Monetary policy and herd behavior in new-tech investment*

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Abstract

This paper studies the interaction between monetary policy and asset prices using a simple general equilibrium model in which asset-price bubbles may form due to herd behavior in investment in a new technology whose productivity is uncertain. The economy is populated with one infinitely lived representative household and overlapping generations of finitely lived entrepreneurs. Entrepreneurs receive private signals about the productivity of the new technology and borrow from the household to publicly invest in the old or the new technology. Monetary policy intervention, by affecting the cost of resources for entrepreneurs, can make the entrepreneurs invest in the new technology if and only if they have received a favourable private signal. In doing so, it reveals this signal and hence prevents herd behavior and the asset-price bubble. We identify conditions under which such a monetary policy intervention is socially desirable.

Key Words : Monetary Policy – Asset Prices – Informational Cascades.

JEL Classification : E52, E32

1 Introduction and Literature Review

Should monetary policy react to perceived asset-price bubbles[1]? This old question has been hotly debated again since the remarkable rise and fall in new-tech equity prices in developed economies in the late 1990s and early 2000s. Today's conventional answer among central bankers is "no". This answer stems from the consideration of the following trade-off. On the one hand, if there is actually a bubble, then such a monetary policy reaction may reduce its size and/or its duration, and hence its welfare costs due to overinvestment. On the other hand, if alternatively there is actually no bubble, then such a monetary policy reaction will be distortive and reduce welfare. Given this trade-off, a monetary policy reaction can be viewed as an insurance-against-bubbles policy, and the two conditions most commonly stressed by central bankers for its desirability are the following ones: (i) the central

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[1]The very definition of an asset-price "bubble" is quite model dependent. We temporarily postpone the exact definition in the context of our model, and want to think of it here as the price of an asset differing from some *benchmark* present discounted value of dividends generated by the asset (possibly using a different pricing kernel than the equilibrium one).

bank should be sufficiently certain that there is actually a bubble; (ii) the bubble should be sufficiently sensitive to modest interest-rate hikes. Because they commonly view these conditions as unlikely to be met in practice (Bernanke, 2002), central bankers thus usually conclude that, in most if not all cases, such a monetary policy reaction is not desirable.

This paper seeks to challenge this view by considering a simple general-equilibrium model in which, because asset-price bubbles are modeled as (rational) herd behavior, these two conditions can easily be met. We focus on bubbles in new-tech equity prices, as our argument rests on some productivity considerations that are not likely to play a key role in the development of other kinds of asset-price bubbles, *e.g.* bubbles in house prices. More precisely, we assume that a new technology becomes available whose productivity will be known with certainty only in the medium term. Entrepreneurs sequentially choose whether to invest in the old or the new technology, each of them on the basis of both the previous investment decisions that she observes and a private signal that she receives about the productivity of the new technology. Herd behavior may then arise as the result of an informational cascade (Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992) that corresponds to a situation in which, because the first entrepreneurs choose to invest in the new technology as they receive encouraging private signals about its productivity, the following entrepreneurs rationally choose to invest in the new technology too whatever their own private signal. The asset we consider in the model is an entrepreneur firm share, and the stock market "bubble" is defined as the difference between the equilibrium share price and the one obtained if investors private signals were public information.

In this context, monetary policy tightening, by making borrowing dearer for the entrepreneurs, can make them invest in the new technology if and only if they receive an encouraging private signal about its productivity. In doing so, it prevents herd behavior and hence the bubble in new-tech equity prices. Under this new explanation of bubbles in new-tech equity prices, the two conditions mentioned above can be met: (i) the central bank can detect herd behavior with certainty, even though it knows less about the productivity of the new technology than each entrepreneur; (ii) given the fragility of informational cascades, a modest monetary policy intervention can be enough to interrupt herd behavior, even though it may not interrupt the new-tech investment craze[2]. As a consequence, under certain conditions, such a monetary policy intervention is *ex ante* preferable, in terms of social welfare, to the laisser-faire policy.

Our way of modeling bubbles in new-tech equity prices has some advantages over each of the following three ways in which they are modeled in the literature on monetary policy and asset-price bubbles. First, bubbles may be modeled as an exogenous boom-and-bust term in the asset-price-

[2]The latter outcome would be expected by many a central banker, *e.g.* Bernanke (2002).

dynamics equation (Bernanke and Gertler, 1999, 2001). This *ad hoc* modeling makes the bubble by construction insensitive to monetary policy. By contrast, our modeling enables monetary policy to affect the bubble. Second, bubbles may be modeled as the result of favourable public news about future productivity that eventually fails to materialize (Gilchrist and Leahy, 2002; Christiano, Ilut, Motto and Rostagno, 2007). In this context, given that expectations are assumed to be rational and that the central bank is assumed to have no informational advantage over the private sector and therefore to be as much surprised by the lower-than-expected eventual productivity level as the private sector, a proper unconditional assessment of the desirability of a given monetary policy requires to consider not only the case where the favourable news does not materialize, but also the case where it does, and to assign an occurrence probability to each case − something this branch of the literature usually does not do[3]. Modeling the asset-price bubble as herd behavior enables us to do just that in a micro-founded way. Third and finally, the asset-price bubble may be modeled as the result of a permanent increase in productivity growth that economic agents gradually recognize afterwards (Gilchrist and Saito, 2006). However, in a new-technology context, this late-recognition assumption may be viewed as less relevant than the early-news assumption that we make.

The other side of the coin, though, is that our way of modeling bubbles in new-tech equity prices and our wish to secure some analytical results compel us to consider a highly stylised model that fails to reproduce some basic characteristics of observed new-tech investment crazes, most notably the concomitant steady growth in consumption and asset prices. Indeed, this model predicts that, during a new-tech investment craze, as long as some uncertainty remains about the productivity of the new technology, consumption should initially jump to a lower level and remain at this level thereafter, while asset prices should initially jump to a higher level and, under the laisser-faire policy, remain at that level thereafter. We therefore view our paper as a first step in building an empirically more relevant model of herd behavior in new-tech investment[4].

Our paper is related to the literature on the role of informational cascades in the business cycle. Within this literature, the paper closest to ours is that of Chamley and Gale (1994), which models investment collapses as the result of herd behavior. A first difference between the two papers is that, unlike them, we consider a general-equilibrium model and conduct policy analysis. A second difference is that they consider an endogenous timing of investment decisions, as they are also interested in modeling strategic investment delay, while in our setup the timing of investment decisions is exogenous.

[3]Gilchrist and Leahy (2002) do actually consider both cases, without needing to assign an occurrence probability to each of them, because the "strong inflation-targeting" monetary policy that they consider is very close to the optimal monetary policy in both cases.

[4]This research agenda would benefit from the works of Beaudry and Portier (2004), Jaimovich and Rebelo (2006) and Christiano, Ilut, Motto and Rostagno (2007), whose models predict an increase in aggregate output, employment, investment and consumption in response to news of future technological improvement.

And a third difference is that, in equilibrium, in their model, an investment surge is always socially optimal, unlike an investment collapse, while in ours, both new-tech and old-tech investment crazes may be socially non-optimal.

The remainder of the paper is structured as follows. Section 2 presents the model. The competitive equilibrium with exogenous information about the productivity of the new technology is described in Section 3. We introduce endogenous information, derive the results about the desirability of policy intervention in a simple case and conduct simulations in more complex cases in Section 4. Section 5 concludes.

2 The model

We consider an economy populated with infinitely lived households, overlapping generations of finitely lived entrepreneurs, and a central bank. For simplicity, we restrict our analysis to equilibria that are symmetric across entrepreneurs and across households, *i.e.* equilibria such that there is one representative household and, in each generation, one representative entrepreneur. Time is discrete, indexed by $t \in \mathbb{Z}$, and there is a single good that is non-storable and can be consumed or invested.

2.1 Technology

A production project requires κ_t units of good at date t, the investment date, and allows to operate a firm that produces $Y_{t+N} = A_{t+N} L_{t+N}^{\alpha}$ units of good at date $t+N$, where $N \in \mathbb{N}^*$, A_{t+N} is a productivity parameter, L_{t+N} is labor services and $0 < \alpha < 1$. A production project needs a newborn entrepreneur to be undertaken, and a newborn entrepreneur cannot undertake more than one project.

To undertake a production project, a newborn entrepreneur needs to choose a technology. We consider altogether three different technologies, which we denote by the real numbers 0, $\overline{z}$ and z, with $0 < \overline{z} < z$. Technology 0 corresponds to the absence of any production project. It is characterized by the investment $\kappa_t = 0$ and the productivity parameter $A_{t+N} = 0$. Technology $\overline{z}$ is characterized by the investment $\kappa_t = \kappa(\overline{z}) > 0$ and the productivity parameter $A_{t+N} = A(\overline{z}) > 0$. Technology z requires more investment than technology $\overline{z}$: $\kappa_t = \kappa(z) > \kappa(\overline{z})$. It may be “good” and lead to the productivity parameter $A_{t+N} = A(z) > A(\overline{z})$, or be “bad” and lead to the same productivity parameter $A_{t+N} = A(\overline{z})$ as technology $\overline{z}$.

We consider two different economies. One is the economy of tranquil times, where at each date $t \in \mathbb{Z}$ the only available technologies are 0 and $\overline{z}$ and this situation is (rightly) expected by households and entrepreneurs to last forever:

$$\forall t \in \mathbb{Z}, \forall k \in \mathbb{N}^*, \mathcal{F}_t = E_{\Omega(h,t)} \mathcal{F}_{t+k} = E_{\Omega(e,t)} \mathcal{F}_{t+k} = \{0, \overline{z}\},$$

where $\mathcal{F}_t$ denotes the set of technologies available at date t, $E_{\Omega(h,t)}$ the expectation operator conditional on the representative household's date t information set $\Omega(h,t)$, and $E_{\Omega(e,t)}$ the expectation operator conditional on the representative newborn entrepreneur's date t information set $\Omega(e,t)$. Endogenous differences in information sets will be the at the core of the model.

The other is the economy with technological change. In the latter, until date 0 included, the only available technologies are 0 and $\overline{z}$ and this situation is (wrongly) expected by households and entrepreneurs to last forever:

$$\forall t \in \mathbb{Z}^{-}, \forall k \in \mathbb{N}^{*}, \mathcal{F}_t = E_{\Omega(h,t)}\mathcal{F}_{t+k} = E_{\Omega(e,t)}\mathcal{F}_{t+k} = \{0, \overline{z}\}.$$

From date 1 onwards, technology z becomes available as well and this situation is (rightly) expected by households and entrepreneurs to last forever:

$$\forall t \in \mathbb{Z}^{+*}, \forall k \in \mathbb{N}^{*}, \mathcal{F}_t = E_{\Omega(h,t)}\mathcal{F}_{t+k} = E_{\Omega(e,t)}\mathcal{F}_{t+k} = \{0, \overline{z}, z\}.$$

We then call $\overline{z}$ the "old technology" and z the "new technology". We assume that whether the new technology is good or bad becomes common knowledge at date $N+1$ whatever the investment decisions taken at dates 1 to N. For each $t \in \{1, ..., N\}$, we note μ_t the probability that the new technology is good conditionally on $\Omega(h,t)$ and $\widetilde{\mu}_t$ the probability that the new technology is good conditionally on $\Omega(e,t)$. The endogeneity of those believes μ and $\widetilde{\mu}$ will allow for herds and therefore asset bubbles.

2.2 Preferences

The representative household supplies inelastically one unit of labor at each date. Her preferences are represented by the following utility function:

$$U_t = E_{\Omega(h,t)} \sum_{j=0}^{\infty} \beta^j \ln(c_{t+j}),$$

where c_t denotes her consumption at date t, and $0 < \beta < 1$. We choose a logarithmic utility function to simplify the algebra.

At each date, one representative entrepreneur is born. She lives for $N+1$ periods and consumes only in her last period of life. The preferences of an entrepreneur born at date t are represented by the following linear utility function:

$$V_t = \beta^N E_{\Omega(e,t)} c^e_{t+N},$$

where c^e_{t+N} denotes her consumption at date $t+N$. We assume that each generation contains a large number of entrepreneurs, so that the representative entrepreneur is price-taker.

2.3 Market organization

There is a good market, a labor market, a bond market and a stock market. All are competitive. The final good is the numéraire. A newborn entrepreneur may want to borrow κ to undertake a production project. The return from this investment will be the profit she will obtain from production N periods onwards. We assume that the only financial market to which the entrepreneurs have access is a market for N-period bonds. Households have also access to this market, and there is secondary market for those bonds. We denote B_{t+N} the number of bonds that pay in period $t+N$, and that has been subscribed by the household in period t. Each of this bond will pay one unit of good in period $t+N$, and its price is denoted q_t. B_t^e is the number of bonds emitted by the entrepreneurs. On the stock market will be traded claims on the future profits of firms. As entrepreneurs (the firms owners) do not have access to the stock market, transactions will always be zero and this market will serve here only as a device to price firms. For that reason, and to facilitate the reading, we will omit firms shares in the household budget constraints.

2.4 Resource constraints

The resource constraint on the good market states that, at each date t, the total number of goods consumed and invested cannot be larger than the total amount of goods available:

$$c_t + c_t^e + \kappa_t \leq Y_t.$$

The resource constraint on the labor market states that, at each date t, labor services cannot exceed the total amount of labor that is supplied:

$$L_t \leq 1.$$

2.5 Monetary policy

We consider a policy that has an effect on the economy only through its effect on the real interest rate, and we interpret it as monetary policy. This amounts in effect to focusing on the real-interest-rate transmission channel of monetary policy. More specifically, we model monetary policy as a tax (or subsidy) on lending together with a positive (or negative) lump-sum transfer to the representative household: at each date t, the representative household lends $q_t B_{t+N}$ to the entrepreneur and gives $(\tau_t - 1)\, q_t B_{t+N}$ to the central bank (when $\tau_t > 1$) or receives $-(\tau_t - 1)\, q_t B_{t+N}$ from the central bank (when $0 < \tau_t < 1$), while the central bank gives a lump-sum transfer $T_t \equiv (\tau_t - 1)\, q_t B_{t+N}$ to the representative household (when $\tau_t > 1$) or receives a lump-sum transfer $T_t \equiv -(\tau_t - 1)\, q_t B_{t+N}$ from the representative household (when $0 < \tau_t < 1$). The budget constraint of the representative household at date t is therefore

$$c_t + \tau_t q_t B_{t+N} \leq B_t + w_t L_t + T_t.$$

We assume that there is no monetary policy intervention before date 1 and after date N: $\forall t \in \mathbb{Z} \setminus \{1, ..., N\}$, $\tau_t = 1$. This assumption will be justified in Section 4.

2.6 Agents programs

The representative household enters period t with a portfolio $\mathcal{S}_{t-1} = (B_t, ..., B_{t+N-1})$ of bonds that pay interest if at maturity. She then decides how much to consume and how much to save, supplying inelastically one unit of labor. Her program can be written in the following recursive way:

$$\mathcal{W}(\mathcal{S}_{t-1}) = \max_{c_t, B_{t+N}} \left\{ \ln(c_t) + \beta E_{\Omega(h,t)} \mathcal{W}(\mathcal{S}_t) \right\}$$
$$\text{subject to } c_t + \tau_t q_t B_{t+N} \leq B_t + w_t L_t^s + T_t \text{ and } L_t^s \leq 1,$$

where w_t is the wage rate at date t. The corresponding optimality conditions are

$$\begin{aligned} \tau_t q_t &= \beta^N E_{\Omega(h,t)} \left[\frac{c_t}{c_{t+N}} \right], \\ L_t^s &= 1, \end{aligned}$$

and a transversality condition.

