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Disaggregating the Chinese annual national accounts to quarterly series

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Abstract

This article develops a methodology to compute up-to-date quarterly macroeconomic data for emerging countries by adapting a well-known method of temporal disaggregation to time series with very small sample size and unstable relationships between them. By incorporating different procedures of structural break detection, the prediction of higher-frequency estimations of yearly official data can be improved. A methodology with a model selection procedure and disaggregation formulas is proposed. Its predictive performance is assessed by using empirical advanced countries data and simulated time series. An application to the Chinese national accounts allows the estimation of the cyclical components of the Chinese expenditure accounts and shows the Chinese economy to have second-order moments more in line with emerging countries than advanced economies like the United States.

*This is the last version of a discontinued project which was initially released in June 2019. Most results have been generalized in a more recent work.

1 Introduction

In order to analyze the macroeconomic fluctuations of a particular economy, time series with a frequency higher than annual are preferable. Unfortunately for both the academic community and the practitioners interested in studying the business cycles of important but opaque emerging countries such as China or Russia, these frequencies are not available for such fundamental aggregates as the ones documented in national accounts. Official free access time series provided by national statistics administrations are usually insufficient: data with a long history are of annual frequency and data with higher frequency have a short history. If the annual data can be easily deflated by finding or approximating price indexes for each account, business cycle analyses still require up-to-date quarterly or monthly series. Chow and Lin (1971) details the interpolation, distribution, and extrapolation of quarterly data using monthly indicators. Stram and Wei (1986) and Wei and Stram (1990) exploits the autocorrelation structure of the initial data without any additional information. While both methods are adaptable for disaggregating annual data, the first method which relies on related higher frequency series fits our case of interest considering the much lower abundance of the observable data. The principle is to incorporate fluctuations from higher-order frequency indicators while retaining the annual aggregated level of the series to disaggregate, and the procedure is straightforward: an annual prediction model can be estimated by linking the annual accounts and the annual aggregations of the quarterly or monthly indicator, then estimated coefficients are applied to the observed quarterly or monthly indicators to estimate a quarterly or monthly disaggregated national account time series which are constrained to add up to the observed yearly data. This method assumes a stable linear relationship between the national accounts series and the indicator series, which is unlikely to be true for emerging markets. As a consequence, the behavior of the macroeconomic aggregates and their relationship may also change over the period, despite the shortness of the series, it is then useful to allow for structural change in the parameters of the linear relationship.

Hansen (1992a) provides a test for parameter stability with an endogenous (or unknown) date of structural break which is only valid if the regressors are stationary. Since annual national accounts data are often found to be non-stationary, especially for emerging markets, a test of parameter instability with non-stationary regressors is more adapted, such as proposed by Hansen (1992b). That being said, the validity of the predicting model resides in the stationarity of the error process, no matter how we decide to specify the structural changes. Although Chow and Lin (1971) imposes the stationarity of the series, which can be attained by differencing the data (Fernandez

(1981)), the residuals may already be stationary if the series are cointegrated, which would avoid unnecessary transformation of the data. Since conventional cointegration tests such as residual-based augmented Dickey-Fuller (ADF) tests lose significant power in the presence of a structural change, Gregory and Hansen (1996) have developed a test of the null of no cointegration against the alternative of cointegration with a structural break, which we will use as a first step to select the linear relationship between the annual national accounts data and the related infra-annual indicators. This test is better used on long series as it shows significant size distortion in small sample (Cook (2006)), we therefore need to assess and adapt its use for time series of small or very small sample sizes beforehand.

Our main objective is to predict quarterly estimates of annual aggregates, which is a different approach than nowcasting procedures which predicts up-to-date annual aggregates from higher frequency and contemporaneous data. Testing for stationary residuals however does not guarantee a small prediction error. Therefore, we confront the cointegration with structural break approach with a prediction-focused selection process that minimizes an observable prediction error such as the root mean square error of the annual predicted aggregates. The first section details the different methodologies we adopt for the disaggregation process. The second section illustrates the competing methodologies by disaggregating two components of the Chinese national accounts. The third section assesses the performance of the methodologies in order to deduce a final disaggregation process, by applying them to the US data where we observe the quarterly series, then to simulated data to study their performance in terms of power and prediction error. The final section gives an example of a business cycle study made possible by the disaggregation of the Chinese national accounts with a basic business cycle stylized facts study and its comparison to former results in the literature for other countries.

2 Methodology

The method of disaggregation introduced by Chow and Lin (1971) uses the contemporary linear relationship between a given time series and the temporal aggregates of higher frequency indicators to predict the former, given the stationarity of the residuals of the relationship. In practice, the test of the stationarity of the residuals of the model linking the series of interest and its aggregated indicator is identical to a cointegration test. When studying emerging countries' data, two problems arise: the available time series have a very small number of observations (less than fifty for annual data), and

the linear relationship is likely to be unstable in the sense that its parameters are subjected to at least one structural change. In order to take into account time-varying parameters in very small samples, we compare two competing methods to modelize and detect structural breaks: the test of cointegration with an unknown structural break of Gregory and Hansen (1996) which minimizes the Augmented Dickey-Fuller (ADF) statistics of the unit root test of the residuals of the model, and a procedure which minimizes the prediction error of the annual series to disaggregate. In this section we detail a two-step methodology for the disaggregation of an annual data series using a related quarterly indicator, by in the first place quickly recalling the case without a structural change derived initially by Chow and Lin (1971), and then expose the case with structural change.

2.1 The case without structural change in very small sample

For the study, we will define T as the length of the low-frequency series and f as the number of observations of the high frequency related series at each date ($f = 4$ for quarterly indicators). Uppercase and lowercase letters respectively stand for low and high-frequency time series. Following Chow and Lin (1971), the underlying high-frequency model can be written as:

$$\mathbf{y} = \mathbf{z}\Gamma + \mathbf{u} \quad (1)$$

where $\mathbf{y} = (y_1, \dots, y_{fT})'$ is the unobserved quarterly series of interest, $\mathbf{z} = (\mathbf{1}_{fT}, \mathbf{x})$ with $\mathbf{1}_p$ a p -dimensional column vector of ones and $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_m)$ the m quarterly indicators, $\mathbf{x}_i = (x_{i,1}, \dots, x_{i,fT})'$ for $i = 1, \dots, m$, and Γ a vector of coefficients. The residuals \mathbf{u} are usually set to follow an AR(1) process such that $u_t = \rho u_{t-1} + \varepsilon_t$ for $t \in \{1, \dots, fT\}$, ε_t being a white noise disturbance. The non stationary case $\rho = 1$ will be given a particular treatment, as in Fernandez (1981).

For the series of interest, only the annual aggregates $\mathbf{Y} = \mathbf{A}\mathbf{y}$ are observed, where $\mathbf{A} = \mathbf{I}_T \otimes \mathbf{1}'_f$ is a $(T \times fT)$ aggregating matrix, with \mathbf{I}_p the p -dimension identity matrix. \mathbf{Y} can be expressed as a linear model of the annual aggregates $\mathbf{Z} = \mathbf{A}\mathbf{z}$ and $\mathbf{U} = \mathbf{A}\mathbf{u}$ such that:

$$\mathbf{Y} = \mathbf{Z}\Gamma + \mathbf{U}. \quad (2)$$

If \mathbf{u} follows an AR(1), it is easily shown that \mathbf{U} follows an ARMA(1,1). If \mathbf{u} is I(1), \mathbf{U} is also I(1). In practice, the choice between the two specifications will depend on the

rejection or not of the stationarity of \mathbf{U} . A simple method is to test for a unit root in \mathbf{U} . If it is rejected, we can obtain the quarterly prediction $\hat{\mathbf{y}}$ for \mathbf{Y} as:

$$\hat{\mathbf{y}} = \mathbf{z}\hat{\Gamma} + (\mathbf{D}'_{fT}\mathbf{D}_{fT})^{-1}\mathbf{A}'(\mathbf{A}(\mathbf{D}'_{fT}\mathbf{D}_{fT})^{-1}\mathbf{A}')^{-1}(\mathbf{Y} - \mathbf{A}\mathbf{z}\hat{\Gamma}) \quad (3)$$

where $\hat{\Gamma} = (\mathbf{z}'\mathbf{A}'\mathbf{A}\mathbf{z})^{-1}\mathbf{z}'\mathbf{A}'\mathbf{Y}$

where \mathbf{D}_p is a $((p-1) \times p)$ matrix that converts a time series to its first differences, such that

$$\mathbf{D}_p = \begin{pmatrix} -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}.$$

If \mathbf{U} is not stationary, the model will be estimated in first difference¹, which we define as the model dO:

$$\mathbf{D}_T\mathbf{Y} = \mathbf{Z}_\Delta\Gamma + \mathbf{V} \quad (4)$$

where $\mathbf{Z}_\Delta = (\mathbf{1}_{T-1}, \mathbf{D}_T\mathbf{A}\mathbf{x})$. If the intercept coefficient in the model in first difference is significant, it implies that there is a significant linear trend component. The disaggregation² formula is therefore (see proof in Appendix A):

$$\hat{\mathbf{y}} = \hat{y}_1\mathbf{1}_{fT} + \mathbf{z}_\Delta\Gamma + (\mathbf{D}'_{fT}\mathbf{D}_{fT})^{-1}\mathbf{A}'(\mathbf{A}(\mathbf{D}'_{fT}\mathbf{D}_{fT})^{-1}\mathbf{A}')^{-1}(\mathbf{D}_T\mathbf{Y} - \mathbf{Z}_\Delta\hat{\Gamma}) \quad (5)$$

where $\hat{\Gamma} = (\mathbf{Z}'_\Delta\mathbf{Z}_\Delta)^{-1}\mathbf{Z}'_\Delta\mathbf{D}_T\mathbf{Y}$

where $\hat{y}_1 = \frac{Y_1}{f} + \left(\frac{f-1}{2f}, (\mathbf{x}_1 - \mathbf{X}_1)'\right)\hat{\Gamma}$ and $\mathbf{z}_\Delta = \{\mathbf{z}'_{\Delta t}\}_t$ with $\mathbf{z}_{\Delta t} = \left(\frac{t-1}{f^2}, (\mathbf{x}_t - \mathbf{x}_1)'\right)'$.

2.2 Taking into account parameter instability

It is known that unit root tests have low power against structural breaks (e.g. Gregory et al. (1996) for the test of the null of no cointegration). Taking into account parameter instability in the regression model can therefore avoid unnecessary differentiation of the variables. Let us define n_b as the annual date of structural break in the parameters, such that some coefficients of the linear relationship change at date $t \geq t_b$. Four types of structural break in the coefficient values are considered³ and can be expressed in our

¹We are less restrictive than Fernandez (1981) by allowing for an intercept in the differentiated model. If the intercept is proven not to be significant, its estimate should be small enough so that the resulting model isn't quantitatively very different.

²We cannot use the disaggregation formula of Fernandez (1981) as the assumption of constancy of the first year values does not hold and would considerably impact the estimates considering our small sample sizes.

³These are the models for which asymptotical properties of the statistics for testing cointegration with structural breaks have been derived by Gregory and Hansen (1996).

framework by redefining the regressors \mathbf{Z} and \mathbf{Z}_Δ and their high-frequency counterparts \mathbf{z} and \mathbf{z}_Δ (see Appendix B for the proof):

- model C: I(0) residuals with a shift in the intercept

$$\mathbf{Z}_t = (1, \mathbf{X}'_t, \mathbb{1}_{t \geq t_b})' \text{ and } \mathbf{z}_t = (f^{-1}, \mathbf{x}'_t, f^{-1} \mathbb{1}_{t \geq f(t_b-1)+1})' \quad (6)$$

- model CS: I(0) residuals with a shift in the intercept and the slope

$$\mathbf{Z}_t = (1, \mathbf{X}'_t, \mathbb{1}_{t \geq t_b}, \mathbf{X}'_t \mathbb{1}_{t \geq t_b})' \text{ and } \mathbf{z}_t = (f^{-1}, \mathbf{x}'_t, f^{-1} \mathbb{1}_{t \geq f(t_b-1)+1}, \mathbf{x}'_t \mathbb{1}_{t \geq f(t_b-1)+1})' \quad (7)$$

- model dC: I(1) residuals with a shift in the intercept

$$\begin{aligned} \mathbf{Z}_{\Delta t} &= (1, (1-L)\mathbf{X}'_t, \mathbb{1}_{t \geq t_b})' \\ &\text{and} \\ \mathbf{z}_{\Delta t} &= \left(\frac{t-1}{f^2}, (\mathbf{x}_t - \mathbf{x}_1)', \frac{t - f(t_b - 1.5) - 0.5}{f^2} \mathbb{1}_{t \geq f(t_b-1)+1} \right)' \end{aligned} \quad (8)$$

- model dCS: I(1) residuals with a shift in the intercept and the slope

$$\begin{aligned} \mathbf{Z}_{\Delta t} &= (1, (1-L)\mathbf{X}'_t, \mathbb{1}_{t \geq t_b}, (1-L)\mathbf{X}'_t \mathbb{1}_{t \geq t_b})' \\ &\text{and} \\ \mathbf{z}_{\Delta t} &= \left(\frac{t-1}{f^2}, (\mathbf{x}_t - \mathbf{x}_1)', \frac{t - f(t_b - 1.5) - 0.5}{f^2} \mathbb{1}_{t \geq f(t_b-1)+1}, \left(\mathbf{x}_t - \frac{\mathbf{X}_{t_b-1}}{f} \right)' \hat{\alpha}_1 \right)' \mathbb{1}_{t \geq f(t_b-1)+1} \end{aligned} \quad (9)$$

The date of structural break t_b is however unknown. We compare two different approaches to select it. One approach is to select the date which yields a stationary model: it is exactly a test of cointegration with an endogenous structural break, as in Gregory and Hansen (1996). Another approach is to select the date which yields the best prediction by minimizing the annual prediction errors.

