W_t)}$ where W_t is a standard brownian motion. Assuming that $\gamma < r$, it is standard to show[17] that

$$\mathbb{E}\left(\int_0^T e^{-rt}\beta_t^3 dt\right) = \frac{1 - e^{-(r-\gamma)T}}{r - \gamma} \tag{42}$$

For simplicity, assume that the monopolist and the regulator agree on the estimation of $\gamma = 0$. In such a case the stochastic process β^3 is a *martingale*[18], i.e., the best prediction of a future value of β^3 is the current value. Let $I = 1$. For a given maturity T, the rate of return of the investment as a function of the regulation parameters ($\zeta = 0.8, k = 3, T$) is equal to

$$Y(0.8; 3; T) = \frac{(1 - e^{-rT})\Pi_3^*(0.8)}{r} - 1 \tag{43}$$

Clearly, *ceteris paribus*, the higher is the maturity of the patent T, the higher is the (expected) rate of return of the investment. However, the higher is T, the higher is the cost of monitoring the monopolist, especially if controls are frequent. To avoid the possible tradeoff between the maturity T of the patent and the cost of monitoring, we suggest a very simple form of monitoring; an audit at a random time. As in e.g., Baron and Besanko (1984), the regulator may annouce at time $t = 0$ a *stochastic audit*, that will be done at a stochastic time $\tau \in \mathbb{R}^+$ defined as follows

$$\tau = \inf\{t \in \mathbb{R}^+ : N_t = 1\} \tag{44}$$

where N_t is a Poisson process with constant intensity λ. Equation (44) just says that τ is the first time t for which the process N_t jumps for the first time. It is well known that the random time τ

[15] For simplicity, we assume that by investing the amount I in R & D, the success is immediate.

[16] If the rate of return is negative, the investment is not undertaken.

[17] To compute this expectation, only the *distributional* properties of the brownian motion are needed. Since $W_t \sim \mathcal{N}(0, \sqrt{t})$, one can replace W_t with $Z\sqrt{t}$, where Z is a standard normal gaussian random variable. By permuting the two integrals, simple computations lead to the desired result.

[18] Note that the stochastic process $\beta = (\beta_t)_{t \in \mathbb{R}^+}$ is a supermartingale.

follows an exponential distribution with parameter λ. Consequently, the regulator can just simulate (at time $t = 0$) one realization $\hat{\tau}$ of τ by Monte Carlo[19]. Of course, the monopolist only knows the probability distribution of τ, not the realization $\hat{\tau}$. The regulator will thus visit the monopolist at time $\hat{\tau}$ (if $\hat{\tau} \leq T$) to check whether or not the regulatory policy is satisfied, i.e., that $k = 3$ and $\zeta = 0.8$ is actually implemented. If the constraints are not satisfied, the patent is broken and the monopolist is not anymore allowed to sell the good. As noted by Laffont and Tirole (1993), for regulated firms in sectors for which substitution is difficult, the regulator may not want to do this. However, in our framework, the decision to shut down the firm is not related to its inefficiency but rather to its fraud. But the monopolist is aware at time $t = 0$ that if she does not respect the regulation, its activity may be stopped. Here is thus the alternative for the monopolist.

1. Charging a continuum[20] of prices and thus realizing a profit at each time t equal to Π^*_{cplte} as long as $\tau > t$.

2. Satisfying the regulatory policy and realizing a profit at each time t equal to $\Pi^*_3(0.8)$ between 0 and T.

By computing the expected net present value of each decision[21], the following proposition provides the simple condition under which the monopolist won't cheat.

Proposition 4 *Let* $V(T) = \frac{1-e^{-rT}}{r}$ *and assume that the monopolist is risk-neutral. When the regulator decides to audit stochastically the monopolist (i.e., at the first time a Poisson process jumps for the first time), then, the monopolist follows the regulatory constraint if*

$$V(T)\Pi^*_3(0.8) - \Pi^*_{\text{cplte}}\Big(V(T)e^{-\lambda T} + H(\lambda, r, T)\Big) \geq 0 \tag{45}$$

where the function $H(\lambda, r, T)$ *represents the expected discounted value of the flow of profit when the regulator will audit the monopolist before time* T.

