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Abstract

We propose a new approach to assess the impact of regulatory changes on the production sector such as competition policies, taxing intermediate goods, robots or AI, trade regulation, production of public firms or environmental standards for firms. Our framework covers multidimensional nonlinear taxation with multiple income sources, General Equilibrium (GE) adjustments and market failures. We clarify under which conditions on the tax system the production sector should be regulated only to increase aggregate output, a recommendation we label the Production Regulation Principle. Under these conditions, regulatory changes can be combined with adequate GE-neutralizing tax reforms to offset the GE effects on taxpayers' utility levels. This ensures that changes in the production sector's regulation that increases aggregate output do not deteriorate individual welfare, thereby resulting in a Pareto improvement. We also provide formulas that balance the effects of regulatory changes on aggregate production and their pre-distributive impact,when a GE-neutralizing tax reform is not feasible. These formulas introduce new GE-multipliers, which also appear in our calculations for the impact of tax reforms, optimal income tax systems and identifying Pareto-improving tax reforms.

Keywords: Production efficiency, Market frictions, Nonlinear income taxation, Several income sources, Endogenous prices.

JEL codes: H21, H22, H23, H24, L5, F13.

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I Introduction

Competition among firms, free trade agreements, and technological advancements – such as artificial intelligence, robotics, and machine learning – drive efficiency gains but may also create adverse distributional consequences (e.g., Buccirossi et al. (2013), Bourlès et al. (2013), Acemoglu and Autor (2011), Autor et al. (2013), Acemoglu and Restrepo (2020)). Recent protectionist trends and debates on taxing robots and AI highlight how regulating the production sector has become central to public and political discourse. This is reinforced by some academic works such as Naito (1999), Guerreiro et al. (2021), Costinot and Werning (2022), Thuemmel (2023). These works question the celebrated Diamond and Mirrlees (1971) production efficiency theorem which advises against distorting the production sector for redistributive purposes. This collection of seemingly disparate results does not provide clear guidance for production policies, i.e. policies that regulates the production sector, such as competition policies, trade agreements, technological policies, taxation of intermediate goods and public production. Should these policies be designed solely to maximize aggregate production – a policy principle we refer to as the “Production Regulation Principle” – or should they also account for potential adverse pre-distributive effects?

We address this question in a model where taxpayers, endowed with heterogeneous characteristics, choose their input supplies – such as effective labor units in a creative job, effective labor units as an entrepreneur, or saving in distinct assets – and allocate their resulting after-tax income to consumption. The production sector is modeled in reduced-form through inverse input demand functions. It may consist of different firms with potential vertical relations and horizontal competition. Firm’s market power, rent-seeking behavior and production externalities, among other phenomena, may prevail. Production policies impact input prices though these inverse demand functions. Input prices are determined endogenously in General Equilibrium (GE).

Our findings rely on identifying, for any multidimensional nonlinear tax system, specific directions of tax reforms that replicate the effects of input price changes, in GE, on agents’ welfare and behavior, that we call “price-replicating directions”. The pre-distributive impact of any production policy reform can be neutralized by a “neutralizing tax reform” along a combination of price replicating directions. We prove that any production policy reform that increases aggregate output is Pareto-improving when it can be combined with a neutralizing tax reform. This is our production regulation principle.

For any tax system, we can determine whether or not neutralizing tax reforms can be implemented. For instance, when there are different types of labor and the tax system is based only on the sum of labor incomes, production policy reforms may not be combined with their neutralizing tax reforms. In such a case, we derive a formula in terms of sufficient statistics to assess the welfare impact of any production policy, comparing its efficiency and pre-distributive effects. We also provide a test that identifies Pareto-efficient tax systems in GE and determine Pareto-improving tax reforms when the tax system is Pareto dominated. Finally, we also provide optimal tax formulas in GE.

We organize the paper around the following key contributions.

First, we present our production regulation principle and detail the key fiscal parameters that any government should carefully asses before implementing a policy intervention. Our contribution is summarized in a decision tree on Figure 2. Production policies that increase efficiency results in a Pareto improvement (i) if the government can reform the tax system along all the price-replicating directions, or, (ii) if the government can reform the tax system in the price-replicating directions of a subset of inputs, provided that the production policies do not alter the relative prices of the other inputs.

Our second contribution are formulas to evaluate the welfare impact of any production policy reform, tax reform or both. These formulas are expressed in terms of familiar elasticity concepts in PE, now systematically extended to account for multiple income sources and cross-base responses, and new statistics, the GE multipliers. Each input has an associated GE multiplier, which captures the impact of price adjustments when aggregate income (accrued from the aggregate supply of that input) increases by one unit. We show that the GE multiplier can be formulated as the sum of two corrective terms: one for market failures and another for the suboptimality of the tax system with respect to tax reforms in price-replicating directions. Our formulas are connected to empirical work measuring parameters such as markup estimates (De Loecker et al., 2020), estimates of the log derivative of inverse demand with respect to production policies (such as entry regulation in Bertrand and Kramarz (2002), Biscourp et al. (2013)) and the familiar welfare weights (Kuziemko et al., 2015, Capozza and Srinivasan, 2024) and estimates of compensated income responses and income effects (e.g., Saez et al. (2012)).

Third, we extend our analysis of GE effects to conditions for a tax reform to be Pareto-improving and to optimal income tax systems. We develop a test to determine whether a given tax system is Pareto efficient, under perfect competition. If it is not, the test identifies potential tax reforms that could achieve a Pareto improvement. To do so, we show that one simply needs to evaluate the revealed marginal welfare weights in PE (as in Bergstrom and Dodds (2024) and Spiritus et al. (2024)), i.e. ignoring the endogeneity of prices. Intuitively, the weights are computed from the optimal tax formula, without imposing restriction on the tax system functional form so that it is optimized along the price-replicating directions. As a result, both corrective terms in GE multipliers are equal to zero. We show that combining a Pareto-improving tax reform in PE – i.e. that weakly decreases tax liabilities for income bundle where the revealed marginal social welfare weights are negative – with a GE-neutralizing tax reform is Pareto-improving in GE. We also show that non-negative revealed weights is sufficient for the non-existence of (local) Pareto-improving reform direction.

Forth, the reduced-form description of the production sector guarantee the robustness of our results to variation in the underlying micro-foundations behind the demand functions. To illustrate the broad applicability of our approach, we also provide, in Appendix B, micro-foundations to analyze several common production policies, including a pro-competitive policy designed to reduce oligopolistic rents, taxation of intermediate goods, automation (robots and AI), trade liberalization, public provision, commodity taxation and environmental regulations for firms. For each of these applications, we prove how the decisions within the production sector are equivalent to those of an hypothetical benevolent "production coordinator". This reformulation of production sector decisions enables us to provide micro-foundations for our reduced forms inverse demand functions. We then retrieve that whether production policies should be designed solely to increase production depends entirely on the availability of neutralizing tax reforms. Hence our approach with reduced forms include many established results in the literature based on micro-founded production functions.

Our paper contributes to a number of literature. The first examines whether production distortion is optimal, when the government is also equipped with a linear income tax system. In these contexts, can distributional concerns justify deviating from production efficiency? Diamond and Mirrlees (1971) demonstrate that if linear income taxes are optimally imposed, the economy should operate on its production frontier. To avoid distorting production decisions, linear tax rates on intermediate goods should therefore be zero. Policymakers often struggle to grasp these concepts – avoiding distortions in the production sector and ensuring the economy operates at its production frontier– as it relies on quite abstract notions. Our paper aims to provide a pedagogical and practical tool for determining when production policies should be regulated only to increase production, focusing on the tax system. We clarify that in Diamond and Mirrlees (1971) and in the long-run model of Saez (2004), since income tax systems distinguish each input, they do incorporate their price-replicating directions for each income source. This neutralizes the pre-distribution effects on individual well-being arising from imperfect input substituability. It clarifies that what matters is not whether tax instruments are linear or nonlinear, but whether the tax system allows for reforms along price-replicating directions. Moreover, we characterize these directions for a large set of tax systems. We make explicit that Diamond and Mirrlees (1971) insights do not require a (fully) optimal tax system but only that the tax system can be reformed along the price-replicating directions. Last but not least, our result does not rely on perfect competition, it extends to settings with market failures.

Second, our results naturally extend to Atkinson and Stiglitz (1976)'s commodity tax problems as well as to the taxes on intermediate goods and factors. Atkinson and Stiglitz (1976) demonstrate that when the government imposes optimal nonlinear income taxation and the utility function is weakly separable between goods and leisure, commodity taxation is unnecessary. In such cases, the government should keep the commodity markets efficient. They prove their results using a fixed-priced model with perfect substitution between different types of production factors, such as low and high-skilled labor. Naito (1999) shows that violating the efficiency of commodity markets can be desirable when workers of different skills are imperfect substitutes in production, and input prices are endogenous.

Naito's reasoning hinges on the fact that the assumed tax schedule does not differentiate between low- and high-skilled labor income. At the same time, the tax instrument can adjust the skilled-to-unskilled wage ratio (Stolper and Samuelson, 1941), creating a positive first-order welfare effect without violating incentive constraints. However, Saez (2004) argues that in a long-run model where workers freely choose occupations, skilled workers can take unskilled jobs, eliminating this first-order effect on welfare. We demonstrate that the no-commodity taxation of Atkinson and Stiglitz (1976) remains robust to endogenous producer prices, whenever the neutralizing tax reforms can be implemented. Taxes on intermediate goods and factors are particularly relevant to the debate on the taxation of robots and AI, which can be seen as specific types of intermediate goods. Costinot and Werning (2022), Guerreiro et al. (2021), Thuemmel (2023), and Beraja and Zorzi (2024) offer insightful derivations of optimal robot tax formulas. We stress that their recommendations to tax or not robots depends solely on the features of the income tax system, rather than on its optimality or on the absence of market failures. Our findings also apply to the trade liberalization debate, as discussed in Diamond and Mirrlees (1971), Dixit and Norman (1980, 1986), Antràs et al. (2017), Hosseini and Shourideh (2018) and Costinot and Werning (2022), as well as to the pricing of inputs for public firms compared to the private sector, as in Little and Mirrlees (1974) and Naito (1999).

Third, we complement the optimal tax literature. Diamond and Mirrlees (1971), Saez (2004) and Saez and Zucman (2023) show that price adjustments in GE do not modify optimal tax formulas. Our analysis highlights that their results stem from two assumptions: perfect competition and a tax system flexible enough to be optimized along all price-replicating directions. In contrast, Stiglitz (1982), Naito (1999), Rothschild and Scheuer (2013, 2014, 2016), Ales et al. (2015), Ales and Sleet (2016), Sachs et al. (2020), Schultz et al. (2023) and Janeba and Schulz (2023) assume that different types of workers supply distinct types of labor, which are imperfect substitutes, while the tax system depends only on the sum of all incomes, i.e. is comprehensive. Due to their comprehensive nature, these optimal income tax systems cannot optimize along their price-replicating directions which depend on the income obtained from each specific input. This is excluded with a comprehensive tax system. As a result, in the previously cited articles, GE effects alter the PE optimal tax formulas. Our paper highlights that whether optimal tax formulas are affected by GE price adjustments depends on whether the tax system allows for improvements along price-replicating directions. This explains why two seemingly unrelated questions–whether production policies should be designed solely to maximize total production and whether the endogeneity of input prices modifies optimal tax formulas–are often framed within the same "production efficiency" result. Under perfect competition, when the optimal tax system also optimizes along all price-replicating directions, as in Diamond and Mirrlees (1971), the long-run model of Saez (2004) and Saez and Zucman (2023), production policies should be designed solely to enhance aggregate production as well as tax formulas unaltered by the endogeneity of input prices.

Fourth, this paper complements the literature on optimal redistribution where individuals can self-select into different sector (e.g., Gomes et al. (2017)) and sector-specific returns depend on the aggregate effort supplies via GE effects (Rothschild and Scheuer, 2013, 2014, Scheuer, 2014, Rothschild and Scheuer, 2016). These papers analyze how GE effects modify the optimal income tax schedule using a mechanism design approach. By assuming weakly separable preferences between effort and consumption, they reduce the multidimensional screening problem to a one-dimensional one. They first solve an "inner" problem, which involves solving a Mirrlees (1971) optimal tax problem for fixed levels of aggregate effort in each sector. This yields optimal income tax formulas that incorporate multipliers corresponding to new feasibility constraints, ensuring that aggregate effort in each sector matches pre-determined levels. They then solve the "outer" problem, optimizing over the fixed sector-specific effort levels, and demonstrate how GE effects influence the multipliers of the feasibility constraints, leading to a difference between the optimal income tax schedule in GE and in PE.

We borrow to Rothschild and Scheuer (2014) by representing all types of market failures, flexibly, through inverse demand functions. In addition, we incorporate the role of production policies into their analysis. Our use of a tax perturbation approach eliminates the need for weakly separable preferences. This also allows us to express how GE effects modify tax reform analyses in an intuitive way through our GE multipliers in terms of market failures and suboptimality of the tax system in price-replicating directions. When the tax system distinguishes incomes from each input, it can be reformed along all the price-replicating directions such that the GE multipliers solely shape the tax system to address market failures. When such failures are absent, as in Scheuer (2014) with a tax system distinguishing salary from entrepreneurial incomes, there is no need to pre-distribute income through input prices (i.e. our GE multipliers are nil). In contrast, when wage and entrepreneurial income are comprehensively taxed, input prices play a pre-distributive role and their adjustments (captured by our GE-multipliers) increase or decrease optimal marginal income tax rates along the income distribution. This arises from the fact that incomes are not predominantly generated by the same inputs at different income levels. This sheds light on the findings of Rothschild and Scheuer (2013, Figure II) and Sachs et al. (2020, Figure 4), where GE price adjustments decrease optimal marginal tax rates at high income levels and increase them at low income levels. Finally, while Rothschild and Scheuer (2013), Rothschild and Scheuer (2014), and Rothschild and Scheuer (2016) derive optimal comprehensive income tax formulas and Scheuer (2014) derives optimal schedular income tax formulas, we derive a multidimensional optimal nonlinear tax formula and explore the conditions under which a tax system is Pareto efficient or Pareto dominated.

Finally, we extend a rich literature that identifies distinct conditions for the Pareto efficiency of a tax system. In PE, Werning (2007), Lorenz and Sachs (2016) and Bierbrauer et al. (2023) show that negative revealed welfare weights indicate a Pareto inefficiency in the observed tax system, when taxpayers earn a single income. Bierbrauer et al. (2023) show that negative revealed welfare weights at some income levels are equivalent to their "revenue function" being increasing around that level, in which case a two-brackets tax reform is Pareto-improving. Spiritus et al. (2024), Bergstrom and Dodds (2024) and Bierbrauer et al. (2024) extend the analysis to multiple income sources in PE. This paper extends these insights by identifying Pareto-improving tax reforms with multiple incomes in GE.

The rest of the paper is organized as follows. The model is presented in the next section. We characterize the tax incidence in GE and the price-replicating directions of tax reforms. In Section III, we present the production regulation principles along with a formula for assessing the impact of any production policy on welfare. We also briefly explain how various micro-founded production policies can be seamlessly and easily integrated into our reduced-form representation of the production sector. The method is presented more formally, for each policy, in Appendix B. In Section IV, we introduce new key statistics, including GE multipliers, and explain how to implement them. In Section V, we provide conditions for the existence of Pareto-improving tax reforms with multiple incomes and GE. In Section VI, we derive optimal tax formulas in GE under tax systems which are unrestricted (multidimensional and nonlinear), schedular and comprehensive. All proofs are gathered in Appendix A.

II The Economic Environment

II.1 Taxpayers

We consider an economy with a unit mass of taxpayers and a production sector that produces a numeraire good using n inputs with $n \geq 2$. Taxpayers are endowed with varying characteristics summarized by an $\hat{n}$-dimensional vector $\boldsymbol{\theta} = (\theta_1, ..., \theta_{\hat{n}})$, with $\hat{n} \geq n$. These types are distributed over a closed and convex type space $\Theta \subset \mathbb{R}^{\hat{n}}$ according to a continuously differentiable density function $f(\cdot)$ which is positive over Θ and a CDF $F(\cdot)$.

Each taxpayer supplies $x_i \geq 0$ units of the i^{th} input and her supply is denoted by $\mathbf{x} = (x_1, ..., x_n)$. For instance, a taxpayer can supply effective units of labor x_1 in a routine job, effective units of labor x_2 in a creative job, effective units of labor x_3 as entrepreneur, investment units in capital x_4, investment units in a specific asset x_5, etc. Each supply of input x_i incurs an effort or a utility cost that depends on the individual type $\boldsymbol{\theta}$.[1]

[1]Our framework can also encompass economies with different sectors, occupations, or industries, as in Rothschild and Scheuer (2013, 2014, 2016), Scheuer (2014) and Gomes et al. (2017), where each x_i stands for the amount of labor supplied in each sector $i = 1, ..., n$. In Rothschild and Scheuer (2013), Scheuer (2014) and Gomes et al. (2017), workers can supply labor

The income generated by supplying input x_i is denoted by $y_i = p_i\, x_i$. For the taxpayers, p_i represents the private return on the i^{th} input they supply and is taken as given. For the firm, it is the price of this input. These input prices are summarized in the vector $\mathbf{p} = (p_1, ..., p_n)$. For instance, if x_1 represents effective labor, then p_1 denotes the wage per unit of effective labor, and y_1 stands for labor income. Similarly, if x_2 corresponds to savings, p_2 represents the gross return on savings, y_2 signifies capital income, and so forth. The various sources of income are concisely represented by the vector $\mathbf{y} = (y_1, ..., y_n)$.

The preferences of type-$\boldsymbol{\theta}$ taxpayer are represented by the utility function $(c, \mathbf{x}; \boldsymbol{\theta}) \mapsto \mathscr{U}(c, \mathbf{x}; \boldsymbol{\theta})$, which is assumed to be twice continuously differentiable over $\mathbb{R}_+^{n+1} \times \Theta$, increasing in the after-tax income c (with partial derivative denoted $\mathscr{U}_c > 0$) and decreasing in the supply of each input (with partial derivative denoted $\mathscr{U}_{x_i} < 0$). The government enforces taxes based on a tax schedule that depends on all sources of income, denoted as: $\mathcal{T} : \mathbf{y} = (y_1, ..., y_n) \mapsto \mathcal{T}(\mathbf{y}) = \mathcal{T}(y_1, ..., y_n)$. After-tax income, hereafter refereed to as consumption, is: $c = \sum_{i=1}^n y_i - \mathcal{T}(y_1, ..., y_n)$.

The marginal rate of substitution between the supply of the i^{th} input x_i and consumption for a taxpayer with type $\boldsymbol{\theta}$, at any bundle $(c, \mathbf{x})$, is given by:

$$\mathcal{S}^i(c, \mathbf{x}; \boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} -\frac{\mathscr{U}_{x_i}(c, \mathbf{x}; \boldsymbol{\theta})}{\mathscr{U}_c(c, \mathbf{x}; \boldsymbol{\theta})}. \tag{1}$$

We assume that the utility function $\mathscr{U}(c, \mathbf{x}; \boldsymbol{\theta})$ is weakly concave in $(c, \mathbf{x})$ and that the indifference sets are convex in $(c, \mathbf{x})$ for all utility levels and all types $\boldsymbol{\theta}$. This implies that matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c\, \mathcal{S}^j\right]_{i,j}$ is positive definite, as shown in Appendix A.1.[2] A $\boldsymbol{\theta}$-taxpayer chooses her supply of inputs $\mathbf{x}$ to solve:

$$U(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \max_{\mathbf{x}=(x_1,...,x_n)} \quad \mathscr{U}\left(\sum_{i=1}^n p_k\, x_k - \mathcal{T}(p_1\, x_1, ..., p_n\, x_n), \mathbf{x}; \boldsymbol{\theta}\right). \tag{2}$$

We assume (relying on usual assumptions presented in Appendix A.2) that, for each taxpayer of type $\boldsymbol{\theta} \in \Theta$, these programs admit a single solution with supplies of inputs denoted by $\mathbf{X}(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} (X_1(\boldsymbol{\theta}), ..., X_n(\boldsymbol{\theta}))$ and incomes denoted by $\mathbf{Y}(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} (Y_1(\boldsymbol{\theta}), ..., Y_n(\boldsymbol{\theta}))$ where $Y_i(\boldsymbol{\theta}) = p_i\, X_i(\boldsymbol{\theta})$. By aggregating the individual input supplies of $X_i(\boldsymbol{\theta})$, we obtain its total quantity, $\mathcal{X}_i$, used in the production process, i.e. $\mathcal{X}_i \stackrel{\text{def}}{\equiv} \int_W X_i(\boldsymbol{\theta})\, \mathrm{d}F(\boldsymbol{\theta})$. The utility achieved by $\boldsymbol{\theta}$-taxpayers is $U(\boldsymbol{\theta}) = \mathscr{U}(C(\boldsymbol{\theta}), \mathbf{X}(\boldsymbol{\theta}); \boldsymbol{\theta})$ where $C(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \sum_{i=1}^n Y_i(\boldsymbol{\theta}) - \mathcal{T}(\mathbf{Y}(\boldsymbol{\theta}))$ is their consumption. The first-order conditions are:

$$\forall i \in \{1, ..., n\} : \quad \mathcal{S}^i(C(\boldsymbol{\theta}), \mathbf{X}(\boldsymbol{\theta}); \boldsymbol{\theta}) = p_i\left(1 - \mathcal{T}_{y_i}(p_1\, X_1(\boldsymbol{\theta}), ..., p_n\, X_n(\boldsymbol{\theta}))\right). \tag{3}$$

For each kind $i = 1, ..., n$ of income, the marginal rate of substitution between the supply of input x_i and consumption is equal to the marginal net-of-tax rate of the i^{th} income times the i^{th} input price.

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only in one sector. In our model, this consists in assuming that $\mathscr{U}(c, \mathbf{x}; \boldsymbol{\theta}) = -\infty$ if more than one supply of factor is positive.

[2] $A_{i,j}$ is a term of matrix A for which the row is i and the column is j.

II.2 Production sector

The production sector can be made of different firms with potential vertical relations and horizontal competition. Firms' market power, rent-seeking behaviors, and production externalities, among other phenomena, can prevail. As in Rothschild and Scheuer (2014), the production side is presented in reduced form. We adopt a highly flexible specification to describe how private returns depend on inputs $(\mathcal{X}_1, ..., \mathcal{X}_n)$ through twice-differentiable inverse demand functions:

$$\forall i \in \{1, ..., n\}: \qquad p_i = \mathscr{P}_i(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}). \tag{4}$$

where $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_L) \in \mathcal{A} \subset \mathbb{R}^L$ is a vector of policies of dimension L, a concept we will elaborate on later. The production function is given by the national accounting equation:

$$\forall(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}): \qquad \mathcal{F}(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) \stackrel{\text{def}}{\equiv} \sum_{i=1}^{n} \mathscr{P}_i(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha})\, \mathcal{X}_i. \tag{5}$$

The GDP on the left-hand side equals the sum of incomes derived from each input on the right-hand side.

A specific case arises under perfect competition where the price, equivalently the *private* return of input i, p_i, coincides with the marginal productivity of the i^{th} input, equivalently the *social* return of input i, $\mathcal{F}_{\mathcal{X}_i}$:

$$\forall i \in \{1, ..., n\}, \forall (\mathcal{X}_1, ..., \mathcal{X}_n, \boldsymbol{\alpha}): \qquad \mathscr{P}_i(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathcal{F}_{\mathcal{X}_i}(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}). \tag{6}$$

Prices are then endogenous whenever inputs are imperfect substitutes.

Profits may occur under imperfect competition or under perfect competition if the production function exhibits decreasing returns to scale. In such a case, to retrieve the national accounting equation (5), let $X_{n+1}(\boldsymbol{\theta})$ denote the share of profits received by taxpayers of type $\boldsymbol{\theta}$ with $\mathcal{X}_{n+1} = \int_\Theta X_{n+1}(\boldsymbol{\theta})\, \mathrm{d}F(\boldsymbol{\theta}) = 1$ and aggregate profits earned by all taxpayers being equal to $p_{n+1}\mathcal{X}_{n+1} = p_{n+1}$. This additional production input $X_{n+1}(\boldsymbol{\theta})$ can be interpreted as an "entrepreneurial input" which is inelastically supplied (McKenzie (1959), and Mas-Colell et al. (1995, pp. 134-135)). Equation (5) then still holds, provided that in the right-hand side of (5), i is summed from 1 to $n+1$ instead of n.

II.3 Production policy reforms

Production policies refer to interventions that impact the productive processes of firms. These policies, represented by the vector $\alpha = (\alpha_1, ..., \alpha_L) \in \mathcal{A} \subset \mathbb{R}^L$, are designed to influence how firms operate and interact on the markets. As a result, they affect aggregate production and the prices of inputs. While these policies do not directly change individual taxpayer behavior at fixed input prices, shifts in input prices can indirectly influence taxpayers' income decisions. Our analysis of production policies extends to examining the impact of various shocks that alter the production set, such as technological advancements or expanded trade opportunities.

A typical example of a production policy is competition policy. This policy aims to promote competition among firms by preventing monopolistic practices, reducing barriers to entry, and encouraging market efficiency. By altering the competitive landscape, competition policy can lead to increased aggregate production and changes in input prices (see e.g. Buccirossi et al. (2013), Bourlès et al. (2013)), as firms adjust their behavior in response to more competitive pressures. However, despite its impact on production, this type of policy does not directly influence the consumption choices, labor supply decisions or any other decisions that affect the various sources of individual income, differentiating it from policies that target final goods or personal income. While competition policy is a typical example of a production policy, other measures, such as intermediate goods taxation, taxes on robots and AI, public production, trade policies, and business regulations, also fall within this category.

To describe how the production sector is affected by marginal changes in the strength of the ℓ^{th} production policy,[3] we differentiate the national accounting equation (5) with respect to direction of the ℓ^{th} production policy α_ℓ:

$$\mathcal{F}_{\alpha_\ell} = \sum_{j=1}^{n} \mathcal{X}_j \frac{\partial \mathscr{P}_j}{\partial \alpha_\ell} = \sum_{j=1}^{n} \mathcal{Y}_j \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell}. \tag{7}$$

where $\partial \log \mathscr{P}_j / \partial \alpha_\ell$ is the log impact on j^{th} input price of a (small) change in the magnitude of the ℓ^{th} production policy, assuming there has been no change in input supplies.

We denote $\mu \lesseqqgtr 0$ as the magnitude of any production policy reform (and later as the magnitude of any tax reform), with each policy variable in the production policy vector $\boldsymbol{\alpha}(\mu) = (\alpha_1(\mu), \ldots, \alpha_L(\mu))$ being a function of μ. By summing over all policy dimensions α_ℓ, we obtain the aggregate marginal changes in production as follows:

$$\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell}\, \alpha'_\ell(0) = \sum_{\ell=1}^{L} \sum_{j=1}^{n} \mathcal{Y}_j \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell}\, \alpha'_\ell(0). \tag{8}$$

II.4 Government

The government's resource constraint is:[4]

$$E \leq \mathscr{B} \stackrel{\text{def}}{\equiv} \int_{\Theta} \mathcal{T}\left(\mathbf{Y}(\boldsymbol{\theta})\right)\, \mathrm{d}F(\boldsymbol{\theta}) \tag{9}$$

where $\mathscr{B}$ represents tax revenue and $E \geq 0$ denotes an exogenous public expenditure requirement.

To assess the impact of reform, we use either the Pareto criterion or a welfare function

$$\mathscr{W} \stackrel{\text{def}}{\equiv} \int_{\Theta} \Phi\left(U(\boldsymbol{\theta}); \boldsymbol{\theta}\right)\, \mathrm{d}F(\boldsymbol{\theta}). \tag{10}$$

[3] There is here a slight abuse of notation since $\boldsymbol{\alpha}$ denotes the vector of production policy and refers here to the direction of the production policy reforms.

[4] According to the national accounting equation (5), the production function $\mathcal{F}$ represents the production net of the budgetary costs of production sector policies.

where $\Phi : (u, \boldsymbol{\theta}) \mapsto \Phi(u, \boldsymbol{\theta})$ may be concave and type-dependent, is increasing in individual utility u and twice continuously differentiable. This specification includes many different social objectives. The objective is utilitarian when $\Phi(U, \boldsymbol{\theta}) = U$ and weighted utilitarian when $\Phi(U; \boldsymbol{\theta}) = \gamma(\boldsymbol{\theta})\, U$. One obtains maximin when $\gamma(\boldsymbol{\theta})$ equal zero for every taxpayer except those with the lowest utility level. When $\Phi(U, \boldsymbol{\theta})$ does not depend on type and is concave in U, one has Bergson-Samuelson preferences. We note that the utility function $\mathscr{U}(\cdot, \cdot; \boldsymbol{\theta})$ is only one possible cardinal representation of type-$\boldsymbol{\theta}$ taxpayers' preferences. Other representations are obtained using an increasing transformation of $\mathscr{U}(\cdot, \cdot; \boldsymbol{\theta})$ such as $\Phi(\mathscr{U}(\cdot; \boldsymbol{\theta}); \boldsymbol{\theta})$. Therefore, the right-hand side of (10) can also be interpreted as a utilitarian objective following a recardinalization of individual utility.

