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Autonomous vehicles, urban deployment, and welfare effects

Zhi-Chun Li[a,*], Wen-Jing Liu[a], André de Palma[b], Yuki Takayama[c], Takao Dantsuji[d]

[a] *School of Management, Huazhong University of Science and Technology, Wuhan 430074, China*

[b] *CY Cergy-Paris University, Thema, France*

[c] *Department of Civil and Environmental Engineering, Institute of Science Tokyo, Tokyo, Japan*

[d] *Department of Civil and Environmental Engineering, Waseda University, Tokyo, Japan*

Abstract

This paper provides a theoretical analysis of the effects of autonomous vehicles (AVs) on the spatial structures of future cities. We consider two types of AVs, private AVs (PAVs) and shared AVs (SAVs). We assume that AVs have a lower marginal travel cost than human-driven traditional vehicles (TVs) due to additional utility caused by free activities in AVs, but PAVs have a lower marginal travel cost than SAVs due to better privacy, convenience and comfort. The land released by SAVs due to exemption of parking land occupancy is dedicated to firm production and household residence. We further assume that the government owns the land, regulates the housing development, and rents out the houses to the households. Two urban spatial models are presented and compared: one focusing on TVs and the other on mixed PAVs and SAVs. Both models account for land competition among firm production, household residence, and parking. The effects of traffic congestion on the TV / AV cities are identified through comparing the solutions of the models with and without considering traffic congestion. The finding shows that after introducing AVs, the city size may expand or shrink, depending on the marginal travel cost of AVs and the SAV market share in the AV market. Social welfare may increase or decrease, depending on the fixed cost of AVs, besides AV marginal travel cost and SAV market share. In addition, the total congestion cost may increase, and ignoring traffic congestion effects will cause overestimates of household utility, city size and social welfare.

Keywords: Private and shared autonomous vehicles; parking; urban spatial model; fixed cost; marginal travel cost; traffic congestion.

JEL classification: R13, R14, R48, R52

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[*]Corresponding author. Tel.: +86-27-8755-6490; Fax: +86-27-8755-6437.

E-mail addresses: smzcli@hust.edu.cn (Z.-C. Li), wenjing.liuro@gmail.com (W.-J. Liu), andre.de-palma@cyu.fr (A. de Palma), takayama.y.cc65@m.isct.ac.jp (Y. Takayama), takao.dantsuji@aoni.waseda.jp (T. Dantsuji).

1. Introduction

The past decade has witnessed the rapid development of auto technologies, particularly autonomous vehicle technologies. Autonomous taxis (or called robotaxis) and autonomous buses have recently been testing in some cities around the world. For instance, in August 2023, Waymo and Cruise were authorized to operate their robotaxi services in San Francisco, US.[1] Baidu company's newly launched robotaxi services, called "Apollo Go", have about 700 robotaxis operating in Wuhan China from 6 am every day to 2 am the next day.[2] Navya, a French company, is currently deploying autonomous shuttle buses in Paris (La Défense) to solve the first- and last-mile problems.[3] Despite of significant advancements in autonomous driving technologies, there may still exist a long way for full automation and widespread applications of autonomous vehicles (AVs).[4] Nevertheless, it is anticipated that as autonomous technologies continue to mature, AVs are becoming part of the mobility landscape, and will probably replace (partially or completely) human-driven traditional vehicles (TVs) as a main mode of transportation in the future.

The emergence of AVs could bring significant changes to the transportation industry and people's lifestyle. On one hand, compared to TVs, AVs can drive themselves automatically without need for drivers, and thus vehicle users can carry out various activities freely in the vehicles, such as eating, sleeping, or meeting. The in-vehicle activities can incur extra utility to the vehicle users and thus reduce their aversions to staying in the vehicles for a long time, particularly under the traffic congestion case (Pudāne, 2020; Wu and Li, 2023). AVs are also likely to reduce commuters' parking cursing time because they could find a parking space automatically. On the other hand, with the development of shared mobility services, people can procure shared AVs (SAVs) services conveniently via APP installed in cellphone. Different from private AVs (PAVs) that need to occupy the parking land at residences and worksites, SAVs can run on the road through the daytime and park outside the city during the nighttime, and thus no (or less) parking land is required for SAVs within the city (Zhang, 2017; Larson and Zhao,

[1] https://en.wikipedia.org/wiki/Robotaxi.
[2] https://www.baiguan.news/p/baidu-apollo-go-robotaxi-wuhan-launch.
[3] https://www.navya.tech/en.
[4] NHTSA (National Highway Traffic Safety Administration), U.S. defines five levels of driving automation: Level 0 (no automation), Level 1 (driver assistance), Level 2 (partial automation), Level 3 (conditional automation), Level 4 (high automation), and Level 5 (full automation). See https://www.nhtsa.gov/document/levels-automation.

2020). The land freed up can be used for other purposes, such as firm production and household residence. It was reported that on average, 20% of land in the US city centers is dedicated to parking, and even 30% in some cities, e.g., 39% for Arlington Texas and 33% for Las Vegas.[5] Therefore, it is plausible that AVs will significantly change the land use configuration of future cities. Naturally, this raises some important and intriguing issues: How will AVs reshape the future urban spatial structure? Will introduction of AVs reduce traffic congestion and increase social welfare? This paper will address these important issues.

To do so, we present an analytical approach in this paper for modeling the effects of AVs on the behavior of all stakeholders (households, firms, and the government), traffic congestion, and social welfare of the urban system. The proposed approach can be used to assess the advantageous and disadvantageous aspects of AVs before widespread adoption, and thus can serve as a useful tool for the ex-ante evaluation and design of various urban and transportation policies in the context of AVs, such that the positive effects of AVs can be completely exerted and the negative effects can be controlled through some proactive regulatory policies. To the best of our knowledge, this paper is the first systematically theoretical study for addressing the effects of AVs on future cities with integrated considerations of land competition among firms, households and parking, government regulation on land development, and traffic congestion effects.

We begin by establishing a benchmark urban spatial model that takes into account the land competition among firm production, TV household residence and parking.[6] The effects of traffic congestion are not considered in the benchmark model, but will be incorporated in the later section. We consider a linear monocentric city, in which all import / export trades occur at the city center. Firms use land and labor to produce goods, households choose residential locations and worksites with parking land demand consideration, and the land of the city is allocated to the highest bidder among households and/or firms. Firms prefer to locate near the central business district (CBD) due to its proximity to the trade market, whereas households

[5] https://bigthink.com/strange-maps/parking-lots-eat-american-cities.

[6] For simplicity of analysis, in this paper we focus on the case of surface parking, in which parking of vehicles needs to occupy the land area, causing a direct effect on the competition for land among households, firms and parking. This assumption has been adopted in some previous studies about parking land use, such as Zakharenko (2016), Franco (2017), and Brueckner and Franco (2018). For land use modeling of other parking facility types, like underground parking and structural parking (i.e., multi-storey car park), please refer to Brueckner and Franco (2017).

prefer to locate near the workplace due to low commuting cost. Using the benchmark urban spatial model, the border between industrial (or employment) area and residential area and their respective size are determined endogenously, together with the wage and land rent gradients. The government owns the land, regulates the housing development, and rent out the houses to the households aiming to maximizing the social welfare of the city system.[7]

The benchmark model presented in this paper is a significant extension to the classical Alonso-type urban models (see Alonso, 1964), in which the wage is exogenously given, and the industrial area is treated as a single point without area such that the change of land use in the industrial area and the land competition between industrial and residential areas cannot be analyzed. In fact, workers' wages are related to their worksites and thus commuting costs, and the size of the industrial area is relevant to the land inputs available to firms. Therefore, it is important to endogenously determine the wage and the size of the industrial area. In this regard, Ogawa and Fujita (1980) and Fujita and Ogawa (1982) made a pioneering contribution. They have been extended to consider other factors in general equilibrium models (see e.g., Koide, 1990; Lucas and Rossi-Hansberg, 2002; Berliant et al., 2002; Berliant and Tabuchi, 2018; Malykhin and Ushchev, 2018; Mossay et al., 2020; Osawa and Akamatsu, 2020; Garrido-da-Silva et al., 2022). However, these studies did not consider the parking land use and the AV role in reshaping urban spatial structure. Moreover, they assumed that household housing size was exogenously given, but not endogenously determined by the model, and thus the social welfare achieved is not maximized. We extend their work to address the issues of the parking land use and the optimal housing size for future AV cities.

We then extend the benchmark urban spatial model to examine the AV effects on the land competition among firms, households and parking, and the social welfare. Two types of AVs, PAVs and SAVs, are considered. For simplicity of analysis, we assume that SAVs do not need to occupy parking land because they run on the road during the day and park outside the city during the night,[8] and the land released by SAVs is used for firm production and household residential uses. Each PAV, similar to a TV, needs a parking space at the industrial area for

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[7] The land in some countries, such as China, is owned by the government, and the property developments are regulated by the government. The households purchase (or rent) the houses from the government.

[8] This assumption is not unreasonable. In fact, an SAV company, similar to a bus (or taxi) company, faces a high investment cost, including SAV purchase cost, operating cost, and maintenance cost etc. Considering high parking charges in the urban central area, the company would rather park the SAVs outside the city during the night.

daytime parking and a parking space at the residential area for nighttime parking.[9] We further assume that AVs have a lower marginal travel cost than TVs due to additional utility caused by free activities in AVs. Moreover, the marginal travel cost of PAVs is lower than that of SAVs due to its better privacy, convenience and comfort. Two extreme cases of full PAVs and full SAVs in the urban system are also discussed in detail. We find that after introducing the AVs, the housing size of PAV households increases, whereas the housing size of SAV households may increase or decrease. The size of the industrial area becomes smaller due to the decreased demand for parking land with SAVs. As a result, the city size may expand or shrink, depending on the marginal travel cost of AVs and the SAV market share in the AV market. We derive the critical condition for city expansion or contraction. The result obtained is an important extension to the previous studies, such as Zakharenko (2016), Larson and Zhao (2020), and Zhong and Li (2023). All of them adopted a numerical simulation method due to analytical intractability, and showed that after introducing AVs, the city would expand. In addition, we find that after introducing the AVs, the social welfare may increase or decrease, relying on the AV fixed cost and marginal travel cost, and SAV market share. The firm production would increase for a full SAV city. We also examine how to determine the optimal SAV market share and the traffic congestion effects on the stakeholders' behavior.

We now summarize the key contributions of this paper relative to the existing literature. First, we present an analytical modeling approach for modeling the behavioral interactions among households, firms, and the government in a TV market through using a general equilibrium framework with firm agglomeration externality consideration. In the proposed approach, the land competition among households, firms, and parking is considered. Second, we present an analytical urban spatial model with mixed PAVs and SAVs to disclose the effects of introducing AVs and the SAV market share on the future urban spatial structure, household utility, firm production, and social welfare. In the proposed model, the PAV / SAV households' residential and work locations, and the firm production site are endogenously determined. The effects of traffic congestion on the TV / AV cities are also identified through comparing the solutions of the models with and without considering traffic congestion. The results show that the introduction of AVs may increase the total congestion cost of the city due to an extended average commuting distance, and ignoring the traffic congestion effects will lead to overestimates of

[9] It should be pointed out that PAVs may be able to find a parking space automatically with AV techniques, e.g., return automatically to the parking space at the residential location. This case will reduce the land for parking, but increase the travel cost due to additional journey. This is left for a future study.

household utility, city size, and social welfare.

The remainder of this paper is organized as follows. In the next section, related literature is reviewed. In Section 3, a benchmark urban spatial model with TV households is presented to endogenously determine the sizes of industrial and residential areas with parking land consideration. The optimal housing size that maximizes the social welfare under the government's housing regulation is also determined. Section 4 extends the benchmark model to account for the PAV/SAV effects on urban land uses (including parking land use), together with determinations of optimal housing size and optimal SAV market share. Section 5 further analyzes two extreme cases of full PAVs and full SAVs. Section 6 extends to the case of incorporating traffic congestion effects. In Section 7, numerical examples are provided to illustrate the model properties. Section 8 concludes this paper and provides suggestions for further studies.

2. Literature review

Our work is closely related to the studies of the urban model with household residential and work location choices. The literature related to this topic can be categorized into two classes in terms of modeling method: monocentric urban models with an exogenous city center location, and urban configuration models with endogenous city center and residential and industrial areas. As far as the first-class method is concerned, Straszheim (1984) presented a segregated model of land and labor markets (with exogenously given city center as import / export trade market) to determine the border between industrial and residential areas, and wage and rent gradients. No closed-form solution was obtained for this formulation because the model is too complex. Sasaki and Kaiyama (1990) extended it to evaluate the effects of transportation system improvements on urban spatial structure. Ross and Yinger (1995) performed analytically comparative static analysis of open city's model considering the firm's and household's competitive behavior (with land and labor as inputs to housing production and consumption goods). Ross (1996) further incorporated capitals as an input to housing production and consumption goods. Although these studies provided a useful approach for modeling household residential location and worksite choices in a monocentric city, they ignored the effect of agglomeration economies (e.g., knowledge and information spillovers) on firm's location. Moreover, there is no analytical solution in general, and thus a simulation method must be adopted.

As far as the second-class method is concerned, Ogawa and Fujita (1980) made a pioneering work. Their model considered the effect of agglomeration economies on firm's location and endogenously determined urban land use patterns, in which neither employment nor residence needs to be specified a priori. Three types of urban land use patterns were obtained: monocentric urban configuration, partially mixed urban configuration, and completely mixed urban configuration. Fujita and Ogawa (1982) extended it to the cases of duocentric and tricentric urban configurations and analyzed the conditions for the structural transition of urban configurations. Both studies assumed that the transaction cost between firms and the household commuting cost are a linear function of distance. The modeling frameworks of Ogawa and Fujita (1980) and Fujita and Ogawa (1982) have been widely extended through assumption relaxations or from other perspectives. For example, Zenou (2000) extended to include the role of unemployment in the formation of a monocentric city and identified the phenomenon of spatial mismatch between residence and worksite. Lucas and Rossi-Hansberg (2002) allowed substitution between land and labor in firm's production technology and substitution between land and goods in household consumption. Berliant et al. (2002) accounted for the uncompensated knowledge spillover in the firm spatial agglomeration. Gaigné et al. (2016) further incorporated the local public good and the agglomeration economies with regard to the number of firms. Berliant and Tabuchi (2018) relaxed the assumption of a linear commuting cost to a nonlinear case. Malykhin and Ushchev (2018) further considered the market interaction between firms and consumers, where consumers have quasilinear quadratic preferences and firms face quadratic transport cost. Mossy et al. (2020) incorporated the role of the demand for final and intermediate goods and the vertical linkages in the presence of increasing returns to scale. Garrido-da-Silva et al. (2022) considered the resident's travel cost of visiting the city center in the model, besides the resident's cost of commuting to work and the firm's cost of shipping industrial goods to the city center. Osawa and Akamatsu (2020) addressed the issue of polycentric city structure as a result of tradeoff between agglomeration economies and congestion effects, based on the theory of potential games. However, these aforementioned studies did not involve the parking land use and the AV effects.

Our work is also closely related to integrated issues of parking land use and urban spatial model. In this regard, only a few studies can be found in the literature. Voith (1998) presented a general equilibrium model to examine the effects of parking and transit subsidy on CBD size, CBD land value, and market shares of cars and transit. However, his model only focused on CBD area, and did not considered the spatial details of the city. Anderson and de Palma (2007) integrated parking in a monocentric city model, in which land can be used for residences or parking lots, and land rents are endogenously determined. They showed that at equilibrium, residents close to CBD walk to work, whereas residents further out drive to parking lot, and then walk to work; and the social optimum is identical to an equilibrium when parking lots are monopolistically competitively priced. Their model did not consider the firm production behavior and the land competition between households and firms. Franco (2017) explored the effects of change in downtown parking supply on welfare, mode choice, and urban spatial structure using a monocentric city model with two transport modes, endogenous residential surface parking, and bottleneck congestion at the CBD. Brueckner and Franco (2017) further investigated the effects of different regimes for provision of parking spaces on urban form, including surface parking, underground parking, and structural parking. However, all these studies did not involve AV issues as well as AV effects on the city, including households, firms, and land use.

