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Children Costs in a One-Headed Household: Empirical Evidence from the UK

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Children Costs in a One-Headed Household: Empirical Evidence from the UK.

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Abstract

There is a growing literature that attempts to assess the cost of children. However, these previous studies exclusively emphasized on children living with couples. The purpose of this present paper is to see to what extent the adaptation of the unitary approach to single-parents can allow estimating the resource sharing in a one-headed household with children. To that end, we use the UK Family Expenditure Survey over the 1978-2007 period. The inferences of the children's cost rest on the assignable goods method, here clothing and the traditional assumption of orthogonality of parents' tastes and demographic change. Our results show that the cost of children increases with the number of children but decreases with family size. Also, we observe that single mothers are more altruists than single fathers in the sense that they devote a larger part of total expenditures to children welfare. However, single fathers try to catch up with their counterparts as the number of children increases. Finally, there is evidence of economies of scale with the presence of same-sex siblings in the household.

JEL Classification: C30, D11, D12, D36, D63, I31, J12, J13

Keywords: Collective Model, Shadow price, Economies of scale, Identification, Resource sharing.

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1 Introduction

In western societies, the nuclear family consisting of two parents and children remains the legal and economic foundation on which the institutions are based. Thus, the single person and even single parent are largely discriminated. In economics, everyone familiar with the literature on equivalence scales is aware of the absence of studies regarding the lone parents despite statistics show that single-parent families are becoming more important in our societies. In the 1970s, traditional family structures are exploding and household behaviors are changing. To quote the sociologist Jean-Claude Kauffman (1993), "Never have marital breakdowns been so numerous, and never have couples been so celebrated on the altar of contemporary values" (My translation). A demographic reconfiguration is set at the expense of married couple families linked to a disruption in ways of living together. This transition into cohabitation is reflected by both an increase in divorce rates and a fall in marriage rates. In this regard, even though married or civil families remain the most common family type, there is a declining trend in the proportion of the married population in most countries in the last sixty years (Browning, Chiappori & Weiss, 2014). Conversely, the proportion of cohabiting couple families and divorced individuals are becoming larger. For example, in the UK over the last 10 years, the proportion of married or civil couple families decreased from 68.6% in 2009 to 66.8% in 2019 while the proportion of cohabiting couple families increased from 15.3% to 18.4% (ONS, 2019). It should be reminded that the couple is not the only way of living as a family. Some people may also prefer to live alone or with their children following a marital breakdown. Some others may be in a transition process on the market of love-seeking for a new love partner. According to the Office for National Statistics (2019), lone parent families in UK were estimated to 2.9 million in 2019, which represents 14.9% of the families, that is a minority certainly but it remains non-negligible. Furthermore, this number has significantly increased between 1999 and 2019, i.e. 14.5% (ONS, 2019).

However, all the related literature on the general equivalence scale focuses on bi-headed households. In other words, the single parent households have been wholly neglected in the literature. See, e.g., (Ray, 1983; Weiss & Willis, 1985; Deaton & Muellbauer, 1986; Gronau, 1988, 1991, Bradbury, 1994, 2008; Bourguignon, 1999; Blundell & al., 2005; Bargain & Donni, 2009, 2012 and Bargain & al., 2010, 2020). Even though many researchers echoed that negative outcomes experienced by children in single-parent families are the result of the economic disparity between one and bi-headed households (Demo & Acock, 1996 and Lamb & al., 1999). Figure 1 illustrates the consumption inequality for one headed and bi-headed households. We can clearly see a rise in consumption inequality for the two groups and for both, that pattern is virtually similar, but nowhere there is a gain to be single parent in terms of consumption. In the early 20th century, the growing economic vulnerability of single mothers has led to the development of public assistance programs (Folbre, 1994). Hence, in this present paper, we suggest estimating what a one-parent household spends on children.

This issue may well fall the third category, say expenditure question of the taxonomy drawn up by Browning (1992) concerning the research question about the cost of children. [1] This perspective seems relevant insofar as it has important policy even legal implications. For instance, it permits at least to determine payments ensuring child's claims to be met for a one-parent family. At some level, since it takes into account the sex/parent differences, it informs politics on which single parent (either father or mother) the children's cost is more important. In other words, which parent spends the most on their children.

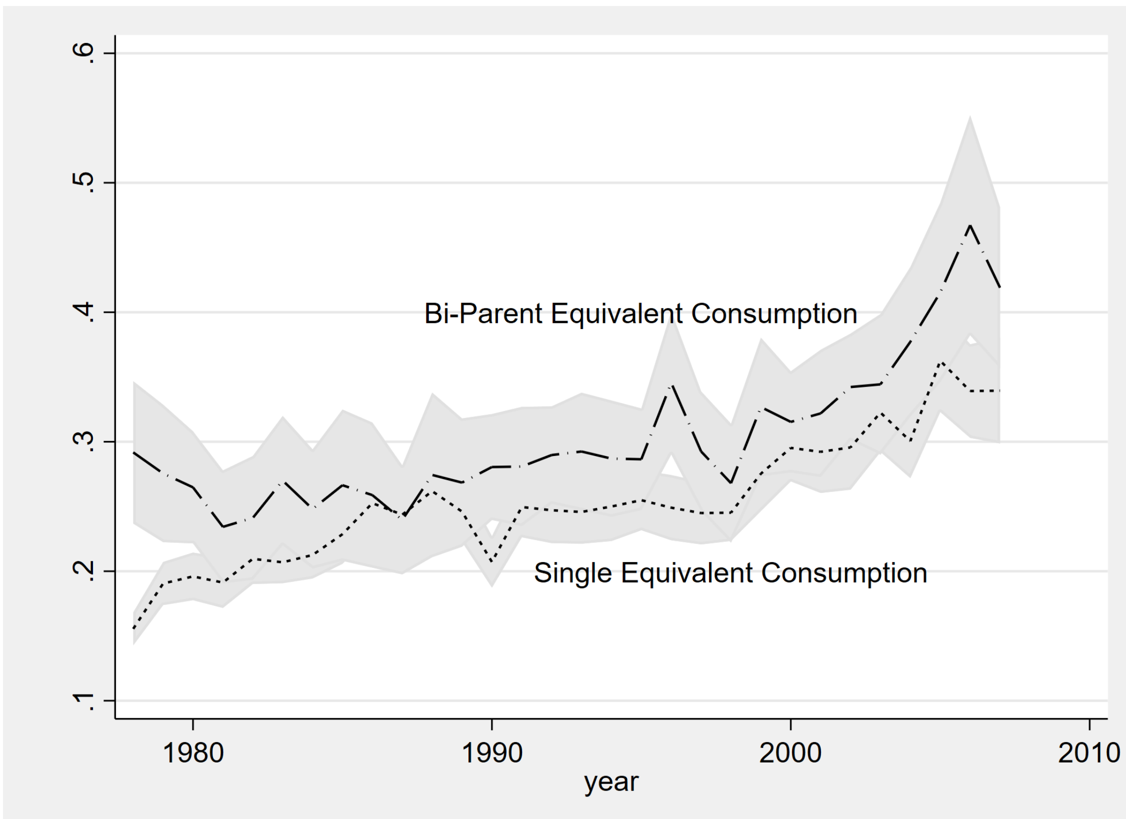

Figure 1: The pattern of inequality by sample

For over two decades, there has been a consensus among researchers that individuals within the household have conflict interests. Then it is unsuitable to treat them as if they were single decision units as usually done in classical microeconomics textbooks. Therefore, recent studies on children costs adopt the collective

[1] Browning (1992) highlights four types of questions that the studies regarding the cost of children bring up. Firstly, the positive question that pose the question of how much children affect the expenditure patterns of a household. Secondly, the needs question that purport to investigate on how much income does a family with children need compared to a childless family. Thirdly, the expenditure question which is interested to how much do parents spend on their children. Finally, the iso-welfare question that addresses the question of how much income does a family with children require to be as well of as a family with no children.

approach previously designed by Chiappori (1988, 1992), Bourguignon & Chiappori (1992), Browning & al. (1994), Browning & Chiappori (1998), Bourguignon (1999) and Chiappori & Donni (2011) to restore the methodological failure in the conventional approach. This new approach has paved the way to modern studies on adult equivalence of a child. However, even if the authors are unanimous in employing the collective model, they still differ on how to estimate the cost of children.

At this point, Gronau (1988, 1991) is somewhat the first that has formally introduced children in the collective household model. Like its predecessors, his paper shows that the separability assumption does matter to ensure the inferences of the children's cost under the collective approach, although he rejects the previous homotheticity assumption. To identify the mechanism of distribution of resources within the household, he estimates the observed effect of environmental variables on the marginal propensity to consume adult goods. In addition, he partially joins the recent paper namely Blundell & al. (2005), Bargain & Donni (2009, 2012) hereafter BD and Bargain & al., (2010, 2020) in modeling children as public goods. However he stands out from Dunbar, Lewbel & Pendakur (2013) hereafter DLP because the latter consider children as consumers who have their own preferences.

