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Abstract

We consider a two-period overlapping generation model with rational altruism *à la* Barro. The government finances public spending with taxes on labor income, capital income, and inheritance. We show that in the long-run, inheritance tax and capital income tax are generally different from zero, even if the optimal tax policy leads to the modified Golden-rule.

Keywords: inheritance tax, capital income tax, altruism.
JEL Classifications: D64, H21, D91.

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1 Introduction

The seminal works by Chamley (1986) and Judd (1985) state that the optimal long-run tax rate on capital income is zero in neoclassical infinite-lived agent models. In the dynastic interpretation of Chamley (1986)'s model, with successive generations (considered for instance in Piketty and Saez, 2013), inheritance tax and capital income tax are equivalent and then equal to zero at the second-best optimum.[1] Indeed, assuming rational altruism *à la* Barro (1974) creates intergenerational links that make the overlapping generation model similar to the infinite-lived agent model. The issue we address is whether optimal inheritance tax and capital income tax are still zero in a model where individuals live for more than one period.

The purpose of this note is then to analyze the second-best linear tax policy in a two-period overlapping generation model with rational altruism, that is, how to finance given public spending with capital income taxation, labor income taxation, and inheritance tax. Of course, the steady-state second-best optimal path corresponds to the modified Golden-rule, implying zero distortion on capital accumulation in the long-run. But, we show that this does not necessarily imply zero inheritance taxation and zero capital income taxation. Indeed, a positive or negative inheritance tax rate means that consumptions of the young and the old are taxed at different rates. In a second-best world, it may be optimal to set capital income and inheritance tax rates with opposite signs in order to achieve the two following goals: (i) reaching the modified Golden-rule; (ii) taxing consumptions in each period at rates driven by standard Ramsey taxation formulas.

To understand these results, one can rely on the definition of the wedges that lead to taxes or subsidies, that is to the difference, for a given pair of goods, between the marginal rate of substitution and the marginal rate of transformation. Let us define the *intertemporal wedge* as the wedge between consumptions when young for two consecutive generations, and the *intratemporal wedge* as the wedge between consumptions of the young and the old who are alive at the same period.[2] Statement (i) corresponds to a zero optimal *intertemporal wedge*, while statement (ii) means that the optimal *intratemporal wedge* is generally different from zero.

Consequently, our work shows that, with altruistic agents, a zero intertemporal wedge does not imply zero inheritance tax and zero capital income tax. The signs of both taxes depend on the intratemporal wedge. If, in practice, it is considered impossible to tax consumptions differently according to consumers' age, the optimal intratemporal wedge can be achieved with taxes and subsidies on inheritance and capital income.

Interestingly, both statements are very close to the one obtained in the standard Diamond model with egoistic agents by Pestieau (1974) and Atkinson and Sandmo (1980). Indeed, with egoistic agents, the second-best optimum has the same characteristics as (i) and (ii). Taxing both consumptions at different rates is achieved with a capital income tax or a capital income subsidy.

[1] See e.g. Boadway et al. (2010), Cremer and Pestieau (2011), Kopczuk (2013).

[2] We consider here usual definitions of intertemporal and intratemporal wedges (see, e.g., Chari et al., 2019)

Public debt then allows to reach the optimal capital stock that makes the capital-labor ratio equal to the optimal level. With altruistic agents, we show that an inheritance tax/subsidy is chosen in order to offset the effect of the capital income tax on the steady-state capital-labor ratio, and then obtain a zero intertemporal wedge in steady state.

Considering rational altruism with agents who live two periods allows us to highlight a new rationale for non-zero inheritance and capital income tax rates. The optimal taxation literature gives other analytical frameworks leading to non-zero optimal tax rates: idiosyncratic labor income shocks, accidental bequests, borrowing constraints, bequest motive different from rational altruism, lack of government commitment. Statement (ii) does not rely on these assumptions and is not a consequence of concern for equity, since it is obtained in a model with a single dynasty and no uncertainty.

