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Precise versus imprecise datasets: revisiting ambiguity attitudes in the Ellsberg paradox

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Precise versus imprecise datasets: revisiting ambiguity attitudes in the Ellsberg paradox*

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Abstract

Most of real-life decision problems are usually characterized by uncertainty regarding the probability distribution of outcomes. This article experimentally investigates individual's attitude towards partial ambiguity, defined by situations where more or less precise sets of observations are available to the agents. Drawing on Ellsberg's 2-urns experiment, I depart from the classic design and describe both urns by datasets with different degrees of precision. As a result, most subjects behave in conformity with the Expected Utility Hypothesis although a significant proportion of choices can still be interpreted as an expression of non-neutral ambiguity attitude. I calculate an individual score of ambiguity-sensitivity which suggests a significant bias towards ambiguity-aversion, but weaker than in the related literature.

Keywords: Preferences for information precision, Ambiguity, Ellsberg paradox, Experiment.

JEL Classification Codes: C91, D81

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1 Introduction

1.1 Motivation

In line with Knight's (1921) distinction between risk and uncertainty, the famous Ellsberg paradox (1961) was a first experimental attempt to illustrate the failure of the Expected Utility hypothesis to predict individual behavior in an uncertain environment. Consider two urns: one urn is filled with 50 red and 50 black balls while the other urn is filled with 100 black and red balls in unknown proportions. Whether the bet is on black or on red, most people prefer betting on the urn with known composition. It is then impossible to infer additive probabilities from these choices. This behavior, known as the Ellsberg paradox, provides evidence of ambiguity aversion since decision makers (henceforth, DMs) are reluctant to bet on events with unknown probabilities.

While the classic Ellsberg's thought-experiment has been replicated several times (Camerer and Weber, 1992 provide an overview), the present research deals with the question of decision making in ambiguous situations when information about the urns is provided *in the form of datasets*. In the classic version of the experiment, exact probabilistic information is provided for one of the urns (the risky urn) whereas no information is given for the other urn (the ambiguous urn). However, this type of information might not be readily available in real-life situations. Indeed, DMs usually observe data generated by the process at hand and have to make a decision based on more or less precise datasets. For instance, in traditional societies, farmers are urged to adapt to climate change by adopting new technologies, for which data are often scarce. Eichberger and Guerdjikova (2012) show that the availability of information on returns of the new technology is crucial for motivating them to modify their practices. In the absence of precise and relevant information and if the share of ambiguity-averse agents is too high, most agents prefer the known prospect (the traditional method) to the unknown alternative (the new technology), hence innovation in the society is slow, resulting in an inefficient equilibrium. Financial trading provides also a relevant example to illustrate the consequences of lack and imprecision of information on portfolio choices.

Indeed, market participants have only partial information regarding expected returns of traded assets. As a result, they might engage in imitative behavior if they believe that other DMs have some important additional information. In particular, Ford *et al.* (2013) prove that herding can be rational for agents with Choquet Expected Utility preferences. However, such behavior contributes to the formation of bubbles, and to the subsequent crisis resulting from the burst of bubbles.

In an attempt to describe decision making in ambiguous environments, I draw on the Ellsberg's experiment with two urns and two colors of balls and I describe both urns by sets of data (Gilboa and Schmeidler, 2001). I investigate the Ellsberg paradox in this set-up of "partial ambiguity". In particular, I ask whether there is still a sizable proportion of ambiguity-averse DMs in this context of partial ambiguity. Consider for instance the following set-up : there are two urns containing 200 balls each. Each of the urns contains an unknown proportion of blue and red balls. Both urns are described by datasets of equal frequencies but different number of observations (i.e., different information precision). In urn 1, 5 blue balls and 5 red balls have been randomly drawn with replacement. In urn 2, 50 blue balls and 50 red balls have been randomly drawn with replacement. Which urn is preferred when betting on blue? On red? A frequentist (or a Bayesian with a prior 50/50 on the composition of the urn) would be indifferent between both urns. In contrast, an ambiguity-averse subject might prefer to bet on the urn with more draws regardless of the color of the ball. Similarly, subjects who choose the least precise urn for both bets exhibit ambiguity-loving preferences. Such preferences are axiomatized in Eichberger and Guerdjikova (2013). The preferences for information precision are investigated for different frequencies of blue and red balls in the datasets describing the urns. There, I ask whether ambiguity attitudes change with the proportion of balls of the winning color in the dataset, which ranges from 0.1 to 0.9 in the experiment.

I conducted a lab experiment to give insights into the individual preferences for information precision. The goal is two-fold: First, I describe individual decision making in the context of partial ambiguity. In particular, I examine whether individual choices can be reconciled with Savage's Subjective Expected Utility (SEU) model or whether non-neutral ambiguity attitudes are needed to explain preferences for data-sources with various degrees of precision. Indeed, the experiment is designed in such a way as to allow a classification of subject' answers into 4 classes of behavior: frequentist, Bayesian, ambiguity-averse and ambiguity-lover (Section 2.2 describes these patterns of preferences in detail). Second, I estimate the extent to which agents' preferences deviate from the SEU predictions by calculating an interpersonal score of ambiguity-sensitivity.

This experiment yields two key findings:

1. Among experimental answers satisfying a standard property of monotonicity, around 2/3 can be explained by the SEU model. The remaining 1/3 contradict the SEU and can be interpreted as an expression of non-neutral ambiguity attitude. Among them, 2/3 displays ambiguity-aversion and 1/3 of answers can be classified as ambiguity-loving.
2. The average score of ambiguity-sensitivity in the experiment is slightly biased towards ambiguity-aversion.

To summarize, when both prospects are partially described, most of experimental choices are compatible with SEU maximization. Nevertheless, non-neutral ambiguity attitudes are still required to explain preferences in the described context of partial ambiguity since I observe a small but non-negligible share of ambiguity-sensitive agents.

The remainder of the paper is organized as follows. The relevant research is reviewed next. Section 2 presents the experimental design and details the different ambiguity attitudes in the current set-up. The results of the experiment are discussed in section 3. Section 4 concludes. Appendix A provides supplementary material for the experiment. Appendix B contains complementary tables and figures.

