THEMA théorie économique, modélisation et applications

THEMA Working Paper n°2022-21
CY Cergy Paris Université, France

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OCTOBER 2023

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October 20, 2023

Abstract

The literature regarding the cost of children exclusively focused on children living with couples. The purpose of this paper is to explore the extent to which tailoring the collective approach to single headed households can facilitate the estimation of the cost of children borne by a single-parent. To that end, I use the UK Family Expenditure Survey over the period 1978-2020. The inferences of children cost rest on the assignable goods method and the assumption of orthogonality of parents' tastes and demographic change. The results show that the costs of supporting children is significantly similar for a representative parent, whether father or mother. However, the average cost of a child amounts to, respectively, 35% and 27% of total expenditures of single fathers and mothers. Overall, the weighted average cost of children for fathers is around 6 percentage points higher than that of mothers. Also, the findings indicate that the resource per child is invariant from the number of children for wealthiest parents, while children from low-income families derive less from their parents' total expenditures with larger family size. Finally, there is evidence of economies of scale with the presence of same-gender siblings.

JEL Classification: C30, D11, D12, D36, D63, I31, J12, J13

Keywords: Collective Model, Shadow price, Economies of scale, Identification, Resource sharing.

*I am indebted to Olivier Donni for his helpful comments along the writing of this paper. Of course, all errors remain mine.

1 Introduction

The cost of child-rearing is credited as one of the main determinants of transfer family policies. Thus, treating equitably single and couple parents through a tax system deduction requires knowing the cost of children in each of these family types. In recent times, economists have developed complex methods to assess the true cost of children, with a specific focus on those residing in couple (Bradbury, 1994, 2008; Bourguignon, 1999; Apps and Rees, 2002; Blundell et al., 2005; Bargain et al., 2010; Bargain and Donni, 2012; Dunbar et al., 2013; Adda et al., 2017; Penglase, 2020; Bargain et al., 2022). The concern is that, if government transfer policies rely on the results of such studies, they are likely to treat parents living as couple and single parents uniformly in terms of the cost of children. Furthermore, if the pattern of intrahousehold inequality is different across parent-type structure, the standard resource shares (computed for 2-parents household) are invalid measures of individual well being of single parents.

Yet, recent studies have documented that one-headed and bi-headed households are in many aspects heterogeneous (DeLeire and Levy, 2005; Nieuwenhuis and Maldonado, 2008).[1] Single-parent families are more likely to have to negotiate the complexities of a triple bind: the interplay between inadequate resources, inadequate employment, and inadequate policies (Nieuwenhuis and Maldonado, 2008). Research conducted by Casper et al. (1994) shows that single mothers and their children are especially vulnerable to poverty. Many researchers echoed that negative outcomes experienced by children in single-parent families are the result of the economic disparity between one and bi-headed households (Demo and Acock, 1996 and Lamb et al., 1999). In an annual publication on "Living Standards, poverty and inequality in the UK", Cribb et al. (2017) report that children living with both parents working have a median (equivalized) income per week that is 24 percentage points greater than that of children raised by a lone working parent. When examining material deprivation, they found that children

[1] See Edin et al. (1997), Grogger (2001), Blank et al. (2003), Meyer and Sullivan (2004, 2008), Winship and Jencks (2004), and DeLeire and Levy (2005) on the changes in the material well-being of single families, speficically single mothers; Meyer and Rosenbaum (2000, 2001), Grogger (2001), Schoeni and Blank (2000), and Blundell and Hoynes (2004) on the labor supply and labor market participation of single mothers; Lazear and Michael (1980) on the material well-being of one and two-earner families, and Nieuwenhuis and Maldonado (2008) on the triple bind of single-parent families. Several key themes emerge from studies that have explored the differences between single-parent and couple parent households: income disparities, labor market participation, child development and well-being, policy implications among others. In the early 20th century, the growing economic vulnerability of single mothers led to the development of public assistance programs (Folbre, 1994).

from households with working lone parents experience the highest poverty rates, with material deprivation rates of 29%.

Statistics point to the growing importance of single-parent family structures in contemporary societies. A glance at Figure 27 shows single parents represent approximately 25% of all households with children in the USA and the UK during recent decades, which is a significant proportion.[2] While several papers exist on the estimation of the children's resource shares, as far as I know, none have focused on the main question this paper addresses, which is the share of total household expenditures going to children in a one-headed household. See, e.g., Browning (1992) for a survey of the cost of children literature. Do the child-rearing costs borne by single fathers differ from that of single mothers? For this purpose, I construct and estimate a static model of intra-household allocation to investigate how changes in the parent and children's characteristics translate into changes in individual-level allocation. The model is estimated on a sample of one-adult households with 3 children from the UK Family Expenditure Survey (henceforth FES) from 1978 to 2020.[3] The estimates are not a direct measure of the well-being of children as they may receive transfers from another parent/agent(government) outside the household.[4] The objective is not to quantify what children receive, rather it centers on the costs incurred by parents in raising children.

This research becomes possible due to relatively large sample data available on single household expenditures. However, given that expenditure surveys commonly provide household-level consumption data, addressing such a question becomes challenging. Economists have long been studying this matter. Initial research about the equivalence of scales dated back to Engel (1895) fits into the so-called unitary approach (also known as the conventional approach).[5] However, for over three decades, there has been a consensus among researchers that individuals within the household have conflict interests. Then it is unsuitable to treat them as if they were single

[2]See Appendix E.

[3]Family Expenditure Survey (FES) has been replaced by Expenditure and Food Survey (EFS) in 2001, then Living Costs and Food Survey from 2008 onwards. For the sake of convenience, I use FES to qualify all three.

[4]See Folbre (2008) for an in-depth analysis of how conceptualizing the cost of children. Research on child development includes numerous articles addressing external investment in children. Those interested in delving deeper into this issue should refer to the works of Costas Meghir on Early Childhood Interventions. Also, the consideration of cognitive skills investment and its lasting effects on children is explored by Cunha et al. (2010), Del Boca et al. (2014, 2016), Agostinelli et al. (2023).

[5]The Rothbarth's approach has shaped the work of Lazear and Michael (1988), Deaton and Muellbauer (1986), Deaton (1989), and Gronau (1991).

decision units as usually done in classical microeconomics textbooks. Therefore, recent studies on children costs (Blundell et al., 2005 hereafter BCM; Bargain and Donni, 2012a; Bargain et al., 2014; Dunbar et al., 2013 hereafter DLP; Penglase, 2020; and Bargain et al., 2022 hereafter BDH) adopt the collective approach previously designed by Chiappori (1988, 1992), Apps and Rees (1988), Bourguignon and Chiappori (1992), Browning et al. (1994), and Browning and Chiappori (1998) to restore the methodological failure in the conventional approach. Nevertheless, these studies differ in the way they model household preferences and estimate the structural parameters.

I develop a structural consumption model for single individual and single-parent families to identify the intrahousehold resource allocation into a variant of the collective approach proposed by Bargain and Donni (2012b). It was tempting to align with the wave using the traditional collective model. However, the collective-type approach seems to be appropriate with the assumption that if the parent behaves as a dictator, then children have no bargaining power in the household. The suggesting household consumption framework with children has three components: an additive utility function, a consumption technology, and a sharing rule.

In DLP, children are included in the model as distinct economic agents and they possess their own utility function with associated resource shares. Although Dauphin et al. (2011) and Cherchye et al. (2009) lend support to the consistency of observed household demand functions with the assumption of children having distinct utility functions, I assume that children bring little income and can have bargaining power only once they reach adulthood. In Dauphin et al. (2011), the considered children are 16 years and older and live in biparental families. It is reasonable to infer that children under 16, as is commonly the case in this study, are not decision-makers. Thus, I incorporate the children's utility in an altruistic way into the adult's utility as previously done by Bargain and Donni (2012b) and BDH.

Most collective household models assume that goods are either purely private or purely public within the household. Following Browning, Chiappori and Lewbel (2004)'s contributions, studies on equivalence of scales have shifted towards a more general type of scale economies. See, e.g., Bargain et al., 2010; Bargain and Donni, 2012a; DLP; Penglase, 2020; BDH. In line with Bargain and Donni (2012a) and BDH, I use a transformation à la Barten to pinpoint economies of scale.

To identify the mechanism of distribution of resources within the household, Gronau (1991) estimates the observed effect of environmental variables on the marginal propensity to consume adult goods. I estimate the individual-level consumption through the identification of the sharing rule. The sharing rule is a function that governs the allocation of total expenditures between parents and children in the model. Since Chiappori's seminal papers on the collective model (1988, 1992), numerous studies have attempted to estimate resource shares.[6] See, e.g., Bourguignon and Chiappori (1994), Browning et al. (1994), Browning and Chiappori (1998), and Vermeulen (2002). However, they fall short of identifying the level of resource shares and have primarily concentrated on childless couples. In contrast, I identify the level of resource shares through the observation of assignable goods, say clothing in my case, using a sample of singles (individual and parent). The use of clothing as an assignable item is now widely accepted in the literature on the cost of children. This approach has found application in various studies, whether employing a non-structural model like Lundberg, Pollak, and Wales (1997) or adopting structural models such as DLP, Penglase (2020) and BDH. Unlike DLP and Penglase (2020), whose the level of resource shares identification relies on excluding total expenditures from the sharing rule, I argue that the sharing rule is a function of total expenditures.

I also pick up the assumptions of separability and preference stability traced back to the work of Gronau (1991) to identify the model.[7] On the one hand, Gronau (1988, 1991) has shown that the separability assumption does matter to ensure the inferences of the children cost.[8] This assumption was taken up in successive studies regarding the equivalence of scales.[9] On the other hand, by preference stability assumption, the tastes of parents are assigned to be independent of demographic change. Both assumptions are still widespread in the field. See, e.g., Bargain et al. (2010), Bargain and Donni (2012a) and BDH. Exceptions include Cherchye et al. (2010), DLP and Penglase (2020), which generalizes the DLP approach to measure

[6] Resource shares stand out as meaningful metrics for individual consumption expenditures when examining household-level data.

[7] At the same time, the preference stability assumption is also used by Deaton et al. (1989) and Tsakloglou (1991).

[8] It is worth noting that Gronau is not the first to use the separability assumption in the studies of equivalence of scales. That, in fact, is also laid down in the research of Lazear and Michael (1988), and Deaton (1989).

[9] See BCM, Bargain et al. (2010), Dauphin et al. (2011), Bargain and Donni (2012a,b), Cherchye et al. (2012), Lise and Seitz (2011) and BDH.

the consumption inequality among children in Malawi. First, Cherchye et al. (2010) adopt a different path, using revealed preference theory to identify the sharing rule in contrast to the methodology assuming preference stability. Their method entails imposing strong restrictions on preferences across households to set identify bounds on the sharing rule. Second, DLP and Penglase (2020) provide a model that semi-parametrically identifies the levels of resource shares given household-level Engel curve on private assignable goods (through what DLP called similar preference across types SAT and similar preference across people SAP).[10]

The contribution of this paper is to mark the initial estimation of the cost of children conducted on single-headed households. Also, it presents an identification result that is tailored to this collective-type approach. Additionally, this study, by accounting for single fathers as well, provides a basis for comparing the relative costs of children between mothers and fathers.[11]

In the empirical application, this study provides new evidence on the measure of the cost of children. First, the costs of children borne by a representative father is not significantly different than that borne by a representative mother; however, fathers allocate a higher average share of resources to children compared to that allocated by mothers.[12] Specifically, the average cost of a child amounts to, respectively, 35% and 27% of total expenditures of single fathers and mothers. This might explain by the fact that the transfer from noncustodial parent - more likely fathers - to custodial one - generally mothers - acts as a discount on the cost of children borne by mothers. Second, the children's cost increases with the number of children but decreases with family size as found in earlier studies. Third, the resource per child remains consistent irrespective of the number of children for wealthier parents, whereas children from low-income families receive a diminishing share of their parents' total expenditures with larger family size. Fourth, the household benefits from economies of scales through two channels: the number of children and the presence of same-gender siblings. The potential policy implications of this work are noteworthy. As an example, it provides a framework for determining payments that guarantee a single-parent household can adequately meet the needs of a child.

[10]DLP and Penglase (2020) do not use distribution factors.

[11]Prior studies on single parents have predominantly centred on mothers. See, e.g., Edin et al. (1997), Meyer and Rosenbaum (2000, 2001), Schoeni and Blank (2000), Grogger (2001), Blank et al. (2003), Blundell and Hoynes (2004), Meyer and Sullivan (2004, 2008), Winship and Jencks (2004) and DeLeire and Levy (2005).

[12]A representative parent is one with characteristics equal to the average in the population.

Furthermore, by accounting for gender-specific differences between parents, it offers insights into policy considerations regarding which single parent carries a more substantial burden of children's costs. Another illustrative scenario involves the ability to calculate child support payments from a noncustodial parent to a custodial parent in case of divorce. A further application lies in directly measuring child poverty.

The remainder of this paper is organized into four following parts. The first presents the theoretical model. The second describes the empirical implementation and data selection. The third reports and discusses the empirical results and the last section concludes.

2 Theoretical Framework

This section presents the consumption household model following Bargain and Donni (2012b) and Bargain et al. (2022). On the one hand, I describe the consumption behavior of single individuals. This introduction is a background for understanding the consumption behavior of a one-headed household with children which will be presented on the other hand.

2.1 The Consumption Behavior of Single Individuals

In this section, I model the consumption behavior of a single-adult household without children acting in a one-period. I assume that each household member has a well-behaved utility function $U_i(x_i, X_i)$, that is twice continuously differentiable, strictly increasing and strictly concave direct utility function over two arguments (an exclusive good x_i and a composite good X_i).[13] The subscript $i = w, m$ denotes respectively women and men. The individual utility is also subject to preference-driven factors that I introduce subsequently into the budget share function in the empirical section. I also assume that individual preferences over consumption bundles are stable so that some prediction could be possible about household behavior. However, household member purchases q_i and Q_i quantities of x_i and X_i respectively. Thus, each individual faces

[13] Here we could have ignored the exclusive good since there is no confusion about the individual consumption in a single household. Then there is apparently no distinction between private and exclusive goods, even private and public goods in such a type of household. However, I keep the distinction to facilitate the understanding in the next section when this will be relevant and more useful. Furthermore, a commodity good can be any other good except an exclusive good.

the budget constraint as follows:

$$q_i p_i + Q_i = y_i \tag{1}$$

where y_i denotes the total household expenditure and p_i the price of the exclusive good. The market price of the composite good is normalized to one.

Household surveys document expenditures rather than consumption per se. If one accounts for durable goods, effective consumption may not reflect expenditures at a given point in time. In this case, I consider only nondurable goods insofar as these tend toward consumption as the fraction of purchased goods that are not consumed is small. Hence, I can assume that:

$$q_i = x_i \quad \text{and} \quad Q_i = X_i \tag{2}$$

At this stage, the optimization program of the household member $i = w, m$ is as follows:

$$\max_{x_i, X_i} \quad u_i(x_i, X_i) \text{ subject to (1) and (2)} \tag{3}$$

The solution of this program allows expressing the demand functions for the exclusive good as:

$$\omega_i = g_i(p_i, y_i) \tag{4}$$

where the subscript $i = w, m$ and $\omega_i = p_i x_i / y_i$. It is worth noting that $U_i(.)$ is strictly increasing, then ω_i must exhaust the consumer's income.

2.2 The Consumption Behavior of One-Headed Households with Children

In this section, I model the consumption behavior of a single-parent household. Each single parent has a well-behaved utility function $W_i[u_i(x_i, X_i), u_c(X_c)]$ that contains two components - the first sub-utility derived from his/her own consumption u_i and the other one from his/her representative child's consumption u_c. The analysis is conducted under the hypothesis that each parent is altruistic toward his children and that, the utility of children is that perceived by his/her parent.[14] Conversely, children are egoistics. I consider an additive utility function taken the form as follows:

$$W_i = u_i(x_i, X_i) + \delta_i(n)u_c(X_c) \tag{5}$$

represents the single parent total utility function, where x_i stands for the adult assignable good, X_i and X_c are respectively the composite goods for parent and children.[15] The utility function of children has one argument for the sake of simplicity and it is associated with a parameter δ_i which measures how the resources devoted to children evolve when the number of children increases.[16] The parameter δ_i could also be seen as the weight given by the adult to the child (Bargain and Donni, 2012b). Alternatively, one might interpret it as the degree of altruism of parent as shown in Appendix A. Note that the previous model is a special case of this one because if $n = 0$, this implies $\delta_i = 0$ and then one goes back to the standard consumption model for a single-adult household.[17] In resume, the single parent cares about the children welfare. I assume that the utility function is endowed with caring preferences. This is to say that parents care about their children's allocation only insofar as it gives them some individualistic welfare. In other words, children matter for the household's choices, but only through the utility their parents derive from their well-being (Browning et al., 2014, p.89).

