UMR 8184

THéorie Économique, Modélisation et Applications

THEMA Working Paper n°2019-05
Université de Cergy-Pontoise, France

BORROWING CONSTRAINTS AND LOCATION CHOICE – EVIDENCE FROM THE PARIS REGION

Sophie Dantan and Nathalie Picard

UNIVERSITÉ de Cergy-Pontoise

CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

January 2019

BORROWING CONSTRAINTS AND LOCATION CHOICE - EVIDENCE FROM THE PARIS REGION

Sophie Dantan[1] and Nathalie Picard[2]

January 2019

ABSTRACT

This paper investigates the determinants of residential segregation using a nested logit model to disentangle household preferences for local amenities, for dwelling type and for homeownership. The model is extended to account for unobservable borrowing constraints which might prevent some households from purchasing a dwelling. A counterfactual distribution of socio-demographic characteristics across the Paris region is then built by relaxing those constraints. The comparison of the actual and counterfactual distributions suggests that if their credit constraints were alleviated, households would tend to locate further from Paris. In particular if constraints were relaxed only on the poorest households, they would not be likely to mix with the richest households.

JEL classification codes : R21, R23, R31
Key words: Homeownership, Tenure choice, Borrowing constraints, Residential segregation, Suburbanization, Urban sprawl, Location choice model, Endogenous choice sets.

---

[1]. Sophie Dantan, Université Cergy-Pontoise, THEMA, Email: Sophie.Dantan@u-cergy.fr
[2]. Nathalie Picard, Université Cergy-Pontoise, THEMA, France, Email: Nathalie.Picard@u-cergy.fr

BORROWING CONSTRAINTS AND LOCATION CHOICE:

EVIDENCE FROM THE PARIS REGION

1. Introduction

Various housing policy measures are implemented to favor the access of poor households to homeownership, in France as in other countries. Measures such as the deductibility of mortgage interests have been rapidly abandoned, while other measures such as the provision of zero-interest-rate loans have been implemented under various forms and restrictions. One of the motives for implementing such measures is to enhance social mobility by enabling the poorest households to cumulate and transmit housing assets.

Little attention has been paid to the effect of such a homeownership-enhancing policy on residential segregation, which is however an important determinant of social mobility (Combes *et al.* 2008, Causa *et al.* 2010, Gobillon *et al.* 2011). In particular, if relaxing constraint on ownership for poor households reinforces residential segregation, the expected positive effect on social mobility might be significantly reduced. Measuring to what extent residential segregation is exacerbated or attenuated by liquidity constraints contributes to determine the relevance of enhancing homeownership for increasing social mobility. The ambiguity comes from the fact that households who are eligible to these measures may prefer buying in the poor suburbs rather than renting in the rich Central part of the city.

We highlight this issue by evaluating the importance of housing liquidity constraints in explaining the social sorting in the Paris region. To do so, we model household preferences for housing characteristics and for tenure status (ownership vs. tenancy) and then extend our model to account for the effect of liquidity constraints on location demand.

The effect of liquidity constraints on segregation is then evaluated by comparing the distribution of households with and without liquidity constraints. In this purely normative exercise, prices and socio-demographic composition are being held constant. The objective of this normative evaluation of policy measures is indeed to evaluate what each household would prefer to do in the observed situations if it were not constrained and if nothing changed for other households. By contrast, a descriptive and predictive analysis of policy measures would require computing their aggregate effects on the endogenous equilibrium of the housing market, that is, changes in socio-demographic composition, in housing prices, and plausible assumptions on real estate supply reactions to changes in demand. Such a predictive analysis is out of the scope of this article.

2. Location choice and tenure status

In the economic literature, the choices of tenure status and of housing consumption have long been studied separately: the former by assuming that tenure choice results from the comparison of the respective costs of owning and renting (Smith *et al.* 1988 for a review), the latter by assuming that tenure status is exogenous and maximizes the utility derived from housing consumption (for instance Artle and Varaiya, 1978). However, as underlined by Lee and Trost (1978), Rosen (1979), and King (1980), housing consumption and tenure choice both result from the same utility maximization process, which implies that they are determined by common variables.

2.1. Tenure choice and life cycle: household decisions and market imperfections

Household decision whether to rent or own a dwelling is the result of a complex mechanism since the acquisition of a dwelling responds to dual motives, namely housing investment and housing consumption.

Households who invest in the ownership of a dwelling often have to engage in a long-term mortgage and then to adapt their consumption path. As a consequence, life-cycle effects might also play an important role in tenure status choice. According to Artle and Varaiya (1978), household tenure choice results from the maximization of their life-cycle consumption of non-housing good. The most patient households should then theoretically first purchase their home at the beginning of their life-cycle, then cumulate housing equity (enlarge or improve their dwelling) at the middle and finally liquidate their housing equity asset by selling their home and renting another one at the end of their life-cycle[3]. By contrast, the most impatient households should favor their current consumption and choose permanent tenancy. Bequest motives or altruism toward their descendants can however attenuate the transition from ownership to renting among the elderly (Megbolugbe *et al* 1997, de Palma *et al* 2015). To account for the fact that housing is also a consumption good, Henderson and Ioannides (1983) develop a two-period model combining housing demand for consumption purpose, on one hand, and housing demand for investment purpose, on the other hand. Both demands are determined under a common budget constraint, which is similar for owners and renters (owner-occupiers rent to themselves). Henderson and Ioannides[4] show that the propensity to own-occupy a dwelling (rather than renting it) does not only depend on the level of wealth but also on the income path: individuals who expect a decrease or a lower increase in their income in the second period (people who are less educated or inherit in the first period) are more likely to own-occupy than to rent.

[3] In Artle and Varaiya (1978), some liquidity-constrained households may however delay their purchase and save enough to afford paying the downpayment. When the delay is very long, their purchase can be cancelled.

[4] In Henderson and Ioannides (1983), housing investment is affected by income path (total wealth held constant) but not by the level of wealth (with income path unchanged). The reversal holds for the housing consumption. This results in a counter-intuitive situation in which the richest individuals tend to be renters. The authors suggests that rental externalities and a progressive tax system tend to make tenancy less desirable to the richest individuals so that they turn to owning-occupancy.

Liquidity constraints, transaction cost and distortive tax modify the return to housing investment and can make it less competitive compared to the return of savings. Consequently, the budget constraint might differ between owner-occupiers and renters when financial markets are imperfect. For instance, the tax on rental income paid by landlords but not by owner-occupiers makes the ownership of a dwelling to rent out less profitable. On the opposite, the deductibility of paid mortgage interest and the zero-mortgage-interest loan make the housing investment more attractive. Similarly, a lower mortgage interest rate or a larger borrowable amount proposed to the wealthiest households might increase their housing investment and help explaining why they are more likely to purchase their home than the poorest households.

Liquidity constraint might influence residential segregation in two ways: it might influence both household decision to move and household location choice when moving. Concerning the mobility decision, Gobillon and Le Blanc (2004, 2008) develop a two-period tenure choice model with an individual-specific borrowable amount and apply it to the case of the French zero-interest loan PTZ (*Prêt à Taux Zéro*). They show that this policy measure has increased ownership among poor households who would have stayed in their previous (rented) dwelling otherwise. However, it mainly benefits households which would anyway have moved and purchased a dwelling even without the PTZ.

We extend their approach and results by modelling the effects of liquidity constraints on household simultaneous choices of tenure and location, in the case of the Paris Region.

2.2. Residential segregation

Residential segregation in Paris region mainly consists in the concentration of the richest households inside Paris and close Western suburbs, while the poorest households tend to live in Northern and Eastern suburbs. Such a phenomenon is consistent with a monocentric model in which household income has a stronger positive effect on the valuation of accessibility of the Central Business District (CBD, here Paris intra muros) than on its demand for dwelling size (Alonso 1964, Mills 1967, Muth 1969, Wheaton 1977). Residential segregation can also be explained by a Tiebout-like (1956) mechanism which leads the richest households to concentrate in the CBD and then to induce an increase in housing prices which excludes poor households from the CBD (Bénabou 1995). In this latter case, social sorting is likely to be exacerbated by financial market imperfections which might prevent poor families from borrowing to acquire their preferred dwelling size and preferred location. Brueckner, Thisse and Zénou (1999) develop an alternative model in which the location choice depends not only on housing price and commuting cost but also on the level of amenities, assumed exogenous in the simplest version of their model. Moreover, they assume that the valuation of amenities increases with income (more rapidly than housing consumption). They show that if the CBD has a great advantage in the provision of amenities, then rich households are likely to concentrate in the center and poor households in the suburbs. By contrast, if the level of amenities slightly decreases or even increases with the distance to the CBD, the reversal might occur. This result explains why the concentration of rich households in the CBD is exacerbated when the CBD concentrates amenities (like in Paris) and is reversed when there are more amenities in suburbs (like in Detroit). They also show that even in the case where the level of some endogenous "modern" amenities increases with the local average income level, the concentration of rich households in the center is the only possible equilibrium when the exogenous amenity advantage of the center is large enough (as may be the case of Paris, for historical reasons).

As suggested above, another potential explanation for residential segregation might rely on household unequal access to ownership. However, the models of residential segregation mentioned above neglect tenure choice decisions. As a consequence, such models cannot be used to evaluate the effect of barriers to ownership on residential segregation.

In Sections 3.1 to 3.2, we propose to extend the standard monocentric model to account simultaneously for the effect of tenure status and liquidity constraint on location choice and, thus, on residential segregation.

Few empirical models relate housing demand and tenure choice (Elder and Zumpano 1991; Rappaport 1996) and none of them explicitly considers the effect of market imperfections on tenure choice. Henderson and Ioannides (1986) evaluate this effect by estimating simultaneously the parameters of the probability to be constrained to rent and the probability to prefer ownership (by maximizing a joint likelihood function). However, they disregard residential location choice.

Following the empirical strategy developed by Berry, Levinshon and Pakes (1995) to model consumers' demand for differentiated goods, Bayer et al (2007) estimate households preferences for residential characteristics. They rely on a market clearing condition to correct for the endogeneity bias induced by unobserved determinants of residential choice and to measure the effect of some determinants of prices and location choice. Their results highlight strong self-segregation and heterogeneity in household preferences for school quality. However, this strategy might not correct for biases induced by credit constraints. Indeed, Rancière and Ouazad (2015) extend the BLP approach of housing market and find a lower price elasticity when mortgage approval and location choice are estimated jointly than when a location choice model is estimated alone.

The approach developed in the present paper differs from the BLP approach in two aspects. First, it does not rely on clearing market conditions, which clearly do not hold in the French dwelling market. Second, the potential sources of endogeneity are explicitly modelled and introduced in the model rather than corrected in a second step. . In Sections 3.3 and 3.4, we explicitly model jointly location choice, tenure choices and liquidity constraint in a nested fully consistent model. The econometric model developed and estimated is used to evaluate the effect of liquidity constraints on location choice and then on residential segregation.

3. Model specification

3.1. Structural monocentric model

In this section, we develop a three-step monocentric model in which households choose their tenure status (*s*) in the first step, their distance (*d*) to the CBD and their level of local amenities (*z*) in the second step, and then their consumption of housing (*H*) and other goods (*C*) in the third step. The only source of heterogeneity between households considered in this section is income. Our model mainly builds upon the amenity-based location model developed by Brueckner et al. (1999), especially concerning endogenous equilibrium prices. Rather than analysing the determinants of equilibrium prices, we borrow their assumptions and conclusions concerning endogenous prices, in order to focus on household decisions conditional on prices, and to generalize their results in terms of household behaviour and heterogeneity.

Consistently with Section 2, our model introduces the distinction between owners and renters. The model is first extended to introduce a *potential* liquidity constraint, consistently with Section 2.1. It is then further extended to a more realistic discrete choice model in which household preferences are heterogeneous and distance *d* to CBD and local amenities *z* are determined by the discrete location *j*. Finally, the model is further extended by considering also dwelling type (*T*, either flat or house) in the first step of the program (at the same time as tenure status *S*). Prices then also depend on dwelling type *T*.

Definition D1)

Household *i* is characterized by income y_i , bounded by a finite value $Y \equiv \max_i y_i$, and by a utility function *U*(.) depending on tenure status *S*, on the amount of local amenities *z*, on floor space H and on consumption C of a composite good, which price is normalized to 1. Local amenities z are valued by an increasing and concave function $\psi(z)$, defined over $\mathbb{R}$ and further specified in Assumption H1). The distance d to the CBD is not valued directly, but only indirectly, through a commuting cost function specified in Assumption H4) and through dwelling price $\pi^S(d,z)$, specified in Assumption H2).

Assumption H1)

The function $\psi(z)$ is continuous and twice derivable on $\mathbb{R}$ and such that: $\psi'(z)>0$ $\forall z\in \mathbb{R}$; $\psi''(z)<0$ $\forall z\in \mathbb{R}$; $\lim_{z\to-\infty}\psi(z)=-\infty$ and $\lim_{z\to+\infty}\psi(z)=\bar{\psi}<\infty$.

The price of a dwelling $\pi^S(d,z)$, further specified in Assumption H2), equals its expected use cost when bought (S = "*own*") and its rental price when rented (S= "*rent*").

In contrast with the models analysed in Brueckner-Thisse-Zenou, the distance d and amount of amenities z are assumed here to entail some degree of independent variation, so that it makes sense to consider partial derivatives with respect to d and to z. Such a hypothesis is consistent with the observation that, at the same distance of Paris, the level of amenities and the concentration of rich households tend to be higher in Westerm suburbs than in Eastern suburbs.

Assumption H2)

The function $\pi^{S}(d,z)$ is continuous and twice derivable on $\mathbb{R}^{+}\times\mathbb{R}$ and such that:

$$\frac{\partial\pi^{S}(d,z)}{\partial d}<0\ \forall(d,z)\in\mathbb{R}^{+}\times\mathbb{R};\ \frac{\partial\pi^{S}(d,z)}{\partial z}>0\ \forall(d,z)\in\mathbb{R}^{+}\times\mathbb{R};$$
$$\lim_{d\to 0}\pi^{S}(d,z)=+\infty;\ \lim_{d\to+\infty}\pi^{S}(d,z)=\overline{\pi}^{S}\geq 0;\ \lim_{z\to-\infty}\pi^{S}(d,z)=0;\ \lim_{z\to+\infty}\pi^{S}(d,z)=+\infty,$$
$$\pi^{S}(d,z)=\pi_{1}^{S}(d)\cdot\pi_{2}^{S}(z).$$

Assumption H2) is consistent with the finding by Brueckner-Thisse-Zenou (1999) that price must decrease with distance to ensure that utility is uniformly-distributed.

The price elasticity to distance *d* and to amenities *z* are denoted, respectively, by

$$\begin{aligned}\xi_{d}^{S}(d)&\equiv\frac{\partial\pi^{S}(d,z)}{\partial d}\times\frac{1}{\pi^{S}(d,z)}=\frac{\pi_{1}^{S'}(d)\pi_{2}^{S}(z)}{\pi_{1}^{S}(d)\pi_{2}^{S}(z)}=\frac{\pi_{1}^{S'}(d)}{\pi_{1}^{S}(d)}<0 \quad \text{and}\\ \xi_{z}^{S}(z)&\equiv\frac{\partial\pi^{S}(d,z)}{\partial z}\times\frac{1}{\pi^{S}(d,z)}=\frac{\pi_{1}^{S}(d)\pi_{2}^{S'}(z)}{\pi_{1}^{S}(d)\pi_{2}^{S}(z)}=\frac{\pi_{2}^{S'}(z)}{\pi_{2}^{S}(z)}>0\end{aligned} \qquad (1)$$

The multiplicative separability assumed in Assumption H2) ensures that the elasticity of price to distance *d* does not depend on amenities *z*, and vice-versa. We further assume increasing price elasticities:

Assumption H3)

$\xi_{d}^{S'}(d)>0$ and $\xi_{z}^{S'}(z)>0$.

Household *i* also incurs an increasing commuting cost $t(d)$, defined over $\mathbb{R}^{+}$ and specified in

Assumption H4).

Assumption H4)

The commuting cost $t(d)$ is continuous and twice derivable on $\mathbb{R}^{+}$ and such that:

$t'(d)>0\ \forall d\in\mathbb{R}^{+}$; $t''(d)\geq 0\ \forall d\in\mathbb{R}^{+}$; $t(0)=0$.

In Bruecker-Thisse-Zenou[5] (1999) the commuting cost is a linear function of the distance ($t''(d)=0$), which is a special case here.

To fix ideas and to obtain closed-form solutions, we consider a specific (Cobb-Douglas) form for household utility as a function of consumption C and floor space H, as specified in Assumption H5).

Assumption H5)

$$U(C,H;z;S) = \beta^S \cdot \psi(z) + (1-\beta^S).\left[\gamma^S.\ln C + \left(1-\gamma^S\right).\ln H\right], \; 0<\gamma^S<1 \text{ and } 0<\beta^S<1. \tag{2}$$

The parameter γ^S measures the preference for consumption C over floor space H, whereas the parameter β^S measures the preference for amenities z over consumption bundle (C,H).

