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Introducing Roy-like Worker Assignment into Computable General Equilibrium Models

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Abstract

This paper develops a new CGE model incorporating a Roy-like worker assignment in which heterogeneous workers endogenously sort into different technologies based on their comparative advantage. The model predicts significantly higher welfare-improving effects of trade liberalization due to technology-upgrading mechanism.

Keywords: Technology upgrading, Heterogeneous firms/workers, Roy model, Computable general equilibrium (CGE), Gains from trade.

JEL codes: C68, D58, F16.

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1 Introduction

Recent firm heterogeneity literature in international trade emphasizes aggregate productivity gains stemming from self-selection of heterogeneous firms following trade liberalization (Melitz, 2003). This branch of models highlights how the exit of the least productive firms and the consequent reallocation of resources to more productive firms lead to welfare gains, in addition to conventional demand-side variety gains (Armington, 1969; Krugman, 1980).¹

All of above approaches, however, do not consider worker-side heterogeneity and the resulting implications. Workers choose occupations based on their comparative advantage and workers' productivity reflects not only their own skill level but also the technology they are employing (Roy, 1951). If technology would exhibit any increasing returns to skill, equilibrium technology-skill assignment would have considerable implications for economic performance and welfare.

In this paper, we develop a new CGE model incorporating a Roy-like worker assignment in which heterogeneous workers in skill endogenously sort into different technologies based on their comparative advantage. Applying the model to Korea-US FTA shows not only significantly higher global welfare gains but also substantially different welfare implications for each country due to technology-upgrading mechanism of the model.

Section 2 describes the base model and compares the welfare implication of trade liberalization with alternative models. Section 3 provides a general equilibrium formulation and Section 4 applies the model to Korea-US FTA. Section 5 concludes.

2 The Theory

2.1 Model Description

We consider two symmetric countries, where households have Dixit-Stiglitz preferences over a continuum of varieties. As usual, demand for individual varieties can readily be derived:

¹See e.g., Arkolakis et al. (2008), Feenstra (2010), and Balistreri and Rutherford (2013) for discussions and analyses of gains from trade in models with firm heterogeneity à la Melitz.

$$x^d(i) = \left[\frac{PC}{p(i)} \right]^\sigma C. \quad (1)$$

Firms are free to enter the market and choose whether to serve only the domestic market or also export. Adopting either strategy requires a strategy-specific technology $n \in \{L, H\}$: L for low-tech and H for high-tech, with associated fixed costs $f_H > f_L$. There is now ample evidence that exporting firms use more productive technologies than domestic firms, as well as pay higher fixed set-up costs to cover both domestic and foreign markets. We assume that exporting firms adopt H -tech while domestic firms are associated with L -tech.

There is a continuum of workers differentiated by their skill level z , with a uniform distribution on support $[z_{\min}, z_{\max}]$. The productivity of a worker depends not only on his/her own skill level z , but also on the technology he/she employs. Let $\varphi_n(z)$ denote the productivity of a worker with skill z when using technology $n \in \{L, H\}$. A higher-skilled worker has an absolute productivity advantage over a less-skilled one at a given technology, and also has a comparative advantage in higher technology. Formally, we assume:

$$0 < \frac{\partial \varphi_L(z)}{\partial z} \frac{1}{\varphi_L(z)} < \frac{\partial \varphi_H(z)}{\partial z} \frac{1}{\varphi_H(z)}, \quad (2)$$

with $\varphi_L(z_{\min}) = \varphi_H(z_{\min})$.

If both firm-types exist in equilibrium, there must be a threshold skill level z^* so that workers with $z \in [z_{\min}, z^*)$ are hired by domestic firms while workers with $z \in (z^*, z_{\max}]$ are employed by exporting firms. In a perfectly competitive labor market, the no-arbitrage condition for the threshold worker z^* leads to:

$$w_L \varphi_L(z^*) = w_H \varphi_H(z^*), \quad (3)$$

where w_n are technology-specific efficiency wage rates. Production requires only labor so that w_n also represent unit production costs of each good. Note from Eqs. (2) and (3) that $w_L > w_H$, exhibiting thus a trade-off between a high-fixed-cost low-marginal-cost technology and a low-fixed-cost high-marginal-cost technology.

We assume monopolistic competition to prevail so that firms charge a constant mark-up over marginal production costs:

$$p_n = \frac{\sigma}{\sigma - 1} w_n, \quad n \in \{L, H\}. \quad (4)$$

The aggregate consumption price index can then be written as:

$$P_C = \left[N_L p_L^{1-\sigma} + N_H p_H^{1-\sigma} + N_H^* \left\{ (1 + \tau^{imp}) p_H^* \right\}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad (5)$$

where N_n denote the number of each firm-type (with an asterisk for foreign exporting firms) and τ^{imp} is an ad-valorem tariff rate on imported goods.

