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Mogens Fosgerauy, Julien Monardoz, André de Palma

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The Inverse Product Differentiation Logit Model*

Mogens Fosgerau[†] Julien Monardo[‡] André de Palma[§]

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Abstract

We introduce the inverse product differentiation logit (IPDL) model, a micro-founded inverse market share model for differentiated products that captures market segmentation according to one or more characteristics. The IPDL model generalizes the nested logit model to allow richer substitution patterns, including complementarity in demand, and can be estimated by linear instrumental variables regression with market-level data. Furthermore, we provide Monte Carlo experiments comparing the IPDL model to the workhorse empirical models of the literature. Lastly, we demonstrate the empirical performance of the IPDL model using a well-known dataset on the ready-to-eat cereals market. (JEL: C26, D11, D12, L)

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[†]University of Copenhagen; mogens.fosgerau@econ.ku.dk.

[‡]University of Bristol; julien.monardo@bristol.ac.uk.

[§]CY Cergy Paris Université; andre.de-palma@cyu.fr.

1 Introduction

We introduce the inverse product differentiation logit (IPDL) model, a micro-founded inverse market share model that captures market segmentation according to one or more characteristics. The IPDL model generalizes the nested logit model to allow richer substitution patterns while retaining its attractive features: it is easy to estimate by linear instrumental variables regression using market-level data, and it is consistent with a model of heterogeneous, utility-maximizing consumers, which makes it useful for analyzing a wide range of economic questions.

The nested logit model is commonly used to estimate demand in differentiated products markets. It can be estimated by linear instrumental variables regression, using that its inverse market share function is linear-in-parameters and in closed form (Berry, 1994). However, it only captures market segmentation according to one or several characteristics treated hierarchically (i.e., partitioning the choice set into nests, nests into subnests, etc.), which imposes strong restrictions on substitution patterns. To avoid these restrictions, several generalized extreme value (GEV) models (McFadden, 1978) have been proposed, for which, however, a closed-form inverse market share function does not exist, preventing the use of regression techniques.[1] Notably, Bresnahan et al. (1997)'s product differentiation logit (PDL) model generalizes the nested logit model by treating the grouping characteristics non-hierarchically. The IPDL model allows the same grouping structure as the PDL model but builds it into a linear-in-parameters, closed-form inverse market share function rather than into a closed-form market share function. The IPDL model is thus not the inversion of the PDL model or any other GEV model but a novel model we introduce in this paper.

The state-of-the-art approach to estimating demand in differentiated products markets is the random coefficient logit (RCL) model with structural error terms to allow for unobserved product characteristics, estimated using the methodology developed by Berry et al. (1995, hereafter BLP).[2] The RCL model allows for rich

[1] See Subsection 3.3 for further details on GEV models.

[2] BLP provide an estimator that allows for rich substitution patterns while handling the endogeneity issues related to the modelling of unobserved product characteristics. BLP also propose an algorithm to compute that estimator. Conlon and Gortmaker (2020) consolidate best estimation

substitution patterns determined by a random coefficients specification of the distribution of unobserved preference heterogeneity. The methodology developed by BLP involves a non-linear, non-convex optimization problem and the simulation and numerical inversion of the market share function. With the IPDL model, in contrast, we specify directly a closed-form, linear-in-parameters inverse market share function, which generates substitution patterns determined by segmentation of the differentiated products, and which can be estimated by linear instrumental variables regression.

By specifying an inverse market share function, we avoid some restrictions embedded in the GEV and the RCL models. In particular, these latter models restrict products to be substitutes in demand, meaning that they have a positive cross-price derivative of market share. By contrast, the IPDL model allows for complementarity in demand. Furthermore, we show that the IPDL model is consistent with utility maximization, which makes it useful for welfare analysis. Specifically, we show the IPDL model is consistent with a representative consumer who chooses a vector of market shares to maximize her quasi-linear direct utility function subject to a budget constraint, which is, in turn, consistent with a population of utility-maximizing, heterogeneous consumers.

However, relying on an inverse market share function entails a cost. Without additional assumptions about the distribution of preferences in the population of consumers, the IPDL model cannot be used to address economic questions at the individual level, such as the distributional effects of any events or policies. Fortunately, many economic questions of interest do not require knowing the distribution of preferences and can thus be addressed using the IPDL model. Prominent examples include the measurement of market power, and the welfare effects of a merger, a new product introduced to the market, or regulatory changes such as tax or trade policies.

We investigate the empirical properties of the IPDL model using Monte Carlo experiments. The IPDL model performs well in approximating the substitution

practices in a Python package. Dubé et al. (2012) propose another algorithm to compute the BLP estimator. Lee and Seo (2015) and Salanié and Wolak (2022) provide approximations of the BLP estimator that are faster and easier to compute.

patterns generated by the PDL model and the RCL model with independent normal random coefficients on dummies for groups defined by market segmentation. Furthermore, even without complementarity in demand, the IPDL model allows substitution patterns that the RCL model cannot replicate. We also find that the IPDL model outperforms the RCL model in approximating the substitution patterns generated by the PDL model. While there is a concern that the IPDL model may generate complementarity in demand when there is none in the data, we do not observe this in our experiments.

We then analyze the empirical performance of the IPDL model using a well-known dataset on the ready-to-eat cereals market, which exhibits segmentation according to the brand of the cereals and the market segment they belong to. We estimate the corresponding IPDL model and compare to the RCL model with independent normal random coefficients on dummies for groups defined by segmentation. We estimate two specifications of both models: one with many markets and another with many products. In both specifications, the IPDL model provides a better out-of-sample fit to the data than the RCL model. Moreover, in both specifications, the IPDL model generates significantly higher substitution between cereals than the RCL model. The IPDL model implies markups in line with the literature (Nevo, 2001; Michel et al., 2023), while the RCL model implies significantly higher markups.

The Monte Carlo experiments and the empirical application suggest that the IPDL model is particularly useful in describing markets that exhibit segmentation. This is the case of many markets: the beers market is segmented by brand and style (e.g., IPA, lager, etc.), the cars market by brand and market segment (e.g., compact, luxury), the ready-to-eat cereals market by brand and market segment (e.g., kids, adults). Often, market segmentation proxies for other (continuous) product characteristics, including prices: lager beers have lower alcohol content than IPA beers; luxury cars are less fuel efficient, larger, and more expensive than compact cars; cereals for kids are more sugary than cereals for adults. In these cases, the IPDL model can generate substitution patterns determined, at least indirectly, by these (continuous) characteristics. When there is no obvious market segmentation, it is still possible to use a clustering algorithm to define groups for the IPDL model.

Other approaches exist for estimating demand in differentiated products markets based on market-level data (e.g., Barnett and Serletis, 2008; Nevo, 2011; Gandhi and Nevo, 2021). The flexible functional form approach (e.g., the AIDS model of Deaton and Muellbauer, 1980) provides rich substitution patterns, including complementarity in demand, and has been successfully applied to many economic questions. However, in this approach, the unobservables enter in a very restrictive way, there are many parameters to estimate, and the introduction of new products cannot be addressed. Other authors propose semi- or non-parametric demand models (e.g., Pinkse and Slade, 2004; Haag et al., 2009; Blundell et al., 2012; Compiani, 2022) or the use of a more flexible specification of the distribution of the random coefficients in the RCL model (e.g., Lu et al., 2022; Wang, 2023). Closest to our paper is Compiani (2022), who non-parametrically estimates inverse market share functions for differentiated products based on market-level data. His approach provides rich substitution patterns but faces the curse of dimensionality that restricts its use to relatively small choice sets. By contrast, the IPDL model can handle very large choice sets.

The paper is organized as follows. Section 2 presents our general setting and discusses the role of demand inversion in estimation. Section 3 introduces the IPDL model, studies its properties, and discusses estimation and identification with market-level data. It further provides Monte Carlo experiments that compare the IPDL model to the workhorse empirical models of the literature and an extended discussion of its relationship to other demand models. Section 4 analyzes the empirical performance of the IPDL model using a well-known dataset on the ready-to-eat cereals markets and compares to the RCL model estimated using the methodology developed by BLP. Section 5 concludes. The supplement provides additional simulation results on the IPDL model.

2 General Setting

We first introduce our setting and discuss the role of demand inversion in estimation. Consider a population of consumers making choices among a set of $J+1$ differentiated products, indexed by $\mathcal{J} = \{0, 1, \ldots, J\}$, where product $j = 0$ is the outside good. We consider data on market shares s_{jt}, prices p_{jt}, and K product/market characteristics $\mathbf{x}_{jt}$ for each product $j = 1, \ldots, J$ in each market $t = 1, \ldots, T$ (Berry, 1994; Berry et al., 1995; Nevo, 2001; Berry and Haile, 2021). For each market t, the market shares s_{jt} are positive and sum to 1, i.e., $\mathbf{s}_t = (s_{0t}, \ldots, s_{Jt})^\intercal \in \Delta^\circ$, where Δ° is the set of positive market share vectors.[3]

Following Berry and Haile (2014), let $\delta_{jt} \in \mathbb{R}$ be an index given by

$$\delta_{jt} = \delta\left(p_{jt}, \mathbf{x}_{jt}, \xi_{jt}; \boldsymbol{\theta}_1\right), \quad j \in \mathcal{J}, \quad t = 1, \ldots, T,$$

where $\xi_{jt} \in \mathbb{R}$ is an unobserved characteristics term for product/market jt, and where $\boldsymbol{\theta}_1$ is a vector of parameters. Consider the system of market share equations

$$s_{jt} = \sigma_j\left(\boldsymbol{\delta}_t; \boldsymbol{\theta}_2\right), \quad j \in \mathcal{J}, \quad t = 1, \ldots, T, \tag{1}$$

which relates the vector of observed market shares, $\mathbf{s}_t$, to the vector of product indexes, $\boldsymbol{\delta}_t = (\delta_{0t}, \ldots, \delta_{Jt})^\intercal$, through the market share function $\boldsymbol{\sigma} = (\sigma_0, \ldots, \sigma_J)$, where $\boldsymbol{\theta}_2$ is a vector of parameters.

Normalize the index of the outside good by setting $\delta_{0t} = 0$ in each market t so that $\boldsymbol{\delta}_t \in \mathcal{D} = \{\boldsymbol{\delta}_t \in \mathbb{R}^{J+1} : \delta_{0t} = 0\}$, and assume that the function $\boldsymbol{\sigma}(\cdot; \boldsymbol{\theta}_2) : \mathcal{D} \to \Delta^\circ$ is invertible. Then, the inverse market share function, denoted by σ_j^{-1}, maps from market shares $\mathbf{s}_t$ to each index δ_{jt} with

$$\delta_{jt} = \sigma_j^{-1}\left(\mathbf{s}_t; \boldsymbol{\theta}_2\right), \quad j \in \mathcal{J}, \quad t = 1, \ldots, T. \tag{2}$$

In addition, assume a linear index

$$\delta_{jt} = \mathbf{x}_{jt}\boldsymbol{\beta} - \alpha p_{jt} + \xi_{jt}, \quad j = 1, \ldots, J, \quad t = 1, \ldots, T,$$

where the vector of parameters $\boldsymbol{\beta} \in \mathbb{R}^K$ captures the consumers' taste for characteristics $\mathbf{x}_{jt}$ and the parameter $\alpha > 0$ is the consumers' marginal utility of income. Then the unobserved product characteristics terms, ξ_{jt}, can be written as a function

[3] Formally, Δ° is the relative interior of $\Delta = \left\{\mathbf{s} \in [0, \infty)^{J+1} : \sum_{j \in \mathcal{J}} s_j = 1\right\}$, the unit simplex in $\mathbb{R}^{J+1}$.

of the data and parameters $\boldsymbol{\theta}_1 = (\alpha, \boldsymbol{\beta})$ and $\boldsymbol{\theta}_2$ to be estimated:

$$\xi_{jt} = \sigma_j^{-1}(\mathbf{s}_t; \boldsymbol{\theta}_2) + \alpha p_{jt} - \mathbf{x}_{jt}\boldsymbol{\beta}, \quad j = 1, \ldots, J, \quad t = 1, \ldots, T. \tag{3}$$

The product characteristics terms, ξ_{jt}, are the structural error terms of the model, as they are observed by consumers and firms but not by the modeler. Prices are likely to be endogenous since firms may consider both observed and unobserved product characteristics when setting prices. Market shares are endogenous as they are defined by the system of Equations (1), where the market share function of each product depends on the entire vector of endogenous prices and unobserved product characteristics. Then, following Berry (1994), we can estimate the market share function $\boldsymbol{\sigma}$ based on the conditional moment restrictions $\mathbb{E}[\xi_{jt}|\mathbf{z}_t] = 0$ for all $j = 1, \ldots, J$ and $t = 1, \ldots, T$, provided that there exist appropriate instruments $\mathbf{z}_t$ for prices and market shares.

Since the seminal papers by Berry (1994) and Berry et al. (1995), the standard practice of the demand estimation literature with market-level data has been to specify a GEV or RCL model. For these models, except for the logit and nested logit models, the implied inverse market share function is not in closed form and must then be computed numerically during estimation, which prevents the use of standard regression techniques. By contrast, we directly specify a closed-form, invertible, linear-in-parameters inverse market share function, for which estimation amounts to linear regression.

Consider as an example the three-level nested logit model, which partitions the choice set into nests and nests into subnests. This is a special case of the IPDL model that we introduce in this paper. Let $\boldsymbol{\theta}_2 = (\mu_1, \mu_2)$ be the vector of grouping parameters, with $\sum_{d=1}^{2} \mu_d < 1$, $\mu_1 \geq 0$ and $\mu_2 \geq 0$ to make the nested logit model consistent with utility maximization. The corresponding inverse market share function is linear in parameters:

$$\sigma_j^{-1}(\mathbf{s}_t; \boldsymbol{\theta}_2) = \left(1 - \sum_{d=1}^{2} \mu_d\right) \ln(s_{jt}) + \sum_{d=1}^{2} \mu_d \ln\left(s_{d(j),t}\right) + c_t = \delta_{jt}, \tag{4}$$

where $s_{1(j),t} = \sum_{k \in 1(j)} s_{kt}$ and $s_{2(j),t} = \sum_{k \in 2(j)} s_{kt}$, with $1(j)$ and $2(j)$ the sets of products belonging the same nest and to the same subnest as product j, respectively,[4] and where $c_t \in \mathbb{R}$ is a market-specific constant determined by the normalization of the vector $\boldsymbol{\delta}_t$. The three-level nested logit model corresponds to the logit model when $\mu_1 = 0$ and $\mu_2 = 0$ and to the two-level nested logit model when either $\mu_1 = 0$ or $\mu_2 = 0$.