I consider a simple model in which household income is given. Moreover, I assume there are neither time-allocation decisions nor household production.[18] Household income is totally spent for purchasing q_i quantities of assignable goods and Q_i quantities of composite goods. Hence, y_i denotes total expenditures instead of total income. The household budget constraint is the following:

$$q_i p_i + Q = y_i \tag{6}$$

where z_i and Z denote respectively the purchased quantities of parent's exclusive goods and of composite ones. Recall that the consumption of children is included in the composite goods.

[14] See Appendix A for technical details.

[15] Assignable good and exclusive good are used interchangeably as well as single parent and lone parent. See Browning et al. (2013) for more details about the notions of exclusive and assignable goods.

[16] See Appendix A for technical details about δ_i.

[17] We see this in more detail along with this subsection.

[18] This broader perspective is addressed by Apps and Rees (2001) and Cherchye et al. (2012).

There are two types of goods. A private assignable good x_i like adult clothing and all other goods that are not privately assignable X_i and X_c such as food.

Exclusive goods are purely private, that is, for any demographic structure of the household, the consumption of exclusive goods reflects exactly what is expended by the household. Thus,

$$q_i = x_i \tag{7}$$

In a household with at least two individuals - an adult and a child - some goods are endowed with public properties. Consequently, the consumption of these goods cannot be accurately captured solely by their purchased quantities. Hence, their purchased quantities are transformed into a higher level of consumption with a transformation rate that depends on all three exogenous variables for now:

$$Q = \pi_i(y_i, p_i, n_i)X_i + X_c \quad \text{with} \quad i = w, m \tag{8}$$

where $\pi_i(y_i, p_i, n_i)$ denotes shadow prices for the parent. I assume the shadow price of children is equal to one. Put equations (7) and (8) into the household budget constraint (6), and one obtains:

$$x_i p_i + \pi_i(y_i, p_i, n_i)X_i + X_c = y_i \tag{9}$$

Parents maximize their own utility subject to the new budget constraint (9). Note that with one adult household, the outcome resulting from the parent's decision is automatically Pareto efficient. This follows from this assumption:

Assumption 1 *The adult acts as a dictator in the household; assuming the role of decision-maker for his/her child.*

In their research, Dauphin et al. (2011) find evidence that children are decision-makers within the household. However, their study is restricted to a sample of children being 16 years old and over. Yet, in this paper, the average age of children is 7 years old. At this age, children do not work, they are unlikely to bring income in the household and they mostly depend on their parent. Therefore, it is reasonable to infer that children have no bargaining power in the household.

The trade-off that needs to be done will happen in the allocation of resources for the parent and children consumption. That is, given the budget and technology constraints, parents cannot make children better off without making themselves worse off. Hence, given Becker's altruistic preferences and the efficiency assumption, the household allocation may be derived from the following optimization program:

$$\begin{aligned} &\max_{x_i, X_i, X_c} u_i(x_i, X_i) + \delta_i(n) u_c(X_c) \\ \text{s.t.} \quad & x_i p_i + \pi_i(y_i, p_i, n_i) X_i + X_c = y_i \end{aligned} \tag{10}$$

where δ_i represents the weight assigned to the child by the parent and depends on the number of children for simplicity. The budget constraint exhibits total expenditures on adult and child consumption.

The adoption of an additive utility function may ease the transition to a decentralized program. The first stage is characterized by the mechanism determining the distribution of resources between parent and children in the household. It results from this maximization program:

$$\max_{\phi_i, \phi_c} \quad \nu_i \left(\frac{p_i}{\pi_i}, y_i \frac{\phi_i}{\pi_i} \right) + \nu_c \left(y_i \frac{\phi_i}{\pi_i} \right) \tag{11}$$

where ν_i and ν_c denote respectively the indirect sub-utility function of parent and children. In the cost of children literature, it is a usual practice to assume that parent's preferences can be segregated between their own consumption and consumption related to their children (Gronau, 1991; Bourguignon, 1999; Cherchye et al., 2012 and BDH, 2022). The second stage leads to the solution of the single-parent decision:

$$\max_{x_i, X_i} \quad u_i(x_i, X_i) \quad \text{s.t.} \quad x_i p_i + \pi_i(y_i, p_i, n_i) X_i = y_i . \phi_i(y_i, p_i, n_i) \tag{12}$$

for some function $\phi_i(y_i, p_i, n_i)$ such that $\phi_i(y_i, p_i, n_i) \leq 1$ and $n > 0$. The total expenditure in multiplying by ϕ_i can be divided into two parts. Thus, ϕ_i represents the single parent share of total expenditure and the remaining, say $\phi_c = 1 - \phi_i$, the fraction of total expenditure that is allocated to children, say, the cost of children. As the budget share equations are homogeneous of degree zero, the solution can be written as:

$$\frac{\omega_i}{\phi_i(y_i, p_i, n_i)} = g_i \left(\frac{p_i}{\pi_i(y_i, p_i, n_i)}, y_i \frac{\phi_i(y_i, p_i, n_i)}{\pi_i(y_i, p_i, n_i)} \right) \tag{13}$$

where $\omega_i = p_i x_i / y_i$. The price of the exclusive goods may affect the share of total expenditure devoted to children, as discussed in Bargain and Donni (2012b). Notice that the stability of adults' preferences upon the exclusive goods means that the presence of children in the family does not alter individual preferences. This process highlights why detailing the child's utility function is not crucial, as it does not dictate the model's outcome. Only the parent's demand function holds significance.

2.3 Identification

An important question in the model of consumer behaviour under study is in regards to the sharing rule. Partly, the answer to this question lies in the hypothesis regarding the state of individual preferences from childless individuals to single parents. The identification of the model rests on the hypothesis of preference stability, the existence of assignable goods and the non-linearity of the Engel curve. As Bargain et al. (2013), I assume that the preferences of individuals do not change with family status, then stable over time. For this study, this means that the preferences of single individuals and single parents are similar. Under such a framework, the estimation of the sharing rule parameters of single parents and children is possible through the information provided by the demand functions of single individuals since indifference curves are unchanged with the occurrence of children in the household. As such, any shifts in consumption among single parents due to the presence of children should be attributed to changes in the household composition rather than shifts in individual preferences from childless individuals to parents.

Chiappori and Eckeland (2009) mentioned that identification requires the estimation of at least three goods. However, Bourguignon (1999) and Bourguignon et al. (2009) demonstrated that having an assignable good suffices to recover the sharing rule and reach identification. Hence, the identification also depends on the existence of assignable goods (clothing) that are observable. Finally, Prais and Houthakker (1955) found evidence to prioritize nonlinear Engle curves and add explicitly socio-demographic characteristics as control variables.

For identification purposes, I need an assumption regarding technology consumption and resource shares. There are currently two approaches in the literature to account for economies of scale. Either they assume the independence of base technology of production (Lewbel and Pendakur, 2008; Bargain and Donni, 2012a; DLP and Bargain and al., 2014), or the transformation à la Barten (Browning et al., 2014 and BDH). In the former approach, the cost of children does not capture variation in prices. Whereas, the latter approach relaxes the independence assumption regarding individual consumption shares and levels of total expenditures, and it allows the exploitation of price variations to achieve economies of scale. The following assumption imposes certain restrictions on the shadow prices and budget shares.

Proposition 1 *Let the demand functions of the exclusive good respectively for single individuals and single parents be:*

$$\begin{aligned} \omega &= g(z_\omega, p, y) \\ \omega &= g(z_\omega, \pi(p, z_\pi, y, n) \cdot p, \phi(p, z_\phi, y, n) \cdot y) \end{aligned}$$

where $\pi(p, z_\pi, y, n)$ denotes the (price) transformation à la Barten and $\phi(p, z_\phi, y, n)$ the sharing rule. Here, z_ω, z_π and z_ϕ denote the sociodemographic variables, p the price of exclusive good, y the total expenditures and n the number of children.

If any of the following conditions are met and under regularity conditions, the functions $\pi(p, z_\pi, y, n)$ et $\phi(p, z_\phi, y, n)$ can be generically identified.

1. *At least one variable in z_ω must be excluded from z_π and z_ϕ.*

2. *π and ϕ are independent of y (see Dunbar et al., 2012 and Penglase, 2020 for the proof).*

3. *π and ϕ are independent of p.*

4. *$\pi(p, z_\pi, y, n)$ and $\phi(p, z_\phi, y, n)$ are known up to a constant (semi-parametric identification)*

5. $\pi(p, z_\pi, y, n) = \pi_1(p, z_\pi, y) \cdot \pi_2(n)$ *with* $\pi_2(1) = 1$ *and* $\phi(p, z_\phi, y, n) = \phi_1(p, z_\phi, y) \cdot \phi_2(n)$ *with* $\phi_2(1) = 1$.

Proof. Why conditions (1)-(3) can allow retrieving the structural components in the model? Let suppose the price is excluded from z_ω. Consider two arbitrary values of prices and presume that the other variables are given. This provides a system of two equations in two unknowns:

$$\omega(p_1, \bar{z}_\omega, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n}) = g(p_1, \bar{z}_\omega, \pi(\bar{z}_\pi, \bar{y}, \bar{n}) \cdot p_1, \phi(\bar{z}_\phi, \bar{y}, \bar{n}) \cdot \bar{y})$$

$$\omega(p_2, \bar{z}_\omega, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n}) = g(p_2, \bar{z}_\omega, \pi(\bar{z}_\pi, \bar{y}, \bar{n}) \cdot p_2, \phi(\bar{z}_\phi, \bar{y}, \bar{n}) \cdot \bar{y})$$

Under some regularity conditions, this system of two equations has generally a unique solution for $\pi(\bar{z}_\pi, \bar{y}, \bar{n})$ and $\phi(\bar{z}_\phi, \bar{y}, \bar{n})$, and for each choice of $(\bar{z}_\omega, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n})$. The same exercice can be done by applying the exclusion restrictions to another variables. The complete proof for the statement 2 is given by Dunbar et al. (2012, online appendix) and Penglase (2020, appendix).

The proof of the fourth statement in this proposition is straightforward. I briefly sketch the proof. Remove the i to make thing simple. Define $\varphi = \phi(\boldsymbol{z}, y)y$ as the share of ressources kept by parents. Further, write

$$\Xi = \frac{\partial\omega/\partial z_k}{\partial\omega/\partial y} = \frac{\partial\varphi/\partial z_k}{\partial\varphi/\partial y}$$

where $\partial(.)/\partial z_k$ and $\partial(.)/\partial y$ are the partial derivatives of ω and φ with respect to distribution factors z_k and total expenditures y. The partials of the ω function are all identified, leading to the conclusion that the partials of $\varphi(\boldsymbol{z}, y)$ are identified as well. ∎

Assumption 2 *For any* (y_i, p_i, n_i),

a) Individual price functions or economies of scale functions are defined as:

$$\pi_i(y_i, p_i, n_i) = \tau_i(p_i, n), \quad \textit{with} \quad \tau_i(p_i, 0) = 1.$$

b) Individual share functions are defined as:

$$\phi_i(y_i, p_i, n_i) = \psi_i(y_i, p_i, n_i), \quad \textit{with} \quad \psi_i(p_i, y_i, 0) = 1.$$

This assumption shows how the presence of children affects the individual price and individual share functions. In the first case, the right-hand side term $\tau_i(pi, n)$ represents the variation in the parent shadow price stemming from the presence of children. If the parent prefers to consume public goods instead of private goods because of children, τ_i should be greater than one and less than one otherwise. This assumption clearly shows as previously argued, the childless household model is a special case of the household model with children. In other words, without children, the market price for the aggregate good is normalized to one for a single household since τ_i is equal to unity.

In the second case, function $\psi_i(y_i, p_i, n_i)$ is the fraction of resources kept by the parent to satisfy their own consumption. The remaining fraction, namely $1 - \psi_i$ is allocated to children. The share of resources accruing to children consumption is positive if and only if the parents' share of resources is less than one. The extreme case where the parent exhausts the total expenditure for themself is excluded. This leads us to an egoistic parent that cares only for themself. Therefore, a parent is expected to care for his/her children. In the absence of children, $\psi_i(y_i, p_i, 0)$ is equal to one, that is the adult individual keeps the entire budget as illustrated in the case of the single-household model. There is an intuitive way to interpret the individual share function. As previously noted, $\phi_i(.)$ is equal to one if there are no children in the household. With children, $\phi_i(.) = 1 * \psi_i(.)$. That is, from the entire budget without children, say one, parents keep for themselves a fraction of ψ_i, and the other fraction $1 - \psi_i$ goes to children.

3 Empirical Implementation

This section is structured as follows. I start with the sample selection process and an overview of the data. Then I present the empirical methodology in two steps: first I specify the model, then I present the issue of endogeneity and the mechanism to resolve it.

3.1 Sample Selection

To measure the cost of children in a single-parent household setting, I use data from the UK *Family Expenditure Survey* (FES) over the period 1978-2020.[19] The FES was closed in 2001 to become *Expenditure and Food Survey* (EFS) and then *Living Costs and Food Survey* (LCF) from 2008 onwards.[20] These surveys provide information on the socio-economic profile of households, along with details about their income and expenditure patterns. Additionally, they gather information about the region in which the households are located.

Initially and throughout the examined timeframe, the sample contains data pertaining to 135,642 households including single individuals, single parents, or bi-headed parents. The adults are aged between 18-60 years and have at most eight children. To perform the empirical analysis, I proceed to some selection. I sample childless adults and single parents aged 55 years with at most six children. BDH harbor some concerns about potential confusion between children's clothing and adult's one. To such queries, I answer that there is no way that an 18-year-old parent is wearing their child's clothes provided that this child can be at most 5 years old. Furthermore, I uphold total expenditures in a positive domain while filtering out outlier values of total expenditures for each household category and observations for which crucial data is lacking. Therefore, I arrive at a sample of 40,079 households with 13,921 single males, 10,726 single females, 1,644 single fathers, and 13,788 single mothers. One noteworthy observation is the relatively low proportion of single fathers, constituting a mere 12% of the total of single mothers. In addition, among parents, there are more than half of them having one child respectively 57% of fathers and 51% of mothers.

In the empirical analysis, I set the budget shares on clothing as dependent variables. As emphasized earlier, my focus remains exclusively on non-durable goods, as expenditures related to durable commodities do not accurately reflect their effective consumption. The demand system encapsulates two exclusive goods – male and female clothing - alongside a composite good representing omitted goods. The latter is strategically designed to maintain the total budget shares at a cumulative value of one. Prices of all goods are measured yearly at the

[19] I thank Olivier Bargain who provided me with the first wave of data that I used in the initial versions of this paper.

[20] I refer to FES hereafter to call these surveys for the sake of simplicity. These surveys are previously used by Lise and Seitz (2011) and BDH.

country level.

With regards to the covariates, I use educational attainment, age, labor force participation, and home ownership as socio-demographic variables pertaining to adults. As for children, I analyze the number of children in the household and their age, in conjunction with the proportion of boys. To leverage the economies of scale, I incorporate a dummy for the presence of siblings of same gender. The level of education is measured in terms of the number of school years completed by the individual. Labor participation and home ownership are indicated using dummies. Additionally, I include year and weekly total expenditures evaluated in pounds. To address regional differences, I draw from the dataset twelve regions of Great Britain which are Nothern, Northern Ireland, York and Humberside, East Midlands, West Midlands, East Anglia, Greater London, South-East, North Western, South Western, Wales and Scotland.

3.2 Econometric Specification

The empirical specification guides the consideration of a demand system quadratic in expenditure, which has been used in earlier studies such as Browning et al. (1994) and BDH. This parameterization overcomes the issue that marginal budget shares are independent of the level of expenditure implied by linearity hypothesis. Additionally, I introduce an error term ϵ_i which encompasses optimization errors and accounts for various factors influencing the budget allocation, but remain unaddressed by the model.

$$\omega_{is} = \alpha_i v_{is} + \beta_i \ln p_{is} + \gamma_i \ln y_{is} + \eta_i (\ln y_{is})^2 + \epsilon_{is} \tag{14}$$

for $i = w, m$, where s denoting household and α_i, β_i, γ_i and η_i represent the parameters to estimate. The vector v_{is} is a linear function of a set of variables such as education level, age, year and its square, labor force participation, home onwership and region of residence. The log price of individual clothing, log total expenditure and its square are additional explanatory variables that are used. It is of particular interest to note that the equations are gender-specific and consequently estimate separately for males and females.