Finally, free entry ensures zero profits for both firm-types so that mark-up revenues exactly cover the fixed costs:

$$\frac{1}{\sigma} p_n x_n = w_n f_n, \quad n \in \{L, H\}. \quad (6)$$

2.2 Trade Liberalization

To understand the basic mechanism how trade liberalization affects the technology-skill assignment, consider the revenue ratio between domestic and exporting firms. From Eqs. (1), (3), (4) and (6), we get:²

$$\frac{w_H}{w_L} = \frac{\varphi_L(z^*)}{\varphi_H(z^*)} = \left[\left\{ 1 + (1 + \tau^{imp})^{-\sigma} \right\} \frac{f_L}{f_H} \right]^{\frac{1}{\sigma}}. \quad (7)$$

From Eqs. (2) and (7), it is then easy to check that a fall in τ^{imp} increases the relative wage w_H/w_L and decreases the equilibrium threshold z^* . A leftward shift of z^* implies that now more firms and workers are matched with high technology (technology-upgrading).

For simplicity, let us assume following linear technologies:³

$$\varphi_n(z) = c + a_n z, \quad n \in \{L, H\}. \quad (8)$$

The shaded area of panel (b) in Figure 1 then shows the economy-wide increased efficiency units of labor due to such technology-upgrading mechanism of trade liberalization.

²Note that $x_L = x_L^d$ and $x_H = x_H^d + x_H^{d*}$, with $(1 + \tau^{imp}) p_H^*$ for the price of imported goods and $p_H^* = p_H$ from the symmetry.

³Any more general functional forms consistent with Eq. (2) can, of course, be adopted.

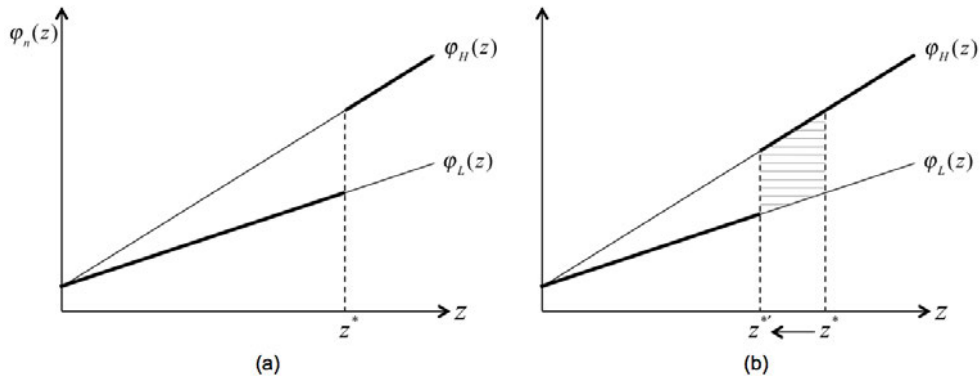


Figure 1: Technology upgrading due to trade liberalization

The aggregate productivity gain as well as the beneficial composition effect – more cheap varieties due to the exit of low-tech firms and entry of high-tech firms – would generate a higher welfare gain.⁴

2.3 Comparison of Alternative Models

To get a feeling of the quantitative effects involved, in this subsection we compare the welfare effects of alternative models. Four models – Armington, Krugman, Melitz, and Jung (the model of this paper) – are calibrated to the following social accounting matrix (SAM).⁵

	Act.	Com.	Fac.	H	Gov	RoW	Total
Act.		1000					1000
Com.				1125		250	1375
Fac.	1000						1000
H			1000		125		1125
Gov		125					125
RoW		250					250
Total	1000	1375	1000	1125	125	250	

Table 1: Social accounting matrix (SAM)

⁴See Jung and Mercenier (2014) for more detailed discussions and analyses in this framework including income effects.

⁵Armington elasticity of 3 is used while a value of 5 is used for the elasticity of substitution between individual varieties in Krugman, Melitz and Jung models, and fixed costs are calibrated to ensure the zero-profit conditions. $a_H/a_L = 1.2$ is used for the technology gap in Jung model. Finally for comparability, a uniform distribution is assumed both in Melitz and Jung models.

Following Figure 2 shows the welfare effects of tariff reduction for alternative models. As can be seen in Figure 2, the alternative models generate positive welfare gains in the order of Jung>Melitz>Krugman>Armington.

