THEMA
théorie économique, modélisation et applications

THEMA Working Paper n°2022-23
CY Cergy Paris Université, France

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December 2022

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Abstract

While carpooling is widely adopted for long travels, it is by construction inefficient for daily commuting, where it is difficult to match drivers and riders, sharing similar origin, destination and time.

To overcome this limitation, we present an Integrated system, which integrates carpooling into transit, in the line of the philosophy of Mobility as a Service. Carpooling acts as feeder to transit and transit stations act as consolidation points, where trips of riders and drivers meet, increasing potential matching.

We present algorithms to construct multimodal rider trips (including transit and carpooling legs) and driver detours. Simulation shows that our Integrated system increases transit ridership and reduces auto-dependency, with respect to current practice, in which carpooling and transit are operated separately. Indeed, the Integrated system decreases the number of riders who are left with no feasible travel option and would thus be forced to use private cars. The simulation code is available as open source.

Keywords: Carpooling, Ride-sharing, Mobility as a Service, Transit, Simulation, Multimodal Transportation

JEL Classification: R41 , R48

1. Introduction

In carpooling systems, a set of drivers accept to pickup and dropoff a set of drivers. Despite its success for inter-city trips, carpooling has not registered similar adoption for daily commuting in urban conurbations. Indeed, matching drivers and riders requires some "sacrifice" from them: they may both need to shift their departure and arrival times in order to "meet" at a time feasible for both; moreover, they have to change their routes, in order to meet at some meeting points. In daily commuting, the interurban time and route adjustments that users are willing to accept are much smaller than for long trips. These makes quite hard to match riders and drivers which have both similar, departure and arrival times and origins and destinations.

In this paper, we propose to overcome this limitations by adopting a Mobility as a Service philosophy. We show that Carpooling has limited benefit if managed independent from transit. Acknowledging the irreplaceable role of transit (Basu, Araldo, Ben-Akiva, et al., 2018), we propose instead to integrate Carpooling into the transit offer. While integration of flexible modes into transit has been recently proposed (Calabrò, Araldo, Ben-Akiva, et al., 2021), the integration of carpooling in particular has not been extensively studied. Few exceptions are (Stiglic et al., 2018) and (Fahnenschreiber et al., 2016). However, the former assumes that riders obey to the matching proposed by the system, even if more convenient travel options were possible, which is unrealistic in practice. Moreover, they limit carpooling only in the First Mile (rider origin to transit station) and not in the Last Mile (station to rider destination). (Fahnenschreiber et al., 2016), instead, can only match one rider per driver.

We propose an Integrated System, which constructs via simple algorithms multimodal rider routes and driver journeys. We show in simulation [1] that such a system would provide a viable solution to private cars to a considerable number of commuters.

2. System model

We consider a suburban area, as in Fig. 1, served by a commuter rail. Users are either *drivers* or *riders*. Drivers are available to pick-up and drop-off other riders. Each vehicle is characterized by a *journey*, which is a sequence of *meeting points*, each visited at a specific time instant. In a meeting point, drivers can let riders alight or board.

Users interact with our system by means of web or smartphone application, through which they declare their trip: origin, destination and departure time. We assume drivers' declarations are done in advance (up to 1h from their departure). Riders' declarations can instead be done on the fly (at the same time than the departure time) or in advance. All requests are processed by a *Controller*, then calculates drivers' detours and feasible transportation options for riders. We compare three systems:

- In the ***No Carpooling System*** riders can just walk and/or use fixed schedule transit.
- ***Current System***: like in current cities, carpooling and transit are handled separately, no multimodal trips are proposed by the system and driver journeys are completely independent from transit.

[1] Code available at https://github.com/YoussefChaabouni/Carpooling

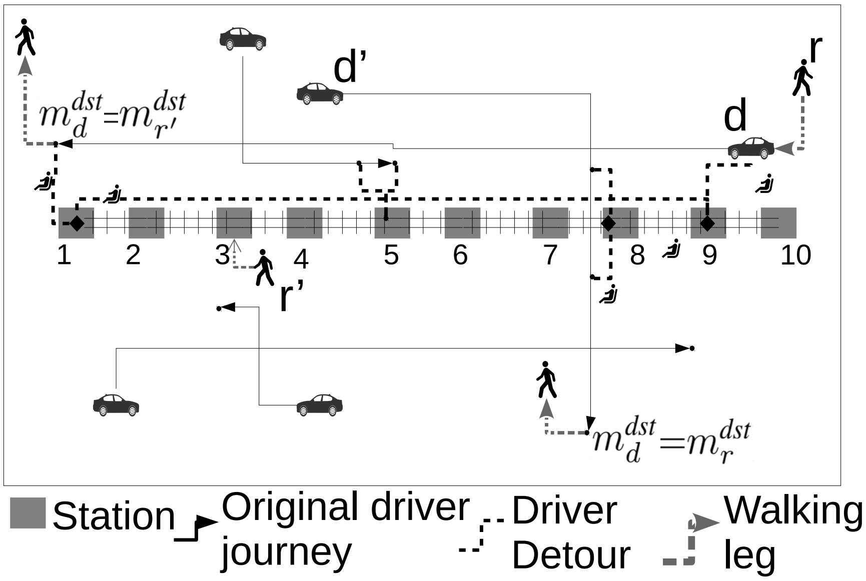


Figure 1: Illustrative scenario

- We propose an ***Integrated System*** in which the transit and carpooling routes are part of the same transportation service. Therefore, a rider can make a part of her trip by carpooling and the rest by transit. Driver journeys are integrated with transit via detours.

