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Ring road investment, cordon tolling, and urban spatial structure: Formulation and a case study

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Abstract

Ring roads, as candidate cordon locations, provide an advantageous condition for implementing cordon tolling schemes. This paper presents a methodology for investigating the ring road investment and cordon tolling problems in a congested ring-radial city. A two-dimensional urban system equilibrium for a ring-radial city is first formulated, in which interactions among stakeholders, including the authorities, property developers, households and commuters, are explicitly considered. Two social welfare maximization models for optimizing the ring road investment and cordon tolling schemes, a short-sighted and a far-sighted one, are then proposed. In the short-sighted model, the ring road investment decision is first made, and then the cordon tolling scheme is optimized based on the determined ring road locations as candidate cordons. However, in the far-sighted model, a simultaneous decision of the ring road investment and cordon tolling is made. The proposed models explicitly incorporate the estimation of the intra-area travel. The case study applied to the city network of Chengdu China shows that ring road investment and cordon tolling can reshape the urban spatial structure as a result of the tug-of-war between the dispersion effects due to ring road investment and the concentration effects due to cordon tolling. There is a large difference in the optimal solutions of the far-sighted model and the short-sighted model (e.g., optimal number of cordons). The former is closer to the social optimum than the latter. The optimal multi-cordon tolling scheme outperforms the optimal single-cordon tolling scheme in terms of the social welfare. However, the gap between them is trivial. Ignoring the household residential relocation behavior in the models leads to underestimates of total cordon toll revenue and social welfare gains.

Keywords: Ring-radial city; ring road investment; cordon tolling; urban spatial structure; urban system equilibrium; social welfare.

JEL classification: R13, R14, R41, R42

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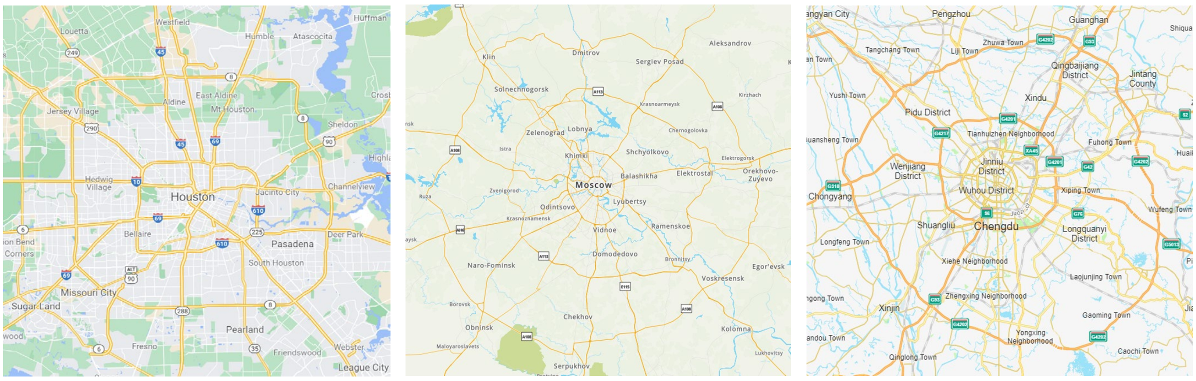
1. Introduction

The past decade has witnessed a dramatic increase in the sizes of some large cities in the world due to rapid urbanization and economic growth, such as Beijing and Seattle. The rapid urban expansion has led to more decentralized urban structure, longer average commuting distance, increased use of private cars, and heavier traffic congestion. In response, local authorities have launched a number of transportation infrastructure investment projects to increase accessibility to the city center and alleviate urban traffic congestion, including constructing ring roads around the city center to expand the capacity of urban road system.



(a) Houston, US (b) Moscow, Russia (c) Chengdu, China

Fig. 1. Some typical examples of ring-radial urban networks.

Thus far, many megacities in the world have introduced ring roads. For example, Houston US has built two ring roads: inner ring (Loop 610) and outer ring (Beltway 8), as shown in Fig. 1a. Moscow, the capital of Russia, has had four ring roads (shown in Fig. 1b). Chengdu, a city located in West China, has had four ring roads under operation (shown in Fig. 1c). Recently, the Chengdu municipal government has launched the construction project of the fifth ring road, which is expected to be completed within this year. Besides, some new city projects around the world have been designed to address contemporary urban challenges, such as population growth, urbanization, and climate change, and to improve the life quality of inhabitants through integrating advanced and sustainable technologies, such as NEOM project of Saudi Arabia, New Songdo city project of Korea, and Masdar city project of the United Arab Emirates.[1] The today's planning of these new cities' road network structures (e.g., radial/ring roads) is very important for their tomorrow's traffic congestion level and

[1] https://www.neom.com/en-us; https://www.kpf.com/project/new-songdo-city; https://masdarcity.ae.

sustainable urban developments, and thus should be carefully made.

Introduction of ring roads in an urban road network, as a supply-side strategy, can provide travelers with more alternative routes to reach their destinations through intersecting the radial major roads, thus creating ring-radial urban network structure with better connectivity and accessibility to suburban areas. Hence, the set of travelers' alternative routes linking to the city center is expanded. As a result, traffic flow is distributed over more routes, and thus traffic congestion levels and en-route travel time costs on the urban radial major roads may be decreased (Saidi et al., 2016). Moreover, as urbanization is promoted, cities become more decentralized, and thus ring roads do play an increasingly important role in traffic congestion reduction via providing a fast road. However, the investment of the ring roads in an urban road network requires a huge capital cost. For instance, the planned length of the fifth ring road in Chengdu China is about 142.8 km, and the expected investment cost is about 34.9 billion RMB, meaning 0.244 billion RMB per km.[2] Naturally, this raises some important and intriguing issues: given the population size of a city, how many ring roads should be invested so as to create the most efficient urban system, in terms of social welfare? Where should the ring roads be located? What effects does the ring road investment bring to the urban spatial structure, in terms of household residential (re)location choice and housing market? The answers to these questions are especially important and urgent for transportation infrastructure investment decisions of many developing cities, such as Chengdu, in which rapid urbanization is currently being carried out.

Some studies have shown that the supply-side strategy, such as the construction of urban roads, is efficient in alleviating traffic congestion in a short term. However, it may induce new traffic demand and thus cause further traffic congestion during peak periods (Goodwin, 1996; Hansen and Huang, 1997; Kono et al., 2007). What's more, the resources that are available for the road construction remain limited in most urban areas. Congestion pricing, particularly cordon tolling, has been suggested as a viable alternative to infrastructure expansion because of its ease of implementation and potential to internalize congestion externalities (Parry, 2002). Typical examples include those adopted in Singapore, London, Hong Kong, Oslo, and Bergen (Wong et al., 2005; Yang and Huang, 2005; Rouwendal and Verhoef, 2006). Recently, feasibility analyses of cordon tolling projects have been made in some major Chinese cities,

[2] "RMB" is the Chinese currency, Renminbi, and US$1 approximates RMB6.94 on March 1, 2023.

such as Beijing, Shanghai, Shenzhen, and Chengdu.[3] Apparently, the urban ring roads can serve as candidate cordon locations because they provide convenient conditions for implementation of cordon tolling schemes. This also raises other important issues: how to design the cordon tolling schemes for a congested ring-radial city in terms of cordon locations and toll levels, particularly for a city with ring roads as candidate cordons? How about the efficiencies of multi-cordon vs. single-cordon tolling schemes in terms of the social welfare?

In light of the above discussions, this paper aims to address the issues of ring road investment and cordon tolling in a congested ring-radial city. As such, both ring road investment and cordon tolling may influence the households' residential location choices and thus the urban spatial structure. In the proposed model, such externality effects of the ring road investment and the cordon tolling on households' residential relocation decisions and the urban spatial structure are incorporated. It is anticipated that the proposed model can serve as a useful tool for long-term planning of ring-radial urban systems, and for evaluation of various transportation infrastructure investment and/or pricing policies.

The main contributions of this paper are twofold. First, an urban system equilibrium model for a congested two-dimensional ring-radial monocentric city is presented, in which the interactions among different stakeholders and the intra-area travel are explicitly considered. Specifically, the authorities determine the optimal number and locations of ring roads to be introduced, cordon locations, and toll levels to maximize total social welfare of the urban system. The households choose their residential locations to maximize their own utilities within income budget constraint. The property developers seek to maximize their own net profit by determining the housing supply. The commuter of each household chooses travel route that minimizes his/her own commuting cost. The boundary contours for commuters' route choices are identified, and the properties of the proposed urban system equilibrium model are analytically explored, together with the difference of the proposed continuum modeling method in this paper and the traditional traffic simulation method. Second, social welfare maximization models are presented to determine the optimal ring road investment and cordon tolling schemes. Two types of decision mechanisms, a short-sighted and a far-sighted one, are investigated and compared. The short-sighted decision mechanism follows a two-stage (or two-step) decision process. In the first stage, the ring road investment decisions

[3] https://www.66law.cn/laws/205007.aspx; http://sc.cnr.cn/sc/2014cd/201412/t20141210_517055571.shtml.

in terms of the number and locations of ring roads are made which temporarily ignores the fact that the infrastructure will be tolled later on. In the second stage, the optimal cordon tolling scheme that considers the ring roads already identified in the first stage as candidate cordon locations is determined. The far-sighted decision mechanism simultaneously determines the number and locations of ring roads and cordon tolling schemes (i.e., a joint decision). The solutions of the models under the two decision mechanisms with and without a consideration of household residential relocation behavior are compared, together with the optimal solutions for the multi-cordon and single-cordon tolling schemes. A case study from Chengdu China is provided to illustrate the properties and applications of the proposed models, together with discussion of policy implications.

The remainder of this paper is organized as follows. Section 2 provides a literature review of previous related studies. Section 3 presents some basic model assumptions. Section 4 formulates the urban system equilibrium problem and its properties. In Section 5, the social welfare maximization models under the short-sighted and far-sighted decisions are proposed for optimizing the ring road investment and cordon tolling schemes. In Section 6, a case study is given for the purpose of model illustration. Finally, Section 7 concludes this paper and provides recommendations for further studies.

2. Literature review

In this section, we conduct a comprehensive literature review of previous related studies. For presentation purpose, we divide the review into two parts according to the themes concerned, namely the ring road investment issue and the cordon tolling issue, given as follows.

2.1. Ring road investment issue

In literature, there are some studies involving ring road investment. For the convenience of readers, we have summarized in Table 1 some main contributions to ring road system research, in terms of decision variable(s), objective function, urban form, modeling approach, and whether considering congestion effects and household residential relocation behavior. Table 1 shows that the previous relevant studies mainly focused on the following topics: routing problems in a symmetric circular city (Tan, 1966; Blumenfeld and Weiss, 1970b; Zitron, 1974), traffic equilibrium assignment problems (Lam and Newell, 1967; D'Este, 1987; Wong, 1994), and ring or radial road location problems (Blumenfeld and Weiss, 1970a; Pearce, 1974; Smith, 1976, 1979; Anas and Moses, 1979; Li, 2004; Li et al., 2013). Particularly, the studies published before the 1980s mainly concerned symmetric and congestion-free (i.e., travel time is independent of traffic flow) circular cities so as to derive analytical solutions. An exception is Lam and Newell (1967), who considered road traffic congestion effects in the route choice equilibrium problem of travelers in a ring-radial city. However, their model focused on a symmetric city structure, and also ignored the household residential relocation behavior due to the ring road investment.

Table 1 Contributions to ring road system research.

Reference	Decision variable(s)	Objective function	Urban form	Modeling approach	Considering congestion effects	Considering household relocations
Tan (1966)	Routing (shortest route)	Min. distance traveled, road space required, or travel time	Symmetric circular city with two ring roads	Analytical	×	×
Lam and Newell (1967)	Equilibrium route flow	Min. route travel time	Symmetric radial-ring city	Analytical + simulations	√	×
Blumenfeld and Weiss (1970a)	Location of an additional ring road	Min. route travel time	Symmetric circular city	Analytical	×	×
Blumenfeld and Weiss (1970b)	Routing (shortest route)	Min. route travel time	Symmetric circular city	Analytical	×	×
Pearce (1974)	Locations of two ring roads	Min. average travel distance	Symmetric circular city	Analytical	×	×
Zitron (1974)	Routing (optimal cost route)	Min. route travel cost	Symmetric circular city	Analytical	×	×
Smith (1976)	Locations of two ring roads	Min. total radial traffic flow	Symmetric circular city	Analytical	×	×
Smith (1979)	Location of a single ring road	Max. traffic relief to radials	Symmetric circular city	Analytical	×	×
Anas and Moses (1979)	Number of radial lines	Max. household welfare	Symmetric radial city	Analytical + simulations	×	√
D'Este (1987)	Equilibrium route flow	Min. route travel time	Asymmetric radial city	Analytical for special case	√	×
Wong (1994)	Equilibrium route flow	Min. route travel time	Asymmetric radial city	Analytical + simulations	√	×
Li (2004)	Location of a single ring road	Min. average travel distance on major roads	Symmetric radial-ring city	Analytical	×	×
Li et al. (2013)	Locations of radial roads	Max. social welfare	Asymmetric radial city	Analytical + simulations	√	√
This paper (2023)	Number and locations of ring roads, cordon locations, and toll levels	Max. social welfare	Asymmetric radial-ring city	Analytical + simulations	√	√

Table 1 also shows that some studies published after the 1980s have relaxed the symmetric and congestion-free assumptions to consider more realistic situations. For example, D'Este (1987) investigated the commuters' radial road choices in a two-dimensional monocentric city with several radial major roads using the ring-radial travel method of Anas and Moses (1979), in which commuters traveling from their home locations to the workplace in the central business district (CBD) first travel along circular dense surface streets to reach a radial major road, and then proceed along the radial major road to reach the city center. Wong (1994) reformulated the traffic assignment model presented in D'Este (1987) as a mathematical programming problem. These two studies (D'Este, Wong) assumed that the household residential distribution in the city is exogenously given and fixed.

Li et al. (2013) further relaxed such an assumption to investigate the optimization problem of the density of radial major roads in a two-dimensional monocentric city using the ring-radial travel method. In their study, the household residential distribution and housing prices are endogenously determined, as done in some studies about the effects of transportation infrastructure improvements on urban configuration (e.g., Getz, 1975; Arnott and MacKinnon, 1977; McDonald and Osuji, 1995). However, the study of Li et al. (2013) considered a radial city without ring roads, and thus cannot properly address the effects of the ring road investment. In reality, more and more cities are ring-radial structures through investing in ring roads, such as Houston, Moscow, and Chengdu. This paper will extend these previous related studies to explore the optimization problem of the number and locations of ring roads in a two-dimensional asymmetric ring-radial monocentric city, while accounting for the traffic congestion effects and household residential relocation choices.

2.2. Cordon tolling issue

In the past decades, significant progress has been made in design of cordon tolling schemes in terms of cordon toll location and/or toll level. The modeling methods adopted in the previous literature can be categorized into two major classes: discrete network modeling approach and continuum modeling approach. The discrete network models are inclined to develop efficient algorithms applicable to actual road networks. Sample studies include May et al. (2002), Verhoef (2002), Shepherd and Sumalee (2004), Sumalee (2004), Zhang and Yang (2004), de Palma et al. (2005), Maruyama and Sumalee (2007), Fujishima (2011), Liu et al. (2014), and Vosough et al. (2020, 2022). The continuum models aim to reveal the fundamental relationships among variables through deriving analytical solutions that are easy to interpret and comprehend, and thus can be regarded as a supplement of the discrete models. The example studies include Mun et al. (2003, 2005), Ho et al. (2005, 2013), Verhoef (2005), Li et al. (2014), Anas and Hiramatsu (2013), Li and Wang (2018), and Tsai and Lu (2018). For comprehensive reviews of congestion tolling issues, readers can refer to Tsekeris and Voß (2010), Lindsey (2010), and de Palma and Lindsey (2011). In this paper, the continuum modelling approach is adopted.