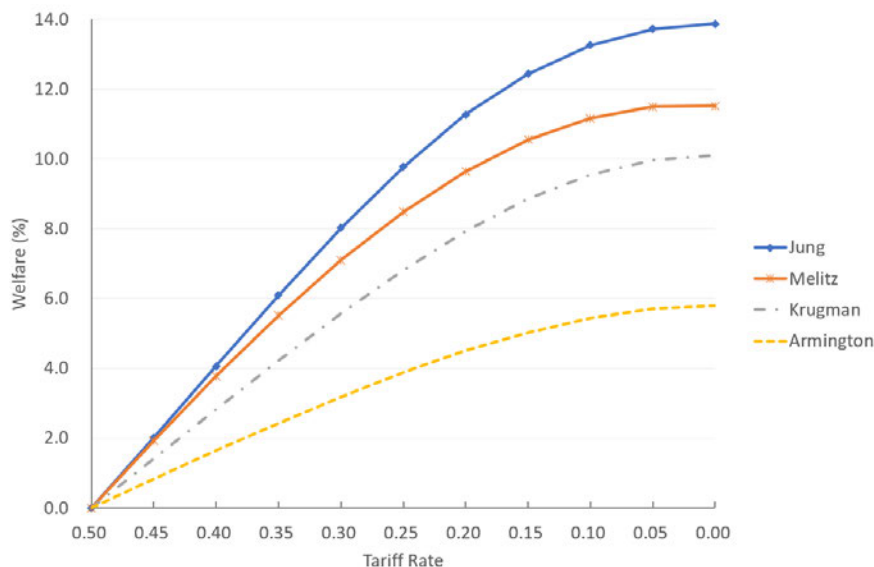


Figure 2: Welfare effects for alternative models

Conventional CGE models are based on a representative agent framework and the gains from trade have mainly been driven by demand-side forces (love-of-variety), whether product differentiations are country-level (Armington) or firm-level (Krugman). On the other hand, recent trade theories focus more on the production-side forces and firm heterogeneity consistent with empirical evidence. In particular, Melitz (2003) highlights the aggregate productivity effects of trade induced by a selection effect of heterogeneous firms: firm-level productivity differences are exogenously given and trade liberalization forces the least productive firms to exit.

As described before, this paper’s framework explores a new source of gains from trade coming from technology-upgrading mechanism. Trade liberalization induces more firms and workers to be matched with high technology, as well as provides more varieties cheaper and efficiently by high-tech firms. These effects lead consequently to significantly higher welfare gains compared to other alternative models.

Having this basic mechanism in mind, we now proceed to formalize and develop the full model.

3 General Equilibrium Formulation

We now incorporate previous framework into a full global CGE model.⁶

As displayed in Eq. (5), in each country there are three groups of consumption goods: supplied by domestic L -firms, domestic H -firms, and foreign H -firms.

In country- i -sector- s , the technology-augmented total efficiency units of labor with each technology are given by:⁷

$$L_{isL}^{sup} = \phi_{is} \left(\int_{z_{\min}}^{z_{is}^*} \varphi_{isL}(z) dz \right) L_{is}^{sh} \quad \text{and} \quad L_{isH}^{sup} = \phi_{is} \left(\int_{z_{is}^*}^{z_{\max}} \varphi_{isH}(z) dz \right) L_{is}^{sh}, \quad (9)$$

where $\varphi_{isn}(z) = c_{is} + a_{isn}z$, $n \in \{L, H\}$; ϕ_{is} is a scale parameter and L_{is}^{sh} is a sectorial labor employment share variable with $\sum_s L_{is}^{sh} = 1$.

The cutoff skill level z_{is}^* in each sector is determined by the no-arbitrage condition for the threshold worker:

$$w_{isL} \varphi_{isL}(z_{is}^*) = w_{isH} \varphi_{isH}(z_{is}^*). \quad (10)$$

The sectorial aggregate employment share L_{is}^{sh} is determined by the no-arbitrage condition of the average worker (average wage balance condition):

$$\frac{\sum_n w_{isn} L_{isn}^{sup}}{L_{is}^{sh}} = \lambda_{ist}^0 \frac{\sum_n w_{itn} L_{itn}^{sup}}{L_{it}^{sh}}, \quad \sum_s L_{is}^{sh} = 1, \quad s \neq t \quad (11)$$

where λ_{ist}^0 is a parameter of the initial average wage difference between sector s and t .

All the other settings follow the conventional multi-country/region multi-sector global trade CGE models with monopolistic competition. To save the space, the full system of equations and determined variables are presented in the Appendix.

