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Managing Road Capacity: Maintenance, Tolls, and Multi-Level Governance

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Abstract

Road congestion and deteriorating infrastructure impose substantial economic and social costs, with estimates reaching up to 1% of GDP in highly congested economies. At the same time, public finances are increasingly constrained, and reliance on the "user-pays" principle has grown, prompting greater use of tolling schemes and private participation in financing, operating, and maintaining road networks. This paper examines the interaction between road maintenance, capacity, and pricing decisions in contexts where different operators share responsibilities. We analyze whether private maintenance and tolling strategies converge toward socially optimal outcomes, and under what conditions misalignments occur. Policy implications for optimal pricing, investment incentives, and the design of capacity and maintenance are discussed.

JEL Classification: R41, R42, R48, H54, Q54, L91

Keywords: Road maintenance; Road capacity; Traffic congestion; Tolls; Perfect Competition; Multi-level governance; Infrastructure finance;

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1 Introduction

Traffic congestion imposes significant economic costs, with national estimates suggesting losses of up to 1% of GDP in highly congested countries ([28]). At the same time, public dissatisfaction with deteriorating road conditions remains widespread.

Moszoro and Soto [26] propose the Mean Speed (MS) score as a novel indicator of road quality, constructed from average travel speeds between major cities using Google Maps data. This metric captures the expeditiousness of road networks and provides a robust, easily replicable alternative to traditional measures such as the World Bank's Rural Access Index and the World Economic Forum's Quality of Road Infrastructure survey, which rely on costly data collection or subjective perceptions. Empirical validation shows that the MS score is strongly correlated with GDP per capita, road density, and perceived road quality, while outperforming existing measures in explanatory power. As such, the MS score offers policy makers a reliable proxy for evaluating road infrastructure, guiding welfare analysis, and prioritizing investment decisions.

With strained public finances and growing acceptance of the "user-pays" principle, policy makers have increasingly turned to strategies that involve direct user charges—such as tolls—and greater private-sector participation in road financing, construction, operation, and maintenance. This raises a central question: to what extent do private operators adopt maintenance and tolling policies that align with socially optimal outcomes?

Climate change adds another layer of complexity. Most scientists agree that global warming is largely driven by human activity, and its consequences—heat waves, droughts, floods, and severe storms ([21])—are particularly acute in urban areas, where infrastructure is dense and often fragile. Rapid urbanization in Asia and Africa, coupled with the expansion of heat islands and impermeable surfaces, amplifies vulnerability (United Nations Human Settlements Programme 2020; [18]). Beyond immediate disruptions, climate change accelerates the deterioration of transport infrastructure, raising maintenance and adaptation costs. Meanwhile, global emissions of greenhouse gases (CO_2, CH_4, N_2O) continue to rise despite reductions in Europe and the United States, as demographic and economic growth in emerging countries outweighs gains from mitigation efforts. The ongoing decline in the world population will also need to be taken into account in decisions regarding the construction and maintenance of infrastructure (see, [23]).

Investment and maintenance are closely related but conceptually distinct stages of infrastructure policy. Investment decisions involve capacity choice: whether to build new infrastructure or upgrade existing assets. Maintenance decisions involve preserving and optimizing the service life of those assets.

Typically, the decision to invest in new capacity precedes the decision to maintain it. Once an asset is built, maintenance must follow to sustain performance and value. In the case, decisions are sequential.

Alternatively, economic appraisal (such as the World Bank's HDM-4, HDM-5 models on road use costs, see [43]) treats investment and maintenance together by evaluating life-cycle costs, including construction, periodic maintenance, and rehabilitation. Thus, optimal decisions require simultaneous consideration of both dimensions. In this case, decisions are simultaneous.

However, the World Bank emphasizes that sustainable capacity choice cannot be separated from maintenance planning: the two must be integrated into a single life-cycle framework (see the review in [7]). We wish in the paper compare sequential and simultaneous capacity and maintenance decisions.

In this paper, we focus on two key feedback mechanisms. First, maintenance and capacity decisions jointly determine travel time (or generalized transport cost). Second, transport costs directly affect social welfare, measured as the sum of consumer surplus and firm profits net of maintenance and construction costs. We integrate these feedbacks in a perfect competition framework.

The literature establishes a clear conceptual distinction between maintenance—understood as asset preservation, planned interventions, and structural upkeep—and operation, which refers to activities such as toll collection, routine management, and emergency repairs ([24]; [11]). This distinction is fundamental, as the long-term sustainability of infrastructure depends as much on adequate maintenance strategies as on its initial construction.

Yet, empirical evidence suggests that investment levels remain insufficient. According to the European Commission ([12]), "Most EU countries have a share of total transport infrastructure investment below 1% of GDP. It is a safe assumption that this does not cover the investment needs (also due to the maintenance requirements) in most countries." Complementing this assessment, the European Construction Industry Federation (FIEC) estimates that annual road maintenance costs are around €$25/m^2$, corresponding to roughly 1% of the initial investment. Critically, postponing maintenance for three years may increase rehabilitation costs by a factor of three to six, thereby underscoring the intertemporal inefficiency of underinvestment.

From an economic evaluation perspective, the valuation of travel time savings continues to constitute a cornerstone of cost–benefit analysis ([27], and [33]). However, the concept of the value of time is not without controversy. In France, official *valeurs tutélaires* are employed in appraisal exercises, rather than econometrically estimated values. This methodological choice raises questions of legitimacy and consistency, particularly in light of critiques such as those formulated by [25], who argues that the valuation of time is often overstated or misapplied.

Against this backdrop, a central issue emerges: what are the costs of road construction and maintenance, and to what extent do these costs, when weighed against the associated benefits, provide a robust justification for infrastructure investment and upkeep? Addressing this question requires not only accurate cost assessment but also a critical reflection on the methodological foundations of benefit valuation, including the contested role of travel time savings.

The detailed road map of the paper follows. In section 2 we consider a standard model to evaluate road capacity with and without pricing. We use a closed (but exact) form) to model congestion in a dynamics framework (congestion depends on the time of the day). We then consider capacity and maintenance sequential choices (first capacity is determined and then maintenance level is chosen). We briefly summarize the main self-financing results in the context of a dynamic model. In Proposition 1, we provide the optimal capacity and maintenance levels and the optimal free flow travel time. In section 3 we consider the standard sequential choice and revisit the self-financing results with optimal road pricing (see Lemma 3). Optimal maintenance and capacity levels are provided in Proposition 2. In section 4, we explore the situation where road capacities and maintenance are computed simultaneously. This is not common practice. We show the self-financing results does not hold in the case; we also evaluate and qualify the social saving of the simultaneously choice (see Proposition 4). In section 5, we introduce a standard model of perfect competition where good are shipped form firms to consumers (iceberg cost), and evaluated how the social welfare depend on travel time (here considered as an exogenous variable). In Section 6 we revisit the standard (sequential) computation of capacity and maintenance in this competitive economy (see Proposition 5). Section 7 concludes. The appendix outlines the foundations of our stationary approach to maintenance (see Proposition 6). Two proofs are relegated to the Appendix.

2 Quality of road and maintenance

The quality of road infrastructure is a decisive factor in shaping travel times, vehicle operating costs, and the long-term efficiency of the transport system. Road deteriorate both from natural causes, such as weather, and from traffic-related wear, requiring maintenance policies that balance usage and investment.

The seminal analysis of Newbery [27] established the principles of optimal maintenance. In parallel papers, other authors including [17] and [37] explored conditions for cost recovery and self-financing. Building on these literature, we develop a framework that links road quality, maintenance, and capacity decisions to user travel times and social welfare, with demand taken as exogenous. We examine in particular cost recovery. This provides the basis for examining how planners can sustain road quality over time and design efficient maintenance strategies.

Users, travelling on the road, can be car drivers (with passenger) or truck drivers (with some load).

2.1 Problem setting and welfare function

The travel time for a road user is a function of the quality of the road, Q:

$$TT^w = g(Q) > 0, g'(Q) < 0 \text{ with: } \overline{TT} < TT^w < 24$$

where $\overline{TT}$ is the minimum trip time (at maximum speed and with no congestion).