1.2 Related literature

The present research builds on the seminal work of Ellsberg (1961), which illustrates the difference between risk and ambiguity. Here, I depart from the classic experiment and describe both urns by sets of observations. The aim of this study is to characterize decision making in ambiguous situations when the information is provided in the form of datasets. In particular, individual decisions in such realistic frameworks have been modelled by Gilboa and Schmeidler (2001) in the Case-Based Decision Theory (CBDT), designed to explain the effect of the available observations (cases) on agents' evaluation of actions. Drawing on Hume's principles (1748), the authors state that:

> "[...] the main reasoning technique that people use is drawing analogies between past cases and the one at hand.
> Applying this idea to decision making, we suggest that people choose acts based on their performance in similar problems in the past." (Gilboa and Schmeidler, 2001, p49)

Indeed, the key idea that underlies the CBDT is the following: information arrives in the form of data, which might be more or less precise and more or less relevant for the decision to be made. As a result, the CBDT provides an original paradigm to model decision making when the probabilities of outcomes are not salient and cannot be easily constructed. Furthermore, Eichberger and Guerdjikova (2013) axiomatize an original version of the alpha-maxmin decision model (Ghirardato *et al.*, 2004) combining the case-based approach with the theoretical literature on decision under ambiguity. In their framework, DMs are characterized by a representation of preferences described by a von-Neumann-Morgenstern subjective utility function over outcomes and non-additive beliefs which associate a set of priors with each data set. More specifically, my experimental design is inspired by their example of betting on a draw from an urn (Eichberger and Guerdjikova, 2013, Example 1, pp1437–1438), which provides a tractable testing setup of the Ellsberg's two-colors experiment when information about both urns consists of a set of observations. In particular, when datasets exhibit identical frequencies but differ in precision, they predict that ambiguity-averse (ambiguity-loving) DMs prefer the more (less) precise dataset, regardless of the bet. This paper is meant to provide experimental assessment of preferences under this framework.

Although numerous experiments have evidenced the existence of Ellsberg paradox (Camerer and Weber, 1992), the authors have principally focused on risk versus full ambiguity, and little is known about individual behavior with intermediate stages of knowledge. Therefore, this research proposes an original version of the Ellsberg's two-colors experiment which builds upon CBDT by providing DMs with samples of observations. Recent research has shown that CBDT provides a relevant framework to model decision making in experimental settings under ambiguity (Grosskopf *et al.*, 2015; Bleichrodt *et al.*, 2017).

Besides, several lab studies have suggested that DMs are ambiguity averse when faced with imprecise information. Arad and Gayer (2012) estimate the degree of confidence in observations that are imprecise. In their experiments, there are as many observations in the precise set as observations in the imprecise set, but some cases in the imprecise dataset are irrelevant for the decision problem. They deduce from experimental results that imprecision of information is a source of ambiguity aversion whereas subjects act as if there was no ambiguity with precise information. The present study departs from their analysis in two main respects: First, in my experiment, all observations in both the precise and imprecise sets are relevant for the choice to be made. The precision of information is determined by the length of the dataset and hence, the precise dataset contains more observations than the imprecise one. Second, they consider only short sequences of observations (8 cases), which is comparable to the imprecise sets in my experiment, whereas the precise sets here contain significantly more observations (100 cases).

In Baillon *et al.* (2017), the authors study the effect of learning new information on decisions to trade options with payoffs dependant on stock prices. They use Initial Public Offerings (for which no prior information on returns is available) that provide an adequate natural framework to study the effect of information on beliefs and ambiguity attitudes. Although they report only little ambiguity-aversion, they find that it does not decrease with information received, yielding them to conclude that ambiguity-aversion is a stable characteristic of DM's preferences. As opposed to their study, I present a lab experiment which allows to control for the frequencies of good/bad outcomes in the datasets. Consequently, it is possible to investigate the influence of reported frequencies on preferences.

Lastly, Chew *et al.* (2017) describe attitudes towards variants of partial ambiguity: two-point ambiguity (two possible compositions of red and black cards in a deck) and disjoint ambiguity (union of disjoint intervals). They observe aversion to increasing size of ambiguity in terms of the number of possible compositions of decks. Although their study provides relevant intuitions, their definition of partial ambiguity differs significantly from the one considered in the present paper. Indeed, they provide partial description about the underlying probabilities of the prospects whereas participants in my experiment have to learn them from observations. Description and statistical inference might induce different behaviors, as evidenced in the literature on the description-experience gap (Barron and Erev, 2003; Hau *et al.*, 2010; Dutt *et al.*, 2014).

In these three experimental papers, the ambiguity attitudes are investigated via Certainty Equivalent measurements. By contrast, participants are confronted with binary choices between prospects in my experiment. Although procedure invariance predicts that normatively equivalent procedures should give the same ranking between options, preference reversals have been widely reported by experimental psychologists and economists (Lichtenstein and Slovic, 1971, 1973; Grether and Plott, 1979). Hence, this paper aims to complement the study of ambiguity attitudes and precision of information with an alternative method for eliciting preferences.

2 Experimental design

2.1 Stimuli

In the spirit of Ellsberg's two-colors urns, I design a short experiment of binary questions on preferences over pairs of bags containing balls. Each bag in the experiment contains 200 balls which can be either blue or red. Each bag is described by a set of observations. The datasets inform participants on previous random draws with replacement from bags and can be either relatively *Precise* (100 draws) or *Imprecise* (10 draws). Keeping the precision (i.e., the length) of the datasets constant makes it simpler for participants to mentally represent frequencies along the experiment. For the sake of brevity, in the following, the bags described by a precise (imprecise) dataset are denominated the precise (imprecise) bags[1]. Given the datasets, the participant is asked to choose his preferred bag to bet on blue and his preferred bag to bet on red.

The proportions of blue balls in datasets (p_B) range approximately from 0.1 to 0.9. In the experiment, p_B can take 5 values: $0.1, 0.3, 0.5, 0.7, 0.9$. These questions deal with simple probabilities that participants can easily mentally represent. For each proportion p_B, there are two pairs of bets. In each pair of bets, datasets describing bags exhibit similar but different frequencies. The imprecise datasets display identical frequencies in both pairs: the number of blue draws is simply given by $p_B * 10$ and the number of red draws is equal to $(1 - p_B) * 10$. On the other hand, the number of blue balls in the precise datasets is given by $p_B * 100 + 1$ in one pair of questions and by $p_B * 100 - 1$ in the second pair of bets[2].

The detailed questionnaire is presented in Table 1. The experiment consists of 10 pairs of questions presented in random order so as to avoid potential order effects. The bags differ in each pair of questions. Therefore, there are 20 bags in this experiment: 10 precise bags and 10 imprecise bags. In questions where the proportion p_B is indexed by a (+), the frequency of blue balls is higher in the precise dataset than in the imprecise one and in questions indexed by a (-) the precise dataset displays a smaller frequency of blue balls. For instance, consider the pairs of bets 0.1^+ and 0.1^-. In q1, the participant is asked to choose between two bags to bet on a blue draw. The precise bag is described by a dataset containing 100 draws, among which 11 are blue and 89 are red; and the imprecise bag is described by a dataset containing 10 draws, among which 1 is blue and 9 are red. In q2, the participant faces the same pair of bags but the winning ball is now red. In q3 and q4, the respondent considers a different pair of bags. The precise dataset contains 9 blue draws and 91 red draws; and the imprecise dataset displays 1 blue draw and 9 red

[1]Note that, during the experimental sessions, the experimenter referred to bags with neutral capital letters (bag A, bag B, bag C...).