The government's Lagrangian is a linear combination of tax revenue $\mathscr{B}$ and welfare $\mathscr{W}$ written as:

$$\mathscr{L} \overset{\text{def}}{\equiv} \mathscr{B} + \frac{1}{\lambda} \mathscr{W}, \tag{11}$$

where the Lagrange multiplier $\lambda > 0$ represents the social value of public funds. We choose to express the Lagrangian in monetary units instead of utility units.

II.5 Equilibrium

We employ two distinct equilibrium concepts: partial equilibrium (PE) with exogenous prices and general equilibrium (GE) with endogenous prices. The GE is defined by:

Definition 1 (General Equilibrium (GE))**.** *Given a tax schedule $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ and production policies $\boldsymbol{\alpha}$, a GE is a set of prices $\mathbf{p} = (p_1, ..., p_n)$, of factor supplies $\mathbf{X}(\theta)$ for each type $\boldsymbol{\theta}$ of taxpayers and aggregate inputs $(\mathcal{X}_1, ..., \mathcal{X}_n)$ such that:*

i) Input supplies $\mathbf{X}(\boldsymbol{\theta})$ maximize $\boldsymbol{\theta}$-taxpayers utility according to (2)*, taking prices $\mathbf{p}$ as given.*

ii) Prices are given by inverse demand functions (4)*, where aggregate inputs sum up individual input supplies according to:*

$$\mathcal{X}_i \overset{def}{\equiv} \int_\Theta X_i(\boldsymbol{\theta})\, \mathrm{d}F(\boldsymbol{\theta}). \tag{12}$$

The PE takes prices as given and thereby omits part *ii*) of Definition 1, as follows.

Definition 2 (Partial Equilibrium (PE))**.** *Given a tax schedule $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ and a set of prices $\mathbf{p} = (p_1, ..., p_n)$, a PE is a set of input supplies $\mathbf{X}(\boldsymbol{\theta})$ for each type $\boldsymbol{\theta}$ of taxpayers that maximize $\boldsymbol{\theta}$-taxpayers utility according to* (2)*, taking prices as given.*

We assume PE and GE exist and are unique.

II.6 Tax system and tax reforms

The tax schedule $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ is assumed to be twice continuously differentiable. A tax reform replaces the prevailing tax schedule $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ by a new one $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y}) - \mu\, R(\mathbf{y})$, μ being the magnitude of the tax reform and the twice-continuously differentiable function $R(\cdot)$ specifying its direction.[5] For a given income vector $\mathbf{y}$, the change in the tax burden due to the reform is therefore given by $\mu\, R(\mathbf{y})$. We add the superscript "PE" to variables evaluated in PE. If a variable has no "PE" superscript, it is evaluated in GE. Additionally, we indicate the direction of the considered reforms as superscripts on the variables: $R(\cdot)$ for a tax reform in direction $R(\cdot)$, and $R(\cdot), \boldsymbol{\alpha}(\cdot)$ for a tax reform in direction $R(\cdot)$ combined with a production policy reform in direction $\boldsymbol{\alpha}(\cdot)$. The magnitude of the reforms is not indicated, as we consistently use μ to represent it. Note that, in PE, when a tax reform affects input prices, we take the determination of prices through the mapping $\mu \mapsto (p_1^{R(\cdot),\boldsymbol{\alpha}(\cdot),PE}(\mu), ..., p^R R(\cdot), \boldsymbol{\alpha}(\cdot), PE_n(\mu))$ as given. At the GE, on the other hand, the mapping $t \mapsto (p_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu), ..., p_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu))$ is endogenous and determined by (4).

After a tax reform in the direction $R(\cdot)$ with magnitude μ, a type-$\boldsymbol{\theta}$ taxpayer solves:[6]

$$U^{R(\cdot),PE}(\boldsymbol{\theta}; \mu, \mathbf{p}) \stackrel{\text{def}}{\equiv} \max_{\mathbf{x}} \mathscr{U}\left(\sum_{i=1}^{n} p_i\, x_i - \mathcal{T}(p_1\, x_1, ..., p_n\, x_n) + \mu\, R(p_1\, x_1, ..., p_n\, x_n), \mathbf{x}; \boldsymbol{\theta}\right). \quad (13)$$

Applying the envelope theorem to (13) leads to:

$$\mathrm{d}U = \left[R(\mathbf{y})\, \mathrm{d}\mu + \sum_{i=1}^{n} (1 - \mathcal{T}_{y_i})\, x_i\, \mathrm{d}p_i\right] \mathscr{U}_c(C(\boldsymbol{\theta}), \mathbf{X}(\boldsymbol{\theta}); \boldsymbol{\theta}).$$

Let

$$g(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \frac{\Phi_U(U(\boldsymbol{\theta}); \boldsymbol{\theta})}{\lambda} \mathscr{U}_c(C(\boldsymbol{\theta}), \mathbf{X}(\boldsymbol{\theta}); \boldsymbol{\theta}) \quad (14)$$

denote the marginal welfare weights for taxpayers of type $\boldsymbol{\theta}$. We therefore get:

$$\frac{\mathrm{d}\, \Phi(U(\boldsymbol{\theta}); \boldsymbol{\theta})}{\lambda} = \left[R(\mathbf{y})\, \mathrm{d}\mu + \sum_{i=1}^{n} (1 - \mathcal{T}_{y_i})\, y_i\, \frac{\mathrm{d}p_i}{p_i}\right] g(\boldsymbol{\theta}). \quad (15)$$

Tax incidence in PE vs GE

When moving from the usual PE environment to GE, the behavioral responses and impact in terms of well-being that prevail in PE are amplified by price adjustments, which in turn modify taxpayers' input supplies and their corresponding incomes. We now describe the effects on input supplies, and their

[5] While optimizing over the highly multidimensional set of tax functions, we can compute partial derivatives along any direction $R(\cdot)$ of tax reforms, allowing for optimization with respect to μ for each tax reform direction.

[6] We assume that the second-order condition associated holds strictly, meaning that the matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i\, p_j\, \mathcal{T}_{y_i y_j}\right]_{i,j}$ is positive definite at $c = C(\boldsymbol{\theta})$, $\mathbf{x} = \mathbf{X}(\boldsymbol{\theta})$, and $\mathbf{y} = \mathbf{Y}(\boldsymbol{\theta})$, for each type $\boldsymbol{\theta} \in \Theta$ and that for each type $\boldsymbol{\theta} \in \Theta$, the program (2) admits a unique global maximum. These properties, further discussed in Appendix A.2, eliminate the need to assume smooth individual responses to tax reforms. Instead, we apply the implicit function theorem to the taxpayers' first-order conditions (3)

corresponding incomes, at GE of a tax reform in the direction $R(\cdot)$, a production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$, or both reforms together.

We first define the standard statistics at the PE: compensated responses and income effects for θ-taxpayers. The compensated responses of their i^{th} income with respect to the j^{th} marginal net-of-tax rate is denoted $\partial Y_i(\boldsymbol{\theta})/\partial \tau_j$ while the income effect on their i^{th} income is $\partial Y_i(\boldsymbol{\theta})/\partial \rho$.[7]

The response to a change in μ at GE includes PE responses, along with the additional behavioral responses to input price adjustments observed in GE. Formally, as shown in Appendix A.3, $\forall i \in \{1, ..., n\}$, the i^{th} input supply of a $\boldsymbol{\theta}$-agent is modified by:

$$\forall \boldsymbol{\theta} \in \Theta : \qquad \frac{\partial X_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} = \frac{\partial X_i^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} + \underbrace{\sum_{j=1}^{n} \frac{\partial X_i(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j} \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}}_{\text{Responses to price changes}}, \tag{16a}$$

which can be rewritten in terms of income responses as:

$$\forall \boldsymbol{\theta} \in \Theta : \qquad \frac{\partial Y_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} = \frac{\partial Y_i^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} + \underbrace{\sum_{j=1}^{n} \frac{\partial Y_i(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j} \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}}_{\text{Responses to price changes}}, \tag{16b}$$

where:

$$p_i \frac{\partial X_i^{R,PE}(\boldsymbol{\theta},\mu)}{\partial \mu} = \frac{\partial Y_i^{R,PE}(\boldsymbol{\theta},\mu)}{\partial \mu} = \underbrace{\sum_{j=1}^{n} \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j} R_{y_j}(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Compensated responses}} + \underbrace{\frac{\partial Y_i(\boldsymbol{\theta})}{\partial \rho} R(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Income effects}}. \tag{16c}$$

Equation (16c) describes that, at PE, after small tax reform of magnitude $d\mu$, taxpayers' decisions are modified, because of changes in the n marginal tax rates by $R_{y_j}(\mathbf{Y}(\boldsymbol{\theta}))\mathrm{d}\mu$ (for $j = 1, ..., n$) or because of a change in the level of tax by $R(\mathbf{Y}(\boldsymbol{\theta}))\mathrm{d}\mu$. These PE responses are encapsulated into the first term on the right-hand side of (16a) and (16b). The second term in the right-hand side of (16a) and (16b) highlights that tax and production policy reforms impact the supply of the i^{th} input X_i (and income Y_i) by a $\boldsymbol{\theta}$-agent not only through changes in its own price p_i but also through variations in the prices of all other inputs $j \neq i$. Responses across different income sources are at play in this new term.

Similarly, the impact on taxpayers' well-being at GE includes the impact at PE, along with the additional behavioral responses to input price adjustments observed in GE. Formally, from (15), we have $\forall \boldsymbol{\theta} \in \Theta$:

$$\frac{1}{\lambda} \frac{\partial \Phi\left(U^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu);\boldsymbol{\theta}\right)}{\partial \mu} = \left[R(Y(\boldsymbol{\theta})) + \sum_{j=1}^{n} (1 - \mathcal{T}_{y_j}(Y(\boldsymbol{\theta}))\, Y_j(\boldsymbol{\theta}) \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} \right] g(\boldsymbol{\theta}). \tag{16d}$$

[7]Formally, we use *compensated* tax reforms of direction $R(\mathbf{y}) = y_j - Y_j(\boldsymbol{\theta})$ and magnitude τ_j to calculate $\partial Y_i(\boldsymbol{\theta})/\partial \tau_j$ that captures only substitution effects. The reform and the responses of θ-taxpayers, around income $Y_j(\boldsymbol{\theta})$, are said to be compensated since the j^{th} marginal net-of-tax rate τ_j is modified while the level of tax is unchanged at $\mathbf{y} = \mathbf{Y}(\boldsymbol{\theta})$. We use *lump sum* tax reforms of direction $R(\mathbf{y}) = 1$ and magnitude ρ to calculate $\partial Y_i(\boldsymbol{\theta})/\partial \rho$ that captures income effects. Strictly speaking, these responses do not just depend on the type θ, but also on the Hessian of the tax function. When the tax function is non-linear, the responses to a tax reform generate changes in the marginal tax rates, which further induce compensated responses to these changes in marginal tax rates, etc. (Jacquet et al., 2013). The behavioral responses encapsulate this "circular process" through the endogeneity of the marginal tax rates.

Price-replicating tax reforms

We now characterize a family of tax reforms that is pivotal to our results. These reforms have as directions:

$$\forall j \in \{1, ..., n\} : \qquad R^{p_j}(\mathbf{y}) \overset{\text{def}}{\equiv} (1 - \mathcal{T}_{y_j}(y_1, ..., y_n))\, y_j. \tag{17}$$

We show, in Appendix A.4, that:

Proposition 1. *The impact, measured at PE, on taxpayers' input supplies and utility of a tax reform in the direction* $R^{p_j}(\mathbf{y}) \overset{def}{\equiv} (1-\mathcal{T}_{y_j}(y_1, ..., y_n))\, y_j$ *with magnitude* μ *is identical to the effect of a log-change in the* j^{th} *input price at GE. Formally,*

$$\forall i : \qquad \frac{\partial X_i^{R^{p_j},PE}(\boldsymbol{\theta}; \mu, \mathbf{p})}{\partial \mu} = \frac{\partial X_i(\boldsymbol{\theta}; \mathbf{p})}{\partial \log p_j} \quad \textit{and} \quad \frac{\partial U^{R^{p_j},PE}(\boldsymbol{\theta}; \mu, \mathbf{p})}{\partial \mu} = \frac{\partial U(\boldsymbol{\theta}; \mathbf{p})}{\partial \log p_j}. \tag{18}$$

Intuitively, the mapping

$$(x_1, \ldots, x_n) \mapsto \sum_{i=1}^{n} p_i\, x_i - \mathcal{T}(p_1\, x_1, \ldots, p_n\, x_n) + \mu\, R(p_1\, x_1, \ldots, p_n\, x_n)$$

between input supplies and after-tax income is the same for all taxpayers. Any tax reform or change in input prices affects taxpayers' program (2) solely through alterations in this mapping. Importantly, this mapping is perturbed *identically* by a log-change in the price of the j^{th} input and by a tax reform in the direction $R^{p_j}(\cdot)$ defined in (17), which we refer to as the "j^{th} price-replicating direction" because it replicates the impact of a log-change in the j^{th} input price. Consequently, each taxpayer's program is affected identically by a log-change in the price of the j^{th} input (at GE) and by a reform in the direction $R^{p_j}(\cdot)$ (at PE), which explains why both impact taxpayers' input supplies and well-being in the same way.

II.7 Price adjustments

In a PE with exogenous input prices (e.g., fixed wages), the effects of a tax change on a specific income of a given agent can be readily derived as a function of behavioral elasticities. However, the main challenge in GE is that this initial response influences prices, which subsequently affects the income decisions of all other agents. These agents may adjust their efforts to earn different types of income, shift income from one source to another, and so forth. Similarly, a change in production policies impact input prices which subsequently impact input supplies which in turn feeds back into prices, and so on. These ripple effects are synthesized in Figure 1. Determining the GE effects of these infinite sequences caused by tax or production policy reforms is a complex task. Our key towards this characterization is a reduced-form production side that we capture in inverse demand functions. According to Definition 1 and inverse demand functions (4), after a tax or production policy reform of magnitude μ and direction, respectively

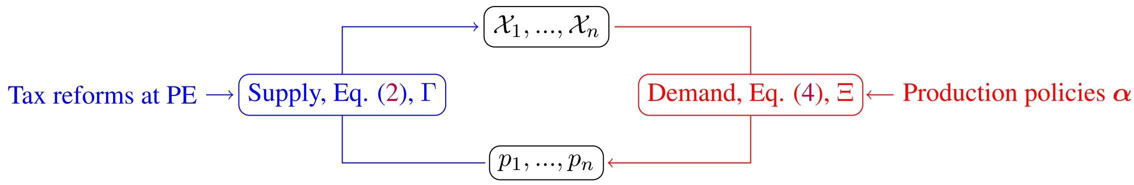

Figure 1: The GE adjustments following a tax or a production policy reform.

$R(\cdot)$ or $\boldsymbol{\alpha}(\cdot)$, the prices $(p_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu), ..., p_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu))$ verify the following fixed-point conditions:

$$\forall \mu, \forall i \in \{1, ..., n\} \qquad p_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu) = \mathscr{P}_i\left(\mathcal{X}_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu), ..., \mathcal{X}_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu)\right) \tag{19}$$

where the i^{th} aggregate input $\mathcal{X}_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\mu)$ is defined from individual i^{th} input $X_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta}, \mu)$ thanks to (12). Let Ξ denote the matrix where the term in the i^{th} line and j^{th} column is the inverse input's demand elasticity of the i^{th} price p_i with respect to the aggregate supply of the j^{th} input $\mathcal{X}_j$:

$$\Xi_{i,j} \overset{\text{def}}{\equiv} \frac{\mathcal{X}_j}{\mathscr{P}_i}\frac{\partial \mathscr{P}_i}{\partial \mathcal{X}_j}. \tag{20a}$$

We denote Γ the matrix of input supply elasticities, where the term $\Gamma_{i,j}$ in the i^{th} row and j^{th} column corresponds to the elasticity of the aggregate supply of the i^{th} input with respect to the price of the j^{th} input,

$$\Gamma_{i,j} \overset{\text{def}}{\equiv} \frac{1}{\mathcal{X}_i}\int_\Theta \frac{\partial X_i(\boldsymbol{\theta}; \mathbf{p})}{\partial \log p_j} \mathrm{d}F(\boldsymbol{\theta}) = \frac{1}{\mathcal{X}_i}\int_\Theta \frac{\partial X_i^{R^{p_j}(\cdot),PE}(\boldsymbol{\theta}; \mu, \mathbf{p})}{\partial \mu} \mathrm{d}F(\boldsymbol{\theta}) \tag{20b}$$

where the second equality follows from Proposition 1. We denote I_n the n-identity matrix and assume that the matrix $I_n - \Xi \cdot \Gamma$ is invertible so that, when we log-differentiates (19), we can apply the implicit function theorem to ensure that equilibrium prices are differentiable with respect to μ.[8] We denote the vector of log-price changes resulting from the production policy reform, assuming no changes in input supplies, as:

$$\frac{\boldsymbol{\partial} \log \mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu} \overset{\text{def}}{\equiv} \left(\sum_{\ell=1}^{L} \frac{\partial \mathscr{P}_1}{\partial \alpha_\ell}\, \alpha'_\ell(0), ..., \sum_{\ell=1}^{L} \frac{\partial \mathscr{P}_n}{\partial \alpha_\ell}\, \alpha'_\ell(0)\right)^T. \tag{21}$$

Thanks to these definitions, in the following lemma proofed in Appendix A.5, we present an equation that formalizes the process of price adjustments.

Lemma 1. *Following a tax reform in direction $R(\cdot)$ or a production policy reform in direction $\boldsymbol{\alpha}(\cdot)$, the price adjustments at GE are given by:*

$$\frac{\boldsymbol{\partial} \log \boldsymbol{p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu} = (I_n - \Xi \cdot \Gamma)^{-1} \cdot \Xi \cdot \frac{\boldsymbol{\partial} \log \boldsymbol{\mathcal{X}}^{R(\cdot),PE}}{\boldsymbol{\partial}\mu} + (I_n - \Xi \cdot \Gamma)^{-1} \cdot \frac{\boldsymbol{\partial} \log \mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu}, \tag{22}$$

[8] Under perfect competition and when the production function is linear, i.e. $\mathcal{F}(\mathcal{X}_1, ..., \mathcal{X}_n) = \sum_{i=1}^n \mathcal{X}_i$, matrix Ξ is nil. Therefore, $I_n - \Xi \cdot \Gamma$ is invertible. Therefore, by continuity, this invertibility remains satisfied as long as the elasticities of substitution between inputs are sufficiently high and competition is not too imperfect. Moreover, using the contracting mapping theorem, the existence and uniqueness of the GE can be shown under the assumption that for all out-of-equilibrium price $\mathbf{p}$ and input vectors $\mathcal{X}_1, ..., \mathcal{X}_n$., matrices $\Xi \cdot \Gamma$ have all eigenvalues with a modulus below a bound strictly lower than 1.

where the required elements are: the matrix of aggregate input supply elasticities Γ*, the matrix of inverse demand elasticities* Ξ*, the vector of input supply responses calculated at the PE – the latter being common sufficient statistics in PE tax models – and the vector of log-price changes (absent changes in input supplies) resulting from the production policy reform.*

Notably, in (22), from (12), $\partial \log p_i^{R(\cdot),PE}/\partial\mu = 0$ which leads to $\partial \log \boldsymbol{\mathcal{X}}^{R(\cdot),PE}/\partial\mu = \partial \log \boldsymbol{\mathcal{Y}}^{R(\cdot),PE}/\partial\mu$. Each i^{th} row of these vectors measures how aggregate input supply or income i reacts to a tax reform of magnitude μ, in the direction R, at the PE. Formally, from (16c), these measures are:

$$p_i \frac{\partial \mathcal{X}_i^{R,PE}(\boldsymbol{\theta},\mu)}{\partial\mu} = \frac{\partial \mathcal{Y}_i^{R,PE}(\boldsymbol{\theta},\mu)}{\partial\mu} = \int_\Theta \left\{ \underbrace{\sum_{j=1}^{n} \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j} R_{y_j}(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Compensated responses}} + \underbrace{\frac{\partial Y_i(\boldsymbol{\theta})}{\partial \rho} R(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Income effects}} \right\} \mathrm{d}F(\boldsymbol{\theta}). \quad (23)$$

In Figure 1, we illustrate the process of price adjustments in GE. After a tax reform, the initial taxpayers' responses in PE impacts the supplies of production factors (through the matrix of supply elasticities Γ) which modifies prices (through the matrix of inverse demand's elasticities Ξ). These price changes again impact the supplies of inputs, creating an ongoing loop of interdependence between prices of inputs and their supplies. The two terms on the right-hand sides of Equations (22)-(23) are these responses in supplies and demands which drive the infinite sequence of input price adjustments. Equation (23) focuses on the supply responses that are already present in PE and are highlighted in blue in Figure 1.

III Production Regulation

In this section, we present three theorems that state our main findings pertaining to what we call the *Production Regulation Principle*. These findings highlight how (*i*) the characteristics of the existing tax system and (*ii*) potential tax reforms lead to drastically different policy recommendations. We will enunciate the Production Regulation Principle in Sub-Section III.2, but first, in Sub-section III.1, we outline the key economic and fiscal parameters that any government should carefully assess before implementing policy interventions. In Subsection III.3, we provide a decision-tree to evaluate whether a production policy reform, efficient in increasing aggregate output, can also lead to a Pareto improvement based on the assumptions satisfied. In Subsection III.4, we discuss the applicability of our framework across various micro-founded examples, showing how they extend or complement existing results in the literature, such as those on intermediate goods taxation (e.g. taxing robots), trade regulation, and business-focused environmental regulation.

III.1 Economic and Fiscal Parameters at a Glance

Assumption 1 (Full Implementability of Price-Replicating Reforms)**.** *The tax authority can reform the tax system in the j^{th} price-replicating direction, $R^{p_j}(\cdot)$ (defined in (17)), for all $j = 1, ..., n$.*

Under Assumption 1, the tax authority is able to differentiate the income derived from each individual input for every taxpayer. Moreover, the structure of the tax system enables reforms along all price-replicating directions.

Assumption 1 does not hold with comprehensive income tax systems, where total income is taxed under a single (potentially nonlinear) schedule, $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y}) = T^c(y_1 + \cdots + y_n)$. With a comprehensive income tax system, Equation (17) implies that the j^{th} price-replicating direction takes the form:

$$R^{p_j}(\mathbf{y}) = \left(1 - T^{c\prime}(y_1 + ... + y_n)\right)\ y_j, \tag{24}$$

which does not depend solely on total income $y_1 + ... + y_n$. Therefore, any reform along a price-replicating direction $R^{p_j}(\cdot)$ would no longer be consistent with a comprehensive tax system, which therefore excludes such reforms.

In contrast, schedular tax systems, which are the sum of several (possibly non-linear) income-specific functions, $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y}) = T_1^s(y_1) + ... + T_n^s(y_n)$, enable reforms along all price-replicating directions $R^{p_j}(\cdot)$. With these systems, Equation (17) implies that the j^{th} price-replicating direction is:

$$\forall j \in \{1, ..., n\}: \qquad R^{p_j}(\mathbf{y}) = \left(1 - T_j^{s\prime}(y_j)\right)\ y_j. \tag{25}$$

Hence, reforming a schedular tax system along price-replicating directions preserves its schedular nature.

As noted by Hourani et al. (2023), schedular tax systems prevail in Costa Rica, Denmark, Finland, Greece, Hungary, Iceland, Israel, Italy, Latvia, Lithuania, Netherlands, Norway, Poland, Slovenia, Spain, Sweden and Turkey. In contrast, tax systems in Switzerland, the United Kingdom and the United States are similar to comprehensive systems (Hourani et al., 2023, Table A1).

Moreover, in practice, incomes arise from various sources, such as labor and capital. Labor incomes can be categorized into routine tasks, manual tasks, and conceptual activities, while capital incomes encompass dividends, interest, capital gains, capital losses, rents, imputed rents, and more. The tax authority is capable of observing the different types of capital income. However, labor incomes are indistinguishable by type. As a result, Assumption 1 holds only when the different types of labor are perfect substitutes. In cases where this condition does not apply, a partial observability scenario may prevail: the government observes only the sum of the first m incomes, $\bar{y} \overset{\text{def}}{\equiv} y_1 + \cdots + y_m$, while fully observing all other income types.[9] The tax system can then be an unrestricted function of incomes,

[9] Partial observability includes cases where several total incomes from different subsets of inputs are observed, but separate incomes from each of these inputs are not distinguished.

$(\bar{y}, ..., y_n) \mapsto \mathcal{T}(\bar{y}, y_{m+1}, ..., y_n)$, or a mixed tax system, as $\mathcal{T}(\mathbf{y}) = T^c(\bar{y}) + \sum_{i=m+1}^{n} T_i(y_i)$. The marginal tax rates for each income from the $m \leq n$ inputs are equal,

$$\text{For } j = 1, ..., m: \quad \frac{\partial \mathcal{T}(\bar{y}, y_{m+1}, ..., y_n)}{\partial y_j} = \mathcal{T}_{\bar{y}}. \tag{26}$$

Partial observability leads us to the following formal assumption:[10]

Assumption 2 (Partial Implementability of Price Replicating Reforms)**.** *The tax authority can reform the tax system in the so-called comprehensive direction:*

$$R^{\bar{y}}(\mathbf{y}) \overset{def}{\equiv} (1 - \mathcal{T}_{\bar{y}}(\bar{y}, y_{m+1}, ..., y_n))\, \bar{y}, \tag{27}$$

where:

$$\bar{y} \overset{def}{\equiv} y_1 + \cdots + y_m \tag{28}$$

and in the j^{th} price-replicating direction, $R^{p_j}(\cdot)$ (defined in (17)), for all $j = m+1, ..., n$.

Assumption 3 (Partial Pre-distributive effects of production policy)**.** *Production policies do not alter the input price ratios $j = 1, ..., m$, with $m \leq n$.*

In Appendix A.6, we give examples of primitives that satisfy Assumption 3. In this paper, we will offer precise policy recommendations for cases when either Assumption 1 or, both Assumptions 2 and 3 are satisfied, as well as when they are not. Therefore, these assumptions are the keys to sound decision-making for policymakers.

III.2 Production Regulation Principle

Now that Assumptions 1 and 2 have highlighted the relevant characteristics to study in a tax system, we identify when a tax reform can improve everyone's situation despite a production policy that increases aggregate output but also has adverse pre-distribution effects. To this end, we first present two lemmas that demonstrate how taxpayers' input supplies and utility levels can be unaltered when a production policy reform is combined with a so-called neutralizing tax reform. These lemmas are demonstrated in Appendices A.7.

Lemma 2. *If Assumption 1 is satisfied, the tax authority can combine any production policy reform in direction $\boldsymbol{\alpha}(\mu) = (\alpha_1(\mu), ..., \alpha_L(\mu))$ with a GE-neutralizing tax reform so that taxpayers' input supplies*

[10] One might think that combining a schedular and a comprehensive tax system, such as $\mathbf{y} \mapsto T^c(y_1 + ... + y_n) + T_1^s(y_1) + ... + T_n^s(y_n)$, would satisfy Assumption 1. However, in the corresponding j^{th} price-replicating direction, $R^{p_j}(\mathbf{y}) = \left(1 - T^{c'}(y_1 + ... + y_n) - T_j^{s'}(y_j)\right) y_j$, the $T^{c'}(y_1 + ... + y_n)y_j$ component is compatible with neither the schedular nor the comprehensive part of this mixed tax system. Assumption 1 is therefore not satisfied. In contrast, this tax system can be reformed in direction $R_{\bar{y}}(\mathbf{y})$ defined in Assumption 2.