2.3 Model selection by rejecting the absence of cointegration with an endogenous structural break

The existing method partly consists in checking the absence of a unit root in the residuals of a model without a structural break. In an approach to consider unit root testing of the residuals as a model selection criterion, a natural extension of the existing method is to take into account a structural break in parameters in the cointegration model. Such a test of no cointegration with an unknown structural break is developed by Gregory and Hansen (1996). The estimated structural break date \hat{t}_b is selected by minimizing the unit root test statistic of the residuals among models where

$t_b \in \mathcal{T}_b = \llbracket 0.15T, 0.85T \rrbracket$, where $\llbracket \cdot \rrbracket$ stands for the integer part function. Since we are interested in small sample series, we compute the ADF statistics $ADF(t_b, M)$ for each possible date t_b and model $M \in \mathcal{M} = \{O, C, CS, dO, dC, dCS\}$, where the number of lags in the ADF model is the smallest for which the residuals of the ADF model are not serially correlated. For each model M considered, we note the resulting statistics testing the null hypothesis of no cointegration against the alternative of cointegration with endogenous structural break $ADF^*(M)$ such that

$$ADF^*(M) = \inf_{t_b \in \mathcal{T}_b} ADF(t_b, M). \quad (10)$$

Gregory and Hansen (1996) compute asymptotical critical values for $ADF^*(M)$, which are reported in Table 1. They also assess the simulated performance of the test statistics and show that the ADF-type statistics are negatively biased with respect to the null in small sample. More precisely, they find that for sample size $T = 100$ and using the asymptotical critical value at level 5% for models C and CS on 2 500 replications of data simulated under the null, the test statistic respectively rejects 8 and 5 percentage points too often the null hypothesis when the latter is true. For a smaller sample size $T = 50$, the size distortion increases to 12 and 8 percentage points respectively.

M	1 %	2.5 %	5 %	10 %	97.5 %
C	-5.13	-4.83	-4.61	-4.34	-2.25
CS	-5.57	-5.19	-4.95	-4.68	-2.55

Table 1: Asymptotical critical values for the cointegration test of Gregory and Hansen (1996) with one regressor ($m = 1$), by model M

The sample sizes we are interested in are much smaller, with $T = 50$ being a high upper bound when it comes to emerging countries' data. We therefore simulate data with sample size $T < 50$, using the following calibration under the null of no cointegration :

$$\begin{cases} Y_t = 1 + 2X_t + U_t, & U_t = U_{t-1} + \varepsilon_t, & \varepsilon_t \sim \text{NID}(0, 1) \\ X_t = -1 + V_t, & V_t = V_{t-1} + \eta_t, & \eta_t \sim \text{NID}(0, 2) \end{cases} \quad (11)$$

The chosen parameter values follow Gregory and Hansen (1996), and are nuisance parameters that do not matter under the null. Table 2 shows the empirical rejection frequencies (ERFs) under the null hypothesis, using the asymptotical critical values at the 5% level for 50 000 replications of data generated by (11). Consistently with Cook (2006), the test expectedly over rejects the null hypothesis as the sample size decreases, and size distortion can be very large for very small sample sizes, up to 28

percentage points for both models C and CS. Using asymptotical critical values would therefore not be adapted in order to select the type of structural break model for our methodology, especially when the amplitude of size distortion is heterogenous between the models.

T	model C	model CS
15	0.28	0.28
20	0.23	0.23
30	0.18	0.17
50	0.12	0.12

Table 2: ERFs at the 5% nominal level using asymptotical critical values from Gregory and Hansen (1996) with one regressor, by model and sample size T

To correct for size distortion, we compute size-adjusted approximate critical values for every model. 50 000 replications of time series of size $T = 13, \dots, 50, 100, 200, 500, 1000$ are simulated using the previous data generating process under the null hypothesis. We also use a response surface method *à la* MacKinnon (1991) where we fit a polynomial of $1/T$ by OLS for each q 'th quantile of the simulated distribution of the test statistics and model M :

$$Crt(T, q, M) = \psi_{\infty} + \sum_{k=1}^K \psi_k T^{-k} + error.$$

The order K of the polynomial is selected by minimizing the corrected Akaike information criterion (AICc hereafter)⁴. The polynomial functions of $1/T$ are mainly of order 3 or 4 for small quantiles, and of lesser order for higher quantiles. This differs with Gregory and Hansen (1996) for whom first-order polynomials are fitted. We can explain it with the fact that they simulate series with $T = 50, 100, 150, 250$, in which cases the tests statistics are located in the flatter part of the functions of $1/T$. As we obtain a function of T to estimate the critical values of each quantile, we can compute an approximate distribution of the test statistic for each model and each sample size. Table 3 reports the resulting size-adjusted critical values for very small sample sizes ($T \leq 50$), as well as the asymptotic ones which correspond to the estimated intercept of the polynomial fit. We naturally observe for every type of structural break model that the critical values increase sharply for very small sample sizes and then converge to the asymptotic values. Moreover, we succeed in replicating the ones obtained by Gregory and Hansen (1996). From the cumulative distribution of the test statistics we obtain

⁴The AICc is the AIC corrected for small sample sizes, such that $AICc = AIC + \frac{2K^2+2K}{T-K-1}$ where K is the number of parameters in the regression.

for every sample size T , we can compute p-values which can be used as a selection criterion for the choice of the model for the regression step of our methodology. More precisely, we can select the model which rejects the unit root in the residuals, i.e. the model which yields a p-value associated with the test statistic lower than a selected level. If several models reject the unit root, we choose one which doesn't differentiate the series. If the candidate models are of the same order of differentiation, the one which fits better the data is selected. The performance of the size-adjusted critical values in terms of power is discussed in Section 3.

		Level				
	model	0.01	0.025	0.05	0.1	0.975
$T = 15$	O	-5.13	-4.47	-3.97	-3.47	-0.37
	C	-7.52	-6.73	-6.16	-5.58	-2.38
	CS	-8.02	-7.18	-6.57	-5.96	-2.55
$T = 20$	O	-4.79	-4.25	-3.81	-3.37	-0.35
	C	-6.75	-6.17	-5.73	-5.25	-2.37
	CS	-7.19	-6.59	-6.11	-5.62	-2.56
$T = 30$	O	-4.48	-4.03	-3.67	-3.28	-0.34
	C	-6.16	-5.72	-5.35	-4.97	-2.35
	CS	-6.55	-6.08	-5.72	-5.3	-2.56
$T = 50$	O	-4.24	-3.85	-3.53	-3.18	-0.32
	C	-5.74	-5.38	-5.08	-4.73	-2.32
	CS	-6.11	-5.73	-5.42	-5.07	-2.55
$T = \infty$	O	-3.9	-3.59	-3.33	-3.04	-0.3
	C	-5.11	-4.83	-4.59	-4.32	-2.26
	CS	-5.41	-5.17	-4.94	-4.66	-2.54

Table 3: Approximate size-adjusted critical values for one regressor

2.4 Model selection by minimizing annual prediction errors

Choosing the model which rejects with the widest margin the unit root in the residuals puts in the model selection more weight on the absence of persistence in the residuals, which can be at the expense of the error variance. Therefore, we consider an alternative method that is more direct than testing for a unit root in the residuals by simply selecting the model and the structural break date which minimizes the error in the prediction of the annual aggregates, here measured by the root mean squared error. For a given model M , if $\hat{\mathbf{Y}}(n_b, M)$ denotes the annual predictions of \mathbf{Y} by the model

M with a structural break in parameters occurring at n_b ⁵, we obtain for each model $M \in \mathcal{M}$ a prediction error criterion $\text{RMSE}(M)$ such that:

$$\text{RMSE}(M) = \min_{t_b \in \mathcal{N}} \sqrt{\frac{1}{T} \sum_{t=1}^T \left(\hat{Y}_t(t_b, M) - Y_t \right)^2}. \quad (12)$$

We then select the model which minimizes $\text{RMSE}(M)$, i.e. yields $\min_{M \in \mathcal{M}} \text{RMSE}(M)$.

3 Monte Carlo simulation

Most emerging and advanced countries disclose their annual national account data for a particular year around the third quarter of the following year. It implies that quarterly indicators are disclosed between 3 and 7 quarters ahead of the annual publications of the national accounts. In order to assess if our method improves the prediction of the data, we look at its ability to predict the quarterly account series of the concomitant period to the indicator (in-sample prediction), as well as up to two years ahead of the last concomitant year (out-of-sample prediction). At first, we use Monte Carlo Simulations to analyze the performance of our two methods: in terms of power and predictive performance for one based on the test of cointegration, and in terms of predictive performance for the other based on minimizing the RMSE. Then we empirically assess the performance of the method by using US data on consumption expenditure from which we compare the quarterly disaggregation of annually aggregated real consumption personal expenditures using retail sales data as an indicator with observed quarterly data.

3.1 Calibration

In order to obtain an assessment of the ability of our method to predict a quarterly series by disaggregating its annual aggregation using a related series, we simulate small and independent quarterly series and a related quarterly indicator, for which we will be able to control for the type of structural break model which generates them. For each replication of the simulated series, we first simulate the indicator \mathbf{x} as a random walk:

$$x_t = 2 + x_{t-1} + \eta_t, \quad \eta_t \sim \mathcal{N}(0, 10).$$

⁵The $\hat{\mathbf{Y}}$ are direct estimates for models estimated in level, or such that $\hat{Y}_t = \widehat{\Delta Y}_t + \hat{Y}_{t-1}$ for $t > 1$ and $\hat{Y}_1 = Y_1$ for models estimated in first difference. The RMSE for all models have then the same magnitude by construction.

In a typical case of temporal disaggregation, the related series \mathbf{x} used as a predictor must be a good indicator of the series \mathbf{Y} we want to disaggregate. As a consequence, most of the variance of the disaggregated values \mathbf{y} must be explained by the variance of \mathbf{x} . Moreover, we want to distinguish the performance of our method with or without a structural change, so we choose parameter values that match the Chinese national accounts data but also imply a sharp structural change. For each model, 10 000 replications of a couple of quarterly series $\{\mathbf{y}, \mathbf{x}\}$ are simulated, following the subsequent data generating processes (DGPs):

$$u_t = \rho u_{t-1} + \varepsilon_t \quad \varepsilon_t \sim \mathcal{N}(0, 1)$$

For the models with I(0) residuals:

$$\begin{aligned} \text{(O)} \quad & y_t = 8 + 0.9x_t + u_t \\ \text{(C)} \quad & \begin{cases} y_t = 3 + 0.9x_t + u_t & t < 4t_b - 3 \\ y_t = 16 + 0.9x_t + u_t & t \geq 4t_b - 3 \end{cases} \\ \text{(CS)} \quad & \begin{cases} y_t = -1 + 1.4x_t + u_t & t < 4t_b - 3 \\ y_t = 19 + 0.9x_t + u_t & t \geq 4t_b - 3 \end{cases} \end{aligned}$$

For the models with I(1) residuals, $v_t = v_{t-1} + u_t$ and consistently with the data, the change in the constant coefficient is of much smaller amplitude since it implies a change of trend in the model in level:

$$\begin{aligned} \text{(dO)} \quad & y_t = 1 + \frac{t-2.5}{16} + 0.8x_t + v_t \\ \text{(dC)} \quad & \begin{cases} y_t = 1 + 0.3\frac{t-2.5}{16} + 0.8x_t + v_t & t < 4t_b - 3 \\ y_t = 1 + 0.3\frac{t-2.5}{16} + 3.3\frac{t-4t_b+5.5}{16} + 0.8x_t + v_t & t \geq 4t_b - 3 \end{cases} \\ \text{(dCS)} \quad & \begin{cases} y_t = 1 + 0.3\frac{t-2.5}{16} + 1.4x_t + v_t & t < 4t_b - 3 \\ y_t = 1 + 0.3\frac{t-2.5}{16} + 3.3\frac{t-4t_b+5.5}{16} + 1.4\frac{X_{t_b-1}}{4} + 0.8(x_t - \frac{X_{t_b-1}}{4}) + v_t & t \geq 4t_b - 3 \end{cases} \end{aligned}$$

We consider $\rho = 0$ and $\rho = 0.5$. For each replication of quarterly series, we can aggregate them to obtain annual series of sample size $T_{total} = T_{in-sample} + 2T_{out-of-sample}$, such that $T_{in-sample} = T \in \{15, 20, 25, 30, 50\}$ and $T_{out-of-sample} = 4$. The date of structural break t_b is random and, if $n \in \{-3, -2, \dots, T, T+1, \dots, T+4\}$, is drawn in from uniform distribution $\mathcal{U}[[0.15T, 0.85T]]$. In the end, we have 10 000 replications of quarterly series $\{\mathbf{y}, \mathbf{x}\}$ of size $T = 92, 112, 132, 152, 232$.