Section 2 describes the intertemporal equilibrium with altruistic and egoistic agents in order to be able to compare the second-best optima in both situations. The second-best optimum with altruistic agents is analyzed in Section 3. In Section 4, we discuss the main results of the altruistic case and compare them to the one obtained in the standard Diamond model with egoistic agents by Pestieau (1974) and Atkinson and Sandmo (1980).

2 Equilibrium

Time is discrete. Population is constant and consists of a unique representative dynasty, where each parent has only one child. Members of the representative dynasty live for two periods, work in the first and retire in the second. They are also altruistic towards their offsprings. When born in t, a household is taxed at rate τ_t^w on labor income, at rate τ_t^x on inherited bequest, and, when old, at rate τ_{t+1}^R on capital income. Consequently, when young, the household born in period t receives an after-tax bequest $(1-\tau_t^x)\,x_t$, chooses labor supply ℓ_t, paid at the net wage $(1-\tau_t^w)\,w_t$, and allocates these resources between consumption c_t and saving s_t. When old, net capital income $\left(1-\tau_{t+1}^R\right)R_{t+1}$ is divided into consumption d_{t+1} and bequests x_{t+1} to the next generation. Requiring that bequests are nonnegative, the above description results in the following constraints

$$
\begin{aligned}
c_t + s_t &= (1-\tau_t^w)\,w_t\ell_t + (1-\tau_t^x)\,x_t && (1)\\
d_{t+1} + x_{t+1} &= \left(1-\tau_{t+1}^R\right)R_{t+1}s_t && (2)\\
x_{t+1} &\geq 0 && (3)
\end{aligned}
$$

Rational altruism implies that utility of the representative household in period t, V_t, depends on consumptions in both periods, labor supply and utility of its offsprings V_{t+1}: $V_t = U\left(c_t, \ell_t, d_{t+1}\right) + \beta V_{t+1}$, where β represents the degree of altruism $(0 \leq \beta < 1)$.[3] If $\beta = 0$, households behave

[3]We assume that U is twice continuously differentiable and strictly concave. First-order derivatives with respect to c and d are positive, while first-order derivative with respect to ℓ is negative.

egoistically as in the Diamond (1965)'s model. If $\beta > 0$, the problem of the dynasty at time zero is to maximize[4]

$$V_{-1} = \sum_{t=-1}^{+\infty} \beta^{t+1} U\left(c_t, \ell_t, d_{t+1}\right) \tag{4}$$

with respect to $(c_t, \ell_t, d_t, x_t)_{t \geq 0}$, subject to

- the law of motion of the bequests (deduced from (1) and (2))

$$x_{t+1} = \left(1 - \tau_{t+1}^R\right) R_{t+1} \left[\left(1 - \tau_t^w\right) w_t \ell_t + \left(1 - \tau_t^x\right) x_t - c_t\right] - d_{t+1},$$

- the budget constraint of the first old

$$d_0 + x_0 = \left(1 - \tau_0^R\right) R_0 \bar{s}_{-1},$$

- the nonnegativity constraint (3) for $t \geq -1$,

given $c_{-1} = \bar{c}_{-1}$ and $\ell_{-1} = \bar{\ell}_{-1}$. From Michel et al. (2006), household optimum satisfies the first-order conditions, for $t \geq 0$,[5]

$$\frac{-U'_{\ell_t}}{U'_{c_t}} = \left(1 - \tau_t^w\right) w_t, \qquad \frac{U'_{d_{t+1}}}{U'_{c_t}} = \frac{1}{\left(1 - \tau_{t+1}^R\right) R_{t+1}} \tag{5}$$

$$-U'_{d_t} + \beta \left(1 - \tau_t^x\right) U'_{c_t} \leq 0, \ = 0 \text{ if } x_t > 0 \tag{6}$$

and the transversality condition

$$\lim_{t \to +\infty} \beta^{t-1} U'_{d_t} x_t = 0 \tag{7}$$

Given $\bar{c}_{-1}$, $\bar{\ell}_{-1}$ and $\bar{s}_{-1}$, conditions (5)-(6) for $t \geq 0$, and (7), associated with budget constraints (1) for $t \geq 0$, and (2) for $t \geq -1$, bring a complete characterization of the household optimum $(c_t, \ell_t, d_t, x_t, s_t)_{t \geq 0}$.

