Estimating Incentive and Welfare Effects of Non-Stationary Unemployment Benefits

Andrey Launov[(*)] and Klaus Wälde[(*), 1]

[(*)]University of Mainz, Université catholique de Louvain and CESifo

May 2012

The distribution of unemployment duration in our equilibrium matching model with spell-dependent unemployment benefits displays time-varying exit rates. Building on Semi-Markov processes, we translate these exit rates into an expression for the aggregate unemployment rate. Structural estimation using a German micro-data set (SOEP) allows us to discuss the effects of a recent unemployment benefit reform (Hartz IV). The reform reduced unemployment by less than 0.1 percentage points. Contrary to general beliefs, we find that the net wage for most skill- and regional groups increased. Taking the insurance effect of unemployment benefits into account, however, we find that the reform is welfare reducing for four out of six labour groups, i.e. for 76% of workers.

JEL Codes: E24, J64, J68, C13
Keywords: Non-stationary unemployment benefits, endogenous effort, matching model, structural estimation, Semi-Markov process

1 Introduction

Continental European unemployment is notorious for its persistence. France, Italy and Germany have had rising unemployment rates from the 1960s up to 2000 and even onward. There seems to be a consensus now that a combination of shocks and institutional arrangements lies at the origin of these high unemployment rates (Ljungqvist and Sargent, 1998; Mortensen and Pissarides, 1999; Blanchard and Wolfers, 2000). Neither institutions nor shocks alone explain the rise in unemployment: institutions have always been there but unemployment has not (at least not at this level) and shocks have hit many countries but not all countries have high unemployment rates. The step from this shock-institutions insight towards finding

a solution to the European unemployment problem seems to be short: As shocks will not go, we need to address the institutions.[2]

A common suggestion to fight unemployment is to reduce long and generous unemployment benefits. This raises other questions, however, as one seems to be faced by a classic efficiency-equity trade-off. While reducing unemployment per se is beneficial, income of the unemployed and the insurance mechanism provided by unemployment benefits should not be neglected.

We examine the employment and welfare effects of a policy reform which reduces the length and level of unemployment benefits. We use Germany as an example of a continental European country for three reasons. First, the unemployment rate in Germany has been rising for many decades, just as e.g. in France or Italy. Second, the German unemployment benefit system has a two-tier structure which is typical of many OECD countries. Third, the so-called "Hartz IV reform" implemented in January 2005 comprises both the reduction of benefit levels and the cut of the duration of entitlement. Reforms of this type were undertaken in many other OECD countries as well (OECD, 2004).

Neglecting minor institutional details, the reform had two main effects. The *maximum* entitlement to unemployment insurance (UI) payments was (almost) uniformly reduced to 12 months (from a former maximum of 32 months). Unemployment assistance (UA) payments, formerly proportional to net earnings before the job loss, were replaced by a uniform benefit level. The effect of this new rule on UA payments on long-term unemployed workers was ambiguous. There are unemployed workers, mainly from low wage groups, whose benefit payments were lower before the reform than afterwards. Those were the "winners" of the reform (47% of long-term unemployed) - in a static sense. On the other hand, there were also long-term unemployed workers with relatively high wages before entering unemployment. These were affected negatively and their income has dropped (53% of long-term unemployed). Even though the fraction of "winners" and "losers" is roughly equal, aggregating gains and losses shows a loss of the average long-term unemployed worker of around 7% (Blos and Rudolph, 2005; OECD, 2007).

At first sight, the reform seems to have worked. The reported unemployment rate dropped from an annual average of 10.5% in 2004 (Bundesagentur für Arbeit, 2009) to 9.0% in 2007. On the other hand, growth rates in Germany were (for German standards) fairly high. While the German economy shrank in 2003, it has recovered since then and probably also created new jobs. The real GDP grew by 0.8% in 2005, by 3.0% in 2006 and by 2.5% in 2007 (Bundesagentur für Arbeit, 2009). The question therefore arises whether the drop in unemployment can be credited to the reform. It is also a priori unclear how strongly various groups were affected by the reform. Did utility of the (short- and long-term) unemployed or employed workers rise or fall? Did firms gain from the reform? What about social welfare?

We provide answers by using a model which combines various strands of the literature and adds some new and essential features. We employ an equilibrium matching framework and extend the standard textbook model for time-dependent unemployment benefits, endogenous effort, risk-averse households, endogenous negative duration dependence of unemployment

---

[2]This conclusion is even stronger for papers which argue that changes in European unemployment can mainly be attributed to shifts in labour market institutions and to a lesser extent to the interaction of institutions with shocks (Nickell et al., 2005).

2

spell and Semi-Markov features. Each of these extensions is crucial. Unemployment benefits in our model need to depend on the length of the unemployment spell as this is a feature of basically all OECD unemployment benefit systems. Letting agents optimally choose their effort to find a job, we can analyze the incentive effects of (reforms of) the unemployment benefit system. Risk-averse households are required as we also want to evaluate insurance effects. Duration dependence, modelled as the outcome of Bayesian learning about a workers type, mirrors observed pattern of individual exit rates beyond the incentive effects induced by the two-tier system. Finally, tools from the Semi-Markov literature are required as they allow us to deduce aggregate (un)employment from individual search behaviour. We can thereby compute macro efficiency effects resulting from micro incentives. Without these Semi-Markov tools, we would not be able to formulate an equilibrium model.

We solve this model numerically by looking at Bellman equations as differential equations. This gives us solutions which are as accurate as numerical precision and which do not require us to approximate the model in any way. Optimal behaviour implies an exit rate out of unemployment which is a function of the time spent in unemployment. We thereby obtain a sufficiently flexible endogenous distribution of unemployment duration which we employ for structural estimation by maximum likelihood.

The main theoretical contribution of our analysis is the explicit treatment of the Semi-Markov nature of optimal individual behaviour due to the presence of spell-dependent unemployment benefits: Optimal exit rates not only depend on whether the individual is unemployed (the current state of the worker) but also on how long an individual has been unemployed. While this Semi-Markov aspect has been known for a while, it has not been fully exploited so far in the search literature. Using results from the applied mathematics literature, we obtain analytic expressions for individual employment probabilities contingent on current employment status and duration of unemployment - equations of the so-called Volterra type. They allow us to compute aggregate unemployment rates using a law of large numbers in our pure idiosyncratic risk economy. Given this link from optimal individual behaviour to aggregate outcomes, we can analyze the distribution and efficiency effects of changes in level and length of unemployment benefits.

The main empirical contribution is the careful structural modelling of exit rates out of unemployment. Falling unemployment benefits imply an increase of search effort and therefore also of *individual* exit rates over time. Empirical evidence shows, however, that *aggregate* exit rates tend to fall with time, which can be both due to unmeasured individual heterogeneity and true negative duration dependence of unemployment duration. We therefore combine individual incentive effects, provided by the benefit system, with an endogenous negative duration dependence that arises in our model due to Bayesian learning of heterogeneous individuals about own unobservable search productivity. We find that along with significant upward pressure on the search effort exercised by the benefit system, endogenous individual spell dependence has a significant downward influence on the dynamics of exit rates. Net dynamics of the exit rate out of unemployment differs across observed characteristics. For some individuals (e.g. high-skilled West Germans), the negative effect of endogenous duration dependence is stronger than the positive effect of the wedge between UI and UA benefits. As a result, the exit rate is monotonically decreasing. For other individuals (e.g. high-skilled East Germans), the situation is exactly the opposite and the exit rate is non-monotonic, increasing up until the expiration of entitlement to UI benefits and decreasing thereafter.

With a policy focus in mind, we emphasize and estimate the trade-off between insurance and incentive effects of labour market policies. The degree of risk-aversion - crucial for understanding the insurance effect - is jointly estimated with all other model parameters. A comparative static analysis, using estimated parameters for the theoretical model, then allows us to derive precise predictions about the employment and distribution effects of changes in the length and level of unemployment benefits.

Providing a short preview of our results, we find that the reform did reduce the unemployment rate - which is the desirable effect - but only by 0.07 percentage points. This (almost negligible) decrease varies considerably, however, across our six regional and skill-groups. For some groups (low-skilled in East and West), unemployment actually went up. We also find that the reform increased net wages for four out of six groups. This can easily be understood from an economic perspective but is somewhat counterintuitive from a policy perspective.

From an economic perspective, net wages rise as the reform induces most individuals to search harder. Harder search makes opening of vacancies more attractive for firms which further contributes to a rise of the exit rate out of unemployment. Moreover, the reduction in benefits lowers the tax rate of a government that operates a balanced budget. Given our Nash bargaining setup for wage setting, the positive effects of the increase in market tightness and the reduction of the tax rate dominate the negative effect of lower benefits on the outside option. As a consequence, net wages rise.

This finding seems counterintuitive from a policy perspective given the discussions of that time and the strong opposition to the reform in the population. Yet, this opposition can easily be understood when we look at intertemporal effects of the reform. Despite the rise in the net wage, most individuals lose from the reform. The value of having a job, of being short- or long-term unemployed and intertemporal expected utility all decrease for all medium- and high-skilled groups. The reason for these intertemporal losses is that the gain in the net wage is overcompensated by expected and anticipated losses in case of long-term unemployment. In short, four out of six groups are opposed to the reform.

Our paper is related to various strands in the literature. From a theoretical perspective, we build on the search and matching framework of Mortensen (1982) and Pissarides (1985). Time-dependent unemployment benefits and endogenous effort have been originally analyzed by Mortensen (1977) in a one-sided job search model. Equilibrium search and matching models with time-dependent unemployment benefits include Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001).[3] In these models, exit rates are constant within each benefit regime. This does not fully capture continuously decreasing exit rates as observed in the data. There also exists a substantial literature that studies optimal insurance allowing for an arbitrary time path of unemployment benefit payments (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997, 2009; Pavoni, 2009; Shimer and Werning, 2007). Our focus is more of a positive nature trying to understand the welfare effects of existing systems which

---

[3]Albrecht and Vroman (2005) and Coles and Masters (2007) also have time-dependent unemployment payments but they do not analyze the implications for individual effort. Albrecht and Vroman focus on the equilibrium wage dispersion and inefficient job rejection. Coles and Masters model aggregate uncertainty implying implicit transfers between firms and the stabilizing effect this has on the unemployment rate over the cycle.

have a simpler benefit structure than the ones resulting from an optimization approach.[4] We also allow for an unlimited number of transitions between employment and unemployment and take equilibrium effects of wages, vacancies and tax rates into account.[5] The modelling of Bayesian learning in continuous time is similar but not identical to Keller, Rady and Cripps (2005).

From an empirical perspective, we estimate a nonstationary structural duration model with discrete distribution of unobserved heterogeneity. Time dependence of the hazard function due to time-dependent benefits and learning is fully described by the equilibrium solution of our theoretical model. The first landmark in the structural econometric estimation of nonstationary job search models is Wolpin (1987), where time dependence of the exit probability is due to finite search horizon. Nonstationary models with time-dependent benefits were originally estimated by van den Berg (1990) and Ferrall (1997). Keane and Wolpin (2002a,b) and Keane and Wolpin (2010) explicitly address time-dependent rules of the evolution of benefits in a life-cycle setting, where benefit levels are perceived to be transitory by forward looking agents. Van den Berg and van der Klaauw (2010) model time dependence of search behaviour via monitoring and sanctions mechanism imposed on benefit recipients.[6] In contrast to our model, this literature deals with one-sided job search, which makes application of its estimates in an equilibrium analysis rather difficult. In addition to that, focus on the incentive effect is only partial (van den Berg and van der Klaauw, 2010) and insurance effect remains largely unaddressed. There also exists a broader empirical equilibrium search literature that deals with unemployment benefit heterogeneity (Bontemps et al., 1999), heterogeneity in workers abilities (Postel-Vinay and Robin, 2002), heterogeneity in workers value of nonparticipation (Flinn, 2006) and heterogeneity in the value of worker-firm match (Eckstein and Wolpin, 1995; Postel-Vinay and Turon, 2010). Unlike in our model, neither of these contributions takes time-dependent unemployment benefits into account.[7]

Semi-Markov methods are taken from the applied mathematical literature, see e.g. Kulkarni (1995) or Corradi et al. (2004). Economic papers which allowed for Semi-Markov features (e.g. Burdett et al., 1985, Aase, 1990, Magnac et al., 1995, Pavoni, 2009) focused on time-varying exit rates but did not exploit their full potential, i.e. they did not use Volterra equations which we need here for equilibrium considerations.

Finally, there is a very small academic literature which discusses the Hartz reforms. Heer (2006) provides a "tentative analysis" which does not explicitly look at the effects of a two-tier system. Fahr and Sunde (2009) focus on aspects of the Hartz reforms (Hartz I-III) which do not affect unemployment benefits. Franz et al. (2007) study the effects of Hartz IV in a CGE model focusing on the impact on various household types. Krebs and Scheffel (2010) present a fairly optimistic picture of the reform based on a calibrated macro model. Krause

---

[4]Galenianos et al. (2011) analyse the welfare effects of flat unemployment benefits (as opposed to our two-tier structure), using a setup with market power of firms.

[5]Acemoglu and Shimer (1999) and Moscarini (2005) use a general equilibrium model, but their setting is restricted to time-invariant benefits only.

[6]Furthermore, Frijters and van der Klaauw (2006) develop a nonstationary search model with nonparticipation, where nonstationarity is due to exogenous stigma effect of unemployment, as well as due to time dependence of the offer distribution. Paserman (2008) provides a structural estimation taking hyperbolic discounting into account. See also Eckstein and van den Berg (2007) for literature review on nonstationary empirical search models.

[7]A general equilibrium model of economic growth is estimated by Lentz and Mortensen (2008).

and Uhlig (2012) also find that the reform was more successful than we do. Their result is based on calibration as well.

The structure of our paper is as follows. Section 2 presents the theoretical model, institutional setting, behaviour of supply and demand sides and the combination of both in economic welfare. Section 3 describes the equilibrium properties of the model. Section 4 illustrates the structural estimation and the underlying data. The simulation results and the evaluation of the institutional reform are presented in section 5. Section 6 concludes.

2 The model

We use a Mortensen-Pissarides type matching model and extend it for time-dependent unemployment benefits, endogenous effort, ex-ante heterogeneous and risk-averse households and an endogenous negative duration dependence resulting from subjective beliefs. To solve it, we use Semi-Markov tools. The separation rate for jobs is constant and there is no search on the job. We focus on steady states in our analysis.

Individuals differ ex-ante by their skills and by their search productivity type. Both skills and type are drawn when born and they are constant throughout life. We let an individual draw her skill group $k = 1...K$ from a discrete probability distribution $\pi^K(k)$. All skills are known to the individual and to the econometrician. Individuals also draw their search productivity type $\chi \in \{0,1\}$ at the beginning of life but do not know the realization. They can learn their type over time in a Bayesian fashion. We denote the share of workers in the population with high search productivity by π^χ. This is also the share within each skill group, i.e.

$$\text{Prob}(\chi = 1|k) = \text{Prob}(\chi = 1) \equiv \pi^\chi. \tag{1}$$

2.1 Workers

We start by considering one skill group. Unemployed workers receive UI benefits b_{UI} and UA benefits b_{UA}. In basically all OECD countries, UI benefits are paid for a certain number of months, after which UA benefits are paid. We denote entitlement length to UI benefits by $\bar{s}$ and assume that it is identical for all individuals (as e.g. in Coles and Masters, 2006).[8] Hence, unemployment payments $b(s)$ are given by

$$b(s) = \begin{cases} b_{UI} & 0 \leq s \leq \bar{s} \\ b_{UA} & \bar{s} < s \end{cases}. \tag{2}$$

We assume $b_{UI} > b_{UA} \geq 0$. Reflecting the institutional setup in most OECD countries and in Germany best, we consider b_{UA} and b_{UI} to be proportional to the net wage w earned at the moment the worker loses the job. With e.g. ξ_{UI} denoting the UI replacement rate, we obtain

$$b_{UI} = \xi_{UI} w. \tag{3}$$

This replacement rate will play a role in the wage setting equation and in the numerical implementation of the reform.

---

[8]Put differently, we do not let $\bar{s}$ be a function of past employment history. See the discussion after (13) for an extension.

An unemployed worker finds a job according to a time-inhomogeneous Poisson process with arrival rate $\mu(.)$. This rate will also be called the job-finding rate, hazard rate or exit rate out of unemployment. We allow this rate to depend on effort $\phi(s)$ an individual exerts to find a job. Effort depends on the length s this individual has been spending in unemployment since her last job. If $s > \bar{s}$, the individual will be called long-term unemployed. In addition to effort, the exit rate of an individual will also depend on skill-specific labour market conditions. Labour market conditions are captured by labour market tightness θ, i.e. the ratio of the number of vacancies V divided by the number of unemployed,

$$\theta \equiv V/(N - L). \tag{4}$$

We will assume that effort and tightness are multiplicative: no effort implies permanent unemployment and no vacancies imply that any effort is in vain. As in Albrecht and Vroman (2002), each of our K groups has its own number of skill-specific vacancies.

Finally, the exit rate is also a function of an individual's search productivity type χ. To save on notation, we will denote the exit rate by $\mu(\phi(s)\theta, \chi)$ but it should be kept in mind that a more complete notation would be $\mu_k(\phi(s)\theta, \chi)$ or $\mu(\phi(s)\theta, \chi, k)$ where k captures individual characteristics or skills, known both to the individual and the econometrician.[9] It should be clear from this discussion that there are two channels through which individuals differ in their exit rates: first, the direct channel where different individuals have different search technologies (captured by k) and, second, the indirect channel where different individuals choose different search effort.