[2]The frequency of blue balls in the precise datasets is different but set as close as possible to the frequency of blue balls in the imprecise datasets. The smallest variation is given by 1 observation since the number of balls can only be described by an integer.

p_B	Question	Number of draws				Bet
		Precise bag		Imprecise bag		
		Blue	Red	Blue	Red	
0.1^+	q1	11	89	1	9	Blue
	q2	11	89	1	9	Red
0.1^-	q3	9	91	1	9	Blue
	q4	9	91	1	9	Red
0.3^+	q5	31	69	3	7	Blue
	q6	31	69	3	7	Red
0.3^-	q7	29	71	3	7	Blue
	q8	29	71	3	7	Red
0.5^+	q9	51	49	5	5	Blue
	q10	51	49	5	5	Red
0.5^-	q11	49	51	5	5	Blue
	q12	49	51	5	5	Red
0.7^+	q13	71	29	7	3	Blue
	q14	71	29	7	3	Red
0.7^-	q15	69	31	7	3	Blue
	q16	69	31	7	3	Red
0.9^+	q17	91	9	9	1	Blue
	q18	91	9	9	1	Red
0.9^-	q19	89	11	9	1	Blue
	q20	89	11	9	1	Red

Table 1: Questionnaire

draws. Choosing the precise (imprecise) bag in the 4 bets provides convincing evidence of strict preference for information precision (imprecision). Therefore, the combination of the two pairs of questions per proportion p_B is used as a robustness test for the elicited preferences. Consequently, an agent whose preferences switch from the precise (imprecise) bag to the other bag for one out of the 4 bets would be interpreted as having only weak preferences for information precision (imprecision). The different attitudes towards ambiguity are described in detail in Section 2.2. Moreover, the slight difference in frequencies allows to constrain subjects to choose between bags, without including an indifference option. Enabling subjects to express indifference would raise technical problems when implementing one choice for real for payment[3].

2.2 Attitudes towards ambiguity

Ambiguity neutrality

Ambiguity-neutral attitude is revealed when the answers of a subject can be explained by SEU maximization. Frequentist and Bayesian DMs fall into this category.

A frequentist sets his beliefs equal to the frequency of observations in the data set. He is therefore indifferent between any two datasets with different precisions but identical frequencies. A frequentist is thus insensitive to the precision of information. For two datasets with different frequencies, he will always chose to bet on the bag with the highest frequency of the winning color whatever the length of the dataset. Table 2 details all the possible combinations of answers for each p_B in the experiment. For the analysis, the questions are here combined by color: for instance, the bets on blue for the proportion 0.1, where the precise dataset displays 11 blue balls in q1 and 9 blue balls in q3, are presented together. Below, the bet on red in q2 is coupled with the bet on red in q4. To bet on blue, a frequentist prefers the precise bag (P) in q1 and the imprecise bag (I) in q3, whereas, to bet on red, he prefers the imprecise bag in q2 and the precise bag in q4.

[3] It would require the experimenter to randomize between bags. However, as first noticed by Raiffa (1961), an ambiguity-averse agent may exhibit a strict preference for randomization and then use the randomization device to hedge. Theoretical debate is still ongoing (see, e.g., Epstein, 2010; Eichberger *et al.*, 2016).

A Bayesian subject is characterized by an updating belief function given by:

$$\lambda = \delta\mu + (1 - \delta)f \tag{1}$$

The updated belief (λ) is a convex combination of the prior regarding the composition of the bag (μ) and the observed frequencies in the dataset (f), weighted by a parameter (δ) that depends on and decreases in the number of observations. The shorter the dataset, the more one relies on his prior: hence, $\delta_I > \delta_P$. A Bayesian DM starts the experiment with a prior regarding the proportion of blue balls in the precise bag ($\mu_P(Blue)$) and a prior regarding the proportion of blue balls in the imprecise bag ($\mu_I(Blue)$). Consider for instance a DM with symmetric prior on the composition of both bags, which are natural and plausible beliefs without information on the composition of bags (Gilboa and Marinacci, 2016, p392). Hence, he assigns the same subjective probabilities to both colors in both bags, i.e., $\mu_P(Blue) = \mu_P(Red) = \mu_I(Blue) = \mu_I(Red) = 0.5$. Intuitively, with equal frequencies in both datasets:

$$\lambda_I(Blue) - \lambda_P(Blue) = (\delta_I - \delta_P)(\mu(Blue) - f(Blue)) \tag{2}$$

$$\lambda_I(Red) - \lambda_P(Red) = (\delta_I - \delta_P)(\mu(Red) - f(Red)) \tag{3}$$

The statistical information corresponds to a negative signal for blue balls if the frequency of blue balls in the datasets is less than the prior, i.e., $f(Blue) < \mu(Blue)$ in (2). Since $\delta_I > \delta_P$, the updated belief is greater in the imprecise bag than in the precise bag, i.e., $\lambda_I(Blue) > \lambda_P(Blue)$. Thus, the DM prefers the imprecise bag to bet on blue. If the frequency of blue balls is less than the prior, the frequency of red balls in the datasets is necessarily higher than the prior. Hence, a negative signal for blue balls translates to a positive signal for red balls for Bayesian DMs. Since $f(Red) > \mu(Red)$ in (3), the updated belief is higher in the precise bag than in the imprecise bag, i.e., $\lambda_P(Red) > \lambda_I(Red)$. Thus, to bet on red, he prefers the precise bag. This reasoning can be extended to the present design with *roughly* equal frequencies. Therefore, to bet on Blue in questions q1 and q3, a Bayesian DM with symmetric prior prefers the imprecise bag[4] (column Bay1 in Table 2). On the other hand, he chooses the precise bag to bet on red in q2 and q4.

In Table 2, the three columns (Freq, Bay1, Bay2) contain all the neutral answers. Column Bay1 displays the answers of Bayesians with priors greater than p_B and Column Bay2 contains the answers of Bayesians with priors smaller than p_B. Note that it is not possible to differentiate between a frequentist and a Bayesian with a prior *equal* to p_B. Indeed, they both prefer to bet on the dataset which displays the highest frequency of the winning color (whatever the precision of the datasets) and hence, their answers coincide in this case. This yields Bayesians to be counted as frequentists and consequently, the measure of frequentist answers might be biased upwards. However, this is of minor importance since both patterns of choice fall into the global category of ambiguity-insensitive preferences and hence it does not affect the partitioning between neutral and non-neutral ambiguity attitudes.