Most of the results obtained here would still hold if Utility were only assumed additively separable, increasing in amenities z and increasing and concave in consumption C and floor space H, with Inada conditions (infinite marginal utilities at zero consumption levels).

We consider a time period long enough to neglect saving and borrowing in the (inter-temporal) budget constraint:

$$y_i = C + \pi^S(d,z)\cdot H + t(d). \tag{3}$$

Under Assumptions H1) to H5), we first show that the city has finite dimension. Different assumptions for limit conditions in H1) and H2) or in Definition D1) could lead to an infinite-dimension city without altering the other conclusions of the model.

Lemma 1

Household i can only select a distance d such that $t(d)<y_i$. The size of the city is finite: there exists a maximal finite distance $D>0$ and a finite maximal commuting cost $T>0$ such that t :

---

[5] We do not make any assumption on the relationship between commuting cost and income, whereas they assume that the value of time and, thus, the slope of the commuting costs is larger for richer households.

$[0;D] \rightarrow [0;T]$ is a one-to-one mapping; its inverse, denoted by $t^{-1}(.)$ is continuous and increasing on $[0;T]$.

Proof: See Appendix 8.1. ■

The model is solved backwards, in three steps.

- In the **third step** of the program, household i maximizes its utility (2) subject to budget constraint (3), given household income y_i, tenure status S, distance d (such that $t(d)<y_i$), and local amenities z, by choosing the optimal levels of housing good $H^*(d,z;S;y_i)$ and of other goods $C^*(d,z;S;y_i)$. This results in the indirect utility of household i with income y_i conditional on tenure status S, on distance d and on amenities z: $U^*(d,z;S;y_i) \equiv U\left(C^*(d,z;S;y_i), H^*(d,z;S;y_i); z; S\right)$.

Lemma 2

Consider $y_i \in \left]0,Y\right], d \in \left[0;t^{-1}(y_i)\right[, z \in \mathbb{R}$ and $S \in \{own, rent\}$. Maximizing utility (2) under budget constraint (3) leads to optimal consumption levels $C^*(d,z;S;y_i) = \gamma^S \cdot (y_i - t(d))$ and $H^*(d,z;S;y_i) = (1-\gamma^S) \cdot \dfrac{y_i - t(d)}{\pi^S(d,z)}$, and to the indirect utility

$$U^*(d,z;S;y_i) = k^S + \beta^S \cdot \psi(z) + (1-\beta^S) \cdot \ln(y_i - t(d)) - (1-\beta^S)(1-\gamma^S) \ln \pi^S(d,z), \qquad (4)$$

where k^S is a non-linear combination of the coefficients β^S and γ^S.

Proof: See Appendix 8.1. ■

- The **second step** of the program consists in choosing the distance d to CBD and the amount of local amenities z so as to maximize the indirect utility $U^*(d,z;S;y_i)$ conditional on tenure status S and on income y_i.

Under assumptions H1) to H5), the optimal distance and amount of local amenities resulting from the second step of the program are shown to be unique (see proofs of Lemmas 3 and 4 in

Appendix 8.1) and are denoted by $d^*(S;y_i)$ and $z^*(S)$. Note that, under Assumption H5)[6], the optimal distance depends on income, whereas the optimal level of amenities does not.

Lemma 3

Under Assumptions H1) to H5), for any tenure status *S*, income y_i and level of amenities *z*, the indirect utility $U^*(d,z;S;y_i)$ is a concave function of *d* on $[0; t^{-1}(y_i)[$ and there exists a unique optimal distance $d^*(S;y_i) \in \left]0;t^{-1}(y_i)\right[$ which maximizes $U^*(d,z;S;y_i)$.

Proof: See Appendix 8.1. ■

Optimal location results from a trade-off between the price, which decreases when moving farther away from the CBD and the transportation cost, which increases when moving farther away from the CBD. Assumption H3) ensures that price decreases faster closer to CBD, whereas Assumption H4) ensures that transportation cost increase faster when farther away from the CBD.

Lemma 4

Under Assumptions H1) to H5), for any tenure status *S*, income y_i and distance $d \in\,]0; t^{-1}(y_i)[$, the indirect utility $U^*(d,z;S;y_i)$ is a concave function of *z* on $\mathbb{R}$ and there exists a unique optimal level of amenities $z^*(S) \in \mathbb{R}$ which maximizes $U^*(d,z;S;y_i)$.

Proof: See Appendix 8.1. ■

The second step of the program results in the "further-indirect" utility $U^{**}(.)$ of household *i* with income y_i conditional on tenure status *S*:

---

[6] With Cobb-Douglas utility, the optimal expenses on *C* and *H* depend on income, but not on prices (respectively 1 and $\pi^S(d,z)$, here).

$$U^{**}(S;y_i) \equiv U^*(d^*(S;y_i), z^*(S); S; y_i)\,. \tag{5}$$

- The **first step** of the program simply consists in choosing, among the two possible tenures, the one which gives the highest "further-indirect" utility: household *i* buys a dwelling if and only if (iff) $U^{**}(own; y_i) > U^{**}(rent; y_i)$ and it rents a dwelling iff $U^{**}(own; y_i) < U^{**}(rent; y_i)$.[7]

3.2. Introduction of a liquidity constraint

The program developed in Section 3.1 neglects the liquidity constraints which, consistently with Section 2, may affect some households if they want to buy a dwelling during the first stage of their life cycle. To formalize the role of such constraints, we introduce in the model of Section 3.1 an upper limit $A_i^{\max}$ on the amount which can be spent on buying a household. According to Lemma 2, without such constraint, the optimal amount spent on buying a dwelling would be $\pi^S(d,z)\cdot H^*(d,z;S;y_i) = (1-\gamma^{own})\cdot(y_i - t(d))$.

Such *potential* constraints do not affect the utility of renting a dwelling and thus does not modify the tenure choice of a household which prefers renting to buying (i.e. $U^{**}(own; y_i) < U^{**}(rent; y_i)$ in the model analysed in Section 3.1). By contrast, if household *i* prefers buying ($U^{**}(own; y_i) > U^{**}(rent; y_i)$), three situations may occur, as illustrated on Figure 1 and shown below. In this example, household *i* prefers ownership to tenancy if unconstrained and its optimal renting location is closer to the CBD (distance $d^*(rent; y_i) \equiv d^{R1*}$) than its optimal buying location ($d^*(own; y_i) \equiv d^{O1*} > d^{R1*}$).

[7] Household *i* is indifferent between renting and buying for only one threshold value of income, which happens with zero probability if the distribution of income is continuous.

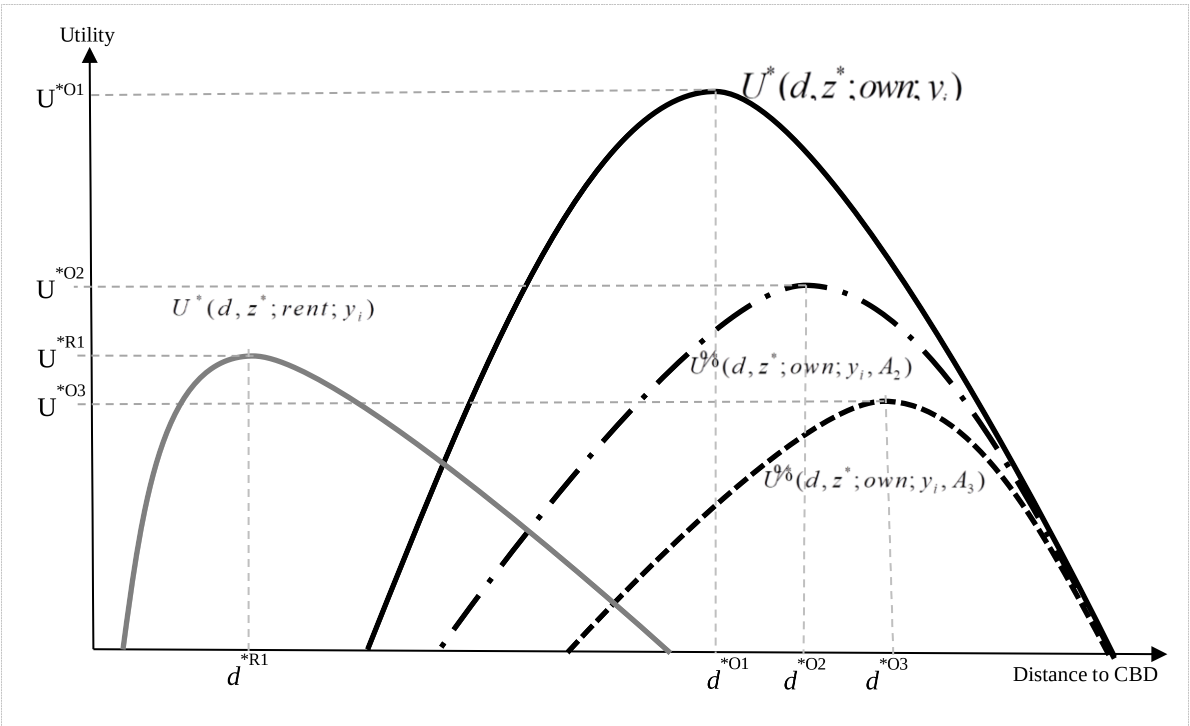
i) If the *potential* constraint is not binding, the tenure choice is the same as in the model without constraint (upper right-hand side curve maximized at d^{O1*} such that $\left(1-\gamma^{own}\right)\cdot\left(y_i - t\left(d^{O1*}\right)\right) < A_i^{\max}$).

ii) If the *potential* constraint is binding and moderate ($A_2 < \left(1-\gamma^{own}\right)\cdot\left(y_i - t\left(d^{O1*}\right)\right)$ on the intermediate right-hand side curve), household *i* is *actually* constrained and buys a cheaper dwelling, located farther away from the CBD: the constrained owning utility $\tilde{U}^*(d, z^*(own); own; y_i, A_2)$ is maximized at $d^{O2*} > d^{O1*}$ such that $\left(1-\gamma^{own}\right)\cdot\left(y_i - t\left(d^{O2*}\right)\right) = A_2$ and $\tilde{U}^*(d^{O2*}, z^*(own); own; y_i, A_2) > U^{**}(rent, y_i)$.

iii) If the *potential* constraint is binding and strong (A_3 very small), household *i* is *actually* constrained and prefers renting than buying its dwelling: the constrained owning utility $\tilde{U}^*(d, z^*(own); own; y_i, A_3)$ is maximized at $d^{O3*} > d^{O1*}$ such that $\left(1-\gamma^{own}\right)\cdot\left(y_i - t\left(d^{O3*}\right)\right) = A_3$ and $\tilde{U}^*(d^{O3*}, z^*(own); own; y_i, A_3) < U^{**}(rent, y_i)$.

Liquidity constraints have no effect on residential segregation in case i); they exacerbate segregation in case ii) and they reduce segregation in case iii). The aggregate effect of liquidity constraints on residential segregation thus depends on the correlation between income y_i and maximum borrowable amount $A_i^{\max}$.

Figure 1: Illustration of the effect of credit constraint on tenure choice and optimal distance



The third step of the program is unchanged for *S=rent*. By contrast, when *S=own*, household *i* maximizes its utility according to:

$$\underset{C,H}{Max}\,U\left(C,H;z;own\right)=\beta^{own}\cdot\psi\left(z\right)+(1-\beta^{own}).\left[\gamma^{own}.\ln C+\left(1-\gamma^{own}\right).\ln H\right]$$
$$\text{subject to } y_i=C+\pi^{own}(d,z)\cdot H+t(d) \qquad . \qquad (6)$$
$$\text{and } H\leq\frac{A_i^{\max}}{\pi^{own}(d,z)}.$$

The constraint is binding iff the optimal expense on housing is larger than $A_i^{\max}$, that is iff:

$$A_i^{\max}<\left(1-\gamma^{own}\right)\cdot\left(y_i-t(d)\right). \qquad (7)$$

Lemma 5

If the maximum borrowable amount $A_i^{\max}$ is larger than $\left(1-\gamma^{own}\right)y_i$, then the potential constraint is never binding. If $A_i^{\max}<\left(1-\gamma^{own}\right)y_i$, then there exists a unique threshold distance $\Delta^*(y_i, A_i^{\max}) \equiv t^{-1}\left(y_i - \frac{A_i^{\max}}{1-\gamma^{own}} \right) \in [0;D]$ such that liquidity constraint is binding iff $d^*(own; y_i) < \Delta^*(y_i, A_i^{\max})$.

The threshold $\Delta^*(y_i, A_i^{\max})$ is increasing in the income level y_i and decreasing in the borrowable amount $A_i^{\max}$; it verifies $0 < \Delta^*(y_i, A_i^{\max}) < t^{-1}(y_i)$.

Proof: If $A_i^{\max} \geq (1-\gamma^{own}) y_i$, then the optimal dwelling expense $H^*(d,z;S;y_i)\pi^S(d,z) = (1-\gamma^S)\cdot(y_i - t(d)) \leq (1-\gamma^S)\cdot y_i \leq A_i^{\max}$, so the potential constraint is not binding. Assume now that $A_i^{\max} < (1-\gamma^{own}) y_i$. Then, given that γ^{own} <1 and using Lemma 1, Eq. (7) can be rewritten:

$$0 < t^{-1}\left(y_i - \frac{A_i^{\max}}{1-\gamma^{own}} \right) < t^{-1}(y_i) \leq D, \tag{8}$$

so there is a unique threshold distance $\Delta^*(y_i, A_i^{\max}) \equiv t^{-1}\left(y_i - \frac{A_i^{\max}}{1-\gamma^{own}} \right) \in [0, D]$ such that the optimal expense at $\Delta^*(y_i, A_i^{\max})$ is : $H^*(\Delta^*(y_i, A_i^{\max}), z; S; y_i)\cdot \pi^S(\Delta^*(y_i, A_i^{\max}), z) = A_i^{\max}$, and the constraint is binding iff $d^*(own; y_i) < \Delta^*(y_i, A_i^{\max})$.

Using Lemma 1 (t^{-1}(.) is increasing), $\Delta^*(y_i, A_i^{\max})$ is increasing in income y_i and decreasing in the borrowable amount $A_i^{\max}$. ■

If the constraint is not binding, then the indirect utility is again given by Eq. (3) (for S = "*own*"). By contrast, when the constraint is binding, the chosen housing expense equals $A_i^{\max}$, the optimal quantities are $\widetilde{H}^*(d,z;own;A_i^{\max}) = \frac{A_i^{\max}}{\pi^{own}(d,z)}$ and

$\tilde{C}^*(d; own; y_i, A_i^{\max}) = y_i - A_i^{\max} - t(d)$ and the program results in the indirect utility of household i with income y_i conditional on distance d, on local amenities z and on homeownership, when liquidity constraints are binding:

$$\begin{aligned}\tilde{U}^*(d, z; own; y_i, A_i^{\max}) &= \beta^{own} \cdot \psi(z) + (1-\beta^{own}) \cdot \gamma^{own} \cdot \ln\left(y_i - t(d) - A_i^{\max}\right) \\ &\quad - (1-\beta^{own}) \cdot (1-\gamma^{own}) \cdot \ln \pi^{own}(d, z) + (1-\beta^{own}) \cdot (1-\gamma^{own}) \cdot \ln A_i^{\max}\end{aligned} \quad (9)$$

To sum up, under liquidity constraints, the indirect utility $\tilde{U}^*$ of household i with income y_i and maximum borrowable amount $A_i^{\max}$ conditional on distance d, on local amenities z and on homeownership is:

$$\begin{aligned}\tilde{U}^*(d, z; own; y_i, A_i^{\max}) &= U\left(y_i - A_i^{\max} - t(d), \frac{A_i^{\max}}{\pi^{own}(d, z)}, z, own\right) && \text{if} \quad d < \Delta^*\left(y_i, A_i^{\max}\right) \\ &= U^*(d, z; own; y_i) && \text{if} \quad d \geq \Delta^*\left(y_i, A_i^{\max}\right)\end{aligned} \quad (10)$$

Proposition 1:

The (owning) utility of Household i is not affected by liquidity constraints in locations which are far enough from the CBD, that is, if $d > \Delta^*\left(y_i, A_i^{\max}\right)$. By contrast, if $d < \Delta^*\left(y_i, A_i^{\max}\right)$, then liquidity constraints induce a loss of utility equal to $U^*(d, z; own; y_i) - \tilde{U}^*(d, z; own; y_i, A_i^{\max})$, which is a positive and decreasing function of d.