We assume the Controller knows exactly the exact instant in which drivers, riders and trains arrive in each visited location. This is an ideal condition, as in reality only estimates of such instants can be computed, which may be imperfect due to external conditions. We will get rid of this assumption in our future work.

Driver journey

The journey of driver d is a sequence of *meeting points* $m \in \mathscr{M}$. She can pickup or dropoff a passenger only in those. The origin and destination of driver d are the first and last meeting points of her journey. All stations $\mathscr{S}$ are also meeting points, and thus a driver can potentially use them to start or to stop his/her journey.

In **No Carpooling** and **Current** System, driver d just drives directly from her origin m^{org} to her destination m^{dst}. In the Current system, she might pick passengers up in m^{org} and drop them off at m^{dst}. In the **Integrated System**, a driver d can make a detour to pass by s_d^{org} or s_d^{dst}, i.e., the stations closest to origin m_d^{org} and destination m_d^{dst}, or to pass by both. Such detours are accepted by driver d only if her journey is no more than 15% longer than the direct trip between m_d^{org} and m_d^{dst}. The detour is realized only if there are riders to pickup or dropoff at the respective station, otherwise it is ignored. The calculation of the driver journey is detailed in Fig. 7.

org_d to her destination dst_d, starting at her departure time and following the shortest path, i.e., $J(d) = \{org_d, dst_d\}$

In the **Current System**, if no riders carpool with d, her journey is the same as before. Otherwise, she picks riders up at m_d^{org} and drops them off at m_d^{dst}.

In both systems, the journey is $J_{cs}(d) = \{m_d^{org}, m_d^{dst}\}$.

In the **Integrated System**, detours are possible. A journey is thus

the subsequence $\{s_d^{org}, \cdots, s_d^{dst}\}$ denote the train station along the journey of d where $s^i \in \mathscr{S}$ are the the subset of the meeting points $\mathscr{S} \subset \mathscr{M}$ that are both connected to transit system and the road network (e.g. a train station, subway station). We design the system to ensures that distance traveled by d never exceeds the shortest distance between between the origin and the destination by more than 15%, $dist(m_d^{org}, m_d^{dst}) < 0.15 \times dist(J_{is}(d))$. Also note that a driver actually passes by a meeting point if there is some rider boarding or alighting there. Otherwise, that meeting point is skipped. Fig. 7 details the calculation of $J_{is}(d)$.

The Controller computes the journey of driver d as in Fig. 7. With half probability the Controller starts by checking if it can add a detour passing by s_d^{org}, increasing the traveled distance no more than 15%. If yes, it then also checks if it is possible to an additional detour via s_d^{dst}. With the other half probability, the order in which the Controller tries to add detours by s_d^{org} and s_d^{dst} is inverted.

We assume the incentive provided to the driver to make the proposed detours (possibly coming from riders' payments) is enough to accept. Incentive schemes (Zhong et al., 2020) are outside our scope.