Assume that the outside good is in a nest by itself, such that $\sigma_0^{-1}(\mathbf{s}_t; \mu_1, \mu_2) = \ln(s_{0t}) + c_t = \delta_{0t}$, and, in turn, as for the logit model, $c_t = -\ln(s_{0t})$ since $\delta_{0t} = 0$. Then, combining with Equation (4), the three-level nested logit model boils down to the linear regression model (Verboven, 1996a)

$$\ln\left(\frac{s_{jt}}{s_{0t}}\right) = \mathbf{x}_{jt}\boldsymbol{\beta} - \alpha p_{jt} + \sum_{d=1}^{2} \mu_d \ln\left(\frac{s_{jt}}{s_{d(j),t}}\right) + \xi_{jt}, \tag{5}$$

for all products $j = 1, \ldots, J$ in each market $t = 1, \ldots, T$, where identification requires at least one instrument for price and two for the endogenous log-share terms.

3 The IPDL Model

The nested logit model can be estimated by linear instrumental variables regression and, due to its parsimony, can handle very large choice sets. However, it imposes strong restrictions on substitution patterns. In this section, we introduce the IPDL model, which generalizes the inverse market share function of the nested logit model while maintaining its desirable features.

Setting Suppose that each market exhibits segmentation according to D discrete product characteristics, indexed by d. Each of these *grouping characteristics* d defines a partition of the choice set, that is, a finite number of *groups* of products such that each product belongs to exactly one group for each grouping characteristic. For example, cars may be grouped by brand, size, and fuel type. We denote by

[4] Setting $\sigma_1 = \mu_1 + \mu_2$ and $\sigma_2 = \mu_1$, where σ_1 and σ_2 refer to as grouping parameters for subnests and nests respectively, we recover Equation (10) of Verboven (1996a) with $0 \leq \sigma_2 \leq \sigma_1 < 1$.

$d(j) \subseteq \{1, \ldots, J\}$ the set of products grouped with product j according to grouping characteristic d. The grouping structure is assumed to be exogenous and common across markets.

Let $\boldsymbol{\theta}_2 = (\mu_1, \ldots, \mu_D)$ be the vector of grouping parameters, with $\sum_{d=1}^{D} \mu_d < 1$ and $\mu_d \geq 0$ for all $d = 1, \ldots, D$ making the IPDL model consistent with utility maximization, as we show below. The IPDL model has an inverse market share function defined by

$$\sigma_j^{-1}(\mathbf{s}_t; \boldsymbol{\theta}_2) = \left(1 - \sum_{d=1}^{D} \mu_d\right) \ln(s_{jt}) + \sum_{d=1}^{D} \mu_d \ln\left(s_{d(j),t}\right) + c_t = \delta_{jt}, \quad j = 1, \ldots, J, \tag{6}$$

where $s_{d(j),t} = \sum_{k \in d(j)} s_{kt}$ is the market share of group $d(j)$ in market t.

Two products are of the same *type* if they belong to the same group according to all grouping characteristics d. We assume that the outside good is the only product of its type, that is,

$$\sigma_0^{-1}(\mathbf{s}_t; \boldsymbol{\theta}_2) = \ln(s_{0t}) + c_t = \delta_{0t}. \tag{7}$$

The index for the outside good is normalized to zero, $\delta_{0t} = 0$, and we find that $c_t = -\ln(s_{0t})$.

By construction, the logit and the nested logit models are special cases of the IPDL model: the logit model is obtained when there is no segmentation, and the nested logit model is obtained when the grouping structure is hierarchical.

The IPDL model generalizes the inverse market share function of the nested logit model by allowing arbitrary, non-hierarchical grouping structures, that is, any partition of the choice set for each grouping characteristic. In Subsection 3.1, we show that the non-hierarchical grouping structure allows the IPDL model to accommodate richer substitution patterns than the nested logit model. The product differentiation logit (PDL) model of Bresnahan et al. (1997) allows the same non-hierarchical grouping structure but is a specific member of the family of GEV models. The IPDL model is different: in general, its inverse market share function does not correspond to any other model. In Subsection 3.1, we show the IPDL model avoids the restriction, inherent in the GEV and RCL models, that all products are substitutes in demand.

Note that the IPDL inverse market share function $\boldsymbol{\sigma}^{-1} = (\sigma_0^{-1}, \ldots \sigma_J^{-1})$ with elements given by Equations (6) and (7) is invertible.[5] That is, any vector of observed market shares $\mathbf{s}_t \in \Delta^\circ$ is rationalized by a unique vector of product indexes $\boldsymbol{\delta}_t \in \mathcal{D}$, which is key for identification purposes. Invertibility here concerns the uniqueness of the market share function that corresponds to the closed-form IPDL inverse market share function; this goes in the opposite direction of the market share invertibility results usually considered in the literature (see, e.g., Berry, 1994; Berry et al., 2013). However, the IPDL market share function is not in closed form. Counterfactual analyses typically require computing the market share function. This can be done by inverting the inverse market share function or by solving the utility maximization problem (see below), numerically after estimation.

Micro-foundation In Appendix B.2, we show that the IPDL inverse market share function defined by Equations (6) and (7) can be derived from a representative consumer's utility, as it is the case for the logit and nested logit (inverse) market share functions (Anderson et al., 1988; Verboven, 1996b). Specifically, the IPDL model is consistent with a representative consumer, endowed with income y, who chooses a vector $\mathbf{s}_t \in \Delta^\circ$ of positive market shares in market t so as to maximize her utility function

$$u(\mathbf{s}_t) = \alpha y + \sum_{j \in \mathcal{J}} \delta_{jt} s_{jt} - \mu_0 \sum_{j \in \mathcal{J}} s_{jt} \ln(s_{jt}) - \sum_{d=1}^{D} \mu_d \left(\sum_{g \in d \cup \{0\}} s_{gt} \ln(s_{gt}) \right), \quad (8)$$

where $\mu_0 = 1 - \sum_{d=1}^{D} \mu_d$, $s_{gt} = \sum_{k \in g} s_{kt}$, and d is identified with the set of groups for grouping characteristic d. The second term in Equation (8) captures the net utility derived from the consumption $\mathbf{s}_t$ of the products absent interaction among them, and the remaining terms express taste for variety. Specifically, the parameter μ_0 measures taste for variety over the entire choice set, while each parameter μ_d measures taste for variety across groups according to characteristic d. A higher value of μ_d puts more weight on variety at the group level, which can be interpreted as meaning that products in the same group according to d are more similar. See Ver-

[5] See Lemma 3 in Appendix B.1.

boven (1996b) for a similar interpretation of the grouping parameter in the nested logit model.

Furthermore, we can show that the utility function (8) belongs to the class of utilities studied by Allen and Rehbeck (2019b), which can be interpreted as representing the behavior of heterogeneous, utility-maximizing consumers. With this interpretation, the grouping parameters capture consumer heterogeneity in taste, like the grouping parameters in the random utility interpretation of the nested logit model. As noted by Allen and Rehbeck (2019b), we do not need to know and even identify the distribution of consumer heterogeneity to identify demand parameters and thus substitution patterns. However, we can make two comments. First, the IPDL model incorporates greater consumer heterogeneity than the logit and nested logit models, as the latter are special cases of the former. Second, as suggested by our simulations below, the IPDL model accommodates similar consumer heterogeneity as the RCL model with independent normal random coefficients on dummies for groups defined by market segmentation.

The IPDL model avoids restrictions inherent in the GEV and RCL models. In particular, these latter models assume that each consumer chooses the product that provides her with the highest utility among all the available products. This assumption, known as the single-unit purchase assumption, restricts products to be substitutes in demand, i.e., as having positive cross-price derivatives of market share. In contrast, we do not retain the single-unit purchase assumption. As a result, the IPDL model allows for complementarity in demand (see Subsection 3.1) and does not rule out multiple choices at the individual level. Some instances of the IPDL model may be consistent with the single-unit purchase assumption, e.g., the IPDL models that are equivalent to the logit or nested logit models.

Identification and Estimation Combining Equations (6) and (7) and using that $\delta_{0t} = 0$ for all $t = 1, \ldots, T$, the IPDL model boils down to the linear regression

$$\ln\left(\frac{s_{jt}}{s_{0t}}\right) = \mathbf{x}_{jt}\boldsymbol{\beta} - \alpha p_{jt} + \sum_{d=1}^{D} \mu_d \ln\left(\frac{s_{jt}}{s_{d(j),t}}\right) + \xi_{jt}, \tag{9}$$

for all products $j = 1, \ldots, J$ in each market $t = 1, \ldots, T$.

Equation (9) has the same form as the logit and nested logit equations, except for the log-share terms. Following the literature, we assume that product/market characteristics $\mathbf{x}_{jt}$ are exogenous and that prices and log-share terms are endogenous. As a consequence, the IPDL model reduces to a linear instrumental variables regression, where identification requires at least one instrument for price and one for each of the log-share terms.

As it is well known, instruments for prices include cost shifters and markup shifters (see, e.g., Berry and Haile, 2014, 2016). The first set of instruments includes the Hausman instruments, i.e., prices in other markets (Hausman et al., 1994; Nevo, 2001). The second set of instruments includes the BLP instruments, i.e., any function of the characteristics of competing products (Berry et al., 1995; Gandhi and Houde, 2023), and exogenous market shocks such as mergers (Miller and Weinberg, 2017). Building on Verboven (1996a) and Bresnahan et al. (1997), the BLP instruments for the IPDL model include, for each grouping characteristic, the sums of characteristics of other products belonging to the same group and the sums of characteristics of other products belonging to different groups or, alternatively, the corresponding squared differences in those characteristics.

Identification of the grouping parameters μ_d requires exogenous variation in the relative shares $s_{jt}/s_{d(j)}$. Intuitively, since they drive substitution patterns among products, their identification requires instruments that provide exogenous variation in the choice set, including changes in prices. Thus, both cost shifters and markup shifters are good candidates for instrumenting the log-share terms.

3.1 Substitution Patterns

The richness of the substitution patterns allowed by the IPDL model can be assessed by analyzing the matrix of own- and cross-price elasticities of market share as well as the matrix of diversion ratios. We derive these in Appendix B.3.

We first focus on the price elasticities of market share. The cross-price elasticity from product j to product k is the percentage change in the market share of product k following a one-percent increase in the price of product j.

To better understand substitution in the IPDL model, consider the cereals market segmented by brand (General Mills, Kellogg's, Quaker) and market segment (all-family, adults, kids). Recall that two products are of the same type if they belong to the same group according to each grouping characteristic, which here means they are of the same brand and market segment. Consider a price decrease of a cereal for kids sold by Kellogg's. We show that this price decrease will reduce the market share of all other cereals of a given type by the same percentage. In other words, the price decrease will draw proportionately from all the cereals of a given type. It will, for example, reduce the market share of each cereal for adults sold by General Mills by the same percentage or of each cereal for all-family sold by Kellogg's by, in general, another percentage. This substitution pattern is a manifestation of the independence from irrelevant alternatives (IIA) property among products of the same type. Furthermore, this price decrease will draw proportionately more or less from cereals of different types depending on the value of the grouping parameters and the number of groups these cereals share. This means that the IIA property does not hold in general for products of different types. There will be as many different cross-price elasticities per product as there are different product types.

Turn now to diversion ratios, which offer a better description of substitution patterns than cross-price elasticities of market share (Conlon and Mortimer, 2021).[6]

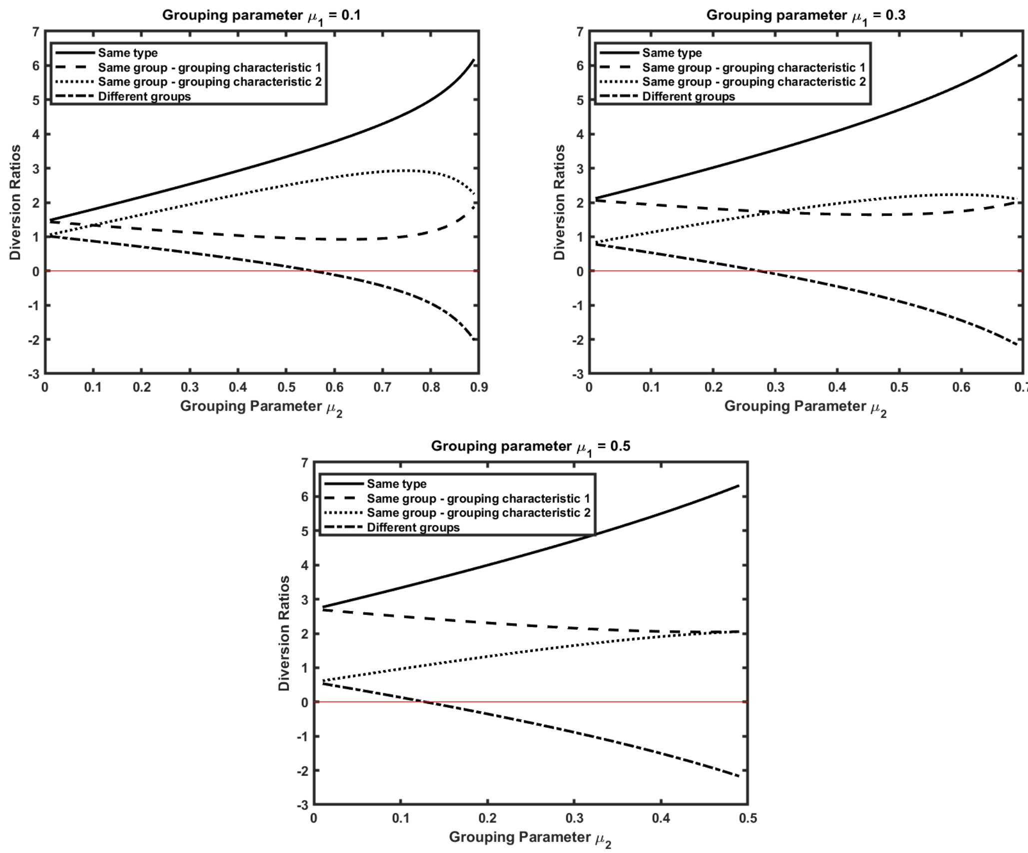
The diversion ratio from product j to product k is the fraction of consumers who leave product j to switch to product k following a price increase of product j.