Proof. See the appendix.

As one can expect, if $\lambda = 0$, i.e., the probability of being auditing is zero, and equation (45) is negative for any value of $T > 0$ and $r > 0$; the monopolist has thus an incentive to cheat. On the contrary, when $\lambda \to \infty$, it is easy to see that equation (45) will be positive. Of course, as the regulator may have many firms to regulate, announcing a high value of λ may be costly because it must be implemented to be credible. Interestingly, even for a very low probability of being auditing, the monopolist's best strategy is to follow the regulatory policy. Consider for example the case in which $r = 4\%$ and $T = 20$. The expected rate of return of the investment when the monopolist

[19] It is very easy to simulate a realization of τ since the distribution function of the exponential random variable is invertible. It suffices indeed to choose randomly a number $u \in]0, 1[$, e.g., via a pseudo random generetor, and then to set $\hat{\tau} = -\frac{\ln(1-u)}{\lambda}$

[20] As a fraud implies the shut down of the firm, if the monopolist decide to cheat, she will charge a continuum of prices. This would not necessarily be the case if the penality was a function of the severity of the fraud.

[21] A possibility that we disregard is the case in which the monopolist does not cheat until the regulator visits, but decide to cheat right after. If regulator does not visit a second time, then, this possibility will be at least prefered to the non cheating solution. However, if the regulator decides to visit the monopolist at the first time of the second jump, then, this possibility is not be interesting.

does not cheat is approximately 8%. When $\lambda = 1\%$, the expected rate of return of the cheating decision is less than 6%; it is thus in the interest of the monopolist to follow the regulatory policy. It is important to realize that an intensity equal to $\lambda = 1\%$ implies that the probability of being auditing over the next 20 years is only equal to $1 - e^{-0.2} \approx 0.18$. When $\lambda = 1\%$, the honest behavior turns out to be optimal for a maturity of 25 or 30 years and/or for a risk-free rate equal to 3% or 2%.

3 Conclusion

In this article, we have examined the choice of the market segmentation and the discriminatory prices as a joint problem. As opposed to the third-degree price discrimination literature in which market segments are given, it is *endogenous* in our framework. The full characterization of the monopolist's the optimal profit policy has been provided both when the monopolist is regulated and unregulated. From a social welfare point of view, we have shown that the usual welfare measures used in the third-degree price discrimination literature typically yield a result which is far below the benchmark, the social welfare under the Ramsey price. One of the main result of this article has been to exhibit a regulatory policy that yield an aggregate Pareto improvement over the uniform pricing profit case, but also over the Malueg and Schwartz's (1994) mixed system. Our result suggests that a small amount of regulated price discrimination can substantially enhance the social welfare.

Many things however remain to be done. Since the analysis has been done for linear demands under an uniform measure, it would clearly be interesting to examine whether or not our results are still valid under a more general model. Another interesting extension would be to consider the case in which information is asymmetric, i.e., when the regulator has less information than the monopolist.

4 Appendix: proofs

Proof of proposition 1

1. First order conditions.

Recall that for $i = 1...k$, the profit function of each segment A_i is equal to

$$\Pi_i(P_i) = \frac{P_i x_{i+1}^2}{2} - x_{i+1}P_i^2 - \frac{P_i x_i^2}{2} + x_i P_i^2 = P_i(x_{i+1} - x_i)\left[\frac{x_{i+1} + x_i}{2} - P_i\right] \quad (46)$$

where $x_{k+1} = \beta$. Using equation (11), it is easy to obtain that the gradient of the aggregate profit function is equal to:

$$\frac{\partial \Pi(\mathbf{x}, \mathbf{P})}{\partial x_1} = -P_1 x_1 + P_1^2 = 0 \quad (47)$$

$$\frac{\partial \Pi(\mathbf{x}, \mathbf{P})}{\partial x_{i+1}} = P_i x_{i+1} - P_i^2 - P_{i+1}x_{i+1} + P_{i+1}^2 = 0 \quad i = 1...k-1 \quad (48)$$

$$\frac{\partial \Pi(\mathbf{x}, \mathbf{P})}{\partial P_i} = \frac{x_{i+1}^2}{2} - 2P_i x_{i+1} - \frac{x_i^2}{2} + 2P_i x_i = 0 \quad i = 1...k \quad (49)$$