$\mathbf{X}(\boldsymbol{\theta}) = (X_1(\boldsymbol{\theta}), ..., X_n(\boldsymbol{\theta}))$ *and utility levels* $U(\boldsymbol{\theta})$ *are unaltered. These tax reforms have the following neutralizing directions:*

$$R^N(\cdot) \overset{def}{\equiv} - \underbrace{\sum_{j=1}^{n} \gamma_j^{\boldsymbol{\alpha}(\cdot)} R^{p_j}(\cdot)}_{\text{Price adjustments replication}} \tag{29a}$$

where the scaling factors $\gamma_j^{\boldsymbol{\alpha}(\cdot)}$ *are given by:*

$$\forall j \in \{1, ..., n\}: \qquad \gamma_j^{\boldsymbol{\alpha}(\cdot)} \overset{def}{\equiv} \sum_{\ell=1}^{L} \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha'_\ell(0). \tag{29b}$$

Neutralizing tax reforms aim at annihilating the effects of any production policy reform on taxpayers' welfare and their incentives to supply inputs. To achieve this, these neutralizing directions incorporate the j^{th} price-replicating directions defined in (17).

Although the tax authority does not distinguish between certain types of income, it can still implement the neutralizing tax reform (29) if Assumptions 2 and 3 are satisfied instead of Assumption 1.

Lemma 3. *If the economy satisfies Assumptions 2 and 3, for any production policy reform with direction* $\boldsymbol{\alpha}(\mu) = (\alpha_1(\mu), ..., \alpha_L(\mu))$*, the GE-neutralizing reform defined by (29a) and (29b) is implementable. Therefore, combining the production policy reform and this GE-neutralizing tax reform leaves unaltered taxpayers' input supplies* $\mathbf{X}(\boldsymbol{\theta}) = (X_1(\boldsymbol{\theta}), ..., X_n(\boldsymbol{\theta}))$ *and utility levels* $U(\boldsymbol{\theta})$*.*

Combining any production policy with the corresponding neutralizing tax reform yields a Pareto improvement, as stated in the following theorems, which are demonstrated in Appendix A.8.

Theorem 1. ***Production regulation Principle – Part I****: If Assumption 1 holds, production policies that increase aggregate output, combined with the neutralizing tax reform in the direction defined in (29), result in a Pareto improvement. Conversely, reducing the use of production policies that decrease aggregate output also leads to a Pareto improvement when combined with a GE-neutralizing tax reform.*

Theorem 1 describes the tax reforms the tax authority should take when, for instance, a reform of the competition policy is implemented that reduces barriers to entry (improving production efficiency), impacts input prices and alters the pre-distribution in the economy. Given that the government can reform the tax system in all price-replicating directions $R^{p_j}(\cdot)$, introducing (small) tax reforms in the neutralizing direction leads to a Pareto improvement. The tax system allows the government to counteract all pre-distributive losses arising from the competition policy reform. Importantly, the tax system does not need to be optimal; it simply needs to enable reforms in the neutralizing direction to be effective (thereby counterbalancing the production policy's impact on welfare and taxpayers' behavioral responses).

When the tax authority is limited to reforming the tax system in a subset of price-replicating directions, and production policy reforms do not modify the input price ratios corresponding to the other inputs, Theorem 2 highlights that combining an efficient production policy reform with a neutralizing tax reform also results in a Pareto improvement.

Theorem 2. ***Production Regulation Principle – Part II****: If Assumptions 2 and 3 hold, production policies that increase aggregate output, combined with the neutralizing tax reform with the neutralizing direction defined in (29), re-expressed as:*

$$R^N(\cdot) = -\sum_{\ell=1}^{L} \alpha_\ell'(0) \frac{\partial \log \mathscr{P}_{\bar{y}}}{\partial \alpha_\ell} R^{\bar{y}}(\cdot) - \sum_{j=m+1}^{n} \sum_{\ell=1}^{L} \alpha_\ell'(0) \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} R^{p_j}(\cdot),$$

result in a Pareto improvement. Conversely, reducing the use of production policies that decrease aggregate output also leads to a Pareto improvement when combined with this neutralizing tax reform.

According to the Production Regulation Principle (Theorems 1 and 2), the choice of production policies, such as competition policy, should be guided solely by efficiency considerations if either all price-replicating directions, $R^{p_j}(\cdot)$, of tax reforms are feasible, or production policies preserve price ratios of the first m inputs, and only the price-replicating comprehensive directions $R^{\bar{y}}(\cdot)$, $R^{p_{m+1}}(\cdot), ..., R^{p_n}(\cdot)$ are feasible. We call this principle the"Production Regulation Principle". Another way of putting it is that the design of any production policy needs not include pre-distribution concerns when the neutralizing tax reforms can be implemented. The production regulation principles can be regarded as some form of "Tinbergen principle": production policies should not be concerned with redistribution, as that role falls within the area of tax policy. Importantly, Theorems 1 and 2 do not require perfect competition, contrasting with the existing tax literature which classically assumes perfect competition, such as Diamond and Mirrlees (1971), Naito (1999), Saez (2004), Rothschild and Scheuer (2013, 2016), Jacobs (2015), Sachs et al. (2020), Costinot and Werning (2022) or Schultz et al. (2023). Moreover, our framework generalizes the usual approach, which assumes an optimal tax schedule and an economy operating on the production possibility frontier, as in Diamond and Mirrlees (1971). We significantly extend these insights by demonstrating that our Production Regulation Principles apply even with suboptimal tax systems that can be reformed through the neutralizing tax reform, and when the economy operates in the interior of the production possibility set.

Underlying Mechanisms

To get an intuitive understanding of Theorem 1 and 2, we clarify the underlying mechanisms. This complements the proofs in Appendix A.8, which takes care of all formal details. When an economy undergoes changes in its production policies so that for some $\ell \in \{1, ..., L\} : \alpha_\ell'(0) \neq 0$, Theorem 1 and 2 suggest that the government assesses whether its current tax system meets Assumption 1 or 2. When it is the case, the GE effects on input supplies and welfare, stemming from production policies, are offset by a tax reform with a GE-neutralizing direction, $R^N(\cdot)$, defined in (29). This direction is calculated based on the opposites of the (rescaled) price-replicating directions defined in (17).

Consequently, any possible deteriorating impact in terms of welfare being nullified, a Pareto improvement is guaranteed if and only if tax revenue is not deteriorated. The impact of the production policies on each price is calculated with fixed input supplies as:

$$\forall j \in \{1, ..., n\} : \qquad \frac{\partial \log p_j^{R^N(\cdot), \boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \sum_{\ell=1}^{L} \frac{\partial \log \mathscr{P}_j^{\boldsymbol{\alpha}(\cdot)}}{\partial \alpha_\ell} \alpha_\ell'(0). \tag{31}$$

Therefore, the impact on tax revenue is modified by $\sum_{j=1}^{n} \sum_{\ell=1}^{L} \mathcal{X}_j (\partial \mathscr{P}_j / \partial \alpha_\ell) \alpha_\ell'(0)$, which is equal to the aggregate marginal changes in production $\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell} \alpha_\ell'(0)$, based on (8). One has a Pareto improvement if and only if $\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell} \alpha_\ell'(0) \ > 0$, meaning that production policies must (solely) be designed to boost aggregate production as the tax system ensures a Pareto-improvement.

If Assumptions 1 and 2 are violated, the policy makers have to check for welfare improvements by comparing efficiency effects with pre-distributive effects, as follows.

Theorem 3. *If Assumption 1 is violated and either Assumption 2 or Assumption 3 is not verified, to determine the welfare impact of any production policy reform in direction* $\boldsymbol{\alpha}(\cdot)$*, we compare its efficiency effects with its pre-distribution effects, as follows:*

$$\frac{\partial \mathscr{L}^{\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \underbrace{\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell} \alpha_\ell'(0)}_{\text{Production efficiency effect}} + \underbrace{\sum_{\ell=1}^{L} \sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \frac{\partial \mathscr{P}_j}{\partial \alpha_\ell} \alpha_\ell'(0)}_{\text{Pre-distributive effects}}, \tag{32}$$

where the first term is the way the production policy reform modifies efficiency and the second term is the pre-distributional loss created by this reform.

Theorem 3 shows that the effects of a production policy reform can be decomposed into two parts: the effects on productive efficiency with unchanged input supplies, and the fact that price changes induced by a production policy trigger behavioral responses that have identical consequences to tax reforms in the price-replicating directions R^{p_j}.

When a competition policy reform (or any other multidimensional reform of the production sector) α_ℓ, for $\ell = 1, ..., L$ is optimal, the net impact in terms of efficiency and pre-distribution is null, resulting in (32) being equal to zero. In this equation, proved in Appendix A.9, the production efficiency effect $\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell} \alpha_\ell'(0)$ represents the aggregate marginal changes in production (as defined in (8)), which, depending on its sign, may lead to either an improvement or deterioration in the government's Lagrangian (11). A competition policy reform (or any other multidimensional reform of the production sector) $\mu \mapsto (\alpha_1(\mu), ..., \alpha_L(\mu))$ is production efficient if:

$$\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell} \alpha_\ell'(\mu) > 0.$$

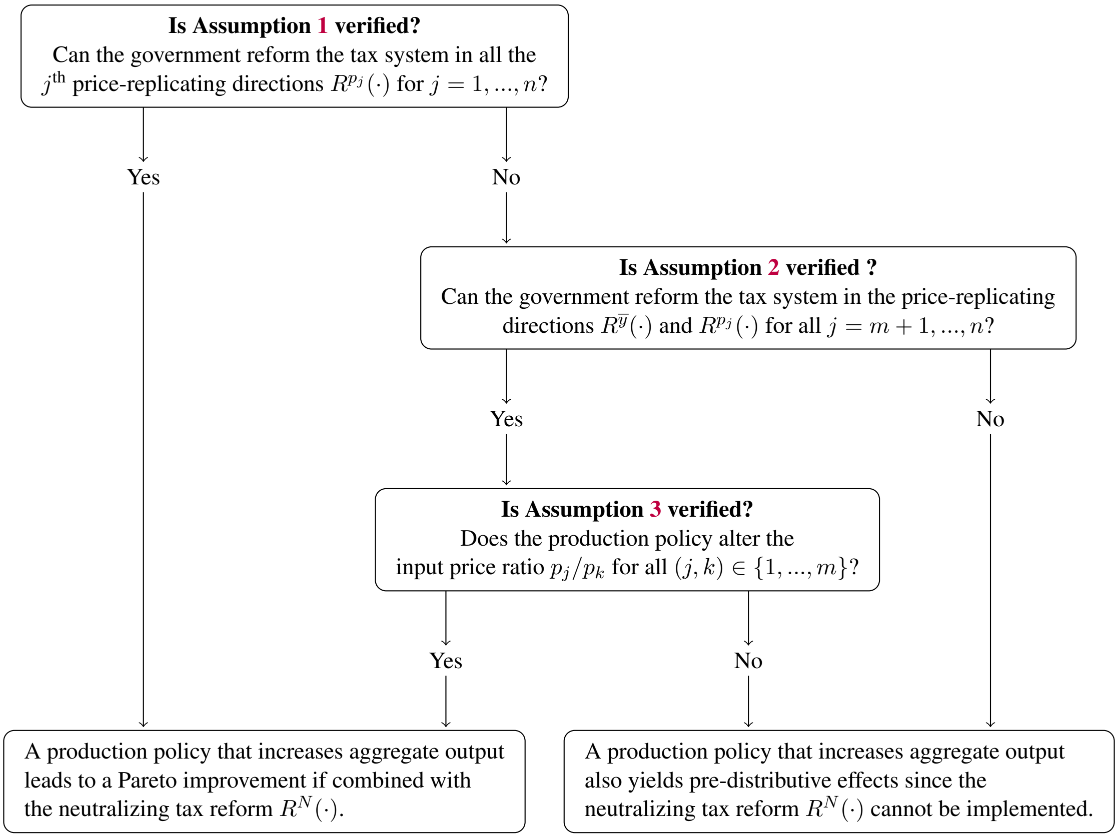

Figure 2: Decision Tree

The government should assess these production efficiency effects in conjunction with the pre-distributive effects, captured in the second term of (32), to evaluate the overall impact of any production policy reform. We show that these *a priori* complex pre-distributive effects are equal, for every change in the policy, to a sum across all inputs prices. This sum consists in the reform's impact on prices, hence on the inverse demands for inputs, $(\partial \mathscr{P}_j/\partial \alpha_\ell)\ \alpha'_\ell(0)$, times an expression $\partial \mathscr{L}^{R^{p_j}(\cdot)}/\partial \mu$ that makes all the technical difficulties linked to the shift from PE to GE vanish. Together, these terms capture the pre-distributional consequences of the competition policy. In the next section, we detail the calculation of the effects on the Lagrangian of a tax reform in the price-replicating directions, $\partial \mathscr{L}^{R^{p_j}(\cdot)}/\partial \mu$.

It is worth noting that Theorems 1, 2 and 3 apply irrespective of perfect competition. They remain valid in the presence of market failures, rent-seeking, externalities in production, or other imperfections in input markets. Furthermore, as already emphasized, Theorems 1 and 2 do not require the tax system to be optimal.

III.3 Decision Tree

Figure 2 presents a decision tree that guides in evaluating whether a given production policy reform–efficient in terms of increasing aggregate production but potentially detrimental to pre-distribution–can be Pareto-improving. The Production Pegulation Principle, which is summarized in the bottom-left box, applies when either Assumption 1 holds (see Theorem 1) or when both Assumptions 2 and 3 are verified (see Theorem 2). If Assumption 2 is violated, Theorem 3, summarized in the bottom-right box, becomes applicable.

III.4 Micro-founded examples

We derived our Production Regulation Principle in Theorems 1 and 2, along with our theorem addressing the impact of production policy reform when these principles do not apply (Theorem 3). This was achieved using a reduced-form description of the production sector that specifies only the inverse demand functions, $\mathscr{P}_i(\cdot)$. This approach has a powerful advantage by ensuring our results remain robust to variations in the underlying micro-foundations behind these inverse demand functions. To illustrate the broad applicability of our reduced-form approach to various policy contexts, we provide a comprehensive exploration of micro-founded examples in Appendix B. We investigate policies impacting the production sector under different contexts, such as competition policy, taxation of intermediate goods, robots and AI, trade liberalization, public production, and firms' environmental regulations.

In all these examples, we show how firms' behaviors in the production sector can be represented by the decision of an hypothetical "production coordinator". Formally, this coordinator allocates aggregate inputs supplies $\mathcal{X}_1, ..., \mathcal{X}_n$ within the production sector to maximize total consumption by taxpayers, $C(\boldsymbol{\theta})$, and the government, E. When formulating the production coordinator's program, we decompose certain resource constraints to replicate the effects of potential market frictions and production policies on resource allocation within the production sector.

This approach allows us to clarify, in each micro-founded examples, which production policies that increase aggregate output can be recommended. We therefore bridge the Production Regulation Principles (Theorems 1 and 2) with key policy implications, including the reduction of sector-specific markups, not taxing intermediate goods, no indirect taxation on commodities, free trade, managing public firms using producer prices in the private sector, etc. Importantly, with these reduced-form examples, we demonstrate that previous results in the literature, which all build upon Diamond and Mirrlees (1971) and rely on micro-founded production functions, can easily be recovered and extended within our framework as special cases. We also reproduce other results from the literature where the production efficiency theorem of Diamond and Mirrlees (1971) does not apply. These departures, including Naito (1999, 2004), Koizumi (2020), Guerreiro et al. (2021), Costinot and Werning (2022) and Thuemmel (2023), arise because neither Assumption 1 nor the combination of Assumptions 2 and 3 are verified. Therefore, production policies must also account for their pre-distributive role, as specified by Equation (32) in Theorem 3 and summarized in Figure 2.

IV New Key Statistics and their Implementation

In this section, we detail two key statistics. The first is the welfare impact of the price-replicating tax reform $\partial \mathscr{L}^{R^{p_j}}/\partial\mu$ which appears in (32). This statistic allows us to obtain the impact on the Lagrangian of all GE changes in input prices. It relies on our second statistic, the "GE-multiplier" that give the welfare impact of price adjustments implied by any tax reform. In this section, we extend our results from Lemma 2 and Theorem 3 to the case where both tax and production policy reforms occur simultaneously. We conclude the section with a practical guide to the empirical implementation of our statistics.

IV.1 GE-Multipliers

The following proposition, proved in Appendix A.9, details the calculation of $\partial \mathscr{L}^{R^{p_j}}/\partial\mu$ which appears in Equation (32). It also introduces the GE multipliers to unlock the gate from the PE to the GE framework.

Proposition 2. *(GE multipliers) At GE, the impact of a tax reform in the j^{th} price-replicating direction $R^{p_j}(\mathbf{y}) \overset{def}{\equiv} (1-\mathcal{T}_{y_j}(y_1,...,y_n))y_j$ is:*

$$\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial\mu} = \int_\Theta \left\{ \underbrace{-(1-g(\boldsymbol{\theta}))R^{p_j}(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Mechanical effects}} + \sum_{i=1}^{n} \left(\mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta}))+\eta_i\right) \underbrace{\frac{\partial Y_i(\boldsymbol{\theta})^{R^{p_j}(\cdot),PE}}{\partial\mu}}_{\text{Responses of incomes at PE}} \right\} \mathrm{d}F(\theta), \tag{33}$$

with:

$$\frac{\partial Y_i(\boldsymbol{\theta})^{R^{p_j}(\cdot),PE}}{\partial\mu} = \underbrace{\sum_{j=1}^{n}\frac{\partial Y_i(\boldsymbol{\theta})}{\partial\tau_j}R^{p_j}_{y_j}(\mathbf{y})}_{\text{Compensated responses}} + \underbrace{\frac{\partial Y_i(\boldsymbol{\theta})}{\partial\rho}R^{p_j}(\mathbf{y})}_{\text{Income effects}},$$

where η_i denote the "GE-multipliers" and are defined as:

$$\forall i \in \{1,...,n\}: \qquad \eta_i \overset{def}{\equiv} \underbrace{\frac{\mathcal{F}_{\mathcal{X}_i}-p_i}{p_i}}_{\text{Market failure correction}} + \underbrace{\sum_{j=1}^{n}\frac{\partial \mathscr{L}^{R^{p_j}}}{\partial\mu}\frac{\Xi_{j,i}}{\mathcal{Y}_i}}_{\text{Correction for "}R^{p_j}\text{-suboptimality" of the tax system}}. \tag{34}$$

If the tax system is optimized along the price-replicating direction $R^{p_j}(\cdot)$, $\forall j \in \{1,...,n\}$, Equation (33), which provides the welfare impact of a tax reform of magnitude μ in the price-replicating direction $R^{p_j}(\cdot)$, is equal to zero, $\forall j \in \{1,...,n\}$. If the tax system is not optimized along the price-replicating directions, we have $\partial \mathscr{L}^{R^{p_j}(\cdot)}/\partial\mu \neq 0$ for at least one $j \in \{1,...,n\}$. The welfare impact can be decomposed into two elements detailed in (33). First, independently of any behavioral change, the tax reform impacts the Lagrangian through changes in tax liabilities $R^{p_j}(\mathbf{Y}(\theta))$. It creates the usual mechanical effects on government revenue and social welfare, $1-g(\boldsymbol{\theta})$. Second, the tax reform affects tax revenue as taxpayers adjust their incomes, represented by $\partial Y_i(\boldsymbol{\theta})^{R^{p_j}(\cdot),PE}/\partial\mu$ for $i=1,\ldots,n$, via compensated responses and income effects (already presented in Equation (16c)). In PE, these behavioral responses impact the Lagrangian solely through their impact on tax revenues. This is why the marginal tax rates $\mathcal{T}_{y_i}$ appear as factors in the PE responses $\partial Y_i^{R(\cdot),PE}(\theta)/\partial\mu$ to tax reforms. In GE, however, the effect of price adjustments must also be considered. Specifically, the i^{th} GE multiplier η_i captures the impact on the Lagrangian of price adjustments resulting from a one unit increase in the i^{th} aggregate income in PE. Consequently, in the right-hand side of (33), the PE responses $\partial Y_i^{R(\cdot),PE}(\theta)/\partial\mu$ to tax reforms should also be multiplied by the GE multipliers η_i.[11]

The GE-multiplier η_i, associated to input i, consists of two elements in (34): a corrective term for market failures and a corrective term for the suboptimality of the tax system in the price-replicating directions, as stated in (34). The corrective term for market failure $(\mathcal{F}_{\mathcal{X}_i} - p_i)/p_i$ assesses whether the social return of input i, $\mathcal{F}_{\mathcal{X}_i}$, differs from its private return, p_i, indicating the absence of perfect competition. If $\mathcal{F}_{\mathcal{X}_i} - p_i$ is strictly positive, the corrective term is positive and, ceteris paribus, depending on the sign of the response of income Y_i, it increases or decreases the Lagrangian, in (33). In a perfectly competitive setting, this term equals zero and (34) simplifies to the corrective term for the suboptimality of the tax system in the price-replicating directions:

$$\forall i \in \{1, ..., n\}: \qquad \eta_i = \sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}}}{\partial \mu} \frac{\Xi_{j,i}}{\mathcal{Y}_i}. \tag{35}$$

The right-hand side of (35) measures the welfare impact of the suboptimality of the tax system along the price-replicating directions. To understand why this term arises, we can recall Proposition 1, which emphasizes that tax reforms in the price-replicating direction R^{p_j} impacts taxpayers' input supply, consumption and utility similarly, at PE, to the effect of a log-change in the price of input j. Therefore, when a tax reform generates a unit increase in the i^{th} aggregate income at the PE, this generates a relative change in the j^{th} price equal to $\Xi_{j,i}/\mathcal{Y}_i$. This, in turn, impacts taxpayers' input supplies and welfare as much as a tax reform in the price-replicating direction $R^{p_j}(\cdot)$. Therefore, the term $\sum_{j=1}^{n}(\partial\mathscr{L}^{R^{p_j}}/\partial\mu)\,(\Xi_{j,i}/\mathcal{Y}_i)$ captures the impact on the Lagrangian of these price changes. When the tax system is optimized in the price-replicating directions, we have $\partial\mathscr{L}^{R^{p_j}}/\partial t = 0\ \forall j \in \{1, ..., n\}$, the tax system fully neutralizes the prices' impact on taxpayers. Let us stress that optimizing along the price-replicating directions does not require the tax system to be optimal, which would impose significantly stricter conditions. Notably, the GE-multipliers η_i depend neither on the direction $R(\cdot)$ nor on the size μ of the reform. The distinct expressions that can be taken by the GE multipliers are summarized, in Table 1.

In economies satisfying Assumption 2, the tax authority is never able to optimize the tax system in the price-replicating directions R^{p_j}, for $j = 1, ..., m$, since it cannot separately observe the related incomes. Therefore, $\sum_{j=1}^{n}(\partial\mathscr{L}^{R^{p_j}}/\partial\mu)\,(\Xi_{j,i}/\mathcal{Y}_i) \neq 0$, for $j = 1, ..., m$. In contrast, the tax system may

[11] The role of GE multipliers η_i in our tax perturbation approach is akin to the role of consistency constraint multipliers in Rothschild and Scheuer (2013, 2014)'s mechanism design approach.

		Is the tax system optimized with respect to all the j^{th} price-replicating directions $R^{p_j}(\cdot)$ for $j = 1, ..., n$?	
		Yes	No
Perfect competition?	Yes	$\eta_i = 0$	$\eta_i = \sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}}}{\partial \mu} \frac{\Xi_{j,i}}{\mathcal{Y}_i}$
	No	$\eta_i = \frac{\mathcal{F}_{\mathcal{X}_i} - p_i}{p_i}$	$\eta_i = \frac{\mathcal{F}_{\mathcal{X}_i} - p_i}{p_i} + \sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}}}{\partial \mu} \frac{\Xi_{j,i}}{\mathcal{Y}_i}$

Table 1: GE multipliers

be optimized along the price-replicating directions $R^{p_j}(\cdot)$ for all $j = m+1, ..., n$ and along the price replication direction $R^{\overline{y}(\cdot)}(\cdot)$ defined in (27). We can extend our insights if we assume inverse demands functions are weakly separable, taking the form:

$$\forall j \in \{1, ..., m\} : \mathscr{P}_j(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathcal{Q}_j(\mathcal{X}_1, ..., \mathcal{X}_m)\, \overline{\mathcal{P}}(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha}) \quad \text{(36a)}$$

$$\forall j \in \{m+1, ..., n\} : \mathscr{P}_j(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathcal{P}_j(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha}). \quad \text{(36b)}$$

This assumption can be microfounded, for instance, under perfect competition and a weakly separable production function of the form $\mathcal{F}(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathscr{F}(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha})$. Under such assumptions, we obtain that for the $n - m + 1$ last incomes, the GE multipliers are solely determined by market failures (see Appendix A.10):

$$\forall i \in \{m+1, ..., n\} : \qquad \eta_i = \frac{\mathcal{F}_{\mathcal{X}_i} - p_i}{p_i}. \quad \text{(37)}$$

Additionally, when the inverse demand elasticities are weakly separable, Assumption 3 is verified, since Equations (36a) imply the following price ratios:

$$\forall (i,j) \in \{1, ..., m\} : \qquad \frac{p_i}{p_j} = \frac{\mathcal{Q}_j(\mathcal{X}_1, ..., \mathcal{X}_m)}{\mathcal{Q}_j(\mathcal{X}_1, ..., \mathcal{X}_m)}$$

do not depend on production policies.

IV.2 Considering Both Tax and Production Policy Reforms

To make our results applicable to more real-world scenarios, we extend the economic environment further by considering an economy where both production policy reforms and tax reforms coexist.

Neutralizing price effects from both reforms

With Equation (29a), we constructed the neutralizing direction of a tax reform which neutralizes the price effects of any production policy $\boldsymbol{\alpha}(\cdot)$ on utility and behavior. We now extend this by constructing tax reforms that neutralize not only the price effects from any production policy $\boldsymbol{\alpha}(\cdot)$ but also the price effects from any initial tax reform $R(\cdot)$. With some abuse of notation, we label this direction $R^N(\cdot)$ as in (29a) and, in Appendix A.7, we show the following lemma.

Lemma 4. *Under Assumption 1, the tax authority can combine any initial tax reform in the direction $R(\cdot)$ and any production policy reform in direction $\boldsymbol{\alpha}(\mu) = (\alpha_1(\mu), ..., \alpha_L(\mu))$ with a GE-neutralizing tax reform so that the effects on taxpayers' input supplies $\mathbf{X}(\boldsymbol{\theta}) = (X_1(\boldsymbol{\theta}), ..., X_n(\boldsymbol{\theta}))$ and utility levels $U(\boldsymbol{\theta})$ are those induced, in PE, by the initial tax reform in direction $R(\cdot)$. These tax reforms have the following neutralizing directions:*

$$R^N(\cdot) \stackrel{def}{\equiv} \underbrace{R(\cdot)}_{\text{Initial tax reform}} - \underbrace{\sum_{j=1}^{n} \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} R^{p_j}(\cdot)}_{\text{Price adjustments replication}} \tag{38a}$$

where the scaling factors are written as $\gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}$ given by:

$$\forall j \in \{1, ..., n\}: \qquad \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} \stackrel{def}{\equiv} \sum_{i=1}^{n} \frac{\Xi_{j,i}}{\mathcal{Y}_i} \frac{\partial \mathcal{Y}_i^{R(\cdot),PE}}{\partial \mu} + \sum_{\ell=1}^{L} \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha_\ell'(0). \tag{38b}$$

Lemma 4 builds upon Lemma 2, which addressed the interplay between any production policy reform and a GE-neutralizing tax reform. It extends this result to encompass the combination of a tax reform and a production policy reform, alongside the relevant GE-neutralizing tax reform. Intuitively, in GE, the impact on taxpayers' utility and input supplies resulting from the combination of a tax reform in the direction $R(\cdot) - \sum_{j=1}^{n} \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} R^{p_j}(\cdot)$, defined in (38a), and a production policy reform $\boldsymbol{\alpha}(\cdot)$ can be decomposed into three main components. First, the PE effects of the tax reform in the direction $R(\cdot)$. Second, the PE effects of the tax reforms $\sum_{j=1}^{N} \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} R^{y_j}(\cdot)$. Third, the tax reform in the direction $R^N(\cdot)$ and the production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$ imply prices changes $\partial p_j^{R^N,\boldsymbol{\alpha}(\cdot)}/\partial\mu$ at the GE. According to Proposition 1, the PE effects of the tax reforms, $-\sum_{j=1}^{N} \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} R^{y_j}(\cdot)$ (the second component), neutralize the responses to price changes, $\partial p_j^{R^N,\boldsymbol{\alpha}(\cdot)}/\partial\mu$ (the third component), whenever $\gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} = \partial p_j^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial\mu$ for all $j \in \{1, \ldots, n\}$. As shown in Appendix A.7, this condition is satisfied when:

$$\forall j \in \{1, \ldots, n\}: \qquad \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} = \sum_{i=1}^{n} \frac{\Xi_{j,i}}{\mathcal{Y}_i} \frac{\partial \mathcal{Y}_i^{R(\cdot),PE}}{\partial \mu} + \sum_{\ell=1}^{L} \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha_\ell'(0) = \frac{\partial \log p_j^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}. \tag{39}$$

Incidence of Tax and Production Policy Reforms

Now, we can extend Theorem 3 and state the welfare impact of both tax reforms and production policy reforms. The proof is in Appendix A.9.