To the best of our knowledge, Zakharenko (2016) was the first to involve the AV effects on urban form from the perspective of urban economics. In his study, the equilibrium solution of household residential and work location choices with parking land use incorporated was endogenously determined. However, his study did not concern the firm behavior and the behavioral difference between PAVs and SAVs. Zakharenko (2023) further considered the travelers' choice equilibrium issue of PAVs and SAVs based on a trade-off between vehicle capital cost and search cost. However, he did not concern the effects of AVs on urban spatial structure. Larson and Zhao (2020) used classic monocentric urban model with fixed CBD land area to model the effects of AVs on urban sprawl, energy consumption, and housing affordability. More recently, Zhong and Li (2023) adopted counterfactual analysis method to examine the effects of AVs on urban expansion for metropolitan areas in the US if autonomous vehicles had been introduced. Dantsuji and Takayama (2024) utilized a bathtub model to analyze the effects of AVs on the spatial structures of a hyper-congested city, without considering firm behavior and auto parking. Using numerical simulation methods, these studies showed that with an assumption of exogenously given worker wage, introduction of AVs would cause urban expansion.

This paper presents analytical models for exploring the effects of AVs on the urban spatial structure in a linear monocentric city with a consideration of interactions among households, firms, and the government. The parking land occupancies for TVs and PAVs and the land released by SAVs for firm production and household residential uses are explicitly considered. The optimal housing size and optimal SAV market share are analyzed. This study provides some new insights into the AV effects on the future urban spatial structure, household utility, firm production, traffic congestion, and social welfare.

3. Urban model with TVs only

3.1. Urban configuration with TVs only

The focus of this paper is on the effects of AVs on urban spatial structure, and thus we attempt to develop simple urban models based on some reasonable assumptions. We first consider a linear, closed, symmetric, monocentric city with TVs only, in which all import / export trades occur in the city center. There is one unit of land available at every location of the linear city. There are three stakeholders in the city: households, firms, and the government. Households provide labor to firms, and conversely, firms pay wages to households. Firms interact with each other, generating agglomeration economies. In the benchmark case, it is assumed that all households travel by TVs, and each of them needs a parking space to be provided at the residential location for nighttime parking and at the work location for daytime parking. Households and firms compete for land for their activities' purposes, including residence and parking for households and production for firms. It is assumed that the government owns the land, and regulates the housing development to maximize the social welfare of the urban system. All households rent the houses from the government, and pay rents to the land owner (i.e., the government). The locational competitions among households and/or firms create an urban configuration of the spatial economy. With the assumption that all import / export trades occur in the city center, the transportation costs of goods and the agglomeration economies among firms make the land at the city central area more attractive to the firms. As a result of the land competition, the firms choose to implement production activity at the CBD area of the city due to its proximity to the trade market, whereas households choose to reside in the outer area of the city. This leads to a segregated city pattern in which employments are located at the CBD area, whereas residents are located at the peripheral area.

For illustration purpose, Fig. 1 shows a symmetric monocentric urban configuration with TVs only. In Figure 1, $-b_1$ and b_1 are the borders between industrial area and residential area, and B is the city boundary. $[-b_1, b_1]$ is the industrial or employment area, and $[-B, -b_1]$ and $[b_1, B]$ are the residential areas. As previously stated, this paper assumes that the type of parking facility provided in the city is surface parking, thus causing parking land occupancy. The land in the industrial area is used for firm production and parking, whereas the land in the residential area is used for household residence and parking. In the following, we determine the equilibrium solutions for households and firms and the optimal housing development for the government. Considering the city's symmetry, we only focus on right-half of the city.

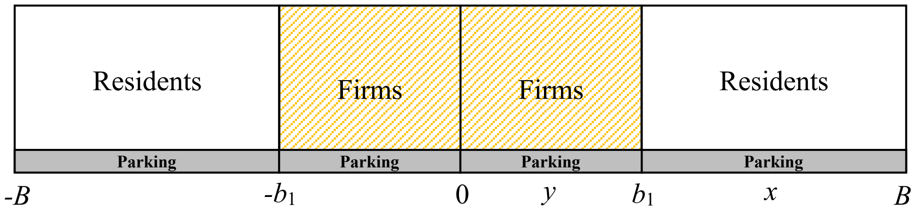


Fig. 1. Symmetric monocentric urban configuration with TVs only.

3.2. Households

Suppose that there are N homogeneous households with identical preferences for land and (numéraire) composite goods. Each household provides one unit of labor to a firm to earn the income in return. The utility of each household is derived from housing consumption and composite non-housing good consumption. For simplicity of analysis, following some previous studies, such as Ogawa and Fujita (1980), Fujita and Ogawa (1982), Berliant and Tabuchi (2018), Regnier and Legras (2018), and Kyriakopoulou and Picard (2021, 2023), we assume that the housing size consumed by each household is a positive uniform size s_h across residential areas. A quasi-linear household utility function is adopted and expressed as

$$U\left(z(x)\right) = z(x) + \gamma \ln s_h, \ x \in [b_1, B], \tag{1}$$

where $U(\cdot)$ is the household utility level, x is the residential location of a household, $z(x)$ is the quantity of numéraire composite non-housing goods consumed by a household residing at location x, and s_h is the quantity of land consumed by a household. The positive constant γ represents household preference for the land. A larger value of γ indicates a stronger preference, and vice versa. The quasi-linear utility function has been adopted in some previous related studies, such as Song and Zenou (2006), Kono et al. (2012), Regnier and Legras (2018), and Li et al. (2024a, b).

A TV has to park at worksite during the day and at residential location during the night, thus causing parking land occupancies at both occasions. We assume that each TV needs s_p units of land for parking at each occasion, which is assumed to be a constant. We further assume $s_h > s_p$, which goes along with usual observation. Household income is spent on housing rent, composite non-housing good consumption, parking fees or rents at residential location and worksite, fixed vehicle ownership cost, and commuting time cost between residence and worksite. The budget constraint for a household residing at location x and working at location y is given as

$$w(y) - \alpha_0^{\mathrm{TV}} - C(x,y) - \left(r(x) + r(y)\right)s_p = z(x) + r(x)s_h,\ x \in [b_1, B], y \in [0, b_1], \tag{2}$$

where the superscript "TV" represents traditional vehicles. $w(y)$ is the wage paid to workers by the firm locating at y. α_0^{TV} is the fixed vehicle ownership cost, including purchase cost, depreciation cost, insurance and registration fees etc. $C(x,y)$ is the commuting time cost of a worker between residence x and worksite y. $r(x)$ and $r(y)$ are the land rents at residence x and workplace y, respectively. The left-hand side of Eq. (2) represents the disposable income of a household residing at x and working at y, which is equal to the household income deducted by the vehicle ownership cost, the commuting cost between residence and worksite, and the parking land rents at both residence and worksite.

For simplicity of analysis, the traffic congestion effects are temporarily ignored at this moment, and will be considered in Section 6. With no congestion effects, $C(x,y)$ can be defined as a function of the commuting distance $x - y$ between residence x and worksite y, expressed as

$$C(x,y) = \alpha_1^{\mathrm{TV}}\left(x - y\right),\ x \in [b_1, B], y \in [0, b_1], \tag{3}$$

where α_1^{TV} is the marginal travel cost per unit of distance by TV.

Each household maximizes its own utility $U(\cdot)$ by choosing the amount of composite goods $z(x)$, residential location x, and worksite y within its budget constraint. From Eqs. (1)-(3), the household utility maximization problem for determining $z(x)$, x and, y can be formulated as

$$\max_{z(x),x,y}\ U\left(z(x), x, y\right) = z(x) + \gamma \ln s_h, \tag{4}$$

$$\text{s.t. } w(y)-\alpha_0^{\mathrm{TV}}-\alpha_1^{\mathrm{TV}}\left(x-y\right)-\left(r(x)+r(y)\right)s_p=z(x)+r(x)s_h,\ x\in[b_1,B], y\in[0,b_1]. \quad (5)$$

Substituting Eq. (5) into Eq. (4), one immediately obtains that household utility $U(\cdot)$ is a function of x and y. From the first-order optimality condition $\partial U(\cdot)/\partial x=0$, one can derive the equilibrium land rent $r(x)$ at residential location x as

$$r_h(x)=R_A+\frac{\alpha_1^{\mathrm{TV}}}{s_h+s_p}(B-x),\ x\in[b_1,B], \tag{6}$$

where the subscript "h" represents "households". R_A is the agricultural land rent with $r_h(B)=R_A$. Eq. (6) shows that the housing rent linearly decreases with an increase in the distance from the city center.

From the first-order optimality condition $\partial U(\cdot)/\partial y=0$, one can derive the relationship between the wage $w_f(y)$ at the best worksite y chosen by the household residing at x and its commuting cost, as follows:

$$w_f(y)=w_f(0)-\alpha_1^{\mathrm{TV}}y-\left(r(0)-r(y)\right)s_p,\ y\in[0,b_1], \tag{7}$$

where the subscript "f" represents "firms". The term $\alpha_1^{\mathrm{TV}}y+\left(r(0)-r(y)\right)s_p$ represents the total commuting cost savings due to choosing to work at location y but not at city center ($y=0$), including travel time cost and parking cost. Eq. (7) means that the wage, $w_f(y)$, at the best worksite y equals the wage at the city center minus the total commuting cost savings due to a decrease in the commuting distance by y between residence and worksite.

Substituting Eqs. (6) and (7) into Eqs. (4) and (5), one obtains the equilibrium composite non-housing good consumption and equilibrium household utility as

$$z=w_f(0)-\alpha_0^{\mathrm{TV}}-\alpha_1^{\mathrm{TV}}B-r(0)s_p-R_A\left(s_h+s_p\right), \tag{8a}$$

$$U^{\mathrm{TV}}=w(0)-\alpha_0^{\mathrm{TV}}-\alpha_1^{\mathrm{TV}}B-r(0)s_p-R_A\left(s_h+s_p\right)+\gamma\ln s_h. \tag{8b}$$

Eqs. (8a) and (8b) show that at equilibrium, all workers are indifferent about their choices of residences and worksites. The equilibrium composite non-housing good consumption z and thus the equilibrium household utility U^{TV} depends on the housing size s_h to be determined by the government.

As previously stated, each TV household needs to occupy s_h units of land for household residence and s_p units of land for parking at the residential area, implying a total of $s_h + s_p$ units of land occupancy per household at the residential area. Hence, the household residential density (i.e., the number of households per unit of residential land area), $n(x)$, at location *x* is given as

$$n(x) = \frac{1}{s_h + s_p}, \text{ for } x \in [b_1, B]; \text{ and } 0, \text{ otherwise}. \tag{9}$$

In the above, we have derived the residential location choice equilibrium of TV households, including the equilibrium land rent $r_h(x)$, the equilibrium composite non-housing good consumption *z*, and the relationship between best worksite $w_f(y)$ and commuting cost savings $\alpha_1^{\text{TV}} y + \left(r(0) - r(y)\right) s_p$. Observe that both $r_h(x)$ and $w_f(y)$ are dependent on the marginal travel cost α_1^{TV}, but are independent of vehicle fixed cost α_0^{TV}, while *z* and thus $U(\cdot)$ is dependent on both α_0^{TV} and α_1^{TV}.

3.3. Firms

We now look at the firm's behavior. Suppose that all firms are homogeneous, implying the same land and labor inputs and the same production technology. They produce the composite goods that are shipped and sold at a unitary price in the trade market using land and labor as inputs. Following Ogawa and Fujita (1980), Fujita and Ogawa (1982), Berliant and Tabuchi (2018), Regnier and Legras (2018), and Kyriakopoulou and Picard (2021, 2023), we assume that the quantities of land and labor used for production for each firm are, respectively, fixed as s_f units of land and l_f units of labor, and there is no unemployment in the city. Therefore, the number of firms, denoted by *M*, can be expressed as

$$M = \frac{N}{l_f}. \tag{10}$$

According to the previous related studies, such as Berliant and Tabuchi (2018), Regnier and Legras (2018), and Kyriakopoulou and Picard (2021, 2023), firm's production function, $F(y)$, depends on spillover, communication, and economic interactions, and can be defined as

$$F(y) = \int_{-\infty}^{+\infty} \left(\beta - \delta|t-y|\right) m(t)dt, \tag{11}$$

where β is a parameter reflecting firm productivity, $m(t)$ is the firm density at location t, and $|t-y|$ is the distance between firms locating at t and y. δ is the transaction cost of unit distance between firms. The term $\beta - \delta|t-y|$ can be interpreted as knowledge or information spillover effects of a firm at location t on a firm at location y, that is, the contribution or agglomeration externality of the firm at t to the production of the firm at y. Clearly, $\beta - \delta|t-y|$ should be positive for any feasible distance in the city under the agglomeration economy effects.[10] This implies a positive interaction among firms, and thus a positive firm production. The higher the mass of firms around y, the higher the production of the firm at y, and vice versa. Empirical studies in regional and urban economics provide evidences of such an agglomeration force (see Ciccone and Hall, 1996; Rosenthal and Strange, 2008).

In the TV era, each worker entails s_p units of land for daytime parking at the worksite, and each firm needs s_f units of land for production. The total quantity of land consumed by each firm with l_f units of labor is the sum of all the land used for the workers' parking and for the firm production, i.e., $s_p l_f + s_f$. Thereby, the firm density (i.e., the number of firms per unit of industrial land area), $m(y)$, at worksite y is

$$m(y) = \frac{1}{s_p l_f + s_f}, \text{ for } y \in [0, b_1]; \text{ and } 0, \text{ otherwise}. \tag{12}$$

Substituting Eq. (12) into Eq. (11), the firm's production function can be calculated as

$$F(y) = \int_{-b_1}^{b_1} \left(\beta - \delta|t-y|\right) m(t)dt = \frac{-\delta y^2 - \delta b_1^2 + 2\beta b_1}{s_p l_f + s_f}, \ y \in [0, b_1]. \tag{13}$$

Eq. (13) shows that the firm production function $F(y)$ is concave on $[0, b_1]$ and reaches its maximum value of $2\beta b_1 - \delta b_1^2$ at the city center (i.e., $y = 0$). This means that the firm production is higher when firms are close to each other due to agglomeration externalities, particularly around the city center.

[10] A sufficient condition is $\beta > 2\delta b_1$. We assume that β is high enough or δ is low enough such that this inequality holds, as assumed in Kyriakopoulou and Picard (2021, 2023).

Each firm maximizes its own net profit by choosing its location. The firm's net profit maximization problem is represented as

$$\max_{y} \; \Pi(y) = F(y) - w_f(y)l_f - r_f(y)s_f, \; y \in [0, b_1], \tag{14}$$

where $\Pi(y)$ is the firm profit at location y, and the production function $F(y)$ is given by Eq. (13).

We assume a perfectly competitive product market, i.e., each firm can freely enter or exit the city. Accordingly, at equilibrium each firm's net profit is equal to 0, i.e., $\Pi(y) = 0$. One can then obtain the bid-rent, $r_f(y)$, of the firm at location y as

$$r_f(y) = \frac{1}{s_f}\left(F(y) - w_f(y)l_f\right), \; y \in [0, b_1]. \tag{15}$$

According to Eq. (15), the firm's bid-rent $r_f(y)$ is positively related to the firm production, but negatively related to the wage.

3.4. Equilibrium

Competition for land among households, among firms, and between households and firms leads to an equilibrium spatial structure of the city, which is described by the following variables: land rent profile at industrial area $r_f(y)$, land rent profile at residential area $r_h(x)$, wage profile $w_f(y)$, border, b_1, between industrial and residential areas, and city boundary B. These variables are determined by the following equilibrium conditions:

$$2\int_0^{b_1} m(y)dy = M, \tag{16a}$$

$$2\int_{b_1}^{B} n(x)dx = N, \tag{16b}$$

$$r_f(b_1) = r_h(b_1), \tag{16c}$$

$$r_h(B) = R_A. \tag{16d}$$

Eqs. (16a) and (16b) are the labor market equilibrium conditions, and Eqs. (16c) and (16d) are the land market equilibrium conditions. Specifically, Eq. (16a) is the conservation constraint about the total number of firms, requiring that all firms are located at the industrial area of the city. Eq. (16b) is the conservation constraint about the total number of households, requiring that all households are accommodated in the residential area of the city. Eq. (16c) represents that the bid-rent of firms equals that of households at border b_1. Eq. (16d) states that the land rent at the city boundary equals the agricultural land rent. The number "2" in Eqs. (16a) and (16b) is due to the city's symmetry.