The representative newborn entrepreneur borrows κ_t at date t, and hires L_{t+N} to produce Y_{t+N} at date $t+N$. Production proceeds are used to pay wages, reimburse the debt and consume. Her budget constraints are therefore

$$\begin{array}{lcll} \kappa_t & \leq & q_t B_{t+N}^e & \text{in period } t, \\ c_{t+N}^e + B_{t+N}^e & \leq & \Pi_{t+N} \equiv A_{t+N} L_{t+N}^{\alpha} - w_{t+N} L_{t+N} & \text{in period } t+N. \end{array}$$

Labor demand L_{t+N} will be set such that marginal productivity of labor equalizes the real wage w_{t+N}:

$$\alpha A_{t+N} L_{t+N}^{\alpha-1} = w_{t+N},$$

while the technology chosen at date t will be

$$z_t = \arg\max_{z_t \in \mathcal{F}_t} \beta^N E_{\Omega(e,t)} \left[\Pi_{t+N} - \frac{\kappa_t}{q_t} \right].$$

We assume entrepreneurs always plays pure strategies, ad do not consider non symmetric equilibria in which entrepreneurs randomize over investment decisions.

2.7 Competitive equilibrium

In this economy, a symmetric competitive equilibrium is a sequence of prices $(q_t, w_t)_{t\in\mathbb{Z}}$, quantities $(B_t, B_t^e, c_t, c_t^e, L_t)_{t\in\mathbb{Z}}$ and technology choices $(z_t)_{t\in\mathbb{Z}}$ such that, for exogenous sequences of actual and expected technological possibilities $(\mathcal{F}_t)_{t\in\mathbb{Z}}$ and $\left(E_{\Omega(h,t)}\mathcal{F}_{t+k} = E_{\Omega(e,t)}\mathcal{F}_{t+k}\right)_{t\in\mathbb{Z},k\in\mathbb{N}^*}$ and for an exogenous sequence of monetary policy interventions $(\tau_t)_{t\in\{1,\ldots,N\}}$, (i) prices and quantities are positive, (ii) the representative household's consumption and bonds holding solve her maximization problem given prices, (iii) the representative newborn entrepreneur's investment decision maximizes her utility given prices, (iv) labor demand maximizes the representative aged $N+1$ entrepreneur's profits given prices, and (v) labor, bonds and good markets clear.

2.8 Discussion

We have made a set of strong assumptions, which are not equally restrictive. Let us first consider preferences. Assuming log utility for the households is crucial for our analytical results, but could be relaxed if we ware to do only numerical analysis. Considering risk-neutral entrepreneurs that consume only in the last period of their life is also crucial in order to solve analytically the model when we introduce endogenous information and potential informational cascades, but is not if we were to do only numerical simulations.

Second, we have introduced bonds of maturity N only. This is not a restriction since other maturity bonds would not be traded.

Third, we have assumed that only non-contingent debt contracts are possible. This assumption is crucial. As entrepreneurs are risk-neutral and have some private information, they would reveal by their net supply of some contingent claims that pay in those state of the world on which they have better information, and informational cascades would then not be possible. What we need here is not the absence of *any* contingent claims, but only of claims contingent on the quality of the new-technology. As we want to think of those episodes as quite infrequent ones, and the quality of a technology being partially soft information in the real life, we think the assumption is a good description of the actual environment. Similarly, the assumption symmetric equilibria is crucial. If entrepreneurs could randomize over investment, this would amount to a case with variable investment size, and the private information of the generation of entrepreneurs would always be revealed by their actions.

Fourth, we consider that entrepreneurs are exogenously ranked (by date of birth), that they cannot wait to invest and that investment projects that pay only N periods ahead. Those assumptions are made to have a simple structure of the model: after exactly N periods, uncertainty is resolved. We can therefore solve the model by backward induction, which happens to be particularly convenient. F

Fifth, monetary policy is modeled as a tax on real interest payments. Although such a policy could (should) be labeled tax policy in our model, we want to think of it as monetary policy for two reasons. First reason, it is possible to write down a (admittedly) particular monetary model whose real allocations are the ones of our current model. In such a model[5], the control variable of the monetary authorities is the inflation rate between period t and period $t+N$. The important assumption we have to make to recover the same real allocations is that the central bank can commit on the inflation rate between period t and period $t+N$. Second reason, the implementation of a fiscal policy that would subsidize or tax individual firm is quite complex, requires a lot of information on who are the agents, where are they, whose turn it is to invest, etc... Monetary policy, by manipulating the cost of funds, requires very little information in the implementation phase. Obviously, it has a cost of distorting not only investors decisions, but also some others agents' ones. This tradeoff is present in the paper as households savings are distorted by real interest rate manipulations.

3 Competitive equilibrium with exogenous information

In this section, we consider economies with exogenous information, *i.e.* we assume that μ_1, ..., μ_N, $\widetilde{\mu}_1$, ..., $\widetilde{\mu}_N$ are exogenous. Our aim is to derive necessary conditions on the parameters for the existence and uniqueness of a competitive equilibrium for all $(\mu_1, ..., \mu_N) \in [0;1]^N$ and $(\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$ and for this equilibrium to have some desirable properties. We first study the existence, uniqueness and local dynamic stability of the steady state in tranquil times. We then turn to the equilibrium path when there is a technological change. The results obtained will be useful for the analysis of the endogenous information case considered in the next section.

3.1 Tranquil times

In tranquil times, the only available technologies are 0 and $\overline{z}$. This case corresponds to $\mu_t = \widetilde{\mu}_t = 0$ for all t. We first study the existence and uniqueness of a steady state:

Proposition 1: *(i) in tranquil times, there exists an equilibrium at which households' consumption level is strictly positive and constant if and only if*

$$\beta^N (1-\alpha) A(\overline{z}) - \kappa(\overline{z}) > 0 \tag{1}$$

$$\text{and } \alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N} \kappa(\overline{z}) > 0; \tag{2}$$

[5] The monetary model is presented is the appendix

(ii) if (1) and (2) hold, then this equilibrium is the unique equilibrium at which households' consumption level is strictly positive and constant, and we call it the steady state.

Proof: suppose that such an equilibrium exists and note $\overline{c} > 0$ households' constant consumption level at this equilibrium. Then, at this equilibrium, $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$. Indeed, otherwise, if there existed $t \in \mathbb{Z}$ such that $z_t = 0$, then we would get $c_{t+N} = 0 \neq \overline{c}$. Moreover, at this equilibrium, $\forall t \in \mathbb{Z}$, $q_t = \beta^N \equiv \overline{q}$, *i.e.* the N-period interest factor is $R_t = \overline{q}^{-1} = \beta^{-N} \equiv \overline{R}$. The labor market equilibrium condition then implies that, at this equilibrium, $\forall t \in \mathbb{Z}$, $w_t L_t = \alpha A(\overline{z})$ and $\Pi_t = (1-\alpha) A(\overline{z})$, from which we deduce $\overline{c} = \alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z})$. Therefore, this equilibrium is the unique equilibrium at which households' consumption level is strictly positive and constant. Moreover, since $\overline{c} > 0$, (2) holds. Finally, the condition that no entrepreneur is willing to deviate from this outcome[6] implies $(1-\alpha) A(\overline{z}) - \beta^{-N}\kappa(\overline{z}) > 0$, so that (1) holds. Conversely, suppose that (2) and (1) hold. Then it is easy to see that the outcome $\forall t \in \mathbb{Z}$, $c_t = \alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z})$, $q_t = \beta^N$ and $z_t = \overline{z}$ is an equilibrium. ■

We then study the local dynamic stability of the steady state, which we define as the existence of some neighborhoods $\mathcal{N}_{\overline{c}}$ of $\overline{c}$ and $\mathcal{N}_{\overline{q}}$ of $\overline{q}$ such that if $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$, $\forall t \in \mathbb{Z}^-$, $c_t \in \mathcal{N}_{\overline{c}}$ and $q_t \in \mathcal{N}_{\overline{q}}$, then $\forall t \in \mathbb{Z}^{+*}$, $c_t \in \mathcal{N}_{\overline{c}}$, $q_t \in \mathcal{N}_{\overline{q}}$ and $(c_t, q_t) \longrightarrow (\overline{c}, \overline{q})$ as $t \longrightarrow +\infty$:

Proposition 2: *if (1) and (2) hold, then: in tranquil times, the steady state is locally, dynamically stable if and only if*

$$\beta^N > \frac{\kappa(\overline{z})}{|\alpha A(\overline{z}) - \kappa(\overline{z})|}. \tag{3}$$

Proof: if $\forall t \in \mathbb{Z}$, $z_t = \overline{z}$, then $\forall t \in \mathbb{Z}$,

$$q_t = \beta^N \frac{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_{t-N}}}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_t}}$$

and hence

$$q_t - \beta^N = \frac{-\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \frac{q_{t-N} - \beta^N}{q_{t-N}},$$

so that there exists a neighborhood of $\overline{q}$ such that any sequence (q_t) originating in this neighborhood will remain in this neighborhood and converge towards $\overline{q}$ if and only if (3) holds. ■

[6] Recall that we restrict to symmetrical equilibrium among entrepreneurs. This condition ensures that such a symmetrical equilibrium exists.

3.2 Technological change

We now consider the response of the economy to the unexpected availability of the new technology z from date 1 onwards. We restrict our analysis to equilibria such that the economy is at its steady state until date 0 included, *i.e.* in particular such that $\forall t \in \mathbb{Z}^-$, $(z_t, c_t, q_t) = (\overline{z}, \overline{c}, \overline{q})$. In words, this mean that if the new technology will always be adopted once i is known to be good, and never once it is known to be bad. We also assume that all these equilibria are such that $\forall t > N$, $z_t = z$ if the new technology turns out to be good and $z_t = \overline{z}$ otherwise, and will check later that this is indeed the case for the values of the parameters that we consider. As technologies z or $\overline{z}$ can be chosen in period $t \leq N$, this implies that, $\forall t > N$,

$$\begin{aligned}
c_t &= \alpha A(z) - \kappa(z) + q_{t-N}^{-1}\kappa(z) \text{ if } z_{t-N} = z \text{ and the new technology is good,} \\
c_t &= \alpha A(\overline{z}) - \kappa(z) + q_{t-N}^{-1}\kappa(z) \text{ if } z_{t-N} = z \text{ and the new technology is bad,} \\
c_t &= \alpha A(\overline{z}) - \kappa(z) + q_{t-N}^{-1}\kappa(\overline{z}) \text{ if } z_{t-N} = \overline{z} \text{ and the new technology is good,} \\
c_t &= \alpha A(\overline{z}) - \kappa(\overline{z}) + q_{t-N}^{-1}\kappa(\overline{z}) \text{ if } z_{t-N} = \overline{z} \text{ and the new technology is bad.}
\end{aligned}$$

Moreover, since the representative entrepreneurs born at dates $-(N-1)$ to 0 have invested in $\overline{z}$ and pay back their debts at dates 1 to N at the interest factor $\overline{R}$, the representative household's consumption at each date $t \in \{1, ..., N\}$ is $c_t = \alpha A(\overline{z}) - \kappa(z_t) + \beta^{-N}\kappa(\overline{z})$. As a consequence, for $t \in \{1, ..., N\}$ and $z_t = \overline{z}$, the Euler equation is written

$$\tau_t q_t = \beta^N \left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{\mu_t}{\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{q_t}} + \frac{1-\mu_t}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{q_t}}\right] \tag{4}$$

and we obtain the following proposition concerning q_t for $t \in \{1, ..., N\}$ when $z_t = \overline{z}$:

Proposition 3: *if (1), (2) and (3) hold, then: (i) there exists a strictly positive real number q_t solution of (4) for all $t \in \{1, ..., N\}$ and $(\mu_1, ..., \mu_N) \in [0;1]^N$ if and only if*

$$\alpha A(\overline{z}) - \kappa(z) > 0 \tag{5}$$

$$\text{and } \forall t \in \{1, ..., N\}, \ \tau_t < \tau(\overline{z}), \tag{6}$$

$$\text{where } \forall x \geq \overline{z}, \ \tau(x) \equiv \frac{\beta^N[\alpha A(\overline{z}) - \kappa(x)] + \kappa(\overline{z})}{\kappa(x)};$$

(ii) if (5) and (6) hold, then $\forall t \in \{1, ..., N\}$, q_t, which we note $q(z, \tau_t, \mu_t, 0)$, is unique, and $\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial \tau_t} < 0$ *and* $\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial \mu_t} > 0$.

Proof: *cf* Appendix A. ■

The result $\frac{\partial q(z,\tau,\mu_t,0)}{\partial \tau} < 0$ is simply due to the fact that a positive tax on lending (*i.e.* a monetary policy tightening) raises the interest rate and therefore lowers q_t. The result $\frac{\partial q(z,\tau,\mu_t,0)}{\partial \mu_t} > 0$ is due to the fact that if entrepreneurs invest in the old technology at date t, then, as μ_t increases, c_t remains unchanged but $E_t\{\frac{1}{c_{t+N}}\}$ increases (because the representative household is expected to lend more, and hence to consume less, at date $t+N$), so that q_t increases.

Alternatively, for $t \in \{1,...,N\}$ and $z_t = z$, the Euler equation is written

$$\tau_t q_t = \beta^N \left[\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{\mu_t}{\alpha A(z) - \kappa(z) + \frac{\kappa(z)}{q_t}} + \frac{1-\mu_t}{\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(z)}{q_t}}\right] \tag{7}$$

and we obtain the following proposition concerning q_t for $t \in \{1,...,N\}$ when $z_t = z$:

Proposition 4: *if (1), (2), (3), (5) and (6) hold, then: (i) there exists a strictly positive real number q_t solution of (7) for all $t \in \{1,...,N\}$ and $(\mu_1,...,\mu_N) \in [0;1]^N$ if and only if*

$$\forall t \in \{1,...,N\},\ \tau_t < \tau(z)\,; \tag{8}$$

(ii) if (8) holds, then $\forall t \in \{1,...,N\}$, q_t, which we note $q(z,\tau_t,\mu_t,1)$, is unique, and $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial \tau_t} < 0$; (iii) if (8) holds, then $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial \mu_t} < 0$ for all $t \in \{1,...,N\}$ if and only if

$$\alpha A(\overline{z}) - \kappa(\overline{z}) < \alpha A(z) - \kappa(z)\,. \tag{9}$$

Proof: *cf* Appendix B. ■

As previously, the result $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial \tau_t} < 0$ is simply due to the fact that a positive tax on lending (*i.e.* a monetary policy tightening) raises the interest rate and therefore lowers q_t. The result $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial \mu_t} \lesseqgtr 0$ is due to the fact that if entrepreneurs invest in the new technology at date t, then, as μ_t increases, c_t remains unchanged but $E_t\{\frac{1}{c_{t+N}}\}$ either increases or decreases depending on the sign of $[\alpha A(z) - \alpha A(\overline{z})] - [\kappa(z) - \kappa(\overline{z})]$ (because the representative household is expected both to lend more, as $\kappa(z) > \kappa(\overline{z})$, and to receive a higher wage, as $\alpha A(z) > \alpha A(\overline{z})$, at date $t+N$), so that q_t either increases or decreases depending on the sign of $[\alpha A(z) - \alpha A(\overline{z})] - [\kappa(z) - \kappa(\overline{z})]$. In the following, we will restrict our analysis to the case where $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial \mu_t} < 0$, which seems more natural to us.

The next proposition shows that, under the conditions so far obtained, the interest rate maximized over $\mu_t \in [0;1]$ that prevails when the entrepreneurs borrow little (as they invest in the old technology) is strictly lower than the interest rate minimized over $\mu_t \in [0;1]$ that prevails when the entrepreneurs borrow much (as they invest in the new technology):

Proposition 5: *if (1), (2), (3), (5), (6), (8) and (9) hold, then:* $\forall t \in \{1, ..., N\}$, $\forall (p, p') \in [0;1]^2$, $q(z, \tau_t, p, 0) > q(z, \tau_t, p', 1)$.