Theorem 4. *The welfare impact of any tax reform in direction $R(\cdot)$ and of any production policy reform* *in direction $\boldsymbol{\alpha}(\cdot)$ on the government's Lagrangian are given by:*

$$\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \int_{\Theta} \left\{ - \left(\underbrace{(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta}))}_{\text{Mechanical effects}} - \sum_{i=1}^{n} (\mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) + \eta_i) \underbrace{\frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu}}_{\text{Responses of incomes at PE}} \right) \right\} \mathrm{d}F(\theta)$$
$$+ \underbrace{\sum_{\ell=1}^{L} \mathcal{F}_{\alpha_\ell}\, \alpha_\ell'(0)}_{\text{Production efficiency effects}} + \underbrace{\sum_{\ell=1}^{L}\sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \frac{\partial \mathscr{P}_j}{\partial \alpha_\ell}\, \alpha_\ell'(0)}_{\text{Pre-distributive effects}}, \tag{40}$$

where the GE-multipliers η_i are defined in (34).

Beside the production efficiency and pre-distributive effects of production policy reforms already in (32), mechanical effects and responses of incomes in PE appear in the first line of (40). These effects are quite similar to those described in Lemma 2, except that, there, direction $R(\cdot)$ was the price-replicating one, $R^{p_j}(\cdot)$. Their interpretation and empirical implementation follow suit, simply with different directions. If the tax and production policy reforms are chosen optimally, the right-hand side of (40) is nil. In the real world, we expect this right-hand side to be often either positive or negative.

IV.3 Empirical Implementation

The welfare impact of any production policy reform can be computed by implementing Equation (32), in conjunction with (33), (34) and (17). Similarly, the welfare impact of a tax reform, or a combination of tax and production policy reforms, can be evaluated using (40). Regardless of the type of reform, the tax authority can weigh the pre-distributive effects against the production efficiency effects. Communicating these effects clearly is crucial to help the public understand the trade-offs involved and fosters informed debate.

For the pre-distributive effect of (32) or (40), we need to calculate $\partial \mathscr{L}^{R^{p_j}(\cdot)}/\partial \mu$. Either the tax system is optimized along all price replication directions, in which case $\partial \mathscr{L}^{R^{p_j}(\cdot)}/\partial \mu = 0$ and $\eta_i = (\mathcal{F}_{\mathcal{X}_i} - p_i)/p_i$ for all $i = 1, ..., n$, according to (34), or this involves implementing each term in Equation (33). The mechanical effects rely on the welfare weights $g(\boldsymbol{\theta})$, which can be calibrated either from normative assumptions (Saez and Stantcheva, 2016) or from survey data (Kuziemko et al., 2015, Capozza and Srinivasan, 2024). The term R^{p_j}, defined in (17), requires the observed marginal tax rate. We also need estimates of compensated responses $\partial Y_i(\boldsymbol{\theta})/\partial \tau_j$ and income effects $\partial Y_i(\boldsymbol{\theta})/\partial \rho$, which can be empirically obtained using quasi-experimental evidences (see, e.g., Saez et al. (2012)).[12]

[12]One common problem in the sufficient statistics approach is that these statistics may depend on the tax system. In our tax incidence analysis, one therefore needs sufficient statistics close to those estimated under the considered tax schedule. Moreover, both compensated responses and income effects are defined with prices held constant. This aligns with the empirical literature, which examines variations in taxpayer response to tax reforms under the assumption that price changes are consistent across all taxpayers

The calibration of the demand-side parameters requires estimating the market failure corrections, $(\mathcal{F}_{\mathcal{X}_i} - p_i)/p_i$, the inverse demand elasticities $\Xi_{i,j}$ with respect to input supplies, and the log-derivatives of the inverse demand functions with respect to production policies, $\partial \mathscr{P}_i/\partial \alpha_\ell$. Market failure corrections $(\mathcal{F}_{\mathcal{X}_i} - p_i)/p_i$ can be calibrated by assuming perfect competition, in which case $(\mathcal{F}_{\mathcal{X}_i} - p_i)/p_i = 0$. Alternatively, one may envision externalities or mark-ups arising from imperfect competition. In the latter case, markup-up estimates from the empirical Industrial Organization literature (see e.g. De Loecker et al. (2020)) can be used to quantify how much input price p_i is under-priced to generate profits. These profits correspond to the income derived from the inelastically supplied entrepreneurial input, which allocates profits among taxpayers (see McKenzie (1959) and Mas-Colell et al. (1995, pp. 134-135)). The matrix of elasticities of inverse demands with respect to input supplies, Ξ, can be calibrated structurally. For instance, assuming two inputs –labor indexed by L and capital indexed by K– under perfect competition, the matrix Ξ can be obtained from the substitution elasticity σ between labor and capital, and the income share α_L and α_K of, labor income and capital income in GDP, respectively. The matrix is given by:

$$\begin{pmatrix} \dfrac{\partial \log \mathscr{P}_L}{\partial \log X_L} & \dfrac{\partial \log \mathscr{P}_L}{\partial \log X_K} \\ \dfrac{\partial \log \mathscr{P}_K}{\partial \log X_L} & \dfrac{\partial \log \mathscr{P}_K}{\partial \log X_K} \end{pmatrix} = \begin{pmatrix} -\dfrac{\alpha_L}{\sigma} & \dfrac{\alpha_L}{\sigma} \\ \dfrac{\alpha_K}{\sigma} & -\dfrac{\alpha_K}{\sigma}. \end{pmatrix}$$

At the macroeconomics level, Antràs (2004) estimates an elasticity of substitution between labor and capital, σ, lower than 0.5 for the US. In the meta-analysis, Knoblach et al. (2020) obtain a long-run elasticity for the aggregate economy in the range of $0.45 - 0.87$. Finally, we need estimates of the log derivative of inverse demand with respect to production policies, $\partial \mathscr{P}_i/\partial \alpha_\ell$, as in Bertrand and Kramarz (2002), Biscourp et al. (2013), who estimate the impact of entry regulation on retail prices using French reforms.

V Pareto-improving tax reforms

We develop, in this section, an approach for the identification of Pareto-improving tax reforms in the presence of multiple incomes and GE adjustments. We provide necessary and sufficient conditions for the existence of Pareto-improving directions of tax reform, with multidimensional nonlinear tax systems and GE effects. We show how to test whether a given tax system can be Pareto improved and whether a given tax reform is Pareto-improving. As a preamble to this exercise, we must establish the optimal tax system when there is no restriction on its form.

For this purpose, we denote Θ_Y the income space, $\partial\Theta_Y$ its smooth boundary. Let $\partial \widehat{Y}_i(\mathbf{y})/\partial \tau_j$, $\partial \widehat{Y}_i(\mathbf{y})/\partial \rho$ and $\widehat{g}(\mathbf{y})$ denote the mean values of $\partial Y_i(\boldsymbol{\theta})/\partial \tau_j$, $\partial Y_i(\boldsymbol{\theta})/\partial \rho$ and $g(\boldsymbol{\theta})$, respectively, among taxpayers with earnings $\mathbf{Y}(\boldsymbol{\theta}) = \mathbf{y}$. The following proposition, proved in Appendix A.11 characterizes the optimal tax system without any restriction on its form.

Proposition 3. *When the tax system for multiple incomes has no restriction on its form, the optimal tax system has to verify the Euler-Lagrange equation:*

$$\left[1-\widehat{g}(\mathbf{y})-\sum_{i=1}^{n}(\mathcal{T}_{y_i}(\mathbf{y})+\eta_i)\frac{\partial\widehat{Y}_i(\mathbf{y})}{\partial\rho}\right]h(\mathbf{y})=-\sum_{j=1}^{n}\frac{\partial\left[\sum_{i=1}^{n}(\mathcal{T}_{y_i}(\mathbf{y})+\eta_i)\frac{\partial\widehat{Y}_i(\mathbf{y})}{\partial\tau_j}h(\mathbf{y})\right]}{\partial y_j},\tag{41}$$

$\forall y\in\Theta_Y$*, and it satisfies the boundary conditions:*

$$\forall y\in\partial W_Y:\sum_{1\le i,j\le n}(\mathcal{T}_{y_i}(\mathbf{y})+\eta_i)\frac{\partial\widehat{Y}_i(\mathbf{y})}{\partial\tau_j}h(\mathbf{y})\phi_j(\mathbf{y})=0\tag{42}$$

where $\phi(\mathbf{y})=(\phi_1(\mathbf{y}),...,\phi_n(\mathbf{y}))$ *is the outward unit vector normal to the boundary at* $\mathbf{y}$*, where the GE multipliers are given by* (34) *with* $\partial\mathscr{L}^{R^{p_j}(\cdot)}/\partial\mu=0$ *for all* $j=1,...,n$*. Under perfect competition,* $\eta_i=0$*, for all* $i=1,...,n$*.*

The Partial Differential Equation (41) is a divergence equation that must hold for any income $\mathbf{y}$. Equations (42) are boundary conditions that must hold at any income $\mathbf{y}\in\Theta_Y$ in the boundary of Θ_Y. Proposition 3 extends to a context with GE effects and market failures the optimal tax formulas of Mirrlees (1976), Golosov et al. (2014), Spiritus et al. (2024), Boerma et al. (2022) and Golosov and Krasikov (2024). The aforementioned tax formulas describe the optimal tax system which is unconstrained on its form, across a large spectrum of economic environments (e.g., with any type of market failure or under perfect competition, with a production factors which are imperfect substitutes or not). Since the system is optimized and not restricted at all on its form, we have $\partial\mathscr{L}^{R^{p_j}}/\partial\mu=0$ for all j. The tax system is optimized along the price-replicating directions defined in (17). Hence, according to (34), GE multipliers are given by (35) to correct for market failures, if any. Under perfect competition, GE multipliers are nil.

We develop a test to determine whether a given tax schedule is Pareto efficient. If it is not, the test identifies potential tax reforms that could achieve a Pareto improvement. To do so, based on (41), one needs to calculate revealed marginal welfare weights, as detailed in Appendix A.11. The literature on the inverse tax problem solves for these weights for which an observed tax system satisfies the first-order conditions of an optimal tax problem, with a single source of income, see, for instance, Bourguignon and Spadaro (2012), Bargain et al. (2014), Lorenz and Sachs (2016), Jacobs et al. (2017), Hendren (2020), Bierbrauer et al. (2023).

In GE, the following lemma highlights that, with multiple incomes and under perfect competition, revealed marginal welfare weights incorporate only PE components, which inherently make their calculation straightforward.

Lemma 5. *In GE, with multiple incomes, revealed marginal social welfare weights depend solely on statistics evaluated in PE. These weights are expressed as:*

$$\widetilde{g}(\mathbf{y}) \overset{def}{\equiv} 1 - \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{y}) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \rho} + \frac{1}{h(\mathbf{y})} \sum_{j=1}^{n} \frac{\partial \left[\sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{y}) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \tau_j} h(\mathbf{y}) \right]}{\partial y_j}, \tag{43}$$

where $h(\cdot)$ denotes the joint income density.

Lemma 5 comes as a surprise if one starts out with the basic intuition that, the revealed marginal welfare weights should reflect GE effects. However, the revealed welfare weights are computed from the optimal tax formula, assuming that the tax system has no restriction on its form and that competition is perfect. Therefore, from Equation (34), the GE multipliers are zero, causing the revealed marginal social welfare weights in GE to coincide with the ones in PE. As a result, the endogeneity of input prices can be ignored when evaluating the revealed welfare weights. They can be inferred from data using (43) and usual estimations of compensated responses $\partial \widehat{Y}_i(\mathbf{y})/\partial \tau_j$, of income responses $\partial \widehat{Y}_i(\mathbf{y})/\partial \rho$ and income density $h(\mathbf{y})$.

In PE, Lorenz and Sachs (2016), Hendren (2020), Bierbrauer et al. (2023) show that negative revealed welfare weights indicate a Pareto inefficiency in the observed tax system, when taxpayers earn a single income ($n = 1$). Bierbrauer et al. (2023) show that negative revealed welfare weights at some income levels are equivalent to their "revenue function" being increasing around that level, in which case a two-brackets tax reform is Pareto-improving.[13] Spiritus et al. (2024, Proposition 2) and Bergstrom and Dodds (2024) extend this result to multiple incomes. With complex and fully flexible tax systems, and following Spiritus et al. (2024, Proposition 2), we can then state that

Lemma 6. *In PE, a tax reform is Pareto-improving if*

- *tax liabilities are (weakly) decreased for income bundles $\mathbf{y}$ where the revealed marginal social welfare weights are negative, $\hat{g}(\mathbf{y}) < 0$,*
- *taxes liabilities are unchanged for income bundles $\mathbf{y}$ where the revealed marginal social welfare weights are non-negative, $\hat{g}(\mathbf{y}) \geq 0$,*
- *the additional tax revenue generated is used to fund a lump-sum transfer.*

In Appendix A.12, we show that combining Lemmas 4 and 6 yields the following proposition.

Theorem 5 (Pareto improving tax reforms)**.** *Under Assumption 1 and perfect competition, if a tax reform in direction $R(\cdot)$ is Pareto-improving in PE (i.e., from Lemma 6, if $\widetilde{g}(\mathbf{y}) < 0$ for some income bundles*

[13] Bierbrauer et al. (2023)'s approach no longer works with multiple incomes because one cannot adjust the vector of marginal tax rates at one point without having to change it elsewhere.

$\mathbf{y}$ then $R(\mathbf{y}) \geq 0$ and if $\widetilde{g}(\mathbf{y}) \leq 0$ then $R(\mathbf{y}) = 0$), implementing, for every income bundle $\mathbf{y}$ such that $\widetilde{g}(\mathbf{y}) < 0$, a small GE-neutralizing tax reform in the direction:

$$R^N(\cdot) = R(\cdot) - \sum_{j=1}^{n} \frac{\partial \log p_j^{R^N}}{\partial \mu} R^{p_j}(\cdot), \tag{44a}$$

where the scaling factors $\gamma_j^{R(\cdot)}$ are:

$$\forall j \in \{1, ..., n\} : \qquad \gamma_j^{R(\cdot)} \stackrel{def}{\equiv} \sum_{i=1}^{n} \frac{\Xi_{j,i}}{\mathcal{Y}_i} \frac{\partial X_i^{R(\cdot),PE}(\boldsymbol{\theta}; \mu, \mathbf{p})}{\partial \mu} = p_j^{R^N(\cdot)} \mu, \tag{44b}$$

is Pareto-improving in GE.

This theorem provides a Pareto-efficiency test and Pareto-improving tax reforms. From Lemma 5, we know the simplicity of empirically implementing $\widetilde{g}(\mathbf{y})$, making it straightforward to verify the sufficiency condition $\widetilde{g}(\mathbf{y}) < 0$. In particular, according to Lemma 4 (with $\boldsymbol{\alpha}(\mu) = \mathbf{0}$), a tax reform in direction $R^N(\cdot)$ defined by Equation (44a) exerts, in GE, the same impact on taxpayers' factor supplies and utilities than the Pareto-improving tax reform $R(\mathbf{y})$ in PE. Since the latter is Pareto-improving, the former achieves Pareto improvement only if the change in price does not reduce tax revenue.[14]

There is a closed analogy between Theorems 1 and 5. Both theorems emphasize how a production policy reform (Theorem 1) or tax reform (Theorem 5) that is Pareto efficient when taxpayers' responses to GE price adjustment are ignored, remains Pareto efficient in GE, provided it is combined with the neutralizing tax reform described in Lemma 4. Importantly, both theorems require the tax system to be sufficiently flexible, as specified in Assumption 1, to ensure the implementability of the neutralizing tax reform.

The following proposition, proofed in Appendix A.13, establishes that positive welfare weights are both necessary and sufficient for the non-existence of a Pareto-improving direction in GE.

Proposition 4. *Under perfect competition, if $\widetilde{g}(\mathbf{y}) \geq 0$ almost everywhere for income bundles $\mathbf{y}$ within the interior of the income bundle space, then there is no Pareto-improving direction neither at the PE, nor at the GE.*

It is noteworthy that, as in Bierbrauer et al. (2023) with a single income, Proposition 4 does not exclude the existence of a Pareto-improving reform which would be non-infinitesimal, i.e. a Pareto improvement resulting from a large magnitude μ.

[14] In Appendix A.12, we show that both reforms have the same budgetary effects. Intuitively, rewriting tax liabilities as $T(\mathbf{Y}(\boldsymbol{\theta})) = \sum_{j=1}^{n} p_j \, X_j(\boldsymbol{\theta}) - C(\boldsymbol{\theta})$, the difference between the effects on tax revenue of a reform in the direction $R^N(\cdot)$ at the GE and a reform in the direction $R(\cdot)$ at the PE is therefore equal to $\sum_{j=1}^{n} \mathcal{X}_j \, \mathrm{d}p_j = \sum_{j=1}^{n} \mathcal{Y}_j \, \mathrm{d}p_j / p_j$. This difference is zero under perfect competition.

VI Optimal Income Tax Systems and GE-multipliers

In this section, we extend the analysis to a Mirrleesian tax model of income taxation extended with several incomes. Proposition 3 characterizes the optimal tax function when the tax system is exhaustive and there is no restriction on its form. In the presence of numerous income types and sources of income, the lack of restrictions on the form of the tax system results in an optimal tax formula expressed as a partial differential equation. However, in practice, policymakers and institutions may imposed constraints on the degree of complexity of the tax system. We argue that realistic tax codes combine many functions (schedules), each of them depending on a single argument (tax base). The imposition of such a realistic restriction on the tax system takes our exploration a step further, revealing that with numerous types and income sources, the optimal tax system must now conform to a system of ordinary differential equations, adopting the ABC form introduced by Diamond (1998) and Saez (2001). This transformation not only enhances the mathematical tractability of the optimal tax model but, critically, introduces a more realistic framework leading to intuitive optimal tax formulas.

VI.1 Schedular tax systems

In this subsection, we investigate the case where the tax system is schedular, i.e. is the sum of n income-specific functions $T_i(\cdot)$, so that:

$$\mathcal{T}(y_1, ..., y_n) = \sum_{i=1}^{n} T_i(y_i).$$

We introduce the possibility that for some incomes, say those for $i > n'$, with $1 \leq n' \leq n$, the corresponding tax schedule is linear i.e. $T_i(y_i) = t_i\ y_i$ where t_i is a real number. Let then denote $h_i(\cdot)$ the density of the i^{th} income and $H_i(\cdot)$ the corresponding CDF. For any variable $Z(\boldsymbol{\theta})$ and for any $i = 0, ..., n$, we denote $\overline{Z(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=y_i}$ the mean of $Z(\boldsymbol{\theta})$ among types $\boldsymbol{\theta}$ for which $Y_i(\boldsymbol{\theta}) = y_i$. The notation $\varepsilon_i(y_i)$ refers to the compensated elasticity of the i^{th} income with respect to its own marginal net-of-tax rate. The corresponding uncompensated elasticity is denoted $\varepsilon_i^u(y_i)$. These means of elasticities are calculated among $\boldsymbol{\theta}$-taxpayers who earn their i^{th} income equal to y_i:

$$\varepsilon_i(y_i) \stackrel{\text{def}}{\equiv} \frac{1 - T_i'(y_i)}{y_i} \left.\overline{\frac{\partial Y_i}{\partial \tau_i}}\right|_{Y_i(\boldsymbol{\theta})=y_i} \qquad \text{and :} \qquad \varepsilon_i^u(y_i) \stackrel{\text{def}}{\equiv} \frac{1 - T_i'(y_i)}{y_i} \left.\overline{\frac{\partial Y_i^u}{\partial \tau_i}}\right|_{Y_i(\boldsymbol{\theta})=y_i}. \tag{45}$$

We thus get the following proposition, which is demonstrated in Appendix A.14.

Proposition 5. *When the tax system is schedular, the GE multipliers $\eta_1, ..., \eta_n$ are given by*

$$\forall i \in \{1, ..., n\} \qquad \eta_i = \frac{\mathcal{F}_{\mathcal{X}_i} - p_i}{p_i}. \tag{46a}$$

at the optimum, which has also to verify:

a) When the i^{th} schedule is nonlinear, i.e. for $i = 1, ..., n'$:

$$\frac{T_i'(y_i) + \eta_i}{1 - T_i'(y_i)} \, \varepsilon_i(y_i) \, y_i \, h_i(y_i) + \sum_{1 \leq k \leq n, k \neq i} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k\right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_i}}\Big|_{Y_i(\boldsymbol{\theta}) = y_i} h_i(y_i)$$
$$= \int_{z=y_i}^{\infty} \left\{ 1 - \overline{g(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=z} - \sum_{k=1}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k\right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_i(\boldsymbol{\theta})=z} \right\} \mathrm{d}H_i(z). \tag{46b}$$

b) When the i^{th} schedule is linear, i.e. for $i = n' + 1, ..., n$:

$$\frac{t_i + \eta_i}{1 - t_i} \int_{\mathcal{W}} \varepsilon_i^u(\boldsymbol{\theta}) \, Y_i(\boldsymbol{\theta}) \, \mathrm{d}F(\boldsymbol{\theta}) + \int_{\mathcal{W}} \sum_{k=1, k \neq i}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k\right) \frac{\partial Y_k^u(\boldsymbol{\theta})}{\partial \tau_i} \, \mathrm{d}F(\boldsymbol{\theta})$$
$$= \int_{\mathcal{W}} [1 - g(\boldsymbol{\theta})] \, Y_i(\boldsymbol{\theta}) \, \mathrm{d}F(\boldsymbol{\theta}). \tag{46c}$$

In the GE multipliers, the corrective term addressing the suboptimality of the tax system disappears in (34), allowing (46a) to be obtained directly. This arises from the fact that according to (25) in Appendix A.14, the price-replicating directions $R^{p_j}(\cdot)$ are part of a schedular tax system. Consequently, an optimal schedular tax system optimizes along all price-replicating directions, i.e. $\partial \mathscr{L}^{R^{p_j}} / \partial t = 0$.

To grasp the economic intuitions behind (46b), consider a small increase in the i^{th} marginal tax rate around income y_i and a uniform increase in tax liabilities for all taxpayers with their i^{th} income above y_i. Given the other tax schedules, the tax schedule specific to the i^{th} income is optimal only if these reforms do not imply any first-order effects on the Lagrangian. In Equation (46b), the costs and gains resulting from these reforms– which are detailed below– are equated.

As reflected in the first term on the left-hand side of (46b), an increase in the i^{th} marginal tax rate around y_i implies direct compensated responses, $\partial Y_i(\boldsymbol{\theta}) / \partial \tau_i$, of the i^{th} income which is proportional to the mean compensated elasticity ε_i of the i^{th} income with respect to its own marginal net-of-tax rate (as emphasized in Equation (45)). A first difference with the one income ABC tax formula is that all behavioral responses have to be averaged across taxpayers who earn the same i^{th} income y_i. Composition effects then take place (Jacquet and Lehmann, 2021). A second difference arises due to the GE price adjustments. Under imperfect competition, the optimal tax formulas include a corrective term which corresponds to the GE multipliers $\eta_1, ..., \eta_n$ given by (46a). Under perfect competition, these corrective terms are nil, as in Saez (2001). A third difference occurs because a rise in the i^{th} marginal tax rate triggers (compensated) cross-base responses of all other tax bases $\partial Y_k(\boldsymbol{\theta}) / \partial \tau_i$ for $k \in \{1, ..., n\} \setminus \{i\}$ (see the second term on the left-hand side of (46b)). For example, taxpayers can report some of their i^{th} income as k^{th} income, with $k \neq i$, when the i^{th} marginal tax rate rises (i.e. the i^{th} marginal net of tax rate τ_i declines), a phenomenon known as income shifting. The compensated increase in the k^{th} income due to income-shifting, i.e. $\partial Y_k(\boldsymbol{\theta}) / \partial \tau_i < 0$, can partly offset the loss due to the compensated responses of the i^{th} income. Conversely, positive cross-base responses ($\partial Y_k / \partial \tau_i > 0$), as in the two-period framework of Lefebvre et al. (2024), can exacerbate the loss due to compensated responses of the i^{th} income.

As usual, on the right-hand side of (46b), a rise in the tax liability above income y_i implies mechanical gains in terms of tax revenue and mechanical welfare losses that are emphasized by the aggregation of $1 - \overline{g(\boldsymbol{\theta})|}_{Y_i(\boldsymbol{\theta})=z}$ for all $z \geq y_i$. It also creates income effects $\partial y_i(\boldsymbol{\theta})/\partial \rho$ on the right-hand side of (46b). Again, compared to the one income optimal income tax formula, welfare weights and incomes responses have first to be aggregated for all income earners with income above y. Second, if competition is imperfect, income responses may be attenuated or exacerbated by GE price adjustments. Third, income response matters for all income sources y_k for $k = 1, ..., n$.

From (46c), we see that, when the tax schedule on the i^{th} income is restricted to be linear, with no restriction on the other tax schedules, similar intuitions than under nonlinear tax schedule apply. There are however several particularities. First, under a linear tax schedule, income effects and compensated effects can be combined and are equivalent to uncompensated responses, as can been verified using the Slutsky Equation (A.6c) in Appendix A.3. Replacing the sum of income and compensated effects by the uncompensated ones implies fewer terms in the right-hand side of (46c) compared to (46b). Second, in the optimal linear tax formula (46c), integrals emphasize that means of sufficient statistics over the whole population need to be estimated instead of means of sufficient statistics at each income level. Third, as expected from the optimal linear tax formula (see e.g. Piketty and Saez (2013)), the mean of welfare weights and uncompensated elasticities are income-weighted. Conversely, the mean of uncompensated cross-base responses $\partial Y_k^u(\boldsymbol{\theta})/\partial \tau_i$ for $k \neq i$ are not income-weighted since they are expressed in derivatives rather than elasticities.

Finally, we provide an order of magnitude of how important GE effects are from a back-to-the envelope calculation. For this exercise, assume there are no cross-base or income responses and fix the right-hand sides of (46b)-(46c). For simplicity, assume there is neither cross-base response nor income responses and fix the right-hand sides of (46b)-(46c). Let $T_i'^{PE}$ denote the optimal marginal tax rate from the right-hand sides of (46b)-(46c), when the GE multipliers are erroneously ignored. The optimal marginal tax rates that take into account GE price adjustments are related to $T_i'^{PE}$ and to the GE multipliers by:[15]

$$T_i' = T_i'^{PE} - \eta_i \left(1 - T_i'^{PE}\right)$$

For example, if $T_i'^{PE} = 0$, the optimal marginal tax rate is equal to minus the GE multipliers. In the absence of a redistributive motive, the marginal tax rate deviates from zero only to correct for market inefficiencies in a Pigou (1920) way. Marginal tax rates then vary one to one with the value of the GE multiplier. However, if the redistributive motive is high enough (which implies larger $T_i'^{PE}$), the effect

[15] Put differently T_i', $T_i'^{PE}$ and η_i are related by:

$$\frac{T_i' + \eta_i}{1 - T_i'} = \frac{T_i'^{PE}}{1 - T_i'^{PE}}$$

where these ratios are equal to the right-hand side of (46b) or (46c).

of the GE multiplier on the optimal marginal tax rate is of a smaller order of magnitude. To illustrate this point, Table 2 shows that the higher the marginal tax rate at the PE (i.e. the higher the redistributive motive) in the first column, the lower the effect of GE multiplier (in the top row) on optimal tax rates.

		η_i				
		-0.10	-0.05	0	0.05	0.10
$T_i^{\prime,PE}$	20%	28%	24%	20%	16%	12%
	40%	46%	43%	40%	37%	34%
	60%	64%	62%	60%	58%	56%
	80%	82%	81%	80%	79%	78%

Table 2: How much GE multipliers matter?

