Long-run inheritance tax and capital income tax with rational altruism

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Abstract

We consider a two-period overlapping generation model with rational altruism à la Barro. The government finances public spending with taxes on labor income, capital income and inheritance. We show that, in the long-run, inheritance tax and capital income tax are generally different from zero, even if the optimal tax policy leads to the modified Golden-rule.

Keywords: inheritance tax, capital income tax, altruism.
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1 Introduction

The seminal works by Chamley (1986) and Judd (1985) state that the optimal long-run tax rate on capital income is zero in neoclassical infinite-lived agent model. In the dynastic interpretation of Chamley (1986)'s model, with successive generations (considered for instance in Piketty and Saez, 2013), inheritance tax and capital income tax are equivalent and then equal to zero at the second-best optimum. Indeed, assuming rational altruism à la Barro (1974) creates intergenerational links that makes the overlapping generation model similar to the infinite-lived agent model. Nevertheless, as soon as one considers more than one-period lifetime, inheritance tax and capital income tax have no longer the same consequences.

In this note, we intent to analyze the second-best linear tax policy in a two-period overlapping generation model with rational altruism. We go back to Chamley (1986)'s problem, that is, how to finance a given public spending with capital income taxation and labor income taxation, adding the possibility of an inheritance tax. Of course, the second-best optimal path converges toward the modified Golden-rule, implying zero distortion on capital accumulation in the long-run. But, we show that this does not necessarily imply zero inheritance taxation and zero capital income taxation. Indeed, a positive or negative inheritance tax rate means that consumptions of the young and the old are taxed at different rates. In a second-best world, it may be optimal to set capital income and inheritance tax rates with opposite signs in order to achieve the two following goals: (i) reaching the modified Golden-rule; (ii) taxing consumptions in each period at rates driven by standard Ramsey taxation formulas.

Section 2 describes the intertemporal equilibrium, while the second-best optimum is analyzed in Section 3.

2 Equilibrium

Time is discrete. Population is constant and consists of a unique representative dynasty, where each parent has only one child. Members of the representative dynasty leave for two periods, work in the first and retire in the second. They are also altruistic towards their offsprings. When born in \( t \), a household is taxed at rate \( \tau^w_t \) on labor income, at rate \( \tau^x_t \) on inherited bequest, and, when old, at rate \( \tau^{R+1}_t \) on capital income. Consequently, when young, the household born in period \( t \) receives an after-tax bequest \( (1 - \tau^x_t) x_t \), chooses labor supply \( \ell_t \), paid at the net wage \( (1 - \tau^w_t) w_t \), and allocates these resources between consumption \( c_t \) and saving \( s_t \). When old, net capital income \( (1 - \tau^{R+1}_t) R_{t+1} \) is divided into consumption \( d_{t+1} \) and bequests \( x_{t+1} \) to the next generation. Requiring that bequests are nonnegative, the above description results in the following

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1 See e.g. Boadway et al. (2010), Cremer and Pestieau (2011), Kopczuk (2013).
constraints
\[ c_t + s_t = (1 - \tau^w_t) w_t \ell_t + (1 - \tau^x_t) x_t \]  
\[ dt_{t+1} + xt_{t+1} = (1 - \tau^R_{t+1}) R_{t+1} s_t \]  
\[ xt_{t+1} \geq 0 \]  
(1)  
(2)  
(3)

Rational altruism implies that utility of the representative household in period t, \( V_t \), depends on consumptions in both periods, labor supply and utility of its offsprings \( \nu_{t+1} \):

\[ V_t = U(c_t, \ell_t, dt_{t+1}) + \beta \nu_{t+1} \]

where \( \beta \) represents the degree of altruism (0 < \( \beta < 1 \)). The problem of the dynasty at time zero is to maximize

\[ V_{-1} = \frac{1}{\beta} U(\bar{c}_{-1}, \bar{\ell}_{-1}, d_0) + \sum_{t=0}^{+\infty} \beta^t U(c_t, \ell_t, dt_{t+1}) \]

subject to (1) for \( t \geq 0 \), (2) and (3) for \( t \geq -1 \), given \( s_{-1} = \bar{s}_{-1}, \bar{c}_{-1} \) and \( \bar{\ell}_{-1} \). From Michel et al. (2006), household optimum satisfies the first-order conditions, for \( t \geq 0,^2 \)

\[ \begin{cases} 
- U_{ct}^t = (1 - \tau^w_t) w_t, & U_{ct}^{t+1} = \frac{1}{(1 - \tau^{R_{t+1}})} R_{t+1} \\
- U_{dt}^t + \beta (1 - \tau^x_t) U_{ct}^t \leq 0, & = 0 \text{ if } x_t > 0 
\end{cases} \]

and the transversality condition

\[ \lim_{t \to +\infty} \beta^{t-1} U_{dt}^t x_t = 0 \]

Conditions (5) and (6) associated with budget constraints (1) for \( t \geq 0 \), and (2) for \( t \geq -1 \), bring a complete characterization of the household optimum \((c_t, \ell_t, dt_t, s_t)_{t \geq 0}\).