Proof: *cf* Appendix C. ■

We then derive a necessary and sufficient condition for a marginal entrepreneur to have no incentive, at dates 1 to N, in any circumstance, to deviate from the other entrepreneurs' common investment decision and invest nothing, as well as a necessary and sufficient condition for the three constraints on the monetary policy instrument to be satisfied in the absence of monetary policy intervention, *i.e.* when $\tau_t = 1$ for all $t \in \{1, ..., N\}$:

Proposition 6: *if (1), (2), (3), (5), (6), (8) and (9) hold, then: (i) a marginal entrepreneur has no incentive at date t to deviate from the other entrepreneurs' common investment decision and invest nothing for all* $t \in \{1, ..., N\}$, *all* $(\mu_1, ..., \mu_N) \in [0;1]^N$ *and all* $(\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$, *if and only if*

$$\forall t \in \{1, ..., N\}, \begin{cases} \text{either } \dfrac{\tau(z)}{1 + \frac{B(z)[\alpha A(\overline{z}) - \kappa(\overline{z})]}{\kappa(z)}} < \tau_t < \dfrac{\tau(\overline{z})}{1 + \frac{\alpha A(\overline{z}) - \kappa(\overline{z})}{(1-\alpha)A(\overline{z})}}, \\ \text{or } B(z) > \dfrac{\kappa(\overline{z})}{(1-\alpha)A(\overline{z})} \text{ and } \tau_t < \dfrac{\tau(\overline{z})}{1 + \frac{\alpha A(\overline{z}) - \kappa(\overline{z})}{(1-\alpha)A(\overline{z})}}, \\ \text{or } B(z) < \dfrac{\kappa(\overline{z})}{(1-\alpha)A(\overline{z})} \text{ and } \tau_t < \dfrac{\tau(z)}{1 + \frac{\kappa(\overline{z})[\alpha A(z) - \kappa(z)]}{\kappa(z)(1-\alpha)A(\overline{z})}} \end{cases}, \tag{10}$$

$$\text{where } B(z) \equiv \frac{\kappa(z) - \kappa(\overline{z})}{(1-\alpha)[A(z) - A(\overline{z})]};$$

(ii) (6), (8) and (10) hold in the absence of monetary policy intervention, i.e. *when* $\tau_t = 1$ *for all* $t \in \{1, ..., N\}$, *if and only if*

$$\begin{cases} \text{either } 1 < \tau(z) < 1 + \dfrac{B(z)[\alpha A(\overline{z}) - \kappa(\overline{z})]}{\kappa(z)}, \\ \text{or } B(z) > \dfrac{\kappa(\overline{z})}{(1-\alpha)A(\overline{z})} \text{ and } 1 < \tau(z), \\ \text{or } B(z) < \dfrac{\kappa(\overline{z})}{(1-\alpha)A(\overline{z})} \text{ and } 1 + \dfrac{\kappa(\overline{z})[\alpha A(z) - \kappa(z)]}{\kappa(z)(1-\alpha)A(\overline{z})} < \tau(z) \end{cases}. \tag{11}$$

Proof: *cf* Appendix D. ■

For each $t \in \mathbb{Z}^{+*}$, let I_t denote the representative newborn entrepreneur's investment decision at date t ($I_t = 1$ when she invests in the new technology and $I_t = 0$ when she invests in the old technology). We also derive necessary and sufficient conditions for a marginal entrepreneur to have no incentive, this time at dates $t > N$, in any circumstance, to deviate from the other entrepreneurs' common investment decision $I_t = 1$ (when the new technology is good) or $I_t = 0$ (when it is bad):

Proposition 7: *if (1), (2), (3), (5), (9) and (11) hold, then: a marginal entrepreneur has no incentive to deviate from the other entrepreneurs' common investment decision $I_t = 1$ (when the new technology is good) or $I_t = 0$ (when it is bad) for all $t > N$, all $(\mu_1, ..., \mu_N) \in [0;1]^N$, all $(\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$ and all $(\tau_1, ..., \tau_N) \in \mathbb{R}^{+*N}$ satisfying (6), (8) and (10), if and only if*

$$\beta^N - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})} \tag{12}$$

$$\text{and } \beta^N \frac{\alpha A(\overline{z}) - \kappa(z)}{\alpha A(z) - \kappa(z)} - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} > \max\left[\frac{\kappa(z)}{(1-\alpha) A(z)}, B(z)\right]. \tag{13}$$

Proof: *cf* Appendix E. ■

We then show that, under the conditions so far obtained, the dynamics of q_t, c_t and c_t^e are well-behaved after date N in the following sense:

Proposition 8: *if (1), (2), (3), (5), (6), (8), (9), (10), (11), (12) and (13) hold, then: $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$, (i) $\forall t \geq 1$, q_t, c_t and c_t^e are strictly positive; (ii) $\lim_{t \longrightarrow +\infty} q_t = \beta^N$,*

$$\lim_{t \longrightarrow +\infty} (c_t, c_t^e) = \left(\alpha A(z) - \kappa(z) + \beta^{-N}\kappa(z), (1-\alpha) A(z) - \beta^{-N}\kappa(z)\right)$$

if the new technology is good and

$$\lim_{t \longrightarrow +\infty} (c_t, c_t^e) = \left(\alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z}), (1-\alpha) A(\overline{z}) - \beta^{-N}\kappa(\overline{z})\right)$$

if it is bad.

Proof: *cf* Appendix F. ■

We finally show that, under the conditions so far obtained, both households and entrepreneurs gain in the long term from a good new technology:

Proposition 9: *if (1), (2), (3), (5), (6), (8), (9), (10), (11), (12) and (13) hold, then: $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$, both households' welfare U_t and entrepreneurs' welfare V_t increase in the long term if the new technology is good.*

Proof: (5) and (13) together imply that $\beta^N > B(z)$ and hence that $(1-\alpha) A(z) - \frac{\kappa(z)}{\beta^N} > (1-\alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{\beta^N}$, so that entrepreneurs' welfare is higher in the long term when the new technology is good than it is initially. Moreover, (9) implies that $\alpha A(z) - \kappa(z) + \frac{\kappa(z)}{\beta^N} > \alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}$, so that households' welfare is also higher in the long term when the new technology is good than it is initially. ■

3.3 List of parameters and conditions

The relevant parameters are α, β, $\kappa(\overline{z})$, $\kappa(\underline{z})$, $A(\overline{z})$, $A(\underline{z})$, N and τ_t for $t \in \{1, ..., N\}$. Given that (5) and (12) imply (2) and (3) and that (8) implies (6), the conditions imposed on these parameters are $0 < \alpha < 1$, $0 < \beta < 1$, $\kappa(\underline{z}) > \kappa(\overline{z}) > 0$, $A(\underline{z}) > A(\overline{z}) > 0$, $N \in \mathbb{N}^*$, $\tau_t > 0$ for $t \in \{1, ..., N\}$, (1), (5), (8), (9), (10), (11), (12) and (13). We will show in the next section that the set of parameter values satisfying all these conditions is not empty.

These conditions are necessary for the existence and uniqueness of an equilibrium with some desirable properties, for all $(\mu_1, ..., \mu_N) \in [0;1]^N$ and $(\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$, among the set of equilibria such that $\forall t > N$, $z_t = \underline{z}$ if the new technology is good, and $z_t = \overline{z}$ otherwise. In addition, the following proposition shows that these conditions ensure that all equilibria are of this type:

Proposition 10: *if (1), (2), (3), (5), (6), (8), (9), (10), (11), (12) and (13) hold, then:* $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$, $\forall t > N$, $z_t = \underline{z}$ *if the new technology is good and* $z_t = \overline{z}$ *if it is bad.*

Proof: *cf* Appendix G. ■

The reason why these conditions are not sufficient for the existence and uniqueness of an equilibrium with desirable properties is that they do not ensure that, at each date between 1 and N, either a marginal entrepreneur has no incentive to deviate from the other entrepreneurs' common investment decision $I_t = 0$ and invest in the new technology, or a marginal entrepreneur has no incentive to deviate from the other entrepreneurs' common investment decision $I_t = 1$ and invest in the old technology, with these two possibilities being mutually exclusive. This will be ensured by an additional condition that we will derive in the next section in the context of endogenous information.

4 Competitive equilibrium with endogenous information

In this section, we assume that the conditions on the parameters listed in Subsection 3.3 are met. We first introduce private signals, study the endogenous dynamics of the information sets and examine the role of monetary policy. We then consider a particular parametrization that enables us to solve the model analytically. We finally run numerical simulations for other parametrizations.

4.1 Information dynamics

We now assume that at each date $t \in \{1, ..., N\}$, the representative new-born entrepreneur, the representative household and the central bank observe the same variables with the only exception that the representative new-born entrepreneur receives a private signal about whether the new technology is good or bad, while the representative household and the central bank receive no such private signal. As a consequence, at each date $t \in \{1, ..., N\}$, the representative household and the central bank's information sets coincide with each other and are included in the representative new-born entrepreneur's information set. We therefore call "public information at date t" the information of the representative household and the central bank at that date. The probability that the new technology is good based on public information available at date t is therefore μ_t.

We assume that, at each date $t \in \{1, ..., N\}$, the timing of events is the following:

- The representative new-born entrepreneur starts with the public information available at date $t-1$. Therefore, she has the prior μ_{t-1} about the probability that the new technology is good. We assume that the initial prior μ_0 is exogenous.

- The representative new-born entrepreneur receives a private signal $S_t \in \{0, 1\}$ about whether the new technology is good or bad. This signal is "good" when $S_t = 1$ and "bad" when $S_t = 0$. We note $\lambda \in \left]\frac{1}{2}; 1\right[$ the probability that a signal, whether good or bad, is right. Bayes' theorem implies that the representative new-born entrepreneur's posterior $\widetilde{\mu}_t$ about the probability that the new technology is good is

$$\widetilde{\mu}_t = S_t \frac{\mu_{t-1}\lambda}{\mu_{t-1}\lambda + \left(1 - \mu_{t-1}\right)(1-\lambda)} + (1 - S_t) \frac{\mu_{t-1}(1-\lambda)}{\mu_{t-1}(1-\lambda) + \left(1 - \mu_{t-1}\right)\lambda}.$$

- The central bank sets τ_t. Her intervention is public information, so that the probability μ_t that the new technology is good based on public information available at date t does take τ_t into account.

- The representative new-born entrepreneur takes her investment decision $I_t \in \{0, 1\}$. This decision is public information, so that the probability μ_t that the new technology is good based on public information available at date t does also take I_t into account. The equilibrium price is then $q_t = q(z, \tau_t, \mu_t, I_t)$.

For each $t \in \{1, ..., N\}$, let $\widetilde{\mu}_t^0$ denote the value taken by $\widetilde{\mu}_t$ when $S_t = 0$ and $\widetilde{\mu}_t^1$ the value taken by $\widetilde{\mu}_t$ when $S_t = 1$. The following proposition shows that there exists at most one equilibrium, derives necessary and sufficient conditions for the existence of this equilibrium, and describes the equilibrium dynamics of I_t and μ_t:

Proposition 11: *(i) there exists an equilibrium if and only if,* $\forall t \in \{1, ..., N\}$, $\forall (S_1, ..., S_t) \in \{0, 1\}^t$, *either (a)* $\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_{t-1}, 0\right) < B(z)$, *or (b)* $\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_{t-1}, 1\right) > B(z)$, *or (c)* $\widetilde{\mu}_t^0 q\left(z, \tau_t, \widetilde{\mu}_t^0, 0\right) < B(z)$ *and* $\widetilde{\mu}_t^1 q\left(z, \tau_t, \widetilde{\mu}_t^1, 1\right) > B(z)$; *(ii) when there exists an equilibrium, this equilibrium is unique; (iii)*

$\forall t \in \{1,...,N\}$, $\forall (S_1,...,S_t) \in \{0,1\}^t$, at most one of the three conditions (a), (b) and (c) is met, and if it is (a) then $\forall S_t \in \{0,1\}$, $I_t = 0$ and $\mu_t = \mu_{t-1}$, if it is (b) then $\forall S_t \in \{0,1\}$, $I_t = 1$ and $\mu_t = \mu_{t-1}$, if it is (c) then $\forall S_t \in \{0,1\}$, $I_t = S_t$ and $\mu_t = \widetilde{\mu}_t$.

Proof: *cf* Appendix H. ■

Proposition 11 implies in particular that $\forall t \in \{1,...,N\}$, $\exists i \in \mathbb{Z}$, $\widetilde{\mu}_t = p_i$ and $\mu_t \in \{p_{i-1}, p_i, p_{i+1}\}$, where $p_0 \equiv \mu_0 \in]0;1[$ and, for $i \in \mathbb{N}^*$,

$$p_i \equiv \frac{p_{i-1}\lambda}{p_{i-1}\lambda + (1-p_{i-1})(1-\lambda)} \quad \text{and} \quad p_{-i} \equiv \frac{p_{-i+1}(1-\lambda)}{p_{-i+1}(1-\lambda) + (1-p_{-i+1})\lambda}.$$

In cases (a) and (b) of Proposition 11, herd behavior arises as the result of an informational cascade (Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992):

Definition 1 (high and low informational cascades): *there is an informational cascade at date $t \in \{1,...,N\}$ when $\forall S_t \in \{0,1\}$, $\mu_t = \mu_{t-1}$; (ii) an informational cascade is high when $I_t = 1$ and low when $I_t = 0$.*

In particular, a high cascade corresponds to a situation in which, because a sufficiently large number of past representative entrepreneurs chose to invest in the new technology as they received encouraging private signals about its productivity, the current representative entrepreneur rationally chooses to invest in the new technology too whatever her own private signal.

The existence of informational cascades is linked to the existence of what we call an stock-market bubble:

Definition 2 (stock market $\mathcal{M}$): *Firms shares (which are claims for future dividends) are traded among households. The stock market is the sum of existing firms value at date t , based on the public information available at date t, of the representative entrepreneur born at date t,* i.e. $\mathcal{M}_t = E_{\Omega(h,t)}\left[\sum_{j=1}^{t} q_j c^e_{j+N}\right]$.

Definition 3 (stock market bubble: *there is an stock market bubble at date $t \in \{2,...,N\}$ when the stock market at date t differs from the value that it would have taken if all previous signals S_i, $1 \leq i \leq t-1$, had been public instead of private.*

Indeed, there is an stock market bubble at date $t \in \{2,...,N\}$ if and only if there exists $i \in \{1,...,t-1\}$ such that there is an informational cascade at date i.

4.2 Monetary policy intervention

Here we do not asses the optimality of monetary policy but show that there exist a policy that can eliminate stock market bubbles. We discuss in the next session of the optimality of such a policy.

From (4) and (7), it is easy to check that, whatever $z \geq \overline{z}$, $\mu \in [0;1]$ and $Q > 0$, there exists a unique $\tau > 0$ such that $q(z,\tau,\mu,0) = Q$ and there exists a unique $\tau > 0$ such that $q(z,\tau,\mu,1) = Q$. Let us note

$$\tau^l(z,\mu,\widetilde{\mu}) \equiv \beta^N \left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{\mu}{[\alpha A(\overline{z}) - \kappa(z)] \frac{B(z)}{\widetilde{\mu}} + \kappa(\overline{z})} + \frac{1-\mu}{[\alpha A(\overline{z}) - \kappa(\overline{z})] \frac{B(z)}{\widetilde{\mu}} + \kappa(\overline{z})}\right]$$

the unique value of τ such that $q(z,\tau,\mu,0) = \frac{B(z)}{\widetilde{\mu}}$ and

$$\tau^u(z,\mu,\widetilde{\mu}) \equiv \beta^N \left[\alpha A(\overline{z}) - \kappa(z) + \frac{\kappa(\overline{z})}{\beta^N}\right] \left[\frac{\mu}{[\alpha A(z) - \kappa(z)] \frac{B(z)}{\widetilde{\mu}} + \kappa(z)} + \frac{1-\mu}{[\alpha A(\overline{z}) - \kappa(\overline{z})] \frac{B(z)}{\widetilde{\mu}} + \kappa(z)}\right]$$

the unique value of τ such that $q(z,\tau,\mu,1) = \frac{B(z)}{\widetilde{\mu}}$. Since $\frac{\partial q(z,\tau,\mu,0)}{\partial \tau} < 0$ and $\frac{\partial q(z,\tau,\mu,1)}{\partial \tau} < 0$ (as implied by Propositions 1 and 2), conditions (a), (b) and (c) of Proposition 11 can then be rewritten in a more policy-oriented form that singles out τ_t: (a) there exists a low cascade at date t if and only if $\tau_t > \tau^l\left(z,\mu_{t-1},\widetilde{\mu}_t^1\right)$; (b) there exists a high cascade at date t if and only if $\tau_t < \tau^u\left(z,\mu_{t-1},\widetilde{\mu}_t^0\right)$; (c) there exists no cascade at date t if and only if $\tau^l\left(z,\widetilde{\mu}_t^0,\widetilde{\mu}_t^0\right) < \tau_t < \tau^u\left(z,\widetilde{\mu}_t^1,\widetilde{\mu}_t^1\right)$.