See proof in Appendix 8.2. ■

Under binding liquidity constraint, the second step of the program is the same as in Section 3.1 except that $U^*(d, z; own; y_i)$ is now replaced by $\tilde{U}^*(d, z; own; y_i, A_i^{\max})$. The optimal distance and quantity of local amenities, now denoted by $\tilde{d}^*\left(own; y_i, A_i^{\max}\right)$ and $\tilde{z}^*(own) = z^*(own)$ maximize the indirect constrained utility $\tilde{U}^*(d, z; own; y_i, A_i^{\max})$ conditional on owning, on income y_i and on maximal amount $A_i^{\max}$. These values are unique (see proof of Proposition 1 in Appendix 8.2). The resulting "further-indirect" constrained utility of household i with income y_i conditional on tenure status S is:

$$\tilde{U}^{**}(own; y_i, A_i^{\max}) \equiv \tilde{U}^*(\tilde{d}^*(own; y_i A_i^{\max}), \tilde{z}^*(own); own; y_i, A_i^{\max}).$$

Obviously, for constrained households, $\tilde{U}^{**}(own; y_i, A_i^{\max}) < U^{**}(own; y_i))$. The proof of this result can easily be derived from Proposition 1. The utility $\tilde{U}^*(\tilde{d}^*(own; y_i A_i^{\max}), \tilde{z}^*(own); own; y_i, A_i^{\max})$ is lower than the utility $U^*(\tilde{d}^*(own; y_i), \tilde{z}^*(own); own; y_i)$ which is itself lower than $U^*(d^*(own; y_i), z^*(own); own; y_i)$ since $d^*(own; y_i)$ and $z^*(own)$ maximize the indirect utility $U^*(.,.; own, y_i)$.

Proposition 2: When the optimal distance to the CBD without liquidity constraints is larger than the threshold distance, that is $d^*(own; y_i) > \Delta^*(y_i, A_i^{\max})$, the optimal distance is the same with and without constraint: $\tilde{d}^*(own; y_i, A_i^{\max}) = d^*(own; y_i)$.

By contrast, when $d^*(own; y_i) < \Delta^*(y_i, A_i^{\max})$, the distance $\tilde{d}^*(own; y_i, A_i^{\max})$ which is optimal under liquidity constraints is comprised between the optimal distance without constraints and the threshold distance: $d^*(own; y_i) < \tilde{d}^*(own; y_i, A_i^{\max}) < \Delta^*(y_i, A_i^{\max})$.

See proofs in Appendix 8.2. ■

Proposition 3: When $d^*(own; y_i) < \Delta^*(y_i, A_i^{\max})$, both the optimal distance $\tilde{d}^*(own; y_i, A_i^{\max})$ and the threshold distance $\Delta^*(y_i, A_i^{\max})$ are decreasing functions of the maximal borrowable amount $A_i^{\max}$.

See proofs in Appendix 8.2. ■

According to Proposition 2, constrained households who buy a dwelling in spite of liquidity constraints are more likely to locate farther away from the CBD than unconstrained ones. Proposition 3 assesses that they locate even farther away when they are more constrained.

Proposition 4: The further indirect utility $\tilde{U}^{**}(own; y_i, A_i^{\max}) \equiv \tilde{U}^{*}(\tilde{d}^{*}(own; y_i A_i^{\max}), \tilde{z}^{*}(own); own; y_i, A_i^{\max})$ is a strictly increasing function of $A_i^{\max}$ when $A_i^{\max} < (1-\gamma^{own}) \cdot (y_i - t(d^{*}(own; y_i)))$.

See proofs in Appendix 8.2. ■

Proposition 4 implies that, when a household prefers buying to renting without constraint, that is $\tilde{U}^{**}(own; y_i, A_i^{\max}) > U^{**}(rent; y_i)$, there exists a unique value $\tilde{A}_i$ which equalizes the further-indirect utility of renting and the further-indirect utility of owning : $U^{**}(rent; y_i) = \tilde{U}^{**}(own; y_i, \tilde{A}_i)$.

It follows that the maximal borrowable amount $A_i^{\max}$ is a crucial determinant of tenure choice, since it has a strong effect on the difference $\tilde{U}^{**}(own; y_i, A_i^{\max}) - U^{**}(rent; y_i)$. Combining the previous propositions leads to the following theorem.

Theorem:

If Household *i* prefers renting than buying, when unconstrained, that is $U^{**}(own; y_i) - U^{**}(rent; y_i) \le 0$, then the tenure choice of Household *i* is not modified by liquidity constraints.

If Household *i* prefers buying to renting without potential constraint, that is $U^{**}(own; y_i) - U^{**}(rent; y_i) > 0$, then its tenure choice is determined by the sign of

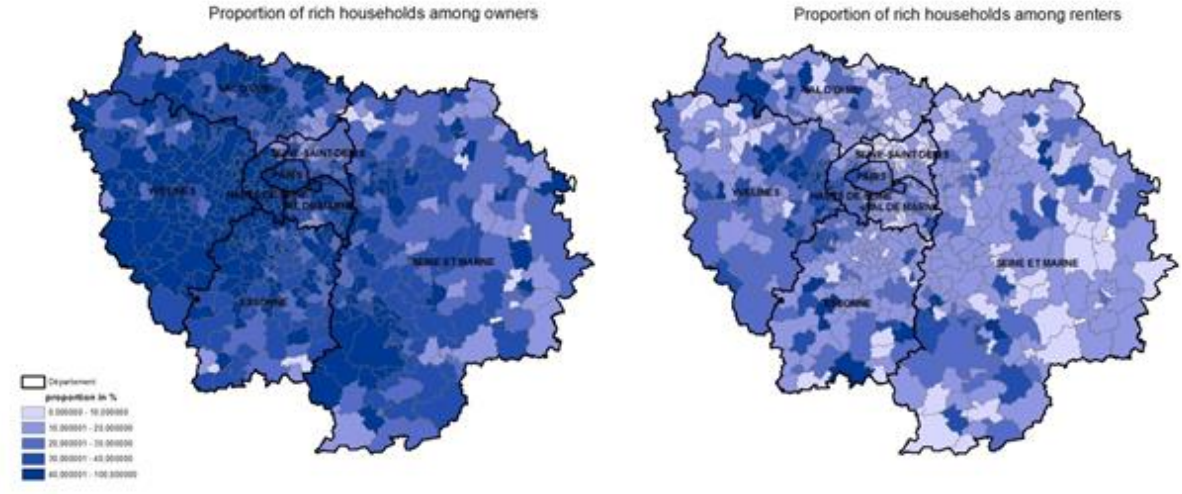
$\tilde{U}^{**}(own; y_i, A_i^{\max}) - U^{**}(rent; y_i)$ which depends on the level of the maximal borrowable amount $A_i^{\max}$.

- If $d^*(own; y_i) > \Delta^*(y_i, A_i^{\max})$, or equivalently $A_i^{\max} > (1-\gamma^{own})\cdot(y_i - t(d^*(own; y_i)))$, the liquidity constraint is not binding so that $\tilde{U}^{**}(own; y_i, A_i^{\max}) = U^{**}(own; y_i) > U^{**}(rent; y_i)$ and the tenure choice is the same as in the model without constraints.
- If $\tilde{A}_i < A_i^{\max} < (1-\gamma^{own})\cdot(y_i - t(d^*(own; y_i)))$, then $U^{**}(own; y_i) > \tilde{U}^{**}(own; y_i, A_i^{\max}) > U^{**}(rent; y_i)$, so Household *i* will buy a less expensive dwelling, located farther away from the CBD.
- If $A_i^{\max} < \tilde{A}_i < (1-\gamma^{own})\cdot(y_i - t(d))$ then $U^{**}(own; y_i) > U^{**}(rent; y_i) > \tilde{U}^{**}(own; y_i, A_i^{\max})$, so Household *i* will decide to rent until it accumulates enough capital to increase $A_i^{\max}$ to reach the optimal level of housing expenses $H^*(d^*(own, y_i), z^*(own); own; y_i)\pi^{own}(d^*(own, y_i), z^*(own)))$.

If the effect of distance to the CBD is less important on renting prices than on selling prices, that is if $-\frac{\partial \pi^{rent}(d,z)}{\partial d} < -\frac{\partial \pi^{own}(d,z)}{\partial d}$, then the optimal location of the rented dwelling is closer to the CBD than the optimal location of the dwelling which would have been bought in the two former cases (for a larger $A_i^{\max}$).

If the borrowable amount $A_i^{\max}$ increases with income, households who would prefer ownership when unconstrained might then sort spatially by income, with the richest households locating close to the CBD and the poorest households locating farther away. Income segregation might then be more severe among owners than among renters.

Figure 2: Proportion of rich households among those who move in 1998, by pseudo-commune and tenure status



Source: French Census of 1999

This is consistent with Figure 2, which shows that rich owners are concentrated in Paris and its Western suburbs. By contrast, the concentration of rich households by pseudo-commune is less stringent among renters.

3.3. Extension to heterogeneous preferences and discrete location choice with unconstrained choice set

We now extend the model in several directions in order to make it more realistic.

First, households have heterogeneous preferences. This means that the parameters β^S and γ^S may depend on household characteristics such as income, household head age and nationality, or household composition (see Table 1 to Table 3). In addition, depending on their characteristics, households may value differently the various components of local amenities *z*. For example, households with children are more sensitive than singles to parks and other green spaces. This implies that the universal function value function $\psi(z)$, which implicitly assumes that all households agree on the way the local amenities can be aggregated in a single unidimensional function $\psi(z)$, has to be replaced with a household-specific function value function $\psi_i(z)$. As a consequence, it will no more be possible to compute the effect of z on equilibrium prices.

Second, distance to CBD is a poor proxy for commuting costs, which also depend on the structure of the (public and private) transportation network.

As a result, it seems more realistic to replace the continuous variables d and $\psi(z)$ by a discrete list of potential locations, namely the different "communes".

Table 1: Distribution of location, tenure status and dwelling type for households which moved in 1998

	Paris	Inner Ring	Outer Ring	Rent flat	Own Flat	Rent House	Own House
Total	**27.58%**	**36.35%**	**36.07%**	**70.79%**	**13.52%**	**5.21%**	**10.48%**
Single	38.23%	34.66%	27.11%	81.08%	13.99%	2.74%	2.20%
couple w/o children	27.13%	35.97%	36.90%	69.53%	15.08%	5.28%	10.10%
couple with children	15.64%	38.61%	45.75%	59.92%	11.74%	8.00%	20.35%
Young	28.92%	36.09%	34.99%	79.07%	10.03%	4.49%	6.41%
middle-age	25.12%	36.99%	37.89%	60.73%	15.79%	6.82%	16.66%
Old	28.54%	35.61%	35.85%	58.30%	25.80%	3.63%	12.27%
Poor	26.90%	39.26%	33.84%	83.92%	8.84%	3.86%	3.38%
medium income	25.99%	35.52%	38.48%	69.93%	13.09%	5.71%	11.28%
Rich	31.03%	33.49%	35.48%	53.35%	20.88%	6.35%	19.43%
French	27.24%	35.16%	37.60%	69.10%	14.46%	5.35%	11.09%
Foreign	29.53%	43.10%	27.37%	80.38%	8.19%	4.37%	7.06%

Table 2: Distribution of location, tenure status and dwelling type for all households

	Paris	Inner Ring	Outer Ring	Rent flat	Own Flat	Rent House	Own House
Total	**23.24%**	**37.04%**	**39.72%**	**49.45%**	**21.98%**	**3.45%**	**25.11%**
Single	35.31%	36.77%	27.92%	60.40%	26.47%	2.11%	11.02%
couple w/o children	20.12%	36.73%	43.15%	40.06%	23.17%	3.04%	33.72%
couple with children	14.15%	37.65%	48.21%	48.02%	16.16%	5.26%	30.56%
young	26.63%	36.96%	36.41%	73.11%	13.90%	4.14%	8.84%
middle-age	20.41%	36.73%	42.87%	46.75%	20.18%	4.22%	28.85%
old	24.47%	37.49%	38.04%	36.61%	29.79%	2.02%	31.58%
poor	24.26%	40.86%	34.88%	63.41%	18.32%	2.89%	15.38%
medium income	20.72%	37.16%	42.12%	51.31%	19.81%	3.89%	24.98%
rich	25.13%	33.10%	41.77%	33.34%	28.17%	3.51%	34.99%
French	22.98%	36.12%	40.90%	46.93%	23.16%	3.42%	26.49%
foreign	25.17%	43.83%	31.00%	68.08%	13.25%	3.75%	14.92%

Third, as mentioned in Section 2, there are imperfections in the financial (and real estate) markets, so that renting and buying prices are not perfectly correlated. Furthermore, prices (per square meter) also vary significantly by dwelling type, in the sense that the prices of flats and houses are not well correlated.

Table 3: distribution of households by tenure status, dwelling type and location

		Rent flat	Own Flat	Rent House	Own House
All households	Total	49.45%	21.98%	3.45%	25.11%
	Paris	66.49%	32.46%	0.45%	0.60%
	Inner Ring	55.61%	23.55%	2.50%	18.33%
	Outer Ring	33.74%	14.39%	6.10%	45.78%
Movers	Total of Movers	70.79%	13.52%	5.21%	10.48%
	Paris	82.55%	16.60%	0.55%	0.30%
	Inner Ring	75.27%	14.26%	3.35%	7.12%
	Outer Ring	57.30%	10.41%	10.63%	21.66%

Households are again assumed to choose their tenure status *S* and their dwelling type *T* in the first step of the program. In second step of the program, households choose location *j* in a discrete set. Location determines distance *d* to CBD and local amenities *z*. Finally, the quantities of housing and other goods are chosen in the third step of the program. Liquidity constraints are neglected in this section, so that the third step of the program is the same as in Section 3.1, with some obvious change in notation.

Location *j* is characterized by a series of tenure-specific prices π_j^{ST}. Eq. (3) is then replaced with the indirect utility for household *i* of choosing location *j*, conditional on tenure *S* and dwelling type *T:*

$$U_{ij}^{ST*} = \alpha_{1i}^{ST} + Z_j.\alpha_{2i}^{ST} + \alpha_{3i}^{ST} \ln \tilde{y}_{ij} - \alpha_{4i}^{ST} \ln \pi_j^{ST} + \varepsilon_{ij}^{ST} \quad (11)$$

where $\alpha_{ki}^{ST}, k = 1,...,4$ are household-specific preference parameters.

Whereas the rental price of a housing unit is observed, the user cost of a purchased housing unit is not and has to be proxied by the purchasing price. Consequently, in Eq. (3), the use cost π_j^S is replaced by the rental price when the dwelling is for rent and by the selling price when for sale. The residual terms ε_{ij}^{ST} reflects unobserved heterogeneity in the preferences and the valuation of unobserved local amenities. By contrast with the theoretical model, Z_j now represents a multidimensional bundle of observed local amenities which can be valued differently by different households. Similarly to capture the effect of commuting cost on disposable income, a vector D_j of measures of accessibility to location *j* is interacted with log-income through the term $\ln(y_i).D_j$ which replace $\ln \tilde{y}_{ij}$ in Eq. (6).

One may assume that each household chooses simultaneously the tenure *S*, the dwelling type *T* and the location *j*, that is, the alternative (*S,T,j*) which provides it with the highest utility. In this case, the probability that alternative (*S,T,j*) is chosen by household *i* is given by:

$$P_i(S,T,j) = \Pr(U_{ij}^{ST*} = \underset{S',T',j'}{Max} U_{ij'}^{S'T'*}). \tag{12}$$

Under the assumption that the residuals are i.i.d. with a Gumbel distribution, the probability that alternative (*S,T,j*) is chosen by household *i* can then be written:

$$P_i(S,T,j) = \frac{\exp\left(V_{ij}^{ST}\right)}{\sum\limits_{S'=\text{own,rent};T'=\text{house,flat};j'\in J} \exp\left(V_{ij'}^{S'T'}\right)}, \tag{13}$$

where *J* denotes the set of locations *j*. The parameters $\alpha_{ki}^{ST}, k = 1,...,4$ measuring marginal utilities in the resulting multinomial logit model can then be estimated using standard maximum likelihood techniques.

The drawback of such a joint model is that it relies on the Independence of Irrelevant Alternatives (IIA) hypothesis which stipulates that the choice between two alternatives is not affected by the availability of other alternatives, not by the utility provided by the alternatives. This hypothesis does not seem plausible when households choose both tenure status *S*, dwelling type *T* and location *j*. It seems more relevant to assume that when the alternative (*S,T,j*) preferred by household *i* is no more available or becomes less attractive, then household *i* will primarily tend to select a different location *j'* but will tend to still select the same tenure status *S* and dwelling type *T*. This tendency is taken into account by estimating a nested logit model (NL) rather than a multinomial logit model (MNL). More details and justifications are provided in Inoa, Picard and de Palma (2015).

Renting a dwelling is often a temporary alternative before buying one, so that the observable and unobservable characteristics that determine the rent of a dwelling might be different from those that determine the decision to buy a similar dwelling. In particular, the expected future sale price of a dwelling, which is part of the unobservable determinants of its purchase, is irrelevant when renting. Moreover, houses and flats differ in their average size, use cost (lower /no condominium fees but larger real estate taxation and maintenance cost for houses) so that some unobservable determinants might be specific to the dwelling type.