The quality of pavements and roadside facilities $Q(t)$ declines over time, exogenously through exposure to environmental conditions (parameter a) and endogenously through use. Sustaining quality requires appropriate maintenance (parameter b), as explained below.

The quality increases with maintenance and decrease with usage and time (see [10]). We denote by t the historical time, and have the following dynamics:

$$\frac{dQ\left(t\right)}{dt}=\left(m\left(t\right)-b\frac{n\left(t\right)}{s}-aQ\left(t\right)\right),Q\left(t_{0}\right)=Q_{o},$$

where $m\left(t\right)$ is usage, s the road capacity and $Q\left(t\right)$ the capacity at time t. We derive this equation from a cumulative-usage formulation, in which cumulative usage serves as the primary explanatory variable for road-quality deterioration. The parameter, a corresponds to natural deterioration (to to with weather conditions, in particular), and parameter b corresponds to deterioration given usage $n\left(t\right)$. [1]

In practice, maintenance is not a continuous variable and the differential equation is not linear and should be solved analytically. Here we consider a stationary state. Constant Quality can be sustained by steady maintenance, denoted by m^{st}, when car usage is fixed, i.e. time independent. The steady state level of quality Q^{st} is unique and explicit:

$$Q^{st}=\frac{1}{a}\max\left\{\left(m^{st}-b\frac{n^{st}}{s}\right),0\right\}. \tag{1}$$

We obtain exactly equation (1) when usage and maintenance are time-invariant (stationary environment). Damage is linear in normalized flow, natural deterioration is proportional to Q at rate a. Quality is bounded below by zero (non negativity constraint). In Appendix 2 we consider a model with cumulative usage and degradation (more standard approach). We show that the solutions boils down to 1.

With to too low or no maintenance, ($m^{st}\leqslant b\frac{n^{st}}{s}$), $Q^{st}=0$. The maintenance to insure that usage does not deteriorate the quality of the road is: $b\frac{n^{st}}{s}$. We consider below low enough maintenance costs, so road quality is positive. Let denote by $Q^{\max}$ the maximum road quality (i.e. the quality of a new road). Then (assuming that quality is smaller than maximum quality $Q^{\max}$):

$$Q^{st}=\frac{1}{a}\left(m^{st}-b\frac{n^{st}}{s}\right)\in\left(0,Q^{\max}\right), \tag{2}$$

with

$$b\frac{n^{st}}{s}<m^{st}<aQ^{\max}+b\frac{n^{st}}{s} \tag{3}$$

Two remarks justify the proposed modelling framework. First, road capacity should be understood as the number of lanes (or total surface area), while the relevant factor for deterioration is the axle load. For simplicity, we adopt a linear degradation process, although the literature (see [16]) typically

[1] For a application, power 4 function (wheight par axel) provides a more realistic approximation. Incidentally, for the rail, power 2 is used.

employs a fourth-power relationship between axle load and pavement damage. The analysis extends naturally to such a non-linear specification. Capacity and maintenance interact: for a given traffic volume, a wider road (ceteris paribus) experiences slower deterioration. Second, this framework enables planners to compare continuous maintenance strategies (e.g., routine patching) with periodic resurfacing, to quantify their economic trade-offs, and to capture the interplay between capacity, traffic, and pavement life. Incorporating the fourth-power law yields a more realistic assessment of heavy-vehicle impacts and optimal cost recovery. (Periodic maintenance strategies are often used as well, particularly in the context of rail maintenance : see [19]). In this presentation, we abstract from the detailed timing of interventions, smoothing these processes to focus on average yearly maintenance costs, average (stationary) road quality, and the annualized financial cost of construction.

The (yearly average) maintenance cost is assumed to be $\psi \cdot m^{st}$, where ψ is the unit maintenance cost.

In our problem usage is given by the number of trips. It is exogenous and constant, and equal to N, so $n^{st} = N$.

2.2 The travel time function

The total travel time is assumed to be the sum of free flow travel time (first term), which depends on quality and on congestion (second term), as follows:

$$TT^{w} = \Phi\left(Q^{st}\right) + g\left(\frac{N}{s}\right), \tag{4}$$

with

$$\Phi\left(Q^{st}\right) = TT^{\min} \bullet \phi\left(\frac{Q^{\max} - Q^{st}}{Q^{\max}}\right), \tag{5}$$

where $TT^{\min} = \nu \cdot N \cdot \overline{TT}$, where ν is the value of time and $\overline{TT}$ is the free flow travel time for a perfect quality of the road (in this case travel time is proportional equal to or slightly less than the inverse of the legal maximum speed) and N is the number of users (inelastic demand). Let $Q^{\max}$ be the quality of a new road. We impose the following conditions: $\phi(\bullet) > 0$; $\phi'(\bullet) > 0, \phi''(\bullet) > 0$; for $Q^{st} = Q^{\max}$, $\phi(0) = 1$ and for $Q^{st} = 0$, $\phi(1) = 1 + \Delta\phi^{\max}/2 >> 1$. The function $\phi(\bullet)$ captures the (convex) increase in free flow travel time due to the deterioration of the quality of the road.

[6] note that road segments deteriorate over time. Importantly, these authors provide causal evidence showing that increasing pavement roughness significantly reduces average vehicle speed, thereby increasing travel time. Specifically, a one standard deviation increase in the International Roughness Index (IRI) leads to an 11% decrease in average speed. [30] developed deterioration models using serviceability indicators (such as PSI and IRI) and found that vehicle operating costs escalate more than proportionally as road conditions worsen—illustrating the convex nature of the relationship between quality deterioration and its economic consequences (here travel time).

For analytical tractability, we shall use a quadratic specification:

$$\phi\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)=1+0.5\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)^2\Delta\phi^{\max},\phi'(\bullet)>0,\phi''(\bullet)>0, \quad (6)$$

which accounts for the convex impact of quality deterioration.

Congestion, expressed by $g(\bullet)>0$ with $g'(\bullet)>0$ is an homogenous function of degree zero in N (usage) and s (capacity). For the standard dynamic bottleneck model (see Vickrey 1969 and Arnott 1990 and 1993 and the review in [22]), congestion depends on the time of the day and congestion occur at bottlenecks. We consider a reduced form for the equilibrium user cost (with no toll); it is given by : $g(N/s)=\delta(N/s)$, with $\delta=(\boldsymbol{B}+\boldsymbol{G})/\boldsymbol{B}.\boldsymbol{G}$, where $\boldsymbol{B}$ is the unit cost of early arrival and $\boldsymbol{G}$ is the unit cost of late arrival. Note that the congestion cost is independent of the value of time ν. The social congestion cost is therefore equal to $Ng(N/s)=\delta N^2/s$.

These authors show that half of the cost is schedule delay cost, while the other half is congestion cost. In particular, the optimal user cost (when roads are optimally tolled) is:

$$g_o\left(\frac{N}{s}\right)=\frac{\delta}{2}\frac{N}{s}=\frac{1}{2}g\left(\frac{N}{s}\right),$$

The social welfare function, if we restrict the benefit to the transportation only (as done in standard transportation economics) is (without road pricing)

$$W\left(Q^{st},s\right)=\mathcal{B}N-TT^{\min}\bullet\phi\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)-\delta\frac{N^2}{s}-\zeta s-\psi m^{st}, \quad (7)$$

where $\mathcal{B}$ is the individual benefit to travel. and ζ is the unit (annualized) construction cost and ψ Is the (annualized) maintenance cost.

With optimal pricing, the user cost (with toll redistributed as an lump sum) is halved, so the welfare is:

$$W\left(Q^{st},s\right)=\mathcal{B}N-TT^{\min}\bullet\phi\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)-\delta\frac{N^2}{2s}-\zeta s-\psi m^{st} \quad (8)$$

We assume that initially the planner (or the local authority) decide the construct of the roads (determined the road capacity). Then with usage and meteorologic condition, the road deteriorates. The planner then decide the maintenance level. We discuss here a stationary state regime, i.e. a situation with constant road quality.

Road capacity differ with or without road pricing, as discussed below.