3.2 Power of the test of cointegration with endogenous structural change

Selecting a model which rejects the presence of a unit root in the residuals can help improve the prediction of the disaggregated estimates. Indeed, the less persistent the

residuals are, the lesser the discrepancies between the model estimates and the observed \mathbf{Y} will affect the accuracy of the predictions. However, using the rejection of the null as a selection criterion requires the corresponding test to have good power for the small samples considered. Table 4 reports the empirical rejection rates at the 5% nominal level when the alternative of no unit root in the residuals is true and the type of regression model is the data generating process, for sample sizes 15, 20, 25, 30 and 50 (the complete tables of the regression of each model against each true model are reported in Appendix C). As expected, the power declines when the serial correlation

T	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.589	0.826	0.94	0.985	0.995	0.351	0.572	0.775	0.899	0.994
C	0.174	0.309	0.537	0.865	1	0.095	0.13	0.307	0.725	1
CS	0.263	0.38	0.572	0.796	1	0.188	0.238	0.363	0.606	0.996
dO	0.203	0.417	0.649	0.83	0.99	0.116	0.233	0.406	0.608	0.973
dC	0.092	0.167	0.268	0.406	0.921	0.064	0.101	0.149	0.237	0.7
dCS	0.093	0.171	0.284	0.444	0.941	0.057	0.088	0.138	0.212	0.696

Table 4: Power of the test: ERFs at the 5% nominal level when there is no unit root in the residuals and the regression model is the true model

ρ increases. More specifically for the models estimated in level with $\rho = 0$, the test without structural break rejects the unit root in the residuals when there is none with 59% probability for the smallest sample size $T = 15$, then with a more reasonable 83% for $T = 20$, and at least 94% for $T > 25$. For the models with structural breaks, the probability is larger than 50% for $T \geq 25$ and larger than 80% for $T \geq 30$. This reveals a low power for the test in level in very small samples, but a reasonable power as the sample size increases. When the models are estimated in first difference, the model without structural break has low power in very small sample (20% probability at $T = 15$, increasing to a reasonable 65% at $T = 25$, 84% at $T = 30$, and larger than 99% for $T \geq 50$), whereas the models with structural break (dC) and (dCS) have even lower power (probability smaller than 50% for $T \leq 40$, and larger than 90% for $T \geq 50$). In general, the test has reasonable power for small sample sizes when there is no structural change, whereas the power is low for very small sample sizes when there is a structural change. Therefore, we cannot expect the method relying on a test of cointegration with endogenous structural break to improve very much the prediction of the disaggregated estimates for very small samples ($T < 30$) for the DGPs with I(0) residuals and for small samples ($T < 50$) for the DGPs with I(1) residuals.

3.3 Predictive performance

The main objective of the procedure is to better predict quarterly series than the usual method which considers neither structural breaks nor very small sample series. We consider three alternatives in selecting the regression model. In the benchmark method, no structural change is considered. Model O is selected if the unit root in the residuals is rejected, model dO if not. In the test-approach method, the best model with or without a structural change which rejects the unit root in the residuals is selected. As we confront both models in level and in first difference, selection by minimizing an information criterion is not possible. We select the most parsimonious one, i.e. in order of preference: O, C, CS, dO, dC, dCS. Model dO is selected when none of the models rejects the unit root. In the RMSE-approach method, we select the model which minimizes $\text{RMSE}(M)$. Table 5 reports the ratio of the quarterly prediction error by using the test-approach or the RMSE-approach on the quarterly prediction error by using the benchmark method.

T	$\rho = 0$									
	test-approach					RMSE-approach				
	15	20	25	30	50	15	20	25	30	50
O	1.002	1	1	1	1	1.164	1.101	1.041	1.029	1.002
C	1.168	0.992	0.901	0.827	0.916	0.495	0.592	0.669	0.727	0.848
CS	1.07	0.92	0.855	0.828	0.892	0.868	0.872	0.873	0.822	0.807
dO	1.049	1.047	1.034	1.038	1.039	1.421	1.307	1.254	1.243	1.142
dC	1.02	1.001	0.984	0.975	0.998	0.937	0.845	0.794	0.77	0.733
dCS	1.04	1.043	1.049	1.057	1.09	1.035	0.947	0.909	0.894	0.816
N	$\rho = 0.5$									
	test-approach					RMSE-approach				
	15	20	25	30	50	15	20	25	30	50
O	1.007	1.002	1	1	1	1.26	1.108	1.091	1.055	1.009
C	1.229	1.087	1.035	0.929	0.984	0.554	0.625	0.688	0.733	0.838
CS	1.118	1.025	0.952	0.904	0.911	0.866	0.902	0.854	0.837	0.81
dO	1.029	1.037	1.03	1.03	1.035	1.384	1.299	1.255	1.24	1.188
dC	1.025	1.026	1.029	1.029	1.043	1.243	1.152	1.105	1.092	1.019
dCS	1.037	1.034	1.033	1.032	1.046	1.25	1.178	1.129	1.122	1.048

Table 5: Quarterly prediction error by method with structural breaks relative to the prediction error of the method without structural breaks

For $\rho = 0$, when the true model is not subject to a structural change (models O and dO), considering a structural change and selecting the type and date by a cointegration

with endogenous break test approach does not improve the prediction. The predictions are in fact almost identical because the procedure would select the models without any structural change. When there is a structural change for the model with $I(0)$ residuals (C and CS), there is a slight loss of accuracy for $T = 15$ but a gain in accuracy of at least 10% for model C and $T \geq 25$ and for model CS and $T \geq 20$. The gain in accuracy is almost inexistent or slightly negative for the cases in first difference for our calibration, mainly because the true change in parameters is of much smaller magnitude in the regression models. However, selecting the type and date of structural change by minimizing the annual prediction error significantly improves the predictions when there is indeed a structural change in the true model. For the models with $I(0)$ residuals, the gain in accuracy is from 15% to 50% for model C, and from 13% to 20% for model CS, and respectively for the models with $I(1)$ residuals from 6% to 26% and from -4% to 18%. The drawback is that when there is no structural change in the true model, considering a structural change in the regression implies a loss of accuracy which becomes very significant for the model in first difference (from -14% to -42% for model dO against from 0% to -16% for model O). For $\rho = 0.5$, all predictions lose accuracy, and there is only a gain for the models with $I(0)$ residuals. However the RMSE-approach yields less accurate predictions for the models with $I(1)$ residuals in very small sample.

In conclusion, updating the disaggregation method by considering a parameter instability in the linear model has different implications depending on the underlying data generating process. When there is a structural change, selecting its type and date with the RMSE-approach improves significantly the accuracy of the prediction, and much more so than selecting them with the test-approach. However, when there is no structural change, the RMSE-approach implies a loss in accuracy while the test-approach doesn't significantly affect it, so it is better not to consider any structural change. As the true generating process isn't known practically, a mixed procedure conciliating all advantages of our methods is proposed as follows: if the test-approach rejects the unit root in any model with a structural change, one should use the RMSE-approach to select the date of the structural break for the corresponding type of model. If the test-approach does not reject the unit root for a model with a structural change, one should not consider any structural change in the model.

3.4 Application and empirical performance on US data

3.4.1 The Data

We try to predict the US quarterly series of national accounts representing national consumption expenditures and net exports of goods and services⁶. For the former, the real personal consumption expenditure series (*rpce*) is considered, for which quarterly observations of seasonally adjusted data are available for the period from 1959Q1 to 2021Q3. The quarterly indicator used is a seasonally adjusted quarterly series of real retail sales (*rretails*), available from 1992Q1 to 2021Q3. For the trade data, the quarterly series of net exports of goods and services (*nxgs*) are available from 1947Q1 to 2021Q2, and we use the quarterly series of net exports of goods (*nxg*) as an indicator, which is available from 1992Q1 to 2021Q2. After aggregating the quarterly national accounts data into annual series, we apply the methodology presented in the previous section.

3.4.2 Predicting the US personal consumption expenditures

Table 6 reports the results from the regression of annual *rpce* on annual *rretails*, for each type of model considered in the previous section. If no structural break is considered as in the benchmark method, the unit root in the residuals is not rejected either in the model in level or in first difference, therefore model dO is selected. If we consider a cointegration model with a structural break in the parameters, there is also no model for which the unit root in the residual is rejected. In this case, we will also select dO. However, the annual prediction errors are minimal for model CS with a break in 2008.

With the selected models, we disaggregate *rpce* into quarterly estimates which we compare to the observed quarterly values. Figure 1 represents the prediction errors.

By comparing the estimates by method $\hat{\mathbf{y}}$ to the observations \mathbf{y} , we assess the actual gain in accuracy by using our new methodology. The average quarterly prediction squared error is calculated for the models selected by each approach:

$$qerr = \sqrt{\frac{1}{119} \sum_{t=1992Q1}^{2021Q3} (\hat{y}_t - y_t)^2}.$$

Not considering structural changes (benchmark) implies a quarterly root mean square error of 22.65 (2.73% of observed standard deviation). For this particular case, considering structural changes but selecting the type and date with the test-based approach

⁶Source: <https://fred.stlouisfed.org/>

	\hat{t}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
Intercept	-1457.49*** (233.17)	-773.25** (260.54)	-516.25 (380.98)	218.96*** (54.47)	155.9. (81.51)	163.17 (285.51)
Retail sales	2.89*** (0.06)	2.59*** (0.09)	2.49*** (0.14)	0.99** (0.32)	1** (0.31)	0.95 (1.95)
\hat{t}_b	NA	2006	2006	NA	2000	2000
Intercept after \hat{t}_b		757.81*** (197.57)	154.27 (680.68)		81.71 (78.68)	74.25 (291.52)
Retail sales after \hat{t}_b			0.17 (0.19)			0.05 (1.97)
$ADF^*(M)$	-2.999	-4.622	-4.821	-2.119	-3.326	-3.33
p-value	0.158	0.18	0.21	0.489	0.718	0.812
	\hat{t}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
Intercept	-1457.49*** (233.17)	-597.01** (180.5)	-674.39** (232.2)	218.96*** (54.47)	118.11 (112.83)	182.95 (707.62)
Retail sales	2.89*** (0.06)	2.53*** (0.06)	2.56*** (0.08)	0.99** (0.32)	0.99** (0.31)	0.52 (5.05)
\hat{t}_b	NA	2008	2008	NA	1996	1996
Intercept after \hat{t}_b		949.49*** (130.49)	1213.84* (505.85)		112.39 (110.15)	47.3 (709.9)
Retail sales after \hat{t}_b			-0.07 (0.13)			0.47 (5.06)
RMSE(M)	10.82	6.21	6.17	9.52	8.16	8.17

Table 6: Model fitting US annual household consumption expenditures and aggregated retail sales (1992:2020), $n = 29$

does not actually detect a structural change, therefore there is no gain in accuracy. However, selecting the type and date by minimizing the annual prediction error implies a quarterly error of 40.29 (4.95% of observed standard deviation), therefore a 77% loss in accuracy (2,16% of observed standard deviation).

3.4.3 Predicting the US net exports of goods and services

Table 7 reports the results from the regression of annualized $nxgs$ by annualized nxg , for the same models as above. As the unit root in residuals is never rejected at the 5% level, model dO is selected in both the benchmark and the test-approach method. If the structural change model and date are selected by minimizing the annual prediction error, model CS with a shift in 2009 is retained. Figure 2 represents the quarterly prediction errors of $nxgs$.