As we see above, without children equation (13) is reduced to equation (4). Now, let define a dummy variable ζ_{is} equal to 1 if the adult is a parent and 0 otherwise. From this, the stochastic structure of the budget share equations for single individuals and single parents can be mathematically captured as follows:

$$\text{If} \quad \zeta_{is} = 0, \quad \text{then} \quad \epsilon_{is} = \omega_{is} - \alpha_i v_{is} - \beta_i \ln p_{is} - \gamma_i \ln y_{is} - \eta_i (\ln y_{is})^2 \tag{15}$$

$$\text{If} \quad \zeta_{is} = 1, \quad \text{then} \quad \epsilon_{is} = \frac{\omega_{is}}{\phi_{is}} - \alpha_i v_{is} - \beta_i \ln\left(\frac{p_{is}}{\pi_{is}}\right) - \gamma_i \ln\left(\frac{\phi_{is} y_{is}}{\pi_{is}}\right) - \eta_i \left[\ln\left(\frac{\phi_{is} y_{is}}{\pi_{is}}\right)\right]^2 \tag{16}$$

Now, let design a logistic function to depict the influence of parental and offspring attributes on child-related costs through the parent resource shares. Given the bounded nature of ϕ_i - the parent's share of total expenditure - ranging from zero to one, its representation as a logistic function aligns well with the approach exemplified in the research by Browning et al. (1994), Lise and Seitz (2011), and BDH (2022) among others.[21]

$$\phi(z, k) = \frac{e^{\psi_{is}(z,k)}}{1 + e^{\psi_{is}(z,k)}} \tag{17}$$

In a reversal of usual trends, I do not call on Taylor expansion to linearize the sharing rule. I express ψ_i as a deterministic function of respectively parents and children attributes, say z and k:

$$\psi(\boldsymbol{z}, \boldsymbol{k}) = \boldsymbol{z}'\Delta_z + \boldsymbol{k}'\Delta_k \tag{18}$$

where Δ_z and Δ_k are vectors of parameters. Here, $\boldsymbol{z}$ contains a constant, the adult's level of education, age, labor market status including the log total expenditures. The vector $\boldsymbol{k}$ encompasses the child-related variables, say the number of children and its square, the age of children and the proportion of children. The two latter is multiplied by the number of children. As a result, the level of resources accruing to children is assumed to depend on both sets of factors, the parent's socio-demographic variables $\boldsymbol{z}$ and the children's one $\boldsymbol{k}$. The vector $\boldsymbol{k}$ is independent of total expenditures as previously shown by BDH which the proof is given in Appendix A. Recall that the decision-making process governing resource allocation is assumed

[21] This function ensures that the share of parent total expenditures transferred to children during estimation cannot be neither negative nor exceed the parent's total expenditures.

not to be subject to children's wishes. In a one-headed household, it is irrelevant to suppose the presence of bargaining power. Nonetheless, there is at least a sharing rule inherent within the model defining the way through which parent and children variables may drive the distribution in the household. For example, I may conjecture that older children cost more. Also, I shall theoretically suppose that expenditures on children rise with the number of children.

Two kinds of economies of scale are modelized in the study. To specify the shadow prices that account for economies of scale between parent and children, I follow BDH (2022) to allow them to vary with total expenditures. As stated earlier, I also use siblings variable to capture economies of scale between children in the household.

3.3 Estimation Strategy and Instruments

The model potentially suffers from two sources of endogeneity issues. The first one is that total expenditures can suffer from measurement error. This is related to the infrequency of purchases that leads to a misrepresentation of actual consumption regarding total expenditures. This is also possibly caused by recall errors from households during surveys. Both would induce a correlation between total expenditures and the error terms in the budget share function.

I tackle two types of endogeneity issues attributable to measurement errors in total expenditures by following the DLP's approach. To do so, I use total income as an instrument. The utility function in this setting applies to a single time period t. Then, I can readily assume that consumption allocation decisions within a given time period are separable from savings decisions across periods. As a result, total income is uncorrelated with consumption allocation errors within a specific period, although exhibits correlation with total expenditures. Thus, it qualifies as a valid instrument to measure error, understood as the gap between total expenditures and actual consumption.

Endogeneity stemming from recall errors can also be dealt with by using total income. The reason is that, even if total income may also be subject to measurement error due to a misevaluation of some assets or misreporting of some others, as long as these measurement errors are orthogonal to consumption recall errors and the correlation between total income and total expenditures hold, then total income can be claimed as a good instrument.

In the fertility studies (Nakamura and Nakamura, 1992) and the demand collective models as well, which generally include the number of children as a nuisance variable, child status variables are often suspected to be correlated with the perturbations. Apps and Rees (2001) suggest that children should be treated as endogenous in the household model. DLP put forward the idea that unobserved preference heterogeneity is connected to both fertility decisions and expenditures on clothing. In short, if the number of children results from a selection process, then the number of children in the household will be endogenous.

In this model, I assume that the number of children is exogenously given. The underlying reasoning for this assumption is straightforward. The marital status of the parent is dissociated from fertility-related choices. When parents decide to have children, they do not prospect (anticipate) their singlehood. Furthermore, a lone parent is assumed to be unable to have children except by artificial insemination or adopting a child which has low probabilities.[22] Thus, a single parent is unlikely to decide how many children to have.

To set the instruments in a suitable way, I write the budget share equations (15) and (16) as a unique budget share equation. To do this, multiply equation (15) by $(1-\zeta_{is})$ if single individual and equation (16) by ζ_{is} if single parent to get:

$$\epsilon_{is} = (1-\zeta_{is})\left[\omega_{is} - \alpha_i v_{is} - \beta_i \ln p_{is} - \gamma_i \ln y_{is} - \eta_i (\ln y_{is})^2\right] + \zeta_{is}\left[\frac{\omega_{is}}{\phi_{is}} - \alpha_i v_{is} - \beta_i \ln\left(\frac{p_{is}}{\pi_{is}}\right) - \gamma_i \ln\left(\frac{\phi_{is} y_{is}}{\pi_{is}}\right) - \eta_i\left(\ln\left(\frac{\phi_{is} y_{is}}{\pi_{is}}\right)\right)^2\right]$$

Rearranging the right-hand side and obtains:

$$\omega_{is} = \alpha_i v_{is} + \beta_i \ln p_{is} + \gamma_i \ln y_{is} + \eta_i (\ln y_{is})^2 + \zeta_{is}\Theta_{is} + \epsilon_{is} \tag{19}$$

with

$$\Theta_{is} = \beta_i \ln\left(\frac{1}{\pi_{is}}\right) + \ln\left(\frac{\phi_{is}}{\pi_{is}}\right)\left[\gamma_i + \eta_i \ln\left(\frac{y_{is}^2 \phi_{is}}{\pi_{is}}\right)\right] - \omega_{is}\frac{1-\phi_{is}}{\phi_{is}}.$$

[22] Other exceptions, the fact that the single parent is able to have children with someone outside the household.

To deal with endogeneity issues, I estimate the system of no simultaneous budget share equations by setting the iterated Two Stage Least Square Method.[23] The nonlinear estimators are iterated until the estimated parameters and error/orthogonality condition covariance matrices settle.

I use as instruments all the exogenous variables, except total expenditures which are instrumented by total income. In order for total income to be a valid instrument, it must be uncorrelated with the error term in the budget share equations and partially correlated with total expenditures as assumed according to DLP. Furthermore, I set as instruments the product ζ_{is} and a second-order polynomial of all the exogenous variables that enter Θ_{is} and total income. This yields 19 instruments for each equation.

4 Estimation Results

This section presents the general findings of the model. I sum up the descriptive statistics of the sample. Then, I present the estimation results.

4.1 Sum up the Data

Table 1 reports descriptive statistics of the sample for the main variables, facilitating a preliminary analysis in the Rothbarth sense. Here are the following analyzes of clothing spending by adults. Descriptive statistics provide evidence of a reduction in adult expenses due to the presence of children, regardless of the adults' gender. As illustrated in the first panel, women and men living alone spend on average respectively £9.4 and £5.3 on clothing per week. These expenditures decrease to £7.4 and £4.3, respectively, for single mothers and single fathers with a child, representing respective declines of 21% and 18%. In addition, note that the more parents have children the less their clothing expenses will be. For instance, the average weekly expenditure on clothing for fathers drops significantly, reaching a minimum of £1.4 (£5.2 for mothers). These findings echo Rothbarth's view since the household size reduces the parents' welfare derived from consumption. Finally, the first panel also presents the percentage of zeros

[23] Recall that the female budget share equation is estimated separately from the male's one as household decisions are unilateraly taken.

Table 1: Descriptive statistics

				Single Mother			Single Father		
				Children					
				1	2	3	1	2	3
Expenditure data									
Female clothing	Weekly expenditure (in \pounds)	9.36	-	7.43	6.10	5.18	-	-	-
		(17.82)		(14.74)	(13.10)	(11.35)			
	Percentage of zeros	0.43	-	0.44	0.47	0.48			
		(0.50)		(0.50)	(0.50)	(0.50)			
Male clothing	Weekly expenditure (in \pounds)	-	5.25	-	-	-	4.30	3.76	1.35
			(15.10)				(11.778)	(10.90)	(4.51)
	Percentage of zeros	-	0.72	-	-	-	0.71	0.71	0.84
			(0.45)				(0.46)	(0.46)	(0.37)
Total weekly expenditure		105.72	111.60	126.32	132.76	135.05	144.94	150.62	143.62
		(73.99)	(82.06)	(86.89)	(86.28)	(86.39)	(90.02)	(96.48)	(75.64)
Individual and household characteristics									
Women's labor participation		0.71	-	0.50	0.43	0.29	-	-	-
		(0.45)		(0.50)	(0.50)	(0.45)			
Men's labor participation		-	0.65	-	-	-	0.55	0.52	0.39
			(0.48)				(0.50)	(0.50)	(0.49)
Women's education (in years)		12.43	-	11.70	11.57	11.27	-	-	-
		(3.40)		(2.39)	(2.25)	(2.04)			
Mens's education (in years)		-	12.28	-	-	-	11.32	11.46	11.31
			(3.44)				(2.18)	(2.19)	(2.11)
Women's age		39.10	-	34.84	33.90	33.33	-	-	-
		(11.15)		(9.17)	(7.02)	(5.92)			
Men's age		-	38.32	-	-	-	38.39	37.10	35.95
			(10.20				(9.14)	(7.90)	(7.05)
House owner		0.52	0.50	0.28	0.28	0.19	0.46	0.46	0.27
		(0.50)	(0.50)	(0.45)	(0.45)	(0.39)	(0.50)	(0.50)	(0.45)
Average age of children		-	-	7.81	7.85	7.82	8.81	8.02	8.04
				(4.84)	(3.73)	(3.09)	(5.26)	(4.10)	(3.27)
Proportion of boys		-	-	0.51	0.50	0.52	0.58	0.52	0.55
				(0.50)	(0.35)	(0.30)	(0.49)	(0.35)	(0.31)
Number of observations		10726	13921	7038	4629	1577	941	505	150

Notes: Expenditures are in 1987 pounds. Standard deviation are in parenthesis.

regarding adults clothing expenses which is quite large. This pattern corresponds with the established understanding that infrequent purchases introduce endogeneity in total expenditure, as discussed by (Keen, 1986).

4.2 Estimations

This section describes and analyzes findings related to the budget share equation detailed above.

4.2.1 Budget Share Equations

Table 2 partially presents the results of the budget share equations.[24] I estimate equation (19) for each gender (men and women) with the iterative two-stage least squares method. At first

[24] The full results of the budget share equations are reported in Table 9 in Appendix D where the estimated parameters for regions are also given.

glance, I notice that socio-demographic preference parameters do not always affect individual budget share in the same way for both adult members. My findings confirm partly what was previously found in the literature by BDH. The clothing budget share of females decreases with education and age, but increases at a certain age. Regarding age estimates, this report is true and highly significant for both genders. Finally, the results suggest that other factors being equal, house owner men spend less on male clothing than those who are not.

Table 2: Results for clothing budget share equations

	Women's budget equation		Men's budget equation	
Parameters	Est. val.	Std. err.	Est. val.	Std. err.
Intercept	0.195***	(0.015)	0.150***	(0.013)
Education	-0.001*	(0.000)	0.000	(0.000)
Age (in years)	-0.004***	(0.001)	-0.004***	(0.001)
Age^2 (in years)	0.004***	(0.001)	0.004***	(0.001)
Year	0.818**	(0.343)	1.049***	(0.381)
$Year^2$	-0.819**	(0.341)	-1.046***	(0.380)
House owner	-0.000	(0.002)	-0.003*	(0.002)
Labor participation	0.002	(0.003)	0.001	(0.002)
Log relative price	-0.003	(0.004)	0.011	(0.007)
Log total expenditures	0.004	(0.0104	0.016*	(0.009)
$(\text{Log total expenditures})^2$	-0.022***	(0.010)	-0.001	(0.006)
Sample size	24 514		15 565	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

4.2.2 Resource Share Equations

The previous findings focus solely on the impact of individual characteristics on clothing budget share. A second and more important feature of this study centers on the influence of the presence of children on parent resource share. The results for resource share equations are exposed in Table 3. It should be convenient to recall that the resource share equations allow recovering the cost of children borne by parent. Remember, resources allocated to children depend on two main sets of factors: the parent characteristics that defined the parent's egoistic or altruistic behavior, along with the total expenditures and children characteristics. As I explained above, ϕ_i represents the level of resources kept by single parents, inevitably $\phi_c = 1 - \psi_i$ the one diverted to children. As such, a negative coefficient should be interpreted as implying a rise in resources allocated to children, as it works to curtail resources taken by parents.

In this sense, the results for individual resource shares indicate that children have an augmenting effect on parent resources. This says that the negative sign of the intercept suggests that the cost of children significantly grows up as the number of children increases. But resources per child fall significantly with family size. Similar findings were previously obtained by Bargain and Donni (2012a), DLP, Penglase (2020) and BDH. Moreover, the results suggest that older children cost parents more. Although only the intercept plays a significant role in explaining the cost of children in the case of single fathers, children estimates for single fathers have the same sign as single mothers. To recall, there are very few single fathers present in the sample compared to single mothers.

I now turn to parent preference parameters. One observes that the children's share is lower as the parents' total expenditures increase. The results suggest that a 10% increase in fathers' total expenditures leads to an increase in their share by around 20%. Nonetheless, it is rational conjecture to make that affluent parents' children derive, in absolute terms, greater benefit from their parents' total expenditures compared to children of less affluent parent. The total expenditure is increasing with parent's total revenue. Thus, the wealthiest parents devote a greater part of their revenue and expenses to financing children welfare.

In summary, the presence of children in the household has a negative impact on parental resources. However, the effect of children is nonlinear on the budget share of parents. I will present a visual depiction of this later on in this paper.[25] Additionally, the parents' resource share is larger in households with a higher level of total expenditures. Finally, the dependence of the sharing functions on male total expenditure stands in contrast to the core identifying assumption that underpins various models (see Bargain and Donni, 2012a; DLP and Penglase, 2020) as outlined by BDH.

In the previous lines, I mentioned the fact that the budget share of children decreases with parent total expenditures. Given the heterogeneity of families regarding total expenditures, I report the per-child resource shares at different point of household total expenditures. To that end, I divide total expenditures in the 20th vigintile. Let concentrate on the second panel of

[25] See Figure 4.