3 Sequential capacity and maintenance

3.1 Optimal road capacity for new roads

Consistent with the standard approach in the literature, we assume that road capacity is exogenously given and that maintenance considerations are disregarded, which also reflects the situation observed in practice.

3.1.1 With road pricing

To compute the optimal capacity—and to decide whether additional capacity should be added—we assume that roads when built are in perfect condition, so that $Q^{st} = Q^{\max}$ and $\phi\left(\left(Q^{\max} - Q^{st}\right)/Q^{\max}\right) = 1$. This assumption corresponds to evaluating the decision regarding the initial size of a newly built road.

For an optimally tooled road, the optimal capacity satisfies:

$$\frac{\partial W\left(Q^{\max}, s\right)}{\partial s} = \delta \frac{N^2}{2s^2} - \zeta = 0.$$

So the optimal road capacity, denoted by s^o is given by:$s_o = N\sqrt{\delta/\left(2\zeta\right)}$,so the user cost (schedule and congestion cost) is $\delta N/\left(2s_o\right) = N\sqrt{\delta\zeta/2}$, and the optimal capacity construction cost is:$s_o = N\sqrt{\delta/\left(2\zeta\right)}$ Note that for the optimal capacity the sum of total congestion cost and total toll cost is: $\delta N/s_o = N\sqrt{2\zeta\delta}$.

At the optimum, congestion is eliminated, and the corresponding reduction in user costs is transformed into toll revenue. The toll revenue, TR—equal to half of the total (social) user cost—is: $TR = \delta/2\left(N^2/s\right)$.

So, for the optimal value of the capacity, we have

$$TR_o = \frac{1}{2}\delta\frac{N^2}{s_o} = N\sqrt{\frac{\delta\zeta}{2}}.$$

This shows that the optimal toll revenue TR^o equals the optimal maintenance cost ζs^o. This is a *self-financing result* (proved in the static congestion literature) is adapted here for the closed form of a dynamic model: if roads are tolled optimally and capacity is set at its optimal level, toll revenues exactly cover construction costs (see Arnott *et al*, 1990 and 1993, for the case of elastic demand).

Observe that the sum of user's cost, construction cost minus toll revenue is equal to $\delta N/s_o$, since toll revenue just covers capacity cost.

One should note that in a network, more capacity does not necessarily imply less congestion if roads are not tolled. This was shown the first time in the seminal article of Wardrop [40]. So more capacity does not imply less congestion, even if demand is inelastic.

3.1.2 Optimal capacity without road pricing and comparison

Without toll pricing, the optimizing problem for the capacity is:

$$\underset{s}{Min}\left(\delta\frac{N^2}{s} + \zeta s\right),$$

which leads to a minimized value of: $s_{oo} = N\sqrt{\delta/\zeta}$. Therefore, the congestion and schedule delay cost is: $\delta N^2/s_{oo} = N\sqrt{\delta\zeta}$. With tolling (compared with the no toll regime, the relative capacity is reduced by about 29%.

$$\frac{s_{oo}-s_o}{s_{oo}} = \frac{N\sqrt{\frac{\delta}{\zeta}} - N\sqrt{\frac{\delta}{2\zeta}}}{N\sqrt{\frac{\delta}{\zeta}}} = 1-\sqrt{0.5} \approx 29,2\,9\%$$

The sum of user's cost, construction cost is $\delta N^2/s_{oo} + \zeta s_{oo} = 2N\sqrt{\delta\zeta}$. The congestion and construction saving with tolling (and with optimized capacities) is:

$$\left(\delta\frac{N^2}{s_{oo}} + \zeta s_{oo}\right) - \frac{1}{2}\delta\frac{N^2}{s_o} = \left(2-\sqrt{\frac{1}{2}}\right) N\sqrt{\delta\zeta} \approx 1.292\,9N\sqrt{\delta\zeta}.$$

So the total relative cost saving when road are tolled and capacity is optimized (compared with the no toll regime) is about 65% :

$$\frac{\left(\delta\frac{N^2}{s_{oo}} + \zeta s_{oo}\right) - \frac{1}{2}\delta\frac{N^2}{s_o}}{\left(\delta\frac{N^2}{s_{oo}} + \zeta s_{oo}\right)} \approx 64,6\,55\%.$$

This saving account for a reduction in the capacity and a reduction in congestion (here congestion is totally cleared out with the toll). We have proved:

Proposition 1 *Consider dynamic congestion and the capacity choice (ignoring maintenance). Capacities are optimized in the toll and no toll regime. Then the relative capacity reduction is about* $29,29\%$ *and the cost saving is about* $64,65\%$*, when the road is tolled, compared to the non toll regime..*

With tolls, congestion is totally cleared (it is a pure deadweight loss) and the capacity is smaller. These two benefits add.

3.2 Optimal maintenance (with or without pricing)

Now we assume that roads have been built and we compute the optimal level of maintenance. The fact that the road are tolled or not plays no role here.

Observe that the optimal maintenance does not depend on the tolling regime. In this case, the capacity, s . The welfare is given (up to an additive constant, k) by

$$W\left(m^{st}\right) = -\Phi\left(Q^{st}\right) - \psi m^{st} + k,$$

where $\Phi\left(Q^{st}\right)$ represents the uncongested travel time (see 5).

The optimal level of maintenance satisfies:

$$\frac{\partial W\left(Q^{st}, s\right)}{\partial m^{st}} = \frac{TT^{\min}}{Q^{\max}} \bullet \phi'\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)\frac{1}{a} - \psi = 0.$$

Note that a solution of the first-order condition is a maximum of the welfare function is convex in m^{st}.

For the quadratic specification:

$$W\left(Q^{st},s\right)=-\Phi_2\left(Q^{st}\right)-\psi m^{st}+k, \tag{9}$$

with

$$\Phi_2\left(Q^{st}\right)=TT^{\min}\bullet\left[1+0.5\left(\frac{Q^{\max}-Q^{st}}{Q^{\max}}\right)^2\Delta\phi^{\max}\right] \tag{10}$$

Then, after substitution of Q^{st} we obtain

$$\frac{\partial W\left(Q^{st},s\right)}{\partial m^{st}}=\frac{1}{a}\frac{TT^{\min}}{\left(Q^{\max}\right)^2}\bullet\left(Q^{\max}-\frac{1}{a}\left(m^{st}-b\frac{N}{s}\right)\right)\Delta\phi^{\max}-\psi. \tag{11}$$

For minimum quality, we require:

$$\left.\frac{\partial W\left(Q^{st},s\right)}{\partial m^{st}}\right|_{m^{st}=bN/s}=\psi^c-\psi>0,$$

that is maintenance unit cost should be small enough:

$$\psi<\frac{1}{a}\frac{TT^{\min}}{Q^{\max}}\bullet\Delta\phi^{\max}:=\psi^c. \tag{12}$$

If this condition is not meet, road are not maintained and free flow travel time is: $TT^{\min}\bullet\left(1+0.5\phi^{\max}\right)$.

If $\psi<\psi^c$, we have a strictly positive optimal maintenance level which satisfies $\partial W\left(Q^{st},s\right)/\partial m^{st}=0$ (see(11)). Therefore, the optimal level of maintenance for the sequential case is:

$$m^{st}_{oE}=b\frac{N}{s}+\left(\psi^c-\psi\right)a^2\frac{\left(Q^{\max}\right)^2}{\Delta\phi^{\max}\bullet TT^{\min}}.$$

Recall the condition 3 ($b\frac{N}{s}<m^{st}_o<aQ^{\max}+b\frac{N}{s}$). The LHS condition is clearly satisfied if $\psi<\psi^c$. The RHS condition (quality less that the maximum quality) is satisfied if $\psi>0$. So we assume that

$$0<\psi<\psi^c. \tag{13}$$

The expression can be rewritten alternatively as (recall 12):

$$m^{st}_{oE}=b\frac{N}{s}+\left(1-\frac{\psi}{\psi^c}\right)aQ^{\max} \tag{14}$$

Note that maintenance increases (via two channels), when the number of users increases. The other comparative statics is left to the reader.