A representative firm that behaves competitively, produces output in period t with capital \( K_t \) and labor \( L_t \). The production function \( F(K, L) \) is linear homogenous and concave, and includes capital depreciation. Marginal products are strictly positive and decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

\[ \begin{cases} 
w_t = F'_L(K_t, L_t) \\
R_t = F'_K(K_t, L_t)
\end{cases} \]

where \( F'_L \) and \( F'_K \) stand for the partial derivatives of \( F \) with respect to labor and capital.

The government finances an exogenous sequence of public spendings with taxes on capital income, labor income and inheritance and has the possibility to issue bonds. Denoting by \( g \), the constant

\[ ^2U_{ct}^t, U_{ct}^t \text{ and } U_{dt}^t \text{ stand respectively for } U_{ct}^t(c_t, \ell_t, dt_{t+1}), U_{ct}^t(c_t, \ell_t, dt_{t+1}) \text{ and } U_{dt}^t(c_t-1, \ell_{t-1}, dt_t). \]
level of public spending, the law of motion of the public debt $\delta_t$ is
\begin{equation}
\delta_{t+1} + \tau^w_t w_t \ell_t + \tau^R_t R_t s_{t-1} + \tau^x_t x_t = R_t \delta_t + g
\end{equation}

The initial public debt $\delta_0$ is given: $\delta_0 = \bar{\delta}_0$. The capital market equilibrium writes
\begin{equation}
s_t = k_{t+1} + \delta_{t+1}
\end{equation}
for $t \geq 0$ and $k_0 = \bar{k}_0 \equiv \bar{s}_{-1} - \bar{\delta}_0$.

Since we are interested in inheritance taxation, we focus on equilibria with nonnegative desired bequests in all periods, that is $U'_{d_t} = \beta (1 - \tau^x_t) U'_{c_t}$, for any $t \geq 0$. Our analysis of the second-best optimum then applies to these situations only.

**Definition 1.** Consider $c_{-1}, \ell_{-1}$ and $k_0$ as given. A competitive equilibrium with tax instruments $(\tau^w_t, \tau^R_t, \tau^x_t)_{t \geq 0}$ is an allocation $(c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}$, a sequence of bequests and savings $(x_t, s_t)_{t \geq 0}$ and a sequence of prices $(w_t, R_t)_{t \geq 0}$ such that:

- $(c_t, \ell_t, d_t, x_t, s_t)_{t \geq 0}$ satisfies the budget constraints (1)-(2), the optimality conditions (5) of the household with $U'_{d_t} = \beta (1 - \tau^x_t) U'_{c_t}$, and the transversality condition (6), for the prices $(w_t, R_t)_{t \geq 0}$ and the tax instruments $(\tau^w_t, \tau^R_t, \tau^x_t)_{t \geq 0}$;
- $(k_t, \ell_t)_{t \geq 0}$ satisfies FOCs (7) of the firms for the prices $(w_t, R_t)_{t \geq 0}$ with $K_t = k_t$ and $L_t = \ell_t$;
- the market clearing conditions for good are satisfied, that is, for $t \geq 0$,
\begin{equation}
c_t + d_t + g + k_{t+1} = F (k_t, \ell_t)
\end{equation}

We do not mention the government budget constraint (8) since, from the Walras law, equilibrium on all markets makes it redundant. The following proposition introduces the characterization of the intertemporal equilibrium using the implementability constraint.