From the equilibrium conditions (16a)-(16d), one can derive border, b_1, between industrial area and residential area, city boundary *B*, wage curve $w_f(y)$, and firm's bid-rent curve $r_f(y)$, as follows:

$$b_1 = \frac{M}{2}\left(s_p l_f + s_f\right),\quad B = \frac{M}{2}\left(s_p l_f + s_f\right) + \frac{N}{2}\left(s_h + s_p\right), \tag{17a}$$

$$w_f(y) = \frac{l_f s_p F(y) + s_f F(b_1) - l_f s_f \alpha_1^{\mathrm{TV}} y}{l_f\left(s_f + l_f s_p\right)} - \frac{s_f}{l_f} R_A,\ y \in [0, b_1], \tag{17b}$$

$$r_f(y) = \frac{F(y) - F(b_1) + \alpha_1^{\mathrm{TV}} l_f y}{s_f + l_f s_p} + R_A,\ y \in [0, b_1]. \tag{17c}$$

Eq. (17b) shows that the wage function $w_f(y)$ is concave and decreasing with regard to *y* on $[0, b_1]$, and reaches its maximum value of $\frac{s_p l_f F(0) + s_f F(b_1)}{l_f\left(s_f + s_p l_f\right)} - \frac{s_f}{l_f} R_A$ at the city center (i.e., $y = 0$). Eq. (17c) shows that the firm's bid-rent function $r_f(y)$ is a concave, quadratic function of *y*.

3.5. Optimal housing size under social welfare maximization

So far, we have formulated the household's and firm's behavior for a TV city. We now look at the behavior of the government. As previously stated, the land and housing development in the city is regulated by the government, and households rent the houses from the government. The government determines the optimal housing size s_h to maximize the social welfare (*SW*) of the system, which is the total benefits of all stakeholders in the urban system, including the consumer surplus (i.e., total utility of all households in the city NU^{TV}), producer surplus (zero profit for perfectly competitive firms), and the profit from land rent. The social welfare maximization problem for optimizing the housing size s_h can be expressed as

$$\max_{s_h}\ SW^{\mathrm{TV}} = NU^{\mathrm{TV}}(s_h) + 2\int_0^{b_1} r_f(y)dy + 2\int_{b_1}^{B} r_h(x)dx - 2\left(R_A + c\right)B\,, \tag{18}$$

where U^{TV} is given by Eq. (8b), and *c* represents the marginal land development cost. The first term on the right-hand side of Eq. (18) is the total utility of all households in the city, the second term is the total land rent from firms, the third term is the total land rent from households, and the last term is the sum of the land development cost and the land opportunity cost.

It is easy to show $\partial^2 SW^{\mathrm{TV}} / \partial s_h^{\ 2} < 0$, meaning that the social welfare is a concave function of the housing size for TV households (see Appendix A), and one can derive the optimal housing size s_h^{TV} as

$$s_h^{\mathrm{TV}} = \frac{4\gamma}{\alpha_1^{\mathrm{TV}} N + 4\left(R_A + c\right)}. \tag{19}$$

It shows that the optimal housing size of households depends on the marginal TV travel cost α_1^{TV}, the agricultural land rent R_A, and the marginal land development cost c.

Note that the marginal TV travel cost α_1^{TV} and the parking land size s_p are two crucial parameters influencing the solution of the urban spatial model. Hence, it is important to carry out the comparative static analyses of these two parameters, as shown in Table 1.

Table 1 Comparative static results.

Parameter	$F(y)$	$r_f(y)$	$r_h(x)$	$w_f(y)$	s_h^{TV}	b_1	B
α_1^{TV}	null	+	?	–	–	null	–
s_p	–	?	+	?	null	+	+

Note: "+" means a positive correlation, "–" means a negative correlation, and "null" means no effect.

The results presented in Table 1 are summarized as follows.

Proposition 1. For the comparative statics of marginal TV travel cost α_1^{TV} and parking land size s_p, we have

(i) Smaller marginal TV travel cost α_1^{TV} causes a decrease in firm land rent $r_f(y)$, an increase in wage $w_f(y)$, household housing size s_h^{TV} and city size *B*, but has no effect on firm production $F(y)$ and industrial area's size b_1.

(ii) Smaller parking land size s_p causes a decrease in residential land rent $r_h(x)$, industrial area's size b_1 and city boundary *B*, but an increase in firm production $F(y)$.

According to Proposition 1, a decrease in the marginal TV travel cost α_1^{TV} through vehicle technological innovation causes a decreased firm land rent. This is because that the lower marginal TV travel cost makes peripheral areas more attractive, leading to a lowered land competition and thus a decreased firm land rent. As a result, firms can afford higher wages to households. With higher wages and reduced marginal TV travel cost, households can enjoy bigger housing spaces, thus causing urban sprawl. On the other hand, a decrease in the parking land size s_p leads more land to be used for firm production and household residence, thus causing a more compact industrial area and city boundary. This centralization of firms enhances agglomeration economy effects, leading to increased firm production. The marginal TV travel cost and parking land use play an important role in understanding the AV effects on the city. With the introduction of AVs, marginal travel cost decreases due to the replacement of human drivers, and the parking land is released, leading to a change in the urban land competition. In the following section, we will further explore the effects of introducing AVs on the future urban spatial structure, household utility, firm production, and social welfare.

4. Urban model with mixed PAVs and SAVs

In this section, we study the effects of introducing AVs on the urban system.

4.1. Urban configuration with mixed PAVs and SAVs

With a rapid progress in artificial intelligent technology and AV technology, it is anticipated that TVs in the city will probably be entirely replaced by AVs in the future. As stated before, AVs can drive themselves automatically without need for drivers, and yield extra utility to vehicle users due to the in-vehicle free activities. It is reasonable to expect that the marginal travel cost of commuters in AVs is lower than that in TVs, thus leading to a significant change in household behavior and in urban system performance.

With convenient ride-hailing services via a shared mobility platform, residents can easily use

SAV services with no need to purchase a vehicle, facing therefore a lower fixed cost than PAVs. By contrast, PAVs can provide better privacy, more convenient and comfortable services due to better accessibility than SAVs, and thus have a lower marginal travel cost than SAVs. Moreover, with adoption of SAVs, the land area required for parking in a city is decreased, which is dedicated to the use of firms and households.

In light of the above discussion, we assume that the marginal travel cost of TVs is the highest, that of PAVs is the lowest, and that of SAV is in between, i.e.,

$$\alpha_1^{\mathrm{PAV}} < \alpha_1^{\mathrm{SAV}} < \alpha_1^{\mathrm{TV}}, \tag{20}$$

where the superscripts "PAV" and "SAV" represent PAVs and SAVs, respectively. α_1^{PAV} and α_1^{SAV} are the marginal travel costs of PAVs and SAVs, respectively.

Consider a linear, closed, symmetric, monocentric city with mixed SAVs and PAVs. Let θ be the proportion of households using SAV services, and thus $1-\theta$ be the proportion of households using PAVs. With the assumption that all goods trades take place in the city center and using condition Eq. (20), it can be shown that at equilibrium, the urban configuration after introducing AVs has the following properties (the proof is relegated to Appendix B).

Proposition 2. With the competition for land between households and firms,

(i) Households are located at the outer area of the city, and PAV and SAV households' residences are segregated, with PAV households residing at the most peripheral area.

(ii) Firms are located at the inner area of the city, and PAV and SAV households' worksites are segregated, with the SAV households' worksites being at the most central area.

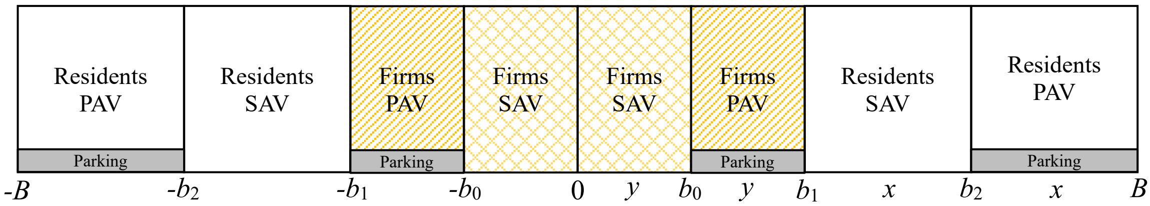


Fig. 2. Symmetric monocentric urban configuration after introducing PAVs and SAVs.

Proposition 2 shows that PAV and SAV households have different residential locations and different worksites. Compared with PAV households, SAV households tend to choose the residential and work locations closer to the city center. This is because that SAVs do not need the land for parking, thus allowing SAV households to reside at more central areas due to waived parking land rent. However, PAVs need the land for parking but have a lower marginal travel cost than SAVs, causing that PAV households choose to reside and work in more peripheral areas with low land competition. Accordingly, their optimal housing sizes, which are determined by the government (denoted as s_h^{PAV} and s_h^{SAV} respectively), should be differentiated from the perspective of the social optimum. Fig. 2 illustrates the urban configuration of the city with mixed PAV and SAV households. In this figure, b_1 and $-b_1$ are the borders between industrial area and residential area, b_2 and $-b_2$ are the borders between residential areas of PAV and SAV households, and b_0 and $-b_0$ are the borders between employment areas of PAV and SAV households. The urban spatial model with mixed PAVs and SAVs aims to determine the following quantities at equilibrium: borders b_0, $-b_0$, b_1, $-b_1$, b_2 and $-b_2$, city boundary B, wage profile $w_f(y)$, land rent profiles $r_h(x)$ and $r_f(y)$, and household utility levels U^{PAV} and U^{SAV}. Again, considering the city symmetry, we focus only on the right-half of the city in the following analysis.

4.2. Households

4.2.1. PAV Households

We first look at the behavior of PAV households. Similar to TVs, PAVs need to park at household residential location during the night and at worksite during the day. The PAV household income is spent on the fixed vehicle ownership cost, commuting time cost, parking cost, composite non-housing good consumption, and housing consumption. Suppose that a PAV has the same vehicle size as a TV, and thus needs an identical land area of s_p for parking at each of trip origin and destination. The housing size of all the PAV households in the city is assumed to be a uniform quantity of s_h^{PAV} (with $s_h^{\mathrm{PAV}} > s_p$). The PAV household utility maximization problem for the choices of composite good consumption $z(x)$, residential location x, and worksite y subject to the budget constraint is given as

$$\max_{z(x),x,y} U\left(z(x),x,y\right) = z(x) + \gamma \ln s_h^{\mathrm{PAV}}, \tag{21}$$

s.t. $w_f(y) - \alpha_0^{\mathrm{PAV}} - \alpha_1^{\mathrm{PAV}}\left(x-y\right) - \left(r(x)+r(y)\right)s_p = z(x) + r(x)s_h^{\mathrm{PAV}},\ x \in [b_2, B], y \in [b_0, b_1],$ (22)

where α_0^{PAV} is the fixed ownership cost per PAV. The main difference between the PAV household's model (Eqs. (21) and (22)) and the TV household's model (Eqs. (4) and (5)) lies in the vehicle fixed cost, marginal travel cost, and the residential and industrial areas.

Solving the maximization problem Eqs. (21) and (22), one obtains the equilibrium residential land rent $r_h^{\mathrm{PAV}}(x)$, equilibrium composite good consumption of PAV households z^{PAV}, and the relationship between the wage $w_f^{\mathrm{PAV}}(y)$ at the best worksite y and the commuting cost savings $\alpha_1^{\mathrm{PAV}}\left(y-b_0\right)+\left(r(b_0)-r(y)\right)s_p$, as follows:

$$r_h^{\mathrm{PAV}}(x)=R_A+\frac{\alpha_1^{\mathrm{PAV}}}{s_h^{\mathrm{PAV}}+s_p}(B-x),\ x\in[b_2,B], \tag{23a}$$

$$w_f^{\mathrm{PAV}}(y)=w_f(b_0)-\alpha_1^{\mathrm{PAV}}\left(y-b_0\right)-\left(r(b_0)-r(y)\right)s_p,\ y\in[b_0,b_1], \tag{23b}$$

$$z^{\mathrm{PAV}}=w_f(b_0)-\alpha_0^{\mathrm{PAV}}-\alpha_1^{\mathrm{PAV}}\left(B-b_0\right)-r(b_0)s_p-R_A\left(s_h^{\mathrm{PAV}}+s_p\right), \tag{23c}$$

$$U^{\mathrm{PAV}}=w_f(b_0)-\alpha_0^{\mathrm{PAV}}-\alpha_1^{\mathrm{PAV}}\left(B-b_0\right)-r(b_0)s_p-R_A\left(s_h^{\mathrm{PAV}}+s_p\right)+\gamma\ln s_h^{\mathrm{PAV}}. \tag{23d}$$

Eq. (23a) shows that the equilibrium PAV household residential land rent, $r_h^{\mathrm{PAV}}(x)$, at location x equals the agricultural land rent plus the total commuting cost savings from a decrease in the commuting distance by $B-x$ between residence and worksite. This means that there exists a compensation rule between household residential land rent and commuting cost for the PAV households, i.e., an increase in residential land rent due to more proximity of residential location to the city center is just equal to a decrease in commuting cost, and vice versa. Eq. (23b) means that the wage, $w_f^{\mathrm{PAV}}(y)$, at the best worksite y equals the wage at location b_0 minus the total commuting cost savings due to a decrease in the commuting distance by $y-b_0$ between residence and worksite. This means that a compensation rule also exists between household wage and commuting cost for the PAV households. That is, the increased commuting cost for a household is just offset by the increased wage paid by a firm. Otherwise, the household has no incentive to choose to work in a farther firm. Eqs. (23c) and (23d) show that at equilibrium, all households are indifferent to their choices of residences and worksites.

4.2.2. SAV Households

SAVs, different from TVs and PAVs, do not occupy land for parking. The SAV household income is spent on the ride fare by SAV, commuting time cost, composite non-housing good consumption, and housing consumption. The utility maximization problem for the SAV households' choices of the amount of composite goods, residence, and worksite is represented as

$$\max_{z(x),x,y} \; U\left(z(x),x,y\right)=z(x)+\gamma\ln s_h^{\mathrm{SAV}}, \tag{24}$$

$$\text{s.t.} \;\; w_f(y)-\alpha_0^{\mathrm{SAV}}-\alpha_1^{\mathrm{SAV}}\left(x-y\right)=z(x)+r(x)s_h^{\mathrm{SAV}},\; x\in[b_1,b_2], y\in[0,b_0], \tag{25}$$

where α_0^{SAV} is the ride price by SAV.

From the maximization problem Eqs. (24) and (25), one can derive the equilibrium residential land rent profile $r_h^{\mathrm{SAV}}(x)$, equilibrium composite good consumption z^{SAV}, and the relationship between the wage $w_f^{\mathrm{SAV}}(y)$ at the best worksite y and the commuting cost savings $\alpha_1^{\mathrm{SAV}}y$, as follows:

$$r_h^{\mathrm{SAV}}(x)=\frac{\alpha_1^{\mathrm{SAV}}}{s_h^{\mathrm{SAV}}}\left(b_2-x\right)+\frac{\alpha_1^{\mathrm{PAV}}}{s_h^{\mathrm{PAV}}+s_p}\left(B-b_2\right)+R_A,\; x\in[b_1,b_2], \tag{26a}$$

$$w_f^{\mathrm{SAV}}(y)=w_f(0)-\alpha_1^{\mathrm{SAV}}y,\; y\in[0,b_0], \tag{26b}$$

$$z^{\mathrm{SAV}}=w_f(0)-\alpha_0^{\mathrm{SAV}}-\alpha_1^{\mathrm{SAV}}b_1-r(b_1)s_h^{\mathrm{SAV}}, \tag{26c}$$

$$U^{\mathrm{SAV}}=w_f(0)-\alpha_0^{\mathrm{SAV}}-\alpha_1^{\mathrm{SAV}}b_1-r(b_1)s_h^{\mathrm{SAV}}+\gamma\ln s_h^{\mathrm{SAV}}. \tag{26d}$$

Eq. (26a) shows that the equilibrium SAV household residential land rent, $r_h^{\mathrm{SAV}}(x)$, at location x equals the land rent at location b_2 (i.e., the last two terms) plus the total commuting cost savings from a decrease in the commuting distance by b_2-x between residence and worksite. Eq. (26b) means that the wage, $w_f^{\mathrm{SAV}}(y)$, at the best worksite y equals the wage at the city center minus the total commuting cost savings. These two equations indicate that the compensation rules between household residential land rent and commuting cost, and between wage and commuting cost also hold for the SAV households. Eqs. (26c) and (26d) show that at equilibrium, all households are indifferent to their choices of residences and worksites.