For the optimal maintenance, road quality is:

$$Q^{st}_o=\frac{1}{a}\left(m^{st}_o-b\frac{N}{s}\right)=\left(1-\frac{\psi}{\psi^c}\right)Q^{\max}>0$$

Note that the capacity clears out, and so he tolling regime is thus irrelevant at this stage. Then:

$$\frac{Q^{\max} - Q_o^{st}}{Q^{\max}} = \left(\frac{\psi}{\psi^c}\right) \in (0, 1) .$$

So with no pricing, any value of the capacity and optimal maintenance, the uncongested user travel time, denote by $\Phi(Q^{st})$ is

$$\Phi_2(Q^{st}) = TT^{\min} \bullet \left[1 + \frac{\Delta\phi^{\max}}{2}\left(\frac{\psi}{\psi^c}\right)^2\right], \psi \in (0, \psi^c), \psi^c \text{ given by 12} \quad (15)$$

We have proved when capacities are priced or not:

Proposition 2 *Consider the travel time function (10). The quality is given by (2). Condition (13) holds. Assume dynamic congestion cost and quadratic quality model. Capacity and maintenance are chosen sequentially. The optimal maintenance, is given by equation (14) and the uncongested user travel time is given by equation (15).*

Note that $\Phi(Q^{st})/TT^{\min} \in (1, \phi(1))$. The welfare function without optimal pricing, after replacement of the optimal capacity and the optimal maintenance level is :

$$\begin{aligned} W\left(Q^{st}, s\right) &= \mathcal{B}N - TT^{\min} \bullet \left[1 + 0.5\left(\frac{\psi}{\psi^c}\right)^2 \Delta\phi^{\max}\right] \\ &\quad - \psi a Q^{\max}\left(1 - \frac{\psi}{\psi^c}\right) - 2N\sqrt{\delta\zeta} - b\psi N\sqrt{\frac{\zeta}{\delta}}. \end{aligned}$$

This expression will be useful in the next section.

4 Simultaneous capacity and maintenance choices

In practice, the decisions to build a road and to maintain it are often made separately, as they typically fall under the authority of different governmental bodies. In the United States, state departments of transportation usually oversee the construction of state and federal highways, while maintenance responsibilities are divided among state, county, and municipal agencies depending on road ownership. In Georgia, road construction may be managed by the Georgia Department of Transportation or by local governments, while maintenance is handled by the entity that owns the roadway—GDOT for state highways, county governments for county roads, and municipal governments for city streets. As another example, according to the World Bank (see, e.g. [41] and [42]) policy makers in Argentina often underfunded maintenance relative to investment.

Typically, construction and maintenance most often depend on different departments, and rely of different funding.

Here we explored a new (hypothetical) situation where these construction and maintenance decisions are made in tandem and harmonized.

We study the minimization problem. Let $(F(s, m^{st}) = -W(Q^{st}, s))$, then the (unique) planner faces the problem:

$$\min_{s,\, m^{st}} \; F(s, m^{st}) := TT^w + \zeta s + \psi m^{st}, \tag{16}$$

with:

$$TT^w = \Phi_2\left(Q^{st}\right) + \delta \frac{N^2}{s}, . \tag{17}$$

where $\Phi_2\left(Q^{st}\right)$ is given by (10) and Q^{st} is given by (2).

We treat N, a, b, δ, ζ, ψ, $TT^{\min}$, $Q^{\max}$, $\Delta\phi^{\max}$ as positive parameters. The decision variables are s and m^{st}.

4.1 Welfare function with respect to maintenance

Let $F = \Phi_2(Q^{st}) + \delta N^2/s + \zeta s + \psi m^{st}$, the first-order condition of maintenance m^{st} is

$$\frac{\partial F}{\partial m^{st}} = \frac{\partial \Phi}{\partial Q^{st}} \cdot \frac{\partial Q^{st}}{\partial m^{st}} + \psi.$$

By substitution we obtain:

$$Q^{\max} - Q^{st} \;= K; K =: \frac{\psi\, a\left(Q^{\max}\right)^2}{TT^{\min}\Delta\phi^{\max}}. \tag{18}$$

Thus the optimal stationary flow level implied by the m^{st}-FOC is $Q^{st} = (Q^{\max} - K,)$ and (2):

$$m^{st} \;=\; a\left(Q^{\max} - K\right) + \frac{bN}{s}. \tag{19}$$

The final expression for maintenance depend on the optimal capacity, dertemined below.

4.2 Welfare function with respect to the capacity

Differentiate $F = \Phi_2(Q^{st}) + \delta N^2/s + \zeta s + \psi m^{st}$ with respect to s we obtain the first-order condition:

$$\frac{\partial F}{\partial s} = \frac{\partial \Phi}{\partial Q^{st}} \cdot \frac{\partial Q^{st}}{\partial s} + \frac{\partial}{\partial s}\left(\delta \frac{N^2}{s}\right) + \zeta = 0,$$

which leads to

$$s^* = N\sqrt{\frac{b\psi + \delta}{\zeta}}. \tag{20}$$

Therefore the optimal level of maintenance is:

$$m_S^{st} = (1 - \frac{\psi}{\psi^c})aQ^{\max} + b\sqrt{\frac{\zeta}{b\psi + \delta}}. \quad (21)$$

No that the first term is the same as for the sequential decision; the second term differs because the capacity is now larger. So both the maintenance cost and the capacities are larger in the simultaneous case. In other words, this suggests that in the sequential case (standard) capacity and maintenance case are too small compared to the simultaneous case.

Recall that with no pricing the optimal capacity in the sequential case is : $s_{oo} = N\sqrt{\delta/\zeta}$, so both expressions coincides when maintenance cost ψ is zero. When maintenance is taken into account in the road capacity decision, the optimal capacity in the simultaneously case is larger than the optimal capacity the standard sequential case: $s_{oo} > s^*$:

$$\frac{s^*}{s_{oo}} = \frac{N\sqrt{\frac{b\psi+\delta}{\zeta}}}{N\sqrt{\frac{\delta}{\zeta}}} = \sqrt{1 + b\psi/\delta} > 1.$$

The same qualitative results can be proved with pricing.

Without pricing, the optimal construction cost is: $\zeta s^* = N\sqrt{(b\psi + \delta)\,\zeta}$ and the social congested time cost is:

$$\frac{\delta N^2}{s^*} = \delta N\sqrt{\frac{\zeta}{b\psi + \delta}}$$

We can now revisit the self-financing theorem with road pricing, in the simultaneous choice of capacity and maintenance.

4.3 Back to self-financing

With toll revenue, we find (with an analogous computation as above):

$$s^{**} = N\sqrt{\frac{b\psi + \delta/2}{\zeta}}$$

so the construction cost is: $\zeta s^{**} = N\sqrt{(b\psi + \delta/2)\,\zeta}$. Since the toll revenue again half of the social cost, we have:$N\sqrt{\frac{b\psi+\delta}{\zeta}}$

$$TR = \frac{1}{2}\delta\frac{N^2}{s^*} = \frac{1}{2}\delta\frac{N^2}{N\sqrt{\frac{b\psi+\delta}{\zeta}}} = \frac{\delta}{2}N\sqrt{\frac{\zeta}{b\psi + \delta}}.$$

and so the maintenance cost are now higher than in the sequential case. Therefore with simultaneous decisions self-financing theorem breaks down. Maintenance cost minus toll revenue is now equal to

$$D(\psi) = N\sqrt{(b\psi + \delta/2)\,\zeta} - \frac{\delta}{2}N\sqrt{\frac{\zeta}{b\psi + \delta}} > 0 \text{ if } \psi > 0$$

We have $D(0) = 0$ and $D(\psi) > 0$ if $\psi > 0$, since $D'(\psi) > 0$. So revenues from road optimal road pricing are insufficient to cover maintenance costs. We have proved

Lemma 3 *Consider a simultaneous computation of capacity and maintenance. The optimal capacity and the optimal level of maintenance are give by (20) and (21), respectively. They are both larger compared with the sequential case. Toll revenues from optimal road pricing do not cover construction cost in the simultaneous case.*

4.3.1 Travel time and welfare

Back-substitution for optimal maintenance and optimal free flow travel time we obtain

$$TT^{w} \;=\; TT^{\min}\left(1 + \frac{\Delta\phi^{\max}}{2}\left(\frac{\psi}{\psi^{c}}\right)^{2}\right) \;+\; \delta\, N\sqrt{\frac{\zeta}{b\psi + \delta}}. \qquad (22)$$

Therefore the uncongested travel time (first term in the above expression) is the same as in the sequential case (See Proposition 1). The second term (the congested travel time) is smaller since the capacity is now larger. As a consequence the total travel time T^{w} is smaller than in the sequential case.