Ambiguity non-neutrality

The columns Pess, WP1, WP2, Opti, WO1 and WO2 in Table 2 describe the non-neutral ambiguity attitudes. Ambiguity non-neutral preferences contradict SEU maximization, meaning that no additive probabilities can be deduced from these patterns of choices. This is the case when, given two particular datasets with (almost) identical observed frequencies, the DM selects the same bag to bet on blue and to bet on red. For instance, choosing the bag associated to the dataset that contains more (less) draws for both bets indicates preference for information precision (imprecision). This is particularly salient because in each pair of questions, one dataset displays a higher frequency of blue balls while the frequency of red balls is higher in the other dataset. Hence, the slight difference in frequencies does not compensate for the difference in the lengths of the datasets. The combination of two pairs of bets with proportions p_B allows to measure the strength of preferences. Pessimistic choices (Pess) consist of strict preferences for information

[4]Formally, for q1: $\lambda_I(Blue) = \delta_I 0.50 + (1-\delta_I)0.10$ and $\lambda_P(Blue) = \delta_P 0.50 + (1-\delta_P)0.11$. $\lambda_I(Blue) - \lambda_P(Blue) = (\delta_I - \delta_P)0.40 - (1-\delta_P)0.01$ is positive for all $\delta_I \in [0;1]$ and all $\delta_P \in [0;1]$ such that $\delta_I > \delta_P$.

precision such that, given a particular p_B, the respondent prefers the precise bag for the 4 bets. WP1 and WP2 stand for weakly-pessimistic choices in the following sense: the subject chooses 3 times the most precise dataset out of 4 bets. Thus, pessimistic and weakly-pessimistic answers compose the general class of ambiguity-averse preferences. On the other hand, optimistic choices (Opti) are interpreted as strict preferences for information imprecision (4 choices in favour of the imprecise bag) and weakly-optimistic choices (WO1 and WO2) consist of weak preferences for information imprecision (3 choices in favour of the imprecise bag). Consequently, optimistic and weakly-optimistic answers define the general class of ambiguity-loving preferences.

Non-monotonic preferences

The last two columns of Table 2 (NM1 and NM2) gather the choices that do not satisfy the property of monotonicity. For instance, monotonicity requires that a DM who prefers the precise bag to bet on blue in q3 should also prefer it in q1 since 11/100 evidence for blue (q1) is always at least as good as 9/100 evidence for blue (q3). If the DM prefers the imprecise bag in q1, a symmetrical argument implies preference for the imprecise bag in q3. Hence, a DM with preference for the imprecise bag in q1 and preference for the precise bag in q3 (as described in the column NM1) violates monotonicity of preferences.

p_B	Q.	Bet	Precise bag		Imprecise bag		Types of answers										
			Blue	Red	Blue	Red	Freq	Bay1	Bay2	Pess	WP1	WP2	Opti	WO1	WO2	NM1	NM2
0.1	q1	Blue	11	89	1	9	*P*	*I*	*P*	*P*	*P*	*P*	*I*	*P*	*I*	*I*	
	q3	Blue	9	91	1	9	*I*	*I*	*P*	*P*	*I*	*P*	*I*	*I*	*I*	*P*	
	q2	Red	11	89	1	9	*I*	*P*	*I*	*P*	*P*	*I*	*I*	*I*	*I*		*P*
	q4	Red	9	91	1	9	*P*	*P*	*I*	*P*	*P*	*P*	*I*	*I*	*P*		*I*
0.3	q5	Blue	31	69	3	7	*P*	*I*	*P*	*P*	*P*	*P*	*I*	*P*	*I*	*I*	
	q7	Blue	29	71	3	7	*I*	*I*	*P*	*P*	*I*	*P*	*I*	*I*	*I*	*P*	
	q6	Red	31	69	3	7	*I*	*P*	*I*	*P*	*P*	*I*	*I*	*I*	*I*		*P*
	q8	Red	29	71	3	7	*P*	*P*	*I*	*P*	*P*	*P*	*I*	*I*	*P*		*I*
0.5	q9	Blue	51	49	5	5	*P*	*I*	*P*	*P*	*P*	*P*	*I*	*P*	*I*	*I*	
	q11	Blue	49	51	5	5	*I*	*I*	*P*	*P*	*I*	*P*	*I*	*I*	*I*	*P*	
	q10	Red	51	49	5	5	*I*	*P*	*I*	*P*	*P*	*I*	*I*	*I*	*I*		*P*
	q12	Red	49	51	5	5	*P*	*P*	*I*	*P*	*P*	*P*	*I*	*I*	*P*		*I*
0.7	q13	Blue	71	29	7	3	*P*	*I*	*P*	*P*	*P*	*P*	*I*	*P*	*I*	*I*	
	q15	Blue	69	31	7	3	*I*	*I*	*P*	*P*	*I*	*P*	*I*	*I*	*I*	*P*	
	q14	Red	71	29	7	3	*I*	*P*	*I*	*P*	*P*	*I*	*I*	*I*	*I*		*P*
	q16	Red	69	31	7	3	*P*	*P*	*I*	*P*	*P*	*P*	*I*	*I*	*P*		*I*
0.9	q17	Blue	91	9	9	1	*P*	*I*	*P*	*P*	*P*	*P*	*I*	*P*	*I*	*I*	
	q19	Blue	89	11	9	1	*I*	*I*	*P*	*P*	*I*	*P*	*I*	*I*	*I*	*P*	
	q18	Red	91	9	9	1	*I*	*P*	*I*	*P*	*P*	*I*	*I*	*I*	*I*		*P*
	q20	Red	89	11	9	1	*P*	*P*	*I*	*P*	*P*	*P*	*I*	*I*	*P*		*I*

Table 2: The 11-groups classification of choices

Note: Freq: Frequentist, Bay1: Bayesian with prior $> p_B$, Bay2: Bayesian with prior $< p_B$, WP1 and WP2: Weakly Pessimistic preferences, WO1 and WO2: Weakly Optimistic preferences, NM1 and NM2: Non-Monotonic preferences.

2.3 Procedures

The experiment was programmed and conducted with the experiment software z-Tree[5] (Fischbacher, 2007). The script was written in French. There were no particular requirements for participation and there was no time limit to answer the questions.

The timing of the experiment is composed of 6 stages:

1. First, I welcome the participants in the lab. Before entering the room, they randomly pick a number that determines where they sit in the room. They are also asked to sign a participation consent before the start of the session.

2. Second, I gave the instructions in the form of an oral presentation with slides[6].

3. Next, the participants are asked to answer a short comprehension test to check their understanding of the experiment[7]. No one can reach the next stage before I have made sure that everybody has answered correctly.