Table 2 Contributions to continuum models for cordon tolling research.

Reference	Decision variable(s)	No. of cordons	Urban form	Modeling approach of urban system	Considering ring road	Household relocation
Mun et al. (2003)	Toll location and toll level	Single	Linear, monocentric	Continuum	×	×
Mun et al. (2005)	Toll location and toll level	Single	Linear, polycentric	Continuum	×	×
Ho et al. (2005)	Toll location and toll level	Multiple	Asymmetric, two-dimensional, monocentric, perfectly divisible dense city	Continuum	×	×
Ho et al. (2013)	Toll level	Single	Asymmetric, two-dimensional, polycentric, perfectly divisible dense city	Continuum	×	×
Verhoef (2005)	Toll location and toll level	Single	Linear, monocentric	Continuum	×	√
de Palma et al. (2011)	Toll location and toll level	Single and multiple	Linear, monocentric	Continuum	×	√
de Lara et al. (2013)	Toll location and toll level	Single	Linear, monocentric	Continuum	×	√
Li et al. (2014)	Toll location and toll level	Single	Symmetric, two-dimensional, monocentric, perfectly divisible dense city	Continuum	×	×
Li and Guo (2017)	Cordon pricing timing, cordon location, and toll level	Single	Symmetric, two-dimensional, monocentric, perfectly divisible dense city	Continuum	×	√
Li and Wang (2018)	Cordon location and toll level	Single	Asymmetric, two-dimensional, monocentric, radial	Continuum + discrete	×	√
Tsai and Lu (2018)	Multi-cordon locations and toll levels	Multiple	Linear, monocentric	Continuum	×	×
This paper (2023)	Number of cordons, cordon location, and toll level	Multiple	Asymmetric, two-dimensional, monocentric, ring-radial city	Continuum + discrete	√	√

Table 2 lists some main contributions of analytical continuum models to cordon tolling problems in the field of urban economics, in terms of decision variable(s), number of cordons, urban form, modeling approach of urban system, and whether considering ring road investment and household residential relocation behavior. It can be seen that the previous continuum models mainly focused on a one-dimensional linear city (e.g., Mun et al., 2003, 2005; Verhoef, 2005; de Palma et al., 2011; de Lara et al., 2013; Tsai and Lu, 2018). The assumption of a linear city can facilitate the model building of the problem concerned and the derivation of the model solution. However, it cannot well reflect the realistic multi-dimensional urban spatial structure and residents' activity and travel choice behavior in the urban network, e.g., the route choice behavior. Some studies have extended the one-dimensional linear urban model to two-dimensional cases (e.g., Ho et al., 2005, 2013; Li et al., 2013, 2014; Li and Guo, 2017; Li and Wang, 2018).

However, these previous related studies usually considered a two-dimensional radial city or a perfectly divisible dense city (i.e., no discrete radial and ring major roads), and did not involve a two-dimensional ring-radial city, which is currently a popular urban structure in many megacities in the world, as shown in Fig. 1. Moreover, the existing studies related to continuum models usually concerned only one charging cordon, except the studies of Ho et al. (2005) and Tsai and Lu (2018), which considered multi-cordon pricing issues. However, the former considered a dense city continuum without radial and ring roads, whereas the latter concerned a one-dimensional linear city. Furthermore, they both did not consider the effects of cordon tolling on the household residential relocation behavior, which has been treated in some relevant studies, such as Verhoef (2005), de Palma et al. (2011), de Lara et al. (2013), Li and Guo (2017), and Li and Wang (2018). In addition, all of the aforementioned studies about the cordon tolling did not consider the effects of ring road investment on design of cordon tolling scheme, e.g., the role of ring roads as candidate cordons.

3. Basic assumptions

To facilitate the presentation of the essential ideas without loss of generality, the following basic assumptions are made in this paper.

A1 The urban system concerned is assumed to be ring-radial, closed and monocentric, in which the population size or the total number of total households is exogenously given and fixed, and all job opportunities are located in a highly compact city center or CBD. All the land within the city boundary is owned by an absentee landlord and the value of the land at/beyond the city boundary equals the agricultural rent or opportunity cost of the land. These assumptions have widely been adopted in urban economics literature (e.g., Alonso, 1964; Muth, 1969; Mills, 1972; Fujita, 1989; O'Sullivan, 2000; Kraus, 2006; McDonald, 2009; Kilani et al., 2010; Li et al., 2013).

A2 There are four types of stakeholders in the urban economy: the authorities, property developers, households and commuters. The authorities determine the number and locations of ring roads, ring road-based cordon locations and toll levels to maximize the urban system's social welfare. The property developers determine the intensity of their capital investments in the land market to maximize their own net profits generated by the housing provisions. Each property developer is assumed to adopt a Cobb-Douglas housing production technology (e.g., Beckmann, 1974; Quigley, 1984; Li et al., 2013).

A3 All households are homogeneous, i.e., the income level and utility function are the same for all households. Each household has a quasi-linear utility function. Such a quasi-linear function can facilitate the analysis of social welfare because the household utility level with the quasi-linear utility is measured in monetary units, leading to the same units with the land rents and toll revenue. The quasi-linear utility function has often been adopted in some previous studies, such as Song and Zenou (2006), Baum-Snow (2007), Kono et al. (2012), Peng et al. (2017), and Li and Wang (2018). The household income is spent on transportation, housing and non-housing goods. Each household aims to maximize its own utility by choosing the residential location, housing floor space and the amount of non-housing goods within the household's budget constraints (e.g., Solow, 1972, 1973; Beckmann, 1969, 1974; Anas, 1982; Fujita, 1989; Kilani et al., 2010; Li et al., 2013).

A4 This study mainly focuses on workers' commuting trips between home locations and workplace, belonging to compulsory trips. Thus, a household's number of trips is not affected by other factors, such as household's income level and trip distance. This means that the potential demand for travel to the CBD per day is pre-given and fixed. An exponential elastic travel demand density function is used to capture the response of travel demand to the travel cost between home location and workplace (Ortuzar and Willumsen, 2001). Such a response includes switching to other travel modes or other departure times. Commuters choose travel routes that minimize their commuting costs.

A5 The ring-radial urban system concerned consists of three types of roads: ring major roads, radial major roads, and dense surface streets (i.e., minor roads). The ring and radial major roads in a city usually comprise the backbone of transportation network of that city, carrying a large number of travel demand. Hence, the congestion effects on the radial and ring major roads cannot be ignored. However, the traffic volume on the dense surface streets or minor roads is usually low, and thus the congestion effects on the minor roads can be trivial and thus neglected. We assume that the vehicle speed on the ring major roads is fastest, that on the minor roads is slowest, and that on the radial major roads is in between. In reality, the radial roads of a city are usually first built, and then ring roads, such as for Chengdu and Paris. In accordance with the real world, we further assume in this paper that radial roads have been build and we are only interested in the construction of ring roads.

4. Urban system equilibrium

As previously assumed, the stakeholders in the city system (the authorities, property developers, households and commuters) interact. The authorities maximize the urban system's social welfare by determining the number and locations of ring roads, ring road-based cordon locations and toll levels. The property developers maximize their own net profits by determining the capital investment intensities. Households maximize their own utilities by determining the residential locations and the consumptions of housing and non-housing goods. Commuters minimize their commuting costs by choosing travel routes. The sequential interactions among these stakeholders lead to Stackelberg equilibria: the commuters' ring-radial route choice equilibrium, the household residential location choice equilibrium, and the housing demand-supply equilibrium. These Stackelberg equilibria jointly determine the urban spatial structure, which are in turn presented as follows.

4.1. Commuters' ring-radial route choice equilibrium

4.1.1. Travel cost

As shown in Fig. 2, *i* represents the *i*th radial major road of the city, and (x, θ_i) represents the polar coordinate of a location in the urban system concerned, where *x* denotes the radial distance of that location from the CBD, and θ_i denotes the angle of that location from the radial major road i. Let φ_i be the angle between adjacent radial major roads i and i+1. $(\varphi_i - \theta_i)$ is thus the angle between location (x, θ_i) and radial major road i+1. Let M_{RAD} be the total number of radial major roads in the city, which is pre-given, and suppose that the government wishes to make an investment plan of the M_{RIN} ring roads in the city in the future, including the locations and number of the ring roads. The subscripts "RAD" and "RIN" represent radial major roads and ring major roads, respectively. Both radial and ring roads may not be evenly spaced.

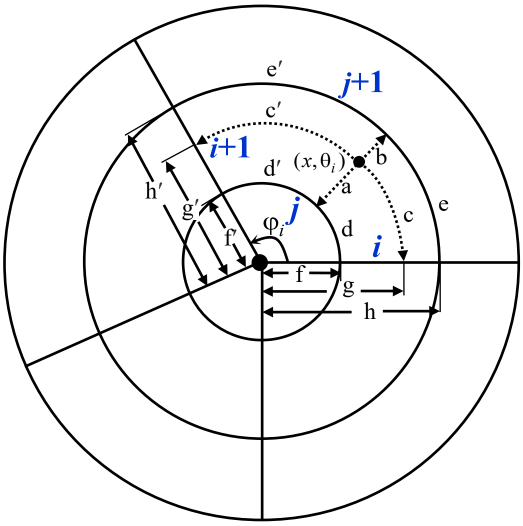


Fig. 2. Route choices of commuters at location (x, θ_i).

Consider a sector area consisting of radial major roads i and i+1 and ring roads j and j+1, represented by $\Theta_{(i,i+1;j,j+1)}$ for ease of presentation. Following the ring-radial travel method of Anas and Moses (1979), D'Este (1987), Wong (1994) and Li et al. (2013), the following routing rules are applied for the commuters residing at any location $(x, \theta_i) \in \Theta_{(i,i+1;j,j+1)}$:

- Travel first along circumferential minor road (a dense surface street without traffic congestion) to radial major road i or i+1, and then travel along the radial major road to get to the CBD (called circumferential-radial routing);
- Travel first along the radial minor road (a dense surface street without traffic congestion) to adjacent ring road j or j+1, and then travel along the ring major road to adjacent radial major road i or i+1, and finally straight along the radial major road to the CBD (called radial-ring-radial routing).

According to the routing rules defined above, one can easily identify six alternative (shortest) routes from any location (x,θ_i) to the CBD. They include: (1) *a-d-f*, (2) *b-e-h*, (3) *c-g*, (4) $a\text{-}d'\text{-}f'$, (5) $b\text{-}e'\text{-}h'$, and (6) $c'\text{-}g'$, where symbols a to g' represent the road segments (as shown in Fig. 3). It can be observed that routes 3 and 6 belong to the circumferential-radial routing, and other routes (i.e., routes 1, 2, 4, and 5) are the radial-ring-radial routing. It should be pointed out that the free-flow travel speeds on the radial major roads, ring major roads and minor roads are different, defined later.

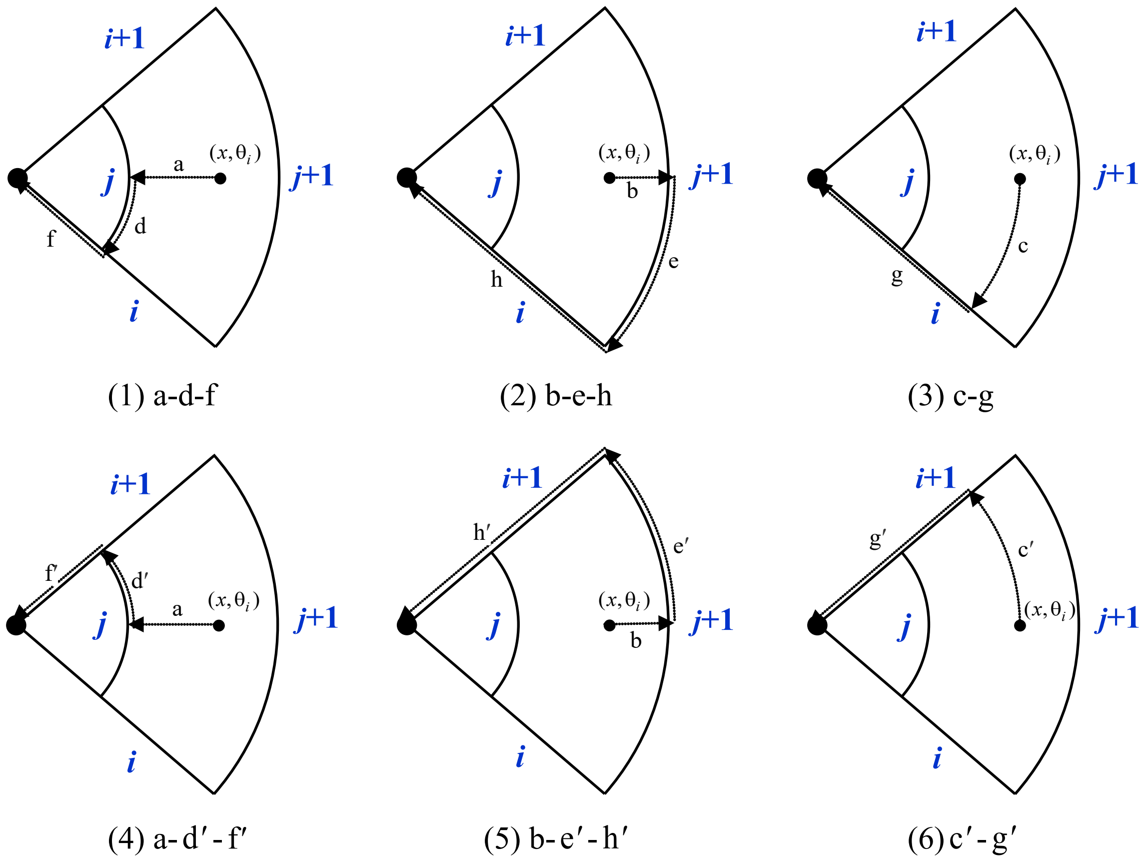


Fig. 3. Six alternative routes.

In order to define the travel costs of all six alternative routes, we first define the travel costs of road segments a to g'. Note that segments a, b, c and c' are the minor roads in the sector area $\Theta_{(i,i+1;j,j+1)}$, and thus have no congestion effects on these segments according to **A5**. Let $C_a(x,\theta_i)$, $C_b(x,\theta_i)$, $C_c(x,\theta_i)$ and $C_{c'}(x,\varphi_i-\theta_i)$ be the travel costs on segments a, b, c and c', respectively. They consist of travel time cost and monetary cost. For ease of presentation, we denote $\bar{\alpha}_1 = \lambda_t / V_0 + \alpha_1$. We thus have

$$\begin{cases} C_a(x,\theta_i) = \lambda_t \dfrac{x - R_j}{V_0} + \alpha_1 (x - R_j) = \bar{\alpha}_1 (x - R_j), \\ C_b(x,\theta_i) = \lambda_t \dfrac{R_{j+1} - x}{V_0} + \alpha_1 (R_{j+1} - x) = \bar{\alpha}_1 (R_{j+1} - x), \\ C_c(x,\theta_i) = \lambda_t \dfrac{x\theta_i}{V_0} + \alpha_1 x\theta_i = \bar{\alpha}_1 \theta_i x, \\ C_{c'}(x,\varphi_i - \theta_i) = \lambda_t \dfrac{x\left(\varphi_i - \theta_i\right)}{V_0} + \alpha_1 x \left(\varphi_i - \theta_i\right) = \bar{\alpha}_1 \left(\varphi_i - \theta_i\right) x, \end{cases} \tag{1}$$

where R_j is the radius of the *j*th ring road, and V_0 is the travel speed of vehicles on minor roads inside the sector area, which is assumed to be a constant due to a congestion-free assumption. λ_t is the value of travel time, which is used to convert the travel time into equivalent monetary units. α_1 is the marginal cost per unit of length (e.g., fuel cost and vehicle maintenance cost) on minor roads. The term with α_1 as coefficient in each equation represents the monetary cost, which is assumed to be a linear function of the length of each road segment, as assumed in Wang et al. (2004), Liu et al. (2009) and Li et al. (2013).

