Produce or Speculate?
Asset Bubble, Occupational Choice and Efficiency

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Abstract

We study the macroeconomic effects of rational asset bubbles in an overlapping-generations economy where asset trading requires specialised financiers and where agents freely choose between working in the production or in the financial sector. Frictions in the market for deposits create rents in the financial sector that affect workers' choice of occupation. When rents are large, the private gains associated with asset bubbles may lead too many workers to become speculators, thereby causing rational bubbles to lose their efficiency properties. Moreover, if speculation can be carried out by skilled labor only then asset bubbles raise income and consumption inequalities, to the benefit of the skilled and the detriment of the unskilled.

Keywords: Rational bubbles; occupational choice; dynamic efficiency

JEL: E22; E44; G21

PRELIMINARY DRAFT

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One of the changes that I would like to see — and I'm going to be talking about in this in weeks to come — is seeing our best and our brightest commit themselves to making things — engineers, scientists, innovators. For so long, we have placed at the top of our pinnacle folks who can manipulate numbers and engage in complex financial calculations. And — and that's good. We need some of that. But you know what we can really use is some more scientists and some more engineers who are building and making things that we can export to other countries. President Barack Obama, Georgetown University Washington, D.C. April 14, 2009

Introduction

The concern that the bubble on housing, securised mortgages and related derivatives in the 2000s may have distorted labor allocation to the benefit of the financial sector and to the detriment of productive and innovative activities, expressed in President Obama's speech, also reflects the view of many professional economists, financial columnists and business leaders.[1] The prospect of large payoffs from trading assets, risky loans and derivatives, the story goes, would have diverted scarce human resources (especially skilled labor) into the financial sector.[2]

The presence of speculative bubbles is a striking feature of periods where the financial sector attracted many skilled workers. In the end of the 1920s and in the recent period, bubbles on housing and related assets have accompanied the expansion of the financial sector. These events induce many people to consider that the expansion of the financial sector sustained by asset bubbles is bad for the economy. However, it turns out that this issue is not so simple, because asset bubbles may also have beneficial consequences. Periods of expansion of the financial sector are accompanied by extreme asset valuations that facilitate

[1] See for instance Duflo (2008) and Krugman.(2009) among many others.

[2] While mostly based on anecdotal evidence, this common assessment of the recent financial history begins to receive empirical support. In their study of carreers of Havard graduates from the 1970s to the 1990s, Goldin and Katz (2008) claim that the most striking changes with regard to occupations concern the ascendancy of finance and management. Among the Harvard students who entered into graduation in 1970, 5 percent of the men were in occupations in finance 15 years after their class graduated. But for cohorts who entered into graduation in 1990, 15 percent were. These findings are consistent with those of Philippon and Reshef (2008) who use detailed information about wages, education and occupations to shed light on the evolution of the U.S. financial sector over the past century.

the emergence of bubbles, but also by vigorous economic expansion (Caballero et al., 2006). Macroeconomic analysis shows that rational bubbles can be efficient (Tirole, 1985, Weil, 2008). Actually, despite the complexity of the relations between financial bubbles, the size of the financial sector and economic efficiency, little is known on this issue that has not been yet analyzed in a consistent framework which accounts for the potential gains and losses associated with bubbles. The aim of this paper is to contribute to fill this gap.

We study the macroeconomic effects of rational bubbles in an economy where firm financing and asset trading require specialized intermediation services provided by financiers. There is a 'production sector', a 'financial sector', and agents choose freely the sector where they work. Market frictions, which limit the possibility to meet trading partners, induce rents which influence the allocation of labor.[3] In this framework, asset bubbles are accompanied by expansions of the financial sector. Nevertheless, asset bubbles are efficient when rents in the financial sector are sufficiently small, because financiers must pass on to lenders much of the productive gains associated with rational bubbles. In that case, asset bubbles, although associated with a large financial sector, may be dynamically efficient. When rents in the financial sector are large, however, bubbles attract so many workers in that sector that the bubbly equilibrium loses its efficiency properties. In this situation, while the first generations always benefit from the early stages of the bubble, the misallocation of labor eventually becomes so severe that all generations born after a certain date are worse off in the bubbly equilibrium. We also show that asset bubbles increase the wage gap between skilled and unskilled workers. Actually, the development of the financial sector in the nineties has been accompanied by a huge increase in top wages[4] in a context where the financial sector uses mainly skilled workers who can choose between becoming a "manager" or a "financier", while most less educated people have industry-specific skills and cannot easily become financier. Accordingly, we extend our basic model to account for "unskilled" individuals, stuck into the productive sector, and "skilled" individuals who can choose to work in the financial or in the

[3] Philippon and Reshef (2008) find that financial jobs were relatively skill intensive, complex, and highly paid until the 1930s and after the 1980s, but not in the interim period. They argue that the deregulation of the financial sector, which began in the 1980s, has been accompanied by the development of complex financial products, by increases in the demand for skills in financial jobs, and by the creation of substantial rents in the financial sector. Philippon and Reshef estimate that rents accounted for 30 percent to 50 percent of the wage differential between the financial sector and the rest of the private sector in the beginning of the 2000s. Philippon (2008) claims that the financial sector, which accounts for 8 percent of GDP in 2006, is probably at least 2 percent above the size required by financial intermediation.

[4] Atkinson, Piketty and Saez (2010), Piketty and Saez (2003).

productive sector. In this framework, asset bubbles attract skilled workers in the financial sector, reduce the supply of skilled labor in the productive sector and then raise the wage gap between skilled and unskilled workers. The flight of skilled workers from the production sector into the financial sector reduces the marginal productivity of unskilled workers, then their wage and their consumption. We find that this phenomenon may induce asset bubbles beneficial to skilled workers to be detrimental to unskilled workers when market frictions in the financial sector are large.

Our paper is related to at least four strands of research.

First, there is a large (and still growing) literature on the existence, dynamics and efficiency properties of rational bubbles (see, for example, Caballero *et al.*, 2006, and Fahri and Tirole, 2008, for recent contributions on this topic). But to the best of our knowledge none of the existing work specifically pertains to the relationship between bubbles and the size of the financial sector. As is well known, in the baseline OLG model of Diamond (1965), where the only friction lies in the demographic structure that prevents agents from participating in all markets, asset bubbles improve welfare (relative to the bubbleless equilibrium) by providing agents with the additional store of values necessary to transfer wealth across periods (see Tirole, 1985, and Weil, 1987, 1989). Asset bubbles can be inefficient, though, when other market imperfections such as capital externalities are added to the OLG structure. For example, Saint-Paul (1992) and Grossman and Yanagawa (1993) show that the crowding out of the capital stock by the bubble loses its efficiency properties under endogenous growth as it lowers growth and the welfare of future generations. Relatedly, Olivier (2000) constructs a model in which households optimally allocate their time endowment between production and research; in this context, bubbles on firms' share favor firm creation and may raise long-run growth. Our paper differs from these three latter studies by ignoring external effects, and is thus closer to the basic (exogenous growth) framework of Diamond, Tirole and Weil. In contrast to Olivier, we focus on the allocation of labor between production and financial intermediation, thereby uncovering a novel source of inefficiency associated with rational bubbles.

Second, the contributions of Philippon (2007, 2008) analyze the determinants of the size of the financial sector in a model where financiers provide monitoring services to entrepreneurs facing borrowing constraints. His framework is very useful to explain historical variations in the income share of the US financial sector and also to study the consequences of corrective taxes when the allocation of human capital across the financial and the non financial sectors is inefficient. However, the consequences of asset bubbles have not been explored in this framework.

Third, some authors have analyzed the inefficient allocation of talents that may follow from the presence of rents. For example, Baumol (1990) draws on historical evidence to argue that the allocation of entrepreneurial resources in society primarily reflects the distribution of individual, rather than economywide, payoffs and may thus be socially inefficient. In a related contribution, Murphy *et al.* (1991) construct a model of occupational choice and show how private returns may draw the marginal talent into rent-seeking, with the consequence of slowing down economic growth. While these authors explicitly refer to "trading" and "speculation" as prominent rent-seeking activities, they do not specifically study the role of bubbles in attracting talents into the financial sector and the potential drain that may result for productive sectors.

Fourth, there is large literature that sheds light on the interactions between financial market imperfections and macroeconomic activity (see Tirole, 2005, for a survey). The papers the most related to our approach are those of Wasmer and Weil (2004), who introduce search frictions on the credit market, and those of Femminis (2002) and Sen (2002) who analyze rational bubbles when there is imperfect competition on the product market. But none of these papers study the interactions between asset bubbles, frictions in the market for deposits, and the allocation of labor.

Our paper is organized as follows. The basic model with frictions in the financial sector is presented in Section 1. The bubbleless and the bubbly equilibria are presented in Sections 2 and 3 respectively. Section 4 is devoted to the analysis of the dynamic efficiency of the bubbly equilibrium. Section 5 deals with the relations between asset bubbles and income inequality. Section 6 provides concluding comments.

1 The model

The economy is populated by overlapping generations of two period-lived, risk-neutral agents who maximize end-of-life consumption. N_t agents are born at date t, and the population grows at rate $n \geq 0$. Every agent is endowed with one unit of labor when young. A newly born agent chooses between working in the production sector or entering the financial sector, and we denote by L_t the number of "workers" in the population at date t (so that $N_t - L_t$ is the number of "financiers" in the population). The central difference between workers and financiers is in the technologies that they have access to. There are two goods: labor, and a numeraire good, which is produced, invested and consumed.

1.1 Technologies

The numeraire good is produced with a constant returns to scale technology, $F(K_t, L_t)$, which is concave, increasing with respect to the quantity of capital, denoted by K_t, and of labor, denoted by L_t. $F(K_t, L_t)$ satisfies Inada conditions. All agents have access to a storage technology that yields $\lambda > 0$ units of the numeraire good at date $t+1$ for 1 unit stored at date t. Only financiers, who are specialized intermediaries, can successfully lend to firms (think of them, for example, as having devoted their first-period labor endowment to the acquisition of unique monitoring skills). Each financier can transform saving into productive capital used by firms at zero marginal cost.[5]

Agents enjoy late-life consumption only. Agents working in the production sector get the productive wage w_t in early life. Financiers get no wage in early life, but make a living out of the intermediation margin they extract by borrowing from lenders and lending to firms. Hence, workers provide for late-life consumption either by storing their wage or by lending it to financiers.

