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Abstract

We analyze a shift from capital income tax towards inheritance tax in a two-period overlapping generation model with rational altruism *à la* Barro, where the population consists of two types of dynasties that differ in altruism. The tax reform is implemented in a way that leaves the capital-labor ratio unchanged in steady state. With inelastic labor supply, the tax reform increases welfare of the less altruistic dynasties, but decreases welfare of the most altruistic ones. We then extend the model introducing elastic labor supply and considering that the old can transfer time to their offspring in order to help them in their domestic tasks. In this context, the tax reform can enhance labor supply and increase aggregate resources for consumption of market goods. A shift from capital income tax towards inheritance tax can then be Pareto improving.

Keywords: altruism, bequests, time transfers, inheritance tax, redistribution.
JEL Classifications: D64, H22, J22.

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1 Introduction

Standard arguments against capital income tax and inheritance tax rely on the discouraging effect on capital accumulation. Nevertheless, both taxes are not equivalent when looking at consequences on redistribution: in most countries, the distribution of inheritance is more concentrated than the distribution of wealth. From this point of view, taxing inheritance in order to subsidize saving may improve welfare of people who do not receive any bequest, and may avoid the discouraging effects on capital accumulation. In this paper, we intend to characterize situations where such a reform is Pareto-improving in a two-period overlapping generation framework. Indeed, inheritance tax and capital income tax do not have the same consequences on individual incentives. In a second-best world, where the economy has not reached a Pareto optimum, shifting the fiscal burden from one tax to the other may create efficiency gains.

Redistributive consequences of the tax reform rely on the fact that the distribution of inheritance is highly concentrated in most developed countries, within a smaller part of the population than life cycle saving. In France in 2010, the bottom 50% poorest with respect to inherited wealth received about 5% of aggregate bequests whereas the top 10% richest received about 60% (see Piketty, 2010). In addition, one quarter of total bequests is transmitted to the top 1% while a third of deceased people leave no bequests. These disparities in terms of bequests are stronger than disparities in terms of wealth. From INSEE (2011), the bottom 50% poorest hold less that 7 % of total wealth, while the top 10% richest own more that 40 % of total wealth. From this point of view, inheritance tax could play a role in reducing inequality.

We consider a two-period overlapping generation model with rational altruism *à la* Barro (1974), where bequests are concentrated on some part of the population. Dynasties have different degrees of altruism, meaning that households within the same generation care differently about their descendants (see Michel and Pestieau, 1998 and Vidal, 1996). Theoretical literature on rational altruism *à la* Barro (1974) with intragenerational heterogeneity suggests that redistributive incidence of inheritance taxation is likely to worsen welfare of every household even those who behave like life-cyclers. As shown by Michel and Pestieau (2005), if households have homothetic preferences, a uniform lump-sum transfer financed through inheritance tax reduces the steady-state welfare of all dynasties which differ in altruism degree. Although inheritance taxation allows to redistribute wealth, the distortive effect of inheritance tax concerning households choice on bequests pushes down the steady-state capital-labor ratio, affects negatively the consumption of all dynasties, and results in a negative impact on steady-state household's welfare.

Nevertheless, a tax reform that consists in implementing an inheritance tax, not to finance a lump-sum transfer, but in order to decrease capital income taxation, attenuates the fall in the capital-labor ratio. Indeed, such a tax reform has two opposite effects on the steady-state capital-labor ratio: positive with the fall in the capital income tax, and negative with the increase in the inheritance tax. In this paper, we focus on tax reforms that neutralize the effects on capital-labor ratio.

Even if the capital-labor ratio is left unchanged, the tax reform needs to modify individual incentives to create efficiency gains and be Pareto-improving. With inelastic labor supply, the tax reform improves welfare of life-cycler households but reduces welfare of individuals who leave bequests. The main reason is that the combination of constant capital-labor ratio and inelastic labor supply involves constant disposable resources in steady state for market good production. We then extend our framework to a case with elastic labor supply that allows to get some change in market good production, even if the capital-labor ratio is constant. The extended framework combines elastic labor supply of the young and family time transfers from the old to the young as an alternative to bequest.[1] We then assume that households face a trade-off between formal work and home production work. By combining an increase in inheritance tax with a decrease in capital income tax, the tax reform encourages individuals to transfer time rather than money to the next generation. To capture this phenomenon, we consider a model that has many similarities with Cardia and Ng (2003) and Cardia and Michel (2004) and is closed to Belan and Moussault (2018).

Kopczuk (2013), or Kindermann et al. (2018), stress that inheritance tax, by reducing wealth transmission, incites the young to work more. Nevertheless, the percentage of people who receive a bequest is quite low, so that the resulting effect on aggregate labor supply may be insufficient. In fact, intergenerational family transfer of time is a strong understated vector for raising aggregate labor supply and, conversely to bequest, concerns a large part of the population. Moreover, time transfers tend to strongly decrease with high earners (see Schoeni et al., 1997), while bequests are highly concentrated and increase with income. To take account of such intragenerational differences, we consider that a small part of population leave bequests, while the others leave time transfers.

To disentangle all the effects, it is fruitful to first consider a situation with elastic labor supply but no time transfer. We give conditions for the tax reform to be Pareto-improving. Shifting capital income taxation towards inheritance taxation creates an incentive to consume more when old in all dynasties. This effect was present with inelastic labor supply and was one of the reason why the reform cannot be Pareto-improving in steady state, since people already were consuming too much when old before the reform. With elastic labor supply, consuming more when old can be obtained, at least partially, through an increase in labor supply when young. We exhibit condition for such a situation to arise. This reinforces the welfare gain of the dynasties that do not leave bequests, and this helps those that leave bequest to compensate the loss due to the rise in inheritance tax. A key element to get a Pareto-improving reform is whether the additional consumption of those who do not leave bequest is fully obtained from their additional labor supply or not.

We then analyze the complete framework with time transfers. The tax reform makes time transfers more attractive. Indeed, higher market-good consumption when old allows to spend less time in home production and leaves more time to transfer to the offspring. This adds another reason for

[1] Numbers of empirical studies indicate that time transfers from parents to their children are on average almost as important as bequests in monetary equivalent in European countries and the United States, such as Schoeni et al. (1997), Cardia and Ng (2003), Attias-Donfut et al. (2005) and Wolff and Attias-Donfut (2007), Ho (2015). Moreover, they confirm that downward transfers in time and money dominate upward transfers.

labor supply of the young to increase: by helping their offspring in domestic production, parents facilitate effort of the young adult in formal work. If such an effect is sufficiently strong, a Pareto-improvement is more likely to be obtained. In this matter, a crucial parameter is the elasticity of substitution between market good consumption and time in home production. The higher it is, the stronger is the increase in labor supply of those who do not leave bequest, allowing to produce additional resources that cover the increase in their consumption. We also develop a numerical example that allows to illustrate the effect of the tax reform in steady state, and also to highlight that the Pareto-improvement can be achieved for all generations along the transitional dynamics.

The paper is organized as follows. Section 2 presents the basic model with inelastic labor supply. In Section 3, we analyze the steady-state effect of tax reform on the welfare of the two types of dynasties in the basic framework. Then in Section 4, the model is extended to elastic labor supply and time transfers. We then study tax reform impacts on both types of dynasties. Final section concludes.

2 Equilibrium

2.1 Dynasties and generations

We consider a two-period overlapping generation model. Time is discrete. The population size is constant and normalized to unity. Each parent has only one child. We consider dynastic altruism *à la* Barro (1974) from parents to children. The economy consists of two types of dynasties (types 1 and 2). All agents that belong to the same type of dynasty i ($i \in \{1, 2\}$), whatever the generation, have the same degree of altruism β_i, and the same level of human capital h_i. We assume $0 \leq \beta_1 < \beta_2 < 1$. We define p_i as the proportion of type $i's$ agents in each generation: $0 < p_i < 1$ and $p_1 + p_2 = 1$.

2.2 Household behavior

An individual born in t that belongs to a type-i dynasty works in period t and retires in period $t + 1$. During its working life, he/she allocates income between market good consumption c^y_{it} and savings s_{it}

$$c^y_{it} + s_{it} = (1 - \tau^w)\, h_i w_t + (1 - \tau^x)\, x_{it} + a_t$$

where w_t is the real wage, x_{it} is the bequest received from his parent and a_t is a lump-sum transfer. Tax rates on labor income τ^w and bequest τ^x are assumed to be constant.

When retired, the individual divides return on savings between market good consumption c^o_{it+1} and bequest to his child x_{it+1},

$$c^o_{it+1} + x_{it+1} = \left(1 - \tau^R\right) R_{t+1} s_{it}$$

where R_{t+1} is the gross interest rate. The tax rate on capital income τ^R is also assumed to be constant. Parents cannot leave negative bequest to their children:

$$x_{it+1} \geq 0$$

Utility U_{it} of a type-i individual born in t is

$$U_{it} = u\left(c_{it}^y, c_{it+1}^o\right) + \beta_i U_{it+1}$$

where the lifetime utility function u is increasing in both arguments.[2] We also assume that both consumptions are normal goods.

Let us rewrite the objective of the households born in $t \geq 0$ as an infinite sum $\sum_{j=t}^{+\infty} \beta_i^t u\left(c_{ij}^y, c_{ij+1}^o\right)$ and substitute c_{ij}^y and c_{ij+1}^o in the objective by their expressions given by the budget constraints in each period. Then, if $\beta_i > 0$, one gets the following optimality conditions with respect to s_{it} and x_{it+1}

$$-u_{c^y}\left(c_{it}^y, c_{it+1}^o\right) + \left(1-\tau^R\right) R_{t+1} u_{c^o}\left(c_{it}^y, c_{it+1}^o\right) = 0 \quad (1)$$

$$-u_{c^o}\left(c_{it}^y, c_{it+1}^o\right) + \beta_i\left(1-\tau^x\right) u_{c^y}\left(c_{it+1}, d_{it+2}\right) \leq 0 \; (=0 \text{ if } x_{it+1} > 0) \quad (2)$$

If $\beta_1 = 0$, then type-1 dynasties consist of life-cyclers. Their bequest is zero and their saving satisfies (1).

The first-old in period 0 allocate after-tax capital income $\left(1-\tau^R\right) R_0 \bar{s}_{i,-1}$ between consumption c_{i0}^o and bequest to their child x_0^i such that

$$-u_{c^o}\left(\bar{c}_{i,-1}^y, c_{i0}^o\right) + \beta_i\left(1-\tau^x\right) u_{c^y}\left(c_{i0}^y, c_{i1}^o\right) \leq 0 \; (=0 \text{ if } x_{i0} > 0)$$

given $\bar{c}_{i,-1}^y$.

2.3 Firms and production

The production sector consists in a representative firm that behaves competitively and combines capital K_t and efficient labor L_t to produce output $F\left(K_t, L_t\right)$. Technology F is linear homogenous, increasing and concave. Profit maximization of the representative firm leads to equality between marginal products and real input prices

$$R_t = F_K\left(K_t, L_t\right) \text{ and } w_t = F_L\left(K_t, L_t\right) \quad (3)$$

[2] We assume that the function $u\left(c^y, c^o\right)$ is strictly concave and twice continuously differentiable over $]0,+\infty[\times]0,+\infty[$. Moreover $u_{c^y}\left(0, c^o\right) = +\infty$, for any $c^o > 0$ and $u_{c^o}\left(c^y, 0\right) = +\infty$ for any $c^y > 0$.

assuming total depreciation of the capital stock in one period. F_K and F_L stand for the partial derivatives of F with respect to capital and efficient labor.