In order to illustrate the mechanism of monetary policy intervention, suppose for a moment that there exists $t \in \{1,...,N\}$ at which there is a high cascade under laisser-faire, *i.e.* that there exists $t \in \{1,...,N\}$ such that $\widetilde{\mu}_t^0 q\left(z,1,\mu_{t-1},1\right) > B(z)$. Then, as implied by Proposition 11, a necessary condition for the monetary policy intervention to get rid of the cascade at date t is $\widetilde{\mu}_t^0 q\left(z,\tau_t,\widetilde{\mu}_t^0,0\right) < B(z)$. Given Proposition 5, $\widetilde{\mu}_t^0 q\left(z,1,\mu_{t-1},1\right) > B(z)$ implies $\widetilde{\mu}_t^0 q\left(z,1,\widetilde{\mu}_t^0,0\right) > B(z)$. Since $\frac{\partial q(z,\tau,\mu,0)}{\partial \tau} < 0$ (as implied by Proposition 3), $\tau_t > 1$ is therefore a necessary condition for the monetary policy intervention to interrupt the cascade at date t. In other words, monetary policy must be tightened to interrupt a high cascade. This is because monetary policy tightening, by making borrowing dearer for the entrepreneurs, can make them invest in the new technology if and only if they receive an encouraging private signal about its productivity. In doing so, it eliminates the high cascade.

Note finally that our assumption that $\tau_t = 1$ for all $t > N$ is not restrictive because, whatever happened until date N, there is no rationale for intervening after date N, as there is then no financial market imperfection anymore.

4.3 An analytically tractable case

We assume here that the functions

$$\begin{array}{c} \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ z \longmapsto \kappa(z) \end{array} \quad \text{and} \quad \begin{array}{c} \mathbb{R}^+ \longrightarrow \mathbb{R}^+ \\ z \longmapsto A(z) \end{array}$$

are twice differentiable at point $z = \overline{z}$, with $\frac{d\kappa}{dz}\big|_{z=\overline{z}} > 0$ and $\frac{dA}{dz}\big|_{z=\overline{z}} > 0$. We also assume that z is arbitrarily close to $\overline{z}$ and that τ_t remains arbitrarily close to 1 at dates 1 to N. The latter conditions are necessary and sufficient for q_t, c_t and c_t^e to remain arbitrarily close to their steady-state values for all $t \in \mathbb{N}^*$, all $p_0 \in]0;1[$ and all $(S_1, ..., S_N) \in \{0,1\}^N$. This, in turn, enables us to linearize the model in the neighborhood of its steady state. We also assume, for simplicity, that $N = 3$. We focus on the case examined in the following proposition:

Proposition 12: *there is no cascade at date* 1 *under laisser-faire* ($\tau_1 = 1$)*, there is a high cascade at date* 2 *when* $S_1 = 1$ *under laisser-faire* ($\tau_2 = 1$)*, and there exists a monetary policy intervention* τ_2 *arbitrarily close to* 1 *that ensures the absence of cascade at date* 2 *when* $S_1 = 1$*, if and only if*

$$\frac{\beta^3 \left[(1-\alpha) \beta^3 p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2 \kappa}{dz^2}\right|_{z=\overline{z}} \right]}{2 \left(\frac{d\kappa}{dz}\big|_{z=\overline{z}} \right)^2} > \frac{1 + \beta^3 (1 - p_1) + \frac{\alpha}{1-\alpha} \frac{p_1}{p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})} \tag{14}$$

$$\textit{and } B(\overline{z}) = p_0 \beta^3, \tag{15}$$

$$\textit{where } B(\overline{z}) \equiv \frac{\frac{d\kappa}{dz}\big|_{z=\overline{z}}}{(1-\alpha) \frac{dA}{dz}\big|_{z=\overline{z}}}.$$

Proof: *cf* Appendix I. ■

The relevant parameters are now α, β, $\kappa(\overline{z})$, $\frac{d\kappa}{dz}\big|_{z=\overline{z}}$, $\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}$, $A(\overline{z})$, $\frac{dA}{dz}\big|_{z=\overline{z}}$, $\left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}}$, p_0, λ, N and $\frac{d\tau_t}{dz}\big|_{z=\overline{z}}$ for $t \in \{1, ..., N\}$. The conditions imposed on these parameters are those corresponding to the conditions listed in Subsection 3.3, to which should be added the following conditions: $0 < p_0 < 1$, $\frac{1}{2} < \lambda < 1$, $N = 3$, (14) and (15). Because, when z is arbitrarily close to $\overline{z}$ and τ_t remains arbitrarily close to 1 at dates 1 to N, (5) and (13) imply (1), (10), (11) and (12), these conditions are altogether equivalent to the following ones: $0 < \alpha < 1$, $0 < \beta < 1$, $\kappa(\overline{z}) > 0$, $\frac{d\kappa}{dz}\big|_{z=\overline{z}} > 0$, $A(\overline{z}) > 0$, $\frac{dA}{dz}\big|_{z=\overline{z}} > 0$, $0 < p_0 < 1$, $\frac{1}{2} < \lambda < 1$, $N = 3$, (14), (15), $\alpha A(\overline{z}) - \kappa(\overline{z}) > 0$, $p_0 \beta^3 < \frac{\alpha}{1-\alpha}$ and

$$\beta^3 - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} > \max \left[\frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})}, p_0 \beta^3 \right]. \tag{16}$$

It is easy to see that the set of parameter values satisfying all these conditions is not empty. As a consequence, neither is the set of parameter values satisfying all the conditions imposed in the general case in Subsection 3.3.

Our aim is to show that there exists a non-empty subset of parameter values satisfying all these conditions and such that the corresponding sequence of monetary policy interventions, characterized by the policy parameters $\left.\frac{d\tau_1}{dz}\right|_{z=\overline{z}}$, $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}}$ and $\left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}}$, is welfare-improving compared to laisser-faire, where the latter is defined as $\left.\frac{d\tau_1}{dz}\right|_{z=\overline{z}} = \left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}} = 0$. To that aim, we consider the following investment-decisions-contingent path of monetary policy interventions: (i) $\left.\frac{d\tau_1}{dz}\right|_{z=\overline{z}} = 0$; (ii) if $I_1 = 0$, then $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = 0$; (iii) if $I_1 = 1$, then $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \min\left\{ \left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}}, \text{there is no cascade at date 2}\right\}$; and (iv) $\forall (I_1, I_2) \in \{0,1\}^2$, $\left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}} = 0$.

The social welfare criterion that we consider is a weighted sum of the utility of the representative household, the utility of the current representative entrepreneur and the expected utilities of the future representative entrepreneurs:

$$W_t = E\left\{ \frac{\kappa(\overline{z}) + \beta^3 \left[\alpha A(\overline{z}) - \kappa(\overline{z})\right]}{\beta^3} U_t + \widehat{V}_t \right| S_1 = 1 \Bigg\},$$

$$\text{where } \widehat{V}_t \equiv E_{\Omega(e,t)} \left\{ \sum_{k=0}^{+\infty} \beta^k V_{t+k} \right\}.$$

The weights are chosen such that the locally linearized social welfare criterion is equal to the discounted sum of current and expected future GDPs. We compute the utilities of households and entrepreneurs conditionally on $S_1 = 1$. This is done without any loss in generality since the intervention considered amounts to laisser-faire when $S_1 = 0$.

Let $\left(U_t^{LF}(z), \widehat{V}_t^{LF}(z), W_t^{LF}(z)\right)$ and $\left(U_t^{I}(z), \widehat{V}_t^{I}(z), W_t^{I}(z)\right)$ denote the values taken by

$$\left(E\left\{U_t \middle| S_1 = 1\right\}, E\left\{\widehat{V}_t \middle| S_1 = 1\right\}, W_t\right)$$

respectively under laisser-faire and under the intervention considered. We first obtain the following proposition:

Proposition 13: *the welfare effects of laisser-faire are characterized by*

$$\begin{aligned}
\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} &> 0, \\
\left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} &> 0 \text{ if } (p_0, \lambda) \text{ is sufficiently close to } (0,1), \\
\left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} &< 0 \text{ if } p_0 \text{ is sufficiently close to } 1, \\
\left.\frac{dW_1^{LF}}{dz}\right|_{z=\overline{z}} &= \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta} \left[\frac{p_1}{(1-\alpha)p_0} - 1 + \beta^3 (1-p_1)\right] > 0.
\end{aligned}$$

Proof: *cf* Appendix J. ■

It is worth noting in particular that we can get $\left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} < 0$ even though each entrepreneur individually gains from investing in the new technology. There are at least two possible reasons for this result. First, the existence of overlapping generations of entrepreneurs may create a negative externality. Indeed, at each date $t \in \mathbb{N}^*$, new-born entrepreneurs do not internalize the possible costs, in terms of interest-rate fluctuations, that their investment decision imposes on the entrepreneurs born at date $t - N$ and on those born at date $t + N$. Second, our simplifying discrete-choice assumption and our focus on symmetric equilibria may play a role. Indeed, if there were only one entrepreneur *per* generation, then she would choose between borrowing little at a low rate or borrowing much at a high rate, and might prefer to borrow little at a low rate. But there are many of them, so that each of them, taking the interest rate as given, has either to choose between borrowing little or much at a low rate, or to choose between borrowing little or much at a high rate. If in both cases she prefers to borrow much, then the only symmetric equilibrium is that all entrepreneurs borrow much at a high rate.

Now let p_A denote the probability of receiving a signal $S_2 = 1$ conditionally on $S_1 = 1$, and p_B the probability of receiving a signal $S_3 = 1$ conditionally on $S_1 = 1$ and $S_2 = 0$, *i.e.* $p_A = p_1\lambda + (1 - p_1)(1 - \lambda)$ and $p_B = p_0\lambda + (1 - p_0)(1 - \lambda)$. We then obtain the following proposition:

Proposition 14: *the welfare effects of the intervention considered are characterized by*

$$\begin{aligned}
\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}} &> 0, \\
\left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} &> 0 \text{ if } (p_0, \lambda) \text{ is sufficiently close to } (0,1), \\
\left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} &< 0 \text{ if } p_0 \text{ is sufficiently close to } 1, \\
\left.\frac{dW_1^I}{dz}\right|_{z=\overline{z}} &= \left\{\left[\frac{p_1}{(1-\alpha)\,p_0} - 1\right] + \frac{\beta^3}{1-\beta}p_1\left[\frac{1}{(1-\alpha)\,p_0} - 1\right]\right. \\
&\quad \left. + \beta(1+\beta)\,p_A\left[\frac{p_2}{(1-\alpha)\,p_0} - 1\right] + \beta^2(1-p_A)\,p_B\left[\frac{p_1}{(1-\alpha)\,p_0} - 1\right]\right\}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \\
&> 0.
\end{aligned}$$

Proof: *cf* Appendix K. ∎

The reasons why we can get $\left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} < 0$ are pretty much the same as under laisser-faire.

We can then examine whether the intervention considered is welfare-improving compared to laisser-faire by computing

$$\left.\frac{dW_1^I}{dz}\right|_{z=\overline{z}} - \left.\frac{dW_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{\beta(1-p_A)}{1-\alpha}\left[\beta(1-p_0)(2\lambda-1) - \alpha\left[1 + \beta(1-p_B)\right]\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}},$$

$$\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{\beta^3 \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\kappa(\overline{z})+\beta^3\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}\left\{\frac{1-p_A}{(1-\alpha)\beta}\left[\frac{p_B(p_1-p_0)}{p_0}\left[\alpha\beta^3+\frac{\left(1-\beta^3\right)\kappa(\overline{z})}{A(\overline{z})}\right]\right.\right.$$
$$\left.-\frac{1+\beta(1-p_B)}{\beta}\left[\alpha\beta^3+\frac{\kappa(\overline{z})}{\alpha A(\overline{z})}\left[(1-\alpha)+\alpha\left(1-\beta^3\right)\right]\right]\right]+\frac{\beta p_0\kappa(\overline{z})}{\alpha A(\overline{z})}$$
$$\left.+\frac{\beta\kappa(\overline{z})\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}{2\alpha A(\overline{z})}\left[\frac{(1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}-\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]\right\},$$

$$\left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}}-\left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} = \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\left\{\frac{1-p_A}{(1-\alpha)\beta}\left[\frac{p_B(p_1-p_0)}{p_0}\left[(1-\alpha)\beta^3-\frac{\left(1-\beta^3\right)\kappa(\overline{z})}{A(\overline{z})}\right]\right.\right.$$
$$\left.+\frac{1+\beta(1-p_B)}{\beta}\frac{\kappa(\overline{z})}{\alpha A(\overline{z})}\left[(1-\alpha)+\alpha\left(1-\beta^3\right)\right]\right]-\frac{\beta p_0\kappa(\overline{z})}{\alpha A(\overline{z})}$$
$$\left.-\frac{\beta\kappa(\overline{z})\left[\alpha A(\overline{z})-\kappa(\overline{z})\right]}{2\alpha A(\overline{z})}\left[\frac{(1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}-\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]\right\}.$$

It is clear that there exist some parameter values satisfying all the conditions listed above and such that p_0 is arbitrarily close to zero, λ is arbitrarily close to one, and $\alpha < \frac{\beta}{1+\beta}$. These results imply that, for those parameter values, the sequence of monetary policy interventions considered increases social welfare W_t relatively to laisser-faire. Of course, when p_0 is low, so is the probability that $S_1 = S_2 = 1$ and therefore so is the probability to intervene at date 2, so that the welfare gain may then not be large.

Note finally that, even when the particular sequence of monetary policy interventions considered is welfare-improving compared to laisser-faire, there are at least two reasons why it might not be the optimal sequence of monetary policy interventions for the welfare criterion that we consider. First, the optimal monetary policy may set $\left.\frac{d\tau_3}{dz}\right|_{z=\overline{z}}$ higher than zero when $I_1 = I_2 = 1$ in order to eliminate the high cascade at date 3. By making S_3 public, this would not benefit future entrepreneurs, since the true productivity of the new technology is common knowledge at date 4 anyway, but it could benefit the representative household at date 3. Second, the optimal monetary policy may set $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}}$ higher than the minimal value to get no cascade at date 2. Indeed, social welfare is maximal under laisser-faire in the fictitious situation where the signals are public instead of private. Any arbitrarily small intervention reduces social welfare in this fictitious situation, *i.e.* there is a neighbourhood of laisser-faire within which increasing intervention decreases social welfare in this situation. But as z becomes arbitrarily close to $\overline{z}$, the size of this neighbourhood may tend towards zero. Hence our minimal intervention for the private signal to be revealed, not to mention interventions of greater size, may fall outside this neighbourhood.

4.4 Optimal Monetary Policy

There are two reasons why equilibrium allocations are inefficient in our setting: private information and incomplete markets. Incomplete markets imply limited risk sharing between agents. If market were complete and information public, risk neutral entrepreneurs would fully insure households against the technological risk. Allocations would therefore imply constant household consumption, and would be supported by the trade of claims contingent on the realization of the new technology (good or bad). As discussed previously, in our simple setup, complete markets would also eliminate informational cascades. In a world with incomplete markets, a second inefficiency arises from the asymmetric information and the assumption that actions but not signals are observed. Revealing her private signal by her action has a social value that the entrepreneur is not internalizing when deciding to invest or not, and might create a stock market bubble. A policy that systematically eliminate stock market bubbles is not optimal. Typically, eliminating a bubble in period $N-1$ provided that there were no bubbles before could be suboptimal, as it does not bring useful extra information (the state of the technology will be revealed the next period) and does distort the entrepreneur decision of period $N-1$. But it implies some redistribution between households and entrepreneurs of period $N-1$, and given the lack of risk sharing caused by incomplete markets, could well improve a social welfare function. This is precisely what does happen in the particular case of the previous section. In order to get some intuition on what an optimal monetary policy could be, we resort to numerical simulations.