The effect of observed household characteristics on the generic preference for a given tenure status and dwelling type (whatever its location) can be imbedded in the parameter α_{1i}^{ST}, both in the MNL and in the NL model. The fact that the local price in location *j* is specific to tenure status and dwelling type is imbedded in the price variable $\ln \pi_j^{ST}$, and the fact that price elasticity may depend on tenure status and dwelling type is imbedded in the coefficients α_{4i}^{ST} (indexed by *i* to reflect the fact that it may depend on observable household characteristics). Similarly, the fact that the willingness to pay for a better accessibility and for local amenities may depend on tenure status, on dwelling type and on observable household characteristics can be imbedded in the coefficients α_{3i}^{ST} and α_{2i}^{ST}, both in the MNL and in the NL model.

To account for the potential correlation between the error terms by dwelling type and tenure status, a type-tenure-specific error term ε_{iT}^{S} and a tenure-specific error term ε_{iS} are added to the equation. They correspond to unobserved heterogeneity of preferences for dwelling type and tenure status. In addition to the type-tenure-specific term α_{i1}^{ST}, a tenure-specific term α_{i1}^{S}, is also included in the indirect utility:

$$U_{ij}^{ST*} = \alpha_{1i}^{ST} + \alpha_{1i}^{S} + Z_j.\alpha_{2i}^{ST} + \alpha_{3i}^{ST} \ln y_i.D_j - \alpha_{4i}^{ST} \ln \pi_j^{ST} + \varepsilon_{ij}^{ST} + \varepsilon_{iT}^{S} + \varepsilon_{iS} \tag{14}$$

A nested logit is then estimated for the choice of location, dwelling type and tenure status (Inoa et al. 2013 detail the interpretation of the nested logit).

The deterministic utility of Equation (44) can be split into three additive deterministic utilities:

$$U_{ij}^{ST*} \quad = V_{ij}^{ST} + V_{iT}^{S} + V_{iS} + \varepsilon_{ij}^{ST} + \varepsilon_{iT}^{S} + \varepsilon_{iS} \tag{15}$$

with

$$\begin{aligned} V_{ij}^{ST} &= Z_j.\alpha_{2i}^{ST} + \alpha_{3i}^{ST} \ln y_i.D_j - \alpha_{4i}^{ST} \ln \pi_j^{ST} \\ V_{iT}^{S} &= \alpha_{i1}^{ST} \\ V_{iS} &= a_{i1}^{S} \end{aligned} \tag{16}$$

V_{ij}^{ST} denotes the deterministic utility provided to household *i* by location *j* conditionally on dwelling type *T* and on tenure status *S*, V_{iT}^{S} denotes the deterministic utility provided by dwelling type *T* conditionally on tenure status *S* (whatever location *j*), and V_{iS} denotes the deterministic utility provided by the tenure status S (whatever location *j* and dwelling type *T*).

Under the standard assumptions of a nested logit model, the probability that household *i* chooses location *j* conditionally on dwelling type *T* and tenure status *S* is given by the usual Multinomial Logit formula:

$$P_i(j|T,S) = \frac{\exp(\mu_{ST} \cdot V_{ij}^{ST})}{\sum_{k \in K(S,T)} \exp(\mu_{ST} \cdot V_{ik}^{ST})} \tag{17}$$

The probability that household *i* chooses a house conditionally on tenure status *S* according to the logistic formula:

$$P_i(T = \text{House}\,|S) = \frac{\exp\left[\lambda_S \cdot \left(V^S_{i,House} + \frac{1}{\mu_{S,House}} .I^S_{i,House}\right)\right]}{\sum_{t\in\{\text{House,Flat}\}} \exp\left[\lambda_S \cdot \left(V^S_{it} + \frac{1}{\mu_{St}} .I^S_{it}\right)\right]} \tag{18}$$

where $I^S_{iT} = \ln\left(\sum_{k\in K(S,T)} \exp(\mu_{ST} \cdot V^{ST}_{it})\right)$ is called the inclusive value of the nest $K(S,T)$ and corresponds to the maximum utility that household *i* can expect conditional on choosing tenure *S* and dwelling type *T*.

Finally, the probability that household *i* chooses a house conditionally on tenure status S:

$$P_i(S = \text{Own}) = \frac{\exp\left[\varphi \cdot (V_{i,Own} + \frac{\varphi}{\lambda_{Own}} .I_{iOwn})\right]}{\sum_{s\in\{\text{Own,Rent}\}} \exp\left[\varphi \cdot \left(V_{is} + \frac{\varphi}{\lambda_S} .I_{is}\right)\right]} \tag{19}$$

where $I^S_{iT} = \ln\left(\sum_{k\in K(S,T)} \exp\left(\lambda_S \cdot V^S_{iT} + \frac{\lambda_S}{\mu_{ST}} \cdot I^S_{iT}\right)\right)$ is the inclusive value of tenure status *S* and can be considered as the maximum utility household *i* can expect conditional on choosing tenure *S*.

The probability for household *i* to choose a dwelling *j* of type *T* with tenure status *S* is the product of the three probabilities defined by Eq. (7) to Eq. (9). To estimate the parameters of those equations, one of the scale parameters must be normalized: we choose to normalize that of the total disturbance: $\varphi = 1$.

3.4. Extension to constrained choice sets

The coefficients estimated from the nested logit might reflect not only household marginal utilities, but also the liquidity constraints they may face. Indeed, as shown by our structural models, the maximum borrowable value $A^{\max}_i$ affects the tenure and location choices when liquidity constraint on household is binding: on one hand, the marginal disutility of the distance decreases; on the other hand, the utility of ownership compared to tenancy decreases.

Moreover, this constraint is likely to modify the choice set faced by households:

i) When the optimal housing consumption that household can afford to buy in location *j* is lower than the minimal buyable housing service in *j* then dwellings for sale in *j* disappears from *i*'s choice set

ii) When $A_i^{\max}$ =0 then *i* can't buy any dwelling and all the dwellings for sale disappear from its choice set

Liquidity constraints are then likely to bias the estimation of the marginal utilities by implicitly reducing each household *i*'s choice set of buyable alternatives to the dwellings whose value is less than $A_i^{\max}$.

Constraints on the choice set can be taken into account by distinguishing several choice sets instead of considering only one. Hence, the obtained model is a discrete choice model with latent (or endogenous) choice sets. In such a model, the probability that a household choose a dwelling is not only the probability that this dwelling provides it with the highest utility but also depends on the probability that this dwelling is available to this household.

Rancière and Ouazad (2015) estimate a location choice model in which the choice set depends on mortgage approval probability, but only consider dwellings for sale. Thus, unlike in the present paper, they implicitly assume that liquidity constraints do not affect tenure choice but only location choice.

In this section, we consider the particular case where some households have a maximum borrowable amount equal to zero and extend the previously described nested logit to account for this case. We assume that some households are constrained to rent their dwelling and, consequently, face a choice set which contains only alternatives to rent. The probability to face this choice set is modeled by a binary logit and integrated to the previous nested logit.

The previous assumptions about the choice between renting/buying and house/flat still hold so that the location choice among the unconstrained choice set can be modeled by the same 3-levels nested logit as before. Eq. (7) to Eq. (9) still hold for unconstrained households. On the opposite, the location choice for constrained households is restricted to the estimation of the two lower levels: the choices of the commune and the type of dwelling. The parameters of these latter choices are assumed to be the same whether the household is constrained or not.

The propensity to be constrained is not observed but inferred from the model by modifying the formula of the probability to buy in the nested logit and maximizing the corresponding likelihood maximization. The probabilities to choose to buy a house become:

$$P_i(S = \text{own}) = P_i(S = \text{own} \mid \text{constraint} = 0) \times P_i(\text{constraint} = 0) \tag{20}$$

with

$$P_i(S = \text{own} \mid \text{constraint} = 0) = \frac{\exp(V_{i\text{"own"}} + \frac{1}{\lambda_S}.I_{i\text{"own"}})}{\sum_{s \in \{\text{own,rent}\}} \exp\left[\lambda_s \cdot (V_{is} + \frac{1}{\lambda_{sT}}.I_{is})\right]} \tag{21}$$

and

$$P_i(\text{constraint} = 0) = \frac{1}{1 + \exp(X_i\delta)} \tag{22}$$

Distinguishing variables' effect on constraints from their effects on choice is made possible by our definition of latent choice: a variable which influences the household constraint will determine which latent choice set this household will face, while a variable which influences its choice will affect its utility. A same variable can affect both constraint and utility, this might be the case of income for instance. The case without constraints is represented in Figure 3 and the case with constraints on Figure 4.

Figure 3: Location choice model without constraints

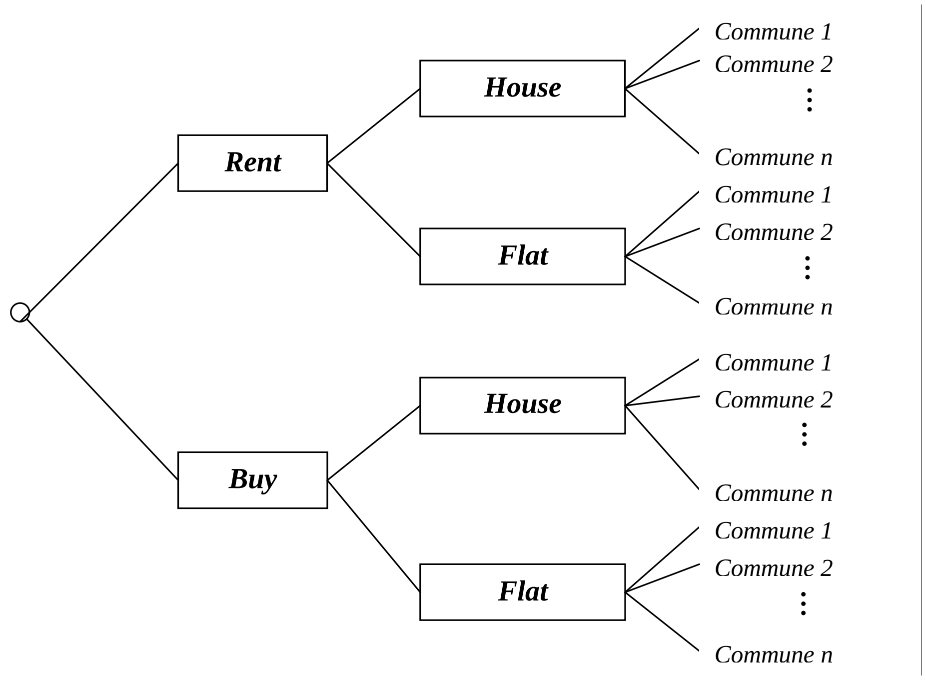
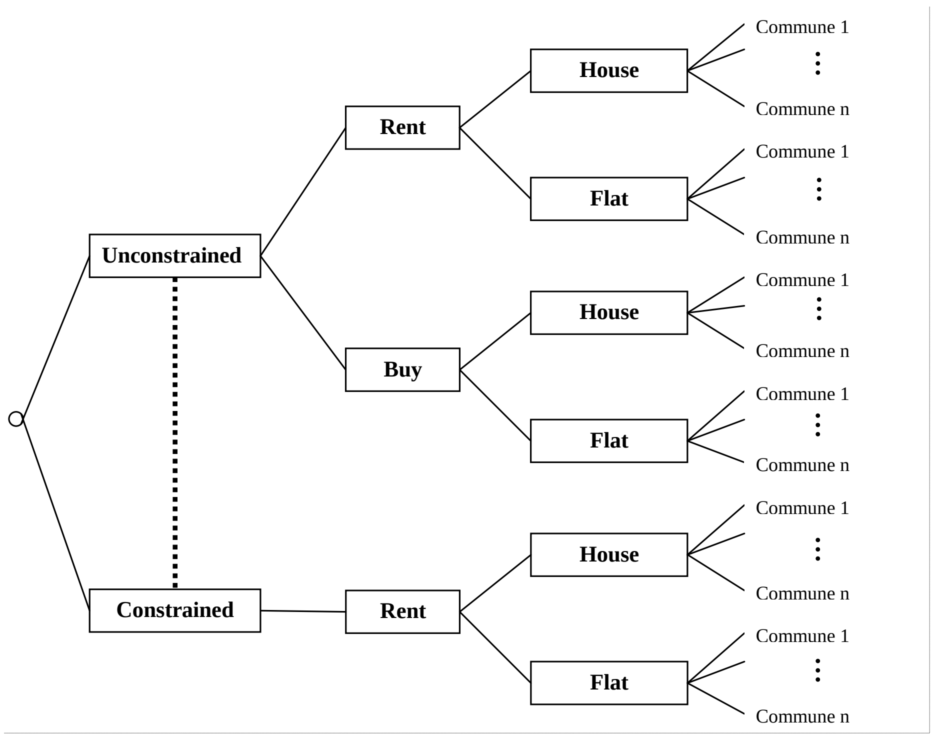


Figure 4: Location choice model with constraints



4. Results

We applied our models to the Paris Region which includes Paris city and its suburbs. The city of Paris contains about 2 million inhabitants for a total of 11 million inhabitants in the whole region. The total number of jobs is 5.1 million. The region spreads over 12,000 sq. Km, which represents 2% of the surface of France, but 19% of the population and 22% of the jobs of the country. There are 3 levels of administrative boundaries in Ile-de-France: 1 "région", 8 "départements" (counties) and 1300 "cities" (communes). In addition, we consider the 3 counties around Paris as close suburb or "inner ring" and the 4 counties far away from Paris as far away suburb or "outer ring".

We use household exhaustive data from the 1999 French Census for Ile-de-France, which represents about 5 million households. In order to study location choice, we restrict our sample to households who moved in 1998. We exclude households freely hosted and households whose head is a student, so that we obtain a sample of 521,132 households. This database contains rich information on households such as household size, number of children, household head gender, occupation, educational attainment, previous county if residence, and so on. Household (per capita) income is not observed directly, but we computed it as a function of household characteristics, with a very good fit.

Each model is estimated following two steps. The first step is common to both model and consists in estimating a discrete location choice model for each of the four nests (T,S) from the sample of households who actually choose this nest. Each nest is constituted of 725 alternatives corresponding the 725 pseudo-communes[8] of the Paris region. In order to form the inclusive value of each nest (T,S) used in Eq. (8), we compute for each household and each nest the utilities of the 725 alternatives from the coefficients obtained in the first step. By summing up the exponential of utilities and computing the logarithm of this sum, we obtained 4 inclusive values (one by nest) that we use to estimate the parameters in the second step.

[8] Pseudo-communes are aggregation of communes which account for their size and importance in the Paris region. Pseudo-communes in Paris city correspond to its partition in arrondissements. In the Inner Ring a pseudo-commune correspond to a commune. By contrast, in the Outer Ring, small far away communes with a weak number of inhabitants are aggregated into bigger pseudo-communes.

The second step consists in estimating simultaneously the equations of dwelling type choice and tenure choice. Eq. (8) and Eq. (9) are then estimated to determine the parameters of the model without constraints whereas Eq. (8), Eq. (11) and Eq. (12) are estimated for the model with constraints. The inclusive values represent the maximum utility a household can obtain from the set of alternatives contained in nest and constitute additional explanatory variables of the dwelling type choice (Eq. (8)). Other determinants consist in variables which are likely to affect the desired size of dwelling and the income path, such as income per capita, family composition and stability of household head employment.

4.1. Location choice

The estimation of the first step requires not only data on households who moved in 1998 but also some descriptive statistics on local amenities. Based on the aggregation of some variables of the Census by pseudo-commune, we computed local characteristics such as the proportions of poor households, of rich households, of households with one member, with 2 members, etc.

The census data was also used to determine the demands for location in each pseudo-commune by type of dwelling and tenure mode in 1998. Combining those demands with the number of vacant dwellings in each pseudo-commune then allow us to measure the supply of dwellings by pseudo-commune and by type of dwelling. Whereas the supply of dwellings in Paris and the close suburbs is mainly constituted of flats, the supply in the further suburbs is more balanced between houses and flats, with a particularly high proportion of flats at the East of the Paris region (see figure 14 in appendix). This reinforced the usual assumption in the canonical model that more land can be consumed when locating far from the central business districts.

Local dwelling prices (per square meter) are edited by the *Editions Callon* in their “yearly guide of venal values” at the commune level, separately available for renting and for buying, for flats and for houses. Unfortunately, this guide concerns only communes with more than

5000 inhabitants, which represent only 287 communes and Parisian *arrondissements* in 1998, the price per square meter in the other communes being missing so that prices have to be predicted from an hedonic price model (Appendix 8.3) and aggregated by pseudo-commune. Dantan (2013) details and interprets hedonic price equations in the Paris region.

We then use the discrete choice model described in Eq. (7) to explain the location of the households who moved in 1998. Since our data contains no information about specific dwelling price but only on average price per square meter at the pseudo-commune level, we use individual-level data from the 1999 census to explain household location choice among the 725 pseudo-communes in Paris Region.

Since we consider the choice between pseudo-communes rather than the choice between specific dwellings, the alternatives considered in the model corresponds to a set of statistically identical actual alternatives. To take into account this aggregation of statistically identical alternatives, the logarithm of the number N of dwellings in the pseudo-commune is added to the list of explanatory variables (Mc Fadden 1978). See de Palma *et al.*, 2005 for details.