2.4 Government

The government has to finance a sequence of public spendings $(G_t)_{t\geq 0}$. Let Δ_t denote the public debt at the beginning of period t. The government budget constraint in period $t \geq 0$ writes

$$\Delta_{t+1} + \tau^R \sum_i p_i R_t s_{it-1} + \tau^w L_t w_t + \tau^x \sum_i p_i x_{it} = R_t \Delta_t + a_t + G_t \tag{4}$$

2.5 Market equilibrium

In period $t \geq 0$, the labor market equilibrium is

$$L_t = \sum_i p_i h_i = \bar{h}$$

where $\bar{h}$ is average productivity. The resource constraint in period t writes

$$F\left(K_t, \bar{h}\right) = \sum_i p_i c_{it}^y + \sum_i p_i c_{it}^o + G_t + K_{t+1} \tag{5}$$

The Walras' law implies equilibrium on the capital market:

$$K_{t+1} + \Delta_{t+1} = \sum_i p_i s_{it} \tag{6}$$

Capital stock K_0 and public debt Δ_0 are given at the beginning of period 0 and satisfy $K_0 + \Delta_0 = \sum_i p_i \bar{s}_{i,-1}$.[3]

The instruments considered allow the social planner to reach a Pareto optimum. Indeed, tax rates on capital income and labor income can be set to zero $\tau^R = \tau^w = 0$. Then, with a zero tax rate on bequest ($\tau^x = 0$), the government budget constraint for all $t \geq 0$ reduces to $\Delta_{t+1} = R_t \Delta_t + a_t + G_t$. This means that an initial public debt and the sequence of public spendings would be shared among all generations and dynasties through uniform lump-sum tax a_t ($t \geq 0$). Inheritance tax would create inefficiency by distorting the household choice on bequests. Nevertheless, inheritance tax may help to reduce wealth inequalities in an economy where dynasties do not have the same accumulation behavior.

[3] Indeed, the capital market equilibrium is obtained from the household budget constraints, the government budget constraint and linear homogeneity of the production function F, which implies $F\left(K_t, \bar{h}\right) = R_t K_t + w_t \bar{h}$.

2.6 Steady state

As stressed by Michel and Pestieau (1998, 2005) and Nourry and Venditti (2001),[4] the dynasties that leave positive bequests in steady-state equilibrium are only those with the highest degree of altruism. Other dynasties behave as life-cyclers and accumulate no wealth. The same result applies in our model. At steady state, optimality conditions (1) and (2) imply

$$\beta_i \left(1-\tau^x\right)\left(1-\tau^R\right) R \leq 1 \;(\, = 1 \text{ if } x_i > 0)$$

Since $\beta_1 < \beta_2$, the preceding condition implies that type-1 dynasties leave no bequest at steady state: $x_1 = 0$. As shown by Nourry and Venditti (2001),[5] bequests of type-2 dynasties are positive iff the capital stock K_M that satisfies $\beta_2 \left(1-\tau^x\right)\left(1-\tau^R\right) F_K\left(K_M, \bar{h}\right) = 1$, is higher than savings that would be obtained if all agents were life-cyclers, with $R = R_M \equiv F_K\left(K_M, \bar{h}\right)$ and $w = w_M \equiv F_L\left(K_M, \bar{h}\right)$. We need to extend this result to take account of fiscal instruments.

In the following, we assume that the government chooses the tax instruments $\left(\tau^R, \tau^w, \tau^x, a\right)$. Then, considering situations with positive bequests of type-2 dynasties ($x_2 > 0$), a steady-state equilibrium is a vector $(c_1^y, c_1^o, c_2^y, c_2^o, K_M, x_2, R_M, w_M)$ such that

$$\begin{aligned}
\beta_2 \left(1-\tau^x\right)\left(1-\tau^R\right) R_M &= 1 && (7)\\
MRS_i^{c^o/c^y} &= \beta_2(1-\tau^x) && (8)\\
c_1^y + \beta_2\left(1-\tau^x\right) c_1^o &= \left(1-\tau^w\right) h_1 w_M + a && (9)\\
c_2^y + \beta_2\left(1-\tau^x\right) c_2^o &= \left(1-\tau^w\right) h_2 w_M + a + (1-\beta_2)\left(1-\tau^x\right) x_2 && (10)\\
w_M &= F_L(K_M, \bar{h}), \quad \text{and} \quad R_M = F_K(K_M, \bar{h}) && (11)\\
G + \sum_i p_i (c_i^y + c_i^o) &= F(K_M, \bar{h}) - K_M && (12)
\end{aligned}$$

where $MRS_i^{c^o/c^y} \equiv u_{c^o}(c_i^y, c_i^o)/u_{c^y}(c_i^y, c_i^o)$ is the marginal rate of substitution of type-i between c^o and c^y, for $i = 1, 2$. The public debt Δ then results from the budget constraint of the government (4) at steady state:

$$\left[1-\left(1-\tau^R\right) R_M\right] \Delta = G + a - \tau^R R_M K_M - \tau^w w_M \bar{h} - \tau^x p_2 x_2 \qquad (13)$$

In order to derive a condition for bequest to be positive in steady state, let us define the consumptions as functions of life-cycle resources for consumption Ω and gross interest rate R:

$$\left(c^y\left(\Omega, R\right), c^o\left(\Omega, R\right)\right) \equiv \arg\max_{(c^y, c^o)} \left\{ u\left(c^y, c^o\right); c^y + \frac{c^o}{R} = \Omega \right\}$$

[4] See also Altig and Davis (1992) or Vidal (1996). Becker (1980) also states the same kind of result in the Ramsey-Koopmans framework. The steady-state capital-labor ratio is determined by the lowest discount rate.

[5] They extend an argument introduced by Thibault (2000) in the case where all dynasties have the same degree of altruism.

We get the following Lemma.

Lemma 1. *Consider the vector of instruments* $\left(\tau^R, \tau^w, \tau^x, a\right)$. *Assume there exists a capital stock* K_M *that satisfies equality (7) and consider an inheritance tax rate* τ^x *close to zero. Then, the steady-state bequest of type-2 agents* x_2 *is positive iff*

$$K_M + \Delta_M > \sum_{i=1}^{2} p_i \left[\mathcal{I}_{i,M} - c^y \left(\mathcal{I}_{i,M}, [\beta_2(1-\tau^x)]^{-1}\right)\right] \tag{14}$$

where $\mathcal{I}_{i,M} \equiv (1-\tau^w) h_i w_M + a$ *and the public debt* Δ_M *satisfies equation* (13) *for* $x_2 = 0$.

Proof. The capital market equilibrium (6) can be rewritten as

$$\Phi(x_2) \equiv K_M + \Delta - \sum_{i=1}^{2} p_i \left[\Omega_i - c^y \left(\Omega_i, [\beta_2(1-\tau^x)]^{-1}\right)\right] = 0$$

where

$$\begin{aligned} \Omega_1 &\equiv (1-\tau^w) h_1 w_M + a \\ \Omega_2 &\equiv (1-\tau^w) h_1 w_M + a + (1-\beta_2)(1-\tau^x) x_2 \end{aligned}$$

From (13), Δ depends on x_2:

$$\Delta = \frac{-G - a + \tau^R R_M K_M + \tau^w w_M \bar{h} + \tau^x p_2 x_2}{\left[\beta_2 (1-\tau^x)\right]^{-1} - 1}$$

and is equal to Δ_M for $x_2 = 0$. Condition (14) is then equivalent to $\Phi(0) > 0$. Therefore, if Φ is decreasing, bequest x_2 is positive at steady state. Derivative of Φ writes

$$\Phi'(x_2) = \frac{p_2 \beta_2 (1-\tau^x)}{1-\beta_2(1-\tau^x)} \left[\tau^x - \frac{1-\beta_2}{\beta_2}\left(1-\beta_2(1-\tau^x)\right)\left(1 - \frac{\partial c_2^y}{\partial \Omega_2}\right)\right]$$

Under the normal good assumption,

$$0 < \frac{\partial c_2^y}{\partial \Omega_2} < 1$$

and the result follows for an inheritance tax τ^x close to zero.

□

3 Fiscal reform at steady state

With coexistence of dynasties that either leave bequest or behave like life-cyclers, the inception of an inheritance tax allows to redistribute wealth. Nevertheless, it also reduces the capital-labor ratio. Indeed, equation (7) leads to a negative relationship between the capital-labor ratio and the inheritance tax rate. Considering homothetic preferences, Michel and Pestieau (2005) have shown that a uniform lump-sum transfer financed through inheritance tax reduces the steady-state lifetime utility of all dynasties.[6] One may explain the result in the following way. Two forces affect welfare of the life-cyclers. First, they receive a lump-sum public transfer. Second, the fall in the capital-labor ratio increases the real interest rate and pushes down the real wage rate. The latter effect on the wage rate overcompensates the other forces leading to a fall in the well-being of the life-cyclers. The main driving force here is the fact that, at a steady state with underaccumulation, any fall in the capital-labor ratio reduces the product disposable for consumption $\left(F\left(K_M, \bar{h}\right) - K_M\right)$. Dynasties that leave bequests also experience a fall in their welfare, for two additional reasons: (i) the inheritance tax creates a distortion in their bequest decision and (ii) the lump-sum transfer they receive is lower than their contribution.

Nevertheless, one can implement fiscal reforms that combine an increase in the inheritance tax with changes in the other tax rates, allowing to attenuate or eliminate the fall in the capital-labor ratio. In the following, we explore the consequence of a tax reform that consists in a switch from capital income taxation towards inheritance taxation. These changes have opposite effects on the capital-labor ratio. A fall in the capital income tax rate τ^R increases the capital-labor ratio while raising the inheritance tax rate τ^x decreases it. Moreover, such a policy still allows to redistribute wealth since the capital income tax is paid by all agents while the inheritance tax is paid only by the dynasties that leave bequests.

We then focus on a fiscal reform that leaves the capital-labor ratio constant. If, additionally, we assume constant labor income tax rate and constant lump-sum transfer, the reduction of τ^R financed through an increase in τ^x is necessarily welfare enhancing for life-cyclers. They do not pay inheritance tax and pay less capital income tax, while wage rate and interest rate remain unchanged. We can then state the following proposition, only using the intertemporal budget constraint (9) of type-1 agents.

Proposition 1. *At steady state, any increase in the inheritance tax τ^x that leaves the first-period income of type-1 agents (i.e. $(1-\tau^w) h_1 w_M + a$) unchanged increases steady-state life-cycle utility of type-1 agents.*

First-period income of type-1 agents is constant if, for instance, the capital-labor ratio is not modified (constant w_M) as well as the instruments τ^w and a. Such a situation can be obtained by setting τ^R in order to keep the product $(1-\tau^x)\left(1-\tau^R\right)$ constant. In this case, the capital stock K_M, characterized by equation (7), is unchanged, as well as the wage rate. Nevertheless, keeping $(1-\tau^x)\left(1-\tau^R\right)$ constant also modifies fiscal receipts. Indeed, the fiscal base of the inheritance tax is $p_2 x_2$ while the fiscal base of the capital income tax is

$$R_M \sum_i p_i s_i = \frac{p_1 c_1^o + p_2\left(c_2^o + x_2\right)}{1-\tau^R}$$

[6] They assume zero public debt, zero public spendings and $\tau^w = \tau^R = 0$, so that the government budget constraint reduces to $\tau^x \sum_i p_i x_{it} = a_t$.

The latter is higher than $p_2 x_2$ at least if the capital income tax rate is positive. Type-2 individuals save not only in order to leave bequests to their offspring, but also to consume during old-age. To keep $(1-\tau^x)\left(1-\tau^R\right)$ constant, the fall in fiscal receipts from capital income tax will be larger than the increase in fiscal receipts from the inheritance tax. This results in a decrease in the steady-state public debt[7] that involves intergenerational redistribution from the first generations towards the ones living at a time where the economy is closed to the steady state.