Furthermore, the recent developments in this strand of literature extend the Rothbarth method by measuring the children's costs which is consistent with economies of scale and parental bargaining. See, e.g., BD (2009; 2012), Bargain & al. (2010; 2020) and DLP (2013). DLP (2013) and BDH (2020) represent an extension of the Browning, Chiappori and Lewbel (2013) hereafter BCL's model (2013) with respective particularity by explicitly introducing children in the household. Their identification is commonly based on exclusive goods, say clothing, except that DLP (2013) relax the assumption of preference stability between single-individuals and couples in favor of similar preferences across people (SAP) and similar preferences across types (SAT).[2] Recall that the hypothesis of preference stability implies that individuals living alone have the same preferences as engaged counterparts. Moreover, DLP (2013) without appealing distribution factors identify the level of members' resources shares by using comparison of Engel curves with demand equations holding prices constant. For their part, BDH (2020) keep the BCL (2013) stability of preferences assumption and identify economies of scale through a transformation à la Barten. Unlike DLP (2013) and BD (2012), they assume a dependence relation between resource shares and total expenditures.

In this present paper, the purpose is twofold. First, it aims at adapting unitary framework to single parents. It is tempting to go with the wave using a collective model. However, the unitary approach seems to be appropriate with the assumption that parent behaves as a dictator, then children has no bargaining power in the household. Second, our paper strives to estimate the resulting model and

[2]From my point of view, the main difference between DLP and BDH is the way in which children is modeled. Children is supposed to be a public good for parents in the paper of BDH (2020) whereas they represent consumers who have their own preferences in the DLP (2013)'s view. The difference of assumptions regarding preferences reflect the particular status of children in each model.

measures the change in the cost of attaining a certain level of welfare as the family composition varies. Meanwhile, it proposes to define the sex/parent differences. To do this, we propose a new model of household consumption behavior with children that has three components. Separate utility functions over goods for each household member, a consumption technology and a sharing rule that defines the rule governing the distribution between parents' and children's resource allocation. We pick up the main assumption of the preference stability traced back to the work of Gronau (1991) to identify the model. By this assumption, we mean that the tastes of parents are assigned to be independent of the demographic change. We use the UK Family Expenditure Survey (FES) over the period 1978-2007. Our findings reveal that the children's cost increases with the number of children but decreases with family size. Also, we observe that single mothers are more altruistic than single fathers in the sense that they devote a larger part of total expenditures to children welfare. However, single fathers try to catch up with their counterparts as the number of children grows up. Finally, there is evidence of economies of scale in the presence of same-sex siblings in the households.

It is worth noting that through the expenditure question we purport to give an estimation of the cost borne by the single parent while the purpose would be to compare the welfare of families with different incomes and demographic structures if we were addressing the adult equivalence of a child in terms of iso-welfare question.

The remainder of this paper is organized in the three following parts. The first presents the theoretical model. The second describes the empirical implementation and data selection. The third reports and discusses the empirical results and the last section concludes.

2 Theoretical Framework

In this section, we present the consumption household model. On the one hand, we describe the consumption behavior of single individuals. This introduction is a background for understanding the consumption behavior of a one-headed household with children which will be presented on the other hand.

2.1 The Consumption Behavior of Single Individuals

Parents do not cease to be so after their children establish their own household. However, the financial and time burden of their children tends to zero at a certain age. In other words, the limit of children cost as children are becoming adults is zero. Thus, the popular one-period model is justified in the evaluation of the cost of children because there are some particular stages of parents' lives where is concentrated the cost of rearing children (Bradbury, 2008). In this framework, we consider a single-adult household without children acting in a one-period. We assume that each household member has a well-behaved utility function $U_i(x_i, X_i)$, that is twice continuously differentiable, strictly increasing and strictly concave direct utility function over two arguments (an exclusive good x_i and a composite good X_i).[3] The subscript $i = w, m$ denotes respectively women and men. We also assume that individual preferences over consumption bundles is stable so that some prediction could be possible about household behavior. However, household member purchases z_i and Z_i quantities of x_i and X_i respectively. Thus, each individual faces the budget constraint as follows:

$$z_i p_i + Z_i = y_i \tag{1}$$

where y_i denotes the total household expenditure. The market price for the composite good is normalized to one.

Household surveys do not report consumption per se, but expenditures. If we account for durable goods, effective consumption may not reflect expenditures at a given point in time. In this case, we consider only nondurable goods insofar as these tend toward consumption as the fraction of purchased goods that is not consumed is small. Hence, we can assume that:

$$z_i = x_i \quad \text{and} \quad Z_i = X_i \tag{2}$$

At this stage, the optimization program of the household member $i = w, m$ is as follows:

$$\max U_i(x_i, X_i) \text{ subject to (1) and (2)} \tag{3}$$

The solution of this program allows expressing the share functions for the exclusive good as:

$$\omega_i = g_i(p_i, y_i) \tag{4}$$

where the subscript $i = w, m$ and $\omega_i = p_i x_i / y_i$. It is worth noting that $U_i(.)$ is strictly increasing, then ω_i must exhaust the consumer's income.

2.2 The Consumption Behavior of One-Headed Households with Children

We model for the case of a single-parent household with children. The single parent has a well-behaved utility function $U_i[u_i(x_i, X_i), u_c(X_c)]$ that contains two components - the first utility derived from their own consumption u_i and the other one from their children's consumption u_c. Our analysis is conducted under the hypothesis that each parent is altruistic toward their children and that, the utility of children is that

[3] Here we could have ignored the exclusive good since there is no confusion about the individual consumption in a single household. Then there is apparently no distinction between private and exclusive goods, even private and public goods in such a type of household. However, we keep the distinction to facilitate the understanding in the next section when this will be relevant and more useful. Furthermore, a commodity good can be any other goods except an exclusive good.

perceived by his/her parent. We consider an additive utility function taken the form as follows:

$$u_i = u_i(x_i, X_i) + \delta_i(n)u_c(X_c) \tag{5}$$

represents the single parent total utility function, where x_i and X_c stands for the adult assignable good and children goods, and X_i the composite goods as previously defined. The utility function of children has one argument for the sake of simplicity, and associated with a parameter δ_i which measures how the resources devoted to children evolve when the number of children increases. [4] The parameter δ_i could also be seen as the weight given by the adult to the child (Bargain & Donni, 2012). Note that the previous model is a special case of this one because if $n = 0$, this implies $\delta_i = 0$ and then we go back to the standard consumption model for a single-adult household. [5] In resume, the single parent cares about the children welfare. We assume that the utility function is endowed with caring preferences. This is to say that parents care about the children's allocation only insofar as it gives them some individualistic welfare. In other words, children matter for the household's choices, but only through the utility their parents derive from their well-being (Browning, Chiappori & Weiss, 2014, p.89).

We consider a simple model in which household income is given. Moreover, we assume there are neither time-allocation decisions nor household production. Household income is totally spent for purchasing z_i quantities of assignable goods and Z quantities of composite goods. Hence, y_i denotes total expenditures instead of total income. The household budget constraint is the following:

$$z_i p_i + Z = y_i \tag{6}$$

where z_i and Z denote respectively the purchased quantities of parent's exclusive goods and of composite ones. We note that the consumption of children is included in the composite goods. There are two types of goods. A private assignable good x like clothing and all other goods that are not privately assignable (x) such as food. i. Exclusive goods are purely private, that is, for any demographic structure of the household, the consumption of exclusive goods reflects exactly what is expended by the household. Thus,

$$z_i = x_i \tag{7}$$

ii. In a household with at least two individuals, an adult and a child, some goods are endowed with public properties. Because of this, the consumption of these goods cannot be reflected in their purchased quantities. Hence, their purchased quantities are transformed into a higher level of consumption with a transformation rate that depends on all three exogenous variables for now:

$$Z = \sum_{j=i,c} A_j(y_i, p_i, n_i)X \quad \text{with} \quad i = w, m \tag{8}$$

[4] See Appendix A for technical details about δ_i.

[5] We see this in more details along with this subsection.

where $A_j(y_i, p_i, n_i)$ denotes shadow prices for the parent and children. Put equations (7) and (8) into the household budget constraint (6), we have:

$$x_i p_i + A_i(y_i, p_i, n_i) X_i = y_i \tag{9}$$

Parents maximize their own utility subject to the new budget constraint (10). Note that with one adult household, the outcome resulting from the parent decision is automatically Pareto efficient. This follows from this assumption:

Assumption 1. *The adult acts as a dictator in the household; he/she makes decision for his/her child.*

The trade-off that needs to be done will happen in the allocation of resources for the parent and children consumption. That is, given the budget and technology constraints, parents cannot make children better off without making themselves worse off. If we decentralize the single parent decision, we can write the optimization program as follows: Hence, given the Becker's altruistic preferences and the efficiency assumption, the household allocation may be derived from the following optimization program:

$$\begin{aligned} \max_{(x_i, X_i, X_c) \in \mathbb{R}^3_+} \quad & u_i(x_i, X_i) + \delta_i(n) u_c(X_c) \\ \text{s.t.} \quad & x_i p_i + A_i(y_i, p_i, n_i) X_i = y_i \\ & X = X_i + X_c \end{aligned} \tag{10}$$

where δ_i represents the child weight in the eyes of his/her parent and depends on the number of children for simplicity. On the right-hand side, the budget constraint exhibits total expenditures on adult and children consumption.