Note that each SAV household at the residential area needs only s_h^{SAV} units of residential land without need for parking land, whereas each PAV household needs s_h^{PAV} units of residential land and s_p units of parking land. This means that each PAV household residing at the most peripheral area $[b_2, B]$ requires a total of $s_h^{\text{PAV}} + s_p$ units of land, whereas each SAV household residing at the area $[b_1, b_2]$ occupies s_h^{SAV} units of land. The household residential density function, $n(x)$, at location *x* can thus be given as

$$n(x) = \begin{cases} 1\big/\left(s_h^{\text{PAV}} + s_p\right), \ x \in [b_2, B], \\ 1\big/s_h^{\text{SAV}}, \ x \in [b_1, b_2], \\ 0, \ \text{otherwise.} \end{cases} \tag{27}$$

4.3. Firms

In order to represent the firm production function, we first define the firm density function. Note that in the future AV era, the SAV commuters do not need to occupy the land for parking due to use of shared services. The land released by the SAVs in the industrial area is used for firm production. But, each of the PAV commuters needs s_p units of land for daytime parking at the worksite, like TV commuters. On the other hand, each firm requiring l_f units of labor needs s_f units of land for production. Therefore, the quantities of land consumed by each firm on $[0, b_0]$ and $[b_0, b_1]$ are, respectively, s_f and $s_p l_f + s_f$ (referring to Fig. 2). One thus obtains the firm density, $m(y)$, at worksite *y* as

$$m(y) = \begin{cases} 1\big/s_f, \ y \in [0, b_0], \\ 1\big/\left(s_p l_f + s_f\right), \ y \in [b_0, b_1], \\ 0, \ \text{otherwise.} \end{cases} \tag{28}$$

According to Eq. (11) and Fig. 2, the production function of firms for a city with mixed PAVs and SAVs can be expressed as

$$\begin{aligned} F(y) &= \int_{-b_1}^{b_1} \left(\beta - \delta\left|t - y\right|\right) m(t) dt \\ &= \begin{cases} \dfrac{1}{s_p l_f + s_f}\left(2\beta\left(b_1 - b_0\right) + \delta\left(b_0^{\,2} - b_1^{\,2}\right)\right) + \dfrac{1}{s_f}\left(-\delta y^2 + 2\beta b_0 - \delta b_0^{\,2}\right), \ y \in [0, b_0], \\ \dfrac{1}{s_p l_f + s_f}\left(-\delta y^2 + 2\delta b_0 y - \delta b_1^{\,2} + 2\beta\left(b_1 - b_0\right)\right) + \dfrac{2b_0}{s_f}\left(\beta - \delta y\right), \ y \in [b_0, b_1]. \end{cases} \end{aligned} \tag{29}$$

Eq. (29) shows that the firm production function is concave and decreasing with regard to *y* on $[0, b_0]$ and $[b_0, b_1]$. It is easy to show $dF(y)/d\theta > 0$ holds, meaning that the firm production increases with the SAV market share θ. This is because the firm density at this moment increases due to more land releases from SAVs.

Again, we assume that all firms are homogeneous with the same inputs of labor and land. Under the perfect competition, each firm's net profit is 0, i.e., $\Pi(y)=0$, where $\Pi(y)$ is defined by Eq. (14). One thus obtains the land rent, $r_f(y)$, of firm at location y as

$$r_f(y)=\frac{1}{s_f}\left(F(y)-w_f(y)l_f\right),\ y\in[0,b_0]\cup[b_0,b_1]. \tag{30}$$

4.4. Equilibrium

Given the market share, θ (or $1-\theta$), of SAV (or PAV) commuters, the spatial structure of the urban system is endogenously determined by these variables: b_0, b_1, b_2, B, $w_f(y)$, $r_f(x)$, and $r_h(x)$. They can be calculated by the following equations which represent the equilibrium conditions of the system:

$$2\left(\int_0^{b_0} m(y)dy+\int_{b_0}^{b_1} m(y)dy\right)=M,\ 2\int_{b_1}^{b_2} n(x)dx=\theta N,\ 2\int_{b_2}^{B} n(x)dx=(1-\theta)N, \tag{31a}$$

$$r_f^{\text{SAV}}(b_0)=r_f^{\text{PAV}}(b_0),\ r_f^{\text{PAV}}(b_1)=r_h^{\text{SAV}}(b_1),\ r_h^{\text{SAV}}(b_2)=r_h^{\text{PAV}}(b_2),\ r_h^{\text{PAV}}(B)=R_A. \tag{31b}$$

Eq. (31a) is the labor market equilibrium conditions for the SAV and PAV households. Eq. (31b) is the land market equilibrium conditions for the industrial and residential areas.

From the equilibrium conditions (31a) and (31b), one can derive

$$b_0=\frac{N\theta s_f}{2l_f},\ b_1=\frac{N}{2l_f}\left((1-\theta)l_f s_p+s_f\right),\ b_2=\frac{N}{2l_f}\left(\theta s_h^{\text{SAV}}l_f+(1-\theta)s_p l_f+s_f\right), \tag{32a}$$

$$B=\frac{N}{2l_f}\left((1-\theta)l_f s_h^{\text{PAV}}+\theta l_f s_h^{\text{SAV}}+2(1-\theta)l_f s_p+s_f\right), \tag{32b}$$

$$r_h(x)=\begin{cases} R_A+\dfrac{\alpha_1^{\text{PAV}}}{s_h^{\text{PAV}}+s_p}\left(B-b_2\right)+\dfrac{\alpha_1^{\text{SAV}}}{s_h^{\text{SAV}}}\left(b_2-x\right),\ x\in[b_1,b_2], \\ R_A+\dfrac{\alpha_1^{\text{PAV}}}{s_h^{\text{PAV}}+s_p}(B-x),\ x\in[b_2,B], \\ R_A,\ x\in[B,+\infty), \end{cases} \tag{32c}$$

$$r_f(y)=\begin{cases}\dfrac{1}{s_f}\left(F(y)+l_f\alpha_1^{\mathrm{SAV}}y\right)-\dfrac{l_f s_p F(b_0)+s_f F(b_1)}{s_f\left(s_p l_f+s_f\right)}+R_A,\ y\in[0,b_0],\\ \dfrac{F(y)-F(b_1)+l_f\alpha_1^{\mathrm{PAV}}\left(y-b_0\right)}{s_f+s_p l_f}+\dfrac{N\theta\alpha_1^{\mathrm{SAV}}}{2}+R_A,\ y\in[b_0,b_1],\end{cases}\tag{32d}$$

$$w_f(y)=\begin{cases}\dfrac{s_p l_f F(b_0)+s_f F(b_1)}{l_f\left(s_f+s_p l_f\right)}-\dfrac{s_f}{l_f}R_A-\alpha_1^{\mathrm{SAV}}y,\ y\in[0,b_0],\\ \dfrac{s_p l_f F(y)+s_f F(b_1)-l_f s_f\alpha_1^{\mathrm{PAV}}\left(y-b_0\right)}{l_f\left(s_f+s_p l_f\right)}-\dfrac{s_f}{l_f}\left(\dfrac{N\theta\alpha_1^{\mathrm{SAV}}}{2}+R_A\right),\ y\in[b_0,b_1].\end{cases}\tag{32e}$$

Eqs. (32a) and (32b) show that for given values of housing sizes s_h^{SAV} and s_h^{PAV}, the borders b_0 and b_2 increase with the SAV market share θ (due to $s_h^{\mathrm{SAV}}>s_p$), while b_1 decreases with θ, i.e., the industrial area becomes more compact due to the decreased parking land demand with SAVs. As θ increases, the city boundary B may decrease or increase, depending on the housing sizes of PAV and SAV households and the parking land occupancy. As $s_h^{\mathrm{SAV}}>s_h^{\mathrm{PAV}}+2s_p$ holds, the city boundary expands with an increase in θ. Eqs. (32c) and (32d) show that the household's land rent $r_h(x)$ is linearly decreasing with the distance from the city center, while the firm's land rent $r_f(y)$ is concave on $[0,b_1]$. Eq. (32e) indicates that the wage function $w_f(y)$ is linearly decreasing on $[0,b_0]$, but concave and decreasing on $[b_0,b_1]$. Therefore, the wage of the SAV workers is higher than that of PAV workers.

4.5. Optimal housing size under social welfare maximization

The government regulates the housing development and the housing is rent out to the PAV and SAV households, aiming to maximize the social welfare. The welfare maximization problem for determining the optimal housing sizes s_h^{PAV} and s_h^{SAV} is

$$\begin{aligned}\max_{s_h^{\mathrm{PAV}},s_h^{\mathrm{SAV}}}\ SW^{\mathrm{AV}}&=\theta NU^{\mathrm{SAV}}\left(s_h^{\mathrm{SAV}}\right)+\left(1-\theta\right)NU^{\mathrm{PAV}}\left(s_h^{\mathrm{PAV}}\right)\\&+2\int_0^{b_0}r_f^{\mathrm{SAV}}(y)dy+2\int_{b_0}^{b_1}r_f^{\mathrm{PAV}}(y)dy+2\int_{b_1}^{b_2}r_h^{\mathrm{SAV}}(x)dx+2\int_{b_2}^{B}r_h^{\mathrm{PAV}}(x)dx-2\left(R_A+c\right)B,\end{aligned}\tag{33}$$

where U^{PAV} and U^{SAV} are given by Eqs. (23d) and (26d), respectively. The first term on the right-hand side of Eq. (33) is the total utility of SAV households, the second term is the total utility of PAV households, the third and fourth terms are the total land rents from firms, the fifth and sixth terms are the total land rents from households, and the last term is the total land cost, including the land opportunity cost and the land development cost.

The welfare function SW^{AV} is concave with regard to the housing sizes for PAV and SAV households (see Appendix C for its derivation), and thus there is a unique solution to the optimization problem. From the first-order optimality condition of maximization problem (33), one can obtain the optimal solutions of s_h^{PAV} and s_h^{SAV} as

$$s_h^{\mathrm{PAV}} = \frac{4\gamma}{(1-\theta)N\alpha_1^{\mathrm{PAV}} + 4\left(R_A + c\right)}, \text{ and } s_h^{\mathrm{SAV}} = \frac{4\gamma}{2\left(1-\theta\right)N\alpha_1^{\mathrm{PAV}} + \theta N\alpha_1^{\mathrm{SAV}} + 4\left(R_A + c\right)}. \quad (34)$$

Eq. (34) shows that an increase in α_1^{PAV} causes a decrease in the optimal housing sizes for both PAV and SAV households. However, as α_1^{SAV} increases, the housing size for SAV households always decreases. In addition, an increase in the SAV market share θ leads to an increase in the optimal housing size s_h^{PAV} for PAV households. Howevere, the optimal housing size s_h^{SAV} for SAV households may increase or decrease, depending on the relationship between α_1^{SAV} and α_1^{PAV}. As $\alpha_1^{\mathrm{SAV}} > 2\alpha_1^{\mathrm{PAV}}$, an increase in θ causes a decrease in s_h^{SAV}. Comparing s_h^{PAV} and s_h^{SAV} in Eq. (34), one can see that the optimal housing size of PAV households is always larger than that of SAV households.

In the above analysis, the SAV market share θ is exogenously given. Naturally, what the government cares about is to find the optimal value of θ to maximize the social welfare. When there is an interior solution, the condition $\partial SW^{\mathrm{AV}}(\theta)/\partial\theta = 0$ should hold. The expression for the optimal value of θ is complicated, and its detailed derivation is provided in Appendix D.

Comparing the results before and after introducing AVs, we obtain the following properties.

Proposition 3. After introducing AVs,

(i) The housing size of PAV households increases, i.e., $s_h^{\mathrm{PAV}} > s_h^{\mathrm{TV}}$; and the housing size of SAV households increases if and only if $\alpha_1^{\mathrm{TV}} > 2(1-\theta)\alpha_1^{\mathrm{PAV}} + \theta\alpha_1^{\mathrm{SAV}}$.

(ii) The industrial area b_1 gets more compact, leading to an increase in the firm production; and the city expands if and only if $(1-\theta)s_h^{\mathrm{PAV}} + \theta s_h^{\mathrm{SAV}} > s_h^{\mathrm{TV}} + 2\theta s_p$.

(iii) The social welfare may increase or decrease, depending on the vehicle fixed cost, marginal travel cost, and SAV market share.

It should be pointed out that the critical condition $\alpha_1^{\text{TV}} > 2(1-\theta)\alpha_1^{\text{PAV}} + \theta\alpha_1^{\text{SAV}}$ in Proposition 3(i) can be derived from $s_h^{\text{SAV}} > s_h^{\text{TV}}$. Proposition 3(i) shows that the housing size of PAV households always increases after the introduction of AVs, while the housing size of SAV households may increase or decrease, depending on the marginal travel costs of AVs and TVs, and the SAV market share. The critical condition of $(1-\theta)s_h^{\text{PAV}} + \theta s_h^{\text{SAV}} > s_h^{\text{TV}} + 2\theta s_p$ in Proposition 3(ii) can be directly obtained by taking a difference of the city boundaries before and after introducing AVs (i.e., Eq. (32a) minus Eq. (17a)). Proposition 3(ii) shows that after introducing AVs, the size of the industrial area becomes smaller because the land demand for parking with SAVs is reduced; and the city boundary may expand or contract, which is a result of trade-off among the following factors: the decreased parking land demand, the increased housing size for PAV households, and the increased or decreased housing size for SAV households. As $(1-\theta)s_h^{\text{PAV}} + \theta s_h^{\text{SAV}} > s_h^{\text{TV}} + 2\theta s_p$ holds, the city enlarges after introducing AVs.

The change of the social welfare before and after introducing the AVs can be defined as $\Delta SW = SW^{\text{AV}} - SW^{\text{TV}}$. Note that ΔSW is a function of fixed vehicle ownership cost, marginal travel cost, and SAV market share. At the early stage of the AV technology development, the vehicle fixed costs of PAVs and SAVs are high. Hence, ΔSW may be negative, meaning a decrease in the social welfare after the introduction of AVs. However, when the AV technologies are mature such that the vehicle fixed costs of PAVs and SAVs are low (e.g., close to that of TVs), but their marginal travel costs are lower than that of TVs, and thus ΔSW is positive, implying that the introduction of AVs increases the social welfare. This highlights the critical role of the maturity level of AV technologies in enabling the widespread adoption of AVs in the real-world applications.

5. Two extreme cases: full PAVs and full SAVs

In this section, we consider two extreme cases: full PAVs ($\theta = 0$) and full SAVs ($\theta = 1$). The case of full PAVs is similar to the TV case, in which all vehicles need to park at residential location and work location, and the main differences are the fixed cost and marginal travel cost of vehicles. Therefore, the expressions for the urban model with the full PAVs are basically consistent with those with TVs. However, for a full SAV system, no land is needed for parking, and all the land of the city is dedicated to industrial production and household residences, as shown in Fig. 3. Hence, the household residential density and the firm density become $m(x) = 1/s_h^{\mathrm{SAV}}$ for $x \in [b_1, B]$, and $m(y) = 1/s_f$ for $y \in [0, b_1]$, respectively.

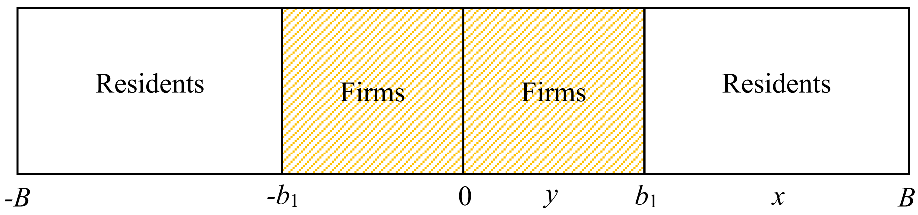


Fig. 3. Symmetric monocentric urban configuration with full SAVs.