Segment d is on ring road j with congestion effects, meaning that as the traffic volume on the segment increases, the travel cost increases. Let $C_{d,j}(R_j,\theta_i)$ be the travel cost on segment d. It consists of travel time cost and monetary cost, given as

$$C_{d,j}(R_j,\theta_i) = \lambda_t T_{\mathrm{RIN}}(R_j,\theta_i) + \alpha_2 R_j \theta_i, \tag{2}$$

where $T_{\mathrm{RIN}}(R_j,\theta_i)$ is the travel time on segment d of ring major road *j* from location (R_j,θ_i) to radial major road *i*. α_2 is the marginal cost per unit of length on major roads. α_2 is assumed to be smaller than α_1 due to a faster free-flow travel speed on major road than on minor road. $R_j\theta_i$ is the length of segment d.

Travel time $T_{\mathrm{RIN}}(R_j,\theta_i)$ in Eq. (2) is related to the traffic congestion level on ring road j. In order to define $T_{\mathrm{RIN}}(R_j,\theta_i)$, one has first to define the unit travel time on the ring road. Let $t\left(Q(R_j,\theta\right)$ be the travel time per unit of length at location (R_j,θ) of ring road j, where $Q(R_j,\theta)$ is the traffic volume (or total number of vehicles) at location (R_j,θ), which will be defined later as an endogenous variable. The flow-dependent travel time $t\left(Q(R_j,\theta\right)$ can be estimated using the following Bureau of Public Roads (BPR) function:

$$t\left(Q(R_j,\theta)\right)=\frac{1}{V_{\mathrm{RIN}}}\left(1.0+\sigma_1\left(\frac{Q(R_j,\theta)}{K_{\mathrm{RIN}}}\right)^{\sigma_2}\right), \tag{3}$$

where V_{RIN} and K_{RIN} are, respectively, the free-flow travel speed and capacity of ring major road, which are assumed to be constants in this paper. σ_1 and σ_2 are positive parameters.

Consequently, the travel time of commuters, $T_{\mathrm{RIN}}(R_j,\theta_i)$, on segment d with congestion effects is an integral of the unit travel time with regard to $\theta\in[0,\theta_i]$ (i.e., from location (R_j,θ_i) to radial major road *i*), represented as

$$T_{\mathrm{RIN}}(R_j,\theta_i)=\int_0^{\theta_i} t\left(Q(R_j,\theta)\right)R_j d\theta, \tag{4}$$

where $t\left(Q(R_j,\theta)\right)$ is given by Eq. (3).

Based on Eqs. (2)-(4), one can define the travel cost on segment e (denoted as $C_{e,j+1}(R_{j+1},\theta_i)$) through replacing R_j in $C_{d,j}(R_j,\theta_i)$ by R_{j+1}. Similarly, the travel cost on segment d', denoted as $C_{d',j}(R_j,\varphi_i-\theta_i)$, can be determined via replacing θ_i in $C_{d,j}(R_j,\theta_i)$ by $\varphi_i-\theta_i$. The travel cost on segment e', denoted as $C_{e',j+1}(R_{j+1},\varphi_i-\theta_i)$, can be defined through replacing θ_i in $C_{e,j+1}(R_{j+1},\theta_i)$ by $\varphi_i-\theta_i$. For more details, please see Appendix A.

In addition, segment g is part of radial major road *i* with congestion effects. The travel cost on segment g, denoted as $C_{g,i}(x)$, can be defined as

$$C_{g,i}(x)=\lambda_t T_{RAD,i}(x)+\alpha_2 x, \tag{5}$$

where $T_{\mathrm{RAD},i}(x)$ is the travel time on segment g of radial major road *i*, expressed as

$$T_{\mathrm{RAD},i}(x)=\int_0^{x} t\left(Q_i(w)\right)dw, \tag{6}$$

where $t\left(Q_i(x)\right)$ is the travel time per unit of length on radial major road *i*, and $Q_i(x)$ is the traffic volume at location *x* of radial major road *i*, defined later as an endogenous variable. The flow-dependent travel time $t\left(Q_i(x)\right)$ can be calculated by

$$t\left(Q_i(w)\right)=\frac{1}{V_{\mathrm{RAD}}}\left(1.0+\sigma_1\left(\frac{Q_i(w)}{K_{\mathrm{RAD},i}}\right)^{\sigma_2}\right), \tag{7}$$

where V_{RAD} is the free-flow travel speed of vehicles on radial major road, which is assumed to be a constant, and $K_{\text{RAD},i}$ is the capacity of radial major road *i*.

Using the same method, one can define the travel costs on segments f and h as $C_{f,i}(R_j)$ and $C_{h,i}(R_{j+1})$, and travel costs on segments f', g' and h', as $C_{f',i+1}(R_j)$, $C_{g',i+1}(x)$ and $C_{h',i+1}(R_{j+1})$. Please also see Appendix A for the details of their definitions.

Based on the above definitions of the travel costs on road segments, one can further define the travel costs on six alternative routes shown in Fig. 3, given as

$$\begin{cases} \psi_1\left(x,\theta_i\right) = C_a(x,\theta_i) + C_{d,j}(R_j,\theta_i) + C_{f,i}(R_j) + \alpha_0 + \sum_{k=1}^{j}\tau_k, \\ \psi_2\left(x,\theta_i\right) = C_b(x,\theta_i) + C_{e,j+1}(R_{j+1},\theta_i) + C_{h,i}(R_{j+1}) + \alpha_0 + \sum_{k=1}^{j+1}\tau_k, \\ \psi_3\left(x,\theta_i\right) = C_c(x,\theta_i) + C_{g,i}(x) + \alpha_0 + \sum_{k=1}^{j}\tau_k, \\ \psi_4\left(x,\varphi_i-\theta_i\right) = C_a(x,\varphi_i-\theta_i) + C_{d',j}(R_j,\varphi_i-\theta_i) + C_{f',i+1}(R_j) + \alpha_0 + \sum_{k=1}^{j}\tau_k, \\ \psi_5\left(x,\varphi_i-\theta_i\right) = C_b(x,\varphi_i-\theta_i) + C_{e',j+1}(R_{j+1},\varphi_i-\theta_i) + C_{h',i+1}(R_{j+1}) + \alpha_0 + \sum_{k=1}^{j+1}\tau_k, \\ \psi_6\left(x,\varphi_i-\theta_i\right) = C_{c'}(x,\varphi_i-\theta_i) + C_{g',i+1}(x) + \alpha_0 + \sum_{k=1}^{j}\tau_k, \end{cases} \tag{8}$$

where α_0 is the fixed component of route travel cost (e.g., parking charges at the CBD area per work trip). τ_k is the cordon toll (if any) for passing through ring road *k*. The commuters using routes 1, 3, 4, and 6 traverse the first *j* cordon ring roads, and thus the total cordon toll to be paid is $\sum_{k=1}^{j}\tau_k$. The commuters using routes 2 and 5 pass through the first *j*+1 cordon ring roads, leading to a total cordon toll of $\sum_{k=1}^{j+1}\tau_k$.

We conduct the comparative static analyses of the route travel costs with regard to angle θ_i and distance *x*, and the results are shown in Table A1 of Appendix A.

Proposition 1. For a given sector area $\Theta_{(i,i+1;j,j+1)}$, as θ_i increases, the travel costs of routes

1, 2 and 3 increase, whereas the travel costs of routes 4, 5 and 6 decrease. As x increases, the travel costs of routes 1, 3, 4 and 6 increase, whereas the travel costs of routes 2 and 5 decrease.

Proposition 1 presents the properties of the route travel costs between any trip location and the CBD. In the following section, we will further look at the market boundary of each alternative route in the ring-radial city.

4.1.2. Market area boundary

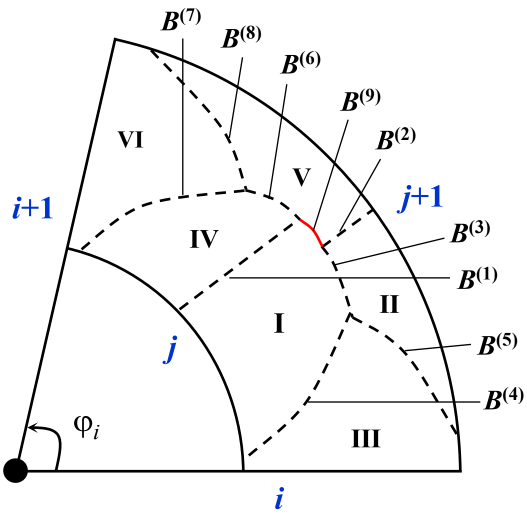
According to **A5**, all commuters choose the route with minimum travel cost for their journeys. Let $\psi_{\min}(x,\theta_i)$ be the minimum travel cost from location (x,θ_i) to the CBD. This means

$$\psi_{\min}(x,\theta_i) = \min\left\{\psi_l(x,\theta_i), l = 1,2,3,4,5,6\right\}. \tag{9}$$

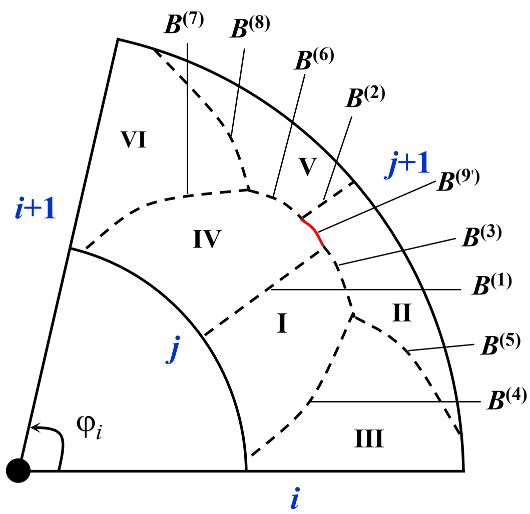
Consider a sector area, e.g., $\Theta_{(i,i+1;j,j+1)}$, there are four major roads, i.e., radial major roads *i* and *i*+1, and ring major roads *j* and *j*+1. Any two of the four major roads compete for travel demand. As a result, there is a watershed boundary (also referred to as market area boundary) between any two major roads that divides the area between those two major roads into two sub-areas, as shown in Fig. 4. Specifically, the competition between radial road *i* and ring road *j* leads to boundary contour $B^{(4)}$, which divides the travel demand in the area $\Theta_{(i,i+1;j,j+1)}$ into those who use radial road *i* and those who use ring road *j*. $B^{(5)}$ is the boundary contour, which divides the travel demand in the area $\Theta_{(i,i+1;j,j+1)}$ into those who use radial road *i* and those who use ring road *j*+1. Similarly, $B^{(7)}$ and $B^{(8)}$ are the boundary contours between radial road *i*+1 and ring road *j* and between radial road *i*+1 and ring road *j*+1, respectively. In addition, the competition between radial roads *i* and *i*+1 leads to boundary contours $B^{(1)}$ and $B^{(2)}$, which divide the travel demand in the area $\Theta_{(i,i+1;j,j+1)}$ into those who use radial major road *i*+1 via ring roads *j* and *j*+1 and those who use radial major road *i* via ring roads *j* and *j*+1. It should be pointed out that $B^{(1)}$ and $B^{(2)}$ may not be in the same line, i.e., discontinuous, when the city is asymmetric. Two possible location relationships exist, i.e., $B^{(1)}$ is on the upper or lower side of $B^{(2)}$, as shown in Fig. 4a and b, respectively. As a result, the boundary contour between ring roads *j* and *j*+1 is divided into three segments, represented as $B^{(3)}$, $B^{(6)}$

and $B^{(9)}$ (or $B^{(9')}$). However, for a symmetric city, $B^{(1)}$ and $B^{(2)}$ in the sector area $\Theta_{(i,i+1;j,j+1)}$ are in a line through the city's radius, and thus the boundary contours $B^{(9)}$ and $B^{(9')}$ are reduced to one point.

As shown in Fig. 4a and b, the boundary contours $B^{(1)}$, $B^{(2)}$, …, $B^{(9)}$ (or $B^{(9')}$) divide the sector area $\Theta_{(i,i+1;j,j+1)}$ into six sub-areas, represented as Ⅰ, Ⅱ, Ⅲ, Ⅳ, Ⅴ and Ⅵ.[4] At equilibrium, the costs of traveling along different routes from any point on a boundary contour to the CBD are equal. For convenience of readers, the equilibrium conditions of all boundary contours are summarized in Table B1 of Appendix B.



(a) With $B^{(9)}$



(b) With $B^{(9')}$

Fig. 4. Route boundary contours.

The following proposition shows a property of the boundary contours $B^{(1)}$ and $B^{(2)}$. Its proof is also provided in Appendix B.

Proposition 2. Given a ring-radial city,

(1) For any sector area $\Theta_{(i,i+1;j,j+1)}$, the boundary contours $B^{(1)}_{(i,i+1;j,j+1)}$ and $B^{(2)}_{(i,i+1;j,j+1)}$ must be straight line segments in the radial directions of the ring-radial city.

(2) For two concentric sector areas $\Theta_{(i,i+1;j,j+1)}$ and $\Theta_{(i,i+1;j-1,j)}$, $B^{(1)}_{(i,i+1;j,j+1)}$ and $B^{(2)}_{(i,i+1;j-1,j)}$

[4] The boundary contours $B^{(1)}$, $B^{(2)}$, …, $B^{(9)}$ (or $B^{(9')}$) and the sub-areas I, II, III, IV, V, VI are associated with the sector area $\Theta_{(i,i+1;j,j+1)}$, consisting of radial major roads *i* and *i*+1, and ring major roads *j* and *j*+1. Thus, (*i*, *i*+1; *j*, *j*+1) should be used as the subscript of these symbols. However, herein it can be omitted without causing any confusion.

must be in the same line and they intersect with ring road j at location $(R_j, \hat{\theta}_i)$ where $\hat{\theta}_i$ is the solution of equation system $\psi_1(R_j, \hat{\theta}_i) = \psi_4(R_j, \varphi_i - \hat{\theta}_i)$ and $\psi_2(R_j, \hat{\theta}_i) = \psi_5(R_j, \varphi_i - \hat{\theta}_i)$.

According to Proposition 2, for a congestion-free and symmetric city, the boundary contours $B^{(1)}$ and $B^{(2)}$ are just the angular bisector of the sector area $\Theta_{(i,i+1;j,j+1)}$. The following proposition further shows the property of the travel costs of the ring roads. Its proof is given in Appendix C.

Proposition 3. For a given ring road *j*, the solution $\hat{\theta}_i$ of equation $\psi_1(R_j, \hat{\theta}_i) = \psi_4(R_j, \varphi_i - \hat{\theta}_i)$ or $\psi_2(R_j, \hat{\theta}_i) = \psi_5(R_j, \varphi_i - \hat{\theta}_i)$ must satisfy

$$\sum_{i=1}^{M_{\text{RAD}}} C_{d,j}(R_j, \hat{\theta}_i) = \sum_{i=1}^{M_{\text{RAD}}} C_{d',j}(R_j, \varphi_i - \hat{\theta}_i), \tag{10}$$

where the left side is the sum of the travel costs on segments d of all ring roads $j = 1, 2, ..., M_{\text{RAD}}$ (i.e., all the circumferences $\hat{\theta}_i R_j$ from $(R_j, \hat{\theta}_i)$ to radial major road *i*). The right side is the sum of the travel costs on segments d' of all ring roads $j = 1, 2, ..., M_{\text{RAD}}$ (i.e., all the circumferences $(\varphi_i - \hat{\theta}_i) R_j$ from $(R_j, \hat{\theta}_i)$ to radial major road *i*+1).