The large number of alternatives (725) raises some computation difficulties in the estimation procedure, namely computation burden and probabilities very close to 0. To avoid such difficulties, we rely on sampling within each nest of our model (see Ben-Akiva and Lerman 1985). Since the IIA assumption holds between the alternatives in the same nest (but not between alternatives in different nests), consistent estimates of the preference parameters can be obtained using random sampling of pseudo-communes at the lower level of our model. We consider 16 pseudo-communes in each household choice set in our empirical application).

We then operate a stratified sampling: 6 pseudo-communes are randomly chosen in the district of the household previous location, 6 in the adjacent district and 4 in the non-adjacent district. The drawing is such that the chosen alternative is part of the sample. To account for the sampling, we add a corrective term in the utility equation (that we remove when computing the inclusive values). Results are presented in Table 4.

As expected, in all samples, the price has a negative effect on location probability. This (absolute value of) price elasticity is reduced in richer households, as illustrated by the positive effect of the interaction between the price and the centred log income per capita. The sign of interaction terms between price and age of household head shows that the (absolute value of) price elasticity increases with the age of household head, potentially because the oldest households are more reluctant to contract mortgage to pay higher prices. By contrast, households with children are less sensitive to prices than households without children, maybe because they are more concerned with bequest motives than non-parents and parents whose children have already left home.

The very positive effect of the dummy "Same County" - which indicates whether a pseudo-commune is located in the same county as the household previous location - indicates that households have a strong tendency to move close to their previous location. This tendency can be explained by households' reluctance to go far from the place where they might have their habits, relatives or friends (Liaw and Frey 2003) but also by large mobility costs. The existence of such costs is consistent with the fact that the effect of "same county" decreases when the income per capita increases. The reluctance to leave the previous county increases as the household head becomes older, reflecting the larger geographical mobility of households at the beginning of their lifecycle.

The accommodation tax rate decreases the probability to choose a house and this tax is all the more disincentive than the household is poor. It has no effect on the location choice when moving to a flat.

The number of subway and railway stations has a negative or insignificant marginal utility which might reflect the negative externalities induced by such infrastructures (noise, crowd…). However this negative effect is attenuated as income per capita increases, probably because the richest households value more accessibility or can afford paying to attenuate the negative externalities. The valuation of accessibility by heterosexual couples depends on each spouse's value of time and bargaining power (Picard *et al.* 2013) so that the difference in valuations of subway and railway stations might reflect differences in intra-household bargaining process.

Airport noise is also found to have a negative effect on the probability to locate in a house but a less significant or even positive effect on the probability to locate in a flat. This can be explained by the fact that house-occupants tend to suffer more from airport vicinity since they are more likely to have a garden in which they are annoyed by the noise. This negative externality overpasses the accessibility benefit of living close to an airport for house-occupants but not for flat-occupants.

The likelihood of benefiting from their own garden might also explain childless house-occupants' reluctance to live in commune where a large fraction of the surface is occupied by public gardens, woods and lakes. By contrast, this variable has an insignificant or positive effect location choice of households who look for a flat. The marginal utility of these green areas increases with the number of children, except the effect of woods when renting a flat.

The population density has a positive effect on the probability to locate in flat but a negative effect on the probability to locate in a house. This confirms house-occupants' reluctance for the externalities of the density.

The provision of public services (measured by the share of the surface devoted to hospitals, infrastructures, sport areas, administration…) has ambiguous effects on utility which might reflect a mix of attraction for these services and of reluctance for paying for them (through local taxation).

Table 4: Location choice

	rent		buy	
	flat	house	flat	house
Seine-et-Marne (77)	0.295***	0.455***	0.424***	2.096***
Yvelines (78)	0.302***	0.446***	0.826***	1.915***
Essonne (91)	0.328***	0.374***	0.565***	2.018***
Hauts-de-Seine (92)	0.090	0.477***	0.186***	1.345***
Seine-Saint-Denis (93)	-0.223***	2.170***	-0.180***	2.760***
Val de Marne (94)	-0.036	1.655***	0.279***	2.328***
Val d'Oise (95)	0.257***	0.449***	0.719***	2.006***
Corrective term	-0.873***	-1.086***	-0.835***	-0.913***
Log(N)	0.958***	0.787***	1.069***	0.811***
Log(price)	-0.699***	-0.073	-0.785***	-1.419***
Log(price)*(age-20)/10	-0.390***	-0.354***	-0.093***	0.012
Log(price)* centered log income	3.762***	0.661***	4.049***	4.078***
Same district	2.125***	2.748***	2.007***	2.452***
Same district *centered log income	-0.783***	-0.521***	-0.715***	-0.428***
Same district*(age-20)/10	0.138***	0.120***	0.149***	0.194***
Accommodation tax rate	-0.003***	-0.017***	0.003	-0.017***
Accommodation tax rate *centered log income	-0.030***	0.022***	-0.006	0.053***
Number railway stations	0.009***	-0.006	-0.008***	-0.048***
Number railway stations*centered log income	0.022***	0.065***	0.024***	0.121***
Number subway stations	0.001	-0.006	-0.016***	-0.008
Number subway stations*centered log income	0.028***	-0.026***	0.003	0.022***
Airport noise	-0.024**	-0.090***	0.181***	-0.020
Density	0.000	-0.022***	0.012***	-0.021***
Fraction of surface with: forest	0.064***	-0.173***	0.216***	-0.020
forest*#children	-0.188***	0.497***	0.374***	-0.066
public gardens	0.212***	-0.454***	0.131	-0.585***
public gardens*#children	0.212***	0.660***	0.247**	0.682***
lake	-0.805***	-0.739***	-0.307**	-0.870***
lake/river*#children	0.236***	0.359	0.585***	-0.424**
urban renewal zone	0.118**	-0.246	0.137	-0.123
public administration	0.504***	-1.357***	0.166	-1.963***
infrastructures	-0.168	-0.290	-2.675***	-0.877*
hospitals	-0.125*	0.639*	-0.665***	-0.325
sport areas	-0.059	-0.166	1.324***	0.623***
% build before 1915	-0.012***	0.004**	-0.013***	-0.001
% build in 1915-1967	-0.001***	0.002***	-0.004***	0.005***
% build after 1989	0.001	0.009***	0.006***	0.009***
Homogamy_poor	0.391***	1.494***	-2.984***	1.840***
Homogamy_middle income	0.682***	1.258***	-0.389*	4.844***
Homogamy_rich	2.591***	3.182***	3.780***	2.901***
Homogamy_young	3.396***	-2.540***	2.613***	-3.152***
Homogamy_middle age	-0.107	-0.423**	-0.516***	-0.205
Homogamy_old	0.625***	0.944	4.029***	1.664***
Homogamy_1-person hh	3.173***	1.828***	3.539***	1.134***
Homogamy_2-person hh	0.256	1.976***	1.499***	3.033***
Homogamy_+2-person hh	2.447***	1.966***	0.406***	0.263**
Homogamy_no-active hh	0.598***	0.750	4.228***	1.778***

Homogamy_1-active hh	1.433	***	-1.159	***	1.324	***	-2.929	***
Homogamy_2-active hh	0.674	***	1.193	***	1.720	***	2.824	***
Homogamy-foreign head	6.601	***	3.968	***	7.079	***	6.251	***
Pseudo-R²	0.2991		0.1862		0.2986		0.2104	
Log-likelihood	-711184		-60810		-136373		-119248	
# observations	368931		27127		70437		54637	

The age of building decreases the probability to choose a commune when looking for a flat probably reflecting the fact that old flats might have a larger use cost (energy, charges…). By contrast, both the percentage of recent buildings (after 1989) and old ones (built before 1967) have a positive effect on house-occupants' utility. The architectural quality and prestige of old dwellings appear to be valued in their location decision.

The estimated effect of the interaction between a household characteristic *x* and the proportion of similar households (denoted by Homogamy_*x* inTable 4) puts the stress on self-attraction between families who share the same income, household composition, number of active members or nationality of the head of household. Exception notably concerns households with one active member since they tend to fly away from their peers when choosing a house and poor households since they might be reluctant to occupy a flat close to other poor households. Similarly, middle-age-headed households are self-repulsed and young households are self-attracted only when choosing a flat, while old households tend to live together.

4.2. Model without liquidity constraints

The simultaneous estimation of Eq. (8) and Eq. (9) are presented in Table 5. Although the decreasing effect of income on the price disutility and its positive effect on the valuation of amenities have already been accounted for in the location choice model, the income per capita is found to influence the probability to choose a house. While it increases the probability to rent a house, it decreases the probability to purchase one.

As expected, the number of children increases the probability to choose a house whatever their age. However, the magnitude of this effect decreases with the children's age from the age of 6, which might reveal a lower interest for dwelling size as the children become older and more likely to leave home.

The coefficients of inclusive values are less than one, which is consistent with the assumptions of the nested logit. The coefficient of the inclusive value is larger for houses than for flats, which suggests more unobserved heterogeneity in the utilities provided by flats than by houses. The coefficients of the inclusive values are smaller for dwellings for sale, suggesting that the unobserved quality of housing services offered by dwelling for sale is less homogeneous than that offered by dwelling for renting.

Turning to tenure choice, the income per capita and the number of children are found to increase the probability to own a dwelling, probably because future bequest to their children might be many households' motive for homeownership. However, this effect disappears as children get older. Since children's births are likely to occur at the beginning of life-cycle, this effect is consistent with the theoretical finding that young households are more likely to own (Artle and Varaiya 1978).

Independently from income, the employment status of the head of household is also found to affect the utility of homeownership: a household is more likely to choose ownership if its head has a permanent employment contract, is self-employed or is retired than if he is unemployed or inactive. Surprisingly, workers in public administration are also found to have a lower probability to purchase than private sector worker, but this difference is weak.

Retired households' behavior is not consistent with the Artle and Varaiya's (1978) findings that older individuals tend to liquidate housing capital and rent at the end of their life-cycle. Bequests might motivate their willingness to purchase a dwelling.

Liquidity constraints constitute another explanation to the effect of employment status. Households with a precarious situation are indeed more likely to be liquidity constrained than household with a permanent contract which could explain why they are less likely to own. On the contrary, retired and self-employed are more likely to obtain favorable loan conditions since they might use their accumulated capital as a guaranty when purchasing their home.

4.1. Model with liquidity constraints

From the inclusive values computed in the first step, we estimate the parameters of Eq. (10) to 13. In this model, the parameters of the probability to be constrained (Eq. (13)) are not obtained from the observation of constraint but inferred from the model by observing only the chosen dwelling. We first estimate a simplified model in which the probability to be constrained is the same for all households. We then generalize the model to include observed heterogeneity in the probability to be constrained.

The drawback of such models is the lack of concavity of the log-likelihood, which may lead to a local maximum, and to wrong coefficients. This is the reason why we implemented an "Expectation-Maximization" algorithm to maximize the log-likelihood[9] of the simplified model

[9] This algorithm consists in iterating an Expectation and a Maximization steps till the convergence of the estimated coefficients. The « Maximization » step consists in estimating the coefficient of the model by maximizing the log-likelihood for a given value of the probability to be constrained. The « Expectation » step consists in deducing a value of this probability from the estimated coefficients.

Table 5: Choice of tenure and dwelling type in the model without constraints

Dwelling type choice	own	rent
Inclusive value (house)	0.451***	0.510***
Inclusive value (flat)	0.176***	0.455***
Intercept (house)	-0.596***	-3.330***
#centered log income	-10.577***	5.662***
# children <3 years	0.772***	0.267***
# children aged 3 to 6	0.861***	0.338***
# children aged 7 to 11	0.725***	0.311***
# children aged 12 to 16	0.601***	0.292***
# children aged 17 to 18	0.501***	0.245***
Tenure choice		
Inclusive value (own)	0.451***	
Inclusive value (rent)	0.316***	
Intercept (own)	-0.694***	
#centered log income	0.529***	
#foreign	-0.402***	
# children <3 years	0.120***	
# children aged 3 to 6	0.149***	
# children aged 7 to 11	0.052***	
# children aged 12 to 16	0.014	
# children aged 17 to 18	-0.009	
# hh head's employment status:		
permanent-contract worker	-	
self-employed	0.158***	
temporary-contract	-0.828***	
public-contract	-0.102***	
retired	0.215***	
unemployed head	-0.778***	
inactive head	-0.054**	
Pseudo-R^2	0.4197	
Log-likelihood	-419249.25	
#observations	521132	

The result of this simplified model is presented in the first two columns of Table 6: compared to those of Table 5, only the coefficients of the inclusive values and the coefficients of the log income per capita are significantly influenced by the inclusion of the uniformly-distributed constraint. Indeed, the log income no longer has a significant effect on the probability to purchase a dwelling. Inclusive values have a more significant effect on the tenure choice. These changes suggest that the heterogeneity in dwellings for sale and the effect of income on the choice between buying a house and buying a flat is underestimated when not accounting for the fact that only a restricted sample of households faces the tenure choice. The decrease in income coefficient in the tenure choice equation shows that income effect is overestimated by the standard nested model because this variable may be a determinant of the liquidity constraint, which is confirmed by the second estimation.

In the extended model the propensity to be constrained depends on the log income per capita and the number of active members in household. Log-likelihood is maximized by taking the coefficients of the simplified model as initial values. The obtained coefficients of the utilities provided by the dwelling type are almost identical to those obtained in the simplified model whereas those of the tenure status change significantly.

The income effect on the probability to choose ownership becomes significantly negative, which suggests that preference for homeownership decreases with income. Actually, as shown by the very significant decreasing effect of income on the propensity to be constrained, the main way through which the income per capita affects the probability to own may not be through preferences but through liquidity constraint. Hence, the richer households are more likely to own their home, not because they have a higher taste for ownership but because they have a higher propensity to have access to homeownership.

The coefficients of the employment status have the inverse signs (except for inactive and self-employed heads of household). For instance, households with a retired head are found to have a lower propensity to purchase while they are found to have a higher one in the model without constraint. Retired households are found to be less interested in homeownership, which is more consistent with Artle and Varaiya (1983). As income, the effect of employment status might not affect the homeownership probability through households' preferences (which depends on life-cycle effects and their income path) but through the probability to be constrained.

The number of children also has a larger effect on the utility, indicating that the asset transmission might be a more important motive of ownership than suggested by the nested model. The coefficient of the inclusive value corresponding to the nest “own” increases compared to its value in the nested logit and exceeds slightly the upper bound of one; the inclusive value of the nest “rent” decreases significantly suggesting that the previously observed heterogeneity in dwellings to rent was mainly due to difference in liquidity constraints.

Last, the estimation of the propensity to be constrained shows that, the propensity to be constrained decrease as income per capita increases. Compared to households whose head has a permanent-employment contract, those with a retired or self-employed head are less likely to be constrained. By contrast, when the head of household is unemployed or employed with a temporary contract, they are more likely to be constrained. This effect of employment status on the propensity to be constrained explains the change in the sign of its effect on the preferences for ownership.

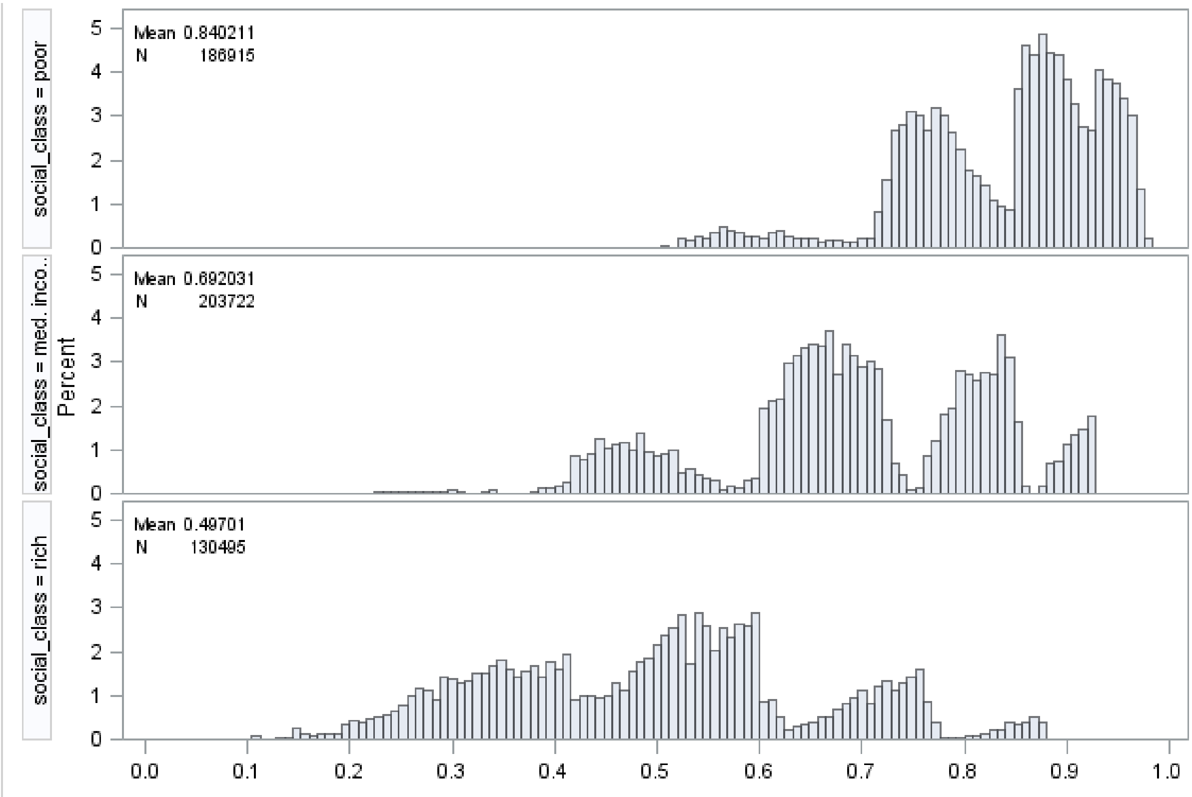
The number of cars owned by the household is also found to decrease the propensity to be constrained, which illustrates the importance of wealth in relaxing liquidity constraints.

