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Mandatory or Voluntary? Optimal Public Good Funding *

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Abstract

I study an economy where a government and heterogeneous individuals can dedicate part of their resources to the funding of public goods. What is the optimal combination between government and voluntary contributions? When individuals make different donation choices only because they have different income, relying on voluntary contributions is Pareto-improving. This is true independently of the efficiency and redistributive properties of the public goods to be funded. When donation choices are influenced by unobserved characteristics (e.g., altruism, ideology), I provide simple policy rules for deciding whether governments or private contributions should be the marginal funding source, based on the price elasticity of giving and the redistributive preferences of the government. In particular, I show that the commonly applied unit-elasticity rule can align with Rawlsian preferences. I study how these policy rules adapt in presence of a market failure (leaky donations) and a government failure (uniform tax subsidies for different causes). Numerical simulations using French data reveal that limiting tax discrimination across causes significantly increases government funding, as direct government contributions compensate for the lack of flexibility of the tax system. These funding rules apply for both optimal and arbitrary provision of public goods.

Keywords: Public good; pptimal taxation; charitable giving; grants; multidimensional heterogeneity

JEL-Codes: H21, H41, D64

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I Introduction

Government spending on public goods is substantial, accounting for 30% of GDP on average in the OECD in 2021.[1] These public services have played a key role in reducing poverty and inequality worldwide (Gethin, 2024). This paper examines how we should fund these valuable public goods in an unequal society.

The funding issue is not trivial, especially in a context of high public debt. In the meantime, private wealth has globally increased. In this context of "rich countries with poor governments" (Chancel et al., 2022), the contribution of the private sector to public good funding appears crucial. This can be achieved in two ways: first, by expanding tax revenue, enabling the government to spend more on public goods provision. This solution relies on mandatory contributions, as individuals are required to pay taxes. The second option is to rely on voluntary contributions, allowing individuals to directly fund public goods. In this case, governments could still participate in public good funding, but only indirectly, by providing incentives for private contributions. The closest real-life equivalent to voluntary contributions to public goods is charitable giving, which is supported by tax incentives in most OECD countries. While relying on voluntary contributions can be less distortive, the low levels of voluntary contributions in most OECD countries and the overrepresentation of wealthy individuals in giving behavior challenge the reliability of this second option.[2]

I analyze the optimal funding problem in a model where both governments and individuals can contribute to various public goods. Individuals are heterogeneous in many unobserved dimensions, creating inequality in both their ability to earn income, their willingness to give to different charitable causes and the utility they get from different public goods. In this second-best world, the government manipulates taxes on income, tax incentives to donations and public spending to maximize social welfare. The economy is large so that individuals neglect the impact of their own donations on public good levels: their donations are motivated by a privately observed warm-glow that can vary across charitable causes.

First, I show that absent transaction costs on donations and with the possibility to tax differently donations to different charitable causes, a public good funding strategy based on tax incentives to donations is likely to dominate a strategy based on direct government contributions. More precisely, if heterogeneity in donation patterns results only from heterogeneity in disposable income, then cutting the grants and relying only on tax incentives to fund public goods is Pareto-improving.Importantly, this result holds regardless of the properties of the public goods being funded and is robust to unobserved, heterogeneous preferences for different public goods. This implies that a public good that generates efficiency gains (e.g., complements labor) and primarily benefits disadvantaged groups (in terms of class, race, gender, or health), as well as a public good that causes efficiency losses (e.g., complements leisure) and primarily benefits the privileged, should both be funded through donations and tax incentives rather than direct government grants. Of course, all these efficiency and equity parameters have to be taken into account when setting the optimal level of these public goods. But they will determine the optimal level of tax incentives, not

[1] OECD (2023), excluding government expenditures on social protection.

[2] The US is an important outlier, where charitable giving from households represents more than 1% of GDP. For the luxury good aspect of charitable giving, see, for instance, (Evans et al., 2017). Several empirical studies actually document a U-shaped pattern in the relationship between income and giving (Hargaden and Duquette, 2024).

justify the use of direct government funding. This Pareto-efficiency condition may not hold when donation heterogeneity not only arises from income heterogeneity but also depends on unobserved characteristics (altruism, ability, ideology...). In this case, I study the social desirability of marginally increasing (or decreasing) the tax subsidy for a specific type of donation while adjusting the corresponding government grant to keep the public good level unchanged. This allows me to derive a (local) optimality condition that depends on three sufficient statistics: the (direct) price elasticity of this type of donation, the associated cross-base elasticities and welfare weights of donors. Absent redistributive motives, tax incentives still dominate grants as long as donations increase when the tax subsidy increase. In other words, tax incentives dominate as long as donations are not Veblen goods. With redistributive preferences, higher level of price elasticity are needed to justify an increase in tax subsidies. If the government is Rawlsian, this elasticity should be higher than 1 (in absolute term) to justify an increase in the tax subsidy, echoing the standard unit-elasticity rule (Feldstein et al. (1980),Roberts (1987), Saez (2004) or Fack and Landais (2010)).

Second, I study the consequence of transaction costs for the optimal funding policy. Unsurprisingly, leakage in private contribution can justify the introduction of direct government funding. If donation choices are income-driven and if leakage is high enough to justify a government grant, the policy rule for a Pareto-efficient policy becomes simple. The tax incentives to this type of donation should be a flat tax credit, equal to 1 net of the leakage.[3] Besides, if preferences for the corresponding public good are also income-driven, then the optimal grant should be set according to a standard Samuelson rule: increase the grant up to the point were the average MRS between the public good and private consumption is equal to 1. Calibrating a two public good economy populated by individuals heterogeneous in both ability and altruism, I show how leakage can justify relatively high level of government funding. In the extreme case, public goods are fully funded by the government, and donations are not subsidized but taxed.

Third, I consider the importance of a limited degree of tax discrimination across charitable causes. The Pareto-efficiency condition discussed above requires that the government can tax donations to different public goods differently. However, in practice, such tax discrimination is limited; most countries apply the same tax subsidy rate to all eligible charitable purposes.[4] In this case, direct funding can be used to compensate the lack of flexibility of the tax system. For example, suppose two public goods, healthcare and education, require the same level of funding, but individuals are more willing to donate to healthcare than to education. With tax discrimination, the government could set a higher tax subsidy for donations to education. However, when constrained to impose the same tax treatment on both types of donations, relying solely on tax incentives would result in either too little funding for education or too much for healthcare. Direct government funding is then necessary to make the appropriate adjustments. To explore this idea, I present a formula to evaluate the social desirability of reforming taxes and grants when tax discrimination is absent. In my numerical exercise, I provide a scenario where the share of direct government funding increases from 0 to 40% when tax discrimination is not allowed.

Fourth, I describe the sufficient statistics required to evaluate reforms of any

[3] In France, the tax credit for donation is 66%. We can rationalize this system with a leakage of 34% (and income-driven donations).

[4] There are exceptions, such as Italy offering a 65% tax credit for cultural donations or France providing a 75% tax credit for limited amounts given to certain social assistance charities.

arbitrary tax system that depends on income and donations. Such a formula can be used to assess the (local) desirability of reforming both tax credit systems (Canada, France, New Zealand) and tax deduction systems (US, Germany, Australia). Such tax incidence formula can be evaluated in three contexts. First, it can be assumed that the government does not optimize its grants provision. In this case, as soon as the tax reform, of either income or donation tax schedule, has an impact on donation behavior, standard tax incidence analysis has to be augmented by an externality term. This externality term depend on the social value of the donation (through the change in the corresponding public good level) and on the endogenous reaction of the donation vector to changes in the public good vector. Second, the tax reform can be evaluated assuming that the government optimizes grants and that grants are needed at the optimum. In this case, the externality parameter takes a simple value of 1 net of leakage. Third, the tax reform can be evaluated for a given level of public good provision. To do so, I construct grant reforms that neutralize the impact of donations on public good provision. This exercise is particularly suited to measure the positive externality of donations relative to the implicit reduction in government spending they enable. Specifically, by focusing on a fixed level of public goods, this approach avoids the need to elicit public good preferences and to deal with the endogenous response of donation levels to changes in public good provision.

In its broad interpretation, this paper deals with the optimal funding of public goods in general. In a more narrow understanding of the model, the paper provides policy recommendation on the optimal mix between private donations and government grants to charities. Indeed, in practice, charities receive money from both individuals and from the government. In countries such as France government grants generally contribute more to the funding of non-profits than individual donations.[5] This paper can provide tools for adjusting the balance between government and donors across different sectors of nonprofits.

Related literature. This paper first relates to the literature on the optimal provision of public goods in presence of distortive tax instruments (Atkinson and Stern (1974), Boadway and Keen (1993),Kaplow (1996)). Because taxes are distortive, providing public goods raise additional efficiency and equity concerns that were not taken into account in the lump-sum tax design of Samuelson (1954). Additional distortions occur when not only governments but also individuals can directly contribute to public good provision (Bergstrom et al. (1986),Blömquist and Christiansen (1998), Saez (2004), Diamond (2006), Aronsson et al. (2024)). My contribution to this literature is twofold. First, I allow for both multidimensional heterogeneity in individuals unobserved characteristics and for nonlinear taxation. This approach allows me to investigate whether deviations from the Samuelson rule are driven by the complexity of individual behavior or limitations in government instruments. Second, I derive policy prescriptions when public good provision is optimal and when it is not. This enables me to investigate whether the policy rules provided by the literature were driven by the design of an optimal provision of public goods or by the design of an optimal funding strategy of a given public goods set.

Because donations to nonprofits are the closest equivalent to voluntary contributions to public goods, I also contribute to the literature on the optimal tax treatment of charitable giving. Assuming a specific form of altruism where the rich care about the poor, Atkinson (1976) derives optimal subsidy formulas for donations. More

[5] An this varies across sectors: grants are equal to households donations for activism nonprofits but 12 times higher for arts nonprofits (Insee, 2018)

recent contributions follow Andreoni (1989, 1990) and assume, as in the present paper, a warm glow motive of giving. Using linear tax instruments, Saez (2004) derives optimal tax formulas in terms of sufficient statistics and redistributive preferences. Using a two-type model with fixed hours of work and additive preferences, Diamond (2006) provides optimal policy analysis, describing how nonlinear subsidies of charitable giving can improve welfare by relaxing incentive compatibility constraints. Introducing charitable giving in Mirrlees (1971), Aronsson et al. (2024) considers the optimal nonlinear tax problem in an economy populated by a continuum of individuals heterogeneous in ability. My main contribution to this strand of the literature is to consider both nonlinear taxation and multidimensional heterogeneity. This again helps in understanding the implications of constraints on individuals' or governments' behavior for optimal policy. For instance, I show that allowing for nonlinear income taxation, even with linear subsidies to donations, can justify a greater use of donations: reducing grants and adjusting both the nonlinear income tax and a linear tax credit can be Pareto-improving, independently of the properties of the public goods to be funded. Another contribution is to consider different types of donations, funding different types of public goods. As one can deduce from Aronsson et al. (2024), most policy prescriptions obtained in a singe good model can extend to a multiple good setting. However, this is no longer true when tax discrimination across causes it limited. Using a reform-based approach, as in Saez (2001), allows me to emphasize the consequence of this empirically plausible limit on government tax instruments. While the model presented here nests most of the important features of this literature, there are aspects that are not taken into account. First, unlike Aronsson et al. (2024), I do not examine the effects of a non-welfarist social welfare function or donations motivated by status considerations. Second, while I allow for leaky private contributions, I consider, as Saez (2004), Diamond (2006) and Aronsson et al. (2024), that this leakage is exogeneous. Muñoz-Sobrado (2023) goes beyond this exogenous leakage assumption by allowing for endogenous cost of fundraising that can interact with government's policy towards the charitable sector.

Eventually this paper also falls within the multidimensional optimal tax literature. Using the tax perturbation approach initiated by Piketty (1997) and Saez (2001) and recently extended by Hendren (2019), Sachs et al. (2020) and Jacquet and Lehmann (2021), I include public goods and charitable givings in a framework with multidimensional unobserved heterogeneity of taxpayers and nonlinear tax instruments. To the best of my knowledge, this paper is the first to feature these two elements in such a general optimal tax framework. In particular, I show that the assumed positive effect of charitable contributions on social welfare enters additively in the optimal nonlinear tax formulas for donations. This additivity is reminiscent of the result of Sandmo (1975) when studying optimal taxation in presence of externalities. Saez (2004) has already noted that this additive property is part of the optimal linear subsidy on charitable contribution. I therefore extend this result to the case of nonlinear tax instruments in a multiple-good environment. This additive property can simplify tax incidence analysis in presence of an externality, even when tax instruments can take arbitrarily complex forms.

The paper is organized as follows. I introduce the general framework in Section II. I study Pareto-efficient funding strategies under separability assumptions in Section III. I consider the social desirability of reforming a funding strategy under general preferences in Section IV. The consequences of limiting tax discrimination are discussed in Section V. I illustrate these results using numerical simulations in Section V. Section VII concludes.

II General Framework

II.1 Taxpayers' program

The economy consists of a unit mass of heterogeneous taxpayer and of a government.

Taxpayers can differ in many individual characteristics summarized in a type vector $\boldsymbol{\theta} = (\theta_1, \theta_2 ... \theta_n) \in \Theta$, where Θ is convex. Types are distributed according to a continuously differentiable density function $f : \boldsymbol{\theta} \mapsto f(\boldsymbol{\theta})$. Importantly, types are only privately observed so that the government cannot directly target these individual characteristics with its policy instruments.

There exists a variety of public goods G_i summarized in a public good vector $\mathbf{G} = (G_1, ..., G_n)$. Taxpayers can decide to fund each of these public goods by making a donation b_i, represented by a donation vector $\mathbf{b} = (b_1, ..., b_n)$. An individual with type $\boldsymbol{\theta}$ chooses labor income y, private good consumption c and $\mathbf{b}$ to maximize a twice continuously differentiable utility function $\mathcal{U} : (c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) \mapsto \mathcal{U}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})$.