Search productivity type χ in the exit rate is assumed to be unknown to an individual. Unemployment is a relatively rare event for most workers and circumstances differ from one unemployment spell to another. Labour market conditions can change and demand might have shifted since the previous unemployment spell (if the worker had one at all). We capture this individual ignorance about search environment by making the search productivity unknown to the individual. We let individuals behave like (passive) Bayesian learners that update some belief $p(s)$ that χ equals one.[10] The information for the update stems from the duration of unemployment. The longer an individual is unemployed, the less likely it is that her search productivity is actually high. Hence, there will be a *subjective* arrival rate $\mu(\phi(s)\theta, p(s))$ which the individual uses for computing optimal effort and there will be an *objective* arrival rate $\mu(\phi(s)\theta, \chi)$, where χ is either zero or one.[11] This setup allows us to obtain endogenous falling exit rates at the individual level.[12]

---

[9]Given our focus on individual search behaviour, we start at the individual level and then derive a matching function (see the discussion following (24)) rather than the other way round. Both ways are of course equivalent.

[10]The Bayesian modelling is inspired by Keller, Rady and Cripps (2005) who study strategic experimentation with two-armed bandits. The setup with χ being either zero or one can easily be generalized to the case where the arrival rate is positive in both cases (Keller and Rady, 2010). The crucial property of this setup for our purposes which we exploit further below (the belief $p(t)$ falls over time) does not change.

[11]The subjective arrival rate can be written in this way due to some linearity in its functional form which we anticipate at this point for expositional convenience. For the functional form, see (24) below.

[12]Our formulation is an alternative to other factors which affect the exit rate out of unemployment. This can include stigma (Vishvanath, 1989), ranking (Blanchard and Diamond, 1994) and gains or losses in individual human capital.

The outcome of our time-varying exit rate will be an endogenous distribution of unemployment duration. Its density is given by (e.g. Ross, 1996, ch. 2)

$$f(s, \chi) = \mu(\phi(s)\theta, \chi) e^{-\int_0^s \mu(\phi(u)\theta, \chi)du}, \tag{5}$$

one for each value of χ. These densities will be crucial later for various purposes including the estimation of model parameters. It is endogenous to the model, as the exit rate $\mu(\phi(s)\theta, \chi)$ is determined by the optimizing behaviour of workers and firms. The two distributions will obviously differ between individuals that have different observed skill levels k.

Households are infinitely lived and do not save. They have a strictly positive time preference rate ρ. The present value of having a job is given by $V(w)$ and depends on the current endogenous wage w only. Employed workers enjoy instantaneous utility $u(w)$. The value $V(w)$ is constant in a steady state as the wage is constant, but differs across steady states. A worker-firm match can be interrupted by exogenous causes which occur according to a time-homogenous Poisson process with a constant arrival rate λ specific to each group. Whenever a worker loses her job, she enters the unemployment benefit system by obtaining insurance payments b_{UI} for the length of $\bar{s}$. Hence, the value of being unemployed when just having lost the job is given by $V(b_{UI}, 0)$ where 0 stands for a spell of length zero. This leads to a Bellman equation for the employed worker of

$$\rho V(w) = u(w) + \lambda[V(b_{UI}, 0) - V(w)]. \tag{6}$$

Given the fact that unemployed workers are Bayesian learners, they use their subjective arrival rate for evaluating the state of being unemployed. The Bellman equation for the unemployed worker therefore reads

$$\rho V(b(s), s) = \max_{\phi(s)} \left\{ u(b(s), \phi(s)) + \frac{dV(b(s), s)}{ds} + \mu(\phi(s)\theta, p(s))[V(w) - V(b(s), s)] \right\}. \tag{7}$$

We explicitly include $b(s)$ and s as state variables for the unemployed worker as the value of being unemployed obviously depends on current income $b(s)$. The spell term s is also included to take two aspects into account. First, it matters for the unemployed worker how long UI payments are paid. The closer $\bar{s}$, the lower one would expect the value of being unemployed is. Second, the subjective belief changes over time. This time dependence is also captured by s.[13]

The instantaneous utility flow of being unemployed, $\rho V(b(s), s)$, is given by three components. The first component shows the instantaneous utility resulting from consumption of $b(s)$ and effort $\phi(s)$. The second component is a deterministic change of $V(b(s), s)$ as the value of being unemployed changes over time. The third component is a stochastic change that occurs at the subjective job-finding rate $\mu(\phi(s)\theta, p(s))$. When a job is found, an unemployed worker gains the difference between the value of being employed $V(w)$ and $V(b(s), s)$.

---

[13] One could add $\bar{s}$ as an explicit argument or as a subscript to stress the dependence of the value of being unemployed (and of optimal effort further below) on remaining entitlement $\bar{s} - s$. Having said this, we opt for simpler notation.

An optimal choice of effort $\phi(s)$ for (7) requires

$$u_{\phi(s)}\left(b\left(s\right),\phi\left(s\right)\right) + \mu_{\phi(s)}\left(\phi\left(s\right)\theta, p\left(s\right)\right)\left[V\left(w\right) - V\left(b\left(s\right), s\right)\right] = 0, \tag{8}$$

where subscripts denote partial derivatives. It states that the utility loss resulting from increasing search effort must be equal to expected utility gain due to higher effort.

As unemployment benefits are discontinuous at $\bar{s}$, the question arises what happens to the value of being unemployed at this point. Value functions measure overall utility from optimal behaviour between now and the end of the planning horizon. The value of being unemployed depends on unemployment benefits and unemployment duration only and is continuous in s. Hence, it holds that the value of being unemployed at $\bar{s}$, where by (2) UI payments are still paid, equals the value an instant thereafter where UA payments are paid. Formally,[14]

$$V\left(b_{UI}, \bar{s}\right) = V\left(b_{UA}, \bar{s}\right). \tag{9}$$

2.2   The firms, the wage and the government

Firms produce under perfect competition on the goods market and each worker-firm match produces output $A_k = Bh_k$.[15] This output depends on labour productivity of the firm's technology B and on individual characteristics h_k. In other words, firms are homogenous ex ante and workers are heterogenous ex ante. The characteristics fixing h_k are the same as the ones which impact on the arrival rate of jobs, $\mu(.)$, or on the separation rates λ_k. These characteristics will play a role in the estimation of the model parameters.

The value of a job J_k to a firm depends on the skill group to which the worker belongs. It is given by instantaneous profits $A_k - w_k/(1-\kappa)$, which is the difference between output and the gross wage $w_k/(1-\kappa)$, reduced by the risk of being driven out of business,

$$\rho J_k = A_k - w_k/(1-\kappa) - \lambda_k\left[J_k - J_{0k}\right]. \tag{10}$$

The interest rate is denoted by $\rho > 0$, which is identical to the discount rate of households. The value of a vacancy is given by J_{0k}.

The rate at which a vacancy is filled depends inter alia on the true rates unemployed individuals find a job. Individual arrival rates are heterogenous within individuals of the same type k due to differences in the length of the unemployment spell. The mean arrival rate for group k, using the endogenous distribution of the unemployment spell $f_k(s, \chi)$ from (5) and the exogenous share π^χ of high-productivity searchers, is given by

$$\bar{\mu}_k = \pi^\chi \int_0^\infty \mu_k\left(\phi\left(s\right)\theta, 1\right) f_k\left(s, 1\right) ds + (1 - \pi^\chi) \int_0^\infty \mu_k\left(\phi\left(s\right)\theta, 0\right) f_k\left(s, 0\right) ds. \tag{11}$$

Vacancies are opened for a specific skill group. The Bellman equation for a vacancy for group k reads (see app. B.1.2) $\rho J_{0k} = -\gamma_k + \theta_k^{-1}\bar{\mu}_k\left[J_k - J_{0k}\right]$. With free entry into vacancy

---

[14]We are aware of the fact that the right-hand side should be written as a limit with s approaching $\bar{s}$ from above.

[15]The analysis so far focused on one particular individual. As stressed above after (4), this allowed us to supress the index k. As this section on firms needs to be explicit about individual skill levels, we will now use the "k-notation" explicitly. Estimates in tab. 1 and parameters and variables in the pre-reform steady states in tab. 2 clearly show which parameters and variables are group-specific.

creation, the value of holding a vacancy is $J_{0k} = 0$, leading to a condition fixing θ_k, i.e. the total number of vacancies given the number of unemployed workers for each group k,

$$\bar{\mu}_k J_k = \gamma_k \theta_k. \tag{12}$$

Modelling wage setting for any country is a big challenge. Looking at Germany, almost two thirds of all wages and salaries are the outcome of negotiations between industry unions and employer federations.[16] Labour income not covered by central negotiations is determined either by individual bargaining, by wage posting or other. As we do not want to model heterogeneity in wage setting in this paper, we assume that the wage for each skill group k is the outcome of collective bargaining. The question then arises what the objective of unions and employer federations are. The main issue thereby is to what extend the interest of unemployed workers are taken into account. As almost all members of unions are employed, we assume here that wages are determined by insiders, i.e. those who currently have a job. Due to its analytical convenience, we also assume that wages are determined by Nash bargaining. We discuss alternatives in a moment.

In case of successful negotiations, the collective value of employed workers of type k is $V(w_k(t)) L_k(t)$. If bargaining fails, workers receive unemployment benefits which - given institutional rules - depend on previous employment history and age. If we make entitlement length $\bar{s}$ from (2) a function of e.g. the employment history, we would obtain a distribution of $\bar{s}$. While modeling this is conceptually straightforward, it is challenging in details.[17] We therefore give the same entitlement $\bar{s}$ to all individuals (estimation does take heterogeneity in $\bar{s}$ into account), independently of their employment history.[18] Hence, if bargaining fails, the collective value of $L_k(t)$ workers is $V(b_k(0), 0) L_k(t)$. The collective contribution of firms to the Nash product is simply $J\left(\frac{w_k(t)}{1-\kappa}\right) L_k(t) - J_{0k} L_k(t)$. The generalized Nash product can therefore be written as $(V(w_k(t)) - V(b_k(0), 0))^{\beta} \left(J\left(\frac{w_k(t)}{1-\kappa}\right) - J_{0k}\right)^{1-\beta} L_k(t)$.

Following the steps as in Pissarides (1985) for risk-neutral or Lehmann and van der Linden (2007) for risk-averse individuals, our setup for collective bargaining with a replacement rate (3) yields (see app. B.1.3)

$$(1-\beta) u(w_k) + \beta m_{w_k}(.) w_k$$
$$= (1-\beta) u(b_{UI,k}, \phi_k(0)) + \beta (1-\kappa) m_{w_k}(.) \left[A_k + \gamma_k \theta_k \frac{\mu(\phi_k(0), 0)}{\bar{\mu}_k}\right], \tag{13}$$

---

[16] This is in contrast to collective bargaining at the firm level as modelled e.g. by Cahuc and Lehmann (2000). Mortensen and Pissarides (1999) consider the case of a monopoly union which sets the share β of the surplus going to the worker. The firm responds by creating and destroying jobs.

[17] The implied distribution of $\bar{s}$ can be described by Fokker-Planck equations (FPEs) of the type recently employed by Bayer and Wälde (2011). As there is maximum and minimum entitlement, however, there are, additionally to standard FPEs, mass points at the bounds of the support of $\bar{s}$. While they could be described by differential equations as well, such a framework is currently much to complex to be used for an evaluation of the type intended in this paper.

[18] We acknowledge the importance of detailed analyses of two-sided heterogeneity and the implications for efficiency of various wage setting mechanisms. See e.g. Gautier et al. (2010). We leave a merging of both approaches for future work.

where

$$m_{w_k}(w_k, b_{UI}, \phi_k(0)) \equiv u_w(w_k) + \frac{\lambda_k}{\rho + \mu(\phi_k(0), 0)} u_w(b_{UI,k}, \phi_k(0)) \qquad (14)$$

is generalized marginal utility from consumption. The first term $u_w(w_k)$ in (14) is marginal utility from consumption as an employed worker. The second term is the generalization due to the fact that $b_{UI,k}$ is proportional to the previously earned wage: An increase in the bargained wage affects (the present value of expected) marginal utility from consumption if unemployed at a later point in time. If UI payments were not proportional to the previously earned wage, $m_{w_k}(.)$ would be given by $u_{w_k}(w_k)$. The left hand side of (13) corresponds to what in models with risk-neutrality is simply the wage rate w_k. On the right hand side, benefits for the unemployed (for risk-neutral households and no time-dependence of effort), are replaced by instantaneous utility from being unemployed, $u(b_{UI,k}, \phi_k(0))$. The contribution of the production side in squared brackets is translated into "utils" by multiplying with generalized marginal utility and takes tax effects into account.

As mentioned before, any real world economy exhibits a multitude of wage setting mechanisms. We are aware of the many alternatives to Nash bargaining and also to the structure of Nash bargaining used here. One alternative to its structure would consist in specifying an outside option where each individual worker would be entitled to UI payments according to past employment history. In the case of individual bargaining, an endogenous wage distribution would arise (see Albrecht and Vroman, 2005). With a distribution of employment history, there would be a distribution of outside options and therefore a distribution of wages. In our case of collective bargaining, however, we would still obtain a unique wage. With l denoting employment history, $\bar{s}(l)$ would denote the length of entitlement to UI payments.[19] One would then replace $V(b_k(0), 0)$ by $\int_0^\infty V(b_k(0), 0, \bar{s}(l)) g(l) dl$, where $g(l)$ is the distribution for employment duration. Clearly, $\int_0^\infty V(b_k(0), 0, \bar{s}(l)) g(l) dl$ is a fixed quantity such that the wage would remain unique.

An alternative to Nash bargaining itself consists in strategic bargaining. Strategic bargaining is the appropriate choice when payoffs change over time as Nash bargaining would correspond to myopic behaviour (Coles and Wright, 1998; Coles and Muthoo, 2003). Strategic bargaining was also used in the analysis of on-the-job search (Cahuc et al., 2006; Shimer, 2006) and in Hall and Milgrom (2008) who stress that employment fluctuations under Nash bargaining are too small. As our collective bargaining setup is the most appropriate assumption for Germany which implies that collective payoffs are stationary and as we do not focus on business cycle issues, we feel justified in using Nash bargaining here.[20]

The economy has a work force consisting of K skill groups, each one of an exogenous size N_k. Employment for each skill group k is endogenous and given by the headcount L_k. Total employment is $L = \sum_{k=1}^n L_k$, and the number of unemployed amounts to $N - L$. Unemployment benefit payments to short- and long-term unemployed are financed by a tax rate κ on gross wages. The flat labour tax κ implies that the net wage is $w_k = (1 - \kappa) w_k^{gross}$.

---

[19] In an empirical implementation, age would be an additional argument for $\bar{s}(l)$.

[20] Despite our focus on a unique wage, we agree that different pay for similar workers (Burdett and Mortensen, 1998; Postel-Vinay and Turon, 2010; Uren and Virág, 2011; Burdett et al., forthcoming) is an important aspect of the real world. We leave an equilibrium analysis of non-stationary unemployment benefits in such a setup for future work.

The number of short-term unemployed workers is

$$U_k^{\text{short}} \equiv \Sigma_{k=1}^n \left(N_k - L_k\right) \int_0^{\bar{s}} f_k\left(s\right) ds \quad \text{and} \quad U_k^{\text{long}} \equiv \Sigma_{k=1}^n \left(N_k - L_k\right) \int_{\bar{s}}^\infty f_k\left(s\right) ds \qquad (15)$$

is the number of the long-term unemployed. The budget constraint of the government therefore equates expenditure on the left-hand side to income on the right,

$$\Sigma_{k=1}^n \left(b_{UI} U_k^{\text{short}} + b_{UA} U_k^{\text{long}}\right) = \Sigma_{k=1}^n \kappa \frac{w_k}{1 - \kappa} L_k. \qquad (16)$$

The government adjusts the wage tax κ such that this constraint holds at each point in time.

2.3 The social welfare function

In addition to the incentive effect of the reform, we would also like to understand the insurance effect. In a world without moral hazard, optimal unemployment insurance would require unemployment benefits to be equal to the net wage. With effort being a function of unemployment benefits, insurance considerations must take into account that effort decreases in unemployment benefits.

We can easily understand whether the insurance effect was taken into account in an appropriate way by computing expected utility of an individual being "behind the Rawlsian veil of ignorance". The individual does not know her skill group k, nor her type χ. It does know probabilities $\pi^K\left(k\right)$ and the population share π^χ, though. This is similar in spirit to social welfare functions employed by Hosios (1990) or Flinn (2006). One can alternatively look at this expected utility as average utility over all (employed and unemployed) workers of type k and type χ. Expected utility conditional of skill and type $EU_{k,\chi}$ is given by

$$EU_{k,\chi} \equiv \frac{L_k}{N_k} V\left(w_k\right) + \frac{N_k - L_k}{N_k} \left(\int_0^{\bar{s}} V_k\left(b_{UI}, s\right) f_k\left(s, \chi\right) ds + \int_{\bar{s}}^\infty V_k\left(b_{UA}, s\right) f_k\left(s, \chi\right) ds\right).$$
$$(17)$$

It adds the share L_k/N_k of employed workers times their welfare $V\left(w_k\right)$ to the share $\left(N_k - L_k\right)/N_k$ of unemployed workers times the average welfare of an unemployed. This average is obtained by integrating over all spells s, where $f_k\left(s, \chi\right)$ is the endogenous density (5) of group k and type χ with exit rates $\mu\left(\phi_k\left(s\right)\theta, \chi\right)$ that follow from the steady state solution of the model, and the $V\left(b_k\left(s\right), s\right)$ are the values of being unemployed with a spell s and benefit payments $b_k\left(s\right)$ from (2).