We put intergenerational redistribution issues aside for the moment to focus on the welfare of type-2 agents in steady state. To do so, we define the intertemporal elasticity of substitution

$$\sigma_i^u \equiv \frac{d\ln\left(c_i^y/c_i^o\right)}{d\ln\left(MRS_i^{c^o/c^y}\right)}, \quad \text{for } i=1,2.$$

Proposition 2. *Let us assume $\beta_2(1-\tau^x)<1$. At steady state, for given τ^w and a, a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant increases the steady-state life-cycle utility of type-1 agents and reduces the one of type-2 agents.*

Proof. Since τ^w and a are not modified, the first-period income of type-1 agents is unchanged. Then applying Proposition 1 allows to state the result for type-1 agents. For type-2 agents, differentiation of lifetime utility $u_2 = u(c_2^y, c_2^o)$ leads to

$$du_2 = u_{c_2^y}\left[dc_2^y + \beta_2(1-\tau^x)\,dc_2^o\right]$$

From the resource constraint (12), one gets

$$dc_2^y + dc_2^o = -\frac{p_1}{p_2}\left(dc_1^y + dc_1^o\right).$$

Moreover, equality (8) implies

$$\frac{dc_i^y}{c_i^y} = \frac{dc_i^o}{c_i^o} - \sigma_i^u \frac{d\tau^x}{1-\tau^x}, \quad i=1,2. \tag{15}$$

[7]Differentiating equation (13) and assuming $d\tau^w=0$, $da=0$ and initially $\tau^x=0$, one gets

$$\left(1-\frac{1}{\beta_2}\right)d\Delta = -R_M\left(K_M+\Delta\right)d\tau^R - p_2 x_2 d\tau^x$$

where, to keep the capital-labor ratio constant: $d\left[\left(1-\tau^R\right)(1-\tau^x)\right]=0$. This implies

$$\left(1-\frac{1}{\beta_2}\right)d\Delta = \left[\left(1-\tau^R\right)R_M\left(K_M+\Delta\right) - p_2 x_2\right]d\tau^x = \left(\sum_{i=1}^{2} p_i c_i^o\right)d\tau^x$$

Then, $d\Delta$ and $d\tau^x$ have opposite signs.

The last two equalities lead to

$$\begin{aligned} & dc_2^y + \beta_2 (1-\tau^x)\, dc_2^o \\ = \; & -(1-\beta_2(1-\tau^x)) \frac{c_2^o}{c_2^y + c_2^o} \sigma_2^u c_2^y \frac{d\tau^x}{1-\tau^x} - \frac{c_2^y + c_2^o \beta_2 (1-\tau^x)}{c_2^y + c_2^o} \frac{p_1}{p_2} (dc_1^y + dc_1^o) \end{aligned} \tag{16}$$

To conclude, one needs to sign $dc_1^y + dc_1^o$. Differentiation of the intertemporal budget constraint of type-1 agents (9) and of the marginal condition $MRS_1^{c^o/c^y} = \beta_2 (1-\tau^x)$ leads to

$$dc_1^y + dc_1^o = \left[\frac{c_1^y + c_1^o}{c_1^y + c_1^o \beta_2 (1-\tau^x)} \beta_2 c_1^o + \frac{c_1^o (1-\beta_2(1-\tau^x))}{c_1^y + c_1^o \beta_2 (1-\tau^x)} \frac{\sigma_1^u c_1^y}{1-\tau^x} \right] d\tau^x > 0$$

since $\beta_2 (1-\tau^x) < 1$. Consequently $dc_2^y + \beta_2 (1-\tau^x)\, dc_2^o < 0$. This concludes the proof.

□

For type-2 agents, the introduction of the inheritance tax reduces the relative price of old-age consumption. This positive effect on their utility is overcompensated by a fall in after-tax bequest $(1-\tau^x)\, x_2$, leading to a reduction in utility.

Leaving the capital-labor ratio constant with the tax reform involves that aggregate resources available for consumption are constant. Thus, any consumption gain for one type of agent is offset by a loss of consumption for the other. For both types of dynasties, the fall in the relative price of old-age consumption leads the agents to shift part of their resources from the youth period to old-age. In addition, the marginal rate of transformation between c^o and c^y ($MRT_i^{c^o/c^y}$) is equal to one whereas the marginal rate of substitution ($MRS_i^{c^o/c^y} = \beta_2 (1-\tau^x)$) is lower than one. As a result, any shift of consumption from c^y to c^o creates an inefficiency in the resource allocation for consumption.

As the utility of type-1 agents increases with the tax reform considered in Proposition 2, the utility of type-2 agents decreases both because of the transfer of resources to type 1-agents and also because of a greater inefficiency in the resource allocation between consumption when young and consumption when old.

The following Proposition states that a tax reform leaving the steady-state capital-labor ratio constant cannot increase lifetime utility of type-2 agents. The tax reform considered allow for changes in the labor income tax rate τ^w or the lump-sum transfer a, that we have kept constant until now.

Proposition 3. *Consider an initial steady-state equilibrium where $\beta_2 (1-\tau^x) < 1$. Assume that government implements a tax reform that consists in a marginal increase in inheritance tax rate ($d\tau^x > 0$) and marginal changes in other tax instruments ($d\tau^R$, $d\tau^w$, da) such that the capital-labor ratio remains constant. If the reform does not reduce the lifetime utility of type-1 agents at steady state, then lifetime utility of type-2 agents necessarily decreases.*

Proof. The fiscal reform $\left(d\tau^x, d\tau^R, d\tau^w, da\right)$ is such that: (i) $d\tau^x > 0$; (ii) the capital-labor ratio remains unchanged, that is $d\left[\left(1-\tau^x\right)\left(1-\tau^R\right)\right]=0$, or equivalently:

$$\frac{d\tau^R}{1-\tau^R} = -\frac{d\tau^x}{1-\tau^x}.$$

Consider the extreme case where the reform does not change lifetime utility of type-1 agents ($du_1 = 0$). We then check whether lifetime utility of type-2 agents can increase ($du_2 > 0$). Recall that du_i has the same sign as

$$dc_i^y + \beta_2\left(1-\tau^x\right) dc_i^o$$

Then, type-1 utility does not change iff $dc_1^y + \beta_2\left(1-\tau^x\right) dc_1^o = 0$. Differentiation of the marginal condition (8) for type-1 agents was given in the proof of Proposition 2 (see equation (15)). Then, straightforward calculations lead to

$$dc_1^y + dc_1^o = \frac{c_1^y c_1^o\left[1-\beta_2\left(1-\tau^x\right)\right]}{c_1^y + \beta_2 c_1^o\left(1-\tau^x\right)}\sigma_1^u \frac{d\tau^x}{1-\tau^x} > 0$$

Replacing in (16) implies that $du_2 < 0$. This concludes the proof.

□

Therefore, the tax reform cannot increase the utility of type-2 agents whereas, as we have seen in Propositions 1 and 2, it is possible to design the reform in a way that increases lifetime utility of type-1 agents.

The crucial point in the preceding result is that disposable resources for consumption cannot vary since we have assumed constant capital-labor ratio and inelastic labor supply. In the following, we drop the latter assumption and assume elastic labor supply in order to examine whether the tax reform can enlarge disposable resources for consumption in market good. Indeed, a tax reform that leaves the capital-labor ratio constant allows for higher quantities of labor and capital in steady state. We distinguish two cases according as the parents can transfer time or not to their children, in addition to bequests. As we shall see, taking account of time transfers introduces an additional effect of the tax reform considered. Time transfers may become more attractive leading potentially the young who do not receive bequest but only time transfers, to work more.

4 Time transfers and elastic labor supply

We now consider a case where the combination of an inheritance tax and a capital income subsidy that leaves the capital-labor ratio unchanged does not imply resources for consumption to be fixed. The tax reform is reconsidered in a framework that combines elastic labor supply of the young and intergenerational time transfers from the old to the young as an alternative to bequest.

4.1 A framework with time transfers

Households of generation t that belongs to type-i dynasties ($i \in \{1, 2\}$) consume a composite good that aggregates market good c^y_{it} when young (resp. c^o_{it+1} when old) and time spent in home production T^y_{it} when young (resp. T^o_{it+1} when old). Labor supply is elastic and the agent's labor supply decision depends on the trade-off between formal work and home production. The lifetime utility function becomes:

$$v_i \left(f^y_i \left(c^y_{it}, T^y_{it} \right), f^o_i \left(c^o_{it+1}, T^o_{it+1} \right) \right) \tag{17}$$

where v_i is strictly quasi-concave and increasing in both quantities of composite goods, f^y_i when young and f^o_i when old.[8] Functions f^y_i and f^o_i are linear homogeneous functions, with positive and decreasing first-order derivatives.

The household's budget constraint during his working life is rewritten as follows:

$$c^y_{it} + s_{it} = (1 - \tau^w) h_i w_t \ell_{it} + (1 - \tau^x) x_{it} + a_t \tag{18}$$

where ℓ_{it} denotes type-i agent's labor supply in the formal sector, and satisfies:

$$\ell_{it} = 1 - T^y_{it} + \mu_i (1 - T^o_{it}) \tag{19}$$

where μ_i represents the productivity parameter of time transfer in home production of the young. It is the same for all dynasties of the same type. It embodies all the factors (health, geographical distance,...) that improves efficiency of the time taken by the grandparent in order to help his offspring.

When retired, type-i agent's budget constraint is the same as in the case with inelastic labor supply:

$$c^o_{it+1} + x_{it+1} = \left(1 - \tau^R \right) R_{t+1} s_{it} \tag{20}$$

The first-order conditions of type-i agent are given in the Appendix section 6.1.

On the production side, the factor prices w_t and R_t of the representative firm are equal to their marginal products (see equation (3)). The budget constraint of the government is the same as equation (4). The labor market equilibrium becomes:

$$L_t = \sum_i p_i h_i \ell_{it}$$

and the resource constraint is the same as equation (5) where $\bar{h}$ has been replaced with $\sum_i p_i h_i \ell_{it}$. The capital market equilibrium (6) is then satisfied as a consequence of the Walras' law.

[8] We follow the formulation with homogenous agents proposed in Belan and Moussault (2018), which is equivalent to the one considered in Cardia and Michel (2004) and close to the model developed in Cardia and Ng (2003).

4.2 Steady state

As shown in Appendix 6.1, only the most altruistic dynasties can leave bequests in steady state. In the following, we assume that type-2 agents make positive bequests. Thus, $x_1 = 0$ and $x_2 > 0$. The gross interest rate satisfies:

$$\beta_2 (1-\tau^x) \left(1-\tau^R\right) R_M = 1$$

The steady-state capital-labor ratio z_M is then characterized by the equality between marginal product of capital $F_K(z_M, 1)$ and the gross interest rate R_M. The resource constraint at steady state then rewrites as

$$\sum_{i=1}^{2} p_i \left(c_i^y + c_i^o\right) = C_M \sum_{i=1}^{2} p_i h_i \ell_i \tag{21}$$

where

$$C_M \equiv F(z_M, 1) - z_M$$

We assume that the productivity parameter of time transfers μ_1 is high enough for type-1 agents to leave time transfers: $T_1^o < 1$. We also consider an initial steady-state equilibrium with no time transfer by type-2 agents ($T_2^o = 1$).