I assume that taxpayers enjoy private consumption (hence $U_c > 0$) and public good consumption (hence $U_{G_1}, U_{G_0} > 0$). Besides, they can enjoy the act of giving (hence $U_b > 0$), through the warm glow motive described in Andreoni (1989, 1990). On the other hand, earning labor income y requires an effort so that $U_y < 0$. Importantly, agents take the level of public goods as given when making their donation decisions. The latter is equivalent to assume that taxpayers are small so that they neglect the impact of their individual contribution to $\mathbf{G}$.[6]

The marginal rate of substitution (MRS) between the private good consumption c and labor income y is given by:

$$\mathbf{M^y}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} -\frac{\mathcal{U}_y(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})}{\mathcal{U}_c(c, y, \mathbf{b}; \mathbf{G}, `)} \tag{1}$$

The MRS between private good consumption c and donations b_i is given by

$$\mathbf{M^{b_i}}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \frac{\mathcal{U}_{b_i}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})}{\mathcal{U}_c(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})} \tag{2}$$

And the MRS between private good consumption c and public good G_i is given by:

$$\mathbf{M^{G_i}}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \frac{\mathcal{U}_{G_i}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})}{\mathcal{U}_c(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta})} \tag{3}$$

The government can tax (or subsidize) labor income y and donations $\mathbf{b}$ through the non-linear tax schedule $T : (y, \mathbf{b}) \mapsto T(y, \mathbf{b})$. The individual's budget constraint

[6] This hypothesis is standard in the optimal tax literature on charitable contribution (see Saez (2004), Diamond (2006) or Aronsson et al. (2024)).

therefore implies $c + \sum_{i=1}^{n} b_i = y - T(y, \mathbf{b})$. Hence an agent with type $\boldsymbol{\theta}$, taking $T(.)$ and $\mathbf{G}$ as given, solves :

$$U(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \max_{y, b_1, \ldots, b_n} \quad \mathcal{U}\left(y - \sum_{i=1}^{n} b_i - T(y, \mathbf{b}), y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}\right) \tag{4}$$

The solution of (4) is denoted $\{y(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta})\}$.

The government can provide a direct grant to public goods. I denote by S_i this governmental subvention to public good G_i. Besides, there can be discrepancy between the amount donated and the actual amount of charitable contributions funding the public good. I denote by μ_i this exogenous "fundraising cost" associated to charity good i. The level of the charity good i is therefore given by:

$$G_i = S_i + (1 - \mu_i) \int_{\Theta} b_i(\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{5}$$

II.2 The Government's program

The government levies taxes to finance the grants S_i and to redistribute resources across agents. Its budget constraint therefore takes the form :

$$\int_{\Theta} T\left(y(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta})\right) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \geq \sum_{i=1}^{n} S_i \tag{6}$$

I suppose that the objective of the government is to maximize the sum over all types θ of a function $\Phi : (U, \theta) \mapsto \Phi\left(U\left(\boldsymbol{\theta}\right), \theta\right)$.

$$SW \stackrel{\text{def}}{\equiv} \int\limits_{\Theta} \Phi\left(U\left(\boldsymbol{\theta}\right); \boldsymbol{\theta}\right) f\left(\boldsymbol{\theta}\right) d\boldsymbol{\theta} \tag{7}$$

Hence the problem of the government is to maximize the generalized social welfare function define in (7) subject to the budget constraint (6). I constrain $\Phi(.)$ to be increasing in individual utility $U(.)$, and to be strictly increasing for at least one type θ. Assuming that $\Phi(.)$ can depend directly on θ allows me to cover a wide range of welfare criteria. For instance, $\Phi(U; \theta) \equiv \phi(\theta) U$, where weights $\phi(\theta)$ directly depend on type θ embeds *weighted utilitarists* views of justice in my framework. Hence standard *utilitarianism* is obtained when $\phi(\theta) = 1$ while a *Rawlsian* objective arises when $\phi(\theta) = 0$ except for the lowest type $\underline{\theta}$ with $\phi\left(\underline{\theta}\right) > 0$.

The problem of the government is therefore to maximize (7) subject to the budget constraint (6). Let λ denote the Lagrange multiplier associated to (6). The problem of the government can therefore be written in monetary units through the Lagrangian.

$$\mathcal{L} = \int_{\Theta} \left[T\left(y(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta})\right) - \sum_{i=1}^{n} S_i + \frac{1}{\lambda} \Phi\left(U\left(\boldsymbol{\theta}\right); \boldsymbol{\theta}\right) \right] dF(\boldsymbol{\theta}) \tag{8}$$

III Pareto improvements under Separable Preferences

The problem here is to find a Pareto condition on the efficient use of tax incentives and direct grants to fund public goods. In particular, I want to know whether both tax incentives and grants are actually needed to guarantee Pareto efficiency. A necessary condition for Pareto efficiency is that there exists no reform that increase tax revenue while leaving taxpayers utility unchanged. Indeed, if such reforms were available, the additional tax revenue could be redistributed in a lump-sum fashion and yield a Pareto-improvement.[7]

Consider a baseline policy mix $\{T(y, \mathbf{b}), \mathbf{S}\}$. Individual optimization under this regime yields for all $\boldsymbol{\theta}$-type a baseline allocation $\{c(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{G}\}$ and a baseline utility level $U(\boldsymbol{\theta})$. Combining public good definition (5) with the individual's budget constraint, government revenue obtained under this baseline allocation is given by:

$$\int_{\Theta} T\left(y(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta})\right) dF(\boldsymbol{\theta}) - \sum_{i=1}^{n} S_i = \int_{\Theta} \left[y(\boldsymbol{\theta}) - c(\boldsymbol{\theta}) - \sum_{i=1}^{n} \mu_i b_i(\boldsymbol{\theta}) \right] dF(\boldsymbol{\theta}) - \sum_{i=1}^{n} G_i \quad (9)$$

A necessary condition for the Pareto-efficiency of this baseline regime $\{T(y, \mathbf{b}), \mathbf{S}\}$ is that the allocation $\{c(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{G}\}$ maximizes (9) for a given set of utility level $U(\boldsymbol{\theta})$. To check this Pareto requirement, we first need to define the feasible alternatives to the baseline regime $\{T(y, \mathbf{b}), \mathbf{S}\}$. In this section, the government is unconstrained in its tax policy instruments so that $T(y, \mathbf{b})$ can depend on income and each type of donations b_i, potentially in nonlinear ways. So the only constraints that the government has to deal with are informational constraints created by private information on individual's type $\boldsymbol{\theta}$ and the resource constraint. Using the revelation principle, the set of alternative policy coincides with the set of incentive-compatible allocations. Formally an allocation $\{c(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{G}\}$ is incentive compatible if it verifies for all $\boldsymbol{\theta}, \boldsymbol{\theta}'$:

$$\mathcal{U}\left(c(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{b}(\boldsymbol{\theta}); \mathbf{G}, \boldsymbol{\theta}\right) \geq \mathcal{U}\left(c(\boldsymbol{\theta}'), y(\boldsymbol{\theta}'), \mathbf{b}(\boldsymbol{\theta}'); \mathbf{G}, \boldsymbol{\theta}\right) \quad (10)$$

The taxation principle (Hammond (1979)) implies that any feasible allocation verifying (10) can be implemented by a policy mix $\{T(.), \mathbf{S}\}$. Maximizing government revenue (9) among the set of allocations verifying the incentive compatibility constraints (10) without changing individual utility would yield a necessary condition for Pareto optimality. To derive explicitly such a condition, I now study special cases of the individual utility function described in (4) that verify some form of separability assumptions

First suppose that preferences for some type of donations are identical across types $\boldsymbol{\theta}$ while others can depend on unobserved individual characteristics. Donations verifying such taste homogeneity are called *consensual* and summarized in a vector $\mathbf{b}^{cs} = b_1^{cs}, ..., b_m^{cs}$. If all donations were consensual, individuals with the same disposable income would make the same donation choices. On the other hand, donations that can depend on individual unobserved characteristics are called *controversial* and

[7] The reasoning applied here is similar to the extension of Atkinson and Stiglitz (1976) provided by Konishi (1995),Laroque (2005) and Kaplow (2006).

summarized in a vector denoted $\mathbf{b}^{ct} = b^{ct}_{m+1}, ..., b^{ct}_{n}$. Second, suppose that individual utility can now be represented as:

$$\mathcal{U}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) = U\left(V\left(c, \mathbf{b}^{cs}\right), \mathbf{b}^{ct}, y; \mathbf{G}, \boldsymbol{\theta}\right) \tag{11}$$

with $V(.)$ a strictly concave, twice continuously differentiable function, strictly increasing in $c, b^{cs}_1, ...b^{cs}_m$. Under (11), preferences for private good consumption c and consensual donations $\mathbf{b}^{cs}$ are (weakly) separable from preferences for both leisure and public goods.

I show in Appendix A.1 that maximizing government revenue (9) while leaving utility, labor supply and public good provision unchanged implies solving this cost minimization program:

$$\begin{aligned} &\min_{c(\boldsymbol{\theta}), b^{cs}_1(\boldsymbol{\theta}), ..., b^{cs}_n(\boldsymbol{\theta})} \int_{\Theta} \left(c(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i b^{cs}_i(\boldsymbol{\theta}) \right) dF(\boldsymbol{\theta}) \\ &\text{subject to : } V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})\right) = \mathcal{V}(\boldsymbol{\theta}), \forall \boldsymbol{\theta} \\ &\qquad G^{cs}_i - (1 - \mu_i) \int_{\boldsymbol{\theta}} b^{cs}_i(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) \geq 0, \forall i \end{aligned} \tag{12}$$

Proposition 1. *If b_i is a consensual donation verifying (11), then:*

i) In absence of leakage ($\mu_i = 0$), setting $S_i = 0$ is Pareto-improving.

ii) If μ_i is large enough so that $S_i > 0$, a Pareto-efficient tax system must verify:

$$T_{b_i}(y, \boldsymbol{b}) = -1 + \mu_i \tag{13}$$

The proof is given in Appendix A.2. The first part of Proposition 1 shows that, as long as donations are consensual and as long as there is no leakage, the government should not provide direct funding to the public goods. Importantly, this result is true independently of the preferences for the public goods to be funded. Suppose that a public good G_i is complementary to work effort and directly benefits more to privileged individuals. On the opposite, suppose that a public good G_j is complementary to leisure, directly benefits more to disadvantaged individuals and on top of this mainly receives donations from high-income earners. In other words, G_i has better efficiency and redistributive properties than G_j. Still, the first part of Proposition 1 implies that is does not justify a higher governmental grants for G_i than for G_j since the grant should be zero for both. The second part considers the case where donations are leaky such that it becomes desirable for the government to directly fund public goods. Consider again the efficient-redistributive public good G_i and the inefficient-unequal public good G_j. Then, if leakage is the same in both sectors such that $\mu_i = \mu_j$, the tax subsidy to donations to these two goods should be the same. Besides, this tax subsidy should be flat so that it should not take into account that rich individual make more donations to G_j than to G_i: the only reason to tax discriminate between b_i and b_j comes from differences in leakage, *i.e* $\mu_i \neq \mu_j$.

Now suppose that utility takes the form:

$$\mathcal{U}(c, y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}) = U\left(V\left(c, \mathbf{b}^{cs}, \mathbf{G}^{cs}\right), \mathbf{b}^{ct}, y; \mathbf{G}^{ct}, \boldsymbol{\theta}\right) \tag{14}$$

where $\mathbf{G}^{cs}$ denotes the vector of public goods receiving consensual donations and $\mathbf{G}^{ct}$ the vector of public goods receiving controversial donations. Contrary to (11), this implies that public goods receiving consensual donations are themselves consensual *i.e* they also verify taste homogeneity and labor separability.

Proposition 2. *If individual preferences verify* (14)*, then:*

i) A Pareto-efficient provision of a consensual public good G_i must verify:

$$\int_{\boldsymbol{\theta}} \mathbf{M}^{G_i}\left(c(\boldsymbol{\theta}), \boldsymbol{b}^{cs}(\boldsymbol{\theta}), \boldsymbol{G}^{cs}\right) dF(\boldsymbol{\theta}) = 1 - \frac{1}{1-\mu_i}\left(\mathbf{M}^{b_i}\left(c(\boldsymbol{\theta}), \boldsymbol{b}^{cs}(\boldsymbol{\theta}), \boldsymbol{G}^{cs}\right) - \mu_i\right) \tag{15}$$

ii) If μ_i is large enough so that $S_i > 0$, this optimality condition boils down to :

$$\int_{\boldsymbol{\theta}} \mathbf{M}^{G_i}\left(c(\boldsymbol{\theta}), \boldsymbol{b}^{cs}(\boldsymbol{\theta}), \boldsymbol{G}^{cs}\right) dF(\boldsymbol{\theta}) = 1 \tag{16}$$

iii) If $\mu_i = 0$ so that $S_i = 0$, a Pareto-efficient tax treatment of consensual donations must verify:

$$T_{b_i}(y, \boldsymbol{b}) = -\int_{\boldsymbol{\theta}} \mathbf{M}^{G_i}\left(c(\boldsymbol{\theta}), \boldsymbol{b}^{cs}(\boldsymbol{\theta}), \boldsymbol{G}^{cs}\right) dF(\boldsymbol{\theta}) \tag{17}$$

Proposition 2 implies that in absence of leakage, the optimal public good provision should be higher than under a standard Samuelson rule. However, if government funding is needed because of high levels of leakage, then the standard Samuelson rule applies.

IV Social desirability under General Preferences

In this section, I consider the social desirability of reforming an arbitrary tax system $T(y, \mathbf{b})$ and grant vector $\mathbf{S}$ in order to fund a given public good vector $\mathbf{G}$. Contrary to the previous section where I delivered necessary conditions for Pareto optimality, I here consider necessary conditions for optimality that can depend on social preferences. Besides, I no longer impose separability assumptions nor constrain the impact of heterogeneous preferences on donations. In particular I now also consider the social desirability of reforming the tax and grant treatment of *controversial* donations.

IV.1 Social Impact of Tax Reforms

I first need to assess the social impact of reforming the tax schedule for a given grant vector $\mathbf{S}$. To do so, I use the perturbation approach initiated by Piketty (1997), Saez (2001) and recently generalized by Sachs et al. (2020) and Jacquet and Lehmann (2021).

Definition 1. *Starting from an initial tax schedule* $T : (y, \boldsymbol{b}) \mapsto T(y, \boldsymbol{b})$*, a tax reform replaces* $T(.)$ *by a new schedule* $\widetilde{T} : (y, b, t) \mapsto \widetilde{T}(y, \boldsymbol{b}, t)$*, with* $t \in \mathbb{R}$ *a scalar measuring the magnitude of the reform.*

Under a reformed tax schedule $\widetilde{T}(.)$, a taxpayer with type $\boldsymbol{\theta}$ enjoys utility :

$$\widetilde{U}(\boldsymbol{\theta}, t) \overset{\text{def}}{\equiv} \max_{y, b_1, \ldots, b_n} \quad \mathcal{U}\left(y - \sum_{i=1}^{n} b_i - \widetilde{T}(y, \mathbf{b}, t), y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}\right) \tag{18}$$

The social desirability of a tax reform depends on the responses of taxpayers. To compute these behavioral responses, I apply the implicit function theorem to the first-order conditions of taxpayers' problem (18). To do so, I impose the following restriction on individual's preferences and the tax function :

Assumption 1.