When we then compute a weighted sum over all skill-groups and types, we obtain overall utility,

$$EU = \Sigma_{k=1}^n \frac{N_k}{N} \left[\pi^\chi EU_{k,1} + \left(1 - \pi^\chi\right) EU_{k,0}\right], \qquad (18)$$

which would be the ultimate aggregate measure of social welfare. We will also report welfare measures of subgroups as computed in (17).

3 Equilibrium properties

3.1 Individual (un)employment probabilities

In models with constant job-finding and separation rates, the number (or measure) of employees can easily be derived by assuming a law of large numbers. Aggregate employment then follows $\dot{L} = \mu [N - L] - \lambda L$. With spell-dependent effort, individual arrival rates $\mu(.)$ are heterogeneous and the number of employees needs to be derived using techniques from the literature on Semi-Markov or renewal processes, e.g. Kulkarni (1995) or Corradi et al. (2004). We need the number of employees in order to compute the unemployment rate and for computing income and expenditure in the government budget constraint. These Semi-Markov tools are therefore essential for any equilibrium model with time-dependent unemployment benefits.

The generalization of Semi-Markov processes compared to continuous time Markov chains consists in allowing the transition rate from one state to another to depend on the time an individual has spent in the current state. We apply this here and let the transition rate from unemployment to employment depend on the time s the individual has been unemployed. Hence, switching from a constant job-finding rate μ to a spell-dependent rate $\mu(s)$ implies switching from Markov to Semi-Markov processes. Processes are called "semi" as the history-dependence of the job finding rate $\mu(s)$ is not Markov. Processes are still called "Markov" as history no longer counts once an individual has found a job. This is also why these processes are related to renewal processes: whenever a transition to a new state occurs, the system starts from the scratch, it is "renewed" and history vanishes.

We start by looking at individual employment probabilities.[21] Let $p_{ij}(\tau, s(t))$ describe the probability with which an individual, who is in state i (either e for employed or u for unemployed) today in t, will be in state $j \in \{e, u\}$ at some future point in time τ, given that her current spell is now $s(t)$. The exit rates that need to use here are the *objective* exit rates.[22] Starting with an individual that just lost her job, i.e. $s(t) = 0$, and taking into account that the separation rate λ remains constant, these expressions read (see app. A.5),

$$p_{uu}(\tau, 0) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) \, dv, \tag{19a}$$

$$p_{eu}(\tau) = \int_t^\tau e^{-\lambda[v-t]} \lambda p_{uu}(\tau - v, 0) \, dv. \tag{19b}$$

Expressions for complementary transitions are given by $p_{ue}(\tau) = 1 - p_{uu}(\tau)$ and $p_{ee}(\tau) = 1 - p_{eu}(\tau)$, respectively.

These equations have a straightforward intuitive meaning. Consider first the case of τ being not very far in the future. Then all integrals (for $\tau = t$) are zero and the probability of being unemployed at τ is, if unemployed at t, one from (19a) and, if employed at t, zero from (19b). For a $\tau > t$, the part $e^{-\int_t^\tau \mu(s(y))dy}$ in (19a) gives the probability of remaining in

---

[21] All of these considerations are specific to a group k. We will use the index k later explicitly again when we talk about the economy-wide unemployment rate.

[22] Hence, $\mu(s(y))$ in what folows is short-hand for $\mu(s(y), \chi)$ where χ either equals zero or one. The numerical implementation computes probabilities p_{eu} and p_{uu} for both types and then computes the weighted average using the estimated value for π^χ.

unemployment for the entire period from t to τ. An individual unemployed today can also be unemployed in the future if she remains unemployed from t to v (the probability of which is $e^{-\int_t^v \mu(s(y))dy}$), finds the job in v (which requires multiplication with the exit rate $\mu(s(v))$) and then moves from employment to unemployment again over the remaining interval $\tau - v$ (for which the probability is $p_{eu}(\tau - v)$). Note that the probability $p_{eu}(\tau - v)$ allows for an arbitrary number of transitions in and out of employment between v and τ (see fig. 7 in app. A.5 for an illustration). As this path is possible for any v between t and τ, the densities for these paths are integrated. The sum of the probability of remaining unemployed all of the time and of finding a job at some v but being unemployed again at τ gives then the overall probability $p_{uu}(\tau, 0)$ of having no job in τ when having no job in t. The interpretation of (19b) is similar. The probability of remaining employed from t to v is simpler, $e^{-\lambda[v-t]}$, as the separation rate λ is constant. The individual then loses the job at v requiring the transition rate λ and then moves back and forth between unemployment and employment to eventually end up in unemployment at τ or earlier. The latter is captured by $p_{uu}(\tau - v, 0)$.

As we can see, these equations are interdependent: The equation for $p_{uu}(\tau, 0)$ depends on $p_{eu}(\tau - v)$ and the equation for $p_{eu}(\tau)$, in turn, depends on $p_{uu}(\tau - v, 0)$. Formally speaking, these equations are integral equations, sometimes called Volterra equations of the first type (19b) and of the second type (19a). Integral equations can sometimes be transformed into differential equations, which will simplify their solution in practice. In our case, however, no transformation into differential equations is known.

After having computed the probability of being unemployed in τ when being unemployed in t for individuals that just became unemployed in t, i.e. who have a spell of length $s(t) = 0$, we will need an expression for $p_{uu}(\tau, s(t))$. This means, we will need the transition probabilities for individuals with an arbitrary spell $s(t)$ of unemployment. Luckily, given the results from (19a and b), this probability is straightforwardly given by

$$p_{uu}(\tau, s(t)) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) p_{eu}(\tau - v) \, dv. \qquad (20)$$

An unemployed with spell $s(t)$ in t has different exit rates $\mu(s(y))$ which, however, are known from our analysis of optimal behaviour at the individual level. Hence, only the integrals in (20) are different, the probabilities $p_{eu}(\tau - v)$ can be taken from the solution of (19a and b).

The notation $p_{uu}(\tau, s(t))$ in (20) and $p_{eu}(\tau)$ in (19b) nicely reflects the Semi-Markov nature of this setup: When employed in t, the probability $p_{eu}(\tau)$ of being unemployed in τ is *not* a function of the past and the only argument of $p_{eu}(\tau)$ is time τ. When unemployed in t, the probability $p_{uu}(\tau, s(t))$ of being unemployed in the future τ as well *is* a function of the past and this is captured by the argument $s(t)$.

3.2 Unemployment within groups

We can now compute the expected number of unemployed for any cross-section distribution of spells $H(s(t))$,

$$E_t[N - L(\tau)] = [N - L(t)] \int_0^\infty p_{uu}(\tau, s(t)) \, dH(s(t)) + p_{eu}(\tau) L(t). \qquad (21)$$

14

We start at the end of this equation, noting that there are $L(t)$ employed workers in t. The expected number of unemployed workers at some $\tau \geq t$ coming from currently employed workers is given by $p_{eu}(\tau) L(t)$. Again, one should keep in mind that the probability $p_{eu}(\tau)$ allows for an arbitrary number of switches between employment and unemployment between t and τ, i.e. it takes permanent turnover into account.

For the unemployed, we compute the mean over all probabilities of being unemployed in the future by integrating over $p_{uu}(\tau, s(t))$ given the current distribution $H(s(t))$. Multiplying this by the number of unemployed today, $N - L(t)$, gives us the expected number of unemployed at τ out of the pool of unemployed in t. The sum of these two expected quantities gives the expected number of unemployed at some future point τ. Dividing by N gives the expected unemployment rate at τ.

When we focus on a steady state, we let τ approach infinity. In a steady state, the cross-section distribution $H(s(t))$ is identical to the distribution $F(s)$ whose density is given in (5). In order to obtain a simple expression for the aggregate unemployment rate, we exploit the pure idiosyncratic-risk structure where micro-uncertainty cancels out at the aggregate level. Hence, we assume that a law of large numbers holds and the population share of unemployed workers equals the average individual probability of being unemployed. This allows us to express (21) for a steady state as $(N - L)/N = [(N - L)/N] \int_0^\infty p_{uu}(s) \, dF(s) + p_{eu}L/N$. We have replaced $L(\tau) = L(t)$ by the steady state employment level L and the individual probabilities by the steady state expressions $p_{uu}(s)$ and p_{eu}. The probability p_{eu} is no longer a function of τ as this probability will not change in steady state, while there will always be a distribution of $p_{uu}(s)$, even in a steady state. Solving for the unemployment rates gives

$$\frac{N - L}{N} = \frac{p_{eu}}{p_{eu} + \left[1 - \int_0^\infty p_{uu}(s) \, dF(s)\right]} = \frac{p_{eu}}{p_{eu} + \int_0^\infty p_{ue}(s) \, dF(s)}, \tag{22}$$

where the second expression is more parsimonious.

If we assumed a constant job arrival rate here, we would get $p_{eu} = p_{uu} = \lambda/(\lambda + \mu)$ and $p_{ue} = \mu/(\lambda + \mu)$. Inserting this into our steady state results would yield the standard expression for the unemployment rate, $(N - L)/N = \lambda/(\lambda + \mu)$. In our generalized setup, the long-run unemployment rate is given by the ratio of individual probability p_{eu} to be unemployed when employed today divided by this same probability plus $\int_0^\infty p_{ue}(s) \, dF(s)$.

3.3   Aggregate unemployment

Let us now return to our K groups and 2 types χ. The arguments leading to (22) can be applied to each individual group. This equation therefore really gives us group-type-specific unemployment rates. In full notation, reintroducing the group and type indices k and χ, we obtain

$$u_{k,\chi} \equiv \frac{p_{eu,k,\chi}}{p_{eu,k,\chi} + \int_0^\infty p_{ue,k,\chi}(s) \, dF_k(s, \chi)}.$$

When we are then interested in group-specific unemployment rates, we simply compute $u_k = \pi^\chi u_{k,1} + (1 - \pi^\chi) u_{k,0}$. The aggregate unemployment rate u is then $u = \Sigma_{k=1}^n \frac{N_k}{N} u_k$.

3.4 Functional forms

Estimation and a numerical solution require functional forms. We assume that the instantaneous utility function of an unemployed worker used e.g. in (7) is

$$u\left(b\left(s\right),\phi\left(s\right)\right)=\frac{b\left(s\right)^{1-\sigma}-1}{1-\sigma}-\phi\left(s\right). \tag{23}$$

Effort is measured in utility terms. The utility function of an employed worker has the same structure only that consumption is given by w and there is no explicit effort. One could therefore look at ϕ as a measure of the difference between disutility from searching and disutility from work.

The *objective* arrival rate of jobs $\mu\left(\phi\left(s\right)\theta,\chi\right)$ is assumed to obey

$$\mu\left(\phi\left(s\right)\theta,\chi\right)=\left(\left(1-\chi\right)\eta_0+\chi\eta_1\right)\left[\phi\left(s\right)\theta\right]^\alpha, \tag{24}$$

where $\chi\in\{0,1\}$ determines the search productivity, unknown to the individual. If $\chi=1$, the search productivity is high, otherwise, it is low. We hence obviously assume that $\eta_1>\eta_0$. Differences in observable skills k imply different search technologies via their impact on η_i and θ. The expression for the objective arrival rate implies that the *subjective* arrival rate reads

$$\mu\left(\phi\left(s\right)\theta,p\left(s\right)\right)=\eta\left(s\right)\left[\phi\left(s\right)\theta\right]^\alpha \tag{25}$$

where

$$\eta\left(s\right)\equiv\left(1-p\left(s\right)\right)\eta_0+p\left(s\right)\eta_1. \tag{26}$$

The functional form of the specification in (24) can easily be made plausible when linking it to a matching function. The matching function represents the aggregate arrival rate and equals the sum over individual arrival rates. For our case, this reads

$$m\left(N-L,V\right)$$
$$=\left(N-L\right)\Sigma_{k=1}^K\left(p_k\int\eta_1\left[\phi\left(s\right)\theta\right]^\alpha f\left(s,1\right)ds+\left(1-p_k\right)\int\eta_0\left[\phi\left(s\right)\theta\right]^\alpha f\left(s,0\right)ds\right)$$
$$=\left(N-L\right)\theta^\alpha\Omega,$$

where $\Omega\equiv\Sigma_{k=1}^K\left(p_k\eta_1\int\phi\left(s\right)^\alpha f\left(s,1\right)ds+\left(1-p_k\right)\eta_0\int\phi\left(s\right)^\alpha f\left(s,0\right)ds\right)$. Using (4), we find $m\left(N-L,V\right)=\Omega\left[N-L\right]^{1-\alpha}V^\alpha$.[23] This shows that we succeed in identifying the elasticity α of vacancies as we assume that both effort and tightness have the same power α in (24).

Given the functional form of the arrival rate in (24), we can now compute the evolution of the subjective probability that one's productivity is high. Starting from some initial belief p_0, the belief follows (see app. A.2 for a derivation) a simple differential equation

$$dp\left(s\right)/ds=-p\left(s\right)\left(1-p\left(s\right)\right)\left(\mu\left(\phi\left(s\right)\theta,1\right)-\mu\left(\phi\left(s\right)\theta,0\right)\right). \tag{27}$$

---

[23]Note that one could argue (see e.g. Cahuc and Lehman, 2000 or Fredriksson and Holmlund, 2001) that the individual arrival rate is a function of the ratio $V/\left(\Omega U\right)$ and not of the ratio $\theta=V/U$ as used here. The former specification would assume a negative externality: If some other unemployed workers search harder, the arrival rate of an individual - ceteris paribus - decreases. Computing the aggregate matching function would then yield $m\left(U,V\right)=\left(\Omega U\right)^{1-\alpha}V^\alpha$. We do not believe that this will make a major quantitative difference and we therefore stick to our specification. For details, see app. B.1.4. We are grateful to Jean-Marc Robin for having raised this point.

As the true exit rate for high-productivity searchers is higher than for low-productivity searchers, this shows that the belief of the unemployed to be a high-productivity searcher falls over time. It falls the faster, the higher search effort. Long-term unemployed therefore believe that their search productivity is low and given by η_0. Obviously, this is just a belief, a subjective probability. It could still be that they have a high search productivity but were just unlucky.

The differential equation (27) turns out to have a convenient analytical solution

$$p(s) = \left(1 + \frac{1-p_0}{p_0} e^{\int_0^s (\eta_1 - \eta_0)[\phi(\tau)\theta]^\alpha d\tau}\right)^{-1}. \tag{28}$$

Keeping p_0 fixed we implicitly assume that an individual does not use an information from the past unemployment spells to update p_0 upon each next entry into unemployment. Otherwise, if some adjustment of the prior p_0 were suggested, the value of unemployment would become dependent on the entire history of past unemployment since the entry into the labour market. This would have led to non-Markov dynamics and rendered the model intractable.

4 Structural estimation

4.1 Econometric model

- Exit rates

Contacts with firms, and hence transitions from unemployment to job, occur with the *objective* arrival rate (24), which is a function of the optimal value of search effort $\phi(s)$ and the individual search productivity type χ. Optimal search effort $\phi(s)$ is implied by the first-order condition (8) given the particular benefit environment (UI and UA payments together with the entitlement length $\bar{s}$), the wage w and the labour market density θ. Since unemployed individuals do not know their search productivity type, this optimal value of search effort $\phi(s)$ is computed using the subjective probability of having a high search productivity, $p(s)$, i.e. using the *subjective* arrival rate. The endogenously downward-sloping profile of $p(s)$ leads to an endogenous negative duration dependence in $\phi(s)$ and hence to an endogenous negative duration dependence in the objective rate of exit out of unemployment. At the same time, explicit presence of the unobservable search productivity type χ in the objective transition rate from unemployment to job means that the resulting structural model is always a two-point mixture over the distribution of χ.

To stress the dependence of search effort on benefits, the wage and labour market tightness, we group these explanatory variables into a vector $\mathbf{z} \equiv \{b_{UI}, b_{UA}, \bar{s}, w, \theta\}$ and write $\phi = \phi(s; \mathbf{z})$. Please note that even though w and θ are endogenous in the theoretical model they are exogenous to the duration of unemployment which is our dependent variable. One can therefore either substitute them out using their theoretical solutions, which depend on the productivity A and vacancy cost γ among others, or one can use the data on w and θ directly. Using the data on w and θ directly simplifies an already complex numerical task of fitting the nonstationary model, making it faster by a substantial factor. Moreover, it lifts the necessity of having employer-side data.

Clearly, along with unobserved search productivity differences indexed by χ and observed characteristics of the market environment summarized in $\mathbf{z}$, there also exist other variables that potentially influence search effort and therefore the exit rate out of unemployment (24). Typical suspects are personal characteristics such as gender, age, education attainment and so on. We group these explanatory variables into a vector $\mathbf{x}$, including an intercept, and suggest that $\mathbf{x}$ enters the exit rate out of unemployment and the worker/firm separation rate with parameters $\boldsymbol{\zeta}_\eta$ and $\boldsymbol{\zeta}_\lambda$, respectively. Finally, we let v denote the difference between η_1 and η_0 in (24).

Given all above, the exit rate out of unemployment (24) for the structural econometric model can be written as

$$\mu_j\left(s;\mathbf{x},\mathbf{z},\nu|\chi\right) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta + \chi\nu\}\left[\phi\left(s;\mathbf{x},\mathbf{z},\nu\right)\theta\right]^\alpha, \qquad j = UI, UA, \tag{29}$$

where for ease of distinction j indicates either the UI or the UA regime. This exit rate is always conditional on the unobserved search productivity type χ. For $\chi = 0$ we obtain $\eta_0 \equiv \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta\}$; similarly $\eta_1 \equiv \exp\{\mathbf{x}'\boldsymbol{\zeta}_\eta + \nu\}$ for $\chi = 1$. As v is unobservable, it is to be estimated along with the rest of the parameters of the model.