Capital depreciates at rate $\delta \in [0,1]$. We denote by $k_t = K_t/N_t$ the amount of capital per worker and by $\ell_t \equiv L_t/N_t$ the share of the working population engaged in the productive sector (i.e., $1-\ell_t$ is the size of the financial sector). With $f(k_t/\ell_t) = F(K_t, L_t)/L_t$ denoting the production function in intensive form, profit maximization by the firm yields:

$$w_t = f(k_t/\ell_t) - (k_t/\ell_t)\, f'(k_t/\ell_t) \equiv \omega(k_t/\ell_t), \tag{1}$$

$$r_t + \delta = f'(k_t/\ell_t)\,. \tag{2}$$

In what follows we refer to $1 + r_t$ as the (gross) "productive" rate, as opposed to the "interest rate" that financiers promise to workers, which we denote by $1 + \rho_t$. Henceforth we will focus on the non trivial case where the gross productive rate is strictly larger than the returns on the storage technology.[6]

[5] This assumption is made for the sake of simplicity. It can easily be checked that our qualitative results remain unchanged if the marginal cost of transforming one unit of saving into one unit of capital is positive but sufficiently small.

[6] The conditions ensuring that this will indeed be the case in equilibrium are given below.

1.2 Labor allocation

Agents choose their occupation (or sector) according to the terminal consumption that they expect from working in either sector. Occupation choice is made at the beginning of life. It is irreversible. In equilibrium, free entry in both sectors will ensure that expected payoffs are equalized and will determine the equilibrium size of each sector. We assume that there are market frictions in the financial sector, which allow financiers to earn a positive unit intermediation margin and thus find this occupation worthwhile.

1.2.1 The financial sector

After workers have decided to engage in the production sector, they start looking for a financier to whom they will lend their wage income at the end of the period. They are randomly matched with financiers according to a standard urn model where each worker has one ball and each financier an urn. A worker looking for a financier sends his ball at random among the urns. In this framework, a worker can be matched at most with one financier at the same time. The number of customers of a financier is a random variable that follows a Poisson distribution with parameter $\ell_t/\left(1-\ell_t\right)$.[7]

After the match has taken place, the worker and the financier bargain over the joint surplus to be earned from the relationship, and we denote by $\tilde{\theta} \in (0,1)$ the share of the surplus that accrues to a financier in a particular match. Importantly, the monopolistic power of the financier at this stage is limited by the fact that workers can decline the deal and restart searching for a financier (with the same random matching process); however, for time constraints they can only search for a finite number of times $\tau \geq 1$. We show in Appendix A that the outcome of this random matching and bargaining process is the following: i) workers strike a deal with the first financier that they meet; ii) the interest rate on which they agree to deal is:

$$1+\rho_{t+1}=\theta\lambda+(1-\theta)\left(1+r_{t+1}\right), \qquad (3)$$

where $\theta \equiv \tilde{\theta}^{\tau} \in (0,1)$. In the remainder of the paper we shall refer to the composite parameter θ as the *market power* of financiers. This market power can be high (low) either because

[7] The probability that a financier receives a ball sent by one worker is $1/(N_t-L_t)$. Then, the probability that a financier receives b balls sent by L_t workers is $\begin{pmatrix} b \\ 1/(N_t-L_t) \end{pmatrix} \left(\frac{1}{N_t-L_t}\right)^{b}\left(1-\frac{1}{N_t-L_t}\right)^{L_t-b}$. This binomial distribution converges towards the Poisson distribution with parameter $L_t/(N_t-L_t)$ when L_t and N_t-L_t are sufficiently large, so that the probability to receive b balls is $e^{-L_t/(N_t-L_t)}\left(\frac{L_t}{N_t-L_t}\right)^{b}/(b!)$.

their bargaining power in a particular match, $\tilde{\theta}$, is high (low), and/or because lenders' ability to meet alternative trading partners, as measured by τ, is high (low).

Equation (3) expresses the interest rate as a weighted sum of the returns on the two underlying technologies, storage and production. When financiers enjoy much market power, they are able to keep the interest rate accruing to workers close to the relatively low storage return (that is, the ultimate outside option for workers). On the contrary, when financiers have little market power their rent is limited and the interest rate must remain close to the relatively high productive rate.

It is convenient to rewrite (3) in terms of the *intermediation margin* that financiers are able to extract from their matches with workers:

$$r_{t+1} - \rho_{t+1} = \theta \left(1 + r_{t+1} - \lambda\right). \tag{4}$$

In equation (4), the return difference $1 + r_{t+1} - \lambda$ is the economywide surplus, per unit of savings, from investing in the production sector rather than storing. Then, the intermediation margin $r_{t+1} - \rho_{t+1}$ is the fraction of this unit surplus that accrues to financiers.

1.2.2 Occupational choice

Agents born at date t must choose at beginning of date t whether to become a worker or a financier, on the basis of the expected date $t+1$ consumption from either occupation. The terminal consumption of a worker born at date t who lends his savings to financiers is:

$$c_{t+1} = w_t \left(1 + \rho_{t+1}\right) \tag{5}$$

Let us now turn to financiers. Their payoff from any match is $w_t \left(r_{t+1} - \rho_{t+1}\right)$. Since they are in number $N_t - L_t$ while workers are in number L_t, and given the assumed matching process, the expected number of matches for a potential financier is $\ell_t / \left(1 - \ell_t\right)$. Hence the total expected consumption from choosing a career in finance is:

$$\mathbb{E}_t(c^f_{t+1}) = \frac{\ell_t w_t}{1 - \ell_t} \left(r_{t+1} - \rho_{t+1}\right). \tag{6}$$

Note that (6) is the *expected* consumption of an agent considering to become a financier, while the *actual* (*ex post*) consumption level of a particular financier depends on his random realized number of matches.

The equilibrium allocation of labor across sectors is determined by free entry. Since agents are risk-neutral, they must get the same expected consumption from either occupation, so that we must have $\mathbb{E}_t(c^f_{t+1}) = c_{t+1}$. In what follows we may thus refer to c_{t+1} as "individual consumption", defined as aggregate consumption divided by the number of old agent at date $t+1$, i.e., N_{t+1}. Equating (5) and (6) and using (4), we find that the equilibrium share of the financial sector is:

$$1 - \ell_t = \theta \left(1 - \frac{\lambda}{1 + r_{t+1}}\right), \tag{7}$$

which is positive provided that gross productive rate, $1 + r_{t+1}$, is larger than the storage return, λ.

The interpretation of equation (7) is straightforward: when the market power of financier, θ, is small, then so is the intermediation margin they are able to extract (see equation (4)) and thus the attractiveness of the financial sector. If, on the contrary, θ is large (i.e., financiers have strong market power), then the large implied margin attracts many agents into the financial sector *ex ante* and hence the number of workers in the production sector is small. A version of the basic overlapping generations model of Diamond (1965) and Tirole (1985) is recovered as a particular case of our framework when we set $\theta = 0$, in which case $\ell_t = 1$ for all t. At the extreme opposite, the crowding out of human resources by the financial sector is maximum when $\theta = 1$, in which case $\ell_t = \lambda / (1 + r_{t+1})$. Similarly, changes in lenders' outside option, λ, alter the rent that financiers can extract and thus the equilibrium size of the financial sector.

The effect of the productive rate, $1 + r_{t+1}$, on labor allocation across sectors also has a straightforward interpretation. Financiers extract a rent from their exclusive access to firms' financing. When the productive rate increases, matched financiers are able to extract some of the additional payoff and hence the intermediation margin rises. This in turns raises the expected payoff from working in the financial sector and reduces the share of producers in the population. Unsurprisingly, this effect is scaled by the market power of financiers, as well as producers' outside investment opportunities, since they both determine how much of the extra surplus financiers can extract from an increase in the productive rate.

2 Bubbleless equilibrium

2.1 Aggregate dynamics and steady state

In the bubbleless equilibrium workers' savings transit through financiers' hands and are then entirely turned into firms' capital. Since workers save their entire wage income, the law of motion for capital is $K_{t+1} = w_t L_t$, which we may rewrite as:

$$(1+n)\, k_{t+1} = \ell_t \omega(k_t/\ell_t). \tag{8}$$

On the other hand, equations (2) and (7) relate current occupational choices to the productive rate, and hence to the stock of capital per producer in the next period:

$$\ell_t = 1 - \theta + \frac{\theta\lambda}{f'\left(k_{t+1}/\ell_{t+1}\right) + 1 - \delta} \tag{9}$$

Equations (8) and (9) define, together with the initial value of capital, k_0, the equilibrium path of (k_t, ℓ_t). It should be noticed that in contrast to capital, the share of financiers is forward-looking because current occupational choices depend on anticipated payoffs and hence on the interest rate that will prevail in the next period. In equation (9), ℓ_t is increasing in k_{t+1}/ℓ_{t+1} since a high value of the latter ratio is associated with a low productive rate, which deters agents from working in the financial sector and thus raises the size of the productive sector. Note also that in the particular case where $\theta = 0$, equation (9) yields $\ell_t = 1$ and hence from (8) the path of k_t is described by the univariate (Diamond-like) dynamics $k_{t+1} = \omega(k_t)/\left(1+n\right)$. When $\theta > 0$, on the contrary, the stock of capital and the allocation of labor across sectors are jointly determined according to (8)–(9).

Let us denote by k^* and ℓ^* the steady state values of capital per worker and the size of the production sector, respectively, in the bubbleless equilibrium. From (8)–(9) we get:

$$\ell^* = 1 - \theta + \frac{\theta\lambda}{f'\left(\gamma^{-1}\left(1+n\right)\right) + 1 - \delta},\ k^* = \gamma^{-1}\left(1+n\right)\ell^* \tag{10}$$

where $\gamma\left(k/\ell\right) \equiv \omega(k/\ell)/\left(k/\ell\right)$. We now make the following assumptions:

$$\gamma'\left(.\right) < 0,\ \gamma\left(0\right) = +\infty,\ \gamma\left(+\infty\right) = 0, \tag{A1}$$

$$f'\left(\gamma^{-1}\left(1+n\right)\right) + 1 - \delta > \lambda. \tag{A2}$$

Assumption (A1) ensures that the steady state defined by (10) exists and is unique. (A2) guarantees that in the bubbleless steady state the value of the productive rate (left hand side) is always greater than the storage return (right hand side); this will imply that in the vicinity of that steady state there will always be a range of interest rates, ρ_t, allowing financiers to extract a positive intermediation margin (i.e., $r_{t+1} - \rho_{t+1} > 0$) while still be able to attract lenders' deposits (i.e., $\rho_{t+1} > \lambda$).[8]

[8] If this condition were not fulfilled the equilibrium share of financiers would go to zero and the value of k_t would be constant and given by $f'(k_t) + 1 - \delta = \lambda$.

Note from (10) that output per worker in the bubbleless steady state, Y/N, is $y^* = \ell^* f(k^*/\ell^*) = \ell^* f(\gamma^{-1}(1+n))$. Since under assumption (A2) the share of the population engaged in production, ℓ^*, decreases with the market power of financiers, θ, higher values of θ reduce output per worker. Finally, from (5) and (10) individual consumption in the bubbleless steady state is given by:

$$c^*(\theta) = \omega(\gamma^{-1}(1+n)) \left[\theta\lambda + (1-\theta)\left(f'\left(\gamma^{-1}(1+n)\right) + 1 - \delta\right)\right], \tag{11}$$

and thus decreases with θ, the market power of financiers.