- *The tax function* $T(.)$ *is twice continuously differentiable.*
- *The second-order conditions associated to* (18) *hold strictly.*
- *Problem* (18) *admits a unique global maximum.*

Assumption 1 corresponds to the sufficient conditions for the tax perturbation approach derived in Assumption 2 of Jacquet and Lehmann (2021). In particular, it prevents any jump in individual decisions after a small tax reform of magnitude t.

Tax reforms can trigger changes in both labor income and donations, through income and substitution effects. I denote by $\frac{\partial x}{\partial \rho}$ the response of variable x (income or donations) due to income effects and by $\frac{\partial x}{\partial \tau_z}$ the compensated response of x to a change in the marginal net-of-tax rate on z. Both income and compensated responses can be measured by studying the impact of lump sum and compensated tax reforms. The details are given in Appendix B.1 where I show that the response of labor income and donations b_i to any reform of magnitude t, evaluated at a given level of public good $\mathbf{G}$ verify:

$$\frac{\partial y(t, \mathbf{G})}{\partial t} = -\frac{\partial y}{\partial \rho} \frac{\partial \widetilde{T}(y, \mathbf{b}, t)}{\partial t} - \frac{\partial y}{\partial \tau_y} \frac{\partial \widetilde{T}_y(y, \mathbf{b}, t)}{\partial t} - \sum_{j=1}^{n} \frac{\partial y}{\partial \tau_{b_j}} \frac{\partial \widetilde{T}_{b_j}(y, \mathbf{b}, t)}{\partial t} \tag{19a}$$

$$\frac{\partial b_i(t, \mathbf{G})}{\partial t} = -\frac{\partial b_i}{\partial \rho} \frac{\partial \widetilde{T}(y, \mathbf{b}, t)}{\partial t} - \frac{\partial b_i}{\partial \tau_y} \frac{\partial \widetilde{T}_y(y, \mathbf{b}, t)}{\partial t} - \sum_{j=1}^{n} \frac{\partial b_i}{\partial \tau_{b_j}} \frac{\partial \widetilde{T}_{b_j}(y, \mathbf{b}, t)}{\partial t} \tag{19b}$$

These direct responses holding $\mathbf{G}$ constant neglect the impact of tax reforms on public good provision. As soon as donations react to tax reforms, this mechanically changes $\mathbf{G}$ through (5). But then individuals can adjust their behavior to these changes in $\mathbf{G}$ and in particular can changes their donation level, which then affect public goods level and so on. Let Ξ denotes the $n \times n$ matrix capturing these direct and cross-base responses of average donations to public goods variations:

$$\Xi \stackrel{\text{def}}{\equiv} \begin{pmatrix} \frac{\partial B_1(\mathbf{G})}{\partial G_1} & \dots & \frac{\partial B_1(\mathbf{G})}{\partial G_n} \\ \vdots & \dots & \vdots \\ \frac{\partial B_n(\mathbf{G})}{\partial G_1} & \dots & \frac{\partial B_n(\mathbf{G})}{\partial G_n} \end{pmatrix} \tag{20}$$

with $B_i = \int_{\boldsymbol{\theta}} b_i(\boldsymbol{\theta}) dF(\boldsymbol{\theta})$ denoting the average donations to cause i.

As one can see from the public good funding equation, to translate changes in donations B_i to changes in public good level G_i, one needs to account for the potential leakage μ_i. Let $\mathcal{X}$ denotes the diagonal matrix capturing this leakage effect:

$$\mathcal{X} \stackrel{\text{def}}{\equiv} \begin{pmatrix} \frac{1}{1-\mu_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{1}{1-\mu_n} \end{pmatrix} \tag{21}$$

Assumption 2. *The matrix $\mathcal{X} - \Xi$ is invertible.*

Under Assumption 2, the response of public good G_i to a tax reform of magnitude t is given by:

$$\frac{\partial G_i(t)}{\partial t} = -\sum_{j=1}^{n} \pi_{i,j} \int_{\theta} \left[\frac{\partial b_j}{\partial \rho} \frac{\partial \widetilde{T}}{\partial t} + \frac{\partial b_j}{\partial \tau_y} \frac{\partial \widetilde{T}_y}{\partial t} + \sum_{k=1}^{n} \frac{\partial b_j}{\partial \tau_{b_k}} \frac{\partial \widetilde{T}_{b_k}}{\partial t} \right] dF(\boldsymbol{\theta}) \tag{22}$$

with $\pi_{i,j}$ the term on the i^{th} line and j^{th} column of the $n \times n$ multiplier matrix $\Pi \stackrel{\text{def}}{\equiv} (\mathcal{X} - \Xi)^{-1}$.

The proof is given in Appendix B.2. To summarize, (22) allows to translate the "micro" change in public good provision resulting from a change in donations to a "macro" change, taking into account the endogeneity of $\mathbf{G}$. To achieve this, one has to multiply the direct response of donations to the tax reform by a coefficient $\pi_{i,j}$ that accounts for the circular relationship between changes in donations and changes in public goods levels.

Equations (19) and (22) describe the responses of endogeneous variables to tax reforms. To assess the social impact of these reforms, we need to measure how these responses feed into both welfare and tax revenue.

To capture the direct welfare impact of tax reforms, I follow Saez (2001) and define marginal social welfare weight as :

$$g(\boldsymbol{\theta}) \stackrel{\text{def}}{\equiv} \frac{\Phi_U \left(U(\boldsymbol{\theta}) ; \theta \right) \; \mathcal{U}_c \left(c, \mathbf{b}, y; \mathbf{G}, \boldsymbol{\theta} \right)}{\lambda} \tag{23}$$

The parameter $g(\boldsymbol{\theta})$ measures the welfare gain in money metric of giving an extra unit of consumption to taxpayers of type θ. I show in Appendix B.3 that if we ignore the public good effect of donations, *i.e* if we hold **G** constant, the social impact of a tax reform of magnitude t is given by:

$$\frac{\partial \widetilde{\mathcal{L}}(t, \mathbf{G})}{\partial t} = \int_\Theta \left[1 - g(\theta) - \frac{\partial y(\boldsymbol{\theta}, \mathbf{G})}{\partial \rho} T_y - \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \mathbf{G})}{\partial \rho} T_{b_i}\right] \frac{\partial \widetilde{T}(y, \mathbf{b}, t)}{\partial t} dF(\theta)$$

$$- \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta}, \mathbf{G})}{\partial \tau_y} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \mathbf{G})}{\partial \tau_y} T_{b_i}\right] \frac{\partial \widetilde{T}_y(y, \mathbf{b}, t)}{\partial t} dF(\theta) \tag{24}$$

$$- \sum_{j=1}^{n} \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta}, \mathbf{G})}{\partial \tau_{b_j}} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \mathbf{G})}{\partial \tau_{b_j}} T_{b_i}\right] \frac{\partial \widetilde{T}_{b_j}(y, \mathbf{b}, t)}{\partial t} dF(\theta)$$

If donations were standard goods that do not generate an externality, measuring (24) would be enough to measure the social impact of tax reforms. The first line describes both welfare and revenue impact of changing tax liability $T(.)$. The second and third lines describe the revenue impact of changing the marginal income tax rate $T_y(.)$ or the marginal tax rate on donation j $T_j(.)$. Both direct responses (how an outcome reacts to its own marginal tax rate) and cross-base responses (how an outcome varies to changes in other marginal tax rates) have to be taken into account to assess the impact of the reform on tax revenue.

Let η_j denotes the externality associated to a donation to a public good j, taking into account all cross-effects between donations level and public goods level.

$$\eta_j \stackrel{\text{def}}{\equiv} \sum_{i=1}^{n} \pi_{i,j} \int_\Theta \left[g(\theta) M^{G_i}(\boldsymbol{\theta}, \mathbf{G}) + \frac{\partial y(\boldsymbol{\theta}, \mathbf{G})}{\partial G_i} T_y + \sum_{k=1}^{n} \frac{\partial b_k(\boldsymbol{\theta}, \mathbf{G})}{\partial G_i} T_{b_k}\right] dF(\boldsymbol{\theta}) \tag{25}$$

We can now measure the total impact of any tax reform, taking into account the endogenous response of public goods.

Proposition 3. *i) The (total) impact of a tax reform of magnitude t on the government Lagrangian* (8) *is given by:*

$$\frac{\partial \widetilde{\mathcal{L}}(t)}{\partial t} = \int_\Theta \left[1 - g(\boldsymbol{\theta}) - \frac{\partial y(\boldsymbol{\theta})}{\partial \rho} T_y - \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \rho} (T_{b_j} + \eta_j)\right] \frac{\partial \widetilde{T}(y, \boldsymbol{b}, t)}{\partial t} dF(\boldsymbol{\theta})$$

$$- \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_y} T_y + \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \tau_y} (T_{b_j} + \eta_j)\right] \frac{\partial \widetilde{T}_y(y, \boldsymbol{b}, t)}{\partial t} dF(\boldsymbol{\theta}) \tag{26}$$

$$- \int_\Theta \sum_{k=1}^{n} \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_{b_k}} T_y + \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \tau_{b_k}} (T_{b_j} + \eta_j)\right] \frac{\partial \widetilde{T}_{b_k}(y, \boldsymbol{b}, t)}{\partial t} dF(\theta)$$

ii) If the Lagrange multiplier associated to the government budget constraint (6) *verifies:*

$$\int_{\Theta} g(\boldsymbol{\theta})dF(\theta) = \int_{\Theta}\left[1 - \frac{\partial y(\boldsymbol{\theta},\mathbf{G})}{\partial \rho}T_y - \sum_{j=1}^{n}\frac{\partial b_j(\boldsymbol{\theta},\mathbf{G})}{\partial \rho}\left(T_{b_j}+\eta_j\right)\right]dF(\boldsymbol{\theta}) \tag{27}$$

then a reform of magnitude $t > 0$ (< 0) *combined with a lump-sum transfer to balance the budget is socially desirable if* $\frac{\partial \tilde{\mathcal{L}}(t)}{\partial t} > 0$ (< 0).

The proof is given in Appendix B.4. The first part of Proposition 3 allows to evaluate the impact of any small reform of a given tax schedule $T(y, \mathbf{b}$. Such a reform is not necessarily budget-balanced. The second part guarantees that a reform with a positive impact on the government Lagrangian combined with a lump-sum transfer to balance the government budget is socially desirable. Compared to the results obtained in Section III, Proposition 3 is valid under general individual preferences, as defined in 4. In particular, it can be used in context where individuals have both unobserved heterogeneous tastes for public goods as well as for donations.

Tax reforms evaluated through (26) take the governmental grant vector $\mathbf{S}$ as given. However, as shown in Section III, the relationship between these two policy instruments is key to understand the optimal way of funding public goods. In particular, when both taxes and the grant vector are optimized, grants might not be desirable, as shown in Proposition 1 and 2.

IV.2 Social Impact of Joint Reforms of Taxes and Grants

I now study the impact of reforming both the tax schedule $T(y, \mathbf{b})$ and the grant vector $\mathbf{S}$. Using a reform-based approach allows me to study this joint optimization problem without constraining individual preferences as in Section III.

It follows from the definition of public good G_i given in (5) that changing the level of the grant S_i directly affects the level of G_i. This direct change in G_i can then affect not only donations given to this specific public good b_i but also labor income y and the whole vector of donations $\mathbf{b}$. This can change the whole vector of public goods $\mathbf{G}$, then changing labor income and donations and so on. I show in Appendix B.5 that a reform of the public grant S_j changes the level of the public good G_i through:

$$\frac{\partial G_i}{\partial S_j} = \frac{\pi_{i,j}}{1-\mu_j} \tag{28}$$

Grant reform not only affect public good provision but also tax revenue, through the direct cost of increasing a grant and through the indirect cost created by labor and donation responses to a change in the level of public goods. As shown in Appendix B.5, the total social impact of increasing a grant S_j is given by:

$$\frac{\partial \mathcal{L}(S_j)}{\partial S_j} = \frac{\eta_j}{1-\mu_j} - 1 \tag{29}$$

Assuming that S_j is strictly positive at the optimum, (29) defined a local optimality condition on grants. Hence the following Proposition:

Proposition 4. *i) If $S_j > 0$ at the optimum, then the grant should be set such that:*

$$\eta_j = 1 - \mu_j \tag{30}$$

ii) If $S_j > 0$ at the optimum $\forall j \in \{1:n\}$, then the incidence of a tax reform of magnitude t, evaluated at the optimal grant vector $\mathbf{S}$ is given by:

$$\begin{aligned}\frac{\partial \widetilde{\mathcal{L}}(t)}{\partial t} = &\int_{\Theta} \left[1 - g(\boldsymbol{\theta}) - \frac{\partial y(\boldsymbol{\theta})}{\partial \rho} T_y - \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \rho}(1 - \mu_j + T_{b_j})\right] \frac{\partial \widetilde{T}(y, \boldsymbol{b}, t)}{\partial t} dF(\boldsymbol{\theta}) \\ &- \int_{\Theta} \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_y} T_y + \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \tau_y}(1 - \mu_j + T_{b_j})\right] \frac{\partial \widetilde{T}_y(y, \boldsymbol{b}, t)}{\partial t} dF(\boldsymbol{\theta}) \\ &- \int_{\Theta} \sum_{k=1}^{n} \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_{b_k}} T_y + \sum_{j=1}^{n} \frac{\partial b_j(\boldsymbol{\theta})}{\partial \tau_{b_k}}(1 - \mu_j + T_{b_j})\right] \frac{\partial \widetilde{T}_{b_k}(y, \boldsymbol{b}, t)}{\partial t} dF(\theta)\end{aligned} \tag{31}$$

The first part of Proposition 4 is obtained by equating (29) to 0, which is a necessary condition for an optimal interior solution. Combining (30) with (26) yields (31).

Proposition 4 generalizes Saez (2004) to a setting with nonlinear taxes and multiple public goods. Yet this result has three important limits. The first one is that to use (31) to evaluate tax reforms, we need to assume that governmental grants have been optimized. The second one is that we need to assume that these optimal grants are strictly positive while propositions 1 and 2 indicates that the nonnegavity constraint on grants can bind at the optimum. The third one is more of a practical issue: assuming it is positive at the optimum, optimizing S_j using (30) requires measuring the total externality parameter η_j. As one can see from the definition of η_j given in (25), this necessarily implies an estimate for the (average) welfare weighted public good preferences $g(\boldsymbol{\theta})M^{G_i}$, which is an empirically challenging task. This third concern also arises when evaluating tax reforms using (26) since η_j directly enters the tax incidence formula. However, except for part ii) and iii) of Proposition 2, the efficiency conditions derived in Section 9 circumvent the complicated task of eliciting public good preferences by looking at the optimal funding mix for a given public good level. The objective is now to find such a condition under general preferences.