The worker/firm separation rate in the structural model becomes simply

$$\lambda\left(\mathbf{x}\right) = \exp\{\mathbf{x}'\boldsymbol{\zeta}_\lambda\}. \tag{30}$$

- Likelihood contributions

We estimate the model using data from the German Socio-Economic Panel (SOEP). Our data are sampled as a flow of entrants into unemployment and employment (see app. A.1).

For those unemployed we have three different types of labour market histories in our data set. The first group of unemployed consists of individuals who enter unemployment with the right to claim UI benefits and exit unemployment before the expiration of the entitlement period ($s \leq \bar{s}$). The second group comprises individuals that enter unemployment with the right to claim UI benefits, but find the job only after the legal right to claim UI benefits has already expired ($s > \bar{s}$). Finally, the third group embarks all individuals that enter unemployment without any right to claim UI benefits from the very beginning ($\bar{s} = 0$). This distinction is important for the reason that German unemployment benefit system hides an additional source of unobserved individual heterogeneity with respect to eligibility to UA benefits once UI benefits expire or may otherwise not be granted. In Germany eligibility to UA benefits is means-tested. At the means test an individual has to provide lengthy information about income sources of the household, number and age of dependents, their health status etc. If means are sufficient, the person becomes ineligible to UA benefits, but might still claim social assistance, which eventually may or may not be provided. In the opposite case a (virtually unlimited) eligibility to UA is secured. Unobserved individual heterogeneity induced by this system obtains from the fact that once exit out of unemployment occurs before the expiration of entitlement an econometrician cannot know about the outcome of the means test. The individuals themselves, however, are very likely to know what the result of the test will be.

In view of this institutional feature, for the first group of individuals, who exit unemployment before the expiration of entitlement to UI benefits, we do not observe the outcome

of the means test for eligibility to b_{UA}. We suggest, though, that individuals know about this outcome even before applying for b_{UA}. Let $\phi\left(s; \mathbf{x}, \mathbf{z}, \nu | b_{UA} = 0\right)$ indicate the search effort given that no b_{UA} is granted, which corresponds to the hypothetical failure at the test. Similarly, let $\phi\left(s; \mathbf{x}, \mathbf{z}, \nu | b_{UA} > 0\right)$ stand for the hypothetical case in which the test will be passed. Finally, let π^{UA} denote the fraction of the individuals that pass the test. Then, given the observed and unobserved characteristics, the individual contribution to the likelihood in this group is

$$
\begin{aligned}
\ell\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi\right) &= \pi^{UA} \left[\mu_{UI}\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi, b_{UA} > 0\right)\right]^{d_u} e^{-\int_0^s \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu | \chi, b_{UA} > 0) du} \\
&+ \left(1 - \pi^{UA}\right) \left[\mu_{UI}\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi, b_{UA} = 0\right)\right]^{d_u} e^{-\int_0^s \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu | \chi, b_{UA} = 0) du}, \quad (31\text{a})
\end{aligned}
$$

where d_u is a dummy variable such that $d_u = 1$ if unemployment spell is uncensored and μ_j, $j = UI, UA$, is as given in (29).

The second group comprises individuals who enter unemployment with the right to claim UI benefits, fail to find a job before entitlement expires, transit to either UA or zero benefit level and thereby reveal the outcome of the means test, and exit unemployment (or not) only after the expiration of entitlement $(s > \bar{s})$. For such individuals, the contribution to the likelihood is given by

$$
\begin{aligned}
\ell\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi\right) &= e^{-\int_0^{\bar{s}} \mu_{UI}(u; \mathbf{x}, \mathbf{z}, \nu | \chi) du} \\
&\times \left(\pi^{UA}\right)^{d_t} \left(1 - \pi^{UA}\right)^{1 - d_t} \left[\mu_{UA}\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi\right)\right]^{d_u} e^{-\int_{\bar{s}}^s \mu_{UA}(u; \mathbf{x}, \mathbf{z}, \nu | \chi) du}, \quad (31\text{b})
\end{aligned}
$$

where d_t is a dummy variable such that $d_t = 1$ if we observe that an individual passes the means test.

The third group embarks all individuals who do not have the right to claim UI benefits $(\bar{s} = 0)$ and enter unemployment receiving lower UA benefits from the very beginning $(d_t = 1)$ or not receiving benefits at all $(d_t = 0)$. Their contribution to the likelihood is therefore

$$
\ell\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi\right) = \left(\pi^{UA}\right)^{d_t} \left(1 - \pi^{UA}\right)^{1 - d_t} \left[\mu_{UA}\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi\right)\right]^{d_u} e^{-\int_0^s \mu_{UA}(u; \mathbf{x}, \mathbf{z}, \nu | \chi) du}. \quad (31\text{c})
$$

As stands in equations (31a)-(31c), individual contributions of all three types of workers are conditional on the unobserved search productivity. Recalling that the share of high-productive individuals is given by π^χ (see p.6 for discussion) the marginal contribution of an unemployed person within any of the groups is finally

$$
\ell\left(s; \mathbf{x}, \mathbf{z}\right) = \pi^\chi \ell\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi = 1\right) + \left(1 - \pi^\chi\right) \ell\left(s; \mathbf{x}, \mathbf{z}, \nu | \chi = 0\right), \quad (32)
$$

which completes the construction of the likelihood contributions of entrants into unemployment.

For entrants into employment the contribution to the likelihood, conditional on the observed characteristics, is simply

$$
\ell\left(l; \mathbf{x}\right) = \left[\lambda\left(\mathbf{x}\right)\right]^{d_j} e^{-\lambda(\mathbf{x})l}, \quad (33)
$$

where d_j is a dummy variable such that $d_j = 1$ if employment spell is uncensored, l is the duration of employment, and $\lambda\left(\mathbf{x}\right)$ is as given in (30).

- Estimation procedure

Estimation of model parameters uses a part of the numerical solution method for the steady state. As described in app. A.3, for a given wage w and vacancy to unemployment ratio θ, the individual exit rate can be computed at any moment of the unemployment spell. Using the individual survey data implies that the wage w for each individual is known and the corresponding θ can be taken from administrative macro data (see app. A.1). Thus for any given parameter vector, the exit rate (29) for any individual in the sample can be computed.

Note that the model is estimated without explicitly specifying the wage setting mechanism. If we used linked employer-employee data, the model could be estimated using the observable productivity data. This would also allow us to estimate the bargaining power parameter β as well as provide more information on the discrepancy between the observed wage and an endogenous wage solution implied by the model. For the rest of the parameters unrelated to wage setting mechanism, however, both approaches must be equivalent, assuming that wage setting in the second approach is correctly specified.

- Identification

The set of parameters to estimate comprises α, σ, $\boldsymbol{\zeta}_\lambda$, $\boldsymbol{\zeta}_\eta$, ν, π^χ and π^{UA}. In addition to bargaining power β, we keep the rate of time preference ρ fixed.

Slope coefficients in $\boldsymbol{\zeta}_\lambda$, $\boldsymbol{\zeta}_\eta$, are identified by covariates, so only intercepts are of interest. With (33), the intercept in $\boldsymbol{\zeta}_\lambda$ is identified by the data on employment duration as a single parameter of the exponential distribution. The share of those passing the means test π^{UA} is identified by observable outcomes of the means test in (31b)-(31c). For identification of the remaining parameters, namely α, σ, ν, π^χ and the intercept in $\boldsymbol{\zeta}_\eta$, it is useful to follow the idea emphasized by Eckstein and van den Berg (2007) and consider identification first in the stationary setting. As nonstationarity frequently brings about additional identification restrictions, identification in the stationary model will in such cases imply identification in the nonstationary one.

Consider first the stationary model where individuals have identical search productivity, so that ν and π^χ drop from the set of the parameters of interest. In such a model, we have only three parameters to estimate, namely α, σ and the intercept in $\boldsymbol{\zeta}_\eta$. By letting $s \to \infty$, the exit rate out of unemployment in this model will be given by the constant μ which solves equation (A.13) in appendix. From this equation it is clear that σ is identified from the variation in the data on unemployment assistance benefits. Furthermore, the middle term in this equation suggests that variation in the value of employment, which itself stems from the variation in wages and benefits, will identify α and the intercept in $\boldsymbol{\zeta}_\eta$ up to scaling, even if θ is a constant. Variation of θ across heterogeneous groups is sufficient to identify α. Imagine there is no cross-sectional variation of market tightness. Even then, α and the intercept in $\boldsymbol{\zeta}_\eta$ are identified by the unemployment duration data, since these data identify μ (see A.13). In the stationary setting, μ is the single parameter of the exponential distribution of unemployment duration. Thus the data on unemployment duration and cross-sectional variation of wages, benefits and tightness are sufficient to identify all parameters of the basic

stationary model.[24]

Consider now the stationary model where individuals are heterogeneous in search productivity. In this model, ν is allowed to shift the intercept of (29), with π^χ being a fraction of high-productive workers. Learning about own productivity type is absent as the model is kept stationary. Again, letting $s \to \infty$, we obtain a mixture of two exponential distributions where ν and 0 are mass points and π^χ is a mixing proportion. Since the parameters of each component-distribution in this mixture are identified by the argument above, ν and π^χ are identified because a class of finite mixtures of exponential distributions is identifiable.

Consider now the entire nonstationary model described in (31a)-(31c). Nonstationarity comes from two sources. The first one is the drop from b_{UI} to b_{UA} at the predetermined duration $\bar{s}$. The second one is learning about own search productivity described by the subjective probability path (28). Nonstationarity induced by the change in benefit regime does not bring any additional parameter into the econometric model. Nonstationarity that stems from learning brings into the model only the initial belief about ones' own productivity p_0. Taking a conservative stance and fixing p_0 to some constant value, identification of the nonstationary model immediately follows. In our empirical application we set $p_0 = 0.5$.

4.2   Estimation results

Table 1 below reports estimation results for two specifications of the structural model. The first one does not condition on the observed individual characteristics, whereas the second one takes them explicitly into account. Numerical complexity of the model makes us restrict attention on only a selected number of key characteristics, which are *sex* (=1 if male), *region* (=1 if an individual comes from East Germany), two education dummies: *medium-skilled* (=1 if middle vocational education) and *high-skilled* (=1 if higher vocational or higher education), and two *age* dummies that define the groups of workers up to 30 years old and 31 to 45 years old.

Both specifications encompass two important dimensions of unobserved heterogeneity in the data. First is the institutional heterogeneity which amounts to impossibility of observing the potential outcome of the means test for all individuals who leave unemployment before expiration of entitlement to UI benefits. Second is the model heterogeneity that rests on the assumption of individuals being Bayesian learners about own search productivity type. As for the estimation results per se, our main finding is the significance of α. This means that changes in the optimal effort in response to any unemployment benefit reform, be it the reform of b_{UA} and $\bar{s}$ or of b_{UI}, will have a significant impact on the exit rate out of unemployment. As effort is a function of unemployment benefits via the first-order condition (8), this finding in particular contributes to the empirical reduced form literature that analyses the

---

[24]One has to admit, though, that identification of σ from cross-sectional variation of benefits implies that intertemporal variation of consumption is not utilized to infer about the degree of risk aversion. This may be viewed as restrictive, since longitudinal data on consumption contain a fair amount of information about σ. Consequently, inference could be improved by explicitly introducing savings decisions into the model (see e.g. Rendon, 2006, Bayer and Wälde, 2011, Lise, 2011, Rendahl, 2012) and using optimal paths of consumption and/or asset accumulation to fit the longitudinal variation of the corresponding data. In our model, however, simultaneous consideration of nonstationarity of search behaviour and optimal consumption decisions in an equilibrium framework quickly leads to intractability.

dependence between unemployment benefits and the probability of leaving unemployment. Evidence on this dependence are somewhat conflicting. Early work by Hujer and Schneider (1989) and Arulampalam and Stewart (1995) finds mostly no significant influence of benefits while later work by Carling et al. (2001) and Røed and Zhang (2003) state the opposite.[25] For German data, Fitzenberger and Wilke (2010) find significant effects of a reduction of both of the level of benefits and the entitlement length, the latter being visible, however, only for entitlements above 12 months. Our entirely structural perspective provides an alternative view. While we do not rule out that for certain types of heterogeneous agents changing benefits may play no role, our result on the significance of α shows that in sufficiently aggregate terms there exists a positive significant relationship between the reemployment probability and a change in the level of unemployment benefit payments. Consequently, a change in the design of the unemployment benefit mechanism will induce a significant response on the macro level.

| | | specification I | | | | specification II | | | |
		Coeff.	SE	z-Stat.	p-Value	Coeff.	SE	z-Stat.	p-Value
ζ_λ:	intercept	-4.4948	0.0566	-79.4364	0.0000	-4.4434	0.1890	-23.5038	0.0000
	sex					0.5378	0.1173	4.5840	0.0000
	region					0.8736	0.1183	7.3849	0.0000
	medium-skilled					-0.3932	0.1466	-2.6830	0.0073
	high-skilled					-0.7568	0.1881	-4.0242	0.0001
	age (up to 30)					-0.3220	0.1598	-2.0148	0.0439
	age (31 to 45)					-0.4163	0.1573	-2.6459	0.0081
ζ_η:	intercept	-4.0928	0.5368	-7.6242	0.0000	-4.4961	0.4577	-9.8237	0.0000
	sex					0.0784	0.1202	0.6524	0.5142
	region					0.4790	0.1824	2.6263	0.0086
	medium-skilled					0.1902	0.1671	1.1380	0.2551
	high-skilled					0.0556	0.2413	0.2304	0.8178
	age (up to 30)					0.5943	0.1793	3.3138	0.0009
	age (31 to 45)					0.4887	0.1620	3.0160	0.0026
α		0.4059	0.1306	3.1085	0.0019	0.4203	0.0954	4.4060	0.0000
σ		0.7639	0.2013	3.7954	0.0001	0.7808	0.1411	5.5329	0.0000
π^{UA}		0.2447	0.0311	7.8666	0.0000	0.2398	0.0300	7.9870	0.0000
ν		1.6974	0.4216	4.0259	0.0001	1.4438	0.3392	4.2569	0.0000
π^χ		0.9246	0.0402	22.9807	0.0000	0.9228	0.0468	19.7155	0.0000
$\log \mathcal{L}$		-2915.05				-2851.67			

Table 1 *Estimation results*

As argued in Section 3.4, parameter α can also be interpreted as the elasticity of the matching function with respect to vacancies. With the estimated value of 0.4 our result

[25]To be fair, the analyses are undertaken for different countries. Moreover, later studies, notably Røed and Zhang (2003), provide more sophisticated treatment of unobserved heterogeneity in comparison to earlier ones.

is perfectly in line with the estimates of the same parameter reported in Petrongolo and Pissarides (2001). Petrongolo and Pissarides (2001) suggest that the aggregate matching function is broadly consistent with constant returns to scale assumption and coefficients in the range of 0.5-0.7 for the unemployment, implying a range of 0.3-0.5 for vacancies.

Our next important finding is on the role of unobserved heterogeneity. As for heterogeneity with respect to the outcome of the means test, tab. 1 shows that π^{UA} is always different from zero and unity at 5% level. Together with the significance of α, this implies that the prospect of not passing the means test significantly increases search effort and exit probability. Similarly, significance of ν provides the evidence of existence of a two-point heterogeneity with respect to individual search productivity. The latter result is remarkable. It is known that importance of unmeasured heterogeneity has been frequently highlighted in the literature on the structural estimation of search models (see e.g. van den Berg and Ridder, 1998 or Frijters and van der Klaauw, 2006). Much fewer attempts, however, have been made to also explicitly introduce the spell dependence. Our structural model addresses these two points in a unified framework, as significance of ν simultaneously implies a significant endogenous downward time dependence of the exit rate out of unemployment. While we see this as an important first step in estimating endogenous time dependence in a structural matching model, our model still does not allow separating time dependence from unobserved heterogeneity, as much of the reduced-form empirical literature does (see e.g. van den Berg and van Ours, 1996, 1999). We would thus call for further research aimed at singling out the influences of these two components in a fully structural setting.

Finally, our estimate of the utility parameter σ is also very comforting. First, the degree of risk aversion is high enough to reject the hypothesis of risk neutrality. Second, significance of σ provides the empirical evidence on the existence of a significant insurance effect of unemployment benefits.

Regarding the fit of the model, for each of six different samples: entire sample, West Germany, East Germany, low-skilled, medium-skilled and high-skilled, we predict the survivor function from the conditional specification and plot it against the Kaplan-Meier estimate of the survival probability in unemployment. These plots can be seen in app. A.4, fig. 5. We find that the model replicates the empirical distribution of the data very well. Except of the third and the fourth months of unemployment duration, where we only slightly overestimate the probability of remaining unemployed, predictions of the model lie within the bands of the 95% confidence interval around the corresponding nonparametric estimate.

We also test the out-of-sample performance of our model and thus draw an external sample.[26] For this external sample, we predict survivor probabilities in unemployment using the structural parameters estimated from the original data. Afterwards we plot these predictions against Kaplan-Meier estimates of the survival probability in unemployment, the latter being computed on the external sample. The plots can be found in app. A.4, fig. 6. Our model performs remarkably well outside the estimation sample. We see that all the predictions, except of the rightmost one at the 30th month of unemployment duration, lie within the

---

[26]The external sample is a flow sample constructed identically to the original sample, except that entries into (un)employment take place between 01.1999 and 12.2001, rather that between 01.1997 and 12.1998 as in the original sample. This means that none of duration observations in the external sample has been used to estimate the parameters. At the same time, both samples relate to identical institutional environment, as no changes to institutions have been made between 01.1997 and 12.2004.

95% confidence bounds for the nonparametric estimates of survivor probability.