Proposition 3 shows that for a given ring road, the travel costs on different-directional segments d and d' of that ring road are interdependent through Eq. (10). It should be pointed out that if the traffic congestion effects on the ring roads can be ignored, then the solution $\hat{\theta}_i$ satisfies Eq. (11):

$$\sum_{i=1}^{M_{\text{RAD}}} \hat{\theta}_i = \pi, \tag{11}$$

which means that the traffic volumes on different radial major roads are interdependent, i.e., an increase in the traffic volume on one radial major road causes a decrease on the other radial major roads, and vice versa.

4.1.3. Travel demand

Define $q_0(x, \theta_i)$ as the potential hourly density of travel demand at location (x, θ_i), measured in number of commuters per unit of area. Let η be the average number of daily trips to the CBD per household, and ξ be the peak-hour factor (i.e., the ratio of peak-hour flow to daily flow), which is used to convert the travel demand from a daily basis to an hourly basis (Li et al., 2013; Li and Guo, 2017). $q_0(x,\theta_i)$ can be given as

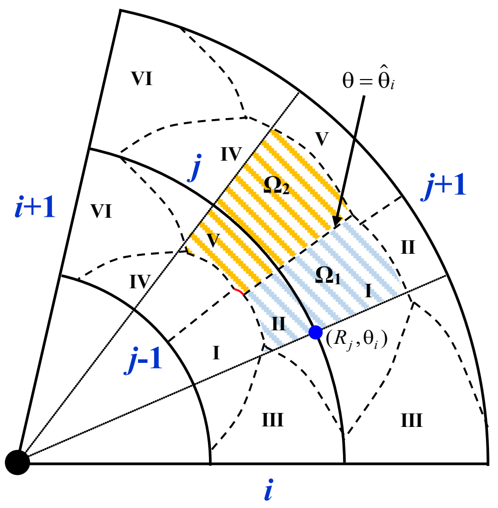
$$q_0(x,\theta_i) = \xi\eta n(x,\theta_i), \tag{12}$$

where $n(x,\theta_i)$ is the household residential density at location (x,θ_i), defined later.

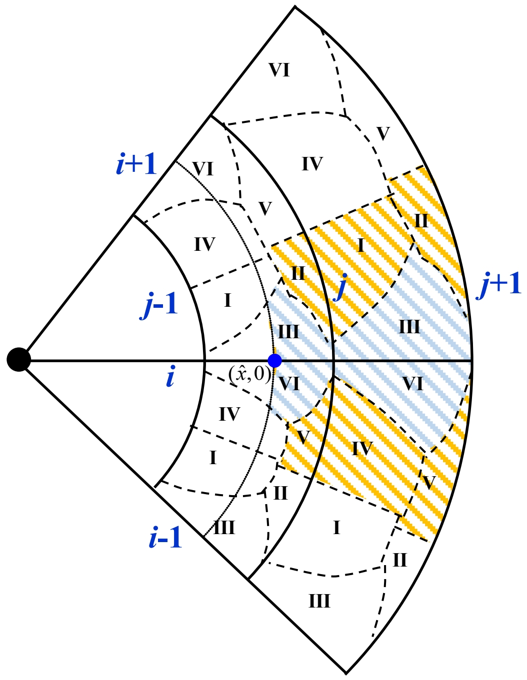
As previously stated, the travel demand is sensitive to the travel cost, and thus is elastic. From **A4**, an exponential elastic demand density function is adopted to capture the effects of the travel demand elasticity. Let $q(x,\theta_i)$ be the actual travel demand density, defined as

$$q(x,\theta_i) = q_0(x,\theta_i)\exp\left(-\omega\psi_{\min}(x,\theta_i)\right), \tag{13}$$

where travel cost $\psi_{\min}(x,\theta_i)$ can be calculated by Eq. (9), and ω is a parameter for measuring the sensitivity of the demand density to the travel cost.



(a) Location (R_j,θ_i) on ring road j



(b) Location $(\hat{x},0)$ on major radial road i

Fig. 5. Catchment area of any location on ring road and major radial road.

In order to calculate the travel times on the ring/radial major roads, $t\left(Q(R_j,\theta\right)$ and $t\left(Q_i(x)\right)$, as defined in Eqs. (3) and (7), one has to determine the travel demand at any location of ring/radial major roads. Let $Q(R_j,\theta_i)$ be the travel demand at location (R_j,θ_i)

(i.e., a location on ring road j with an angle θ_i from radial major road i). According to Figs. 3 and 4, location (R_j,θ_i) may be on segment d or d'. As $\theta_i \le \hat{\theta}_i$, location (R_j,θ_i) is on segment d. From Proposition 2 and Fig. 4, the traffic volume at location (R_j,θ_i) equals the cumulative travel demand from the area encircled by $I_{(i,i+1;j,j+1)}$, $II_{(i,i+1;j-1,j)}$, and $\theta_i \le \theta \le \hat{\theta}_i$, which is denoted as area Ω_1 in Fig. 5a. Contrarily, as $\theta_i > \hat{\theta}_i$, location (R_j,θ_i) is on segment d', and the travel volume at location (R_j,θ_i) is the cumulative travel demand from the area encircled by $IV_{(i,i+1;j,j+1)}$, $V_{(i,i+1;j-1,j)}$, and $\hat{\theta}_i \le \theta \le \theta_i$, which is denoted as area Ω_2 in Fig. 5a. Consequently, $Q(R_j,\theta_i)$ can be expressed as

$$Q(R_j,\theta_i) = \begin{cases} \iint\limits_{\Omega_1=\left\{I_{(i,i+1;j,j+1)} \cup II_{(i,i+1;j-1,j)} \cap \theta_i \le \theta \le \hat{\theta}_i\right\}} q(x,\theta) d\Omega_1, \ \theta_i \le \hat{\theta}_i, \\ \iint\limits_{\Omega_2=\left\{IV_{(i,i+1;j,j+1)} \cup V_{(i,i+1;j-1,j)} \cap \hat{\theta}_i \le \theta \le \theta_i\right\}} q(x,\theta) d\Omega_2, \ \theta_i > \hat{\theta}_i. \end{cases} \tag{14}$$

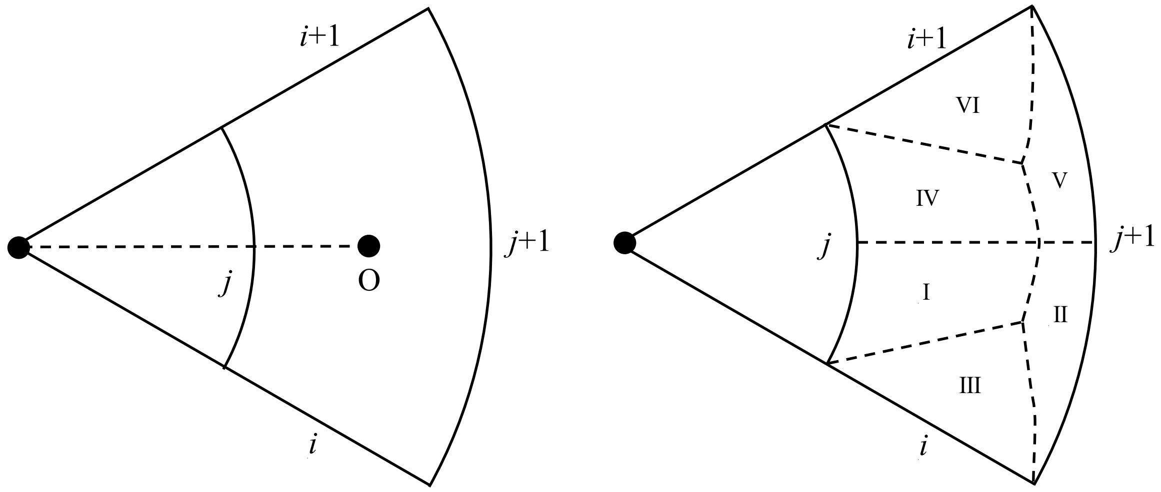
In the following, we define the travel demand at any location of a radial major road. Let $Q_i(\hat{x})$ be the travel demand of location $\hat{x}$ or $(\hat{x},0)$ ($R_{j-1} < \hat{x} \le R_j$) on radial major road i. Referring to Fig. 5b, $Q_i(\hat{x})$ consists of two components: (1) the cumulative travel demand from the catchment areas of radial major road i, namely $\bigcup_{k=j-1}^{M_{\mathrm{RIN}}} \left(III_{(i,i+1;k,k+1)} \cup VI_{(i-1,i;k,k+1)} \right)$, subject to $x > \hat{x}$, which is the light blue area shown in Fig. 5b; and (2) the cumulative travel demand entering radial major road i from all ring roads with $x > \hat{x}$ (i.e., beyond ring road j-1), with the catchment areas $\bigcup_{k=j-1}^{M_{\mathrm{RIN}}} \left(I_{(i,i+1;k+1,k+2)} \cup II_{(i,i+1;k,k+1)} \right)$ and $\bigcup_{k=j-1}^{M_{\mathrm{RIN}}} \left(IV_{(i-1,i;k+1,k+2)} \cup V_{(i-1,i;k,k+1)} \right)$, which is the yellow area shown in Fig. 5b. $Q_i(\hat{x})$ can be mathematically expressed as

$$\begin{aligned} Q_i(\hat{x}) &= \iint\limits_{\Omega=\bigcup_{k=j-1}^{M_{\mathrm{RIN}}} \left(III_{(i,i+1;k,k+1)} \cup VI_{(i-1,i;k,k+1)} \right) \cap x \ge \hat{x}} q(x,\theta) d\Omega + \iint\limits_{\Omega=\bigcup_{k=j-1}^{M_{\mathrm{RIN}}} \left\{ I_{(i,i+1;k+1,k+2)} \cup II_{(i,i+1;k,k+1)} \cup IV_{(i-1,i;k+1,k+2)} \cup V_{(i-1,i;k,k+1)} \right\}} q(x,\theta) d\Omega, R_{j-1} < \hat{x} \le R_j \\ &= \sum_{k=j-1}^{M_{\mathrm{RIN}}} \iint\limits_{\Omega=III_{(i,i+1;k,k+1)} \cup VI_{(i-1,i;k,k+1)} \cap x \ge \hat{x}} q(x,\theta) d\Omega + \sum_{k=j}^{M_{\mathrm{RIN}}} \left(Q(R_k,0) + Q(R_k,\varphi_{i-1}) \right), R_{j-1} < \hat{x} \le R_j, \end{aligned} \tag{15}$$

where $Q(R_k,0)$ is the cumulative travel demand over the catchment area $I_{(i,i+1;k+1,k+2)} \cup II_{(i,i+1;k,k+1)}$, and $Q(R_k,\varphi_{i-1})$ is the cumulative travel demand over the catchment area $IV_{(i-1,i;k+1,k+2)} \cup V_{(i-1,i;k,k+1)}$. Both can be determined by Eq. (14). The first term on the right-hand side of Eq. (15) represents the total travel demand from the catchment areas of radial major road i, and the second term is the total travel demand arriving at radial major road i along all the ring roads.

4.1.4. Comparison of proposed continuum modeling method and traditional method

In the previous subsections, we have finished the description of the disaggregate supply model of the city system using a continuum modeling method. In this subsection, we identify the gap between the proposed continuum modeling method above and the traditional (four-step) traffic simulation method. The traditional method estimates the trip generation and trip distribution through constructing a centroid connector between traffic areas, as shown in Fig. 6(a). That is to say, the centroid node of each traffic area is considered as the trip origin or destination. By contrast, the continuum modeling method proposed in this paper considers any locations in each area as the trip origin or destination.



(a) Traditional method (b) Continuum modeling method of this paper

Fig. 6. Difference between modeling methods.

For quantifying the difference of both methods, we take the area $\Theta_{(i,i+1;j,j+1)}$ as an example, as shown in Fig. 6. Let $N_{(i,i+1;j,j+1)}$ be the total number of households in this area. With the traditional modeling method, the travel distance from the centroid node O of this area to the CBD is $\frac{R_j + R_{j+1}}{2}$, and thus the total travel cost of all the commuters in this area is

$\left(\alpha_2 \frac{R_j + R_{j+1}}{2} + \lambda_t \frac{R_j + R_{j+1}}{2V_{\text{RAD}}} + \alpha_0\right) N_{(i,i+1;j,j+1)}$, where the first term in the bracket is the travel time cost and the other two terms are the monetary cost. However, the total travel cost with the proposed continuum modeling method is $\sum_{i=\text{I}}^{\text{VI}} \iint_{\Omega_i} \psi_{\min}(x,\theta) n(x,\theta) d\Omega_i$. We represent $\Delta\psi_{(i,i+1;j,j+1)}$ as the difference of the total travel costs with the two methods, and thus have

Proposition 4. The travel cost's difference $\Delta\psi_{(i,i+1;j,j+1)}$ is equal to

$$\Delta\psi_{(i,i+1;j,j+1)} = \sum_{i=\text{I}}^{\text{VI}} \iint_{\Omega_i} \psi_{\min}(x,\theta) n(x,\theta) d\Omega_i - \left(\alpha_2 \frac{R_j + R_{j+1}}{2} + \lambda_t \frac{R_j + R_{j+1}}{2V_{\text{RAD}}} + \alpha_0\right) N_{(i,i+1;j,j+1)}. \quad (16)$$

Moreover, $\Delta\psi_{(i,i+1;j,j+1)} \geq 0$ holds, meaning that the traditional modeling method would underestimate the total travel cost of the system compared to the proposed method.

The non-negativity result in Proposition 4 is due to a congestion-free assumption and a rough treatment of the trip generation in the traditional method. Its detailed proof is omitted here, but is available from authors upon request. Therefore, the traditional method can lead to a biased decision. For illustration purpose, we provide an example below.

Example 1. For simplicity, we assume that there are no congestion effects and no cordon tolls in the system. The parameters are set as follows: $R_j = 5.0$ km, $R_{j+1} = 10.0$ km, $V_0 = 20$ km/h, $V_{\text{RAD}} = 80$ km/h, $V_{\text{RIN}} = 100$ km/h, $\alpha_0 = 10$ RMB, $\alpha_1 = 0.8$ RMB/km, $\alpha_2 = 0.6$ RMB/km, $\varphi_i = 60°$, $\lambda_t = 60$ RMB/h, and $\omega = 0$. Under these parameter settings, one can determine the six sub-areas I-VI for area $\Theta_{(i,i+1;j,j+1)}$, as shown in Fig. 6(b). We further assume a uniform population distribution across the area $\Theta_{(i,i+1;j,j+1)}$ with a total size of 200,000 households. Table 3 summarizes the total travel costs and cost components of all commuters under the traditional method and the continuum modeling method proposed in this paper.

It can be seen in Table 3 that the results for sub-areas I, II and III and for sub-areas IV, V and VI are, respectively, symmetric due to the assumptions of congestion-free and uniform population distribution. In the traditional modeling method, the total travel time spent by all commuters is 18.75 thousand hours, and the total monetary cost is 2900 thousand RMB.

However, in the proposed continuum modeling method, the total travel time and total monetary cost are 33.16 thousand hours and 3301.4 thousand RMB, respectively. As a result, the total travel costs of all commuters in these two modeling methods are 4025 thousand RMB and 5291 thousand RMB, respectively. This means that the traditional modeling method underestimates the total travel cost by 1086 thousand RMB, implying a decreased accuracy of about 20.5% for the traditional method.

Table 3 Comparison of results with traditional method and the proposed method in this paper.

Modeling method	Proposed continuum modeling method							Traditional method
	I	II	III	IV	V	VI	Σ	
Total travel time (10^3 hours)	6.55	4.50	5.53	6.55	4.50	5.53	33.16	18.75
Total monetary cost (10^3 RMB)	613.3	452.1	585.3	613.3	452.1	585.3	3301.4	2900
Total travel cost (10^3 RMB)	1006.3	722.1	917.1	1006.3	722.1	917.1	5291	4025

Note: "Σ" represents the sum of I, II, III, IV, V, and VI.

