Leaving the door ajar:
Nonlinear pricing by a dominant firm

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Abstract

An incumbent firm and a buyer agree on a price-quantity schedule before the buyer negotiates with a rival firm. The rival's efficiency and the share of the demand he can address are unknown when the schedule is chosen. Incomplete information yields inefficient exclusion. We link the slope and the curvature of the optimal tariff to the distribution of the uncertainty, and investigate whether foreclosure is complete or partial. When the buyer's disposal costs are finite, he might buy more than needed with the sole purpose of qualifying for rebates. Contingent tariffs allow to overcome the opportunism problem.

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1 Introduction

Nonlinear pricing, an ubiquitous business conduct, keeps attracting the attention of competition agencies.[1] The firms under scrutiny had market power, and antitrust enforcers were concerned that quantitative rebates might have discouraged buyers from switching part of their requirements towards as efficient competitors (for fear of losing rebates). As a result, efficient competitors might have been driven out of the market or marginalized, and, in any case, prevented from selling the efficient quantity.

The nonlinear pricing literature has investigated second-order price discrimination under monopoly or oligopoly, see Wilson (1993). The monopolist's pricing problem has first been studied when consumers differ through a single unobserved characteristic (e.g. Mussa and Rosen (1978) and Maskin and Riley (1984), then extended in a multidimensional settings (Armstrong (1996), Rochet and Choné (1998) and Armstrong (1999)). Nonlinear pricing under oligopoly is studied in Armstrong and Vickers (2001), Martimort and Stole (2009), and Armstrong and Vickers (2010). The literature on competitive price discrimination has mainly explored simultaneous competition and often focused on symmetric equilibria.

In contrast, the present article considers a dominant firm on a market where a smaller firm is present and may challenge its position, at least to a certain extent. We adopt

[1] Among recent antitrust cases, Virgin/British Airways (Commission Decision 2000/74/EC of 14 July 1999), and in the U.S. Virgin v. British Airways, 69 F. Supp. 2d 571, 581, 582 (S.D.N.Y. 1999) as well as 257 F.3d 256 (2nd Cir. 2001), Concord Boat (United States Court of Appeals for the 8th Circuit - 207 F.3d 1039, 8th Cir. 2000), Michelin II (Commission decision of 20 June 2001 COMP/E-2/36.041/PO), Lepage's/3M (324 F.3d 141, 2003), Prokent-Tomra (COMP/E-1/38.113, 2006), and Intel (COMP/C-3 /37.990, 2009).

the basic incumbency setting with three players: an incumbent, a strategic entrant (or a competitive fringe), and a buyer.[2] We assume that the dominant firm and a large buyer commit on a price schedule prior to any negotiation with the competitor. As observed by Aghion and Bolton (1987) in the context of exclusive contracts, "when a buyer and a seller sign a contract, they have a monopoly power over the entrant. They can jointly determine what fee the entrant must pay in order to trade with the buyer." This insight applies under both complete and incomplete information. Nonlinear pricing by a dominant firm under complete information is now well understood.[3]

Our focus is on incomplete information.[4] We consider two dimensions of uncertainty. First, as in Aghion and Bolton (1987), the entrant's cost –or more generally, the surplus he creates with the buyer– is unknown to the buyer and the incumbent when they agree on a price schedule. Second, while we allow the buyer to split his purchases between suppliers, we assume that only a fraction of his requirements can be supplied by the entrant. This fraction constitutes the second characteristic of the entrant that is unknown to the buyer-incumbent pair.

As noticed by competition authorities, it is often unrealistic to assume that a buyer can shift all of his requirements within a relevant time period from the dominant supplier to a competitor. This can be due to demand-side or supply-side considerations. It may be the case that competitors are capacity constrained and cannot serve all of the demand of large customers. It may also be the case that the incumbent's product is a "must-stock" for retailers because only a fraction of final consumers is ready to experiment with competing products (regardless of their price). In both cases, within a relevant time horizon, the entrant can address only a fraction of the buyer's demand, which constitutes the maximum scale of entry. The incumbent faces no competition for the complementary part of his requirements. Ex ante, the size of the "captive market", and consequently that of the "contestable market," are uncertain.

The last important ingredient of our framework is the existence of disposal costs. We allow the buyer to purchase more than his requirements and assume that he can dispose of excess units at some cost. This gives rise to a problem of buyer opportunism. The buyer and the incumbent negotiate a price schedule that places competitive pressure on the entrant and forces him to sell at a low price. We show that this "rent-shifting" strategy involves marginal prices below marginal costs, and may even involve negative marginal prices. A negative marginal price allows the buyer to extract rents from the entrant, but also gives him an ex post incentive to buy more than he needs from the incumbent. While this opportunistic behavior is anticipated ex ante, it constrains the choice of the price schedule by the buyer-incumbent pair. The possibility of buyer opportunism depends on the magnitude of disposal costs, as the buyer has to get rid of units she does not consume

[2]Such a framework has been used to model incumbency and/or a dominance at least since Spence (1977, 1979) and Dixit (1979, 1980).

[3]Marx and Shaffer (1999) looks at two-part tariffs with a focus on below cost pricing, Marx and Shaffer (2004) studies how equilibrium is affected when certain classes of tariffs are forbidden, Marx and Shaffer (2007) focuses on the order of negotiation (Is it better for the buyer to negotiate first with the incumbent or with the entrant?), and Marx and Shaffer (2010) shows how bargaining powers affect profits and when break-up fees are used.

[4]Incomplete information differs from asymmetric information. For instance, in Majumdar and Shaffer (2009), a dominant firm resorts to nonlinear pricing to discriminate a buyer who is informed about the size of demand and who also sells a good provided by a competitive fringe –a situation with asymmetric information.

nor resale. Buyer opportunism is maximal under free disposal and does not exist when disposal costs are infinite.

The contributions of our analysis are threefold. First, we solve a multidimensional screening problem where the number of instruments is smaller than the dimension of unobserved heterogeneity, and we do so for general distributions of the heterogeneity. Second, the generality of the analysis allows to assess the robustness of the Aghion-Bolton framework and to reveal new properties, such as the curvature of optimal price schedules and partial foreclosure. Third, we explain how conditioning the tariff on the quantity purchased from the competing supplier (when feasible) allows to eliminate buyer opportunism, and thus to achieve the same outcome as if disposal costs were infinite. We now explain each of these contributions in more details.

First, we contribute to the nonlinear pricing literature. In our framework, the type of the entrant has two components: the surplus he creates with the buyer and the maximum scale of entry. However, to screen out the entrant's types, the incumbent has only one instrument, namely a price-quantity schedule. This configuration, which generates extensive pooling, has received little attention.[5] Here, the structure of the model makes it possible to characterize the set of implementable allocations, and to construct the solution with few restrictive assumptions on the distribution of the uncertainty. As explained below, the equilibrium pattern of the pooling regions reflects the barriers to entry and to expansion created by the optimal tariff.

Second we link the curvature of optimal price schedules and the form of inefficient exclusion (partial versus full foreclosure) to two structural parameters of the model: the entrant's bargaining power and to the elasticity of entry. The former parameter is zero in the case of a competitive fringe and is, in general, positive in the case of strategic entrants. The latter parameter expresses how entry at a given scale is sensitive to the competitive pressure exerted by the incumbent. It is an key statistics summarizing the two-dimensional distribution of the entrant's characteristics. Our findings can be summarized as follows.

When the size of the contestable demand is known to the buyer and the incumbent (only the entrant surplus is unknown), the pricing problem involves a standard tradeoff between rent extraction and efficiency, and is a mere reformulation of the Aghion-Bolton analysis. Some efficient competitors are foreclosed, and the extent of inefficient exclusion decreases with the entrant's bargaining power and the elasticity of entry. The optimal schedule is a two-part tariff, inefficient exclusion arises in the form of full foreclosure only: no partial foreclosure is observed. These results readily extend under two-dimensional uncertainty, provided that the two unknown parameters are statistically independent or, equivalently, that the elasticity of entry remains constant with the size of the contestable market. We now turn to cases where the elasticity of entry varies with the size of the contestable market.

When the elasticity of entry increases with the size of the contestable share of the demand, the optimal policy of the buyer-incumbent pair is to reduce the competitive pressure exerted on the entrant as the scale of entry increases, which cannot be achieved with two-part tariffs. Optimal tariffs are shown to be concave for high quantities. Here again,

[5] A notable exception is Laffont, Maskin, and Rochet (1987) who solve an example with uniform distributions.

inefficient exclusion arises, in the form of full foreclosure only. Solving the efficiency-rent tradeoff à la Aghion-Bolton separately for any given level of the contestable demand (i.e. solving the "relaxed problem") yields the optimal tariff.

When the elasticity of entry decreases in or is non monotonic with the contestable demand, the solution to the relaxed problem is not incentive compatible as entrants with a large contestable demand would mimic entrants with a smaller one at the relaxed allocation. The incentive compatibility constraints translate into convex parts of the tariff and into partial foreclosure at the optimum. Some efficient entrants sell a positive quantity but are prevented from achieving the maximum scale of entry and serving all of the contestable share of demand. When the elasticity of entry first decreases, then increases as the maximum scale of entry rises, the buyer and the incumbent want to be soft with entrants with small and large contestable markets and to be aggressive with entrants with intermediate contestable markets. This tension generates highly nonlinear tariffs that induce many entrants to choose the same quantity, as is the case with so-called "retroactive rebates" challenged by European competition agencies in recent cases.

Finally, we contribute to the literature on market-share rebates.[6] Specifically, we allow the buyer and the incumbent to condition the negotiated price schedule on the number of units purchased from the entrant, and we show that this instrument allows them to overcome the problem of buyer opportunism. When the price-quantity schedule only depends on the number of units purchased from the incumbent, placing competitive pressure on the buyer may involve negative marginal prices, which, in turn, trigger buyer opportunism if the buyer can freely dispose of excess units and, to a lesser extent, if disposal costs are finite. Our analysis, however, straightforwardly extends to the case where the tariff also depends on the number of units purchased from the entrant. We show that the same quantity allocations are implementable under conditional rebates. There is a huge difference, though: conditional tariffs allow to exert competitive pressure on the buyer without resorting to negative marginal prices (i.e. to negative price for marginal units sold by the incumbent). Competitive pressure can instead come from the implicit price of marginal units sold by the entrant. Thus, conditional rebates, when feasible, allow to eliminate buyer opportunism and to achieve the same outcome as under infinite disposal costs.

The article is organized as follows. For ease of exposition, we assume first that disposal costs are infinite, thus abstracting away from the issue of buyer opportunism. Section 2 introduces the model. Section 3 explains how the negotiation between the buyer and the entrant, which takes place under complete information, is affected by the incumbent's price schedule. Section 4 focuses on the negotiation between the buyer and the incumbent. This negotiation takes place under incomplete information. It introduces the notion of virtual surplus and of elasticity of entry before characterizing the construction of the optimal price schedule. This section also relates the shape of the optimal negotiated price schedule to the primitive of the model. Section 5 introduces finite disposal costs. It extends the previous results, discusses the opportunism of the buyer and shows how a market-share tariff could overcome it. Section 6 we discuss some extensions of the model

[6] Inderst and Shaffer (2010) assume complete information and study a setting with a dominant firm, a competitive fringe and two retailers. They show that market-share rebates are used by the dominant firm to dampen (intra- and inter-brand) competition. Calzolari and Denicolo (2009) address the issue in a duopoly setting (simultaneous game, symmetric firm).

as well as antitrust implications.

2 The model

A buyer, B, may purchase from two firms, E and I. We call firm I the incumbent and firm E the entrant although the game under study does not involve a genuine entry decision.[7] This terminology is convenient to convey the idea that firms are asymmetric. The asymmetry is twofold: the incumbent is first to negotiate a tariff with the buyer; the incumbent can serve all the demand while the entrant can serve only a fraction of it. We call this fraction the "contestable" part of the demand.

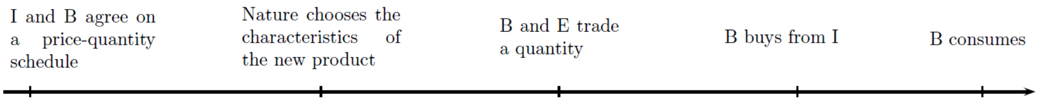

Figure 1: Timing of the game

The timing, sketched in Figure 1, reflects the incumbency advantage of the dominant firm. It is a four stage game which unfolds as follows. First, the buyer and the incumbent negotiate a price-quantity schedule. Formally, if the buyer eventually purchases q_I units from the incumbent, he will pay $T(q_I)$. In Section 5, we solve the game when the buyer and the incumbent can condition the tariff on the quantity, q_E, supplied from the entrant, i.e. they use a tariff of the form $T(q_E, q_I)$. The characteristics of the incumbent's good are common knowledge: its constant marginal cost of production is c_I and the buyer's gross benefit per unit is v_I. Next, the entrant and the buyer observe the characteristics of the new product: its marginal cost of production c_E, the size of s_E of the contestable demand and the gross (per unit) benefit, v_E, for the buyer. Then, the buyer and the entrant, both knowing the terms of the agreement between the buyer and the incumbent, agree on a price and a quantity.[8] This negotiation takes place under complete information and is assumed to be efficient. Finally, the buyer purchases from the incumbent. The buyer's total demand is inelastic, and normalized to one: $q_E + q_I \leq 1$.

The assumption that firm E can address at most a fraction, s_E, of the buyer's demand embodies two interpretations: a supply-side variant in which firm E has capacity s_E and a demand-side variant where the buyer does not value units of good E in excess of s_E. In both cases, the buyer never purchases more than s_E from the entrant: $q_E \leq s_E$.

Given v_E and v_I the buyer's gross benefit per unit of goods E and I, and the quantities $q_E \leq s_E$ and q_I purchased respectively from the entrant and the incumbent, the buyer's gross profit is:

$$V(q_E, q_I) = \begin{cases} v_E q_E + v_I q_I, & \text{if } q_E + q_I \leq 1 \\ -\infty & \text{otherwise.} \end{cases} \tag{1}$$

The above specification assumes infinitely large disposal costs: failing to consume all of the purchased units is infinitely costly, and hence all the purchased units are indeed

[7] In particular it does not feature a sunk entry cost incurred by firm E.

[8] We assume that the buyer and the incumbent cannot renegotiate their agreement once uncertainty is resolved. Otherwise they would agree on a tariff under complete information and appropriate all the surplus (see the end of this section). The contribution of the current paper is, on the contrary, to study the form of the price schedule negotiated under incomplete information.

consumed. This assumption is maintained in Section 3 and 4, and relaxed in Section 5, where finite disposal costs are introduced.

We note $\omega_E = v_E - c_E \geq 0$ the unit surplus generated by good E, and $\omega_I = v_I - c_I \geq 0$ the unitary surplus of good I. At the time of agreeing on the price schedule, the size of the contestable demand, s_E, and the surplus per unit of good E, ω_E, are uncertain. We denote by $[\underline{s}_E, \bar{s}_E]$ and by $[\underline{\omega}_E, \bar{\omega}_E]$ the supports of the random variables s_E and ω_E. The cumulative distribution function of s_E, denoted by G, is assumed to admit a positive and continuous density function g. The distribution of ω_E conditional on s_E is denoted by $F(.|s_E)$ and is assumed to admit a positive and continuous density function, $f(.|s_E)$.

Efficiency benchmark. The total surplus is $W(q_E, q_I) = \omega_E q_E + \omega_I q_I$ if $q_E + q_I \leq 1$ and $-\infty$ otherwise. The first best allocation maximizes W under the constraint $q_E \leq s_E$. Efficiency requires $q_E + q_I = 1$, because W increases with both quantities as long $q_E + q_I \leq 1$. Hence, at the the first best, the quantity purchased from the entrant satisfies

$$\omega_I + \max_{q_E \leq s_E} (\omega_E - \omega_I) q_E, \tag{2}$$

and hence is given by

$$q_E^*(s_E, \omega_E) = \begin{cases} s_E & \text{if } \omega_E \geq \omega_I \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Entry, if efficient, should occur at maximum scale. Hence the maximal value of total surplus is $\omega_E s_E + \omega_I(1 - s_E)$ when $\omega_E \geq \omega_I$, and ω_I when $\omega_I \geq \omega_E$.