Transportation options available to riders

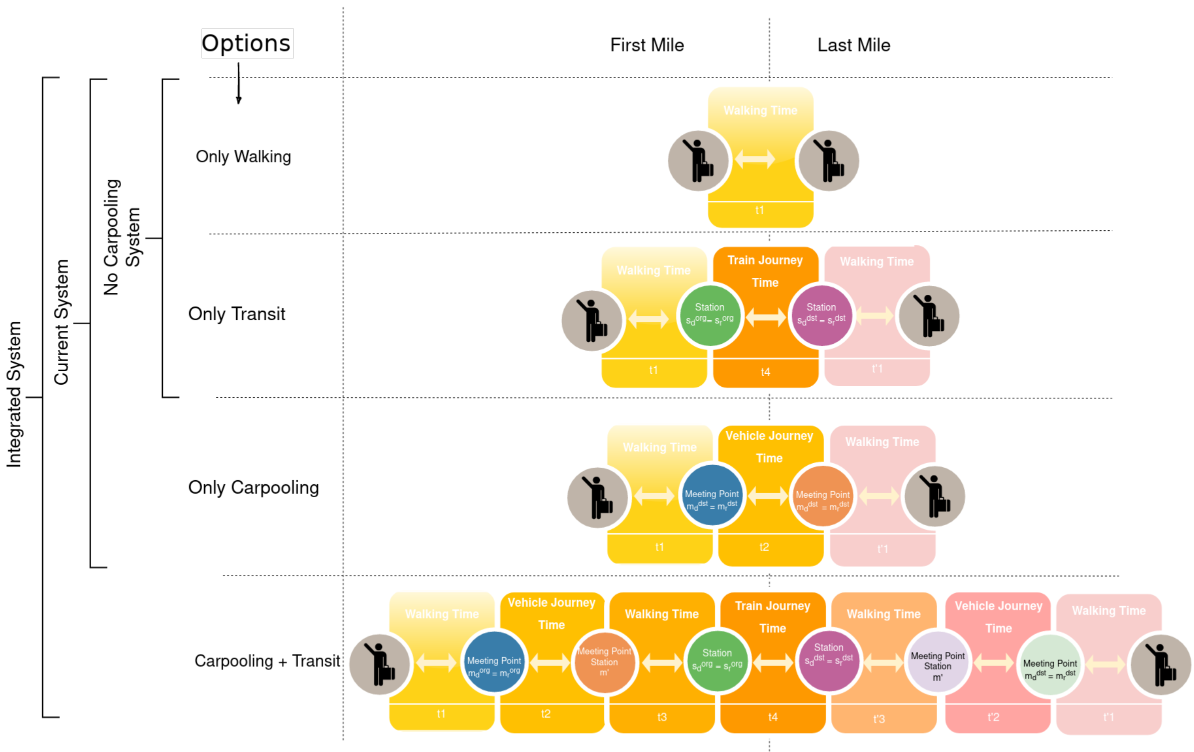


Figure 2: Options available for each of the three considered systems.

Let us assume journeys $J(d)$ of all drivers $d \in \mathscr{D}$ have been defined and consider rider r departing at an origin org_r and willing to arrive to a destination dst_r as soon as possible.

The possible legs of a rider's journey, with the different options available in the three systems are depicted in Fig. 2.

Definition 1. *A transportation option is* feasible *for a rider only if (i) it implies a total waiting time of at most 45 minutes, (ii) a total walking distance of* 2.5 km *and (iii) if the total journey time is less than the one needed to go by foot from origin to destination.*

In the previous definition, if a rider journey is composed by several legs, and she has to wait for several vehicles (either trains or drivers), the total waiting time is the sum of all waiting times. We assume that a rider aims to minimize waiting time by leaving home in order to arrive at a station or meeting point right at the moment where the vehicle she wants to board (carpooling or train) is departing. Similarly, if the rider has multiple walking legs in a single journey, the total walking distance is the sum of their distance.

In the **No Carpooling System**, the rider has two possible *transportation options*:

- *Only walking option*: Rider r walks directly from org_r to dst_r.
- *Only transit option*: Rider r walks from her origin org_r to the closest station $s_r^{org} \in \mathscr{S}$, waits for the next train, travels with that train to the station $s_r^{dst} \in \mathscr{S}$ closest to her destination dst_r, alights there and walks to dst_r.

In the **Current System**, in addition to the previous two, the following *option* is available:

- *Only carpooling option*: Rider r can carpool with a certain driver d only if the origin and destination of d are the closest meeting points to the origin and destination of r. In other words, if we define m_r^{org},m_r^{dst} as the meeting points closest to the origin and destination of rider r, we must have $m_r^{\text{org}} = m_d^{\text{org}}$ and $m_r^{\text{dst}} = m_d^{\text{dst}}$. If rider r and driver d carpool, r first walks from her origin org_r to the origin meeting point m_r^{org}, arriving right at the moment where d is departing. Then, r and d carpool up to the destination meeting point m_d^{dst} and, from there, r walks to her final destination dst_r. Carpooling is possible if condition one holds (Def. 1) and vehicle capacity is not exceeded. Among all the possible drivers with which rider r can carpool, the system proposes the one that brings her to her final destination the earliest, via Fig. 9.

In the **Integrated System**, in addition to the previous 3, the following *option* is available:

- *Carpooling + Transit option*: Rider r carpools (i) with a driver d in the First Mile, i.e., from r's origin org_r to the closest station s_r^{org}, or (ii) with a driver d' in the Last Mile, i.e., from the station s_r^{dst} closest to r's destination up to her destination dst_r, or (iii) with both d in the First and d' in the Last Mile. The system first computes the fastest way for rider r to arrive to her closest station $s_r^o rg$. This can be either by only walking or by combining walking and carpooling. Then, rider r takes the first train up to s_r^{dst}. The system finally computes the fastest way for rider r to reach her final destination, which could be either by only walking or by carpooling with driver d' and then walking. See the algorithm 4 describe in the figure 10 for more details about the driver selection process used in our simulation.

Figure 2 summarizes the different transportation options available for riders. Observe that the trip depicted for Carpooling + Transit may also be shorter, in case the rider carpools only in the First or only in the Last Mile.

Assumption 1. *The system computes the earliest arrival time for each option and selects the one that allows the rider to arrive at her final destination the earliest.*

If all the modes available to a rider are infeasible (in the sense of Def. 1), then we consider her **unserved**: such users cannot use our system and need to resort to their private car.

Observe that the Integrated System offers more options to riders (cf. Fig. 2). As a consequence, less riders will be unserved and the rider travel times decrease with respect to the No Carpooling and Current System. This will be confirmed by the numerical results.

3. Performance Evaluation

Scenario Description

The parameters of the scenario are in Table. 1. Observe that our users are not representative of the entire population of the area, but only of the ones that joined our system. The value of circuity (ratio between actual travelled distance from a point to another and euclidean distance) is taken from (Boeing, 2019).