4.2. Household residential location choice

According to **A3**, each household in the urban system chooses the residential location to maximize its own utility subject to a budget constraint. In this paper, a quasi-linear household utility function is adopted. We represent $U(x,\theta_i)$ as the utility of households at location (x,θ_i). The household utility maximization problem can be expressed as

$$\max_{z,g} \ U(x,\theta_i) = z(x,\theta_i) + \alpha \log g(x,\theta_i), \tag{17}$$

$$\text{s.t.} \ \ z(x,\theta_i) + p(x,\theta_i) g(x,\theta_i) = Y - E(x,\theta_i), \tag{18}$$

where $z(x,\theta_i)$ is the composite non-housing goods consumption per household at location (x,θ_i) and its price is normalized to 1. $g(x,\theta_i)$ is the consumption of housing per household at location (x,θ_i), which is measured in square meters of floor space. $p(x,\theta_i)$ is the annual housing rental price per unit of housing floor area at location (x,θ_i). Y is the annual household income, and $E(x,\theta_i)$ is the annual travel cost from location (x,θ_i) to the CBD in the ring-radial city. α is a positive parameter.

The annual travel cost $E(x,\theta_i)$ can be estimated by

$$E(x,\theta_i) = 2\rho\psi_{\min}(x,\theta_i), \tag{19}$$

where the number "2" denotes a round journey of commuters between home location (x,θ_i) and their workplace located in the CBD, and ρ is the average annual number of trips to the CBD per household. The one-way average travel cost, $\psi_{\min}(x,\theta_i)$, from location (x,θ_i) to the CBD can be determined by Eq. (9).

From the first-order optimality of maximization problem (17) and (18), one can derive

$$g(x,\theta_i)=\frac{\alpha}{p(x,\theta_i)}. \tag{20}$$

When the household residential location choice equilibrium state is reached, all households in the city have the same utility level regardless of their residential location choices. Let u be the common utility level. From Eqs. (17), (18) and (20), one can obtain

$$g(x,\theta_i,u)=\exp\left(\frac{1}{\alpha}(u-Y+E(x,\theta_i)+\alpha)\right), \tag{21}$$

$$p(x,\theta_i,u)=\alpha\exp\left(-\frac{1}{\alpha}(u-Y+E(x,\theta_i)+\alpha)\right),\text{ and} \tag{22}$$

$$z(x,\theta_i)=Y-E(x,\theta_i)-\alpha. \tag{23}$$

Eqs. (21)-(23) define the equilibrium amount of housing floor space per household at location (x,θ_i), the equilibrium housing rental price per unit of housing floor space at (x,θ_i), and the equilibrium consumption of non-housing goods per household at (x,θ_i), respectively.

4.3. Housing market equilibrium

4.3.1. Property developers' housing production behavior

We now consider the housing supply side. According to **A2**, each property developer follows a Cobb-Douglas housing production function, given as

$$h(S(x,\theta_i))=\mu\cdot S(x,\theta_i)^b, 0<b<1, \tag{24}$$

where $h(S(x,\theta_i))$ is the housing production per unit of land area at location (x,θ_i), and $S(x,\theta_i)$ is the capital investment per unit of land area (i.e., capital investment intensity) at location (x,θ_i). μ and b are positive parameters.

In this paper, the property developers aim to maximize their own net profits by determining the capital investment intensity at any location of the urban system, in terms of **A2**. The profit maximization problem can be represented as

$$\max_{S} \ \Lambda(x,\theta_i) = p(x,\theta_i)h\left(S(x,\theta_i)\right) - \left(r(x,\theta_i) + kS(x,\theta_i)\right), \tag{25}$$

where $\Lambda(x,\theta_i)$ is the net profit per unit of housing floor area at location (x,θ_i), $r(x,\theta_i)$ is the rental price per unit of land area at location (x,θ_i), and k is the interest rate of capital. The housing rental price $p(x,\theta_i)$ can be given by Eq. (22). The first term on the right-hand side of Eq. (25) is the total revenue from housing rents, and the last two terms are the land rent cost and the capital cost, respectively.

From the first-order optimality condition of maximization problem (25), one obtains

$$S(x,\theta_i,u) = \left(\alpha\mu b k^{-1} \exp\left(-\frac{1}{\alpha}\left(u - Y + E(x,\theta_i) + \alpha\right)\right)\right)^{\frac{1}{1-b}}. \tag{26}$$

Under perfect competition, each property developer earns zero profit and we thus have

$$r(x,\theta_i) = \mu p(x,\theta_i)\left(S(x,\theta_i)\right)^b - kS(x,\theta_i). \tag{27}$$

Substituting Eqs. (22) and (26) into Eq. (27), one obtains

$$r(x,\theta_i,u) = k\left(\frac{1}{b} - 1\right)\left(\alpha\mu b k^{-1} \exp\left(-\frac{1}{\alpha}\left(u - Y + E(x,\theta_i) + \alpha\right)\right)\right)^{\frac{1}{1-b}}. \tag{28}$$

Eq. (28) shows that given the utility level u, the land rental price monotonically decreases with an increase of the commuting cost or the interest rate, and vice versa.

4.3.2. Housing demand-supply equilibrium

When the housing market equilibrium is reached, the total amount of housing supply equals the total housing demand, expressed as

$$h(S(x,\theta_i,u)) = n(x,\theta_i)g(x,\theta_i,u), \tag{29}$$

where $n(x,\theta_i)$ is the household residential density at location (x,θ_i). The equilibrium amount of housing floor space per household, $g(x,\theta_i,u)$, at location (x,θ_i) is given by Eq. (21). Therefore, the household residential density, $n(x,\theta_i)$, at location (x,θ_i) can be calculated by

$$n(x,\theta_i)=\frac{h\left(S(x,\theta_i,u)\right)}{g(x,\theta_i,u)}=\left(\mu\left(\alpha bk^{-1}\right)^b\exp\left(-\frac{1}{\alpha}\left(u-Y+E(x,\theta_i)+\alpha\right)\right)\right)^{1/(1-b)}. \tag{30}$$

By **A1**, the city concerned in this paper is closed, meaning that all households are exactly inside the city boundary, i.e.,

$$\sum_{i=1}^{M_{\mathrm{RAD}}}\int_0^{\varphi_i}\int_0^{\bar{x}_i(\theta)} n(x,\theta_i)x\,dx\,d\theta=N, \tag{31}$$

where N is the total number of households in the city. $\bar{x}_i(\theta)$ is the distance from the city boundary to the CBD along radial major road i.

On the other hand, the value of the land at/beyond the city boundary equals the agricultural rent or the opportunity cost of the land in terms of **A1**, which means

$$r(\bar{x}_i(\theta),\theta,u)=R_A. \tag{32}$$

Substituting Eq. (28) into Eq. (32) yields

$$k\left(\frac{1}{b}-1\right)\left(\alpha\mu bk^{-1}\exp\left(-\frac{1}{\alpha}\left(u-Y+E(\bar{x}_i(\theta),\theta)+\alpha\right)\right)\right)^{\frac{1}{1-b}}=R_A\cdot \tag{33}$$

In Eqs. (31) and (33), given the household income Y, there are two unknown parameters: household utility u and city boundary $\bar{x}_i(\theta)$. Solving the system of equations (31) and (33) using Gauss-Seidel iterative method, one can obtain the values of u and $\bar{x}_i(\theta)$. The details of the solution method are omitted here, but are available from authors on request.

Note that the household income Y appears in Eqs. (18), (21), (22), (23), (26), (28), (30), (31), and (33). In order to conduct a welfare analysis, it is assumed that the total net land rent revenue and congestion toll revenue are equally redistributed to all residents of the city (see Arnott et al., 1998; Brueckner, 2007), and the ring road investment costs are borne on an equal per capita basis (see Saphores and Boarnet, 2006). Therefore, the household income Y should include the redistributed land rents and congestion tolls minus the borne ring road investment cost, defined as

$$Y=Y_0+Y_{\mathrm{rent}}+Y_{\mathrm{toll}}-Y_{\mathrm{ic}}, \tag{34}$$

where Y_0, Y_{rent}, Y_{toll} and Y_{ic} are the annual work salary, annual redistributed land rents per household, annual redistributed tolls per household and the annual borne ring road investment cost per household, respectively.

Y_{rent}, Y_{toll} and Y_{ic} are defined as

$$Y_{\mathrm{rent}} = \frac{1}{N}\left(\sum_{i=1}^{M_{RAD}} \int_0^{\varphi_i} \int_0^{\bar{x}(\theta_i)} \left(r(x,\theta) - R_A \right) x d\theta dx \right), \tag{35}$$

$$Y_{\mathrm{toll}} = \frac{1}{N}\left(\frac{2\rho}{\eta} \sum_{i=1}^{M_{\mathrm{RAD}}} \sum_{j=1}^{M_{\mathrm{RIN}}} \tau_j \frac{Q_i(R_j)}{\xi} \right), \text{ and} \tag{36}$$

$$Y_{\mathrm{ic}} = \frac{\zeta}{N} \sum_{j=1}^{M_{\mathrm{RIN}}} \Phi_j , \tag{37}$$

where Eq. (35) defines the revenue from the redistributed net land rents per household. In Eq. (36), $(\tau_1, \tau_2, ..., \tau_{M_{\mathrm{RIN}}})$ is the vector of toll levels on all cordons, and $2\rho/\eta$ is the total number of round commuting journeys between home location and the CBD per household per year. $Q_i(R_j)$ is the hourly travel demand passing through ring road j along radial road i, which can be determined by Eq. (15). $Q_i(R_j)/\xi$ is the daily travel demand. In Eq. (37), ζ is a parameter used to convert the total investment cost of all ring roads into average annual cost. Φ_j is the investment cost of ring road j, which is defined as a function of ring road j's length $2\pi R_j$ and capacity K_j (see Yang and Meng, 2000), expressed as

$$\Phi_j = 2\delta\pi R_j K_j , \tag{38}$$

where δ is a positive constant. Eq. (38) indicates that the investment cost of a ring road is proportional to the length and width of that ring road.

Till now, we have formulated the supply side model, the demand side model, and the demand-supply equilibrium model. They provide a useful tool for analyzing the change of urban system performance with a policy made by the authorities.

5. Optimal ring road locations and cordon tolling schemes

In this section, we explore how to determine the optimal concentric ring road locations and the optimal cordon tolling schemes in terms of the cordon locations and toll levels. As previously stated, ring roads have been introduced in many large cities in the world, such as Houston and Chengdu. The ring roads constructed in these cities can serve as candidate cordon locations, because they provide convenient conditions for implementation of cordon tolling schemes. Although such schemes are not implemented in mainland China yet, some local governments are planning to introduce them in near future, such as Beijing, Shanghai, Shenzhen, and Chengdu, as mentioned before.

In this paper, two decision mechanisms are considered for the ring road investment and cordon tolling: one is two-stage (or two-step) decision, and the other is simultaneous decision. In the two-stage model, the first stage is to determine the number and locations of ring roads to be invested in a city. At this moment, the fact that the infrastructure will be tolled later on is ignored. In the second stage, the optimal cordon tolling scheme (including cordon locations and toll level) is determined through considering the ring roads identified in the first stage as the candidate cordon locations. Such a two-stage decision process reflects that the demand-side management lags behind the supply-side investment, which is often the case in reality. In this paper, the government with such a two-stage decision process is called "a short-sighted government". By contrast, a simultaneous decision of the ring road locations and cordon tolling schemes jointly considers both the supply-side and demand-side decisions. Such a decision process can be modeled as the behavior of "a far-sighted government". In the following, we in turn formulate the decision models for the short-sighted and far-sighted governments.

5.1. A short-sighted government

A short-sighted government chooses the ring road locations and the cordon tolling schemes to maximize the social welfare of the urban system in a sequential or two-stage Stackelberg way, in which the number and locations of ring roads first are determined, and then the cordon locations and toll levels are chosen. The social welfare is the total benefits of all stakeholders in the urban system, which is the sum of the total utility of all households in the city, the aggregate net land rent and the total toll revenue minus the total ring road investment cost. As stated before, the net land rents, congestion tolls and the ring road investment cost are uniformly redistributed to the city's households (see Eq. (34)), and thus the social welfare is equal to the total utility of all households in the city (i.e., consumer surplus). The first-stage model for the social welfare maximum problem is thus formulated as

$$\max \ SW\left(M_{\mathrm{RIN}}, R_1, ..., R_{M_{\mathrm{RIN}}}\right) = uN \, , \tag{39}$$

where the decision variables include the number M_{RIN} and locations $\left(R_1, R_2, ..., R_{M_{\mathrm{RIN}}}\right)$ of the ring roads.

Given the number and locations of the ring roads determined in the first stage, the second stage of the model is to determine the cordon locations and toll levels, where the cordon locations are chosen from the set of the locations of all the ring roads. The second-stage problem can be formulated as

$$\max \ SW(\tau_1,\tau_2,...,\tau_{M_{\mathrm{RIN}}}) = uN, \tag{40}$$

where $(\tau_1,\tau_2,...,\tau_{M_{\mathrm{RIN}}})$ is the vector of toll levels on all cordons.

It should be pointed out that as a byproduct, the second-stage problem (40) can be used to examine the optimal solutions of multi-cordon vs. single-cordon tolling schemes through imposing some constraints on the toll level vector $(\tau_1,\tau_2,...,\tau_{M_{\mathrm{RIN}}})$. Specifically, as there are no constraints on $(\tau_1,\tau_2,...,\tau_{M_{\mathrm{RIN}}})$, it leads to a multi-cordon tolling solution. As only one τ_j is not zero (while all others τ_j are zero), a single-cordon tolling solution is incurred. In the next section, we will compare the solutions of the multi-cordon vs. single-cordon tolling schemes.

5.2. A far-sighted government

Different from a short-sighted government that sequentially determines the ring road investment and cordon tolling schemes, a far-sighted government will optimize these variables simultaneously. The simultaneous optimization model for the far-sighted government is expressed as

$$\max \ SW\left(M_{\mathrm{RIN}}, R_1,...,R_{M_{\mathrm{RIN}}}, \tau_1,\tau_2,...,\tau_{M_{\mathrm{RIN}}}\right) = uN. \tag{41}$$

The decision variables of the above model include the number and locations of the ring roads, and cordon locations and toll levels. Again, one can compare the optimal solutions of the multi-cordon vs. single-cordon tolling schemes through imposing the constraints that only one cordon toll is not zero and all others are zero.

The models (39)-(41) are mixed integer programming problems, and are difficult to solve because they are usually non-linear and non-convex. In this paper, the Hooke-Jeeves approach, as a multidimensional search procedure, is applied to solve these models. An advantage of this approach is that it does not require explicit knowledge of the derivative information of objective functions (39)-(41). The basic idea behind the approach is to carry out two kinds of searches sequentially: an exploratory search and a pattern search. Such a solution approach can converge to the optimal solution of the models. For more details of the Hooke-Jeeves approach, readers can refer to Bazaraa et al. (2006).