VI.2 Comprehensive tax systems

Building upon Haig (1921) and Simons (1938), we now turn our attention to comprehensive tax schedules, wherein the tax function depends on the sum of all incomes, so-called comprehensive tax base. Formally, the tax schedule takes the form

$$\mathcal{T}(\mathbf{y}) = T_0\left(y_1 + ... + y_n\right)$$

where $y_0 \stackrel{\text{def}}{\equiv} y_1 + ... + y_n$ and $Y_0(\boldsymbol{\theta}) = Y_1(\boldsymbol{\theta}) + ... + Y_n(\boldsymbol{\theta})$. We denote $h_0(\cdot)$ the density of tax base and $H_0(\cdot)$ the associated CDF. Since marginal tax rate on all incomes is equal to $T_0'(y_1 + ..., y_n)$, the compensated responses with respect to the marginal net of tax rate is given by:

$$\forall i \in \{0, ..., n\} \qquad \frac{\partial Y_i}{\partial \tau_0} = \sum_{j=1}^{n} \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j}, \tag{47}$$

the compensated elasticity of the comprehensive tax base is:

$$\varepsilon_0(y_0) = \frac{1 - T_0'(y_0)}{y_0} \sum_{1 \leq i,j \leq n} \overline{\frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j}\bigg|_{Y_0(\boldsymbol{\theta})=y_0}} \tag{48}$$

which is positive,[16] and the income response of the comprehensive tax base are given by:

$$\frac{\partial Y_0(y_0)}{\partial \rho} = \sum_{k=1}^{n} \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}\bigg|_{Y_0(\boldsymbol{\theta})=y_0}} \tag{49}$$

This elasticity depends on every compensated responses $\partial Y_i(\boldsymbol{\theta})/\partial \tau_j$ to changes in every net-of-marginal tax rate τ_j for $i, j \in \{1, ..., n\}$. The following proposition, which is proved in Appendix A.15, characterizes the optimal comprehensive income tax schedule.

[16]Since the matrix $\left[\frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j}\right]_{i,j}$ is positive definite, the comprehensive tax base's compensated elasticity is positive.

Proposition 6. *When the tax system is comprehensive, the GE multipliers $\eta_1, ..., \eta_n$ are given by* (34) *at the optimum which has also to verify:*

$$\frac{T_0'(y_0)}{1-T_0'(y_0)}\,\varepsilon_0(y_0)\,y_0\,h_0(y_0)+\sum_{1\le k\le n}\eta_k\,\overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_0}}\Big|_{Y_0(\boldsymbol{\theta})=y_i}h_0(y_0) \tag{50}$$
$$=\int_{z=y_0}^{\infty}\left\{1-\overline{g(\boldsymbol{\theta})}|_{Y_0(\boldsymbol{\theta})=z}-T_0'(z)\,\frac{\partial Y_0(z)}{\partial\rho}-\sum_{k=1}^{n}\eta_k\,\overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial\rho}}\Big|_{Y_0(\boldsymbol{\theta})=z}\right\}\mathrm{d}H_0(z).$$

This optimal income tax formula differs from the usual ABC formula since there are several types of income and more importantly, by the presence of GE multipliers η_k. As shown in Appendix A.15, under a comprehensive tax system, the price-replicating directions of tax reforms (given by (24) in the appendix) do not belong to the set of comprehensive tax schedules. Hence the optimal comprehensive tax function does not optimize along all price-replicating directions. This occurs since only the sum of all incomes y_0 determines tax liabilities. Hence the optimal tax system has to solve (45) for all $k = 1, ..., n$ together with (50) for all income levels.

To better understand how GE price adjustments affect the optimal comprehensive tax schedule, we consider a simple economy with two production factors $n = 2$ and perfect competition. In this case, as shown in Appendix A.15, the price-replicating directions simplify to $R^{p_1}(y_1, y_2) = (1-T_0'(y_1+y_2))\,y_1$ and $R^{p_2}(y_1, y_2) = (1-T_0'(y_1+y_2))\,y_2$. The optimal comprehensive tax system optimizes along all comprehensive tax directions, including $(1-T_0'(y_1+y_2))\,(y_1+y_2) = R^{p_1}(y_1, y_2)+R^{p_2}(y_1, y_2)$, but does (generically) not optimize along R^{p_1} or R^{p_2} separately. Optimizing along $(1-T_0'(y_1+y_2))\,(y_1+y_2) = R^{p_1}(y_1, y_2) + R^{p_2}(y_1, y_2)$ leads to $\partial\mathscr{L}^{R^{p_1}}/\partial\mu + \partial\mathscr{L}^{R^{p_2}}/\partial\mu = 0$ by Gateaux differentiability of the Lagrangian with respect to the tax reforms. Denoting σ the elasticity of substitution between the two production factors, one obtains:

$$\eta_1 = -\frac{1}{\sigma\,\mathcal{Y}_1}\frac{\partial\mathscr{L}^{R^{p_1}}}{\partial\mu},\qquad \eta_2 = -\frac{1}{\sigma\,\mathcal{Y}_2}\frac{\partial\mathscr{L}^{R^{p_2}}}{\partial\mu}. \tag{51}$$

Therefore, these two GE multipliers have opposite signs. Let $\mathscr{Y}_k(y_0)$ denote the mean k^{th} income earned by taxpayers with comprehensive tax base y_0. Define:

$$\varepsilon_k^0(y_0)\stackrel{\text{def}}{\equiv}\frac{1-T_0'(y_0)}{\mathscr{Y}_k(y_0)}\,\overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial\tau_0}}\Big|_{Y_0(\boldsymbol{\theta})=y_0}$$

as the elasticity of the mean of the k^{th} income, with respect to the net-of-marginal tax rate τ_0 of the y_0 tax base, among taxpayers earning y_0. Fixing the right-hand side of (50), Equation (51) indicates that the GE price adjustments affect the optimal marginal tax rate at y_0 in proportion to:

$$\sum_{1\le k\le n}\eta_k\,\overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial\tau_0}}\Big|_{Y_0(\boldsymbol{\theta})=y_i}=\frac{1}{\sigma\,(1-T_0'(y_0))}\,\frac{\partial\mathscr{L}^{R^{p_1}}}{\partial\mu}\left[\frac{\mathscr{Y}_2(y_0)}{\mathcal{Y}_2}\,\varepsilon_2^0(y_0)-\frac{\mathscr{Y}_1(y_0)}{\mathcal{Y}_1}\,\varepsilon_1^0(y_0)\right].$$

The impact of GE adjustments on the optimal marginal tax rates at taxable income y_0 relies on the sign of $\partial\mathscr{L}^{R^{p_1}}/\partial\mu$ which is the same across the taxable income distribution. Conversely, the term in square brackets may vary with taxable income. This term compares the two elasticities with respect to the net-of-marginal tax rate scaled by the ratios of average k^{th} income at taxable income y_0 over aggregate k^{th} income $\mathcal{Y}_k$. In particular, if the two elasticities are identical, as is the case for instance in Rothschild and Scheuer (2013) or in Sachs et al. (2020), then the impact of GE price adjustments on the optimal marginal tax rates may be positive at low taxable income levels y_0 and negative at high taxable income levels, as in Figure 2 of Rothschild and Scheuer (2013) and Figure 4 of Sachs et al. (2020). In our framework, this outcome occurs when $\partial \mathscr{L}^{R^{p_1}}/\partial\mu > 0$ and if taxpayers with low (high) taxable income y_0 earn relatively more (less) income 2 and relatively less (more) income 1 than in the overall population.

We derive optimal tax formulas for two polar cases: a schedular tax schedule (Proposition 5) and a comprehensive tax schedule (Proposition 6). In practice tax systems can also fall between these two cases. For instance, a partial observability scenario may prevail where the tax authority observes only the sum of $m < n$ incomes $\overline{y} = y_1 + ... + y_m$, while separately observing the remaining income sources $y_{m+1}, ..., y_n$. This situation can occur when the first m income sources correspond to different types of labor (e.g., routine, manual, conceptual), while the remaining ones represent returns from various forms of investment. Other intermediate cases can also be considered. In all such situations, optimal tax formulas can be derived by combining the key determinants of the optimal schedular system (Proposition 5) with those of the optimal comprehensive system (Proposition 6).

VII Conclusion

A key takeaway from this paper is that the tax system deserves particular attention when assessing the impact of changes in production policy. In multidimensional settings with market failures, we identify the conditions under which production policies that increase aggregate output can be Pareto-improving despite their negative distributional effects. When the tax system can be adjusted through GE-neutralizing tax reforms, government intervention in the production sector is unnecessary.

Moreover, we provide formulas that quantify the impact of any tax and/or production policy reform using standard empirical statistics, along with a key empirical measure for GE effects: the GE multipliers. These GE multipliers highlight the importance of empirical research estimating mark-up, elasticities of subsitution between inputs and the impact of regulation on prices. We then leverage the same statistics to characterize optimal multidimensional, schedular, and comprehensive nonlinear tax systems.

Another key insight of this paper is the practical identification of Pareto-improving tax reforms and Pareto-efficient tax systems, showing that this identification relies on the same condition than in PE.

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Online Appendix

A Proofs

A.1 Convexity of the Indifference Set

Let $\mathscr{C}(\cdot, \mathbf{x}; \boldsymbol{\theta})$ denote the reciprocal of $\mathscr{U}(\cdot, \mathbf{x}; \boldsymbol{\theta})$. Taxpayers of type $\boldsymbol{\theta}$ who supply inputs $\mathbf{x}$ obtain consumption $c = \mathscr{C}(u, \mathbf{x}; \boldsymbol{\theta})$ to enjoy utility $u = \mathscr{U}(c, \mathbf{x}; \boldsymbol{\theta})$. Using (1), we obtain:

$$\mathscr{C}_u(u, \mathbf{x}; \boldsymbol{\theta}) = \frac{1}{\mathscr{U}_c(\mathscr{C}(u, \mathbf{x}; \boldsymbol{\theta}), \mathbf{x}; \boldsymbol{\theta})} \qquad \mathscr{C}_{x_i}(u, \mathbf{x}; \boldsymbol{\theta}) = \mathcal{S}^i(\mathscr{C}(u, \mathbf{x}; \boldsymbol{\theta}), \mathbf{x}; \boldsymbol{\theta}) \tag{A.1}$$

For each type $\boldsymbol{\theta} \in W$ and each utility level u, we assume that the indifference set $\mathbf{x} \mapsto \mathscr{C}(u, x_1, ..., x_n)$ is strictly convex. The i^{th} partial derivative of $\mathbf{x} \mapsto \mathscr{C}(u, x_1, ..., x_n; \boldsymbol{\theta})$ being $\mathcal{S}^i(\mathscr{C}(u, x_1, .., x_n; \boldsymbol{\theta}), , x_1, ..., x_n; \boldsymbol{\theta})$, the Hessian is matrix:

$$\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j\right]_{i,j} = \left[-\frac{\mathscr{U}_{x_i x_j} + \mathcal{S}^j \mathscr{U}_{c x_i} + \mathcal{S}^i \mathscr{U}_{c x_j} + \mathcal{S}^i \mathcal{S}^j \mathscr{U}_{cc}}{\mathscr{U}_c}\right]_{i,j}$$

Therefore, Matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j\right]_{i,j}$ is positive definite if the indifference curves are strictly convex.

The first-order condition associated to (2) are given by:

$$\begin{aligned} 0 \quad = \quad & p_i \left(1 - \mathcal{T}_{y_i}(\mathbf{y})\right) \mathscr{U}_c \left(\sum_{i=1}^n p_k\, x_k - \mathcal{T}\left(p_1\, x_1, ..., p_n\, x_n\right), \mathbf{x}; \boldsymbol{\theta}\right) \\ + \quad & \mathscr{U}_{x_i} \left(\sum_{i=1}^n p_k\, x_k - \mathcal{T}\left(p_1\, x_1, ..., p_n\, x_n\right), \mathbf{x}; \boldsymbol{\theta}\right). \end{aligned}$$

Therefore, using (3), the matrix of the second-order condition is:

$$\left[\mathscr{U}_{x_i x_j} + \mathcal{S}^j \mathscr{U}_{c x_i} + \mathcal{S}^i \mathscr{U}_{c x_j} + \mathcal{S}^i \mathcal{S}^j \mathscr{U}_{cc} - p_i\, p_j\, \mathscr{U}_c \mathcal{T}_{y_i y_j}\right]_{i,j} = -\mathscr{U}_c \left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i\, p_j\, \mathcal{T}_{y_i y_j}\right]_{i,j}.$$

Hence, for taxpayers of type $\boldsymbol{\theta}$, the second-order condition holds strictly if and only if the matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i\, p_j\, \mathcal{T}_{y_i y_j}\right]_{i,j}$ is positive definite, i.e. if and only if the indifference set $\mathbf{x} \mapsto \mathscr{C}(U(\boldsymbol{\theta}), \mathbf{x}; \boldsymbol{\theta})$ is strictly more convex than the budget set $\mathbf{x} \mapsto \sum_{k=1}^n p_k\, x_k - \mathcal{T}(p_1\, x_1, ..., p_n\, x_n)$ at $\mathbf{x} = \mathbf{X}(\boldsymbol{\theta})$.

A.2 Assumptions for the Implicit Function Theorem

To apply the implicit function theorem to the first-order condition associated to the individual maximization program, we assume that:

(i) The initial tax schedule $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ is twice continuously differentiable.

(ii) The second-order condition associated to the individual maximization program (13) holds strictly, i.e. the matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i\, p_j\, \mathcal{T}_{y_i y_j}\right]_{i,j}$ is positive definite at $c = C(\boldsymbol{\theta})$, $\mathbf{x} = \mathbf{X}(\boldsymbol{\theta})$ and at $\mathbf{y} = \mathbf{Y}(\boldsymbol{\theta})$, for each type $\boldsymbol{\theta} \in W$,

(iii) for each type $\boldsymbol{\theta} \in W$, program (13) admits a unique global maximum.

Part (i) ensures that first-order conditions (3) are continuously differentiable in incomes $\mathbf{y}$. It rules out kinks in the tax function, thereby bunching.[17] Parts (i) and (ii) together enable one to apply the implicit function theorem to first-order conditions (3) to ensure that each local maximum of

$$\mathbf{x} \mapsto \mathscr{U}\left(\sum_{k=1}^{n} p_k\, x_k - \mathcal{T}(p_1\, x_1, ..., p_n\, x_n) + \mu\, R(p_1\, x_1, ..., p_n\, x_n), x_1, ..., x_n; \boldsymbol{\theta}\right)$$

is differentiable in type $\boldsymbol{\theta}$, in price $\mathbf{p}$ and in the tax perturbation's magnitude μ of tax reforms. Part (iii) rules out the existence of multiple global maxima. This prevents any incremental tax reform from causing a jump in the taxpayer's choice from one maximum to another. Part (iii) also ensures the allocation changes in a differentiable way with the magnitude of the tax reform and with types.

A.3 Proof of Equations (16a), (16b) and (16c)

The first-order conditions associated to (13) are $\forall i \in \{1, ..., n\}$:

$$\mathcal{S}^i\left(\sum_{i=1}^{n} p_i\, x_i - \mathcal{T}(p_1\, x_1, ..., p_n\, x_n) + \mu\, R(\mathbf{y}), \mathbf{x}; \boldsymbol{\theta}\right) \\ = p_i\left(1 - \mathcal{T}_{y_i}(p_1\, x_1, ..., p_n\, x_n) + \mu\, R_{y_i}(\mathbf{y})\right) \tag{A.2}$$

Differentiating these first-order conditions at $\mu = 0$ and using (A.2) leads to $\forall i \in \{1, ..., n\}$:

$$\sum_{k=1}^{n}\left[\mathcal{S}^i_{x_k} + \mathcal{S}^i_c\, \mathcal{S}^k + p_i\, p_k\, \mathcal{T}_{y_i y_k}\right] \mathrm{d}x_k = \left[p_i\, R_{y_i} - \mathcal{S}^i_c\, R\right] \mathrm{d}\mu \\ + \sum_{j=1}^{n}\left[\mathbb{1}_{i=j}(1 - \mathcal{T}_{y_j}) - p_i\, x_j\, \mathcal{T}_{y_i y_j} - \mathcal{S}^i_c\,(1 - \mathcal{T}_{y_j})\, x_j\right]\, \mathrm{d}p_j. \tag{A.3}$$

Equation (A.3) can be rewritten in matrix form as:

$$\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i p_j \mathcal{T}_{y_i y_j}\right]_{i,j} \cdot \mathrm{d}\mathbf{x}^T = \left[p_i\, R_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) - \mathcal{S}^i_c\, R(\mathbf{Y}(\boldsymbol{\theta}))\right]^T_i \mathrm{d}\mu \\ + \left[(1 - \mathcal{T}_{y_j})\left(\mathbb{1}_{i=j} - x_j\, \mathcal{S}^i_c\right) - p_i\, x_j\, \mathcal{T}_{y_i y_j}\right]_{i,j} \cdot \mathrm{d}\mathbf{p}^T. \tag{A.4}$$

where superscript T denotes the transpose operator$[A_{i,j}]^T_{i,j} = [A_{j,i}]_{i,j}$ and "$\cdot$" denotes the matrix product. Matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i p_j \mathcal{T}_{y_i y_j}\right]_{i,j}$ is the Hessian matrix associated to the maximization program (13). It is therefore symmetric and semi-positive definite. Since the second-order condition associated to the individual maximization program is assumed to hold strictly, matrix $\left[\mathcal{S}^i_{x_j} + \mathcal{S}^i_c \mathcal{S}^j + p_i\, p_j\, \mathcal{T}_{y_i y_j}\right]_{i,j}$ is positive definite, and is therefore invertible. Let $H_{i,j}$ denote the term in the i^{th} row and j^{th} column of the inverse of the Hessian matrix. We obtain by inverting (A.4):

$$\mathrm{d}x_i = \sum_{k=1}^{n} H_{i,k}\left[p_k\, R_{y_k}(\mathbf{Y}(\boldsymbol{\theta})) - \mathcal{S}^k_c\, R(\mathbf{Y}(\boldsymbol{\theta}))\right] \mathrm{d}\mu \\ + \sum_{j=1}^{n}\left\{\sum_{k=1}^{n} H_{i,k}\left[(1 - \mathcal{T}_{y_j})\left(p_j\, \mathbb{1}_{k=j} - y_j\, \mathcal{S}^k_c\right) - p_k\, y_j\, \mathcal{T}_{y_k y_j}\right]\right\} \frac{\mathrm{d}p_j}{p_j} \tag{A.5}$$

[17] In reality, most real world tax schedules are piecewise linear. Theoretically, one should observe bunching at convex kinks and gaps at concave kinks. Empirically, most convex kinks do not cause significant bunching, with the exception of the self-employed in the United States at the first kink point of the EITC (Saez, 2010). Moreover, no gap is observed at concave kinks. These discrepancies between theoretical predictions and empirical evidence can be reconciled by assuming that taxpayers do not optimize with respect to the exact tax schedule but with respect to some smooth approximation of it, e.g. $\mathbf{y} \mapsto \int \mathcal{T}(\mathbf{y} + \mathbf{u})\, \mathrm{d}\Psi(\mathbf{u})$ where $\mathbf{u}$ is an n-dimensional random shock on incomes with joint CDF Ψ, which sastifies part i).

Under a compensated tax reform of the j^{th} marginal tax rate at income $\mathbf{y} = \mathbf{Y}(\boldsymbol{\theta})$ where $R(\mathbf{y}) = y_j - Y_j(\boldsymbol{\theta})$, one has $R(\mathbf{Y}(\boldsymbol{\theta})) = 0$ and $R_{y_k}(\mathbf{Y}(\boldsymbol{\theta})) = \mathbb{1}_{k=j}$. Hence, according to (A.5) compensated responses are given by:

$$\frac{\partial X_i(\boldsymbol{\theta})}{\partial \tau_j} = p_j\, H_{i,j}, \qquad \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j} = p_i\, p_j\, H_{i,j} \tag{A.6a}$$

with $\frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j} = \frac{\partial Y_j(\boldsymbol{\theta})}{\partial \tau_i}$ since the Hessian matrix is symmetric.

Under a lump-sum tax reform where $R(\mathbf{y}) = 1$, one has $R(\mathbf{Y}(\boldsymbol{\theta})) = 1$ and $R_{y_k}(\mathbf{Y}(\boldsymbol{\theta})) = 0$. Hence, according to (A.5), income effects are given by:

$$\frac{\partial X_i(\boldsymbol{\theta})}{\partial \rho} = -\sum_{k=1}^{n} H_{i,k}\, \mathcal{S}_c^k, \qquad \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \rho} = -p_i \sum_{k=1}^{n} H_{i,k}\, \mathcal{S}_c^k. \tag{A.6b}$$

Under an uncompensated tax reform of the j^{th} marginal tax rate at income $\mathbf{y} = \mathbf{Y}(\boldsymbol{\theta})$ where $R(\mathbf{y}) = y_j$, one has $R(\mathbf{Y}(\boldsymbol{\theta})) = Y_j(\boldsymbol{\theta})$ and $R_{y_k}(\mathbf{Y}(\boldsymbol{\theta})) = \mathbb{1}_{k=j}$. Hence, according to (A.5) uncompensated responses are given by the Slutsky equations:

$$\frac{\partial X_i^u(\boldsymbol{\theta})}{\partial \tau_j} = \frac{\partial X_i(\boldsymbol{\theta})}{\partial \tau_j} + Y_j(\boldsymbol{\theta})\, \frac{\partial X_i(\boldsymbol{\theta})}{\partial \rho}, \qquad \frac{\partial Y_i^u(\boldsymbol{\theta})}{\partial \tau_j} = \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \tau_j} + Y_j(\boldsymbol{\theta})\, \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \rho} \tag{A.6c}$$

From (A.5), the input supply responses to log-price changes can be written as:

$$\frac{\partial X_i(\boldsymbol{\theta})}{\partial \log p_j} = \sum_{k=1}^{n} H_{i,k} \left[(1 - \mathcal{T}_{y_j}) \left(p_j\, \mathbb{1}_{k=j} - y_j\, \mathcal{S}_c^k \right) - p_k\, y_j\, \mathcal{T}_{y_k y_j} \right] \tag{A.6d}$$

At the GE, the prices $\mu \mapsto (p_1^R(\mu), \ldots, p_n^R(\mu))$ are affected by any tax or production policy reform of magnitude μ. Therefore, substituting $\mathrm{d}p_j = \left(\partial p_j^{R(\cdot),\alpha(\cdot)} / \partial \mu \right) \mathrm{d}\mu$ into (A.5) results in the following taxpayer responses:

$$\begin{aligned}
\frac{\partial X_i^{R(\cdot),\alpha(\cdot)}(\boldsymbol{\theta}, \mu)}{\partial \mu} \quad &= \quad \sum_{k=1}^{n} H_{i,k} \left[p_k\, R_{y_k}(\mathbf{Y}(\boldsymbol{\theta}), 0) - \mathcal{S}_c^k\, R(\mathbf{Y}(\boldsymbol{\theta}), 0) \right] \\
&\quad + \sum_{j=1}^{n} \left\{ \sum_{k=1}^{n} H_{i,k} \left[(1 - \mathcal{T}_{y_j}) \left(p_j\, \mathbb{1}_{k=j} - y_j\, \mathcal{S}_c^k \right) - p_k\, y_j\, \mathcal{T}_{y_k y_j} \right] \right\} \frac{\partial \log p_j^{R(\cdot),\alpha(\cdot)}}{\partial \mu}.
\end{aligned}$$

Using (A.6a), (A.6b) and (A.6d), we obtain:

$$\frac{\partial X_i^{R(\cdot),\alpha(\cdot)}(\boldsymbol{\theta}, \mu)}{\partial \mu} = \sum_{k=1}^{n} \frac{\partial X_i(\boldsymbol{\theta})}{\partial \tau_k}\, R_{y_k}(\mathbf{Y}(\boldsymbol{\theta})) + \frac{\partial X_i(\boldsymbol{\theta})}{\partial \rho}\, R(\mathbf{Y}(\boldsymbol{\theta})) + \sum_{j=1}^{n} \frac{\partial X_i(\boldsymbol{\theta}; \mathbf{p})}{\partial \log p_j} \frac{\partial \log p_j^R}{\partial \mu} \tag{A.7}$$

which, eventually, leads to (16a), (16b) and (16c).

A.4 Proof of Proposition 1

We take $\mathrm{d}\mu = \mathrm{d}p_j / p_j$. According to (17), we get that for any $k \neq j$, $R_{y_k}^{p_j}(\mathbf{Y}(\boldsymbol{\theta})) = -y_j\, \mathcal{T}_{y_k y_j}$, so:

$$\left[p_k\, R_{y_k}^{p_j}(\mathbf{Y}(\boldsymbol{\theta})) - \mathcal{S}_c^k\, R(\mathbf{Y}(\boldsymbol{\theta})) \right] \mathrm{d}\mu = \left[-(1 - \mathcal{T}_{y_j}) x_j\, \mathcal{S}_c^j - p_k\, x_j\, \mathcal{T}_{y_k y_j} \right] \mathrm{d}p_j.$$

Moreover, we get from (17) that $R_{y_j}^{p_j}(\mathbf{Y}(\boldsymbol{\theta})) = 1 - \mathcal{T}_{y_j} - y_j\, \mathcal{T}_{y_j y_j}$, so:

$$\left[p_k\, R_{y_j}^{p_j}(\mathbf{Y}(\boldsymbol{\theta})) - \mathcal{S}_c^j\, R(\mathbf{Y}(\boldsymbol{\theta})) \right] \mathrm{d}\mu = \left[1 - \mathcal{T}_{y_j} - (1 - \mathcal{T}_{y_j}) x_j\, \mathcal{S}_c^j - p_k\, x_j\, \mathcal{T}_{y_k y_j} \right] \mathrm{d}p_j$$

Therefore, Equation (A.5) ensures that

$$\forall i \in \{1,...,n\}, \forall j \in \{1,...,n\}: \qquad \frac{\partial X_i^{R^{pj},PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} = \frac{\partial X_i(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j}.$$

Finally, we directly obtain the equality:

$$\frac{\partial U^{R^{pj},PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} = \frac{\partial U(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j}.$$

by plugging Equation (17) into Equation (15).

A.5 Proof of Lemma 1

Let $\boldsymbol{\partial \log \mathcal{X}}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\boldsymbol{\partial \mu} \stackrel{\text{def}}{\equiv} (\partial \log \mathcal{X}_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial\mu, ..., \partial \log \mathcal{X}_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial\mu)^T$ denote the column vector of input supply's aggregate responses at GE and let $\boldsymbol{\partial \log p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\boldsymbol{\partial \mu} \stackrel{\text{def}}{\equiv} (\partial \log p_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial\mu, ..., \partial \log p_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial\mu)^T$ denote the column vector of input price responses at GE. Summing (16a) over all types and using (20b) leads to the supply responses equation:

$$\frac{\boldsymbol{\partial \log \mathcal{X}}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} = \Gamma \cdot \frac{\boldsymbol{\partial \log p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} + \frac{\boldsymbol{\partial \log \mathcal{X}}^{R(\cdot),PE}}{\boldsymbol{\partial \mu}} \tag{A.8}$$

The demand response equation si therefore given by:

$$\frac{\boldsymbol{\partial \log p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} = \Xi \cdot \frac{\boldsymbol{\partial \log \mathcal{X}}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} + \frac{\boldsymbol{\partial \log \mathscr{P}}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} \tag{A.9}$$

where $\boldsymbol{\partial \log \mathscr{P}}^{\boldsymbol{\alpha}(\cdot)}/\boldsymbol{\partial \mu} \stackrel{\text{def}}{\equiv} (\partial \log \mathscr{P}_1^{\boldsymbol{\alpha}(\cdot)}/\partial\mu, ..., \partial \log \mathscr{P}_n^{\boldsymbol{\alpha}(\cdot)}/\partial\mu)^T$ denote the column vector of log-derivatives of inverse demands with respect to production policy reforms, holding input supplies fixed. Plugging (A.8) into (A.9) leads to:

$$\frac{\boldsymbol{\partial \log p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} = \Xi \cdot \Gamma \cdot \frac{\boldsymbol{\partial \log p}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}} + \Xi \cdot \frac{\boldsymbol{\partial \log \mathcal{X}}^{R(\cdot),PE}}{\boldsymbol{\partial \mu}} + \frac{\boldsymbol{\partial \log \mathscr{P}}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial \mu}},$$

which eventually leads to (22), whenever Matrix $I_n - \Xi \cdot \Gamma$ is invertible.