So the optimal total cost is (after replacement):

$$F_S(s_o, m_o^{st}) = TT^{\min}\left(1 + \frac{\Delta\phi^{\max}}{2}\left(\frac{\psi}{\gamma^{c}}\right)^{2}\right) + \psi a Q^{\max}\left(1 - \frac{\psi}{\psi^{c}}\right) + 2N\sqrt{\zeta(b\psi + \delta)}$$

The s-FOC has two channels: an indirect congestion channel via $Q^{st}(s)$ and a direct "scale" channel via $\delta N^2/s$. Both scale with s^{-2}, which is why the solution is a simple quadratic in s. The uncongested travel time is the same as in Proposition 1.

We can verify that the sequential optimization is less efficient than the simultaneously optimization;

$$\begin{aligned} F_S(s_o, m_o^{st}) \quad &< \quad F_E(s_o, m_o^{st}), \\ \Leftrightarrow \quad & b\psi + \delta < \delta + b\psi + \frac{1}{\delta}\left(\frac{b\psi}{2}\right)^{2}\frac{1}{\delta} \end{aligned}$$

So we have proved (see also Lemma 3):

Proposition 4 *Consider the travel time function (10). The quality is given by (2). Condition (13) hold. Assume dynamic congestion cost and quadratic quality model. Capacity and maintenance are chosen simultaneously. Then (A) the uncongested travel time is the same and the congested costs is smaller than in the simultaneous case compared with the sequential choice. (B)The optimal capacity and the level of maintenance are larger than in the sequential case. (C) The total welfare is higher in the simultaneous compared with the sequential choice. Road capacity is not fully financed by road pricing.*

The exact value of the saving for the simultaneous optimization is of interest. Let chose for example δ as the parameter of interest. The social saving $SS(\delta)$ (using a simultaneous procedure rather than the sequential decision) is

$$SS(\delta) = \sqrt{\zeta}\left(2\sqrt{\delta} + b\psi\sqrt{\frac{1}{\delta}} - 2\sqrt{b\psi + \delta}\right) \geqslant 0$$

After a long but routine computation, a clean result can be obtained:.

$$\frac{d\,(SS(\delta))}{d\delta} < 0 \text{ iff } \delta < \frac{b\psi}{3}$$

So the saving is decreasing with the congestion parameter δ iff the congestion parameter (and therefore the level of congestion) is small enough ($\delta < b\psi/3$).

5 Perfect Competition with Iceberg Costs

It has been recognized since a long time that economic development is closely related to transportation (see the entertaining article of [5], and its references). Our scope now is to link transport decisions to economic development, i.e. to bridge the gap between transportation and economic development, in the context of capacity and maintenance choices.

This section lays the groundwork for the subsequent analysis of capacity and maintenance decisions, taking into account their broader economic consequences—extending beyond the conventional focus on the value of time. Readers familiar with the topic may wish to proceed directly to the main (standard) results. We refer the reader Helpman and Krugman [20],who apply iceberg costs in competitive (and monopolistic) settings (see also, Behrens and Picard, [4], in economic geography). However, these contributions have thus far overlooked the roles of congestion and maintenance.

Gibbons *et al.* (2019) provide evidence that new road infrastructure in the UK raises firm entry, employment, and productivity, although the benefits are spatially uneven (see [15]). The link between maintenance and econoomic activites is also studied by Gerlter *et al* (2024) , who find that better road maintenance supports job creation, transitions from informal to formal employment, and local welfare gains (see [13]).

We revisit perfect competition when goods are shipped from the firm (workplace) to spatially distributed consumers and the shipping technology features an *iceberg* cost driven by travel time. Firms are price takers and set the factory-gate price equal to marginal cost. With labour as the only input and a unit labour requirement normalized to one, the factory-gate marginal cost equals the competitive wage w, so that $p = w$.

Let TT denote the shipping travel time per unit from the workplace to the consumer, and let v_F be the firm's value of time (VOT) per unit shipped (in currency per unit time). We map the time cost into an iceberg factor by expressing time in *resource* units relative to w:

$$\kappa(TT) \equiv \frac{v_F}{w} TT \qquad (\kappa \geq 0), \tag{23}$$

so that shipping one delivered unit requires producing $(1+\kappa)$ units at the source. The delivered price seen by consumers is then

$$p^{\text{del}}(TT) = (1 + \kappa \bullet TT)\, w = \left(1 + \frac{v_F}{w} TT\right) w = w + v_F\, TT. \tag{24}$$

Hence the iceberg representation based on travel time is *equivalent* to an additive time cost $v_F TT$, while keeping the firm price-taking and the factory-gate price pinned down by w.

Equilibrium with linear demand. For inverse demand $P(Q) = \alpha - \beta Q$ $(\alpha, \beta > 0)$, the competitive equilibrium satisfies $P(Q) = p^{\text{del}}(TT)$, delivering

$$Q^*(TT) = \frac{\alpha - (w + v_F TT)}{\beta} \quad \text{if } \alpha > w + v_F TT, \text{ and } Q^* = 0 \text{ otherwise.} \tag{25}$$

Comparative statics are immediate:

$$\frac{\partial Q^*(TT)}{\partial TT} = -\frac{v_F}{\beta} < 0, \qquad \frac{\partial Q^*}{\partial v_F} = -\frac{TT}{\beta} < 0, \qquad \frac{\partial Q^*}{\partial w} = -\frac{1}{\beta} < 0.$$

Since marginal cost is constant and firms are price takers, pass-through of the time cost to the delivered price is complete: $dp^{\text{del}}/dTT = v_F$. Operating profits remain zero; the resource dissipation in transport equals $v_F TT\, Q^*(TT)$. The resource dissipation in transport increases or decreases with travel time since with larger travel time there is more resources dissipation, but larger travel time decrease production: two opposing forces:

$$\frac{\partial v_F TT\, Q^*(TT)}{\partial TT} = v_F \left(\frac{\alpha - (w + 2 * v_F TT)}{\beta}\right) \lesseqgtr 0.$$

Welfare analysis. Let $\tau \equiv v_F TT$ denote the *per-unit* real resource cost implied by travel time. With linear inverse demand $P(Q) = \alpha - \beta Q$ and competitive factory-gate marginal cost w, the equilibrium quantity is $Q^*(\tau) = (\alpha - w - \tau)/\beta$ (for $\alpha > w + \tau$).∂

Total welfare equals willingness-to-pay minus *real* resource costs of production and shipping:

$$W(\tau) = \int_0^{Q^*(\tau)} (\alpha - \beta q)\, dq \;-\; w\, Q^*(\tau) \;-\; \tau\, Q^*(\tau). \tag{26}$$

Using the equilibrium condition $\alpha - \beta Q^*(\tau) = w + \tau$, we obtain the compact form

$$W(\tau) = \frac{1}{2}\,\beta\,(Q^*(\tau))^2 = \frac{1}{2}\,\frac{(\alpha - w - \tau)^2}{\beta}, \tag{27}$$

which is strictly decreasing and *convex* in τ ($= \tau_F TT$), and thus in TT.