Numerically, the benchmark method provides estimations with a quarterly root mean square error of 4.86 (8.16% of observed standard deviation). No model with structural

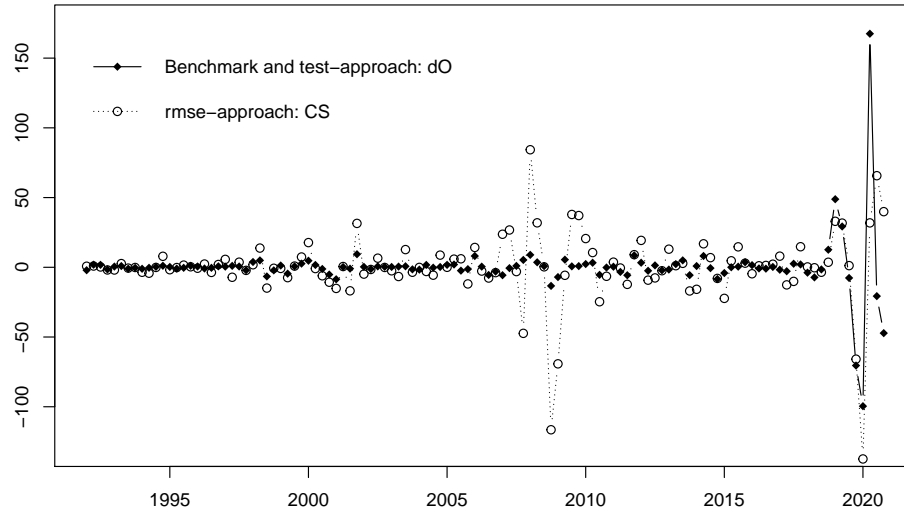


Figure 1: Quarterly prediction errors of US quarterly household consumption expenditures

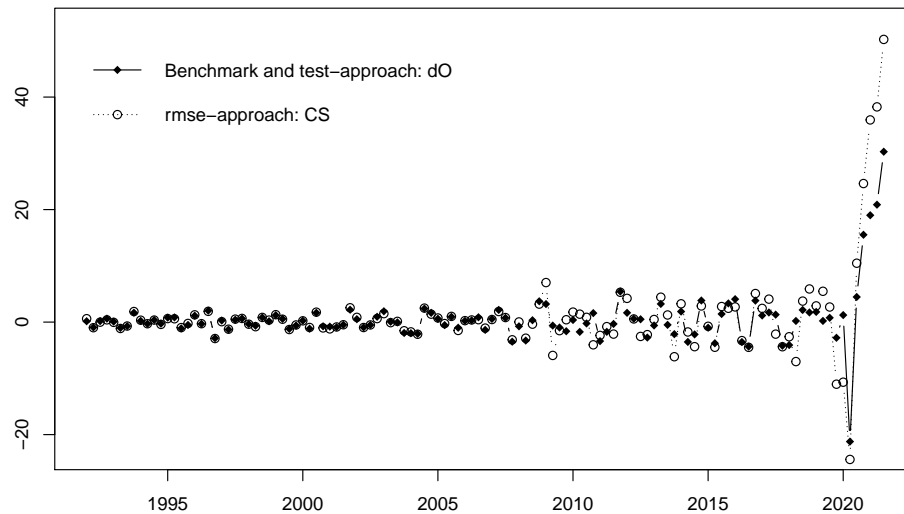


Figure 2: Quarterly prediction errors of US net exports of goods and services

change with stationary residuals is found by the test-approach, so no gain in accuracy is possible. Selecting a structural break model and date by minimizing the annual

	\hat{t}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
Intercept	23.52 (31.83)	68.45*** (14.65)	78.44*** (12.29)	6.76. (3.36)	-4.75 (4.33)	-2.31 (6.17)
Net exports of goods	0.82*** (0.05)	0.99*** (0.03)	1.03*** (0.02)	0.98*** (0.04)	0.96*** (0.03)	1.01*** (0.1)
\hat{t}_b	NA	2012	2009	NA	2005	2005
Intercept after \hat{t}_b		167.55*** (15.76)	-246.83*** (56.41)		19.04** (5.45)	16.49* (7.14)
Net exports of goods after \hat{t}_b			-0.54*** (0.08)			-0.06 (0.1)
$ADF^*(M)$	-2.432	-3.036	-4.409	-2.724	-3.591	-3.79
p-value	0.347	0.832	0.349	0.239	0.591	0.623
	\hat{t}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
Intercept	23.52 (31.83)	71.33*** (14.28)	78.44*** (12.29)	6.76. (3.36)	-3.82 (4.93)	-0.21 (6.78)
Net exports of goods	0.82*** (0.05)	1*** (0.03)	1.03*** (0.02)	0.98*** (0.04)	0.97*** (0.03)	1.06*** (0.13)
\hat{t}_b	NA	2011	2009	NA	2003	2003
Intercept after \hat{t}_b		169.23*** (15.34)	-246.83*** (56.41)		16.14* (5.94)	12.34 (7.71)
Net exports of goods after \hat{t}_b			-0.54*** (0.08)			-0.1 (0.13)
RMSE(M)	30.26	12.69	10.53	28.24	13.18	12.85

Table 7: Model fitting US net exports of goods and services and aggregated net exports of goods (1992:2020), $n = 29$

prediction yields a quarterly RMSE of 7.94 (13.3% of observed standard deviation). Considering a structural break for predicting the US net exports, therefore, implies a loss of prediction accuracy of 63.1% (5.15% relatively to the observed standard deviation).

By failing to detect a model with structural change using the US national accounts, we don't improve the accuracy of the prediction of the quarterly series. Consistently with our simulations, considering a structural change when there is none would worsen the prediction accuracy. In the next section, we apply the methodology to the Chinese national accounts for which we expect to detect a structural change in the parameters.

4 Business cycle stylized facts of the Chinese national accounts

Abeyasinghe and Rajaguru (2004) disaggregate the Chinese GDP using two indicators (nominal M1 money supply and nominal total exports). In this section, we construct the series of the Chinese quarterly GDP by the expenditure approach, namely the sum of consumption, investment, and net exports, by estimating the quarterly series for the national accounts⁷. This will allow us to undertake a business cycle stylized facts analysis *à la* Backus et al. (1992), which quantitatively assesses the relevance of various dynamic and stochastic general equilibrium (DSGE) business cycles models in the case of the United States from 1954Q1 to 1989Q4 by computing second-order moments. In order to compute comparable relative volatility and correlation tables between the components of the Chinese national accounts from 1998Q1 to 2016Q2, we construct quarterly estimates of the Chinese household consumption expenditures, net exports, capital formation, and government consumption expenditures.

4.1 Disaggregating the Chinese national accounts

We detail the procedure for disaggregating the household consumption expenditures and the net exports. The two series differ significantly in terms of volatility around their respective trend, with the consumption data being the most stable one over time. For capital formation and government consumption expenditures, the estimated models are detailed in Appendix D. Figure 3 shows the time series representation of both data series: the scatter points stand for the observed annual national account, the bold lines for their associated quarterly indicator, that is to say, national retail sales for consumption, and net exports of goods for net exports. In order to disaggregate the annual observations into quarterly observations using the quarterly indicator, we want to find the best model that links the annual observations of the national account data to the annual aggregate of their respective quarterly indicator, either in level or in first difference⁸.

Table 8 reports the results for the household consumption expenditures using size-adjusted Gregory and Hansen (1996) test in the upper part, or minimizing the annual prediction errors in the lower part. Not considering structural breaks only rejects the unit root in the residuals for model dO. When considering endogenous structural

⁷Source: <http://www.stats.gov.cn/>

⁸Graphically, we want to find the best fit the scatter of representations in Appendix E.

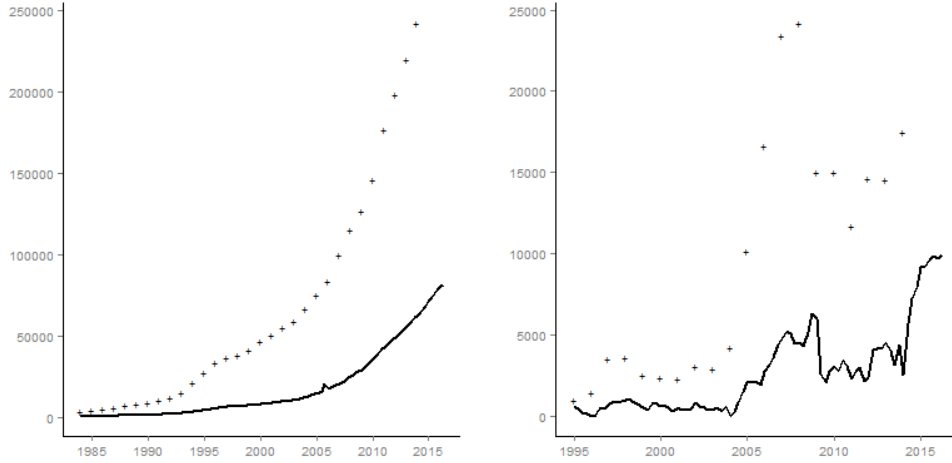


Figure 3: Chinese household consumption expenditure and retail sales of consumer goods(left), net exports of goods and services and net exports of goods (right)

breaks, the presence of a unit root in the residuals is rejected at the 5% level for model CS, dC and dCS. The most parsimonious of them is model CS with a break in 2005 as it does not need differentiating the series. This model is therefore a potential candidate for our data. When minimizing the annual prediction errors for the same model, the structural break date is 2003. Figure 4 represents the fit of the data by the selected model. The quarterly series of household consumption expenditures can then be estimated by applying the disaggregation formula in equation (7). We apply the same procedure to disaggregate the Chinese net exports of goods and services using net exports of goods. The right panel of Figure 3 shows this time series to be more volatile and less driven by a deterministic trend than the data used for disaggregating annual consumption. Table 9 reports the results of the regression of each model. In the benchmark method without considering a structural break, model dO rejects the unit root in the residuals, which is then selected. The unit root is also rejected at the 5% level for models dC and dCS. The model dCS however shows a significant coefficient for the structural change in the slope. Hence it seems reasonable to select model dCS. For such a model, minimizing the annual prediction error selects 2011 as the date of structural change. Figure 5 represents the fitted model for the data.

By applying the same method for the remaining national accounts, we select a model with a break in the intercept and the slope in 2012 for fixed capital formation and a model with a break in the intercept in 2016 for government consumption expenditure (estimation details are provided in Appendix D). All expenditure components of GDP

	\hat{t}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	9779.07*** (1714.66)	4339.36* (1616.06)	-2.23 (1965.65)	1685.21 (1110.86)	345.12 (2121.78)	544.22 (6008.83)
X	0.89*** (0.01)	0.86*** (0.01)	1.31*** (0.08)	0.79*** (0.07)	0.77*** (0.07)	0.54 (6.45)
d		13320.35*** (2458.9)	18955.91*** (3615.34)		1830.86 (2463.36)	1631.36 (6152.04)
dX			-0.46*** (0.08)			0.23 (6.46)
rmse.a	6.88	5	4.57	12.03	11.42	11.42
sb date	NA	1997	2005	NA	1990	1990
inf ADF	-2.624	-4.613	-5.917	-4.99	-4.913	-4.915
pval	0.267	0.157	0.026	0.003	0.095	0.161
	\hat{t}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	9779.07*** (1714.66)	3034.4. (1716.32)	-789.73 (2087.81)	1685.21 (1110.86)	2158.59. (1066.33)	1080.17 (717.68)
X	0.89*** (0.01)	0.86*** (0.01)	1.37*** (0.1)	0.79*** (0.07)	0.66*** (0.08)	0.8*** (0.06)
d		13637.86*** (2403.27)	20257.06*** (3260.01)		6171.56* (2701.44)	36651.8*** (4948.37)
dX			-0.52*** (0.1)			-1.13*** (0.17)
rmse.a	6.88	4.89	4.5	12.03	10.32	3.51
sb date	NA	1995	2003	NA	2014	2014

Table 8: Model fitting Chinese annual household consumption expenditures and aggregated retail sales (1984:2019), $n = 36$

have been disaggregated into quarterly time series, which can then be aggregated⁹ to obtain our quarterly estimates of nominal GDP which we compare to the official GDP data. Figure 6 represents both estimated (in red) and official nominal GDP (in blue) after seasonal adjustment by X13-ARIMA-SEATS, in level and in their cyclical components obtained by Hodrick-Prescott filtering. In terms of level magnitude, our estimates are similar to the official data, which means that our methodology doesn't alter the consistency of the annual national account data. Looking at the cyclical components of log nominal GDP, we reproduce the general fluctuations of the official data, especially troughs around 2003 and before 2010, peaks around 2008 and 2012, and a recession phase since 2012. However, our estimates show more volatile cyclical components¹⁰. We are now able to compute the moments of second-order of the cyclical

⁹We omit the variation of stocks for lack of a related indicator.