A.6 Examples of primitives that satisfy Assumption 3

Assumption 3 specifies that the production policy reform $\boldsymbol{\alpha}(\cdot)$ considered does not affect the ratio of prices between inputs $j = 1, ..., m$. This is satisfied when the inverse demand elasticities take a weakly separable form, as in:

$$\begin{aligned}
\forall j \in \{1, ...m\}: \quad & \mathscr{P}_j(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathcal{Q}_j(\mathcal{X}_1, ..., \mathcal{X}_m)\ \mathscr{P}_{\bar{y}}(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha}) \\
\forall j \in \{m+1, ...n\}: \quad & \mathscr{P}_j(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathcal{P}_j(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha}).
\end{aligned} \tag{A.10}$$

In this case, the price ratio p_i/p_j for $1 \leq i, j \leq m$ depends neither on the other inputs $\mathcal{X}_{m+1}, ..., \mathcal{X}_n$ nor on production policies $\boldsymbol{\alpha}$:

$$\forall i, j \in \{1, ..., m\}: \qquad \frac{p_i}{p_j} = \frac{\mathcal{Q}_i(\mathcal{X}_1, ..., \mathcal{X}_m)}{\mathcal{Q}_j(\mathcal{X}_1, ..., \mathcal{X}_m)}.$$

Equations (A.10) are for instance verified in the case of perfect competition and a weakly separable production function of the form:

$$\mathcal{F}(\mathcal{X}_1, ..., \mathcal{X}_n; \boldsymbol{\alpha}) = \mathscr{F}(\mathcal{A}(\mathcal{X}_1, ..., \mathcal{X}_m), \mathcal{X}_{m+1}, ..., \mathcal{X}_n; \boldsymbol{\alpha}).$$

We note that the above restrictions on the demand elasticities are only one relevant possibility for satisfying Assumption 3, as one only needs that the considered production policy reform does not modify the ratio of prices between different inputs $j = 1, ..., m$.

A.7 Proof of Lemmas 2, 3 and 4

We first consider the general case with a tax reform in the direction $R(\cdot)$ combined with a production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$, as considered in Lemma 4. We then adapt the proof of Lemma 4 to the specific cases considered in Lemmas 2 and 3.

According to (16a), (16c) and (16d), combining a production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$ with a tax reform in the direction $R^N(\cdot) \stackrel{\text{def}}{\equiv} R(\cdot) - \sum_{j=1}^n \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} \, R^{p_j}(\cdot)$ defined in (38a), impacts taxpayers' supplies and utilities though, $\forall \boldsymbol{\theta} \in \Theta$:

$$\begin{aligned}
\forall i \in \{1,...,n\}: \quad \frac{\partial X_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} &= \frac{\partial X_i^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} - \sum_{j=1}^n \frac{\partial X_i^{R^{p_j}(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu}\,\gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} \\
&\quad + \sum_{j=1}^n \frac{\partial X_i(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j}\,\frac{\partial \log p_j^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}. \\
\text{and}: \quad \frac{\partial U^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} &= \frac{\partial U^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu} - \sum_{j=1}^n \frac{\partial U^{R^{p_j}(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu}\,\gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} \\
&\quad + \sum_{j=1}^n \frac{\partial U(\boldsymbol{\theta};\mathbf{p})}{\partial \log p_j}\,\frac{\partial \log p_j^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}.
\end{aligned}$$

According to Proposition 1, if:

$$\forall j \in \{1,...,n\}: \qquad \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} = \frac{\partial \log p_j^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} \tag{A.11}$$

we get that:

$$\begin{aligned}
\forall i \in \{1,...,n\}: \quad \frac{\partial X_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} &= \frac{\partial X_i^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu}. \\
\text{and}: \quad \frac{\partial U^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta};\mu)}{\partial \mu} &= \frac{\partial U^{R(\cdot),PE}(\boldsymbol{\theta};\mu,\mathbf{p})}{\partial \mu}.
\end{aligned}$$

i.e. the combination of the production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$ with a tax reform in the direction $R^N(\cdot)$ have the same effects on taxpayers' input supplies and utility levels at the GE as does a tax reform in the direction $R(\cdot)$ at the PE.

Let $\boldsymbol{\gamma}^{R(\cdot),\boldsymbol{\alpha}(\cdot)} = (\gamma_1^{R(\cdot),\boldsymbol{\alpha}(\cdot)}, ..., \gamma_n^{R(\cdot),\boldsymbol{\alpha}(\cdot)})^T$. Using (22) in Lemma 1, Condition (A.11) can be rewritten in matrix form as:

$$\begin{aligned}
(I_n - \Xi\cdot\Gamma)\cdot\boldsymbol{\gamma}^{R(\cdot),\boldsymbol{\alpha}(\cdot)} &= \Xi\cdot\frac{\boldsymbol{\partial}\log\boldsymbol{\mathcal{X}}^{R^N(\cdot),PE}}{\boldsymbol{\partial}\mu} + \frac{\boldsymbol{\partial}\log\mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu} \\
&= \Xi\cdot\frac{\boldsymbol{\partial}\log\boldsymbol{\mathcal{X}}^{R(\cdot),PE}}{\boldsymbol{\partial}\mu}\Xi\cdot\Gamma\cdot\,\boldsymbol{\gamma}^{R(\cdot),\boldsymbol{\alpha}(\cdot)} + \frac{\boldsymbol{\partial}\log\mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu} \\
&= \Xi\cdot\frac{\boldsymbol{\partial}\log\boldsymbol{\mathcal{Y}}^{R(\cdot),PE}}{\boldsymbol{\partial}\mu} - \Xi\cdot\Gamma\cdot\,\boldsymbol{\gamma}^{R(\cdot),\boldsymbol{\alpha}(\cdot)} + \frac{\boldsymbol{\partial}\log\mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu}
\end{aligned}$$

where the second equality holds because of Equation (20b) and Proposition 1 and the third equality holds because $\boldsymbol{\partial}\log\boldsymbol{\mathcal{X}}^{R(\cdot),PE}/\partial\mu = \boldsymbol{\partial}\log\boldsymbol{\mathcal{Y}}^{R(\cdot),PE}/\partial\mu$ according to (23). Rearranging terms leads to:

$$\boldsymbol{\gamma}^{R(\cdot),\boldsymbol{\alpha}(\cdot)} = \Xi\cdot\frac{\boldsymbol{\partial}\log\boldsymbol{\mathcal{Y}}^{R(\cdot),PE}}{\boldsymbol{\partial}\mu} + \frac{\boldsymbol{\partial}\log\mathscr{P}^{\boldsymbol{\alpha}(\cdot)}}{\boldsymbol{\partial}\mu}. \tag{A.12}$$

which corresponds to Equation (38b), thereby ending the Proof of Lemma 4.

In the absence of an initial tax reform (i.e. when $R(\cdot) \equiv 0$), Equation (A.12) leads to (29b), thereby ending the Proof of Lemma 2.

Finally, under Assumption 3, a production policy reform does not change the price ratio between the m first inputs. This implies that $\partial \log(\mathscr{P}_j/\mathscr{P}_k)/\partial\alpha_\ell = 0$ for all $\ell \in \{1,...,L\}$ and all $(j,k) \in \{1,...,m\}$. Since for all $(j,k) \in \{1,...,m\}$, one has $\partial \log(\mathscr{P}_j/\mathscr{P}_k)/\partial\alpha_\ell = \partial \log \mathscr{P}_j/\partial\alpha_\ell - \partial \log \mathscr{P}_k/\partial\alpha_\ell$, Assumption 3 implies:

$$\forall \ell \in \{1,...,L\}: \qquad \frac{\partial \log \mathscr{P}_1}{\partial \alpha_\ell} = ... = \frac{\partial \log \mathscr{P}_m}{\partial \alpha_\ell} \stackrel{\text{def}}{\equiv} \frac{\partial \log \mathscr{P}_{\overline{y}}}{\partial \alpha_\ell}. \tag{A.13}$$

Under Assumption 2, the government cannot reform the tax system in the price replicating directions $R^{p_1}(\cdot), ..., R^{p_m}(\cdot)$, but can reform the tax system in the price replicating directions $R^{p_{m+1}}(\cdot), ..., R^{p_n}(\cdot)$ and $R^{\overline{y}}(\cdot)$. But Equation (A.13) implies that the GE-neutralizing reform $R^N(\cdot)$ given by (29a) and (29b) can be re-expressed solely in terms of available price replicating directions $R^{p_{m+1}}(\cdot), ..., R^{p_n}(\cdot)$ and $R^{\overline{y}(\cdot)}$ since:

$$\begin{aligned} R^N(\cdot) &= -\sum_{j=1}^n \left(\sum_{\ell=1}^L \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha'_\ell(0) \right) R^{p_j}(\cdot) \\ &= -\left(\sum_{\ell=1}^L \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha'_\ell(0) \right) \sum_{j=1}^m R^{p_j}(\cdot) - \sum_{j=m+1}^n \left(\sum_{\ell=1}^L \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha'_\ell(0) \right) R^{p_j}(\cdot) \end{aligned}$$

According to (27), since the tax system depends on $y_1, ..., y_m$ only though $\overline{y} = y_1 + ... + y_m$ according to Assumption 2, one has that $R^{\overline{y}}(\cdot) = \sum_{j=1}^m R^{p_j}(\cdot)$, so the GE-neutralizing reform can be expressed solely in terms of available price replicating directions $R^{p_{m+1}}(\cdot), ..., R^{p_n}(\cdot)$ and $R^{\overline{y}(\cdot)}$:

$$R^N(\cdot) = -\sum_{\ell=1}^L \alpha'_\ell(0) \frac{\partial \log \mathscr{P}_{\overline{y}}}{\partial \alpha_\ell} R^{\overline{y}}(\mathbf{y}) - \sum_{j=m+1}^n \sum_{\ell=1}^L \alpha'_\ell(0) \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} R^{p_j}(\mathbf{y}), \tag{A.14}$$

which ends the Proof of Lemma 3.

A.8 Proof of Theorems 1 and 2

According to (9) and $\mathcal{T}(\mathbf{Y}(\boldsymbol{\theta})) = \sum_{i=1}^n p_i \, X_i(\boldsymbol{\theta}) - C(\boldsymbol{\theta})$, one gets that tax revenue are equal to:

$$\mathscr{B} = \sum_{i=1}^n p_i \, \mathcal{X}_i(\boldsymbol{\theta}) - \int_\Theta C(\boldsymbol{\theta}) \, \mathrm{d}F(\boldsymbol{\theta}). \tag{A.15}$$

Under Assumption 1, Lemma 2 ensures that combining a production policy reform $\boldsymbol{\alpha}(\cdot)$ with the associated neutralizing tax reform $R^N(\cdot)$ defined in (29a) and (29b) changes neither taxpayers' input supplies $\mathbf{X}(\boldsymbol{\theta})$, nor their consumption levels $C(\boldsymbol{\theta})$. Lemma 3 ensures the same result holds under Assumptions 2 and 3. Therefore, combining a production policy reform $\boldsymbol{\alpha}(\cdot)$ with the neutralizing tax reform $R^N(\cdot)$ affects the Lagrangian only through tax revenue by:

$$\frac{\partial \mathscr{L}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \frac{\partial \mathscr{B}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \sum_{i=1}^n \mathcal{X}_i \frac{\partial p_i^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \sum_{i=1}^n \mathcal{Y}_i \frac{\partial \log \mathscr{P}_i^{\boldsymbol{\alpha}(\cdot)}}{\partial \mu}$$

where we used (29b) and (A.11). Using (7) and (21) yields:

$$\frac{\partial \mathscr{L}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \frac{\partial \mathscr{B}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \sum_{\ell=1}^L \mathcal{F}_{\alpha_\ell} \, \alpha'_\ell(0). \tag{A.16}$$

Therefore, a production policy reform $\boldsymbol{\alpha}(\cdot)$ with the associated neutralizing tax reform does not affect any taxpayer's utility levels but do increases tax revenue if it is production-enhancing, i.e. if $\sum_{\ell=1}^L \mathcal{F}_{\alpha_\ell} \, \alpha'_\ell(0) > 0$.

A.9 Proof of Theorems 3 and 4

Theorem 3 is a particular case of Theorem 4 where the initial tax reform is $R(\cdot) \equiv 0$. We therefore directly prove Theorem 4, by considering the effect on Lagrangian of combining a tax reform in the direction $R(\cdot)$ with a production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$.

According to Equations (9)-(11), we get that:

$$\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \int_{\Theta} \left\{ R(\mathbf{Y}(\boldsymbol{\theta})) + \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) \frac{\partial Y_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta})}{\partial \mu} + \frac{1}{\lambda} \frac{\partial \Phi\left(U^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta});\boldsymbol{\theta}\right)}{\partial \mu} \right\} \mathrm{d}F(\boldsymbol{\theta})$$

Using (16b) and (16d) leads to:

$$\begin{aligned}\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = & \int_{\Theta} \left\{ -(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta})) + \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) \frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu} \right\} \mathrm{d}F(\theta) \\ & + \sum_{j=1}^{n} \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} \int_{\Theta} \left\{ \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \log p_j} + \left(1-\mathcal{T}_{y_j}(\mathbf{Y}(\boldsymbol{\theta}))\right) Y_j(\boldsymbol{\theta})\, g(\boldsymbol{\theta}) \right\} \mathrm{d}F(\boldsymbol{\theta})\end{aligned} \tag{A.17}$$

The first row in the right-hand side of (A.17) corresponds to the PE effects on the Lagrangian.[18]

$$\frac{\partial \mathscr{L}^{R(\cdot),PE}}{\partial \mu} \stackrel{\text{def}}{\equiv} \int_{\Theta} \left\{ -(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta})) + \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) \frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu} \right\} \mathrm{d}F(\boldsymbol{\theta}) \tag{A.18}$$

Moreover, as the term in factor of $\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial \mu$ does not depend on tax reform and since the partial derivatives $\partial \log Y_i^{R(\cdot),\boldsymbol{\alpha}(\cdot)}(\boldsymbol{\theta})/\partial \mu$ and $\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial \mu$ are linear in the direction of tax and production policy reforms according to (16a)-(16d) and (22), so does $\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}/\partial \mu$. We then consider the direction of tax reform $R^N(\cdot)$ defined by (38a) and (38b). We thus get that:

$$\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \frac{\partial \mathscr{L}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} + \sum_{j=1}^{n} \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \gamma_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)} \tag{A.19}$$

Moreover, According to Lemma 4, The combination of a tax reform in the direction $R^N(\cdot)$ defined by $R^N(\cdot)$ defined by (38a) and (38b) have the same effects on taxpayers' input supplies and utility at the GE as does a tax reform in the direction $R(\cdot)$ at the PE. According to (A.15), the GE effects on the Lagrangian of the combination of the tax reform in the direction $R^N(\cdot)$ and of the production policy reform in the direction $\boldsymbol{\alpha}(\cdot)$ therefore differs from the PE effects on the Lagrangian of a tax reform in the direction solely by the effects of price changes at the GE holding factor supplies fixed, i.e.:

$$\frac{\partial \mathscr{L}^{R^N(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \frac{\partial \mathscr{L}^{R(\cdot),PE}}{\partial \mu} + \sum_{j=1}^{n} \mathcal{Y}_j \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}. \tag{A.20}$$

Combining (A.11), (A.19) and (A.20) leads to:

$$\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = \frac{\partial \mathscr{L}^{R(\cdot),PE}}{\partial \mu} + \sum_{j=1}^{n} \left(\mathcal{Y}_j + \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \right) \frac{\partial \log p_j^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu}. \tag{A.21}$$

Plugging (38b) and (A.18) into (A.21) leads to:

$$\begin{aligned}\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} = & \int_{\Theta} \left\{ -(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta})) + \sum_{i=1}^{n} \mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta})) \frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu} \right\} \mathrm{d}F(\theta) \\ + \sum_{j=1}^{n} \left(\mathcal{Y}_j + \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \right) & \sum_{i=1}^{n} \frac{\Xi_{j,i}}{\mathcal{Y}_i} \frac{\partial \mathcal{Y}_i^{R(\cdot),PE}}{\partial \mu} + \sum_{j=1}^{n} \left(\mathcal{Y}_j + \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \right) \sum_{\ell=1}^{L} \frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell} \alpha'_\ell(0)\end{aligned}$$

[18]Recall that production policies affects taxpayers only through change in prices, they have therefore zero impact on the Lagrangian at the PE.

Rearranging terms yields:

$$\begin{aligned}\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} &= \int_{\Theta}\left\{-(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta}))+\sum_{i=1}^{n}\mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta}))\,\frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu}\right\}\mathrm{d}F(\theta) \quad \text{(A.22)}\\ &+\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\mathcal{Y}_j\,\frac{\Xi_{j,i}}{\mathcal{Y}_i}+\sum_{j=1}^{n}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}\frac{\Xi_{j,i}}{\mathcal{Y}_i}\right)\frac{\partial \mathcal{Y}_i^{R(\cdot),PE}}{\partial \mu}\\ &+\sum_{\ell=1}^{L}\sum_{j=1}^{n}\mathcal{Y}_j\frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell}\alpha_\ell'(0)+\sum_{\ell=1}^{L}\sum_{j=1}^{n}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}\frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell}\alpha_\ell'(0)\end{aligned}$$

Differentiating both sides of (5) with respect to $\mathcal{X}_i$ implies:

$$\mathcal{F}_{\mathcal{X}_i}=p_i+\sum_{j=1}^{n}\mathcal{X}_j\,\frac{\partial \mathscr{P}_j}{\partial \mathcal{X}_i}=p_i+\sum_{j=1}^{n}\mathcal{Y}_j\,\frac{\partial \log \mathscr{P}_j}{\partial \mathcal{X}_i}$$

Using that $\partial \log \mathscr{P}_j/\partial \mathcal{X}_i=(p_i/\mathcal{Y}_i)\,\Xi_{j,i}$ according to (20a), we get that

$$\sum_{j=1}^{n}\mathcal{Y}_j\,\frac{\Xi_{j,i}}{\mathcal{Y}_i}=\frac{\mathcal{F}_{\mathcal{X}_i}-p_i}{p_i}$$

Plugging the latter equality and (7) into Equation (A.22), we obtain:

$$\begin{aligned}\frac{\partial \mathscr{L}^{R(\cdot),\boldsymbol{\alpha}(\cdot)}}{\partial \mu} &= \int_{\Theta}\left\{-(1-g(\boldsymbol{\theta}))\, R(\mathbf{Y}(\boldsymbol{\theta}))+\sum_{i=1}^{n}\mathcal{T}_{y_i}(\mathbf{Y}(\boldsymbol{\theta}))\,\frac{\partial Y_i(\boldsymbol{\theta})^{R(\cdot),PE}}{\partial \mu}\right\}\mathrm{d}F(\theta) \quad \text{(A.23)}\\ &+\sum_{i=1}^{n}\left(\frac{\mathcal{F}_{\mathcal{X}_i}-p_i}{p_i}+\sum_{j=1}^{n}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}\frac{\Xi_{j,i}}{\mathcal{Y}_i}\right)\frac{\partial \mathcal{Y}_i^{R(\cdot),PE}}{\partial \mu}\\ &+\sum_{\ell=1}^{L}\mathcal{F}_{\alpha_\ell}\alpha_\ell'(0)+\sum_{\ell=1}^{L}\sum_{j=1}^{n}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}\frac{\partial \log \mathscr{P}_j}{\partial \alpha_\ell}\alpha_\ell'(0)\end{aligned}$$

Using the definition of GE multipliers η_i in Equation (34) finally leads to (40), which ends the Proof of Theorem 4.

A.10 Proof of Equation (37)

According to (17) and (27), one has that $\forall \mathbf{y}:\ R^{\overline{y}}=\sum_{j=1}^{n}R^{p_j}(\mathbf{y})$ which implies that:

$$\frac{\partial \mathscr{L}^{R^{\overline{y}}(\cdot)}}{\partial \mu}=\sum_{j=1}^{m}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}$$

When

$$\frac{\partial \mathscr{L}^{R^{\overline{y}}(\cdot)}}{\partial \mu}=\sum_{j=1}^{m}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}=0 \qquad \text{and} \qquad \forall j\in\{m+1,\dots,n\}: \qquad \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}=0,$$

Equation (34) simplifies to:

$$\eta_i=\frac{\mathcal{F}_{\mathcal{X}_i}-p_i}{p_i}+\sum_{j=1}^{m}\frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}\frac{\Xi_{j,i}}{\mathcal{Y}_i}$$

According to (36a), inverse demand elasticities $\Xi_{j,i}$ have to verify

$$\forall i\in\{m+1,\dots,n\},\forall j\in\{1,\dots,m\}: \qquad \Xi_{j,i}=\frac{\mathcal{X}_i}{\overline{\mathcal{P}}}\frac{\partial \overline{\mathcal{P}}}{\partial \mathcal{X}_i}$$

Hence, we get:

$$\eta_i = \frac{\mathcal{F}_{\mathcal{X}_i} - p_i}{p_i} + \left(\sum_{j=1}^{m} \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu} \right) \frac{1}{p_i \, \overline{\mathcal{P}}} \frac{\partial \overline{\mathcal{P}}}{\partial \mathcal{X}_i}$$

which finally leads to (37) whenever the tax system is also optimized over the price replicating direction $R^{\overline{y}}(\cdot)$, i.e. when:

$$0 = \frac{\partial \mathscr{L}^{R^{\overline{y}}(\cdot)}}{\partial \mu} = \sum_{j=1}^{m} \frac{\partial \mathscr{L}^{R^{p_j}(\cdot)}}{\partial \mu}$$

A.11 Proof of Proposition 3

Rewriting Equation (40) in terms of income $\mathbf{y}$ rather than type $\boldsymbol{\theta}$ and when there is no production policy reform yields:

$$\begin{aligned} \frac{\partial \mathscr{L}^{R(\cdot)}}{\partial t} &= \int_{\mathcal{W}_Y} \left\{ - \left[1 - \widehat{g}(\mathbf{y}) - \sum_{i=1}^{n} \left(\mathcal{T}_{y_i}(\mathbf{y}) + \mu_i \right) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \rho} \right] R(\mathbf{y}) \right. \\ &+ \left. \sum_{1 \leq i,j \leq n} \left(\mathcal{T}_{y_i}(\mathbf{y}) + \eta_i \right) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \tau_j} \, R_{y_j}(\mathbf{y})) \right\} h(\mathbf{y}) \mathrm{d}\mathbf{y}. \end{aligned}$$

Using the divergence theorem on the term of the second line and rearranging, we obtain:

$$\begin{aligned} \frac{\partial \mathscr{L}^{R(\cdot)}}{\partial \mu} &= \oint_{\partial W_Y} \sum_{1 \leq i,j \leq n} \left(\mathcal{T}_{y_i}(\mathbf{y}) + \eta_i \right) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \tau_j} h(\mathbf{y}) \phi_j(\mathbf{y}) R(\mathbf{y}) d\sigma(\mathbf{y}) \\ &- \int_{\mathcal{W}_Y} \left\{ \left[1 - \widehat{g}(\mathbf{y}) - \sum_{i=1}^{n} \left(\mathcal{T}_{y_i}(\mathbf{y}) + \eta_i \right) \frac{\partial \widehat{Y}_i(\mathbf{y})}{\partial \rho} \right] h(\mathbf{y}) \right. \\ &\quad \left. + \sum_{j=1}^{n} \frac{\partial \left[\sum_{i=1}^{n} \left(\mathcal{T}_{y_i}(\mathbf{y}) + \eta_i \right) h(\mathbf{y}) \right]}{\partial y_j} \right\} R(\mathbf{y}) \, d\mathbf{y}. \end{aligned}$$

where $d\sigma(\mathbf{y})$ is the corresponding measure of a surface integral (denoted by $\oint$). If the tax system $\mathbf{y} \mapsto \mathcal{T}(\mathbf{y})$ is optimal, the latest equation has to be equal to zero for all possible directions $R(\cdot)$. This is only possible if both equations given in Proposition 3 are satisfied.

At this optimum, one must have $\partial \mathscr{L}^{\mathcal{R}^{p_j}(\cdot)} / \partial t = 0$ for all $j \in \{1, ..., n\}$. This implies that Equation (34) reduces to $\eta_1, ..., \eta_n = 0$ under perfect competition. Revealed welfare weights $\widetilde{g}(\mathbf{y})$ solve Equation (41) with $\eta_1, ..., \eta_n = 0$ for $\widehat{g}(\mathbf{y})$ for the current tax schedule, which leads to (43).

A.12 Proof of Theorem 5

From the definition of revealed welfare weights, we get that for any direction $R(\cdot)$: $\partial \mathscr{L}^{R}(\mu)/\partial t = 0$. Moreover, since $\eta_1 = ... = \eta_n = 0$, we have that for any direction $R(\cdot)$: $\partial \mathscr{L}^{R}(\mu)/\partial t = \partial \mathscr{L}^{R,PE}(\mu)/\partial t = 0$ from (34) and (A.23). Therefore, using $\partial \mathscr{L}^{R,PE}(\mu)/\partial t = \partial \mathscr{B}^{R,PE}(\mu)/\partial t + (1/\lambda) \partial \mathscr{W}^{R,PE}(\mu)/\partial t$ and Equation (16d) implies:

$$\frac{\partial \mathscr{B}^{R,PE}(\mu)}{\partial t} = - \int_{\mathcal{W}_Y} \widetilde{g}(\mathbf{y}) \, R(\mathbf{y}) \, h(\mathbf{y}) \mathrm{d}\mathbf{y}. \tag{A.24}$$

Therefore, a tax reform with a small positive magnitude μ and a direction $R(\cdot)$ such that $R(\mathbf{y}) = 0$ if $\widetilde{g}(\mathbf{y}) \geq 0$ and $R(\mathbf{y}) \geq 0$ if $\widetilde{g}(\mathbf{y}) < 0$ increases tax revenue at the PE. According to (16d), such a reform also increases at the PE the welfare of taxpayers for which $R(\mathbf{Y}(\boldsymbol{\theta})) > 0$ and leave the welfare of the others unchanged. It is therefore a Pareto-improving tax reform at the PE.

According to Lemma 4, a reform with a small positive t in the direction $R^N(\cdot)$ defined in (38a) and (38b) has the same effects at the GE on taxpayers' utility $U(\boldsymbol{\theta})$ and factor supplies $\mathbf{X}(\boldsymbol{\theta})$ as a reform in the direction $R(\cdot)$ and the same magnitude t at the PE. Since tax revenues are equal to: $\sum_{j=1}^{n} p_j \ \mathcal{X}_j - \int_{\mathcal{W}} \mathscr{C}\left(U(\boldsymbol{\theta}), \mathbf{X}(\boldsymbol{\theta}); \boldsymbol{\theta}\right) \ \mathrm{d}F(\boldsymbol{\theta})$, if a tax reform with a small positive magnitude t and a direction $R(\cdot)$ is Pareto-improving at the PE, which is the case when some revealed welfare are negative and the direction $R(\cdot)$ verifies that , a reform with a small positive magnitude μ and the direction $R^N(\cdot)$ defined by (38a) and (38b) is Pareto-improving if $\sum_j \mathcal{X}_j \ \partial p^{R^N}/\partial t \geq 0$. From (5) we get:

$$\mathcal{F}\left(\mathcal{X}_1^{R,PE}(\mu), ..., \mathcal{X}_n^{R,PE}(\mu)\right) = \sum_{j=1}^{n} p_j^R(\mu) \ \mathcal{X}_j^{R,PE}(\mu)$$

Differentiating both sides with respect to t and using (6) leads to: $\sum_j \mathcal{X}_j \ \partial p^{R^N}/\partial t = 0$. Hence, If a reform with a small positive magnitude μ and a direction $R(\cdot)$ is Pareto-improving at the PE, then, under perfect competition, a reform with a small positive magnitude t and the direction $R^N(\cdot)$ defined by (38a) and (38b) is Pareto-improving at the GE.

A.13 Proof of Proposition 4

We consider the case where revealed welfare weights $\widehat{g}(\mathbf{y}) > 0$ are almost everywhere positive. We first notice that, according to Lemma 4, under perfect competition, there exists a direction $R^N(\cdot)$ such that reforms with positive μ in the direction $R^N(\cdot)$ are Pareto-improving at the GE *if and only* if there exists a direction $R(\cdot)$ such that reforms with positive t in the direction $R^N(\cdot)$ are Pareto-improving at the PE, where $R(\cdot)$ and $R^N(\cdot)$ are related by (38a) and (38b)

Assuming, by contradiction, that there exists a direction of tax reform denoted $R^N(\cdot)$ such that a reform in the direction $R^N(\cdot)$ and a small positive magnitude μ is Pareto-improving at the GE. According to Lemma 4, this implies the existence of a direction of tax reform denoted $R(\cdot)$, such that a reform with this direction and a positive μ is Pareto-improving at the PE. According to (16d), since a reform in the direction $R(\cdot)$ improves taxpayer's welfare at the PE, one must have $R(\mathbf{Y}(\boldsymbol{\theta})) \geq 0$ for all $\boldsymbol{\theta} \in \mathcal{W}$ with a strict inequality for some types. However, according to (A.24), such a reform decreases tax revenues at the PE, leading to a contradiction for a Pareto-improving direction of tax reforms at the PE.