We use simulations to investigate the patterns of diversion ratios in the IPDL model. We randomly generate $1,000$ markets with $J = 45$ products exhibiting segmentation according to two grouping characteristics with corresponding grouping parameters μ_1 and μ_2, each forming two groups so that there are four product types. Figure 1 summarizes the results. For clarity, we group products according to whether they belong to the same group according to both grouping characteristics, belong to the same group only according to the first grouping characteristic, belong to the same group only according to the second grouping characteristic, or do not

[6]Consider Conlon and Mortimer (2021)'s example with three products: the first has a cross-price elasticity of market share with the third of 0.034 and a market share of 0.1, whereas the second has a cross-price elasticity of market share with the third of 0.01 and a market share of 0.35. More consumers switch to the second product than to the first as the price of the third product increases, even though the first product has a larger cross-price elasticity of market share.

belong to the same group according to either grouping characteristic.

Figure 1: Diversion Ratios in the IPDL Model



Notes: The figure displays the mean diversion ratio between products in the IPDL model as a function of μ_2 while keeping the value of μ_1 constant. The figure is based on $1,000$ random samples of markets with 45 products. The red horizontal line corresponds to the threshold between substitutability and complementarity in demand.

Figure 1 exhibits some clear and intuitive patterns. The diversion ratio is the highest between products of the same type. It is the second highest between products of different types but belonging to the same group according to the grouping characteristic with the largest grouping parameter. The diversion ratio is lowest for products of completely different types, i.e., that do not belong to the same group according to either grouping characteristic. Moreover, the diversion ratio between products of the same type increases with the grouping parameters. Conversely, for products of completely different types, the diversion ratio decreases with the grouping parameters and becomes negative when the grouping parameters are sufficiently large, making these products complements in demand. Diversion ratios exhibit a non-monotonic pattern for products that share only one group. This occurs since the cross-price derivative of market share (the numerator of the diversion ratio) and the absolute value of the own-price derivative of market share (the denominator) increase at different rates as the grouping parameter μ_2 increases. The diversion ratio decreases as μ_2 increases when the cross-price derivative of market share increases more slowly than the absolute value of the own-price derivative of market share.

In the supplement, we provide simulation results investigating the substitution patterns of the IPDL model. We find that products of the same type are always substitutes in demand, while products of different types may be substitutes or complements in demand, and that products closer in the characteristics space used to form product types (i.e., higher values of the grouping parameters and/or whether products belong to the same groups or not) have higher diversion ratios.

Complementarity The empirical literature has mainly used two definitions of complementarity (Berry et al., 2014). Products j and k are complements in demand if the cross-price derivative of market share $\partial\sigma_j(\boldsymbol{\delta}_t)/\partial p_{kt}$ is negative. They are complements in utility if the cross-derivative of utility $\partial^2 u(\mathbf{s}_t)/\partial s_{jt}\partial s_{kt}$ is positive.[7]

Products are always substitutes in utility in the IPDL model. As shown in Figure 1, however, they can be complements in demand, depending on the value of the grouping parameters and the grouping structure.

Whether two products are substitutes or complements in demand depends on their relationship to other products (Samuelson, 1974). Assume that all products are substitutes in utility. Then, an increase in the price of product 1, for example, has two opposite effects on the market share of product 2. There is a direct substitution effect that increases the market share of product 2 as the market share of product 1 decreases. Note that the market shares of all the other products, including the outside good, also increase following the increase in the price of product 1. There

[7]These definitions apply to differentiable demand functions and twice differentiable utility functions and in particular to the IPDL demand function.

is also an indirect substitution effect via all the products other than products 1 and 2, including the outside good: substitution between product 2 and these products implies that an increase in the market shares of these products causes the market share of product 2 to decrease. If the indirect effect is larger than the direct effect, then an increase in the price of product 1 leads to a decrease in the market shares of both products 1 and 2, making these products complements in demand, even though they are substitutes in utility.

We can consider products that serve more or less specialized purposes. For example, some European households own both a small car suitable for driving and parking in cities, and a large car more suitable for family vacations, while other households own one or more all-round cars. Then an increase in the price of small cars (product 1) could induce some households to shift to owning only all-round cars and hence the demand for large cars (product 2) could decrease. Another example is guitars, where some guitar players combine more specialized guitars, say, a strat with a jazzbox, while others only use an all-round guitar such as an ES-335. Then strats and jazzboxes could be complements in demand. As a final example, beer drinkers may combine light and dark beers or choose a more middle-of-the-road type, which could make light and dark beers complements in demand.

Ogaki (1990) presents a method for computing the direct substitution effect between products 1 and 2 from the estimates of the price derivatives of market share. He shows that the direct effect can be obtained by removing the effect of the other products (all the products other than products 1 and 2, including the outside good), that is, by considering a change in the market share of product 2 while keeping those of the other products constant. When the direct (resp., indirect) substitution effect is positive, the products are called direct (resp., indirect) substitutes in demand; when it is negative, they are called direct (resp., indirect) complements in demand. See Appendix B.5 for further details.

In the IPDL model, products are necessarily direct substitutes in demand. However, they may be indirect substitutes or complements in demand depending on the value of the grouping parameters and the grouping structure. Therefore, in the IPDL model, complementarity in demand is necessarily due to a negative indirect substitution effect that is larger than a positive direct substitution effect.

To get further intuition on the mechanism generating complementarity in demand in the IPDL model, consider an example with $J = 3$ products and one outside good. We can think of product 1 as a small city car, product 2 as a large vacation-friendly car, and product 3 as an all-round car. Products are grouped according to two characteristics: first whether they are city-friendly or not and second whether they are vacation-friendly or not, so that the grouping is $\{1\}, \{2, 3\}$ for the first characteristic and $\{1, 3\}, \{2\}$ for the second characteristic. This grouping structure induces products 1 and 3 as well as products 2 and 3 to be substitutes in demand. However, depending on the values of the grouping parameters, products 1 and 2 may be substitutes or complements in demand.

With market shares equal to $s_{1t} = s_{2t} = s_{3t} = 1/6$ and price parameter $\alpha = 1$, if $\mu_1 = 1/4$ and $\mu_2 = 1/3$, the direct effect (which equals 0.098) is larger than the indirect effect (which equals -0.077), which makes products 1 and 2 substitutes in demand with a cross-price derivative of market share equal to 0.021. In contrast, if $\mu_1 = 3/5$ and $\mu_2 = 1/3$, the indirect effect (which equals -0.119) is larger than the direct effect (which equals 0.109), which makes products 1 and 2 complements in demand with a cross-price derivative of market share equal to -0.011. A higher μ_1 (from 1/4 to 3/5) makes products 2 and 3 more substitutable as they belong to the same group for the first grouping characteristic, which translates into a larger indirect effect. See Proposition 3 in Appendix B.3 for details.

3.2 Experiments with Simulated Data

The IPDL model is appealing because it generalizes the nested logit model while retaining its computational simplicity. To highlight the advantages of the IPDL model, we consider three Monte Carlo experiments. The experiments have three main goals: (i) to assess the ability of the IPDL model to approximate the true patterns of substitution and implied markups under different data generating processes (DGP); (ii) to compare the IPDL model to the commonly used RCL model; and (iii) to verify that the IPDL model does not generate complementarity in demand when there is none in the data. We assess approximations and compare models in terms of estimated diversion ratios and implied markups using the mean squared error (MSE). We also report the bias and standard error (S.E.) of these estimates.

For each experiment, we generate 50 datasets consisting of $T = 200$ independent markets with $J = 45$ products, where markets exhibit segmentation according to two grouping characteristics, each forming two groups so that there are four product types. We generate a fully structural model of supply and demand, where the supply side is a static price competition model with five multi-product firms, each with nine products. See Appendix C for details.

Experiment 1: Data from the IPDL Model We generate data from the IPDL model and fit the two possible three-level nested logit models and the RCL model with independent normal random coefficients on dummies for groups defined by segmentation. This experiment helps assess the bias that results from imposing a hierarchical grouping structure when the true grouping structure is non-hierarchical. It also allows us to investigate whether the IPDL model allows substitution patterns that the RCL model with independent normal random coefficients on dummies for groups cannot accommodate. We simulate four IPDL models, varying the values of the grouping parameters such that complementarity in demand occurs in the last two models but not in the first two.

We present the results in Table 1. Column (1) shows the true diversion ratios between products of the same type, products that belong to the same group only according to the first grouping characteristic, products that belong to the same group only according to the second grouping characteristic, and products that do not belong to the same group according to either grouping characteristic, as well as the true markups. Columns (2) to (5) compare the estimates of these diversion ratios and markups from the different models we fit in terms of MSE, bias, and standard error. Column (2) provides results for the IPDL model. Column (3) provides results for the nested logit model where the first grouping characteristic defines nests and the second grouping characteristic defines subnests. Column (4) provides results for the nested logit model where the second grouping characteristic defines nests and the first grouping characteristic defines subnests. Finally, Column (5) provides results for the RCL model with independent normal random coefficients on dummies for groups.

We find first that the correctly specified IPDL model produces estimates that are very close to the truth and with small standard errors. Second, the two nested logit models lead to biased estimates with relatively small standard errors. The bias and standard errors increase when the IPDL model exhibits complementarity in demand since the nested logit models shrink the negative diversion ratios towards zero. This means that wrongly imposing a hierarchical grouping structure can substantially affect the estimated patterns of substitution and implied markups. Third, the RCL model also leads to substantially biased estimates with relatively small standard errors. These biases are even larger when the IPDL model exhibits complementarity in demand. This shows that the IPDL model can produce patterns of substitution and implied markups that the RCL model with independent normal random coefficients on dummies for groups may fail to approximate, even when the IPDL model does not produce complementarity in demand.

Table 1: Simulation Results when the DGP is the IPDL Model

	(1)	(2)			(3)			(4)			(5)		
	True	IPDL Model			NL Model 1			NL Model 2			RCL Model		
		Bias	S.E.	MSE	Bias	S.E.	MSE	Bias	S.E.	MSE	Bias	S.E.	MSE
Diversion Ratios	DGP: IPDL Model with $\mu_1 = 0.10$ and $\mu_2 = 0.10$												
Same product type	1.280	-0.005	0.065	0.004	-0.045	0.074	0.008	-0.270	0.177	0.105	-0.702	0.048	0.496
Same group - product characteristic 1	0.867	0.021	0.066	0.005	-0.270	0.105	0.084	-0.344	0.010	0.118	-0.271	0.055	0.076
Same group - product characteristic 2	0.878	0.020	0.066	0.005	-0.338	0.011	0.114	0.003	0.090	0.008	-0.288	0.044	0.085
Different groups	0.420	0.031	0.019	0.001	0.133	0.011	0.018	0.108	0.013	0.012	0.195	0.052	0.041
Markups	37.29	0.052	1.029	1.062	-0.405	1.020	1.204	-0.553	1.067	1.444	-0.433	1.109	1.417
Diversion Ratios	DGP: IPDL Model with $\mu_1 = 0.15$ and $\mu_2 = 0.20$												
Same product type	1.782	-0.007	0.067	0.005	-0.079	0.084	0.013	-0.180	0.200	0.072	-1.327	0.041	1.763
Same group - product characteristic 1	0.908	0.034	0.072	0.006	-0.572	0.121	0.342	-0.536	0.009	0.288	-0.426	0.050	0.184
Same group - product characteristic 2	1.112	0.040	0.072	0.007	-0.710	0.011	0.505	-0.221	0.107	0.060	-0.639	0.039	0.410
Different groups	0.138	0.037	0.025	0.002	0.283	0.010	0.080	0.245	0.011	0.060	0.368	0.050	0.138
Markups	33.04	0.081	0.918	0.850	-0.751	0.896	1.366	-0.574	0.954	1.240	-0.949	1.097	2.103
Diversion Ratios	DGP: IPDL Model with $\mu_1 = 0.20$ and $\mu_2 = 0.30$												
Same product type	2.399	-0.016	0.069	0.005	-0.095	0.098	0.019	-0.014	0.226	0.051	-2.040	0.036	4.163
Same group - product characteristic 1	1.018	0.042	0.077	0.008	-0.887	0.143	0.807	-0.762	0.008	0.580	-0.627	0.045	0.395
Same group - product characteristic 2	1.394	0.058	0.078	0.010	-1.103	0.011	1.216	-0.507	0.126	0.273	-1.014	0.036	1.029
Different groups	-0.162	0.014	0.032	0.001	0.475	0.010	0.226	0.431	0.008	0.186	0.581	0.049	0.340
Markups	28.12	0.075	0.788	0.626	-0.984	0.762	1.549	-0.523	0.826	0.956	-1.313	1.096	2.925
Diversion Ratios	DGP: IPDL Model with $\mu_1 = 0.25$ and $\mu_2 = 0.40$												
Same product type	3.104	0.002	0.067	0.005	-0.039	0.119	0.016	0.303	0.249	0.153	-2.457	0.068	6.042
Same group - product characteristic 1	1.184	0.063	0.080	0.010	-1.177	0.173	1.416	-1.015	0.009	1.031	-0.632	0.067	0.404
Same group - product characteristic 2	1.698	0.078	0.082	0.013	-1.493	0.013	2.228	-0.878	0.144	0.792	-1.342	0.047	1.803
Different groups	-0.463	-0.090	0.041	0.010	0.691	0.013	0.478	0.647	0.010	0.418	0.775	0.035	0.601
Markups	22.26	0.030	0.629	0.397	-1.062	0.616	1.506	-0.385	0.677	0.606	-1.317	1.295	3.411

Notes: Summary statistics across 50 Monte Carlo replications. In the first two DGPs, there is no complementarity in demand. In the last two DGPs, 21% of the pairs of products exhibit complementarity in demand.

Experiment 2: Data from Bresnahan et al. (1997)'s PDL Model We generate data from the PDL model and fit the IPDL model and the RCL model with independent normal random coefficients on dummies for groups. This second experiment helps assess the performance of the IPDL model in predicting the patterns of substitutions and implied markups generated by another model of segmentation. It also allows us to compare the IPDL model to the RCL model with independent normal random coefficients on dummies for groups when both are misspecified. In addition, the experiment tests whether the IPDL model may wrongly generate complementarity in demand in a case where products are substitutes in demand. We simulate four PDL models, varying the values of the grouping parameters μ_1 and μ_2 that control substitution between products.

Table 2 presents the results. Column (1) provides the true diversion ratios between products and the true markups. Columns (2) to (4) compare the estimates of these diversion ratios and markups from the PDL model, the IPDL model, and the RCL model, respectively.