In the case of egoistic agents ($\beta = 0$), bequests are zero in all periods and the household optimum $(c_t, \ell_t, d_t, s_t)_{t \geq 0}$ satisfies conditions (5) and budget constraints (1) for $t \geq 0$ and (2) for $t \geq -1$, with $x_t = x_{t+1} = 0$. Notice also that $\beta > 0$ does not guarantee that condition (6) is satisfied at equality. Altruistic agents do not necessarily leave positive bequests. Weil (1987) and Abel (1987) have given conditions for bequest to be operative (that is, positive) in steady state. This arises in particular if the intensity of altruism is strong enough.

A representative firm that behaves competitively, produces output in period t with capital K_t and labor L_t. The production function $F(K, L)$ is linear homogenous and concave, and includes capital

[4]By induction, utility of the first old V_{-1} can be written as follows: $V_{-1} = \sum_{t=-1}^{T} \beta^{t+1} U_t + \beta^{T+2} V_{T+1}$, where U_t stands for $U\left(c_t, \ell_t, d_{t+1}\right)$. Then we get equation (4) assuming the limit condition: $\lim_{T \to +\infty} \beta^{T+2} V_{T+1} = 0$.

[5]U'_{c_t}, U'_{ℓ_t} and U'_{d_t} stand respectively for $U'_c\left(c_t, \ell_t, d_{t+1}\right)$, $U'_\ell\left(c_t, \ell_t, d_{t+1}\right)$ and $U'_d\left(c_{t-1}, \ell_{t-1}, d_t\right)$.

depreciation. Marginal products are strictly positive and decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$w_t = F'_L(K_t, L_t), \quad R_t = F'_K(K_t, L_t) \tag{8}$$

where F'_L and F'_K stand for the partial derivatives of F with respect to labor and capital.

The government finances an exogenous sequence of public spendings with taxes on capital income, labor income and inheritance, and has the possibility to issue bonds. Denoting by g, the constant level of public spendings, the law of motion of the public debt δ_t is

$$\delta_{t+1} + \tau_t^w w_t \ell_t + \tau_t^R R_t s_{t-1} + \tau_t^x x_t = R_t \delta_t + g \tag{9}$$

The initial public debt δ_0 is given: $\delta_0 = \bar{\delta}_0$. Saving splits between public debt and capital stock. Denoting by k_t, the capital stock in period t, one gets

$$s_t = k_{t+1} + \delta_{t+1} \tag{10}$$

for $t \geq 0$ and $k_0 = \bar{k}_0 := \bar{s}_{-1} - \bar{\delta}_0$. We define the competitive equilibrium for a given sequence of tax rates $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$:

Definition 1. *Consider $c_{-1} = \bar{c}_{-1}$, $\ell_{-1} = \bar{\ell}_{-1}$ and $k_0 = \bar{k}_0$. A competitive equilibrium with tax instruments $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$ is an allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$, a sequence of bequests and savings $(x_t, s_t)_{t\geq 0}$ and a sequence of prices $(w_t, R_t)_{t\geq 0}$ such that:*

- *for the prices $(w_t, R_t)_{t\geq 0}$ and the tax instruments $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$, the path $(c_t, \ell_t, d_t, x_t, s_t)_{t\geq 0}$ satisfies the budget constraints (equation (1) for $t \geq 0$ and equation (2) for $t \geq -1$), the optimality condition (5) for $t \geq 0$, and, if $\beta > 0$, the optimality condition (6) for $t \geq 0$ and the transversality condition (7);*
- *$(k_t, \ell_t)_{t\geq 0}$ satisfies FOCs (8) of the firms for the prices $(w_t, R_t)_{t\geq 0}$ with $K_t = k_t$ and $L_t = \ell_t$;*
- *the market clearing conditions for good are satisfied, that is, for $t \geq 0$,*

$$c_t + d_t + g + k_{t+1} = F(k_t, \ell_t) \tag{11}$$

Equation (10) allows to determine the sequence of public debt that is consistent with such an intertemporal equilibrium. We do not mention the government budget constraint (9) since, from the Walras' law, equilibrium on all markets makes it redundant. The definition of the competitive equilibrium encompasses both altruistic and egoistic cases.