Second best. The negotiation between the buyer and the entrant, studied in Section 3, takes place under complete information and is assumed to be efficient. The buyer and the entrant maximize their joint surplus, which they share according to their outside options and relative bargaining power. Ex ante, the buyer and the incumbent design the price schedule to maximize their expected joint surplus, equal to the total surplus minus the profit left to the entrant, denoted by Π_E:

$$\mathbb{E}\Pi_{BI} = \mathbb{E}\left\{W(q_E, q_I) - \Pi_E\right\}. \tag{4}$$

The sharing of the expected joint surplus between the buyer and the incumbent, and hence the respective bargaining power of each party, play no role in the following analysis.

Complete information. Suppose the entrant is efficient and the buyer and the incumbent know the surplus per unit of good E, $\omega_E > \omega_I$. Then they agree on a two-part tariff with slope slightly above $v_I - \omega_E$, thus offering a surplus slightly below ω_E for each unit of good I. As units of good E create a slightly higher surplus and the negotiation between the buyer and the entrant is efficient, the buyer purchases all contestable units from the entrant. The incumbent sells the remaining units: the allocation is socially efficient. To sell units to the buyer, the entrant must match the incumbent's offer, and thus is left with negligible profit. The buyer and the incumbent therefore appropriate the entire surplus, $\omega_E s_E + \omega_I(1 - s_E)$.

As the slope $v_I - \omega_E$ does not depend on s_E, the above analysis holds when the buyer and the incumbent do not know the size of the contestable market.[9] The complete

[9] The fixed part of the tariff determines the sharing of the surplus between the buyer and the incumbent. When s_E in unknown, the same is true for the expected surplus, $\omega_E \mathbb{E}(s_E) + \omega_I[1 - \mathbb{E}(s_E)]$.

information environment is studied with more general demand functions in Marx and Shaffer (1999) and Marx and Shaffer (2004).

3 Negotiation between the buyer and the entrant

In subsection 3.1, we describe the negotiation between the buyer and the entrant, which takes place under complete information: the parties maximize their joint surplus, knowing the incumbent's price schedule T, and share this surplus according to their relative bargaining power and outside options. In subsection 3.2, we provide a number of examples showing how the quantity purchased from the entrant depends on the shape of the tariff. Finally, in Subsection 3.3, we formally characterize the set of all implementable allocations.

3.1 Maximization of the joint surplus

After having purchased q_E units from the entrant, the buyer chooses q_I to solve

$$U_B(q_E) = \max_{q_I} V(q_E, q_I) - T(q_I). \tag{5}$$

Anticipating the above decision regarding q_I, the buyer and the entrant choose q_E to maximize their joint surplus

$$S_{BE}(c_E, s_E, v_E) = \max_{q_E \leq s_E} U_B(q_E) - c_E q_E, \tag{6}$$

The price schedule $T(.)$ is, on the contrary, key in the definition of this surplus as $U_B(q_E)$ depends on $T(.)$. The buyer and the entrant share S_{BE} according to their respective bargaining power and outside options. The entrant's outside option is normalized to zero. As to the buyer, he may source exclusively from the incumbent, so his outside option is $U_B(0)$. It follows that the surplus created by the relationship between B and E is given by

$$\Delta S_{BE}(c_E, s_E, v_E) = S_{BE}(c_E, s_E, v_E) - U_B(0).$$

Denoting by $\beta \in (0, 1)$ the entrant's bargaining power vis-à-vis the buyer, the entrant gets Π_E and the buyer gets Π_B given by

$$\begin{array}{rcccc} \Pi_E = & 0 & + & \beta & \Delta S_{BE} \\ \Pi_B = & U_B(0) & + & (1-\beta) & \Delta S_{BE}. \end{array}$$

If $\beta = 0$, the entrant has no bargaining power and may be seen as a competitive fringe from which the buyer can purchase any quantity at price c_E. On the contrary, the case $\beta = 1$ happens when the entrant has all the bargaining power vis-à-vis the buyer.

Now we observe that the quantity purchased from the incumbent, solution to (5), is ex post efficient, i.e. maximizes the joint surplus of the buyer-incumbent pair given q_E. In other words, the total quantity purchased by the buyer exactly meets his demand: $q_E + q_I = 1$. On the one hand, the buyer does not purchase more than his total requirements, because disposal costs are assumed to be infinite; hence the solution to problem (5) satisfies $q_I \leq 1 - q_E$ for all q_E. On the other hand, buying less than $1 - q_E$ from the incumbent would destroy surplus as $v_I > c_I$. Lemma A.1 in appendix formally shows that the buyer and the incumbent, when choosing the tariff T, have both the ability and the incentive to make sure that, for any q_E, the buyer will purchase at least $1 - q_E$ from the incumbent after having purchased q_E from the entrant. We may thus conclude that $q_E + q_I = 1$ at the second-best optimum.

Replacing q_I with $1 - q_E$ in (5) and noting that the joint surplus of the buyer and the entrant depends on c_E and v_E only through ω_E, we can write

$$S_{BE}(s_E, \omega_E) = v_I + \max_{q_E \leq s_E} (\omega_E - v_I)q_E - T(1 - q_E).$$

As the buyer's outside option is $U_B(0) = v_I - T(1)$, the surplus from the trade between the buyer and the entrant is

$$\Delta S_{BE}(s_E, \omega_E) = \max_{q_E \leq s_E} (\omega_E - v_I)q_E - T(1 - q_E) + T(1). \tag{7}$$

For any s_E, the function $\Delta S_E(s_E, .)$ is the upper bound of a family of affine functions of ω_E, and hence is convex in ω_E. It follows that $\Delta S_E(s_E, \omega_E)$ is differentiable with respect to ω_E, except possibly at countably many points. By the envelope theorem, its derivative with respect to ω_E is $q_E(s_E, \omega_E)$, solution to (7).[10] Hence, the function $q_E(s_E, \omega_E)$ is nondecreasing in ω_E. We have:

$$\Delta S_{BE}(s_E, \omega_E) = \int_{\underline{\omega}_E}^{\omega_E} q_E(s_E, x)\, \mathrm{d}x. \tag{8}$$

Moreover, it follows from (7) and (8) that q_E and ΔS_{BE} are nondecreasing in s_E. The buyer purchases more units from the entrant as the surplus per entrant's unit, ω_E, and the size of the contestable demand, s_E, rise.

3.2 Examples: Concave, linear, convex tariffs

The problem of the buyer-entrant pair's is not necessarily concave. Specifically, the objective in (7) is convex (concave) if and only if T is concave (convex). In any case, the price schedule is relevant only in the interval $[1 - \bar{s}_E, 1]$, because the entrant cannot sell more than $\bar{s}_E$. This section provides three illustrative examples.

We consider first the case where the tariff T is concave on the relevant range, $[1-\bar{s}_E, 1]$, and hence the objective in (7) is globally convex. The maximum is reached either at $q_E = 0$ or at $q_E = s_E$. The buyer purchases $q_E = s_E$ from the entrant if and only if

$$(\omega_E - v_I)s_E - T(1 - s_E) + T(1) \geq 0$$

or $\omega_E - v_I \geq p^{\mathrm{e}}(s_E)$, where $p^{\mathrm{e}}(s_E)$ is the average price of the last s_E units sold by the incumbent:

$$p^{\mathrm{e}}(s_E) = \frac{T(1) - T(1 - s_E)}{s_E}. \tag{9}$$

[10] For any s_E, the set of solutions to problem (7) is included in the subgradient of the convex function $\Delta S_{BE}(s_E, .)$. At points where $\Delta S_{BE}(s_E, .)$ is differentiable, the subgradient consists of a single point, namely the derivative of ΔS_{BE} with respect to ω_E: the solution of (7) is unique. At points where $\Delta S_{BE}(s_E, .)$ has a convex kink, the subgradient is an interval, see Rockafellar (1997).

Supplying all contestable units from the entrant ($q_E = s_E$) is efficient for the buyer-entrant's pair if and only if the joint surplus thus created, $\omega_E s_E$, exceeds the net surplus foregone by not purchasing the corresponding units from the incumbent, $(v_I - p^{\mathrm{e}}(s_E))s_E$. Geometrically, the effective price $p^{\mathrm{e}}(q_E)$ is the slope of the chord that connects the points $(1, T(1))$ and $(1 - q_E, T(1 - q_E))$, see the left panel of Figure 2.

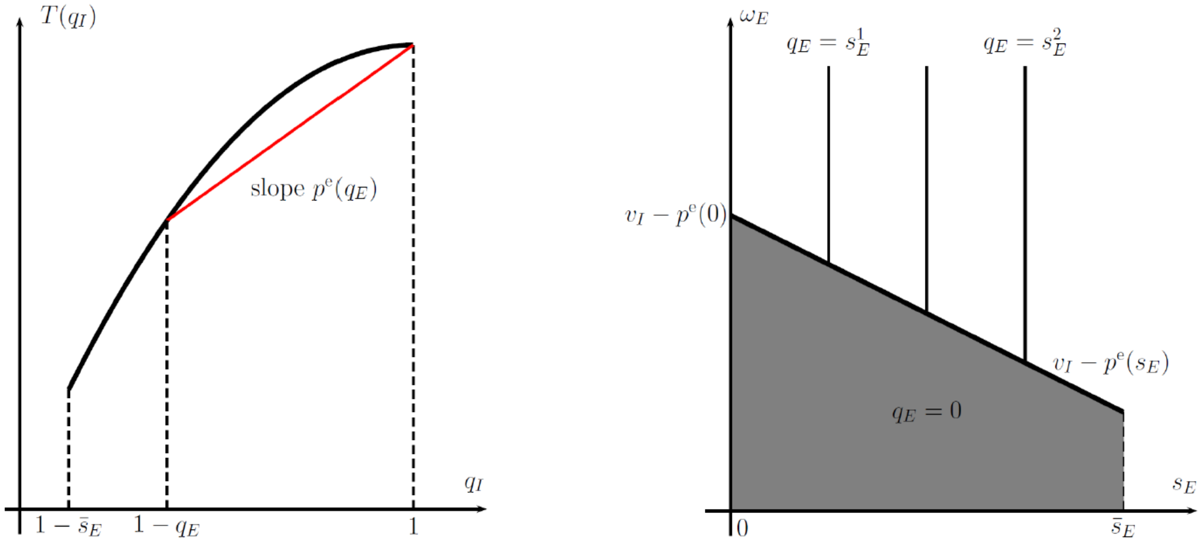

Figure 2: The buyer and the entrant choose q_E under a concave tariff

The right panel of Figure 2 represents the curve with equation $\omega_E = v_I - p^{\mathrm{e}}(s_E)$ in the (s_E, ω_E)-plan. This curve is decreasing by concavity of the tariff. Below the curve (shaded area), the entrant is inactive, $q_E = 0$. Above the curve, the buyer supplies all contestable units from the entrant, $q_E = s_E$, and hence the quantity isolines, i.e. the sets of types for which the quantity is constant, are vertical.

The above analysis holds in particular when the tariff is affine or, equivalently, when the incumbent's effective price $p^{\mathrm{e}}(q_E)$ is constant. This case is represented on Figure 3. Setting the effective price at p^{e} amounts to offering the surplus $v_I - p^{\mathrm{e}}$ per unit of good I. To serve the buyer, the entrant has to match this offer. Hence, entrants with ω_E above (below) $v_I - p^{\mathrm{e}}$ serve all of the contestable demand (are inactive). The efficient quantity, q_E^*, obtains when p^{e} is constant and equal to c_I.

When the price schedule T is strictly convex, the program (7) is concave and has a unique solution, which may or may not be interior. For ω_E higher than $v_I - T'(1 - s_E)$, the solution of (7) is $q_E = s_E$: the entrant serves all of the contestable demand. For ω_E lower than $v_I - T'(1)$, the solution is $q_E = 0$: the entrant is inactive. For ω_E between these two values, the solution is interior, and is given by the first-order condition $\omega_E - v_I + T'(1 - s_E) = 0$: the entrant is active, but serves less than the contestable demand. The right panel of Figure 4 represents in the (s_E, ω_E)-plan the curve with equation $\omega_E = v_I - T'(1 - s_E)$, which is increasing by convexity of the tariff. The quantity isolines are "L"-shaped, with the vertical part above the curve and the horizontal part below.

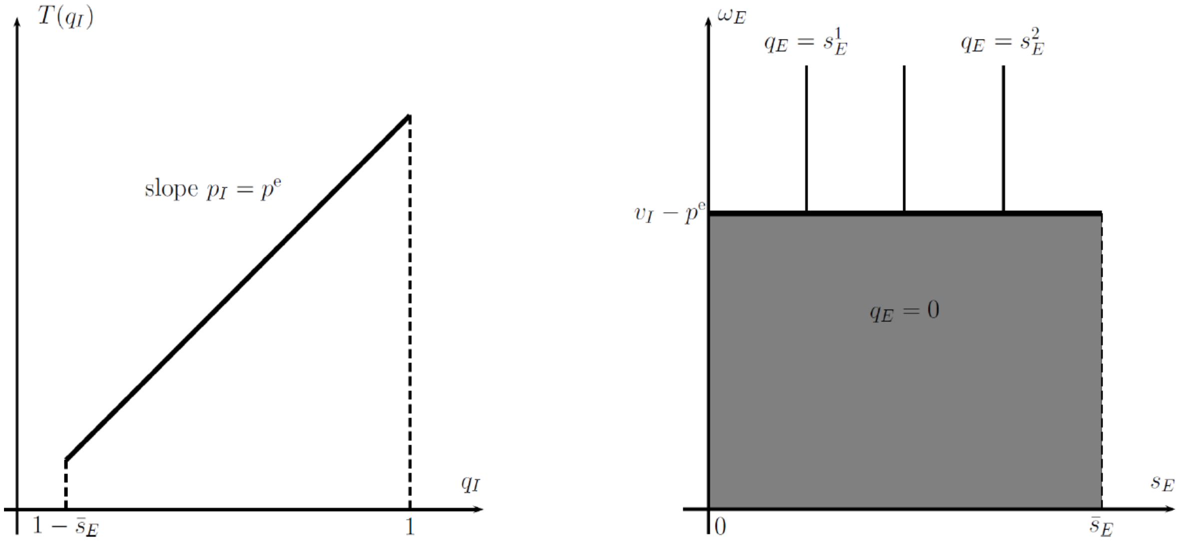

Figure 3: The buyer and the entrant choose q_E under a linear tariff

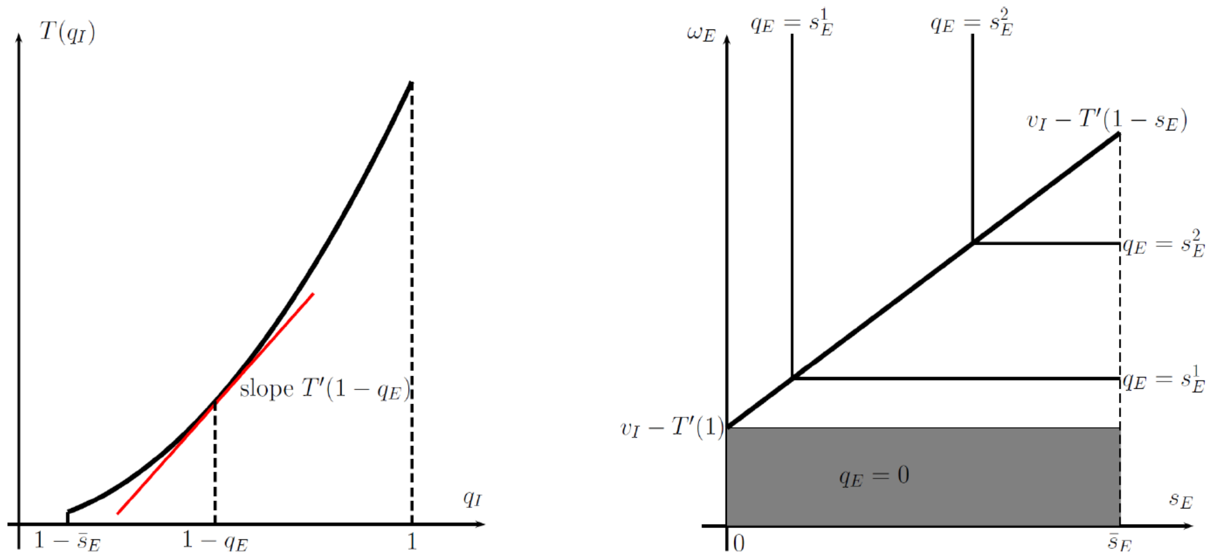

Figure 4: The buyer and the entrant choose q_E under a convex tariff

3.3 Implementable quantity functions

The buyer and the entrant negotiate under complete information and choose a quantity q_E that depends on the entrant's characteristics, (s_E, ω_E). A quantity function $q_E(s_E, \omega_E)$ from $[\underline{s}_E, \bar{s}_E] \times [\underline{\omega}_E, \bar{\omega}_E]$ to $[0, 1]$ is implementable if and only if there exists a tariff T such that q_E is solution to (7) for all (s_E, ω_E).