**Proposition 1.** Consider $k_0$ and $\delta_0$ as given.

- Consider the allocation $(c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}$ of a competitive equilibrium. Then, it satisfies the resource constraint (10) for $t \geq 0$, and the implementability constraint
\begin{equation}
\sum_{t=0}^{+\infty} \beta^t \left[ U'_{c_t} c_t + U'_{\ell_t} \ell_t + \frac{U'_{d_t}}{\beta} d_t \right] = \frac{U'_{d_0}}{\beta} \left( 1 - \tau^R_0 \right) R_0 \bar{s}_{-1}.
\end{equation}

- Conversely, consider an allocation $(c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}$ that satisfies (10) and (11). Then there exist prices $(w_t, R_t)_{t \geq 0}$, bequests and savings $(x_t, s_t)_{t \geq 0}$ and instruments $(\tau^w_t, \tau^R_t, \tau^x_t)_{t \geq 0}$ such that $(c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}$ is the allocation of a competitive equilibrium with tax instruments $(\tau^w_t, \tau^R_t, \tau^x_t)_{t \geq 0}$. 


The implementability constraint (11) requires that the infinite sum converges. In the following, we restrict our analysis to allocations where $U'_c, U'_l$ and $U'_d$ are bounded. This is obtained, for instance, if (i) consumptions and leisure are bounded away from zero in all periods, and (ii) consumptions are bounded above. The latter condition (ii) can result from bounded capital accumulation, that is, the path where, in each period, the entire production with the highest available labor supply is invested in capital accumulation leads to a finite long-run capital stock.3

Proof. First, consider an allocation $(c_t, l_t, d_t, k_{t+1})_{t \geq 0}$ of a competitive equilibrium. It satisfies the resource constraint (10). To show that it also satisfies the implementability constraint (11), we construct the intraperiod budget constraint of the dynasty by eliminating bequest $x_t$ into the budget constraints (1) and (2) of the young and the old leaving in the same period $t$:

$$c_t + s_t = (1 - \tau^w_t) w_t l_t + (1 - \tau^R_t) (1 - \tau^R_t) R_t s_{t-1} - d_t$$

Using the marginal conditions of the household problem (5), one gets

$$U'_c c_t + U'_d d_t + U'_l l_t = U'_c [(1 - \tau^x_t) (1 - \tau^R_t) R_t s_{t-1} - s_t]$$

Then, multiplying by $\beta^t$ and summing over $t = 0, \ldots, T$, leads to

$$\sum_{t=0}^T \beta^t \left[ U'_c c_t + \frac{U'_d}{\beta} d_t + U'_l l_t \right] = \frac{U'_d}{\beta} (1 - \tau^R_0) R_0 s_{-1} - \beta^T U'_c s_T$$

where the last equality uses relationships deduced from the marginal conditions of the households (5)

$$\beta^t U'_c (1 - \tau^x_t) (1 - \tau^R_t) R_t = \beta^{t-1} U'_c, \text{ for } t \geq 1$$

$$U'_c (1 - \tau^x_0) = \frac{U'_d}{\beta}$$

From the first-period budget constraint of the household, one gets

$$\lim_{t \to +\infty} \beta^t U'_c s_t = - \lim_{t \to +\infty} \beta^t \left( U'_c l_t + U'_c c_t \right) + \lim_{t \to +\infty} \beta^{t-1} U'_d x_t = 0 \quad (12)$$

where the last equality results from the transversality condition (6) and the fact that we consider allocations for which $U'_c$ and $U'_l$ are bounded. We then obtain the implementability constraint (11).

For the converse direction, we need to show that, for any allocation $(c_t, l_t, d_t, k_{t+1})_{t \geq 0}$ that satisfies

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3From the resource constraint (10), this remains to assume that the sequence $(k_{t+1}^{\max})_{t \geq 0}$ defined by $k_{t+1}^{\max} = k_0$ and $k_{t+1}^{\max} = F(k_{t+1}^{\max}, 1)$ converges (considering that 1 is the highest available labor supply). A sufficient condition for this is $\lim_{t \to +\infty} F(k_{t+1}) = 0$, which is equivalent to $F(1, 0) = 0$, which means that labor is indispensable to get a positive production.
(10) and (11), there exist tax instruments \((\tau^w_t, \tau^R_t, \tau^x_t)_{t \geq 0}\), bequests and savings \((x_t, s_t)_{t \geq 0}\) and prices \((w_t, R_t)_{t \geq 0}\) for which all equilibrium equations in Definition 1 are satisfied. Indeed, \(\tau^R_0\) is obtained from the implementability constraint (11). Moreover, \((k_t, \ell_t)_{t \geq 0}\) allow to compute \((w_t, R_t)_{t \geq 0}\) with the FOCs (7) of the firms. Then, marginal conditions of the consumer (5) allow to compute \((\tau^x_t, \tau^w_t, \tau^R_{t+1})_{t \geq 0}\). The sequence \((x_t, s_t)_{t \geq 0}\) is obtained from the budget constraints of the household (1)-(2). Finally, the implementability constraint (11) implies that \(\lim_{t \to +\infty} \beta^t U'_c s_t = 0\), which, using (12), leads to the transversality condition (6).