Table 2: Simulation Results when the DGP is the PDL Model

	(1)	(2)			(3)			(4)		
	True	PDL Model			IPDL Model			RCL Model		
		Bias	S.E.	MSE	Bias	S.E.	MSE	Bias	S.E.	MSE
Diversion Ratios		DGP: PDL Model with $\mu_1 = 0.50$ and $\mu_2 = 0.30$								
Same product type	2.932	-0.020	0.077	0.006	-0.257	0.077	0.072	-1.429	0.117	2.056
Same group - product characteristic 1	1.813	-0.000	0.048	0.002	0.059	0.097	0.013	-0.429	0.135	0.202
Same group - product characteristic 2	1.627	-0.005	0.028	0.001	-0.116	0.010	0.023	-0.337	0.129	0.130
Different groups	0.594	0.007	0.031	0.001	-0.136	0.056	0.022	0.599	0.123	0.374
Markups	26.05	-0.081	0.718	0.522	0.002	0.689	0.475	-0.650	0.776	1.024
Diversion Ratios		DGP: PDL Model with $\mu_1 = 0.50$ and $\mu_2 = 0.50$								
Same product type	2.716	0.002	0.087	0.008	-0.144	0.063	0.025	-1.199	0.100	1.447
Same group - product characteristic 1	1.711	0.015	0.112	0.013	-0.025	0.083	0.008	-0.326	0.113	0.119
Same group - product characteristic 2	1.705	-0.011	0.108	0.012	-0.007	0.083	0.007	-0.300	0.113	0.103
Different groups	0.730	-0.005	0.040	0.002	-0.127	0.047	0.018	0.558	0.109	0.323
Markups	29.25	-0.165	0.845	0.741	-0.169	0.819	0.699	-0.588	0.852	1.072
Diversion Ratios		DGP: PDL Model with $\mu_1 = 0.50$ and $\mu_2 = 0.70$								
Same product type	2.469	0.001	0.092	0.009	-0.124	0.058	0.019	-0.954	0.097	0.919
Same group - product characteristic 1	1.931	0.010	0.138	0.019	-0.163	0.077	0.033	-0.474	0.108	0.236
Same group - product characteristic 2	1.463	-0.008	0.115	0.013	0.123	0.075	0.021	-0.002	0.109	0.012
Different groups	0.912	-0.003	0.049	0.002	-0.047	0.043	0.004	0.501	0.106	0.262
Markups	32.93	-0.083	0.925	0.863	-0.074	0.907	0.829	-0.716	0.973	1.459
Diversion Ratios		DGP: PDL Model with $\mu_1 = 0.50$ and $\mu_2 = 0.90$								
Same product type	2.355	0.011	0.085	0.007	-0.144	0.059	0.024	-0.830	0.102	0.700
Same group - product characteristic 1	2.376	-0.007	0.085	0.007	-0.282	0.076	0.085	-0.895	0.111	0.813
Same group - product characteristic 2	1.030	0.009	0.060	0.004	0.196	0.073	0.044	0.438	0.112	0.204
Different groups	0.982	-0.006	0.049	0.002	0.040	0.045	0.004	0.454	0.109	0.218
Markups	34.60	-0.049	0.928	0.864	-0.125	0.911	0.845	-1.455	0.990	3.098

*Notes:*Summary statistics across 50 Monte Carlo replications.

The estimates from the correctly specified PDL model are very close to the true values, with small standard errors. The estimates from the IPDL model are also close to the true values, with small standard errors, but not as close as the correctly specified PDL model. We also find that the IPDL model does not wrongly produce complementarity in demand. Finally, as in the previous experiment, the RCL model leads to substantially biased estimates, and larger standard errors. This experiment provides thus a case where the IPDL model outperforms the RCL model with independent normal random coefficients on dummies for groups.

Experiment 3: Data from the RCL Model with Independent Normal Random Coefficients on Dummies for Groups Defined by Segmentation In this experiment, we generate data from the RCL model with independent normal random coefficients on dummies for groups defined by segmentation. This third experiment helps assess the ability of the IPDL model to approximate the patterns of substitution and implied markups generated by the RCL model with independent normal random coefficients on dummies for groups. It also allows us to check whether the IPDL model wrongly generates complementarity in demand in another case where products are substitutes in demand. We simulate four RCL models, varying the values of the standard deviations of the normal random coefficients RC_1 and RC_2.

Table 3 presents the results. Column (1) provides the true diversion ratios between products and the true markups. Columns (2) and (3) compare the estimates of these diversion ratios and markups from the RCL model and the IPDL model, respectively.

We find first that the correctly specified RCL model produces estimates very close to the true values and with small standard errors. Second, the IPDL model generates estimates reasonably close to the true values with small standard errors. The biases increase with the standard deviations of the normal random coefficients, meaning it is harder to approximate the RCL model as it deviates more from the logit model. However, the biases remain small, except for the diversion ratios between products that belong to only one group according to the first grouping characteristic. These simulations thus show that the IPDL model can approximate reasonably well the rich substitution patterns of the RCL model with independent normal random coefficients on dummies for groups, at least when the standard deviations of the normal random coefficients are not too large. We find, once again, that the IPDL model does not wrongly produce complementarity in demand.

Table 3: Simulation Results when the DGP is the RCL Model

IPDL Model	height	True		RCL Model			
		Bias		S.E.	MSE		
Bias		S.E.	MSE				
Diversion Ratios	DGP: PDL Model with $RC_1 = 0.50$ and $_2 = 1.00$						
Same product type	2.266	-0.020	0.077	0.006	-0.001	0.019	0.000
Same group - product characteristic 1	2.127	-0.020	0.160	0.026	0.023	0.039	0.002
Same group - product characteristic 2	2.141	0.031	0.093	0.010	0.034	0.028	0.002
Different groups	1.930	0.015	0.137	0.019	0.010	0.028	0.001
Different groups	30.16	0.096	0.418	0.184	0.032	0.423	0.180
Diversion Ratios	DGP: RCL Model with $RC_1 = 1.00$ and $RC_2 = 2.00$						
Same product type	2.512	-0.032	0.054	0.004	-0.064	0.018	0.004
Same group - product characteristic 1	1.946	-0.017	0.132	0.018	0.250	0.042	0.064
Same group - product characteristic 2	2.058	0.039	0.072	0.007	0.094	0.027	0.010
Different groups	1.415	0.024	0.099	0.010	0.009	0.031	0.001
Different groups	30.29	0.098	0.423	0.189	-0.092	0.442	0.203
Diversion Ratios	DGP: RCL Model with $RC_1 = 1.50$ and $RC_2 = 3.00$						
Same product type	2.795	-0.033	0.040	0.003	-0.160	0.017	0.026
Same group - product characteristic 1	1.762	-0.008	0.109	0.012	0.604	0.043	0.366
Same group - product characteristic 2	1.912	0.038	0.057	0.005	0.193	0.026	0.038
Different groups	1.008	0.019	0.074	0.006	-0.137	0.032	0.020
Different groups	30.43	0.010	0.428	0.193	-0.234	0.482	0.287
Diversion Ratios	DGP: RCL Model with $RC_1 = 2.00$ and $RC_2 = 4.00$						
Same product type	3.055	-0.031	0.032	0.002	-0.262	0.016	0.069
Same group - product characteristic 1	1.599	-0.001	0.092	0.004	0.983	0.044	0.968
Same group - product characteristic 2	1.744	0.035	0.047	0.003	0.313	0.025	0.099
Different groups	0.735	0.013	0.058	0.004	-0.363	0.033	0.133
Different groups	30.54	0.102	0.432	0.197	-0.386	0.539	0.439

*Notes:*Summary statistics across 50 Monte Carlo replications.

3.3 The IPDL Model versus Other Models

GEV Models The family of GEV models encompasses all additive random utility models in which the vector of random utility terms follow a multivariate extreme value distribution (McFadden, 1978; Anderson et al., 1992; Fosgerau et al., 2013).

Except for the logit model, the nested logit model is the simplest and most popular GEV model. The nested logit market share function is in closed form. It partitions the choice set into nests, nests into subnests, etc. This hierarchical grouping structure imposes restrictions on substitution patterns.[8] Despite these restrictions, the nested logit model is commonly used in applied work with market-level data due to its computational simplicity. Indeed, it is estimated by linear instrumental variables regression, using that its inverse market share function has a linear-in-parameters, closed-form expression (Berry, 1994).

[8]In the three-level nested logit model, for example, there are only three different cross-price elasticities of market share per product: one for products within the same subnest, another lower for products from different subnests within the same nest, and an even lower for products from different nests. The cross-price elasticities are necessarily positive, i.e., all products are substitutes in demand.

Bresnahan et al. (1997) propose the PDL model, a GEV model that accommodates richer substitution patterns than the nested logit model by allowing a non-hierarchical grouping structure. Like the nested logit model, the PDL market share function is in closed form. However, the PDL inverse market share function is not in closed form. The PDL model cannot, therefore, be estimated by linear instrumental variables regression. Instead, estimation of the PDL requires minimizing a non-linear, non-convex generalized method of moments objective, which must be computed by inverting the PDL market share function numerically.

The IPDL model generalizes the nested logit model by allowing the same non-hierarchical grouping structure as the PDL model. In contrast to the PDL model, the IPDL model can be estimated by linear instrumental variables regression but its market share function is not generally in closed form. In contrast to the nested logit and the PDL models, the IPDL model allows complementarity in demand. In summary, the IPDL model allows richer substitution patterns than the nested logit model while retaining its simplicity of estimation. It allows substitution patterns not accommodated by the PDL model while being simpler to estimate with market-level data.

A range of GEV models has been proposed using various grouping structures other than that of the PDL model. Prominent examples include the ordered logit model (Small, 1987), the paired combinatorial logit model (Koppelman and Wen, 2000), the flexible coefficient multinomial logit model (Davis and Schiraldi, 2014), and the ordered nested logit model (Grigolon, 2021). It is straightforward to extend the IPDL model to use the grouping structures of these GEV models. All such models would still have a linear-in-parameter inverse market share function and would, therefore, still be estimated by linear instrumental variables regression.[9] This is, for example, part of the strategy proposed by Monardo (2021), who builds an inverse market share model, which, like Koppelman and Wen (2000) and Davis and Schiraldi (2014)'s models, employs a grouping structure with a group for each pair of products. Finally, using our setting, Hortaçsu et al. (2022) propose a method to estimate the grouping structure from the data.

The generalized nested logit model (Wen and Koppelman, 2001) and the cross-

[9] See Fosgerau et al. (2021) and the supplement for further details.

nested logit model (Vovsha, 1997; Ben-Akiva and Bierlaire, 1999) go further and allow partial group membership, with an additional set of parameters controlling the degrees of group membership. It is possible to extend the IPDL model via a similar construction. In such extensions, the inverse market share function would not be linear in the parameters controlling for the degrees of group membership. Estimation would, therefore, require more complex non-linear instrumental variables regression.

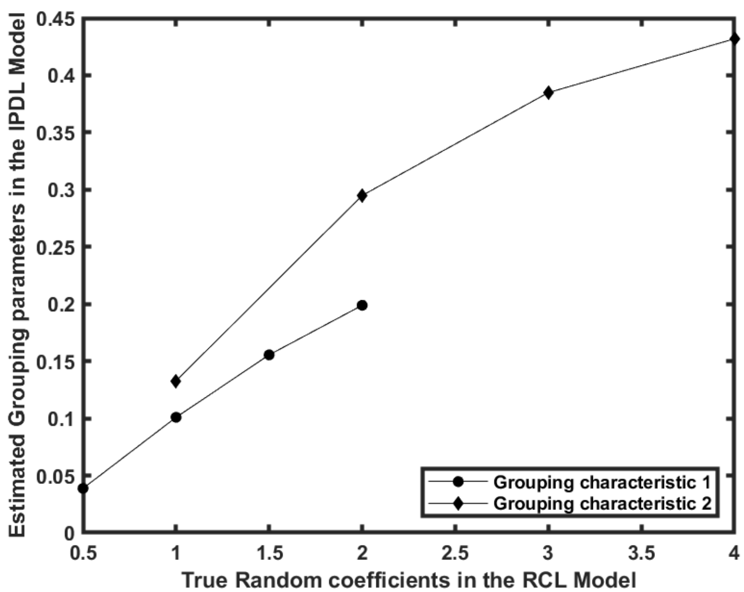
The Random Coefficient Logit Model The RCL model has been the state-of-the-art model in the literature since the seminal paper by Berry et al. (1995). The RCL model extends the logit model by incorporating unobserved preference heterogeneity through the specification of random coefficients on product characteristics, including prices. Most papers that estimate the RCL model with market-level data assume independent normal random coefficients and use the estimation algorithm proposed by Berry et al. (1995), known as the BLP method. The RCL model accommodates rich substitution patterns determined by how close products are in the space of product characteristics that receive a random coefficient. However, the BLP method is computationally demanding as it involves a non-linear, non-convex optimization problem and the simulation and numerical inversion of the market share function (Conlon and Gortmaker, 2020).

Furthermore, when the RCL model has random coefficients on dummies for groups, it produces substitution determined by the random coefficients distribution and the grouping structure. Similarly, the IPDL model delivers substitution determined by the grouping structure and the corresponding grouping parameters.

As shown by Cardell (1997) and further studied by Galichon (2021), the (two-level) nested logit model is an RCL model for which the dummy variables that form the grouping structure receive a random coefficient with a specific distribution. This observation motivates the open question of whether an IPDL model is equivalent to some RCL model. We can immediately rule out IPDL models exhibiting complementarity in demand since products can only be substitutes in demand in the RCL model.

Furthermore, the RCL model, with negative coefficients on prices, satisfies the condition that the higher-order partial derivatives of the market share function σ_i ($i \in \mathcal{J}$) with respect to any set of distinct prices other than p_i are non-negative (McFadden, 1981). This condition rules out complementarity in demand. By contrast, the IPDL model does not necessarily satisfy this condition, even when there is no complementarity in demand. This means that the IPDL model allows behavior that cannot be accommodated by any RCL model, even when all products are substitutes in demand. See Appendix B.4 for details.

Figure 2: Relationship between Grouping Parameters in the IPDL Model and Random Coefficients in the RCL Model



To gain further insights into the comparison between the IPDL model and the RCL model with random coefficients on dummies for groups, we consider again Experiment 3 where we simulate four RCL models with independent normal random coefficients on two dummies for groups, varying the values of the random coefficients, i.e., the standard deviations of the normal random coefficients. Figure 2 shows the mean of the estimated grouping parameters in the misspecified IPDL models against the true value of the random coefficients in the RCL models for the four DGPs.[10] As expected, we find an increasing relationship between the estimated

[10]Grigolon and Verboven (2014) propose a similar figure and discussion for the two-level nested

grouping parameters and the true random coefficients: higher random coefficients in the RCL model means greater deviations from the logit model, and the same does higher grouping parameters in the IPDL model. It suggests, as mentioned above, that the grouping parameters in the IPDL model are consumer heterogeneity parameters, like the random coefficients in the RCL model and the grouping parameters in the nested logit model. This interpretation of the grouping parameters is consistent with the interpretation of the IPDL model as a model of utility-maximizing, heterogeneous consumers. Furthermore, the grouping parameters are increasing in the random coefficients at a decreasing rate such that their values remain low enough to be consistent with the assumptions of the IPDL model (i.e., with two grouping characteristics, $\mu_1 \geq 0$, $\mu_2 \geq 0$, and $\mu_1 + \mu_2 < 1$).