2.2 Stability and local dynamics

We focus on the behavior of the dynamic system in the vicinity of the steady state (k^*, ℓ^*). Log-linearizing (8)–(9) around (k^*, ℓ^*) generates a two-dimensional linear system, the stability of which depends on the number of characteristic roots inside the unit circle and the number of predetermined variables in the system (Blanchard and Kahn, 1980). We show in Appendix B that the characteristic polynomial summarizing the local dynamics of the bubbleless equilibrium has either one or two roots inside the unit circle. Since the system has one predetermined variable (k_t) and one free variable (ℓ_t), this implies that either there is a unique equilibrium trajectory converging towards (k^*, ℓ^*) and indexed by k_0 (determinacy), or this equilibrium is surrounded by an infinity of equilibrium trajectories converging towards (k^*, ℓ^*) and indexed by (k_0, ℓ_0) (indeterminacy). More precisely, we find that the bubbleless steady state is determinate if and only if:

$$\frac{1+\alpha^*}{\epsilon^*}\left[1 + \left(\frac{1-\theta}{\theta}\right)\frac{f'(\gamma^{-1}(1+n)) + 1 - \delta}{\lambda}\right] > 2, \tag{12}$$

where $\alpha^* \in (0,1)$ and $\epsilon^* > 0$ denote the elasticity of the real wage and (minus) that of the productive rate with respect to capital, respectively, evaluated at the bubbleless steady state:

$$\alpha^* = \frac{(k^*/\ell^*)\omega'(k^*/\ell^*)}{\omega(k^*/\ell^*)},\ \epsilon^* \equiv -\frac{(k^*/\ell^*)\, f''(k^*/\ell^*)}{f'(k^*/\ell^*) + 1 - \delta}. \tag{13}$$

Condition (12) is not strong. For example, it is satisfied for all feasible values of the other parameters in the Cobb-Douglas case, where $y = k^\alpha$ and hence $\alpha^* = \alpha$, as long as $\alpha \geq 1/3$. It is also satisfied for any value of α when $\theta \leq 1/2$. In any case, it is satisfied provided that the return on storage, λ, lies sufficiently below the gross productive return $f'(\gamma^{-1}(1+n)) + 1 - \delta$. In the remainder of the paper we shall work out the dynamics of the model for the case where

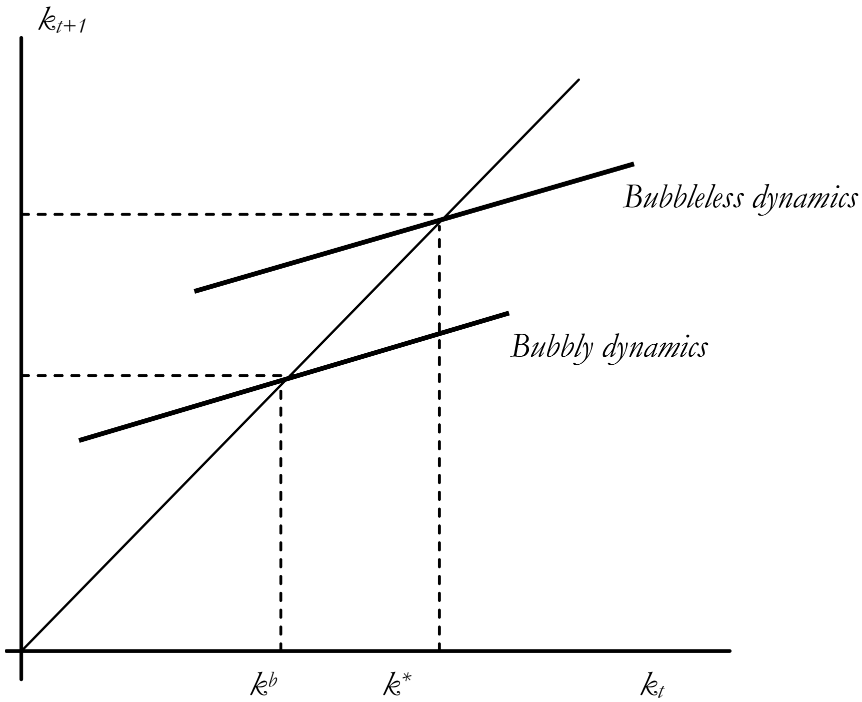

Figure 1: Bubbly and bubbleless equilibria

condition (12) is satisfied, so that k_0 uniquely pins down ℓ_0 and sets the economy on the unique equilibrium trajectory converging towards (k^*, ℓ^*). However, since our results on the welfare impact of rational bubbles follow from the asymptotic properties of the bubbly equilibrium and that indeterminacy also implies convergence towards the steady state, this focus is for expositional clarity and should not be seen as essential in our analysis.

Under condition (12) the local dynamics of k_t in the bubbleless equilibrium is governed by the unique stable root of the system, denoted by p_1, and we have (see Appendix B for details):

$$k_t = (1 - p_1) k^* + p_1 k_{t-1}, \; p_1 \in (0, 1) . \tag{14}$$

This dynamics is represented in Figure 1. Given k_t, equation (14) determines k_{t+1}.

3 Bubbly equilibrium

3.1 Dynamics and steady state

We now derive the bubbly equilibrium of our economy and compare it to the bubbleless equilibrium. We assume that bubbles, like claims to the capital stock, can only be traded by financiers. In the bubbly equilibrium, young financiers buy useless pieces of paper from the old financiers against the numeraire good that they have borrowed from young workers. For expositional clarity we focus on "pure" bubbles with no underlying real asset, but it would be straightforward to introduce a tree with constant payoff and to interpret the bubble as the difference between the trading price of this tree and its fundamental value (as in Tirole, 1985). Moreover, we only study equilibria that are "asymptotically bubbly", that is, equilibria in which the bubble per worker has strictly positive steady state value. We make specific assumption below ensuring the existence of such equilibrium paths, along which the bubble per worker does not vanish asymptotically.

In the bubbly equilibrium, total savings are invested in the production technology as well as in the bubble, i.e., $B_t + K_{t+1} = w_t L_t$. Denoting by $b_t = B_t/N_t$ the value of the bubble per young agent at the end of date t, we have:

$$b_t + (1+n)k_{t+1} = \ell_t \omega(k_t/\ell_t) \tag{15}$$

On the other hand, the absence of arbitrage opportunities requires that from the point of view of financiers the return on trading the bubble be equal to that on investing in production, i.e., $B_{t+1} = (1 + r_{t+1})\, B_t$. From (2), we must thus have:

$$b_{t+1} = \left(\frac{f'\left(k_{t+1}/\ell_{t+1}\right) + 1 - \delta}{1+n} \right) b_t \tag{16}$$

Along an asymptotically bubbly equilibrium the steady state value of the bubble per worker is constant and positive, implying that the ratio in (16) is equal to one. From (7) and (15)–(16) and the properties of $f(.)$ the bubbly steady state (k^b, ℓ^b, b) is unique and given by:

$$\ell^b = 1 - \theta + \frac{\theta\lambda}{1+n}, \quad k^b = f'^{-1}\left(n+\delta\right)\ell^b, \quad b = \ell^b \left[\omega\left(\frac{k^b}{\ell^b}\right) - \frac{(1+n)\,k^b}{\ell^b} \right]. \tag{17}$$

The necessary and sufficient condition for the existence of an equilibrium with asymptotically positive asset bubble is $\omega(k^b/\ell^b)/\left(k^b/\ell^b\right) = \gamma(k^b/\ell^b) > 1 + n$. Since $\gamma'(.) < 0$ (from Assumption (A1)) and $1 + n = \gamma(k^*/\ell^*)$ (see (10)), steady state bubbles will exist if and only if $k^b/\ell^b < k^*/\ell^*$,.i.e., if and only if the bubble asymptotically crowds out capital per worker in the productive sector. Equivalently, steady state bubbles are possible if and only if the real interest rate is higher at the bubbly steady state than at the bubbleless one, i.e., $f'(k^b/\ell^b) = n + \delta > f'(k^*/\ell^*) = \gamma^{-1}(1+n)$. Hence we can make sure that asymptotically bubbly equilibria exist by making the following assumption:

$$n + \delta < f'\left(\gamma^{-1}(1+n)\right) \tag{A3}$$

For example, if the production function is $f(k) = k^\alpha$, this condition boils down to $n+\delta > \alpha(1+n)/(1-\alpha)$.

Note first that even though the existence of a steady state with positive bubble is related to the production technology, to the population growth rate and to the rate of depreciation, it does not depend on the market power of financiers. However, the size of the bubble per worker depends on ℓ^b and thus on θ. Second, the higher productive rate that prevails in the bubbly steady state (relative to that in the bubbleless steady state) is associated with a more attractive financial sector and hence a smaller size of the production sector, i.e., $\ell^b < \ell^*$. Finally, since $k^b/\ell^b < k^*/\ell^*$ while $\ell^b < \ell^*$, it follows that $k^b < k^*$, i.e., the bubble crowds out capital per worker. We summarize these results in the following proposition.

Proposition 1. For all feasible values of θ the bubbly steady state has lower capital per worker, k, greater productive rate, $f'(k/\ell) + 1 - \delta$, and a larger financial sector, $1 - \ell$, than the bubbleless steady state.

The impact of general equilibrium asset bubbles on the capital stock and the rental rate in the long run are well known since the work of Tirole (1985) and Weil (1987). The novelty here is that differences in capital per worker between the bubbly and the bubbleless equilibria affect occupational choices (through their impact on the productive rate) and thus the allocation of labor across sectors. Finally, from (5) and (17) individual consumption in the bubbly steady state is given by:

$$c^b(\theta) = \omega(f'^{-1}(n+\delta))\left[\theta\lambda + (1-\theta)(1+n)\right], \tag{18}$$

and hence decreases with the market power of financiers, θ.

3.2 Stability and local dynamics

We proceed as in Section 2.2 and derive the dynamics of the system in the vicinity of the bubbly steady state. Moreover, since we wish to compare equilibrium trajectories that, for a particular level of initial capital k_0, may converge towards either the bubbleless steady state or the bubbly steady state, we assume that they are close to each other. We can then show (see Appendix C for details) that the steady state (k^b, ℓ^b, b) is determinate provided that condition (12) is satisfied, implying that the bubbly equilibrium (k_t, ℓ_t, b_t) is locally unique.

As is shown in Appendix C, around the bubbly steady states the dynamics of the capital stock can be first-order approximated as follows:

$$k_t = (1 - \tilde{p}_1) k^b + \tilde{p}_1 k_{t-1}, \; \tilde{p}_1 \in (0, 1), \tag{19}$$

where $\tilde{p}_1$ is the (unique) stable root of the bubbly system. Equation (19) is represented in the right-hand quadrant of Figure 1, next to the bubbleless dynamics. Since $k^b < k^*$, the dynamics of k_t along the bubbly equilibrium crosses the 45-degree line below that of the bubbleless equilibrium. Moreover, since k^b and k^* are close to each other (by assumption), an initial level of capital k_0 that is close to one of them is close to both and may set in motion a dynamics converging towards either k^* or k^b. We now analyze the implications of this crowding out of available labor by the financial sector for the dynamic efficiency of rational bubbles.