A.14 Proof of Proposition 5

When the tax system is schedular and linear for $i = n' + 1, ..., n$, we get that:

$$\mathcal{T}(\mathbf{y}) = \sum_{i=1}^{n'} T_i(y_i) + \sum_{i=n'+1}^{n} t_i \ y_i \tag{A.25}$$

the admissible directions of tax reforms must also be schedular, i.e. they must depend only on one type of income and take the form $\mathbf{y} \mapsto R_i(y_i)$. Moreover for $i = n' + 1, ..., n$ the directions specific to the i^{th} income must be linear.

Under Equation (A.25), according to (17) the price-replicating directions are given by $\mathcal{R}^j(\mathbf{y}) = (1 - T'(y_j))y_j$ for $j = 1, ..., n'$ and by $R^j(\mathbf{y}) = (1 - t_j)y_j$ for $j = n' + 1, ..., n$. Perturbing the tax system along the GE-replicating directions thus keeps the tax system being schedular and also linear for $i = n' + 1, ..., n$. Therefore, one has $\partial \mathscr{L}^{\mathcal{R}^j}/\partial t = 0$ for all $j = 1, ..., n$, so, according to (34), the GE multipliers are given by (46a).

Let $R_i(y_i)$ be any direction of a tax reform specific to the i^{th} income. Because the tax schedule is schedular, Equation (40), stating the impact on the Lagrangian of a tax reform at the GE, simplifies to:

$$\frac{\partial \mathscr{L}^{R_i(\cdot)}}{\partial \mu} = \int_{\mathcal{W}} \left\{ - \left[1 - g(\boldsymbol{\theta}) - \sum_{k=1}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho} \right] R_i(Y_i(\boldsymbol{\theta})) \right.$$
$$\left. + \sum_{1=k}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_i} R_i'(Y_i(\boldsymbol{\theta})) \right\} \mathrm{d}F(\boldsymbol{\theta}). \tag{A.26}$$

since $\partial R_i(y_i)/\partial y_j = 0$ whenever $j \neq i$ under a schedular direction of tax reform. Rewritten in terms of the distribution of the i^{th} income leads to:

$$\frac{\partial \mathscr{L}^{R_i(\cdot)}}{\partial \mu} = \int_{\mathbb{R}_+} \left\{ - \left[1 - \overline{g(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=y_i} - \sum_{k=1}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Bigg|_{Y_i(\boldsymbol{\theta})=y_i} \right] R_i(y_i) \right.$$
$$\left. + \sum_{1=k}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_i}}\Bigg|_{Y_i(\boldsymbol{\theta})=y_i} R_i'(y_i) \right\} h_i(y_i) \mathrm{d}y_i.$$

Integrating by parts the first term and rearranging terms using (45) leads to:

$$\frac{\partial \mathscr{L}^{R_i(\cdot)}}{\partial \mu} = \int_{\mathbb{R}_+} \left\{ - \int_{z=y_i}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=z} - \sum_{k=1}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Bigg|_{Y_i(\boldsymbol{\theta})=z} \right] \mathrm{d}H_i(z) \right.$$
$$\left. + \frac{T_i'(y_i) + \eta_i}{1 - T_i'(y_i)} \varepsilon_i(y_i)\, y_i\, h_i(y_i) + \sum_{1 \leq k \leq n, k \neq i} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_i}}\Bigg|_{Y_i(\boldsymbol{\theta})=y_i} h_i(y_i) \right\} R'(y_i) \mathrm{d}y_i.$$
$$- \lim_{y_i \mapsto \infty} \left\{ \int_{z=y_i}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=z} - \sum_{k=1}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Bigg|_{Y_i(\boldsymbol{\theta})=z} \right] \mathrm{d}H_i(z)\, R_i(y_i) \right\}$$
$$+ \lim_{y_i \mapsto 0} \left\{ \int_{z=y_i}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}|_{Y_i(\boldsymbol{\theta})=z} - \sum_{k=1}^{n} \overline{\left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Bigg|_{Y_i(\boldsymbol{\theta})=z} \right] \mathrm{d}H_i(z)\, R_i(y_i) \right\}$$

For $i = 1, ..., n'$, the income specific tax schedule $T_i(\cdot)$ being nonlinear, the above equation must be equal to zero for any non linear direction R_i, which implies (46b).

For $i = n' + 1, ..., n$, the i^{th} income specific tax schedule has to be linear, so the only admissible directions of tax reforms specific to the i^{th} income are proportional to $R_i(y_i) = y_i$. Equation (A.26) then simplifies to:

$$\frac{\partial \mathscr{L}^{R_i(\cdot)}}{\partial \mu} = \int_{\mathcal{W}} \left\{ - \left[1 - g(\boldsymbol{\theta}) - \sum_{k=1}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho} \right] Y_i(\boldsymbol{\theta}) \right.$$
$$\left. + \sum_{1=k}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_i} \right\} \mathrm{d}F(\boldsymbol{\theta}).$$

Using (A.6c), the preceding equation simplifies to:

$$\frac{\partial \mathscr{L}^{R_i(\cdot)}}{\partial \mu} = \int_{\mathcal{W}} \left\{ - \left[1 - g(\boldsymbol{\theta}) \right] Y_i(\boldsymbol{\theta}) + \sum_{1=k}^{n} \left(T_k'(Y_k(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k^u(\boldsymbol{\theta})}{\partial \tau_i} \right\} \mathrm{d}F(\boldsymbol{\theta}).$$

Using (45), the condition $\partial \mathscr{L}^{y_i}/\partial t = 0$ leads to (46c).

A.15 Proof of Proposition 6

When the tax schedule is comprehensive, admissible directions of tax reforms take the form $\mathbf{y} \mapsto R(y_1 + ... + y_n)$. Consequently, Equation (40) simplifies to:

$$\begin{aligned} \frac{\partial \mathscr{L}^{R(\cdot)}}{\partial \mu} &= \int_{\mathcal{W}} \left\{ - \left[1 - g(\boldsymbol{\theta}) - \sum_{k=1}^{n} \left(T_0'(Y_0(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_i(\boldsymbol{\theta})}{\partial \rho} \right] R(Y_0(\boldsymbol{\theta})) \right. \\ &+ \left. \sum_{1 \le j,k \le n} \left(T_0'(Y_0(\boldsymbol{\theta})) + \eta_k \right) \frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_j} R'(Y_0(\boldsymbol{\theta})) \right\} \mathrm{d}F(\boldsymbol{\theta}). \end{aligned}$$

Rewriting this expression in terms of the density $h_0(\cdot)$ and CDF $H_0(\cdot)$ of the taxable income, the last equation becomes:

$$\begin{aligned} \frac{\partial \mathscr{L}^{R(\cdot)}}{\partial \mu} &= \int_{\mathbb{R}_+} \left\{ - \left[1 - \overline{g(\boldsymbol{\theta})}\Big|_{Y_0(\boldsymbol{\theta})=y_0} - \sum_{k=1}^{n} \left(T_0'(y_0) + \eta_k \right) \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_0(\boldsymbol{\theta})=y_0} \right] R(y_0) \right. \\ &+ \left. \sum_{1 \le j,k \le n} \left(T_0'(y_0) + \eta_k \right) \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_j}}\Big|_{Y_0(\boldsymbol{\theta})=y_0} R'(y_0) \right\} h_0(y_0) \mathrm{d}y_0. \end{aligned}$$

Using (47)-(49) leads to:

$$\begin{aligned} \frac{\partial \mathscr{L}^{R(\cdot)}}{\partial \mu} &= \int_{\mathbb{R}_+} \left\{ - \left[1 - \overline{g(\boldsymbol{\theta})}\Big|_{Y_0(\boldsymbol{\theta})=y_0} - T_0'(y_0) \frac{\partial Y_0(y_0)}{\partial \rho} - \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_0(\boldsymbol{\theta})=y_0} \right] R(y_0) \right. \\ &+ \left. \frac{T_0'(y_0)}{1 - T_0'(y_0)} \varepsilon_0(y_0)\, y_0\, R'(y_0) + \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_0}}\Big|_{Y_0(\boldsymbol{\theta})=y_0} R'(y_0) \right\} h_0(y_0) \mathrm{d}y_0. \end{aligned}$$

Integrating by parts the first line yields:

$$\begin{aligned} &\frac{\partial \mathscr{L}^{R(\cdot)}}{\partial \mu} \\ =\ & \int_{\mathbb{R}_+} \left\{ - \int_{z=y_0}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}\Big|_{Y_0(\boldsymbol{\theta})=z} - T_0'(y_0) \frac{\partial Y_0(y_0)}{\partial \rho} - \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_0(\boldsymbol{\theta})=z} \right] \mathrm{d}H_0(y_0) \right. \\ +\ & \left. \frac{T_0'(y_0)}{1 - T_0'(y_0)} \varepsilon_0(y_0)\, y_0\, h_0(y_0) + \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \tau_0}}\Big|_{Y_0(\boldsymbol{\theta})=y_0} h_0(y_0) \right\} R'(y_0) \mathrm{d}y_0 \\ -\ & \lim_{y \mapsto \infty} \int_{z=y_0}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}\Big|_{Y_0(\boldsymbol{\theta})=z} - T_0'(y_0) \frac{\partial Y_0(y_0)}{\partial \rho} - \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_0(\boldsymbol{\theta})=z} \right] \mathrm{d}H_0(y_0)\, R(y_0) \\ +\ & \lim_{y \mapsto 0} \int_{z=y_0}^{\infty} \left[1 - \overline{g(\boldsymbol{\theta})}\Big|_{Y_0(\boldsymbol{\theta})=z} - T_0'(y_0) \frac{\partial Y_0(y_0)}{\partial \rho} - \sum_{k=1}^{n} \eta_k \overline{\frac{\partial Y_k(\boldsymbol{\theta})}{\partial \rho}}\Big|_{Y_0(\boldsymbol{\theta})=z} \right] \mathrm{d}H_0(y_0)\, R(y_0) \end{aligned}$$

At the optimal comprehensive tax schedule, one mus have $\partial \mathscr{L}^R / \partial t = 0$ for all directions, which implies Equation (50).

If there are only two production factors and if the elasticity of substitution between these two factors is denoted σ, one gets:

$$\frac{\mathrm{d}p_1}{p_1} - \frac{\mathrm{d}p_2}{p_2} = \frac{1}{\sigma} \left(\frac{\mathrm{d}\mathcal{X}_2}{\mathcal{X}_2} - \frac{\mathrm{d}\mathcal{X}_1}{\mathcal{X}_1} \right)$$

Under perfect competition, and denoting $\alpha_i = \mathcal{Y}_i / (\mathcal{Y}_1 + \mathcal{Y}_2)$ the i^{th} income share, the differentiation of both sides of (5) lead to:

$$0 = \alpha_1 \frac{\mathrm{d}p_1}{p_1} + \alpha_2 \frac{\mathrm{d}p_2}{p_2} \qquad \Rightarrow \qquad \frac{\mathrm{d}p_1}{p_1} - \frac{\mathrm{d}p_2}{p_2} = \frac{1}{\alpha_2} \frac{\mathrm{d}p_1}{p_1} = -\frac{1}{\alpha_1} \frac{\mathrm{d}p_2}{p_2}$$

Combining the two latter equations leads to:

$$\Xi = \begin{pmatrix} -\frac{\alpha_2}{\sigma} & \frac{\alpha_2}{\sigma} \\ \frac{\alpha_1}{\sigma} & -\frac{\alpha_1}{\sigma} \end{pmatrix}$$

Under perfect competition, the GE multipliers are given by Equation (35), which leads to:

$$\eta_1 = \frac{-\frac{\partial \mathscr{L}^{\mathcal{R}^1}}{\partial t} \alpha_2 + \frac{\partial \mathscr{L}^{\mathcal{R}^2}}{\partial t} \alpha_1}{\sigma\, \mathcal{Y}_1} \qquad \text{and :} \qquad \eta_1 = \frac{\frac{\partial \mathscr{L}^{\mathcal{R}^1}}{\partial t} \alpha_2 - \frac{\partial \mathscr{L}^{\mathcal{R}^2}}{\partial t} \alpha_1}{\sigma\, \mathcal{Y}_2}$$

Using $\partial \mathscr{L}^{\mathcal{R}^1} / \partial t + \partial \mathscr{L}^{\mathcal{R}^2} / \partial t = 0$ eventually yields (51).

B Micro-founded Examples

In the core of the paper, we derive all our results using only the inverse demand functions $\mathscr{P}_i(\cdot)$ to describe the production sector. Relying on these reduced-forms allows us to demonstrate all our results and to show these results are robust to change in the underlying micro-foundations. However, this simplicity hides the large set of problems that can be described by these reduced-forms. We now discuss how our reduced-form description of the production sector is consistent with various micro-founded applications that have been discussed in the literature.

In the following subsections, we provide micro-foundations for several examples (competition policy, the taxation of intermediate goods, public sector pricing rules, commodity taxation, trade policies, and the effects of business-oriented environmental regulations). To ease their presentation, we consider many intermediate goods and sectors and adopt the following notations. There are one final good and S intermediate goods therefore, $S+1$ sectors, indexed by $s = 0, ..., S$. Within each sector s, there exist N_s firms. In sector $s > 0$, firm $\varphi = 1, ..., N_s$ produces the s^{th} intermediate good, employing inputs $\boldsymbol{\mathcal{X}}^{\varphi,s} \stackrel{\text{def}}{\equiv} (\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s})$ and goods $\mathbf{z}^{\varphi,s} \stackrel{\text{def}}{\equiv} (z_0^{\varphi,s}, ..., z_{s-1}^{\varphi,s}, z_{s+1}^{\varphi,s}, ..., z_S^{\varphi,s})$ with the production function $\mathcal{F}^{\varphi,s}(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s})$. Firm $\varphi = 1, ..., N_0$ in sector $s = 0$ produces the final good using inputs $\boldsymbol{\mathcal{X}}^{\varphi,0} \stackrel{\text{def}}{\equiv} \left(\mathcal{X}_1^{\varphi,0}, ..., \mathcal{X}_n^{\varphi,0}\right)$ and goods $\mathbf{z}^{\varphi,0} \stackrel{\text{def}}{\equiv} (z_1^{\varphi,0}, ..., z_S^{\varphi,0})$ with the production function $\mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0}, \mathbf{z}^{\varphi,0}\right)$. In all sectors $s \in \{0, ..., S\}$, let $z_s^{\varphi,s}$ denote the production of firm $\varphi \in \{1, ...N_s\}$. The production functions are differentiable with non-negative partial derivatives and well-behaved.

The market clearing condition for the final goods is:

$$\sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0}, \mathbf{z}^{\varphi,0}\right) = \sum_{s=1}^{S} \sum_{\varphi=1}^{N_s} z_0^{\varphi,s} + \int_{\Theta} C(\boldsymbol{\theta})\, \mathrm{d}F(\boldsymbol{\theta}) + E. \tag{B.1a}$$

It equalizes the total production of firms in the final good sector $s = 0$ in the left-hand side to the demands for the final good $s = 0$ by intermediate goods producers ($z_0^{\varphi,s}$ for $s = 1..., S$ and $\varphi = 1, ..., N_s$), taxpayers ($C(\boldsymbol{\theta})$ for all $\boldsymbol{\theta} \in \Theta$) and the government ($E$), in the right-hand side.

The market clearing condition for input $i = 1, ..., n$ can be expressed as:

$$\forall i \in \{1, ..., n\}: \qquad \mathcal{X}_i = \sum_{s=0}^{S} \sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.1b}$$

i.e., the total supply of the i^{th} input by taxpayers on the left-hand side is equal to the sum of input demands by all firms in all sectors on the right-hand side.

Finally, the market clearing condition in the intermediate goods sector s can be written as:

$$\forall s \in \{1, ..., S\}: \qquad \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s}\right) = \sum_{\substack{s'=0 \\ s'\neq s}}^{S} \sum_{\varphi=1}^{N_{s'}} z_s^{\varphi,s'} \tag{B.1c}$$

i.e., the total production of firms in sector s on the left-hand side is equal to the sum of the demands for good s by firms in all sectors s' other than s on the right-hand side.

In the next subsections, we describe how the resources are allocated within the production sector in different micro-founded contexts. In a nutshell, the problem of resource allocation in the production sector involves determining, for given aggregate supplies of inputs $\mathcal{X}_1, ..., \mathcal{X}_n$, the demand for inputs and goods by each firm $\varphi \in \{1, ..., N_s\}$ in sector $s \in \{0, ..., S\}$. Specifically, this entails determining the input demands $\boldsymbol{\mathcal{X}}^{\varphi,s} = (\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s})$, the demands for goods $\mathbf{z}^{\varphi,s} = \left(z_0^{\varphi,s}, ..., z_{s-1}^{\varphi,s}, z_{s+1}^{\varphi,s}, ..., z_S^{\varphi,s}\right)$, and the firm's production $z_s^{\varphi,s} = \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s}\right)$, subject to the market clearing conditions for intermediate goods (B.1c), for final goods (B.1a) and for inputs (B.1b).

B.1 Competition policy

We first consider an example of a pro-competitive policy designed to reduce oligopolistic rents. Consider that all firms, within each sector, have the same production function with constant returns to scale. There is perfect competition in the final goods sector $s = 0$ and Cournot competition in the intermediate goods sectors $s \in \{1, ..., S\}$. For simplicity, intermediate goods are produced using only inputs and with the same production function denoted $\mathcal{F}^s\left(\boldsymbol{\chi}^{\varphi,s}\right)$. [19] Conversely, the final good is produced using both intermediate goods and inputs according to the following Cobb-Douglas production function:

$$\mathcal{F}_0^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0}, \mathbf{z}^{\varphi,0}\right) = \prod_{s=1}^{S} \left(z_s^{\varphi,0}\right)^{\beta_s} \prod_{i=1}^{n} \left(\mathcal{X}_i^{\varphi,0}\right)^{\gamma_i}$$

where $\beta_s \geq 0$, $\gamma_i \geq 0$ and $\sum_{s=1}^{S} \beta_s + \sum_{i=1}^{n} \gamma_i = 1$. In all sectors $s \in \{0, ..., S\}$, let $z_s \overset{\text{def}}{\equiv} \sum_{\varphi=1}^{N_s} z_s^{\varphi,s}$ denote the total output of the intermediate good s and let $z_s^{-\varphi,s} \overset{\text{def}}{\equiv} z_s - z_s^{\varphi,s}$ denote the amount of intermediate good s produced by the competitors of firm φ. The program of the final good producers $\varphi \in \{1, ..., N_0\}$ is:

$$\max_{\boldsymbol{\mathcal{X}}^{\varphi,0}, \mathbf{z}^{\varphi,0}} \quad \prod_{s=1}^{S} \left(z_s^{\varphi,0}\right)^{\beta_s} \prod_{i=1}^{n} \left(\mathcal{X}_i^{\varphi,0}\right)^{\gamma_i} - \sum_{s=1}^{S} q_s\, z_s^{\varphi,0} - \sum_{i=1}^{n} p_i\, \mathcal{X}_i^{\varphi,0},$$

where q_s denotes the purchasing price of good s, with the normalization $q_0 = 1$ for the final good. With all production functions admitting constant returns to scale and being identical in the final goods sector, the first-order condition of this program leads to the following inverse demand for the s^{th} intermediate good:

$$q_s = \beta_s \left(z_s^0\right)^{\beta_s - 1} \prod_{s'=1, s'\neq s}^{S} \left(z_{s'}^0\right)^{\beta_{s'}} \prod_{i=1}^{n} \left(\mathcal{X}_i^0\right)^{\gamma_i}, \tag{B.2a}$$

where $\mathcal{X}_i^0 \overset{\text{def}}{\equiv} \sum_{\varphi=1}^{N_0} \mathcal{X}_i^{\varphi,0}$ represents the sum of the i^{th} inputs used in the final goods sector, and where $z_s^0 \overset{\text{def}}{\equiv} \sum_{\varphi=1}^{N_0} z_s^{\varphi,0}$ represents the sum of the s^{th} intermediate inputs demanded from the final goods sector. Symmetrically, the first-order condition with respect to the i^{th} input implies:

$$p_i = \gamma_i \left(\mathcal{X}_i^{\gamma_i - 1}\right)^{\gamma_i - 1} \prod_{s=1}^{S} \left(z_s^0\right)^{\beta_s} \prod_{i'=1, i'\neq i}^{n} \left(\mathcal{X}_{i'}^0\right)^{\gamma_{i'}}. \tag{B.2b}$$

[19] Relaxing each of these assumptions do not alter our main results but adds complexity to the analysis.

Since the final goods sector includes many firms and there are numerous intermediate goods sectors, intermediate goods producers take production factor prices $p_1, ..., p_n$ as given, as well as the output of other intermediate goods producers. Noting that only final goods producers purchase intermediate goods, the market clearing condition for the s^{th} intermediate good writes $z_s = z_s^{\varphi,s} + z_s^{-\varphi,s}$. In sector $s = \{1, ..., S\}$, under Cournot competition, firm $\varphi \in \{1, ..., N_s\}$'s maximization program is:[20]

$$\max_{\boldsymbol{\mathcal{X}}^{\varphi,s}, q_s} \quad q_s \, \mathcal{F}^s\left(\mathcal{X}^{\varphi,s}\right) - \sum_{i=1}^{n} p_i \, \mathcal{X}_i^{\varphi,s}$$

$$s.t: \quad q_s = \beta_s \, \left(\mathcal{F}^s\left(\mathcal{X}^{\varphi,s}\right) + z_s^{-\varphi,s}\right)^{\beta_s - 1} \prod_{s'=1, s' \neq s}^{S} \left(z_{s'}^0\right)^{\beta_{s'}} \prod_{i=1}^{n} \left(\mathcal{X}_i^0\right)^{\gamma_i}.$$

At the symmetric Cournot-Nash equilibrium, all producers within a sector make identical choices. Hence, the total production of good z by all firms in sector s is given by $z_s = N_s \, z_s^{\varphi,s}$, while the total production of its competitors is $z_s^{-\varphi,s} = (N_s - 1) \, z_s^{\varphi,s}$. The first-order conditions thus imply:

$$\forall i \in \{1, ..., n\}: \quad p_i = q_s(1 - \alpha_s)\mathcal{F}_{\mathcal{X}_i}^{\varphi,s}, \tag{B.2c}$$

where, $\alpha_s \overset{\text{def}}{\equiv} (1 - \beta_s)/N_s$ measures the extent to which the output price q_s is marked up due to imperfect competition. Since the production functions exhibit constant returns to scale, α_s also denotes the profit share in sector s. Under Cournot competition, this profit share is a decreasing function of the number N_s of firms and an increasing function of the elasticity $1 - \beta_s$ of the inverse demand for the s^{th} intermediate good in absolute value. Competition policies directly set these sector-specific markups α_s. For instance, regulation policies in each sector $s \in \{1, ..., S\}$ affects barriers to entry, entry costs, thereby the number of firms N_s. The allocation of production resources , $(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s})$ (for all firms $\varphi = 1, ..., N_s$ in sector $s = 0, ..., S$), is therefore obtained from a system of prices for inputs $p_1, ..., p_s$ and for intermediate goods $q_1, ..., q_s$, such that it verifies the demands for intermediate goods from final good producers (B.2a), the demands for inputs from final good producers (B.2b), the pricing equation in the intermediate good sectors (B.2c), as well as the market-clearing conditions (B.1a)-(B.1c).

We now demonstrate that this allocation of production resources, $(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s})$ for all firms $\varphi = 1, ..., N_s$ in sector $s = 0, ..., S$, coincides with the choice of an hypothetical "production coordinator". This reformulation will prove useful to easily retrieve the reduced-forms $\mathcal{F}(\cdot)$ and the inverse demand equations $\mathscr{P}_i(\cdot)$ in (4). The production coordinator's objective is the total production of the final good. According to (B.1a), the total production of the final good coincides with the total final good's consumption by taxpayers and the government in (B.3a).[21] The production coordinator's program has to verify resource constraints on production factors (B.1b) and on intermediate goods (B.1c). Crucially, instead of using Equation (B.1c), the production coordinator adopts a reformulation of these resource constraints on intermediate goods, as described by Equations (B.3c) and (B.3d), which together replicate the original constraints (B.1c). However, by incorporating markups α_s and sector-specific profits $\overline{Z}^s$, the coordinator's program replicates the overpricing behaviors described in (B.2c). Thus, the production

[20] Let $X_{n+1}(\boldsymbol{\theta})$ denote the allocation of profits to individuals of type $\boldsymbol{\theta}$ with $\int_\Theta X_{n+1}(\boldsymbol{\theta})\mathrm{d}F(\boldsymbol{\theta}) = 1$. X_{n+1} can be interpreted as an inelastically supplied "entrepreneurial input" (McKenzie (1959), and Mas-Colell et al. (1995, pp. 134-135)) whose presence ensures that Equation (5) holds, provided that i is summed from 1 to $n + 1$ instead of 1 to n.

[21] This is because we assume in this application that the final good is not used as an input by intermediate good producers. Otherwise, the production coordinator's objective would be total production of final good net of its consumption by intermediate good producers, i.e. the GDP $\sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0}, \mathbf{z}^{\varphi,0}\right) - \sum_{s=1}^{S} \sum_{\varphi=1}^{N_s} z_0^{\varphi,s}$.

coordinator's program is:

$$\max_{\{\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\}_{s=0,\dots,S}^{\varphi=1,\dots,N_s}} \quad \sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0},\mathbf{z}^{\varphi,0}\right) \tag{B.3a}$$

$$\forall i \in \{1,...,n\} \quad : \quad \mathcal{X}_i = \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.3b}$$

$$\forall s \in \{1,...,S\} \quad : \quad \alpha_s\,\overline{Z}_s + (1-\alpha_s)\sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}\right) = \sum_{\varphi=1}^{N_0} z_s^{\varphi,0}, \tag{B.3c}$$

where

$$\forall s \in \{1,...,S\} : \quad \overline{Z}_s \overset{\text{def}}{\equiv} \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\right), \tag{B.3d}$$

the sector specific production, is taken as given by the production coordinator. Let $p_i^\star$ denote the Lagrange multiplier associated with constraint (B.3b) and let $q_s^\star$ the Lagrange multiplier associated to (B.3c). The Lagrangian of Program (B.3a)-(B.3c) is written as:

$$\begin{aligned}\mathcal{L} \quad = \quad & \sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0},\mathbf{z}^{\varphi,0}\right) + \sum_{i=1}^{n} p_i^\star\left[\mathcal{X}_i - \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s}\mathcal{X}_i^{\varphi,s}\right] \\ + \quad & \sum_{s=1}^{s} q_s^\star\left[\alpha_s\,\overline{Z}_s + (1-\alpha_s)\sum_{\varphi=1}^{N_s}\mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}\right) - \sum_{\varphi=1}^{N_0} z_s^{\varphi,0}\right].\end{aligned}$$

The first-order conditions of the production coordinator's program are:

$$\forall s \in \{1,...,S\} : \qquad q_s^\star \quad = \quad \beta_s\,\left(z_s^0\right)^{\beta_s-1}\prod_{s'=1,s'\neq s}^{S}\left(z_{s'}^0\right)^{\beta_{s'}}\prod_{i=1}^{n}\left(\mathcal{X}_i^0\right)^{\gamma_i} \tag{B.4a}$$

$$\forall i \in \{1,...,n\} : \qquad p_i^\star \quad = \quad \gamma_i\,\left(\mathcal{X}_i^{\gamma_i-1}\right)^{\gamma_i-1}\prod_{s=1}^{S}\left(z_s^0\right)^{\beta_s}\prod_{i'=1,i'\neq i}^{n}\left(\mathcal{X}_{i'}^0\right)^{\gamma_{i'}}. \tag{B.4b}$$

Here, we use that the production functions, in the final good sector are Cobb-Douglas with constant returns to scale. Finally, the first-order condition with respect to the i^{th} input $i \in \{1,...,n\}$ for firm $\varphi \in \{1,...,N_s\}$ in sector $s \in \{1,...,S\}$ is:

$$p_i^\star = q_s^\star(1-\alpha_s)\mathcal{F}_{\mathcal{X}_i}^{\varphi,s}. \tag{B.4c}$$

Since the production coordinator takes the sector-specific profits $\overline{Z}_s$ as given, the first-order conditions (B.4a)-(B.4c) are equivalent to, respectively, the demands for intermediate goods by final good producers (B.2a), the demands for inputs by final good producers (B.2b) and the pricing equation in the intermediate good sectors (B.2c). Moreover, we note that in all intermediate good sectors $s \in \{1,...,S\}$, the combination of constraints (B.3c) and (B.3d) implies the market-clearing condition (B.1c). Therefore, the production coordinator allocates resources within the production sector as firms do in the decentralized equilibrium. This reformulation of production sector decisions through the program of the hypothetical production coordinator enables to define the inverse demand equations $\mathscr{P}_i(\cdot)$ as the Lagrange multipliers associated to (B.3b), as functions of the vector of input supplies $(\mathcal{X}_1,...,\mathcal{X}_n)$, and of the vectors of sector-specific mark-ups $(\alpha_1,...,\alpha_S)$. Moreover, the (aggregate) production function defined in (5) $\mathcal{F}(\cdot)$ is the value function of Program (B.3a)-(B.3c).[22]

[22]The allocation of resources actually solves a fixed-point problem since the production coordinator's program also depends on sector-specific profits $(\overline{Z}_1,...,\overline{Z}_s)$ that are taken as given by the production coordinator. However, $(\overline{Z}_1,...,\overline{Z}_s)$ is determined by the solution of the production coordinator's program through (B.3d).