From equation (47), we get directly that $x_1 = P_1$. Simplifying equations (48) and (49), we get

$$x_{i+1} - (P_{i+1} + P_i) = 0 \quad \text{for } i = 1...k-1 \quad (50)$$

$$x_{i+1} + x_i - 4P_i = 0 \quad \text{for } i = 1...k \quad (51)$$

It is easy to check that $x_i = \frac{2i-1}{2k+1}\beta$ and $P_i = \frac{i}{2k+1}\beta$, for $i = 1, 2...k$ solve the above system of equations.

2. Second order conditions

Let $H(\mathbf{P}, \mathbf{x})$ be the Hessian matrix of the profit function $\Pi(\mathbf{x}, \mathbf{P})$. To show that $(\mathbf{P}^*, \mathbf{x}^*)$ is a (local) maximum, we have to show that $H(\mathbf{P}^*, \mathbf{x}^*)$ is negative definite. Let $H^* \equiv H(\mathbf{P}^*, \mathbf{x}^*)$.

$$H^* = \begin{pmatrix} A^*_{\mathbf{PP}} & B^*_{\mathbf{Px}} \\ C^*_{\mathbf{xP}} & D^*_{\mathbf{xx}} \end{pmatrix}$$

where $A^*_{\mathbf{PP}} = \left(\frac{\partial^2 \Pi(\mathbf{P}^*, \mathbf{x}^*)}{\partial P_i \partial P_j}\right)_{i,j}$, $B^*_{\mathbf{Px}} = \left(\frac{\partial^2 \Pi(\mathbf{P}^*, \mathbf{x}^*)}{\partial P_i \partial x_j}\right)_{i,j}$, $B^*_{\mathbf{P,x}}$ is the transpose of $C^*_{\mathbf{xP}}$, and $D^*_{\mathbf{xx}} = \left(\frac{\partial^2 \Pi(\mathbf{P}^*, \mathbf{x}^*)}{\partial x_i \partial x_j}\right)_{i,j}$. Each of these matrices are $k \times k$ matrices. Using equations (47), (48) (49) to obtain the Hessian matrix, it easy to show that :

$$A^*_{\mathbf{PP}} = \frac{\beta}{2k+1}.A \quad (52)$$

$$C^*_{\mathbf{PP}} = \frac{\beta}{2k+1}.C \quad (53)$$

$$B^*_{\mathbf{PP}} = \frac{\beta}{2k+1}.C^T \tag{54}$$

$$D^*_{\mathbf{PP}} = \frac{\beta}{2k+1}.D \tag{55}$$

where C^T is the transpose of C and

$$A = \begin{pmatrix} -4 & 0 & \dots & 0 \\ 0 & -4 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -4 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 0 & \dots & \dots & 0 \\ -1 & -1 & \dots & \dots & 0 \\ 0 & -1 & \ddots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & -1 \end{pmatrix}$$

and

$$D = \begin{pmatrix} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -1 \end{pmatrix}$$

Note that the matrices A and D are diagonal and that the matrix C is bi-diagonal, the main diagonal and the sub-diagonal, i.e., the diagonal just below the main one. Of course, $B = C^T$ is bi-diagonal, the main diagonal and the super-diagonal, i.e., the diagonal which is just above the main one. Let

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

It thus follows that

$$H^* = \frac{\beta}{2k+1}M \tag{56}$$

Let Q be a matrix such that

$$\bar{Q} = -Q \tag{57}$$

To show that M is negative definite, we shall show that

$$\bar{M} = \begin{pmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{pmatrix}$$

is positive definite.

Definition (see Axelson p 92). Let Q be a matrix partitioned into two by two block form

$$Q = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix} \tag{58}$$

where $A_{i,j}$, $i = 1, 2$ $j = 1, 2$ are square matrices. If $A_{2,2}$ is non-singular, the Schur complement (of Q with respect to $A_{2,2}$) is defined as

$$A_{1,1} - A_{1,2}.A_{2,2}^{-1}.A_{2,1} \tag{59}$$

Theorem (see Axelsson corollary 3.8' p 94). Let Q be a symmetric matrix and partitioned as in (58). Then, Q is positive definite if and only if the Schur complement $A_{1,1} - A_{1,2}.A_{2,2}^{-1}.A_{2,1}$ and $A_{2,2}$ are positive definite.