Models with Complementarity The literature has allowed for complementarity in two main ways. The first strand of empirical literature incorporates complementarity in demand through micro-founded demand systems. Prominent examples include the AIDS model of Deaton and Muellbauer (1980), the EASI model of Lewbel and Pendakur (2009), and the linear demand model (see, e.g., Pinkse and Slade, 2004; Thomassen et al., 2017; Lewbel and Nesheim, 2019). The IPDL model belongs to this strand. The IPDL model, however, differs from these models regarding how unobservables enter. In all these models, the unobservables enter in a very restrictive way: the unobservables of a given product affect only its own demand, whereas the IPDL model, via the terms ξ_{jt}, allows the unobservables of a given product to affect both its own demand (market share) and those of all its competing products.

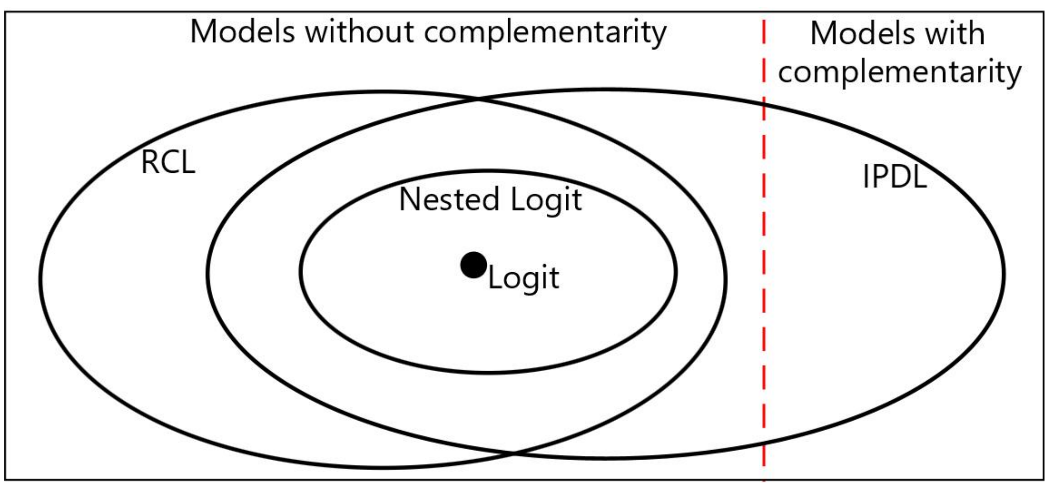
Another strand of literature incorporates complementarity by building demands for baskets of products. Prominent examples include Gentzkow (2007), Iaria and Wang (2020), and Ershov et al. (2021), who directly extend the RCL model to allow each consumer to choose among baskets of products rather than products alone. In their models, complementarity arises from positive demand synergies: products are complements in utility if the utility that a consumer derives from consuming the basket is higher than the sum of the utilities she derives from consuming the prod-

logit model.

ucts separately. Products can also be complements in demand. Another example is Iaria and Wang (2022), who, similarly to us, build a demand model based on a grouping structure, which can be inverted to obtain an inverse market share function that can easily be estimated even with large choice sets. Our paper thus differs from these papers regarding the channel through which products are complements in demand. Further, we only need to observe purchases at the product level, whereas these approaches rely on purchases at the basket level.

Summary We illustrate the relationship between the logit, nested logit, RCL and IPDL models in Figure 3. As is well known, the logit model is a special case of the nested logit model. Besides, all nested logit models are both IPDL models and RCL models. Otherwise, the sets of RCL models and IPDL models are generally different. We cannot exclude that some IPDL models are equivalent to some RCL models. Finding general conditions under which this is the case, as for the nested logit model, is a hard mathematical problem that we leave for future research.

Figure 3: Relationships between logit, nested logit, RCL, and IPDL models



4 Empirical Application

In this section, we use the IPDL model to estimate the demand in the ready-to-eat cereals market, which has been studied extensively (Nevo, 2000, 2001; Backus et al., 2021; Michel et al., 2023). We have three main goals: (i) to show how the IPDL model works with a well-known dataset; (ii) to investigate the computational performance of the IPDL model when there are many markets or many products; (iii) to compare the computational performance and goodness-of-fit of the IPDL model and the RCL model estimated using the methodology developed by BLP. Details for this section are provided in Appendix D.

4.1 Data

Data Sources We use data from the Dominick's Dataset, which is made publicly available by the James M. Kilts Center, University of Chicago Booth School of Business. This is weekly store-level scanner data, comprising information on 30 categories of packaged products at the universal product code (UPC) level for all Dominick's Finer Foods chain stores in the Chicago metropolitan area over the period 1989-1997. The data are supplemented by store-specific information, including average household size and daily store traffic.

For our analysis, we consider the ready-to-eat cereals category during the period 1991 – 1996. We use data from 25 Dominick's stores, and we aggregate UPCs into what we call cereals (e.g., Kellogg's Special K). We select 45 cereals from 6 national manufacturers (General Mills, Kellogg's, Nabisco, Post, Quaker, and Ralston), representing around 75% of each manufacturer's total sales on the period.[11] We define three market segments, namely adults, kids, and all-family, according to the classification provided by the website cerealfacts.org.

For all products in all markets, we compute prices as market volume-weighted average retail prices per ounce of the UPCs that form the product, deflated by the monthly consumer price index for all urban consumers in the Chicago-Naperville-Elgin area from the U.S. Bureau of Labor Statistics. We compute the potential market size by multiplying the total number of persons in a market by the monthly per capita consumption of cereals. We compute the total volume of a product sold in a market, which we divide by the potential market size to obtain the product's

[11] Only package sizes between 10 and 32 ounces are included. The 45 cereals account for around 58% of the national market. See, e.g., Corts (1996) for national figures.

market share. The market share of the outside good is then the difference between one and the sum of the products' market shares.

We supplement the Dominick's Dataset with information on the nutrient content (fiber, sugar, fat, protein, and sodium) of the cereals from the USDA Nutrient Database for Standard Reference (release SR11, year 1996) and on the type of grains (rice, wheat, corn, and oats) using manufacturers' websites and different websites collecting nutritional information. We also use monthly input prices from the websites indexmundi.com (corn, rice, sugar, and wheat) and macrotrends.net (oats) to construct cost-based instruments.

Descriptive Statistics Table 4 presents descriptive statistics on market shares and retail prices of cereals by brand and market segment. Kellogg's and General Mills are the largest two brands and are active in all market segments. Market segments have about equal market shares, and cereals for kids have higher prices on average. Taken together, Kellogg's and General Mills account for around 73 percent of the market, excluding the outside good. Furthermore, cereals for kids tend to be more expensive than their competitors, cereals for adults tend to be cheaper, General Mills and Ralston set higher prices, and Quaker lower prices.

Table 4: Shares and prices by brand and market segment

	All-family		Adults		Kids		Total	
	shares	prices	shares	prices	shares	prices	shares	prices
General Mills	3.39	20.12	2.04	20.18	3.23	21.04	8.66	20.48
Kellogg's	1.29	17.04	6.10	16.84	6.34	18.39	13.73	17.57
Nabisco	–	–	0.69	18.07	–	–	0.69	18.07
Post	0.84	16.44	1.63	15.98	0.97	21.91	3.44	17.77
Quaker	2.09	15.79	1.19	14.45	–	–	3.28	15.30
Ralston	0.75	20.91	–	–	0.19	24.55	0.94	21.65
Total	8.36	18.26	11.65	17.13	10.73	19.61	30.74	18.69
Outside good							69.26	

Notes: Shares and prices refer to overall sample average market shares in percent and prices in cents per ounce, respectively.

Table 5 shows the average nutrient content of the cereals by brand and market segment. Cereals offered by Nabisco contain, on average, less sugar, fat and sodium, and more fiber and protein than those of its competitors. In contrast, cereals offered by General Mills and Post are, on average, more sugary, cereals offered by Quaker have more fat and are rather sugary, and those offered by Nabisco and Ralston have less fat. Cereals offered by Kellogg's are rather sugary. Furthermore, cereals for kids contain more sugar than cereals of the other segments; they also contain less fiber and protein. By contrast, cereals for adults tend to have less sugar and sodium but more fiber.

Table 5: Nutrient content by brand and market segment

	Sugar g/ounce	Fiber g/ounce	Fat g/ounce	Protein g/ounce	Sodium mg/ounce	#
Brands						
General Mills	8.50	1.68	0.25	0.59	194.82	10
Kellogg's	7.85	1.32	0.23	0.59	149.59	15
Nabisco	0.24	3.25	0.057	0.83	1.98	2
Post	8.68	1.91	0.28	0.57	170.48	9
Quaker	7.85	1.50	0.76	0.69	143.56	5
Ralston	5.02	1.10	0.11	0.54	239.91	4
Segments						
All-family	7.04	1.64	0.23	0.62	195.68	12
Adults	5.17	2.19	0.32	0.77	135.56	19
Kids	11.29	0.77	0.29	0.36	177.43	14
All	7.57	1.60	0.29	0.60	164.62	45

Notes: Nutrient content refers to (unweighted) averages across cereals, by brand and market segment. Column # gives the number of by brand and market segment.

Overall, we can view the brands and market segments as proxying, at least partially, the nutrient content of the cereals as well as their prices. As a result, an IPDL model that groups cereals according to the brands and market segments would produce substitution patterns depending on this grouping structure and, thus, indirectly on the nutrient content and prices.

4.2 Specification and Identification

Specification We specify an IPDL model with two grouping characteristics: (i) the market segment the cereals belong to (F for all-family, A for adults, and K for kids), and (ii) the brand the cereals belong to (G for General Mills, K for Kellogg's, N for Nabisco, P for Post, Q for Quaker, and R for Ralston). We estimate this

IPDL model using the linear instrumental variables regression (9) with $D = 2$ grouping characteristics, where $\mathbf{x}_{jt}$ includes a constant and fixed effects mentioned just below, and where $d_1 = \{F, A, K\}$ and $d_2 = \{G, K, N, P, Q, R\}$.

We compare the IPDL model to the RCL model. We specify an RCL model with independent normal random coefficients on a constant and on the dummies for the groups *F*, *K*, *K*-*G*, and *P*-*Q*.[12] We estimate this RCL model using the methodology developed by BLP and implementing the best practices as advocated by Conlon and Gortmaker (2020).

We estimate two specifications of the IPDL and RCL models, varying the definition of products and markets. In the first specification (large T), we define a product as a cereal and a market as a store-month pair. As a result, the sample covers $J = 45$ products in $T = 1,675$ markets. In the second specification (large J), we define a product as a cereal/store pair and a market as a month. As a result, the sample covers $J = 1,125$ products in $T = 67$ markets. In both specifications, we include fixed effects for products, months, and years. We further include fixed effects for stores in the first specification.

Identification To identify the substitution patterns, we rely on two sets of instruments. The first set uses a second-order polynomial of cost shifters and continuous exogenous characteristics. The cost shifters are the input prices (sugar, corn, oats, rice, and wheat) multiplied by the corresponding characteristics so that they vary by cereal and across time. The continuous product characteristics are sugar, fiber, fat, protein, and sodium content.

The second set consists of BLP-type instruments. Specifically, we use the quadratic version of Gandhi and Houde (2023)'s differentiation instruments. For each exogenous continuous product characteristic x^k and for each pair (i, j) of products, we compute the differences $d^k_{i,j,t} = x^k_{jt} - x^k_{it}$. The differentiation instruments we consider are all quadratic interactions of these differences, summed over products belonging to the same brand, to competing brands, to the same market segment, and to competing segments. Furthermore, we compute exogenous price

[12] For the sake of parsimony, we divide brands into three groups according to their popularity measured in terms of market shares: General Mills and Kellogg's, Post and Quaker, and Nabisco and Ralston.

indices, $\hat{p}_{jt}$, as the predicted values from a linear regression of the price variable on the two sets of instruments and a constant. We then compute the differences $d^{\hat{p}}_{i,j,t} = \hat{p}_{jt} - \hat{p}_{it}$, which we interact with itself and the other differences to construct additional differentiation instruments.

For the IPDL models, we use the exogenous price index as an instrument for the price and construct instruments for the two log-share terms as the predicted values from regressions of these terms on the two sets of instruments described just above, and a constant.[13] For the RCL models, we first estimate using the exogenous price index, the set of second-order polynomial bases of cost shifters and continuous exogenous characteristics, the set of BLP-type instruments, and a constant as instruments. Then, based on these estimates, we compute the optimal instruments and estimate using these instruments.

4.3 Results

Table 6 presents the estimation results based on the two-step generalized method of moments (GMM) estimator using a weighting matrix clustered at the month-store level. Columns (1) and (2) provide the results for the large T and the large J specifications of the IPDL model. Columns (3) and (4) provide the results for the large T and the large J specifications of the RCL model.

Demand Parameters The estimated parameter on the negative of price (α) has the expected sign and is significantly different from zero in both specifications of the IPDL and the RCL models. For both the IPDL and the RCL models, the estimates of α have the same magnitude in the two specifications.

For both specifications of the IPDL model, the grouping parameters are precisely estimated and satisfy the assumptions of the IPDL model (i.e., $\mu_1 \geq 0$, $\mu_2 \geq 0$, and $\mu_1 + \mu_2 < 1$).[14] Furthermore, the grouping parameter for brand is

[13] A potential problem is weak identification, which occurs when instruments are only weakly correlated with the endogenous variables. In both specifications, the Sanderson and Windmeijer (2016)'s F-statistics to test whether each endogenous variable is weakly identified are far above 10, the rule-of-thumb usually used for linear instrumental variables regressions, thereby suggesting that instruments are not weak.

[14] We impose no constraints on the parameters during the estimation.

higher than that for market segment (i.e., $\mu_1 > \mu_2$), which indicates that brand reputation confers more protection from substitution than does the market segment, i.e., cereals of the same brand are more protected from cereals from other brands than cereals of the same market segment are from cereals from other market segments.

In both specifications of the RCL model, only two of the five random coefficients are precisely estimated. This is consistent with the literature, which has found that it may be hard in practice to identify random coefficients on dummies when product fixed effects are included in the model (see, e.g., Conlon and Mortimer, 2013). Moreover, random coefficients do not have the same magnitude in the two specifications.

Computational time We compare the computational time of the IPDL and the RCL models, measured as the execution time from the computation of the instruments to the estimation of the model. We find that the IPDL model is much faster to estimate than the RCL model. The IPDL model takes 8 seconds for the large T specification and 2.5 minutes for the large J specification, against 1h50 and 2h20, respectively, for the RCL model.