Similar to the previous sections, from the urban equilibrium conditions and the social welfare maximization model for determining the optimal housing size, one can derive the border, b_1, between industrial and residential areas, city boundary *B*, firm's bid-rent curve $r_f(y)$, wage curve $w_f(y)$, housing rent curve $r_h(x)$, and the optimal housing size s_h. Table 2 summarizes the optimal solutions of urban system under the two extreme cases of full PAVs and full SAVs.

Table 2 Solutions for full PAVs ($\theta = 0$) and full SAVs ($\theta = 1$).

	Full PAVs ($\theta = 0$)	Full SAVs ($\theta = 1$)
Optimal housing size s_h	$\dfrac{4\gamma}{N\alpha_1^{\mathrm{PAV}} + 4(R_A + c)}$	$\dfrac{4\gamma}{N\alpha_1^{\mathrm{SAV}} + 4(R_A + c)}$
Border between industrial and residential areas b_1	$\dfrac{N}{2l_f}(s_f + s_p l_f)$	$\dfrac{N s_f}{2l_f}$
City boundary B	$\dfrac{N}{2l_f}\left(s_f + l_f\left(s_h^{\mathrm{PAV}} + 2s_p\right)\right)$	$\dfrac{N}{2l_f}\left(s_f + l_f s_h^{\mathrm{SAV}}\right)$
Firm production $F(y)$, $y \in [0, b_1]$	$\dfrac{2\beta b_1 - \delta\left(y^2 + b_1^2\right)}{s_p l_f + s_f}$	$\dfrac{2\beta b_1 - \delta\left(b_1^2 + y^2\right)}{s_f}$
Firm land rent $r_f(y)$, $y \in [0, b_1]$	$\dfrac{F(y) - F(b_1) + l_f \alpha_1^{\mathrm{PAV}} y}{s_p l_f + s_f} + R_A$	$\dfrac{F(y) - F(b_1) + l_f \alpha_1^{\mathrm{SAV}} y}{s_f} + R_A$
Wage $w_f(y)$, $y \in [0, b_1]$	$\dfrac{l_f s_p F(y) + s_f F(b_1) - y l_f s_f \alpha_1^{\mathrm{PAV}}}{l_f\left(s_p l_f + s_f\right)} - \dfrac{s_f}{l_f} R_A$	$\dfrac{F(b_1) - s_f R_A}{l_f} - y\alpha_1^{\mathrm{SAV}}$
Household land rent $r_h(x)$, $x \in [b_1, B]$	$R_A + \dfrac{\alpha_1^{\mathrm{PAV}}}{s_h^{\mathrm{PAV}} + s_p}(B - x)$	$R_A + \dfrac{\alpha_1^{\mathrm{SAV}}}{s_h^{\mathrm{SAV}}}(B - x)$
Household utility U	$w_f(0) - \alpha_0^{\mathrm{PAV}} - \alpha_1^{\mathrm{PAV}} B - r(0)s_p - R_A\left(s_h^{\mathrm{PAV}} + s_p\right) + \gamma \ln s_h^{\mathrm{PAV}}$	$w_f(0) - \alpha_0^{\mathrm{SAV}} - \alpha_1^{\mathrm{SAV}} B - R_A s_h^{\mathrm{SAV}} + \gamma \mathrm{In} s_h^{\mathrm{SAV}}$

In order to look at the effects of introducing PAVs, we compare the solutions with TVs and with full PAVs, summarized as follows.

Proposition 4. Given the total number, *N*, of households, compared to the TV city, for a city with full PAVs

(i) The size of industrial area and the firm production remains unchanged. However, the firm land rent decreases, leading to an increased wage level.

(ii) The household land rent decreases, the housing size increases, and the size of the city with full PAVs expands.

(iii) The social welfare may increase or a decrease, depending on the vehicle fixed cost and marginal travel cost of PAVs and TVs.

Proposition 4 shows that the full PAVs lead to a decreased firm land rent, but a higher wage level, which benefits all workers. On the other hand, a decrease in the marginal travel cost of PAVs and an increase in the wage allow households to afford larger housing spaces, thus causing urban sprawl. The high vehicle ownership cost of PAVs, particularly at the early stage of the AV technology development, may initially lead to a decrease in the social welfare. As AV technologies mature and the vehicle ownership cost of PAVs decreases, the introduction of AVs is expected to enhance the social welfare.

Similarly, by comparing the results before and after introducing SAVs, we can identify the effects of SAVs on the future urban spatial structure as follows.

Proposition 5. Given the total number, *N*, of households, compared to the TV city, for a city with full SAVs

(i) The size of industrial area decreases, and thus the firm production rises.

(ii) The housing size of SAV households would increase, but the city expands if and only if $s_p < \dfrac{2\gamma N\left(\alpha_1^{\mathrm{TV}} - \alpha_1^{\mathrm{SAV}}\right)}{\left(N\alpha_1^{\mathrm{SAV}} + 4\left(R_A + c\right)\right)\left(N\alpha_1^{\mathrm{TV}} + 4\left(R_A + c\right)\right)}$.

(iii) The social welfare may increase or a decrease, depending on the ride fare by SAV and the marginal travel costs of SAVs and TVs.

Proposition 5 shows that introduction of SAVs leads the size of the industrial area to contract, inducing a higher firm density within this area. This enhances the agglomeration externalities, such as the knowledge or information spillover effects. As a result, the firm production increases.

On the other hand, the optimal housing size always increases due to introduction of SAVs. However, the city size with full SAVs may increase or decrease, depending on two opposite effects: the increased housing size of SAV households and the decreased parking land demand. Specifically, as the increased housing size outweighs the decreased parking land size, the city expands after introducing the full SAVs. As the ride price by SAV is relatively low, the introduction of SAVs will lead to an increase in the social welfare due to its lower marginal travel cost.

6. Incorporating traffic congestion effects

Traffic congestion was ignored so far. This section extends the analysis to include the effects of traffic congestion. Let $c(Q(x))$ be the commuting time cost per unit of distance at location *x*. Following Mun et al. (2003), Chu and Tsai (2008), and Li et al. (2014), the commuting time cost at location *x* can be estimated by a linear function of the traffic volume (i.e., the mass of commuters) crossing that location, $Q(x)$, expressed as

$$c(Q(x)) = \alpha_1^i + \alpha_2^i Q(x),\ i = \text{TV, PAV, SAV}, \tag{35}$$

where α_1^i is the free-flow travel time cost per unit of distance by mode *i*, and α_2^i is the marginal congestion time cost. AVs enable commuters to engage in productive / enjoyable in-vehicle activities. The disutility of traffic congestion and thus the marginal congestion time costs of AVs can be reduced significantly. Therefore, it is reasonable to assume that the marginal congestion time cost of TVs is the highest, that of PAVs is the lowest, and that of SAVs is in between, i.e., $\alpha_2^{\text{PAV}} < \alpha_2^{\text{SAV}} < \alpha_2^{\text{TV}}$.

The commuting time cost, $C(x, y)$, between residence *x* and worksite *y* is then given by

$$C(x, y) = \int_y^x c(Q(t))dt, \tag{36}$$

where $c(\cdot)$ is determined by Eq. (35).

We first examine the case with TVs only (i.e., before introducing AVs). The number of commuters crossing location *x*, $Q(x)$, equals the number of commuters living at any locations on the right-hand side of *x* and working on the left-hand side of *x*, given by

$$Q(x)=\begin{cases}\int_0^x m(x)l_f dx=\dfrac{l_f x}{s_p l_f+s_f}, x\in[0,b_1],\\ \int_x^B n(x)dx=\dfrac{B-x}{s_h+s_p}, x\in[b_1,B].\end{cases} \tag{37}$$

From Eqs. (35)-(37), the commuting time cost function for full TV city can be calculated as

$$C(x,y)=\alpha_1^{\mathrm{TV}}\left(x-y\right)+\frac{\alpha_2^{\mathrm{TV}}B}{s_h+s_p}\left(x-b_1\right)-\frac{\alpha_2^{\mathrm{TV}}}{2\left(s_h+s_p\right)}\left(x^2-b_1^2\right)+\frac{\alpha_2^{\mathrm{TV}}l_f}{2\left(s_p l_f+s_f\right)}\left(b_1^2-y^2\right). \tag{38}$$

We now examine the case after the introduction of AVs (i.e., mixed PAVs and SAVs). With the assumptions of $\alpha_1^{\mathrm{PAV}}<\alpha_1^{\mathrm{SAV}}$ and $\alpha_2^{\mathrm{PAV}}<\alpha_2^{\mathrm{SAV}}$, PAV households are incentivized to reside and work in more peripheral areas with lower land competition. However, SAV households tend to choose the residential and work locations closer to the city center, as they do not occupy parking land. As a result, a spatial segregation emerges between the residences and worksites for PAV and SAV households, as shown in Fig. 2. The number of commuters, $Q(x)$, crossing location x then becomes

$$Q(x)=\begin{cases}\int_0^x m(x)l_f dx=\dfrac{l_f x}{s_f}, x\in[0,b_0],\\ \int_0^{b_0} m(x)l_f dx+\int_{b_0}^x m(x)l_f dx=\dfrac{l_f b_0}{s_f}+\dfrac{l_f\left(x-b_0\right)}{s_p l_f+s_f}, x\in[b_0,b_1],\\ \int_x^{b_2} n(x)dx+\int_{b_2}^B n(x)dx=\dfrac{b_2-x}{s_h^{\mathrm{SAV}}}+\dfrac{B-b_2}{s_h^{\mathrm{PAV}}+s_p}, x\in[b_1,b_2],\\ \int_x^B n(x)dx=\dfrac{B-x}{s_h^{\mathrm{PAV}}+s_p}, x\in[b_2,B].\end{cases} \tag{39}$$

Substituting Eqs. (35) and (39) into Eq. (36), one can derive the commuting time cost function for the AV city with mixed PAVs and SAVs as follows.

$$C(x,y)\begin{cases}=\alpha_1^{\mathrm{SAV}}\left(x-y\right)+\alpha_2^{\mathrm{SAV}}\left(\dfrac{l_f}{s_f}\left(b_0b_1-0.5b_0^{\,2}-0.5y^2\right)+\dfrac{l_f\left(b_1-b_0\right)^2}{2\left(s_p l_f+s_f\right)}\right)\\ \quad+\alpha_2^{\mathrm{SAV}}\left(\dfrac{b_2x-0.5x^2-b_2b_1+0.5b_1^{\,2}}{s_h^{\mathrm{SAV}}}+\dfrac{B-b_2}{s_h^{\mathrm{PAV}}+s_p}\left(x-b_1\right)\right), y\in[0,b_0], x\in[b_1,b_2],\\ =\alpha_1^{\mathrm{PAV}}\left(x-y\right)+\alpha_2^{\mathrm{PAV}}\left(\dfrac{l_f b_0\left(b_1-y\right)}{s_f}+\dfrac{l_f\left(0.5b_1^{\,2}-b_0b_1-0.5y^2+b_0y\right)}{s_p l_f+s_f}\right)\\ \quad+\alpha_2^{\mathrm{PAV}}\left(\dfrac{\left(b_2-b_1\right)^2}{2s_h^{\mathrm{SAV}}}+\dfrac{b_1\left(b_2-B\right)+Bx-0.5x^2-0.5b_2^{\,2}}{s_h^{\mathrm{PAV}}+s_p}\right), y\in[b_0,b_1], x\in[b_2,B].\end{cases} \tag{40}$$

Substituting Eqs. (38) and (40) into the models proposed in the previous sections, one can obtain the urban model solutions with traffic congestion consideration (for more details, please see Appendix E). It can easily be shown that as the marginal congestion travel costs of TVs and AVs equal zero, i.e., $\alpha_2^i = 0$, $i = \text{TV, PAV, SAV}$, the solutions under the congestion scenario are reduced to those under no congestion scenario. As expected, the no congestion scenario is a special case of the congestion scenario.

The total commuting distances of all households for the full TV city and the mixed AV city, represented as TD^{TV} and TD^{AV}, can, respectively, be calculated as

$$TD^{\text{TV}} = \int_{b_1}^{B} (t - b_1) n(t) dt + \int_0^{b_1} (b_1 - t) l_f m(t) dt = \frac{NM \left(l_f \left(s_h^{\text{TV}} + 2s_p \right) + s_f \right)}{8}, \tag{41a}$$

$$\begin{aligned} TD^{\text{AV}} &= \int_0^{b_0} (b_1 - t) l_f m(t) dt + \int_{b_0}^{b_1} (b_1 - t) l_f m(t) dt + \int_{b_1}^{b_2} (t - b_1) n(t) dt + \int_{b_2}^{B} (t - b_1) n(t) dt \\ &= \frac{NM}{8} \left((2-\theta)\theta s_h^{\text{SAV}} l_f + (1-\theta) l_f \left((1-\theta) s_h^{\text{PAV}} + s_p \right) + s_f \right). \end{aligned} \tag{41b}$$

One can then obtain the total congestion costs, TC^{TV} and TC^{AV}, for the full TV city and the mixed AV city as

$$TC^{\text{TV}} = \int_0^{b_1} \int_y^{b_1} \alpha_2^{\text{TV}} N(t) dt m(y) l_f dy + \int_{b_1}^{B} \int_{b_1}^{x} \alpha_2^{\text{TV}} N(t) dt n(x) dx = \frac{\alpha_2^{\text{TV}} M N^2 \left(l_f \left(s_h^{\text{TV}} + 2s_p \right) + s_f \right)}{24}, \tag{42a}$$

$$\begin{aligned} TC^{\text{AV}} &= \int_0^{b_0} \int_y^{b_1} \alpha_2^{\text{SAV}} N(t) dt m(y) l_f dy + \int_{b_0}^{b_1} \int_y^{b_1} \alpha_2^{\text{PAV}} N(t) dt m(y) l_f dy \\ &\quad + \int_{b_1}^{b_2} \int_{b_1}^{x} \alpha_2^{\text{SAV}} N(t) dt n(x) dx + \int_{b_2}^{B} \int_{b_1}^{x} \alpha_2^{\text{PAV}} N(t) dt n(x) dx \\ &= \frac{N^2 M}{48} \Big\{ \alpha_2^{\text{SAV}} \theta \left(s_h^{\text{SAV}} (3-\theta)\theta l_f + \left(3 - \theta^2\right) s_f + 3\left(1-\theta^2\right) s_p l_f \right) \\ &\quad + \alpha_2^{\text{PAV}} (1-\theta) \left(3\theta(2-\theta) s_h^{\text{SAV}} l_f + 2(1-\theta)^2 l_f \left(s_h^{\text{PAV}} + s_p \right) + (2+\theta)(1-\theta)\left(s_f + s_p l_f \right) \right) \Big\}. \end{aligned} \tag{42b}$$

In order to investigate the effects of traffic congestion on the urban spatial structure, we conduct comparative static analyses of the marginal congestion time costs of TVs, PAVs, and SAVs with respect to the optimal housing size, city boundary, and total commuting distance. In the full TV city, an increase in the marginal congestion time cost, α_2^{TV}, of TVs leads to a decrease in the optimal housing size for TV households. This is because the higher congestion cost raises the commuting cost, thereby reducing the housing consumption. Consequently, both the city size and total commuting distance decrease in α_2^{TV}. In the mixed AV city, an increase in the marginal congestion time cost, α_2^{PAV}, of PAVs results in a reduction in the optimal housing sizes for both PAV and SAV households. The marginal congestion time cost, α_2^{SAV}, of SAVs decreases the optimal housing size for SAV households, but has no impact on the optimal housing size for PAV households. As a result, both the city size and the total commuting distance decrease with α_2^{PAV} and α_2^{SAV}.

Comparing the solutions with and without the congestion effects in the full TV city and the mixed AV city, we obtain the following proposition.

Proposition 6. For a full TV city or a mixed AV city, ignoring traffic congestion effects will overestimate the housing size, household utility, city size, and social welfare, compared to the congestion scenario.

Proposition 6 shows that traffic congestion externality can significantly influence the urban spatial structure, and ignoring its effects may lead to biased decision-making. Therefore, it is important to incorporate the congstion effects in the evaluation and design of urban policies, particularly for heavily congested cities.

To examine the effects of AVs on the urban spatial structure with congestion effect consideration, we compare the solutions before and after introducing the AVs, and obtain the following properties.