In this section, we show that any implementable quantity function $q_E(s_E, \omega_E)$ may be represented by a boundary line in the (s_E, ω_E)-plan such that $q_E = s_E$ above the boundary and q_E does not depend on s_E below the boundary. Such boundary lines have equations of the form $\omega_E = \Psi(s_E)$, where Ψ is called a boundary function. We demonstrate below the existence of a one-to-one map between quantity functions $q_E(s_E, \omega_E)$

and boundary functions $\Psi(s_E)$. To solve the two-dimensional problem, it turns out to be convenient to work with boundary functions rather than directly with quantity functions.

As q_E is nondecreasing in ω_E, there exists, for any $s_E > 0$, a threshold $\Psi(s_E)$ such that the buyer supplies all contestable units from the entrant, $q_E(s_E, \omega_E) = s_E$, if and only if $\omega_E > \Psi(s_E)$. We define the *boundary function* $\Psi(s_E)$ associated to the quantity function $q_E(s_E, \omega_E)$ by

$$\Psi(s_E) = \inf\{x \in [\underline{\omega}_E, \bar{\omega}_E] \mid q_E(x, s_E) = s_E\},$$

with the convention $\Psi(s_E) = \bar{\omega}_E$ when the above set is empty. Because the quantity function $q_E(s_E, \omega_E)$ is nondecreasing in s_E and constant below the boundary, we have:

$$q_E(s_E, \omega_E) = \begin{cases} \min\{\ x \leq s_E \mid \Psi(y) \geq \omega_E \text{ for all } y \in [x, s_E]\} & \text{if } \Psi(s_E) > \omega_E, \\ s_E & \text{if } \Psi(s_E) \leq \omega_E. \end{cases} \tag{10}$$

For type A (resp. B) on Figure 5, we have $\Psi(s_E) < \omega_E$ (resp. $\Psi(s_E) > \omega_E$) and the solution of the problem (7) is unique and equal to s_E^2. In contrast, type C is indifferent between s_E^2 and s_E^3 and, by convention, is assumed to choose s_E^3. In other words, when (7) has multiple solutions, equation (10) selects the highest.

The quantity q_E is continuous (discontinuous) when crossing increasing (decreasing) parts of the boundary $\omega_E = \Psi(s_E)$. Alternatively put, the constraint $q_E \leq s_E$ in problem (7) is binding (slack) on decreasing (nondecreasing) parts of the boundary. In Appendix B.1, we explain how to recover the price schedule T from the boundary function Ψ, thus proving the sufficient part, and thus prove next result.

Lemma 1. *A quantity function $q_E(.,.)$ is implementable if and only if there exists a boundary function $\Psi(.)$ defined on $[0, 1]$ such that (10) holds.*

Pooling areas and foreclosure (partial versus complete) The pooling sets, i.e. the sets on which the quantity $q_E(s_E, \omega_E)$ is constant, can be one- or two-dimensional. As shown on Figure 5, one-dimensional pooling sets can be of two types: (i) vertical lines above points on the boundary line where that line decreases; (ii) "L"-shaped unions of vertical lines above and horizontal lines at the right of points where the boundary line increases. There always exists a two-dimensional pooling area, namely the region where the quantity is zero. Other two-dimensional pooling sets exist in regions where the boundary line increases and has a vertical part, see e.g. Figures 13a and 13b.

Increasing parts of the boundary function thus translate into horizontal pooling segments or two-dimensional pooling areas, and hence into partial foreclosure: $0 < q_E(s_E, \omega_E) < s_E$ for some types located below the boundary. In such regions, the constraint $q_E \leq s_E$ is slack: increasing s_E does not allow the competitor to enter at a larger scale and q_E does not depend on s_E.

Shape of the boundary line and curvature of the tariff As formally stated in Lemma B.1, flat parts of the boundary line correspond to linear parts of the tariff (see Figure 3) and increasing parts of the boundary line correspond to convex parts of the tariff (see Figure 4). In both cases, the constraint $q_E \leq s_E$ in the buyer-entrant pair's problem (7) is not binding.

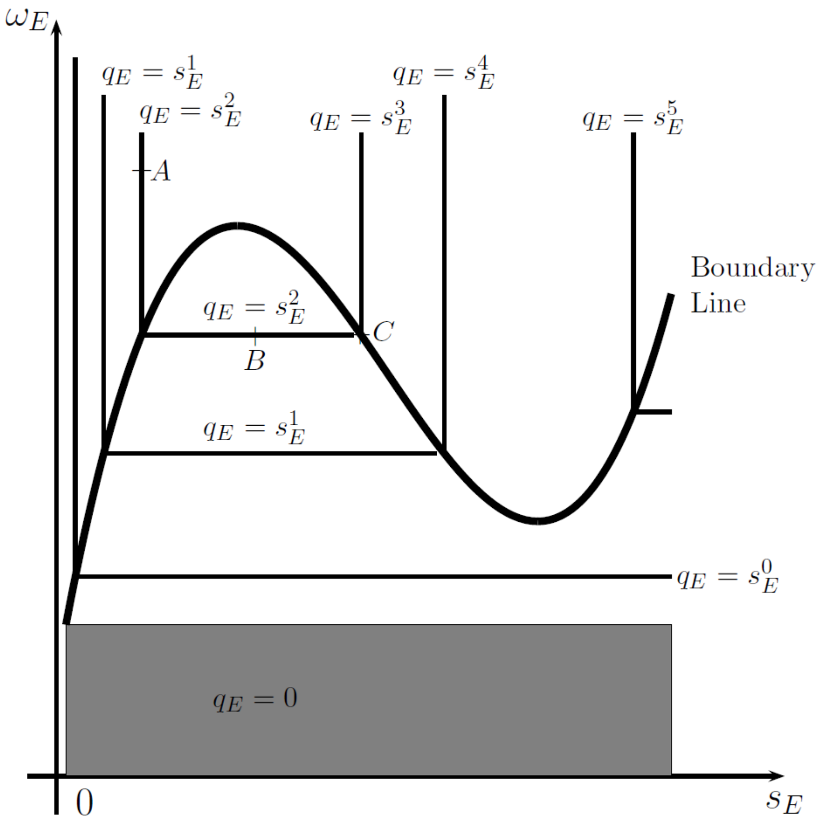

Figure 5: Implementable quantity function (isolines)

In contrast, the curvature of the tariff may change along decreasing parts of the boundary: the tariff is concave near local maxima of the boundary line and convex near local minima, see equation (B.3) in appendix and Figures 8a, 8b, 9a, and 9b. Local maxima of the boundary line thus correspond to inflection points of the tariff. An example is the point $A3$ on Figures 12a and 12b.

Finally, it is worthwhile noticing that upward and downward discontinuities in the boundary line have different interpretations in terms of price schedule. Upward discontinuities of the boundary line correspond to convex kinks in the tariff, see Figures 13a and 13b. Downward discontinuities of the boundary correspond to upward discontinuities of the tariff, see Figures 14a and 14b.

4 Designing the price schedule

The buyer and the incumbent design the price schedule so as to maximize their joint surplus, given by (4). Using $q_I = 1 - q_E$ and replacing Π_E with the value derived in Section 3.1, we can rewrite their common objective as

$$\mathbb{E}\Pi_{BI} = \omega_I + \mathbb{E}\left\{(\omega_E - \omega_I)q_E - \beta\Delta S_{BE}\right\}. \tag{11}$$

To solve the buyer-incumbent pair's problem, we rely on the duality, exposed in Section 3, between the incumbent's price schedule, T, and the quantity purchased from the entrant, q_E. We look for the quantity function q_E, then we recover the price schedule T from this function.[11] Section 4.1 expresses the problem in terms of q_E, by introducing the notion of virtual surplus. The maximization of the virtual surplus, ignoring incentive compatibility, gives rises to a relaxed problem. Section 4.2 presents cases where the solution of the relaxed problem is incentive compatible. etc.

4.1 Virtual surplus and elasticity of entry

Expanding (11), we write the joint expected surplus of the buyer-incumbent pair as:

$$\mathbb{E}\Pi_{BI} = \omega_I + \int_{s_E}\int_{\underline{\omega}_E}^{\bar{\omega}_E} \left\{(\omega_E - \omega_I)q_E - \beta\Delta S_{BE}\right\} \mathrm{d}F(\omega_E|s_E)\,\mathrm{d}G(s_E).$$

Using (8) and integrating the rent term $\beta\Delta S_{BE} f$ by parts with respect to ω_E, for each s_E, yields

$$\mathbb{E}\Pi_{BI} = \omega_I + \int_{s_E}\int_{\underline{\omega}_E}^{\bar{\omega}_E} S^{\mathrm{v}}(q_E; s_E, \omega_E)\,\mathrm{d}F(\omega_E|s_E)\,\mathrm{d}G(s_E), \tag{12}$$

where, following Jullien (2000), we have defined the "virtual surplus" S^{v} as

$$S^{\mathrm{v}}(q_E, s_E, \omega_E) = \left[\omega_E - \omega_I - \beta\,\frac{1 - F(\omega_E|s_E)}{f(\omega_E|s_E)}\right] q_E.$$

The virtual surplus is the total surplus $W(q_E, 1 - q_E)$ adjusted for the informational rents $\beta q_E\left(1 - F(\omega_E|s_E)\right)/f(\omega_E|s_E)$ induced by the self-selection constraints. The virtual surplus depends linearly on the quantity q_E. Hereafter, the bracketed term in the above equation is called "virtual surplus per unit" and denoted by s^{v}.

[11] In fact, the tariff will be determined only up to an additive constant, which reflects the sharing of the expected surplus between the buyer and the incumbent.

As observed in Section 3.2, setting a constant effective price p^{e} amounts to offering the surplus $v_I - p^{\mathrm{e}}$ per unit of good I. Entrants with ω_E above (below) this value serve all of the contestable demand (are inactive). The fraction of active entrants, for a given size of the contestable demand, s_E, is thus $1 - F(v_I - p^{\mathrm{e}}|s_E)$. Decreasing the effective price, i.e. increasing the offered surplus, places more competitive pressure on the entrant, and hence reduces the fraction of active entrants. This leads us to define the elasticity of entry by

$$\varepsilon(\omega_E|s_E) = \frac{\omega_E f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}. \tag{13}$$

The virtual surplus per unit can be rewritten as $s^{\mathrm{v}} = \omega_E(1 - \beta/\varepsilon) - \omega_I$. Throughout the paper, we maintain the following assumption.

Assumption 1. *For any given size of the contestable demand s_E, the elasticity of entry, $\varepsilon(\omega_E|s_E)$, is nondecreasing in ω_E and its limit at $\bar{\omega}_E$ is greater than one.*

Assumption 1 holds in particular when the hazard rate $f(\omega_E|s_E)/(1 - F(\omega_E|s_E))$ is nondecreasing in ω_E, a usual assumption in the nonlinear pricing literature. It is also true in the limit case where the elasticity does not depend on ω_E; this happens when ω_E, conditionally on s_E, follows a Pareto distribution, given by $1 - F(\omega_E|s_E) = (\omega_E/\underline{\omega}_E)^{-\varepsilon(s_E)}$: the elasticity of entry is then constant in ω_E and equal to $\varepsilon(s_E)$.

The variations of the elasticity of entry with s_E are related to the statistical link between the random variables s_E and ω_E. The relationship is stated in Lemma 2, proved in Appendix C.1.

Lemma 2. *The elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E if and only if the random variables s_E and ω_E are independent.*

If the elasticity of entry increases (decreases) with s_E, then ω_E first-order stochastically decreases (increases) with s_E.

The buyer and the incumbent maximize the expected virtual surplus, given by (12), over all implementable quantity function q_E. To solve this problem, we first ignore the implementability conditions derived in Section 3.3 and maximize (12) over all quantity functions. This is what we call the "relaxed problem". We denote by q_E^{r} its solution. If q_E^{r} is implementable, then it is the solution of the complete problem. To avoid uninteresting corner solutions, we assume hereafter that the entrant may *a priori* be more or less efficient than the incumbent.

Proposition 1. *Assume $\underline{\omega}_E < \omega_I < \bar{\omega}_E$. The solution of the relaxed problem is given by*

$$q_E^{\mathrm{r}}(s_E, \omega_E) = \begin{cases} 0 & \text{if } \omega_E \le \hat{\omega}_E(s_E) \\ s_E & \text{otherwise,} \end{cases}$$

where $\hat{\omega}_E(s_E)$ is the unique solution to

$$\frac{\hat{\omega}_E(s_E) - \omega_I}{\hat{\omega}_E(s_E)} = \frac{\beta}{\varepsilon(\hat{\omega}_E(s_E)|s_E)}. \tag{14}$$

Proof. By linearity, the solution to the relaxed problem is s_E (zero) when the virtual surplus per unit, s^{v}, is positive (negative). The equation $s^{\mathrm{v}} = 0$ is equivalent to (14). The virtual surplus per unit is negative for $\omega_E = \omega_I$ and positive for $\omega_E = \bar{\omega}_E$, hence the existence of $\hat{\omega}_E$. The left-hand side of (14) increases in $\hat{\omega}_E$, and the right-hand side is nonincreasing by Assumption 1, which yields uniqueness. □

The threshold $\hat{\omega}_E(s_E)$ summarizes the tradeoff between efficiency and rent extraction at a given level of s_E. Equation (14) shows an analogy with the textbook monopoly pricing formula. The buyer-incumbent pair indeed has a monopoly power over entry, or more precisely over the quantity produced by the smaller rival. The buyer and the incumbent jointly act like a monopoly towards the rival, setting $\hat{\omega}_E$ to extract rent at the cost of reducing the probability of entry. When the threshold $\hat{\omega}_E$ is higher, the efficiency-rent tradeoff pushes towards less entry. The higher ε, the more reactive the entrant: the buyer and the incumbent cannot easily extract rents and cannot place strong competitive pressure on the entrant, hence a lower $\hat{\omega}_E$, and more entry.

The buyer has two tools to extract surplus from the entrant. First, his bargaining power $1-\beta$. Second, the tariff negotiated with the incumbent which determine both the size of the surplus created by the entry and the outside option of the buyer. They are very different in nature. First, the former is exogenous and the latter is endogenous. Second, whereas β does not directly impact the efficiency (if entry creates a positive surplus its sharing is irrelevant), the price schedule can deter efficient entry. Equation (14) shows that they are related. The larger the bargaining power of the buyer (i.e. the lower β) and the lower the threshold $\hat{\omega}_E(s_E)$; the efficiency-rent tradeoff pushes towards more entry. In the limit case where the buyer has all the bargaining power vis-à-vis the entrant ($\beta = 0$), there is no tradeoff, and hence no inefficient exclusion: $\hat{\omega}_E(s_E)$ coincides with the efficient threshold ω_I. On the contrary, the lower the bargaining power of the buyer and the higher $\hat{\omega}_E(s_E)$; the efficiency-rent tradeoff pushes towards less entry.