¹⁰Our results may be underestimated because of the omission of the variation of stocks.

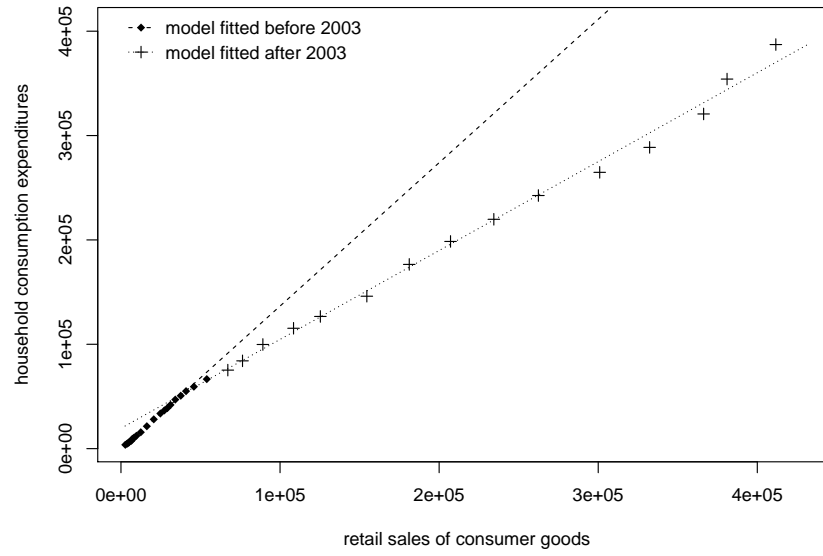


Figure 4: Fitted models for predicting the Chinese annual household consumption expenditures from the annualized retail sales

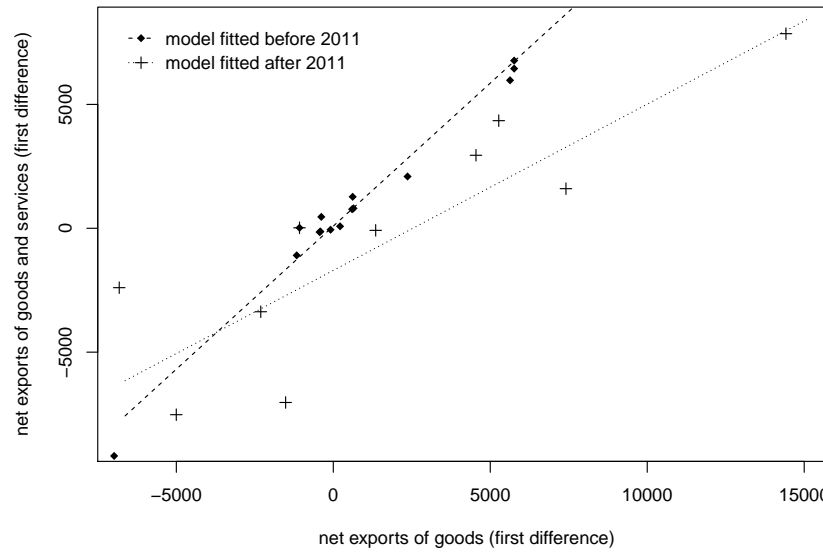


Figure 5: Fitted models for predicting the Chinese annual net exports of goods and services from the annualized net exports of goods

	\hat{t}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	3761.9* (1493.57)	748.31 (828.6)	881.63 (838.56)	-492.77 (473.29)	662.61 (537.27)	-70.67 (430.45)
X	0.5*** (0.08)	1.04*** (0.08)	0.97*** (0.07)	0.78*** (0.1)	0.76*** (0.08)	1.12*** (0.13)
d		-1.7e+04*** (2042.11)	-2e+04** (7033.7)		-2474.59** (774.48)	-1812.54* (806.95)
dX			0.11 (0.23)			-0.46* (0.16)
rmse.a	61.54	30.08	28.93	103.01	28.55	23.94
sb date	NA	2014	2015	NA	2009	2013
inf ADF	-0.98	-4.575	-4.228	-4.314	-6.294	-10.657
pval	0.909	0.205	0.435	0.018	0.013	0.001
	\hat{t}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	3761.9* (1493.57)	780.72 (798.6)	154.6 (800.9)	-492.77 (473.29)	338.43 (512.56)	86.33 (447.18)
X	0.5*** (0.08)	0.98*** (0.07)	1.11*** (0.08)	0.78*** (0.1)	0.81*** (0.09)	1.15*** (0.14)
d		-1.7e+04*** (1950.82)	-5918.51 (5211.08)		-2311.32* (836.87)	-1786.32* (738.49)
dX			-0.42* (0.18)			-0.48** (0.16)
rmse.a	61.54	29.1	26.88	103.01	23.8	15.09
sb date	NA	2015	2014	NA	2011	2011

Table 9: Model fitting Chinese net exports of goods and services and aggregated net exports of goods (1995:2016), $n = 22$

components of the Chinese national accounts.

4.2 Second-order moments of the cyclical components

Business cycle stylized facts are computed from the cyclical components of the logarithmic transformation of seasonally adjusted and deflated time series, except for net exports which are considered the non-logarithmic share of GDP. Considering what has been done in the previous sections, we still have to deflate the series in such a way that the components of the GDP are consistent with the GDP data in volume. A common use is to apply the GDP deflator to the nominal data in levels. An implied GDP deflator can be retrieved from the official data by dividing the quarterly nominal GDP by the quarterly real GDP. The latter is constructed by applying the quarterly year-over-year real growth rates (available for 1992Q1-2016Q2) to the quarterly real GDP in level (available for 2011Q1-2016Q2). However, we showed that the official

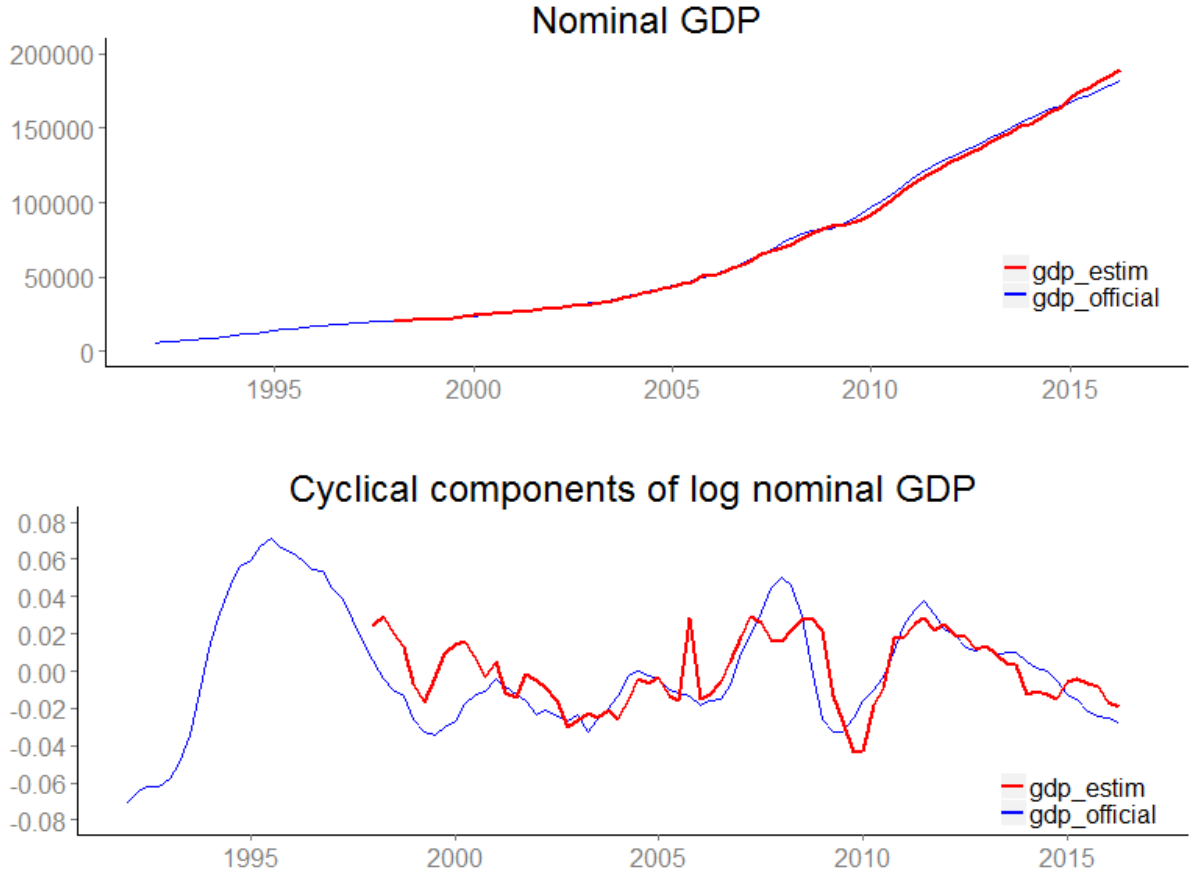


Figure 6: Estimated Chinese quarterly nominal GDP

GDP quarterly data show less volatility than the fluctuations implied by our quarterly estimates of the national accounts data. If we want to be agnostic about the use of the official data, we would rather not apply the implied GDP deflator to our estimates. Therefore, we consider two alternatives as GDP quarterly data and deflator: (i) the official GDP quarterly data and implied deflator, (ii) our GDP estimates and the official quarterly Consumer Price Index (CPI)¹¹ as a deflator.

Having disaggregated the nominal annual Chinese national accounts into nominal quarterly estimates, we deflate the series with the implicit deflator of the Chinese GDP or the CPI, then we seasonally adjust the resulting real series and finally recover the cyclical components by Hodrick-Prescott (HP) filtering (with smoothing parameter $\lambda = 1600$). Figure 7 represents the cyclical components of the selected disaggregated national accounts, compared to the cyclical components of the Chinese GDP for the

¹¹Computed as the quarterly average of the official monthly CPI.

two cases of GDP estimates previously mentioned. Table 10 reports the moments of

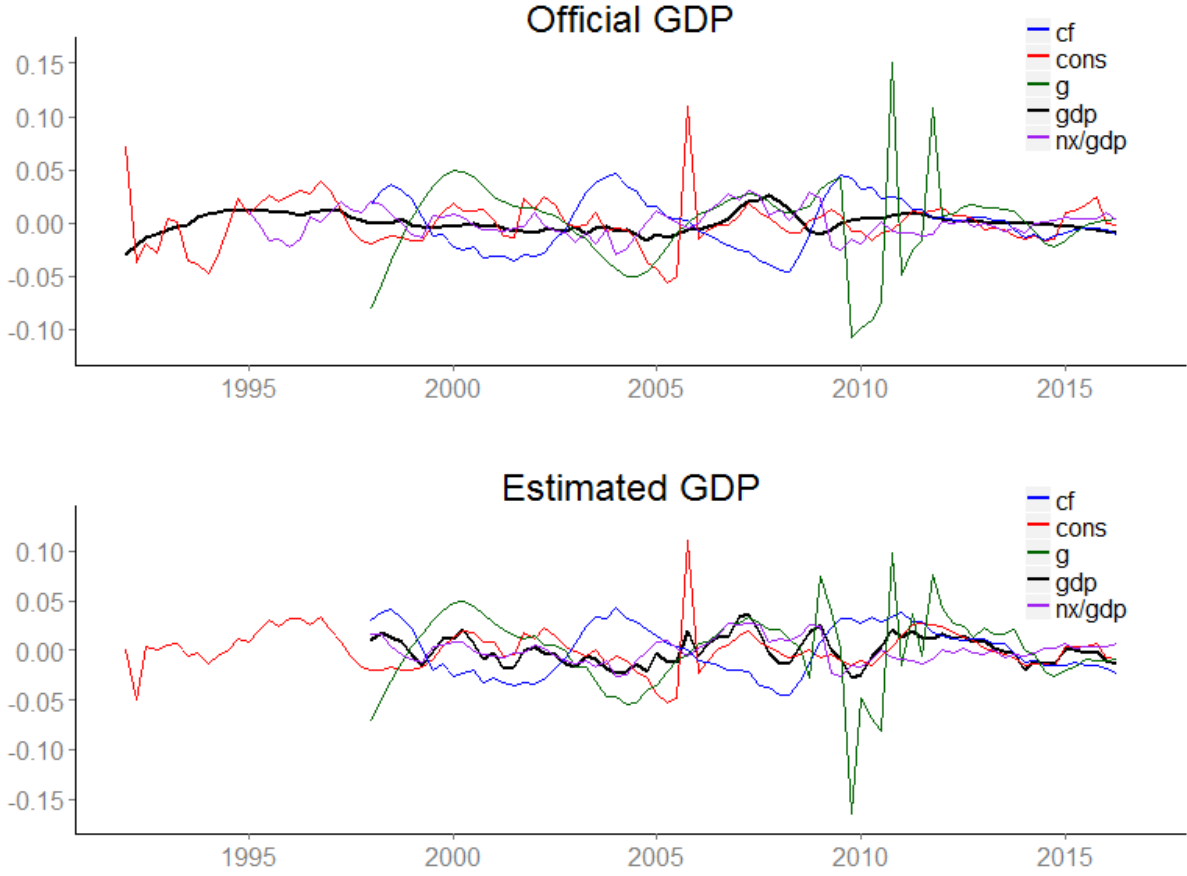


Figure 7: Cycle components of the estimated Chinese quarterly national accounts, by GDP estimates and deflator