Proposition 7. With congestion effect consideration, after introducing the AVs

(i) The housing size of PAV households increases, i.e., $s_h^{\mathrm{PAV}} > s_h^{\mathrm{TV}}$, while the housing size of SAV households increases if and only if $6\alpha_1^{\mathrm{TV}} + 2\alpha_2^{\mathrm{TV}} N > 12(1-\theta)\alpha_1^{\mathrm{PAV}} + 3\alpha_2^{\mathrm{PAV}}(1-\theta)(2-\theta)N + 6\theta\alpha_1^{\mathrm{SAV}} + \theta\alpha_2^{\mathrm{SAV}} N(3-\theta)$.

(ii) The industrial area b_1 gets more compact, leading to an increase in the firm production; and the city expands if and only if $\theta s_h^{\mathrm{SAV}} + (1-\theta)s_h^{\mathrm{PAV}} > s_h^{\mathrm{TV}} + 2\theta s_p$.

(iii) The total commuting distance and total congestion cost of the city system may increase or decrease, depending on the SAV market share, the marginal congestion time costs and marginal travel time costs of TVs, PAVs, and SAVs.

(iv) The social welfare may increase or decrease, depending on the vehicle fixed cost, marginal travel time cost, marginal congestion time cost, and SAV market share.

It should be pointed out that the results in Proposition 7 are similar to those without congestion effects. Proposition 7 shows that the housing size of PAV households increases due to their lower marginal travel time cost, which encourages higher housing consumption. However, the housing size of SAV households may increase or decrease, relying on the marginal travel time cost, marginal congestion time cost, and the SAV market share. The critical condition in Proposition 7(i) is derived from $s_h^{\mathrm{SAV}} > s_h^{\mathrm{TV}}$. It is because the release of parking spaces and the lower marginal travel time cost of SAVs encourage SAV households to consume larger housing. After introducing the AVs, the size of the industrial area decreases due to reduced parking land demand with SAVs. This means that the industrial area becomes more contact, leading to increased firm production due to the agglomeration economy effects. The city boundary may expand or contract after introducing AVs, depending on the trade-off of the following effects: (i) the decreased parking land demand, (ii) the increased housing size for PAV household, and (iii) the increased or decreased housing size for SAV households. As $\theta s_h^{\mathrm{SAV}} + (1-\theta) s_h^{\mathrm{PAV}} > s_h^{\mathrm{TV}} + 2\theta s_p$, the city expands, resulting in the occupation of more artificially developed land.

Compared to TVs, AVs have a lower marginal congestion time cost due to the additional in-vehicle utility. However, the introduction of AVs may contribute to city expansion, resulting in longer commuting distances, which could lead to a higher commuting time consumption due to congestion. As a result, the total congestion cost in the city may decrease or increase after introducing AVs, depending on the trade-off between two opposite effects: the decreased marginal congestion time cost and the increased commuting time under congestion. When the negative effects of the increased commuting time outweigh the reduction in the marginal congestion time cost, the total congestion cost increases after the introduction of AVs. Otherwise, the AVs can help reduce the total congestion cost. Similar to the no congestion case, the introduction of AVs may lead to a reduction in the social welfare under congestion case, particularly at the early stages of AV technology development, where the vehicle fixed costs of PAVs and SAVs are quite high. As the AV technologies become mature, the vehicle fixed costs of PAVs and SAVs decrease. Therefore, the social welfare increases after introducing AVs.

In light of the above discussions, we find that if AVs are introduced hastily or without careful planning, they may impose significant negative effects instead, including increased vehicular miles traveled, exacerbated traffic congestion, excessive urban sprawl, and decreased social welfare. Therefore, it is crucial for the authorities to make cautiously a plan of AV implementation. Some regulatory policies may also be made to tackle these potential negative effects so as to make full use of the AV advantages to improve the social welfare and urban efficiency.

7. Numerical illustrations

In this section, numerical examples are provided to further illustrate the economic consequences of AVs. We examine the effects of AVs on the future city size, traffic congestion, and social welfare; and compare the land rent, wage, and firm production profiles before and after introducing AVs.

7.1. Parameter specifications

We assume a population size of 2,000 households (i.e., $N = 2,000$) in the linear monocentric city corridor.[11] Each firm has an average of 32 workers and 250 meters of land, i.e., $l_f = 32$ and $s_f = 250$. The parameters in the firm production function are set as: $\beta = \$40,000/\text{year}$ and $\delta = \$1/m/\text{year}$. The annual agricultural land rent R_A is \$1,000 per meter. The marginal land development cost *c* is \$23/m. Households have a land preference valued at \$15,000, i.e., $\gamma = \$15,000$. The parking land occupancy per vehicle s_p is 2.5 meters. The annaul ownership cost of a TV α_0^{TV} is \$2600 per year. The AV cost may be extremely high, particularly at the early stage of AV technology development (Zakharenko, 2016). In this paper, we assume that the annual ownership costs of a PAV α_0^{PAV} is \$24000 per year, and the annual ride price by SAV α_0^{SAV} is \$21000 per year. The annual marginal travel time costs, α_1^{TV}, α_1^{PAV}, and α_1^{SAV}, of TVs, PAVs, and SAVs are assumed to be \$0.3, \$0.05, and \$0.25 per meter, respectively. The annual marginal congestion time costs, α_2^{TV}, α_2^{PAV}, and α_2^{SAV}, of TVs, PAVs, and SAVs are

[11] In this example, we consider a linear urban corridor with a length of 25 km and a total of 2,000 households. Thereby, for a circular symmetric city with such a corridor as the radius, the total number of households is $2000 \times 25 \times 2\pi = 314000$ households.

\$0.00028/veh-m, \$0.000275/veh-m, and \$0.000278/veh-m, respectively. The values of some parameters defined above are from Brueckner (1997), Zakharenko (2016), Reginer and Legras (2018), Li et al. (2022), and Kyriakopoulou and Picard (2023). Table 3 further summarizes the baseline values of all the input parameters.

Table 3 Values of input parameters.

Parameter	Description	Baseline value
N	Number of households	2,000
γ	Household preference for land (\$)	15,000
s_p	Land for parking (m)	2.5
α_0^{TV}	Annual ownership cost of TVs (\$/veh/year)	2600
α_1^{TV}	Marginal travel time cost of TVs (\$/m/year)	0.3
α_2^{TV}	Marginal congestion travel time cost of TVs (\$/veh-m/year)	0.00028
α_0^{PAV}	Annual ownership cost of PAVs (\$/veh/year)	24,000
α_1^{PAV}	Marginal travel time cost of PAVs (\$/m/year)	0.05
α_2^{PAV}	Marginal congestion travel time cost of PAVs (\$/veh-m/year)	0.000275
α_0^{SAV}	Annual ride price by SAVs (\$/year)	21,000
α_1^{SAV}	Marginal travel time cost of SAVs (\$/m/year)	0.25
α_2^{SAV}	Marginal congestion travel time cost of SAVs (\$/veh-m/year)	0.000278
s_f	Land size for production per firm (m)	250
l_f	Units of labor for production per firm	32
β	Parameter reflecting firm productivity (\$/year)	40,000
δ	Transaction cost of unit distance between firms (\$/m/year)	1.0
R_A	Agricultural land rent (\$/m/year)	1,000
c	Land development cost (\$/m)	23

7.2. Effects of AVs on future urban spatial structure and traffic congestion cost

To illustrate the AV effects on the urban spatial structure, we conduct a sensitivity analysis of city boundary B with regard to SAV market share θ for non-congestion and congestion cases, as shown in Fig. 4. Without the consideration of congestion effects, the city boundary with full TVs is 25.6 km. The city boundary for AV city decreases by 6.25 km from 27.13 km to 20.88 km with the change of SAV market share θ from 0 to 100%. There exists a critical vaule of $\theta = 28.64\%$ such that the city expands at its left-hand side, and shrinks at its right-hand side, compared to the city with full TVs (i.e., before introducing AVs). When the congestion effects are considered, the city boundary decreases for both TV city and AV city. The critical value of θ becomes 26.31%, as shown in Fig. 4. This implies that as the SAV market share θ exceeds 26.31%, the city contracts after introducing AVs, and vice versa. These observations are the results of the trade-off between the decreased marginal travel time cost after the introduction of AVs and the reduced parking land demand due to increased SAV market share. As the matter of fact, the introduction of AVs will induce some households to move outwards to enjoy large housing size due to a lower marginal travel time cost. Meantime, it also causes a reduced parking land demand. As the increased housing size exceeds the reduced parking land size, the city expands, and contracts otherwise. These results are consistent with those of Propositions 3 and 7. Fig. 4 also shows that the city size curve under congestion is always below that under no congestion, implying that ignoring traffic congestion effects will overestimate the city size.

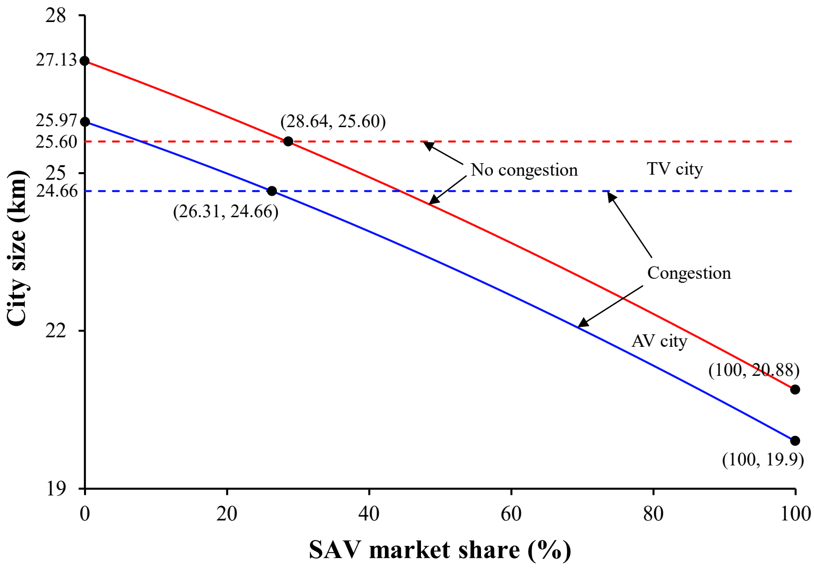


Fig. 4. Change of city size B with SAV market share θ (the blue solid and dotted lines represent AV and TV cities under no congestion, and the red solid and dotted lines represent AV and TV cities under congestion, respectively).

In order to examine the AV effects on traffic congestion, we carry out a sensitivity analysis of the total congestion cost of the city with the SAV market share θ, as shown in Fig. 5. The total congestion cost for the TV city is \$2.301 million. After introducing AVs, the total congestion cost of the city may decrease or increase, depending on the SAV market share. Specifically, at a higher SAV market share, i.e., $\theta > 8.57\%$, AVs can reduce the total congestion cost due to the lower marginal congestion time cost. However, at a low level of the SAV market share, i.e., $\theta \leq 8.57\%$, the total congestion cost increases after introducing AVs. This is because in this case, the city expands significantly after introducing AVs, resulting in longer commuting distance and commuting time, thus incurring higher total congestion cost.

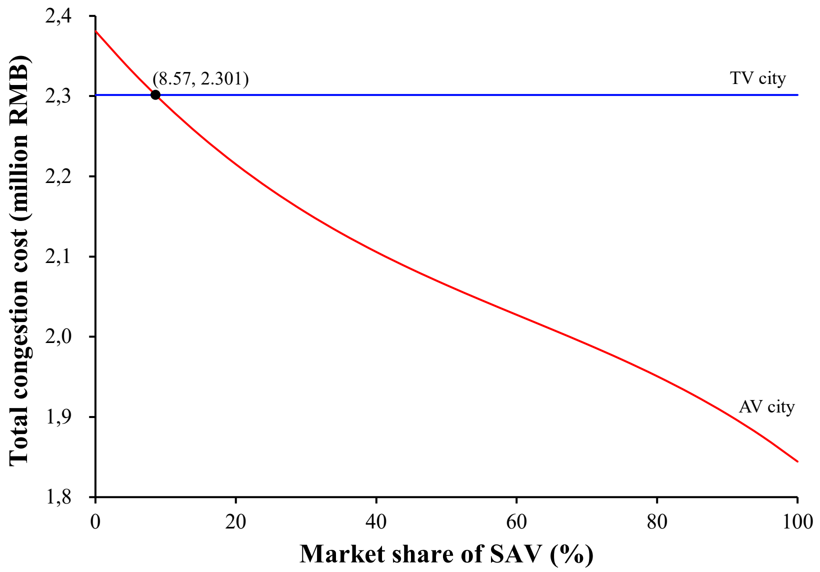


Fig. 5. Total congestion cost before and after introducing AVs (the blue line represents TV city, and the red line represents AV city).

7.3. Effects of AVs on social welfare

Fig. 6 shows the changes of the social welfare with the SAV market share θ under non-congestion and congestion cases before and after introducing AVs. It can be seen in Fig. 6 that the social welfare curve under congestion is always below that under no congestion. This means that ignoring the congestion effects will overestimate the social welfare. Fig. 6 also shows that despite of whether the congestion effects are considered or not, the introduction of AVs may cause an increase or a decrease in the social welfare. Specifically, for the SAV market share $\theta \geq 84.9\%$ under no congestion and $\theta \geq 84\%$ under congestion, the social welfare after introducing AVs decreases, implying a welfare loss. Moreover, the welfare curve is concave with regard to θ, and thus there exists a unique optimal solution of θ that maximizes the social welfare of the AV city (this is consistent with the solution of Eq. (D.3)). Specifically, the optimal SAV market share θ is 27.24% under no congestion, and 37.11% under congestion. These findings suggest that if AVs are deployed haphazardly, they may increase or decrease the social welfare. Therefore, it is important for authorities to implement targeted regulatory measures, such as optimizing the SAV market share, to guide the deployments of AVs in a way that balances technological advancement and social welfare.

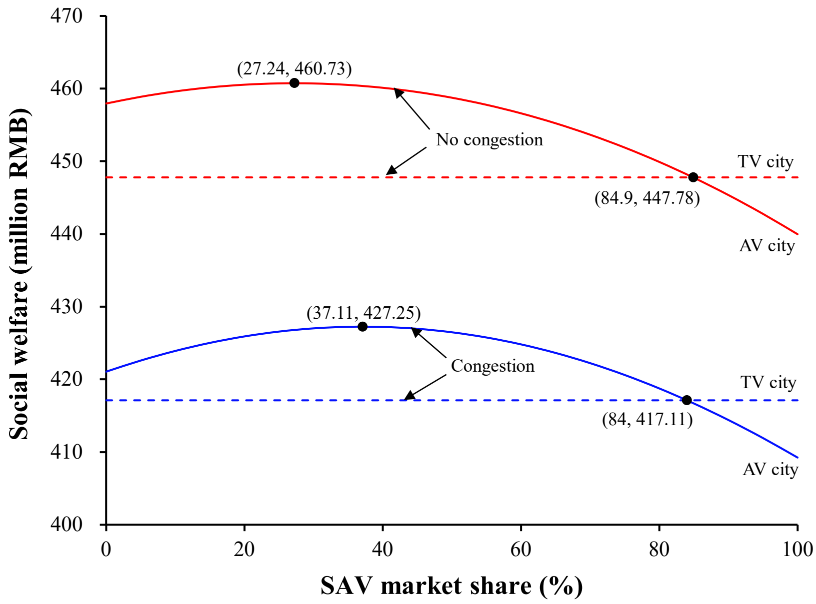
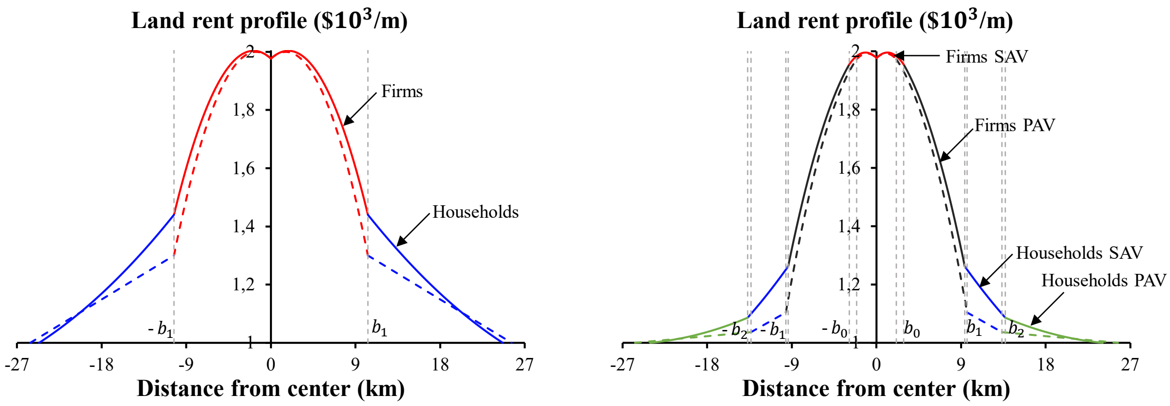


Fig. 6. Social welfare before and after introducing AVs (the blue solid and dotted lines represent AV and TV cities without congestion, while the red solid and dotted lines represent AV and TV cities with congestion, respectively).