The most efficient allocation of resources within the production sector consists in maximizing the total production of final goods subject to the resource constraints on inputs (B.1b) and intermediate goods (B.1c). This coincides with the production coordinator's program only when $\alpha_1 = ... = \alpha_S = 0$. We conclude that the production regulation principles (Theorems 1 and 2) recommend nullifying the markups $\alpha_1, ..., \alpha_S$.

In practice, whenever the tax system can be improved along all the price-replicating directions, no mark-up should persist in the markets. This policy implication extends beyond the specific case of Cournot competition. Any competition policy that reduces markups α_s is desirable provided that the tax system can be reformed by neutralizing tax reforms to offset the welfare impact of such reforms. We posit that this reasoning extends to policies like merger regulations in the case of horizontal or vertical integration, as well as to corporate law reforms.

B.2 Taxation of intermediate goods and taxing robots and AI

The advent of robots and AI raises the question of relevant tax policy responses. Our approach also addresses this issue and extends more broadly to the taxation of intermediate goods. We consider that all firms operate under constant or decreasing returns to scale, and intermediate goods (e.g. robots, AI or any other intermediate good or service) are subject to the sector-specific ad-valorem tax rates α_s, for $s = 1, ..., S$, with the normalization $\alpha_0 = 0$ for the final good. Again, q_s denotes the purchasing price of good s, with the normalization $q_0 = 1$ for the final good. In this scenario, firm $\varphi = 1, ..., N_s$ in sector $s = 0, ..., S$ solves:

$$\pi^{\varphi,s} \stackrel{\text{def}}{\equiv} \max_{\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}} \quad q_s(1-\alpha_s)\, \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s}\right) - \sum_{i=1}^{n} p_i\, \mathcal{X}_i^{\varphi,s} - \sum_{\substack{s'=0 \\ s'\neq s}}^{S} q_{s'}\, z_{s'}^{\varphi,s}, \tag{B.5}$$

where $\pi^{\varphi,s}$ denotes the profit of firm φ in sector s. Since firms operate under perfect competition, profit $\pi^{\varphi,s}$ is positive if the production function of the firm φ in sector s has decreasing returns to scale. Let $X_{n+1}(\boldsymbol{\theta})$ denote the exogenous share of firms' profits earned by $\boldsymbol{\theta}$-taxpayers so that $p_{n+1}X_{n+1}(\boldsymbol{\theta})$ is the profits earned by $\boldsymbol{\theta}$-taxpayers. Program (B.5) leads to the following conditions:

$$\forall i \in \{1, ..., n\}: \quad p_i = q_s(1-\alpha_s)\mathcal{F}^{\varphi,s}_{\mathcal{X}_i} \qquad \text{and} \qquad \forall s' \neq s: \quad q_{s'} = q_s(1-\alpha_s)\mathcal{F}^{\varphi,s}_{z_{s'}}. \tag{B.6}$$

The competitive allocation of the production resources is a vector $(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s})$ for all firms $\varphi = 1, ..., N_s$ in sector $s = 0, ..., S$, a vector of intermediate goods' prices $(q_1, ..., q_S)$ (with normalization $q_0 = 1$) and a vector of input prices $(p_1, ..., p_n)$. These vectors must verify the market clearing conditions (B.1b) and (B.1c), as well as the optimality conditions (B.6), for all firms, in all sectors.

As in Subsection B.1, we determine the optimization program of a hypothetical production coordinator whose solution coincides with the competitive allocation of production resources, $(\boldsymbol{\mathcal{X}}^{\varphi,s}, \mathbf{z}^{\varphi,s})$ (for all firms $\varphi = 1, ..., N_s$ in sector $s = 0, ..., S$). Here, its program consists in maximizing the total production of the final good net of the final good demands by the firms producing intermediate goods. According to (B.1a), this coincides with the total consumption of final good by taxpayers and the government, which corresponds to the objective function (B.7a). The production coordinator's program has to verify resource constraints on inputs (B.1b), rewritten as (B.7b), and on intermediate goods (B.1c). Instead of the latter equation, the production coordinator considers (B.7c) and (B.7d) where the government collects a fraction α_s of the production of each intermediate good, as described by Equations (B.7c) and (B.7d).

The program for the production coordinator is:

$$\max_{\{\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\}_{s=0,\dots,S}^{\varphi=1,\dots,N_s}} \quad \sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0},\mathbf{z}^{\varphi,0}\right) - \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} z_0^{\varphi,s} \tag{B.7a}$$

$$\forall i \in \{1,\dots,n\} \quad : \quad \mathcal{X}_i = \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.7b}$$

$$\forall s \in \{1,\dots,S\} \quad : \quad \alpha_s\,\overline{Z}_s + (1-\alpha_s)\sum_{\varphi=1}^{N_s}\mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\right) = \sum_{\substack{s'=0\\ s'\neq s}}^{S}\sum_{\varphi=1}^{N_{s'}} z_s^{\varphi,s'} \tag{B.7c}$$

where

$$\forall s \in \{1,\dots,S\}: \quad \overline{Z}_s \stackrel{\text{def}}{\equiv} \sum_{\varphi=1}^{N_s}\mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\right) \tag{B.7d}$$

is taken as given by the production coordinator.

Let $p_i^\star$ denote the Lagrange multiplier associated to (B.7b), and let $q_s^\star$ denote the Lagrange multiplier associated to (B.7c). Adopting the normalization $q_0^\star = 1$ and $\alpha_0 = \overline{Z}_0 = 0$, the Lagrangian of (B.7a)-(B.7c) is:

$$\mathcal{L} = \sum_{s=0}^{S} q_s^\star \left[\alpha_s\,\overline{Z}_s + (1-\alpha_s)\sum_{\varphi=1}^{N_s}\mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\right) - \sum_{\substack{s'=0\\ s'\neq s}}^{S}\sum_{\varphi=1}^{N_{s'}} z_s^{\varphi,s'}\right] + \sum_{i=1}^{n} p_i^\star\left[\mathcal{X}_i - \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s}\mathcal{X}_i^{\varphi,s}\right].$$

The first order conditions are:

$$\forall i \in \{1,\dots,n\}: \quad p_i^\star = q_s^\star(1-\alpha_s)\mathcal{F}_{\mathcal{X}_i}^{\varphi,s} \qquad \text{and} \qquad \forall s' \neq s: \quad q_{s'}^\star = q_s^\star(1-\alpha_s)\mathcal{F}_{z_{s'}}^{\varphi,s}. \tag{B.8}$$

These conditions coincide with (B.6), provided that $p_i^\star = p_i$ for all $i = 1,\dots,n$ and $q_s^\star = q_s$ for all $s = 0,\dots,S$. Therefore, the production coordinator allocates resources within the production sector identically to how firms do in the decentralized equilibrium. This reformulation of production sector decisions through the program of the hypothetical production coordinator enables to define the inverse demand equations $\mathscr{P}_i(\cdot)$ as the Lagrange multipliers associated with (B.7c) as functions of the vector of input supplies $(\mathcal{X}_1,\dots,\mathcal{X}_n)$, and of the vectors $(\alpha_1,\dots,\alpha_S)$. Moreover, the (aggregate) production function defined in (5) $\mathcal{F}(\cdot)$ is the value function of Program (B.7a)-(B.7c).

Again, the most efficient allocation of resources within the production sector consists in maximizing the total production of final goods subject to the resource constraints on inputs (B.1b) and intermediate goods (B.1c). This coincides with the production coordinator's program only when $\alpha_1 = \dots = \alpha_S = 0$, i.e. when intermediate goods are untaxed. We conclude that the production regulation principles (Theorems 1 and 2) recommend not taxing intermediate goods $\alpha_1,\dots,\alpha_S = 0$.

Diamond and Mirrlees (1971)'s production efficiency theorem already recommends not taxing intermediate goods. However, our regulation principles (Theorems 1 and 2) imply that this recommendation applies also under a suboptimal tax schedule, or under imperfect competition. Actually, increasing aggregate output leads to a Pareto improvement without the need to be on the production possibility frontier or to fully optimize the tax system. A neutralizing tax reform is all we need. In our example, production functions that exhibit decreasing returns, as in Dasgupta and Stiglitz (1971, 1972), imply that the neutralizing tax reform optimized the tax system along, in particular, the $n+1^{\text{th}}$ price-replicating direction associated with the $n+1^{\text{th}}$ entrepreneurial factor. Profits are fully taxed, as shown in Dasgupta and Stiglitz (1971, 1972).

This formulation of our framework also enables us to address the question of taxing robots and AI, as Koizumi (2020), Guerreiro et al. (2021), Costinot and Werning (2022) and Thuemmel (2023) do, by simply considering them as particular intermediate goods. This literature typically finds optimal to tax robots, because their tax authorities are assumed to be unable to distinguish between various (imperfectly substitutable) types of labor, such as routine and non-routine tasks. In such cases, the neutralizing tax system cannot be implemented (i.e. the tax system cannot optimized along all its price-replicating directions). As Assumption 1 is violated, the Production Regulation Principle in Theorem 1 does not apply. Moreover, since taxing robots affects the wage ratio between routine and non-routine labor, Assumption 3 is violated, preventing the Production Regulation Principle (Part II) in Theorem 2. In such a case, the pre-distributive effects of taxing robots matter and the optimal tax on robots is determined by Equation (32) in Theorem 3 (See also Figure 2).

B.3 Commodity taxation

Using the framework employed to analyze the taxation of intermediate goods in Subsection B.2, we can study whether the taxation of final goods should be uniform when the utility is weakly separable in leisure and consumption, as examined by Atkinson and Stiglitz (1976). Their theorem considers that each taxpayer has preference over factor $\mathbf{x}$ and commodities $\mathbf{z} = (z_1, ..., z_S)$, according to a weakly separable utility function of the form $\mathcal{U}(\mathscr{V}(z_1, ..., z_S), \mathbf{x}; \boldsymbol{\theta})$. We can align our model with theirs by interpreting our intermediate goods as their commodities $(z_1, ..., z_s)$. Additionally, assume that all taxpayers in Section B.2 produce and consume one final good z_0 using the same production function $z_0 = \mathscr{V}(z_1, ..., z_n)$ so that this corresponds to the sub-utility obtained from commodities in Atkinson and Stiglitz (1976). We assume constant returns to scale in the production functions of the intermediate good sectors $s \in 1, ..., S$ and that final goods are not employed as production factors (thus, $z_0^{\varphi,s} = 0$ for all firms $\varphi \in 1, ..., N_s$ in sectors $s = 1, ..., S$). Upon this reinterpretation, our taxation of intermediate goods in Section B.2 is taxation of commodities in Atkinson and Stiglitz (1976). Therefore, the no-tax result on intermediate goods discussed in Section B.2 translates to a no-tax result on commodities, or equivalently, uniform commodity tax rates, in Atkinson and Stiglitz (1976).

This reinterpretation shows that the production regulation principles (Theorems 1 and 2) imply that the no-commodity taxation result of Atkinson and Stiglitz (1976) remains robust to endogenous producer prices, whenever the neutralizing tax reform can be implemented. This applies, for instance, in the long-run model of Saez (2004) where taxation is occupation-specific so that Assumption 1 holds. Conversely, in frameworks such as Naito (1999), the short-run model of Saez (2004), or in Jacobs (2015), the income tax system does not discriminate between the different types of labor, thereby violating Assumption 1. The same level of income drives the same tax rate, even when earned by different labor types. In this type of framework, the tax systems can therefore not be reformed along the price-replicating direction specific to each type of labor. Commodity taxation should then not be uniform and may have a pre-distributive role, which is described in Equation (32) in Theorem 3. It is worth mentioning that our reinterpretation of Atkinson and Stiglitz (1976)'s theorem leading to no-tax on intermediate goods does not hold when taxpayers have different preferences $\mathcal{V}(\cdot)$ over commodities, as in e.g. Saez (2002) and Ferey et al. (2024).

B.4 Trade policy

We now adapt our multi-sector framework to discuss the desirability of trade liberalization policies such as the reduction of tariff measures or of technical barriers. For this purpose, we assume that, in each sector $s \in \{0, ..., n\}$, certain firms operate abroad. Regardless of whether firms are domestic or foreign, the arguments of the production function refer only to goods or production factors from the home country. Foreign firms $\varphi \in \{1, ..., N_s\}$ in sector $s \in \{0, ..., S\}$ do not use domestic factors of production, so $\mathcal{X}_i^{\varphi,s} = 0$ for all $i \in \{1, ..., n\}$, but these foreign firms export goods $z_{s'}^{\varphi,s}$ from sector $s' \neq s$. Their imports of goods s are given by $\mathcal{F}^{\varphi,s}(\boldsymbol{z}^{\varphi,s}; \alpha)$, where the vector α captures the impact of trade frictions. In particular, α_s captures various costs associated with the imports or exports of foreign producers in sector s, costs that trade policies can diminish, so that $\mathcal{F}_\alpha^{\varphi,s} < 0$ for foreign firms. Conversely, trade policies do not impact the production possibilities of domestic firms, hence, $\mathcal{F}_\alpha^{\varphi,s} = 0$ for domestic firms. As in the previous subsection, the allocation of production resources within the production sector can be described as the maximization program of an hypothetical production coordinator. Assuming perfect competition, the competitive allocation of resources within the production sector coincides with the solution of the following production coordinator's program:

$$\max_{\{\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\}_{s=0,\ldots,S}^{\varphi=1,\ldots,N_s}} \quad \sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0},\mathbf{z}^{\varphi,0};\alpha\right) - \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} z_0^{\varphi,s} \tag{B.9a}$$

$$\forall i \in \{1,...,n\} \quad : \quad \mathcal{X}_i = \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.9b}$$

$$\forall s \in \{1,...,S\} \quad : \quad \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s};\alpha_s\right) = \sum_{\substack{s'=0\\ s'\neq s}}^{S}\sum_{\varphi=1}^{N_{s'}} z_s^{\varphi,s'}. \tag{B.9c}$$

For each vector of factor supply $(\mathcal{X}_1,...,\mathcal{X}_n)$ and each vector $(\alpha_0,...,\alpha_S)$ of sector-specific trade costs, the inverse demands $\mathscr{P}_i(\cdot)$ are defined as the Lagrange multipliers associated to (B.9b) and the production function $\mathcal{F}(\cdot)$ is the value function associated to Program (B.9a)-(B.9c). A policy that reduces trade costs therefore unambiguously improves aggregate production.[23] The desirability of trade liberalization policies thus depends solely on whether or not a neutralizing tax reform can take place. This type of reform is feasible in Diamond and Mirrlees (1971), Dixit and Norman (1980, 1986) where the tax system includes sector-specific and linear taxes on labor. Hence Assumption 1 is verified, ensuring that the Production Regulation Principle (Part 1) (in Theorem 1) applies, thereby, supporting the case for free trade. The multi-country Ricardian model of trade proposed by Hosseini and Shourideh (2018) also aligns with the free trade recommendation for the same reasons. Conversely, in Costinot and Werning (2022), the different types of labor are imperfect substitutes but generate incomes that the tax administration cannot distinguish and therefore must tax comprehensively, in accordance with Assumption 2. However, as along as trade policies impacts the wage ratios between the different types of labor, which means that Assumption 3 is violated, Theorem 2 does not apply and the Production Regulation principle does not hold (See Figure 2). Consequently, the impact of trade liberalization policies should be evaluated thanks to Theorem 3, where production efficiency effects have to be balanced against pre-distributive effects, as described by Equation (32). In this context, protectionist measures become desirable when their pre-distibutive effects are more beneficial than their detrimental effects on aggregate production.

B.5 Public production

Consider the government owns the public firm $\varphi^\star$ in sector $s^\star$. Within this framework, the production policies are the public firm's demand of inputs and the demand of goods, i.e. $\boldsymbol{\alpha} \stackrel{\text{def}}{\equiv} (\boldsymbol{\mathcal{X}}^{\varphi^\star,s^\star},\mathbf{z}^{\varphi^\star,s^\star})$. The private firms solve (B.5) and their behaviors are described by (B.6). Therefore, the allocation of production resources coincides now with the solution of the following production coordinator's program:

$$\max_{\{\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\}_{s=0,\ldots,S}^{\varphi=1,\ldots,N_s,(\varphi,s)\neq(\varphi^\star,s^\star)}} \quad \sum_{\varphi=1}^{N_0} \mathcal{F}^{\varphi,0}\left(\boldsymbol{\mathcal{X}}^{\varphi,0},\mathbf{z}^{\varphi,0}\right) - \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} z_0^{\varphi,s} \tag{B.10a}$$

$$\forall i \in \{1,...,n\} \quad : \quad \mathcal{X}_i = \sum_{s=0}^{S}\sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.10b}$$

$$\forall s \in \{1,...,S\} \quad : \quad \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\mathbf{z}^{\varphi,s}\right) = \sum_{\substack{s'=0\\ s'\neq s}}^{S}\sum_{\varphi=1}^{N_{s'}} z_s^{\varphi,s'}. \tag{B.10c}$$

[23] Applying the envelope theorem with respect to the α's to the Lagrangian of (B.9a)-(B.9c) yields $\mathcal{F}_{\alpha_s} < 0$, since $\mathcal{F}_{\alpha_s}^{\varphi,s} < 0$ for foreign firms.

Again, for each vector of factor supply $(\mathcal{X}_1, ..., \mathcal{X}_n)$ and each vector of production policy, $(\boldsymbol{\mathcal{X}}^{\varphi^\star,s^\star}, \mathbf{z}^{\varphi^\star,s^\star})$, the inverse demands $\mathscr{P}_i(\cdot)$ are defined as the Lagrange multipliers associated to constraints (B.10b) and the production function $\mathcal{F}(\cdot)$ is the value function associated to program (B.10a)-(B.10c).

According to the Production Regulation Principles (Theorems 1 and 2), if either Assumption 1 holds or the combination of Assumptions 2 and 3, the government sets the production plan $(\boldsymbol{\mathcal{X}}^{\varphi^\star,s^\star}, \mathbf{z}^{\varphi^\star,s^\star})$ of the public firm $\varphi^\star$ in sector $s^\star$ to maximize the total production of the final good (minus its consumption by producers of intermediate goods), as detailed in (B.10a). This amounts to solving program (B.10a)-(B.10c) with respect to the production plan of private firms (as in (B.10a)-(B.10c)) and of the public firm $\varphi^\star$ in sector $s^\star$. In such a case, private and public firms face the same first-order conditions:

$$\forall i \in \{1, ..., n\}: \quad p_i = q_s\, \mathcal{F}^{\varphi,s}_{\mathcal{X}_i} \qquad \text{and} \qquad \forall s' \neq s: \quad q_{s'} = q_s\, \mathcal{F}^{\varphi,s}_{z_{s'}}.$$

This has the implication, that in evaluating public projects prices used to value factors purchased (or sold) in the market by the public sector should be producer prices (Diamond and Mirrlees, 1971, Little and Mirrlees, 1974). Again, we do not need to assume optimality of the tax schedule, i.e. optimality with respect to all directions $R(\cdot)$. We only need that Assumption 1 or the combination of Assumptions 2 and 3 are verified. However, as soon as neither Assumption 1 nor the combination of Assumptions 2 and 3 are verified, it is desirable to use a different price system for public firms as emphasized in Naito (1999).

B.6 The effects of business-focused environmental regulations

Consider now the scenario where the production sector is polluting, e.g. with carbon emissions and firms have the option to mitigate emissions by adopting cleaner technologies. Production policy consists in taxing carbon emissions. Here, intermediate good producers not only produce intermediate goods according to the production function $\mathcal{F}^{\varphi,s}(\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s}; \beta^{\varphi,s})$ but also emit carbon according to $\mathcal{E}^{\varphi,s}(\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s}; \beta^{\varphi,s})$ where $\beta^{\varphi,s}$ is the degree of cleanliness in the technology adopted by firm $\varphi \in \{1, ..., N_s\}$ in sector $s \in \{1, ..., S\}$. Employing more production factor increases both production and pollution, thus $\mathcal{F}^{\varphi,s}_{\mathcal{X}_i} > 0$ and $\mathcal{E}^{\varphi,s}_{\mathcal{X}_i} > 0$. Production is concave in $\beta^{\varphi,s}$ with a maximum at a level normalized to zero. Hence $\mathcal{F}^{\varphi,s}_{\beta} < 0$ if $\beta^{\varphi,s} > 0$ and $\mathcal{F}^{\varphi,s}_{\beta} > 0$ if $\beta^{\varphi,s} < 0$. Conversely, carbon emissions decrease when firms adopt greener technology, thus $\mathcal{E}^{\varphi,s}_{\beta} < 0$. For simplicity, we assume that intermediate good producers do not use intermediate goods or the final good as input.

We assume that the government can observe each firm's carbon emissions and tax them at a rate denoted by α. Assuming perfect competition and a constant returns to scale production functions, firm $\varphi \in \{1, ..., N_s\}$ in sector $s \in \{0, ..., S\}$ solves:

$$\max_{\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s}, \beta^{\varphi,s}} \qquad q_s\, \mathcal{F}^{\varphi,s}\left(\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s}; \beta^{\varphi,s}\right) - \sum_{i=1}^{n} p_i\, \mathcal{X}_i^{\varphi,s} - \alpha\, \mathcal{E}^{\varphi,s}\left(\mathcal{X}_1^{\varphi,s}, ..., \mathcal{X}_n^{\varphi,s}; \beta^{\varphi,s}\right).$$

This leads to the following first-order conditions:

$$\forall i \in \{1, ..., n\}: \qquad q_s\, \mathcal{F}^{\varphi,s}_{\mathcal{X}_i} = p_i + \alpha\, \mathcal{E}^{\varphi,s}_{\mathcal{X}_i} \qquad \text{and}: \qquad q_s\, \mathcal{F}^{\varphi,s}_{\beta} = \alpha\, \mathcal{E}^{\varphi,s}_{\beta} \tag{B.11}$$

As in B.3, each taxpayer produces a final good through the same production function, which is denoted $\mathcal{F}_0(\cdot)$. Moreover, pollution exerts a negative externality. Hence $\mathcal{F}_0$ is decreasing in aggregate emissions $\mathcal{E} \stackrel{\text{def}}{\equiv} \sum_{s=1}^{S} \sum_{\varphi=1}^{N_s} \mathcal{E}^{\varphi,s}\left(\mathcal{X}^{\varphi,s}; \beta^{\varphi,s}\right)$, so we have $\mathcal{F}^0\left(z_1, ..., z_S, \mathcal{E}\right)$, with $\mathcal{F}^0_{z_i} > 0 > \mathcal{F}^0_{\mathcal{E}}$. For tractability, we assume that the final good production function exhibits constant returns to scale with respect to intermediate goods consumption $(z_1, ..., z_S)$. This leads to the intermediate goods demand conditions:

$$\forall s \in \{1, ..., S\}: \qquad q_s = \mathcal{F}^0_{z_s}\left(z_1^0, ..., z_S^0, \mathcal{E}\right), \tag{B.12}$$

where

$$z_s^0 \stackrel{\text{def}}{\equiv} \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s}, \beta^{\varphi,s}\right)$$

denotes the total production of the s^{th} intermediate good.

The competitive allocation of resources within the production sector is the same as the one chosen by an hypothetical production coordinator whose program consists in:

$$\max_{\{\mathcal{X}^{\varphi,s},\beta^{\varphi,s}\}_{s=1,\dots,S}^{\varphi=1,\dots,N_s},z_1^0,\dots,z_S^0} \quad \mathcal{F}^0\left(z_1^0,\dots,z_S^0,\overline{\mathcal{E}}\right) - \alpha \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} \mathcal{E}^{\varphi,s}\left(\mathcal{X}^{\varphi,s};\beta^{\varphi,s}\right) + \alpha\,\overline{\mathcal{E}} \tag{B.13a}$$

$$\forall i \in \{1,\dots,n\}: \quad \mathcal{X}_i = \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} \mathcal{X}_i^{\varphi,s} \tag{B.13b}$$

$$\forall s \in \{1,\dots,S\}: \quad \sum_{\varphi=1}^{N_s} \mathcal{F}^{\varphi,s}\left(\boldsymbol{\mathcal{X}}^{\varphi,s},\beta^{\varphi,s}\right) = z_s^0. \tag{B.13c}$$

where the production coordinator takes aggregate emissions

$$\overline{\mathcal{E}} = \sum_{s=1}^{S}\sum_{\varphi=1}^{N_s} \mathcal{E}^{\varphi,s}\left(\mathcal{X}^{\varphi,s};\beta^{\varphi,s}\right) \tag{B.13d}$$

and carbon tax revenue $\alpha\,\overline{\mathcal{E}}$ as given.[24] Tax revenue $\alpha\,\overline{\mathcal{E}}$ enters the production coordinator's objective because it is distributed to all taxpayers in a lump-sum way.

For each vector of input supply, $(\mathcal{X}_1,\dots,\mathcal{X}_n)$, each carbon tax rate α and each carbon tax revenue $\alpha\ \mathcal{E}$, the inverse demands $\mathscr{P}_i(\mathcal{X}_1,\dots,\mathcal{X}_n;\alpha,\overline{\mathcal{E}})$ are defined as the Lagrange multipliers associated to constraints (B.13b) and the production function $\mathcal{F}(\mathcal{X}_1,\dots,\mathcal{X}_n;\alpha,\overline{\mathcal{E}})$ is the value function associated to program (B.13a)-(B.13c).

To determine the carbon tax that maximizes total production, one must choose α to maximize (B.13a), subject to (B.13b)-(B.13d) and taking into account the effects on aggregate emissions $\overline{\mathcal{E}}$. This implies that the carbon tax maximizing aggregate production satisfies the Pigouvian rule $\mathcal{F}^0_{\overline{\mathcal{E}}} = -\alpha$, which corrects for the externality.[25]

The Pigouvian rule $\mathcal{F}^0_{\overline{\mathcal{E}}} = -\alpha$ is optimal in two cases: (i) when the tax system satisfies Assumption 1, in which case Theorem 1 applies, or (ii) when both Assumptions 2 and 3 hold, in which case Theorem 2 applies. In both cases, the production regulation principle reduces to applying the Pigouvian rule $\mathcal{F}^0_{\overline{\mathcal{E}}} = -\alpha$. Conversely, if Assumption 1 and either 2 or 3 are violated, the optimal carbon tax must also account for the pre-distributive effects of taxation, as characterized by Equation (32) in Theorem 3.

[24] Denoting p_i the Lagrange multiplier associated to the i^{th} equation (B.13b) and q_s the Lagrange multiplier associated to s^{th} equation (B.13c), the first-order conditions of (B.13) with respect to $\mathcal{X}^{\varphi,s},\beta^{\varphi,s}$ and z_s^0 leads to (B.11) and (B.12). Since the production coordinator is constrained by the same resource constraints (B.13c) as the competitive economy, the production allocation chosen by the production coordinator coincides with that of the competitive economy. Finally, since revenue from carbon tax $\alpha\,\overline{\mathcal{E}}$ shows up in the production coordinator's objective (B.13a), the Walras Law ensures that the value function associated to the production coordinator's program (B.13) verifies the accounting equation (5).

[25] Applying the envelope theorem to Program (B.13) with r««««espect to α and taking (B.13d) into account leads formally to $\mathcal{F}_\alpha = 0$. Applying the envelope theorem with respect to $\overline{\mathcal{E}}$ leads to $\mathcal{F}_{\overline{\mathcal{E}}} = \mathcal{F}^0_{\overline{\mathcal{E}}} + \alpha$.