Definition 2. *For any tax reform of magnitude t, a grant neutralizing reform replaces an initial grant vector $\mathbf{S}$ by a new grant vector $\widetilde{\mathbf{S}} : t \mapsto \widetilde{S}_1(t), ..., \widetilde{S}_n(t)$ such that:*

$$\widetilde{S}_j(t) = S_j + (1 - \mu_j)\left(B_j(\mathbf{G}) - B_j(t, \mathbf{G})\right) \tag{32}$$

If $\widetilde{S}_j(t) \geq 0$, one can neutralize the impact a reform of magnitude t has on the corresponding public good G_j.

Proposition 5. *If grant neutralizing reforms are available such that $\widetilde{S}_j(t) \geq 0 \; \forall j \in \{1:n\}$, then:*

i) The impact of implementing a tax reform of magnitude t combined with the corresponding grant neutralizing reforms $\widetilde{\mathbf{S}}(t)$ is given by:

$$\frac{\partial \mathscr{L}^N(t)}{\partial t} = \int_\Theta \left[1 - g(\theta) - \frac{\partial y(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \rho} T_y - \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \rho}(1 - \mu_i + T_{b_i})\right] \frac{\partial \widetilde{T}(y, \boldsymbol{b}, t)}{\partial t} dF(\theta)$$

$$- \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \tau_y} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \tau_y}(1 - \mu_i + T_{b_i})\right] \frac{\partial \widetilde{T}_y(y, \boldsymbol{b}, t)}{\partial t} dF(\theta)$$

$$- \sum_{j=1}^{n} \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \tau_{b_j}} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta}, \boldsymbol{G})}{\partial \tau_{b_j}}(1 - \mu_i + T_{b_i})\right] \frac{\partial \widetilde{T}_{b_j}(y, \boldsymbol{b}, t)}{\partial t} dF(\theta) \tag{33}$$

ii) If the Lagrange multiplier associated to the government budget constraint verifies (27), then a reform of magnitude $t > 0$ (< 0) combined with a lump-sum transfer to balance the budget is socially desirable if $\frac{\partial \mathscr{L}^N(t)}{\partial t} > 0$ (< 0).

V Social desirability under constrained tax instruments

Consider a linear schedular system for donations: the income tax is nonlinear, separated for the donation tax schedule. Besides, the tax schedule on donation is linear and schedular: each donations can be submitted to a different linear tax rate t_i. In such a system a "tax credit" system, the tax schedule $T(y, \mathbf{b})$ is constrained to take the form:

$$T(y, \mathbf{b}) = T(y) + \sum_{i=1}^{n} t_i \, b_i \tag{34}$$

With such a system, there exists two ways of funding a given level of public good G_i: either by using the governmental grant S_i or the tax rate of the corresponding donations t_i. Using grant neutralizing reforms as described in definition 2, it is possible to assess whether substituting a grant funding with a donation funding is socially desirable.

The tax credit system described by (34) allows for different tax rate for different types of donations. In other words, it allows for tax discrimination across charitable causes. In practice, the degree of tax discrimination across types of donations is limited so that tax credit system are often uniform, *i.e* apply the same tax rate to each type of donations:

$$T(y, \mathbf{b}) = T(y) + t_0 \, b_0 \tag{35}$$

with $b_0 = \sum_{i=1}^{n} b_i$ the sum of donations across charitable causes.

Proposition 6. *If grant neutralizing reforms are available such that $\widetilde{S}_j(t) \geq 0 \; \forall j \in \{1 : n\}$, then:*

i) The impact of decreasing the linear tax rate t_j by a magnitude t with the corresponding grant neutralizing reforms $\widetilde{\mathbf{S}}(t)$ is given by:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[g(\boldsymbol{\theta}) - 1 - \epsilon_j^u(\boldsymbol{\theta}) \frac{1 - \mu_j + t_j}{1 + t_j} \right] b_j(\boldsymbol{\theta}) dF(\boldsymbol{\theta})$$
$$+ \int_\Theta \left[\frac{\partial y^u(\boldsymbol{\theta})}{\partial \tau_{b_j}} T_y + \sum_{\substack{i=1 \\ i \neq j}}^{n} \frac{\partial b_i^u(\boldsymbol{\theta})}{\partial \tau_{b_j}} (1 - \mu_i + t_i) \right] dF(\boldsymbol{\theta}) \tag{36}$$

with $\epsilon_{b_j}^u(\boldsymbol{\theta}) = -\frac{1+t_j}{b_j(\boldsymbol{\theta})} \frac{\partial b_j^u(\boldsymbol{\theta})}{\partial \tau_{b_j}}$ denote the uncompensated elasticity of donation b_j to its marginal tax rate $1 + t_{b_j}$.

ii) Suppose that crowding out is the same across sector $\mu_i = \mu$. In absence of tax discrimination, the impact of decreasing the uniform linear tax rate t_0 by a magnitude t with the corresponding grant neutralizing reforms $\widetilde{\mathbf{S}}(t)$ is given by:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[b_0(\boldsymbol{\theta}) \left(g(\theta) - 1 - \epsilon_0^u(\boldsymbol{\theta}) \frac{(1 - \mu + t_0)}{1 + t_0} \right) + \frac{\partial y^u(\boldsymbol{\theta})}{\partial \tau_{b_0}} T_y \right] dF(\boldsymbol{\theta}) \tag{37}$$

with $\epsilon_0^u(\boldsymbol{\theta}) = -\frac{1+t_c}{b_0(\boldsymbol{\theta})} \frac{\partial b_0^u(\boldsymbol{\theta})}{\partial \tau_{b_0}}$ the uncompensated elasticity of total donations b_0 to their marginal tax rate rate $1 + t_0$.

The proof is given in Appendix C.1. Consider a donation type j such that both labor income and other donations $i \neq j$ does not respond to changes in t_j. In this case, G_i with $i \neq j$ are not going to be affected by the reform. In this case, decreasing t_j(hence increasing the marginal subsidy to donations j) while decreasing the grant S_j (potentially to 0) is socially desirable if:

$$-\overline{\epsilon_j} > \frac{1 + t_j}{1 - \mu_j + t_j} (1 - \overline{g}_j) \tag{38}$$

with $\overline{\epsilon_j} = \frac{\int_\Theta b_j(\boldsymbol{\theta}) \epsilon_j^u(\boldsymbol{\theta})}{B_j(\boldsymbol{\theta})}$ and $\overline{g}_j = \frac{\int_\Theta b_j(\boldsymbol{\theta}) g(\boldsymbol{\theta})}{B_j(\boldsymbol{\theta})}$ the average price elasticity and welfare weights weighted by donations b_j.

Suppose that there is no leakage on sector i ($\mu_i = 0$) and consider two scenarios for the government objective. Suppose first that the least advantaged person in society does not make donations to public good j so that $b_j(\underline{\boldsymbol{\theta}}) = 0$. A Ralwsian social planner only values the well-being of the least advantaged so in this scenario, $\overline{g}_j = 0$. Then it is socially desirable to rely more on donations and less on grants to fund G_j if $|\overline{\epsilon}| > 1$. The idea is that $|\overline{\epsilon}| > 1$ implies that lowering the tax rate on donations j while decreasing the grant S_j has a net positive impact on government revenue. Since public goods levels are unchanged, this additional source of revenue, that can be redistributed to the lowest type, is the only relevant parameter for the desirability of the joint reforms.[8] Now consider the opposite scenarios, where individual utility is linear

[8] Boadway and Jacquet (2008) shows the equivalence between maximizing a Ralwsian objective and maximizing government revenue in the standard Mirrlees (1971) model.

in private good consumption ($U_c = 1$) and the government is utilitarian. This implies $\overline{g} = 1$ and shuts down any redistributive motive for the government: the choice between government and donation-based funding can only be driven by efficiency concerns. In this case, it is socially desirable to fund G_j with donations as long as $|\overline{\epsilon_j}| > 0$. In other words, unless donations are Veblen goods, it is always socially desirable to fund the public good with tax incentives. The intuition is that absent redistributive motives, a mechanical loss in tax revenue exactly compensate the mechanical gain in welfare. Absent leakage, a 1 dollar increase in donations b_j allows for a 1 dollar decrease in grant S_j. As long as $t_j > -1$, this gain in government revenue through a reduction in S_j is higher than the loss in tax revenue from the tax subsidy.

Both policy rules (36) and (37) are expressed in terms of sufficient statistics that can in principle provide empirically grounded policy recommendations. Yet, given the limited degree of tax discrimination across charitable causes, we have limited information on both direct and cross-base elasticites of different types of donations to different tax subsidies. However the uniform tax credit system has been investigated and we do have estimates on the elasticity of total donations to the uniform credit rate $-t_0$.[9]

Proposition 7. *Suppose that aggregate labor income does not respond to reforms of the uniform tax credit and that leakage is uniform across sectors such that $\mu_i = \mu$ for all i. Then a small decrease in t_0 is socially desirable as long as:*

i)

$$-\overline{\epsilon}_0 > \frac{1 - \mu + t_0}{1 + t_0}(1 - \overline{g}_0) \tag{39}$$

ii) Grant neutralizing reforms are available for all public goods, i.e $\widetilde{S}_j(t) \geq 0 \; \forall j \in \{1 : n\}$

Proposition 7 directly derives from (37) with $\frac{\partial Y^u}{\partial \tau_{b_0}}$, which is a standard assumption.[10] The first part emphasizes that, in absence of leakage ($\mu = 0$), even an elasticity lower than 1 in absolute value can justify a use of tax incentives instead of grants. Indeed, absent redistributive motives ($\overline{g}_0 = 1$), it is always locally desirable to decrease t_0 as long as the elasticity is strictly positive. The standard unit elasticity rule only applies to the extreme case of a Rawlsian social planner ($\overline{g}_0 = 0$) which indeed $|\overline{\epsilon_0}| > 1$.[11] However, limiting the degree of tax discrimination implies that the government necessarily changes the tax subsidy to every donations when implementing a tax reform. Therefore, for all donations that a price sensitive, a reform of the uniform tax credit necessarily requires a reform of all elements of the grant vector **S**. The second part of Proposition 7 emphasizes the importance of this condition: even if (39) is always verified, as soon as one of the grant S_j hits the zero lower bound, such that G_j is funded only through donations, it is no longer possible to fund the other public goods with donations while keeping G_j unchanged.

[9] See for instance Fack and Landais (2010).

[10] Note that contrary to assumption 2 in Saez (2004), there is no need to constrain the response of income to public goods.

[11] See for instance Feldstein et al. (1980), Roberts (1987), Saez (2004) or Fack and Landais (2010) for the usual rationale behind this rule.

VI Numerical Simulations

I calibrate the model on French 2018 Income Tax Data (POTE[12]. The dataset is exhaustive and provides all the information, regarding income, age, family composition, filled by taxpayers on their personal income tax report. It also provides the amount each taxpayer reported as donations. [13] I consider a sample of singles, reporting a strictly positive amount of donations, excluding donations to political parties. This yields a sample of 2 289 179 individuals, earning on average 29 281 € of taxable income and donating on average 390 € to non-political general interest organizations.[14]

I consider a tax credit system as described in (34) and calibrate a two public goods economy where individuals differ $ex - ante$ in both productivity and altruism. I consider the following functional form for the utility function:

$$\mathcal{U}(c, b_1, b_2; w, \beta, G_1, G_2) = (1-\beta)\ln\left(c - \frac{\varepsilon}{1+\varepsilon} y^{\frac{1+\varepsilon}{\varepsilon}} w^{-\frac{1}{\varepsilon}}\right) + \beta_s * \beta \ln(b_1) + (1-\beta_s) * \beta \ln(b_2) + \alpha_1 \ln(G_1) + \alpha_2 \ln(G_2) \quad (40)$$

This type of Cobb-Douglas preferences combined with iso-elastic preferences for labor allows me to both rule out income effect on labor supply (as standard in optimal taxation exercise since Diamond (1998)) while allowing income effects on donations. I assume direct elasticity of labor income of $\varepsilon = 0.2$, consistent with estimates provided by Saez et al. (2012). The productivity parameter w of each individual is then obtained by inverting the first-order condition of (40) with respect to y, using the actual nonlinear marginal income tax schedule in France in 2018. Since I cannot clearly distinguish between different donations types in the data (except for donations to political parties) and therefore only observes $b_0 = b_1 + b_2$, the calibration of donation preferences is performed in two step. First, I consider a one good version of (40) where agents optimize over total donations b_0. Inverting the first-order condition associated to b_0, using the 66% tax credit on donation in France, allows me recover the (total) altruism parameter β. Given this individual altruism parameter β, I then split total donations b_0 to b_1 and b_2 by specifying the share β_s.

The policy exercise conduct here is the following: the government is adjusting the linear tax rate on donations t_1 and t_2, the grants S_1 and S_2 and the demogrant R (or the intercept of the nonlinear tax schedule) to maximize social welfare, given an arbitrary level of public goods G_1 and G_2. To do so, I use (36) to evaluate the impact of jointly reforming the tax rate applied to donation i and the corresponding grant S_i to keep G_i constant. I do this until the welfare gain associated to this joint reform is nil or the grant S_i hits the zero lower bound so that G_i becomes fully funded through donations.

I label donation preferences as *unbiased* when individuals value equally do-

[12] *Fichier permanent des occurences et des traitements*, accesible via *Centre d'accès sécurisée aux données* (CASD)

[13] Note that income is third-party reported while donations amounts are reported by taxpayers.

[14] Here I aggregate donation to *organismes d'aide aux personnes en difficulté* (7UD), *autres organismes d'intérêt général* (7UF) and to European Union NGOs (7VA and 7VC).

nations to cause 1 and to cause 2 so that $\beta_s = 0.5$. In this baseline scenario, this implies that both G_1 and G_2 receives 195 euros in donations per capita. To match the prominent role of governmental grant compared to household donations in France, I assume $S_1 = S_2 = 300$ at baseline. This yields a baseline target of public good spending per capita $G_1 = G_2 = 500$.

I assume a standard utilitarian social planner so that the only redistributive motive comes from the decreasing marginal utility of consumption in (40). This allows me to isolate the importance of both the market failure (leakage) and the government failure (uniform tax credit) on the optimal funding problem.

VI.1 Unbiased Donation Preferences

Consider a first scenario where leakage is uniform across sectors $\mu_1 = \mu_2 = \mu$ and where preferences across donation types are unbiased so that $\beta_s = 0.5$. This implies that at baseline, both public goods receive an equal amount of donations B_1 and B_2 and grants S_1 and S_2. Figure 1 describes how the optimal policy mix evolves when leakage increases.