Relative to the frictionless benchmark $\tau = 0$, the welfare loss decomposes into a *resource* rectangle and a *distortion* triangle. It is the sum of transport cost ($\tau\, Q^*(\tau)$) and deadweight (quantity) loss ($0.5\tau \left(Q^0 - Q^*(\tau)\right)$).

$$\Delta W_{\text{total}} \equiv W(0) - W(\tau) \;=\; \frac{(\alpha - w)\, v_F TT - \frac{1}{2}(v_F TT)^2}{\beta}, \tag{28}$$

where $Q^0 = (\alpha - w)/\beta$ is the quantity when $TT = 0$. Equivalently, the pure deadweight "triangle" is $\frac{1}{2}\tau^2/\beta = \frac{1}{2}(v_F TT)^2/\beta$, while the rectangle $\tau Q^*(\tau) = v_F TT\,(\alpha - w - v_F TT)/\beta$ captures real resources dissipated in transport. Observe that the welfare loss increases with travel time TT.

$$\frac{\partial \Delta W_{\text{total}}}{\partial TT} = \frac{(\alpha - w - v_F)\, TT)}{\beta} > 0$$

Endogenous wage under perfect competition. So far the wage w was taken as given. We now pin it down by labour-market clearing. With iceberg costs $\kappa(TT) = (v_F/w)TT$, producing one delivered unit requires $1 + \kappa(TT) = 1 + \frac{v_F}{w}TT$ units at the workplace, so labour demand per delivered unit equals $1 + \frac{v_F}{w}TT$.

Let labour supply be upward sloping: $L^s(w)$ with $L'_w > 0$. Market clearing requires

$$\left(1 + \frac{v_F}{w} TT\right) Q = L^s(w). \tag{29}$$

On the product market, price equals delivered marginal cost, hence $P(Q) = p^{\text{del}} = w + v_F TT$. For linear inverse demand $P(Q) = \alpha - \beta Q$ we obtain

$$w = \alpha - \beta Q - v_F TT. \tag{30}$$

Equations (29)–(30) jointly determine (w, Q) for any given TT.

Comparative statics (general case). Define the equilibrium by $F(w, TT) = \left(1 + \frac{v_F}{w} TT\right) \frac{\alpha - (w + v_F TT)}{\beta} - L^s(w) = 0$. Assuming $F_w < 0$ (standard stability), the Implicit Function Theorem gives

$$\frac{dw^*}{dTT} \;=\; -\frac{F_{TT}}{F_w}. \tag{31}$$

Because $F_{TT} = \frac{v_F}{w}\frac{\alpha-(w+v_F TT)}{\beta} - \left(1+\frac{v_F}{w}TT\right)\frac{v_F}{\beta} - \frac{v_F TT}{w^2}\frac{\alpha-(w+v_F TT)}{\beta}$, we have $F_{TT} < 0$ whenever demand is interior ($\alpha > w + v_F TT$). Hence $dw^*/dTT > 0$ if $F_w < 0$, i.e. *equilibrium wages rise with shipping time.* Intuition: longer TT raises the labour requirement per delivered unit (via the iceberg factor), shifting labour demand up and bidding up w; at the same time, higher delivered marginal cost reduces Q, partially offsetting the effect. The net effect on w is positive, as expected, under standard slopes.

Closed form with linear labour supply. Let $L^s(w) = \lambda w$ with $\lambda > 0$. Combining (29) and (30) yields a single equation in w:

$$\left(1+\frac{v_F}{w}TT\right)\frac{\alpha-(w+v_F TT)}{\beta} \;=\; \lambda w. \tag{32}$$

This quadratic has a unique economically relevant root $w^*(TT) \in (0, \alpha - v_F TT)$ for $\alpha > v_F TT$. Differentiating (32) implicitly verifies $\frac{dw^*}{dTT} > 0$ under $\lambda > 0$, and recovers the special case of a perfectly elastic labour supply ($\lambda \to \infty$) in which w is pinned at its outside option and our earlier results obtain.

Let

$$\left(1+\frac{v_F}{w}TT\right)\frac{\alpha-(w+v_F TT)}{\beta} \;-\; \lambda w = 0$$

The solution is:

$$w^*\left(TT\right) = \frac{\alpha - 2v_F TT + \sqrt{\alpha^2 + 4\beta\lambda v_F TT\left(\alpha - v_F TT\right)}}{2\left(1+\beta\lambda\right)} \tag{33}$$

Delivered price and quantities. Given $w^*(TT)$, the delivered price is $p^{\mathrm{del}}(TT) = w^*(TT) + v_F TT$, and

$$Q^*(TT) = \frac{\alpha - p^{\mathrm{del}}(TT)}{\beta} = \frac{\alpha - (w^*(TT) + v_F TT)}{\beta}, \tag{34}$$

which declines in TT both directly (via $v_F TT$) and indirectly (via $w^*(TT)$).

Welfare with endogenous w. All welfare expressions from the previous subsection remain valid after replacing w by $w^*(TT)$. Because $\frac{dw^*}{dTT} > 0$, the welfare loss from travel time is *larger* than under an exogenous wage: higher wages amplify the delivered marginal cost and further reduce Q^*. So:

$$W(TT) = \frac{1}{2}\beta\left(Q^*(TT)\right)^2 = \frac{1}{2\beta}\left(\alpha - \left(w^*(TT) + v_F TT\right)\right)^2 \tag{35}$$

and

$$\frac{dW(TT)}{dTT} = -\frac{1}{\beta}\left(\alpha - \left(w^*(TT) + v_F TT\right)\right)\left(\frac{dw^*(TT)}{dTT} + v_F\right) < 0,$$

i.e. with larger travel time costs, TT, welfare decreases because wage increase and iceberg transport cost of good increase.

Spatial interpretation. If travel time depends on distance d and congestion, we may write $TT = TT(d)$ (e.g., $TT'(d) > 0$). Then the delivered price to a consumer at distance d is

$$p^{\text{del}}(d) = w + v_F\, TT(d), \tag{36}$$

which rises with d. Higher TT (due to longer distances or congestion) shrinks the market area served from a given workplace and lowers total traded quantity. Any reduction in TT (infrastructure, logistics, or congestion management) shifts the supply schedule downward in delivered-price space and increases Q^*.

Under perfect competition with competitively set wages, transportation frictions that scale with travel time enter as an iceberg factor $\kappa(TT) = (v_F/w)TT$. They do not create markups but raise delivered prices one-for-one with the firm's VOT times shipping time, depressing quantities and welfare.

6 Capacity and Maintenance in Competition

We consider here the standard optimization (sequential choice) and treat the case of no pricing (pricing and simultaneous choices can be treated analogously).

We consider a competitive economy with the following elements (recall for clarify): Linear inverse demand $P(Q) = \alpha - \beta Q$, with $\alpha, \beta > 0$. Factory-gate price**:** Equal to the inelastic wage p. Value of time**:** Each firm values time at $v_F > 0$, so the delivered price is $p^{del}(TT) = p + v_F TT$. Travel time (capacity channel):

$$TT = TT^{\min} + \delta\,\frac{N^2}{s}, \quad \delta > 0,$$

where N is traffic, s is capacity, and $TT^{\min}$ is free-flow time. Construction cost**:** ζs with $\zeta > 0$. Maintenance (quality channel)**:** Road quality satisfies

$$Q^{st} = \frac{1}{a}\left(m^{st} - b\tfrac{N}{s}\right), \quad a, b > 0,$$

with stationary maintenance expenditure m^{st} (cost ψm^{st}). Quality impact on free-flow time:

$$\Phi(Q^{st}) = TT^{\min}\left[1 + \frac{\Delta\phi^{\max}}{2}\left(\frac{Q^{\max} - Q^{st}}{Q^{\max}}\right)^2\right],$$

where $Q^{\max}$ is maximal quality, $\Delta\phi^{\max} > 0$ scales deterioration.

Proposition 5 *Consider the sequential procedure without pricing: first choose capacity s for a new road with $Q^{st} = Q^{\max}$, then choose stationary maintenance m^{st}. Under the setup above, we obtain:*

1. ***Capacity.*** *The welfare FOC for s reduces to a depressed cubic with (at most) two positive roots. When two roots exist, the larger maximizes welfare. A sufficient existence condition is $4C_1^2C_2^3 > 27$, with*

$$C_1 = \sqrt{\tfrac{\delta}{\zeta}\tfrac{v_F}{\beta}}\,\sqrt{N}, \qquad C_2 = \alpha - (p + v_F TT^{\min}) > 0.$$

The optimal capacity s^* increases with usage N, the value of time v_F, and the congestion index δ, and decreases with construction cost ζ.