The first-order conditions of type-i agents can be rewritten as follows[9]

$$\begin{aligned}
MRS_i^{c^o/c^y} &= \beta_2 (1-\tau^x) \equiv P^R, \text{ for } i = 1,2 && (22)\\
MRS_i^{T^y/c^y} &= (1-\tau^w) h_i w \equiv P_i^y, \text{ for } i = 1,2 && (23)\\
MRS_1^{T^o/c^o} &= \frac{\beta_1 \mu_1 (1-\tau^w) h_1 w}{\beta_2 (1-\tau^x)} \equiv P_1^o && (24)
\end{aligned}$$

where P^R, P_i^y and P_i^o denote the relative prices respectively between c_i^o and c_i^y, between T_i^y and c_i^y, and between T_i^o and c_i^o. In order to disentangle the effects of the reform, we successively consider the two following cases:

- Positive bequests for type-2 agents only ($x_2 > 0$ and $x_1 = 0$) and no time transfer for all dynasties ($T_i^o = 1$, for $i = 1,2$).
- Positive bequests for type-2 agents only ($x_2 > 0$ and $x_1 = 0$) and positive time transfers only for type-1 agents ($T_1^o < 1$ and $T_2^o = 1$).

[9] We use the following definitions of the marginal rates of substitution:

$$MRS_i^{c^o/c^y} \equiv \frac{v_{i,f_i^y} f_{i,c_i^y}^y}{v_{i,f_i^o} f_{i,c_i^o}^o}, \text{ and } MRS_{T_i^j/c_i^j} \equiv \frac{f_{i,T_i^j}^j}{f_{i,c_i^j}^j}, \text{ for } j = y, o$$

where partial derivatives have the same definition as in Appendix 6.1.

4.3 Tax reform without time transfer

We consider the same tax reform as with inelastic labor supply: a marginal increase in the inheritance tax rate from $\tau^x = 0$ and a marginal decrease in the capital income tax rate τ^R such that the capital-labor ratio remains constant, i.e.

$$\frac{d\tau^R}{1-\tau^R} = -\frac{d\tau^x}{1-\tau^x}.$$

Other instruments τ^w and a remain unchanged.

The consequence on type-1 utility in steady state will be qualitatively the same since they only experience a fall in the price of the second period consumption P^R. Therefore, considering the intertemporal budget constraint of type-1 agents

$$c_1^y + P_1^y T_1^y + P^R c_1^o = P_1^y + a,$$

the same argument as in Proposition 1 applies and leads to the following result:

Proposition 4. *At steady state, any increase in the inheritance tax τ^x, that leaves both the net wage P_1^y and the lump-sum transfer a constant, increases steady-state life-cycle utility of type-1 agents.*

We now turn to the change in utility for type-2 agents. With inelastic labor supply, the tax reform has two negative effects on welfare of the type-2 agents: (i) resources consumed by type-1 agents increase, leaving less to type-2 agents; (ii) the fall in P^R increases the gap between $MRS_i^{c^o/c^y}$ and $MRT_i^{c^o/c^y}$, leading type-2 agents to consume more in their second period of life, whereas they already consume too much.

With home production, the fall in P^R still incites agents to shift resources from the first to the second period of life. But, this also involves a fall in the time devoted to home production when young and a rise in labor supply. Introducing elastic labor supply can then attenuate, even reverse, the two above negative effects on type-2 agent welfare, allowing for a Pareto-improving reform.

The next proposition gives necessary and sufficient conditions for the tax reform (i) to increase labor supply of both types of agents, (ii) to increase resources disposable for type-2 agents, and (iii) to be Pareto-improving. It is important here to define what we mean by resources disposable for type-2 agents. We consider that the maximum production of market good in the economy is obtained if both types of agents work their entire unit time when young, that is $C_M\bar{h}$. Then resources used by type-1 young and old in period t are the sum of the consumptions $(c_1^y + c_1^o)$ and of the domestic time valued with the marginal rate of transformation $(C_M h_1)$. From the resource constraint of the economy (equation (21)), the resources left to type-2 agents are $C_M\bar{h} - E_1$, where $E_1 \equiv c_1^y + C_M h_1 T_1^y + c_1^o$.

To state the next proposition, we introduce some additional notations. Let us define the intertemporal elasticity of substitution σ_i^v between the composite goods when young f_i^y and old f_i^o:

$$\sigma_i^v = \frac{d\ln\left(f_i^y/f_i^o\right)}{d\ln\left(v_{i,f_i^o}/v_{i,f_i^y}\right)} \tag{25}$$

where v_{i,f_i^o} and v_{i,f_i^y} stands for the marginal utilities of both composite goods. We also define the elasticity of substitution σ_i^o between c_i^o and T_i^o

$$\sigma_i^o = \frac{d\ln\left(c_i^o/T_i^o\right)}{d\ln\left(f_{i,T_i^o}^o/f_{i,c_i^o}^o\right)}. \tag{26}$$

Since P_i^y's are fixed (τ^w and w do not change), the ratio c_i^y/T_i^y does not change with the tax reform and we have $dc_i^y/c_i^y = dT_i^y/T_i^y$. Let us also define the shares

$$\alpha_i^y \equiv \frac{c_i^y}{c_i^y + MRS_i^{T^y/c^y}T_i^y},\ \alpha_i^o \equiv \frac{c_i^o}{c_i^o + MRS_i^{T^o/c^o}T_i^o} \text{ and } \alpha_i^{yM} \equiv \frac{c_i^y}{c_i^y + C_M h_i T_i^y}$$

that are not affected by the tax reform.

Proposition 5. *At steady state, for given τ^w and a, consider a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio unchanged.*

(i) Labor supply of type-1 agents (ℓ_1) increases iff

$$\frac{1-\alpha_1^o}{\sigma_1^o} + \frac{\alpha_1^o}{\sigma_1^v} < 1 \tag{27}$$

(ii) Resources used by type-1 agents ($E_1 \equiv c_1^y + C_M h_1 T_1^y + c_1^o$) decrease iff

$$\frac{1}{\sigma_1^v}\left(\alpha_1^o + \frac{\alpha_1^{yM}c_1^o}{c_1^y}\right) + \frac{1-\alpha_1^o}{\sigma_1^o} + \frac{\alpha_1^{yM}}{P^R\alpha_1^y} - 1 < 0 \tag{28}$$

(iii) Labor supply of type-2 agents (ℓ_2) increases iff

$$\left(\frac{\alpha_2^o}{\sigma_2^v} + \frac{1-\alpha_2^o}{\sigma_2^o}\right)[-p_1 dE_1] < p_2 c_2^o\left(\frac{-dP^R}{P^R}\right) \tag{29}$$

(iv) Steady-state utility of type-2 agents increases iff

$$-p_1 dE_1 + \frac{p_2}{D_2}\frac{c_2^y}{\alpha_2^y P^R}\left(\frac{\alpha_2^y P^R}{\alpha_2^{yM}} - 1\right)\left(\frac{-dP^R}{P^R}\right) > 0 \tag{30}$$

where $D_2 \equiv \frac{c_2^y}{\alpha_2^y P^R c_2^o}\left(\frac{\alpha_2^o}{\sigma_2^v} + \frac{1-\alpha_2^o}{\sigma_2^o}\right) + \frac{1}{\sigma_2^v} > 0$.

Proof. See Appendix 6.2. □

To interpret the results in Proposition 5, it is useful to see that the inequalities $\alpha_i^{yM} < \alpha_i^y P^R$, for $i = 1, 2$, give possibilities for efficiency gains. Indeed, in the case with inelastic labor supply (Proposition 3), the assumption $P^R < 1$ implies misallocation of consumption between periods (c_i^y vs c_i^o). The tax reform incites agents to shift consumption from the first to the second period of life, whereas $MRS_i^{c^o/c^y}$ $(= P^R)$ is lower than $MRT_i^{c^o/c^y}$ $(= 1)$. With elastic labor supply, this effect is still detrimental for welfare and inefficient in terms of resource allocation.

However, the leisure-consumption trade-off also transforms the condition $P^R < 1$ into $\alpha_i^{yM} < \alpha_i^y P^R$, for $i = 1, 2$. It then adds another misallocation of resources between time T_i^y and private good c_i^y in the first period of life. This inequality implies that substituting consumption for domestic time improves efficiency at the steady-state equilibrium. People allocate too much time to home production. An increase in labor supply can improve the allocation of resources.

If $\alpha_i^{yM} < \alpha_i^y P^R$, the latter effect overcompensates the shift of consumption from the first to the second period of life. Therefore, in the case with elastic labor supply, the tax reform still exacerbates the distortion in the intertemporal allocation of consumption, but now reduces the distortion in the allocation between consumption and time devoted to home production.

We deduce from (30) that the tax reform is Pareto improving if the two following conditions are satisfied:

- $\alpha_2^{yM} < \alpha_2^y P^R$ which implies efficiency gains (comparison of $MRS_2^{c^o/c^y}$ and $MRT_2^{c^o/c^y}$).
- and E_1 decreases, which is likely to be obtained if (i) $\alpha_1^{yM} < \alpha_1^y P^R$, and (ii) both elasticities σ_1^v and σ_1^o are sufficiently high.

Figure 1 gives a graphical representation in the plane (σ_1^v, σ_1^o) of inequalities (27) and (28) that allows to represent situations where type-1 agents work more, and use a lower quantity of aggregate resources. There are thresholds A_1^o and B_1^o on both elasticities below which E_1 always increases. Indeed, if σ_1^v is high, the tax reform involves a large fall in T_1^y enhancing the rise in labor supply. On the contrary, if σ_1^v is low, complementarity between consumptions in both periods implies a small increase in labor supply. Another key element is the elasticity of substitution between time and market good in home production when old, that is σ_1^o. As c_1^o grows, domestic production increases, but the rise is higher if time and market good are highly substitutable into home production. By contrary, if they are complement, higher market good consumption does not bring much utility.

Moreover, from inequality (29), it appears that change in labor supply of type-2 agents depends on the variations of E_1. Indeed, for a negative change in E_1, labor supply of type-1 agents increases; this allows for situations where type-2 agents work less and consume more, despite the fact that they pay an inheritance tax.

Nevertheless, realistic values of the intertemporal elasticities of substitution σ_i^v are generally lower than one. Consequently, there is some ambiguity on the sign of dE_1, that raises the possibility