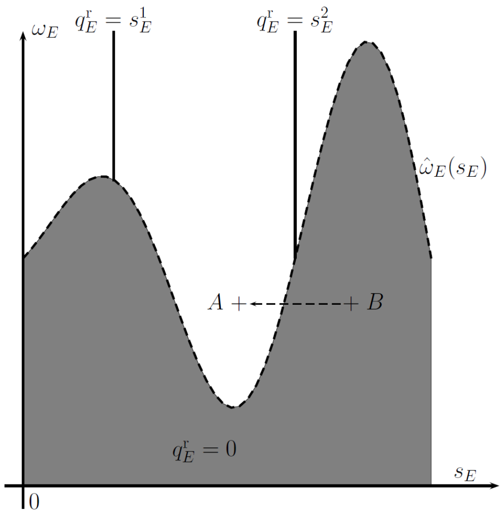

Figure 6: ERT line (dashed) and solution to the relaxed problem (here not implementable)

Hereafter, we call the curve with equation $\omega_E = \hat{\omega}_E(s_E)$ in the (s_E, ω_E)-plan the ERT line.[12] As shown on Figure 6, the solution to the relaxed problem is zero below this line and s_E above. In the represented case, the quantity function q_E^r is not implementable, because implementable functions are nondecreasing in s_E and q_E^r decreases from s_E to zero when crossing increasing parts as the ERT line. For example, the type represented at point B, who sells $q_E^r = 0$ and earns zero rent, would have an incentive to mimic

[12]The acronym ERT stands for Efficiency Rent Tradeoff.

type A, who sells all of the contestable demand and earns a positive rent. The relaxed quantity function, shown on Figure 6, is not consistent with the pattern of implementable quantity allocations, represented on Figure 5.

4.2 Nondecreasing elasticity of entry

In this section we assume that the elasticity of entry does not decrease with the size of the contestable demand, s_E. We consider first the case where $\varepsilon(\omega_E|s_E)$ does not depend on s_E, i.e. s_E and ω_E are independent. Then we examine the case where $\varepsilon(\omega_E|s_E)$ increases with s_E, i.e. ω_E first-order stochastically decreases with s_E. In both cases, the solution of the relaxed problem is incentive compatible and is therefore the solution of the buyer-incumbent pair's problem.

Proposition 2. *When the elasticity of entry, $\varepsilon(\omega_E|s_E)$, does not depend on s_E, the second best can be achieved through a two-part tariff with slope: $v_I - \hat{\omega}_E$. The equilibrium features inefficient exclusion. Partial foreclosure is not present.*

Proof. The ERT threshold given by (14) does not depend on s_E, because the elasticity ε does not. The solution of the relaxed problem, given by Proposition 1, is implementable with a constant boundary function $\Psi(s_E) = \hat{\omega}_E$, see Figure 7a.

The second best tariff is obtained as follows. From (8), the gain from trade between the buyer and the entrant is given by $\Delta S_{BE}(s_E, \omega_E) = (\omega_E - \hat{\omega}_E)s_E$ for $\omega_E > \hat{\omega}_E$. By definition of ΔS_{BE}, we have: $\Delta S_{BE}(s_E, \omega_E) = (\omega_E - v_I)s_E + T^{\text{SB}}(1) - T^{\text{SB}}(1 - s_E)$, hence

$$T^{\text{SB}}(1) - T^{\text{SB}}(1 - s_E) = (v_I - \hat{\omega}_E)s_E.$$

The effective price, defined by (9), is constant and equal to $v_I - \hat{\omega}_E$. The second best allocation is achieved by a two-part tariff, see Figure 7b. □

To make sure that the competitor serves all of the contestable demand if $\omega_E \geq \hat{\omega}_E$ and is inactive otherwise, the buyer and the incumbent set the effective price at $v_I - \hat{\omega}_E$. The smaller the elasticity of entry, ε, the larger the ERT threshold, $\hat{\omega}_E$, the smaller the slope of the two-part tariff, the stronger the competitive pressure put on the entrant. The slope of the optimal price schedule is negative whenever v_I is lower than $\hat{\omega}_E$. In such a case, the buyer would be better off purchasing more than $1 - s_E$ units from the incumbent. Yet the buyer cannot take advantage of the negative marginal price offered by the incumbent because doing so would leave him with unconsumed units and disposal costs are assumed to be infinite (see Section 5 for finite disposal costs).

As pictured in Figure 7a, the tradeoff between efficiency and rent extraction results in some efficient entrants being fully foreclosed in equilibrium. Inefficient foreclosure arises due to incomplete information as in Aghion and Bolton (1987). The fraction of efficient types that are inactive increases with the entrant's bargaining power vis-à-vis the buyer as $\hat{\omega}_E$ increases with β.

From now on, we consider cases where the elasticity of entry is not constant with s_E. This implies that a two-part tariff is no longer optimal. In this section, we start with the case where the elasticity increases with s_E.

Proposition 3. *When the elasticity of entry $\varepsilon(\omega_E|s_E)$ increases with s_E, the effective price, $p^{\text{e}}(q_E)$, increases with q_E. The price schedule is concave in a neighborhood of*

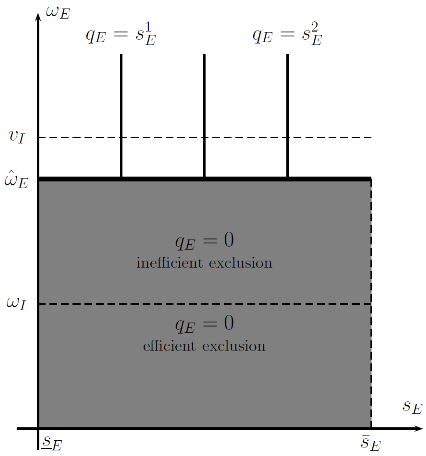

Figure 7a: Second best with $\varepsilon(\omega_E|s_E)$ constant in s_E

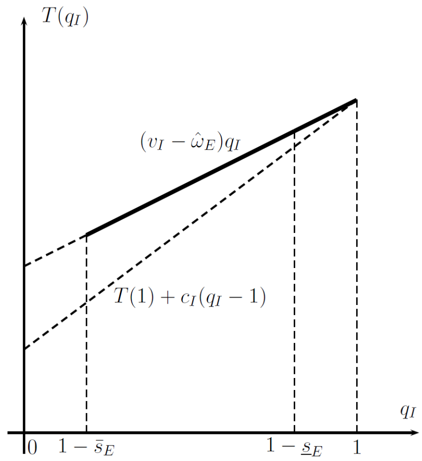

Figure 7b: Optimal price schedule (case $v_I > \hat{\omega}_E$)

$q_I = 1$. It is globally concave if $\hat{\omega}_E$ is concave or moderately convex in s_E. The equilibrium features inefficient exclusion. Partial foreclosure is not present.

Proof. When $\varepsilon(\omega_E|s_E)$ increases with s_E, the ERT threshold, $\hat{\omega}_E$, given by (14), decreases with s_E, and the solution of the relaxed problem is implementable. Its associated boundary function has equation $\Psi(s_E) = \hat{\omega}_E(s_E)$, see Figure 8a.

By the same reasoning as in Section 4.2, the surplus gain from the trade between the buyer end the entrant, $\Delta S_{BE}(s_E, \omega_E)$, equals $(\omega - \hat{\omega}_E(s_E))s_E$ above the ERT line and zero below, and the second best tariff is given by

$$T(1) - T(1 - s_E) = (v_I - \hat{\omega}_E(s_E))s_E.$$

In other words, the effective price $p^e(s_E)$ is set at $v_I - \hat{\omega}_E(s_E)$, and is thus increasing in s_E. To prove that T is concave in a neighborhood of $q_I = 1$, we compute $T(q_I) = T(1) + (v_I - \hat{\omega}_E(1 - q_I))(q_I - 1)$, then $T'(q_I) = (v_I - \hat{\omega}_E(1 - q_I)) + \hat{\omega}'_E(1 - q_I)(q_I - 1)$ and $T''(q_I) = 2\hat{\omega}'_E(1 - q_I) - \hat{\omega}''_E(1 - q_I)(q_I - 1)$. The term $\hat{\omega}'_E$, which is negative for any q_I, tends to make the tariff concave. Assuming that $\hat{\omega}''_E(0)$ is not infinite, we get $T''(1) = 2\hat{\omega}'_E(0) < 0$, hence the concavity at the top. □

As shown on Figures 8a and 9a, the entrant is either inactive ($q_E = 0$) or serves all the contestable demand ($q_E = s_E$). For a given ω_E the jump from zero to s_E can never occur if ω_E is not large enough. The jump occurs when s_E is large enough for intermediate values of ω_E. Finally, if ω_E is large enough, $q_E = s_E$ for any s_E. On the other hand, for a given s_E, $q_E = 0$ if ω_E is small (below $\hat{\omega}_E(s_E)$) while $q_E = s_E$ when ω_E is large (above $\hat{\omega}_E(s_E)$).

Some efficient entrants are foreclosed. As the elasticity of entry increases with s_E, the ERT results in a lower $\hat{\omega}_E$ as s_E increases. Consequently, the optimal effective price $p^e(q_E) = v_I - \hat{\omega}_E(q_E)$ increases with q_E: the larger the contestable market-share, the lower the competitive pressure. If $v_I \geq \hat{\omega}_E(\underline{s}_E)$, the effective price is positive for any quantity, as shown on Figures 8a and 8b. If $v_I < \hat{\omega}_E(\underline{s}_E)$, the effective price is negative for small values of q_E, as represented on Figures 9a and 9b.

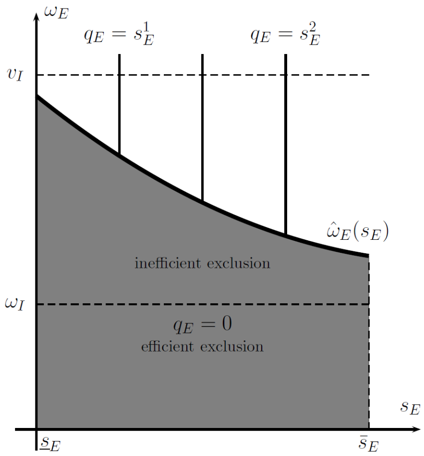

Figure 8a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

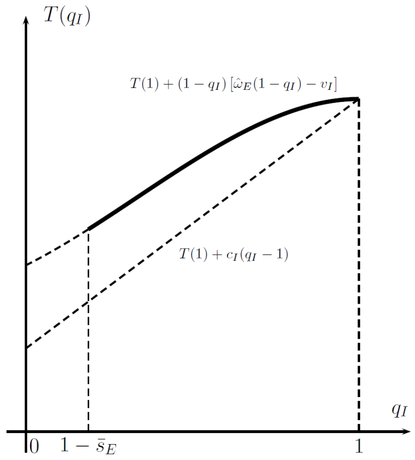

Figure 8b: Optimal price schedule (case $\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(0)$)

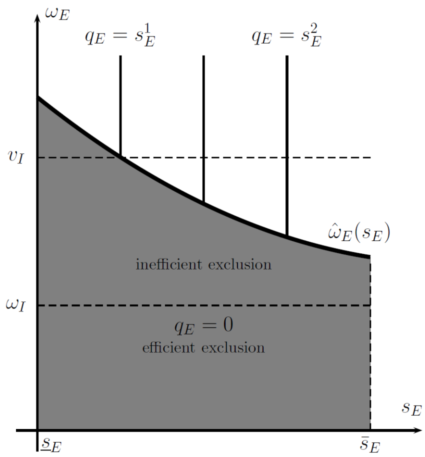

Figure 9a: Second best with $\varepsilon(\omega_E|s_E)$ increasing in s_E

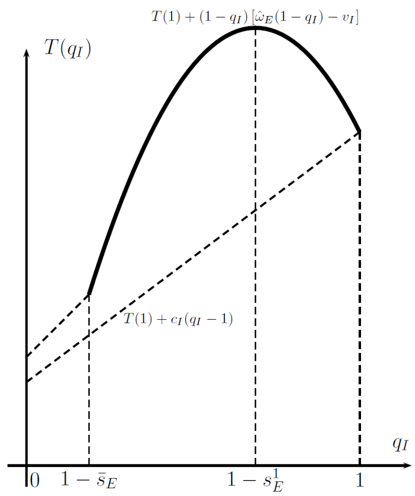

Figure 9b: Optimal price schedule (case $\underline{s}_E = 0$ and $v_I < \hat{\omega}_E(0)$)

4.3 The general case

As explained at the end of Section 4.1, solving the problem separately for each s_E generally yields non implementable quantity functions. We must therefore consider the complete problem, which consists in maximizing the expected virtual surplus

$$\iint s^{\mathrm{v}}(s_E, \omega_E) q_E(s_E, \omega_E)\, \mathrm{d}F(\omega_E|s_E)\, \mathrm{d}G(s_E)$$

over all implementable quantity functions q_E. In this section, we show that the problem can be solved separately for each ω_E provided some monotonicity constraints are satisfied.[13] The heart of the construction is the characterization of horizontal pooling intervals.

Consider an implementable quantity function q_E. For any ω_E, the function of one variable $q_E(., \omega_E)$ is nondecreasing on $[0, 1]$, being either constant or equal to the identity map: $q_E = s_E$. By convention, we call regions where it is constant "odd intervals", and regions where $q_E = s_E$ "even intervals".

We are thus led to consider any partition of the interval $[0, 1]$ into "even intervals" $[s_{2i}, s_{2i+1})$ and "odd intervals" $[s_{2i+1}, s_{2i+2})$, where (s_i) is a finite, increasing sequence with first term zero and last term one.[14] We associate to any such partition the function of one variable that coincides with the identity map on even intervals, is constant on odd intervals, and is continuous at odd extremities. We denote by K the set of the functions thus obtained.

For any implementable quantity function q_E, the functions of one variable, $q_E(., \omega_E)$, belong to K for all ω_E. Conversely, any quantity function such that $q_E(., \omega_E)$ belong to K for all ω_E is implementable if and only if even (odd) extremities do not increase (decrease) as ω_E rises. Even (odd) extremities constitute decreasing (increasing) parts of the boundary line. We call the conditions on the extremities the "monotonicity constraints". The problem is to maximize the expected virtual surplus under these constraints.

Lemma 3. *Let $a(.)$ be a continuous function on $[0, 1]$. Then the problem*

$$\max_{r \in K} \int_0^1 a(s) r(s) \, \mathrm{d}s$$

admits a unique solution r^* characterized as follows. For any interior even extremity s_E^{2i}, the function a equals zero at s_E^{2i} and is negative (positive) at the left (right) of s_E^{2i}. For any interior odd extremity s_E^{2i+1}, the function a is positive at s_E^{2i+1} and satisfies

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} a(s) \, \mathrm{d}s = 0. \tag{15}$$

If $a(1) > 0$, then $r^*(s) = s$ at the top of the interval $[0, 1]$. If $a(1) < 0$, then r^* is constant at the top of the interval.

Applying Lemma 3 with $a(s_E) = s^{\mathrm{v}}(s_E, \omega_E)$ for any given ω_E, we find that the virtual surplus is zero at candidate even extremities: $s^{\mathrm{v}}(x_{2i}(\omega_E), \omega_E) = 0$ and is negative (positive) at the left (right) of these extremities. In other words, candidate even extremities belong to decreasing parts of the ERT line. Thus, as regards even extremities, the monotonicity constraints are never binding.