4 Dynamic efficiency

In the limit case where the financial sector is perfectly competitive (i.e., $\theta = 0$), the size of the financial sector is zero and our model collapses into a version of Diamond's (1969). Consequently, the standard results applies that rational bubbles can exist only to the extend that they restore dynamic efficiency (Tirole, 1985). The question that we ask here is: Do bubbles keep their efficiency properties when the market power of financiers allows them to seize part of the free lunch generated by bubbles?

It would seem, at first sight, that the answer should be "yes": since the bubbly equilibrium is associated with a higher productive rate than the bubbleless equilibrium, and that the overall surplus associated with this higher rate is shared between financiers and workers, agents in both sectors should benefit (or at least not suffer) from the bubble. In short, it would seem that the size of rent extraction by the financial sector should affect the way the efficiency gains associated with the bubble are shared amongst agents, but not the dynamic efficiency of the bubble *per se*.

This reasoning is wrong, however, for it ignores the effects of rent extraction by the financial sector on occupational choices and the implied distortion in the allocation of labor

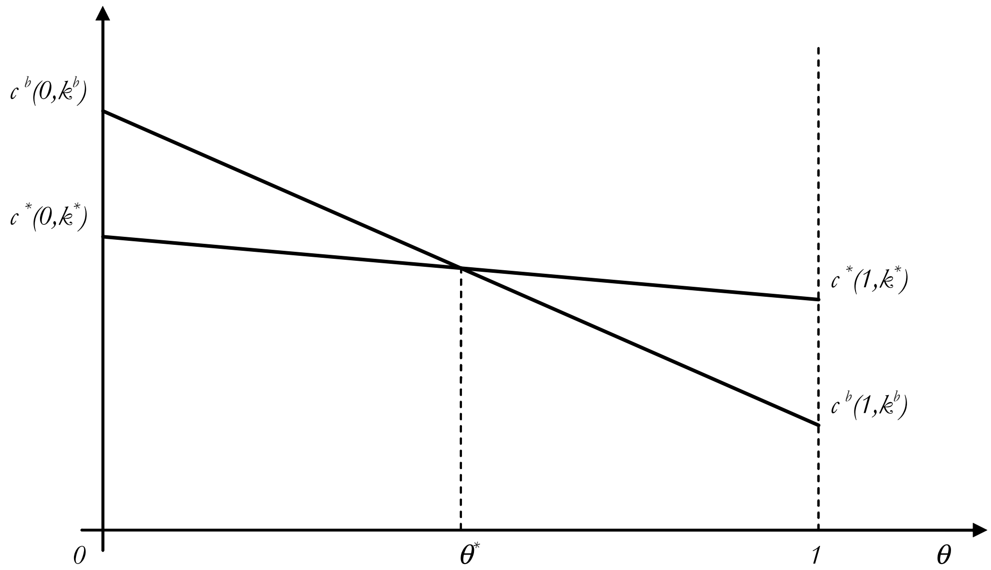

Figure 2: $c^*(\theta, k^*)$ and $c^b(\theta, k^b)$

across sectors. As a first illustration of this potential welfare loss, assume that financiers are able to extract the largest possible rent, i.e., $\theta = 1$. Then, equations (11) and (18) give $c^*(1) = \lambda\omega(k^*/\ell^*)$, while $c^b(1) = \lambda\omega(k^b/\ell^b)$. Since $k^b/\ell^b < k^*/\ell^*$ and $\omega'(k/\ell) < 0$, it follows that $c^b(1) < c^*(1)$ and hence rational bubbles cannot be dynamically efficient (i.e., some generations, possibly located far into the future, are better off without rather than with bubbles). The following proposition generalizes this reasoning for the cases where $\theta < 1$.

Proposition 2. The bubbly steady state has higher individual consumption than the bubbleless steady state if and only if rent extraction by the financial sector is not too large, i.e., if and only if $\theta < \theta^*$, where $\theta^* \in (0, 1)$.

Proof. We must compare $c^*(\theta)$ (defined by equation (11)) and $c^b(\theta)$ (given by equation (18)) for $\theta \in [0, 1]$. For δ, n and λ given, $c^*(\theta)$ and $c^b(\theta)$ are continuous and linearly decreasing in $\theta \in [0, 1]$, while $c^b(0) > c^*(0)$ and $c^b(1) < c^*(1)$ (see Figure 2). The first inequality is necessarily true from the asymptotic efficiency of bubbles in the Diamond-Tirole economy (which we recover when $\theta = 0$). The second inequality is equivalent to $\omega(f'^{-1}(n + \delta)) < \omega(\gamma^{-1}(1 + n))$, which is also true under Assumption (A3) since $\omega'(.) = -f''(.) > 0$. Hence there is a unique $\theta^* \in (0, 1)$ such that $c^b(\theta^*) = c^*(\theta^*)$, to the left (right) of which $c^b(\theta) > (<) c^*(\theta)$. *QED*

The central implication of Proposition 2 is that the bubbleless equilibrium cannot be dynamically inefficient when rent extraction by the financial sector is too serious. This is because, given $\theta > \theta^*$ and k_0 close to both k^b and k^*, convergence towards k^b necessarily implies a consumption loss in finite time, relative to convergence towards k^*. This obviously does not imply that all generations necessarily suffer from the bubble, since along the bubbly path the crowding out of productive labor by the financial sector is maximum only in the long-run, while the usual beneficial effects of bubbles may be dominant the short- and the medium-run.

To illustrate the effects that asset bubbles may have at different points in time, let us use the following example, which allows for a simple comparison of the differences in consumption paths across equilibria. Assume that $k_0 = K_0/N_0 = k^*$, so that $c_t = c^*$ in (11) for all $t \geq 1$ if the economy settles on the bubbleless dynamics at date 0. This path is represented by the bold horizontal lines in Figure 3. From equation (11), as θ is raised, the c^*-line shifts down and workers expected consumption at all dates falls.

Now consider what happens in the bubbly equilibrium. From equation (18), the asymptotic consumption level of workers, represented by the c^b-line (dotted line), shifts downwards as θ increases; and by Proposition 2, it shifts more than the c^*-line whenever $\theta > \theta^*$. In this situation, some generations (possibly located far into the future) are bound to incur a welfare loss if the economy settles on the bubbly equilibrium.

The opposite occurs in the short-run. Indeed, use (5) and (9) to write the consumption of workers born at date 0 as follows:

$$c_1(\theta) = \omega\left(\frac{k_0}{\ell_0}\right) \cdot \left[\theta\lambda + (1-\theta)\left(f'\left(\frac{k_1}{\ell_1}\right) + 1 - \delta\right)\right] = \omega\left(\frac{k_0}{\ell_0}\right) \cdot \left(\frac{\lambda\ell_0}{\ell_0 + \theta - 1}\right).$$

Since $\omega'(k/\ell) > 0$ while k_0 is given this last equation implies that consumption of individuals born at date 0 decreases with the size of the productive sector, ℓ_0. We can then show that the size of the productive sector at date zero is smaller in the bubbly equilibrium than in the bubbleless equilibrium, so that the consumption of individuals born at date zero is always higher in the bubbly equilibrium than in the bubbleless equilibrium. This can be proven by contradiction. First, let us define the variable $x_t \equiv k_t/\ell_t$, so that $\hat{x}_t = \hat{k}_t - \hat{\ell}_t$, and note that for $k_0 = k^*$ we have $x_1^* = k_0/\ell_0^* = x^*$ in the bubbleless equilibrium.[9] In the bubbly equilibrium, we have $x_1^b = x^b + \tilde{p}_1\left(k_0/\ell_0^b - x^b\right)$.[10] Now, suppose that $\ell_0^b > \ell_0^*$. From equation

[9] The proof also works when k_0 is in the vicinity of k^*.

[10] The latter expression directely follows from local equilibrium dynamics of the bubbly equilibrium, in

(9), this would imply that $x_1^b > x^*$, that is,

$$x^b + \tilde{p}_1 \left(\frac{k_0}{\ell_0^b} - x^b \right) > x^* \Leftrightarrow \tilde{p}_1 \left(x^* \frac{\ell_0^*}{\ell_0^b} - x^b \right) > x^* - x^b. \quad (20)$$

We know from equation (19) that $\tilde{p}_1 < 1$ and from Proposition 1 that $x^* - x^b > 0$. Thus, inequality (20) cannot hold for $\ell_0^b > \ell_0^*$. Hence it must thus be the case that $\ell_0^b < \ell_0^*$ and $c_1^b(\theta) > c_1^*(\theta)$.

After date 0, factor payments in the bubbly equilibrium gradually (and monotonically) adjust toward their steady state values (since convergence is monotonic in k_t). Given this gradual adjustment, several generations may enjoy the consumption boom generated by the early stages of the bubble. When $\theta < \theta^*$ (i.e., rents are small), $c^b(\theta) > c^*(\theta)$, and individual consumption may at all dates be higher in the bubbly equilibrium than in the bubbleless equilibrium (this situation is depicted in the left hand panel of Figure 3); when such is the case, the standard result that the bubbly equilibrium is dynamically efficient while the bubbleless equilibrium is not applies. However, when $\theta > \theta^*$, $c^b(\theta)$ lies below $c^*(\theta)$ and the bubbly equilibrium is bound to lose its welfare-improving properties (right hand panel of Figure 3). To summarize, when the initial stock of capital is close to the steady state value of the bubbleless equilibrium, the bubbly path is associated with higher consumption per worker in the short run, but bubbles reduce welfare in the long run when the market power of financiers is too large.

5 Asset bubble and income inequalities

In this section, we study the impact of bubbles on income inequality. In the model developed up to now, all individuals could become financier. Actually, bubble-prone economies seem to be associated with complex financial systems needing professionals whose skills are transferable across sectors (i.e., so that they can freely choose between becoming a "managers" or a "banker"), while less educated people have industry-specific skills and cannot easily become financier. Therefore, we introduce "unskilled" individuals, stuck into the productive sector, while "skilled" individuals can (like those of the previous sections) become financier. This distinction allows us to show that skilled individuals can benefit more from asset bubbles than unskilled people. The bubble triggers a flight of skilled workers from the production

which $\hat{k}_t = \tilde{p}_1 \hat{k}_{t-1}$ and $\hat{\ell}_t = \mu \hat{k}_t$, where μ is a constant. This implies that $\hat{x}_{t+1} = \tilde{p}_1 \hat{x}_t$, and hence $x_1^b = x^b + \tilde{p}_1 \left(x_0^b - x^b \right)$, where $x_0^b = k_0 / \ell_0^b$.