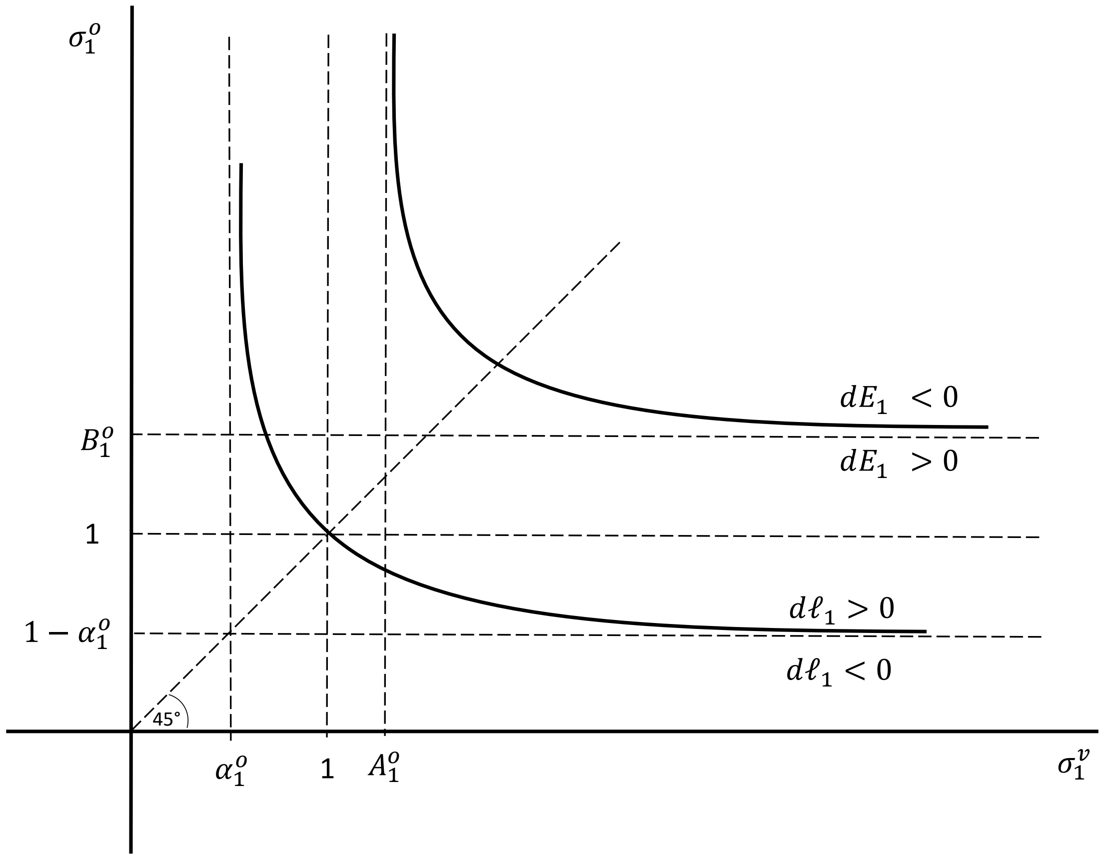

Figure 1: Fiscal incidence without time transfer

Note: The curve $dE_1 = 0$ is plotted under the assumption that $\alpha_1^{yM} < \alpha_1^y P^R$. If $\alpha_1^{yM} \geq \alpha_1^y P^R$, dE_1 is always positive. Values A_1^o and B_1^o that determine asymptotes of the curve $dE_1 = 0$ are respectively higher than α_1^o and $1 - \alpha_1^o$, but not necessarily higher than 1.

of a fall in steady-state utility of type-2 agents. We now want to check whether there is still an ambiguity if we consider time transfers of type-1 agents. Indeed, the tax reform may in this case incite the old to give more time to the young and then enhance their labor supply.

4.4 Tax reform with time transfers by type-1 agents

In the preceding section, by assuming that the old spend all their time to produce home production goods ($T_1^o = T_2^o = 1$), we have overlooked the effect of the tax reform on the allocation by the old between market good consumption and time for home production. We now complete the framework considering that type-1 agents can leave time transfers to their offspring. Labor supply of the young also depends on time transfer from the old, for childcare or any other purpose. As we have seen, these transfers tend to be decreasing with income, while bequests are increasing with income. To introduce this dimension, we assume that type-2 agents (high-income agents) leave bequests but no time, while type-1 agents (low-income agents) give time but no bequest.

With time transfers, from equations (22)-(24), the tax reform still reduces the relative price P^R between both consumptions, but also increases the relative price P_1^o for type-1 agents between time devoted to home production and consumption when old. The latter effect makes time transfers more attractive and then is likely to enhance labor supply of type-1 agents when young. We now want to explore this mechanism and analyze under which condition it makes the tax reform Pareto-improving. To discuss the consequences of the tax reform, it is useful to distinguish interperiod and intraperiod effects:

1. *Interperiod effect.* The introduction of an inheritance tax decreases the relative price of the second-period market-good consumption P^R for both types of agents. The fall in P^R is an *interperiod effect* which involves a negative effect on the consumption in the composite good when young (negative effect on c_i^y and T_i^y, $i = 1, 2$) whereas the effect is positive on the composite good consumed when old (positive effect on c_i^o, $i = 1, 2$, and T_1^o for type-1 agents). The effect on young's labor supply of type-1 agents $\ell_1 = 1 - T_1^y + \mu_1 (1 - T_1^o)$ is ambiguous since the fall in T_1^y and the rise in T_1^o have opposite effects on labor supply. The magnitude of interperiod effects depends on the intertemporal elasticity of substitution σ_i^v.

2. *Intraperiod effect for type-1 agents.* The tax reform also increases the relative prices P_1^o between market good and time used in home production when old. This *intraperiod effect* has a positive impact on c_1^o and a negative effect on T_1^o. The negative effect on T_1^o affects positively time transfers and, therefore, the young's labor supply. Type-1 agents are incited to increase the young's labor supply through higher time transfers. The magnitude of this effect depends on the elasticity of substitution σ_1^o.

4.4.1 Relative change in type-1 agents variables and utility in steady state

Let us first compute the relative variations of consumptions (c_1^y, c_1^o) and times devoted to home production (T_1^y, T_1^o) for type-1 agents. For a given capital-labor ratio, the steady-state allocation $(c_1^y, T_1^y, c_1^o, T_1^o)$ is solution of a system of 4 equations that includes equations (22)-(24) and the intertemporal budget constraint of type-1 agents

$$c_1^y + P_1^y T_1^y + P^R (c_1^o + P_1^o T_1^o) = P_1^y [1 + \mu_1 (1 - (1 - \beta_1) T_1^o)] + a \tag{31}$$

In Appendix 6.3, we give conditions to sign the relative changes in each of the four variables according to the values of σ_1^v and σ_1^o. Figure 2 gives a representation in the plane (σ_1^v, σ_1^o) of the loci where these variables increase or decrease. Whatever the elasticities, old age consumption of type-1 agents rises. An upward sloping line separates situations where c_1^y and T_1^y increase from situations where they both decrease. Moreover, another upward sloping line separates situations where domestic time when old increases or decreases. This allows to identify an area where labor supply of type-1 agents necessarily increases ($dT_1^y < 0$ and $dT_1^o < 0$). The line $d\ell_1 = 0$ goes through the point $\sigma_1^v = \sigma_1^o = 1$ and remains in areas where one of the domestic times T_1^y and T_1^o is reduced while the other is increased. Notice that this line has not necessarily a positive slope. The net effect on the resources used by type-1 agents (E_1) is ambiguous. E_1 increases in the zone on the left of $dT_1^y = 0$ and below the line $d\ell_1 = 0$.

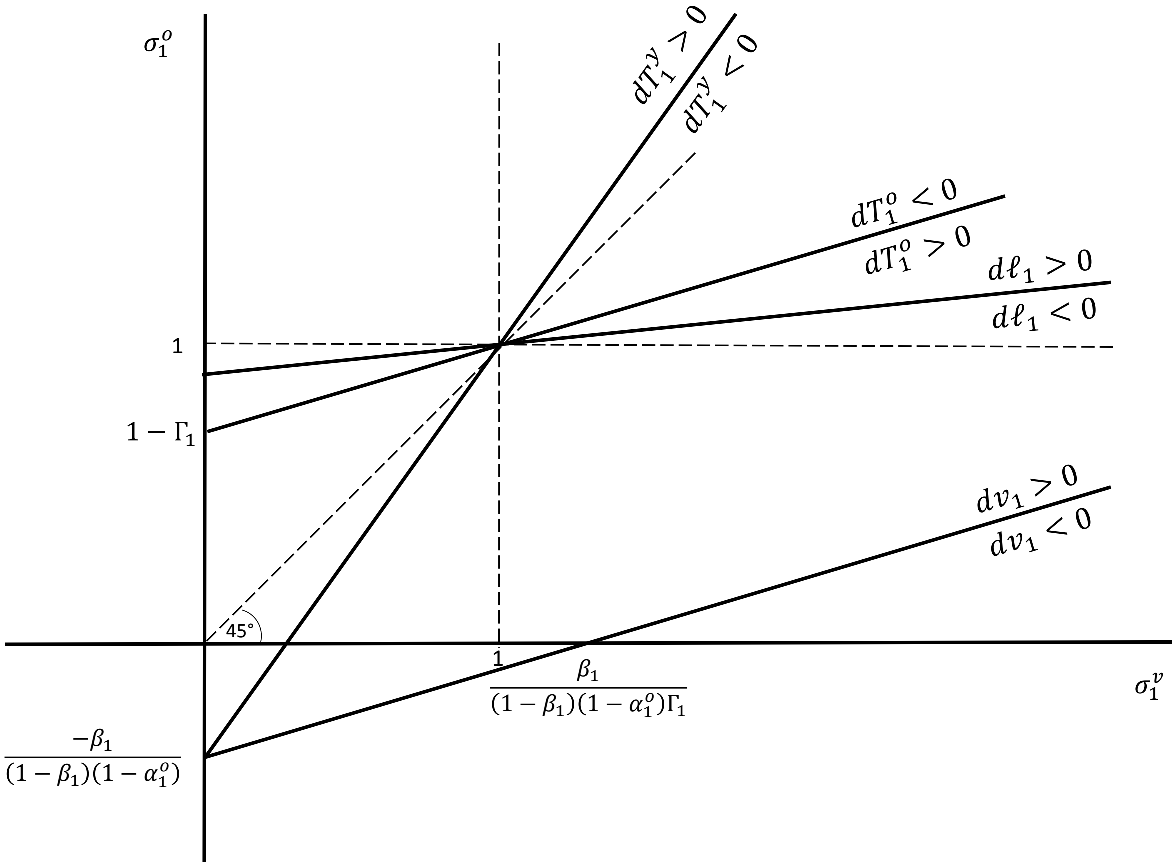

Figure 2: Fiscal incidence when type-1 agents leaves positive time transfer

Realistic values of the intertemporal elasticity σ_i^v are lower than one, while estimations of the intraperiod elasticity σ_i^o between domestic time and consumption stand between 1.4 and 2.5.[10] Thus the relevant locus in Figure 2 is $\{(\sigma_1^v, \sigma_1^o)\,;\sigma_1^v \leq 1, \sigma_1^o > 1\}$ where dE_1 can a priori be positive or negative.

Let us now consider the consequence on lifetime utility of type-1 agents.

Proposition 6. *For given τ^w and a, a shift from capital income tax towards inheritance tax, that*

[10] A number of studies have estimated the elasticity of substitution between market good and time for home production. They report values in the range of 1.4 to 2.5 (see e.g. Rogerson and Wallenius, 2016, Aguiar and Hurst, 2007, Chang and Schorfheide, 2003, McGrattan et al., 1997).

leaves the capital-labor ratio unchanged, increases the steady-state welfare of type-1 agents iff

$$\sigma_1^o > \Gamma_1 \sigma_1^v - \frac{\beta_1}{(1-\beta_1)(1-\alpha_1^o)}$$

where $\Gamma_1 \equiv \frac{c_1^y}{\alpha_1^y \Omega_1}$ *and* $\Omega_1 \equiv \frac{c_1^y}{\alpha_1^y} + P^R \frac{c_1^o}{\alpha_1^o}$.

Proof. See Appendix 6.3. □

Utility of type-1 agents does not necessarily increase. The intertemporal budget constraint (31) allows to describe the opposite effects on utility. Indeed, old age domestic time T_1^o can increase or decrease depending on the relative size of the interperiod and intraperiod effects. In Figure 2, the lines $dT_1^o = 0$ and $dv_1 = 0$ have the same slope. Then the cases where utility decreases are necessarily with $dT_1^o > 0$, so lower time transfers. These cases also correspond to reductions in T_1^y and c_1^y. Any rise in T_1^o reduces the right-hand side in the intertemporal budget constraint leading to an ambiguous effect on utility. Nevertheless, realistic values of σ_1^v and σ_1^o implies that $\sigma_1^o \geq \sigma_1^v$. Under this inequality, one can easily verify that type-1 agents utility increases.