The goodness of fit analysis just described implies that our theoretical model accurately replicates the structure of the data generation process and assures that all our simulations based on the estimated parameters are well-grounded.

4.3   Prediction of demand side parameters

One can partition the complete list of all model parameters into parameters that are estimated, α, σ, ζ_λ, ζ_η, ν, π^χ and π^{UA}, parameters that are exogenously fixed, β and ρ (for which we undertake robustness checks below), and parameters that are predicted. This section now turns to the latter.

After having estimated all the parameters, we are left with determining labour productivity A_k, vacancy cost γ_k and the equilibrium tax rate κ. Wages w_k and tightness θ_k were taken as exogenous in this first part of the estimation which was built on the household side of the model. As wage and tightness are endogenous in equilibrium, we now take the estimated parameters and compute A_k, γ_k and the corresponding tax rate κ using the full equilibrium structure of our economy in the steady state.[27] We compute A_k, γ_k and κ such that the government budget is balanced and equilibrium endogenous variables w_k and θ_k equalize with the average wage and labour market tightness from our data. See tab. 2 for results.

5   Evaluating the labour market reform

We now use the structurally estimated parameters in order to describe the steady state equilibrium of 2004 and to evaluate the reform effective as of January 2005.

5.1   The pre-reform steady state

Data is heterogeneous in many respects. The above estimation allows us to construct up to 36 groups. Given our priors on which characteristics appear the most important, we chose six groups k around which we organize our discussion of the results. Individuals are distinguished by where they live (East and West) and their skill level (High, Medium and Low), i.e. $k \in \{EH, EM, EL, WH, WM, WL\}$.

All parameters used in this paper, apart from the estimated ones in specification II of tab. 1, plus selected endogenous variables are provided in tab. 2. The first bit of information in this table comprises the shares of each group in the entire population, N_k/N, and the share of individuals eligible to UI benefits within each group, π_k^{UI}. These are directly observed in the data.

The next set of parameters are policy parameters: Entitlement length $\bar{s}_k$ to UI payments and UA payments b_{UA}. SOEP does not contain information on the length of entitlement to UI benefits. Using statutory rules, however, allows computing the length of entitlement once we know the length of previous job durations and the age of an individual.[28] For this

---

[27]See app. A.3 for a description of the numerical solution procedure.

[28]See e.g. www.oecd.org/dataoecd/9/54/29730499.pdf

reason, for every person that enters unemployment we also retrieve her job history. This job history also provides us with the record of the latest wage earned. Mean entitlement to UI payments for groups k ranges from 11 to 15 months (conditional on being entitled to UI payments). Turning to UA payments, as only an estimated share of $\pi^{UA} = 24.0\%$ passes the means test (see tab. 1), mean UA payments for each cell k in tab. 2 are computed as the product of the statutory replacement rate ρ_{UA}, the previous wage and the shares π_k^{UI} and π^{UA}, $b_{UA,k} = \pi_k^{UI}\pi^{UA}\rho_{UA}w_k$.[29]

In the block on predicted parameters, we use results of specification II of tab. 1 to calculate the average separation rate λ for each group. The average of η_0 for each group is computed by using the individual η_0 as presented below eq. (29). Building on the mean wage and tightness per group, we predict A_k and γ_k as described in sect. 4.3.

		WH[a)]	WM	WL	EH	EM	EL
observed ratios	N_k/N	17.9%	44.0%	19.2%	6.0%	11.2%	1.7%
	π_k^{UI}	47.2%	54.1%	41.1%	73.5%	62.5%	66.7%
policy parameters	$\bar{s}_k$	13	12	11	15	12	12
	$b_{UA,k}$ [b)]	95.23	93.16	53.58	125.19	84.12	78.46
predicted parameters	λ_k	0.0054	0.0081	0.0120	0.0150	0.0200	0.0290
	$\eta_{0,k}$	0.0194	0.0227	0.0193	0.0272	0.0352	0.0314
	A_k	1885	1686	1465	1748	1439	1253
	γ_k	11612	14126	13052	16529	17635	5686
equilibrium values	w_k	1528.9	1304.7	988.9	1290.7	1020.4	892.1
	θ_k	0.54	0.31	0.19	0.15	0.10	0.24
	$\bar{\mu}_k$	0.15	0.13	0.08	0.10	0.11	0.13
	$\frac{N_k-L_k}{N_k}$	4.2%	6.8%	15.1%	14.5%	17.2%	20.0%
aggregate equilibrium values	κ	2.7%	exogenous		ρ	2.4%	
	$\frac{N-L}{N}$	9.8%	parameters		β	0.5	

[a)] WH ... EL: skill groups from West and East with High, Medium and Low skills

[b)] Unconditional values. Values conditional on entitlement are $\left(\pi^{UI}\pi^{UA}\right)^{-1}$ times higher (see text)

Table 2 *Parameters and selected equilibrium values of the pre-reform steady state*

Selected equilibrium values appear next in tab. 2. Our equilibrium values fit perfectly by construction for average wages and average labour market tightness within each of six cells. The time-averages of matching rates within groups, $\bar{\mu}$, also take reasonable values. A value of 0.08 for low-skilled individuals in West Germany (WL) means that the average probability to find a job per month (as we use monthly data) is given by 8%. It increases to

---

[29] UI payments in group k are determined by $\xi_{UI,k}$ as in (3). Quantitatively, the replacement rate is given by $\xi_{UI,k} = \pi_k^{UI}\rho_{UI}$ where $\rho_{UI} = 0.65$ is the statutory replacement rate and π_k^{UI} is the share of individuals who are entitled to UI payments. We do not report b_{UI} here as it is not a policy parameter for our analysis further below.

15% for high-skilled individuals in West Germany. The group-specific unemployment rates match aggregate statistics also very well.

Concerning aggregate equilibrium values, our tax rate κ is sufficiently close to the actual social security contribution rate (this is the only purpose of taxes in our model). The aggregate unemployment rate in our model is 9.8%, whereas the official aggregate number is 10.5% (Bundesagentur für Arbeit, 2009). This is remarkably close, as our estimation is based on the flow sample and we do not target the unemployment rates in the econometric model explicitly. Finally, as in the estimation part, the time preference rate is chosen to fit an annual interest rate of 2.4%. The bargaining power β is set equal to 0.5.

For comparative statics below, we will take the exogenous parameters, the estimated and the predicted parameters and the replacement rate for short-term unemployed ρ_{UI} as given. We will then change long-term benefits b_{UA} and the entitlement period $\bar{s}$ to understand the effects on equilibrium values.

Although the economy is in the steady state, there are still dynamics on the micro level as illustrated in fig. 1. At any point in time, individuals of all six groups we consider lose jobs and search for jobs.

Figure 1 *The subjective belief $p(s)$, the value of being unemployed $V(b_k(s), s)$, effort $\phi(s)$ and true exit rates as a function of the spell s (in months)*

The upper left panel shows the estimated subjective belief $p(s)$ as specified in (27). We use $p(0) = 0.5$ throughout the paper as initial condition (see p. 21 for discussion). The value of being unemployed, normalized by $V(b_{UI}, 0)$, unambiguously falls over time as shown by the upper middle panel. The intuition is simple: If there was a constant belief $p(s)$, a

long-term unemployed would live in a stationary world and the value of being a long-term unemployed worker would be stationary as well. With an ever decreasing belief, the job finding rate - taking optimally chosen effort into account - goes down and the value of being unemployed approaches a lower limit determined by the lower limit of $p(s)$. Both with a constant and falling belief, the value of a *short*-term unemployed worker falls as the point in time where lower UA benefits are paid comes closer over time.

The optimal search effort of the unemployed worker is shown by the upper right panel. Effort paths depend crucially on which group one looks at. For some it decreases monotonically (e.g. high-skilled individuals in the west), for others effort increases during the first 12 months and then starts falling (e.g. high-skilled workers in the east). The maximum of optimal effort does not necessarily need to lie at $\bar{s}$ as it is the outcome of the interaction of the negative duration dependence (lower $p(s)$ reduces optimal effort) and the incentive-effect, i.e. the potential gain from finding a job. As gains increase due to a falling value of being unemployed, this second effect tends to increase effort. This can be seen from the first-order condition in (8) or, more directly, from (A.4) in the appendix. The initial increase of effort clearly reflects the rising incentive to search harder the closer $\bar{s}$ approaches. The fact that unemployed workers finally "give up" is ultimately the effect of the belief that falls further and further. Irrespective of whether individuals are good or bad searchers, once an individual is unemployed long enough, she is "convinced" (i.e. the subjective belief is close to zero) that she actually *is* a bad searcher.

The lower left and middle panels display true exit rates for high- and low-productive searchers, respectively. As true exit rates are power functions of the optimal effort scaled by search productivity constant, their dynamics repeats the one of effort. We see that true exit rates can be monotone decreasing as well as non-monotone. As for the difference between two types, high-productive searchers exit on average four times faster than low-productive ones.

5.2 The effects of the reform

The reform was characterized by adjustments to UA benefits b_{UA} (which are given levels after the reform and no longer proportional to the previous wage) and entitlement length $\bar{s}$. The effect of the reform is visible in fig. 2. The horizontal axis of fig. 2 plots "Hartz steps". The "1" represents the situation before the reform. The "0" represents the situation after the reform and "-1" to "-4" shows the effects of a stronger reform. By stronger we mean that in each additional step $\bar{s}$ is reduced by another month and b_{UA} is reduced by further 10%. The replacement rate ξ_{UI} remains unchanged and UI payments are paid according to (3).

We see that the Hartz reform reduced entitlement length $\bar{s}$ for all groups but actually led to improvements in the level of UA payments for two groups. These are the low-skilled in the East and the low-skilled workers in the West. This feature has been one of the main criticisms against the reform. The groups which have the highest region-specific unemployment rates (20.0% and 15.1%, respectively, according to tab. 2) are the groups where incentives to search more intensively are actually reduced.
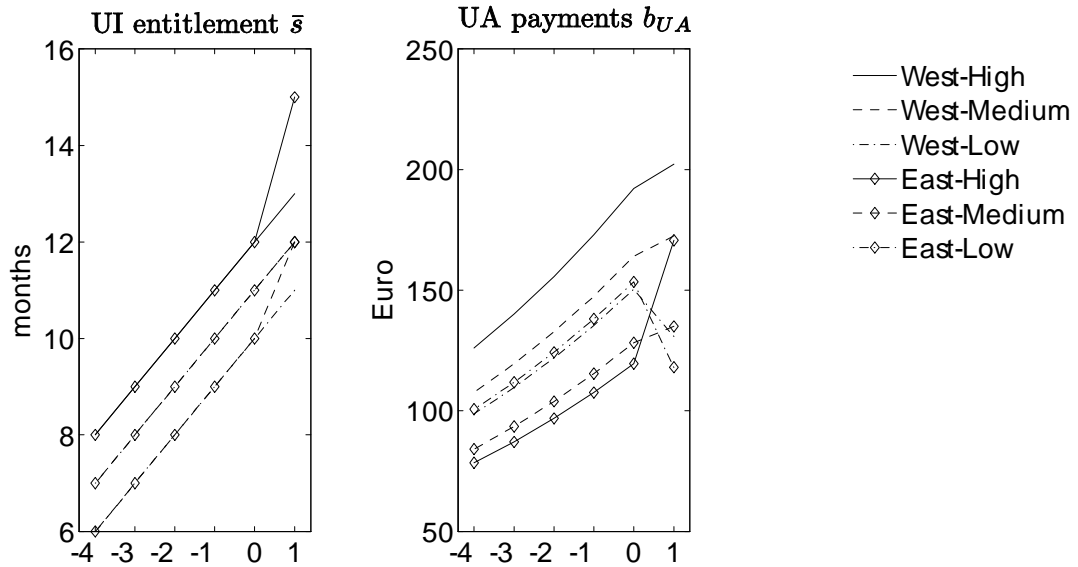
Figure 2 *Hartz steps: Adjustments of b_{UA} and $\bar{s}$ due to the reform (from 1 to 0) and in case of stronger reforms (steps -1 to -4)*

The effects of these changes can be seen in fig. 3. The horizontal axis of fig. 3 plots "Hartz steps" as in fig. 2. The vertical axes plot changes relative to the pre-reform steady state which is normalized (for levels, see tab. 2) to 1. The pre-reform steady state is therefore always given by the point $(1, 1)$.

As a general pattern, we see a lot of heterogeneity due to the reform, i.e. the step from 1 to 0, and more homogenous predictions for the subsequent, hypothetical Hartz steps. The heterogeneity due to the reform is visible in the entire first row of fig. 3. Effort, tightness, the average exit rate and the unemployment rate of low-skilled workers in the East and West move in opposite directions to the medium- and high-skilled group. This can easily be understood, however, from the opposite movements of UA payments to these groups visible in the right panel of fig. 2.

The effects of more (or less) generous UA payments go in the expected direction. Groups that benefit from the reform reduce their effort, fewer vacancies per unemployed worker are created, the exit rate goes down and the unemployment rate increases. The opposite is true for the medium- and high-skilled groups where the unemployment benefit system became less generous. For these groups, the reform had the desired effect in that it reduced their unemployment rates.

When we consider the subsequent, hypothetical Hartz steps from 0 to -1 and further to -4, we find that all groups now increase their search effort and there are more vacancies for each unemployed worker. As a consequence, the exit rate goes up and the unemployment rate falls for all groups.
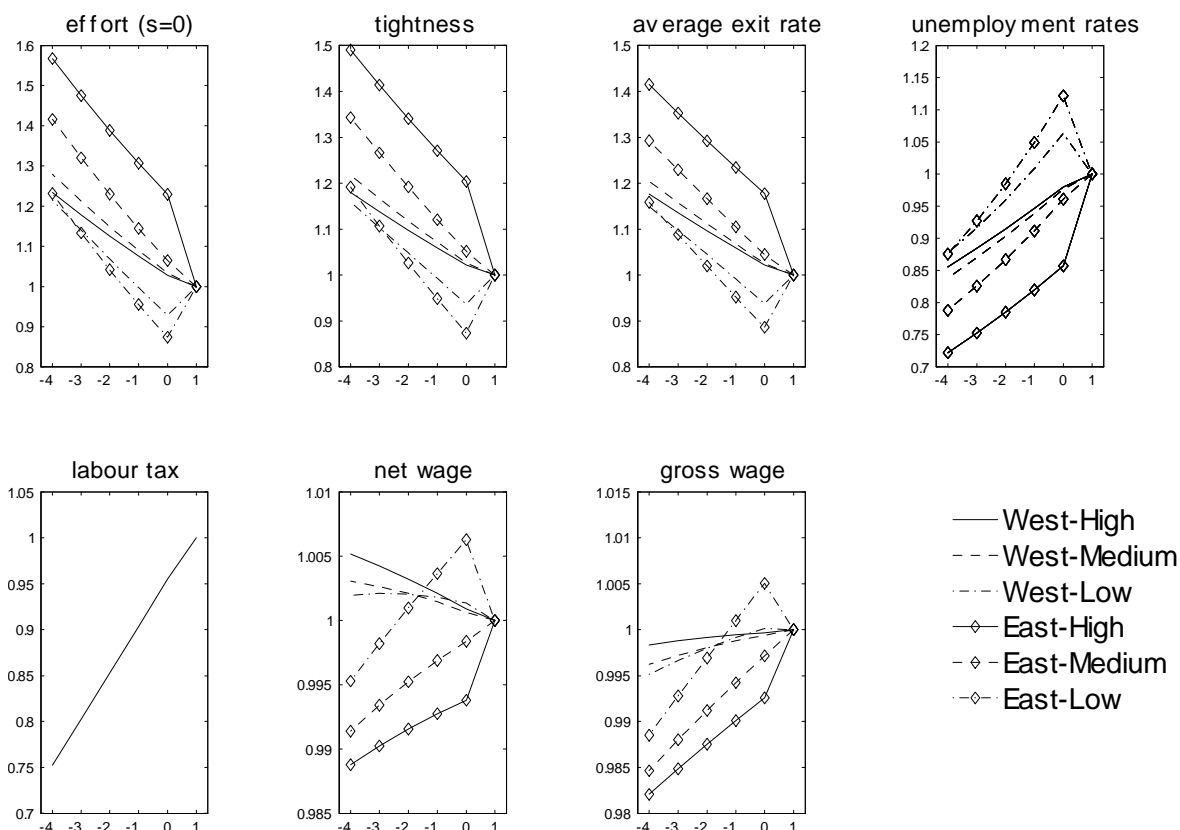
Figure 3 *The effects of changes in UA payments b_{UA} and entitlement length $\bar{s}$ (Hartz steps)*

Generally speaking, we find very weak effects of the reform. The qualitative effects of the reform are as intended by policy makers. Effort $\phi(0)$ when becoming unemployed rises the stronger the reform is, i.e. the more benefits and entitlement length are reduced. Labour market tightness θ rises too, as long as the benefit system becomes less generous. The increase in θ, i.e. the number of vacancies per unemployed worker, is crucial for our discussion further below. Understanding this effect is simple, however: More effort by unemployed workers makes opening a vacancy more attractive. Hence, lower benefits induce a higher number of vacancies per unemployed worker. The quantitatively weak effect of the reform becomes visible when looking at the reduction of the unemployment rates in tab. 3.

West-High	W-Medium	W-Low	East-High	E-Medium	E-Low
0.08	0.15	-0.80	1.81	0.56	-1.90

Table 3 *Reduction in unemployment due to the Hartz reform (percentage points; minus sign means 'increase')*

Unemployment falls for medium- and high-skill groups from the pre-reform steady state "1" to the reform level at "0" by levels between 0.08 and 1.81 percentage points. For the low-skilled groups, unemployment *rises*. Starting at an aggregate unemployment rate of 9.8%, the reform decreases the unemployment rate to 9.7%, i.e. by (less then) 0.1 percentage point.