Applying this theorem to $\bar{M}$, we thus have to show that the matrices $\bar{D}$ and its Schur complement $\bar{A} - \bar{B}.\bar{D}^{-1}.\bar{C}$ are positive definite. Since

$$\bar{D} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

it is obviously invertible and positive definite. Since $\bar{D}$ is the identity matrix and as C is the transpose of B, the Schur complement in our case reduces to:

$$\bar{A} - \bar{C}^T \bar{C} \tag{60}$$

It thus follows that

$$\bar{C}^T.\bar{C} = \begin{pmatrix} 2 & 1 & 0 & \dots & 0 \\ 1 & 2 & 1 & \dots & 0 \\ 0 & 1 & 2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 1 \\ 0 & 0 & \dots & 1 & 1 \end{pmatrix}$$

is a (real symmetric) tri-diagonal matrix (the main diagonal, the sub-diagonal, and the super-diagonal) so that the Schur complement

$$\bar{A} - \bar{C}^T.\bar{C} = \begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -1 \\ 0 & 0 & \dots & -1 & 3 \end{pmatrix}$$

is also a tri-diagonal matrix. Before we show that the Schur complement is positive definite, we have to briefly present the notion of *irreducible matrix*. Our presentation follows from Warga (2000). Let

Q be a $n \times n$ matrix and let $P_1...P_n$ be distinct points in the plane which are called "nodes". For each $q_{i,j} \neq 0$, P_i is connected to P_j by means of a *directed arc* $P_i \curvearrowright P_j$ denoted $\overrightarrow{P_iP_j}$. One can thus associate a finite directed graph $G(Q)$ to the matrix Q.

Definition (Warga p 19). A directed graph with n nodes is strongly connected if, for any ordered pair (P_iP_j) of nodes, with $1 \leq i,j \leq n$, there exists a directed graph consisting of abutting directed arcs

$$\overrightarrow{P_iP_{l_1}}, \overrightarrow{P_{l_1}P_{l_2}}...\overrightarrow{P_{l_{r-1}}P_{l_r=j}} \tag{61}$$

connecting P_i to P_j.

Theorem (Warga p 20). An $n \times n$ matrix Q is irreducible if and only if its directed graph $G(Q)$ is strongly connected.

Since the Schur complement $\bar{A} - \bar{C}^T.\bar{C}$ is tri-diagonal (main diagonal sub-diagonal and super-diagonal), its directed graph $G(\bar{A} - \bar{C}^T.\bar{C})$ is strongly connected. To see this, consider for example $\overrightarrow{P_iP_j}$, with $j > i$. Since the super-diagonal is given by $a_{i-1,i}$, we just have to use it to connect P_i and P_j. If $j < i$, we just have to use the sub-diagonal. See e.g., example 4.4 of Axelsson p 127 for a similar example.

Definition (Axelsson p 133). The matrix Q is said to be irreducibly diagonally dominant if Q is irreducible and if $|q_{i,i}| \geq \sum_{j \neq i} |q_{i,j}|$ with a strict inequality for at least one i.

Theorem (Axelsson p 133). If Q is a real symmetric with positive diagonal dominant entries and is irreducibly diagonally dominant, then, Q is positive definite.

The main diagonal of the Schur complement is equal to 2 except the last component which is equal to 3 so that it is irreducibly diagonal dominant. Since the Schur complement is also a real symmetric matrix, the above theorem allows us to conclude that it is definite positive. We have thus shown that both the Schur complement $\bar{A} - \bar{C}^T.\bar{C}$ and $\bar{D}$ are definite positive. As a consequence, $\bar{M}$ is positive definite so that M is negative definite $\square$ Note that by unicity of the candidate point, it is the global maximum.