4.5 Numerical simulations

In this subsection we plan to run numerical simulations for a parametrization that satisfies the conditions listed in Subsection 3.3 but is less peculiar than those considered in the previous subsection. We will in particular relax the assumptions that $N=3$, that z is arbitrarily close to $\overline{z}$ and that parameters are such that monetary policy interventions can be arbitrarily small to eliminate a cascade. Our aim is to characterize numerically the investment-decisions-contingent path of monetary policy interventions that maximizes an *ex ante* welfare criterion.

On the one hand, we expect the relaxation of the assumption $N=3$ to spread the gains of a monetary policy intervention over more periods and therefore to increase the desirability of this intervention, *i.e.* to make the conditions for its desirability less stringent. Moreover, the relaxation of the assumption that z is arbitrarily close to $\overline{z}$ will make households' risk-aversion matter in welfare computations and may therefore also be expected to increase the desirability of a monetary policy intervention. On the other hand, the relaxation of the assumption that parameters are such that monetary policy interventions can be arbitrarily small to eliminate cascades will increase the distortion caused by this intervention and will therefore tend to decrease its desirability.

5 Conclusion

The first contribution of this paper has been to develop a dynamic general equilibrium model in which informational cascades can occur in equilibrium. In the model, entrepreneurs receive private information about the quality of a news technology, and invest or not in that new technology, borrowing from households. Although entrepreneurs information is private, entrepreneurs actions are publicly observable. Because investment is lumpy (invest or not in the new technology), it is not always possible for households and other entrepreneurs to infer private signal from actions. In such a case, social learning stops and the economy enters an informational cascade. We call such a situation a stock price bubble, as the stock market value is endogenously different from what is would be under public information. The second contribution has to introduce some monetary policy (defined as manipulation of the real interest rate), and to show that such a policy was capable of eliminating stock market bubbles, as defined previously.

Although the numerical simulations mentioned above are still to be done, some conclusions can already be drawn from this work at that stage. The most important one is that, if booms in new-tech equity prices are best modeled as the result of herd behavior, then the conditions most commonly stressed by central bankers for the desirability of a monetary policy reaction to these booms may prove less demanding than they seem at first sight.

6 References

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7 Appendix

A Proof of Proposition 3

Let us note, for all $z > \overline{z}$ and $\tau_t > 0$,

$$D_0(\tau_t) \equiv \frac{\beta^N}{\tau_t}\left[\alpha A(\overline{z}) - \kappa(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^N}\right],\ F_0(z) \equiv \alpha A(\overline{z}) - \kappa(z),\ G_0 \equiv \alpha A(\overline{z}) - \kappa(\overline{z}) \text{ and } H_0 \equiv \kappa(\overline{z}),$$

so that (4) corresponds to

$$q_t = D_0(\tau_t)\left[\frac{\mu_t}{F_0(z) + \frac{H_0}{q_t}} + \frac{1-\mu_t}{G_0 + \frac{H_0}{q_t}}\right].$$

Note that conditions (2) and (3) together imply $G_0 > 0$.

Suppose first that (4) admits a strictly positive solution q_t for all $\mu_t \in [0;1]$. Then, (4) admits a strictly positive solution q_t in particular for $\mu_t = 0$, which implies that $D_0(\tau_t) > H_0$, and for $\mu_t = 1$, which implies that $F_0(z) > 0$ (given that $D_0(\tau_t) > H_0$). These two inequalities correspond to conditions (5) and (6) in Proposition 3. Now suppose conversely that $D_0(\tau_t) > H_0$ and $F_0(z) > 0$. Then, when $\mu_t \in \{0;1\}$, (4) admits a unique solution q_t and this solution is strictly positive. When $\mu_t \notin \{0;1\}$, (4) is equivalent to

$$\Phi_0(z)\, q_t^2 + \Psi_0(z,\tau_t,\mu_t)\, q_t + \Omega_0(\tau_t) = 0$$

where, for all $z > \overline{z}$, all $\tau_t > 0$ and all $\mu_t \in]0;1[$, $\Phi_0(z) \equiv F_0(z)\, G_0$, $\Psi_0(z,\tau_t,\mu_t) \equiv [F_0(z) + G_0]\, H_0 - D_0(\tau_t)[G_0\mu_t + F_0(z)(1-\mu_t)]$ and $\Omega_0(\tau_t) \equiv H_0[H_0 - D_0(\tau_t)]$. We have: $\forall \mu_t \in]0;1[$, $[\Psi_0(z,\tau_t,\mu_t)]^2 - 4\Phi_0(z)\,\Omega_0(\tau_t) \geq -4\Phi_0(z)\,\Omega_0(\tau_t) > 0$, so that (4) admits two distinct real-number solutions and, since $\frac{\Omega_0(\tau_t)}{\Phi_0(z)} < 0$, one solution is strictly negative and the other strictly positive. Point (i) of Proposition 3 follows.

From the previous paragraph, we also get that if $D_0(\tau_t) > H_0$ and $F_0(z) > 0$, then (4) admits a unique strictly positive solution q_t for all $\mu_t \in [0;1]$, which we note $q(z,\tau_t,\mu_t,0)$. When $\mu_t \in]0;1[$, the derivation of $\Phi_0(z)\,q(z,\tau_t,\mu_t,0)^2+\Psi_0(z,\tau_t,\mu_t)\,q(z,\tau_t,\mu_t,0)+\Omega_0(\tau_t)=0$ with respect to $x\in\{\tau_t,\mu_t\}$ leads to

$$[2\Phi_0(z)\,q(z,\tau_t,\mu_t,0)+\Psi_0(z,\tau_t,\mu_t)]\,\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial x}+q(z,\tau_t,\mu_t,0)\,\frac{\partial\Psi_0(z,\tau_t,\mu_t)}{\partial x}=0,$$

where $2\Phi_0(z)\,q(z,\tau_t,\mu_t,0)+\Psi_0(z,\tau_t,\mu_t)>0$ by definition of $q(z,\tau_t,\mu_t,0)$. Given that $\frac{\partial\Psi_0(z,\tau_t,\mu_t)}{\partial\tau_t}=\frac{D_0(\tau_t)}{\tau_t}[G_0\mu_t+F_0(z)(1-\mu_t)]>0$ and $\frac{\partial\Psi_0(z,\tau_t,\mu_t)}{\partial\mu_t}=D_0(\tau_t)[F_0(z)-G_0]<0$, we therefore obtain that $\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial\tau_t}<0$ and $\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial\mu_t}>0$ for $\mu_t\in\,]0;1[$ and by continuity for $\mu_t\in\{0;1\}$ as well. Point (ii) of Proposition 3 follows.

B Proof of Proposition 4

Let us note, for all $z>\overline{z}$ and $\tau_t>0$,

$$D_1(z,\tau_t)\equiv\frac{\beta^N}{\tau_t}\left[\alpha A(\overline{z})-\kappa(z)+\frac{\kappa(\overline{z})}{\beta^N}\right],\ F_1(z)\equiv\alpha A(z)-\kappa(z),\ G_1\equiv\alpha A(\overline{z})-\kappa(\overline{z})\ \text{ and }\ H_1(z)\equiv\kappa(z),$$

so that (7) can be rewritten as

$$q_t=D_1(z,\tau_t)\left[\frac{\mu_t}{F_1(z)+\frac{H_1(z)}{q_t}}+\frac{1-\mu_t}{G_1+\frac{H_1(z)}{q_t}}\right].$$

Note that conditions (2) and (3) together imply $G_1>0$ and that condition (5) implies $F_1(z)>0$.

Suppose first that (7) admits a strictly positive solution q_t for all $\mu_t\in[0;1]$. Then, (7) admits a strictly positive solution q_t in particular for $\mu_t=0$, which implies that $D_1(z,\tau_t)>H_1(z)$. The latter inequality corresponds to condition (8) in Proposition 4. Now suppose conversely that $D_1(z,\tau_t)>H_1(z)$. Then, when $\mu_t\in\{0;1\}$ or $F_1(z)=G_1$, (7) admits a unique solution q_t and this solution is strictly positive. When $\mu_t\notin\{0;1\}$ and $F_1(z)\neq G_1$, (7) is equivalent to

$$\Phi_1(z)\,q_t^2+\Psi_1(z,\tau_t,\mu_t)\,q_t+\Omega_1(z,\tau_t)=0$$

where, for all $z>\overline{z}$, all $\tau_t>0$ and all $\mu_t\in\,]0;1[$, $\Phi_1(z)\equiv F_1(z)\,G_1$, $\Psi_1(z,\tau_t,\mu_t)\equiv[F_1(z)+G_1]\,H_1(z)-D_1(z,\tau_t)[G_1\mu_t+F_1(z)(1-\mu_t)]$ and $\Omega_1(z,\tau_t)\equiv H_1(z)[H_1(z)-D_1(z,\tau_t)]$. We have: $\forall\mu_t\in\,]0;1[$, $[\Psi_1(z,\tau_t,\mu_t)]^2-4\Phi_1(z)\,\Omega_1(z,\tau_t)\geq-4\Phi_1(z)\,\Omega_1(z,\tau_t)>0$, so that (7) admits two distinct real-number solutions and, since $\frac{\Omega_1(z,\tau_t)}{\Phi_1(z)}<0$, one solution is strictly negative and the other strictly positive. Point (i) of Proposition 4 follows.

From the previous paragraph, we also get that if $D_1(z,\tau_t)>H_1(z)$, then (7) admits a unique strictly positive solution q_t for all $\mu_t\in[0;1]$, which we note $q(z,\tau_t,\mu_t,1)$. When $\mu_t\in\,]0;1[$, the derivation of $\Phi_1(z)\,q(z,\tau_t,\mu_t,1)^2+\Psi_1(z,\tau_t,\mu_t)\,q(z,\tau_t,\mu_t,1)+\Omega_1(z,\tau_t)=0$ with respect to $x\in\{\tau_t,\mu_t\}$ leads to

$$[2\Phi_1(z)\,q(z,\tau_t,\mu_t,1)+\Psi_1(z,\tau_t,\mu_t)]\,\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial x}+q(z,\tau_t,\mu_t,1)\,\frac{\partial\Psi_1(z,\tau_t,\mu_t)}{\partial x}=0,$$

where $2\Phi_1(z)\, q(z,\tau_t,\mu_t,1) + \Psi_1(z,\tau_t,\mu_t) > 0$ by definition of $q(z,\tau_t,\mu_t,1)$. Given that $\frac{\partial\Psi_1(z,\tau_t,\mu_t)}{\partial\tau_t} = \frac{D_1(z,\tau_t)}{\tau_t}\left[G_1\mu_t + F_1(z)(1-\mu_t)\right] > 0$ and $\frac{\partial\Psi_1(z,\tau_t,\mu_t)}{\partial\mu_t} = D_1(z,\tau_t)\left[F_1(z) - G_1\right] < 0$, we therefore obtain that, for $\mu_t \in \left]0;1\right[$ and by continuity for $\mu_t \in \{0;1\}$ as well, $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial\tau_t} < 0$, $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial\mu_t} < 0$ if $F_1(z) > G_1$, $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial\mu_t} = 0$ if $F_1(z) = G_1$, and $\frac{\partial q(z,\tau_t,\mu_t,1)}{\partial\mu_t} > 0$ if $F_1(z) < G_1$. Since inequality $F_1(z) > G_1$ corresponds to condition (9) in Proposition 4, points (ii) and (iii) of Proposition 4 follow.

C Proof of Proposition 5

From (4) and (7) we easily get, using the notations of appendices A and B,

$$q(z,\tau_t,0,0) = \frac{D_0(\tau_t) - H_0}{G_0} \text{ and } q(z,\tau_t,0,1) = \frac{D_1(z,\tau_t) - H_1(z)}{G_1}.$$

Since $D_0(\tau_t) > D_1(z,\tau_t)$, $H_0 < H_1(z)$ and $G_0 = G_1 > 0$, we obtain that

$$q(z,\tau_t,0,0) > q(z,\tau_t,0,1).$$

Now Propositions 3 and 4 imply

$$\frac{\partial q(z,\tau_t,\mu_t,0)}{\partial\mu_t} > 0 \text{ and } \frac{\partial q(z,\tau_t,\mu_t,1)}{\partial\mu_t} < 0.$$

We therefore conclude that $\forall (p,p') \in [0;1]^2$, $q(z,\tau_t,p,0) > q(z,\tau_t,p',1)$. Proposition 5 follows.

D Proof of Proposition 6

The necessary and sufficient condition for a marginal entrepreneur to have no incentive to deviate from the other entrepreneurs' common investment decision and invest nothing is that (a) if this decision is to invest in the new technology and if a marginal entrepreneur has no incentive to deviate from this decision and invest in the old technology, then a marginal entrepreneur has no incentive to deviate from this decision and invest nothing, and (b) if this decision is to invest in the old technology and if a marginal entrepreneur has no incentive to deviate from this decision and invest in the new technology, then a marginal entrepreneur has no incentive to deviate from this decision and invest nothing. Therefore, a marginal entrepreneur has no incentive at date t to deviate from the other entrepreneurs' common investment decision and invest nothing for all $t \in \{1,...,N\}$, all $(\mu_1,...,\mu_N) \in [0;1]^N$ and all $(\widetilde{\mu}_1,...,\widetilde{\mu}_N) \in [0;1]^N$, if and only if (a) $\forall t \in \{1,...,N\}$, $\forall (\mu_1,...,\mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1,...,\widetilde{\mu}_N) \in [0;1]^N$,

$$(1-\alpha)A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z,\tau_t,\mu_t,0)} > \widetilde{\mu}_t(1-\alpha)A(z) + (1-\widetilde{\mu}_t)(1-\alpha)A(\overline{z}) - \frac{\kappa(z)}{q(z,\tau_t,\mu_t,0)}$$
$$\Longrightarrow (1-\alpha)A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z,\tau_t,\mu_t,0)} > 0,$$

and (b) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$,

$$\widetilde{\mu}_t (1-\alpha) A(z) + (1-\widetilde{\mu}_t)(1-\alpha) A(\overline{z}) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t, 1)} > (1-\alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z, \tau_t, \mu_t, 1)}$$
$$\Longrightarrow \widetilde{\mu}_t (1-\alpha) A(z) + (1-\widetilde{\mu}_t)(1-\alpha) A(\overline{z}) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t, 1)} > 0,$$

which is equivalent to (a) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$,

$$\widetilde{\mu}_t q(z, \tau_t, \mu_t, 0) < B(z) \Longrightarrow q(z, \tau_t, \mu_t, 0) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})},$$

and (b) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$, $\forall (\widetilde{\mu}_1, ..., \widetilde{\mu}_N) \in [0;1]^N$,

$$\widetilde{\mu}_t q(z, \tau_t, \mu_t, 1) > B(z) \Longrightarrow q(z, \tau_t, \mu_t, 1) > \frac{\kappa(z)}{\widetilde{\mu}_t (1-\alpha) A(z) + (1-\widetilde{\mu}_t)(1-\alpha) A(\overline{z})},$$

which is in turn equivalent to (a) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$,

$$q(z, \tau_t, \mu_t, 0) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})},$$

and (b) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$,

$$q(z, \tau_t, \mu_t, 1) > B(z) \Longrightarrow q(z, \tau_t, \mu_t, 1) > \frac{\kappa(z)}{\frac{B(z)}{q(z,\tau_t,\mu_t,1)} (1-\alpha) A(z) + \left(1 - \frac{B(z)}{q(z,\tau_t,\mu_t,1)}\right)(1-\alpha) A(\overline{z})},$$

which, given Proposition 3, is in turn equivalent to (a) $\forall t \in \{1, ..., N\}$,

$$q(z, \tau_t, 0, 0) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})},$$

and (b) $\forall t \in \{1, ..., N\}$, $\forall (\mu_1, ..., \mu_N) \in [0;1]^N$,

$$q(z, \tau_t, \mu_t, 1) > B(z) \Longrightarrow q(z, \tau_t, \mu_t, 1) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})},$$

which, given (4) and Proposition 4, is in turn equivalent to (a) $\forall t \in \{1, ..., N\}$,

$$\tau_t < \frac{\tau(\overline{z})}{1 + \frac{\alpha A(\overline{z}) - \kappa(\overline{z})}{(1-\alpha) A(\overline{z})}},$$

and (b) $\forall t \in \{1, ..., N\}$,

$$\left[q(z, \tau_t, 0, 1) > B(z) \text{ and } B(z) < \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})} \right] \Longrightarrow q(z, \tau_t, 1, 1) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})}.$$