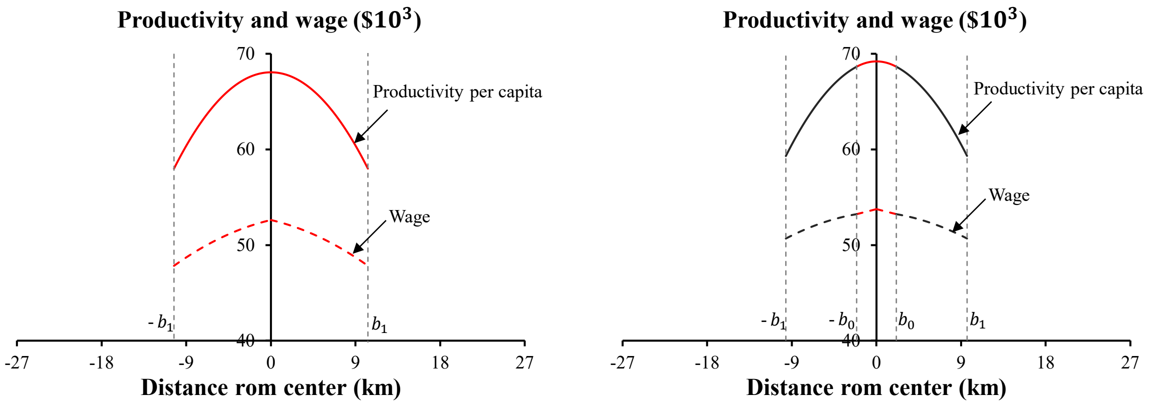
7.4. Effects of AVs on land rent, wage, and firm production with no congestion and congestion

In order to examine the effects of AVs and traffic congestion on the land rent, wage, and firm production, we plot the land rent, wage, and production profiles for the full TV city and the mixed AV city with optimal SAV market share under no congestion and congestion, as shown in Figs. 7 and 8. Fig. 7a shows that in the TV city, despite of whether the congestion effects are considered or not, firms are concentrated at the CBD area, while households are located at the outer areas. The firm's land rent profile is concave on $[-b_1, 0]$ and $[0, b_1]$. However, the shapes of the households' land rent curves on $[-B, b_1]$ and $[b_1, B]$ differ significantly: in the absence of congestion, household land rent is linearly decreasing with the distance from the city center, but it is concavely decreasing under congestion. Fig. 7b shows that in the AV city, firms remain at the inner area of the city, which is partitioned into the worksites of SAV and PAV households from the CBD outwards. PAV and SAV households reside at the outer areas of the city, particularly with PAV households being at the most peripheral area. These results are consistent with those of Proposition 2.



(a) (b)

Fig. 7. Land rent profiles: (a) TV city with solid lines being land rents of firms (in red) and households (in blue) under no congestion, and dotted lines being those under congestion; (b) AV city with optimal SAV market share ($\theta = 27.24$ under no congestion and $\theta = 37.11$ under congestion): solid lines represent land rents of firms with SAV (in red) and PAV (in black) workers, and of SAV (in blue) and PAV (in green) households under no congestion, and dotted lines represent those under congestion.



(a) TV city (b) AV city ($\theta = 27.24$)

Fig. 8. Wage and production profiles: (a) TV city with solid line being the productivity per capita of firms and dotted line being the wage; (b) AV city with optimal SAV market share $\theta = 27.24$: solid lines represent the productivity per capita of firms with SAV (in red) and PAV (in black) workers, and dotted lines represent wages.

As for the wage or productivity profile under no congestion and congestion, we find that both are basically consistent (please refer to Appendix E). For illustration purpose, Fig. 8 plots the productivity and wage profiles for the city with full TVs and with mixed AVs of $\theta = 27.24$ (optimal SAV market share) under no congestion. Fig. 8 shows that the productivity and wage profiles for the TV city are concave and decreasing on $[-b_1, 0]$ and $[0, b_1]$. After introducing AVs, the wage profile for the SAV workers on $[-b_0, b_0]$ decreases linearly with the distance from the city center. Meanwhile, the industrial area $[-b_1, b_1]$ becomes smaller. This is because SAVs do not occupy parking land, and the land released is used for firm production. Hence, the density of firms in the industrial area increases, which enhances the knowledge or information spillover effects due to agglomeration externalities. As a result, the productivity per capita and thus the wage level enhances through comparing Figs. 8a and b.

8. Conclusion and further studies

8.1. Conclusion

In this paper, a theory for analyzing the AV effects on future cities was presented. Two types of AVs, namely PAVs and SAVs, were considered. The land released by SAVs is dedicated to firm production and household residential uses. The housing development in the city is regulated by the government, aiming to maximize the social welfare of the city. A benchmark urban model, accounting for the competition of land among firm production, TV household residences, and parking, was presented to determine the sizes of industrial and residential areas and the wage and land rent profiles. In order to reveal the AV effects on the urban spatial structure, an urban model, incorporating the competitive behavior of firms and PAV / SAV households, was presented and compared with the benchmark urban model. The AV regulation issue was discussed through determining the optimal SAV market share. The effects of traffic congestion on the TV / AV cities were also identified through comparing the solutions of the models with and without considering traffic congestion. The findings showed that after introducing AVs, the housing size of PAV households increases, while the housing size of SAV households may increase or decrease. The size of industrial area contracts due to the reduced parking land demand with SAVs. The firm production rises for a full SAV city. The city may become more centralized or decentralized, depending on the marginal travel costs of PAVs/SAVs and the SAV market share. Social welfare may increase or decrease, depending on the fixed cost of AVs, besides AV marginal travel cost and SAV market share. The introduction of AVs may increase the total congestion cost, and ignoring congestion effects will overestimate the city size and social welfare.

8.2. Policy implications

The findings obtained in our paper contribute to a deeper understanding of how AVs will reshape the urban spatial structure of the future cities. They have important practical implications for policymakers and urban planners to develop effective transportation and urban policies in the context of AVs. As previously shown, the introduction of AVs may cause too high urban sprawl, leading to excessively high land artificialization in suburban areas and inefficient land use. To mitigate these spatial externalities, policymakers should implement well-designed strategies for land use and urban planning that regulate urban expansion while maximizing the benefits of AV deployment. Some measures, such as controlling urban growth boundaries, can help regulate excessive suburbanization and housing development to achieve sustainable urban growth. Additionally, the land released from decreased parking land demand (particularly with SAV adoption) should be well planned and used for firm production, residential development, and transportation infrastructure investment so as to improve the land-use efficiency.

We also find in this paper that AVs may offer sinificantly potential benefits, but also encourage more vehicular miles traveled, exacerbate traffic congestion, and cause higher traffic congestion cost, even for a fixed demand case, as assumed in this paper. It is anticipated that AVs could induce new trip demands (a rebound effect) due to a lower marginal travel cost of AVs, which further aggravate the traffic congestion. To alleviate traffic congestion and improve transport system efficiency with AVs, the transportation sector may jointly adopt some transportation demand management strategies in AV implementation, such as congestion pricing, reasonable taxation on AVs, high-occupancy vehicle lanes, intelligent public transportation, and smart parking system. With these demand management strategies, the congestion externalities in the AV implementation are internalized, and thus the total congestion cost is reduced.

In addition, we find that introduction of AVs may lead to a decline in household utility and social welfare, depending on the AV cost and the SAV market share. Particularly, at the early stage of AV technological development, AV performance is immature and AV cost may be extramely high. Thererfore, during the process of AV development, some incentive policies, such as project funding or subsidy, may be provided to accelerate the technological research. On the other hand, there are an optimal market share of PAVs and SAVs, as previously disclosed. Therefore, specification of a proper SAV market share is essential for preventing excessively high PAV ownership and promoting efficient sharing services of SAVs. These measures will help stimulate AV development and control the AV cost to achieve a welfare-maximizing urban system.

8.3. Limitations and future research

Although this paper provides some new insights into AV effects on future cities, some directions for further extensions are listed as follows. First, we considered an auto city, in which the auto is the main mode of transportation. This is the case in many U.S. cities. However, in many Asian and European cities (e.g., Beijing, Hong Kong, Tokyo, Paris), public transportation has a big market share in the urban transportation. Therefore, it is important to explore the evolving effects of AV technology development on the modal share. Second, we focused on a linear monocentric urban configuration. However, many realistic cities have radial or circular structures (Li et al., 2013, 2024a) or polycentric urban configurations (Fujita and Ogawa, 1982). Therefore, it is worthwhile to extend the proposed framework to account for other urban forms. Third, the AV effects on industrial pollution and traffic emission were not considered. However, it is meaningful to extend to take into account such effects in the context of AVs, which is left for a future study. Finally, for simplicity, parking of PAV commuters was assumed to be identical with that of TV commuters. Actually, PAVs may be able to find a parking space automatically away from the worksite, thus significantly reducing the commuter parking search time. In this regard, it would further favor AVs due to decreased parking cruising time (Anderson and de Palma, 2004; Fagnant and Kockelman, 2015), but probably increase the vehicle street cruising time. There is, therefore, a need to incorporate the automatic parking behavior of PAVs in the model in future study.

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Appendix A: Derivation of optimal housing size for TV households

The social welfare maximization problem for determining the optimal housing size for TV households can be formulated as

$$\begin{aligned}\max_{s_h}\ SW^{\mathrm{TV}} &= NU^{\mathrm{TV}}(s_h)+2\int_0^{b_1} r_f(t)dt+2\int_{b_1}^{B} r_h(t)dt-2\left(R_A+c\right)B \\ &= N\left(w(0)-\alpha_0^{\mathrm{TV}}-\alpha_1^{\mathrm{TV}}B-r(0)s_p-R_A\left(s_h+s_p\right)+\gamma\ln s_h\right) \\ &\quad +2\int_0^{b_1} r_f(t)dt+2\int_{b_1}^{B} r_h(t)dt-2\left(R_A+c\right)B.\end{aligned} \tag{A.1}$$

Taking the second-order derivative of SW^{TV} with regard to s_h yields

$$\frac{\partial^2 SW^{\mathrm{TV}}}{\partial s_h^{\ 2}}=-\frac{\gamma N}{s_h^{\ 2}}<0\,. \tag{A.2}$$

Eq. (A.2) shows that SW^{TV} is a concave function of the housing size, and thus the optimal housing size s_h can be uniquely determined by setting $\partial SW^{\mathrm{TV}}/\partial s_h=0$, yielding

$$s_h=\frac{4\gamma}{\alpha_1^{\mathrm{TV}}N+4\left(R_A+c\right)}\,. \tag{A.3}$$

Appendix B: Proof of Proposition 2

First, we identify the locations of firms and households. With the assumption that all good trades take place in the city center, firms are located at the CBD area (i.e., industrial area), whereas households are located at the peripheral area (i.e., residential area), leading to border b_1 (or $-b_1$) between industrial area and residential area, as shown in Fig. 2.

Then, we look at the residential locations of PAV and SAV households. With the assumption of $\alpha_1^{\mathrm{PAV}} < \alpha_1^{\mathrm{SAV}}$, PAV households have incentives to reside at more peripheral locations in order to enjoy bigger housing spaces, cheaper housing / land rents and lower parking rents, compared to SAV households. This creates a critical location, represented by b_2 (or $-b_2$), such that the SAV households reside at the locations close to the industrial area, whereas the PAV households reside at the peripheral area, as indicated in Fig. 2.

Finally, we analyze the work locations of PAV and SAV households. Considering that PAVs need to occupy land for parking at both residence and worksite, whereas SAVs do not occupy parking land. For a given household residing at location *x* and working at location *y*, the disposable income of choosing to use PAVs and SAVs can, respectively, be defined as

$$\Lambda^{\mathrm{PAV}}(y|x) = w_f(y) - \alpha_0^{\mathrm{PAV}} - \alpha_1^{\mathrm{PAV}}(x-y) - \big(r(x)+r(y)\big)s_p, \tag{B.1}$$

$$\Lambda^{\mathrm{SAV}}(y|x) = w_f(y) - \alpha_0^{\mathrm{SAV}} - \alpha_1^{\mathrm{SAV}}(x-y). \tag{B.2}$$

Owing to $\alpha_0^{\mathrm{SAV}} < \alpha_0^{\mathrm{PAV}}$ and $\alpha_1^{\mathrm{PAV}} < \alpha_1^{\mathrm{SAV}}$, for a work location at the CBD area (implying a short commuting distance), the disposable income of SAV households is higher than that of PAV households, i.e., $\Lambda^{\mathrm{SAV}}(y|x) > \Lambda^{\mathrm{PAV}}(y|x)$. It is just reversed for a work location at the outer area (meaning a long commuting distance), i.e., $\Lambda^{\mathrm{SAV}}(y|x) < \Lambda^{\mathrm{PAV}}(y|x)$. By the intermediate value theorem, there exists a critical location, represented by b_0 (or $-b_0$), such that the SAV households choose to work at a more central area than the PAV households, as shown in Fig. 2. This completes the proof of Proposition 2.