Goodness-of-fit We compare the goodness-of-fit of the IPDL and the RCL models. Like Compiani (2022), we measure goodness-of-fit based on the cross-validated MSE to evaluate how models perform out-of-sample. Building on insights from Rivers and Vuong (2002) and following Komiyama and Shimao (2018)'s procedure, we also measure goodness-of-fit based on the cross-validated GMM J-statistic. In both specifications, we find a lower MSE for the IPDL model than for the RCL model, meaning that the IPDL model fits the data better than the RCL model according to this measure of fit. We similarly find a lower GMM J-statistic for the IPDL model than for the RCL model for the large T specification, while the GMM J-statistics are about the same for the large J specification.[15]

[15] We have also tested more usual specifications of the RCL model, where continuous product characteristics (sugar, fiber, and fat) and price receive a random coefficient and are interacted with demographics (income and child). None of these specifications provided a better fit to the data.

Table 6: Estimation Results

	(1) IPDL Model Large T	(2) IPDL Model Large J	(3) RCL Model Large T	(4) RCL Model Large J
Prices ($-\alpha$)	-17.886	-17.439	-13.330	-14.407
	(1.430)	(1.333)	(0.7859)	(1.6778)
Grouping Parameters				
Brand (μ_1)	0.4146	0.4387	–	–
	(0.0331)	(0.0261)	–	–
Market segment (μ_2)	0.1727	0.1569	–	–
	(0.0415)	(0.0387)	–	–
Random Coefficients				
Constant	–	–	0.0098	0.3965
	–	–	(0.7140)	(0.8243)
Kellogg's-General Mills	–	–	0.0364	1.106
	–	–	(0.6099)	(0.4680)
Post-Quaker	–	–	0.2522	0.2068
	–	–	(0.4807)	(0.5247)
Kids	–	–	1.192	1.498
	–	–	(0.5178)	(2.163)
All-family	–	–	0.7552	1.236
	–	–	(0.4335)	(0.3416)
Product fixed effects	45	1125	45	1125
Month fixed effects	11	11	11	11
Store fixed effects	24	–	24	–
Year fixed effects	5	5	5	5
Time	8sec	$\sim$ 2.5min	$\sim$1h50	$\sim$2h20
Cross-validated MSE	0.202	0.226	0.375	0.305
Cross-validated GMM J-statistic	0.0141	0.0226	0.0157	0.0223
Complementarity				
95% CI	1.31%	1.89%	0%	0%
99% CI	0.30%	0.77%	0%	0%
99.5% CI	0%	0.002%	0%	0%
99.9% CI	0%	0%	0%	0%
Mean Diversion Ratios				
Same product type	9.165	0.342	1.070	0.071
Same group - brand	7.932	0.298	0.776	0.047
Same group - market segment	2.979	0.123	1.064	0.059
Different groups	1.118	0.041	0.725	0.032
Towards outside good	31.26	28.10	66.92	59.66
Mean markup	35.49%	36.54%	55.45%	54.83%

Notes: The number of observations is 75,375. Standard errors clustered at the month-store level are shown in parentheses. A constant is included. For complementarity, 95% CI refers to the percentage of pairs of complements when using the 95% confidence interval for the cross-price derivatives of market share.

Substitution Patterns and Markups We provide the diversion ratios between cereals averaged across markets and products according to whether they are of the same type, belong to the same brand, belong to the same market segment, or do not belong to either the same brand or market segment. We find significantly different diversion ratios between the IPDL and the RCL models. For the large T specification, both the IPDL and RCL models generate diversion ratios depending on segmentation. However, they lead to different qualitative results. For example, the IPDL model predicts that cereals from the same brand are closer substitutes than cereals belonging to the same market segment, and conversely for the RCL model. For the large J specification, in the IPDL model, diversion ratios are determined by segmentation, whereas in the RCL model, they seem to be determined by segmentation only weakly. Furthermore, in both specifications, the IPDL model generates significantly higher substitution between cereals than the RCL model. Lastly, the RCL model generates twice as much substitution towards the outside as the IPDL model.

We also provide the percentage of pairs of complements in demand in the IPDL model, determined by the percentage of significantly negative cross-price derivatives of market shares. This takes into account that some pairs of cereals exhibiting negative cross-price derivatives of market shares may actually be independent and not complements in demand. We find that the IPDL model generates a small amount of complementarity in demand. To verify that complementarity in demand is not a model artifact, we compute the cross-price derivatives of market shares using different values of the grouping parameters from the estimated values. We find that parameter values exist such that the model does not exhibit complementarity in demand.

Finally, we provide share-weighted average markups, averaged across markets and products, computed using a static oligopolistic price competition model for which a pure-strategy Nash equilibrium in prices is assumed to exist. We find that the IPDL model yields markups that have the same magnitude across the two specifications and are in line with the literature (Nevo, 2001; Michel et al., 2023). By contrast, the RCL model yields much higher markups as it generates lower substitution between cereals.

5 Concluding Comments

We have introduced the IPDL model, a micro-founded inverse market share model for differentiated products. The IPDL model generalizes the nested logit model to allow richer substitution patterns, including complementarity in demand. Like the nested logit model, the IPDL model can be estimated by linear instrumental variables regression using market-level data, and it is consistent with a model of heterogeneous, utility-maximizing consumers.

Our Monte Carlo experiments show that the IPDL model can reasonably approximate the substitution patterns generated by the leading models of the literature. Our empirical application, using a well-known dataset on the ready-to-eat cereals market, shows that the IPDL model fits the data better than a similar RCL model while being much faster to estimate. With the IPDL model, we find (weak) evidence of complementarity in demand due to the indirect substitution effect, a result that would not be possible with the RCL model. These results suggest that the IPDL model can be useful for describing markets that exhibit segmentation.

This paper opens several avenues for future work. First, it would be interesting to see applications of the IPDL model to different markets and economic issues. The IPDL model may lead to qualitatively different conclusions from the workhorse models of the literature, particularly when there is complementarity in demand. Second, it seems worthwhile to extend the IPDL model to allow for unobserved heterogeneity in preferences through random coefficients, in analogy with what has been done with the logit and nested logit models. Third, a natural next step would be to develop an estimation method for the IPDL model using individual-level data rather than market-level data. Finally, it would be interesting to incorporate forward-looking consumer behavior in the IPDL model and to develop a corresponding estimation procedure.

Appendix

A Mathematical Appendix

Notation We use italics for scalar variables and real-valued functions, boldface for vectors, matrices and vector-valued functions, and calligraphic for sets. $\mathbb{R}_+$ is the set of non-negative real numbers, $\mathbb{R}_{++}$ is the set of positive real numbers, $\mathbb{R}_+^{J+1} = [0, \infty)^{J+1}$, and $\mathbb{R}_{++}^{J+1} = (0, \infty)^{J+1}$. As default, vectors are column vectors: $\mathbf{s} = (s_0, \ldots, s_J)^\intercal \in \mathbb{R}^{J+1}$.

$\Delta \subset \mathbb{R}^{J+1}$ is the unit simplex: $\Delta = \left\{\mathbf{s} \in [0, \infty)^{J+1} : \sum_{j \in \mathcal{J}} s_j = 1\right\}$, and $\Delta^\circ = \left\{\mathbf{s} \in (0, \infty)^{J+1} : \sum_{j \in \mathcal{J}} s_j = 1\right\}$ is its relative interior.

Let $\mathbf{G} = (G_0, \ldots, G_J) : \mathbb{R}^{J+1} \to \mathbb{R}^{J+1}$ be a vector function composed of functions $G_j : \mathbb{R}^{J+1} \to \mathbb{R}$. The matrix $\mathbf{J}_{\mathbf{G}}^{\mathbf{s}}(\bar{\mathbf{s}}) \in \mathbb{R}^{(J+1)\times(J+1)}$ with entries $(i + 1, j + 1)$ given by $\frac{\partial G_i(\bar{\mathbf{s}})}{\partial s_j}$ denotes the Jacobian matrix of $\mathbf{G}$ with respect to s at point $\bar{\mathbf{s}}$.

A univariate function $\mathbb{R} \to \mathbb{R}$ applied to a vector is a coordinate-wise application of the function, e.g., $\ln(\mathbf{s}) = (\ln(s_0), \ldots, \ln(s_J))$. $\mathbf{1} = (1, \ldots, 1)^\intercal \in \mathbb{R}^{J+1}$ is a vector consisting of ones, and $\mathbf{I} \in \mathbb{R}^{(J+1)\times(J+1)}$ denotes the identity matrix.

Preliminaries This section provides some mathematical definitions and a result used in the proofs that follow.

Definition 1. $G_j : \mathbb{R}_{++}^{J+1} \to \mathbb{R}^{J+1}$ is linearly homogeneous if $G_j(\lambda \mathbf{s}) = \lambda G_j(\mathbf{s})$ for all $\lambda > 0$ and $\mathbf{s} \in \mathbb{R}_{++}^{J+1}$. $\mathbf{G}$ is homogeneous if each of its components G_j is.

Definition 2. A matrix $\mathbf{A} \in \mathbb{R}^{(J+1)\times(J+1)}$ is positive quasi-definite if its symmetric part, defined by $\frac{1}{2}(\mathbf{A} + \mathbf{A}^\intercal)$, is positive definite.

It follows that a symmetric and positive definite matrix is positive quasi-definite.

Lemma 1 (Gale and Nikaido 1965, Theorem 6)**.** If a differentiable mapping $\mathbf{F} : \Theta \to \mathbb{R}^{J+1}$, where Θ is a convex region of $\mathbb{R}^{J+1}$, has a Jacobian matrix that is everywhere quasi-definite in Θ, then $\mathbf{F}$ is injective on Θ.

B Properties of the IPDL Model

Recall first that $d(j)$ is the set of products that are grouped with product j according to grouping characteristic d and that $s_{d(j)} = \sum_{k \in d(j)} s_k$ denotes the market share of group $d(j)$. To ease exposition, we omit the notation for parameters $\boldsymbol{\theta}_2$ and markets t. Recall then that the IPDL model is defined by

$$\sigma_j^{-1}(\mathbf{s}) = \ln G_j(\mathbf{s}) + c = \delta_j, \quad j \in \mathcal{J}, \tag{10}$$

where the function $\mathbf{G} : \mathbb{R}_{++}^{J+1} \to \mathbb{R}_{++}^{J+1}$ is defined by

$$\ln G_j(\mathbf{s}) = \left(1 - \sum_{d=1}^{D} \mu_d\right) \ln(s_j) + \sum_{d=1}^{D} \mu_d \ln\left(s_{d(j)}\right), \quad j = 1, \ldots, J \tag{11}$$

$$\ln G_0(\mathbf{s}) = \ln(s_0), \tag{12}$$

with $\sum_{d=1}^{D} \mu_d < 1$ and $\mu_d \geq 0$, $d = 1, \ldots, D$.

Lemma 2. Let $\ln \mathbf{G} = (\ln G_0, \ldots, \ln G_J)$.

1. The Jacobian matrix $\mathbf{J}_{\ln \mathbf{G}}^{\mathbf{s}}(\mathbf{s})$ of the function $\ln \mathbf{G}$ with respect to s has entries

$$\frac{\partial \ln G_i(\mathbf{s})}{\partial s_j} = \begin{cases} \dfrac{1 - \sum_{d=1}^{D} \mu_d}{s_i} + \sum_{d=1}^{D} \dfrac{\mu_d}{s_{d(i)}} & \text{if } i = j > 0 \\ \sum_{d=1}^{D} \dfrac{\mu_d}{s_{d(i)}} \mathbf{1}\{j \in d(i)\} & \text{if } i \neq j,\ i > 0,\ j > 0 \\ \dfrac{1}{s_0} & \text{if } i = j = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

   It is positive definite for all $\mathbf{s} \in \mathbb{R}_{++}^{J+1}$.

2. The function

$$\mathbf{s} \to -\mathbf{s}^{\intercal} \ln \mathbf{G}(\mathbf{s}) = -\sum_{j \in \mathcal{J}} s_j \ln G_j(\mathbf{s})$$

   is strictly concave on Δ°.

3. The function $\mathbf{s} \to \mathbf{G}(\mathbf{s})$ is linearly homogeneous.

Proof of Lemma 2.
1. $\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathbf{s})$ is positive definite for all $\mathbf{s} \in \mathbb{R}^{J+1}_{++}$, as it is the sum of a positive definite diagonal matrix and a block diagonal positive semi-definite matrix.
2. Consider $\mathbf{s} \in \Delta^{\circ}$. The Hessian of $-\mathbf{s}^{\intercal} \ln \mathbf{G}(\mathbf{s})$ is $-\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathbf{s})$, which is negative definite (by part 1).
3. Note that $G_0(\mathbf{s}) = s_0$ and

$$G_j(\mathbf{s}) = s_j^{1-\sum_{d=1}^{D} \mu_d} \prod_{d=1}^{D} \left(s_{d(j)}\right)^{\mu_d}, \quad j = 1, \ldots, J.$$

$\mathbf{G}$ is linearly homogeneous since for any $\lambda > 0$ and $\mathbf{s} \in \mathbb{R}^{J+1}_{++}$, we have

$$\begin{aligned}
G_j(\lambda \mathbf{s}) &= (\lambda s_j)^{1-\sum_{d=1}^{D} \mu_d} \prod_{d=1}^{D} \left(\sum_{k \in d(j)} \lambda s_k \right)^{\mu_d} \\
&= \left[\lambda^{1-\sum_{d=1}^{D} \mu_d} \prod_{d=1}^{D} \lambda^{\mu_d} \right] \left[s_j^{1-\sum_{d=1}^{D} \mu_d} \prod_{d=1}^{D} \left(s_{d(j)}\right)^{\mu_d} \right] \\
&= \lambda^{1-\sum_{d=1}^{D} \mu_d + \sum_{d=1}^{D} \mu_d} G_j(\mathbf{s}) \\
&= \lambda G_j(\mathbf{s}),
\end{aligned}$$

for all $j = 1, \ldots, J$, and $G_0(\lambda \mathbf{s}) = \lambda s_0 = \lambda G_0(\mathbf{s})$. □

B.1 Invertibility

As stated in the following proposition, the function is $\ln \mathbf{G}$ injective and hence invertible on its range.

Proposition 1. The function $\ln \mathbf{G}$ defined by Equations (11) and (12) is injective, with range equal to $\mathbb{R}^{J+1}$.

Proof of Proposition 1. The function $\ln \mathbf{G}$ is differentiable on the convex region $\mathbb{R}^{J+1}_{++}$. The Jacobian matrix $\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathbf{s})$ is positive quasi-definite since it is symmetric and positive definite by Lemma 2. Then, $\ln \mathbf{G}$ is injective by Lemma 1.