Table 6: Choice of tenure and dwelling type in the model with constraints

	Simplified model		Extended model	
Dwelling type choice	own	rent	own	rent
Inclusive value (house)	0.444 ***	0.515 ***	0,397 ***	0,519 ***
Inclusive value (flat)	0.188 ***	0.458 ***	0,283 ***	0,454 ***
Intercept (house)	-0.528***	-3.348***	0,135 ***	-3,407***
#centered log income	-9.767***	5.675 ***	-4,462***	5,625 ***
# children <3 years	0.782 ***	0.261 ***	0,878 ***	0,243 ***
# children aged 3 to 6	0.870 ***	0.333 ***	0,928 ***	0,325 ***
# children aged 7 to 11	0.731 ***	0.306 ***	0,763 ***	0,300 ***
# children aged 12 to 16	0.601 ***	0.288 ***	0,619 ***	0,284 ***
# children aged 17 to 18	0.496 ***	0.243 ***	0,514 ***	0,234 ***
Tenure choice				
Inclusive value (own)	0.620***		1.837***	
Inclusive value (rent)	0.411***		1.560***	
Intercept (own)	-0.162***		3.899***	
#centered log income	0.018		-10.376***	
#foreign	-0.440***		-0.815***	
# children <3 years	0.144***		0.836***	
# children aged 3 to 6	0.184***		1.400***	
# children aged 7 to 11	0.068***		0.692***	
# children aged 12 to 16	0.026*		0.739***	
# children aged 17 to 18	0.006		0.702***	
# hh head's employment status:				
permanent-contract worker	-		-	
self-employed	0.252***		0.118**	
temporary-contract	-0.906***		0.167**	
public-contract	-0.118***		0.066	
retired	0.226***		-1.382***	
unemployed head	-0.856***		0.811***	
inactive head	-0.061**		-1.200***	
Probability to be constrained				
Intercept	-0.978***		1.628***	
#centered log income			-1.418***	
# hh head's employment status:				
permanent-contract worker			-	
self-employed			-0.070***	
temporary-contract			0.802***	
public-contract			0.088***	
retired			-0.886***	
unemployed head			0.811***	
inactive head			-0.395***	
#1 car in hh			-0.754***	
#2 cars in hh			-1.485***	
Pseudo-R²	0.4201		0.4326	
Log-likelihood	-418929.871		-409880,988	
#observations	521132		521132	

From the previous estimations, we compute households' probability to be constrained and represent its distribution in Figure 5 and Figure 6. As shown on Figure 5, the income per capita does not perfectly discriminate constrained households from unconstrained ones but as income increases, the distribution of the probability to be constrained tends to move towards weaker values. Then, whereas the probability to be constrained is comprised between 55 and 100% among the poor households, it is only comprised between 35 and 90% for medium-income households and lower than 85% among the rich ones. These values are largely higher than the uniform probability of 27.32% obtained from the first estimation of the latent choice set model, which shows that this model tend to underestimate the importance of the constraint. In the rest of the study, the reference to the latent choice set model or model with constraints will always concern the "extended" model with a household-specific probability to be constrained.

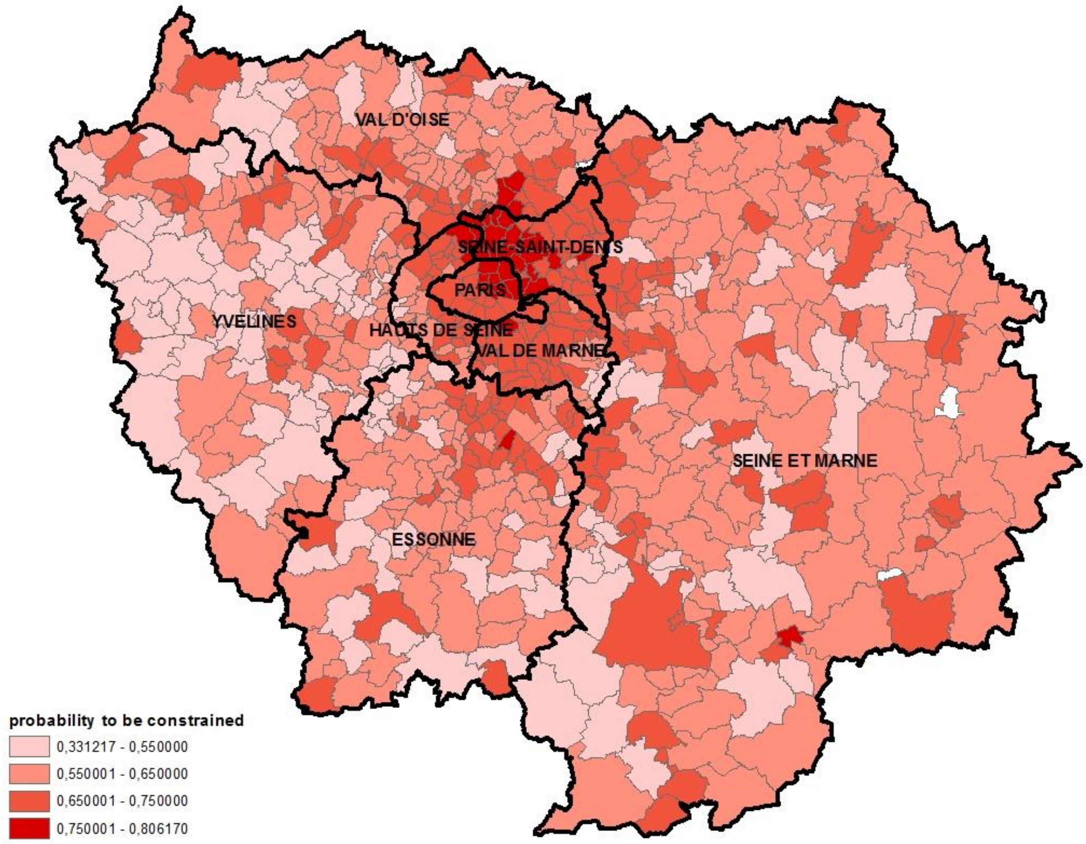
Figure 5: Distribution of the probability to be constrained among rich, medium-income and poor households



The distribution of the constrained households among the households is far from being geographically uniform and is very close to the distribution of poor households. As the map in Figure 4 shows, the percentage of constrained households among the movers (which is equivalent to the mean probability to be constrained) is higher in Paris and the cities at the north-east of Paris (particularly the Seine-Saint-Denis district). The Seine-Saint-Denis district is known to be one of the poorest in France and to concentrate many poor, mono-parental or foreign families, so the high level of constraints among households who choose to move there is not surprising. In Paris however, the high level of constraint among Paris immigrants might be due a strong proportion of low- and middle-income singles who can't afford buying their home but can afford renting in Paris.

Further from Paris, the distribution of constrained movers is less clear: at the west, households which installed in 1998 are less constrained whereas at the east, pseudo-communes with high and low proportions of constrained immigrant households are mixed.

Figure 6: Proportion of constrained households among movers by pseudo-commune



5. Simulations

5.1. Changes in destinations

A cancellation of the probability to be constrained is simulated (from the model with constraints) so as to evaluate what the allocation of moving households would be in that case, under the unrealistic hypothesis that the induced changes in demand would not affect prices. Such a cancellation modifies the probability to own a dwelling and consequently the choice probability of each pseudo-commune $\sum_{\substack{T \in \{\text{house,flat}\} \\ S \in \{\text{own,rent}\}}} P_i(j|T,S)$. By modifying the allocation of households among house-owners, house-tenants, flat-owners and flat-tenants, the cancellation of the latent constraints modify the valuation (and then the demand) for a pseudo-commune.

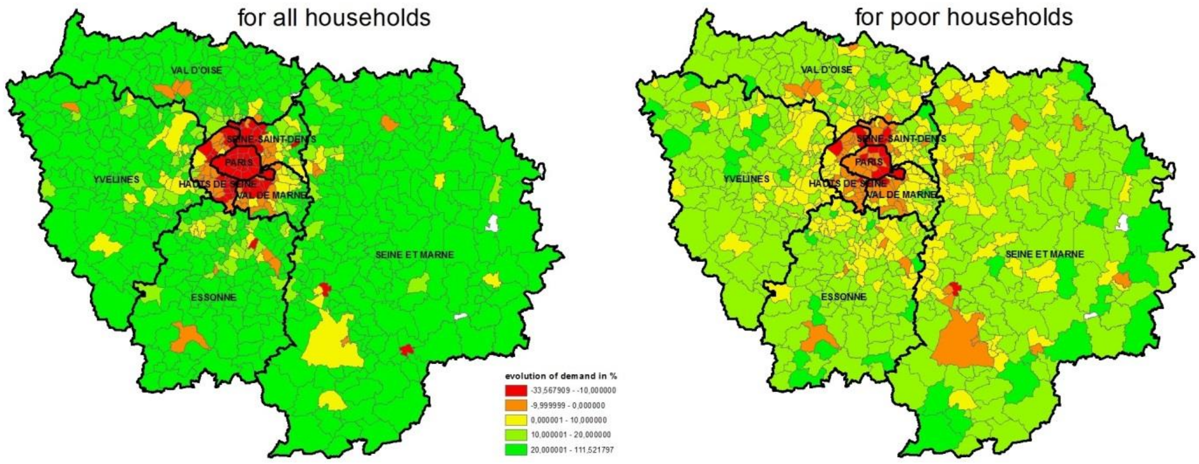
This simulation must be interpreted with care since it doesn't take into account the effect of the resulting change in demand on dwelling prices and, in longer term, on other local characteristics such as the social composition and school quality. It is certain that changes in demand would have induced changes in price in the same direction and would have partially cancelled the effect of removing the latent constraint. Then, the partial-equilibrium simulations might not perfectly reflect the demand which could result from an increased access to the financial market.

Demands for a pseudo-commune are simulated by aggregating households' predicted probability to choose it. Predictions are achieved first under the assumption that the actually probability to be constrained equals its predicted value and then under the hypothesis that this probability equals zero for poor households. Comparing the two corresponding demands indicates strong changes in the demand for some pseudo-commune when the latent constraint is cancelled.

As shown on Figure 7, if the latent constraint was cancelled for all households, prices and socio-demographic composition held constant, the demand for Paris and some of its close suburbs would decrease. On the contrary, it would increase slightly for the further pseudo-communes of the Inner Ring and more and more significantly as the pseudo-commune is located far from Paris. More precisely the predicted demand for the Outer Ring would increase by less than 10% when a pseudo-commune is close to the central counties, by a percentage comprised between 10 and 20% in western and southern pseudo-communes further, by more than 20% in eastern pseudo-communes further from Paris. Higher prices in the West of Paris than in the East might explain this dissymmetry.

If latent constraints were cancelled only on the moving households of the lowest income (per capita), prices and socio-demographic composition held constant, the decrease in the demand for Paris would be slightly lower and the increase in the demand for the farther away suburbs weaker. This indicates that poor households would not be the only ones to locate further from the central district in the absence of liquidity constraints. To explain this repulsive effect of the central districts and the attractiveness of the Outer Ring, let's observe the distribution of the households by county, dwelling type and tenure status for each income category in Table 5.

Figure 7: Evolution in the demand (%) when probability to be constrained is cancelled



The obtained strong suburbanization phenomenon would be mainly due to a decrease in the demand for dwellings to rent which would be only partially compensated for by an increase in the demand for dwellings to purchase in Paris. The decrease would particularly effect the demand for flats to rent, whereas the increase in demand for ownership would both concern flats and houses (except in Paris where houses are extremely rare goods). In the other districts, the same phenomenon would be observed but, since there is a higher supply of houses in those counties, the increase in demand for houses for sale would largely exceed the decreasing demand for dwellings to rent. Consequently, the demand would slightly decrease in Paris and strongly increase elsewhere, in particular in the Eastern pseudo-communes which count more houses among their supplied dwellings at a lower price per square meter.

That poor households increase their demand for further suburbs clearly indicates that relaxing the liquidity constraints would enhance their land consumption more than for their demand for the local determinants of life-quality. Although such a change could reduce the proportion of poor households in the pseudo-communes close to Paris, it could also increase it in the

Eastern pseudo-communes so that its effect on residential segregation in the whole Paris region is ambiguous and need to be studied more deeply.

In all counties, the decrease in the demand for renting would concern all income categories whatever the type of dwelling, but it is not the case for the increase in demand for ownership. Indeed, while poor and middle-income households' demand for purchasing flats and houses would both increase - in stronger proportions for poor households - the rich households' demand for flats for sale would remain quite unchanged whereas the demand for purchasing houses would strongly increase. Then the latent constraint not only restricts the poor and middle-income households demand for homeownership but also reduces the rich households' consumption of housing services and increase their land consumption.

Table 7: Simulated destinations of households from model with constraints by district, dwelling type and tenure status

			Actual choices				Preferred choices				Changes in poor households' demand	
			all	Poor	Med.	Rich	all	Poor	Med.	Rich		
Paris (75)	flat	rent	22.90	25.01	22.46	20.58	8.20	8.43	8.12	8.00	-5.95%	2.33%
		own	3.69	1.79	3.20	7.16	12.58	11.90	11.69	14.94	3.63%	
	house	rent	0.24	0.26	0.20	0.27	0.06	0.06	0.05	0.09	-0.07%	
		own	0.12	0.05	0.07	0.32	0.32	0.24	0.22	0.60	0.07%	
Hauts-de-Seine (92)	flat	rent	11.28	6.67	5.21	2.92	1.38	1.77	1.27	1.00	-2.95%	-0.53%
		own	2.44	0.63	0.88	1.00	3.03	4.01	2.78	2.03	2.27%	
	house	rent	0.44	0.77	1.19	1.17	0.23	0.16	0.22	0.34	-0.10%	
		own	0.71	0.78	3.12	3.85	6.94	4.22	9.20	7.28	0.25%	
Seine-St-Denis (93)	flat	rent	7.73	8.00	6.25	5.07	1.82	2.13	1.59	1.73	-2.88%	-0.23%
		own	1.16	1.17	1.52	2.84	5.96	7.37	4.87	5.64	2.37%	
	house	rent	0.58	0.65	1.00	1.58	0.24	0.14	0.20	0.47	-0.16%	
		own	0.95	0.63	2.06	4.19	5.48	3.35	6.00	7.73	0.90%	
Val-de-Marne (94)	flat	rent	7.27	6.80	5.19	3.17	1.42	1.79	1.30	1.07	-2.31%	0.07%
		own	1.57	0.84	1.08	1.35	3.96	5.40	3.42	2.73	2.04%	
	house	rent	0.46	0.47	0.76	0.95	0.16	0.10	0.15	0.28	-0.10%	
		own	0.86	0.54	1.98	3.03	4.73	2.93	5.77	5.67	0.44%	
Essonne (91)	flat	rent	5.26	11.94	11.21	10.46	3.68	3.70	3.51	3.92	-1.79%	0.56%
		own	1.06	1.20	2.11	4.71	7.82	7.53	7.09	9.38	1.64%	
	house	rent	0.71	0.35	0.42	0.60	0.11	0.08	0.09	0.18	-0.13%	
		own	1.73	0.18	0.50	1.78	1.68	0.87	1.47	3.17	0.86%	
Seine-et-Marne (77)	flat	rent	5.16	10.82	7.54	3.60	2.05	2.78	1.89	1.23	-1.76%	0.47%
		own	0.82	1.13	1.34	0.92	5.04	7.75	4.53	1.94	1.21%	
	house	rent	1.03	0.56	0.66	0.48	0.12	0.11	0.11	0.13	-0.22%	
		own	2.47	0.53	1.27	1.03	3.13	3.04	3.91	2.04	1.23%	
Yvelines (78)	flat	rent	6.58	8.93	7.31	4.82	2.11	2.50	2.01	1.70	-2.11%	0.91%
		own	1.73	1.07	1.61	2.22	5.63	6.76	5.31	4.50	2.23%	
	house	rent	1.02	0.36	0.48	0.56	0.10	0.07	0.09	0.16	-0.18%	
		own	2.08	0.29	0.93	1.57	2.34	1.51	2.74	2.94	0.97%	
Val d'Oise (95)	flat	rent	4.58	5.78	4.62	2.81	1.19	1.46	1.09	0.94	-1.55%	0.61%
		own	0.99	0.73	1.02	1.29	3.65	4.74	3.30	2.61	1.44%	
	house	rent	0.73	0.53	0.81	0.89	0.16	0.11	0.15	0.25	-0.15%	
		own	1.67	0.54	1.97	2.81	4.69	2.98	5.87	5.30	0.87%	

Table 8: Social composition of new households' population

	Actual choices			Preferred choices		
	Poor	Med.	Rich	Poor	Med.	Rich
Paris (75)	36.07%	37.60%	26.32%	30.04%	**41.16%**	**28.81%**
Hauts-de-Seine (92)	32.96%	37.47%	29.56%	30.48%	**38.86%**	**30.66%**
Seine-St Denis (93)	44.91%	40.57%	14.52%	**46.08%**	39.71%	14.21%
Val de Marne (94)	37.61%	39.78%	22.61%	**38.03%**	39.51%	22.45%
Seine-et-Marne (77)	33.47%	42.89%	23.63%	**36.64%**	40.85%	22.51%
Yvelines (78)	32.84%	37.13%	30.03%	**37.82%**	34.38%	27.80%
Essonne (91)	35.45%	40.26%	24.29%	**39.36%**	37.82%	22.82%
Val d'Oise (95)	34.17%	41.34%	24.48%	**38.86%**	38.40%	22.74%

6. Conclusion

We examine how residential location, tenure status and dwelling type are chosen by households and how liquidity constraint might affect their choice. It shows that households' characteristics do not only influence the location choice but also the choices of dwelling type and tenure status. Hence, the probability to choose a house is strongly influenced by the household size whereas tenure status choice essentially depends on the head's characteristics. Concerning this latter point, we observe a significant change in the coefficients values when we introduced constraints in the discrete choice model: poor households' preferences for buying is found significantly larger than suggested by the unconstrained model.