3 Second-best optimum with altruistic agents

To solve the government program in an economy with altruistic households ($\beta > 0$), we leave aside the non-negativity constraint on bequests. This means that the path $(x_t)_{t\geq 0}$ that will result from the optimality conditions will be the sequence of desired bequests. Then the allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ that satisfies optimality conditions of the government program is implementable if the associated desired bequests are positive in all periods.

In the following proposition, we introduce a characterization of the set of available competitive equilibria using the implementability constraint. So doing, we follow the primal approach introduced by Lucas and Stokey (1983) and used by Atkeson et al. (1999) for analyzing the optimal capital income taxation.

Proposition 1. *Consider $c_{-1} = \bar{c}_{-1}$, $\ell_{-1} = \bar{\ell}_{-1}$ and $k_0 = \bar{k}_0$. Let us assume that households are altruistic ($\beta > 0$).*

- *Consider the allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ of a competitive equilibrium with operative bequests. Then, it satisfies the resource constraint (11) for $t \geq 0$, and the implementability constraint*

$$\sum_{t=0}^{+\infty} \beta^t \left[U'_{c_t} c_t + U'_{\ell_t} \ell_t + U'_{d_{t+1}} d_{t+1} \right] = \frac{U'_{d_0}}{\beta} \left[\left(1 - \tau_0^R\right) R_0 \bar{s}_{-1} - d_0 \right] \tag{12}$$

- *Conversely, consider an allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ that satisfies (11) and (12). Then there exist prices $(w_t, R_t)_{t\geq 0}$, desired bequests and savings $(x_t, s_t)_{t\geq 0}$ and instruments $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$ such that $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ is the allocation of a competitive equilibrium with tax instruments $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$.*

Proof. First, consider the allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ of a competitive equilibrium. It satisfies the resource constraint (11). To show that it also satisfies the implementability constraint (12), we deduce the intertemporal budget constraint of the household born in t from the budget constraints (1) and (2):

$$c_t + \frac{d_{t+1}}{(1 - \tau_{t+1}^R) R_{t+1}} = (1 - \tau_t^w) w_t \ell_t + (1 - \tau_t^x) x_t - \frac{x_{t+1}}{(1 - \tau_{t+1}^R) R_{t+1}}, \tag{13}$$

and use the marginal conditions of the household problem (5)-(6) to get

$$U'_{c_t} c_t + U'_{\ell_t} \ell_t + U'_{d_{t+1}} d_{t+1} = \frac{U'_{d_t} x_t}{\beta} - U'_{d_{t+1}} x_{t+1}$$

Then, multiplying by β^t and summing over $t = 0, ..., T$, one obtains

$$\sum_{t=0}^{T} \beta^t \left[U'_{c_t} c_t + U'_{\ell_t} \ell_t + U'_{d_{t+1}} d_{t+1} \right] = \frac{U'_{d_0} x_0}{\beta} - \beta^T U'_{d_{T+1}} x_{T+1}$$

For $T \to +\infty$, the transversality condition (7) allows to get the implementability constraint (12).

For the converse direction, we need to show that, for any allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ that satisfies (11) and (12), there exist tax instruments $\left(\tau_t^w, \tau_t^R, \tau_t^x\right)_{t\geq 0}$, desired bequests and savings $(x_t, s_t)_{t\geq 0}$ and prices $(w_t, R_t)_{t\geq 0}$ for which all equilibrium equations in Definition 1 are satisfied. Indeed, τ_0^R is obtained from the implementability constraint (12). Moreover, factor prices $(w_t, R_t)_{t\geq 0}$ are deduced from the sequence with the FOCs (8) of the firms. Then, marginal conditions of the consumer (5)-(6) allow to compute $\left(\tau_t^x, \tau_t^w, \tau_{t+1}^R\right)_{t\geq 0}$. Finally, the sequence $(x_t, s_t)_{t\geq 0}$ is obtained from the budget constraints of the households (1) for $t \geq 0$ and (2) for $t \geq -1$, and given $\bar{s}_{-1}$.■