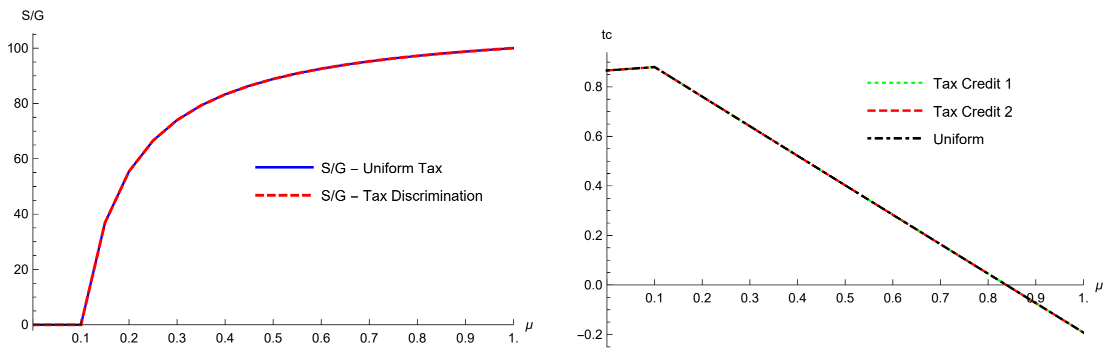

Figure 1: Optimal government funding (left) and tax credit rates (right) as a function of leakage μ with unbiased donation preferences ($\beta_s = 0.5$).

Given the Cobb-Douglas structure of preferences (40), there are no cross-base responses and the direct price elasticity of giving for both b_1 and b_2 is equal to -1. Besides, we assume a standard utilitarian social planner so that welfare weights are strictly positive. Absent leakage, the policy rule (38) therefore implies that both tax rates t_1 and t_2 are decreased up to the points where both grants S_1 and S_2 hit the zero lower bound. This is why at $\mu = 0$, $S/G = 0$ on the left graph of Figure 1. When leakage increases, the desirability of relying only donations becomes ambiguous and we see that government starts to directly contribute to public good funding around $\mu = 0.1$. In the meantime, the tax credit rates $-t_1$ and $-t_2$ starts falling as described in the left graph of Figure 1. For very high levels of leakage ($\mu > 0.8$), public good funding becomes fully governmental and donations are no longer subsidized but taxed as tax credits become negatives.

When leakage is uniform and when donation preferences are unbiased, $t_1 = t_2 = t_0$ so that the funding policy with and without tax discrimination coincides, as one can see from Figure 1. Now consider a scenario where preferences are still unbiased but leakage is different across sectors.

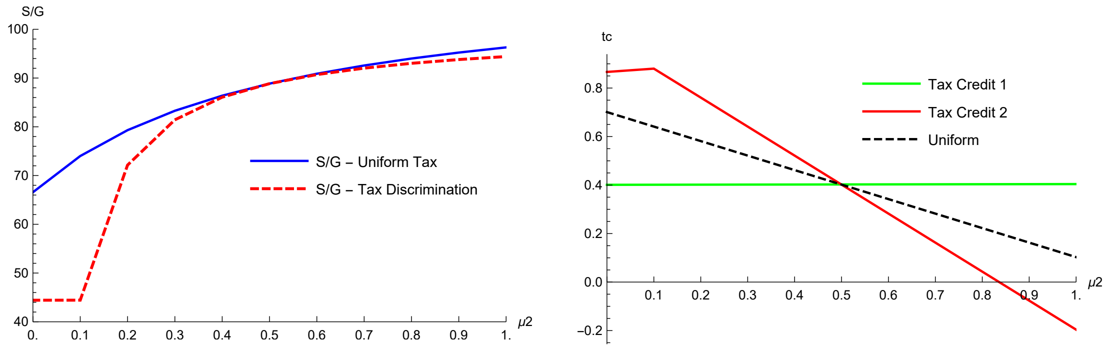

Figure 2: Optimal government funding (left) and tax credit rates (right) as a function of leakage μ_2, with $\beta_s = 0.5$ and $\mu_1 = 0.5$.

Figure 2 displays the optimal policy mix for different values of leakage in the second sector μ_2, while μ_1 is fixed at 0.5. In this case, the optimal funding policy varies depending on the degree of tax discrimination. If we allow for tax discrimination, then the tax credit rate for sector 1 does not depend on changes in leakage in sector 2, as shown in the right graph of Figure 2. The tax credit on sector 2 naturally decreases as leakage increases, displaying a similar pattern then in the right part of Figure 1. However, when tax discrimination is not allowed, the uniform tax credit (the dotted black curve) averages the two tax rates. The idea is that when μ_2 is low, the government wants to impose a higher tax subsidy for donations to public good 2 compared to donations to public good 1. This is not feasible when tax discrimination is not allowed. With a uniform tax credit, the subsidy to b_1 is higher (lower) than it should for low (high) values of μ_2 while the subsidy to b_2 is lower (higher) than it should for high (low) values of μ_2. This lack of flexibility of the tax system reflects in the share of direct government funding. The gap between the blue and red curves in the right graph of Figure 2 shows how limiting tax discrimination can increase the desirability of grants. For low values of μ_2, grants only compensate for leakage in sector 1 in a tax discrimination system while they also compensate for too low level of donations to sector 2, due to $-t_0 < -t_2$, in a uniform tax system. For $\mu_1 = 0.1$, this leads to a 30 percentage point increase in the share of government funding. This gap then reduces when μ_2 gets closer to 0.5, *i.e* when the leakage differential between the two sectors reduces.

VI.2 Biased Donation Preferences

Figures 1 and 2 display optimal funding strategies under the assumption that individuals may differ in the share of their income they spend donations (heterogeneous altruism) but not on the allocation across sector 1 and sector 2. I now consider biases in donation patterns, such that one sector might receive a different amount of donations than the other at baseline. Using (40), a bias in favor of sector 1 (sector 2) is introduced when $\beta_s > 0.5 (< 0.5)$. To better understand the specific role of such biases, I first rule out leakage in both sectors such that $\mu_1\mu_2 = 0$. Figure 3 describes how the funding strategy changes when individuals donate more to one of the two sector.

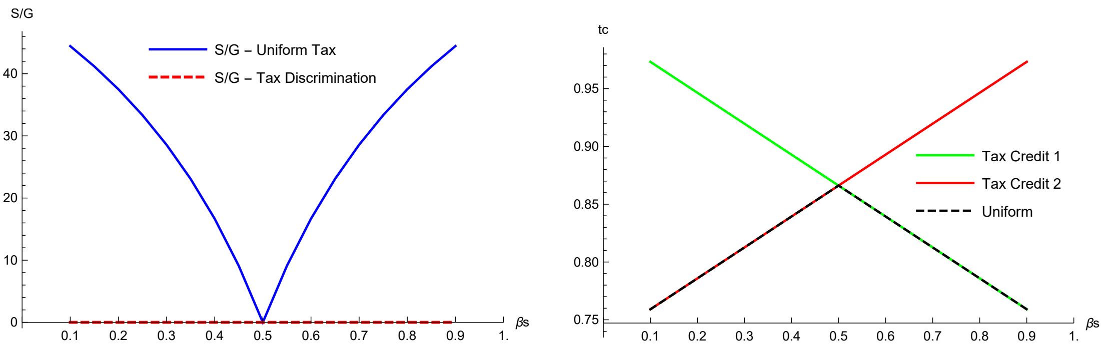

Figure 3: Optimal government funding (top) and tax credit rates (bottom) as a function of the bias parameter β_s, without leakage ($\mu = 0$)

In absence of leakage, the existence of such biased preferences are not important in a tax discrimination system: the government should not directly contribute to public good funding for all values of β_s, as shown in the red curve in the left part of Figure 3. When people donate less (more) to sector 1, the tax credit to sector 1 is higher (lower) than the one to sector 2, as shown in the right part of Figure 3. However, in absence of tax discrimination, tax incentives are not sufficient to fund the public goods. The idea is that because public goods 1 and 2 receives different amount of donations, it is not possible to completely fund both public goods with donation with only one tax credit rate. Using (39), decreasing t_0 is desirable until one of the two grants S_1 or S_2 hits the zero lower bound. When sector 1 receives less donations, S_2 is going to hit the zero lower bound before S_1, so that G_2 is fully funded by donations and $t_0 = t_2$. An opposite patterns occur when sector 2 receives less donation. This is why the black curve ($-t_0$) coincides with the red curve ($-t_2$) for $\beta_s < 0.5$ and with the green curve for $\beta_s > 0.5$ on the right part of Figure 3. In the extreme cases where donations are heavily biased, the left part of Figure 3 indicates that the share of government funding can go from 0 to 40% when limiting tax discrimination.

Eventually, consider the case where not only donation preferences but also leakage can be different between sectors. Suppose for instance that the donation technology of sector 1 is less efficient than the one of sector 2, such that $\mu_1 > \mu_2$. In Figure 4, I plot the optimal funding strategy when donations are more less biased towards the efficient or the leaky sector. In this scenario, tax discrimination can be used to set a high tax credit to the efficient sector and a low tax credit to the leaky sector. These two tax credit do not depend on donors preferences, as shown in the two red lines of the bottom graph. However, grants are sensitives to donors preferences and the red line on the top graph shows that the more people give to the leaky sector, the more the government should fund the public good. At the limit where the charitable cause most favored by individuals is also the one with higher leakage, the share of public goods funded directly by the government is above 90%. Again, the absence of tax discrimination reinforces this pattern, as both tax rates and grants depend on donors preferences. The blue curve on the bottom graph illustrates that under a uniform tax regime, public good become almost fully funded by governments as soon as individuals tend to give more to the leaky sectors.

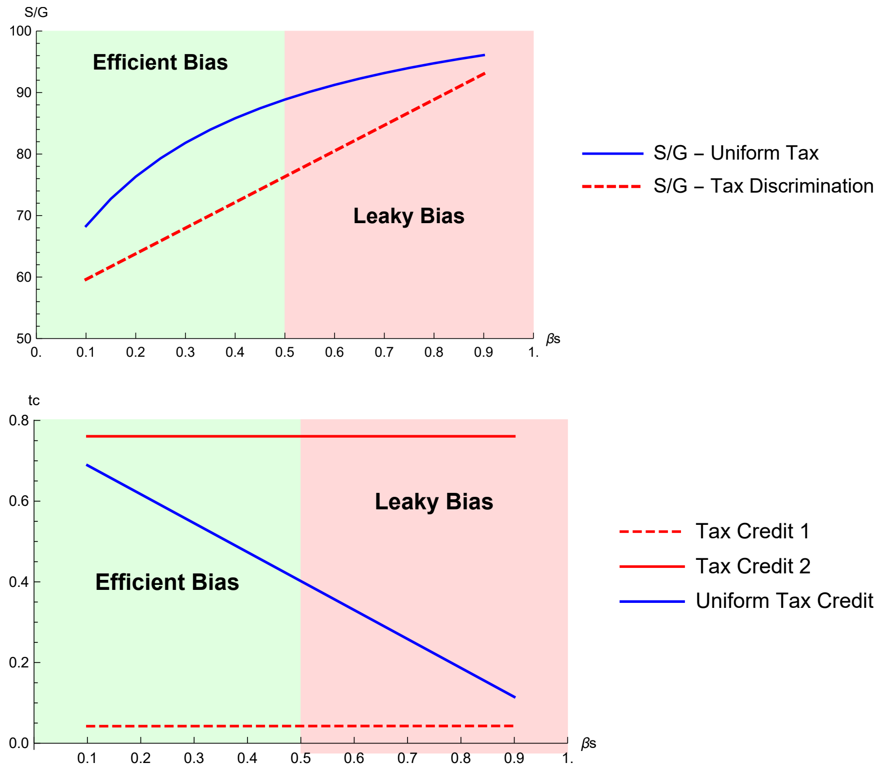

Figure 4: Optimal government funding (top) and tax credit rates (bottom) as a function of the bias parameter β_s, with $\mu_1 = 0.8$ and $\mu_2 = 0.2$

VII Conclusion

In this paper I show how governments should rely on private contributions to fund public goods. In absence of leakage and with unlimited tax discrimination, relying on tax incentives to donations has unambiguous advantages over direct government funding. However, introducing leakage and limiting the degree of tax discrimination across donation types significantly increases the social desirability of direct government funding.

Although the policy instruments considered here can match most of the OECD countries' tax treatment of charitable giving, the matching system used for instance in the UK is left out of the analysis. This system where the government tops up individual's contribution is an alternative to tax credits and deductions. Although studied by the empirical literature on giving[15], this mechanism has not been introduced in a formal optimal tax exercise.

[15] See for instance Peter and Lideikyte Huber (2022), chapter 9, part 3.2.1 for a brief review of the literature.

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A Proofs of the Results of Section III

A.1 Proof Equation (12)

Consider a baseline incentive-compatible allocation $\{c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{G}\}$. Under this baseline allocation, an individual of type $\boldsymbol{\theta}$ gets a baseline utility level $\mathcal{U}(\boldsymbol{\theta}) = U\left(V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}, \boldsymbol{\theta}\right)$. Individual utility maximization then implies:

$$\mathcal{U}(\boldsymbol{\theta}) = \max_{y, \mathbf{b}^{ct}} U\left(V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})), \mathbf{b}^{ct}, y; \mathbf{G}, \boldsymbol{\theta}\right) = \max_{y, \mathbf{b}^{ct}} U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}, y; \mathbf{G}, \boldsymbol{\theta}\right) \tag{41}$$

with $\mathcal{V}(\boldsymbol{\theta}) = V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})\right)$ the baseline subutility level from consumption and consensual donations.

By definition the baseline allocation is incentive-compatible so it verifies:

$$U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}, \boldsymbol{\theta}\right) \geq U\left(\mathcal{V}(\boldsymbol{\theta}'), \mathbf{b}^{ct}(\boldsymbol{\theta}'), y(\boldsymbol{\theta}'); \mathbf{G}, \boldsymbol{\theta}\right) \tag{42}$$

Eventually, the baseline allocation should verify the nonnegativity constraint on government grants $S_i \geq 0$. Using the definition of G_i in (5), this implies that baseline public goods and donations levels should verify:

$$G_i - (1 - \mu_i) \int_{\boldsymbol{\theta}} b_i(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) \geq 0, \forall i \tag{43}$$

Now consider an alternative allocation $\{\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{ct}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}), \widehat{G}\}$. Suppose that the subutility from consumption and consensual donations is the same under the baseline and the alternative allocation:

$$V(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta})) = V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})) = \mathcal{V}(\boldsymbol{\theta}) \tag{44}$$

Besides suppose that public goods level is the same under both allocation: $\widehat{\mathbf{G}} = \mathbf{G}$.