Table 3: Estimated paramaters of the individual resource shares and individual prices

		Without Siblings		With Siblings	
		Women	Men	Women	Men
	Parent characteristics				
$\boldsymbol{z}$	Intercept	1.840***	0.725	1.854***	0.775
		(0.459)	(1.059)	(0.448)	(1.065)
	Education	0.009	-0.026	0.008	-0.022
		(0.017)	(0.064)	(0.016)	(0.063)
	Age (in years)	0.005	0.036**	0.004	0.037**
		(0.006)	(0.018)	(0.006)	(0.017)
	Labor participation	-0.083	0.236	-0.075	0.286
		(0.126)	(0.244)	(0.123)	(0.248)
	Log total expenditures	0.359	0.937**	0.445	1.990***
		(0.547)	(0.410)	(0.549)	(0.580)
	Children characteristics				
$\boldsymbol{k}$	Intercept	-0.856***	-0.730**	-0.882***	-0.876**
		(0.237)	(0.367)	(0.230)	(0.413)
	Number of children	0.101***	0.041	0.107***	0.068
		(0.032)	(0.069)	(0.031)	(0.077)
	Age (in years)	-0.010*	-0.001	-0.010*	-0.002
		(0.006)	(0.017)	(0.005)	(0.017)
	Proportion of boys	-0.034	-0.122	-0.043	-0.111
		(0.031)	(0.131)	(0.032)	(0.139)
	Same-sex siblings			0.049**	0.075
				(0.025)	(0.116)
	Shadow prices				
$\boldsymbol{\tau}$	Log total expenditures	-0.298	1.000	-0.226	1.077
		(0.387)	(0.713)	(0.417)	(0.696)
Sample size		24 514	15 565	24 514	15 565
Sargan statistics		25.51	24.24	29.79	6.62
(Nb of free parameters, Instruments)		(32, 42)	(32, 42)	(33, 43)	(33, 43)

Notes: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Figure 1. At the bottom of the distribution, the resource shares per child diverge from around 29%, (32% to 50%) respectively for one child, (two, and three children) families. That is, single-child families with limited financial means provide better conditions for their child compared to families with multiple children. Nonetheless, as parent total expenditures rise, the resource allocation per child converges to around 12%. This indicates that, regardless of the number of children, the resources per child are homogeneously distributed in the families at the top of the

distribution. This graph is rich with instructive contents. On the one hand, it conveys the idea that there exists a minimal expenditure threshold to sustain impoverished parents. This also implies a baseline level of consumption, irrespective of the number of children. Let's illustrate with an example. Suppose a single father earns the minimum wage, which is £1100 per month. Assume no family allowances are provided, etc. Additionally, suppose his subsistence total expenditures amount to £1000. For such a parent, if he has one child, that child will receive £100. However, if he has two or more children, they would divide £100 among themselves, as the parent's minimum subsistence total expenditures are £1000. In the other hand, this graph highlights that children living in affluent households experience nearly uniform levels of material well-being in terms of financial ressource they get, irrespective of family size. In essence, the number of children does not matter for wealthiest parents.

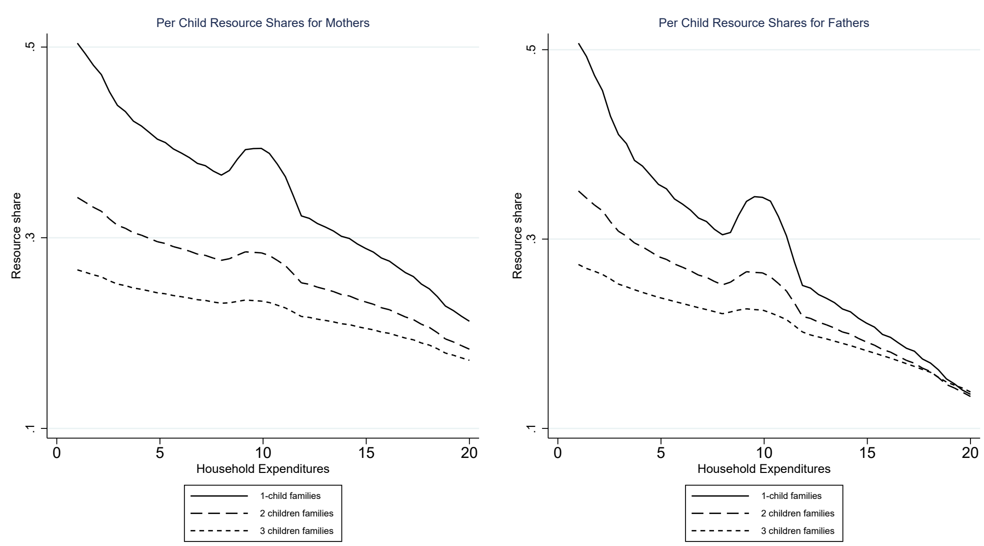

Figure 1: Children resource share by total expenditures

4.2.3 Economies of Scale

I now delve into the scenario involving the control for the presence of siblings in the model. The sign of the coefficient of the number of children provides insights into the potential existence of economies of scale generated by variations in family size. The ensuing estimates shed light on a different kind of economies of scale. In some families, siblings of same gender and close-in-age tend to share clothing. For instance, in specific family contexts, the clothings of older children are passed down to subsequent offspring, indicating a form of economies of scale among siblings. The inclusion of the siblings variable in the model facilitates an exploration of these economies of scale. This variable effectively captures both the aspect of family size and the gender-composition-related impact. It is important to bear in mind that the sibling variable is a dummy variable, denoting whether the sibling shares the same gender or not. The coefficient associated with sibling reveals that, the share of total expenditures kept by mothers is larger when the household comprises siblings of same gender.

Figure 2 provides us with valuable insights into economies of scale generated by same-gender siblings. Respectively, the straight blue and red lines indicate siblings of mixed and same gender. Figure 2 clearly depicts that, over the years, the average cost of children is lesser in families made up of siblings of same gender. Overall, families comprised of children with distinct genders experience a higher parental cost for child-rearing than families comprised of children sharing the same gender as illustrated by Figure 3.

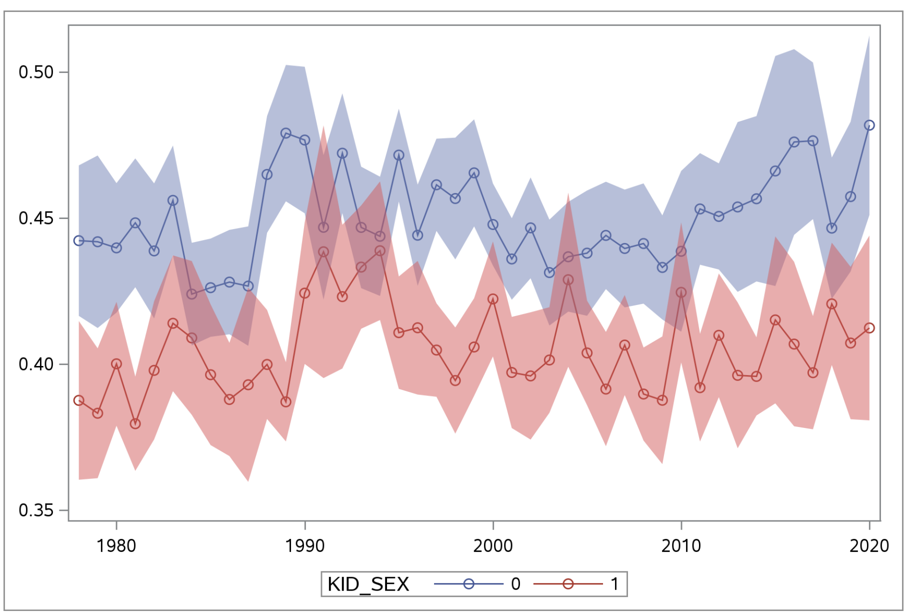

Figure 2: Share of total expenditures allocated to siblings by year

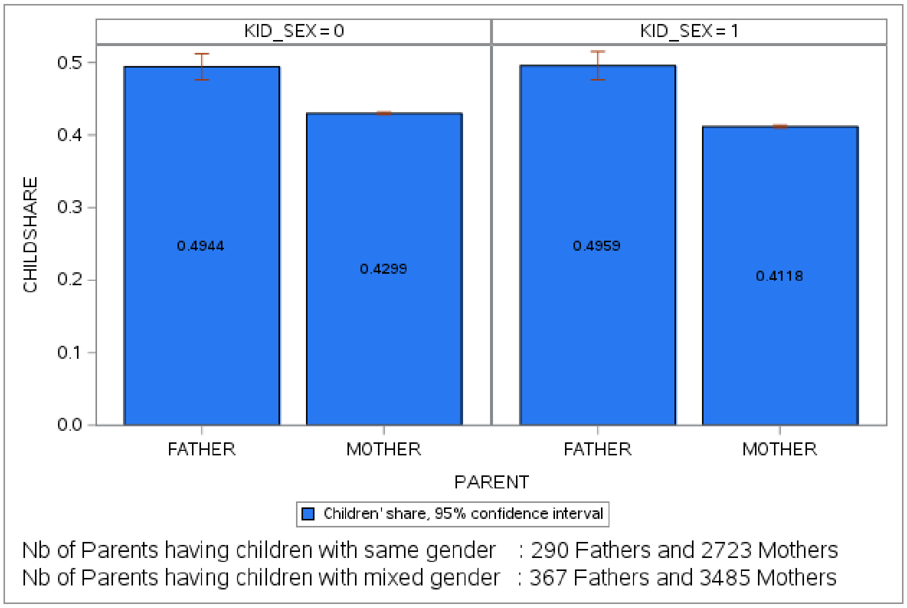

Figure 3: Share of parents resources devoted to children by the gender Composition of Siblings

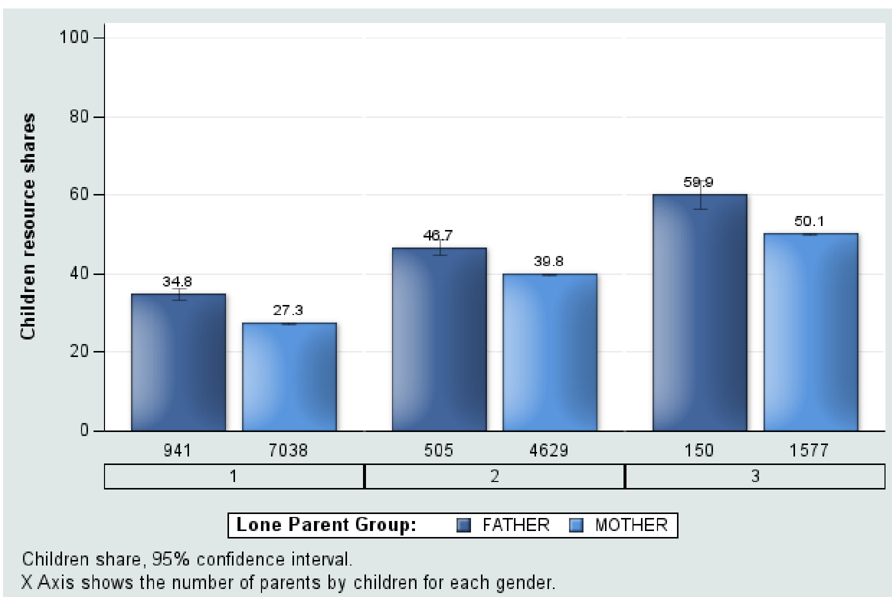

Figure 4: Share of parents total expenditures devoted to children

4.2.4 Intra-household Resource Allocation

To determine the cost of children, BDH have computed the cost of children at the average point of the sample. This approach seeks to deduce the cost of children for a representative household. In this paper, I also compute the average cost of children for fathers and mothers. The results will allow us to answer the following questions. How do the cost of children evolve with the number of children? Do children affect the resources of both single parents equally? Is the cost of children linear?

Table 4: Children resource share estimates

	$\bar{\phi_c}$						ϕ_c(mean of data)					
	Single Mothers			Single Fathers			Single Mothers			Single Fathers		
Nb. of children	Mean	Lower bound	Upper bound	Mean	Lower bound	Upper bound	Est.	Lower bound	Upper bound	Est.	Lower bound	Upper bound
1	0.284	0.282	0.286	0.348	0.333	0.363	0.316	0.248	0.445	0.303	0.187	0.367
	(0.101)	(0.099)	(0.102)	(0.236)	(0.226)	(0.247)	(0.090)			(0.058)		
2	0.397	0.394	0.401	0.467	0.446	0.488	0.524	0.309	0.726	0.454	0.380	0.538
	(0.118)	(0.115)	(0.120)	(0.241)	(0.227)	(0.256)	(0.125)			(0.049)		
3	0.497	0.491	0.503	0.599	0.563	0.636	0.682	0.375	0.932	0.620	0.527	0.711
	(0.125)	(0.121)	(0.129)	(0.224)	(0.201)	(0.253)	(0.153)			(0.046)		
Sample size		13 244			1 596			13 244			1 596	

Notes: Standard deviation and standard errors are in parenthesis. In the first part of the table, I use the structural parameters associated to z and k to compute ϕ_c for each parent, then I take the average to obtain $\bar{\phi_c}$. In the second part, I evaluate ϕ_c at the mean of z and k_{-1} for one, two and three children. k_{-1} indicates that I exclude the number of children in k

Table 4 reports the average cost of children and the cost of children for a representative parent as well. I begin with the point estimates, the second part of table 4. A representative parent is defined as a (mean) individual in the sample. The results suggest that there is significantly no difference between the cost of children incurred by a representative single mother and that of a representative single father. Whereas, regardless the number of children, the average cost of children is significantly larger for father than mother as illustrated in the first part. At first sight, this result seem unexpected.[26] But such a result may be attributed to the fact that single mothers receive support from external members of the household, such as fathers' children. In general, child custody is typically entrusted to the mother. Various factors are put forward to explain this: domestic and parental work are still predominantly undertaken by women today, women in couples with children more frequently work part-time than fathers,

[26] Cherchye et al. (2012) found that empowering fathers is most beneficial to children than empowering mothers.

and so forth. Data itself highlights this fact, with single fathers accounting for only 12% of the total of single mothers. The provision of child support and other payments by the father to the mother (typically the custodial parent) may explain this gap as it serves as a child's cost discount.

Additionally, for both techniques, the resource share allocated to children increases with the size of households. It is noteworthy that the cost of children exhibits nonlinear patterns. Figure 4 offers a clearer perspective on this aspect. For instance, the cost attributed to children for a father with tree children falls short of doubling the cost for a father with a unique child. The same pattern is observed for mothers. This might suggest a substantial drain on the income per child. As depicted in Figure 1, this aspect is particularly pronounced for children raised by economically modest households. The distribution of income per child exhibits notable disparities among impoverished households. In parallel, Figure 4 is quite striking as it may indicate significant economies of scales among children within the households. It further allows comparison between the cost borne by each single parents. As discussed earlier, one shows that fathers allocate a substantial share of their budget to children as opposed to mothers. In that respect, the average cost of children for single fathers that raises one child, (two and three children) is around 35% (47% and 60%) compared to 27% (40% and 50%) for single mothers. This difference is statiscally significant as illustrated by confidence intervals. A lingering question is: do fathers, as a whole, dedicate a larger part of their total expenditure to children than mothers? The answer is affirmative, as indicated in Table 11 in Appendix D. The weighted average cost of children for fathers is around 6 percentage points higher than that of mothers.[27] Overall, single mothers spend less on their children than their counterparts in terms of the share of total expenditures reserved for children. As fathers demonstrate greater total expenditures than mothers, one might infer that fathers are more generous than mothers. Put it in more explicit terms, fathers, in absolute value, allocate a larger portion of their resources to their children than mothers do. Table 5 test whether or not the variance between each sample is equal. The output indicates that the hypothesis of variances equality is strongly rejected.

[27] I weight by the number of children. Keep in mind that the unweighted test draws the same conclusion. See Table 11 in Appendix D.

Table 5: Test for equality of variances

	Equality of variances			
Method	Num DF	Den DF	F Value	Pr>F
Folded F	1 595	13 243	6.12	<.0001

Notes: DF for Degree of Freedom.

Table 6: Robustness tests

Models		Female				Male			
		Sargan statistics	LR-type statistics	Degrees of freedom	p-value	Sargan statistics	LR-type statistics	Degrees of freedom	p-value
Reference model		20.79		10		6.63		10	
Models with	linear time trend in z	17.46	3.33	1	0.07	5.62	1.00	1	0.32
	linear time trend in k	19.06	1.73	1	0.19	4.19	2.44	1	0.12
	prices of clothing in z	18.45	2.34	1	0.13	3.50	3.13	1	0.08
	prices of clothing in k	19.58	1.21	1	0.27	3.49	3.14	1	0.08
	cubic term in Engel curves	20.65	0.14	1	0.93	3.12	3.51	1	0.06
Models without	economies of scale	29.19	0.60	1	0.44	9.02	2.39	1	0.12
	log total expenditures in z	23.51	2.72	1	0.10	10.45	3.83	1	0.05

Notes: The first column in each panel for both females and males shows up the Sargan statistics, which are the objective function value times the number of observations. The LR-type statistics in the second column in each panel are computed as the absolute value of the difference between the Sargan statistics of the baseline model and those of the respective alternative model. It is worth noting that the objective function calculation for the alternative models is conducted using the identical baseline model weighting matrix.

4.2.5 Sensitivity Analysis

I set three procedures to check the robustness of the results. Firstly, I set up seven variants of the model. Secondly, I test overidentifying restrictions. Thirdly and finally, I estimate the model on a restricted sample of households. The core results exhibit qualitative consistency, albeit less pronounced in significance.