The lower left panel of fig. 3 shows that the tax rate falls. This has the following reasons: Lower benefit payments and a lower number of recipients reduce overall expenditure. This reduced expenditure is paid by more workers who earn higher gross wages.

One of the most surprising results is the increase in the net wage in West Germany. The increase in the net wages becomes clear from the wage equation (13) when taking into account that the positive tightness effect dominates the negative effort effect. This is an interesting feature of this wage bargaining setup with endogenous effort which stands in strong contrast to perfectly competitive setups, to bargaining setups with exogenous effort and to search setups where the reservation wage is a simple decreasing function of the outside option. Here, the outside option (utility from being an unemployed worker) decreases as well but this is overcompensated, given our estimates, by the positive effect of more vacancies per unemployed worker.
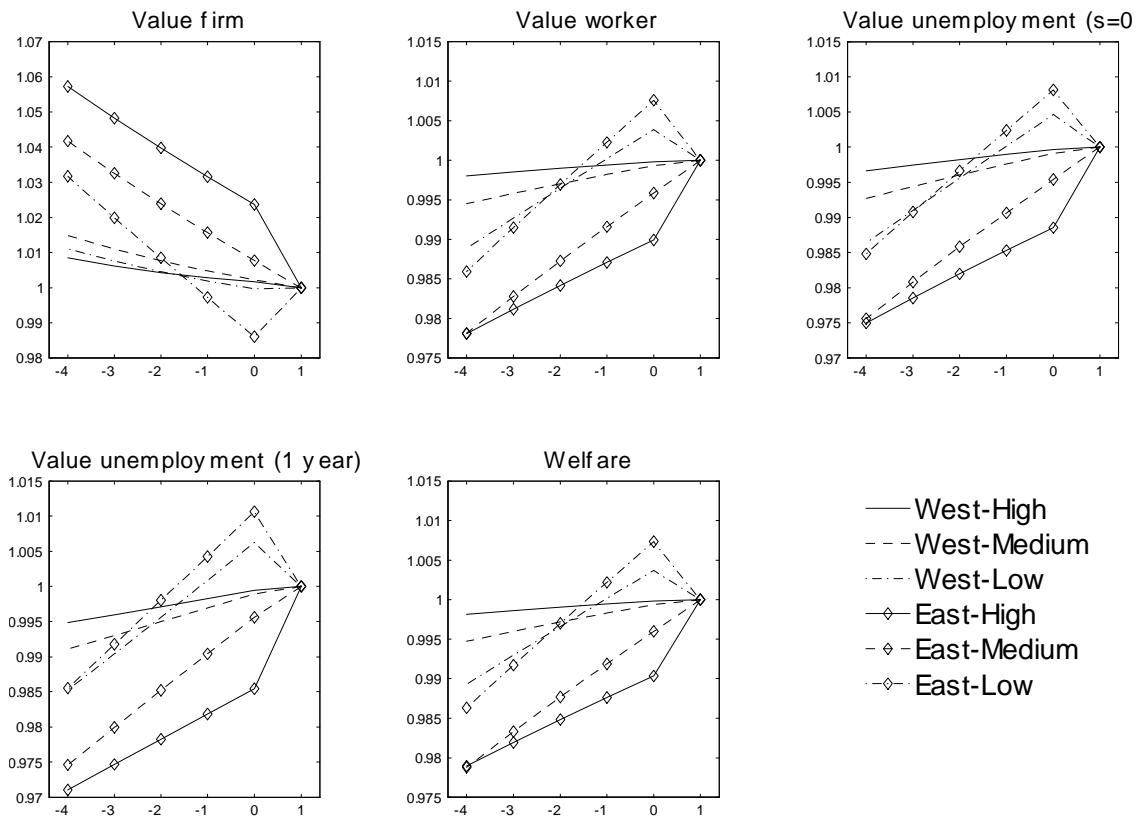

Figure 4 *Intertemporal effects of Hartz steps*

When we look at the intertemporal effects of the reform in fig. 4, we see that the direct effect of the reform from fig. 2 shapes the intertemporal effect. Low-skilled workers in the

West and in the East that gain in terms of UA payments, as visible in fig. 2, also gain in an intertemporal sense. Since their net wage has experienced a sufficiently high increase, their value of working goes up. The same happens to the value of unemployment when just entering the state of unemployment, i.e. at $s = 0$. The value of short-term unemployment rises as UI payments did not change due to the reform and the future always looks good: When finding a job the workers enjoy a higher wage and when staying unemployed they face a prospect of an increased b_{UA}. Apparently, the drop in the entitlement period $\bar{s}$ was not strong enough to overturn these positive effects. Finally the value of unemployment at $s = 12$ months has also gone up.

The same dominant effect of the design of the reform from fig. 2 is visible for the medium- and high-skilled workers in East Germany. They were hit the most by the reduction, both in terms of UA payments and in terms of the reduction of entitlement. As their net wage decreases, it is not surprising that the value of having a job decreases as well. Furthermore, given that long-term unemployment becomes much less attractive now, the value of being a short-term unemployed worker is also reduced.

Considering the intertemporal effects of the reform for medium- and high-skilled workers in the West, an interesting parallel to the public opinion about this reform in Germany can be made. While the increase in the net wage is intuitive from an economic perspective, as discussed above, it completely contradicts public perception at the time Hartz IV was phased in. There were huge demonstrations in Germany as a whole against the reform. It was widely felt that the Hartz reforms are really "the end of the welfare state" and entirely in favour of firms. Representatives of the middle-class expressed their fear that unemployment is now "the direct road to poverty".

If the net-wage goes up, most individuals (in the West) should have been actually happy about the reform. However, when we look at the intertemporal effects of the reform, i.e. when we look at the value functions in fig. 4, we see that public perception was right after all. The value of a firm goes up, at least for those 4 out of 6 groups where the gross-wage decreases. Even more importantly, the values of employment, short- and long-term unemployment fall, despite the rise in the net wage. Apparently the rise in the net wage was not strong enough to compensate for anticipated losses in case of long-term unemployment. With this intertemporal view, the fears of the public, especially of the middle-class, can be well understood.

The values of being employed and unemployed can all be aggregated to expected utility expressions from (17). This shows that, in the end, the design of the reform seems to dominate. The low-skilled workers in the East and West gain, the rest loses. In this intertemporal expected utility sense, four out of six groups, amounting to 76% of all workers (see tab. 2), would reject the reform. The insurance mechanism was not sufficiently taken into account.

5.3   Going further and robustness

- Going further

Let us now ask what the results would have been if the reform had been "tougher". More precisely speaking, we reduce UI entitlement and UA payments further as shown by steps -1

to -4 in fig. 2. Here as well, we find only small effects. If we reduce entitlements and benefits by 1/3, the unemployment rate reduces by 1/5 only.

These weak effects can be made plausible with a back-of-the-envelope calculation. As the unemployment rate is approx. 10% and only 1/3 becomes long-term unemployed, only 3.3% of the entire labour force are affected. Of these 3.3% only $\pi^{UA} = 24.0\%$ pass the means test. In an intertemporal sense, income of the representative household is reduced only during $24.0\% * 3.3\% \approx 1\%$ of ones' lifetime. The duration of unemployment insurance payments is reduced by 10.7%, the level of the payments by 7%. Let this add up - to make this simple and high - to 18%. If 1% of lifetime income is reduced by 18%, overall lifetime income reduces by $1\% * 18\% \approx 0.2\%$. No surprise that quantitative effects are weak.

These weak effects do not mean, however, that a tougher reform is desirable. As the welfare figure in fig. 4 clearly shows, expected utility of all groups would decrease. We therefore conclude that alternative policy instruments are required to reduce unemployment than a simple cut in generosity of entitlements.

- Economic growth

It is often argued that unemployment falls in times the economy experiences a positive productivity shock. In contrast, all the results above were obtained by keeping productivities A_k unchanged. Our structural model allows considering both the pure effect of the reform and the combined effect of the reform and productivity growth. Having found fairly small effects of the reform alone, it would be interesting to look into such a combined effect.

TFP in Germany grew by 4.0% from 2005 to 2007 (Ameco, 2010). Over the same period (annual averages of) unemployment in Germany fell from 10.5% to 9.0%. We take productivity growth into account by letting output of a worker-firm match increase by 4.0%. Once growth is accounted for, we find that the aggregate unemployment rate decreases from 9.8% prior the reform to 9.3%, i.e. by 0.5 percentage points. Compared to the effect of the reform alone, the influence of productivity growth is much stronger, explaining about 80% of the entire reduction in the unemployment rate.

In tab. 4 we show the combined effect on heterogeneous groups. We see that negative effect on the low-skilled in the West now turns positive. Negative effect on low-skilled in the East becomes weaker and positive effect on the rest of unemployed stronger.

West-High	W-Medium	W-Low	East-High	E-Medium	E-Low
0.22	0.40	0.03	2.31	1.34	-0.78

Table 4 *Reduction in unemployment due to the Hartz reform combined with growth (percentage points; minus sign means 'increase')*

Studying the effects of the reform joint with economic growth, effort at $s = 0$, tightness and average exit rates go up for all groups, except of low-skilled in East Germany where the

reduction of b_{UA} was too sharp.[30] Furthermore, unlike in fig. 3, net and gross wages now uniformly increase for all groups due to increase in output. Consequently, the value of the firm decreases uniformly for all groups, which is also in contrast to the dynamics shown in fig. 4. Finally, welfare of all workers improves and the aggregate welfare goes up as a result. This welfare result suggests that productivity growth was very likely more important than the reform itself.

- The effects of bargaining power

Before we conclude, we undertake several robustness checks with respect to changes in the bargaining parameter of unions, β. We first ask how our predictions would have looked like if we had set β equal to 0.3 or 0.7 instead of the standard symmetric value of 0.5. The reduction of the unemployment rate turned out to be very robust to these variations. Instead of a reduction of 0.065 percentage points for $\beta = 0.5$ (we have reported this result as "(less then) 0.1 percentage point"), we obtain a reduction of 0.059 for $\beta = 0.3$ and 0.069 for $\beta = 0.7$. We conclude from this that our basic insight – the effect of the reform is smaller than 0.1 percentage points – is not driven by our exogenous choice of β.[31]

We can ask another interesting question, however, concerning changes of β over time. One can wonder what the effects of the reform would have been if, in addition to reductions of entitlement and UA payments, bargaining power of unions had dropped. This question makes sense if one takes into account that the Hartz IV reforms were preceded by Hartz I - III reforms. One can easily argue that these preceding reforms affected bargaining power of unions.

We start from the same pre-reform steady state as described above in tab. 2 and then reduce β to the values shown in tab. 5.

β	West-High	W-Medium	W-Low	East-High	E-Medium	E-Low	Germany (aggregate)
0.50	0.08	0.15	-0.80	1.81	0.56	-1.90	*0.1*
0.45	0.38	0.62	0.27	2.69	1.74	-0.31	*0.7*
0.40	0.64	1.03	1.15	3.44	2.7	0.98	*1.3*

Table 5 *Hartz reform and bargaining power (percentage points; minus sign means 'increase')*

We see that by assuming that bargaining power of unions dropped sufficiently due to the reform, one can generate the effect of the reform that matches the difference between pre- and post-reform unemployment rate observed in the official statistics.

---

[30]Because of space constraints, all corresponding graphs are relegated to app. B.3.

[31]Our results are similarly robust for variations in the time-preference rate ρ.

This prediction of the model points to interesting avenues for research. On the one hand, one would explore in more detail what the determinants of bargaining power are. This would have to be included into the theoretical setup. One the other hand, one would then definitely also need firm data in order to be able to estimate this parameter and potentially even its change over time.

6   Conclusion

Our project started by inquiring into the effects of the Hartz IV labour market reform on incentives and insurance mechanisms for the workforce. At the macro level, we investigate the effects on the unemployment rate and social welfare. We have developed an estimable search and matching model with endogenous effort under time-dependent unemployment benefits. The main extension compared to the existing search and matching literature is the endogenous distribution of unemployment duration that arises due to individual choice of search intensity in a nonstationary environment. A link between these micro-dynamics and macro quantities like the unemployment rate was developed using tools from the literature on Semi-Markov processes. The theoretical model describes the density of unemployment duration of an individual as a function of various model parameters. This density provides the basis for structural estimation via maximum likelihood. Equilibrium policy analyses were performed using the parameter estimates of the best fitting specification.

We find that the aggregate unemployment rate did decrease due to the reform. Unemployed workers on average have stronger incentives to search hard. This induces firms to open more vacancies per unemployed worker and the exit rate out of unemployment increases. The reduction of the unemployment rate was quantitatively very small, however, amounting to less than 0.1 percentage points. Looking at the average effect on welfare, we find that social welfare falls. The efficiency effect of more output is therefore overcompensated by the reduced insurance effect. The reform therefore can be given some credit due to these (small) employment effects. It does not take insurance effects of unemployment benefit systems sufficiently into account, however.

When we look at various regional and skill groups, we discover considerable heterogeneities. First of all, the reform by design actually increased unemployment assistance payments to low-skilled groups, both in the East and the West. As a consequence, the unemployment rate of these groups increased. Looking at the effects of the reform on the net wage, it is actually beneficial to four out of six groups. This is (mostly) due to the fact that the tax rate needed to finance unemployment assistance payments fell.

Despite this increase in the net wage, however, the value of being employed actually *falls* for four out of six groups. This is due to the anticipation effect. Workers could become unemployed at some point which could turn into long-term unemployment. In a present-value sense, the increase in the net wage therefore is not high enough to compensate for the possibility of losses as a long-term unemployed worker. Looking at the standard expected utility welfare measure, this negative evaluation of the reform is confirmed. Welfare falls for four out of six groups. The two groups which gain are those whose unemployment assistance payments rise.

When we go beyond Hartz IV and reduce the generosity of the system further, we find that

the losses from the reduction of the insurance effect become stronger and stronger. Expected utility decreases for all six groups. This is strong evidence for the fact that the Hartz reforms are not desirable from the perspective of each individual worker. This explains the strong public opposition to the reform when it was worked out and when it was implemented.

A   Appendix

A.1   Data

The data comes from the German Socio-Economic Panel (SOEP). The SOEP is a panel surveying households on an annual basis (see www.gsoep.de for details). We draw a flow sample of entrants to employment and unemployment at each month of years 1997-98. The choice of the year of sampling is determined by the fact that no changes to either benefit levels or the entitlement length were made between the 1 January 1997 and 1 January 2005, when the Hartz IV reform was implemented. With December 2005 being the latest month of our observation period, we end up with a sample that describes a stationary entitlement-benefit environment and provides a fairly reliable information on long-term unemployment (only 9.12% of unemployment durations in our sample are right-censored). For each entrant we retrieve the duration of stay in the current state since the moment of entry. Following van den Berg and Ridder (1998, p.1194), we exclude individuals with transitions to states other than full-time employment and unemployment.

Units of measurement are months for the duration data and real EUR (in prices of 2004) for the wage and benefits data. Wage is the average monthly wage for the months of employment within a year prior to job loss, as these are the wage bases that conform with the observed benefit levels. Descriptive statistics for the entire sample can be found in tab. 6. Descriptive statistics for selected cell-specific variables are in tab. 2.

| | **Unemployment** | | | **Employment** | | | **Sample characteristics** | | |
	Mean	StD		Mean	StD		Mean	StD
Duration (s)	8.81	13.16	Duration (l),			Males	0.5380	0.4988
UI benefits (b_{UI})	727.46	294.94	censored	57.55	25.73	East Germans	0.4227	0.4942
Entitlement ($\bar{s}$)	12.18	5.48	Duration (l),			Medium-skill	0.5961	0.4909
Wage (w)	1166.26	538.07	all sample	40.68	30.13	High-skill	0.2090	0.4068
Share of entitled						Age up to 30	0.3843	0.4866
to UI (π^{UI})	0.5657	0.4963	obs: total/cens.		694 / 392	Age 31 to 45	0.4264	0.4948
Share of $\bar{s} = 12$								
among entitled	0.4882	0.5010				obs: total		1067
Observed share of								
passing the test	0.2850	0.4525						
obs: total/cens.		373 / 34						

Table 6 *Descriptive statistics (months and EUR)*

A.2 Evolution of the belief

The derivation is similar but not identical to Keller, Rady and Cripps (2005). The contents is similar but in its modelling not identical to Keller and Rady (2010).

In the Bayesian tradition, we let the individual hold a belief about χ being equal to 1. We denote this subjective probability by $p(s)$. The probability that the individual does not find a job over a small time interval dt assuming that $\chi = 1$ and with (24) is given by $1 - \eta_1 [\phi(s)\theta]^\alpha ds$. For notational simplicity, we denote this by $1 - \mu_1 ds$ in this appendix. The probability over ds of no jump for $\chi = 0$ is with (24) given by $1 - \eta_0 [\phi(s)\theta]^\alpha ds$. We denote this by $1 - \mu_0 ds$ in this appendix, i.e.

$$\mu_i \equiv \eta_i [\phi(s)\theta]^\alpha. \tag{A.1}$$

We will also use, only in this appendix,

$$\Delta(s) \equiv \mu_1(s) - \mu_0(s). \tag{A.2}$$

Now imagine the individual does indeed not find a job during ds. The update of the belief when not finding a job during ds is given by

$$p(s) + dp(s) = \frac{p(s)[1 - \mu_1 ds]}{p(s)[1 - \mu_1 ds] + (1 - p(s))[1 - \mu_0 ds]}. \tag{A.3}$$

This expression directly follows from Bayes rule: the posterior probability for having a high search productivity on the left-hand side, given the evidence of no jump, is given by the prior $p(s)$ times the likelihood of the event 'no jump' conditional on having high search productivity, $1 - \mu_1 ds$, divided by the unconditional probability that there is no jump.