2. ***Maintenance.*** *Given s^*, the optimal stationary maintenance level is*

$$m^{st*} = \frac{bN}{s^*} + a\left(Q^{\max} - K\,\varphi\right),$$

where

$$K = \frac{\psi\, a\,(Q^{\max})^2}{TT^{\min}\,\Delta\phi^{\max}}, \qquad \varphi = \frac{\beta}{v_F}\cdot\frac{1}{\alpha - (p + v_F TT)}.$$

Maintenance rises with N (through $\frac{bN}{s^*}$) and with the value of time v_F (since a higher v_F raises the return to reducing TT).

3. ***Welfare.*** *With perfect competition (no markups), welfare is strictly decreasing and convex in TT. Therefore, an increase in v_F raises the marginal value of reducing TT, which weakly increases both s^* and m^{st*}.*

See proof in the Appendix 2

The limiting cases are discussed below:

If $v_F \to 0$, then $p^{del}(TT) \to p$ and capacity reverts to the "transport-only" optimum from the previous section; the maintenance rule simplifies with $\varphi \to 0$.

If $N \to 0$, congestion vanishes so capacity becomes irrelevant and only the free-flow time and maintenance channel matter.

If $\psi \to 0$, maintenance is costless, so $Q^{st} \to Q^{\max}$ and free-flow time is minimized.

If $\zeta \to \infty$, capacity is prohibitively costly, so the equilibrium tends to the minimal feasible s.

This demonstrated how capacity and maintenance decisions can be consistently embedded within an economic framework by introducing iceberg costs into a model of perfect competition. Travel time, expressed as an iceberg factor proportional to the firm's value of time, translates directly into higher delivered prices, lower equilibrium quantities, and welfare losses that are strictly increasing and convex in travel time. When wages are endogenized through labour-market clearing, these effects are amplified: longer shipping times raise labour demand per delivered unit, bid up wages, and further increase delivered marginal costs.

By integrating transport frictions with product and labour market equilibria, the analysis provides a more satisfactory economic foundation for studying capacity and maintenance choices. This approach extends beyond the conventional valuation of time by explicitly linking infrastructure performance, congestion, and maintenance policies to welfare outcomes and market equilibrium.

7 Conclusions

Future research could extend the competitive model developed here to a dynamic duopoly (or oligopolist) framework in which road capacity and maintenance are chosen strategically, recognizing that reconstruction is an infrequent, capital-intensive decision, whereas preservation activities can be adjusted flexibly over time. Embedding these asymmetric adjustment speeds in a multi-stage game would clarify how competing operators balance short-term pricing with long-term quality management, and how such interactions affect welfare, investment incentives, and regulatory outcomes.

A promising avenue for future research is to extend the analysis to a strategic setting in which toll and maintenance decisions are determined within a game-theoretic framework. In the literature on congestible facilities, duopolistic competition between roads has been modelled extensively, both as one-stage toll-setting games and as two-stage capacity–toll games in which capacity is chosen in the first stage and tolls in the second. However, such literature omits maintenance.

In [8] the authors model road competition as a two-stage game where roads set capacity and tolls sequentially. They show how capacity decisions made first can influence toll competition. In [9] the authors analyze a two-stage game in which competing firms choose capacities and subsequently compete in prices under network externalities and congestion. Recent growth in the literature applying game-theoretic methods to transportation can be attributed, at least in part, to processes of deregulation and intensified competition. We refer the reader to the recent monograph of Small, Verhoef and Lindsey ([32]°

When the strategic variables are capacity and tolls, it is natural to assume that capacity is fixed in the initial stage, with tolls adjusted subsequently. By contrast, the appropriate sequencing is less obvious when maintenance and tolls are the decision variables. Major reconstruction or rehabilitation projects occur infrequently and involve large, sunk costs, whereas pavement preservation and other maintenance activities can be scheduled more flexibly over time. Toll changes, on the other hand, can be implemented rapidly or gradually depending on tolling technology, user acceptance, and institutional or regulatory constraints.

Incorporating these asymmetries in decision timing into a dynamic or multi-stage duopoly model could help clarify how firms balance short-term pricing flexibility with long-term infrastructure quality management. Such an approach would provide richer insights into the joint optimization of tolls and maintenance under competition, with implications for welfare, investment incentives, and regulatory policy.

In [35] the authors shows that when different governmental authorities compete in setting road charges and choosing capacity, this strategic interaction can lead to inefficient outcomes, such as under- or over-investment in capacity and suboptimal toll levels, compared to coordinated decision-making. In Evaluating Urban Transport Improvements: Cost–Benefit Analysis in the Presence of Agglomeration and Income Taxation Venables [36] develops a theoretical urban equilibrium model to demonstrate that transport improvements can expand city size, thereby amplifying agglomeration economies and boosting productivity for both new and existing urban workers. When these productivity gains are factored into cost–benefit analysis—and combined with distortionary income taxation—the resulting welfare benefits of transport investments are substantially greater than those captured by conventional appraisal methods. Agglomeration forces and taxation effects remain to be explicitly incorporated into the proposed model. Here we show that the market structure in the economy plays also a key role.

Future research should address several open questions at the interface of pavement quality, vehicle dynamics, and user costs. An important issue is whether vehicle-induced pavement damage is a monotonic function of speed, or if non-linear thresholds exist. Similarly, the relationship between pavement roughness, as captured by the International Roughness Index (IRI), and vehicle operating costs—including fuel consumption, wear, and maintenance—requires more systematic investigation. Another area of inquiry is whether road capacity itself depends on IRI, as surface conditions may directly affect traffic flow and safety margins. Finally, identifying which pavement quality measure most effectively captures the combined impacts on driving speed, accident risks, fuel and other user costs, and vehicle damage would provide a more reliable basis for both engineering standards and economic evaluation of infrastructure investments (see[43] and ([34]). Potholes exert a significant influence on overall road quality and performance Ben-Edigbe's study [3], based on traffic surveys in Nigeria, shows that adverse road surface conditions such as potholes and edge subsidence reduce optimum speeds by about 50%, demonstrating that such conditions significantly disrupt otherwise uninterrupted traffic flow.

When the value of time is proxied by the average net wage, the benefits are often insufficient to cover the full costs of infrastructure (construction and maintenance). By contrast, if time is valued at the gross wage, including employer contributions and production taxes, and if firms are treated at the level of their opportunity cost of time, the resulting benefit–cost ratio is more likely to cover both investment and maintenance costs. While this distinction is not explicitly discussed in the literature, it provides a plausible methodological hypothesis consistent with cost–benefit logic.

Finally, we wish to point out that the literature on capacity and maintenance often fails to adopt a sufficiently broad perspective. These decisions span several time scales, as sketched below. (1) The benefits of expanding road capacity depend on several factors: Mode choice: some users may shift from public transport to car travel.(2) Induced demand: shorter travel times may encourage additional car use. (3) Demographic growth: population tends to increase over the lifetime of road infrastructure.(4) Spatial dynamics: improved accessibility may attract new residents and firms. While items (1) and (2) can be modeled through activity- or mode-choice frameworks, items (3) and (4) can be captured with origin–destination (O–D) matrices.

A promising avenue for future research is to integrate a maintenance model with a dynamic traffic model ([14]). In such a framework, the traffic model would generate daily flows (and their composition) on the different roads, while its inputs would be road capacities and quality levels, both determined by maintenance decisions. This approach would allow us to test the conjectures advanced in this article on an actual transport network.

8 Appendices

8.1 Appendix 1: Setup and travel time mapping

Let $Q(t) \in \mathbb{R}_+$ denote the *quality* of a road segment at time t. A representative user's door-to-door travel time is

$$TT^w(t) = g(Q(t)), \qquad g'^w(t). \tag{37}$$

Here TT is the physical lower bound (free-flow, no congestion).