Appendix C: Derivations of optimal housing sizes for PAV and SAV households

The social welfare maximization problem for determining the optimal housing sizes for PAV and SAV households can be formulated as

$$\begin{aligned}\max_{s_h^{\mathrm{PAV}},s_h^{\mathrm{SAV}}} SW^{\mathrm{AV}} &= \theta N U^{\mathrm{SAV}}\left(s_h^{\mathrm{SAV}}\right)+\left(1-\theta\right)NU^{\mathrm{PAV}}\left(s_h^{\mathrm{PAV}}\right)\\ &\quad +2\int_0^{b_0} r_f^{\mathrm{SAV}}(t)dt+2\int_{b_0}^{b_1} r_f^{\mathrm{PAV}}(t)dt+2\int_{b_1}^{b_2} r_h^{\mathrm{SAV}}(t)dt+2\int_{b_2}^{B} r_h^{\mathrm{PAV}}(t)dt-2\left(R_A+c\right)B\\ &= \theta N\left(w_f(0)-\alpha_0^{\mathrm{SAV}}-\alpha_1^{\mathrm{SAV}}b_1-s_h^{\mathrm{SAV}}r(b_1)+\gamma\ln s_h^{\mathrm{SAV}}\right)\\ &\quad +\left(1-\theta\right)N\left(w_f(b_0)-\alpha_0^{\mathrm{PAV}}-\alpha_1^{\mathrm{PAV}}\left(B-b_0\right)-r(b_0)s_p-R_A\left(s_h^{\mathrm{PAV}}+s_p\right)+\gamma\ln s_h^{\mathrm{PAV}}\right)\\ &\quad +2\int_0^{b_0} r_f^{\mathrm{SAV}}(t)dt+2\int_{b_0}^{b_1} r_f^{\mathrm{PAV}}(t)dt+2\int_{b_1}^{b_2} r_h^{\mathrm{SAV}}(t)dt+2\int_{b_2}^{B} r_h^{\mathrm{PAV}}(t)dt-2\left(R_A+c\right)B.\end{aligned} \tag{C.1}$$

Taking the second-order derivatives of SW^{AV} with regard to s_h^{PAV} and s_h^{SAV}, one can obtain its Hessian matrix, represented by $H\left(s_h^{\mathrm{PAV}},s_h^{\mathrm{SAV}}\right)$, as follows:

$$H\left(s_h^{\mathrm{PAV}},s_h^{\mathrm{SAV}}\right)=\begin{bmatrix}-\dfrac{\gamma\theta N}{s_h^{\mathrm{PAV}2}}, & 0\\ 0, & -\dfrac{\gamma\left(1-\theta\right)N}{s_h^{\mathrm{SAV}2}}\end{bmatrix}. \tag{C.2}$$

It is easy to show that the Hessian matrix $H\left(s_h^{\mathrm{PAV}},s_h^{\mathrm{SAV}}\right)$ is negative definite, meaning that SW^{AV} is concave. There is thus a unique solution for s_h^{PAV} and s_h^{SAV}, which can be determined by setting $\partial SW^{\mathrm{AV}}\big/\partial s_h^{\mathrm{PAV}}=0$ and $\partial SW^{\mathrm{AV}}\big/\partial s_h^{\mathrm{SAV}}=0$, leading to

$$s_h^{\mathrm{PAV}}=\frac{4\gamma}{(1-\theta)N\alpha_1^{\mathrm{PAV}}+4\left(R_A+c\right)},\text{ and } s_h^{\mathrm{SAV}}=\frac{4\gamma}{4\left(R_A+c\right)+2\alpha_1^{\mathrm{PAV}}\left(1-\theta\right)N+\theta N\alpha_1^{\mathrm{SAV}}}. \tag{C.3}$$

Appendix D: Derivation of optimal SAV market share

The social welfare maximization problem for determining the optimal SAV market share θ can be formulated as

$$\begin{aligned}\max_{\theta}\ SW^{\mathrm{AV}} &= \theta NU^{\mathrm{SAV}}(\theta)+(1-\theta)NU^{\mathrm{PAV}}(\theta)\\ &\quad +2\int_0^{b_0} r_f^{\mathrm{SAV}}(y)dy+2\int_{b_0}^{b_1} r_f^{\mathrm{PAV}}(y)dy+2\int_{b_1}^{b_2} r_h^{\mathrm{SAV}}(x)dx+2\int_{b_2}^{B} r_h^{\mathrm{PAV}}(x)dx-2(R_A+c)B\\ &=\theta N\left(w_f(0)-\alpha_0^{\mathrm{SAV}}-\alpha_1^{\mathrm{SAV}}b_1-s_h^{\mathrm{SAV}}r(b_1)+\gamma\ln s_h^{\mathrm{SAV}}\right)\\ &\quad +(1-\theta)N\left(w_f(b_0)-\alpha_0^{\mathrm{PAV}}-\alpha_1^{\mathrm{PAV}}(B-b_0)-r(b_0)s_p-R_A\left(s_h^{\mathrm{PAV}}+s_p\right)+\gamma\ln s_h^{\mathrm{PAV}}\right)\\ &\quad +2\int_0^{b_0} r_f^{\mathrm{SAV}}(y)dy+2\int_{b_0}^{b_1} r_f^{\mathrm{PAV}}(y)dy+2\int_{b_1}^{b_2} r_h^{\mathrm{SAV}}(x)dx+2\int_{b_2}^{B} r_h^{\mathrm{PAV}}(x)dx-2(R_A+c)B.\end{aligned} \tag{D.1}$$

Taking the first-order derivative of SW^{AV} with regard to θ, we have

$$\begin{aligned}\frac{\partial SW^{\mathrm{AV}}}{\partial\theta} &= N\left(U^{\mathrm{SAV}}-U^{\mathrm{PAV}}\right)+\theta N\left(\frac{\partial w_f(0)}{\partial\theta}-\alpha_1^{\mathrm{SAV}}\frac{\partial b_1}{\partial\theta}-\frac{\partial r(b_1)}{\partial\theta}s_h^{\mathrm{SAV}}-r(b_1)\frac{\partial s_h^{\mathrm{SAV}}}{\partial\theta}+\frac{\gamma}{s_h^{\mathrm{SAV}}}\frac{\partial s_h^{\mathrm{SAV}}}{\partial\theta}\right)\\ &\quad +(1-\theta)N\left(\frac{\partial w_f(b_0)}{\partial\theta}-\alpha_1^{\mathrm{PAV}}\left(\frac{\partial B}{\partial\theta}-\frac{\partial b_0}{\partial\theta}\right)-\frac{\partial r(b_0)}{\partial\theta}s_p-R_A\frac{\partial s_h^{\mathrm{PAV}}}{\partial\theta}+\frac{\gamma}{s_h^{\mathrm{PAV}}}\frac{\partial s_h^{\mathrm{PAV}}}{\partial\theta}\right)+2R_A\frac{\partial B}{\partial\theta}.\end{aligned} \tag{D.2}$$

Setting $\partial SW^{\mathrm{AV}}/\partial\theta=0$, and after some calculations, we obtain

$$\begin{aligned}&N\left(U^{\mathrm{SAV}}-U^{\mathrm{PAV}}\right)+\frac{NM^2\left(1-\theta^2\right)s_p\delta}{2}-\frac{(1-\theta)NM\alpha_1^{\mathrm{SAV}}s_f}{2}+R_AM\left(s_f+\left(s_h^{\mathrm{SAV}}-s_h^{\mathrm{PAV}}-2s_p\right)l_f\right)\\ &+\frac{N^2}{2}\left(\alpha_1^{\mathrm{SAV}}\left((2\theta-1)s_p-\theta s_h^{\mathrm{SAV}}\right)+\alpha_1^{\mathrm{PAV}}\left((1-\theta)\left(2s_p+s_h^{\mathrm{PAV}}\right)+(2\theta-1)s_h^{\mathrm{SAV}}\right)\right)=0.\end{aligned} \tag{D.3}$$

The optimal value of θ satisfies Eq. (D.3). It is difficult to derive a closed-form solution of θ from this equation. One has to use a numerical method to solve it, such as bisection method.

Appendix E: Solutions of the model with traffic congestion consideration

Solutions	TV city	AV city with PAVs and SAVs
Optimal housing size s_h	$s_h^{\text{TV}}=\dfrac{12\gamma}{3\alpha_1^{\text{TV}}N+\alpha_2^{\text{TV}}N^2+12\left(R_A+c\right)}$	$s_h^{\text{PAV}}=\dfrac{12\gamma}{3(1-\theta)N\alpha_1^{\text{PAV}}+\alpha_2^{\text{PAV}}N^2\left(1-\theta\right)^2+12\left(R_A+c\right)}$ $s_h^{\text{SAV}}=\dfrac{24\gamma}{24\left(R_A+c\right)+12(1-\theta)N\alpha_1^{\text{PAV}}+3\alpha_2^{\text{PAV}}\left(1-\theta\right)\left(2-\theta\right)N^2+6\theta N\alpha_1^{\text{SAV}}+\theta\alpha_2^{\text{SAV}}N^2\left(3-\theta\right)}$
Borders b_0, b_1, b_2	$b_1=\dfrac{N}{2l_f}\left(s_f+s_pl_f\right)$	$b_0=\dfrac{N\theta s_f}{2l_f}$; $b_1=\dfrac{N}{2l_f}\left(s_f+\left(1-\theta\right)s_pl_f\right)$; $b_2=\dfrac{N}{2l_f}\left(s_f+\left(1-\theta\right)s_pl_f+\theta s_h^{\text{SAV}}l_f\right)$
City boundary B	$B=\dfrac{N}{2l_f}\left(s_f+l_f\left(s_h^{\text{TV}}+2s_p\right)\right)$	$B=\dfrac{N}{2l_f}\left(s_f+\theta s_h^{\text{SAV}}l_f+(1-\theta)l_f\left(s_h^{\text{PAV}}+2s_p\right)\right)$
Firm production $F(y)$	$F(y)=\dfrac{-\delta y^2-\delta b_1^{\,2}+2\beta b_1}{s_pl_f+s_f},y\in\left[0,b_1\right]$	$F(y)=\begin{cases}\dfrac{1}{s_pl_f+s_f}\left(2\beta\left(b_1-b_0\right)+\delta\left(b_0^{\,2}-b_1^{\,2}\right)\right)+\dfrac{1}{s_f}\left(-\delta y^2+2\beta b_0-\delta b_0^{\,2}\right),\ y\in[0,b_0]\\ \dfrac{1}{s_pl_f+s_f}\left(-\delta y^2+2\delta b_0y-\delta b_1^{\,2}+2\beta\left(b_1-b_0\right)\right)+\dfrac{2b_0}{s_f}(\beta-\delta y),\ y\in[b_0,b_1]\end{cases}$
Firm land rent $r_f(y)$	$r_f(y)=\dfrac{F(y)-F(b_1)-l_f\alpha_1^{\text{TV}}\left(b_1-y\right)}{s_f+s_pl_f}-\dfrac{\alpha_2^{\text{TV}}l_f^{\,2}\left(b_1^{\,2}-y^2\right)}{2\left(s_pl_f+s_f\right)^2}+\dfrac{N\alpha_1^{\text{TV}}}{2}+\dfrac{\alpha_2^{\text{TV}}N^2}{8}+R_A,y\in\left[0,b_1\right]$	$r_f(y)=\begin{cases}\dfrac{F(y)+l_f\alpha_1^{\text{SAV}}y}{s_f}-\dfrac{l_fs_pF(b_0)+s_fF(b_1)}{s_f\left(s_f+l_fs_p\right)}-\dfrac{N^2\theta\left(1-\theta\right)\left(\alpha_2^{\text{PAV}}-\alpha_2^{\text{SAV}}\right)}{4}+\dfrac{\alpha_2^{\text{SAV}}l_f^{\,2}y^2}{2s_f^{\,2}}+R_A,y\in[0,b_0],\\ \dfrac{F(y)-F(b_1)+l_f\alpha_1^{\text{PAV}}\left(y-b_0\right)}{s_f+l_fs_p}-\dfrac{\alpha_2^{\text{PAV}}\theta\left(1-\theta\right)N^2}{4}-\dfrac{l_f\alpha_2^{\text{PAV}}\left(b_0-y\right)}{2\left(s_f+s_pl_f\right)}\left(N\theta-\dfrac{l_f\left(b_0-y\right)}{s_pl_f+s_f}\right)+\dfrac{\alpha_1^{\text{SAV}}\theta N}{2}+\dfrac{\alpha_2^{\text{SAV}}\theta N^2\left(2-\theta\right)}{8}+R_A,y\in[b_0,b_1]\end{cases}$
Wage $w_f(y)$	$w_f(y)=\dfrac{s_fF(b_1)+s_pl_fF(y)+l_fs_f\alpha_1^{\text{TV}}\left(b_1-y\right)}{l_f\left(s_f+s_pl_f\right)}+\dfrac{\alpha_2^{\text{TV}}l_fs_f\left(b_1^{\,2}-y^2\right)}{2\left(s_pl_f+s_f\right)^2}-\dfrac{s_f}{l_f}\left(\dfrac{N\alpha_1^{\text{TV}}}{2}+\dfrac{\alpha_2^{\text{TV}}N^2}{8}+R_A\right),y\in\left[0,b_1\right]$	$w_f(y)=\begin{cases}\dfrac{l_fs_pF(b_0)+s_fF(b_1)}{l_f\left(s_f+l_fs_p\right)}-\alpha_1^{\text{SAV}}y+\dfrac{N^2\theta\left(1-\theta\right)s_f\left(\alpha_2^{\text{PAV}}-\alpha_2^{\text{SAV}}\right)}{4l_f}-\dfrac{\alpha_2^{\text{SAV}}l_fy^2}{2s_f}-\dfrac{s_f}{l_f}R_A,y\in[0,b_0]\\ \dfrac{s_pl_fF(y)+s_fF(b_1)+l_fs_f\alpha_1^{\text{PAV}}\left(b_0-y\right)}{l_f\left(s_f+l_fs_p\right)}+\alpha_2^{\text{PAV}}\left(\dfrac{l_fb_0\left(b_0-y\right)}{s_f+s_pl_f}+\dfrac{s_f\theta\left(1-\theta\right)N^2}{4l_f}-\dfrac{s_fl_f\left(y-b_0\right)^2}{2\left(s_pl_f+s_f\right)^2}\right)-\dfrac{\alpha_1^{\text{SAV}}\theta Ns_f}{2l_f}-\dfrac{N^2\theta\alpha_2^{\text{SAV}}s_f\left(2-\theta\right)}{8l_f}-\dfrac{s_fR_A}{l_f},y\in[b_0,b_1]\end{cases}$
Household land rent $r_h(x)$	$r_h(x)=R_A+\dfrac{\alpha_1^{\text{TV}}}{s_p+s_h^{\text{TV}}}\left(B-x\right)+\dfrac{\alpha_2^{\text{TV}}\left(x-B\right)^2}{2\left(s_h^{\text{TV}}+s_p\right)^2},\ x\in[b_1,B]$	$r_h(x)=\begin{cases}\dfrac{1}{s_h^{\text{SAV}}}\left(\alpha_1^{\text{SAV}}\left(b_2-x\right)+\alpha_2^{\text{SAV}}\left(\dfrac{\left(x-b_2\right)^2}{2s_h^{\text{SAV}}}+\dfrac{B-b_2}{s_h^{\text{PAV}}+s_p}\left(b_2-x\right)\right)\right)+\dfrac{\alpha_1^{\text{PAV}}}{s_p+s_h^{\text{PAV}}}\left(B-b_2\right)+\dfrac{\alpha_2^{\text{PAV}}\left(B-b_2\right)^2}{2\left(s_h^{\text{PAV}}+s_p\right)^2}+R_A,x\in[b_1,b_2]\\ \dfrac{\alpha_1^{\text{PAV}}}{s_p+s_h^{\text{PAV}}}\left(B-x\right)+\dfrac{\alpha_2^{\text{PAV}}\left(B-x\right)^2}{2\left(s_h^{\text{PAV}}+s_p\right)^2}+R_A,x\in[b_2,B]\\ R_A,\ x\in[B,+\infty)\end{cases}$
Household utility U	$U^{\text{TV}}=w_f(0)-\alpha_0^{\text{TV}}-\alpha_1^{\text{TV}}B-r(0)s_p-R_A\left(s_p+s_h^{\text{TV}}\right)-\dfrac{\alpha_2^{\text{TV}}\left(b_1-B\right)^2}{2\left(s_h+s_p\right)}-\dfrac{\alpha_2^{\text{TV}}l_fb_1^{\,2}}{2\left(s_pl_f+s_f\right)}+\gamma\ln s_h^{\text{TV}}$	$U^{\text{PAV}}=w(b_0)-\alpha_0^{\text{PAV}}-\alpha_1^{\text{PAV}}\left(B-b_0\right)-r(b_0)s_p-R_A\left(s_h^{\text{PAV}}+s_p\right)-\alpha_2^{\text{PAV}}\left(\dfrac{l_fb_0\left(b_1-b_0\right)}{s_f}+\dfrac{l_f\left(b_1-b_0\right)^2}{2\left(s_pl_f+s_f\right)}\right)-\alpha_2^{\text{PAV}}\left(\dfrac{\left(b_2-b_1\right)^2}{2s_h^{\text{SAV}}}+\dfrac{B+b_2-2b_1}{2\left(s_h^{\text{PAV}}+s_p\right)}\left(B-b_2\right)\right)+\gamma\ln s_h^{\text{PAV}}$ $U^{\text{SAV}}=w_f(0)-\alpha_0^{\text{SAV}}-\alpha_1^{\text{SAV}}b_2-R_As_h^{\text{SAV}}-\dfrac{\alpha_1^{\text{PAV}}s_h^{\text{SAV}}}{s_p+s_h^{\text{PAV}}}\left(B-b_2\right)-\dfrac{\alpha_2^{\text{PAV}}s_h^{\text{SAV}}\left(B-b_2\right)^2}{2\left(s_h^{\text{PAV}}+s_p\right)^2}-\alpha_2^{\text{SAV}}\left(\dfrac{l_fb_0}{2s_f}\left(2b_1-b_0\right)+\dfrac{l_f\left(b_1-b_0\right)^2}{2\left(s_pl_f+s_f\right)}\right)-\alpha_2^{\text{SAV}}\left(\dfrac{\left(b_1-b_2\right)^2}{2s_h^{\text{SAV}}}+\dfrac{B-b_2}{s_h^{\text{PAV}}+s_p}\left(b_2-b_1\right)\right)+\gamma\ln s_h^{\text{SAV}}$