second-order of the cyclical components of the Chinese national accounts, also when considering the two alternatives measures of GDP values and deflator. We consider only the period 2000Q1:2014Q4 in order to mitigate the effects of the estimation in the border period dates from the HP filtering and in the most recent dates which are subject to regular revisions. By looking at the standard deviations, we observe that after deflating the series, our estimates of the quarterly GDP have almost twice as volatile cyclical components as the official data. This difference is also translated into the relative volatilities between GDP and the expenditure components. When we consider the official GDP data, Chinese consumption is 2.17 times as volatile as the GDP, which implies no consumption smoothing at the national level, therefore no precautionary savings. It shows very little persistence (autocorrelation of 0.15) and

procyclicality (contemporary correlation of 0.19 with GDP). Investment is 3 times as volatile as GDP, highly persistent and countercyclical ($\rho(cf_t, gdp_t) = -0.32$). Government consumption expenditure are even more volatile (4.89 times as volatile as GDP) and slightly procyclical ($\rho(g_t, gdp_t) = 0.16$), while net exports are 1.34 times as volatile as GDP and procyclical ($\rho(nx/gdp_t, gdp_t) = 0.29$). Our GDP estimates deflated by the CPI show the same results qualitatively but with lesser volatilities, and higher correlations to GDP for the national accounts, except for capital formation which have a lower correlation to GDP.

Official GDP with implied GDP deflator												
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(gdp)}$	$\rho_1(x)$	$\rho(x_t, gdp_{t+k}), \text{ where } k=$								
				-4	-3	-2	-1	0	1	2	3	4
gdp	0.87	1	0.9	0.28	0.52	0.74	0.9	1	0.9	0.74	0.52	0.28
cons	2.17	2.5	0.15	0.02	0.08	0.06	0.1	0.19	0.18	0.2	0.13	0.06
cf	2.55	2.94	0.94	-0.21	-0.32	-0.4	-0.39	-0.32	-0.24	-0.14	-0.08	-0.03
g	4.24	4.89	0.32	0.34	0.3	0.2	0.16	0.16	0.19	0.19	0.14	0.08
nx/gdp	1.34	...	0.72	0.26	0.34	0.33	0.32	0.29	0.36	0.44	0.47	0.43

Estimated GDP with the official CPI as deflator												
x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(gdp)}$	$\rho_1(x)$	$\rho(x_t, gdp_{t+k}), \text{ where } k=$								
				-4	-3	-2	-1	0	1	2	3	4
gdp	1.53	1	0.69	-0.1	0.13	0.4	0.69	1	0.69	0.4	0.13	-0.1
cons	2.24	1.46	0.21	0.05	0.31	0.31	0.35	0.54	0.23	0.13	0.05	-0.06
cf	2.51	1.64	0.95	-0.36	-0.26	-0.2	-0.15	-0.1	-0.12	-0.08	-0.01	0.1
g	4.13	2.7	0.41	0.12	0.24	0.54	0.58	0.6	0.35	0.13	-0.05	-0.16
nx/gdp	1.3	...	0.76	0.12	0.03	0.13	0.38	0.55	0.56	0.42	0.15	-0.06

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

gdp : Gross domestic product, *cons*: household personal consumption expenditures, *cf*: fixed capital formation (private investment), *g*: government consumption expenditures, *nx*: net exports of goods and services

Table 10: Second order moments of the business cycles of the Chinese economy 2000Q1:2014Q4

As a matter of comparison, we proceed to compute the stylized facts of the US business cycles for the same period in Table 11. The second-order moments computed when considering the official GDP data take values very far from what we usually find for other countries, where in general investment is procyclical and net exports countercyclical. Also in developed countries, the relative volatility of consumption is much smaller and implies consumption smoothing (relative volatility of 0.86 for the US in the same period). However, Aguiar and Gopinath (2007) calculate the business cycles stylized facts

for various emerging markets in the last decades of the last century and our computed relative volatility of consumption is of the same magnitude as Brazil in 1991Q1:2002Q1 (2.01), Ecuador in 1980Q1:2002Q2 (2.39) or Slovakia in 1993Q1:2003Q1 (2.04). The

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k})$, where $k=$								
				-4	-3	-2	-1	0	1	2	3	4
gdp	1.29	1	0.89	0.31	0.51	0.72	0.89	1	0.89	0.72	0.51	0.31
cons	1.11	0.86	0.87	0.32	0.5	0.69	0.86	0.93	0.84	0.63	0.39	0.15
cf	7.5	5.83	0.91	0.22	0.44	0.68	0.86	0.94	0.86	0.73	0.57	0.42
g	1.3	1.01	0.9	-0.29	-0.38	-0.46	-0.48	-0.48	-0.53	-0.58	-0.59	-0.52
nx/gdp	0.49	...	0.79	-0.03	-0.26	-0.51	-0.71	-0.76	-0.72	-0.56	-0.39	-0.26

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 11: Second order moments of the business cycles for the United States 2000Q1:2014Q4

cyclical components of our quarterly estimates have a low contemporaneous correlation with the cyclical component of the official quarterly GDP. Since GDP is the aggregation of the national accounts, it indicates that the cyclical components of the official GDP data are inadequate for our estimated fluctuation data. Therefore using our own estimates of GDP data, computed as the sum of the quarterly estimates of the national accounts data, as well as the CPI as an aggregate price deflator, allows us to perform a more reliable analysis of the business cycle stylized facts. Indeed, when we consider our estimates of the quarterly GDP, the contemporaneous correlation of consumption, government expenditures, and share of net exports to GDP are significantly higher, with respective values of 0.54, 0.6, and 0.55. The correlation of investment to GDP however drops from -0.32 to -0.1, which can indicate either inadequacy of the CPI as a deflator for investment or the investment of fixed assets as an indicator of the fluctuations, but we note that all components of GDP retain the same sign of contemporaneous correlation as previously. Consistently with the higher volatility of our GDP estimates, the relative volatilities of the expenditure accounts to GDP drop to 1.46 for consumption, 1.64 for investment, and 2.7 for government expenditures. The lower but still high relative volatility of consumption which implies an absence of consumption smoothing at the national level is more in line with mid-range emerging markets (in terms of consumption smoothing) such as Argentina in 1993Q1:2002Q4 (1.38), Israel in 1980Q1:2003Q1 (1.6) or South Africa in 1980Q1:2003Q1 (1.61). It is however still almost twice as large as its US analog for the same period.

Concluding remarks

We develop in this chapter a methodology to construct quarterly data for emerging economies with an application to the Chinese economy based on a mixed procedure of endogenous structural break test and prediction error minimization. Simulations confirm that non-linear parametrization improves the accuracy of the quarterly disaggregation of time series which behave like recent data from emerging markets such as China. It allows us to compute the so-called stylized facts of its quarterly business cycles. We find that the Chinese business cycles fluctuate in a different way from advanced economies such as the US, but have similarities with other emerging countries, namely exacerbated volatilities of the expenditure accounts relative to GDP and an absence of consumption smoothing at the national level. In addition, we find the low persistence of the cyclical components of both private and public consumption, as well as a much lower procyclicality of consumption and a countercyclical capital formation and procyclical government spending and net exports. We haven't taken any instability in the volatility into account. Even though we find it reasonable for the very small samples considered, it might be a good research topic for the future. Direct extensions of our methodology would be to consider multiple indicators and structural breaks while considering the limits due to the small sample size of the time series we are interested in. Also, a more thorough parametrization of the alternative hypothesis could provide a better assessment of the power of the test procedure. This will be investigated in the next chapters.

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A Estimating the trend component in the case of models estimated in first difference

When estimating a model in first difference, a significant intercept is translated into a trend component for the annual estimates in level, thus also for the disaggregated estimates. Estimating model (dO) yields predicted first difference values :

$$\Delta \hat{Y} = (\mathbf{1}_{T-1} \quad \Delta X) \begin{pmatrix} \hat{\mu} \\ \hat{\alpha} \end{pmatrix}$$

where μ is the intercept and α the vector of slope coefficients. The equation implies that the predicted level values are defined up to an initial value \hat{Y}_1 . We set the latter to the initial observed annual value Y_1 , therefore :

$$\begin{cases} \hat{Y}_1 &= Y_1 \\ \hat{Y}_t &= \hat{Y}_{t-1} + \hat{\mu} + (X_t - X_{t-1})' \hat{\alpha} \quad \text{for } t = 2, \dots, T \end{cases}$$

or equivalently,

$$\begin{cases} \hat{Y}_1 &= Y_1 \\ \hat{Y}_t &= Y_1 + (t-1)\hat{\mu} + (X_t - X_1)' \hat{\alpha} \quad \text{for } t = 2, \dots, T \end{cases} \quad (13)$$

which we write matricially

$$\hat{Y} = (\mathbf{1}_T \quad \mathbf{T}(T) \quad X) \begin{pmatrix} Y_1 - \hat{\mu} - X_1' \hat{\alpha} \\ \hat{\mu} \\ \hat{\alpha} \end{pmatrix}$$

where $\mathbf{T}(T) = (1 \quad \dots \quad T)'$. The presence of an annual trend component implies also that the predicted values at the disaggregated level also have a trend component:

$$\hat{y}_t = \hat{y}_{t-1} + (g(t) - g(t-1)) \hat{\mu} + (x_t - x_{t-1})' \hat{\alpha} \quad (14)$$

for $t = 1, \dots, fT$, where $g(t)$ is a quarterly trend component, i.e for a constant ν

$$g(t) = g(t-1) + \nu \quad \text{for } t \in \{2, \dots, fT\}$$

therefore

$$g(t) = g(1) + (t-1)\nu \quad \text{for } t \in \{2, \dots, fT\}. \quad (15)$$

By backward induction we can rewrite (14) up to an initial value \hat{y}_1 .

$$\hat{y}_t = \hat{y}_1 + (g(t) - g(1)) \hat{\mu} + (x_t - x_1)' \hat{\alpha}$$

therefore

$$\hat{y}_t = \hat{y}_1 + (t-1)\nu \hat{\mu} + (x_t - x_1)' \hat{\alpha}. \quad (16)$$

The first difference of the annual aggregation must sum up to the first difference of the annual estimates, i.e. in the case of a disaggregation to a higher frequency f :

$$\sum_{s=f(t-1)+1}^{ft} \hat{y}_s - \sum_{s=f(t-2)+1}^{f(t-1)} \hat{y}_s = \hat{Y}_t - \hat{Y}_{t-1} \text{ for } t \in \{2, \dots, T\}.$$

It implies for $t = 2$:

$$\begin{aligned} \hat{\mu}\nu \left(\sum_{s=f+1}^{2f} (s-1) - \sum_{s=1}^f (s-1) \right) + \left(\sum_{s=f+1}^{2f} x_s - \sum_{s=1}^f x_s \right)' \hat{\alpha} &= \hat{Y}_2 - \hat{Y}_1 \\ \hat{\mu}\nu \left(\sum_{s=f}^{2f-1} s - \sum_{s=0}^{f-1} s \right) + (X_2 - X_1)' \hat{\alpha} &= Y_1 + \hat{\mu} + \hat{\alpha}(X_2 - X_1) - Y_1 \end{aligned}$$

therefore

$$\nu = \left(\sum_{s=f}^{2f-1} s - \sum_{s=0}^{f-1} s \right)^{-1} = \left(\sum_{s=0}^{2f-1} s - 2 \sum_{s=0}^{f-1} s \right)^{-1} = \left(\frac{(2f-1)2f}{2} - 2 \frac{(f-1)f}{2} \right)^{-1} = f^{-2}.$$

The higher frequency estimates become, for $t = 1, \dots, fT$:

$$\hat{y}_t = \hat{y}_1 + \hat{\mu} \frac{t-1}{f^2} + (x_t - x_1)' \hat{\alpha}. \quad (17)$$

Now we want the first annual aggregation to equal the first annual observation Y_1 , therefore:

$$\begin{aligned} \sum_{s=1}^f \hat{y}_1 + \hat{\mu} \sum_{s=1}^f \frac{s-1}{f^2} + \sum_{s=1}^f (x_t - x_1)' \hat{\alpha} &= Y_1 \\ f(\hat{y}_1 - x_1' \hat{\alpha}) - \frac{f-1}{2f} \hat{\mu} + X_1' \hat{\alpha} &= Y_1 \\ \hat{y}_1 &= \frac{1}{f} \left(Y_1 - X_1' \hat{\alpha} - \frac{f-1}{2f} \hat{\mu} \right) + x_1' \hat{\alpha}. \end{aligned}$$

The quarterly estimates \hat{y}_t when the model is estimated in first difference are therefore :

$$\hat{y}_t = \frac{Y_1}{f} + \hat{\mu} \frac{t - 0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f} \right)' \hat{\alpha} \quad \forall t \in \{1, \dots, fT\}. \quad (18)$$

B Structural break variables and disaggregation formulas

We demonstrate the case $m = 1$. The generalization to the matricial form is straightforward.