The adoption of an additive utility function makes possible the transition to a decentralized program. The reasoning behind the two-stage budgeting program is quite simple. In the first stage, parent agrees on a sharing rule driving the distribution of resources in the household. The sharing rule to which we shall come back below can be seen as transfers from parent to children which we consider in that framework as always positive. In the second stage, each household member decides independently on their own consumption level with respect to her respective budget constraint. Formally, the decentralization process can be summarized as the two stages: The first stage is characterized by the mechanism determining the distribution of resources. It results from this maximization program:

$$\begin{aligned} \max_{\phi_i \in \mathbb{R}_+, \phi_c \in \mathbb{R}_+} \quad & \nu_i(\frac{p_i}{A_i}, \frac{y_i \phi_i}{A_i}) + \delta_i \nu_c(A_i, \frac{y_i \phi_c}{A_i}) \\ \text{s.t.} \quad & \phi_i + \phi_c = 1 \end{aligned} \tag{11}$$

where ν_i and ν_c denotes respectively the indirect sub-utility function of parent and children. The second stage leads to the solution of the single-parent decision:

$$\max_{x_i \in \mathbb{R}_+} u_i(x_i) \quad \text{subject to} \quad x_i p_i + A_i(y_i, p_i, n_i^*) X_i = y_i . \Phi_i(y_i, p_i, n_i) \tag{12}$$

for some function $\Phi_i(y_i, p_i, n_i)$ such that $\Phi_i(y_i, p_i, n_i) \leq 1$ and $n > 0$. The total expenditure in multiplying by Φ_i can be divided into two parts. Thus, Φ_i represents the single parent share of total expenditure and the remaining, say $\Phi_c = (1 - \Phi_i(y_i, p_i, n_i))$, the fraction of total expenditure that is allocated to children, in other words the cost of children. As the budget share equations are homogeneous of degree zero, the solution can be written as:

$$\frac{\omega_i}{\Phi_i(y_i, p_i, n_i)} = g_i \left(\frac{p_i}{A_i(y_i, p_i, n_i^*)}, y \frac{\Phi_i(y_i, p_i, n_i)}{A_i(y_i, p_i, n_i^*)} \right) \tag{13}$$

where $\omega_i = p_i x_i / y_i$. The price of the exclusive goods may affect the share of total expenditure devoted to children, as discussed in Bargain and Donni (2012). Notice that the stability of adult's preferences upon the exclusive goods means that the presence of children in the family does not alter the individual preferences.

2.3 Identification

An important question in our model of consumer behaviour concerns the response we should expect in individuals clothing budget share when family size changes. The answer to this question lies in the hypothesis regarding the state of individual preferences from childless individuals to single parents. The identification of our model rests on the hypothesis of preference stability, the existence of assignable goods and the non-linearity of the Engel curve. We assume as BCL (2013) that the preferences of individuals do not change with family status, then stable over time. This means that preferences of single individuals and single parents are similar. Under such a framework, the estimation of the sharing rule parameters of single parents and children is possible through the information provided by the demand functions of single-individuals since indifference curves are unchanged with the occurrence of children in the household. Thus, change in single parent consumption that would occur should fall to the presence of children in the household and not to the change of individual preferences from childless individual to parent.

Chiappori & Eckeland (2009) evoked that identification requires the estimation of at least three goods. However, Bourguignon (1999) and Bourguignon, Browning and Chiappori (2009) showed that having an assignable good suffices to recover the sharing rule and reach identification. Hence, our identification also depends on the existence of assignable goods (clothing) which are observable for each individual in the household. Finally, Prais & Houthakker (1955) found evidence to prioritize non-linear Engle curves and adding explicitly socio-demographic characteristics as control variables.

For identification purposes, we need an assumption regarding technology consumption and resource shares. There are currently two approaches in the literature to account for economies of scale. Either we assume the independence of base technology of production (Lewbel & Pendakur, 2008; BD, 2012; DLP, 2013 and Bargain & al., 2014), or the transformation à la Barten (Browning & al., 2014 and BDH, 2020). In the former approach, the cost of children does not capture variations in prices. Whereas in the latter approach, the relaxation of the independency assumption regarding individual consumption shares and levels of total spending allows for economies of scale to exploit price variations. The following assumption presents the shadow price and budget share by taking into account price variation.

Assumption 2. *For any* (y_i, p_i, n_i),

a) Individual price functions or scale economies functions are defined as:

$$A_i(y_i, p_i, n_i) = a_i(n_i), \text{ with } \quad a_i(0) = 1.$$

b) Individual share functions are defined as:

$$\Phi_i(y_i, p_i, n_i) = \psi_i(p_i, y_i, n_i; Z), \text{ with } \quad \psi_i(p_i, y_i, 0) = 1.$$

This assumption shows how the presence of children affects the individual price and individual share functions. In the first case, the right-hand side term $a_i(n_i)$ represents the variation in the parent shadow price stemming from the presence of children. If the parent prefers to consume public goods instead of private goods because of children, $a_i(n_i)$ should be greater than one and less than one otherwise. This assumption clearly shows as previously argued, the childless household model is a special case of household model with children. In other words, without children the market price for the aggregate good is normalized to one for a single household since $a_i(n_i)$ is equal to unity. As we can see, the function $a_i(.)$ is not conditional on y_i and p_i. We assume that children affect the shadow price of commodity goods for parents regardless of income. Furthermore, the preference of parent to consume exclusive goods is invariant to the decision of consuming commodity goods. Then the price of assignable good does not affect the economies of scale. Under this assumption, the individual price function is identified.

In the second case, function $\psi_i(y_i, p_i, n_i; Z)$ is the fraction of resources kept by the parent to satisfy their own consumption. The remaining fraction, namely $1 - \psi_i(y_i, p_i, n_i; Z)$ is allocated to children. The share of resources accruing to children consumption is positive if and only if the parents' share of resources is less than one. We exclude the extreme case where the parent exhausts the total expenditure for themself. This case leads us to an egoistic parent that cares only for herself.

Remember that we assumed that parent cares about his children. In the absence of children, $\psi_i(y_i, p_i, 0)$ is equal to one, that is the adult-individual keeps the entire budget as illustrated in the case of the single-household model. There is an intuitive way to interpret the individual share function. As we previously noted, $\Phi_i(.)$ is equal to one if there are no children in the household. With children, $\Phi_i(.) = 1 * \psi_i(.)$. That is, from the entire budget without children, say one, parents keep for themselves a fraction of ψ_i and the other fraction $1 - \psi_i(y_i, p_i, n_i; Z)$ goes to children.

3 Empirical Implementation

This section is structured as follows. We start with the sample selection process, and an overview of the data. We present the empirical strategy in two steps: first we specify the model, then we present the mechanism for resolving the issue of endogeneity.

3.1 Data and Sample Selection

To measure the cost of children in a single parent-household setting, we use data from the UK Family Expenditure Survey (FES) on the period 1978-2007.[6] It collects information on single, one-headed and bi-headed household expenditures on durable and nondurable goods, on the household income and labor force participation, and household socio-economic characteristics. The database also provides information about the region where the households reside.

Initially, the sample contains data on some 103,182 households that may be single individuals, single parents or bi-headed parents. The adults are aged between 18-60 years and have at most eight children. To perform the empirical analysis, we proceed to some selection. We sample single females, single males, and single parents with at most six children. We make no exclusion about the age of adults. Some may worry about the confusion between the children's clothing and adult's one. To this we answer that there is no way that an 18 year old parent is wearing their child's clothes since we suppose this child can be at the most 5 years old. We keep only positive household total expenditures. Therefore, we arrive at a sample of 35,704 households with 12 651 single males, 10,732 single females, 1,477 single fathers and 10,844 single mothers. Among parents, there are more than half of them that have one child respectively 57.89% of fathers and 50.27% of mothers. There are also only 12.25% of fathers raising more than 2 children against 16% of mothers.

In the empirical analysis, we set the budget shares on clothing as dependent variables. We do not work on durable goods since expenditures associated with these goods fail to represent their effective consumption. Thus, as stressed above we handle only nondurable goods. Our demand system has three exclusive goods – male, female and children clothing – and a composite good which stands for the omitted goods

[6]We thank Olivier Bargain that provided us all data we use as part of this study.

since budget shares sum up to one. Prices of all goods are measured yearly at the country level.

Two things should be noted. On the one hand, the issue of irregular spending is treated through a questionnaire which reports clothing expenditures throughout a recall period of the last 3 months. On the other hand, we record positive expenditures on male clothing made by a marginal proportion of single women, and vice versa. These spending are interpreted as gifts and we discount them since clothing is considered as an assignable good.

With regards to the independent variables, we use the level of education, age and labor force participation as socio-demographic variables for parents. For children, we utilize the number of children in the household and their age, as well as the proportion of boys. In order to exploit the economies of scale, we also include a dummy for the presence of siblings. The education level is measured in terms of number of school years completed by the individual. The labor participation is a dummy variable which takes the value of one if the individual works and zero otherwise. We also include weekly total expenditures evaluated in pounds. To account for residence, we use twelve regions of Great Britain from the dataset which are Nothern, York and Humberside, East Midlands, East Anglia, Greater London, South-East except Greater London, South Western, Wales, West Midlands, North Western, Scotland and Northern Ireland.