Now rewrite the denominator as

$$
\begin{aligned}
& p(s)[1 - \mu_1 ds] + (1 - p(s))[1 - \mu_0 ds] \\
=\; & 1 - p(s)\mu_1 ds - (1 - p(s))\mu_0 ds \\
=\; & 1 - ((1 - p(s))\mu_0 + p(s)\mu_1) ds \\
=\; & 1 - (\mu_0 + p(s)(\mu_1 - \mu_0)) ds
\end{aligned}
$$

Using this, we can write the posterior (A.3) as

$$
\begin{aligned}
p(s) + dp(s) \; & = \; \frac{p(s)[1 - \mu_1 ds]}{1 - (\mu_0 + p(s)(\mu_1 - \mu_0)) ds} \\
& = \; p(s)\frac{[1 - \mu_1 ds](1 + (\mu_0 + p(s)(\mu_1 - \mu_0)) ds)}{1 - ((\mu_0 + p(s)(\mu_1 - \mu_0)) ds)^2} \\
& = \; p(s)\frac{1 - \mu_1 ds + [1 - \mu_1 ds](\mu_0 + p(s)(\mu_1 - \mu_0)) ds}{1 - ((\mu_0 + p(s)(\mu_1 - \mu_0)) ds)^2} \\
& = \; p(s)\frac{1 - \mu_1 ds + (\mu_0 + p(s)(\mu_1 - \mu_0)) ds - \mu_1 ds(\mu_0 + p(s)(\mu_1 - \mu_0)) ds}{1 - ((\mu_0 + p(s)(\mu_1 - \mu_0)) ds)^2}.
\end{aligned}
$$

As $ds^2 = 0$, we get

$$
\begin{aligned}
p(s) + dp(s) &= p(s)(1 - \mu_1 ds + (\mu_0 + p(s)(\mu_1 - \mu_0)) ds) \\
&= p(s)(1 - (\mu_1 - \mu_0) ds + p(s)(\mu_1 - \mu_0) ds) \\
&= p(s)(1 - (1 - p(s))(\mu_1 - \mu_0) ds) \Leftrightarrow \\
dp(s) &= -p(s)(1 - p(s))(\mu_1 - \mu_0) ds \\
&= -p(s)(1 - p(s)) \Delta(s) ds
\end{aligned}
$$

where the last equality used the definition for $\Delta(s)$ in (A.2). As $\eta_1 > \eta_2$ implies from (A.1) that $\Delta(s) > 0$, this shows that the prior falls over time.

Now replace $\Delta(s)$ by $\Delta(s) = \mu_1(s) - \mu_0(s) = (\eta_1 - \eta_0)[\phi(s)\theta]^\alpha$ from (A.2) and (A.1). Using the true exit rates $\mu(\phi(s)\theta, 0)$ or $\mu(\phi(s)\theta, 1)$ from (24), this can be written as $\Delta(s) = \mu(\phi(s)\theta, 1) - \mu(\phi(s)\theta, 0)$ and we find (27) in the main text.

A.3 Steady state solution

We solve for the steady state of the model by separating the model into two "blocks".

Block 1 describes the behaviour of an individual of type k. We suppress the index k here. Given the functional forms for utility, the spell-effect in (23) and (24) and the definition of $\eta(s)$ in (26), the first-order condition for effort (8) reads (see app. B.1.1)

$$
\phi(s) = \{\alpha\eta(s)\theta^\alpha [V(w) - V(b(s), s)]\}^{1/(1-\alpha)}, \tag{A.4}
$$

This condition holds for both short- and long-term unemployed. Plugging this into the Bellman equation for the unemployed (7), using (24) and expressing it as a differential equation in s gives (see app. B.2.1)

$$
\frac{dV(b(s), s)}{ds} = \rho V(b(s), s) - \frac{b(s)^{1-\sigma} - 1}{1 - \sigma} - \frac{1 - \alpha}{\alpha}(\alpha\eta(s)\theta^\alpha)^{1/(1-\alpha)}[V(w) - V(b(s), s)]^{1/(1-\alpha)}, \tag{A.5}
$$

which is again valid for both short- and long-term unemployed. As the value of being unemployed an instant before and an instant after becoming a long-term unemployed is identical, we use

$$
V(b_{UI}, \bar{s}) = V(b_{UA}, \bar{s}) \tag{A.6}
$$

from (9) when solving this differential equation.

Both for (A.4) and (A.5) we need $\eta(s)$ which from (26) is given by

$$
\eta(s) \equiv (1 - p(s))\eta_0 + p(s)\eta_1.
$$

The belief is described by

$$
\frac{dp(s)}{ds} = -p(s)(1 - p(s))(\mu(\phi(s)\theta, 1) - \mu(\phi(s)\theta, 0)) \tag{A.7}
$$

from (27) where the initial condition is arbitrary, i.e. determined by the optimism of the individual about how likely she considers to be a high-productivity searcher. For completeness sake, the arrival rates in (A.7) are the objective ones from (24), i.e.

$$
\mu(\phi(s)\theta, 1) = \eta_1 [\phi(s)\theta]^\alpha, \quad \mu(\phi(s)\theta, 0) = \eta_0 [\phi(s)\theta]^\alpha.
$$

As $\eta_1 > \eta_0$, the belief $p(s)$ falls over time and approaches zero.

Finally, since for an infinite unemployment spell all quantities are stationary, we get the terminal condition for (A.5) by using $\lim_{s \to \infty} \dot{V}(b_{UA}, s) = 0$,

$$\rho V(b_{UA}) = \frac{b_{UA}^{1-\sigma} - 1}{1 - \sigma} + \frac{1 - \alpha}{\alpha} \{\alpha \eta_0 \theta^\alpha\}^{1/(1-\alpha)} [V(w) - V(b_{UA})]^{1/(1-\alpha)}. \qquad (A.8)$$

The Bellman equation for the employed worker (6) can be written with the explicit utility function as

$$(\rho + \lambda) V(w) = \frac{w^{1-\sigma} - 1}{1 - \sigma} + \lambda V(b_{UI}, 0). \qquad (A.9)$$

Now imagine we insert $V(w)$ from (A.9) into (A.5) and (A.8). Imagine further that we know all parameters and assume, for the time being, some values for w and θ. Then guess some value $V^{\text{ini}}(b_{UA})$. From (A.8) with (A.9), this fixes $V(b_{UI}, 0)$ (see app. B.2.2 for an explicit expression used for the numerical implementation). With this $V(b_{UI}, 0)$ and some initial $p(0)$, solve the differential equation system (A.5) and (A.7) from 0 to $\bar{s}$. Then, using (A.6), continue solving this system from $\bar{s}$ to "infinity", which practically speaking is a high number (240 months in our implementation). If for "infinity" we find that $\lim_{s \to \infty} V(b_{UA}, s) = V^{\text{ini}}(b_{UA})$ holds, we are done. If not, we adjust $V^{\text{ini}}(b_{UA})$ and continue searching.

Hence, with some exogenous w and θ, we have obtained the time path of effort over the unemployment spell, $\phi(b(s), s)$, the spell-path of the value of being unemployed, $V(b(s), s)$, and the value of a job $V(w)$. With these quantities, we can then compute the true exit rates from (24) and (29) in the empirical application. We can also compute the number of employed and unemployed individuals. We can compute these from (22), using (19a,b) and (20) and the group-specific exit rates.

Block 2 allows to endogenize the wages w_k, one for each group k, the number of vacancies per unemployed worker, θ and the tax rate κ. The equations that fix these variables are the wage setting equation (13), an equation for θ, and the budget constraint of the government (16). The equation for θ builds on the Bellman equation for the firm (10) which for $J_{0k} = 0$ can be written as

$$J_k = \frac{A_k - w_k/(1 - \kappa)}{\rho + \lambda_k}. \qquad (A.10)$$

Using this for (12), we obtain

$$\bar{\mu}_k \frac{A_k - w_k/(1 - \kappa)}{\rho + \lambda_k} = \gamma_k \theta_k. \qquad (A.11)$$

Hence, the wages, tightness and the tax rate need to be such that (13), (A.11) and (16) hold.

Numerically speaking, given the equilibrium values $\{\phi(b(s), s), V(b(s), s), V(w)\}$ resulting from block 1, we compute w_k and θ_k from (13) and (A.11) and κ from (16) written as

$$\frac{1 - \kappa}{\kappa} = \frac{\Sigma_{k=1}^n w_k L_k}{\Sigma_{k=1}^n \left(\xi_{UI} w_k U_k^{\text{short}} + \xi_{UA} w_k U_k^{\text{long}}\right)} \quad \Leftrightarrow$$

$$\kappa = \left(\frac{\Sigma_{k=1}^n w_k L_k}{\xi_{UI} \Sigma_{k=1}^n w_k U_k^{\text{short}} + \xi_{UA} \Sigma_{k=1}^n w_k U_k^{\text{long}}} + 1\right)^{-1} \qquad (A.12)$$

using (3) for b_{UI} and b_{UA} and (15). If the wages and θ do not coincide with the guesses for block 1, we start all over until they do so.

Finally, one useful by-product of the of the steady state solution is the exit rate out of unemployment in the stationary environment μ. Assuming workers are identical in search productivity ($\eta_1 = \eta_0$) and letting $s \to \infty$ we can use (A.8) to show that μ solves the equation

$$(1 - \alpha)\,\mu - \alpha\,[\eta_0\theta^\alpha]^{1/\alpha}\,\mu^{1-1/\alpha}\left[\rho V\left(w\right) - \frac{b_{UA}^{1-\sigma} - 1}{1 - \sigma}\right] + \rho = 0. \qquad (A.13)$$

This result provides the basis for identification of the structural econometric model from wage, benefit and duration data.

A.4   Estimated and predicted survivor functions

Fig. 5 shows the predicted survivor functions (solid lines) for heterogeneous population groups joint with the Kaplan-Meier survivor probabilities (circles). Corresponding 95% confidence intervals are depicted by triangles. For a discussion, see towards the end of sect. 4.2.
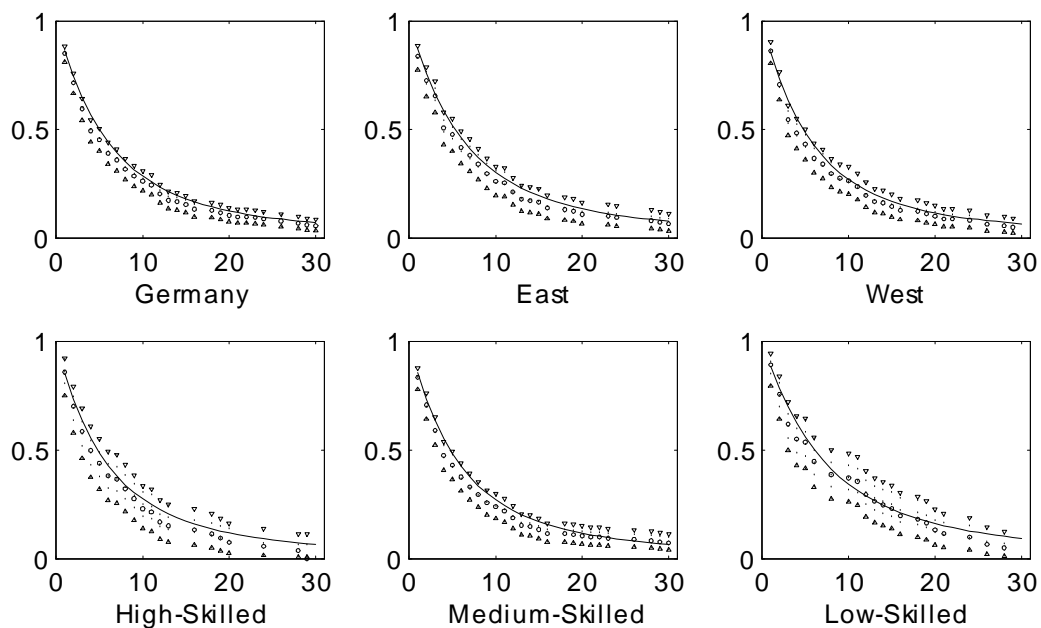

Figure 5 *Kaplan-Meier and predicted survivor functions*

The next figure shows the survivor functions (solid lines) for heterogeneous population groups joint with the Kaplan-Meier survivor probabilities (circles) predicted for the external sample. Corresponding 95% confidence intervals are depicted by triangles.
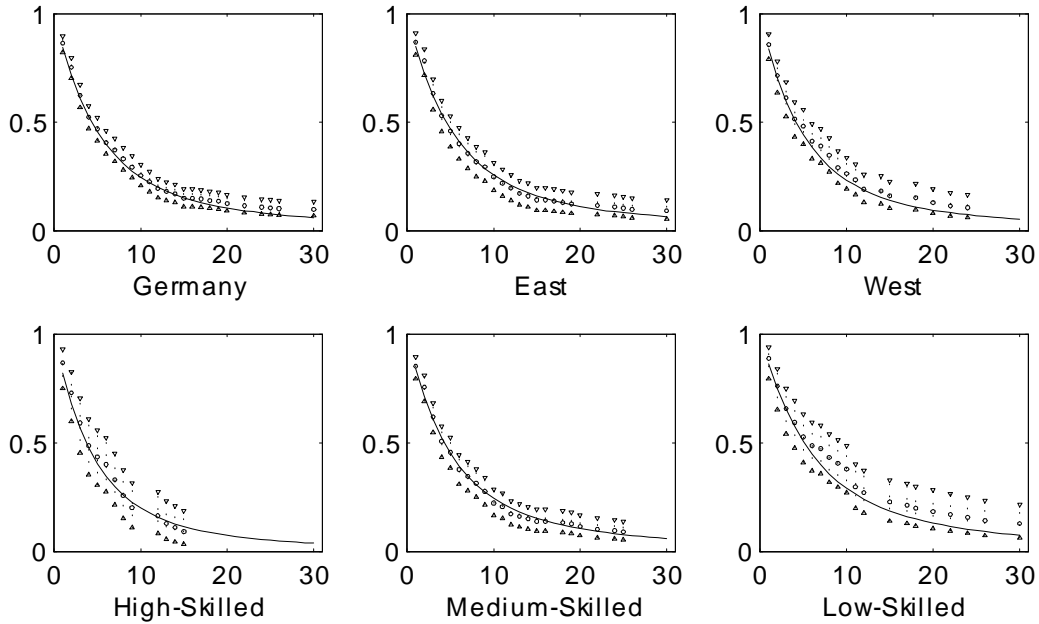
Figure 6 *Kaplan-Meier and predicted survivor functions on the external sample*

A.5   A Semi-Markov process

This section provides a short introduction into Semi-Markov processes. Technically, it follows Kulkarni (1995) and Corradi et al. (2004). The original work is by Pyke (1961a,b). Due to their technical nature, these papers are less accessible and we hope that this appendix helps that these very useful methods become more widely used. To the best of our knowledge, this is the first application of these concepts in economics. For more details and the numerical implementation, see Schumm (2010, ch. 4). The first subsection describes the general approach to Semi-Markov processes while the second adapts it to our question.

A.5.1   The general approach

Let Y_n denote the state of a system after the nth transition. Let this state be i. Let the point in time of the nth transition be denoted by S_n. Define the probability that the system after the next transition is in j and that this transition takes place within a period of length x or shorter, conditional on the system being in i after the nth transition, as

$$Q_{ij}(x) \equiv P\left\{Y_{n+1} = j, S_{n+1} - S_n \leq x \mid Y_n = i\right\}.$$

The probability that any transition takes place is then given by summing up the probabilities for each j, $Q_i(x) = \Sigma_{j \neq i} Q_{ij}(x)$, not taking into account transitions from i to i.[32]   The

---

[32]We differ from the notation in the cited literature in that we explicitly write $j \neq i$ here or $k \neq i$ below. This is equivalent to setting the transition rate from i to i to zero. As our application does not

probability that the system will be in j in τ is given by

$$p_{ij}(\tau) = (1 - Q_i(\tau))\delta_{ij} + \Sigma_{k \neq i} \int_0^\tau p_{kj}(\tau - x) \, dQ_{ik}(x). \tag{A.14}$$

The interpretation of this integral equation is as follows: the first part of the right hand side gives the probability that the system, being currently in state i, never leaves state i until τ. In this case $j = i$ and $\delta_{ij} = 1$, so $1 - Q_i(\tau)$ is the survival probability in state i. If $j \neq i$, $\delta_{ij} = 0$. The second part of the right hand side collects all cases in which the transition from i to j (which includes i) occurred via another state $k \neq i$. First, we take the probability that the process stayed in state i for a period of length x and passed to state k then (captured by $Q_{ik}(x)$). Then we need the probability that the process which is in state k after x will be in state j at τ (captured by $p_{kj}(\tau - x)$). As the transition from i to k can be anywhere between 0 and τ, we have to integrate over x in order to cover all possible transitions.

Equation (A.14) can slightly be rewritten, provided that $Q_{ik}(x)$ is once differentiable (which holds for our case), as

$$p_{ij}(\tau) = (1 - Q_i(\tau))\delta_{ij} + \Sigma_{k \neq i} \int_0^\tau p_{kj}(\tau - x) \frac{dQ_{ik}(x)}{dx} dx. \tag{A.15}$$

The derivative $dQ_{ik}(x)/dx$ now gives the density of going from i to k after duration x. Multiplied by the probability $p_{kj}(\tau - x)$ of subsequently going from k to j (which may include many intermediate transitions to other states) gives the density of ending up in j after having gone to k after x. Integrating over all durations x gives the probability of starting in i and being in j at τ.