Lemma 3 also implies that the virtual surplus is positive at odd extremities. These extremities therefore lie above the ERT line. By the first-order condition (15), the expected virtual surplus is zero on horizontal pooling intervals:

$$\mathbb{E}(s^{\mathrm{v}} | H) = 0, \tag{16}$$

[13] If the monotonicity constraints are violated, two-dimensional pooling occurs, as explained in Appendix C.4.

[14] For notational consistency, we denote the first term of the sequence by $s_0 = 0$ if the first interval is even and by $s_1 = 0$ if the first interval is odd. Similarly, we denote the last term by $s_{2n} = 1$ if the last interval is odd and by $s_{2n+1} = 1$ if the last interval is even.

where H is a horizontal pooling interval with extremities s_E^{2i+1} and s_E^{2i+2}. The virtual surplus on a pooling interval is first positive, then negative as s_E rises, and its mean on the interval is zero. The segment $[AB]$ on Figure 10b is an example of horizontal pooling interval (in fact the horizontal part of an "L"-shaped pooling set). Unfortunately, the first-order condition (16) does not imply that candidate odd extremities $x_{2i+1}(\omega_E)$ are nondecreasing with ω_E: odd extremities could decrease with ω_E in some regions, generating two-dimensional pooling. In Appendix C.3, we check that the monotonicity constraints regarding the odd extremities hold under fairly mild conditions, as stated in Proposition 4 below.

Proposition 4. *Maximizing the expected virtual surplus separately for each ω_E gives rise to a boundary line whose decreasing parts coincide with the ERT line and increasing parts generate horizontal pooling segments with zero expected virtual surplus, as expressed by equation (16). This construction yields the solution of the complete problem in the following circumstances:*

1. *the conditional density $f(\omega_E|s_E)$ is nondecreasing in ω_E;*
2. *the hazard rate, $f/(1-F)$, is nondecreasing in ω_E and (C.1) holds;*
3. *the elasticity of entry is nondecreasing in ω_E (Assumption 1) and (C.3) holds.*

The conditions (C.1) and (C.3) depend on the entrant's bargaining power vis-à-vis the buyer, β, and the range of values for the elasticity of entry, ε. These conditions are not very restrictive. They allow for large variations of ε as the maximum scale of entry, s_E, varies. For instance, if the rival's bargaining power, β equals one, the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 3.98$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 26.64$. If β equals .75, then the elasticity of entry may vary freely between $\underline{\varepsilon} = 1.2$ and $\bar{\varepsilon} = 5.99$, or between $\underline{\varepsilon} = 5$ and $\bar{\varepsilon} = 33.59$.

When none of the assumptions set out in Proposition 4 is satisfied, two-dimensional pooling may occur. The treatment of two-dimensional pooling is presented in Appendix C.4.

To construct the optimal boundary, we proceed as follows. We first draw the ERT line $\omega_E = \hat{\omega}_E(s_E)$. We start with $s_E = 1$ and then consider lower and lower values of s_E. For $s_E = 1$, we know that $\Psi(1) = \hat{\omega}_E(1)$. If the ERT line decreases at $s_E = 1$, the boundary coincides with the ERT line, as long as it remains decreasing. When the ERT line starts increasing (possibly at $s_E = 1$), we know that there is horizontal pooling. Equation (16) provides a unique candidate value for $\Psi(s_E)$. This candidate is the solution provided that it increases with s_E and remains above $\hat{\omega}_E(s_E)$. If the candidate boundary hits the ERT line at some value of s_E, it must be on a decreasing part of that line and, from that value on, the optimal boundary coincides with the ERT line (as long as $\hat{\omega}_E$ remains decreasing).

The ERT threshold $\hat{\omega}_E(s_E)$ is smaller than $\bar{\omega}_E$. The above construction shows that the optimal boundary is located below the maximal value of $\hat{\omega}_E(s_E)$. Hence, above this maximum, the quantity is efficient: $q_E(s_E, \omega_E) = s_E = q_E^*(s_E)$ for all s_E: there is no distortion at the top of the distribution of ω_E.

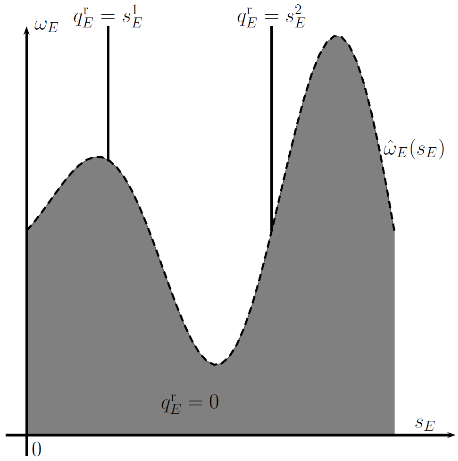

Figure 10a: Non implementable $q_E^{\rm r}$

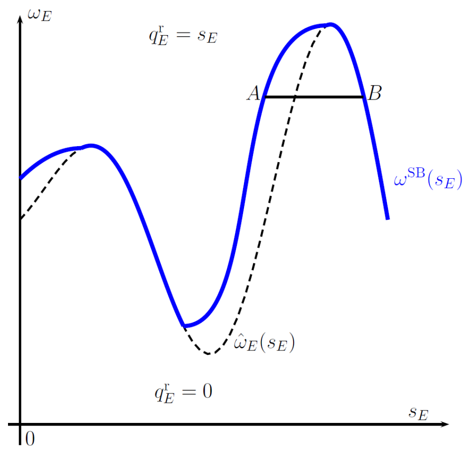

Figure 10b: ERT line $\hat{\omega}_E(s_E)$ (dashed). Optimal boundary $\omega_E^{\rm SB}$ (solid)

4.4 Decreasing elasticity of entry

We now turn to the case where the elasticity of entry is decreasing with s_E (ω_E first-order stochastically increases with s_E). A two-part tariff cannot be optimal.

Proposition 5. *Assume that $\varepsilon(\omega_E|s_E)$ decreases with s_E, then the optimal tariff is convex. The equilibrium outcome exhibits inefficient exclusion, in the form of both full and partial foreclosure.*

Proof. When $\varepsilon(\omega_E|s_E)$ decreases with s_E, then $\hat{\omega}_E$ is monotonically increasing and therefore it is not implementable. As seen in Section 4.3, the optimal boundary line $\omega_E^{\rm SB}$ is increasing and given by Proposition **??**. To obtain the optimal tariff, we follow the lines of Section **??**. For all $s_E \in [\underline{s}_E, \bar{s}_E]$, let $s'_E > s_E$ then the solution of (7) is interior for the type $(s'_E, \omega_E^{\rm SB}(s_E))$ and the solution is $q_E = s_E$. Therefore $T'(1 - s_E) = v_I - \omega_E^{\rm SB}(s_E)$ or

$$T'(q_I) = v_I - \omega_E^{\rm SB}(1 - q_I)$$

now, $\partial \omega_E^{\rm SB}(1 - q_I)/\partial q_I = (\omega_E^{\rm SB})'(1 - q_I) > 0$ as $\omega_E^{\rm SB}$ is increasing, which proves that T is convex. □

As depicted in Figure 11a, when the entrant's type lies in the hatched triangle below the boundary line and above the horizontal line $\omega_E^{\rm SB}(0)$, the entrant produces a quantity strictly lower than s_E. That is, entry is partially foreclosed. Here, the price schedule plays the role of a barrier to expansion in addition to a barrier to entry. Some efficient types of entrant are active but prevented to serve all the contestable demand (they face a barrier to expansion). Otherwise, and as in sections 4.2 and **??**, some efficient types remain inactive (they face a barrier to entry).

For a given ω_E, q_E can be null for all s_E (whenever ω_E is lower than $\omega_E^{\rm SB}(0)$), when ω_E is intermediate, $q_E = s_E$ when s_E is small and q_E is constant when s_E is above a threshold (formally $(\omega_E^{\rm SB})^{-1}(\omega_E)$. Finally, if ω_E is large enough (formally above $\omega_E^{\rm SB}(\bar{s}_E)$), then

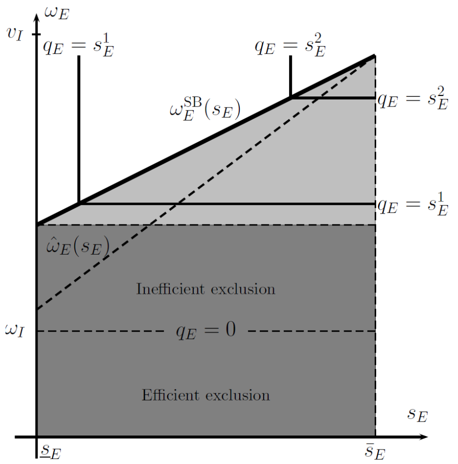

Figure 11a: Second best with $\varepsilon(\omega_E|s_E)$ decreasing in s_E

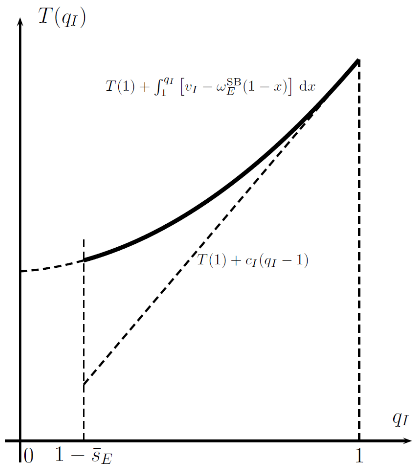

Figure 11b: Optimal price schedule ($\underline{s}_E = 0$ and $v_I > \hat{\omega}_E(1)$)

$q_E = s_E$ for all s_E. Fixing s_E and letting ω_E increase from $\underline{\omega}_E$ to $\bar{\omega}_E$, q_E is null until ω_E reaches $\omega_E^{SB}(0)$, then q_E increases with ω_E, and finally $q_E = s_E$ for all ω_E above $\omega_E^{SB}(s_E)$.

A small market share of E can, therefore, reflect either a small s_E (this is the case for an efficient enough E who sells at full capacity) or a large s_E with partial foreclosure (this is the case when E is sufficiently efficient to enter but not enough to break the ω_E^{SB} line and sell at full capacity).

Qualitatively these situations are very different. In the first one, E is frustrated because he had to abandon a fraction of his surplus to the buyer. However, depending on the interpretation of s_E, either he cannot produce more or the buyer is not interesting in buying more from the entrant. In the second case (partial foreclosure), E is similarly deprived of some surplus, but in addition he is also frustrated because he cannot sell all the units that the buyer would like to acquire in the absence of T.

As $\varepsilon(\omega_E|s_E)$ is decreasing, the buyer-incumbent pair would like to extract more surplus from a large entrant and hence put more competitive pressure on him. Yet, a large entrant can always mimic a smaller one. To take an example, the efficiency rent tradeoff would imply that for some ω_E it would be optimal to ask a a type with a large s_E to sell nothing while a type with a small s_E to sell at full capacity. Obviously this is not possible with only T as an instrument.

In Figure 11b, the optimal price schedule is increasing because it is drawn under the assumption that v_I is larger than $\omega_E^{SB}(s_E)$ for all s_E. If, however, ω_E^{SB} becomes larger than v_I for s_E large enough, then the slope of T is negative for the small q_I (as $q_I = 1 - s_E$).

4.5 Non monotonic elasticity of entry

We now turn to a case where the elasticity of entry in non monotonic with the size of the contestable demand, s_E. This encompasses many situations but a particular case, namely the U-shape elasticity, illustrates the nonlinearity of the price schedule.

Assume that the elasticity of entry is first decreasing, then increasing with s_E. That is, when the size of the contestable market is small or large, the elasticity is rather high while it is low when the size of the contestable demand is average. According to (??) a U-shape elasticity leads to an inverted U-shape $\hat{\omega}_E$.

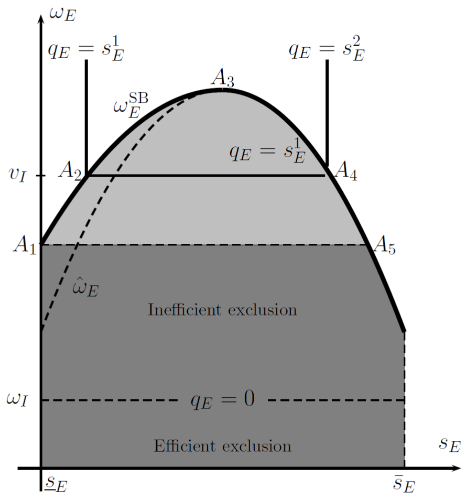

Figure 12a: 2nd best with U shaped $\varepsilon(\omega_E|s_E)$ in s_E

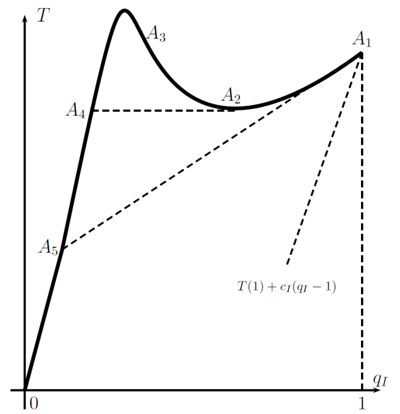

Figure 12b: Optimal price schedule ($\underline{s}_E = 0$ and $\bar{s}_E = 1$)

We rely on Figures 12a and 12b to explain the shape of the optimal price schedule in this instance. Between A_1 and A_3, ω_E^{SB} is increasing, therefore (as already seen in Section 4.4) the quantity negotiated between the buyer and the entrant is given by the first order condition: $T'(1-s_E) = v_I - \omega_E^{\text{SB}}(s_E)$ (i.e. $T'(q_I) = v_I - \omega_E^{\text{SB}}(1-q_I)$) and T is convex. In particular at A_2, $v_I = \omega_E^{\text{SB}}(s_E)$ and $T' = 0$. Between A_3 and A_5, we can recover T by using the pooling condition: in the grey area q_E is constant along horizontal lines. For exemple, if the entrant is at A_4 the buyer-entrant pair is indifferent between buying s_E^2 or s_E^1. That is (abstracting from the fact that on the Figure $\omega_E = v_I$ for A_2 and A_4):

$$(\omega_E - v_I)s_E^1 - T(1-s_E^1) = (\omega_E(s_E^2) - v_I)s_E^2 - T(1-s_E^2)$$

As $T(1-s_E^1)$ is known, one can infer $T(1-s_E^2)$. Rewriting the above expression, it comes that

$$T(1-s_E^2) = T(1-s_E^1) + (\omega_E(s_E^2) - v_I)(s_E^2 - s_E^1)$$

In particular, using the fact that for A_2 and A_4 $\omega_E = v_I$, it comes that $T(1-s_E^1) = T(1-s_E^2)$ as shown on Figure 12b. It is readily confirmed that $T'' = 0$ at A_3, i.e. T has an inflexion point. After A_5, the same indifference condition applies but $s_E^1 = 0$. Therefore $T(1-s_E^2) = T(1) + (\omega_E(s_E^2) - v_I)s_E^2$.

Thus, an inverted U-shape ω_E^{SB} is associated to a price schedule which is convex at the end (small values of s_E), concave in the middle and either concave or convex for the small values of q_I. In addition to characterizing the shape of T, Figures 12a and 12b are also helpful to show what happens when v_I is first above ω_E^{SB} then below and finally above. Under this assumption, T is reminiscent to a retroactive rebate. The buyer has a strong incentive to buy up to A_2 as T is decreasing.

Notice that every point on T is used by the buyer depending on the type of the entrant. For example, at A_2, the tariff $\left(1 - s_E^1, T(1 - s_E^1)\right)$ is used for all (s_E, ω_E) on the isoline $q_E = s_E^1$ which is depicted in bold on Figure 12a. The choice of the buyer is similar between A_1 and A_3. A point $(q_I, T(q_I))$ before A_3 is chosen by the buyer whenever the type of the entrant is $s_E = 1 - q_I$ and $\omega_E \geq \hat{\omega}_E(1 - q_I)$. That is, along vertical isolines like $q_E = s_E^2$ starting from A_4 on Figure 12a.