4. Stage 4 consists of filling out the questionnaire on choices of bags.

5. In a next stage, participants are asked to fill out a complementary questionnaire on socio-economic characteristics.

6. Lastly, participants are paid according to a Random-Lottery Incentive System (see section 2.4).

Regarding the technical aspect of the composition of the bags, I wrote a 3-step program on MATLAB: first, I generate 199 bags with composition ranging from (1 blue, 199 red), (2 blue, 198 red), ... to (199 blue, 1 red); second, for a random half of them, I randomly draw 10 balls with replacement, for the second half, 100 balls are randomly drawn with replacement; lastly, I keep the bags for which I obtain datasets with frequencies of interest. Detailed bags composition is provided in Appendix B (Table B1).

[5] `http://www.ztree.uzh.ch/en.html`

[6] Appendix A.1 provides a full set of the instructions.

[7] See Appendix A.2 for a screenshot of the comprehension questionnaire.

2.4 Incentives

The payment scheme includes a show-up fee of 5€ and in addition each participant plays out one of his choices for real according to a Random Incentive System (RIS). In concrete terms, when the subject has completed the whole questionnaire, one question is randomly selected and displayed on his computer. Hence, the question used for payment is not necessarily the same for all participants. The answer of the respondent is reminded and the subject is asked to reach the experimenter's office. There, he faces the pair of bags corresponding to the selected question and he has to randomly draw a ball from the bag chosen during the experiment. If he wins the bet, he is paid 18€ (including show-up fee); if he loses, he gets 6€ (including show-up fee). Participants implement real draws with real bags in order to persuade subjects that the procedure is truly not-manipulated by experimenters. This has an organisational cost since I display as many real bags as questions, i.e., 20 different bags in total.

Popularized by Savage (1954), the use of the RIS induces subjects to consider all questions of the experiment to be equally relevant while only paying one of them, which avoids potential portfolio effects resulting from the payment of all questions in the experiment. Hence, for a given research budget, this method allows to collect a large number of observations from each subject. Moreover, the RIS is easy to explain, to understand and to implement in the lab. For these reasons, it has been extensively used to incentivize experimental choices. The RIS requires that subjects perceive each decision as a single real choice (i.e., in isolation). Rather, they may perceive the whole experiment as a single choice problem involving compound lotteries, in which case the RIS does not allow to elicit true preferences (Holt, 1986; Karni and Safra, 1987; Bade, 2015). However, several experimental studies have concluded that isolation is verified when decision problems consist of simple binary choices (Starmer and Sugden, 1991; Hey and Lee, 2005a, 2005b). In the present experiment, the subject answers 20 binary questions, therefore I use the RIS which provides an appropriate mechanism to incentivize decisions.

p_B	**Neutral**		**Non-neutral**		
	Freq	Baye	AA	AL	NM
0.1	33	26	19	10	12
0.3	28	29	25	6	12
0.5	42	12	19	15	12
0.7	27	33	17	5	18
0.9	33	23	21	9	14
Total	*33*	*25*	*20*	*9*	*13*
	58		***29***		***13***

Table 3: Choices by categories (in %)

Note: Baye contains Bay1 and Bay2 subgroups; AA stands for ambiguity-averse and contains Pess, WP1 et WP2 subgroups; AL stands for ambiguity-loving and contains Opti, WO1 and WO2 subgroups; NM contains NM1 and NM2 subgroups.

3 Results

3.1 Neutral and non-neutral ambiguity attitudes

The percentages of answers falling in the 4 general classes of ambiguity attitudes are given in Table 3. The detailed frequencies of answers within the 11-groups classification are displayed in Table B2 in Appendix B.

On average, 87% of answers satisfy the standard property of monotonicity. Among them, 2/3 of choices are ambiguity-neutral and can be explained by SEU maximization. They are distributed among frequentist and Bayesian answers. The remaining 1/3 of answers satisfying monotonicity contradict the Expected Utility hypothesis and can be interpreted as an expression of non-neutral ambiguity attitude. Among them, 2/3 displays ambiguity-aversion and only 1/3 of the answers can be classified as ambiguity-loving.

Regarding non-neutral ambiguity attitudes, a share of 20% of the whole sample of answers displays ambiguity-aversion (pessimistic and weakly-pessimistic answers), suggesting that the difference in frequencies does not compensate for the precision of the datasets. For half of them, the bag associated with the most precise dataset is chosen for the two complementary bets even when the frequencies of the datasets are slightly changed (pessimistic). The other half prefers the precise dataset for 3 bets out of 4 (weakly-pessimistic answers). On the other side, only 9% of answers can be classified as ambiguity-loving (optimistic and weakly-optimistic answers).

Non-monotonic answers amount to 13% of response patterns in the experiment: 60.4% of respondents do not violate monotonicity in any of the 5 pairs of bets, 18.7% violate monotonicity only once, 13.2 % twice and 7.7% thrice.

The shares of respondents falling into the 5 reported categories are pretty stable across the proportions p_B apart from the questions related to frequencies 1/2–1/2: except for the proportion 0.5, where they amount to 42%, frequentist answers represent approximately a share of 30% of the total answers. At 0.5, only 12% of answers correspond to Bayesian preferences, while they represent at least 23% in the other proportions. This may be due to the fact that a significant share of subjects behave as Bayesians with a prior on p_B equals to 1/2, because, as stated previously, it is not possible to distinguish between frequentists and Bayesians with prior *equal* to p_B. This is confirmed by the fact that significant percentages of answers fall into Bay1, i.e., prior greater than p_B, for questions related to proportions smaller than 0.5 and the tendency is reversed for proportions greater than 0.5 with a substantial share of answers falling in Bay2, i.e., prior less than p_B (see Table B2 in Appendix B). Apart from this difference due to classification of neutral answers, it is not possible to conclude on a particular effect of frequencies of blue and red balls in the datasets on ambiguity attitudes.