3.2 Functional Forms

The empirical specification leads us to consider a parameterization that combines flexibility and empirical tractability. We specify the budget shares of single individuals by a second-form demand function.

$$\omega_i = \alpha V_i + \beta log p_i + \gamma log y_i + \eta (log y_i)^2 + \epsilon_i \tag{14}$$

for $i = w, m$ and where α, β, γ and η represent the parameters to estimate. The vector V is a linear function that gathers a set of individual characteristic variables such as education, age, labor force participation, region of residence. In fine, the error term ϵ_i captures all other factors that may explain the budget share, but that are not included in the model.

As we see above, without children the equation (13) is reduced to equation (4). Hence, we define a dummy variable equal to 1 if adult is a parent and 0 otherwise. From this, the stochastic structure of the budget share equation (12) can be mathematically captured as follows:

$$\text{If } D_i = 0, \text{ then } \epsilon_i = \omega_i - \alpha V_i - \beta log p_i - \gamma log y_i - \eta (log y_i)^2 \tag{15}$$

$$\text{If } D_i = 1, \text{ then } \epsilon_i = \frac{\omega_i}{\phi_i} - \alpha V_i - \beta log\left(\frac{p_i}{A_i}\right) - \gamma log\left(\frac{\phi_i y_i}{A_i}\right) - \eta\left[log\left(\frac{\phi_i y_i}{A_i}\right)\right]^2 \tag{16}$$

We now turn to the empirical specification of individual's share of total expenditures and economies of scale consistent with Assumption 1 that sets conditions for identification. We design a logistic function to understand how the characteristic of parents and children affect the cost of children through the parent resource shares. Since Φ_i - the parent's share of total expenditure - is bounded between zero and one, it is suitable to express it as a logistic function as made in Browning & al. (1994) and BDH (2020):

$$\psi_i = \frac{exp[\xi_i(S,K)]}{1+\exp[\xi_i(S,K)]} \tag{17}$$

with S and K constituting vectors of sociodemographic parameters for parents and children respectively. In a reversal of usual trends, we do not call on Taylor expansion to linearize the sharing rule. We express the sharing rule as a deterministic function:

$$\xi_i = \alpha_0 + \alpha_s S_i + \alpha_k K_i \tag{18}$$

Again S_i and K_i stand for, respectively, parent and children characteristics with associated parameters α_s and α_k. The vector κ_i is independent from total expenditures as previously shown by BDH (2012) which the proof is given in the appendix A.

In equation (16), we assume the level of resources that accrues to children depends on both sets of factors, the parent's socio-demographic variables S_i on the one hand and the socio-demographic characteristics of children K_i on the other. In other words, α_s should not be interpreted as bargaining power because children are assumed to have no power. In other words, the parents' resource allocation decision is assumed not to be subject to the children's wishes. Nevertheless, there is at least a sharing rule in the model that is defined through the way in which parents and children variables may drive the distribution in the household. For example, we may conjecture that the older children cost more. Also, we can theoretically suppose that the expenditures on children rise with the number of children. In a single family, it is irrelevant to suppose the presence of the sharing rule or bargaining power. Indeed, the so-called parent egotistic/caring preferences depend on a linear function containing the log expenditure and a set of socio-demographic variables such as education level, age, labor market participation.

Furthermore, the vector of parameters $\alpha_{i,k}$ in the equation (16) shows how children characteristics play a role in determining the cost of children. This part of contribution contains children-specific variables K_i (i.e., number of children, squared number of children, average age of children, proportion of boys). We do not use constant to avoid having many constants in the sharing rule equation that may account for some econometric issues.

Two kinds of economies of scale are modelized in our study. To specify the shadow prices that account for economies of scale between parent and children, BHD (2020) allow them to vary with total expenditures. We might express them as a function of prices and individual characteristics, but we doubt of its empirical relevance in our study. For the sake of simplicity, we fix shadow prices to the inverse of the square root of the number of children. As we will see later, we have other variables that also allow us to capture economies of scale between children in the household.

3.3 Estimation Strategy and Instruments

Our model potentially suffers from two sources of endogeneity issues. The first one is that total expenditures can suffer from measurement error. This is related to the infrequency of purchases that leads to a misrepresentation of actual consumption regarding total expenditures. This is also possibly caused by recall errors from households during surveys. Both would induce a correlation between total expenditures and the error terms in the share budget function.

We tackle two types of endogeneity issues attributable to measurement errors in total expenditures by following the DLP (2013)'s approach. To do so, we use total income as instruments. The utility function in our setting applies to a single time period t. Then, we can readily assume that consumption allocation decisions within a time period are separable from savings decisions across periods. As a result, total income is uncorrelated with consumption allocation errors within a period. Since total income is uncorrelated with within period consumption allocation errors but correlated with total expenditures, then it can be considered as a valid instrument to measure error understood as the gap between total expenditures and actual consumption.

Endogeneity stemming from recall errors can also be dealt with by using total income. The reason is that even if total income may also be subject to measurement error due to a misevaluation of some assets or misreporting of some others, as long as these measurement errors are orthogonal to consumption recall errors and the correlation between total income and total expenditures hold, then total income can be claimed as a good instrument.

In the fertility studies (Nakamura & Nakamura, 1992) and the demand collective models as well, which generally include the number of children as a nuisance variable, child status variables are often suspected to be correlated with the perturbations. DLP (2013) put forward the idea that unobserved preference heterogeneity is connected to both fertility decisions and clothing expenditures. In short, if the number of children results from a selection process, then the number of children in the household will be endogenous.

In our model, we assume that the number of children is exogenously given. The rationale behind this is quite simple. The parent celibate status is separated from the decisions of fertility. When parents decide to have children, they do not prospect (anticipate) their singlehood. Furthermore, a lone parent is assumed to be unable to have children except by artificial insemination or adopting a child which have low probabilities.[7] Thus, a single parent is unlikely to decide how many children to have.

[7] Another exceptions, the fact that the single parent is able to have children with someone outside the household.

To set the instruments in a suitable way, we write the budget share equations (15) and (16) as a unique budget share equation. To do this, multiply equation (15) by $(1 - D_i)$ if single individual and equation (16) by D_i if lone parent:

$$\epsilon_i = (1 - D_i)\left[\omega_i - \alpha V_i - \beta logp_i - \gamma logy - \eta(logy)^2\right] + D_i\left[\frac{\omega_i}{\phi_i} - \alpha V - \beta log\left(\frac{p_i}{A_i}\right)\right.$$
$$\left. - \gamma log\left(\frac{\phi_i y}{A_i}\right) - \eta\left(log\left(\frac{\phi_i y}{A_i}\right)\right)^2\right]$$

and rearranging the second member of the equation above leads to the following result:

$$\epsilon_i = \left[\omega_i - \alpha V_i - \beta logp_i - \gamma logy - \eta(logy)^2 + D_i Q_i\right] \tag{19}$$

with

$$Q_i = \omega_i \frac{1 - \phi_i}{\phi_i} - \beta log\left(\frac{1}{A_i}\right) - log\left(\frac{\phi_i}{A_i}\right)\left[\gamma + \eta log\left(\frac{y^2\phi_i}{A_i}\right)\right].$$

To deal with endogeneity issues, we estimate the system of no simultaneous budget share equations by setting the iterated Two Stage Least Square Method. The nonlinear estimators are iterated until the estimated parameters and error/orthogonality condition covariance matrices settle.

We use as instruments all the exogenous variables, except total expenditures which are instrumented by total income. In order for total income to be a valid instrument, it must be uncorrelated with the error term in the budget share equations as we assume this in following DLP (2013) and partially correlated with total expenditure. Furthermore, we set as instruments the product D_i and a second-order polynomial of all the exogenous variables that enter Q_i and total income. This yields 19 instruments for each equation.

4 Empirical Results

This section presents the general findings of the model. We sum up the descriptive statistics of the sample. Then, we present the estimation results.

4.1 Sum up the Data

We have a sample with 87 469 households including 10 565 single males, 7 831 single females, 1 355 single fathers and 9 794 single mothers. Among parents, there are 58.6%, 31.4% and 10% of fathers with one, two and three children respectively; and on the other hand 50.8%, 36.2% and 13% of mothers with one, two and three children, respectively. Table 1 provides the descriptive statistics of the sample which allow to do a first analysis in the Rothbarth sense. Here are the following analyzes spending

Table 1: Descriptive statistics

		Single Women	Single Men	Single mother			Single father		
				Children					
				1	2	3	1	2	3
Expenditure data									
Female clothing	Weekly expenditure (in £)	10.7	-	8.5	6.6	5.0	-	-	-
Male clothing	Weekly expenditure (in £)	-	5.9	-	-	-	4.8	4.0	2.2
	Percentage of zeros		-				-	-	-
Total weekly expenditure (in £)									
Individual and household characteristics									
Women's labor participation		.7	-	.5	.4	.3	-	-	-
Men's labor participation		-	.7	-	-	-	.6	.5	.4
Women's education (in years)		12.4	-	11.4	11.3	11.1	-	-	-
Mens's education (in years)		-	12.3	-	-	-	11.2	11.3	11.2
Women's age		39.3	-	35.7	34.1	33.2	-	-	-
Men's age		-	38	-	-	-	38.5	37.0	36.2
House owner		.6	.5	.3	.3	.2	.5	.5	.3
Number of children		0	0	1	2	3	1	2	3
Average age of children		-	-	8.4	8.2	7.9	8.8	8.1	8.1
Proportion of boys		-	-	.5	.5	.5	.6	.5	.6
Number of observations		7831	10565	4971	3548	1275	794	426	135

on clothing items by adults with children. The results show that the presence of children decreases the parent clothing expenditures. By example, women and men living alone spend on average 10.7 pounds and 5.9 pounds by week, respectively. While the clothing expenditures for a single mother and a single father with one child are 8.5 pounds and 4.8 pounds, respectively. In addition, we note that the more the parents have children the less their clothing expenditures will be. These findings echo Rothbarth's view since the household size reduces the parents' welfare derived from consumption. Finally, these descriptive results also show that children are more expensive for single mothers since they affect her budget share more heavily than that of a single father.