Also, by Lemma 2, the function $\mathrm{s} \rightarrow \sum_{j\in\mathcal{J}} s_j \ln G_j(\mathbf{s})$ is strictly convex. Hence, for any $\boldsymbol{\delta} \in \mathbb{R}^{J+1}$, the maximization problem $\sup_{\mathbf{s}\in\mathbb{R}_+^{J+1}} \left\{\sum_{j\in\mathcal{J}} s_j(\delta_j - \ln G_j(\mathbf{s}))\right\}$ has a unique solution. Lastly, note that $|\ln \mathbf{G}(\mathbf{s})| \rightarrow \infty$ whenever $\mathrm{s} \rightarrow \mathrm{s}^0$, where s^0 is on the boundary of $\mathbb{R}_+^{J+1}$, which means that at least one component of $\ln \mathbf{G}$ tends to infinity as s approaches the boundary of $\mathbb{R}_+^{J+1}$ and ensures that the solution is interior. Then, the solution is given by the first-order condition, which is $\boldsymbol{\delta} = \ln \mathbf{G}(\mathbf{s})$. □

Invertibility of $\ln \mathbf{G}$ is equivalent to invertibility of the IPDL inverse market share function. Consider any vector of market shares $\mathrm{s} \in \Delta^\circ$. Then, holding $\delta_0 = 0$, the invertibility of the IPDL inverse market share function ensures that there exists a unique vector of indexes $\boldsymbol{\delta} \in \mathcal{D}$ that rationalizes that vector $\mathbf{s}$ with the market share function $\boldsymbol{\sigma}$, i.e., $\mathrm{s} = \boldsymbol{\sigma}(\boldsymbol{\delta})$.

Lemma 3. The IPDL inverse market share function $\boldsymbol{\sigma}^{-1}$ defined by Equations (10) – (12) is invertible.

Our invertibility result relates to Berry et al. (2013), who show invertibility for demand functions that satisfy their "connected substitutes" conditions: (i) products are weak gross substitutes, that is, everything else equal, an increase in δ_i weakly decreases the market shares σ_j for all other products $j \neq i$; and (ii) the “connected strict substitution” condition holds, i.e., there is sufficient strict substitution between products to treat them in one demand system. Their result also applies to any demand function for which a suitable transformation can be found such that the transformed demand satisfies the connected substitutes conditions. Whether or not their result could be applied to the IPDL inverse market share is a difficult question, as it requires finding such a transformation. We instead provide a direct proof of invertibility, which does not require such a transformation to be found.

Let $\mathbf{H} = \mathbf{G}^{-1}$ denote the inverse of $\mathbf{G}$: $\mathbf{H}(e^{\boldsymbol{\delta}}) = (H_0(e^{\boldsymbol{\delta}}), \ldots, H_J(e^{\boldsymbol{\delta}})) = \mathbf{G}^{-1}(e^{\boldsymbol{\delta}})$. We show that $\mathbf{H}$ is linearly homogeneous.

Lemma 4. The function $e^{\boldsymbol{\delta}} \rightarrow \mathbf{H}(e^{\boldsymbol{\delta}})$ is linearly homogeneous.

Proof. Let $\mathbf{G}(\mathbf{s}) = e^{\boldsymbol{\delta}}$ or equivalently $\mathbf{s} = \mathbf{H}(e^{\boldsymbol{\delta}})$. Then, for any $\lambda > 0$ and $e^{\boldsymbol{\delta}} \in \mathbb{R}^{J+1}_{++}$,

$$\mathbf{H}(\lambda e^{\boldsymbol{\delta}}) = \mathbf{H}(\lambda \mathbf{G}(\mathbf{s})) = \mathbf{H}(\mathbf{G}(\lambda \mathbf{s})) = \lambda \mathbf{s} = \lambda \mathbf{H}(e^{\boldsymbol{\delta}}).$$

B.2 Micro-foundation

Consider a representative consumer facing the choice set of differentiated products, $\mathcal{J}$, and a homogeneous numéraire good, with demands for the differentiated products summing to one. Let p_j and v_j be the price and the quality of product $j \in \mathcal{J}$, respectively. The price of the numéraire good is normalized to 1, and the representative consumer's income y is sufficiently high ($y > \max_{j \in \mathcal{J}} p_j$) to guarantee that consumption of the numéraire good is positive.

In this subsection, we show that the IPDL inverse market share function is consistent with a representative consumer who chooses a vector $\mathbf{s} \in \Delta$ of market shares of the differentiated products and a quantity $z \geq 0$ of the numéraire good so as to maximize her direct utility function

$$\alpha z + \sum_{j \in \mathcal{J}} v_j s_j - \left[\left(1 - \sum_{d=1}^{D} \mu_d \right) \sum_{j \in \mathcal{J}} s_j \ln(s_j) + \sum_{d=1}^{D} \mu_d \left(\sum_{g \in d \cup \{0\}} s_g \ln(s_g) \right) \right], \tag{14}$$

subject to the budget constraint and the constraint that the market share vector sums to one,

$$\sum_{j \in \mathcal{J}} p_j s_j + z \leq y \quad \text{and} \quad \sum_{j \in \mathcal{J}} s_j = 1, \tag{15}$$

where $\alpha > 0$ is the marginal utility of income, $s_g = \sum_{k \in g} s_k$, and d is identified with the set of groups for grouping characteristic d.

The first two terms of the direct utility (14) describe the utility that the representative consumer derives from the consumption $(\mathbf{s}, z)$ of the differentiated products and the numéraire in the absence of interaction among them. The third term (in square brackets) is a strictly concave function of $\mathbf{s}$ that expresses her taste for variety (Lemma 2 above).

We further show that the direct utility function (14) gives the indirect utility function

$$\alpha y + \ln\left(\sum_{k\in\mathcal{J}} H_k\left(e^{\boldsymbol{\delta}}\right)\right), \tag{16}$$

where the second term is, up to an additive constant, the consumer surplus $CS(\boldsymbol{\delta})$.

We summarize these results as follows.

Proposition 2. The IPDL model (10) – (12) is consistent with a representative consumer who maximizes her direct utility (14) subject to constraints (15). Further, the direct utility (14) gives the indirect utility (16), where the second term is the convex consumer surplus function.

Proof. Consider the representative consumer maximizing utility (14) subject to constraints (15). The budget constraint is always binding since $\alpha > 0$ and $y > \max_{j\in\mathcal{J}} p_j$. Substituting the budget constraint into the direct utility (14), the representative consumer then chooses $\mathbf{s} \in \Delta$ to maximize

$$u(\mathbf{s}) = \alpha y + \sum_{j\in\mathcal{J}} \delta_j s_j - \left[\left(1 - \sum_{d=1}^{D}\mu_d\right)\sum_{j\in\mathcal{J}} s_j \ln(s_j) + \sum_{d=1}^{D}\mu_d\left(\sum_{g\in d\cup\{0\}} s_g \ln(s_g)\right)\right]$$

where $\delta_j = v_j - \alpha p_j$. The Lagrangian of the utility maximization problem given by

$$\mathcal{L}(\mathbf{s}, \lambda) = u(\mathbf{s}) + \lambda\left(1 - \sum_{j\in\mathcal{J}} s_j\right)$$

yields $\sum_{j\in\mathcal{J}} s_j = 1$ as well as the first-order conditions

$$\delta_j - \left[\left(1 - \sum_{d=1}^{D}\mu_d\right)(\ln(s_j) + 1) + \sum_{d=1}^{D}\mu_d\left(\ln\left(s_{d(j)}\right) + 1\right)\right] - \lambda = 0$$

which can be simplified as

$$\delta_j - \left[\left(1 - \sum_{d=1}^{D}\mu_d\right)\ln(s_j) + \sum_{d=1}^{D}\mu_d \ln\left(s_{d(j)}\right) + 1\right] - \lambda = 0,$$

for all $j = 1, \ldots, J$, and $\delta_0 - (\ln(s_0) + 1) - \lambda = 0$ for the outside good.

The first-order conditions for an interior solution have a unique solution since the objective is strictly concave by Lemma 2; hence, the utility-maximizing demand exists uniquely. Setting $c = 1 + \lambda$, we show that the representative consumer model leads to the IPDL inverse market share function.

Further, exponentiating and applying $\mathbf{H}$ on both sides of Equation (10) leads to

$$\mathbf{s} = \mathbf{H}(e^{\boldsymbol{\delta}} e^{-c}) = \mathbf{H}(e^{\boldsymbol{\delta}}) e^{-c},$$

where the last equality uses the homogeneity of $\mathbf{H}$ (Lemma 4). Using that demands sum to 1, we find that

$$e^c = \sum_{k \in \mathcal{J}} H_k \left(e^{\boldsymbol{\delta}}\right), \tag{17}$$

so that the IPDL market share function is given by

$$\sigma_j(\boldsymbol{\delta}) = \frac{H_j\left(e^{\boldsymbol{\delta}}\right)}{\sum_{k \in \mathcal{J}} H_k\left(e^{\boldsymbol{\delta}}\right)}, \quad j \in \mathcal{J}. \tag{18}$$

Finally, substituting the market share functions (18) with the market shares s_j gives the indirect utility function (16).

By Roy's identity, the Hessian of the consumer surplus is $\mathbf{J}^{\boldsymbol{\delta}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta})$, which by Proposition 3 (part 2.) is positive semi-definite. Convexity of the consumer surplus then follows. □

Anderson et al. (1988) and Verboven (1996b) show that the logit and nested logit models are both consistent with a utility-maximizing representative consumer model. Proposition 2 extends these results to the IPDL model.

Furthermore, as shown by Allen and Rehbeck (2019b), utility (14) can be obtained, by aggregating across heterogeneous, utility-maximizing consumers, from the class of latent utility models with additively separable unobservable heterogeneity called perturbed utility.[16] This implies that the IPDL model embodies consumer heterogeneity and can be rationalized by a model with heterogeneous,

[16] See Hofbauer and Sandholm (2002), McFadden and Fosgerau (2012), and Fudenberg et al. (2015) for more details on perturbed utility models. Allen and Rehbeck (2019a) show that some perturbed utility models allow for complementarity in demand.

utility-maximizing consumers.

B.3 Substitution Patterns

Proposition 3. The IPDL model has the following properties.

1. The independence from irrelevant alternatives (IIA) property holds for products of the same type; but does not hold in general for products of different types.

2. The matrix of price derivatives of market share $\mathbf{J}^{\mathbf{p}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta})$ with entries $\partial\sigma_i(\boldsymbol{\delta})/\partial p_j$ is equal to
$$\mathbf{J}^{\mathbf{p}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta}) = -\alpha\left(\left[\mathbf{J}^{\mathbf{s}}_{\ln\mathbf{G}}(\mathrm{s})\right]^{-1} - \mathbf{s}\mathbf{s}^{\intercal}\right), \tag{19}$$
with $\mathrm{s} = \boldsymbol{\sigma}(\boldsymbol{\delta})$ and where $\mathbf{J}^{\mathbf{s}}_{\ln\mathbf{G}}(\mathrm{s})$ has entries given by Equation (13). In the absence of income effects, the matrix of price derivatives of market share is the Slutsky matrix. It is symmetric and positive semi-definite, which implies that the IPDL market share functions are non-decreasing in their own index δ_j, $\partial\sigma_j(\boldsymbol{\delta})/\partial\delta_j \geq 0$. The cross-price elasticity from product j to product k is given by $(\partial\sigma_k(\boldsymbol{\delta})/\partial p_{jt})(p_{jt}/\sigma_k(\boldsymbol{\delta}))$. The diversion ratio from product j to product k is given in percentage terms by $100(\partial\sigma_k(\boldsymbol{\delta})/\partial p_j)/|\partial\sigma_j(\boldsymbol{\delta})/\partial p_j)|$.

3. Products can be substitutes or complements in demand.

4. Products are substitutes in utility.

Proof of Proposition 3.
1. Using Equation (6), for any pair of products j and k we have

$$\frac{\sigma_j\left(\boldsymbol{\delta}\right)}{\sigma_k\left(\boldsymbol{\delta}\right)} = \exp\left(\frac{\delta_j - \delta_k}{1-\sum_{d=1}^{D}\mu_d} + \sum_{d=1}^{D}\frac{\mu_d}{1-\sum_{d=1}^{D}\mu_d}\ln\left(\frac{\sigma_{d(k)}\left(\boldsymbol{\delta}\right)}{\sigma_{d(j)}\left(\boldsymbol{\delta}\right)}\right)\right). \tag{20}$$

For products j and k of the same type (i.e., with $d\left(k\right) = d\left(j\right)$ for all d), Equation (20) reduces to $\frac{\sigma_j(\boldsymbol{\delta})}{\sigma_k(\boldsymbol{\delta})} = \exp\left(\frac{\delta_j-\delta_k}{1-\sum_{d=1}^{D}\mu_d}\right)$, which is independent of the characteristics or existence of all other products, that is, IIA holds for products of the same type. When products are of different types, the ratio can depend on the characteristics of other products, which means that IIA does not hold in general.

2. Recall that the IPDL model is defined by

$$\ln G_j(\mathbf{s}) + CS(\boldsymbol{\delta}) = \delta_j, \quad j \in \mathcal{J}, \tag{21}$$

where $\ln G_j$ is given by Equations (11) and (12) and where we have used that $c = \ln\left(\sum_{k\in\mathcal{J}} H_k\left(e^{\boldsymbol{\delta}}\right)\right) = CS(\boldsymbol{\delta})$.

Differentiate this equation with respect to $\boldsymbol{\delta}$, then $\mathbf{I} = \mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\,\mathbf{J}^{\boldsymbol{\delta}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta}) + \mathbf{1}\mathrm{s}^{\intercal}$, with $\mathrm{s} = \boldsymbol{\sigma}(\boldsymbol{\delta})$, and where we have used Roy's identity. $\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})$ is invertible. Then, $\mathbf{J}^{\boldsymbol{\delta}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta}) = \left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{-1}\left[\mathbf{I} - \mathbf{1}\mathrm{s}^{\intercal}\right] = \left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{-1} - \left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{-1}\mathbf{1}\mathrm{s}^{\intercal}$. Finally, note that $\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\,\mathrm{s} = \mathbf{1}$ so that $\left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{-1}\mathbf{1}\mathrm{s}^{\intercal} = \mathrm{s}\mathrm{s}^{\intercal}$.