The government is looking for the allocation $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ that maximizes welfare (4) among the paths that satisfy the resource constraints (11) in all periods $t \geq 0$ and the implementability constraint (12). Let us define

$$W\left(c_t, \ell_t, d_{t+1}\right) := U_t + \lambda\left[U'_{c_t} c_t + U'_{\ell_t}\ell_t + U'_{d_{t+1}} d_{t+1}\right]$$

where λ is the multiplier of the implementability constraint in the government's problem; the arguments of utility U_t and marginal utilities U'_{c_t}, U'_{ℓ_t} and $U'_{d_{t+1}}$ are (c_t, ℓ_t, d_{t+1}) for $t \geq 0$, and $\left(\bar{c}_{-1}, \bar{\ell}_{-1}, d_0\right)$ for $t = -1$. The capital income tax in period 0 is a lump-sum tax since it applies to a basis that the household cannot modify. We consider it as given: $\tau_0^R = \bar{\tau}_0^R$. The problem of the government then remains to maximize

$$SW_{-1} := U_{-1} + \left(d_0 - \left(1 - \bar{\tau}_0^R\right) F'_K\left(\bar{k}_0, \ell_0\right)\bar{s}_{-1}\right)\lambda U'_{d_0} + \sum_{t=0}^{+\infty} \beta^{t+1} W\left(c_t, \ell_t, d_{t+1}\right)$$

with respect to $(c_t, \ell_t, d_t, k_{t+1})_{t\geq 0}$ subject to the resource constraint in each period.

Proposition 2. *If the second-best optimum allocation converges towards a steady state, the corresponding capital-labor ratio reaches the modified Golden-rule level ($\beta F'_K(k, \ell) = 1$) and optimal tax rates satisfy:*

$$\left(1 - \tau^R\right)\left(1 - \tau^x\right) = 1. \tag{14}$$

Then, assuming $\tau^x < 1$ and $\tau^R < 1$,

$$\left.\begin{array}{rcl} \tau^x > 0 \quad \Leftrightarrow \quad \tau^R < 0 & \Leftrightarrow & H^c > H^d \\ \tau^w > 0 & \Leftrightarrow & H^c > H^\ell \\ \tau^x > \tau^w & \Leftrightarrow & H^\ell > H^d \end{array}\right\} \tag{15}$$

where H^z corresponds to the sum of elasticities of the marginal utility U'_z with respect to each of the quantities c, ℓ and d

$$H^z := \frac{-U''_{zc}c}{U'_z} + \frac{-U''_{z\ell}\ell}{U'_z} + \frac{-U''_{zd}d}{U'_z}$$

Proof. We consider the infinite Lagrangian

$$SW_{-1} + \sum_{t=0}^{+\infty} \beta^{t+1} q_t \left[F\left(k_t, \ell_t\right) - c_t - d_t - g - k_{t+1} \right] \tag{16}$$

where $\beta^{t+1} q_t$ is the multiplier of the resource constraint in period t. The first-order conditions with respect to c_t, ℓ_{t+1}, d_{t+1} and k_{t+1}, for $t \geq 0$, write

$$W'_{c_t} = q_t, \qquad W'_{\ell_{t+1}} + q_{t+1} F'_{\ell_{t+1}} = 0, \qquad W'_{d_{t+1}} = \beta q_{t+1}, \qquad q_t = q_{t+1} \beta F'_{k_{t+1}} \tag{17}$$

where $F'_{k_t} := F'_K\left(k_t, \ell_t\right)$ and $F'_{\ell_t} := F'_L\left(k_t, \ell_t\right)$. Since we focus on the steady-state optimal tax rates, we leave aside the first-order conditions with respect to d_0 and ℓ_0 for which some additional terms from the first-old utility should be added. In steady state, q_t is constant and the capital-labor ratio then satisfies the modified Golden-rule: $\beta F'_k = 1$. Additionally, from (5)-(6) and (8), we deduce that, at a steady-state equilibrium with operative bequest: $\beta\left(1 - \tau^x\right)\left(1 - \tau^R\right) F'_k = 1$. Then optimal tax rates satisfy equation (14), which implies that their signs are opposite.