4.2 Estimations

This section describes and analyses findings related to the budget share equation detailed above.

4.2.1 Budget Share Equations

Table 2 presents the results of the budget share equations. We estimate both equations (13) and (14) simultaneously with the iterative two stage least squares method. At first glance, we notice that socio-demographic characteristics do not always affect individual budget share in the same way for both single parents. Our findings confirm partly what was previously found in the literature by BDH (2020). Namely that the budget share of mothers decreases with education and age, but increases at a certain age. Regarding age estimates, this report is true and highly significant for both genders. We observe that other factors being equal, women in the labor market spend more in female clothing than those who are outside. Furthermore, the impact of log total expenditure and its square on budget shares is the same for both parents even if the coefficient associated with the last one turns out to be insignificant for mothers. The budget shares of women are negatively and significantly related to relative prices.

4.2.2 Resource Share Equations

The previous findings focus solely on the impact of individual characteristics on clothing budget share. A second and more important feature of our estimation centers on the influence of the presence of children on parent resource share. The results for resource share equations are exposed in table 3. It should be convenient to recall that the resource share equations allow recovering the parent contribution to children welfare. Remember, resources allocated to children depend on two main set of factors: the parent characteristics that defined the parent egoistic or altruistic behavior and children characteristics. As we explained above, ϕ_i represents the level of resources kept by single parents, inevitably $1-\phi_i$ the one diverted to children. Thus, a negative sign should be interpreted as an increase of children resources since it tends to reduce the resources taken by parents.

In this sense, the results for individual resource shares imply that children have an augmenting effect on parent resources. This says that the negative sign of the intercept suggests that the cost of children significantly grows up as the number of children increases. But resources per child fall significantly with the family size. Moreover, older children cost parents more. Furthermore, estimates for single fathers have mostly the same sign than single mothers. However, they are statistically significant for only the number of children.

One observes that the most educated parents (significant for mothers and not for fathers) spend less in children's clothing. At first sight, that seems surprising. However, it is not that surprising because we may conjecture that the most educated parents invest more in cultural goods like books, toys, cinema, etc. In addition, socialization theories argue that two-parent families are the optimal structure for socialization of children. Then mother's investment in cultural goods can be seen as an (imperfect) substitute for the role that the (absent) parent could have played in the socialization of children. This is consistent with the idea according to which the children's quality (Becker & Lewis 1973; Willis 1973) is expected to increase with parents' income and schooling (Gronau, 1991). We also know the price for raising a child of a given quality is expensive. On the other hand, since the total income is an increasing function of education. Then the resources allotted to children is higher in absolute terms for most educated parents even if the share of resources devoted to children is smaller.

Table 2: Results for Budget Share Equations

	Women's budget equation		Men's budget equation	
Parameters	Est.val.	Std. err.	Est.val.	Std. err.
Intercept	0.2100***	(0.0185)	0.1116***	(0.0172)
Education	-0.0011**	(0.0003)	0.0001	(0.0002)
Age (in years)	-0.0037***	(0.0006)	-0.0036***	(0.0005)
Ageˆ2 (in years)	0.0000***	(0.0000)	0.0000***	(0.0000)
Year	-0.0078*	(0.0046)	0.0041	(0.0050)
Yearˆ2	-0.0012	(0.0014)	0.0024	(0.0015)
House owner	0.0002	(0.0020)	-0.0029	(0.0019)
Labor participation	0.0079**	(0.0031)	-0.0015	(0.0025)
Region:				
Nothern	0.0021	(0.0052)	-0.0050	(0.0052)
York & Humberside	-0.0020	(0.0047)	-0.0140***	(0.0048)
East Midlands	0.0044	(0.0050)	-0.0209***	(0.0050)
East Anglia	-0.0053	(0.0054)	-0.0149***	(0.0053)
Greater London	-0.0007	(0.0045)	-0.0119***	(0.0046)
South-East	0.0012	(0.0046)	-0.0171***	(0.0045)
South West	-0.0013	(0.0048)	-0.0197***	(0.0049)
Wales	-0.0104*	(0.0053)	-0.0118**	(0.0054)
Nort-West	-0.0037	(0.0045)	-0.0139***	(0.0046)
West Midlands	0.0027	(0.0048)	-0.0121***	(0.0048)
Scotland	-0.0046	(0.0045)	-0.0131***	(0.0047)
Log relative price	-0.0281***	(0.0078)	0.019	(0.0149)
Log total expenditures	-0.0208**	(0.0097)	-0.0484***	(0.0148)
(Log total expenditures)ˆ2	-0.0024	(0.0028)	-0.0166***	(0.0040)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

Table 3: Estimated Parameters of the Individual Resource Shares

	Women's budget equation		Men's budget equation	
Parameters	Est.val.	Std. err.	Est.val.	Std. err.
Parent characteristics				
Intercept	2.9054***	(1.0440)	0.5340	(1.2694)
Education	0.0394**	(0.0193)	0.0997	(0.0823)
Age (in years)	0.0042	(0.0057)	0.0529**	(0.0224)
Labor participation	-0.0661	(0.1265)	-0.0967	(0.2925)
Log total expenditures	0.9813***	(0.3644)	1.2129*	(0.6741)
Children characteristics				
Number of children	-0.8028***	(0.1757)	-0.8569**	(0.4067)
(Number of children)ˆ2	0.0722***	(0.0265)	0.0638	(0.0743)
Age (in years)	-0.0276**	(0.0138)	-0.0171	(0.0321)
Proportion of boys	-0.0645	(0.0705)	0.2925	(0.2525)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

One also observes that the children's share is lower as the parents' total expenditure share increases. The same hypothesis can be stated here. At first glance, we may suppose that children in the wealthiest households, in absolute terms, record higher expenditures than those in the poorest ones. Second, total expenditure share is increasing with mother's total revenue and level of education. Thus, wealthiest parents devote a greater part of their revenue and expenses to financing the cultural goods of their offspring. There is a simple way we can explain this. Consider the quality of children as a discrete variable of range 0 to 10. Say that the more the quality of children approaches 10, the more it is becoming a luxury good. Otherwise, the limit of a child as its quality tends to 10 is a luxury good. And we know since Engel that the proportion of income spent on the consumption of essential goods decreases with income. Thus, wealthier parents are likely to have the best quality of children, consequently investing more in goods capable of improving the quality of their children. Since the quality of children can be a production function with arguments level of education, cultural level, health status, etc. For instance, wealthier parents can use their income to meet children claims in terms of attending better schools not only for having better education but also to meet people with a given social category, for social networking.

In summary, the presence of children in the household has a negative impact on parent resources. However, the effect of children is nonlinear on the budget share of parents. Additionally, the coefficients associated with parent education and total expenditures turn out to be negative. These results possibly mean that as income and education level rise the desired "quality" of children rises, and then investing in non-cultural goods becomes less desirable.

Table 4: Estimated Parameters of Scale Economies

	Women's budget equation		Men's budget equation	
Parameters	Est.val.	Std. err.	Est.val.	Std. err.
Parent characteristics				
Intercept	2.8695***	(1.0611)	0.6283	(1.3983)
Education	0.0394**	(0.0195)	0.0997	(0.0832)
Age (in years)	0.0035	(0.0075)	0.0537**	(0.0232)
Labor participation	-0.0753	(0.1296)	-0.1100	(0.3071)
Log total expenditures	1.0772***	(0.3895)	1.2319*	(0.6946)
Children characteristics				
Number of children	-0.7097***	(0.1700)	-0.8911*	(0.4556)
(Number of children)ˆ2	0.0604**	(0.0261)	0.0681	(0.0795)
Age (in years)	-0.0281*	(0.0142)	-0.0173	(0.0323)
Proportion of boys	-0.0752	(0.0730)	-0.2830	(0.2527)
Siblings of same sex	0.1475*	(0.0823)	-0.0385	(0.2386)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

In the previous lines, we evoked the fact that the budget share of children decreases with parent total expenditures. Given the heterogeneity of families regarding total expenditures, we report the per child resource shares at different point of household total expenditures. To that end, we divide total expenditures in 20th vigintile. We may note that the number of children does not matter for the wealthiest parents. Let concentrate on the second panel of the figure 2. As we can see, at the bottom of the distribution the resource shares per child diverge from 30, 45 to 68 percent respectively for one, two and three children families. That is, children in one-children families are better-off than those in families with multiple children. While the resource shares per child converge to 12 percent as families total expenditures grow up. That is, regardless the number of children, the resources per child is homogeneously distributed in the families at the top of the distribution. In other words, in terms of resource shares they kept, children living in wealthy families have almost the same level of welfare regardless the family size.