3 Second-best optimum

The government is looking for an allocation \((c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}\) that maximizes welfare (4) subject to the resource constraints (10) in all periods \(t \geq 0\) and the implementability constraint (11). Let us define

\[
\begin{align*}
W(c_t, \ell_t, d_{t+1}) &\equiv U_t + \lambda \left[ U'_c c_t + U'_\ell \ell_t + U'_d d_{t+1} \right] \\
\end{align*}
\]

where \(\lambda\) is the multiplier of the implementability constraint. The arguments of \(U_t, U'_c, U'_\ell\) and \(U'_d\) are \((c_t, \ell_t, d_{t+1})\) for \(t \geq 0\), and \((\bar{c}_{-1}, \bar{\ell}_{-1}, d_0)\) for \(t = -1\). Since the capital income tax in period 0 applies to a basis that the household cannot modify, it can be considered as a lump-sum tax. We consider it as given: \(\tau^R_0 = \bar{\tau}^R_0\). The government then maximizes

\[
SW_{-1} = \frac{1}{\beta} \left[ U_{-1} + (d_0 - (1 - \bar{\tau}^R_0) R_0 s_{-1}) \lambda U'_d d_0 \right] + \sum_{t=0}^{+\infty} \beta^t W(c_t, \ell_t, d_{t+1})
\]

with respect to \((c_t, \ell_t, d_t, k_{t+1})_{t \geq 0}\) subject to the resource constraint in each period, given \(k_0\) and \(s_{-1}\).

**Proposition 2.** If the second-best optimum allocation converges towards a steady state, the corresponding capital-labor ratio corresponds to the modified Golden-rule

\[
\beta F'_k(k, \ell) = 1
\]

Tax rates \(\tau^x\) and \(\tau^R\) have opposite signs and the following equivalences apply

\[
\begin{align*}
\tau^x > 0 &\iff H^c > H^d \\
\tau^w > 0 &\iff H^c > H^\ell \\
\tau^x > \tau^w &\iff H^\ell > H^d
\end{align*}
\]

where \(H^z\) corresponds to the sum of elasticities of the marginal utility \(U'_z\) with respect to each of

\[\footnote{From now on, we define \(F'_k\) (resp. \(F'_\ell\)) as the derivative of \(F\) with respect to \(K\) (resp. \(L\)) when \(K = k_t\) and \(L = \ell_t\).}
the quantities \(c, \ell\) and \(d\)

\[
H^z \equiv \frac{-U''_{zc}}{U'_z} + \frac{-U''_{z\ell}}{U'_z} + \frac{-U''_{zd}}{U'_z}
\]

**Proof.** We consider the infinite Lagrangian

\[
SW_{-1} + \sum_{t=0}^{\infty} \beta^t q_t \left[ F(k_t, \ell_t) - c_t - d_t - g - k_{t+1} \right]
\]

where \(\beta^t q_t\) is the multiplier of the resource constraint in period \(t\). The first-order conditions with respect to \(c_t, \ell_t, d_{t+1}\) and \(k_{t+1}\), for \(t \geq 0\), write

\[
W'_c = q_t, \quad W'_{\ell_{t+1}} + q_{t+1}F'_{\ell_{t+1}} = 0, \quad W'_{d_{t+1}} = \beta q_{t+1}, \quad q_t = q_{t+1}\beta F'_{k_{t+1}} \quad (13)
\]

Since we focus on the steady-state optimal tax rates, we leave aside the first-order conditions with respect to \(d_0\) and \(\ell_0\) for which some additional terms from the first-old utility should be added.