Moreover, optimality conditions (17) in steady state imply

$$\frac{-W'_\ell}{W'_c} = F'_\ell, \qquad \frac{W'_d}{W'_c} = \beta, \qquad \frac{-W'_\ell}{W'_d} = \frac{F'_\ell}{\beta} \tag{18}$$

In equilibrium, we have

$$\frac{-U'_\ell}{U'_c} = \left(1 - \tau^w\right) F'_\ell, \qquad \frac{U'_d}{U'_c} = \beta\left(1 - \tau^x\right), \qquad \frac{-U'_\ell}{U'_d} = \frac{\left(1 - \tau^w\right) F'_\ell}{\beta\left(1 - \tau^x\right)}$$

Combining with (18), one gets

$$\left(1 - \tau^w\right) \frac{1 + \lambda - \lambda H^\ell}{1 + \lambda - \lambda H^c} = 1, \qquad \left(1 - \tau^x\right) \frac{1 + \lambda - \lambda H^d}{1 + \lambda - \lambda H^c} = 1, \qquad \frac{1 - \tau^w}{1 - \tau^x} \, \frac{1 + \lambda - \lambda H^\ell}{1 + \lambda - \lambda H^d} = 1$$

From the definition of H^z, we deduce

$$W'_z = U'_z \left[1 + \lambda - \lambda H^z\right], \text{ for } z = c, \ell, d.$$

Since q_t is the marginal social cost of an increase in public spendings in period t, it is necessarily positive at the optimum. Then, the terms W'_z have the same sign as U'_z. Consequently, $1 + \lambda - \lambda H^z > 0$. We then obtain the equivalences given in the statement.■

4 Discussion

In steady state, the second-best optimum leads to a capital-labor ratio that corresponds to the modified Golden-rule level (zero intertemporal wedge). On this matter, the two-period overlapping generation model with altruistic agents leads to the same result as infinite-lived agent models considered by Chamley (1986) and Judd (1985), OLG models with egoistic agents (see e.g. Pestieau, 1974, Atkinson and Sandmo, 1980), and successive-generation models with altruistic agents (see *e.g.* Boadway et al., 2010, Cremer and Pestieau, 2011, Kopczuk, 2013).

Nevertheless, by contrast to successive-generation models, Proposition 2 shows that reaching the modified Golden-rule does not necessarily imply that the inheritance tax rate and the capital income tax rate should be zero in steady state. The reason is that, with two-period lifetime, second-best optima can lead the social planner to tax differently consumption when young and consumption when old (non-zero intratemporal wedge). Precisely, the inheritance tax rate, if positive, acts as a subsidy toward old-age consumption. From the point of view of the parent, one additional unit given to the offspring will have a higher cost in term of current consumption. Inheritance tax affects the allocation of resources between generations that live in the same period. By contrast, the capital income tax modifies the allocation of the representative household between consumptions along the life-cycle. Taxing capital income increases the price of old-age consumption into the intertemporal budget constraint of the households.

Equation (14) in Proposition 2 implies that, in steady state with altruistic agents and operative bequests, the inheritance tax rate and the capital income tax rate should have opposite signs in order to reach the optimal capital-labor ratio. A positive inheritance tax should be associated with a negative capital income tax, both leading to a decrease in the relative price of second-period consumption to first-period consumption.