4.4.2 Change in lifetime utility of type-2 agents in steady state

Condition for type-2 agents utility to increase is the same as the one derived in the model without time transfer (inequality (30)). The efficiency condition $\alpha_2^y P^R > \alpha_2^{yM}$ still plays a role. Recall it implies that an increase in type-2 agent labor supply compensated by a fall in their consumption can raise their lifetime utility. The only difference comes from the term dE_1 which now includes the change in T_1^o:

$$dE_1 = dc_1^y + dc_1^o + h_1 C_M \left(dT_1^y + \mu_1 dT_1^o\right)$$

Clearly situations where dT_1^o is negative are favourable for lower E_1. Figure 2 shows that strong intraperiod and interperiod complementarities no longer imply a fall in labor supply. When the straight line $d\ell_1 = 0$ is decreasing, labor supply increases as soon as one of the two elasticities is high enough. When it is increasing, we need to have strong intraperiod substitutability (T_1^o is likely to decrease). Consequently, a Pareto-improving reform should be obtained if the intraperiod elasticity is high enough. We develop a numerical example in order to show that reasonable values of intraperiod elasticity σ_1^o can lead to a rise in type-2 agents lifetime utility.

To set the parameters, we assume that the initial steady state matches the characteristics presented in Table 1. We also assume that there is no inheritance tax ($\tau^x = 0$) and that labor income and capital income tax rates are equal ($\tau^w = \tau^R = \frac{G+a}{Y}$). Parameters that represent efficiency of time transfers to the next generation are $\mu_1 = 0.9$ and $\mu_2 = 0.5$. This allows to reach a steady state where type-1 agents are the only one to make time transfers. We consider CES form for the production function of private firms, the home production functions and utility (see footnote of

Table 1: Targeted variables

Variable[1]	Value	Variable[1]	Value
$\sum_i (c_i^y + c_i^o)/Y$	0.535	p_1	0.9
G/Y	0.25	$\sum_i p_i \ell_i$	0.45
K/Y	0.215	x_2/Y	0.15
a/Y	0.1	$p_2 s_2/K$	0.4
Δ/Y	0	h_1	1
RK/Y	0.34	h_2	3
wL/Y	0.66	$p_2 h_2 \ell_2/L$	0.3

[1] $Y \equiv F(K, L)$.

Table 2). The time preference parameter γ_i is set in order to reach 1% per year, assuming that the periods have a 30-years length: $\gamma_i = (1,01)^{30} = 0.74$. We assume standard values of the elasticity of substitution between capital and efficient labor in the production function of the private sector ($\sigma^F = 0.5$) and of the intertemporal elasticity of substitution ($\sigma_i^v = 0.75$). Finally the intraperiod elasticities of substitution between consumption and domestic time are set to 2.5. Many papers have given estimations of this elasticity (see footnote 10): they belong to the interval $[1.4, 2.5]$. We consider the upper bound of this interval to show the existence of a set of parameters that yields a Pareto-improving reform. From the steady-state equations, we then deduce the values of the other parameters (see Table 2): technological parameters A and a^F, degrees of altruism β_i, and share parameters a_i^y and a_i^o, for $i = 1, 2$.

With these parameter values, the lines $dT_1^o = 0$, $dT_1^y = 0$, $d\ell_1 = 0$ and $dE_1 = 0$ are plotted in the plane (σ_1^v, σ_1^o) on Figure 3. In this example, the line $d\ell_1 = 0$ and $dE_1 = 0$ are increasing. Therefore, whatever σ_1^v, there exists a lower bound on σ_1^o above which the resources used by type-1 agents E_1 decreases with the tax reform.

With the values of the parameters considered in the numerical example, the point (σ_1^v, σ_1^o) is $(0.75, 2.5)$ and stands just below the line $dE_1 = 0$. Consequently, the tax reform leads type-1 agents to use more resources. Nevertheless, the efficiency conditions, $\alpha_i^{yM} < \alpha_i^y P^R$ for $i = 1, 2$, are satisfied leaving the possibility of a Pareto-improving reform.

In the following, we consider a tax reform that consists in an increase in the inheritance tax rate (from 0 to 0.05%), associated with a fall in the capital income tax rate (from 0.35% to 0.347%) that leaves the steady-state capital-labor ratio unchanged. The lump-sum transfer a is also unchanged, while the public debt adjusts in order to balance the government budget.

As shown in Table 3, both utilities increase with the tax reform despite the fact that E_1 increases. Bequest of type-2 agents is strongly reduced, as well as their savings. By contrast, savings by type-1 agents increase, leading to a higher capital stock. Labor supply of both types of agents also increase. For type-1 agents, this comes from the rise in time transfers (lower T_1^o). For type-2 agents, higher consumption when old leads people to consume less when young and then also to use less time for domestic production.

Table 2: Structural parameters

Parameter		Value
Production function[a]		
Elasticity of substitution between production factors	σ^F	0.5
Scale parameter	A	2.25
Share parameter of physical capital	a^F	0.16
Representative household		
Home production function when young[b]		
Elasticity of substitution between c_i^y and T_i^y	σ_i^y	2.5
Share parameter of market good c_1^y (type-1 agents)	a_1^y	0.48
Share parameter of market good c_2^y (type-2 agents)	a_2^y	0.34
Home production function when old[c]		
Elasticity of substitution between c_i^o and T_i^o	σ_i^o	2.5
Share parameter of market good c_1^o (type-1 agents)	a_1^o	0.47
Share parameter of market good c_2^o (type-2 agents)	a_2^o	0.71
Preferences		
Degree of altruism of type-1 agents	β_1	0.8
Degree of altruism of type-2 agents	β_2	0.97
Efficiency of time transfer (type-1 agents)	μ_1	0.9
Efficiency of time transfer (type-2 agents)	μ_2	0.5
Elasticity of substitution[d] between f_i^y and f_i^o	σ_i^v	0.75
Time preference	γ_i	0.74

Note: We consider CES production and utility functions:

[a] $F(K,L) = A\left(a^F K^{\rho^F} + (1-a^F)L^{\rho^F}\right)^{\frac{1}{\rho^F}}$, with $\rho^F = 1 - \frac{1}{\sigma^F}$.

[b] $f_i^y\left(c_i^y, T_i^y\right) = \left(a_i^y\left(c_i^y\right)^{\rho_i^y} + (1-a_i^y)\left(T_i^y\right)^{\rho_i^y}\right)^{\frac{1}{\rho_i^y}}$, with $\rho_i^y = 1 - \frac{1}{\sigma_i^y}$.

[c] $f_i^o\left(c_i^o, T_i^o\right) = \left(a_i^o\left(c_i^o\right)^{\rho_i^o} + (1-a_i^o)\left(T_i^o\right)^{\rho_i^o}\right)^{\frac{1}{\rho_i^o}}$, with $\rho_i^o = 1 - \frac{1}{\sigma_i^o}$.

[d] $v_i(x,y) = \left(1 - \frac{1}{\sigma_i^v}\right)^{-1}\left(x^{1-\frac{1}{\sigma_i^v}} + \gamma_i y^{1-\frac{1}{\sigma_i^v}}\right)$.

Table 3: Relative changes in the steady-state variables

Variable	Change (%)	Variable	Change (%)
c_1^y	0.03	c_2^y	-0.11
c_1^o	1.08	c_2^o	0.35
T_1^y	0.03	T_2^y	-0.11
T_1^o	-0.18	x_2	-71.21
ℓ_1	0.27	ℓ_2	0.08
s_1	0.58	s_2	-12.23
U_1	0.02	U_2	0.01
E_1	7.10^{-4}	K	0.22

Notice that the fall in bequests of type-2 agents has no dramatic consequence on their welfare, since it increases with the tax reform. Nevertheless, increase in utility is stronger for type-1 agents in terms of relative change but also in absolute value ($+1.46.10^{-3}$ for U_1 and $+5.10^{-4}$ for U_2). The

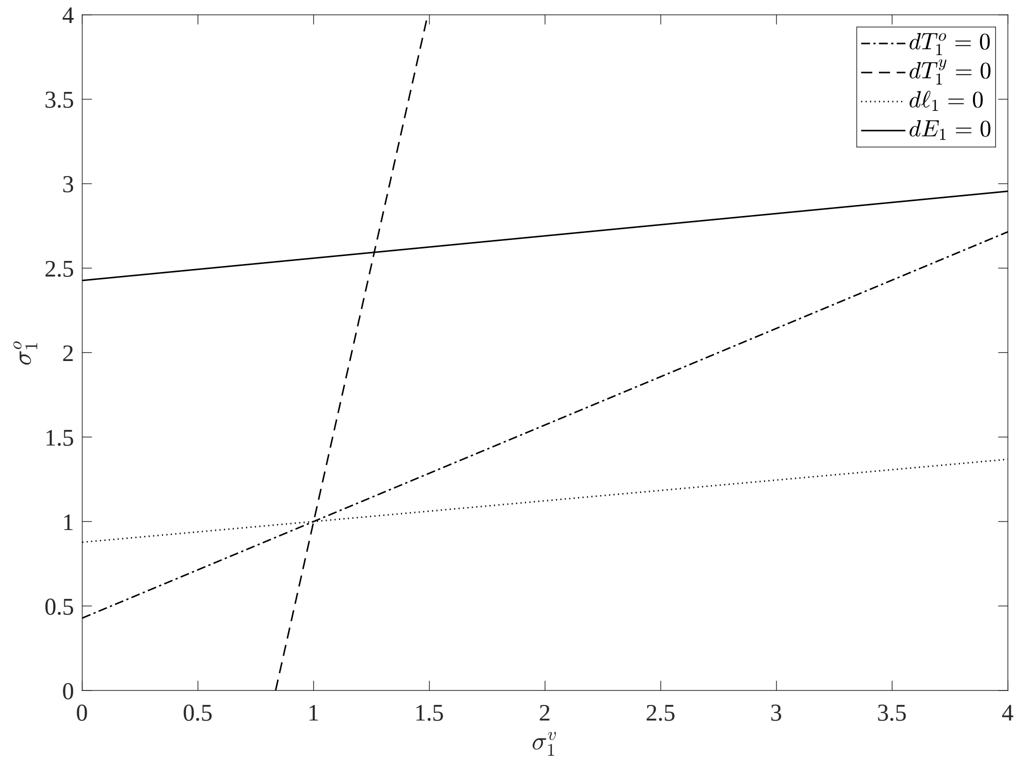

Figure 3: Consequence of the tax reform on resources used by type-1 agents

Note: Parameters are set to the values given in Tables 1 and 2. Utility of type-1 agents increases for all values of the elasticities of substitution considered on the graph (between 0 and 4). As represented in Figure 2, dT_1^o is positive under the line $dT_1^o = 0$, dT_1^y is positive on the left of the line $dT_1^y = 0$, $d\ell_1$ is positive above the line $d\ell_1 = 0$ and, finally, dE_1 is negative above $dE_1 = 0$.

tax reform then allows for a fall in wealth inequalities (strong reduction in bequest and capital income of type-2 dynasties; increase in capital income of type-1 agents) and a slight reduction in welfare inequalities.

4.4.3 Pareto improvement along the transitional dynamics

We now intend to highlight that long-term efficiency gains are not a resource cutback on early generations. To do so, we need to compute the welfare impact of the tax reform along the transitional dynamics. Welfare of a type-i agent who belongs to generation t corresponds to the infinite sum

$$W_{it} = \sum_{\theta=t}^{+\infty} \beta_i^{\theta-t} v_{i\theta}$$

where $v_{i\theta}$ is type-i lifetime utility in generation $\theta \geq t$. Then a Pareto improvement is achieved if the tax reform does not reduce W_{it}, for any agent from generation $t \geq -1$, and increases W_{it} for at least one agent.