A.5.2 Our two-state system

We now need to adjust the notation such that it suits our purposes. We look at a worker who just moved in t (like today) into either employment e or unemployment u. Define $Q_{eu}(\tau)$ as the probability that a worker who just found a job in t "jumps" to u in a period of time shorter or equal to $\tau - t$. With a duration s dependent arrival rate $\lambda(s(v))$, this is then simply given by

$$Q_{eu}(\tau) = 1 - e^{-\int_t^\tau \lambda(s(v))dv}, \tag{A.16}$$

where $s(v) = v - t$ is the duration in her current state. In perfect analogy and using a spell-dependent arrival rate $\mu(s(v))$, we get $Q_{ue}(\tau) = 1 - e^{-\int_t^\tau \mu(s(v))dv}$. For the complementary events - remaining in a given state - the probabilities are simply $Q_{ee}(\tau) = 1 - Q_{eu}(\tau)$ and $Q_{uu}(\tau) = 1 - Q_{ue}(\tau)$. The probabilities that a transition takes place at all in this two state process are

$$Q_e(\tau) \equiv Q_{eu}(\tau), \ Q_u(\tau) \equiv Q_{ue}(\tau). \tag{A.17}$$

With two possible states, we have four transition probabilities for the future: an unemployed (employed) person can either be unemployed or employed at some future point in time

---

have transitions from i to i (i.e. transition rates from i to i are zero), we find using $j \neq i$ explicitly more intuitive for our purpose. We are indebted to Ludwig Fahrmeir for various communications on Semi-Markov processes. For an excellent introduction in German, see Fahrmeir et al. (1981).

τ. Two are redundant as the probability of e.g. an unemployed worker of being employed is complementary to the probability of being unemployed, $p_{ue}(\tau) = 1 - p_{uu}(\tau)$, and similarly $p_{ee}(\tau) = 1 - p_{eu}(\tau)$. Hence, we only focus on $p_{uu}(\tau)$ and $p_{eu}(\tau)$. These probabilities are, using the general equation (A.15),

$$p_{uu}(\tau) = 1 - Q_u(\tau) + \int_t^\tau p_{eu}(\tau - v) \frac{dQ_{ue}(v)}{dv} dv, \tag{A.18a}$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v) \frac{dQ_{eu}(v)}{dv} dv. \tag{A.18b}$$

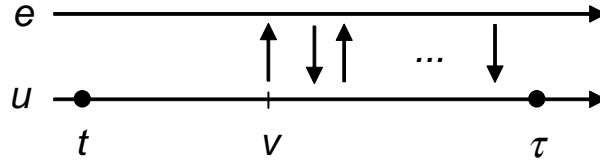These equations can be most easily be understood by looking at the following figure.

Figure 7 *Illustrating transition probabilities*

Let's consider $p_{uu}(\tau)$: An individual unemployed in t can be unemployed in τ by always remaining unemployed. This is the term $1 - Q_u(\tau)$. The individual can be unemployed in τ by remaining unemployed until v where she jumps into employment, the density for which is $dQ_{ue}(v)/dv$. After v, the probability of returning to unemployment in the remaining time span of $\tau - v$ is $p_{eu}(\tau - v)$. Note that this probability includes an arbitrary number of transitions larger than zero in this remaining period $\tau - v$. In contrast to integrating over x as in (A.15), we integrate over the point in time v here simply as this is more intuitive. The interpretation for $p_{eu}(\tau)$ is in perfect analogy.

As a last step, we need to determine the two derivatives $dQ_{ue}(v)/dv$ and $dQ_{eu}(v)/dv$. Given duration-dependent arrival rates, the derivatives of (A.16) are,

$$\frac{dQ_{ue}(v)}{dv} = e^{-\int_t^v \mu(s(y))dy} \frac{d}{dv} \int_t^v \mu(s(y)) dy = e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) \tag{A.19a}$$

$$\frac{dQ_{eu}(v)}{dv} = e^{-\int_t^v \lambda(s(y))dy} \frac{d}{dv} \int_t^v \lambda(s(y)) dy = e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)). \tag{A.19b}$$

Given (A.17) and the derivatives, the equations (A.18) become

$$p_{uu}(\tau) = e^{-\int_t^\tau \mu(s(y))dy} + \int_t^\tau p_{eu}(\tau - v) e^{-\int_t^v \mu(s(y))dy} \mu(s(v)) dv,$$

$$p_{eu}(\tau) = \int_t^\tau p_{uu}(\tau - v) e^{-\int_t^v \lambda(s(y))dy} \lambda(s(v)) dv.$$

The final adjustment we need to make is to replace $\lambda(s(v))$ by λ as the separation rate is assumed to be constant. This then gives equations (19) in the main text.

B   Appendix

All references to appendices starting with B refer to the Referees' appendix. It is available upon request (and will be made available in the web).

References

Aase, K. K. (1990): "Unemployment insurance and incentives," The Geneva Papers on Risk and Insurance - Theory, 15, 141 – 157.

Acemoglu, D., and R. Shimer (1999): "Efficient Unemployment Insurance," Journal of Political Economy, 107, 893–928.

Albrecht, J., and S. Vroman (2002): "A matching model with endogenous skill requirements," International Economic Review, 43(1), 283–305.

Albrecht, J., and S. Vroman (2005): "Equilibrium Search with Time-varying Unemployment Benefits," Economic Journal, 115, 631–648.

Ameco (2010): "Annual macro-economic database," European Commission DG ECFIN. http://ec.europa.eu/economy_finance/db_indicators/ameco/index_en.htm.

Arulampalam, W., and M. Stewart (1995): "The Determinants of Individual Unemployment Duration in an Era of High Unemployment," Economic Journal, 105, 321–332.

Bayer, C., and K. Wälde (2011): "Describing the Dynamics of Distributions in Search and Matching Models by Fokker-Planck Equations," mimeo - available at www.waelde.com/pub.

Blanchard, O., and P. Diamond (1994): "Ranking, Unemployment Duration, and Wages," Review of Economic Studies, 61, 417–434.

Blanchard, O., and J. Wolfers (2000): "The Role of Shocks And Institutions In the Rise Of European Unemployment: The Aggregate Evidence," Economic Journal, 110, C1–C33.

Blos, K., and H. Rudolph (2005): "Verlierer, aber auch Gewinner," IAB Kurzbericht, 17, 1–6.

Bontemps, C., J.-M. Robin, and G. van den Berg (1999): "An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms," International Economic Review, 40, 1039–1072.

Bundesagentur für Arbeit (2009): "Analytikreport der Statistik - Juli 2009," http://www.pub.arbeitsagentur.de/hst/services/statistik/interim/analytik/reports/zentral.shtml.

Burdett, K., C. Carrillo-Tudela, and M. G. Coles (forthcoming): "Human Capital Accumulation and Labour Market Equilibrium," International Economic Review.

Burdett, K., N. M. Kiefer, and S. Sharma (1985): "Layoffs and duration dependence in a model of turnover," Journal of Econometrics, 28, 51 – 69.

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," International Economic Review, 39, 257–273.

Cahuc, P., and E. Lehmann (2000): "Should unemployment benefits decrease with the unemployment spell?," Journal of Public Economics, 77, 135–153.

Cahuc, P., F. Postel-Vinay, and J.-M. Robin (2006): "Wage Bargaining with On-the-job Search: Theory and Evidence," Econometrica, 74, 323–365.

Carling, K., B. Holmlund, and A. Vejsiu (2001): "Do Benefit Cuts Boost Job Findings," Economic Journal, 111, 766–790.

Coles, M., and A. Masters (2006): "Optimal Unemployment Insurance in a Matching Equilibrium," Journal of Labor Economics, 24, 109–138.

——— (2007): "Re-entitlement effects with duration-dependent unemployment insurance in a stochastic matching equilibrium," Journal of Economic Dynamics and Control, 31, 2879–2898.

Coles, M. G., and A. Muthoo (2003): "Bargaining in a non-stationary environment," Journal of Economic Theory, 109, 70–89.

Coles, M. G., and R. Wright (1998): "A Dynamic Equilibrium Model of Search, Bargaining, and Money," Journal of Economic Literature, 78, 32–54.

Corradi, G., J. Janssen, and R. Manca (2004): "Numerical Treatment of Homogeneous Semi-Markov Processes in Transient Case - a Straightforward Approach," Methodology and Computing in Applied Probability, 6, 233 – 246.

Eckstein, Z., and G. van den Berg (2007): "Empirical Labor Search: A Survey," Journal of Econometrics, 136, 531–564.

Eckstein, Z., and K. I. Wolpin (1995): "Duration to First Job and the Return to Schooling: Estimates from a Search-Matching Model," Review of Economic Studies, 62, 263–286.

Fahr, R., and U. Sunde (2009): "Did the Hartz Reforms Speed-Up the Matching Process? A Macro-Evaluation Using Empirical Matching Functions," German Economic Review, 10, 284–316.

Fahrmeir, L., H. L. Kaufmann, and F. Ost (1981): Stochastische Prozesse. Eine Einführung in Theorie und Anwendungen. Hanser Fachbuchverlag.

Ferrall, C. (1997): "Unemployment Insurance Eligibility and the School-to-Work Transition in Canada and the United States," Journal of Business & Economic Statistics, 15, 115–129.

Fitzenberger, B., and R. Wilke (2010): "New Insights into Unemployment Duration and Post Unemployment Earnings in Germany," Oxford Bulletin of Economics and Statistics, 72, 794–826.

Flinn, C. (2006): "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates," Econometrica, 74, 1013–1062.

Franz, W., N. Guertzgen, S. Schubert, and M. Clauss (2007): "Reformen im Niedriglohnsektor - eine integrierte CGE-Mikrosimulationsstudie der Arbeitsangebots- und Beschäftigungseffekte," ZEW Discussion Paper No. 07-085.

Fredriksson, P., and B. Holmlund (2001): "Optimal Unemployment Insurance in Search Equilibrium," Journal of Labor Economics, 19, 370–399.

Frijters, P., and B. van der Klaauw (2006): "Job Search with Nonparticipation," Economic Journal, 116, 45–83.

Galenianos, M., P. Kircher, and G. Virág (2011): "Market Power and Efficiency in a Search Model," International Economic Review, 52(1), 85–103.

Gautier, P., C. Teulings, and A. van Vuuren (2010): "On-the-Job Search, Mismatch and Efficiency," Review of Economic Studies, 77, 245–272.

Hall, R. E., and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," American Economic Review, June 2008.

Heer, B. (2006): "Should Unemployment Benefits Be Related to Previous Earnings?," FinanzArchiv, 62, 530 – 550.

Hopenhayn, H. A., and J. P. Nicolini (1997): "Optimal Unemployment Insurance," Journal of Political Economy, 105, 412–438.

——— (2009): "Optimal Unemployment Insurance and Employment History," Review of Economic Studies, 76, 1049–1070.

Hosios, A. J. (1990): "On the Efficiency of Matching Models and Related Models of Search and Unemployment," Review of Economic Studies, 57, 279–298.

Hujer, R., and H. Schneider (1989): "The Analysis of Labour Market Mobility Using Panel Data," European Economic Review, 33, 530–536.

Keane, M., and K. Wolpin (2002a): "Estimating Welfare Effects Consistent With Forward-Looking Behavior. Part I: Lessons from a Simulation Exercise," Journal of Human Resources, 37(2), 570–599.

——— (2002b): "Estimating Welfare Effects Consistent With Forward-Looking Behavior.Part II: Empirical Results," Journal of Human Resources, 37(2), 600–622.

——— (2010): "The Role of Labor and Marriage Markets, Preference Heterogeneity, and the Welfare System in the Life Cycle Decisions of Black, Hispanic and White Women," International Economic Review, 51(1), 851–892.

Keller, G., and S. Rady (2010): "Strategic experimentation with Poisson bandits," Theoretical Economics, 5(2), 275–311.

Keller, G., S. Rady, and M. Cripps (2005): "Strategic Experimentation with Exponential Bandits," Econometrica, 73(1), 39–68.

Krause, M. U., and H. Uhlig (2012): "Transitions in the German labor market: Structure and crisis," Journal of Monetary Economics, 59, 64–79.

Krebs, T., and M. Scheffel (2010): "A Macroeconomic Model for the Evaluation of Labor Market Reforms," mimeo University of Mannheim.

Kulkarni, V. G. (1995): Modeling and Analysis of Stochastic Systems. Chapman & Hall.

Lehmann, E., and B. van der Linden (2007): "On the Optimality of Search Matching Equilibrium When Workers Are Risk Averse," Journal of Public Economic Theory, 9, 867–884.

Lentz, R., and D. T. Mortensen (2008): "An Empirical Model of Growth Through Product Innovation," Econometrica, 76, 1317–1373.

Lise, J. (2011): "On-the-Job Search and Precautionary Savings: Theory and Empirics of Earnings and Wealth Inequality," mimeo UCL.

Ljungqvist, L., and T. J. Sargent (1998): "The European Unemployment Dilemma," Journal of Political Economy, 106(3), 514–550.

Magnac, T., J.-M. Robin, and M. Visser (1995): "Analysing Incomplete Individual Employment Histories Using Indirect Inference," Journal of Applied Econometrics, 10, S153–S169.

Mortensen, D. (1977): "Unemployment Insurance and Job Search Decisions," Industrial and Labor Relations Review, 30, 505–517.

Mortensen, D. T. (1982): "Property Rights and Efficiency in Mating, Racing, and Related Games," American Economic Review, 72, 968–79.

Mortensen, D. T., and C. A. Pissarides (1999): "New developments in models of search in the labor market," in Handbook of Labor Economics, ed. by O. Ashenfelter, and D. Card, chap. 39, pp. 2567–2627. Elsevier.

Moscarini, G. (2005): "Job Matching and the Wage Distribution," Econometrica, 73(2), 481–516.

Nickell, S., L. Nunziata, and W. Ochel (2005): "Unemployment in the OECD since the 1960s: What Do We Know?," Economic Journal, 115, 1–27.

OECD (2004): Benefits and Wages. OECD, Paris.

——— (2007): Benefits and Wages. "Daten und Grafiken", downloaded from www.oecd.org/de/benefitsandwages on 12 June 2008.

Paserman, M. D. (2008): "Job Search and Hyperbolic Discounting: Structural Estimation and Policy Evaluation," Economic Journal, 118, 1418–1452.

Pavoni, N. (2009): "Opitmal Unemployment Insurance, with Human Capital Depreciation, and Duration Dependence," International Economic Review, 50, 323–362.

Petrongolo, B., and C. A. Pissarides (2001): "Looking into the Black Box: A Survey of the Matching Function," Journal of Economic Literature, 39(2), 390–431.

Pissarides, C. A. (1985): "Short-run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages," American Economic Review, 75, 676–90.

Postel-Vinay, F., and J.-M. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," Econometrica, 70, 2295–2350.

Postel-Vinay, F., and H. Turon (2010): "On-the-Job Search, Productivity Shocks, and the Individual Earnings Process," International Economic Review, 51(3), 599–629.

Pyke, R. (1961a): "Markov Renewal Processes: Definitions and Preliminary Properties," The Annals of Mathematical Statistics, 32, 1231–1242.

——— (1961b): "Markov Renewal Processes with Finitely Many States," The Annals of Mathematical Statistics, 32, 1243–1259.

Rendahl, P. (2012): "Asset Based Unemployment Insurance," International Economic Review, 53, forthcoming.

Rendon, S. (2006): "Job Search and Asset Accumulation under Borrowing Constraints," International Economic Review, 47, 233–263.

Ross, S. M. (1996): Stochastic processes, 2nd edition. Academic Press, San Diego.

Røed, K., and T. Zhang (2003): "Does Unemployment Compensation Affect Unemployment Duration?," Economic Journal, 113, 190–206.

Schumm, I. (2010): "Lessons learned from Germany's 2001 to 2006 labour market reforms," Dissertation, University of Würzburg. Chapter 4 available at www.waelde.com/pub, full dissertation available at http://www.opus-bayern.de/uni-wuerzburg/volltexte/2010/4370/.

Shavell, S., and L. Weiss (1979): "The Optimal Payment of Unemployment Insurance Benefits over Time," Journal of Political Economy, 87, 1347–1362.

Shimer, R. (2006): "On-the-Job Search and Strategic Bargaining," European Economic Review, pp. 811–830.

Shimer, R., and I. Werning (2007): "Reservation Wages and Unemployment Insurance," Quarterly Journal of Economics, 122, 1145–1185.

Uren, L., and G. Virág (2011): "Skill Requirements, Search Frictions, and Wage Inequality," International Economic Review, 52(2), 379–406.

van den Berg, G. J. (1990): "Nonstationarity in Job Search Theory," Review of Economic Studies, 57, 255–277.

van den Berg, G. J., and G. Ridder (1998): "An Empirical Equilibrium Search Model of the Labor Market," Econometrica, 66, 1183–1221.

van den Berg, G. J., and B. van der Klaauw (2010): "Structural Empirical Evaluation of Job Search Monitoring," Mimeo, Free University of Amsterdam.

van den Berg, G. J., and J. C. van Ours (1996): "Unemployment Dynamics and Duration Dependence," Journal of Labor Economics, 14, 100–125.

——— (1999): "Duration Dependence and Heterogeneity in French Youth Unemployment Durations," Journal of Population Economics, 12, 273–285.

Vishvanath, T. (1989): "Job Search, Stigma Effect and Escape Rate from Unemployment," Journal of Labor Economics, 7, 487–502.

Wolpin, K. (1987): "Estimating a Structural Job Search Model: The Transition from School to Work," Econometrica, 55, 801–817.