6. A case study: Application to Chengdu

In this section, a case study is provided to illustrate the applications of the proposed models and the contributions of this paper. The case study aims to: (i) ascertain the effects of the ring road investment and the cordon tolling schemes on the urban system with a given number of radial major roads, (ii) identify the differences between the short-sighted and far-sighted decisions, and (iii) examine the effects of ignoring household residential relocation behavior. To do so, some performance measures are defined as follows:

$$\text{City size or area} = \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} x dx d\theta, \tag{42a}$$

$$\text{Average household residential density} = N / \text{city area}, \tag{42b}$$

$$\text{Average housing space per family} = \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} g(x,\theta) n(x,\theta) x dx d\theta \Big/ N, \tag{42c}$$

$$\text{Average housing price} = \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} p(x,\theta) h(S(x,\theta)) x dx d\theta \Big/ \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} h(S(x,\theta)) x dx d\theta, \tag{42d}$$

$$\text{Average land value} = \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} r(x,\theta) x dx d\theta \Big/ \text{city area}, \tag{42e}$$

$$\text{Average capital investment intensity} = \sum_{i=1}^{M_{\mathrm{RAD}}} \int_0^{\varphi_i} \int_0^{\bar{x}_i(\theta)} S(x,\theta) x dx d\theta \Big/ \text{city area}. \tag{42f}$$

Applying the first-best toll scheme to a city can yield the social optimal solution in terms of social welfare. The social-optimal or first-best toll scheme can hardly be implemented in reality due to its continuous change over space and/or time (causing high technical requirement and low public acceptability). It can provide an upper bound of urban system performance, and can thus serve as a benchmark for evaluating the efficacy of the ring road investment. At first-best toll scheme, the road tolls equal the congestion externalities that an additional auto trip imposes on the existing vehicles in the system. By **A5** and marginal cost pricing principle, the first-best toll level at any location x of radial major road i is

$$\tau_i(x) = \lambda_t \int_0^x \frac{\partial t_i(\cdot)}{\partial Q_i(w)} Q_i(w) dw = \lambda_t \frac{\sigma_1 \sigma_2}{V_{\mathrm{RAD}}} \int_0^x \left(\frac{Q_i(w)}{K_{\mathrm{RAD},i}} \right)^{\sigma_2 - 1} \frac{Q_i(w)}{K_{\mathrm{RAD},i}} dw. \tag{43}$$

Eq. (43) represents the marginal effects of an additional auto trip at location x of radial major road i on the total travel cost of all auto users between location x on that radial major road and the CBD.

6.1. Parameter specifications

In this case study, Chengdu network, as shown in Fig. 1c, is adopted. Chengdu, as the capital city of Sichuan province, is located at the West China. Chengdu network can be simplified as an asymmetric urban system with five radial major roads connecting the CBD and the city boundary, as shown in Fig. 7.[5] The angles among these five radial major roads are 65°, 85°, 35°, 65° and 110°, with capacities of 20000, 23000, 20000, 21000 and 25000 vehicles per hour, respectively. Without loss of generality, it is assumed that the capacities of all ring roads are identical, with 12000 vehicles per hour. Given the five radial major roads, we employ the proposed models in the previous sections to determine the optimal ring road investment scheme and the optimal cordon tolling scheme with ring roads as candidate cordons.

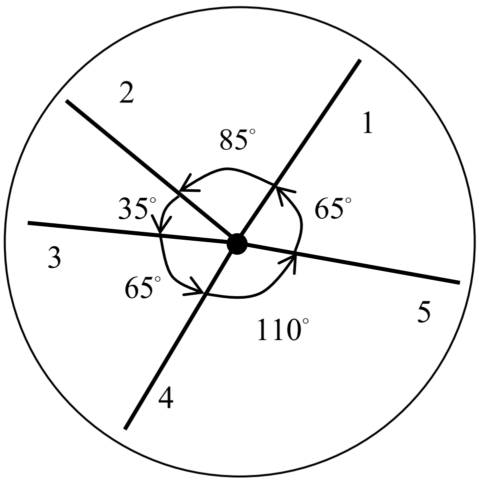


Fig. 7. Chengdu road network with five radial major roads.

The total number of households N in the urban area is about 3.5 million. The annual income per household Y is about RMB100,000. The parameter α in the household's utility function (17) is set as RMB20,000, implying that the annual housing consumption accounts for 20% of the annual income. This is currently the case in many large Chinese cities, such as Beijing, Shanghai, and Chengdu.[6] The agricultural rent R_A at the city boundary is set as RMB300,000 per square kilometer.

[5] https://map.baidu.com/@11585451,3556256.75,12z.
[6] https://research.ke.com/121/ArticleDetail?id=480.

Table 4 Input parameters for the numerical illustration.

Parameter	Definition	Baseline value
σ_1, σ_2	Parameters of BPR function	0.15, 4.0
α_0	Fixed component of monetary travel cost (RMB)	10
α_1, α_2	Variable component of monetary travel cost (RMB/veh-km)	0.8, 0.6
k	Interest rate	5%
N	Total number of households in the city	3,500,000
R_A	Agricultural rent at the city boundary (RMB/km^2)	300,000
V_{RAD}	Free-flow travel speed on radial major roads (km/h)	1/80
V_{RIN}	Free-flow travel speed on ring roads (km/h)	1/100
V_0	Free-flow travel speed on minor roads (km/h)	1/20
Y_0	Annual household income (RMB/year)	100,000
α	Parameter in household's utility function (RMB/year)	20,000
b, μ	Parameters in housing production function	0.75, 6.0×10^{-3}
λ_t	Value of travel time (RMB/h)	60
η	Average daily number of trips to the CBD per household	1.0
ρ	Average annual number of trips to the CBD per household	365
ξ	Peak-hour factor	10%
δ	A positive parameter in road construction cost function	3.5×10^{3}
ω	A parameter that reflects demand sensitivity to travel cost	3×10^{-3}
ζ	A parameter for converting total construction cost into annual cost	1/30

Note: The data are mainly from Li and Guo (2017) and Li and Wang (2018).

The free-flow travel speeds on the ring roads, radial major roads and minor roads, V_{RIN}, V_{RAD}, and V_0, are 100, 80 and 20 km per hour, respectively. The value of travel time λ_t is RMB60 per hour. The average daily number of trips to the CBD per household, η, is assumed to be 1.0, and thus the total annual number of trips to the CBD per household, ρ, is 365. The peak-hour factor, ξ, is set as 10%. The interest rate k is set as 5%. The parameters, σ_1 and σ_2, in the BPR travel time functions (3) and (7) are 0.15 and 4.0, respectively. The fixed component of the monetary travel cost, α_0, is RMB10 per trip, and the variable components, α_1 and α_2, of the monetary travel cost on the minor roads and the major roads are RMB0.8 and RMB0.6 per km, respectively. The parameters ω in the travel demand function (13) is 0.003. The parameters δ in the road investment cost function (38) is 3500. The parameter ζ, used to convert the total investment cost of all ring roads into average annual cost, is 1/30. The parameters b and μ in the housing production function (24) are set as

0.75 and 6.0×10^{-3}, respectively. The baseline values of the model input parameters are summarized in Table 4. In the following analyses, unless specifically stated otherwise, the input data are identical to the baseline values. The computer program codes in Java language for the numerical study are available from the authors on request.

6.2. Comparison of models' solutions

Fig. 8 shows the changes of the social welfare with the number of ring roads under the short-sighted and far-sighted decision mechanisms (two-stage vs. simultaneous decisions). It can be seen that the social welfare curve with the far-sighted model is always above that with the short-sighted model, meaning that the far-sighted decision would be superior to the short-sighted decision, in terms of the social welfare. As the number of the ring roads increases, the social welfare first increases and then decreases regardless of the decision mechanisms adopted. The maximum social welfare, respectively, occurs at 3 and 5 ring roads for the short-sighted and far-sighted decision mechanisms, causing social welfares of 373.126 and 374.34 billion RMB per year.

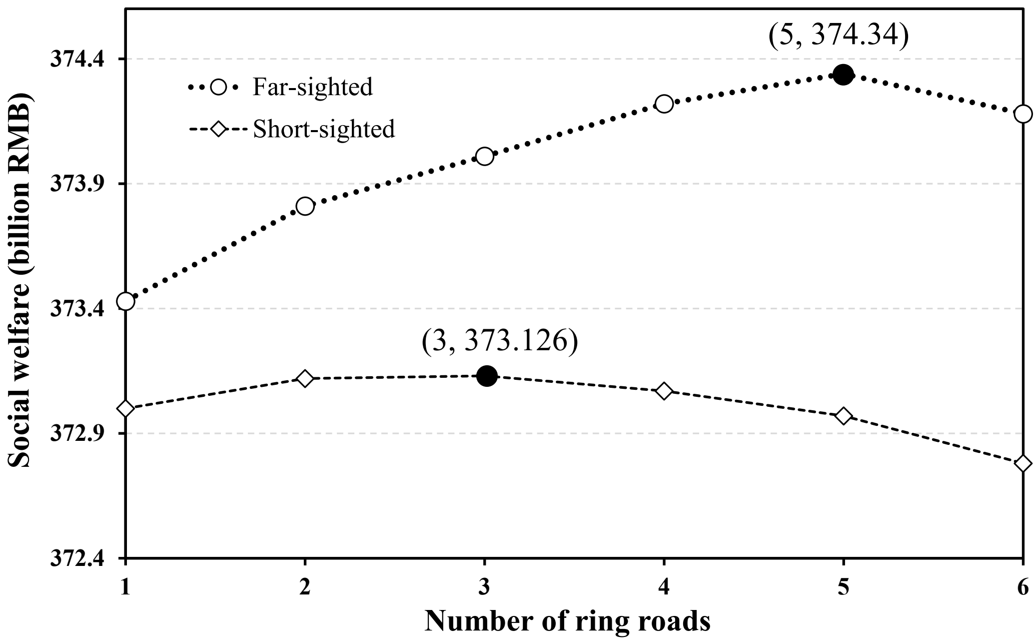


Fig. 8. Changes of social welfare with number of ring roads under short-sighted and far-sighted decisions.

Table 5 further summarizes the solutions of the models under different scenarios. It can be seen that for the short-sighted (or two-stage) decision, the optimal locations of 3 ring roads to be introduced in Stage I (i.e., ring road investment stage) are 8.8, 11.3 and 14.6 km from the CBD, respectively. Given the 3 ring roads generated in Stage I as candidate cordon locations, the optimal cordon tolls in Stage II (i.e., cordon charging stage) are RMB3.86, RMB0.25 and RMB0.34 for the 3 ring roads, respectively. This means that the commuters living beyond the third ring road need to pay a total toll of RMB4.45 for a journey from their home locations to the CBD, while the commuters between Rings 2 and 3 and between Rings 1 and 2 have to pay tolls of RMB4.11 and RMB3.86 for their CBD trips, respectively. For the far-sighted (simultaneous) decision, the optimal locations of the 5 ring roads to be introduced are 5.8, 7.5, 9.6, 11.8 and 15.0 km from the CBD, with tolls of RMB5.09, RMB0.3, RMB0.75, RMB0.57 and RMB0.21 under the cordon tolling scheme, respectively. Thereby, the commuters living beyond the fifth ring road need to pay a total toll of RMB6.92 for their CBD commutes, while the commuters between Rings 5 and 4, Rings 4 and 3, Rings 3 and 2, and Rings 2 and 1 need to pay tolls of RMB6.71, RMB6.14, RMB5.39, RMB5.09, respectively.

Table 5 also shows that the do-nothing case (i.e., no ring road investment) leads to the lowest social welfare of RMB372.587 billion per year, whereas the first-best tolling scheme yields the highest social welfare of RMB375.565 billion per year (i.e., social optimal solution), implying a welfare gain of RMB2.978 billion per year compared to the do-nothing case. This verifies that the first-best tolling scheme is the most efficient scheme that leads to the social optimal urban system. The welfare gains of the short-sighted and far-sighted decisions are RMB1.276 billion and RMB1.753 billion per year with regard to the do-nothing case, which are 42.8% and 58.9% of the welfare gain of the first-best scheme, respectively. This means that the far-sighted decision approximates to the first-best scheme (or social optimum) more closely than the short-sighted decision.

In order to compare the efficiencies of the multi-cordon vs. single-cordon tolling schemes, the optimal solutions for the single-cordon tolling schemes under the short-sighted and far-sighted decisions are also shown in Table 5. It can be seen that for each decision mechanism, the optimal multi-cordon tolling scheme outperforms the optimal single-cordon tolling scheme in terms of the social welfare. However, the gap of the welfare gains between the single-cordon and multi-cordon tolling schemes is trivial. Specifically, the gaps of the welfare gains are, respectively, 1.0% and 2.0% for the short-sighted and far-sighted decisions. It should be mentioned that the toll collection cost is not considered here due to data unavailability. Such a cost includes the investment cost of toll-collection equipment, the installation, operating and maintenance costs of the equipment, and the tolling transaction cost (Persad et al., 2007; Odeck and Welde, 2010; Li and Guo, 2017). Therefore, when choosing the number of tolling cordons in practice, it is necessary for the authorities to tradeoff the benefit of the cordon tolling schemes and the toll collection cost.

In addition, Table 5 shows that the ring road investment leads to an expanded city, whereas congestion tolling causes a decrease in the city size. Specifically, compared to the do-nothing case, the city size under the first-best scheme decreases by 47 square kilometers (from 1285 to 1238). The city size under the two-stage short-sighted decision first increases in Stage I and then decreases in Stage II: the ring road investment in Stage I leads to an expansion in the city size by 170 square kilometers (from 1285 to 1455), due to provision of more alternative routes. However, after implementing the cordon tolling scheme in Stage II, the city size becomes 1316 square kilometers, which is 139 square kilometers (from 1455 to 1316) smaller than before cordon tolling implementation (i.e., Stage I), but 31 square kilometers (from 1285 to 1316) larger than the do-nothing case. This is because the dispersion effect caused by the ring road investment in Stage I exceeds the concentration effect caused by the cordon tolling in Stage II. Different from the two-step short-sighted decision, the far-sighted decision causes a slight decrease in the city size by 24 square kilometers (from 1285 to 1261). Again, this is a result of trade-off between city decentralization effect due to ring road investment and city concentration effect due to cordon tolling.

To sum up, the ring road investment and the cordon tolling can enhance not only the social welfare of the system, but also the household utility level under the redistributions of the net land rents and cordon tolls. A "win-win" situation takes place for the city system and residents.

Table 5 Comparison of solutions of the models under different decision mechanisms.

Performance index	Before investing in ring roads (do-nothing)	Short-sighted (two-step) decision			Far-sighted (simultaneous) decision		First-best toll scheme
		Stage I: introducing ring roads	Stage II: introducing cordon tolling	Stage II': a single-cordon tolling scheme	All ring roads as candidate cordons (multi-cordon)	A single-cordon tolling scheme	
Number of ring roads	—	3	3	3	5	5	—
Locations of ring roads / cordons (km)	—	(8.8, 11.3, 14.6)	(8.8, 11.3, 14.6)	(8.8, 11.3, 14.6)	(5.8, 7.5, 9.6, 11.8, 15.0)	(5.8, 7.7, 11.1, 13.1, 16.6)	—
Toll level (RMB)	—	—	(3.86, 0.25, 0.34)	(3.5, 0, 0)	(5.09, 0.3, 0.75, 0.57, 0.21)	(5.57, 0, 0, 0, 0)	—
City size (km^2)	1285	1455	1316	1353	1261	1385	1238
Average household residential density (households/km^2)	2723	2405	2659	2588	2775	2525	2825
Average housing rental price (RMB/m^2)	2452	2298	2433	2406	2516	2462	2516
Average housing space per family (m^2/household)	8.11	8.65	8.19	8.27	7.93	8.10	7.90
Average land value (million RMB/km^2)	13.6	12.0	13.3	12.9	13.9	12.6	14.1
Average capital investment intensity (million RMB/km^2)	817	722	790	771	818	746	847
Aggregate land rent (billion RMB/year)	17.11	17.06	17.11	17.10	17.12	17.08	17.13
Total toll revenue (billion RMB/year)	—	—	2.00	1.83	4.40	4.28	6.06
Average annual ring road investment cost (million RMB/year)	—	305	305	305	437	478	—
Household utility level (RMB/year)	106453	106607	106818	106808	106954	106937	107304
Annual social welfare (billion RMB/year)	372.587 (0%)	373.126 (18.1%)	373.863 (42.8%)	373.831 (41.8%)	374.340 (58.9%)	374.281 (56.9%)	375.565 (100%)

Note: The percentage in the last row represents the ratio of welfare gain to the first-best tolling scheme.