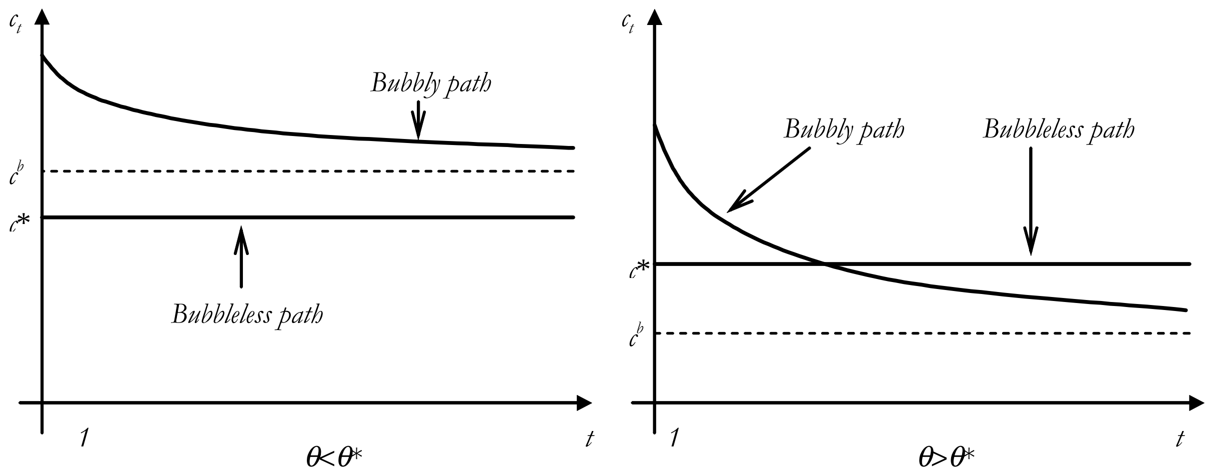

Figure 3: Consumption dynamics

sector into the financial sector which reduces the marginal productivity of unskilled workers, then, their wage and their consumption. Accordingly, asset bubbles beneficial to skilled workers, may hurt unskilled workers.

5.1 Labor allocation under skill heterogeneity

There are N_t skilled households and $L_{u,t} = uN_t$ unskilled households in the population (hence total population is now $(1+u)\,N_t$). We now use L_t to denote the number of skilled workers engaged in production. With this notation, which allows us to directly generalize the previous model, $\ell_t = L_t/N_t$ and $1-\ell_t$ are now the shares of skilled workers engaged in production and finance, respectively, while skilled and unskilled workers are in proportion $1/\left(1+u\right)$ and $u/\left(1+u\right)$ in the population, respectively. Similarly, we now use w_t to denote the wage of skilled workers and $w_{u,t}$ that of unskilled workers.

Production now requires both labor types, and for simplicity we assume that the production function is of the form:

$$Y_t = AK_t^{\alpha}L_t^{\beta}L_{u,t}^{1-\alpha-\beta}, \tag{21}$$

or, in intensive form, $y_t = \Omega k_t^{\alpha}\ell_t^{\beta}$, with $\Omega \equiv Au^{1-\alpha-\beta}$. First, equating the marginal product of capital to the user cost of capital gives the following gross productive rate:

$$1 + r_t = \alpha\Omega k_t^{\alpha-1}\ell_t^{\beta} + 1 - \delta. \tag{22}$$

Second, equating the marginal product of each labor type to the corresponding real wages, we find that the equilibrium skill premium in this economy is:

$$\frac{w_t}{w_{u,t}} = \frac{\beta u}{(1-\alpha-\beta)\,\ell_t}. \tag{23}$$

Finally, since $\ell_t \in [0,1]$ we can make sure that skilled workers always earn a higher wage than unskilled workers by assuming that

$$u > (1-\alpha-\beta)/\beta \tag{A4}$$

If (A4) did not hold, unskilled labor could be so scarce, and consequently well remunerated in equilibrium, that skilled workers would prefer to go for unskilled jobs in the productive sector than working in the financial sector; this would lead the size of the latter to shrink to zero.

Equation (23) indicates that the skill premium is increased as the share of skilled workers in the productive sector goes down. In other words, increases in the size of the financial sector raise wage inequalities. This occurs for two reasons. First, for a given quantity of capital and unskilled labor in production, a reduction in the availability of skilled labor in that sector raises its relative price, w_t. Second, a lower level of skilled labor in the productive sector reduces the productivity of unskilled labor and hence the corresponding real wage, $w_{u,t}$.

As is shown in Appendix A, the bargaining outcome that generates the equilibrium interest rate, ρ_{t+1}, as a function of the productive rate, r_{t+1}, is independent of the size of individual savings brought to the financier (equal to w_t or $w_{u,t}$ here); hence equations (3)–(4) also apply to the economy with two labor types and uniquely determines the interest rate ρ_{t+1}. We denote by $c_{u,t+1} = w_{u,t}(1+\rho_{t+1})$ and $c_{t+1} = w_t(1+\rho_{t+1})$ the terminal consumption of an unskilled worker and that of a skilled worker engaged in production, respectively. There are $N_t - L_t$ financiers, who extract the intermediation margin $r_{t+1} - \rho_{t+1}$ and meet depositors according to the same random matching process as before. For any particular financier, the expected number of matches with skilled worker engaged in production is $\ell_t/(1-\ell_t)$, while any match with a skilled leads to a deposit collection of w_t. On the other hand, the expected number of matches with unskilled workers is $u/(1-\ell_t)$, while any match with an unskilled worker leads to the collection of w_{ut} units of savings. Hence, using (23) we find that the (expected) terminal consumption of a skilled worker in the financial sector is:

$$\begin{aligned}\mathbb{E}_t(c^f_{t+1}) &= \left(\frac{\ell_t w_t + u w_{u,t}}{1-\ell_t}\right)(r_{t+1}-\rho_{t+1}) \\ &= \left(\frac{1-\alpha}{\beta}\right)\frac{\ell_t}{1-\ell_t} w_t\,(r_{t+1}-\rho_{t+1}),\end{aligned}$$

which generalizes equation (6) above.

Since by assumption the demand for unskilled labor by the financial sector is zero, the absence of arbitrage opportunities across alternative career choices applies to skilled workers only. Equating c_{t+1} and $\mathbb{E}_t(c^f_{t+1})$, we find that in equilibrium the share of skilled workers choosing to work in the financial sector is

$$1-\ell_t = \frac{(1-\alpha)\,\theta\,(1+r_{t+1}-\lambda)}{(1-\alpha-\beta)\,\theta\,(1+r_{t+1}-\lambda)+\beta\,(1+r_{t+1})}, \tag{24}$$

which generalizes equation (7) above. It is easy to check from (24) that $\partial\ell_t/\partial r_{t+1} < 0$, that is, a higher productive rate attracts more skilled workers into finance, due to the greater intermediation margin to be earned there. Taken together, equations (23) and (24) indicate that a higher productive rate will be associated with a greater skill premium. Moreover, since the interest rate is the same for all workers, we have $c_{t+1}/c_{u,t+1} = w_t/w_{u,t}$, so that wage inequalities are directly reflected into consumption inequalities.

5.2 Bubbleless equilibrium

Using the expressions for w_t and $w_{s,t}$ derived from (21) and rearranging, we find that total savings are $w_{u,t}L_{u,t} + w_tL_t = (1-\alpha)\,w_tL_t/\beta$. In the bubbleless equilibrium all these savings are invested into next period's capital stock, K_{t+1}. Hence the capital accumulation equation can be written as:

$$(1+n)\,k_{t+1} = (1-\alpha)\,\Omega k_t^{\alpha}\ell_t^{\beta}. \tag{25}$$

The dynamics of the bubbleless equilibrium is described by a two-dimensional system formed by the labor allocation equation (24) (with the productive rate r_{t+1} given by (22)) and the capital accumulation equation (25).

We solve the model with skill heterogeneity in the same way as we solved the basic model. We first compute the steady state of the bubbleless equilibrium. It will then be compared to the bubbly analogue, with particular attention being paid to asymptotic levels of capital and consumption per worker. Second, we examine the local stability of this equilibrium to show that it exists and is unique under condition (12). This second step is detailed in Appendix C.

From equations (22) and (25), the value of the productive rate at the bubbleless steady state is:

$$1+r^* = \frac{\alpha\,(1+n)}{(1-\alpha)} + 1 - \delta.$$

The steady state value of the other variables can then be computed sequentially: the share of skilled labor in production, ℓ^*, is uniquely determined by (24) and the value of r^*, while capital per worker, k^*, can be computed from r^* and ℓ^* using (22). Finally, note that assumption (A2) is still assumed to hold here, i.e., $\alpha(1+n)/(1-\alpha)+1-\delta>\lambda$.

5.3 Bubbly equilibrium

In the bubbly equilibrium, aggregate savings, $(1-\alpha)\,w_t L_t/\beta$, are used to finance the purchase of capital stock, K_{t+1}, and the aggregate bubble B_t. Hence the capital accumulation equation becomes:

$$b_t + (1+n)\,k_{t+1} = (1-\alpha)\,\Omega k_t^{\alpha} \ell_t^{\beta}, \tag{26}$$

where, by our normalization, $b_t = B_t/N_t$ now denotes the bubble per skilled worker. The absence of arbitrage opportunities for speculators implies that the dynamics of the bubble must be

$$b_{t+1} = \left(\frac{1+r_{t+1}}{1+n}\right) b_t. \tag{27}$$

Equation (26)–(27), together with (22) and (24), fully describe the dynamics of the bubbly equilibrium.

As usual, the steady state of the bubbly equilibrium satisfies $b_{t+1} = b_t$ and hence the "golden-rule" relation $r^b = n$. The existence of asymptotically bubbly equilibria is ensured if the value of the bubble per skilled worker in the steady state, b, is positive. From (22) and (26), this is the case if and only if the productive rate in the bubbleless steady state lies below the golden rule interest rate, i.e.,

$$r^* = \frac{\alpha(1+n)}{(1-\alpha)} - \delta < r^b = n. \tag{28}$$

Again, the bubbly steady state (k^b, ℓ^b, b) can be computed sequentially as follows. Substituting n for r_{t+1} in (24) gives ℓ^b. With ℓ^b and r^b known, the steady state counterpart of (22) uniquely determine k^b. Finally, k^b and ℓ^b can be substituted into the steady state counterpart of (26) to find b. Moreover, and as is shown in Appendix C, the bubbly steady state is determinate under condition (12) provided that k^b is sufficiently close to k^* (or, equivalently, that b is sufficiently small). This establishes the local uniqueness of the asymptotically bubbly equilibrium.

5.4 Dynamic efficiency

The central implication of the heterogenous skill model is that asset bubbles affect relative wages and consumption levels through their effect on the allocation of skilled workers across sectors. We focus here on the comparison of steady state consumption levels, and rely on the local stability of both equilibria to argue that, starting from k_0 sufficiently close to k^* and k^b, these consumption levels will asymptotically converge towards their steady state value. The following proposition establishes that, as a result of rising income inequalities, unskilled workers are the first to bear the cost of the misallocation of labor generated by asset bubbles.