Individual utility maximization therefore implies:

$$\max_{y, \mathbf{b}^{ct}} U\left(V\left(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta})\right), \mathbf{b}^{ct}, y; \widehat{\mathbf{G}}, \boldsymbol{\theta}\right) = \max_{y, \mathbf{b}^{ct}} U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}, y; \mathbf{G}, \boldsymbol{\theta}\right) \tag{45}$$

Combining (45) with (41) implies:

$$U\left(V\left(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta})\right), \widehat{\mathbf{b}^{ct}}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}); \widehat{\mathbf{G}}, \boldsymbol{\theta}\right) = U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}, \boldsymbol{\theta}\right) = \mathcal{U}(\boldsymbol{\theta}) \tag{46}$$

This implies that individual utility, labor supply and controversial donations levels are the same under the alternative and baseline allocations. Using (42) this implies that the allocation $\{\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{ct}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}), \widehat{G}\}$ is incentive compatible.

To yield a Pareto-improvement over the baseline allocation, the alternative must increase government revenue. Using (9) this would imply:

$$\begin{aligned} &\int_{\Theta} \left[\widehat{y}(\boldsymbol{\theta}) - \widehat{c}(\boldsymbol{\theta}) - \sum_{i=1}^{m} \mu_i \widehat{b}_i^{cs}(\boldsymbol{\theta}) - \sum_{i=m+1}^{n} \mu_i \widehat{b}_i^{ct}(\boldsymbol{\theta}) \right] dF(\boldsymbol{\theta}) - \sum_{i=1}^{n} \widehat{G}_i \geq \\ &\int_{\Theta} \left[y(\boldsymbol{\theta}) - c(\boldsymbol{\theta}) - \sum_{i=1}^{m} \mu_i b_i^{cs}(\boldsymbol{\theta}) - \sum_{i=m+1}^{n} \mu_i b_i^{ct}(\boldsymbol{\theta}) \right] dF(\boldsymbol{\theta}) - \sum_{i=1}^{n} G_i \end{aligned} \tag{47}$$

Using (46) this actually boils down to:

$$\int_{\Theta} \left[\widehat{c}(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i \widehat{b}_i^{cs}(\boldsymbol{\theta}) \right] dF(\boldsymbol{\theta}) \leq \int_{\Theta} \left[c(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i b_i^{cs}(\boldsymbol{\theta}) \right] dF(\boldsymbol{\theta}) \tag{48}$$

Eventually, the alternative allocation must verify the nonnegativity constraints on governmental grants. Public goods levels are the same under the alternative and the baseline. Besides, controversial donations levels are the same under the alternative and the baseline. It therefore follows from the public good funding equation (5) that the grants given to the public goods receiving controversial donations $\mathbf{G}^{ct}$ is the same under baseline than under the alternative. So I only need to ensure that the grants given to public goods with consensual funding, denoted $\mathbf{G}^{cs}$ verify the nonnegativity constraint.

Hence the problem of choosing, among all incentive-compatible allocations, the one that maximizes government revenue (9) while leaving public goods provision and individual utility unchanged boils down to (12).

A.2 Proof Proposition 1

The Lagrangian associated to problem (12) is:

$$\begin{aligned} L = &\int_{\boldsymbol{\theta}} \left[c(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i b_i^{cs}(\boldsymbol{\theta}) - \phi(\boldsymbol{\theta}) \left(V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})\right) - \mathcal{V}(\boldsymbol{\theta}) \right) \right] dF(\theta) \\ &- \sum_{i=1}^{m} \lambda_i \left(G_i - (1 - \mu_i) \int_{\boldsymbol{\theta}} b_i^{cs}(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) \right) \end{aligned} \tag{49}$$

with $\phi(\theta)$ and λ_i the Lagrange multipliers associated to the subutility and nonnegativity constraints.

The F.O.C with respect to $c(\theta)$ yields :

$$\phi(\theta) = \frac{1}{V_c\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})\right)} \tag{50}$$

The F.O.C with respect to $b_i(\boldsymbol{\theta})$ yields :

$$\mu_i - \phi(\boldsymbol{\theta}) V_{b_i^{cs}}\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta})\right) + \lambda_i\left(1 - \mu_i\right) = 0 \tag{51}$$

Combining (50) with (51) yields :

$$\lambda_i = \frac{1}{1-\mu_i}\left(\frac{V_{b_i^{cs}}\left(c(\theta), \mathbf{b}(\theta)\right)}{V_c\left(c(\theta), \mathbf{b}(\theta)\right)} - \mu_i\right) \tag{52}$$

If $\mu_i = 0$, $\lambda_i > 0$ so the nonnegativity constraint is binding and $S_i = 0$. This proves the first part of proposition 1.

If μ_i is large enough so that $S_i > 0$, then complementary slackness implies $\lambda_i = 0$. This implies:

$$\begin{aligned} \frac{V_{b_i}\left(c(\theta), \mathbf{b}(\theta)\right)}{V_c\left(c(\theta), \mathbf{b}(\theta)\right)} &= \mu_i \\ 1 + T_{b_i}(y, \mathbf{b}) &= \mu_i \end{aligned} \tag{53}$$

where the last line follows for the F.O.C for b_i of the individual maximization program (4). This proves the second part of Proposition 1.

A.3 Proof Proposition 2

Consider a baseline incentive-compatible allocation $\{c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}), \mathbf{G}^{cs}, \mathbf{G}^{ct}\}$. Under this baseline allocation, an individual of type $\boldsymbol{\theta}$ gets a baseline utility level $\mathcal{U}(\boldsymbol{\theta}) = U\left(V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}^{ct}, \boldsymbol{\theta}\right)$. Individual utility maximization then implies:

$$\mathcal{U}(\boldsymbol{\theta}) = \max_{y, \mathbf{b}^{ct}} U\left(V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}), \mathbf{b}^{ct}, y; \mathbf{G}^{ct}, \boldsymbol{\theta}\right) = \max_{y, \mathbf{b}^{ct}} U\left(V(\boldsymbol{\theta}), \mathbf{b}^{ct}, y; \mathbf{G}^{ct}, \boldsymbol{\theta}\right) \tag{54}$$

with $\mathcal{V}(\boldsymbol{\theta}) = V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right)$ the baseline subutility level from consumption, consensual donations and consensual public goods.

By definition the baseline allocation is incentive-compatible so it verifies:

$$U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}^{ct}, \boldsymbol{\theta}\right) \geq U\left(\mathcal{V}(\boldsymbol{\theta}'), \mathbf{b}^{ct}(\boldsymbol{\theta}'), y(\boldsymbol{\theta}'); \mathbf{G}^{ct}, \boldsymbol{\theta}\right) \tag{55}$$

Eventually, the baseline allocation should verify the nonnegativity constraint on government grants $S_i \geq 0$. Using the definition of G_i in (5), this implies that baseline public goods and donations levels should verify:

$$G_i - (1-\mu_i)\int_{\boldsymbol{\theta}} b_i(\boldsymbol{\theta})dF(\boldsymbol{\theta}) \geq 0, \forall i \tag{56}$$

Now consider an alternative allocation of consumption, consensual donations and consensual public goods: $\{\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{ct}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}), \widehat{\mathbf{G}}^{cs}, \widehat{\mathbf{G}}^{ct}\}$.

Suppose that the subutility from consumption and consensual donations is the same under the baseline and the alternative allocation:

$$V(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{G}}^{cs}) = V(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}) = \mathcal{V}(\boldsymbol{\theta}) \tag{57}$$

Besides suppose that controversial public goods levels are the same under both allocations: $\widehat{\mathbf{G}}^{ct} = \mathbf{G}^{ct}$

Individual utility maximization therefore implies:

$$\max_{y,\mathbf{b}^{ct}} U\left(V\left(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{G}}^{cs}\right), \mathbf{b}^{ct}, y; \widehat{\mathbf{G}^{ct}}, \boldsymbol{\theta}\right) = \max_{y,\mathbf{b}^{ct}} U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}, y; \mathbf{G}^{ct}, \boldsymbol{\theta}\right) \tag{58}$$

Combining (58) with (54) implies:

$$U\left(V\left(\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{G}}^{cs}\right), \widehat{\mathbf{b}}^{ct}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}); \widehat{\mathbf{G}}^{ct}, \boldsymbol{\theta}\right) = U\left(\mathcal{V}(\boldsymbol{\theta}), \mathbf{b}^{ct}(\boldsymbol{\theta}), y(\boldsymbol{\theta}); \mathbf{G}^{ct}, \boldsymbol{\theta}\right) = \mathcal{U}(\boldsymbol{\theta}) \tag{59}$$

So individual utility, labor supply and controversial donations levels are the same under the alternative and baseline allocations. Using (55) this implies that the allocation $\{\widehat{c}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{cs}(\boldsymbol{\theta}), \widehat{\mathbf{b}}^{ct}(\boldsymbol{\theta}), \widehat{y}(\boldsymbol{\theta}), \widehat{G}^{cs}, \widehat{G}^{ct}\}$ is incentive compatible.

To yield a Pareto-improvement over the baseline allocation, the alternative must increase government revenue. Using (9) this would imply:

$$\begin{aligned}&\int_{\Theta}\left[\widehat{y}(\boldsymbol{\theta}) - \widehat{c}(\boldsymbol{\theta}) - \sum_{i=1}^{m}\mu_i\widehat{b}_i^{cs}(\boldsymbol{\theta}) - \sum_{i=m+1}^{n}\mu_i\widehat{b}_i^{ct}(\boldsymbol{\theta})\right]dF(\boldsymbol{\theta}) - \sum_{i=1}^{n}\widehat{G}_i \geq \\ &\int_{\Theta}\left[y(\boldsymbol{\theta}) - c(\boldsymbol{\theta}) - \sum_{i=1}^{m}\mu_i b_i^{cs}(\boldsymbol{\theta}) - \sum_{i=m+1}^{n}\mu_i b_i^{ct}(\boldsymbol{\theta})\right]dF(\boldsymbol{\theta}) - \sum_{i=1}^{n}G_i\end{aligned} \tag{60}$$

Using (59) this actually boils down to:

$$\int_{\Theta}\left[\widehat{c}(\boldsymbol{\theta}) + \sum_{i=1}^{m}\mu_i\widehat{b}_i^{cs}(\boldsymbol{\theta})\right]dF(\boldsymbol{\theta}) + \sum_{i=1}^{m}\widehat{G}_i^{cs} \leq \int_{\Theta}\left[c(\boldsymbol{\theta}) + \sum_{i=1}^{m}\mu_i b_i^{cs}(\boldsymbol{\theta})\right]dF(\boldsymbol{\theta}) + \sum_{i=1}^{m}G_i^{cs} \tag{61}$$

Eventually, the alternative allocation must verify the nonnegativity constraints on governmental grants. Controversial public goods levels are the same under the alternative and the baseline. Besides, controversial donations levels are the same under the alternative and the baseline. It therefore follows from the public good funding equation (5) that the grants given to $\mathbf{G}^{ct}$ are the same under baseline than under the alternative. So I only need to ensure that the grants $\mathbf{G}^{cs}$ verify the nonnegativity constraint.

Hence the problem of choosing, among all incentive-compatible allocations, the one that maximizes government revenue (9) while leaving controverisal public goods provision and individual utility unchanged boils down to:

$$\begin{aligned} &\min_{c(\boldsymbol{\theta}),\mathbf{b},\mathbf{G}^{cs}} \int_{\Theta} \left(c(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i b_i^{cs}(\boldsymbol{\theta}) \right) dF(\boldsymbol{\theta}) + \sum_{i=1}^{m} G_i^{cs} \\ &\text{subject to : } V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right) = \mathcal{V}(\boldsymbol{\theta}), \forall \boldsymbol{\theta} \\ &\qquad G_i^{cs} - (1-\mu_i) \int_{\boldsymbol{\theta}} b_i^{cs}(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) \geq 0, \forall i \end{aligned} \tag{62}$$

The Lagrangian associated to this problem is:

$$\begin{aligned} L = &\int_{\boldsymbol{\theta}} \left[c(\boldsymbol{\theta}) + \sum_{i=1}^{m} \mu_i b_i^{cs}(\boldsymbol{\theta}) - \phi(\boldsymbol{\theta}) \left(V\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right) - \mathcal{V}(\boldsymbol{\theta}) \right) \right] dF(\theta) \\ &+ \sum_{i=1}^{m} \left[G_i^{cs} - \lambda_i \left(G_i - (1-\mu_i) \int_{\boldsymbol{\theta}} b_i^{cs}(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) \right) \right] \end{aligned} \tag{63}$$

F.O.C with respect to $c(\boldsymbol{\theta})$ yields:

$$\phi(\boldsymbol{\theta}) = \frac{1}{V_c\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right)} \tag{64}$$

F.O.C with respect to $b_i^{cs}(\boldsymbol{\theta})$ yields for all i:

$$\mu_i - \phi(\boldsymbol{\theta}) V_{b_i^{cs}}\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right) + \lambda_i (1-\mu_i) = 0 \tag{65}$$

If $\mu_i = 0$, combining (64) with (65) implies $\lambda_i > 0$ hence $S_i = 0$. If μ_i is large enough so that $S_i > 0$ ($\lambda_i = 0$), combining (65) with the individual first order yields $T_{b_i} = -1 + \mu_i$. This proves the first part of Proposition 2.

F.O.C with respect to G_i^{cs} yields for all i:

$$1 - \lambda_i - \int_{\boldsymbol{\theta}} \phi(\boldsymbol{\theta}) V_{G_i^{cs}}\left(c(\boldsymbol{\theta}), \mathbf{b}^{cs}(\boldsymbol{\theta}), \mathbf{G}^{cs}\right) dF(\boldsymbol{\theta}) = 0 \tag{66}$$

Combining (64), (65) and (66) yields (15) yields (15).

If μ_i is large enough so that $S_i > 0$, complementary slackness implies $\lambda_i = 0$. Using (66), this yields (16) and proves the last part of Proposition 2.