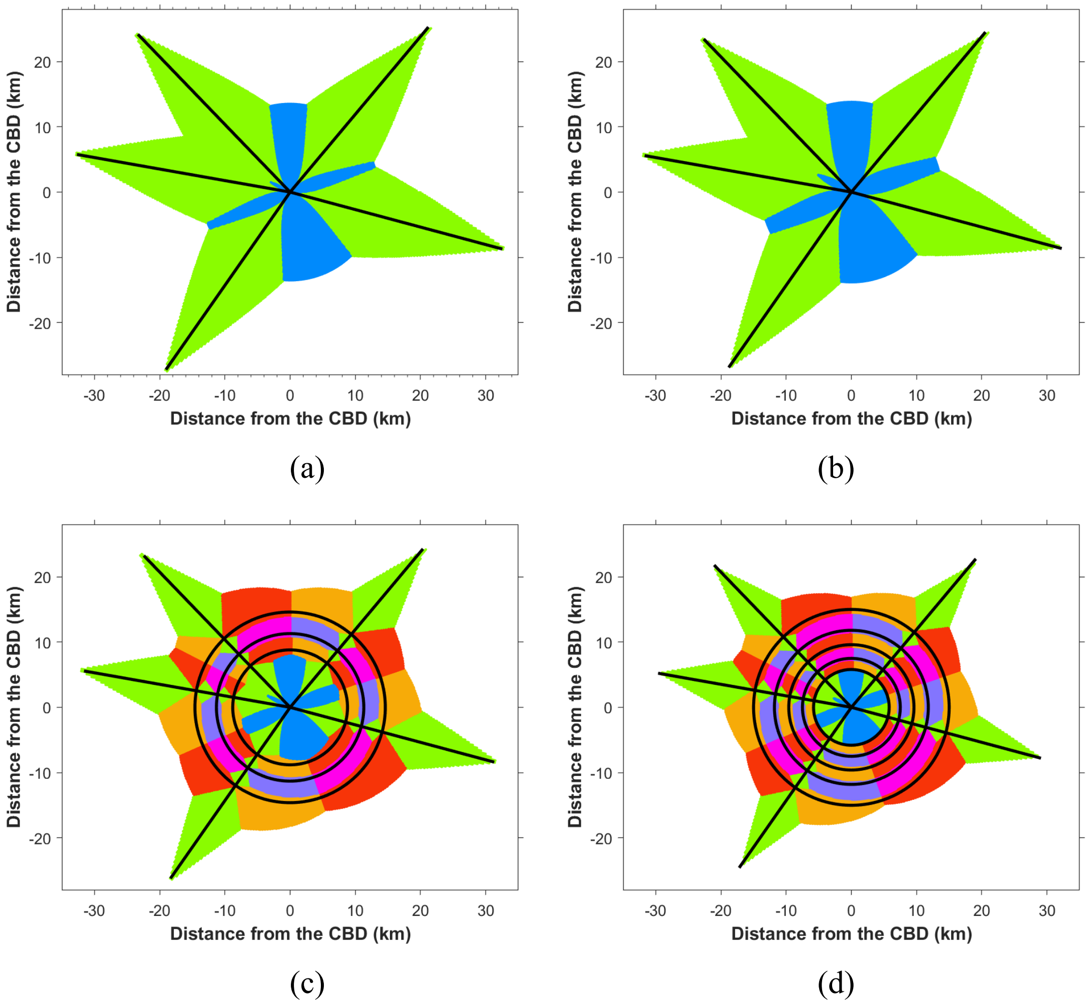


Fig. 9. Catchment areas of ring/radial roads under different cases: (a) do-nothing; (b) first-best tolling; (c) stage I of short-sighted decision (i.e., ring road investment stage); (d) far-sighted decision. Different colors represent the catchment areas of associated ring/radial road segments.

6.3. Effects of ring road investment and cordon tolling on commuters' route choices

In order to intuitively look at the effects of the ring road investment and cordon tolling on commuters' route choices, we adopted a graphical approach to show the commuters' route choices under different cases, as shown in Fig. 9a-d. These different cases include: (a) do-nothing (no ring road investment); (b) first-best tolling; (c) stage I of short-sighted decision; and (d) far-sighted decision. In Fig. 9a and b, the blue part represents a residential area in which the commuters go to the CBD through dense minor roads directly. The green part represents a residential area in which the commuters first travel along a minor circumferential road to get to the radial major road, and then proceed along the radial major road to reach the CBD. The part between any two radial major roads is divided into three areas: blue area in the middle and green areas on both sides. Comparing Fig. 9a and b, we can find that the blue area becomes bigger after introducing the first-best tolling scheme. This means that the first-best tolling scheme will lead more commuters to arrive directly at the CBD using the minor roads.

In Fig. 9c and d, various colors (e.g., red, yellow, pink, blue, green) represent the catchment areas of associated ring road or radial major road. It can be observed that after introducing the ring roads, the city boundary between any two radial major roads becomes bulge outward, and the areas of the blue and green parts within the innermost ring road significantly decrease, compared to Fig. 9a and b. Moreover, the ring spacing in Fig. 9c with three ring roads is bigger than that in Fig. 9d with five ring roads. These observations imply that the ring road investment plays an important role in reshaping the urban spatial structure.

6.4. Effects of ring road investment and cordon tolling on urban spatial structure

In order to examine the effects of ring road investment and cordon tolling on the urban spatial structure, Fig. 10a-d and a'-d' further plot the contours of the household residential density and housing price across the city under the cases of do-nothing, first-best tolling, Stage I of short-sighted decision, and far-sighted decision, respectively. It can be seen that after implementing the first-best toll, the household residential density at any location of the city becomes denser, particularly, the household residential density in the CBD increases by 5963 (from 34895 to 40858) households per square kilometer (see Fig. 10a and b). The residential density in the CBD decreases by 2474 (from 34895 to 32421) households per square kilometer after introducing three ring roads (see Fig. 10a and c), but increases by 4903 (from 34895 to 39798) households per square kilometer after introducing five ring roads and cordon tolling schemes (see Fig. 10a and d), respectively.

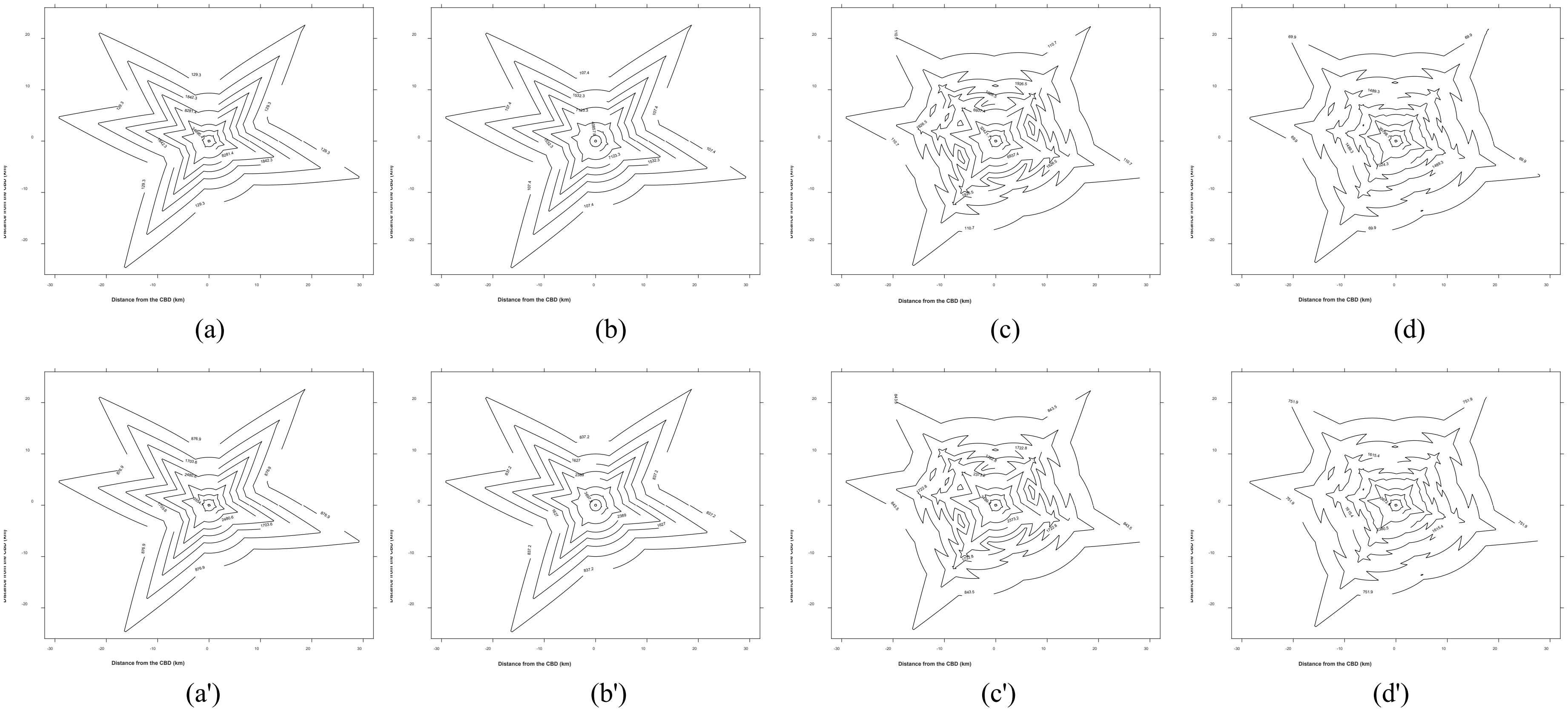


Fig. 10. (a)-(d) represent household residential density (household/m^2) under the cases of do-nothing, first-best tolling, Stage I of short-sighted decision, and far-sighted decision, respectively. (a')-(d') represent associated housing rental price (RMB/m^2).

It can also be seen that for the no ring road cases (i.e., do-nothing and first-best tolling), the household residential density curves and housing price curves are monotonically decreasing with the distance from the CBD, as shown in Fig. 10a, a', b and b'. However, after investing in the ring roads, such a monotonicity is destroyed. Specifically, the household residential density and housing price first decrease from the CBD outward, then increase nearby the ring major roads and decrease with the distance from the ring major roads outward, and reach the peaks (locally maximum) on the ring major roads, as shown in Fig. 10c, c', d and d'. These results further illustrate the effects of the ring road investment on the urban spatial structure.

6.5. Effects of ignoring household residential relocation behavior

Finally, we look at the effects of ignoring household residential relocation behavior in the model, which can be considered as a short-run case. Ignoring the household residential relocation behavior means that the urban spatial structures (including household residential distributions, housing spaces and housing prices) before and after ring road investment and cordon tolling remain unchanged. Table 6 summarizes the solutions of the models under different scenarios when the household residential relocation behavior is not considered. It can be seen that the optimal number of ring roads under both the short-sighted and far-sighted decisions is 3. The optimal locations of the three ring roads to be introduced in Stage I of the short-sighted decision are, respectively, 7.7, 10.0 and 13.0 km from the CBD, and the associated optimal cordon tolls in Stage II of the short-sighted decision are RMB0.19, 0, 0.2, respectively. For the far-sighted decision, the optimal ring road locations are 7.8, 9.8 and 12.6 km from the CBD, and the optimal cordon tolls are RMB0.15, 0.35 and 0.15, respectively.

It can also be seen that comparing Tables 5 and 6, after household residential relocation behavior is ignored, the total toll revenue significantly decreases for both the short-sighted and far-sighted decisions due to underestimating the number of suburban residents, but significantly increases for the first-best toll scheme due to overestimating the number of suburban residents. In addition, the average household utility and the social welfare decrease. This is because ignoring household relocation behavior is equivalent to adding some extra constraints in the model, including the constraints on residential location, housing and non-housing goods consumption. As a result, the feasible region of the model solution is reduced, thus leading to a sub-optimal solution.

Table 6 Solutions of models without considering household residential relocation behavior.

	Before investing in ring roads (do-nothing)	Short-sighted (two-step) decision		Far-sighted (simultaneous) decision	First-best toll scheme
		Stage I: introducing ring roads	Stage II: introducing cordon tolling		
Number of ring roads	—	3	3	3	—
Locations of ring roads / cordons (km)	—	(7.7, 10.0, 13.0)	(7.7, 10.0, 13.0)	(7.8, 9.8, 12.6)	—
Toll level (RMB)	—	—	(0.19, 0, 0.2)	(0.15, 0.35, 0.15)	—
Total toll revenue (billion RMB/year)	—	—	0.16	0.27	7.18
Average annual ring road investment cost (million RMB/year)	—	270	270	265	—
Average household utility level (RMB/year)	106453	106651	106655	106656	107134
Annual social welfare (billion RMB/year)	372.587 (0%)	373.278 (29.0%)	373.293 (29.7%)	373.297 (29.8%)	374.968 (100%)

7. Conclusion and policy implications

This paper investigated the issues of ring road investment and cordon toll pricing in a two-dimensional city. The urban system concerned includes four types of stakeholders: local authorities, property developers, households and commuters. The interdependences among these parties via some interrelated equilibria were considered: housing demand-supply equilibrium, household residential location choice equilibrium and commuters' ring-radial route choice equilibrium. Based on the urban system equilibria analysis, two models for determining the number and locations of ring roads and the cordon tolling locations and levels, a short-sighted and a far-sighted decision model, were proposed for maximizing the social welfare of urban system. In the proposed models, the household residential distribution, capital investment intensity, land values, housing prices and housing space can all be endogenously determined. The proposed models allowed for an understanding of intra-area travel that is often poorly treated in traffic simulation models. A case study of Chengdu, China was given to illustrate the applications of the proposed models in the ring road investment and cordon toll design. The effects of ignoring the household relocation behavior due to ring road investment and cordon toll pricing on the urban system were also examined.

There are some important policy implications from the findings of this study. First, the decision mechanisms have significant impacts on the optimal solutions for the ring road investment and cordon tolling schemes. For example, for the case of Chengdu China, investment in three ring roads is enough under the short-sighted decision. However, it needs to invest in five ring roads under the far-sighted decision. The far-sighted decision (i.e., simultaneous decision on ring road investment and cordon tolling) outperforms the short-sighted decision (a two-step decision) in terms of the social welfare, and more approximates the best-first tolling scheme, and thus can be used as a substitute for the best-first scheme. Second, the ring road investment and cordon tolling can reshape the urban spatial structure. Specifically, the ring road investment can lead to urban sprawl, but the cordon tolling can cause a decrease in city size. The interaction between the dispersion effects caused by the ring road investment and the concentration effects caused by the cordon tolling jointly affects the urban form. The ring road investment and cordon tolling can also lead to a win-win situation for the city system and the residents, in terms of social welfare and household utility. Third, the optimal multi-cordon tolling scheme is superior to the optimal single-cordon tolling scheme in terms of the social welfare. However, the gap of the welfare gains between the single-cordon and multi-cordon tolling schemes is not significant. In practice, it is necessary for the authorities to tradeoff the benefit of the multi-cordon tolling schemes and the cost of the toll collection system. Finally, ignoring the household residential relocation behavior in the models could lead to significantly biased decisions on the ring road investment and cordon tolling, e.g., underestimates of the average household utility, total toll revenue and the social welfare. Therefore, the authorities should take into account the externalities caused by transport infrastructure investment and congestion tolling in the decision model.