Parameter	Value
simulation area	$15\times 8\,\text{km}^2$
# train station	10
avg. distance between station	1.5 km
average speed	
walking	$4.5\,\text{km}\,\text{h}^{-1}$
car (source: `statista.com`)	$38\,\text{km}\,\text{h}^{-1}$
train	$60\,\text{km}\,\text{h}^{-1}$
arrival density	
rider	$8.3\,\text{rider}/\text{km}^2/\text{h}$
driver	$4.8\,\text{driver}/\text{km}^2/\text{h}$
number of users	
riders	2988
drivers	1728
max. vehicle occupancy	4 seats
network circuity	1.2

Table 1: Simulation parameters

We generated uniformly distributed meeting points $m_i \in \mathscr{M}$, with average density of $3.55\,\text{meeting point}/\text{km}^2$. Then we added the train stations $s \in \mathscr{S}$ and also 4 to 5 meeting points uniformly distributed inside a circle of 300 m radius centered on each station

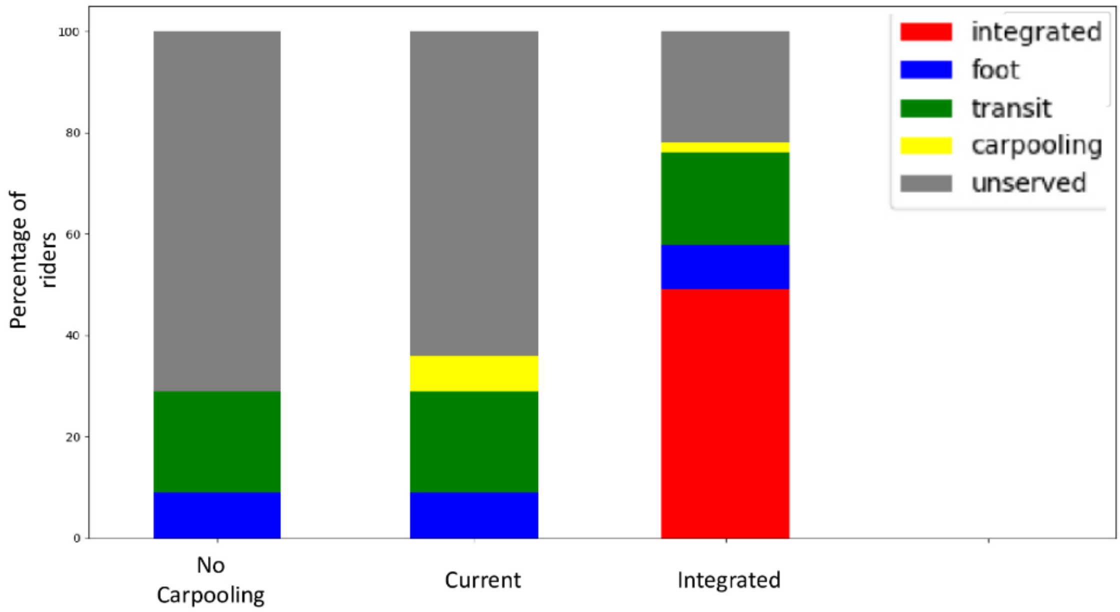


Figure 3: Breakdown of riders' transportation modes

$s \in \mathscr{S}$, to account for the higher population density therein. To generate origin m_d^{org} and destination m_d^{dst} of driver $d \in \mathscr{D}$, we select two random meeting points from the set $\mathscr{M} \setminus \mathscr{S}$.

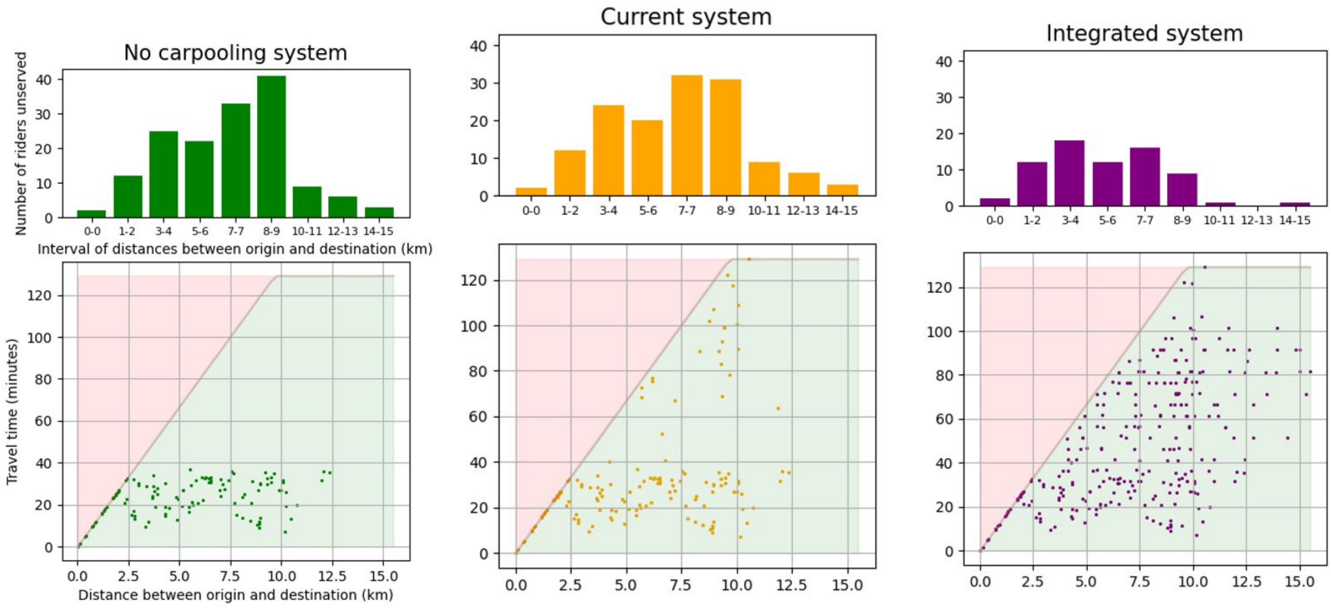
We simulate driver and drivers departing in a 3h interval, but we only measure our metrics on the ones departing in the 1st hour, in order to avoid typical simulation boundary effects.