One of the main goal of this paper is to determine whether the sharing rule function can generate an accurate estimate of the cost of children over time. I have assumed that the sharing rule is function of both children and parent characteristics. The first two specifications I estimate is whether the results change to the inclusion of time successively in $\boldsymbol{z}$ and $\boldsymbol{k}$ parts. Table 6 reports the results of Sargan's test and LR-type statistics. The null hypothesis that the sharing rule is not affected by linear time trend neither in $\boldsymbol{z}$ nor in $\boldsymbol{k}$ is not rejected at usual significance levels. The drivers of $\boldsymbol{z}$ and those of $\boldsymbol{k}$ remain fairly steady over time. As such, potential shifts in child resources are unlikely to be mediated by time through parent or children characteristics.

The second check I consider allows for differences in the sharing rule parameters by introducing the price of clothing into the sharing rule function. The results suggest that the prices play an insignificant role in the variability of individual resource shares. The next specification test examines the sensitivity of the results obtained to the integration of a third order term in Engel curves. The p-values (0.93 and 0.06 respectively for female and male) lead to a failure in rejecting the null hypothesis in the sharing rule equation at standard significance levels.

Table 7: Estimated paramaters of the individual resource shares: further results

		I-Simplified		II-Only Mixed Gender Siblings		III-Only Working Individuals	
		Women	Men	Women	Men	Women	Men
	Parent characteristics						
$\boldsymbol{z}$	Intercept	8.458	1.796	1.831***	-0.819	1.859***	0.051
		(6.338)	(2.547)	(0.477)	(1.245)	(0.657)	(2.315)
	Education	-0.002	-0.022	0.013	0.039	0.008	0.002
		(0.018)	(0.079)	(0.017)	(0.096)	(0.025)	(0.172)
	Age (in years)	0.002	0.037*	0.002	0.068***	-0.003	0.047
		(0.006)	(0.020)	(0.006)	(0.024)	(0.008)	(0.039)
	Labor	0.046***	0.327	-0.086	0.740**	-	-
		(0.111)	(0.340)	(0.133)	(0.290)	-	-
	Log total expenditures	0.900	1.904***	0.549	2.637***	0.289	2.675***
		(0.442)	(0.727)	(0.525)	0.764	(1.129)	(0.859)
	Children characteristics						
$\boldsymbol{k}$	Intercept	-8.569***	-2.086	-0.804***	-1.083**	-0.752**	0.290
		(7.475)	(3.373)	(0.213)	(0.506)	(0.321)	(1.377)
	Number of children	2.089***	0.348	0.098***	0.126	0.074*	-0.197
		(2.081)	(0.823)	(0.030)	(0.087)	(0.042)	(0.348)
	Age (in years)	-0.014**	-0.007	-0.009*	-0.047*	-0.009	-0.003
		(0.006)	(0.023)	(0.005)	(0.025)	(0.009)	(0.040)
	Proportion of boys	-0.098	-0.113	-0.120**	0.527	-0.017	-0.218
		(0.067)	(0.168)	(0.056)	(0.374)	(0.059)	(0.455)
	Same-sex siblings	0.469	0.189	-	-	0.071*	-0.412
		(0.202)	(0.274)	-	-	(0.043)	(0.396)
	Shadow prices						
$\boldsymbol{\tau}$	Log total expenditures	0.147	-5.787	-0.142	1.407*	-0.154	1.246
		(0.563)	(0.810)	(0.413)	(0.793)	(0.796)	(0.528)
Sample size		24 514	15 565	21 713	15 268	13 685	9 823
Sargan statistics		4.56	5.60	16.10	9.36	14.11	11.77
(Nb of free parameters, Instruments)		(33, 38)	(33, 38)	(32, 42)	(32, 42)	(31, 38)	(31, 38)

Notes: $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

The results of the final set of specifications are reported in the second panel of Table 6. First, I test empirically the hypothesis of economies of scale. To this end, I implement the LR-type statistics, which is defined as the difference between the Sargan statistics of the unconstrained and the constrained models. Under the null hypothesis, both models (with or without economies of scales) are significantly equivalent. Under the alternative, assuming the shadow prices of the composite goods are equal to one is rejected, then validate my approach. The findings does not support the theoretical hypothesis of economies of scale within household setting. The economies of scale function potentially suffers from functional form misspecification. Finally, I find evidence for introducing the log total expenditures in $\boldsymbol{z}$ part of the sharing rule function, even though this approach lacks empirical backing regarding male data.

Table 7 presents further results of robustness. Due to the relatively small sample size, especially when considering single fathers, the estimates are prone to biaises arising from a high degree of overidentification. To check the sensitivity of the estimates to the number of instruments used, I estimate the model with a reduced number of instruments. This entails the removal of the second-order polynomials for the exogenous variables entering in Θ_{is} as previously done by BDH. As illustrated by the results of Model I, the main conclusions remain consistent, although the estimated coefficient parameters and standard errors are becoming large. In Model II, I use a sample that exludes parents with children of the same gender. The findings closely mirror those obtained from the benchmark model. The fifth and final columns report the estimated coefficients of the model where all individuals are in the labor market. While less significant, the conclusions maintain their qualitative similarity.

5 Conclusion

Several models attempted to assess the cost of children for parents. However, these are focused only on children living in bi-headed households, while most OECD countries have experienced a demographic reconfiguration towards single parenthood. In this paper, I refined the collective approach to fit single parent decisions. The fundamental aim is to measure the cost of children borne by each single parent. To this purpose, I assume that parents' resources devoted to children depend both on parent and children characteristics. The model also allows to identify the presence of economies of scale between children. To test the validity of the model, I use data from the UK Family Expenditure Survey (FES) over the period 1978-2020. The results confirm partly what was found in the existing literature for couple-parents, namely, the cost of children increases with the number of children but decreases with family size. Interestingly, I found that the average cost of a child amounts to respectively 35% and 27% of the total expenditures of lone fathers and mothers. As such, children cost more to single fathers than single mothers, which is potentially explain by the transfer from single fathers to single mothers. Globally and significantly, the weighted average cost differential of children supported by parents is 6 percentage point, favoring fathers. Whereas the cost of children incurred by a representative parent, whether father or mother, is not significantly different. Furthermore, the findings reveal that the number of children does not matter for affluent parents, as the resource per child is invariant from the number of children. On the contrary, children from low-income families derive less from their parents' total expenditures with larger family size.

To conclude, it seems important to stress some limits of this paper. First, a more in-depth theoretical analysis is required to establish complete identification of shadow prices. Regarding endogeneity issues, my instruments are not sufficiently exogenous to strengthen the relevance of the results even if they are highly correlated with total expenditures. Potential critics might raise questions about plausible correlation between the number of children in the household and the residuals in the clothing equations even though I fail to find sufficient evidence for that claim. Moreover, my model overlooks the aspect of single-parenthood, while the causes of such a status remain undisclosed. Meanwhile, most countries legally acknowledge shared custody. If the single parent is not widowed, child custody can be jointly held by both parents. In this respect, a wide avenue for further research emerges.

An aspect worth delving into, whether in single-parent or dual-parent households, involves quantifying the value of parental time dedicated to children. This avenue of research would provide a mechanism for capturing certain subjective elements - such as parental affection and the conveyance of love to children - that might not be quantifiable in conventional metrics. Such a framework would enable us to further explore potential economies of scale. Ultimately, an analysis could have been conducted to assess how the presence of siblings of same gender or children close-in-age influences parental resources. Regrettably, the empirical testing of these further economies of scale has been hampered by the inadequacy of data related to children's age.

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6 Appendix

A A parametric Example

Consider the case of a single parent possessing a Cobb-Douglas utility function $u_i = \alpha_i ln(x_i) + (1-\alpha_i)ln(X_i)$, with $i = m, w$, where α_i is a parameter, and whose properties are well known. Also, let embed the preferences of children in a simple logarithmic utility function $u_c = ln(X_c)$. Recall that the two-stage budgeting process simply means that on the one hand, the parent defines a sharing rule that governs the distribution of resources between her and his children. On the second hand, each household member decides alone, based on his share of income how to reach the maximum level of utility through the consumption of goods. Thus, solve separately the parent and children optimization program consisting to:

Solving the latter problem gives the sharing rules as a function of the children's weight, $\phi_i = \frac{1}{1+\mu_c}$ and $\phi_c = \frac{\mu_c}{1+\mu_c}$. Since I have assumed that children have no bargaining power. The parameter μ_c should be interpreted as the weight that the parent assigns to children. In other words, it represents how children are perceived by their parents. The more the characteristics of children are well perceived by the parent, the higher the weight of children μ_c will be and the lower the parent share in total expenditures will be. the value of children weight the parents act as if they are a dictator. The parameter μ_c may be seen as a degree of altruism of the parent. The more the parent is altruistic, the lower his share in total expenditures will be.

I have previously assumed that the function κ_i does not depend on total expenditures. BDH have shown that this is related to the altruistic behavior of parents. In equation (5) representing the additive utility function of the household, I said that the parameter δ_i may be seen as a weight the parent gives to the child. Let's re-express equation (5) by weighting the sub-utility functions of each member by a parameter δ_i, whose $(1-\delta_i)$ represents the degree of altruism of the parent.

As we can see, the parameter δ_i is independent of total expenditures. The resources allocated to children depend on the degree of altruism of the parent $(1-\delta_i)$. This simple Cobb-Douglas parametric example proves that there is theoretical support for the assumption of the independence of total expenditures in the κ_i function.

B Proof for Identification

The budget share functions for individuals living alone are of the form:

$$\omega_i = g_i(p_i, y_i) \tag{20}$$

For single parents, the budget share functions are given by:

$$\omega_i = \phi_i g_i \left(\frac{p_i}{\pi_i}, \frac{y_i \phi_i}{\pi_i} \right) \tag{21}$$

for each individual $i = w, m$. The assumption of preference stability I made states that the budget share functions $g_i(.)$ are the same for single individuals and lone parents of gender i. Since the prices and total expenditures are observable by the econometrician, then the function $g_i(.)$ is known from the estimations on single individuals. Let $F(\omega_i, \phi_i)$ is C^1 in an open set B containing (ω_0, ϕ_0), with

$$F(\omega_i, \phi_i) = \phi_i g_i \left(\frac{p_i}{\pi_i}, \frac{y_i \phi_i}{\pi_i} \right)$$

Assume

$$\frac{\partial ln g_i}{\partial ln(y \frac{\phi_i}{g \pi_i})} + 1 \neq 0 \quad \text{everywhere,} \tag{22}$$

that is, the elasticity of g_i with respect to $(y_i \frac{\phi_i}{\pi_i})$ is different from -1 or, equivalently the income-elasticity of the demand for the exclusive good is different from zero (a rather weak condition). Then $F_2 \neq 0$, that is the derivatives of F with respect to ϕ_i are different from zero. Then, from the implicit function theorem I can write:

$$\Phi_i = G_i \left(\frac{p_i}{\pi_i}, \frac{y_i}{\pi_i}, \omega_i \right), \tag{23}$$

where G_i is the inverse of (21) with respect to ϕ_i. The individual share of each adult is thus identified up to a function $\pi_i(y_i, p_i, n)$.

C Informal Investigation

I present a linear regression model to estimate the share of total resources devoted to children on both parent and children characteristics. The objective is simply to explore and confirm the existing correlation between parental preferences and the average cost of children.

Table 8: Estimates of the average cost of children

		Women		Men	
	Parameters	Est. value	Std. Err.	Est. value	Std. Err.
$\boldsymbol{z}$	Intercept	0.055***	(0.001)	0.338***	(0.012)
	Education	-0.002***	(0.000)	0.004***	(0.001)
	Age (in years)	-0.001***	(0.000)	-0.007***	(0.000)
	Labor	0.017***	(0.000)	-0.062***	(0.002)
	Log total expenditures	-0.096***	(0.000)	-0.367***	(0.002)
$\boldsymbol{k}$	Number of children	0.209***	(0.001)	0.182***	(0.011)
	(Number of children)^2	-0.021***	(0.000)	-0.013***	(0.003)
	Age (in years)	0.003***	(0.000)	0.001**	(0.000)
	Proportion of boys	0.013***	(0.000)	0.027***	(0.003)
	Same-sex siblings	-0.026***	(0.000)	-0.028***	(0.003)
Sample size		13 244		1 596	

Notes: $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

While caution is needed in interpreting these results as causal effects, it remains valid to assert that these findings do validate a highly pronounced correlation between individual characteristics (parent and children) and the average cost of children. Furthermore, these estimates corroborate the signs of the different coefficients obtained in the structureal model estimation.

D Additional Estimation Results

Table 9: Results for budget share equations

	Women's budget equation		Men's budget equation	
Parameters	Est. val.	Std. err.	Est. val.	Std. err.
Intercept	0.190***	(0.013)	0.138***	(0.010)
Education	-0.001**	(0.000)	0.000	(0.000)
Age (in years)	-0.004***	(0.001)	-0.003***	(0.001)
Age2 (in years)	0.004***	(0.001)	0.003***	(0.001)
Year	0.721**	(0.322)	0.952***	(0.338)
year2	-0.722**	(0.321)	-0.949***	(0.337)
House owner	-0.002	(0.002)	-0.004**	(0.002)
Labor participation	0.002	(0.003)	0.002	(0.002)
Region:				
Norhern	0.002	(0.004)	-0.008*	(0.004)
York & Humberside	0.000	(0.004)	-0.015***	(0.004)
East Midlands	0.004	(0.004)	-0.021***	(0.004)
East Anglia	-0.002	(0.004)	-0.018***	(0.004)
Greater London	0.002	(0.004)	-0.016***	(0.004)
South-East	0.000	(0.004)	-0.019***	(0.004)
South-West	-0.002	(0.004)	-0.021***	(0.004)
Wales	-0.002	(0.004)	-0.014***	(0.004)
West-Midlands	0.002	(0.004)	-0.016***	(0.004)
North_West	-0.000	(0.004)	-0.017***	(0.004)
Scotland	-0.003	(0.004)	-0.014***	(0.004)
Log relative price	-0.005	(0.004)	0.011*	(0.006)
Log total expenditures	-0.002	(0.010)	0.010	(0.007)
(Log total expenditures)2	-0.026***	(0.007)	-0.004	(0.005)
Sample size	24 514	15 565	24 514	15 565

Notes: $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.

Table 10: Estimates of the difference of the average cost of children by parent

Parents	Method	N	95% LC Mean	Mean	95% UC Mean	95% LC SDV	SDV	95% UC SDV
Panel 1: Unweighed mean								
I-Fathers		1596	0.397	0.409	0.421	0.242	0.250	0.259
II-Mothers		13244	0.342	0.344	0.346	0.095	0.096	0.097
Diff(I-II)	Pooled	-	0.059	0.065	0.071	0.121	0.122	0.123
Diff(I-II)	Satterthwaite	-	0.053	0.065	0.077	-	-	-
Panel 2: Weighed mean								
I-Fathers		1596	0.433	0.445	0.457	0.300	0.310	0.321
II-Mothers		13244	0.378	0.379	0.381	0.124	0.125	0.127
Diff(I-II)	Pooled	-	0.059	0.066	0.072	0.154	0.156	0.158
Diff(I-II)	Satterthwaite	-	0.053	0.066	0.078			

Notes: N, LC, UC and SDV mean respectively sample size, Lower Confidence, Upper confidence and Standard Deviation. DF for Degree of Freedom.

Table 11: Estimates of the difference of the average cost of children by gender of children

Parents	Method	N	95% LC Mean	Mean	95% UC Mean	95% LC SDV	SDV	95% UC SDV
Panel 1: Cost of boys								
I-Fathers		1596	0.410	0.423	0.435	0.244	0.253	0.262
II-Mothers		13244	0.350	0.352	0.353	0.098	0.099	0.100
Diff(I-II)	Pooled	-	0.065	0.071	0.078	0.124	0.125	0.126
Diff(I-II)	Satterthwaite	-	0.059	0.071	0.084	-	-	-
Panel 2: Cost of girls								
I-Fathers		1596	0.380	0.392	0.404	0.237	0.246	0.254
II-Mothers		13244	0.335	0.336	0.338	0.091	0.092	0.093
Diff(I-II)	Pooled	-	0.050	0.056	0.062	0.117	0.118	0.120
Diff(I-II)	Satterthwaite	-	0.044	0.056	0.068			

Notes: LC, UC and SDV mean respectively sample size, nLower Confidence, Upper confidence and Standard Deviation. DF for Degree of Freedom.