Proof of Lemma 2. Consider a given consumer $\omega \in A_i \equiv]x_i, x_{i+1}]$, where $P_i < x_i$ is the price charged. Given the demand function, the surplus of consumer ω is equal to $\frac{(\omega - P_i)^2}{2}$ so that $S_i = \int_{x_i}^{x_{i+1}} \frac{(\omega-P_i)^2}{2} d\omega = \frac{(x_{i+1}-P_i)^3-(x_i-P_i)^3}{6}$. Evaluated at the optimal profit policy, it follows that $S_i^*(P_i^*) = \frac{6i^2+2}{6(2k+1)^3}$. Let $CS_k^* = \sum_{i=1}^{k} S_i(P_i^*)$ be the total surplus. It thus follows that

$$CS_k^* = \frac{1}{3}\left(\frac{k(k+1)(2k+1)+2k}{2(2k+1)^3}\right) \tag{62}$$

Combining now equations (15) and (62) yields the desired result $\square$

Proof of proposition 3

Let $Q_{A_i^*}(P_i)$ be the demand function of the segment $A_i^* =]x_i^*; x_{i+1}^*]$ (see proposition 1) with $P_i < x_i^*$. Recall from equation (2) that $Q_i(P_i) = (x_{i+1} - x_i)\left(\frac{x_{i+1}+x_i}{2} - P_i\right)$ so that under the optimal segmentation policy given by $A_i^* = \left]\frac{2i-1}{2k+1}; \frac{2i+1}{2k+1}\right]$, the demand function, denoted $Q_{A_i^*}(P_i)$ of each segment is given by

$$Q_{A_i^*}(P_i) = \frac{2}{2k+1}\left(\frac{2i}{2k+1} - P_i\right) \tag{63}$$

Since the demand is evaluated in $P_i = \zeta P_i^* = \frac{\zeta i}{2k+1}$, it follows that $Q_{A_i^*}(\zeta P_i^*) = \frac{2i(2-\zeta)}{(2k+1)^2}$. As a consequence, the weight $w_i^*(\zeta P_i^*)$ is such that

$$w_i^*(\mathbf{P}^*(\zeta)) = \frac{Q_{A_i^*}(\zeta P_i^*)}{\sum_{i=1}^{k} Q_{A_i^*}(\zeta P_i^*)} \tag{64}$$

so that $w_i^*(\mathbf{P}^*(\zeta)) = \frac{2i}{k(k+1)}$. It is easy to show that

$$\sum_{i=1}^{k} \zeta P_i^* \, w_i^*(\mathbf{P}^*(\zeta)) = \sum_{i=1}^{k} \frac{\zeta 2i^2}{k(k+1)(2k+1)} = \frac{\zeta}{3} \tag{65}$$

Since $P^* = \frac{1}{3}$, the constraint is binding and satisfied for all value of ζ. Consequently, $\mathbf{P}^*(\zeta) \in \mathcal{P}_{\mathbf{x}^*}(\zeta)$. The profit of a given segment A_i^* is equal to $\Pi_{A_i^*}(\zeta P_i^*) = \zeta P_i^* Q_{A_i^*}(\zeta P_i^*) = \frac{\zeta(2-\zeta)2i^2}{(2k+1)^3}$, so that, for a given fixed value of k, the sum is equal to $\sum_{i=1}^{k} \Pi_{A_i^*}(\zeta P_i^*) = \dfrac{\zeta(2-\zeta)k(k+1)}{3(2k+1)^2}$. It is easy to show that the consumers surplus of a given group A_i^* is equal to $S_i^*(\zeta P_i^*) = \frac{6i^2(2-\zeta)^2+2}{6(2k+1)^3}$ so that $CS_k^*(\zeta) = \frac{2k+(2-\zeta)^2 k(k+1)(2k+1)}{6(2k+1)^3}$ □

Optimal pricing policy under the optimal segmentation with a ζ-average price constraint