We show that the benefits of carpooling are marginal if, as in real cities nowadays, it is operated independent from transit (i.e., Current System). Such benefits only emerge when carpooling is integrated with transit. By doing this, our proposed Integrated System greatly improves transportation accessibility, i.e., the easiness for a traveler to move from a location to another. In particular, our Integrated System offers a feasible transportation option to 40% more travelers (which would otherwise remain unserved and would have no other choice apart from private car), with limited detours to drivers.

We contrast the No Carpooling and the Current systems with our Integrated System. To allow for direct comparison, we provide the same input (i.e., the same set of rider origin-destination pairs and departure times and the same set of driver origin-destination pairs and departure times) to all the three systems.

Served Transportation Demand

In Fig. 3, we divide riders based on the selected multimodal transportation option. The percentage of unserved users in the Current System shows that carpooling itself is not beneficial without integrating it with transit. Indeed, only very few riders r find a driver d whose origin and destination meeting points correspond to hers ($m_d^{org} = m_r^{org}$ and $m_d^{dst} = m_r^{dst}$) and whose departure time is compatible with hers. We see instead that carpooling is an excellent feeder for transit: many riders find drivers to carpool with to reach a transit station in the First Mile or to go from a station to their destination in the Last Mile. In fact, transit stations take the role of demand consolidation points (Araldo et al., 2019), which are easily served by carpooling. A considerable amount of drivers, who were left



(a) No Carpooling System

(b) Current System

(c) Our Integrated System

Figure 4: Travel time against origin-destination distance. Each point represents a rider, the affine function is an average person walking. The upper bar plots represent the number of unserved riders, per each origin-destination distance value

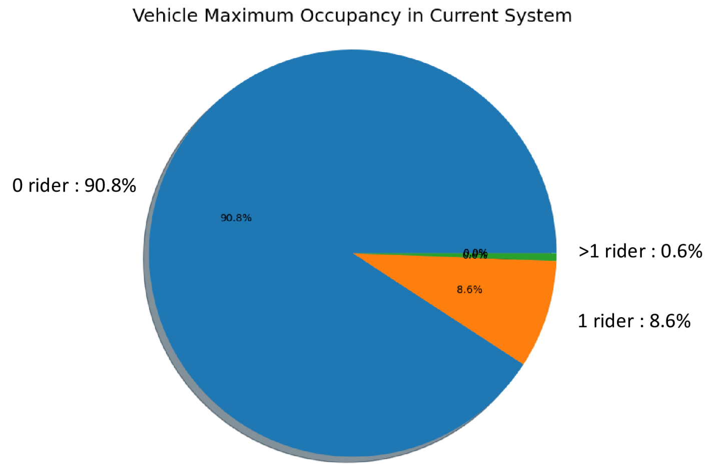
with no feasible options in the Current system, thus being forced to take their private cars, can instead in the Integrated System perform their trip combining transit and carpooling.

Travel Times

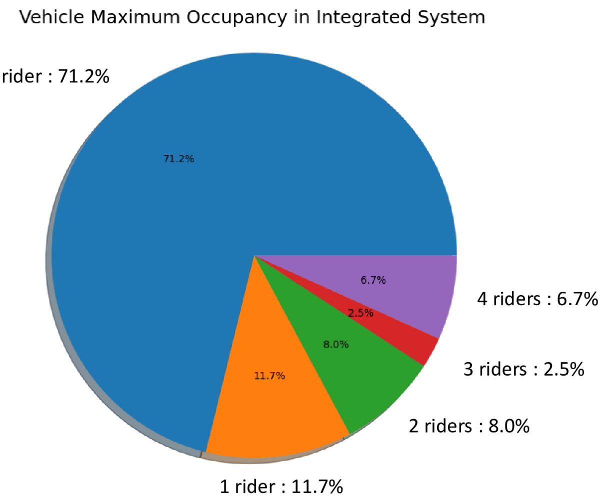
In Fig. 4b, for each rider r, we plot her origin-destination distance against her travel time. As expected, almost all travelers manage to perform their trips for very short distances. However, as the distance increases, only a smaller part of them can do it. In the No Carpooling System, no traveler can perform a trip of more than 15 km. Introducing carpooling, as in the Current System, creates feasible options for longer trips. However, only few "lucky" riders have such options, as the others do not find any driver with compatible origin, destination and departure time. The Integrated System, instead, provides feasible transportation options for much more travelers, which is particularly visible for longer trips. In general, for any distance, Fig. 4c shows much more feasible trips in the Integrated System than in the other systems. The difference is given by the riders that were unserved in the other systems (upper bar plots) and have instead feasible options in the Integrated System.