Proposition 3. There exists a threshold level of the market power of financiers, denoted by $\theta_u^* \in (0,1)$, such that the consumption of unskilled workers is lower in the bubbly than in the bubbleless steady state whenever $\theta > \theta_u^*$. In the vicinity of $\theta = \theta_u^*$ the steady state consumption level of skilled workers is higher in the bubbly steady state than in the bubbleless steady state.

Proof. In Appendix D.

Proposition 3 identifies a new source of breakdown of dynamic efficiency under endogenous occupational choice, namely, the fact that the bubble may be harmful to unskilled workers even when it benefits skilled workers. This is notably the case when θ is higher than, but close to θ_u^*. Note also that the opposite cannot occur: because all depositors are paid the same interest rate while the bubble raises wage inequalities, it cannot be that the bubble raises the consumption of the unskilled while lowering that of the skilled (relative to the bubbleless equilibrium). Importantly, the proposition does not establish an upper threshold of θ above which skilled workers would lose; in fact, one can easily construct examples in which the consumption of the skilled is higher in the bubbly than in the bubbleless equilibrium *for all possible values of* θ (see below); in contrast, there always is such a threshold for the unskilled.

To get further insight into the redistributive effects of asset bubbles under heterogenous skills, it may be useful to draw the values of key steady state variables as a function of θ, the market power of financiers. Our first example, which uses $A = u = \delta = 1$, $\alpha = 1/3$, $\beta = 1/2$, $\lambda = 0.5$ and $n = 0.1$, is depicted in Figure 4. For all values of θ, the bubbly steady state (bold curves) is associated with a larger financial sector, lower wages and greater wage inequalities than the bubbleless steady state. Crucially, there are now two threshold levels for θ (instead of one as in Figure 2): θ_u^*, above which unskilled workers asymptotically suffer from the bubble, and θ_s^*, above which *skilled* workers asymptotically suffer from the bubble. Since $\theta_u^* < \theta_s^*$ (as is consistent with Proposition 3), there is a range of market powers within which the bubbly steady state is beneficial to the skilled but detrimental to the unskilled, relative to the bubbleless steady state.

θ_s^* need not be strictly smaller than one, as Figure 5 illustrates. This second example uses the same parameters as those of the first example except for the fact that we set $\beta = 1/3$ and $u = 1.2$. In this situation, we still have a threshold $\theta_u^* \in (0, 1)$ for the unskilled, but no such a threshold for the skilled: these always benefit from the bubble asymptotically. A central difference with the previous example, and one that is responsible for this result, is that here bubbles turn out to raise the *wage income* of skilled workers for sufficiently high values of θ. To understand why this is the case, recall that the skilled wage is $w_t = \beta\Omega k_t^\alpha \ell_t^{\beta-1}$ in equilibrium. On the one hand, the bubbly steady state has lower capital per worker than the bubbleless steady state, which pushes this wage down; on the other hand, the bubbly steady state has fewer skilled workers in the production sector, which raises their marginal product and hence pushes up the wage of skilled workers. Ultimately the impact of the bubble on the (steady state) equilibrium wage of skilled workers depends on this two forces. When the market power of financiers is sufficiently strong, the brain drain from production to speculation that takes place in the bubbly equilibrium may cause the second effect to dominate, resulting in higher wages and higher consumption levels. In other words, the model explains not only why bubbles raise income inequalities, but also how they may lead to an *absolute* increase in both the wage income and capital income of skilled workers. Hence our model is consistent with the observed rise of U.S. top incomes that occurred from the mid-1990s to the late 2000s (Atkinson, Piketty and Saez, 2010) and the related increase in wage inequalities on the U.S. labor market (Autor, Katz and Kearney, 2006) which precisely occurred at a time when the financial sector attracted much skilled labor.

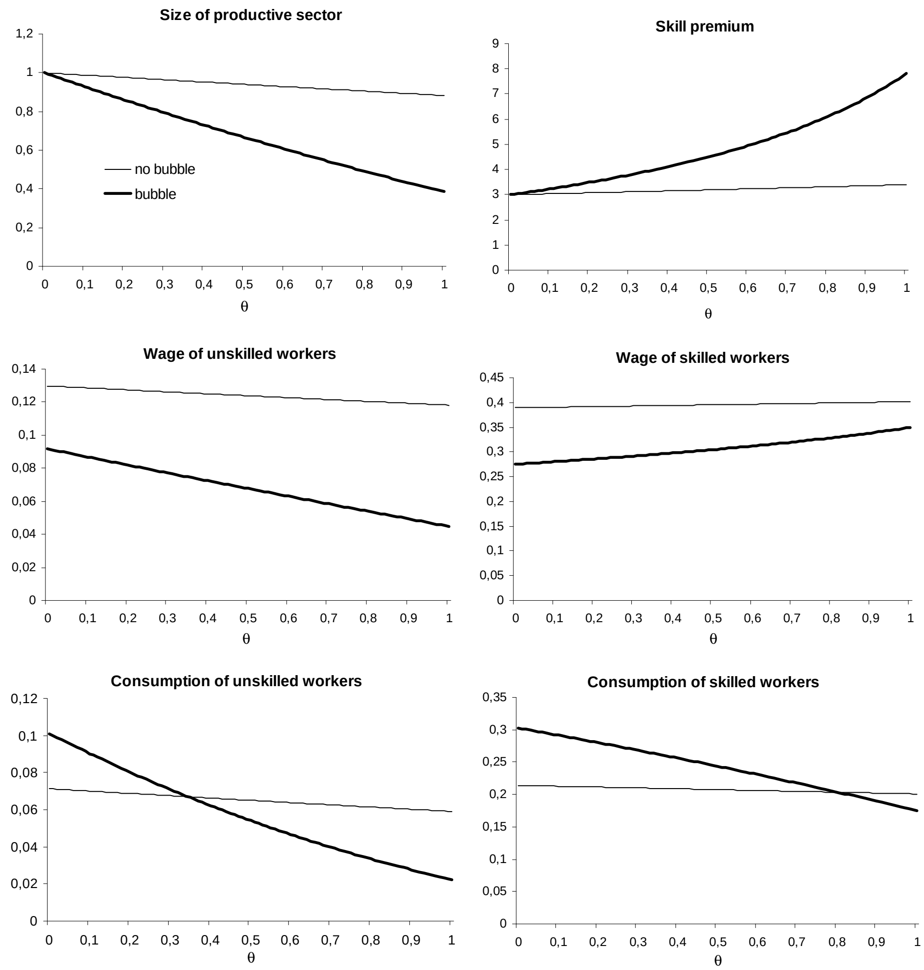

Figure 4: Example of steady state with $A = u = \delta = 1$, $\alpha = 1/3$, $\beta = 1/2$, $\lambda = 0.5$ and $n = 0.1$

6 Concluding remarks

Our paper shows that when the financial sector is sufficiently competitive then people are better off in an economy with a large financial sector that produces and manage asset bubbles than in an economy without asset bubble and with a smaller financial sector. However, when financial market frictions are too severe, asset bubbles are associated with such a large financial sector that bubbles lose their traditional efficiency properties. Moreover, asset bubbles increase wage inequalities and are primarily detrimental to low-skilled workers. From this point of view, the concern that asset bubbles are detrimental to productive and innovative activities can be justified. However, our paper suggests that the main concern should be the regulation of the financial sector, rather than the existence of asset bubbles *per se*.

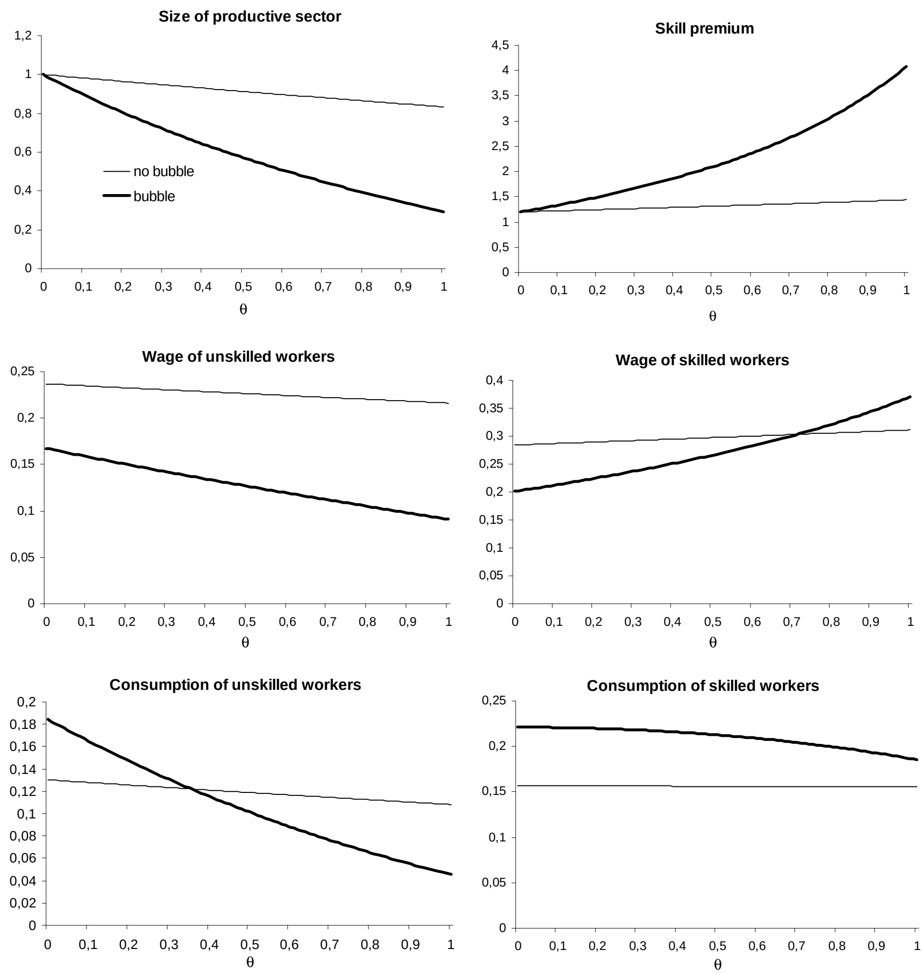

Figure 5: Example of steady state with $A = \delta = 1$, $\alpha = 1/3$, $\beta = 1/3$, $\lambda = 0.5$, $u = 1.2$ and $n = 0.1$.