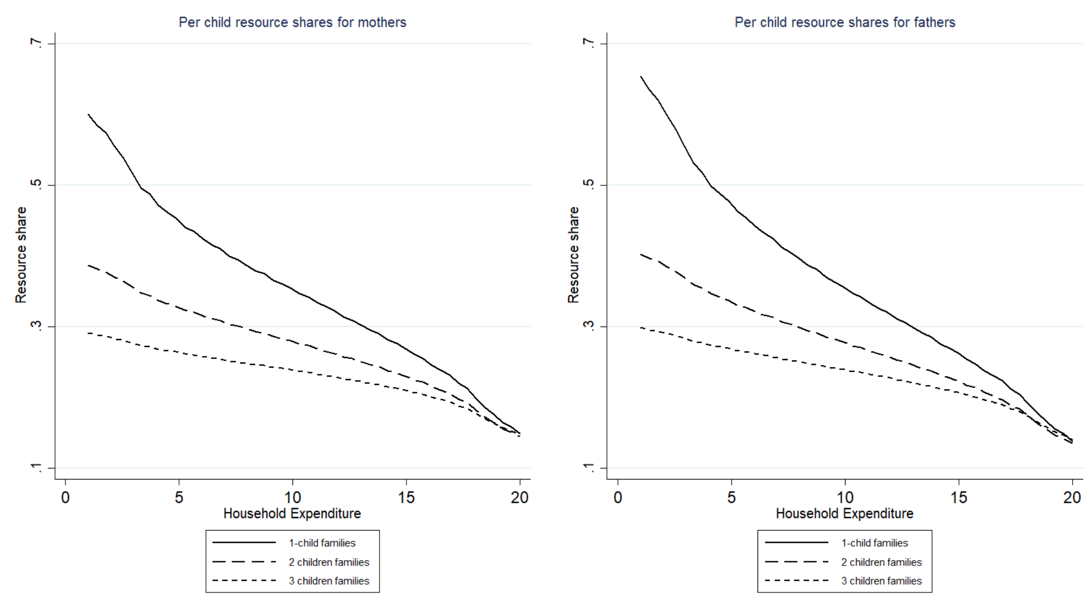

Figure 2: Children resource shares by total expenditures

4.2.3 Economies of Scale

We now turn our attention to the case of controlling for the presence of siblings in our model. The coefficient of the squared number of children from table 3 allows us to comment on the potential existence of economies of scale generated by the family size. The following estimates confirm this evidence. In some households, siblings of same sex and close-in-age are accustomed to wearing the same clothes. In some families, the clothes of the elders remained for the next offspring. Adding the siblings variable to the model allows us to explore these economies of scale. The siblings variable allows us to capture simultaneously the size and sex effects. Recall that sibling is a dummy variable indicating whether sibling is of same sex or not. The regression coefficient of sibling shows that children of same sex, on average, affects the total resources of single mothers less than those of the opposite sex. That said, the mothers share of total expenditures is larger when household is composed of same-sex siblings. It is worth noting that the positive sign must be interpreted as a positive impact of siblings of same gender on the share of total expenditures kept by single mothers. Thus, the presence of siblings of the same sex in the household decreases the resources budgeted for a child even though the total cost of children increases as shown by the number of children's coefficient.

Figure 3 provides us with valuable insights about economies of scale generated by same-sex siblings. Respectively, the straight blue and red lines indicate different sex and same-sex siblings. The figure clearly depicts that the total cost of children is lesser in families made up of siblings of different sexes. Unfortunately, in the current state of the data, we cannot evaluate the interaction effect of the variable age and sibling on the resources allocated to children. That would be helpful to analyze how close-in-age siblings of same sex influence the cost of clothing share of children. We guess that there would be place for further scale economies.

4.2.4 Intra-household Resource Allocation

To determine the cost of children, we compute the individual shares at the average point of the sample. The results will allow us to answer some questions. How do resources allocated to children evolve with the number of children? Do children affect resources of both single parents equally? Is the cost of children linear? As we can see in table 5, the resource share allocated to children increases with the size of households. Thus, the children's cost is twice as high for a single mother with three children than with one child. This probably constitutes a heavy drain on the income per child. Our findings further point towards two important things. Overall, single fathers spend less on their children than their counterparts in terms of the share of total expenditures reserved for children. In that respect, a single father that raises one child devotes on average 25% of his total expenditures to the child compared to 30% for a single mother. Additionally, the children's share for a single father with three children is of 58% while it is of 59% for a single mother. In other words, single fathers keep a larger part of total expenditures than the single mothers.

It should also be stressed though, that as the number of children goes up the share of the fathers resources allocated to children grows faster than that of the mother. Figure 4 is a good illustration of this. Here, the report is unequivocal: fathers broadly underinvest in children comparatively to mothers. However as stressed by the graphic, fathers attempt to fill the gap in terms of investment in children as its number goes way up. The share of fathers' resources devoted to children have even outreached that of their counterparts for four children families. The presence of three children in

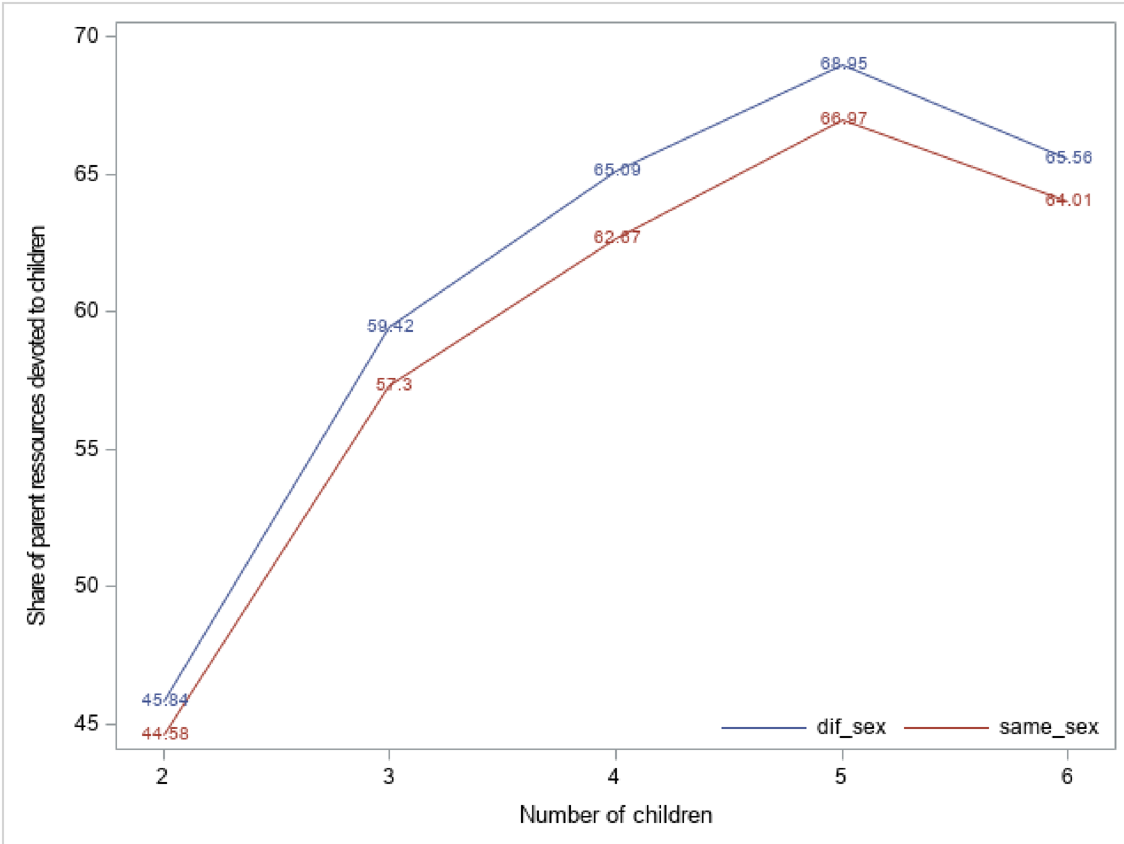

Figure 3: Evolution of the cost of children in families with siblings

the household is worth respectively and roughly 59 and 58 percent of total resources of lone mothers and fathers. However, the arrival of a new child shrinks total allocated resources to meet children needs to 59 and 64 percent of single mother's and father's total expenditures. It is important to keep in mind that any conclusion from this paper is based on the share of parent total expenditures devoted to children.