Given (7), condition (b) holds if and only if $\forall t \in \{1, ..., N\}$,

$$\text{either } \frac{\tau(z)}{1 + \frac{B(z)[\alpha A(\overline{z}) - \kappa(\overline{z})]}{\kappa(z)}} < \tau_t, \text{ or } B(z) > \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})},$$
$$\text{or } \left[\tau_t < \frac{\tau(z)}{1 + \frac{B(z)[\alpha A(\overline{z}) - \kappa(\overline{z})]}{\kappa(z)}}, \; B(z) < \frac{\kappa(\overline{z})}{(1-\alpha) A(\overline{z})} \text{ and } \tau_t < \frac{\tau(z)}{1 + \frac{\kappa(\overline{z})[\alpha A(z) - \kappa(z)]}{\kappa(z)(1-\alpha) A(\overline{z})}} \right].$$

Now given conditions (5), (9) and $\tau(z) < \tau(\overline{z})$, we have

$$B(z) < \frac{\kappa(\overline{z})}{(1-\alpha)A(\overline{z})} \Longrightarrow \left[\frac{\tau(z)}{1+\frac{\kappa(\overline{z})[\alpha A(z)-\kappa(z)]}{\kappa(z)(1-\alpha)A(\overline{z})}} < \frac{\tau(z)}{1+\frac{B(z)[\alpha A(\overline{z})-\kappa(\overline{z})]}{\kappa(z)}} \text{ and } \frac{\tau(z)}{1+\frac{\kappa(\overline{z})[\alpha A(z)-\kappa(z)]}{\kappa(z)(1-\alpha)A(\overline{z})}} < \frac{\tau(\overline{z})}{1+\frac{\alpha A(\overline{z})-\kappa(\overline{z})}{(1-\alpha)A(\overline{z})}}\right].$$

Point (i) of Proposition 6 follows.

Note that (1) and (5) imply

$$1 < \frac{\tau(\overline{z})}{1+\frac{\alpha A(\overline{z})-\kappa(\overline{z})}{(1-\alpha)A(\overline{z})}},$$

which in turn implies $1 < \tau(\overline{z})$, and that (5) and

$$1 < \frac{\tau(z)}{1+\frac{\kappa(\overline{z})[\alpha A(z)-\kappa(z)]}{\kappa(z)(1-\alpha)A(\overline{z})}}$$

imply $1 < \tau(z)$. Point (ii) of Proposition 6 follows.

E Proof of Proposition 7

In this appendix, for simplicity, for any pair $(x, x') \in \{\overline{z}, z\} \times \{0, \overline{z}, z\}$ such that $x \neq x'$, by "a marginal entrepreneur has no incentive to deviate from x to x'" we mean that a marginal entrepreneur has no incentive to deviate from the other entrepreneurs' common investment decision $I_t = 0$ (when $x = \overline{z}$) or $I_t = 1$ (when $x = z$), in order to invest in the old technology (when $x' = \overline{z}$) or to invest in the new technology (when $x' = z$) or not to invest (when $x' = 0$).

First, it is straightforward that $\forall t > N$, a marginal entrepreneur has no incentive to deviate from $\overline{z}$ to z when the new technology is bad. Then, we have that $\forall t \in \{N+1, ..., 2N\}$,

$$q_t = \beta^N \frac{\alpha A(\overline{z}) - \kappa(z)}{\alpha A(z) - \kappa(z)} + \frac{1}{\alpha A(z) - \kappa(z)} \left[\frac{\beta^N}{q(z, \tau_{t-N}, \mu_{t-N}, 0)} \kappa(\overline{z}) - \kappa(z)\right]$$

if $z_{t-N} = \overline{z}$ and the new technology is good,

$$q_t = \beta^N + \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \left[\frac{\beta^N}{q(z, \tau_{t-N}, \mu_{t-N}, 1)} - 1\right]$$

if $z_{t-N} = z$ and the new technology is good,

$$q_t = \beta^N + \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{\beta^N}{q(z, \tau_{t-N}, \mu_{t-N}, 0)} - 1\right]$$

if $z_{t-N} = \overline{z}$ and the new technology is bad, and

$$q_t = \beta^N + \frac{1}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left[\frac{\beta^N}{q(z, \tau_{t-N}, \mu_{t-N}, 1)} \kappa(z) - \kappa(\overline{z})\right]$$

if $z_{t-N} = z$ and the new technology is bad. Since τ_{t-N} can be arbitrarily close to zero, $q\left(z, \tau_{t-N}, \mu_{t-N}, 0\right)$ and $q\left(z, \tau_{t-N}, \mu_{t-N}, 1\right)$ can be arbitrarily large and therefore q_{t-N} can be arbitrarily large in equilibrium. Moreover, since $\widetilde{\mu}_{t-N}$ can be arbitrarily close to zero, we can have in equilibrium both z_{t-N} being equal to $\overline{z}$ and $q_{t-N} = q\left(z, \tau_{t-N}, \mu_{t-N}, 0\right)$ being arbitrarily large. As a consequence, $\forall t \in \{N+1, ..., 2N\}$,

$$\inf_{\tau_{t-N}, \mu_{t-N}} q_t = \beta^N \frac{\alpha A\left(\overline{z}\right) - \kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)} - \frac{\kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)}$$

if the new technology is good and

$$\inf_{\tau_{t-N}, \mu_{t-N}} q_t = \beta^N - \frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)}$$

if the new technology is bad. As a consequence, $\forall t \in \{N+1, ..., 2N\}$, a marginal entrepreneur has no incentive to deviate from $\overline{z}$ to 0 when the new technology is bad, nor from z to 0 or $\overline{z}$ when the new technology is good, if and only if conditions (12) and (13) are met. Moreover, $\forall t > 2N$,

$$q_t = \beta^N + \frac{\beta^N \kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N}\right)$$

if the new technology is good, and

$$q_t = \beta^N + \frac{\beta^N \kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N}\right)$$

if the new technology is bad. Therefore, condition (5) and the fact that q is always strictly positive in equilibrium together imply that $\forall t > 2N$,

$$q_t > \beta^N - \frac{\kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)}$$

if the new technology is good, and

$$q_t > \beta^N - \frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)}$$

if the new technology is bad. If conditions (12) and (13) are met, then, $\forall t > 2N$,

$$q_t > \max\left[\frac{\kappa\left(z\right)}{\left(1-\alpha\right) A\left(z\right)}, B\left(z\right)\right]$$

if the new technology is good, and

$$q_t > \frac{\kappa\left(\overline{z}\right)}{\left(1-\alpha\right) A\left(\overline{z}\right)}$$

if the new technology is bad. As a consequence, $\forall t > 2N$, a marginal entrepreneur has no incentive to deviate from $\overline{z}$ to 0 when the new technology is bad, nor from z to 0 or $\overline{z}$ when it is good. Proposition 7 follows.

F Proof of Proposition 8

Propositions 3 and 4 imply that, $\forall t \in \{1, ..., N\}$, $q_t > 0$. Moreover, as shown in Appendix E, $\forall t \in \{N+1, ..., 2N\}$,

$$q_t > \beta^N \frac{\alpha A(\overline{z}) - \kappa(z)}{\alpha A(z) - \kappa(z)} - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)}$$

if the new technology is good, and

$$q_t > \beta^N - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}$$

if the new technology is bad. Therefore, conditions (12) and (13) imply that $\forall t \in \{N+1, ..., 2N\}$, $q_t > 0$. Finally, using the equations

$$q_t = \beta^N + \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N} \right)$$

if the new technology is good, and

$$q_t = \beta^N + \frac{\beta^N \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N} \right)$$

if the new technology is bad, which hold $\forall t > 2N$, we get by recurrence that $\forall t > 2N$, $q_t > 0$. To sum up, we get that $\forall t \geq 1$, $q_t > 0$. Together with (5), this implies in turn that $\forall t \geq 1$, $c_t > 0$. Besides, condition (1) and Propositions 6 and 7 imply that $\forall t \geq 1$, $c_t^e > 0$. Point (i) of Proposition 8 follows.

If the new technology is good, then $\forall t > 3N$,

$$q_t - \beta^N = \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2 q_{t-N} q_{t-2N}} \left(q_{t-2N} - \beta^N \right).$$

Using the results: (a) $\forall t > N$, $q_t > 0$; (b) $\forall t > 2N$,

$$q_t - \beta^N = \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N} \right);$$

and (c)

$$\beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} > 0,$$

which follows from (13), we get that $\forall t > 3N$,

$$\begin{aligned}
&\left[\beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] \left[q_{t-2N} + \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] > 0 \\
\Longrightarrow &\left[\beta^N - \frac{\kappa(z)}{\alpha A(z) - \kappa(z)} \right] q_{t-2N} + \frac{\beta^N \kappa(z)}{\alpha A(z) - \kappa(z)} > \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2} \\
\Longrightarrow &\; q_{t-N} q_{t-2N} > \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2} \\
\Longrightarrow &\left| \frac{[\kappa(z)]^2}{[\alpha A(z) - \kappa(z)]^2 q_{t-N} q_{t-2N}} \right| < 1,
\end{aligned}$$

from which we conclude that $\lim_{t\longrightarrow+\infty} q_t = \beta^N$. Alternatively, if the new technology is bad, then $\forall t > 3N$,

$$q_t - \beta^N = \frac{[\kappa(\overline{z})]^2}{[\alpha A(\overline{z}) - \kappa(\overline{z})]^2 q_{t-N} q_{t-2N}} \left(q_{t-2N} - \beta^N\right).$$

Using the results: (a) $\forall t > N$, $q_t > 0$; (b) $\forall t > 2N$,

$$q_t - \beta^N = \frac{\beta^N \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left(\frac{1}{q_{t-N}} - \frac{1}{\beta^N}\right);$$

and (c)

$$\beta^N - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} > 0,$$

which follows from (12), we get that $\forall t > 3N$,

$$\begin{aligned}
&\left[\beta^N - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\right]\left[q_{t-2N} + \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\right] > 0 \\
\Longrightarrow &\left[\beta^N - \frac{\kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})}\right] q_{t-2N} + \frac{\beta^N \kappa(\overline{z})}{\alpha A(\overline{z}) - \kappa(\overline{z})} > \frac{[\kappa(\overline{z})]^2}{[\alpha A(\overline{z}) - \kappa(\overline{z})]^2} \\
\Longrightarrow &\; q_{t-N} q_{t-2N} > \frac{[\kappa(\overline{z})]^2}{[\alpha A(\overline{z}) - \kappa(\overline{z})]^2} \\
\Longrightarrow &\left|\frac{[\kappa(\overline{z})]^2}{[\alpha A(\overline{z}) - \kappa(\overline{z})]^2 q_{t-N} q_{t-2N}}\right| < 1,
\end{aligned}$$

from which we conclude that $\lim_{t\longrightarrow+\infty} q_t = \beta^N$. To sum up, we get that $\lim_{t\longrightarrow+\infty} q_t = \beta^N$ whether the new technology is good or bad. Then, since $\forall t > 2N$,

$$\begin{aligned}
(c_t, c_t^e) &= \left(\alpha A(z) - \kappa(z) + q_{t-N}^{-1}\kappa(z), (1-\alpha) A(z) - q_{t-N}^{-1}\kappa(z)\right) \text{ if the new technology is good,} \\
(c_t, c_t^e) &= \left(\alpha A(\overline{z}) - \kappa(\overline{z}) + q_{t-N}^{-1}\kappa(\overline{z}), (1-\alpha) A(\overline{z}) - q_{t-N}^{-1}\kappa(\overline{z})\right) \text{ if the new technology is bad,}
\end{aligned}$$

we get that $\lim_{t\longrightarrow+\infty} (c_t, c_t^e) = \left(\alpha A(z) - \kappa(z) + \beta^{-N}\kappa(z), (1-\alpha) A(z) - \beta^{-N}\kappa(z)\right)$ if the new technology is good and $\lim_{t\longrightarrow+\infty} (c_t, c_t^e) = \left(\alpha A(\overline{z}) - \kappa(\overline{z}) + \beta^{-N}\kappa(\overline{z}), (1-\alpha) A(\overline{z}) - \beta^{-N}\kappa(\overline{z})\right)$ if it is bad. Point (ii) of Proposition 8 follows.

G Proof of Proposition 10

First, there exists no equilibrium such that entrepreneurs choose to invest nothing at some date $t > N$. Indeed, if entrepreneurs chose to invest nothing at some date $t > N$, then q_t would be infinite, so that a marginal entrepreneur would prefer to deviate from the other entrepreneurs' common decision and invest in the old or the new technology.

Second, there exists no equilibrium such that entrepreneurs choose to invest in the new technology at some date $t > N$ when this technology is bad. Indeed, if entrepreneurs chose to invest in the new technology at some date $t > N$ when this technology is bad, then a marginal entrepreneur would prefer to deviate from the other entrepreneurs' common decision and invest in the old technology, as the latter requires less investment and leads to the same productivity.

Third, if there existed an equilibrium such that entrepreneurs choose to invest in the old technology at some date $t > N$ when the new technology is good, then at this equilibrium we would have

$$q_t = \beta^N \frac{\alpha A\left(z_{t-N}\right) - \kappa\left(\overline{z}\right) + \frac{\kappa(z_{t-N})}{q_{t-N}}}{\alpha A\left(\overline{z}\right) - \kappa\left(z_{t+N}\right) + \frac{\kappa(\overline{z})}{q_t}},$$

which would then imply

$$\begin{aligned} q_t &= \frac{\beta^N \left[\alpha A\left(z_{t-N}\right) - \kappa\left(\overline{z}\right) + \frac{\kappa(z_{t-N})}{q_{t-N}}\right] - \kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(z_{t+N}\right)} \\ &\geq \frac{\beta^N \left[\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)\right] - \kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)} = \beta^N - \frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)} \end{aligned}$$

since $\frac{\kappa(z_{t-N})}{q_{t-N}} > 0$ in equilibrium. Now, a marginal entrepreneur would prefer to deviate from the other entrepreneurs' common decision and invest in the new technology if and only if

$$(1-\alpha) A\left(z\right) - \frac{\kappa\left(z\right)}{q_t} > (1-\alpha) A\left(\overline{z}\right) - \frac{\kappa\left(\overline{z}\right)}{q_t},$$

that is to say if and only if $q_t > B\left(z\right)$. Therefore, a sufficient condition for the non-existence of an equilibrium such that entrepreneurs choose to invest in the old technology at some date $t > N$ when the new technology is good is

$$\beta^N - \frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)} > B\left(z\right).$$

Now, the latter condition is met since (13) implies

$$\beta^N \frac{\alpha A\left(\overline{z}\right) - \kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)} - \frac{\kappa\left(z\right)}{\alpha A\left(z\right) - \kappa\left(z\right)} > B\left(z\right),$$

which, given (5), implies in turn

$$\begin{aligned} \beta^N &> \frac{\kappa\left(z\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(z\right)} + B\left(z\right) \frac{\alpha A\left(z\right) - \kappa\left(z\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(z\right)} \\ &> \frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right) - \kappa\left(\overline{z}\right)} + B\left(z\right). \end{aligned}$$

Proposition 10 follows.