Although this paper provides a new method for modeling the interactions among land use, housing market, transportation infrastructure improvements and cordon tolling in a general city network, further extensions are necessary. First, this paper focused on auto mode, and thus ignored the competition and substitution effects between auto and public transit. To incorporate the interaction and competition between different modes, the single-mode transport system can be extended to consider a multi-modal transport system (Capozza, 1976; Anas and Moses, 1979; Li and Wang, 2018). Second, all households in this paper were assumed to be homogenous. However, an empirical study by Kwon (2003) showed that household income level may significantly vary with their residential location choices. Therefore, the proposed models should be extended to consider the behavioral difference of heterogeneous households with different income levels and demographic characteristics in a further study. Third, in this paper, only traffic congestion externality was considered and other externalities such as environmental externalities were ignored (Yin et al., 2013; Li et al., 2014; Vosough et al., 2020, 2022). Further studies can be conducted to incorporate the effects of vehicle emissions externalities in a dynamic traffic model (de Palma et al., 2005) to create an environmentally sustainable urban system. Finally, parking issue was not considered in this paper. This is an important factor influencing the transport expenses of urban residents and thus the residential location and travel choices (Arnott et al., 1991; Fosgerau and de Palma, 2013). Therefore, there is indeed a need to extend the proposed models to incorporate the parking issue.

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Appendix A: Definitions of travel costs and proof of Proposition 1

(1) Travel costs of segments e, d', e', f, h, f', g' and h'

Based on Fig. 3 and Eqs. (2)-(4), we can define the travel cost of segment e (denoted as $C_{e,j+1}(R_{j+1},\theta_i)$) through replacing R_j in $C_{d,j}(R_j,\theta_i)$ by R_{j+1}, as follows:

$$C_{e,j+1}(R_{j+1},\theta_i)=\lambda_t T_{\mathrm{RIN}}(R_{j+1},\varphi_i)+\alpha_2 R_{j+1}\theta_i\,. \tag{A.1}$$

The travel cost of segment d', denoted as $C_{d',j}(R_j,\varphi_i-\theta_i)$, can be defined by replacing θ_i in $C_{d,j}(R_j,\theta_i)$ by $\varphi_i-\theta_i$ below:

$$C_{d',j}(R_j,\varphi_i-\theta_i)=\lambda_t T_{\mathrm{RIN}}(R_j,\varphi_i-\theta_i)+\alpha_2 R_j(\varphi_i-\theta_i)\,. \tag{A.2}$$

The travel cost of segment e', denoted as $C_{e',j+1}(R_{j+1},\varphi_i-\theta_i)$, can be defined through replacing θ_i in $C_{e,j+1}(R_{j+1},\theta_i)$ by $\varphi_i-\theta_i$, given as

$$C_{e',j+1}(R_{j+1},\varphi_i-\theta_i)=\lambda_t T_{\mathrm{RIN}}(R_{j+1},\varphi_i-\theta_i)+\alpha_2 R_{j+1}(\varphi_i-\theta_i)\,. \tag{A.3}$$

Based on Fig. 3 and Eqs. (5)-(7), the travel costs of segments f and h, denoted as $C_{f,i}(R_j)$ and $C_{h,i}(R_{j+1})$, can, respectively, be defined through replacing *x* in $C_{g,i}(x)$ by R_j and R_{j+1} as

$$C_{f,i}(R_j)=\lambda_t T_{RAD,i}(R_j)+\alpha_2 R_j\,, \tag{A.4}$$

$$C_{h,i}(R_{j+1})=\lambda_t T_{RAD,i}(R_{j+1})+\alpha_2 R_{j+1}\,. \tag{A.5}$$

Similarly, the travel costs of segments f', g' and h', denoted as $C_{f',i+1}(R_j)$, $C_{g',i+1}(x)$ and $C_{h',i+1}(R_{j+1})$, can, respectively, be defined as

$$C_{f',i+1}(R_j)=\lambda_t T_{RAD,i+1}(R_j)+\alpha_2 R_j\,, \tag{A.6}$$

$$C_{g',i+1}(x)=\lambda_t T_{RAD,i+1}(x)+\alpha_2 x\,, \tag{A.7}$$

$$C_{h',i+1}(R_{j+1})=\lambda_t T_{RAD,i+1}(R_{j+1})+\alpha_2 R_{j+1}\,. \tag{A.8}$$

(2) Proposition 1: Comparative static results of route travel costs

Table A1 Comparative static analysis of route travel costs.

Route no.	Derivatives with regard to angle θ_i	Derivatives with regard to distance x from CBD
1	$\frac{\partial\psi_1(x,\theta_i)}{\partial\theta_i}=\left(\lambda_t t(Q(R_j,\theta_i)+\alpha_2\right)R_j>0$	$\frac{\partial\psi_1(x,\theta_i)}{\partial x}=\overline{\alpha}_1>0$
2	$\frac{\partial\psi_2(x,\theta_i)}{\partial\theta_i}=\left(\lambda_t t(Q(R_{j+1},\theta_i))+\alpha_2\right)R_{j+1}>0$	$\frac{\partial\psi_2(x,\theta_i)}{\partial x}=-\overline{\alpha}_1<0$
3	$\frac{\partial\psi_3(x,\theta_i)}{\partial\theta_i}=\overline{\alpha}_1 x>0$	$\frac{\partial\psi_3(x,\theta_i)}{\partial x}=\overline{\alpha}_1\theta_i+\lambda_t t(Q_i(x))+\alpha_2>0$
4	$\frac{\partial\psi_4(x,\varphi_i-\theta_i)}{\partial\theta_i}=-\left(\lambda_t t(Q(R_j,\theta_i)+\alpha_2\right)R_j<0$	$\frac{\partial\psi_4(x,\varphi_i-\theta_i)}{\partial x}=\overline{\alpha}_1>0$
5	$\frac{\partial\psi_5(x,\varphi_i-\theta_i)}{\partial\theta_i}=-\left(\lambda_t t(Q(R_{j+1},\theta_i))+\alpha_2\right)R_{j+1}<0$	$\frac{\partial\psi_5(x,\varphi_i-\theta_i)}{\partial x}=-\overline{\alpha}_1<0$
6	$\frac{\partial\psi_6(x,\varphi_i-\theta_i)}{\partial\theta_i}=-\overline{\alpha}_1 x<0$	$\frac{\partial\psi_6(x,\varphi_i-\theta_i)}{\partial x}=\overline{\alpha}_1(\varphi_i-\theta_i)+\lambda_t t(Q_{i+1}(x))+\alpha_2>0$

Appendix B: Equilibrium conditions of boundary contours and Proof of Proposition 2

(1) Equilibrium conditions of boundary contours

Table B1 Equilibrium conditions of all boundary contours in the ring-radial city.

Boundary contours	Competing routes	Equilibrium condition
$B^{(1)}$	Routes 1 and 4	$\psi_1(x,\theta_i)=\psi_4(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(1)}$
$B^{(2)}$	Routes 2 and 5	$\psi_2(x,\theta_i)=\psi_5(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(2)}$
$B^{(3)}$	Routes 1 and 2	$\psi_1(x,\theta_i)=\psi_2(x,\theta_i),\ \ (x,\theta_i)\in B^{(3)}$
$B^{(4)}$	Routes 1 and 3	$\psi_1(x,\theta_i)=\psi_3(x,\theta_i),\ \ (x,\theta_i)\in B^{(4)}$
$B^{(5)}$	Routes 2 and 3	$\psi_2(x,\theta_i)=\psi_3(x,\theta_i),\ \ (x,\theta_i)\in B^{(5)}$
$B^{(6)}$	Routes 4 and 5	$\psi_4(x,\varphi_i-\theta_i)=\psi_5(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(6)}$
$B^{(7)}$	Routes 4 and 6	$\psi_4(x,\varphi_i-\theta_i)=\psi_6(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(7)}$
$B^{(8)}$	Routes 5 and 6	$\psi_5(x,\varphi_i-\theta_i)=\psi_6(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(8)}$
$B^{(9)}$	Routes 1 and 5	$\psi_1(x,\theta_i)=\psi_5(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(9)}$
$B^{(9')}$	Routes 2 and 4	$\psi_2(x,\theta_i)=\psi_4(x,\varphi_i-\theta_i),\ \ (x,\theta_i)\in B^{(9')}$

(2) Proof of Proposition 2

We first prove that boundary contour $B^{(1)}_{(i,i+1;j,j+1)}$ must be a straight line. To do so, one needs to prove that the angle $\hat{\theta}_i$ of any location on $B^{(1)}_{(i,i+1;j,j+1)}$ is identical, or equivalently, the angle $\hat{\theta}_i$ of the intersection between $B^{(1)}_{(i,i+1;j,j+1)}$ and R_j has a sole solution. Note that for any point $(\hat{x},\hat{\theta}_i)\in B^{(1)}_{(i,i+1;j,j+1)}$, $\psi_1(\hat{x},\hat{\theta}_i)-\psi_4(\hat{x},\varphi_i-\hat{\theta}_i)=0$ holds. Combing it and Eq. (8), one can obtain

$$C_a(\hat{x},\hat{\theta}_i)+C_{d,j}(R_j,\hat{\theta}_i)+C_{f,i}(R_j)-\left(C_a(\hat{x},\varphi_i-\hat{\theta}_i)+C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)+C_{f',i+1}(R_j)\right)=0\,. \tag{B.1}$$

According to Eq. (1), $C_a(\hat{x},\hat{\theta}_i)=C_a(\hat{x},\varphi_i-\hat{\theta}_i)$ holds. Substituting it into Eq. (B.1) yields

$$C_{d,j}(R_j,\hat{\theta}_i)+C_{f,i}(R_j)-\left(C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)+C_{f',i+1}(R_j)\right)=0\,. \tag{B.2}$$

Denote $\Delta_{14}(\hat{\theta}_i)$ as the left-hand side of Eq. (B.2), i.e.,

$$\Delta_{14}(\hat{\theta}_i)=C_{d,j}(R_j,\hat{\theta}_i)+C_{f,i}(R_j)-\left(C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)+C_{f',i+1}(R_j)\right). \tag{B.3}$$

One can then derive

$$\frac{\Delta_{14}(\hat{\theta}_i)}{\partial\hat{\theta}_i} = 2\left(\lambda_t t(Q(R_j, \hat{\theta}_i) + \alpha_2 R_j\right) > 0. \tag{B.4}$$

Eq. (B.4) implies that $\Delta_{14}(\hat{\theta}_i)$ is monotonically increasing with regard to angle $\hat{\theta}_i$. In addition, $\Delta_{14}(0) < 0$ and $\Delta_{14}(\varphi_i) > 0$. According to the zero-point theorem, $\hat{\theta}_i$ in Eq. (B.2) has one root (or solution) only within the interval $[0, \varphi_i]$.

Similarly, for any point $(\tilde{x}, \tilde{\theta}_i) \in B^{(2)}_{(i,i+1;j,j+1)}$, $\psi_2(\tilde{x}, \tilde{\theta}_i) - \psi_5(\tilde{x}, \varphi_i - \tilde{\theta}_i) = 0$ holds. From Eqs. (1) and (8), we have

$$C_{e,j+1}(R_{j+1}, \tilde{\theta}_i) + C_{h,i}(R_{j+1}) - \left(C_{e',j+1}(R_{j+1}, \varphi_i - \tilde{\theta}_i) + C_{h',i+1}(R_{j+1})\right) = 0. \tag{B.5}$$

Denote $\Delta_{25}(\tilde{\theta}_i)$ as the left-hand side of Eq. (B.5). One can derive

$$\frac{\partial\Delta_{25}(\tilde{\theta}_i)}{\partial\tilde{\theta}_i} = 2\left(\lambda_t t(Q(R_{j+1}, \tilde{\theta}_i) + \alpha_2 R_{j+1}\right) > 0. \tag{B.6}$$

This means that $\Delta_{25}(\tilde{\theta}_i)$ is monotonically increasing with regard to angle $\tilde{\theta}_i$. In addition, $\Delta_{25}(0) < 0$ and $\Delta_{25}(\varphi_i) > 0$. Therefore, $\tilde{\theta}_i$ in Eq. (B.5) has a sole solution within the interval $[0, \varphi_i]$. This means that the angle $\tilde{\theta}_i$ of any location on $B^{(2)}_{(i,i+1;j,j+1)}$ is identical.

In light of the above, $B^{(1)}_{(i,i+1;j,j+1)}$ and $B^{(2)}_{(i,i+1;j-1,j)}$, which are from two concentric sector areas $\Theta_{(i,i+1;j,j+1)}$ and $\Theta_{(i,i+1;j-1,j)}$, are straight line segments. For any point $(R_j, \hat{\theta}_i)$ on the boundary contour $B^{(1)}_{(i,i+1;j,j+1)}$, $\hat{\theta}_i$ is the solution to Eq. (B.2). For any point $(R_j, \tilde{\theta}_i)$ on the boundary contour $B^{(2)}_{(i,i+1;j-1,j)}$, from Eqs. (1) and (8), $\tilde{\theta}_i$ is the solution to the following equation

$$C_{e,j}(R_j, \tilde{\theta}_i) + C_{h,i}(R_j) - \left(C_{e',j}(R_j, \varphi_i - \tilde{\theta}_i) + C_{h',i+1}(R_j)\right) = 0. \tag{B.7}$$

Note that segments e, h, e' and h' in the sector area $\Theta_{(i,i+1;j-1,j)}$ are just segments d, f, d', and f' in the sector area $\Theta_{(i,i+1;j,j+1)}$, respectively. Hence, Eq. (B.7) is actually equivalent to Eq. (B.2). Therefore, $\hat{\theta}_i = \tilde{\theta}_i$ holds, and thus $B^{(1)}_{(i,i+1;j,j+1)}$ and $B^{(2)}_{(i,i+1;j-1,j)}$ are in the same line.

Appendix C: Proof of Proposition 3

Suppose that $\hat{\theta}_i$ is the solution of equation $\psi_1(R_j,\theta_i)=\psi_4(R_j,\varphi_i-\theta_i)$. From Eqs. (1) and (8), one can obtain

$$C_{d,j}(R_j,\hat{\theta}_i)+C_{f,i}(R_j)=C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)+C_{f',i}(R_j)\,. \tag{C.1}$$

Summing both sides of Eq. (C.1) from i = 1 to M_{RAD} yields

$$\sum_{i=1}^{M_{\mathrm{RAD}}}\left(C_{d,j}(R_j,\hat{\theta}_i)+C_{f,i}(R_j)\right)=\sum_{i=1}^{M_{\mathrm{RAD}}}\left(C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)+C_{f',i+1}(R_j)\right). \tag{C.2}$$

According to Fig. 3 and Appendix A, $C_{f,i+1}(R_j)=C_{f',i+1}(R_j)$ and $C_{f',M_{\mathrm{RAD}}+1}(R_j)=C_{f,1}(R_j)$ hold. Accordingly, we obtain

$$\sum_{i=1}^{M_{\mathrm{RAD}}}C_{f,i}(R_j)=\sum_{i=1}^{M_{\mathrm{RAD}}}C_{f',i+1}(R_j)\,. \tag{C.3}$$

From Eqs. (C.2) and (C.3), we have

$$\sum_{i=1}^{M_{\mathrm{RAD}}}C_{d,j}(R_j,\hat{\theta}_i)=\sum_{i=1}^{M_{\mathrm{RAD}}}C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)\,. \tag{C.4}$$

If the traffic congestion effects on the ring roads can be ignored, from the definitions of $C_{d,j}(R_j,\hat{\theta}_i)$ and $C_{d',j}(R_j,\varphi_i-\hat{\theta}_i)$ (see Eqs. (2) and (A.2)), we have

$$\sum_{i=1}^{M_{\mathrm{RAD}}}\left(\lambda_t\frac{R_j\hat{\theta}_i}{V_{RIN,j}}+\alpha_2R_j\hat{\theta}_i\right)=\sum_{i=1}^{M_{\mathrm{RAD}}}\left(\lambda_t\frac{R_j(\varphi_i-\hat{\theta}_i)}{V_{RIN,j}}+\alpha_2R_j(\varphi_i-\hat{\theta}_i)\right). \tag{C.5}$$

From Eq. (C.5), one can immediately obtain

$$\sum_{i=1}^{M_{\mathrm{RAD}}}\hat{\theta}_i=\sum_{i=1}^{M_{\mathrm{RAD}}}\frac{\varphi_i}{2}=\pi\,. \tag{C.6}$$