E Additional Figures

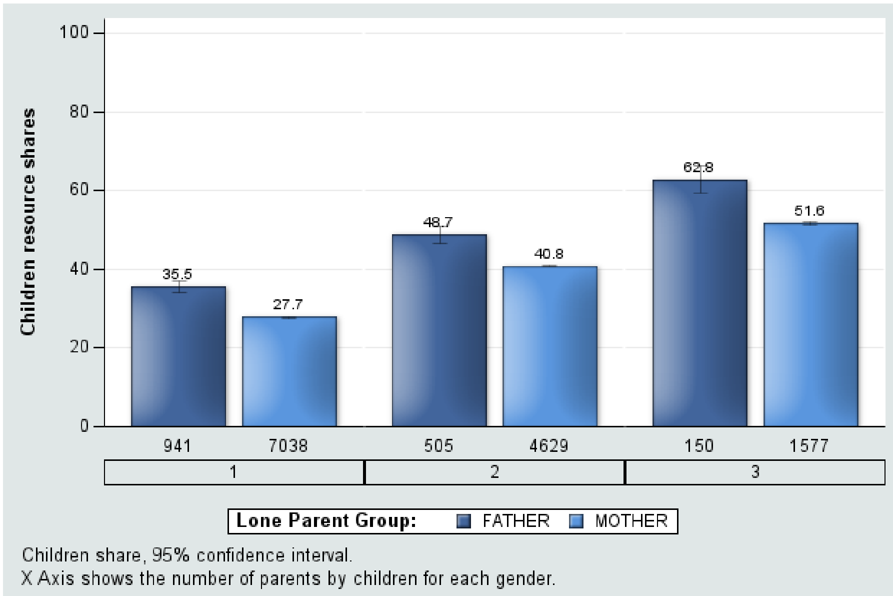

Figure 5: Share of parents' total expenditures devoted to boys

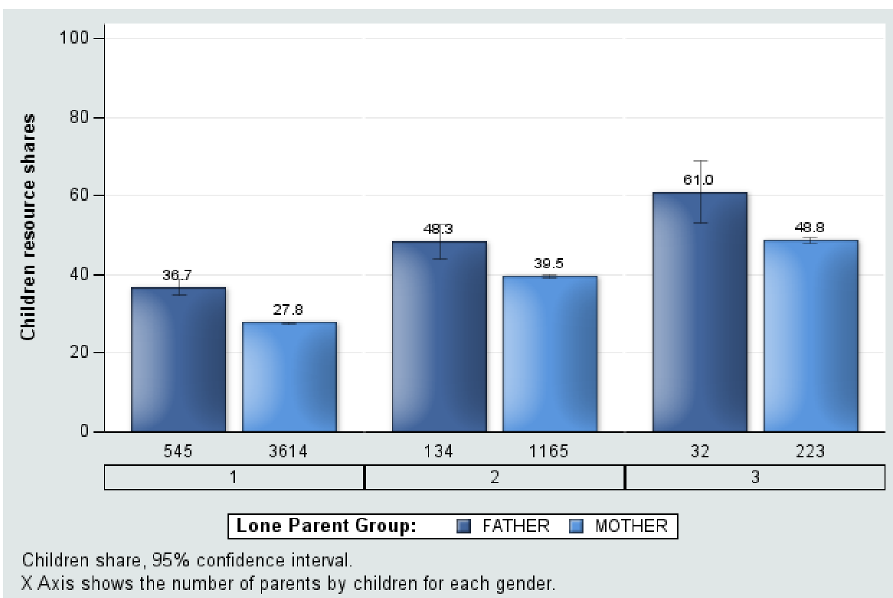

Figure 6: Share of parents' total expenditures devoted to children in families with only boys

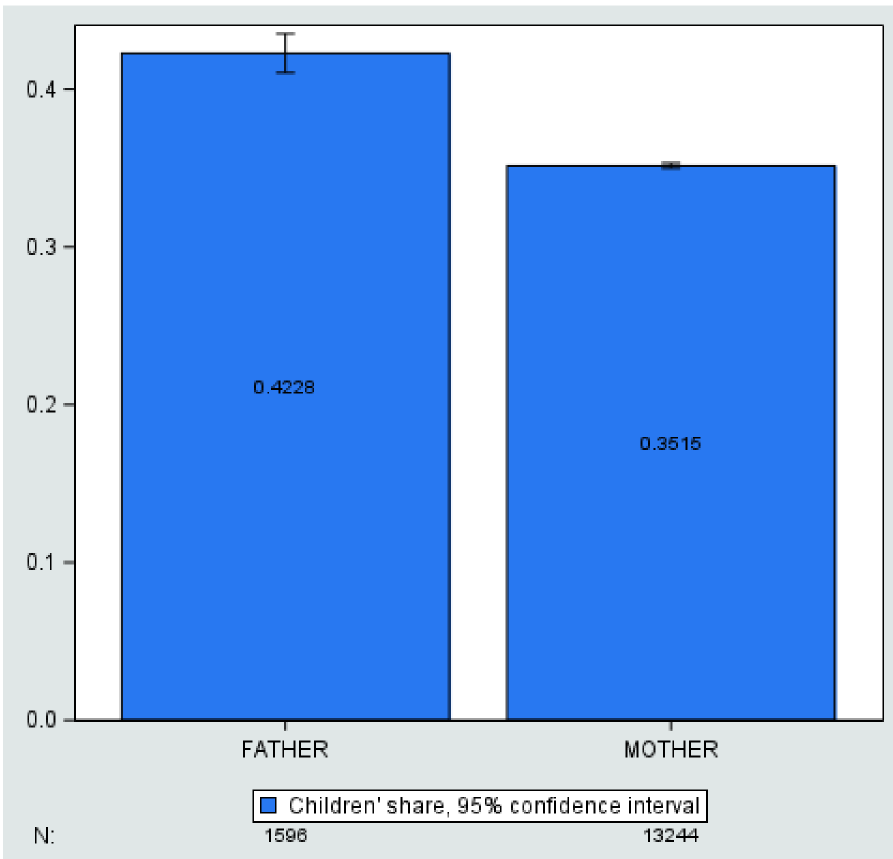

Figure 7: Cost of boys borne by each single parent

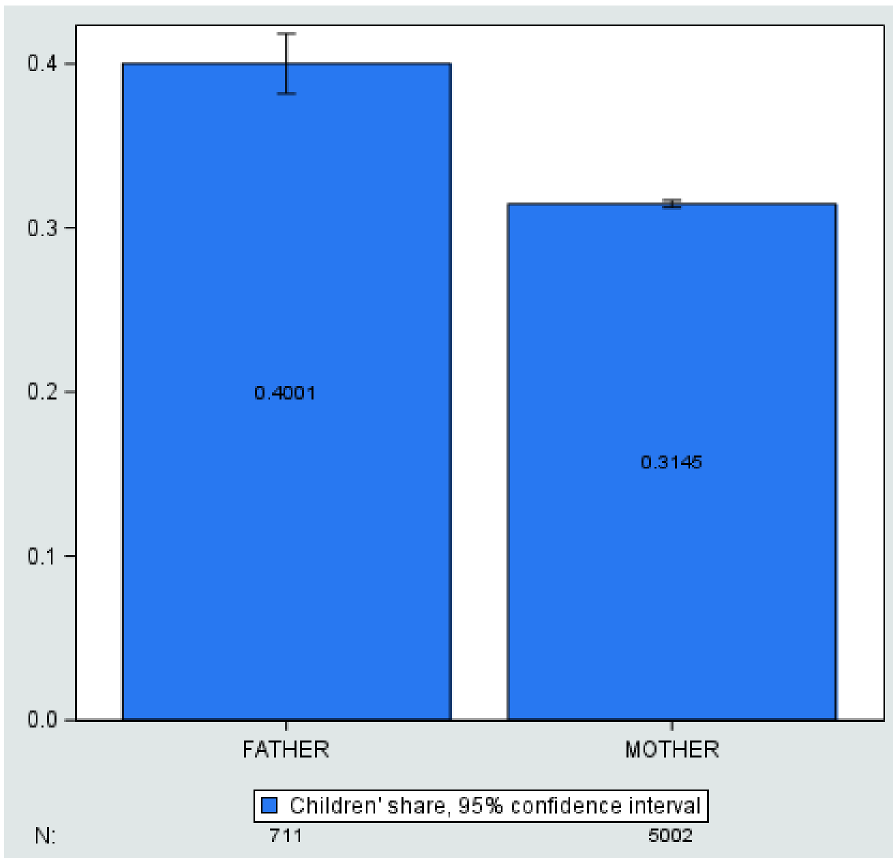

Figure 8: Cost of children borne by each single parent in families with only boys

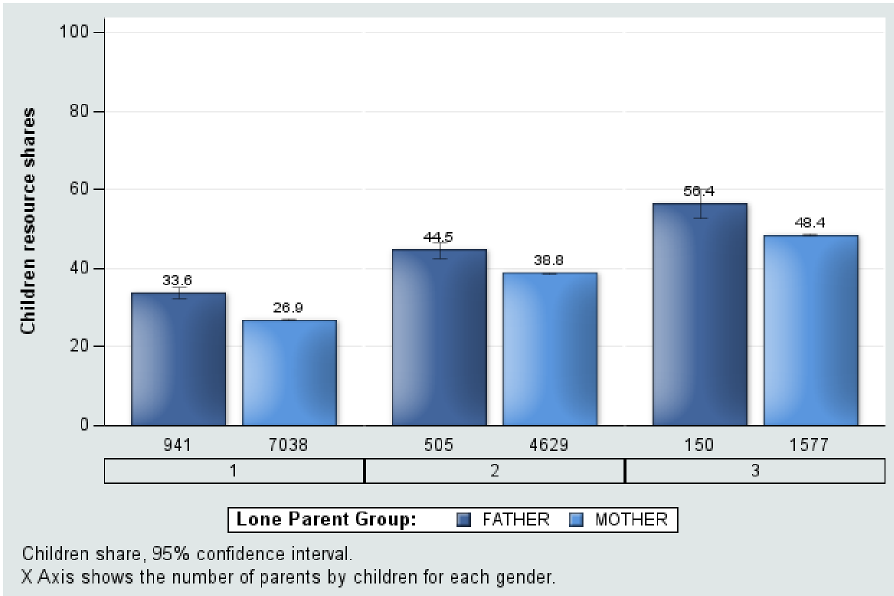

Figure 9: Share of parents' total expenditures devoted to girls

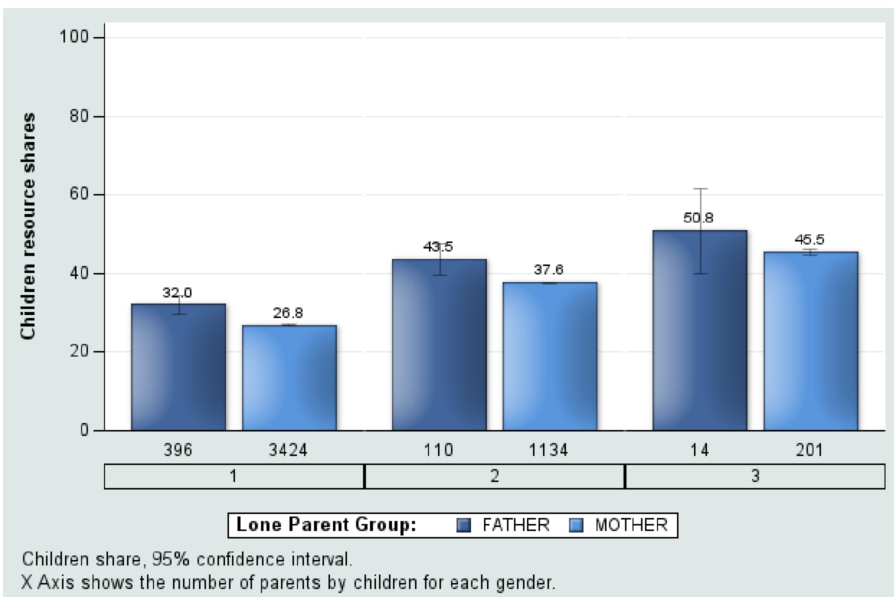

Figure 10: Share of parents' total expenditures devoted to children in families with only girls

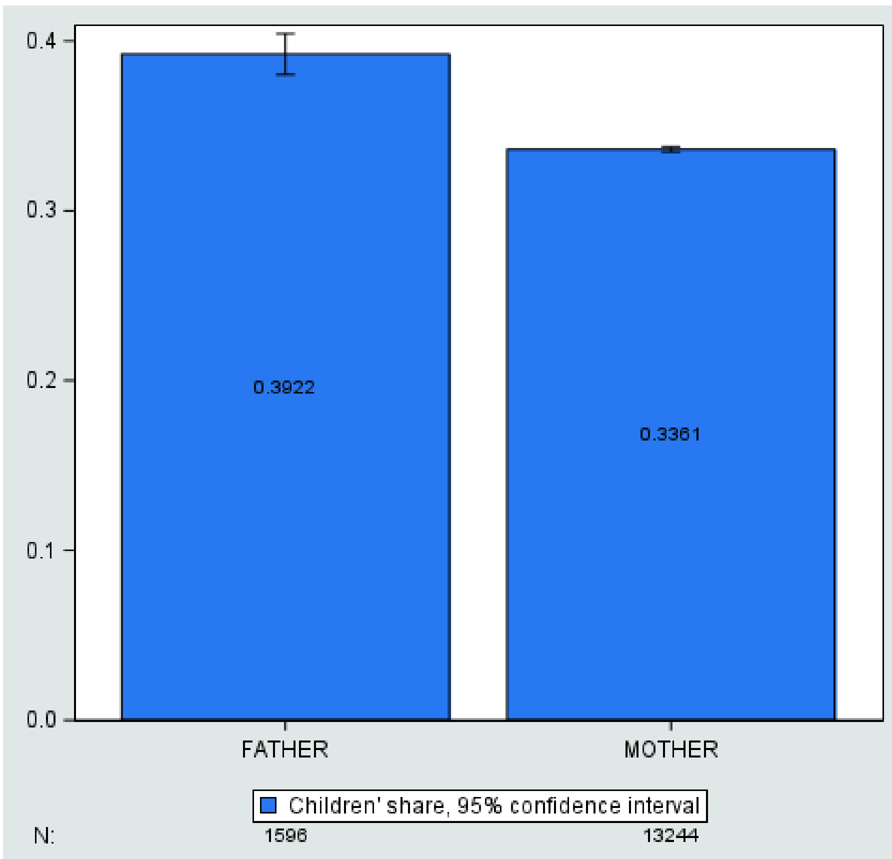

Figure 11: Cost of girls borne by each single parent

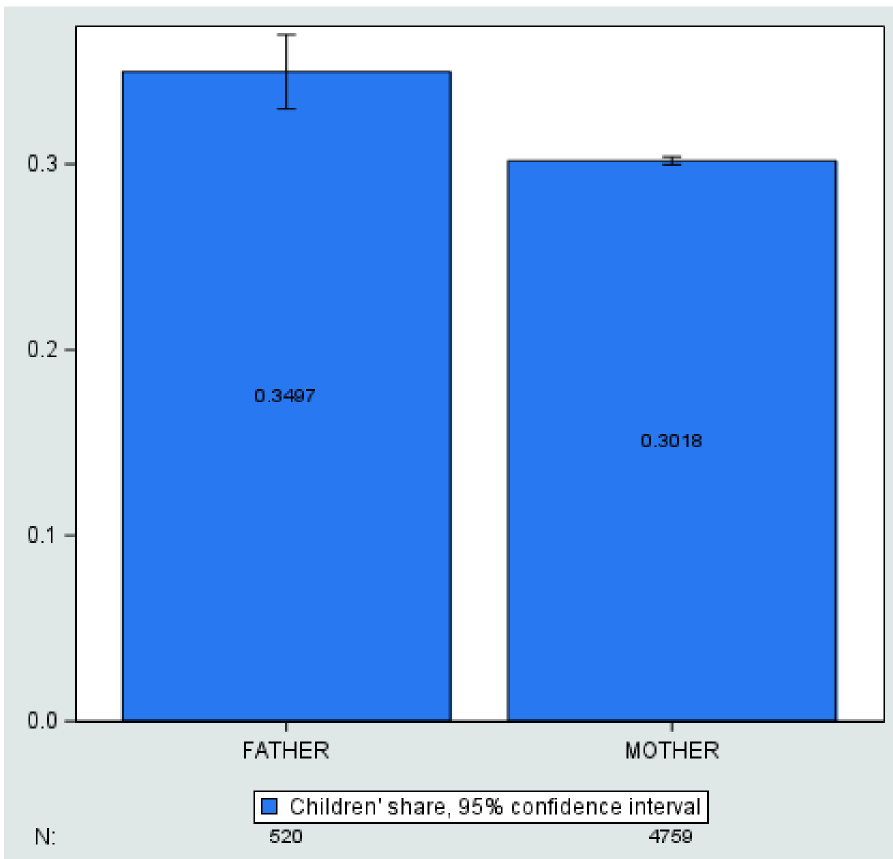

Figure 12: Cost of children borne by each single parent in families with only girls