Table 5: Individual Shares at the Average Point of the Sample

	Single parent with one child		Single parent with two child		Single parent with three child	
Parameters	Est.val.	Std. err.	Est.val.	Std. err.	Est.val.	Std. err.
Mother's share	0.7028***	(0.0690)	0.5415***	(0.0545)	0.4057***	(0.0427)
Children's share	0.2971***	(0.0690)	0.4584***	(0.0545)	0.5942***	(0.0427)
Father's share	0.7494***	(0.1036)	0.5716***	(0.0980)	0.4230***	(0.0980)
Children's share	0.2505**	(0.1036)	0.4283***	(0.0980)	0.5769***	(0.0980)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

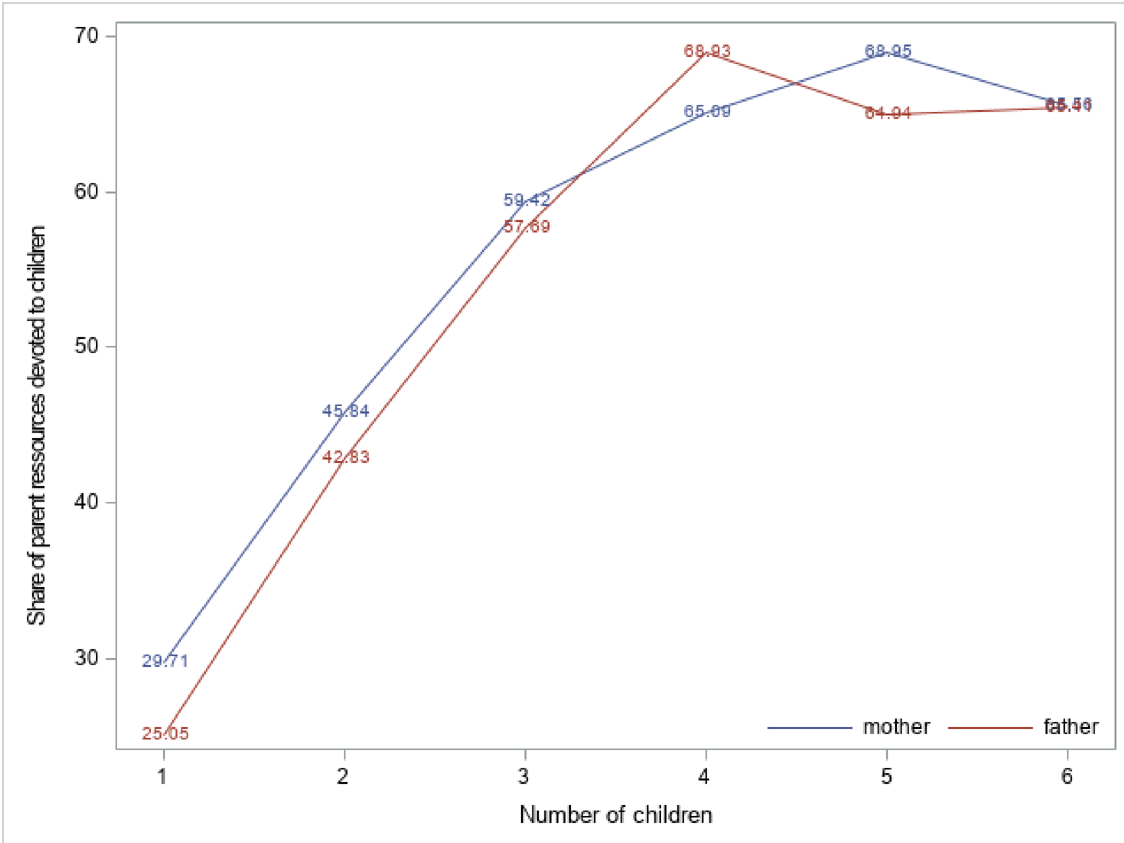

Figure 4: Evolution of the cost of children with family size

Table 6: Individual Shares at the Average Point of the Sample

		Benchmark model		Cubic Log Exp		Parents with 4 children		Working Parents	
Parameters		Est.val.	Std. err.	Est.val.	Std. err.	Est.val.	Std. err.	Est.val.	Std. err.
Mother	Number of children	-0.8028***	(0.1757)	-0.8017***	(0.1810)	-0.9389***	(0.2234)	-0.9657***	(0.3670)
	(Number of children)^2	0.0721***	(0.0265)	0.0721***	(0.0271)	0.1059***	(0.0394)	0.0659	(0.0494)
Father	Number of children	-0.8569**	(0.4067)	-1.4546	(0.7567)	-1.0150*	(0.5763)	-0.3920	(0.8106)
	(Number of children)^2	0.0638	(0.1057)	0.1997	(0.1504)	0.1057	(0.1149)	-0.0563	(0.1818)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

4.2.5 Robustness

We set three procedures to check the robustness of our results. Firstly, we set up four variants of our model. Secondly, we test of overidentifying restrictions. Thirdly and finally, we estimate our model on sample of parents having at most 4 children, then on working parents only. The general conclusion is pretty clear about the estimates in women budget share equations. In comparison with the benchmark model, the parameters of the number of children increase in absolute value and are quite stable.

Table 7 reports the results of Hansen's test and LR type statistics. We note that the large value of the objective function in the benchmark women budget share equations lead us to reject the null hypothesis that all instruments are valid. However, overindentification restrictions are not rejected in the case of men budget share. For males, the p-value of the variants confirm the robustness of our base-model except when we introduce a cubic term in Engel curves. Including a cubic term for log expenditures affecting both budget shares in a meaningful way. According to the p-value, there is sufficient evidence to reject the null hypothesis. For women, all specifications are rejected at usual significance levels. Except for the third model where the prices of clothing have statistically no effect on the resource shares. The model without scale economies are strongly rejected regarding the p-value. This evidence supports the hypothesis of scale economies.

Table 6 presents further results of robustness. We note that the regression coefficient of the squared number of children turns out to be negative and insignificant for the sample of working fathers and still negative but statistically insignificant for number of children. The rejection of children status variables was predictable since the exclusion of nonworking fathers significantly reduces the sample to only 598 fathers. The lack of information due to the small size of the sample might explain such findings. In that respect, not much can be inferred by estimating the budget share of only working men since they represent virtually half of the few samples of single fathers. Our results suggest that we should be modest. At this level, we may cautiously deduce that children seem to be weighed more on single mother's budgets in terms of share of total expenditures they deprive their parents.

5 Conclusion

Several models attempted to assess the cost of children for parents. However, these are focused only on children living in bi-headed households while most OECD countries have experienced a demographic reconfiguration towards single parenthood. In this paper, we adapted the BDH's model to single parents. Our fundamental aim is to measure the extent to which single parents privately invest in their children. To this purpose, we assume that parents' investment in children depends both on parent characteristics that define their caring preferences (altruist or egoistic) and children's ones. Our model also allows us to identify the presence of economies of scale between children. To test the validity of our model, we use data from UK Family Expenditure Survey (FES) over the period 1978-2007. The results confirm what was found by BDH, namely the cost of children increases with the number of children but decreases with family size. The presence of three children in the household is worth respectively and roughly 59 and 58 percent of total resources of lone mother's and father's. However, the arrival of a new child shrinks total allocated resources to meet children's needs to 59 and 64 percent of single mother's and father's total expenditures. Our results show that children globally cost more to single mothers than single fathers. But the latter reduces their backwardness compared to single mothers as the number of children increases.

Finally, it seems important to stress some limits of our paper. Technically speaking, the resource share equations need to be more flexible. Moreover, shadow prices need further theoretical consideration so that they have been fully identified. Regarding endogeneity issues, our instruments are not sufficiently exogenous to strengthen the relevance of our results even if they are highly correlated with total expenditures. Some critics might raise a potential correlation between the number of children in the household and the residuals in the clothing equations even though we fail to find sufficient evidence for that claim. Moreover, our model does not take into account the single parent status. We have no information about the causes of single parenthood. Meanwhile, most countries legally recognize shared custody. In this sense, if the single parent is not widowed, the custody of the children is shared between both parents. In this regard, there is a broad avenue to conduct further research. An issue that could be interesting to analyze either in one-headed or bi-headed households is to measure the cost of time that parents spend with their children. This allows to recover at least some subjective aspects which are not necessarily measurable such as parent love and affection transmitted to children. In such a model, the economies of scale could also be explored. In fine, we could have also tested how the presence of siblings of same sex and of close-aged children impacts the parent's resources. Unfortunately, the empirical testing of these further economies of scale has been hampered by the inadequacy of data on children's age. In the database, the age of children is the mean age of overall children in the household. We need the distribution of age by child so that we can explore if the close-in-age of children generates economies of scale. We guess it is the case, but we cannot empirically test it which leaves the way open for future research.

6 References

6.1 Papers

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7 Appendix

A A parametric Example

Consider the case of a single parent possessing a Cobb-Douglas utility function - $\mu_i = \alpha_i ln(x_i) + (1-\alpha_i)ln(X_i)$, with $i = m, w$, where α_i is a parameter, and whose properties are well known. Also, let embed the preferences of children in a simple logarithmic utility function $u_c = ln(X_c)$. Recall that the two-stage budgeting process simply means that on the one hand, the parent defines a sharing rule that governs the distribution of resources between her and his children. On the second hand, each household member decides alone, based on his share of income how to reach the maximum level of utility through its consumption of goods. Thus, solve separately the parent and children optimization program consisting to:

$$\begin{aligned} \max_{x_i \in \mathbb{R}_+, X_i \in \mathbb{R}_+} \quad & u_i = \alpha_i ln(x_i) + (1-\alpha_i)ln(X_i) \\ \text{s.t.} \quad & p_i x_i + A_i X_i = y\phi_i \end{aligned} \tag{20}$$

and

$$\begin{aligned} \max_{X_c \in \mathbb{R}_+} \quad & u_c = ln(X_c) \\ \text{s.t.} \quad & X_c = y\phi_c \end{aligned} \tag{21}$$

to get the solutions as: $x_i = \alpha_i \frac{y\phi_i}{p_i}$ and $X_i = (1-\alpha_i)\frac{y\phi_i}{A_i}$ for parents and $X_c = y\phi_c$ for children. Using these results as an input to the decentralized problem of the equation () to obtain the decentralized parametric optimization program as follows:

$$\begin{aligned} \max_{\phi_i \in \mathbb{R}_+, \phi_c \in \mathbb{R}_+} \quad & \mu_i ln(\phi_i) + \mu_c ln(\phi_c) + ln(y_i) + \mu_i \Lambda_i \\ \text{s.t.} \quad & \phi_i + \phi_c = 1 \\ \text{with} \quad & \mu_i + \mu_c = 1, \;\; \text{where} \;\; \Lambda = \alpha_i ln\left(\frac{\alpha_i}{p_i}\right) + (1-\alpha_i) ln\left(\frac{1-\alpha}{A_i}\right) \end{aligned} \tag{22}$$

Solving the latter problem gives the sharing rules as a function of bargaining weights, namely $\phi_i = \frac{1}{1+\delta_i}$ and $\phi_c = \frac{\delta_i}{1+\delta_i}$. The parameter δ_i may be seen as a degree of altruism of the parent. The more the parent is altruistic, the lower his share in total expenditures will be.