From the estimated models, demands were simulated and compared under different assumptions. Simulations enable to judge the importance of liquidity constraints by what would be the location of each household assuming off liquidity constraint for this specific household. Such approach is relevant for normative matters, but the range of the effects estimated is too large to be neglected in a predictive model, for three reasons.

The first reason is that alleviating liquidity constraints would dramatically change the (dis)equilibrium prices. The second reason is that it would dramatically change the social mix of the population in each location. The third reason is that it would change the local share of houses and flats, and request to build, for example, a large number of houses in the Outer Ring. At the same time, some existing flats and houses would become vacant according to unconstrained demand. The local changes in the demand by dwelling type predicted by unconstrained demand are by far too large to assume that an additional supply will meet such a demand, given the imperfections existing in real estate and land development markets.

Since both prices and social mix are important factors of the demand, the unconstrained demand would be significantly affected by such externalities if liquidity constraints were alleviated. Computing the resulting equilibrium without liquidity constraints would require developing an algorithm in the vein of the one used by de Palma et al (2007), which is left for future research.

In case liquidity constraints were alleviated, the demand for flats to rent in Paris, for example, would decrease dramatically, which would decrease the price for flats to rent in Paris. This decrease in renting prices would be so large that it would significantly change the trade-off between renting and buying a flat inside Paris, and consequently reduce the selling prices inside Paris. In addition, our results neglecting externalities show that alleviating liquidity constraints would significantly change the social mix and density, especially inside Paris and in the far away suburbs.

Nevertheless, the simulations suggest that removing the liquidity constraint would not necessarily improve the social mix. Indeed, prices and social composition held constant, poor households benefiting from a larger access to the financial market would not necessarily choose to live closer to the highly-endowed counties but rather to increase their land consumption or to change their tenure choice in the farther away counties.

Finally, if the latent constraint was cancelled for all households, the increased proportions of rich household which would choose to live in the Outer Ring of Paris region would be balanced by the arrival of new poor households so that the residential segregation would change marginally. If poor households were the only ones to benefit from the cancellation of the constraint, their flight for the East would not be balanced by the arrival of middle-income and rich households in this region. Then, policy measures such as the null-interest loan (PTZ) would have little effect on residential segregation in the Paris region when it can be used by all households but could also result in poor households' suburbanization if they are the only eligible households. In the latter case, social sorting close to Paris would be mitigated but the one that occurs between the Eastern and Western suburbs of the Paris region would be reinforced. In that sense, residential segregation appear to be mainly driven by households' tastes for land and public good so that relaxing liquidity constraint would have a limited or even negative efficiency on mitigating the social sorting. As suggested by Bénabou (1995), a better way to enhance social mix would then be to enforce it by building public housing in rich pseudo-communes or to subsidize the arrival of wealthy households in poor neighborhoods.

7. References

Alonso, W. (1964). *Location and land use*, Cambridge: Harvard University Press.

Artle, R. Varaiya, P. (1978). Life cycle consumption and homeownership. *Journal of Economic Theory*, Elsevier, 18(1), 38-58.

Bayer, P., Ferreira, F., McMillan, R. (2007). A Unified Framework for Measuring Preferences for Schools and Neighborhoods, *Journal of Political Economy*, University of Chicago Press, vol. 115(4), pages 588-638.

Ben-Akiva M., Lerman, S. (1985) Discrete Choice Analysis: Theory and Application to Travel Demand, MIT Press

Bénabou, R. (1995). Equity and efficiency in human capital investment: the local connection. *Review of Economic Studies*, 63(2), 237-264.

Berry, S., Levinsohn, J. Pakes, A. (1995) Automobile Prices in Market Equilibrium. *Econometrica*, 63(4), pages 841-890.

Brueckner, J.K. Thisse, J.-F. Zenou, Y. (1999). Why is Central Paris rich and downtown Detroit poor? An amenity-based theory. *European Economic Review*, 43, 91-107

Combes, P. Ph. Duranton, G. Gobillon, L. (2008). Spatial Wage Disparities: Sorting Matters! *Journal of Urban Economics*, 63(2), 723-742.

Dantan, S. (2013). The intergenerational transmission of inequality: the role of public policies, Thesis, Université de Cergy-Pontoise, Ch. 3.

de Palma A., K. Motamedi, N. Picard et P. Waddell (2005). A Model of Residential Location Choice with Endogenous Housing Prices and Traffic for the Paris Region, European Transport, 31, 67-82.

de Palma, A. de Lapparent, M. Picard, N. (2015). Modeling real estate investment decisions in households, in *Integrated transport and land use modeling for sustainable cities*, Bierlaire, M. de Palma, A. Hurtubia, R. Waddell, P. (eds.), Ch. 8, Ecole Polytechnique Fédérale de Lausanne Press.

Elder, H. Zumpano, L. (1991). Tenure choice, housing demand and residential location. *The Journal of Real Estate Research*, American Real Estate Society 6(3), 341-356.

Gobillon, L. Le Blanc, D. (2004). L'effet des contraintes d'emprunt sur la mobilité résidentielle et les choix entre location et propriété. *Annales d'Économie et de Statistique*, 74, 15-45.

Gobillon, L. Le Blanc, D. (2008). Economic Effects of Upfront Subsidies to Ownership: the case of the Prêt à Taux Zéro in France. *Journal of Housing Economics*, 17(1), 1-33.

Gobillon, L. Magnac, T. Selod, H. (2011). The effect of location on finding a job in the Paris region, *Journal of Applied Econometrics*, 26(7), 1079-1112.

Henderson, J. Ioannides, Y. (1986). Tenure choice and the demand for housing. *Economica*, 53, 231-246.

Henderson, J. Ioannides, Y. (1983). A model of housing tenure choice. *American Economic Review*, 73, 98-113.

Inoa, I. Picard, N. de Palma, A. (2015) Effect of an Accessibility Measure in a Model for Choice of Residential Location, Workplace, and Type of employment. *Mathematical Population Studies*, 22(1), 4-36.

Lee, L-F. Trost, R. (1978). Estimation of some limited dependent variable models with application to housing demand. *Journal of Econometrics*, Elsevier, 8(3), 357-382.

Liaw, K.L. Frey, W. (2003). Location of adult children as an attraction for black and white elderly return and onward migrants in the United States: Application of a three-level nested logit model with census data. *Mathematical Population Studies*, 10(2), 75-98.

Mc Fadden, D. (1978) Modelling the choice of residential location, in *Spatial Interaction Theory and Planning Models*, A. Karlquist et al. (eds;), North-Holland Publishing Company.

Megbolugbe, I. Linneman, P. Wachter, S. Cho, M. (1997). Do borrowing constraints change US homeownership rates? *Journal of Housing Economics,* 6, 318-333

Mills, E. (1967). An Aggregate Model of Resource Allocation in a Metropolitan Area. *American Economic Review*, 57, 197-211.

Muth, R. (1969). *Cities and housing,* University of Chicago Press, Chicago.

Ouazad, A. Rancière, R. (2015). Structural Demand Estimation with Borrowing Constraints, *PSE Working Papers* halshs-01207997, HAL.

Picard, N. de Palma, A. Dantan, S. (2013). Intra-household choice models of mode choice and residential location. *International Journal of Transport Economics*, XL (3), 419-445.

Rappaport, C. (1997). Housing Demand and Community Choice: An Empirical Analysis. *Journal of Urban Economics*, 42(2).

Rosen, H. (1979) Housing decisions and the U.S. income tax: An econometric analysis. *Journal of Public Economics*, Elsevier, 11(1), 1-23.

Tiebout, C. (1956). A pure theory of local expenditures. *Journal of Political Economy*, 64(5), 416-424.

Wheaton, W.C. (1977). Income and urban residence: an analysis of consumer demand for location. *American Economic Review*, 620-632.

8. Appendix:

8.1. Structural model without liquidity constraint

Proof of Lemma 1

Household *i*'s budget constraint (3) can be written: $y_i - t(d) = C + \pi^S(d,z) \cdot H$. All the right-hand terms are >0, which implies $y_i - t(d) > 0$. Assumption H4) implies that $t : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a one-to-one mapping and that its inverse $t^{-1}(.)$ is defined, continuous and increasing on $\mathbb{R}^+$. It also implies that $t^{-1}(Y)$ is finite. Let $D \leq t^{-1}(Y)$ represent the (finite) maximal distance selected by individuals living in the city and $T \equiv t(D)$. Then $t(.)$ is also a one-to-one mapping from [0;*D*] to [0;*T*].■

Proof of Lemma 2

Consider $y_i \in \left]0, Y\right], d \in \left[0; t^{-1}(y_i)\right[, z \in \mathbb{R}$ and $S \in \{own, rent\}$. The program:

$$\underset{C,H}{Max}\, U(C;H;z;S) = \beta^S \cdot \psi(z) + (1-\beta^S).\left[\gamma^S.\ln C + (1-\gamma^S).\ln H\right]$$

$$\text{Subject to } y_i - t(d) = C + \pi^S(d,z) \cdot H$$

can be solved analytically and leads to the following solutions:

$$C^*(d,z;S;y_i) = \gamma^S \cdot (y_i - t(d)) \quad \text{(A1)}$$

$$H^*(d,z;S;y_i) = (1-\gamma^S) \cdot \frac{y_i - t(d)}{\pi^S(d,z)} \quad \text{(A2)}$$

Introducing (A1) and (A2) in the direct utility *U*(*C,H;z;S*) leads to the indirect utility:

$$U^*(d,z;S;y_i) = k^S + \beta^S \psi(z) + (1-\beta^S)\ln(y_i - t(d)) - (1-\beta^S)(1-\gamma^S)\ln \pi^S(d,z), \quad \text{(A3)}$$

where $k^S = (1-\beta^S)\left[\gamma^S \ln\gamma^S + \left(1-\gamma^S\right)\ln\left(1-\gamma^S\right)\right]$.

Proof of Lemma 3

Using (1) and (A3), the first- and second-order derivatives of the indirect utility with respect to distance *d* can be written:

$$\begin{aligned}\frac{\partial U^*}{\partial d}(d,z;S;y_i) &= \left(1-\beta^S\right)\cdot\left[\frac{-t'(d)}{y_i-t(d)} - \left(1-\gamma^S\right)\cdot\frac{\partial\pi^S(d,z)}{\partial d}\times\frac{1}{\pi^S(d,z)}\right] = \left(1-\beta^S\right)\cdot\left[\frac{-t'(d)}{y_i-t(d)} - \left(1-\gamma^S\right)\xi_d^S(d)\right]\\ \frac{\partial^2 U^*}{\partial d^2}(d,z;S;y_i) &= \left(1-\beta^S\right)\cdot\left[-\frac{\left(y_i-t(d)\right)\cdot t''(d)+\left(t'(d)\right)^2}{\left(y_i-t(d)\right)^2} - \left(1-\gamma^S\right)\cdot\xi_d^{S'}(d)\right]\end{aligned} \tag{A4}$$

According to Assumption H4), $t''(d)\geq 0$ and $t'(d)>0$, so $\frac{\left(y_i-t(d)\right)\cdot t''(d)+\left(t'(d)\right)^2}{\left(y_i-t(d)\right)^2}>0$.

According to Assumption H3), $\xi_d^{S'}(d)<0$. As a result, $\frac{\partial^2 U^*}{\partial d^2}(d,z;S;y_i)<0$, so $U^*(.)$ is a concave function of d on the interval $[0; t^{-1}(y_i)]\subset[0;D[$, and it has a unique maximum $d^*(S;y_i)$ on $[0; t^{-1}(y_i)]$. Based on Equation (A3) and Assumption H2), $U^*(d,z;S;y_i)\underset{d\to 0}{\longrightarrow}-\infty$ and $U^*(d,z;S;y_i)\underset{d\to t^{-1}(y_i)}{\longrightarrow}-\infty$, which excludes corner solutions. Therefore, $0<d^*(S;y_i)<t^{-1}(y_i)$ and $U^*(d,z;S;y_i)$ increases with d when $d<d^*(S;y_i)$ and decreases when $d>d^*(S;y_i)$. Note that the optimal distance $d^*(S;y_i)$ does not depend on amenities z because z does not appear in the first order condition of (A4).

Proof of Lemma 4

Based on (A3), the first- and second-order derivatives of the utility with respect to amenities *z* can be written:

$$\begin{aligned}\frac{\partial U^*}{\partial z}(d,z;S;y_i) &= \beta^S\psi'(z)-\left(1-\beta^S\right)\left(1-\gamma^S\right)\frac{\partial\pi^S(d,z)}{\partial z}\times\frac{1}{\pi^S(d,z)} = \beta^S\psi'(z)-\left(1-\beta^S\right)\left(1-\gamma^S\right)\cdot\xi_z^S(z)\\ \frac{\partial^2 U^*}{\partial z^2}(d,z;S;y_i) &= \beta^S\psi''(z)-\left(1-\beta^S\right)\left(1-\gamma^S\right)\cdot\xi_z^{S'}(z)<0\end{aligned} \tag{A5}$$

Assumption H1) implies that $\psi''(z)<0$ and assumption H3) implies that $\xi_z^{S'}(z)>0$. As a result, $\frac{\partial^2 U^*}{\partial z^2}(d,z;S;y_i)<0$, so $U^*(.)$ is a concave function of z on $\mathbb{R}$, and it has a unique maximum $z^*(S)$ on $\mathbb{R}$. The optimal level of amenities only depends on S, not on d or y_i because S and z are the only variables appearing in the first order condition of (A5). The indirect utility increases with z when $z<z^*(S)$ and decreases when $z>z^*(S)$.

8.2. Structural model with liquidity constraint

Assume, as in Section 3.2, that a household i who buy cannot spend more than a maximal amount $A_i^{\max}$.

$$\underset{C,H}{Max}\, U_i\left(C;H;z;S\right)=\beta^S\cdot\psi(z)+(1-\beta^S).\left[\gamma^S.\ln C+\left(1-\gamma_i^S\right)\ln H\right]$$ s.t.

$$y_i=C+\pi^{own}(d,z)\times H+t(d)$$ and

$$\pi^{own}(d,z).H\leq A_i^{\max}$$

In the rest of this appendix, we will not consider the case where $A_i^{\max}>\left(1-\gamma^{own}\right)y_i$ since it corresponds to the model without constraint.

The unicity of the optimal amount $\tilde{z}^*(S)$ of amenities which maximizes the indirect utility $\tilde{U}^*(d,\cdot;S;y_i,A_i^{\max})$ can be proved as that of $z^*(S)$ in the proof of Lemma 2. The first-order condition of the maximization by respect to z is the same as in Eq. (A4), so, the level of amenities is the same as in the model without constraint: $\tilde{z}^*(own)=z^*(own)$

The unicity of the optimal distance $\tilde{d}^*\left(S;y_i,A_i^{\max}\right)$ is proved at the end of the proof of Proposition 1.

Proof of Proposition 1:

When $d \le \Delta^*\left(y_i, A_i^{\max}\right)$, the optimal consumption values $\tilde{H}^*(d,z;own;A_i^{\max})$ and $\tilde{C}^*(d;own;y_i,A_i^{\max})$ differ from the values $H^*(d,z;own)$ and $C^*(d;own;y_i)$ which maximize the utility $U_i\left(C;H;z;S\right)$.