In steady state, \(q_t\) is constant and the capital-labor ratio then satisfies the modified Golden-rule: \(\beta F'_{k} = 1\). Since in steady-state equilibrium, from (5), we have \(\beta (1 - \tau^x)(1 - \tau^R) F'_{k} = 1\). Then optimal tax rates satisfy

\[
(1 - \tau^x)(1 - \tau^R) = 1
\]

which implies that their signs are opposite. With the definition of \(H^z\), one deduces

\[
W'_{z} = U'_z [1 + \lambda - \lambda H^z], \quad \text{for } z = c, \ell, d.
\]

Since \(q_t\) is the marginal social cost of an increase in public spendings in period \(t\), it is necessarily positive at the optimum. Then, (13) implies that the terms \(W'_z\) have the same sign as \(U'_z\). This implies \(H^z < \frac{1}{\lambda} + 1\). Moreover, optimality conditions (13) in steady state imply

\[
\frac{-W'_c}{W'_c} = F'_\ell, \quad \frac{W'_d}{W'_c} = \beta, \quad \frac{-W'_d}{W'_d} = \frac{F'_\ell}{\beta}
\]

In equilibrium, we have

\[
\frac{-U'_c}{U'_c} = (1 - \tau^w) F'_\ell, \quad \frac{U'_d}{U'_c} = \beta (1 - \tau^x), \quad \frac{-U'_d}{U'_d} = \frac{(1 - \tau^w) F'_\ell}{\beta (1 - \tau^x)}
\]

Then, straightforward calculations lead to the equivalences given in the statement. □

In steady state, as in the infinite-lived agent models considered by Chamley (1986) and Judd (1985), the second-best optimum leads to a capital-labor ratio that corresponds to the modified Golden-rule level. The two-period overlapping generation model with rational altruism is often presented as equivalent to an infinite-lived agents model. It is then natural that the same result about the steady-state capital-labor ratio applies.
Nevertheless, Proposition 2 shows that reaching the modified Golden-rule does not necessarily imply that the inheritance tax rate and the capital income tax rate should be zero. With two-period lifetime, second-best optima can lead the social planner to tax differently consumptions when young and consumption when old. Yet, it turns out that inheritance tax allows to do so. This is the reason why the optimal inheritance tax can be positive or negative. Consequently, the capital income tax allows to keep the capital-labor ratio at the modified Golden-rule level. Then, a positive (resp. negative) inheritance tax implies a negative (resp. positive) capital income tax, so that \((1 - \tau^x) (1 - \tau^R) = 1\). This result crucially depends on the length of life. If people would live for one period, optimal inheritance tax would be zero as in the Chamley-Judd framework.

As stated in Proposition 2, the sign of the inheritance tax is the same as the sign of \(H^c - H^d\). With an additively separable utility function, we get \(H^z = \frac{-U''_{zz}}{U'_z}\), for \(z = c, \ell, d\). Then, the signs of \(\tau^x\) and \(\tau^w\), and their relative levels depend on the elasticities of marginal utilities. In fact, the case for zero inheritance tax and zero capital income tax becomes very narrow. For instance, with a CES utility function of the form

\[
U(c, \ell, d) = \frac{c^{1-1/\sigma}}{1-1/\sigma} + \gamma_1 \frac{(1-\ell)^{1-1/\sigma}}{1-1/\sigma} + \gamma_2 \frac{d^{1-1/\sigma}}{1-1/\sigma},
\]

the optimal tax rates \(\tau^x\) and \(\tau^R\) are zero. But, they differ from zero as soon as one departs from a common function for \(c, d\) and leisure \(1 - \ell\). A direct consequence of Proposition 2 is also that separability between consumption and leisure has no key importance. Unlike Atkinson and Stiglitz (1976)’s well-known result, quasi-separability between leisure and consumption does not imply zero capital income taxation.

It is also interesting to notice that our results do not depend on who pays the tax. The way bequests are introduced into the Barro model leaves the possibility to interpret them as inter vivos monetary transfers. In this case, it seems reasonable to consider that the parent pay the tax. If the inheritance tax is paid by the old, budget constraints of the household rewrite as

\[
c_t + s_t = (1 - \tau^w_t) w_t \ell_t + x_t
\]
\[
d_{t+1} + (1 + \tau^x_{t+1}) x_{t+1} = (1 - \tau^R_{t+1}) R_{t+1} s_t
\]

where \(\tau^x_{t+1}\) is now the tax paid by the parents when the children receive \(x_{t+1}\). Marginal conditions of the household problem imply in steady state

\[
\frac{-U'_\ell}{U'_c} = (1 - \tau^w) F'_\ell, \quad \frac{U'_d}{U'_c} = \frac{\beta}{1 + \tau^x}, \quad \frac{-U'_\ell}{U'_d} = (1 - \tau^w)(1 + \tau^x) \frac{F'_\ell}{\beta}
\]

Then, to get the modified Golden-rule, tax rates must satisfy \(\frac{1-\tau^R}{1+\tau^x} = 1\), which leads to the same kind of equivalence as in Proposition 2.
References