Driver vehicle Occupancy

The number of occupied seats in a driver car changes with time, as riders board and alight. In figure 5, we focus on the *maximum occupancy*, i.e., the maximum number of riders that have simultaneously been in driver d's vehicle. For instance, if driver d picks up one rider at her origin meeting point m_d^{org}, then other two riders at a station, and all riders alight at the meeting point m_d^{dst}, the maximum occupancy of driver d's car is $M_d = 3$. The Integrated System allows to more efficiently exploit the capacity offered by



(a) Current system



(b) Integrated system

Figure 5: Vehicle maximum occupancy in current and integrated systems

carpooling. Indeed, in the Current System only very "lucky" riders r find a feasible rider d matching, i.e., with corresponding origin and destination meeting points ($m_d^{org} = m_r^{org}$ and $m_d^{dst} = m_r^{dst}$) and compatible departure times (neither too late nor too early). In the Integrated system, instead, transit stations are consolidation points, and the probability to find a driver passing by a station at the "right" time is relatively high. Observe also that this increase in rider-vehicle matching is also boosted by the fact that we purposely construct vehicle detours in order to preferentially pass through transit stations, around which we consolidate demand (riders) and offer (drivers), who can thus more easily be matched.

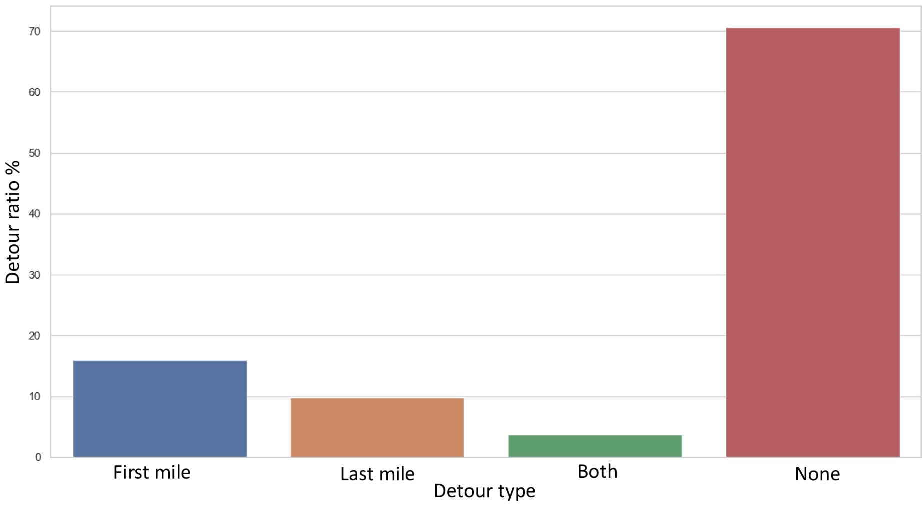


Figure 6: Percentage of detours in the Integrated System

Detours

Fig. 6 shows that the Integrated System requires detours only to a relatively small percentage of drivers, either in the first or last mile (i.e., through the station closest to the origin or the destination of the driver). Even fewer drivers make a detour in both first and last mile. This indicates that the Integrated System does not impose a high dis-utility to drivers. On the contrary, by just requiring relatively few driver detours, we are able to achieve high accessibility improvements for riders. This successful result is due to the demand consolidation operated around few meeting points and, more importantly, around transit stations.

4. Conclusion

We have proposed an Integrated System in which carpooling and transit are offered as a unified mobility service. By requiring relatively small detours to drivers, our system greatly increases accessibility and richer feasible travel options, which would allow to reduce the need for using private cars, with societal and environmental benefits.

Much remain to do, on the basis of this first building block. In this work, we have not considered congestion on the road and in transit, which would instead impact and be impacted by our system. Moreover, we have assumed each user is either rider or driver, while in reality they can change from a day to another. Endogenous use of car and endogenous number of trips (e.g. due to teleworking, teleshopping) are likely also to affect congestion patterns. Finally, a future cost-benefit analysis can guide the design of incentive strategies for drivers, such as privileged parking, special transit fares, and possibly High-Occupancy Toll (HOT) or High-Occupancy Vehicle (HOV) Express Lanes.

Acknowledgement

Supported by the Paris Ile-de-France Region (DIM RFSI) and AFINITE project.

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5. Appendix

We report now Fig. 7, 8, 9 and 10 run by the Controller.

Algorithm 1: Generation of driver d's journey in the Integrated System.