5 Disposal costs, buyer opportunism, and conditional tariffs

We have assumed so far that the buyer incurs an infinite cost if he does not consume all of the purchased units. The magnitude of disposal costs, however, varies substantially across industries (e.g. chips for microprocessors, tyres for trucks, or heavy pieces of machineries) and might depend on the existence of a second-hand market for the raw input.[15] In Section 5.1 we show how the previous analysis extends to the case of finite disposal costs. In Section 5.2, we link infinite disposal costs to market-share tariffs.

5.1 Finite disposal costs and opportunism

This section is devoted to the case where disposal costs are finite. We note γ the cost of purchased units in excess of consumption. We treat γ as an exogenous parameter. When $\gamma = 0$, the buyer can freely dispose of units he does not need. When $\gamma > 0$, the buyer pays a cost γ for each non-consumed purchased units. Finally, $\gamma < 0$ would mean that the buyer can resale, as it is, the good bought from the incumbent, for example on a second hand market. Admittedly, $-c_I < \gamma$ otherwise this second hand market would be profitable.

As previously, the buyer's total consumption is inelastic, and normalized to one: $x_E + x_I \leq 1$. Thus, having purchased quantities $q_E \leq s_E$ and q_I from the buyer and the incumbent, the buyer chooses consumption levels so as to maximize

$$V(q_E, q_I) = \max_{(x_E, x_I) \in X} v_E x_E + v_I x_I - \gamma(q_E - x_E) - \gamma(q_I - x_I), \tag{17}$$

where the set X is defined by the constraints $x_E \leq q_E$, $x_I \leq q_I$, and $x_E + x_I \leq 1$: the buyer cannot consume more than he has purchased and more than his total requirement, normalized to one. In appendix D.1, we extend Lemma A.1 to the case of finite disposal costs, thus showing that the buyer's total purchases are not lower than his total demand.

The buyer, however, could purchase *more* than his requirement, as disposal costs are now assumed to be finite, with the sole purpose of benefiting from a rebate offered by the incumbent. We call such a behavior opportunistic. The expression of V given in (17) shows that if the slope of $T(q_I)$ is lower than $-\gamma$ for a given q_I it can create buyer's opportunism. Imagine the buyer already bought q_E and that $T'(1 - q_E) < -\gamma$, then he buys more than $1 - q_E$ (to reduce the total price paid at the incumbent) while consuming only $x_I = 1 - q_E$. These excess purchases are costly from the point of view of the incumbent because of production costs.

[15] Depending on the industry, the seller can verify more or less easily what the buyer does with the purchased goods. Disposal costs can also be seen as costs to avoid the monitoring of the incumbent.

We first establish an optimality result that holds irrespective of the informational structure, i.e. whether or not the buyer and the incumbent know the entrant's characteristics when signing the contract.

Proposition 6. *There is no buyer opportunism at equilibrium: the buyer does not buy more than its total requirements,* $q_E + q_I \leq 1$.

Proof. We show in Appendix D that the buyer and the incumbent are better off using a tariff with slope greater than or equal to $-\gamma$. □

Lemma A.1 in appendix shows that, in equilibrium, $q_E + q_I \geq 1$. Combining this result with Proposition 6, we conclude that the buyer purchases the exact quantity necessary to meet its requirements: $q_E + q_I = 1$, and consumes all purchased units: $x_E = q_E$, $x_I = q_I$. Hence, the buyer incurs no disposal costs. It follows that the expressions (7) and (11), respectively for the surplus created by the trade between the buyer and the entrant, ΔS_{BE}, and the expected profit of the buyer-incumbent pair, $\mathbb{E}\Pi_{BI}$, still hold. The buyer-incumbent's with γ finite is the same as with $\gamma = +\infty$ with the additional constraint that $T'(q_I) > -\gamma$. Consequently, and regardless of the informational structure, the expected profit of the buyer-incumbent coalition is weakly lower when the disposal costs are finite than when they are infinite.

Throughout this section, we say that the entrant is *super-efficient* if $\omega_E \geq v_I + \gamma$. It follows from Proposition 6 and from the buyer-entrant problem (7) that, in equilibrium, whatever the informational structure, a super-efficient entrant serves all the contestable demand: $q_E(s_E, \omega_E) = s_E$ for all $\omega_E > v_I$. Indeed, for such an entrant, the function $(\omega_E - v_I)q_E - T(1 - q_E)$ is nondecreasing on $(0, s_E)$.

Proposition 6 prompts us to extend the notion of admissibility to the case with finite disposal cost. We say that a quantity function is admissible if it can be obtained as solution to (7), where T is a tariff satisfying $T' \geq -\gamma$ for all q. Lemma 1 must be adapted as follows.

Lemma 4. *A quantity function* $q_E(.,.)$ *is admissible if and only if there exists a boundary function* $\Psi(.)$ *defined on* $[0, 1]$*, with* $\omega_I \leq \Psi(s_E) \leq v_I + \gamma$ *for all* s_E*, such that (10) holds.*

The new condition on the boundary, $\Psi(s_E) \leq v_I + \gamma$, expresses that super-efficient entrants serve all of the contestable demand: $q_E = s_E$. The sufficient part of the lemma is proved in Section D. The analysis with infinite disposal costs (Sections **??** and **??**) carries over without any change when there are no super-efficient entrants: $\bar{\omega}_E \leq v_I + \gamma$.

Inefficient entry never occurs at the second best (Appendix D.3 extends Proposition **??** to the case of finite disposal costs.

Under perfect information, only super-efficient entrants earn a positive profit. The equilibrium configuration can be obtained with a constant effective price, $p^{\mathrm{e}}(q) = \max(v_I - \omega_E, -\gamma)$. Super-efficient entrants earns $\beta(\omega_E - v_I - \gamma)s_E$. Other entrants earn zero profit.

Under one-dimensional uncertainty (ω_E unknown), the expression (**??**) applies to entrants that are not super-efficient. For those entrants, it must be replaced with s_E. If the incumbent's cost function is affine or concave, the second-best quantity is s_E for $\omega_E \geq \min(\hat{\omega}_E, v_I + \gamma)$, where $\hat{\omega}_E$ is given by (14), and zero otherwise. The second best can be achieved with a two-part tariff, by setting the constant effective price schedule at $p^{\mathrm{e}}(q) = \max(v_I - \hat{\omega}_E, -\gamma)$. Efficient entrants with $v_I - c_I^{\mathrm{e}}(s_E) < \omega_E \leq \min(\hat{\omega}_E, v_I + \gamma)$ are excluded.

Under two-dimensional uncertainty, the construction of the optimal quantity is modified as follows: First apply the procedure exposed in Section **??**, then replace Ψ with $\min(\Psi, v_I + \gamma)$.

5.2 Conditional tariffs

Proposition 6 extends to the case where the tariff T can be made conditional on the quantity purchased from the entrant. Applying the same reasoning as in Appendix D to each value of q_E, one may show that the buyer and the incumbent are better off using a tariff $T(q_E, q_I)$ such that the marginal price of an extra unit of good I, $T_{q_I}(q_E, q_I)$, is greater than or equal to $-\gamma$, and that the buyer purchases the quantity necessary to meet its requirement: $q_E + q_I = 1$. These properties, however, do not imply that the effective price be greater than $-\gamma$ and that super-efficient entrants serve all of the contestable demand. Indeed, the effective price is now given by $[T(0, 1) - T(q_E, 1 - q_E)]/q_E$, which can be lower than $-\gamma$, and the objective of the buyer-entrant coalition, $(\omega_E - v_I)q_E - T(q_E, 1 - q_E)$, is not necessarily monotonic for super-efficient entrants.

Proposition 7. *Conditioning the tariff on the quantity purchased from the entrant allows the buyer and the incumbent to earn the same profit as if disposal costs were infinite.*

Proof. The set of admissible quantity functions with conditional tariffs does not depend on $\gamma \in [0, +\infty]$. Moreover, with $\gamma = +\infty$ this set is the set of quantity functions implementable with unconditional tariffs. □

6 Discussion

The chief concern of antitrust enforcers as regards abuses of dominant position is inefficient exclusion. In its guidelines on exclusionary conducts by dominant undertakings, the European Commission advocates the "as-efficient competitor test", which consists in checking that efficient rivals are not foreclosed. This test is presented as a first step in the legal assessment: if the test is violated, the dominant firm may have the burden of justifying its pricing policy, for instance by putting forward efficiency considerations.

We study nonlinear pricing by a dominant firm who competes with a smaller rival, focusing on exclusionary effects. We exclude any efficiency reasons for the dominant firm to use nonlinear pricing as well as any predation purposes. We examine the consequences of the incumbent's monopoly power over the rival, in the spirit of Aghion and Bolton (1987). In our model, the common distinction in the antitrust doctrine between *exploitative* and *exclusionary* abuses is blurred because it is the exploitation of the incumbency advantage, combined with incomplete information which yields inefficient exclusion.[16] That is, the two aspects are intertwined in the tradeoff between rent extraction and efficiency.

The exploitative part of the mechanism is sometimes called "rent-shifting": the existence of the tariff enhances the buyer's bargaining position vis-à-vis the entrant by altering her outside option in the negotiation. Under incomplete information, the buyer and the incumbent adjust the competitive pressure placed on the entrant to solve the efficiency-rent tradeoff. Our analysis places no *a priori* restriction on the shape of the incumbent's tariff. Depending on the distribution of the uncertainty, tariffs, in equilibrium, may be locally increasing or decreasing, and locally convex, linear or concave. The exclusionary part comes from the fact that in equilibrium, efficient competitors may be barred from serving the demand (complete foreclosure) or prevented to expand (partial foreclosure).[17] In any case, those who enter are forced to grant favorable conditions to attract buyers.

The competitive pressure placed on the rival firm translates into the amount of rebates that the buyer gives up by supplying units from the rival, hence the importance of the incumbent's "effective price" emphasized by the European Commission.[18] At the second best, the effective price is always below the incremental cost, because the buyer and the incumbent only want to shift rents from a rival more efficient than the incumbent. If the Commission could enforce its "as-efficient competitor test", then any exploitative attempt would be eliminated and hence there would be no exclusion of efficient rivals.

Yet, in practice, enforcing the as-efficient competitor test is by no means trivial; cost measurements are imprecise in nature. To our knowledge, the *Intel* decision contains the

[16] Under complete information, only the exploitative abuse is at play as the second-best allocation is efficient, as observed at the end of Section 2.

[17] In several cases, Virgin/British Airways, Michelin, and Intel (See references in footnote 1). The defendant argued that his market share declined during the year under scrutiny. This could happen in our model, for a given s_E, if ω_E increases but remain below ω_E^{SB}. The rival remains partially foreclosed but less and less when ω_E increases.

[18] When the tariff only depends on the quantity purchased from the incumbent, the effective price is simply the average price of the last units offered by the incumbent. The computation must be adapted when the tariff also depends on the quantity purchased from the entrant, as buying more from the entrant (as opposed to buying less from the incumbent) can in itself affect the effective price.

first and, to date, the sole attempt to implement the test in an antitrust case.[19]

The incumbent can take advantage of its position only if he can commit on the price schedule (either under complete or incomplete information). In practice, it seems, therefore, crucial to check this point. In two recent cases[20] the European Commission stressed that the dominant firm used a long reference period to calculate the rebates (one year). That is both cases, the dominant firm was able to commit on a price schedule for the whole year.

Finally, our analysis also explains how low disposal costs limit the ability of the dominant firm to exploit its position. When the rival is expected to be much more efficient than the incumbent, the price schedule can exhibit decreasing parts. These lower prices for larger quantity allow the buyer to be opportunistic by purchasing more than his needs simply to cash in on the tariff.[21] As these non consumed units have to be dispose of at a cost, the price schedule can be decreasing but at rate no larger than the disposal cost.

To counter this opportunistic behavior of the buyer, the incumbent has two strategies: first she could monitor the buyer, making sure that he purchases only up to his needs. Second, the dominant firm may want to condition her prices on quantities purchased from rivals. We show that resorting to a market-share tariff is equivalent to impose an infinite disposal cost to the buyer.

References

Philippe Aghion and Patrick Bolton. Contracts as a Barrier to Entry. *The American Economic Review*, 77(3):388–401, June 1987.

Mark Armstrong. Multiproduct Nonlinear Pricing. *Econometrica*, 64(1):51–75, January 1996.

Mark Armstrong. Price Discrimination by a Many-Product Firm. *Review of Economics Studies*, 66(1):151–168, January 1999.

Mark Armstrong and John Vickers. Competitive Price Discrimination. *RAND Journal of Economics*, 32(4):579–605, Winter 2001.

Mark Armstrong and John Vickers. Competitive Nonlinear Pricing and Bundling. *Review of Economics Studies*, 77(1):30–60, January 2010.

Giacomo Calzolari and Vincenzo Denicolo. Competition with exclusive contracts and market-share discounts. CEPR discussion paper No 7613, December 2009.

Avinash Dixit. A Model of Duopoly Suggesting a Theory of Entry Barriers. *Bell Journal of Economics*, 10(1):20–32, Spr. 1979.

[19] Admittedly, the Commission has run the test for a single value of q, namely $q = 1 - s_E$, where s_E was a "realistic" value for the size of the contestable demand. This approach requires to agree, *ex post*, on the magnitude of the contestable demand. This issue has proved highly contentious in the *Intel* case.

[20] Virgin/British Airways and Michelin. See references in footnote 1.

[21] The price schedule agreed upon between the incumbent and the buyer can be seen (in broad terms) as a specific investment by the incumbent. As often with specific investment one party can (in some state of nature) take advantage (hold-up) of the fact that the other party is committed to this investment.

Avinash Dixit. The Role of Investment in Entry-Deterrence. *Economic Journal*, 90(357): 95–106, March 1980.

Roman Inderst and Greg Shaffer. Market-Share contracts as facilitating practices. *RAND Journal of Economics*, 41(4):709–729, Winter 2010.

Bruno Jullien. Participation Constraints in Adverse Selection Models. *Journal of Economic Theory*, 93(1):1–47, July 2000.

Jean-Jacques Laffont, Eric Maskin, and Jean-Charles Rochet. Optimal non linear pricing with two dimensional characteristics. In R. Radner T. Groves and S. Reiter, editors, *Information, Incentives and Economic Mechanisms*, pages 256–266. 1987.

Adrain Majumdar and Greg Shaffer. Market-Share Contracts with Asymmetric Information. *Journal of Economics and Management Strategy*, 18(2):393–421, Summer 2009.

David Martimort and Lars Stole. Market participation in delegated and intrinsic common-agency games. *RAND Journal of Economics*, 40(1):78–102, Spring 2009.

Leslie M. Marx and Greg Shaffer. Predatory Accommodation: Below-Cost Pricing without Exclusion in Intermediate Goods Markets. *RAND Journal of Economics*, 30(1): 22–43, Spring 1999.

Leslie M. Marx and Greg Shaffer. Rent Shifting, Exclusion, and Market-Share Discounts. June 2004.

Leslie M. Marx and Greg Shaffer. Rent shifting and the order of negotiations. *International Journal of Industrial Organization*, 25(5):1109–1125, October 2007.

Leslie M. Marx and Greg Shaffer. Break-up fees and bargaining power in sequential contracting. *International Journal of Industrial Organization*, 28(5):451–463, September 2010.

Eric Maskin and John Riley. Monopoly with incomplete information. *RAND Journal of Economics*, 15(2):171–96, Summer 1984.

Michael Mussa and Sherwin Rosen. Monopoly and Product Quality. *Journal of Economic Theory*, 18(2):301–17, Aug. 1978.

Jean-Charles Rochet and Philippe Choné. Ironing, Sweeping, and Multidimensional Screening. *Econometrica*, 66(4):783–826, July 1998.

R. Tyrell Rockafellar. *Convex analysis*. Princeton University Press, Princeton, New Jersey, 1997.

A. Michael Spence. Entry, Capacity, Investment and Oligopolistic Pricing. *Bell Journal of Economics*, 8(2):534–44, Autmn. 1977.