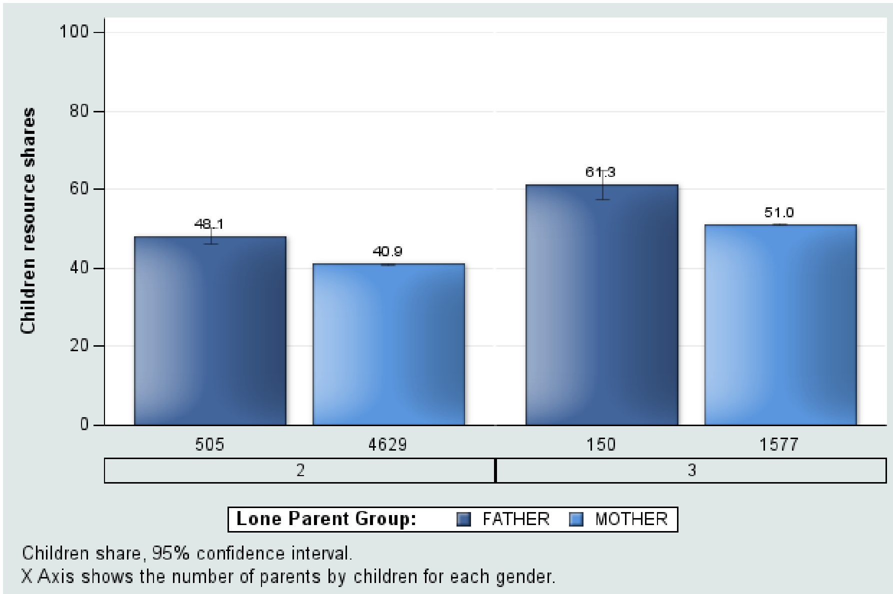

Figure 13: Share of parents' total expenditures devoted to children of mixed-gender

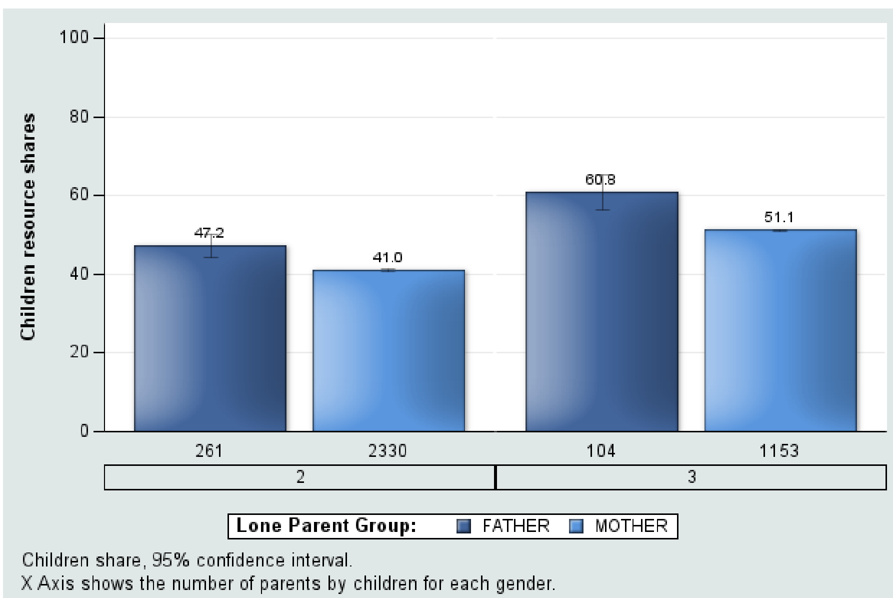

Figure 14: Share of parents' total expenditures devoted to children in families with only mixed-gender siblings

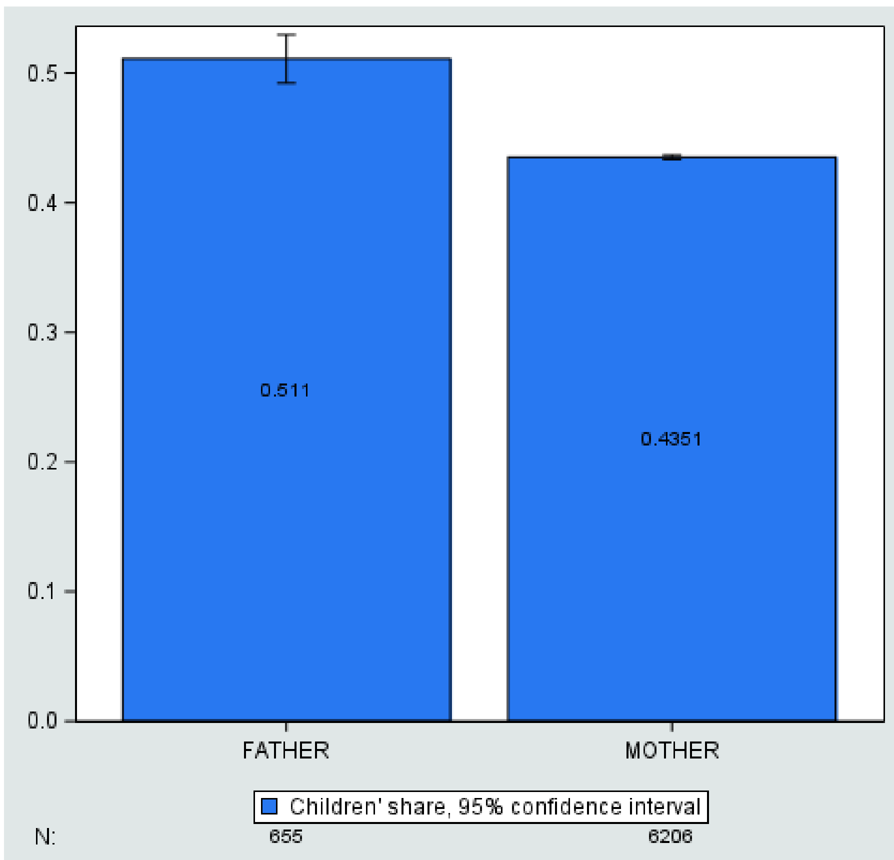

Figure 15: Cost of children of mixed-gender borne by each single parent

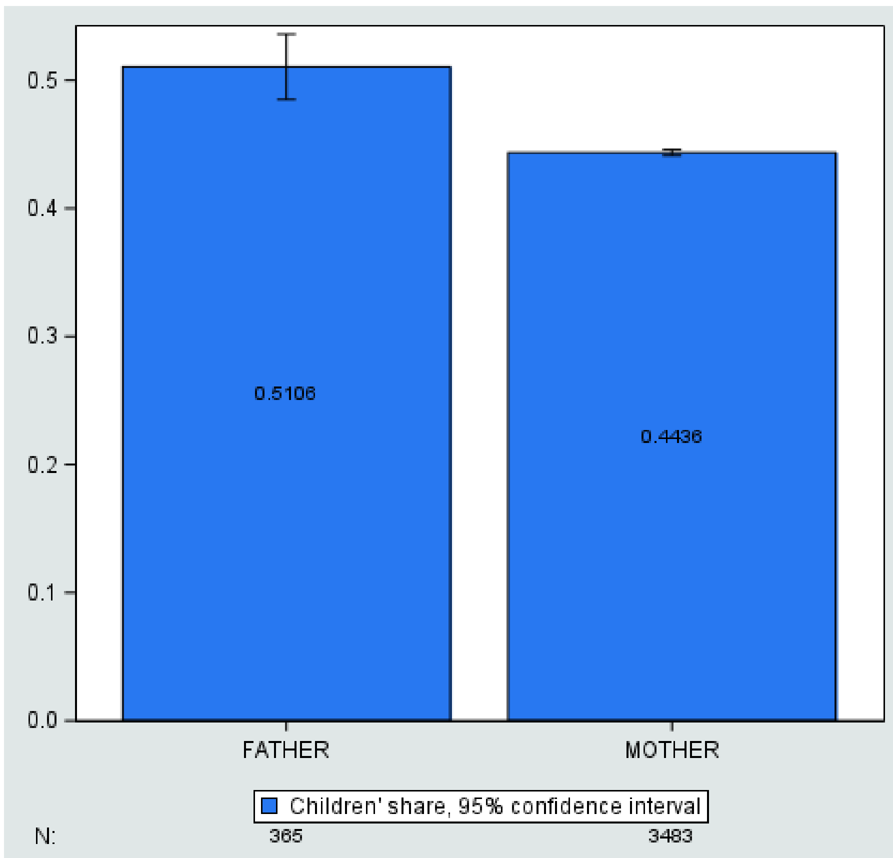

Figure 16: Cost of children borne by each single parent in families with only mixed-gender siblings

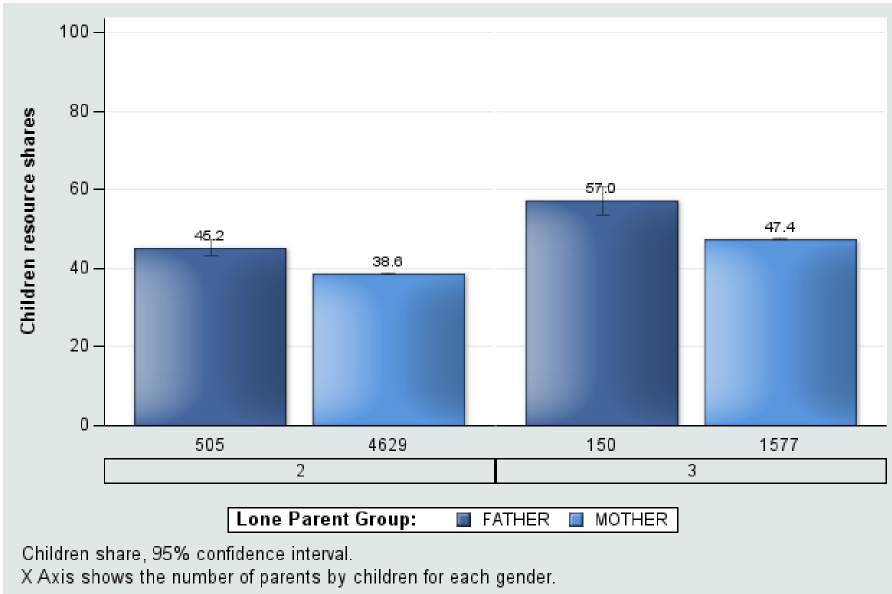

Figure 17: Share of parents' total expenditures devoted to children of same-gender

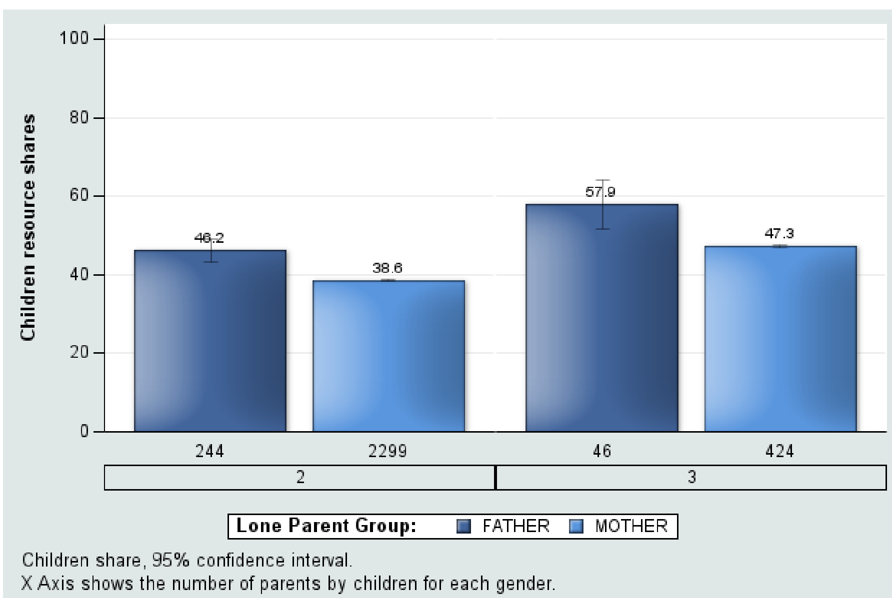

Figure 18: Share of parents' total expenditures devoted to children in families with only same-gender siblings

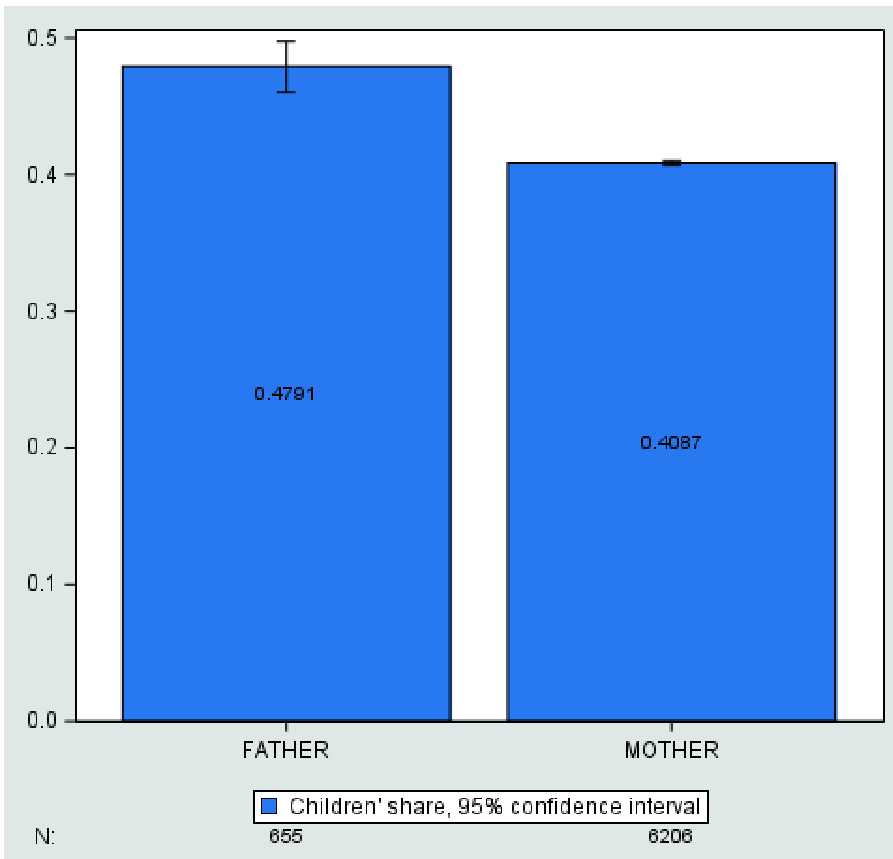

Figure 19: Cost of children of same-gender borne by each single parent

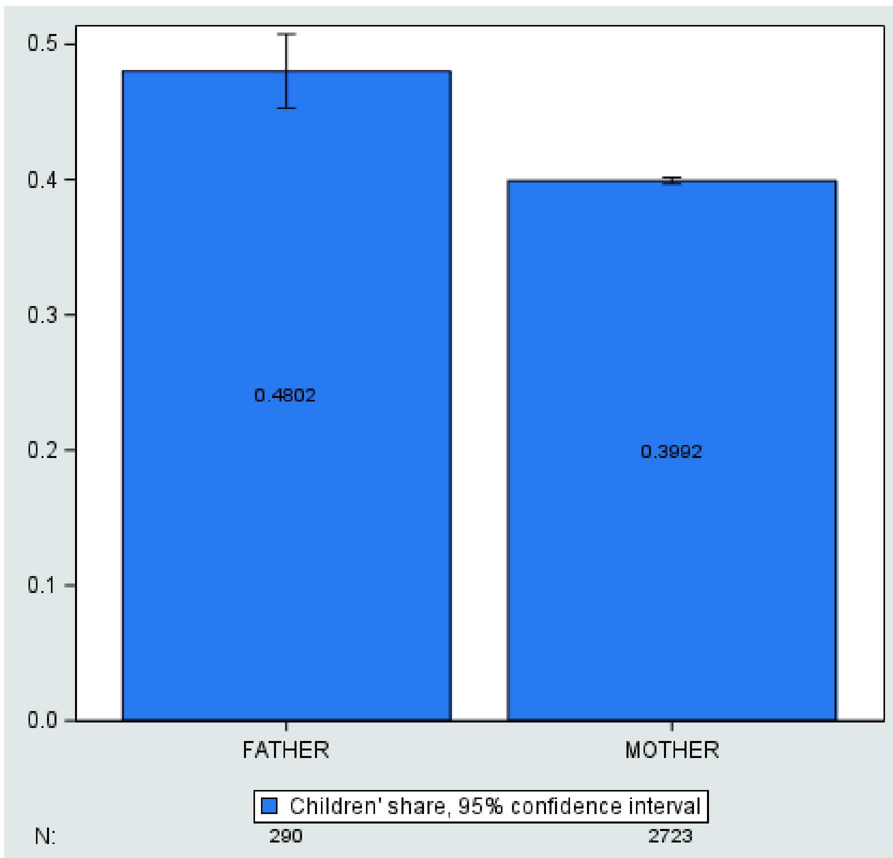

Figure 20: Cost of children borne by each single parent in families with only same-gender siblings