B.1 Model C

$$\begin{aligned} \hat{y}_t &= \frac{\hat{\mu}}{f} + x'_t \hat{\alpha} + \frac{\hat{\mu}_1}{f} \mathbb{1}_{t \geq f(t_b-1)+1}, \quad t = 1, \dots, fT \\ \Rightarrow \quad \hat{Y}_t &= \hat{\mu} + X'_t \hat{\alpha} + \hat{\mu}_1 \mathbb{1}_{t \geq t_b}, \quad t = 1, \dots, T \end{aligned}$$

Proof:

It can be easy shown that for $t = 1, \dots, T$, $\sum_{i=-f+1}^0 \hat{y}_{fT+i} = \hat{Y}_t$.

B.2 Model CS

$$\begin{aligned} \hat{y}_t &= \frac{\hat{\mu}}{f} + x'_t \hat{\alpha} + \left(\frac{\hat{\mu}_1}{f} + x'_t \hat{\alpha}_1\right) \mathbb{1}_{t \geq f(t_b-1)+1}, \quad t = 1, \dots, fT \\ \Rightarrow \quad \hat{Y}_t &= \hat{\mu} + X'_t \hat{\alpha} + (\hat{\mu}_1 + X'_t \hat{\alpha}_1) \mathbb{1}_{t \geq t_b} \quad t = 1, \dots, T \end{aligned}$$

Proof:

It can be shown again that for $t = 1, \dots, T$, $\sum_{i=-f+1}^0 \hat{y}_{fT+i} = \hat{Y}_t$.

B.3 Model dC

$$\begin{aligned} \hat{y}_t &= \frac{Y_1}{f} + \hat{\mu} \frac{t - 0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f}\right)' \hat{\alpha} + \hat{\mu}_1 \frac{t - f(t_b - 1.5) - 0.5}{f^2} \mathbb{1}_{t \geq f(t_b-1)+1}, \quad t = 2, \dots, fT \\ \Rightarrow \quad \Delta \hat{Y}_t &= \hat{\mu} + \Delta X'_t \hat{\alpha} + \hat{\mu}_1 \mathbb{1}_{t \geq t_b} \quad t = 1, \dots, T \end{aligned}$$

Proof:

The quarterly estimates in level have an implicit quarterly trend like in the case in first difference without a structural break but that changes at the structural break, so have the following form for $t = 1, \dots, fT$:

$$\hat{y}_t = \frac{Y_1}{f} + \hat{\mu} \frac{t - 0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f}\right)' \hat{\alpha} + \hat{\mu}_1 h(t, t_b)$$

where

$$h(t, t_b) = \begin{cases} 0 & \text{for } t < f(t_b - 1) + 1 \\ g(t) & \text{for } t \geq f(t_b - 1) + 1 \end{cases} = g(t) \mathbb{1}_{t \geq f(t_b-1)+1}$$

with

$$\begin{aligned} g(t) &= g(t-1) + \nu \quad \text{for } t \in \{f(t_b - 1) + 1, \dots, fN\} \\ &= g(f(t_b - 1) + 1) + (t - f(t_b - 1) - 1)\nu \end{aligned}$$

$$\text{and} \quad \sum_{s=f(t_b-1)+1}^{ft} g(s) = t - t_b + 1 \quad \text{for } t \in \{t_b, \dots, T\}$$

. It can be shown again that $\nu = f^{-2}$. At the structural break date $t = t_b$:

$$\begin{aligned}
& \sum_{s=f(t_b-1)+1}^{ft_b} g(s) = 1 \\
\Leftrightarrow & \sum_{s=f(t_b-1)+1}^{ft_b} \left(g(f(t_b-1)+1) + \frac{s-f(t_b-1)-1}{f^2} \right) = 1 \\
\Leftrightarrow & fg(f(t_b-1)+1) + f^{-2} \left(\sum_{s=1}^{ft_b} s - \sum_{s=1}^{f(t_b-1)} s - f^2(t_b-1) - f \right) = 1 \\
\Leftrightarrow & fg(f(t_b-1)+1) + f^{-2} \left(\frac{ft_b(ft_b+1)}{2} - \frac{f(t_b-1)(f(t_b-1)+1)}{2} - f^2(t_b-1) - f \right) = 1 \\
\Leftrightarrow & fg(f(t_b-1)+1) + f^{-2} (0.5f^2 - 0.5f) = 1 \\
\Leftrightarrow & fg(f(t_b-1)+1) + 0.5 - \frac{0.5}{f} = 1.
\end{aligned}$$

Hence

$$g(f(t_b-1)+1) = \frac{0.5f + 0.5}{f^2}.$$

Therefore, for $t = 1, \dots, fT$:

$$h(t, t_b) = \frac{t - f(t_b - 1.5) - 0.5}{f^2} \mathbb{1}_{t \geq f(t_b-1)+1}. \quad (19)$$

B.4 Model dCS

$$\begin{aligned}
& \hat{y}_t = \frac{Y_1}{f} + \hat{\mu} \frac{t - 0.5(f+1)}{f^2} + \left(x_t - \frac{X_1}{f} \right)' \hat{\alpha} + \left(\hat{\mu}_1 \frac{t - f(t_b - 1.5) - 0.5}{f^2} + (x_t - \frac{X_{t_b-1}}{f})' \hat{\alpha}_1 \right) \mathbb{1}_{t \geq f(t_b-1)+1}, \\
& t = 2, \dots, fT \\
& \Rightarrow \Delta \hat{Y}_t = \hat{\mu} + \Delta X_t' \hat{\alpha} + (\hat{\mu}_1 + \Delta X_t' \hat{\alpha}_1) \mathbb{1}_{t \geq t_b}, \quad t = 1, \dots, T
\end{aligned}$$

Proof:

$$\sum_{s=f(t-1)+1}^{ft} \hat{y}_t - \sum_{s=f(t-2)+1}^{f(t-1)} \hat{y}_s = \hat{Y}_t - \hat{Y}_{t-1} \quad \text{for } t \in \{2, \dots, T\}.$$

For $t > t_b$:

$$\begin{aligned}
& \sum_{s=f(t_b-1)+1}^{ft_b} \hat{y}_s - \sum_{s=f(t_b-2)+1}^{f(t_b-1)} \hat{y}_s = \Delta \hat{Y}_{t_b} \\
& \hat{\mu} + \Delta X_t' \hat{\alpha} + \hat{\mu}_1 + \left(\Delta X_t + \sum_{s=f(t_b-1)+1}^{ft_b} h(s, t_b) - \sum_{s=f(t_b-2)+1}^{f(t_b-1)} h(s, t_b) \right)' \hat{\alpha}_1 = \hat{\mu} + \Delta X_t' \hat{\alpha} + \hat{\mu}_1 + \Delta X_t' \hat{\alpha}_1.
\end{aligned}$$

Therefore

$$\sum_{s=f(t_b-1)+1}^{ft_b} h(s, t_b) = \sum_{s=f(t_b-2)+1}^{f(t_b-1)} h(s, t_b) \quad (20)$$

which means that for $t > t_b$ the annual sum of the adjustment to x_t must be equal to the preceding annual sum. At the break location $n = t_b$:

$$\begin{aligned} \sum_{t=f(t_b-1)+1}^{ft_b} \hat{y}_t - \sum_{t=f(t_b-2)+1}^{f(t_b-1)} \hat{y}_t &= \Delta \hat{Y}_{t_b} \\ \hat{\mu} + \Delta X'_{t_b} \hat{\alpha} + \hat{\mu}_1 + \left(X_{t_b} - \sum_{t=f(t_b-2)+1}^{f(t_b-1)} h(t, t_b) \right)' \hat{\alpha}_1 &= \hat{\mu} + \Delta X'_{t_b} \hat{\alpha} + \hat{\mu}_1 + \Delta X'_{t_b} \hat{\alpha}_1. \end{aligned}$$

Therefore

$$\sum_{t=f(t_b-2)+1}^{f(t_b-1)} h(t, t_b) = X_{t_b-1} \quad (21)$$

which means that for $t > t_b$ the annual sum of the adjustment to x_t must be equal to the annual sum of the indicator at the year preceding the structural break location. A simple condition satisfying (20) and (21) would be :

$$h(t, t_b) = \frac{X_{t_b-1}}{f} \quad \text{for } t \in \{1, \dots, fT\}. \quad (22)$$

C Power of the cointegration test

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.589	0.826	0.94	0.985	0.995	0.351	0.572	0.775	0.899	0.994
C	0.017	0.01	0.007	0.005	0.006	0.019	0.013	0.01	0.007	0.009
CS	0.04	0.039	0.038	0.041	0.049	0.039	0.04	0.04	0.047	0.057
dO	0.039	0.041	0.04	0.042	0.051	0.051	0.058	0.053	0.055	0.066
dC	0.02	0.016	0.018	0.016	0.017	0.03	0.028	0.025	0.023	0.021
dCS	0.031	0.031	0.03	0.029	0.032	0.035	0.036	0.035	0.033	0.037

Table 12: Power of the test by true model, for regression model (O)

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.307	0.521	0.731	0.871	0.997	0.175	0.294	0.449	0.613	0.969
C	0.174	0.309	0.537	0.865	1	0.095	0.13	0.307	0.725	1
CS	0.145	0.174	0.279	0.512	0.615	0.108	0.132	0.247	0.482	0.612
dO	0.04	0.037	0.032	0.035	0.037	0.041	0.04	0.041	0.041	0.052
dC	0.02	0.013	0.014	0.01	0.012	0.024	0.022	0.018	0.019	0.017
dCS	0.033	0.036	0.036	0.037	0.045	0.032	0.027	0.029	0.032	0.036

Table 13: Power of the test by true model, for regression model (C)

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.304	0.492	0.691	0.837	0.997	0.181	0.276	0.42	0.559	0.954
C	0.247	0.349	0.541	0.797	1	0.167	0.207	0.344	0.637	1
CS	0.263	0.38	0.572	0.796	1	0.188	0.238	0.363	0.606	0.996
dO	0.048	0.046	0.04	0.044	0.047	0.047	0.045	0.043	0.047	0.063
dC	0.032	0.034	0.035	0.037	0.054	0.032	0.034	0.036	0.031	0.042
dCS	0.04	0.045	0.042	0.048	0.076	0.036	0.032	0.034	0.036	0.052

Table 14: Power of the test by true model, for regression model (CS)

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.916	0.994	0.998	0.999	1	0.812	0.98	0.997	0.998	1
C	0.342	0.944	0.998	1	1	0.377	0.922	0.996	1	1
CS	0.314	0.793	0.934	0.973	0.986	0.347	0.813	0.951	0.983	0.992
dO	0.203	0.417	0.649	0.83	0.99	0.116	0.233	0.406	0.608	0.973
dC	0.084	0.136	0.193	0.282	0.673	0.082	0.157	0.244	0.389	0.852
dCS	0.199	0.356	0.532	0.698	0.953	0.121	0.239	0.403	0.586	0.962

Table 15: Power of the test by true model, for regression model (dO)

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.534	0.878	0.984	0.998	0.999	0.368	0.728	0.938	0.992	0.999
C	0.183	0.257	0.59	0.917	1	0.178	0.283	0.6	0.896	1
CS	0.219	0.349	0.592	0.855	0.997	0.218	0.346	0.609	0.871	0.999
dO	0.096	0.161	0.264	0.415	0.915	0.068	0.093	0.148	0.239	0.695
dC	0.092	0.167	0.268	0.406	0.921	0.064	0.101	0.149	0.237	0.7
dCS	0.11	0.207	0.322	0.489	0.934	0.071	0.115	0.177	0.265	0.76