Cumulative usage and degradation

Let $n(t)$ be traffic flow (vehicles per hour) and s the practical capacity (veh/h). Define a cumulative usage (or damage) index

$$U(t) = \int_{t_0}^{t} \psi\left(\frac{n(\tau)}{s}\right) \mathrm{d}\tau, \qquad \psi'(\cdot) > 0, \tag{38}$$

with the "fourth-power" choice $\psi(x) = x^\gamma$ ($\gamma \simeq 4$) being common in pavement engineering. See an analogous approach (but for rail) in [19].

Assume marginal quality loss per unit cumulative usage is constant $b > 0$, so that $\mathrm{d}Q/\mathrm{d}U = -b$. By the chain rule,

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = -\,b\,\psi\left(\frac{n(t)}{s}\right). \tag{39}$$

Natural deterioration and maintenance

Add (i) natural deterioration at rate $a > 0$ and (ii) maintenance effort $m(t) \geq 0$ that instantaneously raises Q (in reduced form). The reduced-form continuous-time dynamics are

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = m(t) \;-\; a\,Q(t) \;-\; b\,\psi\left(\frac{n(t)}{s}\right), \qquad Q(t_0) = Q_0 \geq 0. \tag{40}$$

Linear-in-usage specialization

To match the linear specification used in your note, set $\psi(x) = x$. Then

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = m(t) - a\,Q(t) - b\,\frac{n(t)}{s}. \tag{41}$$

Steady state and conditions for equivalence with your formula

We seek a stationary quality level Q^{st} under constant usage $n(t) \equiv n^{\mathrm{st}}$ and constant maintenance $m(t) \equiv m^{\mathrm{st}}$.

Proposition 6 (Stationary quality) *Suppose:$a > 0$, $b > 0$. Usage is time-invariant: $n(t) \equiv n^{\mathrm{st}}$. Maintenance is time-invariant: $m(t) \equiv m^{\mathrm{st}}$. The linear-in-usage degradation holds: $\psi(x) = x^\gamma$. Quality is constrained to be nonnegative: $Q(t) \in \mathbb{R}_+$.*

Then the unique stationary solution of (41) *is*

$$Q^{\mathrm{st}} = \max\left\{0,\ \frac{1}{a}\left(m^{\mathrm{st}} - b\left(\frac{n^{\mathrm{st}}}{s}\right)^{\gamma}\right)\right\}. \tag{42}$$

Proof. Under (A1)–(A4), (41) is a linear ODE with constant coefficients and constant right-hand side. The unconstrained fixed point solves $0 = m^{\text{st}} - aQ^{\text{st}} - b(n^{\text{st}}/s)$, giving $Q^{\text{st}} = (m^{\text{st}} - b(n^{\text{st}}/s))/a$. Under (A5), we project onto $\mathbb{R}_+$, yielding (42). Uniqueness follows from linearity and $a > 0$ (global asymptotic stability). □ ■

With $\psi(x) = x^\gamma$ ($\gamma > 1$), the steady state becomes

$$Q^{\text{st}} = \max\left\{0,\ \frac{1}{a}\left(m^{\text{st}} - b\right)\left(n^{\text{st}}/s\right)^{\gamma}\right\}, \tag{43}$$

with the same stability logic. In the paper we consider $\gamma = 1$.

Transition dynamics

When (A2)–(A4) hold with constants $m^{\text{st}}, n^{\text{st}}$, the closed-form solution of (41) is

$$Q(t) = Q^{\text{st}} + \left(Q_0 - Q^{\text{st}}\right) e^{-a\,(t-t_0)}. \tag{44}$$

Thus $Q(t)$ converges monotonically to Q^{st} at rate a.

From quality to travel time To link Q to congestion-sensitive travel time, let the *effective capacity* be an increasing function of quality, $s_{\text{eff}}(Q) = s\,f(Q)$ with $f'(Q) > 0$. A parsimonious choice is linear scaling $f(Q) = Q/Q_{\max}$ on $[0, Q_{\max}]$ (cap at $f(Q) \in [0,1]$). Then a BPR-style travel time is

$$TT^w(t) = TT\left[1 + \alpha\left(\frac{n(t)}{s_{\text{eff}}(Q(t))}\right)^{\beta}\right], \qquad \alpha > 0,\ \beta \in [3,5]. \tag{45}$$

Under constant $(m^{\text{st}}, n^{\text{st}})$, combine (44) and (45) to obtain an explicit time path $TT^w(t)$ as $Q(t)$ relaxes to Q^{st}.

Maintenance policies beyond constants If maintenance occurs in discrete interventions $\{t_k\}$ of sizes $\{\Delta_k\}$,

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -aQ - b\,\psi\!\left(\frac{n}{s}\right) \quad \text{for } t \neq t_k, \qquad Q(t_k^+) = Q(t_k^-) + \Delta_k. \tag{46}$$

Between interventions, $Q(t)$ decays exponentially; at each t_k it jumps up by Δ_k. A stationary cycle exists when the average uplift per period equals average losses, reproducing the steady formula with m^{st} interpreted as the long-run average maintenance rate.

Empirical note: We need to estimate: a: natural deterioration (weathering, oxidation, aging). b: marginal damage per unit usage; larger for heavier axles. With axle-load sensitivity, b implicitly scales γ in $\psi(x) = x^\gamma$.[2] $m(t)$: effective maintenance (resurfacing, sealing, rehabilitation), in quality units per unit time.

[2] Engineering practice often uses a "fourth-power law", i.e., $\gamma \approx 4$, reflecting axle-weight effects.

8.2 Appendix 2 Proof (sketch) of Proposition 5.

(i) *Capacity.* With $Q^{st} = Q^{\max}$, $TT = TT^{\min} + \delta N^2/s$ and delivered price $p^{del}(TT) = p + v_F TT$, quantity is $Q^*(TT) = [\alpha - (p + v_F TT)]/\beta$. Welfare is

$$W(s) = \frac{1}{2\beta}\left(\alpha - (p + v_F TT)\right)^2 - \zeta s.$$

FOC:

$$\frac{1}{\beta}\left(\alpha - (p + v_F TT)\right) v_F\, \delta\, \frac{N^2}{s^2} = \zeta.$$

Substitute $TT = TT^{\min} + \delta N^2/s$ and define

$$C_1 = \sqrt{\frac{\delta}{\zeta}\frac{v_F}{\beta}}\sqrt{N}, \qquad C_2 = \alpha - (p + v_F TT^{\min}),$$

to obtain a depressed cubic in s. Two positive roots exist when $4C_1^2 C_2^3 > 27$; the larger maximizes welfare. Comparative statics follow: s^* rises with N, v_F, δ, and falls with ζ.

(ii) *Maintenance.* For given s, $TT = \Phi(Q^{st}) + \delta N^2/s$, with $\Phi(Q^{st})$ as in the setup. Maximizing

$$\frac{1}{2\beta}\left(\alpha - (p + v_F TT)\right)^2 - \psi m^{st}$$

gives FOC

$$\frac{1}{\beta}\left(\alpha - (p + v_F TT)\right)(-v_F)\frac{\partial TT}{\partial m^{st}} = \psi.$$

With

$$\frac{\partial TT}{\partial m^{st}} = -\frac{TT^{\min}\Delta\phi^{\max}}{(Q^{\max})^2}(Q^{\max} - Q^{st})\frac{1}{a},$$

let $K = \frac{\psi\, a\,(Q^{\max})^2}{TT^{\min}\Delta\phi^{\max}}$ and $\varphi = \frac{\beta}{v_F} \cdot \frac{1}{\alpha - (p + v_F TT)}$. The FOC implies $Q^{st} = Q^{\max} - K\varphi$, hence

$$m^{st*} = \frac{bN}{s} + a(Q^{\max} - K\varphi).$$

(iii) *Welfare.* For fixed p, $W(TT) = \frac{1}{2\beta}\left(\alpha - (p + v_F TT)\right)^2$ is strictly decreasing and convex in TT. Thus higher v_F increases the marginal value of reducing TT, raising both s^* and m^{st*}.

(iv) *Limiting cases.* The statements follow directly: when $v_F \to 0$, the pricing channel vanishes, recovering the transport-only model; when $N \to 0$, congestion disappears; when $\psi \to 0$, maintenance is costless, so roads are always at $Q^{\max}$; when $\zeta \to \infty$, capacity is not expanded.

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