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Appendix

A. Matching and bargaining outcomes

We work out the solution to the matching and bargaining process backwards. Consider the match between a financier and a depositor having reached the τth bargaining round, and call $\rho_{\tau,t+1}$ the bargained interest rate resulting from the match. The depositor's payoff from the match is $w_t(1+\rho_{\tau,t+1})$ and his outside option $w_t\lambda$, so his surplus from the match is $S^w_{\tau,t+1} = w_t(1+\rho_{\tau,t+1}-\lambda)$. Since the financier's outside option is zero, his surplus from this match is $S^f_{\tau,t+1} = w_t(r_{t+1}-\rho_{1,t+1})$. With a surplus share $\tilde{\theta}$, the financier extracts the payoff $\tilde{\theta}(S^w_{\tau,t+1}+S^f_{\tau,t+1}) = \tilde{\theta}w_t(1+r_{t+1}-\lambda)$, which by definition is equal to $w_t(r_{t+1}-\rho_{\tau,t+1})$. Hence the unit intermediation margin that a financier extracts from a match with a depositor in his τth bargaining round is:

$$r_{t+1}-\rho_{\tau,t+1} = \tilde{\theta}(1+r_{t+1}-\lambda)$$

Now consider what happens in the $(\tau-1)$th match of a depositor, and call $\rho_{\tau-1,t+1}$ the interest that results from the bargaining process. The depositor's gain from the match is $w_t(1+\rho_{\tau-1,t+1})$ while the outside option is now $w_t(1+\rho_{\tau,t+1})$ (i.e., what can be extracted from the following match), so the surplus from this $(\tau-1)$th match is $S^w_{\tau-1,t+1} = w_t(\rho_{\tau-1,t+1}-\rho_{\tau,t+1})$. On the other hand the financier's outside option on a particular match is still 0, so his surplus from this match is $S^f_{\tau-1,t+1} = w_t(r_{t+1}-\rho_{\tau-1,t+1})$. The condition that $S^f_{\tau-1,t+1} = \tilde{\theta}(S^f_{\tau-1,t+1}+S^w_{\tau-1,t+1})$ gives the following intermediation margin to the financier:

$$r_{t+1}-\rho_{\tau-1,t+1} = \tilde{\theta}(r_{t+1}-\rho_{\tau,t+1}) = \tilde{\theta}^2(1+r_{t+1}-\lambda)$$

Note that for both parties the gain from this $(\tau-1)$th match is higher than their outside options (i.e., $\rho_{\tau-1,t+1} > \rho_{\tau,t+1}$ and $r_{t+1}-\rho_{\tau-1,t+1} > 0$), so both agree to strike a deal at this stage. By the same logic, repeated τ times, both parties agree to strike a deal at the first match, giving the financier an intermediation margin:

$$\begin{aligned} r_{t+1}-\rho_{1,t+1} &= \tilde{\theta}(r_{t+1}-\rho_{2,t+1}) = \tilde{\theta}^2(r_{t+1}-\rho_{3,t+1}) = ... \\ &= \tilde{\theta}^{\tau-1}(r_{t+1}-\rho_{\tau,t+1}) = \tilde{\theta}^{\tau}(1+r_{t+1}-\lambda). \end{aligned}$$

This is exactly equation (4) in the body of the paper since ρ_{t+1}, the prevailing interest rate, is the one that result from (all) depositors' unit gain from their first bargaining round, $\rho_{1,t+1}$. Equation (3) is a rewriting of (4).

B. Local dynamics of the basic model

Bubbleless equilibrium

We use hatted variables to denote proportional deviations of the corresponding variables from the steady state (e.g., $\hat{k}_t = (k_t - k^*)/k^*$). Linearising (8) and (9) around (k^*, l^*) yields:

$$\hat{k}_{t+1} = \alpha^* \hat{k}_t + (1 - \alpha^*) \hat{\ell}_t, \ A^* \hat{\ell}_t = \hat{k}_{t+1} - \hat{\ell}_{t+1},$$

where

$$\alpha^* \equiv \frac{(k^*/\ell^*)\omega'(k^*/\ell^*)}{\omega(k^*/\ell^*)}, \ \epsilon^* \equiv -\frac{(k^*/\ell^*) f''(k^*/\ell^*)}{f'(k^*/\ell^*) + 1 - \delta}, \text{ and}$$
$$A^* \equiv \frac{1}{\epsilon^*}\left[1 + \left(\frac{1-\theta}{\theta}\right)\left(\frac{f'(\gamma^{-1}(1+n)) + 1 - \delta}{\lambda}\right)\right].$$

We write the linearized dynamics of the model as $\hat{x}_{t+1} = M\hat{x}_t$, where $\hat{x}_t = [\ \hat{k}_t \quad \hat{\ell}_t\ ]'$ and

$$M = \begin{bmatrix} \alpha^* & 1 - \alpha^* \\ \alpha^* & 1 - \alpha^* - A^* \end{bmatrix}.$$

The characteristic polynomial of M is $P(p) = p^2 - (1 - A^*) p - \alpha^* A^*$ and has roots:

$$p_{1,2} = \frac{1}{2}\left(1 - A^* \pm \sqrt{(1 - A^*)^2 + 4\alpha^* A^*}\right).$$

Note that both roots are real, that $p_1 \in (0, 1)$ and that $p_2 < -1$ if and only if $A^*(1 + \alpha^*) > 2$, which is inequality (12) in the body of the paper (when (12) is not satisfied we have $p_2 \in (-1, 0)$ and hence indeterminacy). The general solution of the linearized system is

$$\hat{k}_t = c_1 \frac{p_1 + \alpha^* + A^*}{\alpha^*} p_1^t + c_2\ p_2^t \tag{29}$$
$$\hat{\ell}_t = c_1 \frac{p_2 + \alpha^* + A^*}{\alpha^*} p_1^t + c_2\ p_2^t \tag{30}$$

where c_1 and c_2 are two numbers whose value is determined by the initial value of $\hat{k}_t$ and the terminal value of $\hat{\ell}_t$. Since $\hat{\ell}_\infty = 0$ and $p_2 < -1$, one has $c_2 = 0$. Then:

$$c_1 = \frac{\alpha^* \hat{k}_0}{p_1 + \alpha^* + A^*}.$$

Substituting this expression of c_1 into (29) yields

$$\hat{k}_t = p_1^t \hat{k}_0,$$

which gives (14) in the body of the paper.

Bubbly equilibrium

Let us first define α^b and ϵ^b as the same elasticities as those in (13) but evaluated at the bubbly steady state. The linearization of (15) around (k^b, ℓ^b, b) gives:

$$\hat{k}_{t+1} = \nu\alpha^b\hat{k}_t + \nu\left(1-\alpha^b\right)\hat{\ell}_t + (1-\nu)\,\hat{b}_t,$$

where

$$\alpha^b \equiv \frac{(k^b/\ell^b)\omega'(k^b/\ell^b)}{\omega(k^b/\ell^b)},\ \epsilon^b \equiv -\frac{\left(k^b/\ell^b\right)f''\left(k^b/\ell^b\right)}{f'\left(k^b/\ell^b\right)+1-\delta},$$
$$\text{and } \nu \equiv \frac{\omega(k^b/\ell^b)}{(1+n)\left(k^b/\ell^b\right)} = \frac{\gamma\left(f'^{-1}\left(n+\delta\right)\right)}{(1+n)}.$$

Note that $\nu > 1$ since $\gamma(k^*/\ell^*) = 1+n$, $\gamma(.)$ is decreasing in k/ℓ (by assumption) and $k^b/\ell^b < k^*/\ell^*$ (i.e., the bubble asymptotically crowds out capital per producer). Next, linearizing (9) gives:

$$\hat{k}_{t+1} - \hat{\ell}_{t+1} = A^b\hat{\ell}_t,$$

where:

$$A^b \equiv \frac{1}{\epsilon^b}\left[1+\left(\frac{1-\theta}{\theta}\right)\frac{f'\left(k^b/\ell^b\right)+1-\delta}{\lambda}\right] = \frac{1}{\epsilon^b}\left[1+\left(\frac{1-\theta}{\theta}\right)\frac{1+n}{\lambda}\right].$$

Finally, linearizing (16) yields:

$$\hat{b}_{t+1} = \hat{b}_t - \epsilon^b(\hat{k}_{t+1} - \hat{\ell}_{t+1}).$$

From the three linearized difference equations we can write the bubbly equilibrium in matrix form as $[\ \hat{k}_{t+1}\ \ \hat{\ell}_{t+1}\ \ \hat{b}_{t+1}\ ]' = N[\ \hat{k}_t\ \ \hat{\ell}_t\ \ \hat{b}_t\ ]'$, with:

$$N = \begin{bmatrix} \nu\alpha^b & \nu\left(1-\alpha^b\right) & 1-\nu \\ \nu\alpha^b & \nu\left(1-\alpha^b\right)-A^b & 1-\nu \\ 0 & -\epsilon^b A^b & 1 \end{bmatrix}.$$

The characteristic polynomial of N is:

$$\tilde{P}\left(p\right) = -p^3 + \left[1+\nu-A^b\right]p^2 + \left[A^b\nu\alpha^b - \nu + A^b + \epsilon^b A^b\left(\nu-1\right)\right]p - A^b\nu\alpha^b.$$

We determine the location of the roots of $\tilde{P}\left(p\right)=0$ by drawing $\tilde{P}\left(p\right)$ over $(-\infty,+\infty)$. First, note that $P\left(0\right) = -A^b\nu^b\alpha^b < 0$, $P\left(-\infty\right) = +\infty$ and $P\left(+\infty\right) = -\infty$. Moreover, we have that $P\left(1\right) = \epsilon^b A^b\left(\nu-1\right) > 0$, which implies that one of the roots (say $\tilde{p}_1$) lies between

0 and 1, while another (say $\tilde{p}_2$) lies in $(1, +\infty)$. The third root, $\tilde{p}_3$, is below -1 if and only if:

$$P(-1) = 2\left(1 + \nu^b - A^b(1 + \nu\alpha^b\right) + \epsilon^b A^b (1 - \nu) < 0$$

When k^b is close to k^*, our assumption throughout, ν is close to 1 and A^b, α^b and ϵ^b are close to A^*, α^* and ϵ^*, respectively. At $k^b = k^*$ the latter inequality becomes:

$$P(1) = 2(2 - A^*(1 + \alpha^*) < 0,$$

and is thus satisfied under condition (12). This implies that there is a neighborhood of k^* such that when k^b lies in this neighborhood the dynamics of the bubbly system has exactly one stable root. Then, the dynamics of capital in this neighborhood $\hat{k}_{t+1} = \tilde{p}_1 \hat{k}_t$, which gives (19) in the body of the paper.