In the numerical example considered above, steady-state public debt decreases from 0 to -0.01 as a percentage of current GDP. This means that some resources have been taken to previous generations. We consider a tax reform that consists in an increase in the inheritance tax rate from zero and a fall in the capital income tax rate from period 1, in order to keep the capital-labor ratio unchanged in the long run. Starting from no inheritance tax and a uniform tax rate on labor and capital incomes ($\tau^R = \tau^w = \bar{\tau} \equiv \frac{G+a}{Y}$), we consider the following values of the tax instruments after the reform:

- $\tau_t^x = \tau^x = 0.05$ for any $t \geq 0$
- $\tau_t^R = \tau^R$ for any $t \geq 1$, where $\tau^R = 1 - \frac{1-\bar{\tau}}{1-\tau^x}$
- τ^w is unchanged (equal to $\bar{\tau}$) for any $t \geq 0$.

We consider a tax reform where the government wants to stabilize the public debt to a new steady state value from period 0. The lump-sum transfer a_t adjusts to balance government budget from period to period and converges towards its value at the initial steady state ($a = 0.1\ Y$, see Table 1). Numerical results are presented in Figures 4, 5 and 6. From Figure 6, one can see that all generations benefit from the tax reform. Nevertheless, type-1 agents in generation 0 experience a loss in their lifetime utility, mainly because the wage rate when young is reduced.

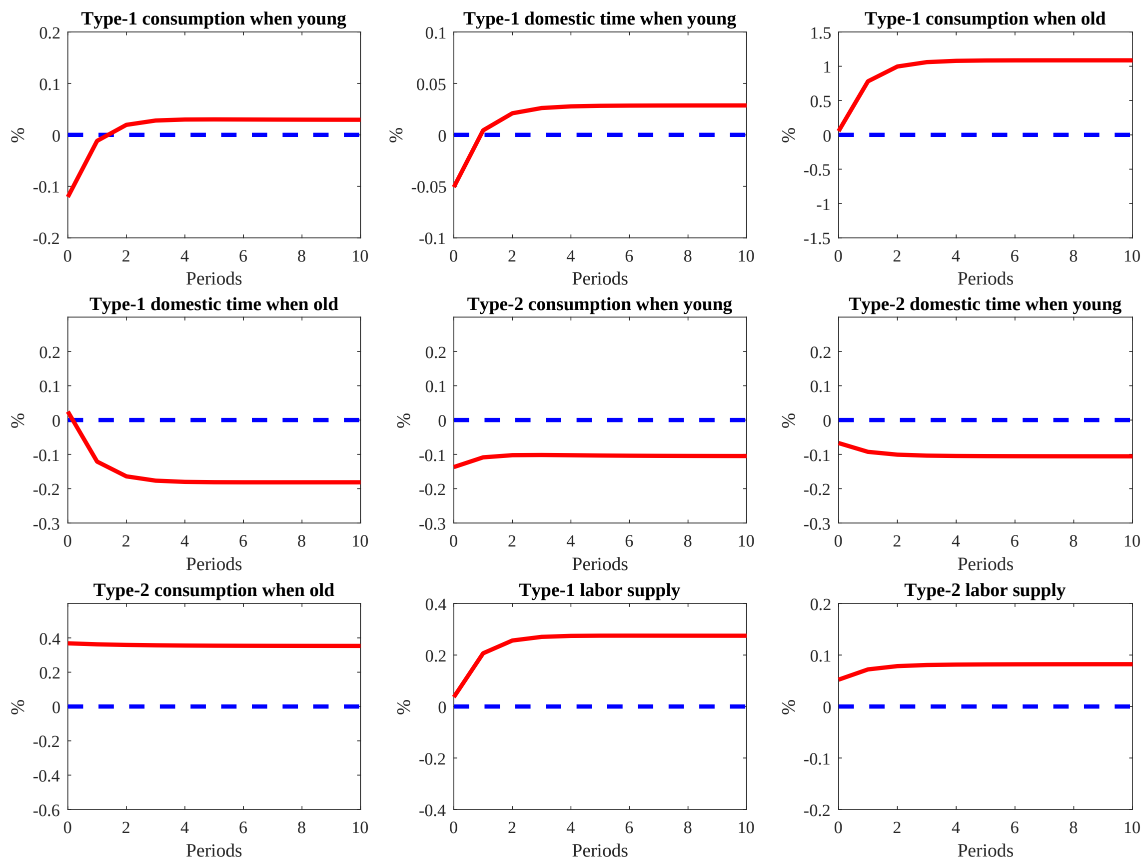

Figure 4: Tax reform: Consumptions, domestic times and labor supply

Note: Bold lines correspond to relative change in the variable considered.

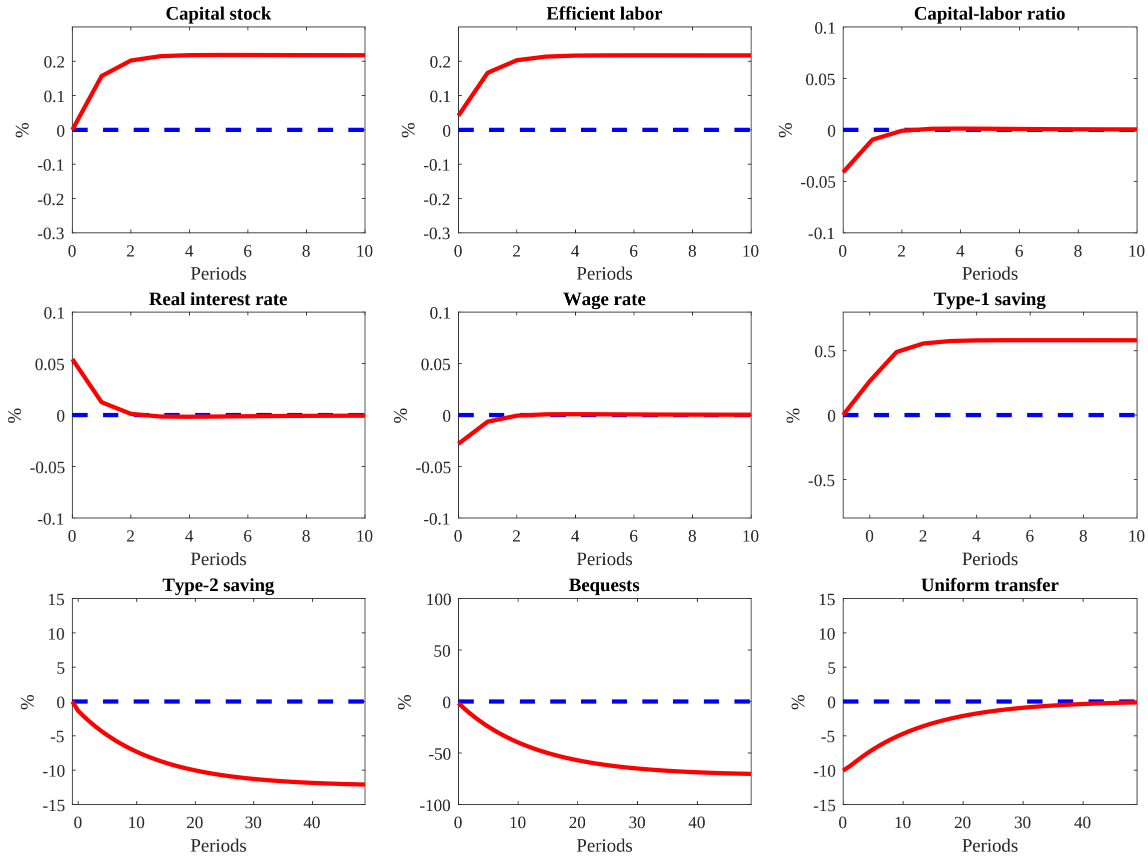

Figure 5: Tax reform: Capital, efficient labor, savings, bequests and uniform transfer
Note: Bold lines correspond to relative change in the variable considered.

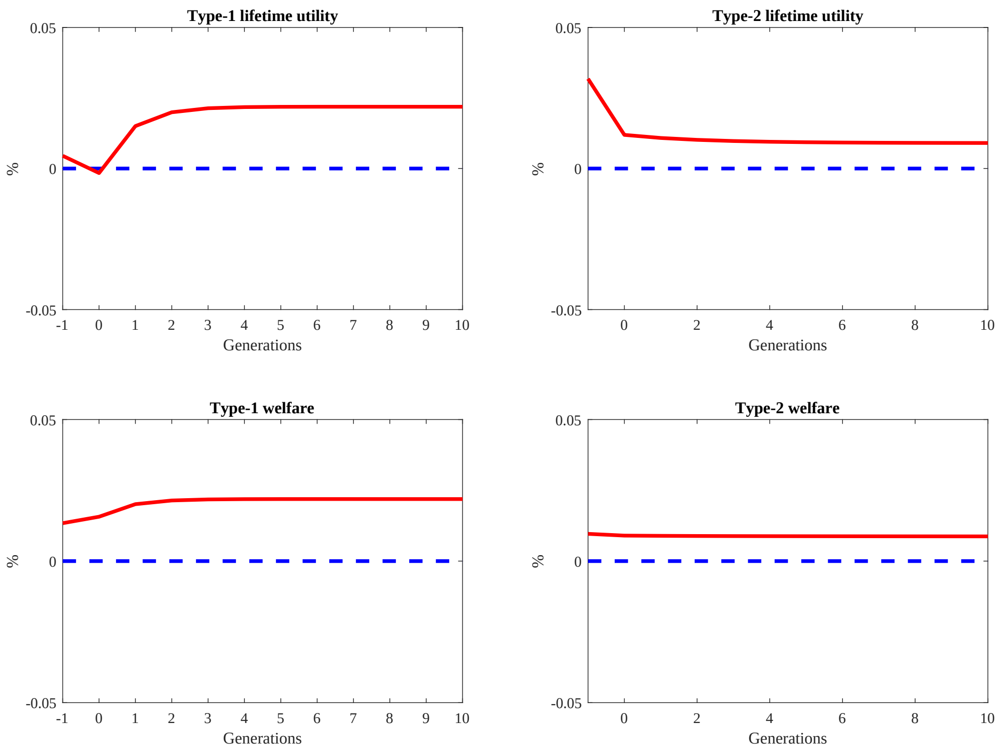

Figure 6: Tax reform: Lifetime utilities and individual welfare

Note: Bold lines correspond to relative change in the variable considered.

5 Conclusion

We have considered a fiscal reform that combines increase in inheritance taxation and decrease in capital income taxation, leaving the capital-labor ratio unchanged. Such a tax reform allows to separate capital accumulation issues from distributional issues within and between generations. However, with inelastic labor supply, the reform benefits to life-cyclers and is detrimental for dynasties that leave bequests. Keeping the capital-labor constant implies that disposable resources for consumption also remain constant in steady state. The reform then only modifies resource allocation without efficiency gain.

Introducing elastic labor supply and family time transfers allows to take account of potential change in aggregate resources for market-good production. This makes Pareto-improving reform possible. We give a numerical example where utilities of both types of dynasties increase in the long-run and also along the transitional dynamics. Moreover, inequalities in terms of wealth and utility are also reduced.