H Proof of Proposition 11

For each $t \in \{1, ..., N\}$, let μ_t^0 denote the value taken by μ_t when $I_t = 0$ and μ_t^1 the value taken by μ_t when $I_t = 1$. Since entrepreneurs take the interest rate as given when deciding in which technology to invest, $I_t = 0$ is supported by an equilibrium only if

$$(1-\alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z, \tau_t, \mu_t^0, 0)} > \widetilde{\mu}_t \left[(1-\alpha) A(z) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t^0, 0)}\right] + (1-\widetilde{\mu}_t)\left[(1-\alpha) A(\overline{z}) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t^0, 0)}\right],$$

that is to say only if

$$\widetilde{\mu}_t q(z, \tau_t, \mu_t^0, 0) < B(z). \tag{17}$$

Similarly, $I_t = 1$ is supported by an equilibrium only if

$$(1-\alpha) A(\overline{z}) - \frac{\kappa(\overline{z})}{q(z, \tau_t, \mu_t^1, 1)} < \widetilde{\mu}_t \left[(1-\alpha) A(z) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t^1, 1)}\right] + (1-\widetilde{\mu}_t)\left[(1-\alpha) A(\overline{z}) - \frac{\kappa(z)}{q(z, \tau_t, \mu_t^1, 1)}\right],$$

that is to say only if

$$\widetilde{\mu}_t q(z, \tau_t, \mu_t^1, 1) > B(z). \tag{18}$$

Proposition 5 implies that $\forall\left(\mu_t^0, \mu_t^1\right) \in [0;1]^2$, conditions (17) and (18) cannot hold for the same values of the parameters. This implies that at most one of the following four cases can occur in equilibrium at each date $t \in \{1, ..., N\}$: $S_t = 0 \Longrightarrow I_t = 0$ and $S_t = 1 \Longrightarrow I_t = 0$ (case a), $S_t = 0 \Longrightarrow I_t = 1$ and $S_t = 1 \Longrightarrow I_t = 1$ (case b), $S_t = 0 \Longrightarrow I_t = 0$ and $S_t = 1 \Longrightarrow I_t = 1$ (case c), $S_t = 0 \Longrightarrow I_t = 1$ and $S_t = 1 \Longrightarrow I_t = 0$ (case d).

Note first that case d is in fact impossible, as it would require $\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_t^1, 1\right) > B(z)$ and $\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_t^0, 0\right) < B(z)$, where $\widetilde{\mu}_t^0 < \widetilde{\mu}_t^1$, which contradicts Proposition 5. Note then that cases a and b both lead to $\mu_t = \mu_{t-1}$, while case c leads to $\mu_t = \widetilde{\mu}_t$. As a consequence, case a is supported by an equilibrium if and only if $\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_{t-1}, 0\right) < B(z)$ and $\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_{t-1}, 0\right) < B(z)$, that is to say if and only if

$$\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_{t-1}, 0\right) < B(z); \tag{19}$$

case b is supported by an equilibrium if and only if $\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_{t-1}, 1\right) > B(z)$ and $\widetilde{\mu}_t^1 q\left(z, \tau_t, \mu_{t-1}, 1\right) > B(z)$, that is to say if and only if

$$\widetilde{\mu}_t^0 q\left(z, \tau_t, \mu_{t-1}, 1\right) > B(z); \tag{20}$$

and case c is supported by an equilibrium if and only if

$$\widetilde{\mu}_t^0 q\left(z, \tau_t, \widetilde{\mu}_t^0, 0\right) < B(z) \text{ and } \widetilde{\mu}_t^1 q\left(z, \tau_t, \widetilde{\mu}_t^1, 1\right) > B(z). \tag{21}$$

Given that $\widetilde{\mu}_t^0 < \widetilde{\mu}_t^1$, Proposition 5 implies that at most one of the three conditions (19), (20) and (21) holds for some given values of the parameters. Proposition 11 follows.

I Proof of Proposition 12

Given Proposition 11, there is a high cascade at date 2 when $S_1 = 1$ under laisser-faire ($\tau_2 = 1$) if and only if $p_0 q(z, 1, p_1, 1) > B(z)$. Moreover, since z is arbitrarily close to $\overline{z}$, $q(z, 1, p_1, 1)$ and $B(z)$ are arbitrarily close to β^3 and $B(\overline{z})$ respectively. As a consequence, there is a high cascade at date 2 when $S_1 = 1$ under laisser-faire ($\tau_2 = 1$) and there exists a monetary policy intervention τ_2 arbitrarily close to 1 that ensures the absence of cascade at date 2 when $S_1 = 1$ if and only if

$$p_0 \beta^3 = B(\overline{z})$$

and

$$p_0 \left. \frac{\partial q(z, 1, p_1, 1)}{\partial z} \right|_{z=\overline{z}} > \left. \frac{dB}{dz} \right|_{z=\overline{z}}, \tag{22}$$

where the first of these two conditions correspond to (15). The partial derivative of (7) at date 2 for $\tau_2 = 1$ and $\mu_2 = p_1$ with respect to z, taken at point $z = \overline{z}$, and the use of (15) lead to

$$\left. \frac{\partial q(z, 1, p_1, 1)}{\partial z} \right|_{z=\overline{z}} = -\frac{\left[1 + \beta^3 (1 - p_1)\right] + \frac{\alpha p_1}{(1-\alpha) p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})} \left. \frac{d\kappa}{dz} \right|_{z=\overline{z}}.$$

Besides, using (15), we also get

$$\left. \frac{dB}{dz} \right|_{z=\overline{z}} = \frac{\beta^3 p_0 \left[\left. \frac{d^2 \kappa}{dz^2} \right|_{z=\overline{z}} - (1-\alpha) \beta^3 p_0 \left. \frac{d^2 A}{dz^2} \right|_{z=\overline{z}} \right]}{2 \left. \frac{d\kappa}{dz} \right|_{z=\overline{z}}}. \tag{23}$$

These last two results can then be used to rewrite (22) as (14). Therefore, there is a high cascade at date 2 when $S_1 = 1$ under laisser-faire ($\tau_2 = 1$) and there exists a monetary policy intervention τ_2 arbitrarily close to 1 that ensures the absence of cascade at date 2 when $S_1 = 1$ if and only if (14) and (15) hold. Now, given Proposition 11, there is no cascade at date 1 if and only if

$$\begin{aligned} p_{-1} q(z, 1, p_{-1}, 0) &< B(z), \\ \text{and } p_1 q(z, 1, p_1, 1) &> B(z). \end{aligned}$$

If (15) holds, then these two conditions hold as well, since $p_{-1} < p_0 < p_1$ and $q(z, 1, p_{-1}, 0)$, $q(z, 1, p_1, 1)$ and $B(z)$ are arbitrarily close to β^3, β^3 and $B(\overline{z})$ respectively. Proposition 12 follows.

J Proof of Proposition 13

Concerning households, we have

$$\begin{aligned} U_1^{LF}(z) = &(1 + \beta + \beta^2) \ln \left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z) \right] + p_1 \beta^3 \textstyle\sum_{i=0}^{+\infty} \beta^i \ln \left[\alpha A(z) + \frac{\kappa(z)}{q_{i+1}^{(1)}(z)} - \kappa(z) \right] \\ &+ (1 - p_1) \beta^3 \textstyle\sum_{i=0}^{2} \beta^i \ln \left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_{i+1}^{(2)}(z)} - \kappa(\overline{z}) \right] \\ &+ (1 - p_1) \beta^3 \textstyle\sum_{i=3}^{+\infty} \beta^i \ln \left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i+1}^{(2)}(z)} - \kappa(\overline{z}) \right], \end{aligned}$$

where superscripts (1), resp. (2), indicates that the new technology turns out to be good, resp. bad. Computations then lead to

$$\begin{aligned}
q_i^{(1)}(z) &= q_i^{(2)}(z) = q(z,1,p_1,1) \text{ for } i\in\{1,2,3\},\\
q_i^{(1)}(z) &= \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_{i-3}^{(1)}(z)}-\frac{1}{\beta^3}\right] \text{ for } i\geq 4,\\
q_i^{(2)}(z) &= \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)}-\frac{1}{\beta^3}\right]+\frac{\kappa(z)-\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})} \text{ for } i\in\{4,5,6\},\\
q_i^{(2)}(z) &= \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)}-\frac{1}{\beta^3}\right] \text{ for } i\geq 7.
\end{aligned}$$

Using (15) and the fact that $\forall i\geq 1$, $q_i^{(1)}(\overline{z})=q_i^{(2)}(\overline{z})=\beta^3$, we get

$$\begin{aligned}
\left.\frac{dq_j^{(k)}}{dz}\right|_{z=\overline{z}} &= \frac{-1}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[1+\beta^3(1-p_1)+\frac{\alpha p_1}{(1-\alpha)p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \text{ for } j\in\{1,2,3\} \text{ and } k\in\{1,2\},\\
\left.\frac{dq_{3i+j}^{(1)}}{dz}\right|_{z=\overline{z}} &= \left[\frac{-\kappa(\overline{z})}{\beta^3[\alpha A(\overline{z})-\kappa(\overline{z})]}\right]^i\left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}} \text{ for } i\geq 1 \text{ and } j\in\{1,2,3\},\\
\left.\frac{dq_{3i+j}^{(2)}}{dz}\right|_{z=\overline{z}} &= \left[\frac{-\kappa(\overline{z})}{\beta^3[\alpha A(\overline{z})-\kappa(\overline{z})]}\right]^{i-1}\left\{\frac{1}{[\alpha A(\overline{z})-\kappa(\overline{z})]}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}-\frac{\kappa(\overline{z})}{\beta^3[\alpha A(\overline{z})-\kappa(\overline{z})]}\left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}}\right\}\\
&\quad \text{for } i\geq 1 \text{ and } j\in\{1,2,3\}.
\end{aligned}$$

We end up with

$$\begin{aligned}
\left.\frac{dU_1^{LF}}{dz}\right|_{z=\overline{z}} &= \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{(1-\beta)\left[\kappa(\overline{z})+\beta^3[\alpha A(\overline{z})-\kappa(\overline{z})]\right]}\left\{\frac{\alpha\beta^3p_1}{(1-\alpha)p_0}+\frac{\kappa(\overline{z})\left(1-\beta^3\right)}{\alpha A(\overline{z})}\left[1+\frac{\alpha p_1}{(1-\alpha)p_0}\right]\right\}\\
&> 0.
\end{aligned}$$

Concerning entrepreneurs, we have

$$\begin{aligned}
&\widehat{V}_1^{LF}(z) = p_1\textstyle\sum_{i=0}^{+\infty}\beta^{3+i}\left[(1-\alpha)A(z)-\dfrac{\kappa(z)}{q_{i+1}^{(1)}(z)}\right]\\
&\quad+(1-p_1)\textstyle\sum_{i=0}^{2}\beta^{3+i}\left[(1-\alpha)A(\overline{z})-\dfrac{\kappa(z)}{q_{i+1}^{(2)}(z)}\right]+(1-p_1)\textstyle\sum_{i=3}^{+\infty}\beta^{3+i}\left[(1-\alpha)A(\overline{z})-\dfrac{\kappa(\overline{z})}{q_{i+1}^{(2)}(z)}\right],
\end{aligned}$$

from which we get, using (15),

$$\left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta}\left\{1-\beta^3+\beta^3p_1-\frac{p_1}{p_0}+\frac{\kappa(\overline{z})\left(1-\beta^3\right)}{\alpha A(\overline{z})\beta^3}\left[1+\frac{\alpha p_1}{(1-\alpha)p_0}\right]\right\}.$$

The coefficient of $\frac{p_1}{p_0}$ in this expression linear in $\frac{p_1}{p_0}$ is

$$\frac{-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta}\left[-1+\frac{\kappa(\overline{z})\left(1-\beta^3\right)}{(1-\alpha)A(\overline{z})\beta^3}\right]>\frac{\beta^3\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta}>0,$$

given the conditions $\alpha A(\overline{z}) - \kappa(\overline{z}) > 0$ and (16), so that we get

$$\lim_{(p_0,\lambda)\longrightarrow(0,1)} \left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} = \lim_{\frac{p_1}{p_0}\longrightarrow+\infty} \left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} = +\infty.$$

Moreover,

$$\lim_{p_0\longrightarrow 1} \left.\frac{d\widehat{V}_1^{LF}}{dz}\right|_{z=\overline{z}} = \frac{-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{1-\beta}\left[\frac{\kappa(\overline{z})\left(1-\beta^3\right)}{\alpha\left(1-\alpha\right)A(\overline{z})\beta^3}\right] < 0.$$

Proposition 13 follows.

K Proof of Proposition 14

Let us first derive the intervention $\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}}$ and the corresponding interest rates. Since $\tau^l(\overline{z}, p_0, p_0) = 1 < \tau^u(\overline{z}, p_2, p_2)$, we get

$$\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \left.\frac{\partial\tau^l(z, p_0, p_0)}{\partial z}\right|_{z=\overline{z}}$$

which, using (23), leads to

$$\left.\frac{d\tau_2}{dz}\right|_{z=\overline{z}} = \frac{\beta^3 \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\beta^3\left[\alpha A(\overline{z}) - \kappa(\overline{z})\right] + \kappa(\overline{z})}\left[p_0 - \frac{\left[\alpha A(\overline{z}) - \kappa(\overline{z})\right]\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}} - (1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right]}{2\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]. \tag{24}$$

Moreover, Proposition 11 and (23) imply

$$\left.\frac{dq(z, \tau_2(z), p_0, 0)}{dz}\right|_{z=\overline{z}} = \frac{1}{p_0}\left.\frac{dB}{dz}\right|_{z=\overline{z}} = \frac{\beta^3\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}} - (1-\alpha)\beta^3 p_0 \left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right]}{2\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}. \tag{25}$$

Finally, the total derivative of (7) at date 2 for $\tau_2 = \tau_2(z)$ and $\mu_2 = p_2$ with respect to z, taken at point $z = \overline{z}$, and the use of (24) lead to

$$\left.\frac{dq(z, \tau_2(z), p_2, 1)}{dz}\right|_{z=\overline{z}} =$$

$$-\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\left\{\frac{1+\beta^3(1+p_0-p_2)+\frac{\alpha}{1-\alpha}\frac{p_2}{p_0}}{\alpha A(\overline{z}) - \kappa(\overline{z})} + \frac{\beta^3\left[(1-\alpha)\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}\right]}{2\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right\}. \tag{26}$$

Concerning households, we have

$$
\begin{aligned}
U_1^I(z) = {} & \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right] \\
& + p_A \Bigg\{ \sum_{i=1}^{2} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right] + p_2 \sum_{i=3}^{+\infty} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(1)}(z)} - \kappa(z)\right] \\
& + (1-p_2) \Bigg[\sum_{i=3}^{5} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_{i-2}^{(2)}(z)} - \kappa(\overline{z})\right] \\
& + \sum_{i=6}^{+\infty} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(2)}(z)} - \kappa(\overline{z})\right] \Bigg] \Bigg\} \\
& + (1-p_A) \Bigg\{ \beta \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(\overline{z})\right] + p_B \Bigg[\beta^2 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(z)\right] \\
& + p_1 \Bigg[\sum_{i \in \mathbb{N} \setminus \{0,1,2,4\}} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(3)}(z)} - \kappa(z)\right] + \beta^4 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_2^{(3)}(z)} - \kappa(z)\right] \Bigg] \\
& + (1-p_1) \Bigg[\sum_{i \in \{3,5\}} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_{i-2}^{(4)}(z)} - \kappa(\overline{z})\right] \\
& + \sum_{i \in \mathbb{N} \setminus \{0,1,2,3,5\}} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(4)}(z)} - \kappa(\overline{z})\right] \Bigg] \Bigg] \\
& + (1-p_B) \Bigg[\beta^2 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{\beta^3} - \kappa(\overline{z})\right] + p_{-1} \Bigg[\beta^3 \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_1^{(5)}(z)} - \kappa(z)\right] \\
& \sum_{i=4}^{5} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(5)}(z)} - \kappa(z)\right] + \sum_{i=6}^{+\infty} \beta^i \ln\left[\alpha A(z) + \frac{\kappa(z)}{q_{i-2}^{(5)}(z)} - \kappa(z)\right] \Bigg] \\
& + (1-p_{-1}) \Bigg[\beta^3 \ln\left[\alpha A(\overline{z}) + \frac{\kappa(z)}{q_1^{(6)}(z)} - \kappa(\overline{z})\right] \\
& + \sum_{i=4}^{+\infty} \beta^i \ln\left[\alpha A(\overline{z}) + \frac{\kappa(\overline{z})}{q_{i-2}^{(6)}(z)} - \kappa(\overline{z})\right] \Bigg] \Bigg] \Bigg\},
\end{aligned}
$$

where superscripts (1) to (6) correspond to the following cases:

Superscript	S_2	S_3	Technology
(1)	1	0 or 1	good
(2)	1	0 or 1	bad
(3)	0	1	good
(4)	0	1	bad
(5)	0	0	good
(6)	0	0	bad

Computations then lead to

$$q_1^{(1)}(z) = q(z,1,p_1,1),\ q_2^{(1)}(z) = q(z,\tau_2(z),p_2,1),\ q_3^{(1)}(z) = q(z,1,p_2,1),$$
$$q_i^{(1)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_{i-3}^{(1)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 4,$$
$$q_1^{(2)}(z) = q(z,1,p_1,1),\ q_2^{(2)}(z) = q(z,\tau_2(z),p_2,1),\ q_3^{(2)}(z) = q(z,1,p_2,1),$$
$$q_i^{(2)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z)-\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})} \text{ for } 4 \leq i \leq 6,$$
$$q_i^{(2)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(2)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 7,$$
$$q_1^{(3)}(z) = q(z,1,p_1,1),\ q_2^{(3)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(3)}(z) = q(z,1,p_1,1),$$
$$q_i^{(3)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_{i-3}^{(3)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 4 \text{ and } i \geq 6,$$
$$q_5^{(3)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_2^{(3)}(z)} - \frac{1}{\beta^3}\right] - \frac{\kappa(z)-\kappa(\overline{z})+\beta^3\left[\alpha A(z)-\alpha A(\overline{z})\right]}{\alpha A(z)-\kappa(z)},$$
$$q_1^{(4)}(z) = q(z,1,p_1,1),\ q_2^{(4)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(4)}(z) = q(z,1,p_1,1),$$
$$q_i^{(4)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(4)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z)-\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})} \text{ for } i \in \{4,6\},$$
$$q_i^{(4)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(4)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 5 \text{ and } i \geq 7,$$
$$q_1^{(5)}(z) = q(z,1,p_1,1),\ q_2^{(5)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(5)}(z) = q(z,1,p_{-1},0),$$
$$q_i^{(5)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_{i-3}^{(5)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i = 4 \text{ and } i \geq 7,$$
$$q_i^{(5)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(z)-\kappa(z)}\left[\frac{1}{q_{i-3}^{(5)}(z)} - \frac{1}{\beta^3}\right] - \frac{\kappa(z)-\kappa(\overline{z})+\beta^3\left[\alpha A(z)-\alpha A(\overline{z})\right]}{\alpha A(z)-\kappa(z)} \text{ for } i \in \{5,6\},$$
$$q_1^{(6)}(z) = q(z,1,p_1,1),\ q_2^{(6)}(z) = q(z,\tau_2(z),p_0,0),\ q_3^{(6)}(z) = q(z,1,p_{-1},0),$$
$$q_4^{(6)}(z) = \beta^3 + \frac{\beta^3\kappa(z)}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_1^{(6)}(z)} - \frac{1}{\beta^3}\right] + \frac{\kappa(z)-\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})},$$
$$q_i^{(6)}(z) = \beta^3 + \frac{\beta^3\kappa(\overline{z})}{\alpha A(\overline{z})-\kappa(\overline{z})}\left[\frac{1}{q_{i-3}^{(6)}(z)} - \frac{1}{\beta^3}\right] \text{ for } i \geq 5.$$

Using (15), (25) and (26), we get

$$\left.\frac{dq_1^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_1)+\frac{\alpha p_1}{(1-\alpha)\,p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{1,...,6\}\,,$$

$$\left.\frac{dq_2^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A(\overline{z})-\kappa\left(\overline{z}\right)}\left[1+\beta^3\left(1+p_0-p_2\right)+\frac{\alpha}{1-\alpha}\frac{p_2}{p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\,,$$
$$+\frac{\beta^3}{2\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}-(1-\alpha)\,\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right] \quad \text{for } k\in\{1,2\}\,,$$

$$\left.\frac{dq_2^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{\beta^3}{2\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}\left[\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}-(1-\alpha)\,\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}\right] \quad \text{for } k\in\{3,...,6\}\,,$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_2)+\frac{\alpha p_2}{(1-\alpha)\,p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{1,2\}\,,$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{-1}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left[1+\beta^3(1-p_1)+\frac{\alpha p_1}{(1-\alpha)\,p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{3,4\}\,,$$

$$\left.\frac{dq_3^{(k)}}{dz}\right|_{z=\overline{z}} = \frac{\beta^3 p_{-1}}{\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \quad \text{for } k\in\{5,6\}\,,$$

$$\left.\frac{dq_{3i+j}^{(1)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(1)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{1,2,3\}\,,$$

$$\left.\frac{dq_{3i+j}^{(2)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left[\left.\frac{dq_j^{(2)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right] \quad \text{for } i\geq 1 \text{ and } j\in\{1,2,3\}\,,$$

$$\left.\frac{dq_{3i+j}^{(3)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(3)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{1,3\}\,,$$

$$\left.\frac{dq_{3i+2}^{(3)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_2^{(3)}}{dz}\right|_{z=\overline{z}}+\frac{\beta^3}{\kappa\left(\overline{z}\right)}\left[1+\frac{\alpha}{(1-\alpha)\,p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(4)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left[\left.\frac{dq_j^{(4)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right] \quad \text{for } i\geq 1 \text{ and } j\in\{1,3\}\,,$$

$$\left.\frac{dq_{3i+2}^{(4)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_2^{(4)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+1}^{(5)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_1^{(5)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(5)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_j^{(5)}}{dz}\right|_{z=\overline{z}}+\frac{\beta^3}{\kappa(\overline{z})}\left[1+\frac{\alpha}{(1-\alpha)\,p_0}\right]\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\}$$
$$\text{for } i\geq 1 \text{ and } j\in\{2,3\}\,,$$

$$\left.\frac{dq_{3i+1}^{(6)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left\{\left.\frac{dq_1^{(6)}}{dz}\right|_{z=\overline{z}}-\frac{\beta^3}{\kappa(\overline{z})}\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right\} \quad \text{for } i\geq 1,$$

$$\left.\frac{dq_{3i+j}^{(6)}}{dz}\right|_{z=\overline{z}} = \left[\frac{-\kappa(\overline{z})}{\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\right]^i\left.\frac{dq_j^{(6)}}{dz}\right|_{z=\overline{z}} \quad \text{for } i\geq 1 \text{ and } j\in\{2,3\}\,.$$

Using $p_1 = p_A p_2 + (1-p_A)\, p_0$, we end up with

$$\begin{aligned}
\left.\frac{dU_1^I}{dz}\right|_{z=\overline{z}} &= \frac{\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}}{\kappa\left(\overline{z}\right)+\beta^3\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}\left\{\frac{\kappa\left(\overline{z}\right)}{\alpha A\left(\overline{z}\right)}\left[1+p_0\beta^4+p_A\beta\left(1+\beta\right)+\left(1-p_A\right)p_B\beta^2\right]\right.\\
&\quad+\frac{\kappa\left(\overline{z}\right)}{\left(1-\alpha\right)p_0 A\left(\overline{z}\right)}\left[\left(1+\beta^4+\beta^5\right)p_1+p_A p_2\beta\left(1+\beta\right)\left(1-\beta^3\right)+\left(1-p_A\right)p_B p_1\beta^2\left(1-\beta^3\right)\right]\\
&\quad+\frac{\beta^3\alpha}{\left(1-\beta\right)\left(1-\alpha\right)p_0}\left[p_1\left(1-\beta+\beta^3\right)+p_A p_2\beta\left(1-\beta^2\right)+\left(1-p_A\right)p_B p_1\beta^2\left(1-\beta\right)\right]\\
&\quad\left.+\frac{\beta^4\kappa\left(\overline{z}\right)\left[\alpha A\left(\overline{z}\right)-\kappa\left(\overline{z}\right)\right]}{2\alpha A\left(\overline{z}\right)}\left[\frac{\left(1-\alpha\right)\beta^3 p_0\left.\frac{d^2A}{dz^2}\right|_{z=\overline{z}}-\left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2}\right]\right\}\\
&> 0
\end{aligned}$$

given (14).

Concerning entrepreneurs, we have

$$\begin{aligned}
\widehat{V}_1^I\left(z\right) &= p_A\beta^3\left\{p_2\sum_{i=0}^{+\infty}\beta^i\left[\left(1-\alpha\right)A(z)-\frac{\kappa(z)}{q_{i+1}^{(1)}\left(z\right)}\right]\right.\\
&\quad\left.+\left(1-p_2\right)\left[\sum_{i=0}^{2}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(z)}{q_{i+1}^{(2)}\left(z\right)}\right]+\sum_{i=3}^{+\infty}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(\overline{z})}{q_{i+1}^{(2)}\left(z\right)}\right]\right]\right\}\\
&\quad+\left(1-p_A\right)\beta^3\left\{p_B\left[p_1\left[\sum_{i\in\mathbb{N}\smallsetminus\{1\}}\beta^i\left[\left(1-\alpha\right)A(z)-\frac{\kappa(z)}{q_{i+1}^{(3)}\left(z\right)}\right]\right.\right.\right.\\
&\quad\left.\left.+\beta\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(\overline{z})}{q_2^{(3)}\left(z\right)}\right]\right]+\left(1-p_1\right)\left[\sum_{i\in\{0,2\}}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(z)}{q_{i+1}^{(4)}\left(z\right)}\right]\right.\\
&\quad\left.\left.+\sum_{i\in\mathbb{N}\smallsetminus\{0,2\}}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(\overline{z})}{q_{i+1}^{(4)}\left(z\right)}\right]\right]\right]\\
&\quad+\left(1-p_B\right)\left[p_{-1}\left[\sum_{i\in\mathbb{N}\smallsetminus\{1,2\}}\beta^i\left[\left(1-\alpha\right)A(z)-\frac{\kappa(z)}{q_{i+1}^{(5)}\left(z\right)}\right]\right.\right.\\
&\quad\left.+\sum_{i=1}^{2}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(\overline{z})}{q_{i+1}^{(5)}\left(z\right)}\right]\right]\\
&\quad\left.\left.+\left(1-p_{-1}\right)\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(z)}{q_1^{(6)}\left(z\right)}+\sum_{i=1}^{+\infty}\beta^i\left[\left(1-\alpha\right)A(\overline{z})-\frac{\kappa(\overline{z})}{q_{i+1}^{(6)}\left(z\right)}\right]\right]\right]\right\}.
\end{aligned}$$

Using (15) and $p_1 = p_A p_2 + (1-p_A) p_0$, we end up with

$$\begin{aligned}
\left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} &= \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{-1}{1-\beta} \left[(1-\beta) + p_1\beta^3 + (1-p_A) p_B \beta^2 (1-\beta) + p_A \beta \left(1-\beta^2\right)\right] \right. \\
&+ \frac{1}{p_0 (1-\beta)} \left[p_1 \left(1-\beta+\beta^3\right) + p_A p_2 \beta \left(1-\beta^2\right) + (1-p_A) p_B p_1 \beta^2 (1-\beta)\right] \\
&- \frac{\kappa(\overline{z})}{A(\overline{z}) \alpha \beta^3} \left[1 + p_0 \beta^4 + p_A \beta (1+\beta) + (1-p_A) p_B \beta^2\right] \\
&- \frac{\kappa(\overline{z})}{A(\overline{z})(1-\alpha) p_0 \beta^3} \left[\left(1+\beta^4+\beta^5\right) p_1 + p_A p_2 \beta (1+\beta) \left(1-\beta^3\right) \right. \\
&\left. + (1-p_A) p_B p_1 \beta^2 \left(1-\beta^3\right)\right] \\
&\left. - \frac{\beta\kappa(\overline{z}) \left[\alpha A(\overline{z}) - \kappa(\overline{z})\right]}{2\alpha A(\overline{z})} \left[\frac{(1-\alpha)\beta^3 p_0 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right] \right\}.
\end{aligned}$$

The coefficient of $\frac{p_1}{p_0}$ in this expression linear in $\frac{p_1}{p_0}$ is

$$\begin{aligned}
& \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{1}{(1-\beta)} \left[\left(1-\beta+\beta^3\right) + \lambda\beta\left(1-\beta^2\right) + (1-p_A) p_B \beta^2 (1-\beta)\right] \right. \\
& \left. - \frac{\kappa(\overline{z})}{A(\overline{z})(1-\alpha)\beta^3} \left[\left(1+\beta^4+\beta^5\right) + \lambda\beta(1+\beta)\left(1-\beta^3\right) + (1-p_A) p_B \beta^2 \left(1-\beta^3\right)\right] \right\} \\
> & \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{1}{(1-\beta)} \left[\left(1-\beta+\beta^3\right) + \lambda\beta\left(1-\beta^2\right) + (1-p_A) p_B \beta^2 (1-\beta)\right] \right. \\
& \left. - \left[\left(1+\beta^4+\beta^5\right) + \lambda\beta(1+\beta)\left(1-\beta^3\right) + (1-p_A) p_B \beta^2 \left(1-\beta^3\right)\right] \right\} \\
= & \left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \frac{\beta^3}{(1-\beta)} \left[\left(1-\beta+\beta^3\right) + \lambda\beta\left(1-\beta^2\right) + (1-p_A) p_B \beta^2 (1-\beta)\right] \\
> & \; 0,
\end{aligned}$$

where the first inequality comes from the conditions $\alpha A(\overline{z}) - \kappa(\overline{z}) > 0$ and (16), so that we get

$$\lim_{(p_0,\lambda)\longrightarrow(0,1)} \left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} = \lim_{\frac{p_1}{p_0}\longrightarrow+\infty} \left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} = +\infty.$$

Moreover,

$$\begin{aligned}
\lim_{p_0\longrightarrow 1} \left.\frac{d\widehat{V}_1^I}{dz}\right|_{z=\overline{z}} &= -\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}} \left\{ \frac{\kappa(\overline{z})}{A(\overline{z})\alpha\beta^3} \left[1+\beta^4+\lambda\beta(1+\beta)+(1-\lambda)\lambda\beta^2\right] \right. \\
&+ \frac{\kappa(\overline{z})}{A(\overline{z})(1-\alpha)\beta^3} \left[\left(1+\beta^4+\beta^5\right) + \lambda\beta(1+\beta)\left(1-\beta^3\right) + (1-\lambda)\lambda\beta^2\left(1-\beta^3\right)\right] \\
&\left. + \frac{\beta\kappa(\overline{z})\left[\alpha A(\overline{z}) - \kappa(\overline{z})\right]}{2\alpha A(\overline{z})} \left[\frac{(1-\alpha)\beta^3 \left.\frac{d^2 A}{dz^2}\right|_{z=\overline{z}} - \left.\frac{d^2\kappa}{dz^2}\right|_{z=\overline{z}}}{\left(\left.\frac{d\kappa}{dz}\right|_{z=\overline{z}}\right)^2} \right] \right\} \\
&< 0,
\end{aligned}$$

given the conditions $\alpha A(\overline{z}) - \kappa(\overline{z}) > 0$ and (14). Proposition 14 follows.