3.2 Score of ambiguity-sensitivity

From a within-subject perspective, I calculate an individual score S of ambiguity-sensitivity: for a given color in a given proportion p_B, it takes the value 1 if the DM exhibits ambiguity-averse preferences, 0 if neutral and -1 if ambiguity-loving. The total score for a given individual is obtained by summing up the scores for the 10 pairs of questions. Hence, a score of 10 refers to extreme ambiguity-aversion and -10 denotes extreme ambiguity-loving. Formally, $S = \sum_{p_B,j} s_{p_B,j}$, where $p_B \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$,

$j \in \{B, R\}$ denotes the color of the bet and

$$s_{p_B,j} = \begin{cases} +1 & \text{if the precise bag is chosen twice } (PP), \\ 0 & \text{if if each bag is chosen once } (IP \text{ or } PI), \\ -1 & \text{if the imprecise bag is chosen twice } (II). \end{cases}$$

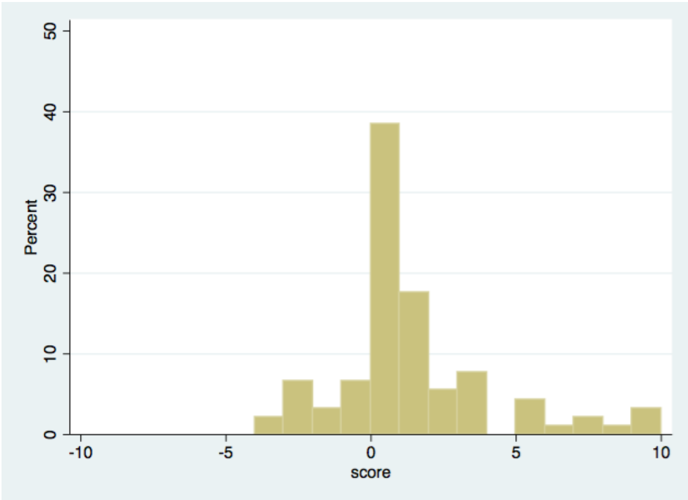

Figure 1: Histogram of individual score

In the data, scores range from -4 to 10 (see Table 4 and Figure 1). 43% of participants have a positive score, 38% have a null score and the remaining 19% have a score below 0. The average score in the experiment is .96, significantly different from 0 (one-sample $t(90) = 3.2$, $p < 0.01$). This number includes non-monotonic answers for which $s_{p_B,j}$ takes value 0 (since each dataset is chosen once). Excluding non-monotonic answers, the average score equals 1.04.

Since null scores can be obtained with ambiguity-neutral answers as well as with combinations of ambiguity-loving and ambiguity-averse answers, further investigations are needed to distinguish between scores resulting from unstable ambiguity-sensitive preferences and scores due to regular decision patterns. For $p_B \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $s_{p_B,\cdot}$ is the individual score in threshold p_B obtained by summing up the score for bets on blue and bets on red: $s_{p_B,\cdot} = s_{p_B,B} + s_{p_B,R}$ hence $s_{p_B,\cdot} \in [-2; 2]$. Excluding non-monotonic answers, the maxima and minima $s_{p_B,\cdot}$ are given in Table 5[8]. The individuals on the bold diagonal have the same decision mode along the experiment since their *per threshold* score is constant. For instance, the 4 subjects in group (2,2) choose the precise dataset at each question. They exhibit extreme ambiguity-aversion. The farther from the diagonal, the more the DM exhibits different choice behaviors. The largest group is the (0,0) group with 35 subjects: such participants obtain a zero score at each threshold, this denotes clear ambiguity-neutrality. Looking more in detail at the data: almost half of them (16) behave as pure frequentists and the rest (19) exhibits a mix of frequentist and Bayesian answers. The second largest group is the (0,1) group. These two findings are in line with the previous result showing a weak bias towards ambiguity-aversion. It is worth noting that only one subject obtains a negative maximum score -1 and no one stands in the other groups with negative maximum scores. This suggests that no regularity can be found in the patterns of ambiguity-loving answers.

3.3 Discussion

In order to keep the experiment as simple as possible, the precision of the datasets is kept constant in this study and the imprecise (precise) dataset contains always 10 (100) draws. Ert and Trautmann (2014) propose an experiment where participants can learn about an ambiguous prospect by sampling unlimitedly at no cost. A sample returns the result of an independent draw from the relevant distribution. They report that median numbers of samples are small and lie between 11 and 15 (depending on the decision problem). Hence, the reported neutral ambiguity attitudes in my experiment might result from the fact that DMs consider the observation of 10 draws to be sufficient to form a confident judgment regarding the composition of the bag. Consequently, agents would not process differently the dataset containing 10 draws and the dataset containing 100 draws.

[8]Keeping the non-monotonic answers does not significantly change the reported frequencies in the table.

This finding is also is in line with the "law of small numbers" of Tversky and Kahneman (1971). According to them, DMs have a tendency to regard limited samples of observations as highly representative. The reason is that they believe random samples to be very similar to one another and to the population from which they are drawn. Hence, agents make confident inference about the true distribution based on the results of short samples, as if the law of large numbers applied to them. The results of Experiment 1 in Arad and Gayer (2012) support this prediction. Participants are asked to consider betting on a draw from an opaque two-color Ellsberg's urn. In the control group, participants know the composition of the urn whereas subjects of the treatment group are provided with a sample of observations from the urn. The urn in the treatment group contains 90 balls and the dataset consists of 8 random draws with replacement. The distribution of Certainty Equivalents do not differ significantly across treatments. Therefore, the authors conclude that there should be no great difference between the beliefs in both informative conditions, yielding agents to value the prospects equally.

S	N	Freq	CumFreq
-4	2	2.2	2.2
-3	6	6.6	8.8
-2	3	3.3	12.1
-1	6	6.6	18.7
0	35	38.4	57.1
1	16	17.6	74.7
2	5	5.5	80.2
3	7	7.7	87.9
4	0	0.0	87.9
5	4	4.4	92.3
6	1	1.1	93.4
7	2	2.2	95.6
8	1	1.1	96.7
9	0	0.0	96.7
10	3	3.3	100.0
Total	*91*	*100.0*	

Table 4: Score of ambiguity-sensitivity

Note: N gives the number of subjects, Freq stands for Frequency, CumFreq for Cumulative Frequency.

$s_{p_B,\cdot}$ min $\backslash s_{p_B,\cdot}$ max	-2	-1	0	1	2	*Total*
-2	**0**	1	2	8	4	*15*
-1	-	**0**	5	2	6	*13*
0	-	-	**35**	13	8	*56*
1	-	-	-	**1**	2	*3*
2	-	-	-	-	**4**	*4*
Total	*0*	*1*	*42*	*24*	*24*	*91*

Table 5: Maxima and minima *per* p_B individual scores

Note: Excluding non-monotonic answers.

4 Conclusion

I present an adaptation of the Ellsberg's two-colors experiment allowing to explore decision making under partial ambiguity. In this study, one bag is described by a precise set of observations containing a large number of draws and the other bag is described by an imprecise set of observations containing a small number of draws. I report that a majority of choices in the experiment are consistent with the SEU model when datasets are available for both bags. Indeed, most answers correspond to frequentist and Bayesian preferences. On the other hand, non-neutral ambiguity attitudes cannot be removed from the analysis. More precisely, ambiguity-non-neutral choices consist of a majority of ambiguity-averse answers whereas ambiguity-loving attitudes can be interpreted as a marginal and unstable trait of preferences. This is confirmed by the estimation of a score of ambiguity-sensitivity which is slightly biased towards ambiguity-aversion on average.