C. Local dynamics of the model with skill heterogeneity

Bubbleless equilibrium

The bubbleless equilibrium is given by equations (22), (24) and (25). Defining $R_t \equiv 1 + r_t$ and linearizing (22) around the bubbleless steady states gives:

$$\hat{R}_t = (\alpha - 1)\varkappa^* \hat{k}_t + \beta \varkappa^* \hat{\ell}_t, \quad \text{with } \varkappa^* = \frac{\alpha \Omega k^{*\alpha-1} \ell^{*\beta}}{\alpha \Omega k^{*\alpha-1} \ell^{*\beta} + 1 - \delta} \in (0, 1].$$

Linearizing (24) gives:

$$\hat{\ell}_t = -\psi^* \hat{R}_{t+1}, \quad \text{with } \psi^* = \frac{(1-\alpha)\lambda\theta R^*}{[(1-\theta)R^* + \theta\lambda]((1-\alpha-\beta)\theta(R^* - \lambda) + \beta R^*)} > 0.$$

Finally, linearizing (25) gives:

$$\hat{k}_{t+1} = \alpha \hat{k}_t + \beta \hat{\ell}_t.$$

We obtain a two-dimensional system by substituting $\hat{R}_t$ (first equation) into the linearized expression for $\hat{\ell}_t$ (second equation). In matrix form, we have $\hat{x}_{t+1} = M_s \hat{x}_t$, where:

$$M_s = \begin{bmatrix} \alpha & \beta \\ \alpha(1-\alpha)/\beta & 1 - \alpha - 1/\beta\psi^*\varkappa^* \end{bmatrix}.$$

The characteristic polynomial of M_s is:

$$P_s(p) = p^2 - (1 - 1/\beta\psi^*\varkappa^*)\,p - \alpha/\beta\psi^*\varkappa^*,$$

so that $P_s(+\infty) = P_s(-\infty) = +\infty$, $P_s(0) = -\alpha/\beta\psi^* \varkappa^* < 0$ and $P_s(1) = (1-\alpha)/\beta\psi^*\varkappa^* > 0$. Hence, $P_s(p) = 0$ has one root, p_{s1}, that belongs to $(0,1)$. The other one is strictly less than -1 if and only if $P_s(-1) < 0$, that is, if and only if:

$$(1+\alpha)/\beta\psi^*\varkappa^* > 2.$$

Since $\varkappa^* \leq 1$, a sufficient condition for the inequality to be satisfied is $(1+\alpha)/\beta\psi^* > 2$, that is, after rearranging,

$$\frac{1+\alpha}{1-\alpha}\left[1+\left(\frac{1-\theta}{\theta}\right)\frac{R^*}{\lambda}\right]\left(1+\frac{(1-\alpha-\beta)\,\theta\,(R^*-\lambda)}{\beta R^*}\right) > 2.$$

This inequality is necessarily satisfied since, with the production function (21), the determinacy condition (12) (our assumption throughout) gives:

$$\frac{1+\alpha}{1-\alpha}\left[1+\left(\frac{1-\theta}{\theta}\right)\frac{R^*}{\lambda}\right] > 2,$$

while $(1-\alpha-\beta)\,\theta\,(R^*-\lambda)/\beta R^* > 0$. Thus, in the vicinity of the steady state the bubbleless dynamics has exactly one root inside the unique circle, p_{s1}. This implies that the bubbleless equilibrium exists and is locally unique.

Bubbly equilibrium

The bubbly equilibrium is given by equations (22), (24), (26) and (27). Linearizing (22) and (24) around the bubby steady states gives:

$$\hat{R}_t = (\alpha-1)\,\varkappa^b\hat{k}_t + \beta\varkappa^b\hat{\ell}_t, \text{ with } \varkappa^b = \frac{\alpha\Omega\left(k^b\right)^{\alpha-1}\ell^{b\beta}}{\alpha\Omega\left(k^b\right)^{\alpha-1}\left(\ell^b\right)^{\beta}+1-\delta} \in (0,1], \text{ and}$$
$$\hat{\ell}_t = -\psi^b\hat{R}_{t+1}, \text{ with } \psi^b = \frac{(1-\alpha)\,\lambda\theta R^b}{\left[(1-\theta)\,R^b+\theta\lambda\right]\left((1-\alpha-\beta)\,\theta\left(R^b-\lambda\right)+\beta R^b\right)} > 0.$$

Second, linearizing equation (26) around (k^b, ℓ^b) gives:

$$\hat{k}_{t+1} = \nu_s\alpha\hat{k}_t + \nu_s\beta\hat{\ell}_t + (1-\nu_s)\,\hat{b}_t,$$

where $\nu_s \equiv (1-\alpha)(n+\delta)/\alpha(1+n) > 1$ under inequality (28). Finally, (27) gives:

$$\hat{b}_{t+1} = \hat{b}_t + \hat{R}_{t+1}$$

The linearized bubbly system is three-dimensional (since $\hat{R}_{t+1}$ can be eliminated from the system). One can thus write the bubbly equilibrium under skill heterogeneity in matrix form as $[\ \hat{k}_{t+1}\ \ \hat{\ell}_{t+1}\ \ \hat{b}_{t+1}\ ]' = N_s[\ \hat{k}_t\ \ \hat{\ell}_t\ \ \hat{b}_t\ ]'$, with:

$$N_s = \begin{bmatrix} \nu_s \alpha & \nu_s \beta & 1-\nu_s \\ (1-\alpha)\,\nu_s \alpha/\beta & (1-\alpha)\,\nu_s - 1/\beta \varkappa^b \psi^b & (1-\alpha)\,(1-\nu_s)\,/\beta \\ 0 & -1/\psi^b & 1 \end{bmatrix}$$

The characteristic polynomial of N_s is:

$$\tilde{P}_s(p) = -p_s^3 + \left[1 + (1-\alpha)\,\nu_s - \frac{1}{\beta \varkappa^b \psi^b} + \nu_s \alpha\right] p^2$$
$$+ \left[-\left(\frac{1-\alpha}{\beta}\right) \frac{(1-\nu_s)}{\psi^b} - (1-\alpha)\,\nu_s + \frac{1}{\beta \varkappa^b \psi^b} - \nu_s \alpha + \frac{\nu_s \alpha}{\beta \varkappa^b \psi^b}\right] p - \frac{\nu_s \alpha}{\beta \varkappa^b \psi^b}.$$

Here again, the location of the roots of $\tilde{P}_s(p) = 0$ can be found by drawing $\tilde{P}_s(p)$ over $(-\infty, +\infty)$. Note that $\tilde{P}_s(-\infty) = +\infty$, $\tilde{P}_s(+\infty) = -\infty$, while

$$\tilde{P}_s(0) = -\frac{\nu_s \alpha}{\beta \varkappa^b \psi^b} < 0, \qquad \tilde{P}_s(1) = -\left(\frac{1-\alpha}{\beta}\right) \frac{(1-\nu_s)}{\psi^b} > 0.$$

This establishes the location of the first two roots, $\tilde{p}_{s1} \in (0,1)$ and $\tilde{p}_{s2} \in (1, +\infty)$. The third root, $\tilde{p}_{s3}$, is necessarily negative. A necessary and sufficient condition for $\tilde{p}_{s3} < -1$ (so that the equilibrium is locally unique) is that $\tilde{P}_s(-1) < 0$, that is,

$$1 + (1-\alpha)\,\nu_s - \frac{1}{\beta \varkappa^b \psi^b} + \nu_s \alpha + \left(\frac{1-\alpha}{\beta}\right) \frac{(1-\nu_s)}{2\psi^b} - \frac{\nu_s \alpha}{\beta \varkappa^b \psi^b} < 0.$$

This inequality is true provided that k^b is sufficiently close to k^* (or, equivalently, provided that b is sufficiently small). Indeed, as k^b approaches k^*, ν_s and ψ^b approach 1 and ψ^*, respectively, and the right hand side of the latter inequality approaches $2 - (1+\alpha)\,/\beta \varkappa^* \psi^*$. We know from the local dynamics of the bubbleless equilibrium above that $2 - (1+\alpha)\,/\beta \varkappa^* \psi^*$ is negative under condition (12).

D. Proof of Proposition 3

In steady state, the consumption of unskilled workers is $c_u(\theta) = w_u[\theta\lambda + (1-\theta)(1+r)]$, where $(w_u, r) = (w_u^*, r^*)$ or $\left(w_u^b, r^b\right)$, while $c_u(\theta)$ is continuous in θ over $[0,1]$. The first part of the proposition requires us to show that $c_u^b(1) < c_u^*(1)$, while $c_u^b(0) > c_u^*(0)$ (in case the $c_u^b(\theta)$-curve crosses the $c_u^*(\theta)$-curve more than once, θ_u^* is the crossing that is closest to $\theta = 1$). The first inequality holds if and only if $w_u^b < w_u^*$. Using the steady state counter part of (21) and (22) and rearranging, we can rewrite the steady state wage of unskilled workers,

w_u, as follows:

$$w_u = \left(\frac{1-\alpha-\beta}{u}\right)\Omega k^{\alpha}\ell^{\beta} = \left(\frac{1-\alpha-\beta}{\alpha u}\right)(\alpha\Omega)^{\frac{1}{1-\alpha}} \times \left(\frac{\ell^{\beta}}{(r+\delta)^{\alpha}}\right)^{\frac{1}{1-\alpha}},$$

where $(\ell, r) = (\ell^*, r^*)$ or $\left(\ell^b, r^b\right)$. We have that $r^b > r^*$ (our condition for the bubbly steady state to exist, assumed throughout), which in turn implies that $\ell^b < \ell^*\ \forall\theta \in [0,1]$ (see (24)). Hence $w_u^b < w_u^*$, which implies that $c_u^b(1) < c_u^*(1)$.

We now need to show that $c_u^b(0) > c_u^*(0)$. When $\theta = 0$, $\ell = 1$ (see (24) again) and hence

$$c_u(0) = w_u(1+r) = \left(\frac{1-\alpha-\beta}{\alpha u}\right)(\alpha\Omega)^{\frac{1}{1-\alpha}} \times \frac{1+r}{(r+\delta)^{\frac{\alpha}{1-\alpha}}}.$$

Computing the derivative $\partial c_u(0)/\partial r$, we find that it is positive whenever $(r+\delta)/(1+r) > \alpha/(1-\alpha)$. This inequality is satisfied at $r = r^*$, while $(r+\delta)/(1+r)$ is increasing in r; hence $c_u(0)$ is increasing in r over $\left[r^*, r^b\right]$, which implies that $c_u^b(0) > c_u^*(0)$.

Let us turn to second part of the proposition, which bears upon the asymptotic consumption level of skilled workers in the vicinity of θ_u^*. From (23) and the fact that the interest rate paid to a worker does not depend on whether he is skilled or not, we know that

$$\frac{c^*(\theta)}{c_u^*(\theta)} = \frac{\beta u}{(1-\alpha-\beta)\,\ell^*} < \frac{c^b(\theta)}{c_u^b(\theta)} = \frac{\beta u}{(1-\alpha-\beta)\,\ell^b},\ \forall\theta \in [0,1].$$

By definition, at $\theta = \theta_u^*$ we have that $c_u^*(\theta) = c_u^b(\theta)$, and hence $c^b(\theta_u^*) > c^*(\theta_u^*)$. Since both $c^b(\theta)$ and $c^*(\theta)$ are continuous in θ, $c^b(\theta) > c^*(\theta)$ provided that θ is sufficiently close to θ_u^*.