B Proofs of the Results of Section IV

B.1 Proof of equation 19

The first-order-condition of (18) with respect to y and b_i yields :

$$M^y\left(y - \sum_{j=1}^{n} b_j - \widetilde{T}\left(y, \mathbf{b}, t\right), y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}\right) = 1 - \widetilde{T}_y(y, \mathbf{b}, t) \tag{67}$$

$$M^{b_i}\left(y - \sum_{j=1}^{n} b_j - \widetilde{T}\left(y, \mathbf{b}, t\right), y, \mathbf{b}; \mathbf{G}, \boldsymbol{\theta}\right) = 1 + \widetilde{T}_{b_i}(y, \mathbf{b}, t) \tag{68}$$

Differentiating (67) we get :

$$\left[\left(1 - \widetilde{T}_y\right) M^y_c + M^y_y + \widetilde{T}_{y,y}\right] dy + \sum_{j=1}^{n}\left[M^y_{b_j} + \widetilde{T}_{y,b_j} - (1 + \widetilde{T}_{b_j}) M^y_c\right] db_j =$$
$$\left[\frac{\partial \widetilde{T}}{\partial t} M^y_c - \frac{\partial \widetilde{T}_y}{\partial t}\right] dt - \sum_{i=0}^{n} M^y_{G_i}\, dG_i$$

Using (67) and (68) this can be rewritten as :

$$\left[M^y M^y_c + M^y_y + \widetilde{T}_{y,y}\right] dy + \sum_{j=1}^{n}\left[M^y_{b_j} + \widetilde{T}_{y,b_j} - M^{b_j} M^y_c\right] db_j = \left[\frac{\partial \widetilde{T}}{\partial t} M^y_c - \frac{\partial \widetilde{T}_y}{\partial t}\right] dt - \sum_{j=1}^{n} M^y_{G_j}\, dG_j$$

Differentiate (68), for all i we have :

$$\left[\left(1 - \widetilde{T}_y\right) M^{b_i}_c + M^{b_i}_y + \widetilde{T}_{b_i,y}\right] dy + \sum_{j=1}^{n}\left[M^{b_i}_{b_j} + \widetilde{T}_{b_i,b_j} - (1 + \widetilde{T}_{b_j}) M^{b_i}_c\right] db_j =$$
$$\left[\frac{\partial \widetilde{T}}{\partial t} M^{b_i}_c - \frac{\partial \widetilde{T}_i}{\partial t}\right] dt - \sum_{j=1}^{n} M^{b_i}_{G_j}\, dG_j$$

And using (67) and (68) :

$$\left[M^y M^{b_i}_c + M^{b_i}_y + \widetilde{T}_{b_i,y}\right] dy + \sum_{j=1}^{n}\left[M^{b_i}_{b_j} + \widetilde{T}_{b_i,b_j} - M^{b_j} M^{b_i}_c\right] db_j = \left[\frac{\partial \widetilde{T}}{\partial t} M^{b_i}_c - \frac{\partial \widetilde{T}_i}{\partial t}\right] dt - \sum_{j=1}^{n} M^{b_i}_{G_j}\, dG_j$$

We can sum up in matrix form:

$$\mathbf{A} \cdot \begin{pmatrix} dy \\ db_1 \\ \vdots \\ db_n \end{pmatrix} = \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} M_c^y - \frac{\partial \widetilde{T}_y}{\partial t} \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_1} + \frac{\partial \widetilde{T}_{b_1}}{\partial t} \\ \vdots \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_n} + \frac{\partial \widetilde{T}_{b_n}}{\partial t} \end{pmatrix} dt - \begin{pmatrix} M_{G_1}^y & \dots & M_{G_n}^y \\ M_{G_1}^{b_1} & \dots & M_{G_n}^{b_1} \\ \vdots & \ddots & \vdots \\ M_{G_1}^{b_n} & \dots & M_{G_n}^{b_n} \end{pmatrix} \cdot \begin{pmatrix} dG_1 \\ \vdots \\ dG_n \end{pmatrix} \tag{69}$$

$$\text{with } \mathbf{A} = \begin{pmatrix} M^y\, M_c^y + M_y^y + \widetilde{T}_{y,y} & M_{b_1}^y + \widetilde{T}_{y,b_1} - M^{b_1} M_c^y & \dots & M_{b_n}^y + \widetilde{T}_{y,b_n} - M^{b_n} M_c^y \\ M^y\, M_c^{b_1} + M_y^{b_1} - \widetilde{T}_{b_1,y} & M_{b_1}^{b_1} + \widetilde{T}_{b_1,b_1} - M^{b_1} M_c^{b_1} & \dots & M_{b_n}^{b_1} + \widetilde{T}_{b_1,b_n} - M^{b_n} M_c^{b_1} \\ \vdots & \vdots & \ddots & \vdots \\ M^y\, M_c^{b_n} + M_y^{b_n} - \widetilde{T}_{b_n,y} & M_{b_1}^{b_n} + \widetilde{T}_{b_n,b_1} - M^{b_1} M_c^{b_n} & \dots & M_{b_n}^{b_n} + \widetilde{T}_{b_n,b_n} - M^{b_n} M_c^{b_n} \end{pmatrix}$$

Assuming that the matrix **A** is invertible, one can rewrite (69) as:

$$\begin{pmatrix} \frac{dy}{dt} \\ \frac{db_1}{dt} \\ \vdots \\ \frac{db_n}{dt} \end{pmatrix} = \mathbf{A}^{-1} \cdot \left\{ \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} M_c^y - \frac{\partial \widetilde{T}_y}{\partial t} \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_1} + \frac{\partial \widetilde{T}_{b_1}}{\partial t} \\ \vdots \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_n} + \frac{\partial \widetilde{T}_{b_n}}{\partial t} \end{pmatrix} - \begin{pmatrix} M_{G_1}^y & \dots & M_{G_n}^y \\ M_{G_1}^{b_1} & \dots & M_{G_n}^{b_1} \\ \vdots & \ddots & \vdots \\ M_{G_1}^{b_n} & \dots & M_{G_n}^{b_n} \end{pmatrix} \cdot \begin{pmatrix} \frac{dG_1}{dt} \\ \vdots \\ \frac{dG_n}{dt} \end{pmatrix} \right\} \tag{70}$$

The direct response of labor income and donations b_i to a reform of magnitude t, at a given level of public good $\mathbf{G}$ is obtained by ignoring the $\frac{d\mathbf{G}}{dt}$ vector in (70). This yields:

$$\begin{pmatrix} \frac{\partial y(t,\mathbf{G})}{\partial t} \\ \frac{\partial b_1(t,\mathbf{G})}{\partial t} \\ \vdots \\ \frac{\partial b_n(t,\mathbf{G})}{\partial t} \end{pmatrix} = \mathbf{A}^{-1} \cdot \begin{pmatrix} \frac{\partial \widetilde{T}}{\partial t} M_c^y - \frac{\partial \widetilde{T}_y}{\partial t} \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_1} + \frac{\partial \widetilde{T}_{b_1}}{\partial t} \\ \vdots \\ \frac{\partial \widetilde{T}}{\partial t} M_c^{b_n} + \frac{\partial \widetilde{T}_{b_n}}{\partial t} \end{pmatrix} \tag{71}$$

A lump-sum tax reform of magnitude ρ can be defined as :

$$\widetilde{T}(y, \mathbf{b}, \rho) = T(y, \mathbf{b}) - \rho \tag{72}$$

Such a reform changes tax liability uniformly without changing the marginal tax rate on y and b so that $\frac{\partial \widetilde{T}(y,b,\rho)}{\partial \rho} = -1$ and $\frac{\partial \widetilde{T_y}(y,b,\rho)}{\partial \rho} = \frac{\partial \widetilde{T_{b_i}}(y,\mathbf{b},\rho)}{\partial \rho} = 0$. Hence there would be no substitution effects in taxpayers responses and this captures only income effect. Using (71), the matrix of income effects is given by:

$$\begin{pmatrix} \frac{\partial y}{\partial \rho} \\ \frac{\partial b_1}{\partial \rho} \\ \vdots \\ \frac{\partial b_n}{\partial \rho} \end{pmatrix} = -\mathbf{A}^{-1} \cdot \begin{pmatrix} M_c^y \\ M_c^{b_1} \\ \vdots \\ M_c^{b_n} \end{pmatrix} \tag{73}$$

Let $X(\theta)$ denotes the initial optimal choice of $x \in \{y, b_1, ..., b_n\}$ for a taxpayer of type θ. To compute substitution effects, consider compensated reforms that leave tax liability at this initial choice unchanged. A compensated reform of the net of tax rate on x is given by:

$$\widetilde{T}(y, \mathbf{b}, \tau_x) = T(y, \mathbf{b}) - \tau_x \left(x - X(\theta)\right) \tag{74}$$

This implies $\frac{\partial \widetilde{T}(y,\mathbf{b})}{\partial \tau_x} = \frac{\partial \widetilde{T}(y,\mathbf{b})_{-x}}{\partial \tau_x} = \frac{\partial \widetilde{T}(y,\mathbf{b})_x}{\partial \tau_{-x}} = 0$ and $\frac{\partial \widetilde{T}_x}{\partial \tau_x} = -1$. Hence compensated reforms only affect the marginal tax rate of x and thus can modify y and $\mathbf{b}$ only through substitution effects. Using (71), the matrix of compensated responses is given by :

$$\begin{pmatrix} \frac{\partial y}{\partial \tau_y} & \frac{\partial y}{\partial \tau_{b_1}} & \cdots & \frac{\partial y}{\partial \tau_{b_n}} \\ \frac{\partial b_1}{\partial \tau_y} & \frac{\partial b_1}{\partial \tau_{b_1}} & \cdots & \frac{\partial b_1}{\partial \tau_{b_n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial b_n}{\partial \tau_y} & \frac{\partial b_n}{\partial \tau_{b_1}} & \cdots & \frac{\partial b_n}{\partial \tau_n} \end{pmatrix} = \mathbf{A}^{-1}. \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{pmatrix} \tag{75}$$

Plugging (73) and (75) into (70), any direct response to a tax perturbation t can be rewritten in terms of substitution and income effects :

$$\begin{pmatrix} \frac{\partial y(t,\mathbf{G})}{\partial t} \\ \frac{\partial b_1(t,\mathbf{G})}{\partial t} \\ \vdots \\ \frac{\partial b_n(t,\mathbf{G})}{\partial t} \end{pmatrix} = \mathbf{A}^{-1} \begin{pmatrix} M_c^y \\ M_c^{b_1} \\ \vdots \\ M_c^{b_n} \end{pmatrix} \frac{\partial \widetilde{T}}{\partial t} + \mathbf{A}^{-1} \cdot \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -1 \end{pmatrix} \cdot \begin{pmatrix} \frac{-\partial \widetilde{T}_y}{\partial t} \\ \frac{-\partial \widetilde{T}_{b_1}}{\partial t} \\ \vdots \\ \frac{-\partial \widetilde{T}_{b_n}}{\partial t} \end{pmatrix}$$

$$= - \begin{pmatrix} \frac{\partial y}{\partial \rho} \\ \frac{\partial b_1}{\partial \rho} \\ \vdots \\ \frac{\partial b_n}{\partial \rho} \end{pmatrix} \frac{\partial \widetilde{T}}{\partial t} - \begin{pmatrix} \frac{\partial y}{\partial \tau_y} & \frac{\partial y}{\partial \tau_{b_1}} & \cdots & \frac{\partial y}{\partial \tau_{b_n}} \\ \frac{\partial b_1}{\partial \tau_y} & \frac{\partial b_1}{\partial \tau_{b_1}} & \cdots & \frac{\partial b_1}{\partial \tau_{b_n}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial b_n}{\partial \tau_y} & \frac{\partial b_n}{\partial \tau_{b_1}} & \cdots & \frac{\partial b_n}{\partial \tau_n} \end{pmatrix} \cdot \begin{pmatrix} \frac{\partial \widetilde{T}_y}{\partial t} \\ \frac{\partial \widetilde{T}_{b_1}}{\partial t} \\ \vdots \\ \frac{\partial \widetilde{T}_{b_n}}{\partial t} \end{pmatrix} \tag{76}$$

which leads to (19).

B.2 Proof of Equation 22

From the equilibrium condition (5), the level of public good i after a small tax reform of magnitude t is described by the fixed-point condition:

$$G_i(t) = S_i + (1 - \mu_i) \int_{\Theta} b_i(\boldsymbol{\theta}, t, G_1(t), ..., G_n(t)) dF(\boldsymbol{\theta}) \tag{77}$$

Differentiating (77) with respect to t yields:

$$\frac{1}{1-\mu_i} \frac{\partial G_i(t)}{\partial t} - \sum_{j=1}^{n} \frac{\partial G_j(t)}{\partial t} \int \frac{\partial b_i(\boldsymbol{\theta}, \mathbf{G})}{\partial G_j} dF(\boldsymbol{\theta}) = \int_{\theta} \frac{\partial b_i(\boldsymbol{\theta}, t, \mathbf{G})}{\partial t} dF(\boldsymbol{\theta}) \tag{78}$$

Using (78), the responses of public goods G_i for all $i \in \{1, 2..., n\}$ can be summarized in matrix form by:

$$\begin{pmatrix} \frac{1}{1-\mu_1} - \frac{\partial B_1(\mathbf{G})}{\partial G_1} & -\frac{\partial B_1(\mathbf{G})}{\partial G_2} & \cdots & -\frac{\partial B_1(\mathbf{G})}{\partial G_n} \\ -\frac{\partial B_2(\mathbf{G})}{\partial G_1} & \frac{1}{1-\mu_2} - \frac{\partial B_2(\mathbf{G})}{\partial G_2} & \cdots & -\frac{\partial B_2(\mathbf{G})}{\partial G_n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial B_n(\mathbf{G})}{\partial G_1} & \cdots & & \frac{1}{1-\mu_n} - \frac{\partial B_n(\mathbf{G})}{\partial G_n} \end{pmatrix} \times \begin{pmatrix} \frac{\partial G_1(t)}{\partial t} \\ \frac{\partial G_2(t)}{\partial t} \\ \vdots \\ \frac{\partial G_n(t)}{\partial t} \end{pmatrix} = \begin{pmatrix} \frac{\partial B_1(t,\mathbf{G})}{\partial t} \\ \frac{\partial B_2(t,\mathbf{G})}{\partial t} \\ \vdots \\ \frac{\partial B_n(t,\mathbf{G})}{\partial t} \end{pmatrix} \tag{79}$$

Using the definition of the leakage matrix (21) and the macro matrix (20), (79) can be rewritten as:

$$(\mathcal{X}-\Xi)\cdot\begin{pmatrix}\frac{\partial G_1(t)}{\partial t}\\ \frac{\partial G_2(t)}{\partial t}\\ \vdots\\ \frac{\partial G_n(t)}{\partial t}\end{pmatrix}=\begin{pmatrix}\frac{\partial B_1(t,\mathbf{G})}{\partial t}\\ \frac{\partial B_2(t,\mathbf{G})}{\partial t}\\ \vdots\\ \frac{\partial B_n(t,\mathbf{G})}{\partial t}\end{pmatrix}$$

Under Assumption 2, this yields:

$$\left[\frac{\partial G_i(t)}{\partial t}\right]_i=\Pi\cdot\left[\frac{\partial B_i(t,\mathbf{G})}{\partial t}\right]_i$$

So for all $i\in\{1,...,n\}$:

$$\frac{\partial G_i(t)}{\partial t}=\sum_{j=1}^{n}\pi_{i,j}\int_\theta\frac{\partial b_j(t,\theta,\mathbf{G})}{\partial t}dF(\boldsymbol{\theta})\tag{80}$$

Plugging (19b) into (80) yields (22).