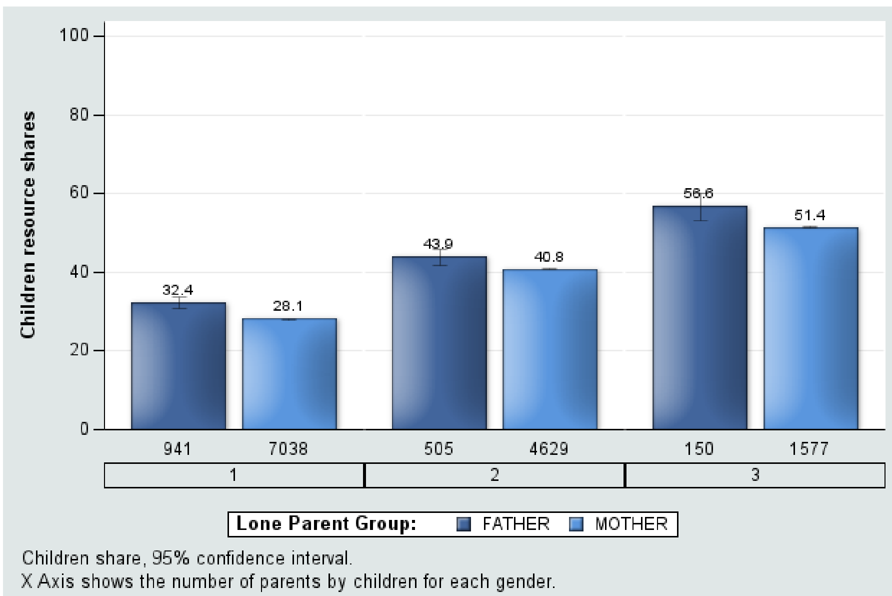

Figure 21: Share of working parents' total expenditures devoted to children

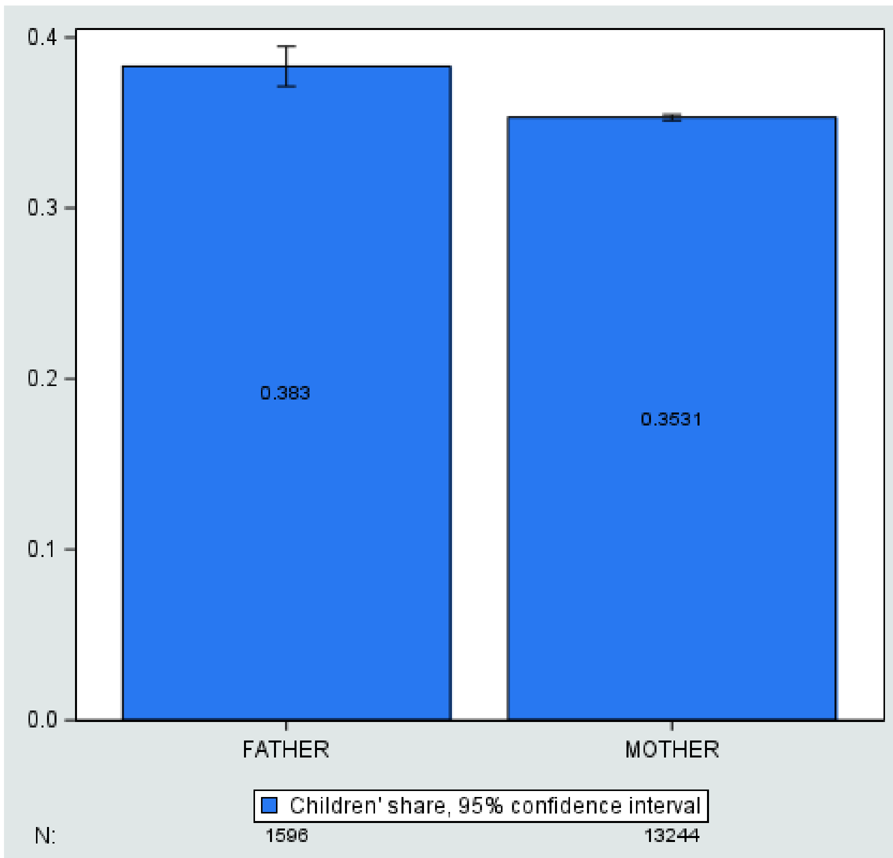

Figure 22: Cost of children borne by working parent

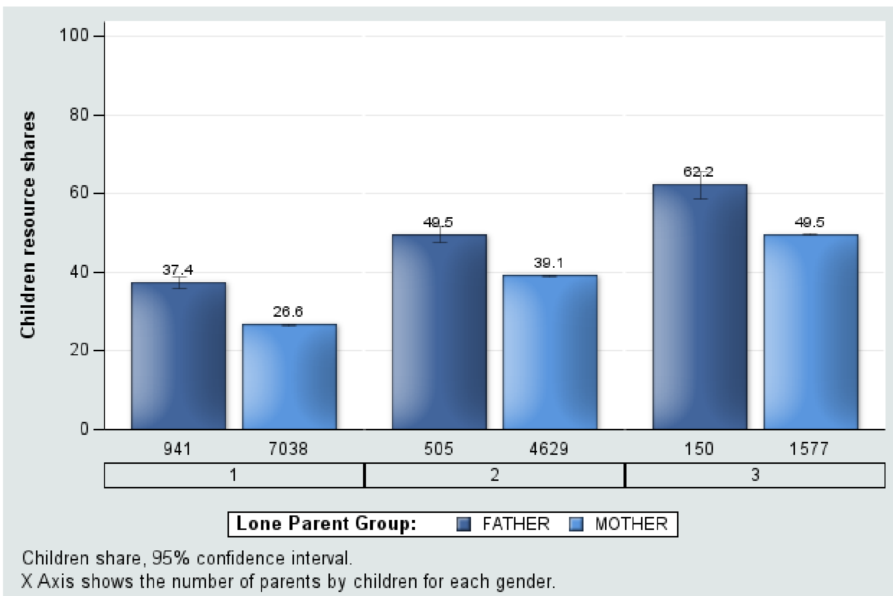

Figure 23: Share of non-working parents' total expenditures devoted to children

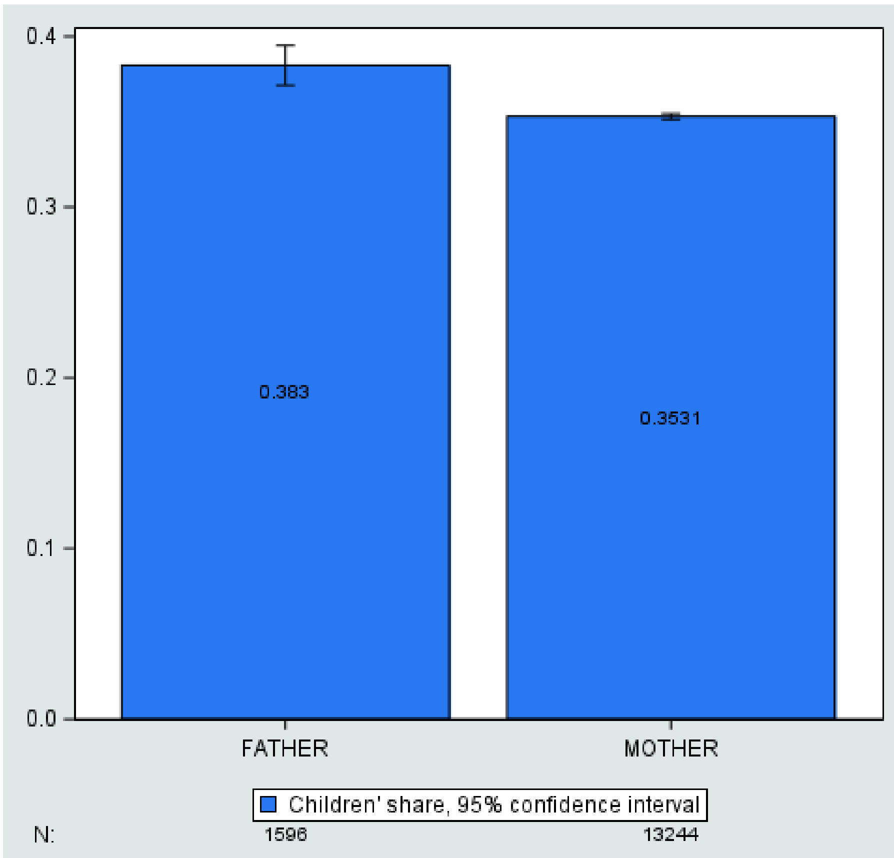

Figure 24: Cost of children borne by non-working parent

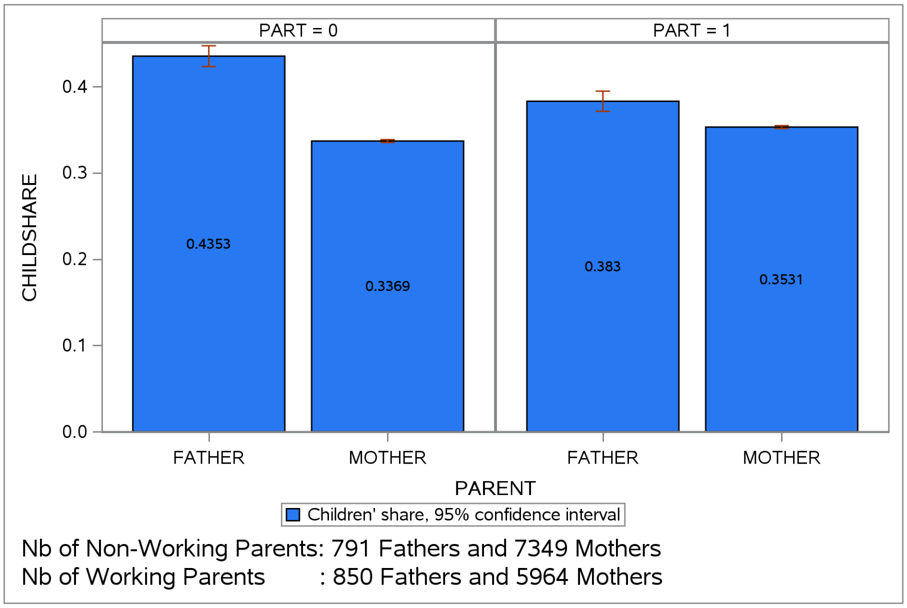

Figure 25: Cost of children borne by parent by labor status

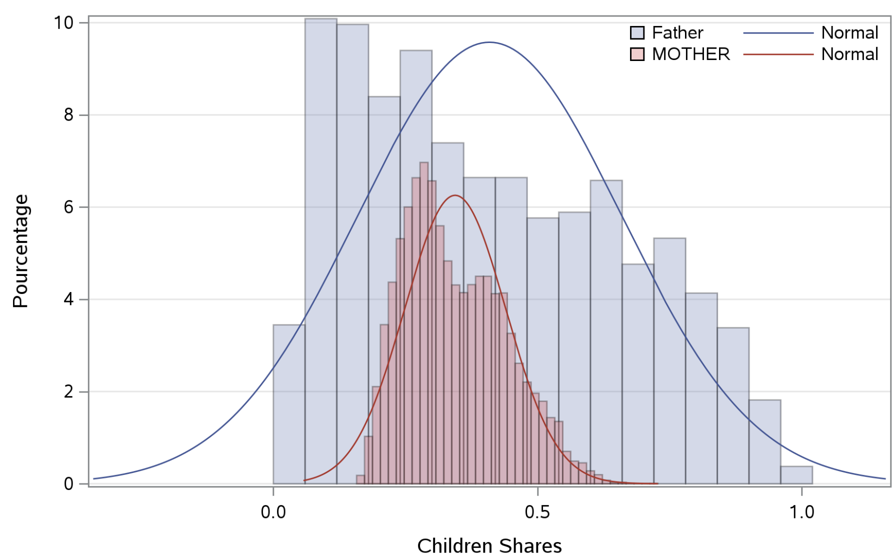

Figure 26: Nonparametric distribution of children's share

Note: Based on the sharing rule estimates, the mean share of children is 0.34 and 0.40 respectively for mothers and fathers.

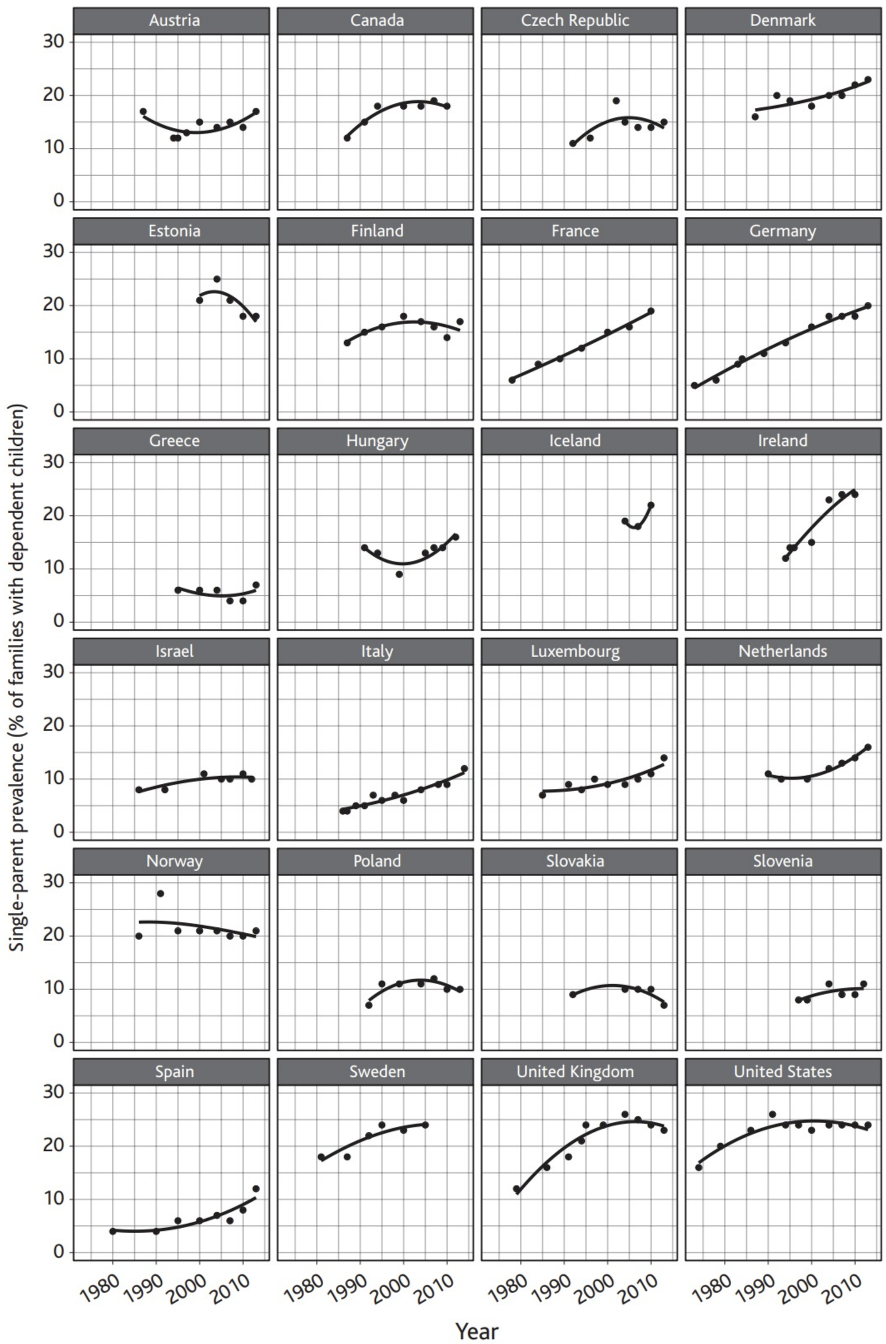

Figure 27: Trends in single parenthood