In order to reach a general assessment of preferences under partial ambiguity, it is necessary to conduct additional experiments with different lengths of datasets. In particular, it might be worthwhile to compare the case of full ambiguity to partial ambiguity when very few observations are available since there is empirical evidence of ambiguity-aversion under full ambiguity and of ambiguity-neutrality when ambiguous prospects are described by short sequences of information. Both findings suggest that ambiguity-averse DMs consider small samples informative enough to resolve the ambiguity characterizing the decision situation. This question will be addressed in the subsequent chapters.

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A Supplementary material for the experiment

A.1 Instructions (translated from French)

Note: The sentences in brackets are not included in the instructions but inform the reader on the moment an action is performed.

— Start of Instructions —

Hello everybody, and thank you for accepting our invitation to participate in this experiment. My name is Roxane Bricet and I am pursuing my research at the Economics department of the University of Cergy-Pontoise. I am going to briefly present the outline of the experiment, please pay close attention to this short presentation.

First, some important rules which must be respected: It is important that the experiment takes place in silence, you are not allowed to communicate with other participants. The use of mobile phones and calculators is forbidden. No particular preliminary knowledge is needed to participate. As soon as the experiment starts, if you have any questions, please raise your hand and I will assist you.
Thank you in advance for your cooperation.

What do we do?
You are invited to participate in an economic experiment on decision theory.
Formally, in each question you will consider two opaque bags containing balls. In each bag, balls can be either blue or red.

Consider the following example:

The exact proportions of blue and red balls in each bag are unknown. However, we have partial information on the content of the bags. Indeed, we have randomly drawn one ball from bag Y and reported its color, **then we replaced it in bag Y**. This operation has been replicated several times in bag Y and in bag Z. Note that the number of draws from bag Y may be different from the number of draws in bag Z.

(a) Bag Y. Bag Y contains 200 balls, which can be either blue or red.

(b) Bag Z. Bag Z contains 200 balls, which can be either blue or red.

[At this point, we use real bags Y and Z to randomly draw with replacement 2 balls from one of the two bags.] These **random draws with replacement** are given for your information.

Example:

From bag Y, we have drawn a blue ball 15 times and a red ball 85 times.

From bag Z, we have drawn a blue ball 2 times and a red ball 8 times.

Your choices of bags:

In the experiment, you will be asked to answer questions of the following type:

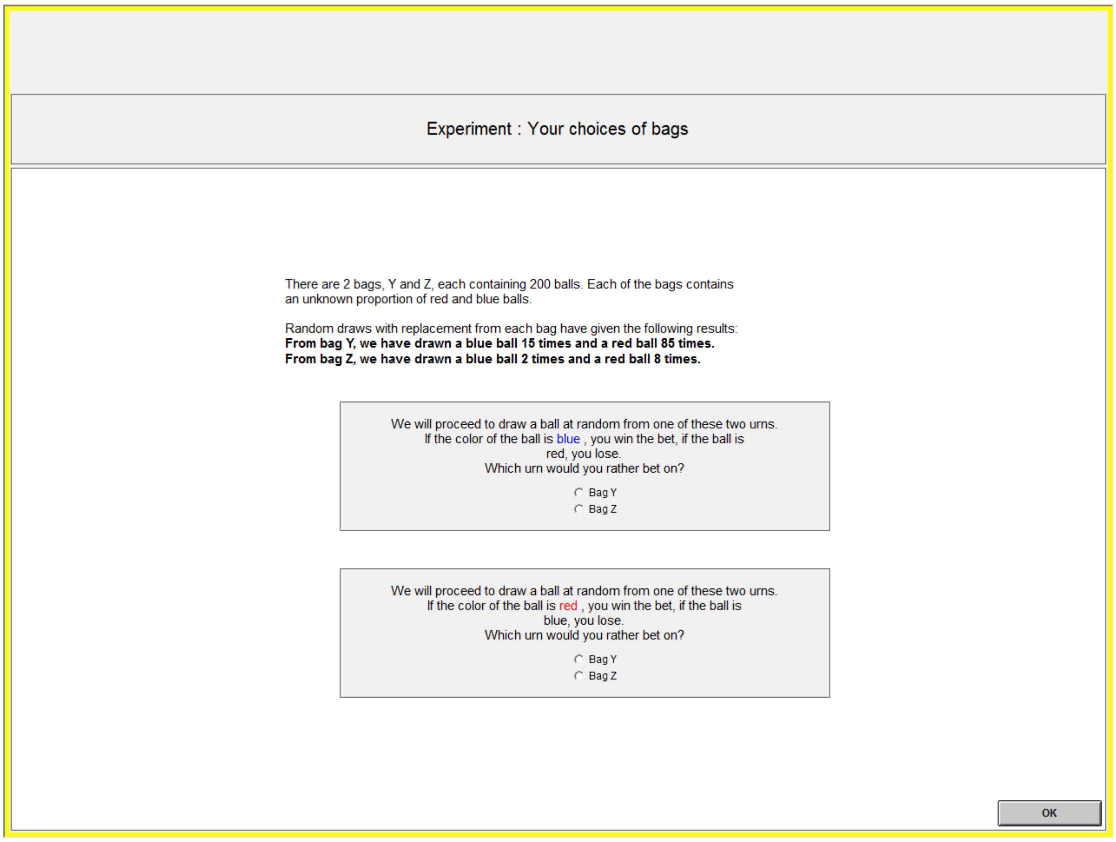
We will proceed to draw a ball at random from one of these two bags. If the color of the ball is blue, you win the bet, if the ball is red, you lose.

Which bag would you rather bet on?

☐ Bag Y ☐ Bag Z

Important: There are no "right" or "wrong" answers in this experiment, it is only a matter of preferences.

Consider the following example of screen that you will face during the experiment:



Your payment:

- For your participation in the experiment, each of you will receive **5 euros**.
- Moreover, you have the opportunity to win **an additional 13 euros**. Indeed, at the end of the session, one of the questions in the experiment will be randomly selected. Each participant will be rewarded depending on his choice to the selected question. Let us return to the previous example: suppose that you prefer to draw a new ball from bag Y, you will proceed to draw a ball at random from bag Y. If the ball is blue, you win the bet and you obtain 13 additional euros; if the ball is red, you lose the bet and you only receive one extra euro. Therefore, it is in your own interest to make choices according to your true preferences.

Altogether, you can win up to 18 euros if you draw the right ball from the selected bag!

Plan of the experiment:

In practice, you will answer the questions of the experiment using the computer in front of you.