Input:

- m_d^{org}, m_d^{dst}: the meeting points closest to the origin and destination of d, respectively.
- s_d^{org}, s_d^{dst}: the stations closest to the origin and destination of d, respectively.
- $t^+(d, org_d)$: Time instant at which the driver starts.

Output: $J(d)$

Initialize $J(d) := \{org_d, m_d^{org}, m_d^{dst}, dst_d\}$
Throw r uniformly at random in $[0,1]$
if $m_d^{org} = dst_d$ **then**
    $J(d) := \{org_d, dst_d\}$
else
    if $r \leq 0.5$ **then**
        // Try to add a detour close to the origin
        if $d(m_d^{org}, s_d^{org}) + d(s_d^{org}, m_d^{dst}) \leq 1.15 \cdot d(m_d^{org}, m_d^{dst})$ **then**
            // Add a detour through station s_d^{org}
            $J(d) := \{org_d, m_d^{org}, s_d^{org}, m_d^{dst}, dst_d\}$
            if $d(m_d^{org}, s_d^{org}) + d(s_d^{org}, s_d^{dst}) + d(s_d^{dst}, m_d^{dst}) \leq 1.15 \cdot d(m_d^{org}, m_d^{dst})$ **then**
                //Also add a detour through the station close to the driver destination.
                $J(d) := \{org_d, m_d^{org}, s_d^{org}, s_d^{dst}, m_d^{dst}, dst_d\}$
    else
        // Try to add a detour close to the destination
        if $d(m_d^{org}, s_d^{dst}) + d(s_d^{dst}, m_d^{dst}) \leq 1.15 \cdot d(m_d^{org}, m_d^{dst})$ **then**
            // Add a detour through station s_d^{dst}
            $J(d) := \{org_d, m_d^{org}, s_d^{dst}, m_d^{dst}, dst_d\}$
            if $d(m_d^{org}, s_d^{org}) + d(s_d^{org}, s_d^{dst}) + d(s_d^{dst}, m_d^{dst}) \leq 1.15 \cdot d(m_d^{org}, m_d^{dst})$ **then**
                //Also add a detour through the station close to the driver destination.
                $J(d) := \{org_d, m_d^{org}, s_d^{org}, s_d^{dst}, m_d^{dst}, dst_d\}$
// Now compute the time instants at which the driver visit all the meeting points.
Let the journey be $J(d) = \{z_0, z_1, \ldots, z_k\}$, where $z_0 = org_d$, $z_k = dst_d$ and all the others are the intermediary meeting points.
$t^-(d, z_i) := t^+(d, z_{i-1}) + drive(z_{i-1}, z_i)$, for $i = 1, \ldots, k$
$t^+(d, z_i) := t^-(d, z_i) + 1$ min for $i = 1, \ldots, k-1$

Figure 7: Pseudo-code for the algorithm 1

Algorithm 2: Computation of the arrival time of rider r, if she carpools with driver d.

Input:

- z: Location from which rider r starts
- z': Location that rider r wishes to reach
- t: Time at which the rider starts from z
- d: Driver with which rider r carpools
- m: Meeting point in which rider r boards d's vehicle.
- m': Meeting point at which rider r alights.

Output:

- t': Arrival time of r in z' (if $t' = \infty$, it means that r cannot be matched with d)
- wt: Waiting time experienced by r.
- wd: Distance rider r has to walk.

Initialize $t' := wt := wd := \infty$
// Check if there is a seat available
foreach *meeting point* m'' *in driver* d*'s journey from* m *to* m' **do**
    if $c^+(d, m'') \leq 0$ **then**
        return
$t' := t + walk(z, m)$ // First, rider r needs to walk to m
$wd := d(z, m)$
if $t' > t^+(d, m)$ **then**
    // The rider would arrive after the driver has departed
    return
else
    $wt := t^+(d, m) - t'$ //The rider waits for driver d's departure
    $t' := t^-(d, m')$ // The rider carpools up to m'
    $wd := wd + d(m', z')$ // Then she finally walks to z'
    $t' := t' + walk(m', z')$

Figure 8: Pseudo-code for the algorithm 2

Algorithm 3: Driver selection for rider r in the Only Carpooling option.

Output: Arrival time of r in z' (if $t' = \infty$, it means that r cannot be matched with any driver).

Initialize $t' := wt := wd := \infty$;
foreach *driver* $d \in \mathscr{D}$ **do**
    // In the Only Carpooling option, a rider and a driver can carpool only if they have the same origin and destination meeting points
    if $m_d^{org} = m_r^{org}$ *and* $m_d^{dst} = m_r^{dst}$ **then**
        //Compute the arrival time of rider r if she carpools with d, by calling Alg.2.
        $\hat{t}, \hat{wt}, \hat{wd} :=$ Alg.2($z = org_r, z' = dst_r, d, m = m_d^{org}, m' = m_d^{dst}$)
        if $\hat{wd} \leq 2.5$ *Km and* $\hat{wt} \leq 45$ *min and* $\hat{t}' < t'$ **then**
            $t' := \hat{t}$; $wd := \hat{wd}$; $wt := \hat{wt}$;

Figure 9: Pseudo-code for the algorithm 3

Algorithm 4: Driver selection for rider r in the Carpooling+Transit option.