The condition we get for positive inheritance taxation can be related to the one obtained by Pestieau (1974) and Atkinson and Sandmo (1980) for negative capital income tax rate in the OLG model with egoistic agents. Results by Pestieau (1974) and Atkinson and Sandmo (1980) are derived using the dual approach, that is, taking prices rather than quantities as control. To highlight the link between conditions (15) and the conditions derived with egoistic agents, it is convenient to also write the second-best problem in the latter case using the primal approach. With $\beta = 0$, the intertemporal budget constraint is the same as (13) except that bequests in t and $t+1$ are zero. For generation-t household, using marginal conditions (5), the implementability constraint writes

$$U'_{c_t} c_t + U'_{\ell_t} \ell_t + U'_{d_{t+1}} d_{t+1} = 0 \tag{19}$$

Let us define γ as the social discount factor, not necessarily equal to the degree of altruism β. The social planner maximizes the discounted sum of utilities under the resource constraint (11) and the implementability constraint for generation t (19). The infinite Lagrangian is the same as in equation (16) with γ instead of β as the discount factor, and replacing the function W by

$$W_t \left(c_t, \ell_t, d_{t+1}\right) := U_t + \mu_t \left[U'_{c_t} c_t + U'_{\ell_t} \ell_t + U'_{d_{t+1}} d_{t+1}\right]$$

where $\gamma^t \mu_t$ is the multiplier of the implementability constraint (19). Therefore, optimality conditions in steady state are the same as (18), except for the degree of altruism β which is replaced by the social discount factor γ, recalling that the steady-state value of the multiplier μ replaces λ into the derivatives of W. Since in steady-state equilibrium with egoistic agents, we have

$$\frac{U_c'}{U_d'} = \left(1-\tau^R\right) F_k', \quad \frac{-U_\ell'}{U_c'} = \left(1-\tau^w\right) F_\ell',$$

it is then straightforward to see that equivalences (15) become

$$\left.\begin{array}{rcl} \tau^R < 0 & \Leftrightarrow & H^c > H^d \\ \tau^w > 0 & \Leftrightarrow & H^c > H^\ell \\ \left(1-\tau^R\right)\left(1-\tau^w\right) > 1 & \Leftrightarrow & H^\ell > H^d \end{array}\right\}$$

If the terms H^z have the same values with altruistic agents and egoistic agents, then conditions for the signs of τ^R and τ^w are exactly the same. Of course, such a sameness can be obtained in very particular cases that combine additive separability and isoelasticity (*e.g.* $U(c,\ell,d) = c^{\rho_0} + \gamma_1 \ell^{\rho_1} + \gamma_2 d^{\rho_2}$). More generally, apart from these particular cases, H^z depends on the steady-state quantities (c,ℓ,d,k) characterized by the set of four equations that includes two optimality conditions among (18), the resource constraint (11) in steady state and the characterization of the capital-labor ratio $\beta F_k' = 1$. All these equations are exactly the same if the degree of altruism β and the social discount factor γ are equal. Consequently, conditions that characterize the sign of the tax rates τ^R and τ^w under assumptions of egoistic or altruistic agents become identical if generations are discounted in the same way.

Nevertheless, the way the social planner implements the modified Golden-rule in the long-run is not the same. With altruistic agents and operative bequests, positive inheritance tax and negative capital income tax can compensate one another to put the capital-labor ratio at the modified Golden-rule level. But equilibrium saving and equilibrium labor supply (obtained with the optimal tax instruments) do not necessarily lead to the optimal capital-labor ratio. The social planner can use public debt to fill the gap between saving and capital stock (see equation (10)). By contrast, with egoistic agents, the negative capital income tax rate cannot be compensated by a positive inheritance tax rate to reach the optimum steady-state capital-labor ratio. But, the optimal level of capital stock can be achieved using public debt.

We have shown that a zero optimal intertemporal wedge does not imply zero inheritance and capital income tax rates. Both depend on the intratemporal wedge, that is, on the fact that the social planner may prefer to apply different tax rates to the consumptions of the young and the old living in the same period. This result is obtained on efficiency ground and is not linked to any equity concern. Undoubtedly, a non-zero intratemporal wedge should still be obtained in a model with heterogeneous agents, that is, considering different wages, different preferences, different altruism behavior, etc. In this note, we have isolated this property in the simplest possible model, but the reasons why we get a non-zero optimal intratemporal wedge should still be present in more general frameworks and should affect the optimal tax rates on inheritance and capital income.

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