Consequently, $\mathbf{J}^{\boldsymbol{\delta}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta})$ is symmetric. As $\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})$ is positive definite, the square-root matrix $\left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{1/2}$ exists and is also positive definite. Then

$$\left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{1/2}\mathbf{J}^{\mathbf{s}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta})\left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{1/2} = \left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{-1/2}(\mathbf{I} - \mathbf{1}\mathrm{s}^{\intercal})\left[\mathbf{J}^{\mathbf{s}}_{\ln \mathbf{G}}(\mathrm{s})\right]^{1/2}$$

is symmetric and idempotent and thus positive semi-definite. Then $\mathbf{J}^{\boldsymbol{\delta}}_{\boldsymbol{\sigma}}(\boldsymbol{\delta})$ is positive semi-definite.

3. Suppose there are $J = 3$ products and one outside good. Products are grouped according to two grouping characteristics: the grouping is $\{1\}, \{2,3\}$ for the first characteristic and $\{1,3\}, \{2\}$ for the second characteristic.

Let $\boldsymbol{\sigma}(\boldsymbol{\delta}) = \mathrm{s}$. Using Proposition 3 (part 2.), we show that

$$\frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2} = s_1 s_2\left[1 + \frac{\mu_1\mu_2 s_3}{D}\right], \tag{22}$$

where we have set $\alpha = 1$ and where $D = -(1-\mu_1-\mu_2)(s_1+s_3)(s_2+s_3) - \mu_1\mu_2 s_3(1-s_0) < 0$.

Products 1 and 2 are complements in demand if and only if $\frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2} < 0$, that is, if and only if $(1-\mu_1-\mu_2)(s_1+s_3)(s_2+s_3) - \mu_1\mu_2 s_0 s_3 < 0$. With market shares equal to $s_1 = s_2 = s_3 = 1/6$, if $\mu_1 = 1/4$ and $\mu_2 = 1/3$, products 1 and 2 are substitutes in demand. In contrast, if $\mu_1 = 3/5$ and $\mu_2 = 1/3$, they are complements

in demand.
4. The IPDL model restricts products to be substitutes in utility, since

$$\frac{\partial u(\mathbf{s})}{\partial s_i \partial s_j} = -\sum_{d=1}^{D} \frac{\mu_d}{s_{d(j)}} \mathbf{1}\{i \in d(j)\} - \frac{\mu_0}{s_0}. \tag{23}$$

is negative. □

B.4 Higher-Order Partial Derivatives of Demand in the RCL Model

Consider an additive random utility model (ARUM) in which the vector of random utility terms follows a joint distribution with finite means that is absolutely continuous and independent of $\boldsymbol{\delta}$. The logit and nested logit models are special cases of this.

The ARUM market share functions (or choice probabilities) satisfy a range of general conditions (McFadden, 1981), one of which, sometimes referred to as non-negativity, is that the partial derivatives of the market share function σ_i ($i \in \mathcal{J}$) with respect to any set of distinct prices other than p_i are non-negative if the coefficients on prices are negative, i.e.,

$$\frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2} \geq 0; \quad \frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2 \partial p_3} \geq 0; \quad \frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2 \partial p_3 \partial p_4} \geq 0, \text{ etc.} \tag{24}$$

In particular, this condition rules out complementarity in demand in the ARUM.

The RCL market share function is a mixture of logit market share functions, and thus it also satisfies the non-negativity condition when price coefficients are negative (almost surely).

By contrast, the IPDL model does not necessarily satisfy the non-negativity condition. To show this, we consider, again, the example of Subsection 3.1 with $J = 3$ products and one outside good. Market shares are equal to $s_1 = s_2 = s_3 = 1/6$ and $\alpha = 1$. Products are grouped according to two grouping characteristics: the grouping is $\{1\}, \{2, 3\}$ for the first characteristic and $\{1, 3\}, \{2\}$ for the second characteristic.

When $\mu_1 = 1/4$ and $\mu_2 = 1/3$, all products are substitutes in demand, but the non-negativity condition does not hold for the second-order partial derivatives

$$\frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2} = 0.021 > 0; \quad \frac{\partial^2 \sigma_1(\boldsymbol{\delta})}{\partial p_2 \partial p_3} = -0.020 < 0. \tag{25}$$

When $\mu_1 = 3/5$ and $\mu_2 = 1/3$, products 1 and 2 are complements in demand, but the non-negativity condition holds for the second-order partial derivatives

$$\frac{\partial \sigma_1(\boldsymbol{\delta})}{\partial p_2} = -0.011 < 0; \quad \frac{\partial^2 \sigma_1(\boldsymbol{\delta})}{\partial p_2 \partial p_3} = 0.005 > 0. \tag{26}$$

B.5 Direct and Indirect Substitution Effects

We follow Ogaki (1990) to decompose the substitution effect between products i and j into the indirect substitution effect and the direct substitution effect.

Let $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}$ be the matrix of price derivatives of market share. Recall that $\mathcal{J} = \{0, 1, \ldots, J\}$ denotes the choice set. Then, $\mathcal{J}_{-(i,j)}$ denotes the choice set without products i and j.

The indirect substitution effect between product i and j, S^{i}_{ij}, is equal to

$$S^{\mathrm{i}}_{ij} = \mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[i, \mathcal{J}_{-(i,j)}] \left[\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[\mathcal{J}_{-(i,j)}, \mathcal{J}_{-(i,j)}]\right]^{-1} \mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[\mathcal{J}_{-(i,j)}, j], \tag{27}$$

where $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[\mathcal{J}_{-(i,j)}, \mathcal{J}_{-(i,j)}]$ denotes the matrix $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}$ after removing the rows and columns involving products i and j, $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[i, \mathcal{J}_{-(i,j)}]$ denotes the matrix $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}$ after keeping the row involving product i and removing the columns involving products i and j, and $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}[\mathcal{J}_{-(i,j)}, j]$ denotes the matrix $\mathbf{J}^{\mathrm{p}}_{\boldsymbol{\sigma}}$ after keeping the column involving product j and removing the rows involving products i and j.

The direct substitution effect between product i and j, S^{d}_{ij}, is then equal to

$$S^{\mathrm{d}}_{ij} = \frac{\partial \sigma_j(\boldsymbol{\delta})}{\partial p_i} - S^{\mathrm{i}}_{ij}. \tag{28}$$

Products i and j are direct complements in demand if $S^{\mathrm{d}}_{ij} < 0$, and direct substitutes in demand if $S^{\mathrm{d}}_{ij} > 0$.

C Details on the Experiments

C.1 Simulated Data

For each experiment, we generate 50 datasets consisting of $T = 200$ independent markets with $J = 45$ products, where markets exhibit segmentation according to two grouping characteristics $d = 1, 2$, each forming two groups, denoted g_{d0} and g_{d1}, so that there are four product types. The grouping structure is simulated using binomial distributions and is common across markets.

In each experiment, we simulate a fully structural model of supply and demand, where the observed characteristic x_{jt} and the cost-shifter z_{jt} are i.i.d. $\mathcal{U}(0, 1)$. The unobserved product characteristic term is $\xi_{jt} = u_{1t} + u_{2t}$ and the unobserved cost component is $\omega_{jt} = u_{1t} + u_{3t}$, where u_{1t}, u_{2t}, and u_{3t} are i.i.d. $\mathcal{U}(-0.5, 0.5)$. Prices and market shares are determined endogenously. The supply side is a static price competition model among five multi-product firms, each with nine products and with constant marginal cost given by $c_{jt} = 2 + x_{jt} + z_{jt} + w_{jt}$.

Experiment 1 We simulate four IPDL models, varying the values of the grouping parameters μ_1 and μ_2 (Table 1). In each model, we set $\delta_{jt} = -3+2x_{jt}-0.5p_{jt}+\xi_{jt}$.

Experiment 2 We simulate four PDL models, varying the values of the grouping parameters μ_1 and μ_2 (Table 2). The PDL model is a GEV model. Its market share function is given by $\sigma_j(\boldsymbol{\delta}) = e^{\delta_j}(\partial G_j(e^{\boldsymbol{\delta}})/\partial e^{\delta_j})/G(e^{\boldsymbol{\delta}})$, where $G(e^{\boldsymbol{\delta}}) = a_1 \left[\sum_{h\in\{0,1\}} \left(\sum_{j\in g_{1h}} e^{\delta_j/\mu_1}\right)^{\mu_1}\right] + a_2 \left[\sum_{h\in\{0,1\}} \left(\sum_{j\in g_{2h}} e^{\delta_j/\mu_2}\right)^{\mu_2}\right]$ with $a_1 = (1-\mu_1)/(2-\mu_1-\mu_2)$ and $a_2 = 1 - a_1$. In each model, we set $\delta_{jt} = -1 + 2x_{jt} - 0.5p_{jt} + \xi_{jt}$.

Experiment 3 We simulate four RCL models with independent normal random coefficients on dummies for groups defined by segmentation, varying the values of the standard deviations of the normal random coefficients RC_1 and RC_2 (Table 3). In each model, the mean utility of product j in market t is given by $\delta_{jt} = 3 - p_{jt} + x_{jt} + \xi_{jt}$. We use the package PyBLP by Conlon and Gortmaker (2020) to simulate the RCL models. We use 200 Halton draws over the standard normal distribution to integrate the market share functions numerically. Each dimension of integration of Halton draws uses a different prime, discards the first 1,000 points, and then scrambles the sequence.

C.2 Estimation Procedures

Nested Logit and IPDL Models We estimate the nested logit and IPDL models using the two-stage least square estimator. We compute instruments as the predicted values from regressions of the endogenous variables (i.e., the price variable and two log-share terms) on a constant, the product characteristic x_{jt}, the cost shifter z_{jt}, and Gandhi and Houde (2023)'s differentiation instruments in x_{jt} $\sum_{k\in d_1(j)} \left(d^x_{j,k,t}\right)^2$, $\sum_{k\notin d_1(j)} \left(d^x_{j,k,t}\right)^2$, and $\sum_{k\in d_2(j)} \left(d^x_{j,k,t}\right)^2$ where $d^x_{j,k,t} = x_{kt} - x_{jt}$.

PDL Models We estimate the PDL models using the following two-step estimation procedure, which is standard in the literature. First, we solve for the error term ξ_{jt} as a function of the parameters μ_1 and μ_2. That is, given values for μ_1 and μ_2, we numerically compute the values $\delta_{jt}(\mu_1, \mu_2)$ of δ_{jt} that equate the observed to the predicted market shares, and compute $\hat{\xi}_{jt} = \delta_{jt}(\mu_1, \mu_2) - \hat{\beta}_0 - \hat{\beta} x_{jt} + \hat{\alpha} p_{jt}$, where $\hat{\alpha}$ and $\hat{\beta}$ are the estimates from Berry (1994)'s regression $\delta_{jt} = \beta_0 + \beta x_{jt} - \alpha p_{jt} + \xi_{jt}$. Second, we interact $\hat{\xi}_{jt}$ with instruments to form the GMM objective function that we minimize over μ_1 and μ_2. We use the following instruments: (i) the product characteristic x_{jt} and the cost shifter z_{jt}, (ii) the predicted values $\hat{p}_{jt}$ from a linear regression of the price variable on a constant, x_{jt}, z_{jt} and the same differentiation instruments in x_{jt} as for the nested logit and IPDL models, and (iii) the differentiation instruments in x_{jt}. Given the estimates $\hat{\mu}_1$ and $\hat{\mu}_2$ of μ_1 and μ_2, we can compute the values $\delta_{jt}(\hat{\mu}_1, \hat{\mu}_2)$ and recover the estimates $\hat{\beta}_0$, $\hat{\beta}$, and $\hat{\alpha}$ from the linear instrumental variables regression of $\delta_{jt}(\hat{\mu}_1, \hat{\mu}_2)$ on a constant, x_{jt} and p_{jt} using $\hat{p}_{jt}$ as an excluded instrument. We use the GMM estimator with a homoskedastic weighting matrix.

RCL Models We estimate the RCL models using the nested-fixed point approach proposed by BLP. We use the package PyBLP by Conlon and Gortmaker (2020) and implement their best practices. That is, we numerically integrate the market share functions using 200 Halton draws over the standard normal distribution; we numerically compute the δ_{jt}'s that equate the observed to the predicted market shares using the SQUAREM accelerated fixed point algorithm; we minimize the GMM objective function using the Knitro 13.1 Interior/Direct algorithm; and we use the "approximate" version of the feasible optimal instruments. To be specific, we use the following two steps: we first estimate the RCL model using the same set of instruments as for the PDL model; second, based on these estimates, we compute the optimal instruments and re-estimate the model using these instruments. In both steps, we use the GMM estimator with a homoskedastic weighting matrix.

D Details on the Empirical Application

Potential market size We compute the potential market size by multiplying the total number of persons in a market by the monthly per capita consumption of cereals. For each store in a month, we compute the total number of persons as the weekly average number of households who visited that store in that given month, multiplied by the average household size. We compute the weekly average number of households using the information on the daily traffic store and assuming that consumers visit stores twice a week. We compute the monthly per capita consumption of cereals using the information from the USDA's Economic Research Service that per capita US consumption of cereals was equal to 13.4 pounds in 1991, 13.9 in 1992, 14.6 in 1993, 14.8 in 1994, 14.6 in 1995 and 14.3 in 1996.

Estimations We estimate the IPDL and RCL models using the procedures described in Appendix C.2 but using the two-step GMM estimator with a weighting matrix clustered at the month-store level. For the RCL models, we also absorb the fixed effects using the package PyHDFE.

Computational Time We run all the estimations on a personal computer with 2.50 GHz Intel Core i7-11850H CPU and 16.0 GB RAM, Windows operating system (Windows 10), Python version 3.9. We obtain the computational time using

Python's timeit() function that returns the number of seconds it took to execute the code.

Goodness-of-fit We measure goodness-of-fit using a ten-fold cross-validated procedure: (i) we randomly split the sample at the market level into ten sub-samples of (approximately) equal size; (ii) we estimate both specifications of the IPDL and RCL models using nine of the sub-samples and compute the MSE for the tenth sub-sample; (iii) we repeat (ii) for each of the ten sub-samples; (iv) we compute the average of the ten MSE. The model that obtains the lowest average MSE, called the cross-validated MSE, fits the data best. We also implement this procedure with the GMM J-statistic in place of the MSE.

Complementarity in Demand We compute the percentage of pairs of complements in demand while accounting for the fact that some pairs of cereals exhibiting negative cross-price derivatives of market shares may be independent and not complements in demand. For this purpose, we use 95%, 99%, 99.5%, and 99.9% confidence intervals for the cross-price derivatives of market share. We compute them using a parametric bootstrap: we repeatedly draw from the estimated joint distribution of parameters; for each draw, we compute the average (over markets) cross-price derivatives of market share for all pairs of products, thus generating a bootstrap distribution. We use 999 draws.

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