A. Michael Spence. Investment Strategy and Growth in a New Market. *Bell Journal of Economics*, 10(1):1–19, Spr. 1979.

Robert Wilson. *Nonlinear pricing*. Oxford University Press, Oxford; New York; Toronto and Melbourne, 1993.

Appendix

A The buyer's total purchases are not lower than his total demand

This section presents an optimality result that holds irrespective of the information structure, i.e. whether the buyer and the incumbent know the entrant's characteristics at the time of contracting. It is in the buyer's and incumbent's common interest to agree on a price schedule that induces the former to purchase at least $1 - q_E$ units from the latter, once he has purchased q_E from the entrant, for any value of q_E. Hence, in equilibrium, the buyer's total purchases are equal to, or exceed, his total demand.[22]

Lemma A.1. *The buyer and the incumbent are better off using a tariff with slope T' smaller than or equal to v_I. Consequently, we may assume, with no loss of generality, that the buyer does not buy less than its total requirements: $q_E + q_I \geq 1$.*

Proof: We start from any price schedule T. Let $\tilde{T}$ be defined by

$$\tilde{T}(q_I) = \inf_{q \leq q_I} T(q) + v_I(q_I - q). \tag{A.1}$$

The tariff $\tilde{T}$ is derived from the tariff T as follows. When the incumbent offer q units at price $T(q)$, she also offers to buy more units than q, say $q_I > q$, at price $T(q) + v_I(q_I - q)$. The additional units are offered at the monopoly price v_I. By construction, the slope of $\tilde{T}$ is lower than or equal to v_I.

Let $\tilde{U}_B(q_E)$ denote the buyer's net utility after he has purchased q_E units from the entrant under the price schedule $\tilde{T}$

$$\tilde{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \tilde{T}(q_I). \tag{A.2}$$

As $\tilde{T} \leq T$, we have: $\tilde{U}_B \geq U_B$. Suppose that, under $\tilde{T}$, it is optimal for the buyer to purchase $\tilde{q}_I$ from the incumbent if he has purchased q_E from the entrant. By construction of $\tilde{T}$, there exists $q_I \leq \tilde{q}_I$ such that $\tilde{T}(\tilde{q}_I)$ equals or is arbitrarily close to $T(q_I) + v_I(\tilde{q}_I - q_I)$. We have:

$$\begin{aligned} \tilde{U}_B(q_E) = V(q_E, \tilde{q}_I) - \tilde{T}(\tilde{q}_I) &= V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \\ &= V(q_E, q_I) - T(q_I), \end{aligned} \tag{A.3}$$

which implies $\tilde{U}_B(q_E) \leq U_B(q_E)$, and hence $\tilde{U}_B(q_E) = U_B(q_E)$ for all q_E. As the problem of the buyer-entrant pair depends only on the functions $U_B(.)$ and $\tilde{U}_B(.)$, they agree on the same quantity q_E and the entrant earns the same profit under T and $\tilde{T}$ for all (c_E, s_E, v_E).

We now examine the quantity purchased from the incumbent. Suppose that the buyer, having purchased q_E from the entrant, chooses to purchase q_I from the incumbent

[22] Lemma A.1 is stated and proved under infinite disposal costs. Appendix D.1 extends the result to the case of finite disposal costs.

under the original price schedule T. As $\tilde{T}(q_I) \leq T(q_I)$, the buyer may choose to purchase the same quantity from the incumbent under the new tariff $\tilde{T}$:

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \tilde{T}(q_I).$$

Yet, under the tariff $\tilde{T}$, if $q_I < 1-q_E$, the buyer may as well choose to purchase $1-q_E$ from the incumbent. Indeed, by definition of $\tilde{T}$, we have $\tilde{T}(1-q_E) \leq T(q_I) + v_I(1-q_E-q_I)$ and hence

$$\begin{aligned} U_B(q_E) = \tilde{U}_B(q_E) &= V(q_E, q_I) - T(q_I) \\ &\leq V(q_E, q_I) + v_I(1-q_E-q_I) - \tilde{T}(1-q_E) \\ &= V(q_E, 1-q_E) - \tilde{T}(1-q_E). \end{aligned} \tag{A.4}$$

As $v_I > c_I$, the change from q_I to $1-q_E > q_I$ increases the total surplus:

$$\begin{aligned} W(q_E, 1-q_E) &= V(q_E, 1-q_E) - c_E q_E - c_I(1-q_E) \\ &= V(q_E, q_I) - c_E q_E - c_I q_I + (v_I - c_I)(1-q_E-q_I) \\ &\geq W(q_E, q_I). \end{aligned} \tag{A.5}$$

In sum, the change from T to $\tilde{T}$ does not alter the entrant's profit and does not decrease the total surplus. We conclude from (4) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

B Implementation

B.1 Recovering the tariff from the boundary line

We prove here the sufficient part of Lemma 1. Starting from any boundary function Ψ defined on $[0,1]$, we define the quantity function $q_E(s_E, \omega_E)$ by equation (10), and the profit function $\Delta S_{BE}(s_E, \omega_E)$ by equation (8). We observe that the functions thus defined $q_E(s_E, \omega_E)$ and $\Delta S_{BE}(s_E, \omega_E)$, are nondecreasing in both arguments, and the latter function is convex in ω_E. Next, we notice that the expression

$$(\omega_E - v_I) q_E(s_E, \omega_E) - \Delta S_{BE}(s_E, \omega_E)$$

is constant on q_E-isolines. Indeed, both $q_E(., \omega_E)$ and $\Delta S_{BE}(., \omega_E)$ are constant on horizontal isolines (located below the boundary Ψ). On vertical isolines (above the boundary), $\Delta S_{BE}(s_E, .)$ is linear with slope s_E, guaranteing, again, that the above expression is constant. We may therefore define $T(q)$, up to an additive constant, by

$$T(1) - T(1-q) = (v_I - \omega_E)q + \Delta S_{BE}(s_E, \omega_E), \tag{B.1}$$

for any (s_E, ω_E) such that $q = q_E(s_E, \omega_E)$. Equation (B.1) unambiguously defines $T(1) - T(1-q)$ on the range of the quantity function $q_E(.,.)$. This range contains zero, but may have holes when $\bar{\omega}_E$ is finite and Ψ is above $\bar{\omega}_E$ on some intervals. Specifically, if Ψ is above $\bar{\omega}_E$ on the interval $I = [s_E^1, s_E^2]$, then q_E does not take any value between s_E^1 and s_E^2. In this case, we define T by imposing that it is linear with slope $v_I - \bar{\omega}_E$ on the corresponding interval: $T(1-s_E^1) - T(1-q) = (v_I - \bar{\omega}_E)(q - s_E^1)$ for $q \in I$.

We now prove that the buyer and the entrant, facing the above defined tariff T, agree on the quantity $q_E(s_E, \omega_E)$. We thus have to check that

$$\Delta S_{BE}(s_E, \omega_E) \geq (\omega_E - v_I)q' + T(1) - T(1 - q') \tag{B.2}$$

for any $q' \leq s_E$. When q' is the range of the quantity function, we can write $q' = q_E(s'_E, \omega'_E)$ for some (s'_E, ω'_E), with $q' \leq s'_E$. Observing that $q' = q_E(q', \omega'_E)$ and using successively the monotonicity of ΔS_{BE} in s_E and its convexity in ω_E, we get:

$$\begin{aligned}\Delta S_{BE}(s_E, \omega_E) &\geq \Delta S_{BE}(q', \omega_E) \\ &\geq \Delta S_{BE}(q', \omega'_E) + (\omega_E - \omega'_E)q',\end{aligned}$$

which, after replacing $T(1) - T(1 - q')$ with its value from (B.1), yields (B.2). To check (B.2) when q' is not in the range of the quantity function (q' belongs to a hole $[s^1_E, s^2_E]$ as explained above), use (B.2) at s^1_E and the linearity of the tariff between s^1_E and q'.

B.2 Shape of the boundary function and curvature of the tariff

Lemma B.1 relates the shape of the boundary function Ψ to the curvature of the price schedule T.

Lemma B.1. *The following properties hold:*

1. *If Ψ is increasing (resp. constant) around s_E, then the tariff is strictly convex (resp. linear) around $1 - s_E$.*
2. *If Ψ decreases and is concave around s_E, then the tariff is concave around $1 - s_E$.*
3. *If Ψ decreases and is convex around s_E and s_E is close to a local minimum of Ψ, then the tariff is convex around $1 - s_E$.*
4. *If Ψ has a local maximum at s_E, then the tariff has an inflection point at $1 - s_E$.*

Proof. First, suppose that Ψ is nondecreasing on a neighborhood of s_E. Let s'_E slightly above s_E. Then $q_E = s_E$ is an interior solution of the buyer-entrant pair's problem (7) for s'_E and $\omega_E = \Psi(s_E)$. It follows that the first order condition $\Psi(s_E) - v_I + T'(1 - s_E) = 0$ holds, implying property 1 of the lemma. The property holds when Ψ has an upward discontinuity at s_E, in which case the tariff has a convex kink at $1 - s_E$. To illustrate, Figures 13a and 13b consider the case where the boundary line is a nondecreasing step function with two pieces.

Next, suppose that the boundary line decreases around s_E. Here we assume that Ψ is twice differentiable. We denote by $[\sigma(s_E), s_E]$ the set of value s'_E such that $q_E(s'_E, \omega_E) = \sigma(s_E)$, where $\omega_E = \Psi(s_E)$. The buyer-entrant surplus $\Delta S_{BE}(s_E, \omega_E)$ is convex and hence continuous in ω_E. It can be computed slightly below or above $\Psi(s_E)$. At $(s_E, \Psi(s_E))$, the buyer and the entrant are indifferent between quantities s_E and $\sigma(s_E)$:

$$\Delta S_{BE}(s_E, \Psi(s_E)) = [\Psi(s_E) - v_I]\sigma(s_E) - T(1 - \sigma(s_E)) = [\Psi(s_E) - v_I]s_E - T(1 - s_E).$$

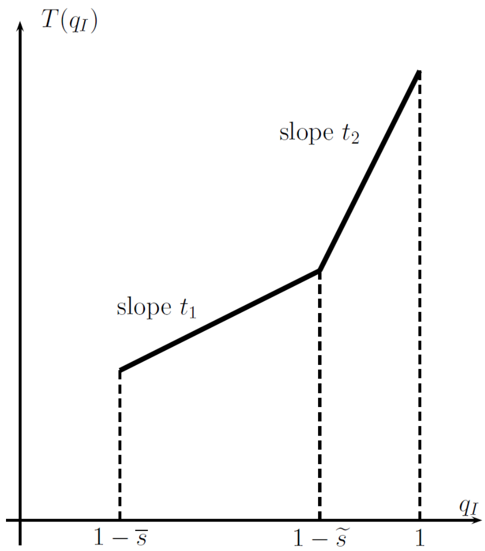

Figure 13a: Convex kink in the price schedule

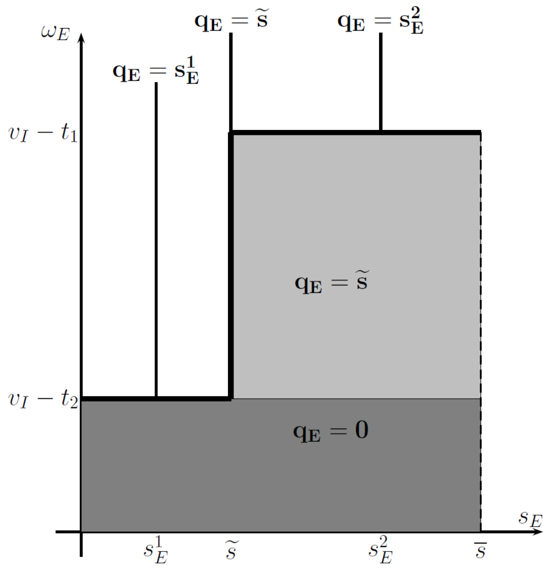

Figure 13b: Two-step increasing boundary line

Differentiating and using the first-order condition at $\sigma(s_E)$ yields

$$T'(1-s_E) = -\Psi'(s_E)[s_E - \sigma(s_E)] - \Psi(s_E) + v_I.$$

Differentiating again yields

$$T''(1-s_E) = \Psi''(s_E)[s_E - \sigma(s_E)] + \Psi'(s_E)[2 - \sigma'(s_E)]. \tag{B.3}$$

In the above equation, the two bracketed terms are nonnegative (use $\sigma' \leq 0$), and the slope Ψ' is negative by assumption, which yields item 2 of the lemma. Around a local minimum of Ψ, Ψ' is small, and the first term is positive, hence property 3. Property 4 follows from items 1 and 2. □

Finally note that when Ψ has a downward discontinuity at s_E, the tariff has an upward discontinuity at $1 - s_E$. To illustrate, Figures 14a and 14b consider the case where the boundary line is a nonincreasing step function with two pieces.

C Derivation of the optimal quantity function

C.1 Elasticity of entry and distribution of (s_E, ω_E)

In this section, we prove Lemma 2.

Proof. The elasticity of entry varies with s_E in the same way as the hazard rate h given by

$$h(\omega_E|s_E) = \frac{f(\omega_E|s_E)}{1 - F(\omega_E|s_E)}.$$

We have

$$\int_{\underline{\omega}_E}^{\omega_E} h(x|s_E)\,\mathrm{d}x = -\ln[1 - F(\omega_E|s_E)].$$

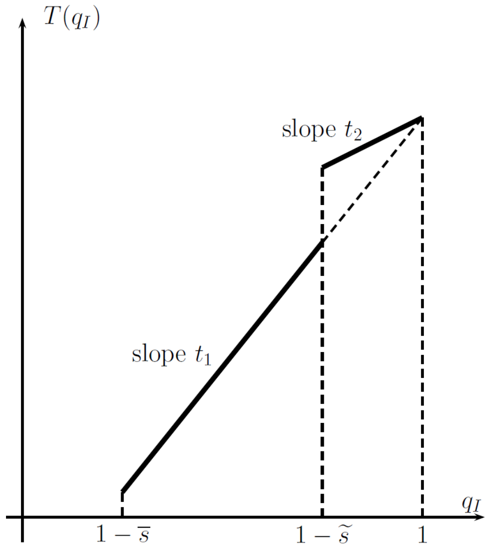

Figure 14a: Upward jump in the price schedule

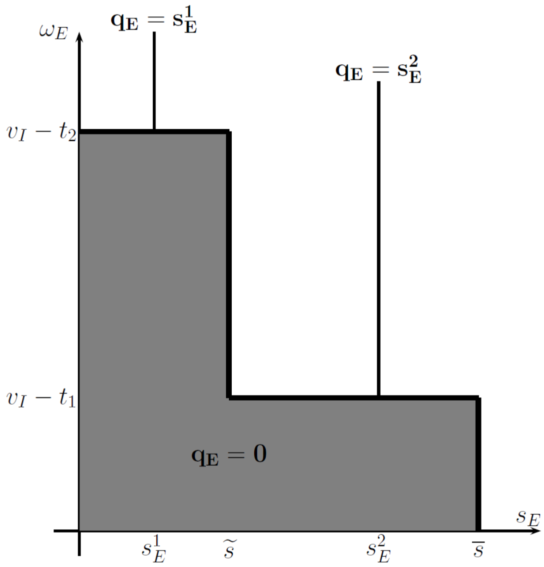

Figure 14b: Two-step decreasing boundary line

If the elasticity of entry does not depend on (increases with, decreases with) s_E, the same is true for the hazard rate, and hence also for the cdf $F(\omega_E|s_E)$, which yields the results.[23] □

C.2 Proof of Lemma 3

Proof Letting $I(r) = \int_0^1 a(x)r(x)\,\mathrm{d}x$, we have

$$I(r) = \sum_i \int_{x_{2i}}^{x_{2i+1}} x a(x)\,\mathrm{d}x + \sum_i x_{2i+1} \int_{x_{2i+1}}^{x_{2i+2}} a(x)\,\mathrm{d}x,$$

where the index i in the two sums goes from either $i = 0$ or $i = 1$ to either $i = n-1$ or $i = n$, in accordance with the conventions exposed in Footnote 14. Differentiating with respect to an interior even extremity yields

$$\frac{\partial I}{\partial x_{2i}} = a(x_{2i}).[x_{2i-1} - x_{2i}].$$

The first-order condition therefore imposes $a(x^*_{2i}) = 0$. The second-order condition for a maximum shows that a must be negative (positive) at the left (right) of x^*_{2i}.