We have previously assumed that the function κ_i does not depend on total expenditures. We will show that this is related to the altruistic characteristics of the utility function. In the equation (5) representing the additive utility function of the household, we said that the parameter δ_i may be seen as a weight the parent gives to the child. Let re-express the equation (5) by sharing the weight on each household member.

$$U_i = \delta_i[\alpha_i ln(x_i) + (1-\alpha_i)ln(X_i)] + (1-\delta_i)ln(X_c) \tag{23}$$

As before, we can obtain the associated value function given the optimal x_i^*, X_i^* and X_c^* as follows:

$$V_i = \delta_i[\alpha_i ln(\phi_i) + \delta_i(1-\alpha_i)ln(\phi_c)] + ln(y) + \delta_i(\Lambda).$$

Substituting the indirect utility function into the equation () gives rise to the following program:

$$\max_{\phi_i \in \mathbb{R}_+, \phi_c \in \mathbb{R}_+} \quad \mu_i[\delta_i ln(\phi_i) + (1-\delta_i)ln(\phi_c) + ln(y_i) + \delta_i \Lambda_i] \tag{24}$$

As we can see, the weight that parent associated to each utility function δ_i is independent from total expenditures. This simple Cobb-Douglas parametric example proves that there is a theoretical support to the assumption of the independence of total expenditures in the κ_i function.

B Proof for Identification

In the case of a one-headed household, the budget share functions are the form:

$$\omega_i = \Phi_i g_i \left(\frac{p_i}{A_i}, \frac{y_i \Phi_i}{A_i} \right) \tag{25}$$

for each individual $i = w, m$, where the function $g_i(.)$ is known from the estimations on single individuals. Also assume that

$$\frac{\partial ln g_i}{\partial ln(y \frac{\Phi_i}{g A_i})} + 1 \neq 0 \quad \text{everywhere,} \tag{26}$$

that is, the elasticity of g_i with respect to $(y_i \frac{\Phi_i}{A_i})$ is different from -1 or, equivalently the income-eslasticity of the demand for the exclusive good is different from zero (a rather weak condition). Then from the implicit function theorem we can write:

$$\Phi_i = G_i \left(\frac{p_i}{A_i}, \frac{y_i}{A_i}, \omega_i \right), \tag{27}$$

where G_i is the inverse of (20) with respect to Φ. The individual share of each adult is thus identified up to a function $A_i(y_i, p_i, n)$.

C Additional Estimation Results

Table 8: Resource Shares of Parents with one child for different percentiles of Total Expenditures

Resource shares	Mother's share		Father's share	
	Est. val.	Std. err.	Est. val.	Std. err.
Parents out of the labor market				
5th percentile of total expenditure	0.4624***	(0.0819)	0.3864***	(0.0963)
25th percentile	0.5719***	(0.0685)	0.5624***	(0.0740)
50th percentile	0.6452***	(0.0685)	0.6699***	(0.1008)
75th percentile	0.7263***	(0.0744)	0.7867***	(0.1251)
95th percentile	0.8438***	(0.0773)	0.8806***	(0.1168)
99th percentile	0.9117***	(0.0638)	0.9421***	(0.0840)
Parents on the labor market				
5th percentile of total expenditure	0.4624***	(0.0819)	0.3912***	(0.1177)
25th percentile	0.5719***	(0.0685)	0.5674***	(0.0739)
50th percentile	0.6452***	(0.0685)	0.6744***	(0.0871)
75th percentile	0.7263***	(0.0744)	0.7901***	(0.1099)
95th percentile	0.8438***	(0.0773)	0.8827***	(0.1057)
99th percentile	0.9156***	(0.0638)	0.9432***	(0.0775)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

Table 7: General Tests

Models		Mother				Father			
		Sargan statistics	LR-type statistics	Degrees of freedom	p-value	Sargan statistics	LR-type statistics	Degrees of freedom	p-value
Reference model		28.35		16	0.005	19.15		16	0.261
Model with	linear time trend in S	22.44	5.91	1	0.015	18.75	0.4	1	0.527
	linear time trend in K	22.32	6.03	1	0.014	19.06	0.09	1	0.764
	prices of clothing in S	26.77	1.58	1	0.209	18.78	0.37	1	0.543
	cubic term in Engel curves	36.29	7.94	1	0.005	8.89	10.26	1	0.001
Model without scale economies		6.68	21.67	1	0.000	4.07	15.08	1	0.000

Table 9: Resource Shares of Parents with two children for different percentiles of Total Expenditures

Resource shares	Mother's share		Father's share	
	Est. val.	Std. err.	Est. val.	Std. err.
Parents out of the labor market				
5th percentile of total expenditure	0.4693***	(0.0786)	0.2421***	(0.0759)
25th percentile	0.5787***	(0.0689)	0.4151***	(0.0512)
50th percentile	0.6516***	(0.0713)	0.5155***	(0.0903)
75th percentile	0.7318***	(0.0780)	0.6251***	(0.1376)
95th percentile	0.8474***	(0.0796)	0.8139***	(0.1677)
99th percentile	0.9177***	(0.0647)	0.8802***	(0.1467)
Parents on the labor market				
5th percentile of total expenditure	0.3277***	(0.0677)	0.2325**	(0.0925)
25th percentile	0.4236***	(0.0528)	0.4031***	(0.0473)
50th percentile	0.4939***	(0.0515)	0.5031***	(0.0691)
75th percentile	0.5794***	(0.0634)	0.6134***	(0.1154)
95th percentile	0.7249***	(0.0905)	0.8062***	(0.1564)
99th percentile	0.8382***	(0.0936)	0.8749***	(0.1406)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

Table 10: Resource Shares of Parents with three children for different percentiles of Total Expenditures

Resource shares	Mother's share		Father's share	
	Est. val.	Std. err.	Est. val.	Std. err.
Parents out of the labor market				
5th percentile of total expenditure	0.2460***	(0.0563)	0.2106***	(0.0594)
25th percentile	0.3215***	(0.0462)	0.2901***	(0.0589)
50th percentile	0.3806***	(0.0441)	0.3615***	(0.0807)
75th percentile	0.4557***	(0.0544)	0.4502***	(0.1232)
95th percentile	0.5890***	(0.0890)	0.5769***	(0.1825)
99th percentile	0.6934***	(0.1110)	0.7527***	(0.2183)
Parents on the labor market				
5th percentile of total expenditure	0.2424***	(0.0577)	0.2040***	(0.0675)
25th percentile	0.3173***	(0.0463)	0.2819***	(0.0593)
50th percentile	0.3760***	(0.0421)	0.3523***	(0.0704)
75th percentile	0.4508***	(0.0500)	0.4403***	(0.1060)
95th percentile	0.5843***	(0.0841)	0.56712***	(0.1642)
99th percentile	0.6892***	(0.1072)	0.7452***	(0.2068)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.

Table 11: Resource Shares of Parents with four children for different percentiles of Total Expenditures

Resource shares	Mother's share		Father's share	
	Est. val.	Std. err.	Est. val.	Std. err.
Parents out of the labor market				
5th percentile of total expenditure	0.1821***	(0.0526)	0.1540***	(0.0537)
25th percentile	0.2856***	(0.0407)	0.2119***	(0.0528)
50th percentile	0.3411***	(0.0409)	0.3141***	(0.0852)
75th percentile	0.3986***	(0.0505)	0.3979***	(0.1306)
95th percentile	0.5305***	(0.0888)	0.6250***	(0.2369)
99th percentile	0.6731***	(0.1237)	0.7737***	(0.2444)
Parents on the labor market				
5th percentile of total expenditure	0.1804***	(0.0533)	0.1510***	(0.0575)
25th percentile	0.2833***	(0.0406)	0.2080***	(0.0545)
50th percentile	0.338***	(0.0395)	0.3090***	(0.0787)
75th percentile	0.3959***	(0.0479)	0.3922***	(0.1205)
95th percentile	0.5276***	(0.0857)	0.6194***	(0.2270)
99th percentile	0.6706***	(0.1214)	0.7696***	(0.2388)

Notes: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parenthesis.