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6 Appendix

6.1 First-order conditions of the dynastic problem with time transfers

Plugging c_{it}^y, ℓ_{it} and c_{it}^o from the budget constraints (18)-(20) into the utility function (17), we get the following marginal conditions, for $i = 1, 2$:

- with respect to s_{it}

$$-v_{i,f_{it}^y} f_{i,c_{it}^y}^y + \left(1 - \tau^R\right) R_{t+1} v_{i,f_{it+1}^o} f_{i,c_{it+1}^o}^o = 0$$

- with respect to T_{it}^y (assuming interior solution $\mu_i \left(1 - T_{it}^o\right) < T_{it}^y < 1 + \mu_i \left(1 - T_{it}^o\right)$)

$$-\left(1 - \tau^w\right) h_i w_t f_{i,c_{it}^y}^y + f_{i,T_{it}^y}^y = 0$$

- with respect to x_{it}

$$-v_{i,f_{it}^o} f_{i,c_{it}^o}^o + \beta_i \left(1 - \tau^x\right) v_{i,f_{it}^y} f_{i,c_{it}^y}^y \leq 0, \ = 0 \text{ if } x_{it} > 0$$

- with respect to T_{it}^o (assuming $T_{it}^o > 0$)

$$v_{i,f_{it}^o} f_{i,T_{it}^o}^o - \beta_i \mu_i \left(1 - \tau^w\right) h_i w_t v_{i,f_{it}^y} f_{i,c_{it}^y}^y \geq 0, \ = 0 \text{ if } T_{it}^o < 1$$

At steady state, marginal conditions with respect to x_{it} imply

$$\frac{v_{i,f_i^o} f_{i,c_i^o}^o}{v_{i,f_i^y} f_{i,c_i^y}^y} \geq \beta_2 \left(1 - \tau^x\right) > \beta_1 \left(1 - \tau^x\right)$$

Therefore, type-1 agents don't leave positive bequests.

6.2 Proof of Proposition 5

Step 1. From the definition of σ_i^v (25) and equation (22) for both types of agents, we get

$$\sigma_i^v d \ln \left(P^R \frac{f_{i,c_i^y}^y \left(1, \frac{T_i^y}{c_i^y}\right)}{f_{i,c_i^o}^o \left(c_i^o, 1\right)} \right) = d \ln \left(\frac{c_i^y f_i^y \left(1, \frac{T_i^y}{c_i^y}\right)}{f_i^o \left(c_i^o, 1\right)} \right)$$

where T_i^y / c_i^y is constant. Differentiation of the latter equation leads to

$$\frac{dc_i^y}{c_i^y} = \sigma_i^v \frac{dP^R}{P^R} + \left(\alpha_i^o + \left(1 - \alpha_i^o\right) \frac{\sigma_i^v}{\sigma_i^o}\right) \frac{dc_i^o}{c_i^o} \tag{32}$$

using the following relationships deduced from linear homogeneity of f_i^y and f_i^o:

$$\frac{df^y_{i,c_i^y}}{f^y_{i,c_i^y}} = 0, \quad \frac{df^o_{i,c_i^o}}{f^o_{i,c_i^o}} = \frac{f^o_{i,c_i^o c_i^o} c_i^o}{f^o_{i,c_i^o}} \frac{dc_i^o}{c_i^o}, \quad \text{and} \quad \frac{-c_i^o f^o_{i,c_i^o c_i^o}}{f^o_{i,c_i^o}} = \frac{1-\alpha_i^o}{\sigma_i^o}$$

where $\alpha_i^o = c_i^o f^o_{i,c_i^o} \,/\, f_i^o$.

Step 2. We differentiate

- the resource constraint (21), rewritten as

$$\sum_{i=1}^{2} p_i \left(\frac{c_i^y}{\alpha_i^{yM}} + c_i^o \right) = C_M \bar{h}$$

- the intertemporal budget constraint of type-1 agents, rewritten as

$$\frac{c_1^y}{\alpha_1^y} + P^R c_1^o = P_1^y + a$$

We get

$$\begin{aligned} p_2 \left(\frac{dc_2^y}{\alpha_2^{yM}} + dc_2^o \right) &= -p_1 \left(\frac{dc_1^y}{\alpha_1^{yM}} + dc_1^o \right) = -p_1 dE_1 \\ \frac{dc_1^y}{\alpha_1^y} + P^R dc_1^o + c_1^o dP^R &= 0 \end{aligned}$$

Combined with (32), one gets

$$\begin{aligned} dc_1^y &= \frac{c_1^y}{D_1} \left(\frac{\alpha_1^o}{\sigma_1^v} + \frac{1-\alpha_1^o}{\sigma_1^o} - 1 \right) \left(\frac{-dP^R}{P^R} \right) \\ dc_1^o &= \frac{c_1^o}{D_1} \left(\frac{c_1^y}{\alpha_1^y P^R c_1^o} + \frac{1}{\sigma_1^v} \right) \left(\frac{-dP^R}{P^R} \right) \\ dc_2^y &= \frac{c_2^y}{p_2 c_2^o D_2^M} \left[\left(\frac{\alpha_2^o}{\sigma_2^v} + \frac{1-\alpha_2^o}{\sigma_2^o} \right) [-p_1 dE_1] - p_2 c_2^o \left(\frac{-dP^R}{P^R} \right) \right] \\ dc_2^o &= \frac{1}{p_2 D_2^M} \left[\frac{1}{\sigma_2^v} [-p_1 dE_1] + \frac{p_2 c_2^y}{\alpha_2^{yM}} \left(\frac{-dP^R}{P^R} \right) \right] \end{aligned}$$

where, for $i = 1, 2$,

$$\begin{aligned} D_i &\equiv \frac{c_i^y}{\alpha_i^y P^R c_i^o} \left(\frac{\alpha_i^o}{\sigma_i^v} + \frac{1-\alpha_i^o}{\sigma_i^o} \right) + \frac{1}{\sigma_i^v} > 0 \\ D_i^M &\equiv \frac{c_i^y}{\alpha_i^{yM} c_i^o} \left(\frac{\alpha_i^o}{\sigma_i^v} + \frac{1-\alpha_i^o}{\sigma_i^o} \right) + \frac{1}{\sigma_i^v} > 0 \end{aligned}$$

Step 3. Result (i)-(iii) are then deduced from

$$\begin{aligned} d\ell_i &= -dT_i^y = -\frac{T_i^y}{c_i^y} dc_i^y, \text{ for } i = 1, 2 \\ dE_1 &= dc_1^y + C_M h_1 dT_1^y + dc_1^o = \frac{dc_1^y}{\alpha_1^{yM}} + dc_1^o \end{aligned}$$

Finally, we derive the condition for type-2 agent utility to increase. Assuming $T_i^o = 1$ in (17), one gets

$$dv_i = \left(f_{i,c_i^y}^y dc_i + f_{i,T_i^y}^y dT_i^y \right) v_{i,f_i^y} + f_{i,c_i^o}^o dc_i^o \ v_{i,f_i^o}$$

Using the first-order conditions of the consumer problem (22)-(24), dv_i has the same sign as

$$dc_i^y + P_i^y dT_i^y + P^R dc_i^o = \frac{dc_i^y}{\alpha_i^y} + P^R dc_i^o$$

Then straightforward calculations lead to result (iv).

6.3 Incidence of the tax reform on type-1 agents variables in steady-state

Step 1. From equation (22) and the definition of σ_1^v (25), we get

$$\sigma_1^v d\ln \left(P^R \frac{f_{1,c_1^y}^y \left(1, \frac{T_1^y}{c_1^y}\right)}{c_1^o f_{1,c_1^o}^o \left(1, \frac{T_1^o}{c_1^o}\right)} \right) = d\ln \left(\frac{c_1^y f_1^y \left(1, \frac{T_1^y}{c_1^y}\right)}{f_1^o \left(1, \frac{T_1^o}{c_1^o}\right)} \right) \tag{33}$$

where T_1^y / c_1^y is constant, while T_1^o / c_1^o changes with P_1^o. Since $d\ln P_1^o = d\ln P_1^y - d\ln P^R$ and $dP_1^y = 0$, the definition of σ_1^o (see equation (26)) implies

$$\frac{dc_1^o}{c_1^o} = \frac{dT_1^o}{T_1^o} + \sigma_1^o \left(\frac{-dP^R}{P^R} \right). \tag{34}$$

Then (33) implies

$$\frac{dc_1^y}{c_1^y} = \frac{dT_1^y}{T_1^y} = \frac{dT_1^o}{T_1^o} + \alpha_1^o \left(\sigma_1^o - \sigma_1^v \right) \left(\frac{-dP^R}{P^R} \right) \tag{35}$$

using the following relationships deduced from linear homogeneity of f_1^y and f_1^o:

$$\frac{df_{1,c_1^y}^y}{f_{1,c_1^y}^y} = 0, \ \frac{df_{1,c_1^o}^o}{f_{1,c_1^o}^o} = \frac{f_{1,c_1^o c_1^o}^o c_1^o}{f_{1,c_1^o}^o} \left(\frac{dc_1^o}{c_1^o} - \frac{dT_1^o}{T_1^o} \right) \text{ and } \frac{-c_1^o f_{1,c_1^o c_1^o}^o}{f_{1,c_1^o}^o} = \frac{1 - \alpha_1^o}{\sigma_1^o}$$

where $\alpha_1^o = c_1^o f_{1,c_1^o}^o \, / \, f_1^o$.

Step 2. Furthermore, differentiating the intertemporal budget constraint of type-1 agents

$$c_1^y + P_1^y T_1^y + P^R c_1^o + \mu_1 P_1^y T_1^o = P_1^y (1 + \mu_1) + a$$

leads to (using equations (34) and (35)):

$$\frac{dT_1^o}{T_1^o} = \frac{\alpha_1^o (1 - \Gamma_1 - \sigma_1^o + \Gamma_1 \sigma_1^v)}{1 + (1 - \Gamma_1) \frac{(1-\beta_1)(1-\alpha_1^o)}{\beta_1}} \left(\frac{-dP^R}{P^R} \right) \tag{36}$$

where

$$\Gamma_1 \equiv \frac{c_1^y}{\alpha_1^y \Omega_1} \text{ and } \Omega_1 \equiv \frac{c_1^y}{\alpha_1^y} + P^R \frac{c_1^o}{\alpha_1^o}$$

Thus, $dT_1^o > 0$ is equivalent to

$$\sigma_1^o > \Gamma_1 \sigma_1^v + 1 - \Gamma_1.$$

Then, from (34), one easily checks that $dc_1^o > 0$. Moreover, from (35), one gets that $dc_1^y > 0$ (or $dT_1^y > 0$) is equivalent to

$$\sigma_1^o > \sigma_1^v \left(\frac{\beta_1}{(1 - \beta_1)(1 - \alpha_1^o)} + 1 \right) - \frac{\beta_1}{(1 - \beta_1)(1 - \alpha_1^o)}$$

Finally, one can compute the change in labor supply of type-1 agents $d\ell_1 = -dT_1^y - \mu_1 dT_1^o$ which is positive iff

$$\sigma_1^o > 1 + \frac{\frac{1}{1-\alpha_1^y} - \frac{\beta_1}{1-\alpha_1^o} - (1 - \beta_1)}{\frac{1}{1-\alpha_1^y} - \Gamma_1 (1 - \beta_1)} \Gamma_1 (\sigma_1^v - 1)$$

Step 3. We now derive the result in Proposition 6 about type-1 agents lifetime utility. Differentiating steady-state life-cycle utility v_1 and using marginal conditions (22)-(24), dv_1 has the same sign as:

$$dc_1^y + P_1^y dT_1^y + P^R dc_1^o + \beta_1 \mu_1 P_1^y dT_1^o \tag{37}$$

Replacing (34)-(35) in (37) implies that dv_1 has the same sign as

$$\frac{dT_1^o}{T_1^o} + (\sigma_1^o - \Gamma_1 \sigma_1^v) \alpha_1^o \left(\frac{-dP^R}{P^R} \right)$$

Then, one replaces $\frac{dT_1^o}{T_1^o}$ by its expression in equation (36). Since the tax reform implies $dP^R < 0$, one gets that life-cycle utility of type-1 agents increases iff

$$1 + (\sigma_1^o - \Gamma_1 \sigma_1^v) \frac{(1 - \beta_1)(1 - \alpha_1^o)}{\beta_1} > 0$$

which leads to the result.