Table 16: Power of the test by true model, for regression model (dC)

N	$\rho = 0$					$\rho = 0.5$				
	15	20	25	30	50	15	20	25	30	50
O	0.503	0.856	0.978	0.999	1	0.347	0.691	0.917	0.99	1
C	0.262	0.324	0.491	0.781	1	0.234	0.322	0.516	0.793	1
CS	0.276	0.437	0.639	0.856	1	0.266	0.415	0.635	0.858	1
dO	0.082	0.131	0.221	0.346	0.869	0.057	0.069	0.119	0.179	0.595
dC	0.07	0.116	0.189	0.299	0.845	0.049	0.073	0.107	0.172	0.578
dCS	0.093	0.171	0.284	0.444	0.941	0.057	0.088	0.138	0.212	0.696

Table 17: Power of the test by true model, for regression model (dCS)

D other accounts

	\hat{n}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	34879.01*** (5541.04)	25937.52*** (5639.2)	16742.53* (7144.65)	7370.84. (3844.41)	7209.48* (3412.17)	2744.09 (3225.7)
X	0.52*** (0.02)	0.49*** (0.02)	0.76*** (0.1)	0.35** (0.1)	0.01 (0.16)	0.55*** (0.11)
d		26468.75** (9096.84)	54152.51** (13941.66)		18059.77* (7295.83)	38984.4** (10783.29)
dX			-0.31** (0.1)			-0.88** (0.23)
rmse.a	12.79	10.64	9.33	16.53	9.06	5.29
sb date	NA	2005	2009	NA	2007	2012
inf ADF	-3.377	-3.846	-5.425	-2.305	-4.346	-7.516
pval	0.096	0.506	0.115	0.411	0.311	0.005
	\hat{n}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	34879.01*** (5541.04)	25951.41*** (4403.89)	26928.98*** (4531.34)	7370.84. (3844.41)	7209.48* (3412.17)	2744.09 (3225.7)
X	0.52*** (0.02)	0.61*** (0.02)	0.61*** (0.02)	0.35** (0.1)	0.01 (0.16)	0.55*** (0.11)
d		-5.6e+04*** (12267.72)	-9.7e+04* (45447.17)		18059.77* (7295.83)	38984.4** (10783.29)
dX			0.07 (0.08)			-0.88** (0.23)
rmse.a	12.79	8.86	8.65	16.53	9.06	5.29
sb date	NA	2014	2014	NA	2007	2012

Table 18: Model fitting Chinese fixed capital formation and investment in fixed assets (1998:2014)

	\hat{n}_b selected by cointegration test					
	O	C	CS	dO	dC	dCS
(Intercept)	8308.5** (2726.93)	5792.74* (2325.52)	12705.1*** (1616.99)	2611.42 (1906.91)	2575.41. (1365.79)	2220.6 (1385.65)
X	0.59*** (0.02)	0.73*** (0.05)	0.49*** (0.03)	0.43** (0.14)	0.22. (0.11)	0.26* (0.11)
d		-2.4e+04** (7189.16)	-8.2e+04*** (12417.87)		9468.51*** (2169.98)	16803.91* (6629.18)
dX			0.5*** (0.07)			-0.44 (0.38)
rmse.a	16.91	13.33	8.21	35.32	10.79	8.68
sb date	NA	2011	2014	NA	2015	2015
inf ADF	-0.936	-3.038	-3.634	-3.065	-6.071	-6.843
pval	0.914	0.846	0.709	0.154	0.026	0.015
	\hat{n}_b selected by annual prediction error					
	O	C	CS	dO	dC	dCS
(Intercept)	8308.5** (2726.93)	12456.3*** (1731.53)	13065.65*** (1217.29)	2611.42 (1906.91)	2039.3. (1103.07)	2220.6 (1385.65)
X	0.59*** (0.02)	0.49*** (0.02)	0.48*** (0.01)	0.43** (0.14)	0.28** (0.08)	0.26* (0.11)
d		26614.66*** (4254.76)	-7.2e+04** (21663.04)		11156.07*** (1782.43)	16803.91* (6629.18)
dX			0.47*** (0.1)			-0.44 (0.38)
rmse.a	16.91	9.67	6.57	35.32	8.69	8.68
sb date	NA	2016	2016	NA	2016	2015

Table 19: Model fitting Chinese government consumption expenditures and government expenditures (1998:2014)

E Scatterplot for the Chinese household consumption expenditures (resp. net exports of goods and services) and retail sales of consumer goods (resp. net exports of goods)

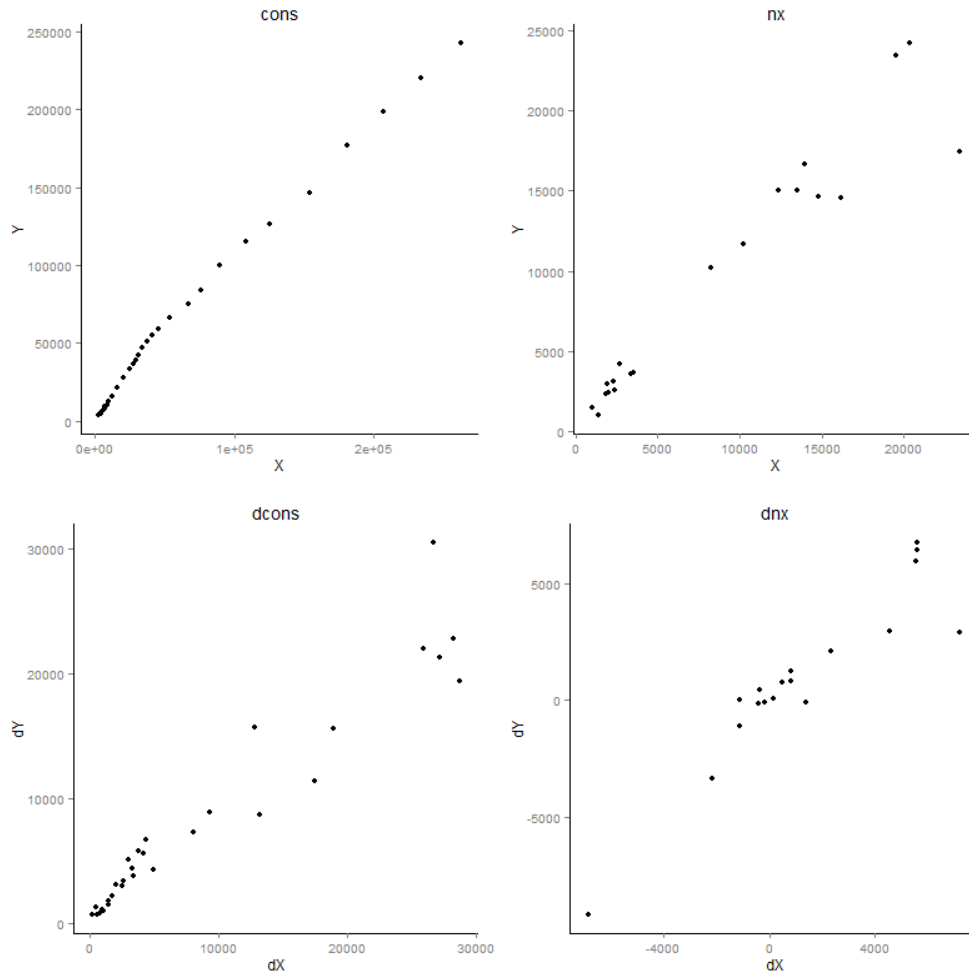


Figure 8: Scatter representation of the Chinese data, in level and in first difference

F Cyclical components of the Chinese indicators

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k}), \text{ where } k=$										
				-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	0.8	1	0.9	0.12	0.28	0.52	0.74	0.9	1	0.9	0.74	0.52	0.28	0.12
cons	2.76	3.44	0.32	-0.18	-0.28	-0.23	-0.24	-0.17	-0.05	0.01	0.12	0.24	0.36	0.5
cf	3.95	4.93	0.75	-0.12	-0.23	-0.32	-0.31	-0.23	-0.14	-0.01	0.08	0.11	0.2	0.25
g	9.17	11.45	0	-0.01	0.04	0.05	0.02	0.01	0	0.03	0.03	0	-0.03	-0.07
nx/gdp	1.23	1.54	0.71	0.11	0.23	0.27	0.23	0.23	0.21	0.27	0.37	0.4	0.35	0.26

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 20: CHN indicators 1998Q1:2016Q2

G Cyclical components of other series for the Chinese economy

x	$\sigma(x)$ $\frac{\sigma(x)}{\sigma(\text{gdp})}$ $\rho_1(x)$			$\rho(x_t, \text{gdp}_{t+k}), \text{ where } k=$									
	-5	-4	-3	-2	-1	0	1	2	3	4	5		
gdp	1.53	1	0.69	-0.1	0.13	0.4	0.69	1	0.69	0.4	0.13	-0.1	
cons	2.24	1.46	0.21	0.05	0.31	0.31	0.35	0.54	0.23	0.13	0.05	-0.06	
cf	2.51	1.64	0.95	-0.36	-0.26	-0.2	-0.15	-0.1	-0.12	-0.08	-0.01	0.1	
g	4.13	2.7	0.41	0.12	0.24	0.54	0.58	0.6	0.35	0.13	-0.05	-0.16	
nx/gdp	1.3	...	0.76	0.12	0.03	0.13	0.38	0.55	0.56	0.42	0.15	-0.06	

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 21: CHN cycles with estimated gdp, deflated by CPI 1998Q1:2016Q2

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k}), \text{ where } k=$										
				-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	1.38	1	0.55	-0.06	-0.27	-0.13	0.17	0.55	1	0.55	0.17	-0.13	-0.27	-0.06
cons	2.04	1.48	0.2	-0.11	-0.13	0.13	0.2	0.23	0.54	0.17	-0.1	-0.17	-0.16	-0.04
cf	2.43	1.77	0.94	-0.22	-0.22	-0.15	-0.09	-0.08	-0.11	-0.22	-0.27	-0.26	-0.17	0
g	4.08	2.97	0.39	-0.02	-0.09	-0.01	0.3	0.37	0.4	0.23	0.06	-0.03	-0.09	-0.05
nx/gdp	1.22	0.89	0.75	0.17	0.05	-0.05	-0.02	0.22	0.54	0.6	0.5	0.24	0.03	0

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 22: CHN cycles with estimated gdp, deflated by official implied gdp.defl 1998Q1:2016Q2

x	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	$\rho(x_t, \text{gdp}_{t+k}), \text{ where } k=$										
				-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	2.52	1	0.84	-0.43	-0.18	0.2	0.56	0.84	1	0.84	0.56	0.2	-0.18	-0.43
cons	3.04	1.21	0.62	-0.42	-0.18	0.21	0.5	0.7	0.82	0.66	0.47	0.2	-0.1	-0.3
cf	3.77	1.5	0.91	-0.38	-0.17	0.12	0.4	0.59	0.64	0.55	0.36	0.13	-0.09	-0.27
g	4.27	1.7	0.27	-0.09	0.03	0.13	0.34	0.41	0.42	0.29	0.14	-0.04	-0.21	-0.25
nx/gdp	1.22	0.49	0.75	0.2	0.13	0.02	-0.04	-0.03	0.06	0.1	0.11	0.04	-0.05	-0.08

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 23: CHN cycles with estimated gdp, deflated by reconstructed implied gdp.defl 1998Q1:2016Q2

H US business cycle correlations (1947:2016)

x				$\rho(x_t, \text{gdp}_{t+k})$, where k=										
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(\text{gdp})}$	$\rho_1(x)$	-5	-4	-3	-2	-1	0	1	2	3	4	5
gdp	1.62	1	0.85	-0.07	0.12	0.35	0.62	0.85	1	0.85	0.62	0.35	0.12	-0.07
cons	1.21	0.75	0.79	-0.11	0.02	0.18	0.42	0.64	0.79	0.75	0.61	0.43	0.25	0.07
cf	7.66	4.72	0.8	-0.26	-0.09	0.15	0.42	0.66	0.83	0.74	0.56	0.32	0.11	-0.03
g	3.16	1.95	0.89	0.3	0.35	0.33	0.28	0.23	0.15	0.04	-0.04	-0.08	-0.08	-0.05
nx/gdp	0.43	0.26	0.77	0.23	0.15	0.05	-0.11	-0.26	-0.34	-0.38	-0.35	-0.29	-0.23	-0.2

Note: Statistics are calculated on HP-filtered season-adjusted deflated data. Except for net exports/output, all series are in logarithms.

Table 24: US 1947Q1:2016Q2