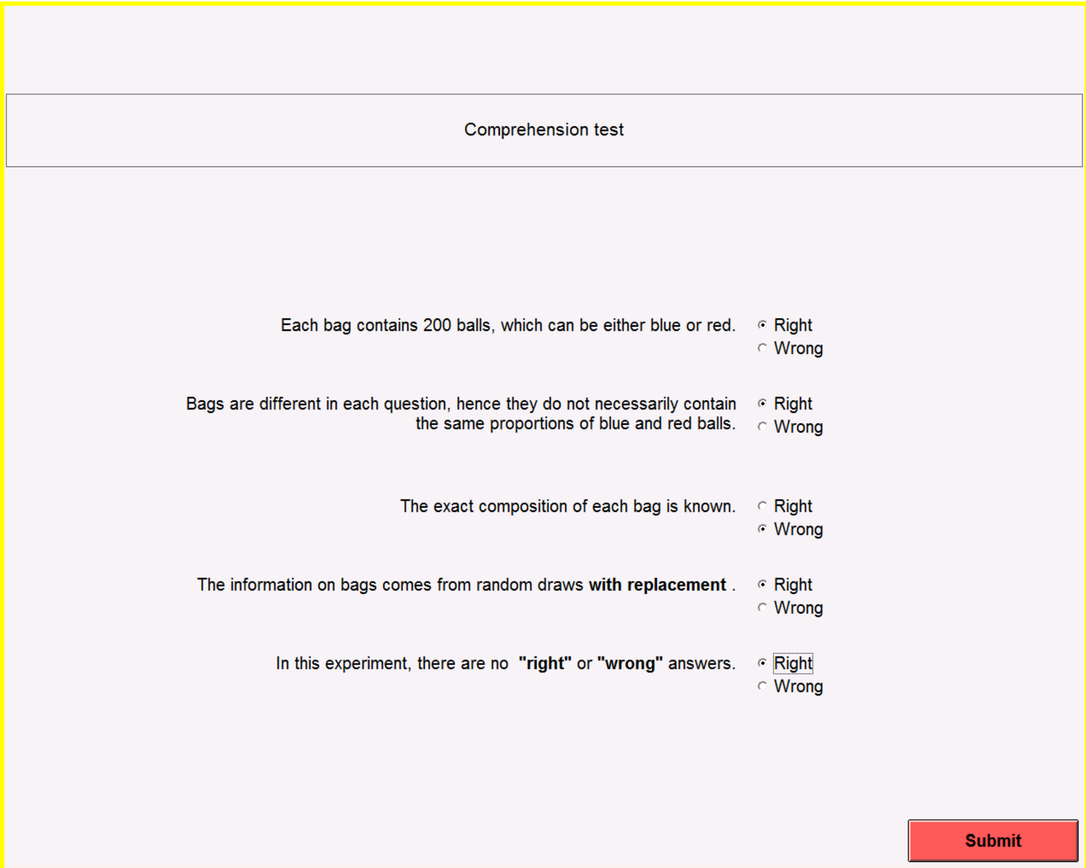
- The first screen will remind you briefly of the outline of the experiment.
- Then, you will be asked 5 comprehension questions to check your understanding of the instructions. Your answers to these questions do not affect your payment.
- Next, the experiment on YOUR CHOICES OF BAGS will start. **The question that determines your payment will be randomly selected among them**. This part will consist of 10 consecutive screens for a total of 20 questions.
- Finally, you will be asked to fill out a short complementary questionnaire to get to know you better. Your answers to these questions do not affect your payment.
- The last screen will inform you of the randomly selected question and you will be reminded of your answer to this question.

Your answers will be kept strictly confidential and anonymous, henceforth, feel free to answer as you like. Moreover, there is no time limit, so take all the time you need to read the instructions and answer the questions.

Do you have any questions?
If everything is clear, you can now start the experiment!

— End of Instructions —

A.2 Comprehension test


Comprehension test

Each bag contains 200 balls, which can be either blue or red. ◉ Right ○ Wrong

Bags are different in each question, hence they do not necessarily contain the same proportions of blue and red balls. ◉ Right ○ Wrong

The exact composition of each bag is known. ○ Right ◉ Wrong

The information on bags comes from random draws **with replacement** . ◉ Right ○ Wrong

In this experiment, there are no **"right"** or **"wrong"** answers. ◉ Right ○ Wrong

Submit

B Complementary tables and figures

p_B	Question	Bag	Dataset		Bag	
			Blue	Red	Blue	Red
0.1^+	q1, q2	P	11	89	38	162
		I	1	9	98	102
0.1^-	q3, q4	P	9	91	26	174
		I	1	9	61	139
0.3^+	q5, q6	P	31	69	62	138
		I	3	7	58	142
0.3^-	q7, q8	P	29	71	68	132
		I	3	7	50	150
0.5^+	q9, q10	P	51	49	88	112
		I	5	5	93	107
0.5^+	q1, q12	P	49	51	102	98
		I	5	5	106	94
0.7^+	q13, q14	P	71	29	133	67
		I	7	3	105	95
0.7^-	q15, q16	P	69	31	145	55
		I	7	3	158	42
0.9^+	q17, q18	P	91	9	188	12
		I	9	1	163	37
0.9^-	q19, q20	P	89	11	175	25
		I	9	1	170	30

Table B1: Bags composition

Lecture: In the pair of questions (q1,q2), participants are informed that 100 balls have been randomly drawn with replacement from the precise bag, among them 11 were blue and 89 were red; 10 balls have been randomly drawn with replacement from the imprecise bag, among them 1 was blue and 9 were red (public information). The precise bag contains actually 38 blue balls and 162 red balls and the imprecise bag contains actually 98 blue balls and 102 red balls (non-public information).

p_B	Freq	Bay1	Bay2	Pess	WP1	WP2	Opti	WO1	WO2	NM1+NM2
0.1	32.9	20.9	5.5	8.8	4.4	5.5	3.3	3.3	3.3	12.1
0.3	28.5	19.8	8.8	15.4	7.7	2.2	2.2	2.2	1.1	12.1
0.5	41.7	7.7	4.4	9.9	1.1	7.7	7.7	6.6	1.1	12.1
0.7	27.4	9.9	23.1	7.7	4.4	4.4	2.2	2.2	1.1	17.6
0.9	32.9	6.6	16.5	11.0	4.4	5.5	4.4	3.3	1.1	14.3
Total	*32.7*	*11.6*	*13.0*	*10.5*	*4.4*	*5.1*	*4.0*	*3.5*	*1.5*	*13.6*

Table B2: Detailed frequencies of choices

Note: Freq: Frequentist, Bay1: Bayesian with prior $> p_B$, Bay2: Bayesian with prior $< p_B$, WP1 and WP2: Weakly Pessimistic preferences, WO1 and WO2: Weakly Optimistic preferences, NM1 and NM2: Non-Monotonic preferences.