Output: t': Arrival time of r in dst_r (if $t' = \infty$, it means that Carpooling+Transit is infeasible for the rider).

Initialize $t' := t^+(r, org_r)$
// Walking distance and waiting time accumulated by the rider.
Initialize $wd := wt := 0$
////////// FIRST MILE (to reach s_r^{org})
Initialize $t_{\text{First}} = \infty$ // Instant in which the rider arrives at s_r^{org}
foreach $d \in \mathscr{D}$ **do**
    if $m_d^{org} = m_r^{org}$ ***and*** *d passes by* s_r^{org} **then**
        $\hat{t}, \hat{wt}, \hat{wd}$ = Alg. 2 $(z = org_r, z' = s_r^{org}, t = t', d = d, m = m_d^{org}, m' = s_r^{org})$
        if $wd + \hat{wd} \leq 2.5\ Km$ ***and*** $wt + \hat{wt} \leq 45\ min$ ***and*** $\hat{t} < t_{First}$ **then**
            $t_{\text{First}} := \hat{t}$; $wd := wd + \hat{wd}$; $wt := wt + \hat{wt}$;
if $t_{First} = \infty$ **then**
    // It is not possible to bring r directly to the station. Let m' be the meeting point closest to s_r^{org}. Find a driver that can bring the rider in m' and let the rider walk from there to the station.
    foreach $d \in \mathscr{D}$ **do**
        if $m_d^{dst} = m'$ **then**
            $\hat{t}, \hat{wt}, \hat{wd}$ = Alg. 2 $(z = org_r, z' = s_r^{org}, t = t', d = d, m = m_d^{org}, m' = m')$
            if $\hat{wd} \leq 2.5\ Km$ ***and*** $\hat{wt} \leq 45\ min$ ***and*** $\hat{t} < t_{First}$ **then**
                $t_{\text{First}} := \hat{t}$; $wd := \hat{wd}$; $wt := \hat{wt}$;
if $t_{First} = \infty$ **then**
    // The last resort for the rider is to walk to the station
    $\hat{wd} := d(org_r, s_r^{org})$
    if $\hat{wd} < 2.5Km$ **then**
        $t_{\text{First}} := t' + walk(org_r, s_r^{org})$; $wd := \hat{wd}$
if $t_{First} = \infty$ **then**
    // The rider cannot reach the origin station s_r^{org}
    $t' := \infty$; **return**
$t' = t_{\text{First}}$
////////////////// TRAIN
$t' = t' + 1$ min // After reaching the station, we assume 1 minute is needed to reach the platform
Increment wt by the time the rider waits for the next train after t'
t':= instant in which the train arrives at station s_r^{dst}, i.e., the closest to the destination
////////////// LAST MILE (from s_r^{dst} to dst_r)
Initialize $t_{\text{Last}} = \infty$ // Instant in which the rider arrives at dst_r
Get $\mathscr{D}^{s_r^{dst}}$ // Drivers passing by s_r^{dst}.
foreach $d \in \mathscr{D}^{s_r^{dst}}$ **do**
    $\hat{t}, \hat{wt}, \hat{wd}$ = Alg. 2 $(z = s_r^{dst}, z' = dst_r, t = t', d = d, m = s_r^{dst}, m' = m_d^{dst})$
    if $wd + \hat{wd} \leq 2.5\ Km$ ***and*** $wt + \hat{wt} \leq 45\ min$ ***and*** $\hat{t} < t_{Last}$ **then**
        $t_{\text{Last}} := \hat{t}$; $wd := wd + \hat{wd}$; $wt := wt + \hat{wt}$;
if $t_{Last} = \infty$ **then**
    // It is not possible to carpool starting from s_r^{dst}. Let m'' the stop point closest to s_r^{dst}. Find a driver that starts from there.
    foreach $d \in \mathscr{D}$ **do**
        if *d passes by* s_r^{dst} *and* $m_r^{dst} = m_r^{dst}$ **then**
            if $m_d^{org} = m''$ **then**
                $\hat{t}, \hat{wt}, \hat{wd}$ = Alg. 2 $(z = s_r^{dst}, z' = dst_r, t = t', m = m'', m' = m_d^{dst})$
                if $wd + \hat{wd} \leq 2.5\ Km$ ***and*** $wt + \hat{wt} \leq 45\ min$ ***and*** $\hat{t} < t_{Last}$ **then**
                    $t_{\text{Last}} := \hat{t}$; $wd := wd + \hat{wd}$; $wt := wt + \hat{wt}$;
if $t_{Last} = \infty$ **then**
    // The last resort for the rider is to walk from the station to her destination
    $\hat{wd} := d(s_r^{dst}, dst_r)$
    if $wd + \hat{wd} < 2.5Km$ **then**
        $t_{\text{Last}} := t' + walk(s_r^{dst}, dst_r)$; $wd := wd + \hat{wd}$
if $t_{First} = \infty$ **then**
    // The rider cannot reach her final destination
    $t' := \infty$; **return**
$t' = t_{\text{Last}}$

Figure 10: Pseudo-code for the algorithm 4