Assume that the segmentation is fixed as given by proposition 1 and let $k = 2$. The optimal pricing policy is the solution of the following optimization problem, with $\zeta < 1$

$$\max_{P_1, P_2} \Pi_{A_1^*}(P_1) + \Pi_{A_2^*}(P_2) \tag{66}$$

subject to the average-price constraint given by

$$\frac{\Pi_{A_1^*}(P_1) + \Pi_{A_2^*}(P_2)}{Q_{A_1^*}(P_1) + Q_{A_2^*}(P_2)} \leq \zeta P^* \tag{67}$$

where $\Pi_{A_i^*}(P_i) = P_i Q_{A_i^*}(P_i)$, and where $Q_{A_i^*}(P_i)$ is given by equation (63). We then form the Lagrangian given by

$$\mathcal{L}(P_1, P_2, \lambda) = \Pi_{A_1^*}(P_1) + \Pi_{A_2^*}(P_2) - \lambda\left(\frac{\Pi_{A_1^*}(P_1) + \Pi_{A_2^*}(P_2)}{Q_{A_1^*}(P_1) + Q_{A_2^*}(P_2)} - \zeta P^*\right) \tag{68}$$

where λ is the multiplier. It is easy but cumbersome to show that the gradient of the Lagrangian with respect to P_1, P_2 leads to $P_2^\diamond = P_1^\diamond + 0.2$. Assuming that the constraint is binding, i.e., $\lambda > 0$, we then reinject $P_2^\diamond = P_1^\diamond + 0.2$ in the constraint to obtain the following quadratic equation in $P_1^\diamond$.

$$-2(P_1^\diamond(\zeta))^2 + P_1^\diamond(\zeta)\left(\frac{12+10\zeta}{15}\right) + \left(\frac{9-25\zeta}{75}\right) = 0 \qquad \zeta \in [0,1] \tag{69}$$

It is easy to check that when $\zeta = 1$, $P_1^\diamond(\zeta) = 0.2$, as expected. Consider now the case in which $\zeta = 0.8$. Then, the two roots in $P_{\zeta,1}$ are given by

$$P_1^\diamond(0.8) = \frac{(-20/15) \pm \sqrt{(20/15)^2 - (88/75)}}{-4} \tag{70}$$

The optimal pricing policy gives thus $P_1^\diamond(0.8) = \frac{(-20/15)+\sqrt{(20/15)^2-(88/75)}}{-4} \approx 0.139$, so that $P_2^\diamond(0.8) \approx 0.339$ whereas the suboptimal pricing policy gives $P_1^*(0.8) = 0.16$ and $P_2^*(0.8) = 0.32$. The total profit under the optimal pricing policy is equal to 0.0770 while it is equal to 0.0768 under the sub-optimal pricing policy.

Proof of proposition 4

Let

$$V(x) = \frac{1-e^{-rx}}{r} \qquad x \geq 0 \tag{71}$$

When the monopolist follows the regulatory constraint, the net present value is equal to

$$\mathbb{E}(Z) = V(T)\Pi_3^*(0.8) - 1 \tag{72}$$

We implicitly assume that we restrict the set of parameters (r, T) so that $\mathbb{E}(Z) > 0$. Consider now the case in which the flow of profits is stopped at the first time a standard Poisson process jumps for the first time. Let $\mathbf{1}_{\tau > T}$ be an indicator. The net present value when the monopolist does not follow the regulation is given by the following random variable

$$Z_{\text{cheat}} = \Pi^*_{\text{cplte}}\left(V(T)\mathbf{1}_{\tau>T} + V(\tau)\mathbf{1}_{\tau\leq T}\right) - 1 \tag{73}$$

Since τ follows a exponential distribution with parameter λ, it thus follows that, since the monopolist is risk-neutral

$$\mathbb{E}(Z_{\text{cheat}}) = \Pi^*_{\text{cplte}}\left(V(T)\mathbb{E}(\mathbf{1}_{\tau>T}) + \mathbb{E}(V(\tau)\mathbf{1}_{\tau\leq T})\right) - 1 \tag{74}$$

$$\Pi^*_{\text{cplte}}\left(V(T)e^{-\lambda T} + \int_0^T V(\tau)\lambda e^{-\lambda\tau} d\tau\right) - 1 \tag{75}$$

$$\Pi^*_{\text{cplte}}\left(V(T)e^{-\lambda T} + \underbrace{\frac{1-e^{-\lambda T}}{r} + \frac{\lambda}{r(\lambda+r)}(e^{-(r+\lambda)T} - 1)}_{H(\lambda,r,T)}\right) - 1 \tag{76}$$

Taking now $\mathbb{E}(Z) - \mathbb{E}(Z_{\text{cheat}})$ and the result follows $\square$

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