Thus $U^*(d,z;own;y_i) \equiv U\left(C^*\left(d,z;own;y_i\right), H^*\left(d,z;own;y_i\right); z;own\right)$ is still larger than the utility $\tilde{U}^*(d,z;own;y_i,A_i^{\max}) \equiv U\left(\tilde{C}^*\left(d,z;S;y_i\right), \tilde{H}^*\left(d,z;S;y_i\right); z;own\right)$. So the difference between the indirect utility of owning without and with constraints conditionally on the distance and the level of amenities is positive when $d \le \Delta^*\left(y_i, A_i^{\max}\right)$ and equal to zero otherwise:

$$\begin{array}{lll} U^*(d,z;own,y_i) - \tilde{U}^*(d,z;own,y_i,A_i^{\max}) > 0 & if & d < \Delta^*\left(y_i, A_i^{\max}\right) \\ U^*(d,z;own,y_i) - \tilde{U}^*(d,z;own,y_i,A_i^{\max}) = 0 & if & d \ge \Delta^*\left(y_i, A_i^{\max}\right) \end{array} \qquad \text{(A7)}$$

Deriving the utility $\tilde{U}^*(d,z;own,y_i,A_i^{\max})$ by respect to *d* leads to the following result:

$$\frac{\partial \tilde{U}^*}{\partial d}(d,z;own;y_i,A_i^{\max}) = -\frac{\left(1-\beta^{own}\right)\gamma^{own}}{y_i - t\left(d\right) - A_i^{\max}} \cdot t'\left(d\right) - \frac{\left(1-\beta^{own}\right)\left(1-\gamma^{own}\right)}{\pi^{own}(d,z)} \cdot \frac{\partial \pi^{own}(d,z)}{\partial d} \qquad \text{(A8)}$$

Combining Eq. (A8) and Eq. (A4) gives the following result:

$$\frac{\partial U^*}{\partial d}(d,z;own,y_i) - \frac{\partial \tilde{U}^*}{\partial d}(d,z;own,y_i;A_i^{\max}) = \left(1-\beta^{own}\right) \cdot t'\left(d\right) \cdot \left(\frac{A_i^{\max} - \left(1-\gamma^{own}\right)\left(y_i - t(d)\right)}{\left(y_i - t\left(d\right) - A_i^{\max}\right)\left(y_i - t(d)\right)}\right)$$

$$\begin{array}{lll} < 0 & if & d < \Delta^*\left(y_i, A_i^{\max}\right) \\ = 0 & if & d \ge \Delta^*\left(y_i, A_i^{\max}\right) \end{array} \qquad \text{(A9)}$$

Thus, the loss of utility $U^*(d,z;own,y_i) - \tilde{U}^*(d,z;own,y_i,A_i^{\max})$ induced by credit constraint, when a household chooses a location $d < \Delta^*\left(y_i, A_i^{\max}\right)$, is a positive and decreasing function of *d*.

Unicity of the optimal distance $\tilde{d}^*\left(S; y_i, A_i^{\max}\right)$

As in the model without constraint, the indirect utility tends to $-\infty$ both when the distance tends to 0 and to the upper bound $t^{-1}(y_i)$. Moreover, according to Eq. (A7) and Eq. (A9), $\tilde{U}^*(d, z; own, y_i, A_i^{\max})$ and its derivative are continuous functions of the distance in $\Delta^*\left(y_i, A_i^{\max}\right)$ so the function $\tilde{U}^*(., z; own, y_i, A_i^{\max})$ is continuous on the interval (0, $t^{-1}(y_i)$):

$$\begin{aligned} \frac{\partial \tilde{U}^*}{\partial d}(d, z; own; y_i, A_i^{\max}) &< \frac{\partial U^*}{\partial d}(d, z; own; y_i) \qquad if \qquad d < \Delta^*\left(y_i, A_i^{\max}\right) < t^{-1}\left(y_i\right) \\ &= -\frac{\partial U^*}{\partial d}(d, z; own; y_i) \qquad if \qquad \Delta^*\left(y_i, A_i^{\max}\right) \leq d < t^{-1}\left(y_i\right) \end{aligned} \tag{A10}$$

Under hypotheses H1, as in the proof of Lemma 1, the indirect utility is a concave function of the distance and, consequently, there exists a unique optimal distance $\tilde{d}^*\left(S; y_i, A_i^{\max}\right)$ which maximizes it. This indirect utility increases with the distance when $d < \tilde{d}^*\left(S; y_i, A_i^{\max}\right)$ and decreases otherwise.

Proof of Proposition 2:

Applying Eq. (A9) to $d = d^*\left(own; y_i\right)$ and recalling that $\frac{\partial U^*}{\partial d}(d^*(own; y_i), z; own, y_i) = 0$, the partial derivative $\frac{\partial \tilde{U}^*}{\partial d}$ is shown to be positive when $d = d^*\left(own; y_i\right)$:

$$\frac{\partial \tilde{U}^*}{\partial d}(\tilde{d}^*\left(S; y_i, A_i^{\max}\right), z; own, y_i, A_i^{\max}) > 0$$

Applying Eq. (A9) to $d = \Delta^*\left(y_i, A_i^{\max}\right)$ and recalling that $\frac{\partial U^*}{\partial d}(\Delta^*\left(y_i, A_i^{\max}\right), z; own, y_i,) < 0$, the partial derivative $\frac{\partial \tilde{U}^*}{\partial d}$ is shown negative when $d = \Delta^*\left(y_i, A_i^{\max}\right)$:

$$\frac{\partial \tilde{U}^*}{\partial d}(\Delta^*\left(y_i, A_i^{\max}\right), z; own, y_i, A_i^{\max}) = \frac{\partial U^*}{\partial d}(\Delta^*\left(y_i, A_i^{\max}\right), z; own, y_i,) < 0 .$$

Since the indirect utility without constraint is a concave increasing function of the distance *d*, its derivative $\frac{\partial U^*}{\partial d}(.)$ is decreasing. $\frac{\partial U^*}{\partial d}(.)$ is positive when $d^*(own; y_i)$ and negative when *d*= $\Delta^*\left(y_i, A_i^{\max}\right)$. Therefore, the continuity and the monotonicity of $\frac{\partial U^*}{\partial d}(.)$ implies that $\frac{\partial U^*}{\partial d}(.)$ equals zero for a unique value comprised between $d^*(own; y_i)$ and $\Delta^*\left(y_i, A_i^{\max}\right)$.This value corresponds to the optimal distance $\tilde{d}^*\left(own; y_i; A_i^{\max}\right)$

Proof of Proposition 3:

Consider a maximum borrowable amount A^1 such that $d^*(own; y_i) < \Delta^*\left(y_i, A^1\right)$ and a lower value A^0, $A^0 < A^1$. Thus, according to Proposition 2 and knowing that the threshold distance decreases as the borrowable amount increases (Lemma 3), the corresponding optimal distances with constraint $\tilde{d}^*\left(own; y_i; A^k\right)$, *k=0,1* verify: $d^*(own; y_i) < \tilde{d}^*\left(own; y_i; A^k\right) < \Delta^*\left(y_i, A^k\right) \leq \Delta^*\left(y_i, A^0\right) \qquad k = 0,1$.

Applying Eq. (A8) to $A_i^{\max} = A^0$ and $A_i^{\max} = A^1$, we can define the difference between the derivatives of the two corresponding indirect utilities and show that:

$$\frac{\partial \tilde{U}^*}{\partial d}(d, z; own, y_i; A^0) - \frac{\partial \tilde{U}^*}{\partial d}(d, z; own, y_i; A^1) = \left(1 - \beta^{own}\right) \cdot t'(d).\gamma^{own}.\left(\frac{A^1 - A^0}{\left(y_i - t(d) - A^1\right)\left(y_i - t(d) - A^0\right)}\right) > 0$$
$$if \qquad d < \Delta^*\left(y_i, A^1\right) \leq \Delta^*\left(y_i, A^0\right) \; k = 0,1$$

(A10)

Applying Eq. (A10) to the particular case where $d = \tilde{d}^*\left(own; y_i; A^1\right)$ and recalling that $\frac{\partial \tilde{U}^*}{\partial d}(d, z; own, y_i; A^1) = 0$, the derivative of the indirect utility for $A_i^{\max} = A^0$ is found to be positive:

$$\frac{\partial \tilde{U}^*}{\partial d}(d,z;own,y_i;A^0)-\underbrace{\frac{\partial \tilde{U}^*}{\partial d}(d,z;own,y_i;A^1)}_{=0}=\frac{\partial \tilde{U}^*}{\partial d}(\tilde{d}^*\left(own;y_i;A^1\right),z;own,y_i,A^0)>0 \qquad \text{(A11)}$$
$$if \quad d<\Delta^*\left(y_i,A^1\right)\le\Delta^*\left(y_i,A^0\right) \qquad k=0,1$$

Recalling that the derivative $\frac{\partial U^*}{\partial d}(d,z;own,y_i,A^0)$ is decreasing in d and cancels for $d=\tilde{d}^*\left(own;y_i;A^0\right)$ (see proof of the unicity of $d=\tilde{d}^*\left(own;y_i;A^0\right)$) , Eq. (A11) implies that $\tilde{d}^*\left(own;y_i;A^1\right)<\tilde{d}^*\left(own;y_i;A^0\right)$

This illustrates that, when the constraint is binding, the optimal distance which maximizes the utility of buying increases as the borrowable amount decreases.

Proof of Proposition 4 :

The further-indirect utility $\tilde{U}^{**}(own;y_i,A_i^{\max})\equiv\tilde{U}^*(\tilde{d}^*\left(own;y_iA_i^{\max}\right),\tilde{z}^*\left(own\right);own;y_i,A_i^{\max})$ of buying a dwelling can be derived by respect to the maximum borrowable amount as follows:

$$\begin{aligned}\frac{\partial \tilde{U}_i^{**}}{\partial A_i^{\max}}(own;y_i,A_i^{\max})&=\frac{\partial \tilde{U}_i^*}{\partial A_i^{\max}}(\tilde{d}^*,\tilde{z}^*;own;y_i,A_i^{\max})\\&+\underbrace{\frac{\partial \tilde{U}_i^*}{\partial \tilde{d}^*}(\tilde{d}^*,\tilde{z}^*;own;y_i,A_i^{\max})}_{=0}.\frac{\partial \tilde{d}^*}{\partial A_i^{\max}}\left(own;y_i;A_i^{\max}\right).\end{aligned} \qquad \text{(A12)}$$

Reminding that $\tilde{d}^*\left(own;y_iA_i^{\max}\right)$ maximizes $\tilde{U}^*(.,\tilde{z}^*\left(own\right);own;y_i,A_i^{\max})$, the second term of the sum is equal to zero and deriving Eq. (A6) by respect to $A_i^{\max}$ shows that the first term equals:

$$\frac{\partial \tilde{U}_i^*}{\partial A_i^{\max}}(d,\tilde{z}^*;own;y_i,A_i^{\max})=\frac{(1-\beta^{own})\cdot\left[(1-\gamma^{own})\cdot\left(y_i-t\left(d\right)\right)-A_i^{\max}\right]}{A_i^{\max}.\left(y_i-t\left(d\right)-A_i^{\max}\right)}$$

which is positive when $A_i^{\max}<\left(1-\gamma^{own}\right)\cdot\left(y_i-t\left(d\right)\right)$

Thus, when $A_i^{\max} < \left(1-\gamma^{own}\right)\cdot\left(y_i - t\left(d^*\left(own;y_i\right)\right)\right)$, the derivative of Eq. (A12) is positive and then the further-indirect utility $\tilde{U}^{**}(own;y_i,A_i^{\max}) \equiv \tilde{U}^*(\tilde{d}^*\left(own;y_i A_i^{\max}\right),\tilde{z}^*\left(own\right);own;y_i,A_i^{\max})$ is an increasing function of $A_i^{\max}$

$$\frac{\partial \tilde{U}_i^{**}}{\partial A_i^{\max}}(own;y_i,A_i^{\max}) = \frac{(1-\beta^{own})\cdot\left[(1-\gamma^{own})\cdot\left(y_i - t(d)\right) - A_i^{\max}\right]}{A_i^{\max}.\left(y_i - t(d) - A_i^{\max}\right)} > 0 \qquad \text{(A13)}$$

8.3. Estimation of housing prices

In the *Côte Callon*, flats' prices per square meter are available in only 287 cities and *arrondissements,* while houses' prices are available in only 267 cities (houses in Paris are scarce). We estimated hedonic price regression in order to predict dwelling prices in the 1300 cities and Parisian administrative units (20 *arrondissements*) of the Paris Region. To predict the missing prices for the rest of the 1300 communes in Ile-de-France, hedonic price equations are estimated by regressing the logarithm of available prices on local characteristics. In addition to local amenities, the rate of firms' local taxation (*taxe professionnelle*) and the share of this tax devoted to the commune (the rest being devoted to the biggest administrative unit *Region* and *Département)* are included in the explanatory variable of the hedonic equation. Since dwellings and firms compete for the surface in a city, those two variables are likely to influence the dwelling prices although they have no effect on the location choice. This approach is then quite similar to the two-stage least squares in standard regressions and may partially circumvent the endogeneity of the prices.

Table 9 presents the estimates of dwelling price equation by type of dwelling, by tenure status and by age of the building. We assumed that dwelling prices can be proxied by the prices of old dwellings. Then, to obtain prices in each pseudo-commune, we compute the average of the predicted prices weighted by the supply of dwellings.

Table 9: Price equations

	house				flat			
	buy		rent		buy		rent	
	new	old	new	old	new	old	new	old
Intercept	9.6646***	9..4384***	4.5525***	4.4506***	9.5602***	9.2617***	4.4093***	4.3432***
Paris	-	-	-	-	0.1840	0.1968	-0.0071	-0.0421
Essonne (91)	-0.0638**	-0.0740***	-0.0220	-0.0567***	-0.0306	-0.0712***	-0.0385**	-0.0537***
Hauts-de-Seine (92)	0.0917***	0.1730***	0.0907***	0.1229***	0.1962***	0.1808***	0.1187***	0.1428***
Seine-St-Denis (93)	-0.1676***	-0.1880***	0.0083	0.0055	-0.0820**	-0.1535***	0.0333	0.0063
% build before 1915	0.0032**	0.0030**	0.0040***	0.0039***	0.0292	0.0031***	0.0037***	0.0032***
% build in 1915-1967	0.0008	-0.0007	0.0010**	-0.0001	0.0051***	0.0003	0.0003	-0.0001
% build after 1989	0.0000	-0.0008	-0.0005	0.0002	0.0005	0.0020*	-0.0002	0.0014*
Noisy	0.0032	0.0084	-0.0317	-0.0132	-0.0001	-0.0203	-0.0055	-0.0218
Density	0.0160***	0.0097***	0.0100***	0.0091***	0.0053**	0.0026	0.0066***	0.0052***
Fraction of surface with:								
Urban renewal zone	-0.0265	-0.1642	0.0178	-0.0841	-0.3423**	-0.2916**	-0.1937*	-0.1885*
Public gardens	0.3403***	0.3452***	0.2609***	0.2217***	0.3443***	0.4277***	0.2627***	0.2573***
Water	0.1663	0.1314	-0.0435	-0.0712	0.2348*	0.2084*	-0.0026	-0.0101
Forest	0.1403***	0.1721***	0.0672*	0.0954***	0.1117**	0.1429***	0.0909***	0.0878***
Public administration	0.1666	0.4495*	0.1796	0.1603	0.5823***	0.6198***	0.2398*	0.3053**
Infrastructures	0.9251	0.4603	0.4544	0.5816	0.6649	0.4037	0.3892	0.5347
Hospitals	0.6645**	0.1069	0.3563	0.2301	0.3762	0.1498	0.1966	0.2822
Sport areas	0.3420*	0.1291	0.1737	0.1024	0.2337	0.0994	0.1091	0.0973
Accommodation tax rate	-0.0109***	-0.0073***	-0.0067***	-0.0028	-0.0065**	-0.0074***	-0.0035**	-0.0031*
# Railway stations	0.0000	-0.0003	0.0026	0.0043	0.0078*	0.0045	0.0076***	0.0058**
# Subway stations	0.0005	0.0041	0.0030	0.0064*	0.0038	0.0049	0.0075**	0.0056*
Distance_close	-0.0008	-0.0045	0.0102	0.0065	-0.0054	-0.0042	0.0028	0.0061
Distance_far	0.0070	-0.0009	0.0022	-0.0006	0.0056	-0.0021	0.0006	0.0009
Firm tax rate	0.0062**	0.0059**	0.0058***	0.0055***	0.0025	0.0048*	0.0053***	0.0062***
% of the firm tax devoted to municipality	-0.7003***	-0.6200***	-0.5371***	-0.6517***	-0.4737**	-0.5847***	-0.5477***	-0.6936***
R^2	0.5922	0.6015	0.6059	0.6629	0.7528	0.6788	0.7051	0.6469
R^2 adjusted	0.5536	0.5638	0.5685	0.6310	0.7301	0.6493	0.6780	0.6146
#observations	267	267	267	267	287	287	287	287