Differentiating with respect to an interior odd extremity yields

$$\frac{\partial I}{\partial x_{2i+1}} = \int_{x_{2i+1}}^{x_{2i+2}} a(x)\,\mathrm{d}x.$$

The first-order condition therefore imposes $\int_{x^*_{2i+1}}^{x^*_{2i+2}} a(x)\,\mathrm{d}x$. The second-order condition for a maximum imposes that a is nonnegative at x^*_{2i+1}.

[23] The variable ω_E first-order stochastically decreases (increases) with s_E if and only if $F(\omega_E|s_E)$ increases (decreases) with s_E.

If $a(1) > 0$, then it is easy to check that $r^*(x) = x$ at the top, namely on the interval $[x^*_{2n}, x^*_{2n+1}]$ with x^*_{2n} being the highest zero of the function a and $x^*_{2n+1} = 1$. If the function a admits no zero, it is everywhere positive and hence $r^*(x) = x$ on the whole interval $[0, 1]$.

If $a(1) < 0$, then r^* is constant at the top, namely on the interval $[x^*_{2n-1}, x^*_{2n}]$, with $x^*_{2n} = 1$ and $\int_{x^*_{2n-1}}^1 a(x)\,\mathrm{d}x = 0$. If the integral $\int_y^1 a(x)\,\mathrm{d}x$ remains negative for all y, then r^* is constant and equal to zero on the whole interval $[0, 1]$.

C.3 Monotonicity constraints for odd extremities

We now investigate the monotonicity constraint regarding the odd extremities $s_E^{2i+1}(\omega_E)$, i.e. we check whether it is nondecreasing in ω_E.

$$A(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} s^{\mathrm{v}}(s, \omega_E) f(\omega_E|s) g(s)\,\mathrm{d}s = 0$$

This can also be written as

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f(\omega_E|s) - \beta(1 - F(\omega_E|s))\right] g(s)\,\mathrm{d}s = 0$$

or

$$\int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[\omega_E(1 - \beta/\varepsilon) - \omega_I\right] f(\omega_E|s) g(s)\,\mathrm{d}s = 0$$

Recall that the elasticity of entry, ε, may vary with the maximum scale of entry, s, in the above integral. A more compact way to express the same condition is

C.3.1 Nondecreasing hazard rate in ω_E

When the hazard rate is nondecreasing, the corresponding condition is weaker:

$$\beta \le 4\underline{\varepsilon} * \bar{\varepsilon}/(\Delta\varepsilon)^2. \tag{C.1}$$

C.3.2 Nondecreasing elasticity in ω_E

The function A is nonincreasing in s_E^{2i+1} as the virtual surplus is nonnegative at this point:

$$\frac{\partial A}{\partial s_E^{2i+1}}(s_E^{2i+1}, \omega_E) = -s^{\mathrm{v}}(s_E^{2i+1}, \omega_E) f(\omega_E|s_E^{2i+1}) g(s_E^{2i+1}) \le 0.$$

Differentiating with respect to ω_E, we get

$$\frac{\partial A}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[(\omega_E - \omega_I) f'(\omega_E|s) + f(\omega_E|s) + \beta f(\omega_E|s)\right] g(s)\,\mathrm{d}s,$$

where we denote by f' the derivative of f in ω_E. When f is nondecreasing in ω_E, or $f' \ge 0$, we have $\partial A/\partial\omega_E \ge 0$, and hence the odd extremities are nondecreasing in ω_E.

We now want to extend this result under the general assumption that the elasticity of entry is nondecreasing in ω_E. Using the monotonicity of ε in ω_E, we have:

$$\frac{\partial \varepsilon(\omega_E|s_E)}{\partial \omega_E}(s_E^{2i+1}, \omega_E) = \frac{\partial}{\partial \omega_E}\left[\frac{\omega_E f(\omega_E|s_E)}{1-F(\omega_E|s_E)}\right] \geq 0$$

hence $f' \geq -(1+\varepsilon)f/\omega_E$. Using $\omega_E \geq \omega_I$, we find that

$$\frac{\partial A}{\partial \omega_E} \geq \int_{s_E^{2i+1}}^{s_E^{2i+2}} \left[\frac{\omega_I}{\omega_E} - \varepsilon\left(1 - \frac{\beta}{\varepsilon} - \frac{\omega_I}{\omega_E}\right)\right] f(\omega_E|s)g(s)\,\mathrm{d}s.$$

Recalling (16), we can rewrite the above inequality as

$$\frac{\partial A}{\partial \omega_E} \geq \mathbb{E}\left(1 - \frac{\beta}{\varepsilon}\middle| H\right) - \operatorname{cov}\left(\varepsilon, 1 - \frac{\beta}{\varepsilon}\middle| H\right).$$

This inequality holds as an equality when the elasticity of entry is constant in ω_E. We now look for a sufficient condition for the right-hand side to be nonnegative for any distribution of ε. Noting $m = \mathbb{E}(\varepsilon|H)$ the expectation of ε on H, the condition can be rewritten as

$$\mathbb{E}\left[(\varepsilon - m - 1)\left(1 - \frac{\beta}{\varepsilon}\right)\middle| H\right] \leq 0.$$

The function $(\varepsilon - m - 1)(1 - \beta/\varepsilon)$ is convex in ε. We denote by $[\underline{\varepsilon}, \bar{\varepsilon}]$ the support of the distribution of ε. For given values of $\underline{\varepsilon}$, $\bar{\varepsilon}$ and $m = \mathbb{E}(\varepsilon|H)$, the expectation of this convex function is maximal when the distribution of ε has two mass points at $\underline{\varepsilon}$ and $\bar{\varepsilon}$, associated with the respective weights $\frac{\bar{\varepsilon}-m}{\bar{\varepsilon}-\underline{\varepsilon}}$ and $\frac{m-\underline{\varepsilon}}{\bar{\varepsilon}-\underline{\varepsilon}}$. We thus need to make sure that

$$(\bar{\varepsilon} - m)(\underline{\varepsilon} - m - 1)\left(1 - \frac{\beta}{\underline{\varepsilon}}\right) + (m - \underline{\varepsilon})(\bar{\varepsilon} - m - 1)\left(1 - \frac{\beta}{\bar{\varepsilon}}\right) \leq 0, \qquad \text{(C.2)}$$

for any $m \in [\underline{\varepsilon}, \bar{\varepsilon}]$. The above function is the sum of two quadratic functions of m. The first is convex with roots $\underline{\varepsilon} - 1$ and $\bar{\varepsilon}$; the second is concave with roots $\underline{\varepsilon}$ and $\bar{\varepsilon} - 1$. Both quadratic functions have zero derivative at $m = (\underline{\varepsilon} + \bar{\varepsilon} - 1)/2$. The sum of the two functions is concave as $\underline{\varepsilon} < \bar{\varepsilon}$.

When $\bar{\varepsilon} \leq \underline{\varepsilon} + 1$, the concave quadratic function is negative on the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$, and hence the inequality (C.2) holds on that interval. When $\bar{\varepsilon} > \underline{\varepsilon} + 1$, we need to make sure that the maximum value of the concave quadratic function is lower than the minimum value of the convex quadratic function. This is is the case if and only if

$$\left(1 - \frac{\beta}{\bar{\varepsilon}}\right)(\Delta\varepsilon - 1)^2 \leq \left(1 - \frac{\beta}{\underline{\varepsilon}}\right)(\Delta\varepsilon + 1)^2.$$

or, equivalently

$$\beta \leq \frac{\bar{\varepsilon}}{1 + (1 + \Delta\varepsilon)^2/4\underline{\varepsilon}}. \qquad \text{(C.3)}$$

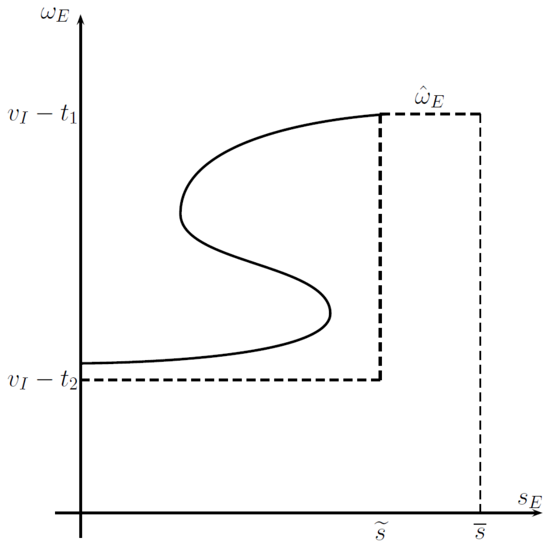

Figure 15a: Non monotonic odd extremities

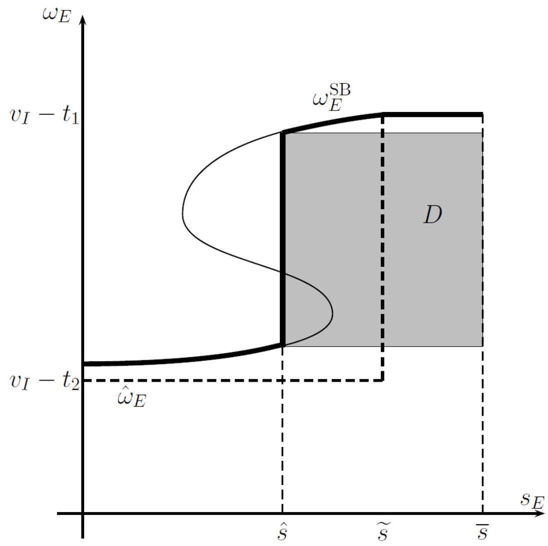

Figure 15b: Two-dimensional pooling area: $q_E = \hat{s}$ on D.

C.4 Two-dimensional pooling

D Finite disposal costs

D.1 The buyer's total purchases are not lower than his total demand

In this section, we extend Lemma A.1 to the case of finite disposal costs. Using the definition of V, equation (17), and applying the envelope theorem, we get

$$\frac{\partial V}{\partial q_I}(q_E, q_I) = -\gamma + \mu = v_I - \nu,$$

where μ and ν are the respective Lagrange multipliers for the constraints $x_I \leq q_I$ and $x_E + x_I \leq 1$ in the buyer's problem (1). It follows that the derivative of V with respect to q_I equals v_I when $q_I < 1 - q_E$ (as ν must be zero in this case) and $-\gamma$ when $q_I > 1 - q_E$ (recall that $x_E = q_E$, and hence x_I must be smaller than q_I if $q_I > 1 - q_E$, which yields $\mu = 0$). In any case, this partial derivative does not exceed v_I.

The proof of Lemma A.1 follows the same route as in Section A. The only needed modification in the proof consists in replacing equality (A.3) with the inequality

$$V(q_E, \tilde{q}_I) - T(q_I) - v_I(\tilde{q}_I - q_I) \leq V(q_E, q_I) - T(q_I),$$

where we have used $\partial V/\partial q_I \leq v_I$. This inequality is enough to guarantee $U_B(q_E) = \tilde{U}_B(q_E)$ for all q_E. Equations (A.4) and (A.5) continue to hold as equalities because $\partial V/\partial q_I = v_I$ in the region where q_I is below $1 - q_E$.

D.2 The slope of the tariff is above $-\gamma$

In this section, we prove Proposition 6

Starting from any tariff T, we define $\hat{T}$ as

$$\hat{T}(q_I) = \inf_{q \geq q_I} T(q) + \gamma(q - q_I).$$

Starting from any quantity level q, the incumbent offers the incumbent the opportunity to buy less units than q, $q_I \leq q$, in return for the payment $T(q) + \gamma(q - q_I)$. This option allows the buyer to avoid disposal costs, and is relevant only if γ is finite. The slope of the new tariff is larger than or equal to $-\gamma$.

Let $\hat{U}(q_E)$ the buyer's net utility after he has purchased units q_E units from the entrant under the price schedule $\hat{T}$:

$$\hat{U}_B(q_E) = \max_{q_I} V(q_E, q_I) - \hat{T}(q_I). \tag{D.1}$$

As $\hat{T} \leq T$, we have: $\hat{U}_B \geq U_B$. Suppose that, under $\hat{T}$, it is optimal for the buyer to purchase $\hat{q}_I$ from the incumbent if he has purchased q_E from the entrant. By construction of $\hat{T}$, there exists $q_I \geq \hat{q}_I$ such that $\hat{T}(\hat{q}_I)$ equals or is arbitrarily close to $T(q_I)+\gamma(q_I - \hat{q}_I)$. Using the definition of V, we get:

$$\begin{aligned} \hat{U}_B(q_E) &= V(\hat{q}_I, q_E) - \hat{T}(\hat{q}_I) \\ &= V(\hat{q}_I, q_E) - \gamma(q_I - \hat{q}_I) - T(q_I) \\ &\leq V(q_I, q_E) - T(q_I). \end{aligned}$$

It follows that $\hat{U}_B(q_E) \leq U_B(q_E)$, and hence $\hat{U}_B(q_E) = U_B(q_E)$. The buyer and the entrant agree on the same quantity q_E as their choice only depends on U_B and $\hat{U}_B$, which coincide. The entrant's profit, $\beta \Delta S_{BE}$ is the same under T and $\hat{T}$.

Suppose that the buyer has purchased q_E from the entrant and let q_I be the optimal quantity purchased from the incumbent under tariff T. As $\hat{T}(q_I) \leq T(q_I)$, the buyer may always choose to purchase the same quantity from the incumbent ($\hat{q}_I = q_I$) under the tariffs $\hat{T}$ and T:

$$U_B(q_E) = \tilde{U}_B(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, q_I) - \hat{T}(q_I).$$

Yet consider the special case where $q_I > 1 - q_E$. We know from Footnote **??** that $V(q_E, q_I)$ is nonincreasing and linear in q_I with slope $-\gamma$ on $(1 - q_E, q_I)$. By definition of $\hat{T}(1 - q_E)$, we get

$$V(q_E, q_I) - V(q_E, 1 - q_E) = -\gamma[q_I - (1 - q_E)] \leq T(q_I) - \hat{T}(1 - q_E)$$

or

$$U_B(q_E) = \hat{U}(q_E) = V(q_E, q_I) - T(q_I) \leq V(q_E, 1 - q_E) - \hat{T}(1 - q_E).$$

It follows that the buyer may purchase $\hat{q}_I = 1 - q_E$ from the incumbent. The change from q_I to $\hat{q}_I$ does not decrease the total surplus. On the contrary, it avoids production and disposal costs:

$$V(q_E, \hat{q}_I) - c_E q_E - C_I(\hat{q}_I) \geq V(q_E, q_I) - c_E q_E - C_I(q_I).$$

In sum, the change from T to $\hat{T}$ does not alter the entrant's profit and does not decrease the total surplus. We conclude from (11) that the change does not decrease the expected payoff of the buyer-incumbent coalition.

D.3 Inefficient entry never arises in equilibrium

In this section, we extend Proposition **??** to the case of finite disposal costs.

D.4 Proof of lemma 4

Conversely, assume that $\Psi(s_E) \leq v_I + \gamma$, and define the quantity function by (10), the profit function $\Delta S_{BE}(s_E, \omega_E)$ by equation (**??**) and the tariff by (B.1). Differentiating the latter equation with respect to ω_E below the boundary – a region where q_E increases with ω_E– yields $T'(q) = v_I - \omega_E \geq -\gamma$.