B.3 Proof of Equation 24

To measure the impact of the reform on the government's program, we can define the perturbed Lagrangian as:

$$\widetilde{\mathcal{L}}\left(t,\mathbf{G}(t)\right)=\int_\Theta\left[T\left(y\left(\boldsymbol{\theta},t,\mathbf{G}(t)\right),\mathbf{b}\left(\boldsymbol{\theta},t,\mathbf{G}(t)\right),t\right)-\sum_{i=1}^{n}S_i+\frac{1}{\lambda}\Phi\left(U\left(\boldsymbol{\theta},t,G(t)\right);\boldsymbol{\theta}\right)\right]dF(\boldsymbol{\theta})\tag{81}$$

The impact of a tax reform of magnitude t is then given by:

$$\frac{\partial\widetilde{\mathcal{L}}\left(t,G(t)\right)}{\partial t}=\int_\Theta\left[\frac{\partial\widetilde{T}(y,\mathbf{b},t)}{\partial t}+\frac{\partial y(\boldsymbol{\theta},t,\mathbf{G})}{\partial t}T_y+\sum_{j=1}^{n}\frac{\partial b_j(\boldsymbol{\theta},t,\mathbf{G})}{\partial t}T_{b_j}+\frac{\Phi_U}{\lambda}\frac{\partial U(\boldsymbol{\theta},t,\mathbf{G})}{\partial t}\right]dF(\boldsymbol{\theta})$$
$$+\sum_{i=1}^{n}\frac{\partial G_i(t)}{\partial t}\frac{\partial\mathcal{L}\left(\mathbf{G}\right)}{\partial G_i}\tag{82}$$

Applying the envelope theorem to (18), the impact of a perturbation of magnitude t social welfare for a given $\mathbf{G}$ verifies:

$$\frac{1}{\lambda}\frac{\partial\Phi\left(U\left(\boldsymbol{\theta},t,\mathbf{G}\right);\boldsymbol{\theta}\right)}{\partial t}=-\frac{\partial\widetilde{T}\left(y,\mathbf{b},t\right)}{\partial t}g(\boldsymbol{\theta}) \tag{83}$$

Plugging (19) and (83) in (82), holding **G** constant, yields (24).

B.4 Proof of Proposition 3

Combining (81) with (23) and (3), the impact of a change in public good G_i on the Government Lagrangian is:

$$\frac{\partial\mathcal{L}\left(\mathbf{G}\right)}{\partial G_i}=\int_{\Theta}\left[\frac{\partial y(\boldsymbol{\theta},\mathbf{G})}{\partial G_i}T_y+\sum_{j=1}^{n}\frac{\partial b_j(\boldsymbol{\theta},\mathbf{G})}{\partial G_i}T_{b_j}+g(\theta)M^{G_i}(\boldsymbol{\theta},\mathbf{G})\right]dF(\boldsymbol{\theta}) \tag{84}$$

Combining (84) with (22) yields:

$$-\sum_{i=1}^{n}\frac{\partial G_i(t)}{\partial t}\frac{\partial\mathcal{L}\left(\mathbf{G}\right)}{\partial G_i}=$$
$$\sum_{i=1}^{n}\sum_{j=1}^{n}\pi_{i,j}\int_{\theta}\left[\frac{\partial b_j}{\partial\rho}\frac{\partial\widetilde{T}}{\partial t}+\frac{\partial b_j}{\partial\tau_y}\frac{\partial\widetilde{T}_y}{\partial t}+\sum_{k=1}^{n}\frac{\partial b_j}{\partial\tau_{b_k}}\frac{\partial\widetilde{T}_{b_k}}{\partial t}\right]\left[\frac{\partial y(\boldsymbol{\theta},\mathbf{G})}{\partial G_i}T_y+\frac{\partial b_j(\boldsymbol{\theta},\mathbf{G})}{\partial G_i}T_{b_j}+g(\theta)M^{G_i}(\boldsymbol{\theta},\mathbf{G})\right]dF(\boldsymbol{\theta})$$

Inverting summation order and using (25) yields:

$$-\sum_{i=1}^{n}\frac{\partial G_i(t)}{\partial t}\frac{\partial\mathcal{L}\left(\mathbf{G}\right)}{\partial G_i}=\sum_{j=1}^{n}\eta_j\int_{\theta}\left[\frac{\partial b_j}{\partial\rho}\frac{\partial\widetilde{T}}{\partial t}+\frac{\partial b_j}{\partial\tau_y}\frac{\partial\widetilde{T}_y}{\partial t}+\sum_{k=1}^{n}\frac{\partial b_j}{\partial\tau_{b_k}}\frac{\partial\widetilde{T}_{b_k}}{\partial t}\right]dF(\boldsymbol{\theta}) \tag{85}$$

Note that (82) can be rewritten as:

$$\frac{\partial\widetilde{\mathcal{L}}\left(t,\mathbf{G}(t)\right)}{\partial t}=\frac{\partial\widetilde{\mathcal{L}}\left(t,\mathbf{G}\right)}{\partial t}+\sum_{i=1}^{n}\frac{\partial G_i(t)}{\partial t}\frac{\partial\mathcal{L}\left(\mathbf{G}\right)}{\partial G_i} \tag{86}$$

Plugging (85) and (24) into (86) yields (26).

A lump-sum reform of magnitude ρ changes only tax liability by $\frac{\partial T}{\partial\rho}=-1$. Equating (26) to 0 for $\frac{\partial T}{\partial\rho}=-1$ and $\frac{\partial\widetilde{T}_y(y,b,\rho)}{\partial\rho}=\frac{\partial\widetilde{T}_{b_i}(y,\mathbf{b},\rho)}{\partial\rho}=0$ yields (27).

B.5 Proof of Equations 28 and 29

The equilibrium level of a public good G_i after a change in the grant S_j, for $\{i,j\}\in\{1,...,n\}$, is defined by the fixed point condition:

$$G_i(\mathbf{S}) = S_i + (1-\mu_i)\int_\Theta b_i(\boldsymbol{\theta}, G_1(\mathbf{S}), ..., G_n(\mathbf{S}))dF(\boldsymbol{\theta}) \tag{87}$$

Differentiating (87) yields:

$$\frac{1}{1-\mu_i}\frac{\partial G_i(\mathbf{S})}{\partial S_j} - \sum_{k=1}^{n}\frac{\partial G_k(S_j)}{\partial S_j}\frac{\partial B_i(\mathbf{G})}{\partial G_k} = \mathbb{1}_{i=j} \times \frac{1}{1-\mu_i} \tag{88}$$

This n-conditions can be summarized in matrix form as:

$$\begin{pmatrix} \frac{1}{1-\mu_1} - \frac{\partial B_1(\mathbf{G})}{\partial G_1} & -\frac{\partial B_1(\mathbf{G})}{\partial G_2} & \cdots & -\frac{\partial B_1(\mathbf{G})}{\partial G_n} \\ -\frac{\partial B_2(\mathbf{G})}{\partial G_1} & \frac{1}{1-\mu_2} - \frac{\partial B_2(\mathbf{G})}{\partial G_2} & \cdots & -\frac{\partial B_2(\mathbf{G})}{\partial G_n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial B_n(\mathbf{G})}{\partial G_1} & \cdots & & \frac{1}{1-\mu_n} - \frac{\partial B_n(\mathbf{G})}{\partial G_n} \end{pmatrix} \times \begin{pmatrix} \frac{\partial G_1(S_j)}{\partial S_j} \\ \frac{\partial G_2(S_j)}{\partial S_j} \\ \vdots \\ \frac{\partial G_n(S_j)}{\partial S_j} \end{pmatrix} \tag{89}$$

$$= \begin{pmatrix} \frac{1}{1-\mu_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{1-\mu_n} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \vdots \\ \mathbb{1}_{i=j} \\ \vdots \\ 0 \end{pmatrix}$$

Under Assumption 2, we can use (21) and (20) to rewrite it as:

$$\left[\frac{\partial G_i(\mathbf{S}}{\partial S_j}\right]_i = \Pi \cdot \mathcal{X} \cdot \left[\mathbb{1}_{i=j}\right]_i \tag{90}$$

which yields (28).

This measure the impact of the reform of S_j on social welfare, define the reformed Lagrangian:

$$L(S_j) = \int_\Theta \left[T\left(y(\boldsymbol{\theta}, \mathbf{G}(S_j)), \mathbf{b}(\boldsymbol{\theta}, \mathbf{G}(S_j))\right) - \sum_{i=1}^{n} S_i + \frac{1}{\lambda}\Phi\left(U\left(\boldsymbol{\theta}, \mathbf{G}(S_j)\right); \boldsymbol{\theta}\right)\right] dF(\boldsymbol{\theta}) \tag{91}$$

Differentiate (91) and using (25) yields (29).

B.6 Proof of Proposition 5

To measure the impact of the joint reform on the government's program, we can define the neutralized Lagrangian as:

$$\mathscr{L}^N\left(t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t))\right) = -\sum_{i=1}^{n} \widetilde{S}_j(t)$$
$$+ \int_{\Theta} \left[T\left(y\left(\boldsymbol{\theta}, t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t)\right), \mathbf{b}\left(\boldsymbol{\theta}, t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t)\right), t\right) + \frac{1}{\lambda}\Phi\left(U\left(\boldsymbol{\theta}, t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t)\right); \boldsymbol{\theta}\right)\right] dF(\boldsymbol{\theta})$$

So the impact of the joint reform is given by:

$$\frac{\partial \mathscr{L}^N\left(t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t)\right)}{\partial t} = \frac{\partial \widetilde{\mathcal{L}}\left(t, \mathbf{G}\right)}{\partial t} + \sum_{i=1}^{n} \left(\frac{\partial G_i(t, \mathbf{S}))}{\partial t} + \sum_{j=1} \frac{\partial \widetilde{S}_j(t)}{\partial t} \frac{\partial G_i(\mathbf{S})}{\partial S_j} \right) \frac{\partial \mathcal{L}\left(\mathbf{G}\right)}{\partial G_i} - \sum_{j=1}^{n} \frac{\partial \widetilde{S}_j(t)}{\partial t} \tag{92}$$

Using (22), (28) and Definition 2 we obtain:

$$\sum_{i=1}^{n} \left(\frac{\partial G_i(t, \mathbf{S}))}{\partial t} + \sum_{j=1} \frac{\partial \widetilde{S}_j(t)}{\partial t} \frac{\partial G_i(\mathbf{S})}{\partial S_j} \right) = \sum_{i=1}^{n} \sum_{j=1}^{n} \pi_{i,j} \left(\frac{\partial B_j(t, \mathbf{G})}{\partial t} + \frac{\partial \widetilde{S}_j(t)}{\partial t} \frac{1}{1-\mu_j} \right) = 0$$

So the impact of the joint reform is given by:

$$\frac{\partial \mathscr{L}^N\left(t, \mathbf{G}(t, \widetilde{\mathbf{S}}(t)\right)}{\partial t} = \frac{\partial \widetilde{\mathcal{L}}\left(t, \mathbf{G}\right)}{\partial t} + \sum_{j=1}^{n} (1-\mu_j) \frac{\partial B_j(t, \mathbf{G})}{\partial t} \tag{93}$$

Plugging (24) in (93) yields (33).

C Proofs of the Results of Section V

C.1 Proof of Proposition 6

Consider a reform of the linear tax rate on the j-th donation type is the tax credit system 34:

$$\widetilde{T}(y, \mathbf{b}, t) = \mathcal{T}(y) + \sum_{\substack{i=1 \\ i \neq j}}^{n} t_i\, b_i + (t_j - t)\, b_j \tag{94}$$

Such a reform changes tax liability: $\frac{\partial \widetilde{T}(y, \mathbf{b}, t)}{\partial t} = -b_j$ and the marginal tax rate on donation j : $\frac{\partial \widetilde{T}_j(y, \mathbf{b}, t}{\partial t} = -1$. Hence using (33) this yields:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[g(\boldsymbol{\theta}) - 1 + \frac{\partial y(\boldsymbol{\theta})}{\partial \rho} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta})}{\partial \rho} \left(1 - \mu_i + t_i)\right) \right] b_j(\boldsymbol{\theta}) dF(\boldsymbol{\theta}) + \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_{b_j}} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta})}{\partial \tau_{b_j}} (1 - \mu_i + t_i) \right] dF(\theta) \tag{95}$$

The Slutsky equation implies:

$$\frac{\partial x_i^U}{\partial \tau_j} = \frac{\partial x_i}{\partial \tau_j} + x_j(\theta) \frac{\partial x_i}{\partial \rho} \tag{96}$$

So we can rewrite (95) as:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[b_j(\boldsymbol{\theta})(g(\boldsymbol{\theta}) - 1) + \frac{\partial y^u(\boldsymbol{\theta})}{\partial \tau_{b_j}} T_y + \sum_{i=1}^{n} \frac{\partial b_i^u(\boldsymbol{\theta})}{\partial \tau_{b_j}} (1 - \mu_i + t_i) \right] dF(\boldsymbol{\theta}) \tag{97}$$

Using the definition of $\epsilon_j(\boldsymbol{\theta})$ yields (36).

Now consider a reform of the tax rate on donations in a uniform tax credit system as described in (35). With such a system, the only available reforms are: $\widetilde{T}(y, \mathbf{b}, t) = T(y) + (t_0 - t) b_0$, with $t > 0 (t < 0)$ to uniformly reduce(increase) the price of every donations. Such a reform changes tax liability: $\frac{\partial \widetilde{T}(y,\mathbf{b},t)}{\partial t} = -b_0(\boldsymbol{\theta})$ and the marginal tax rate on every donation $i \in [1:n]$: $\frac{\partial \widetilde{T}_{b_i}(y,\mathbf{b},t)}{\partial t} = -1$. Hence using (33) this yields:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[1 - g(\theta) - \frac{\partial y(\boldsymbol{\theta})}{\partial \rho} T_y - \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta})}{\partial \rho} (1 - \mu_i + t_0) \right] b_0(\boldsymbol{\theta}) dF(\theta) + \int_\Theta \left[\frac{\partial y(\boldsymbol{\theta})}{\partial \tau_{b_0}} T_y + \sum_{i=1}^{n} \frac{\partial b_i(\boldsymbol{\theta})}{\partial \tau_{b_0}} (1 - \mu_i + t_0) \right] dF(\theta) \tag{98}$$

Using the Slutsky equation this can be rewritten as:

$$\frac{\partial \mathscr{L}^j(t)}{\partial t} = \int_\Theta \left[b_0(\boldsymbol{\theta}) \left(g(\theta) - 1 \right) + \sum_{i=1}^{n} \frac{\partial b_i^u(\boldsymbol{\theta})}{\partial \tau_0} (1 - \mu_i + t_0) + \frac{\partial y^u(\boldsymbol{\theta})}{\partial \tau_{b_0}} T_y \right] dF(\theta) \tag{99}$$

Using $\mu_i = \mu$ yields (37).