One additional calibration issue in this framework is the calibration of a_{isn} , the sectorial technological parameters. The best way would, of course, be to estimate them directly if data are available. Otherwise, though indirect, they can also be calibrated easily using the information on such as sectorial employment and wage share between

⁶Used indexes are: country i, j ; sector s, t ; factor f (other than labor); technology (and/or firm-type) $n \in \{L, H\}$.

⁷Here we assume a uniform skill distribution. It is, however, straightforward to incorporate various more general skill distributions. See Jung (2015) introducing log-normal skill distribution and analyzing technology-augmented skill distribution in a North-South offshoring context.

domestic and exporting firms.

4 Application

Before concluding and for a real world application of the model, in this section we run simulations for the case of Korea-US FTA: using GTAP 9 data base, four scenarios are simulated. Following Tables 2 reports the calculated Equivalent Variation (EV) and Compensating Variation (CV) welfare measures.⁸

(Billions of dollars)

		SCN1		SCN2		SCN3		SCN4	
		S2 50% Tariff Cut		S1 & S2 50% Tariff Cut		S2 100% Tariff Cut		S1 & S2 100% Tariff Cut	
		Korea	US	Korea	US	Korea	US	Korea	US
Jung	EV	0.677	2.189	1.415	2.079	1.320	5.296	3.032	5.030
	CV	0.679	2.192	1.418	2.081	1.327	5.312	3.049	5.045
Armington	EV	1.216	0.517	2.412	0.498	2.534	1.047	5.292	1.006
	CV	1.217	0.517	2.416	0.498	2.537	1.047	5.301	1.006

Table 2: Welfare effects of Korea-US FTA

As can be seen in Table 2, introducing the technology-upgrading mechanism not only yields significantly higher global welfare gains (sum of the two countries), but also changes substantially the welfare implications for each country. For example in SCN4, Armington model predicts welfare gains of about 5.3 and 1.0 billions of dollars for Korea and US respectively, but Jung model predicts about 3.0 and 5.0 billions of dollars for Korea and US respectively. Thus, any policy without considering the close interplay between technology and skill might lead to different results not only quantitatively but also even qualitatively.

⁸Three aggregate sectors are considered: primary (S1), manufacturing (S2), and Service (S3), where perfect competition is assumed for S1 and S3. For a_H/a_L in the manufacturing sector, 1.142 and 1.144 are used for Korea and US, respectively (Aw et al., 2000; Bernard and Jensen, 1999). For the elasticity of substitution between individual varieties, 12.6 is used from Broda and Weinstein (2006).

5 Concluding Remarks

Technology and skill and the interplay between them have long been one of the main concerns in economics and economic policy. As this paper shows, the quantitative predictions of most conventional CGE models that do not consider their close links might be significantly misleading. The technology-skill links should, in particular, be important in international trade context where engaged countries face different technological and labor market environments one another.

This paper's new CGE framework incorporating the technology up- and/or downgrading mechanisms can be easily extended and applied to various large-scale CGE models for richer predictions on productivity, income distribution, welfare, etc.

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Appendix: Equations and determined variables

<Household>

$$\begin{aligned}
 (Inc_i^H) \quad Inc_i^H &= \sum_{s,n} w_{isn} L_{isn}^{sup} + \sum_f r_{if} \overline{F_{if}^{sup}} \\
 (Sav_i^H) \quad Sav_i^H &= \mu_i^H (1 - \tau_i^{Inc}) Inc_i^H \\
 (Con_i) \quad P_i^{Con} Con_i &= (1 - \mu_i^H) (1 - \tau_i^{Inc}) Inc_i^H \\
 (C_{is}) \quad P_{is}^C C_{is} &= \alpha_{is}^C P_i^{Con} Con_i \\
 (P_i^{Con}) \quad \ln P_i^{Con} &= \sum_s \alpha_{is}^C \ln P_{is}^C
 \end{aligned}$$

<Firms>

$$\begin{aligned}
 (X_{isn}) \quad X_{isn} &= \alpha_{isn}^X Z_{isn} \\
 (Q_{isn}) \quad Q_{isn} &= \alpha_{isn}^Q Z_{isn} \\
 (Z_{isn}) \quad P_{isn}^Z Z_{isn} &= P_{isn}^X X_{isn} + P_{isn}^Q Q_{isn} \\
 (XS_{itsn}) \quad XS_{itsn} &= \alpha_{itsn}^{XS} \left(\frac{P_{isn}^X}{P_{it}^C} \right)^{\sigma_{isn}^X} X_{isn} \\
 (P_{isn}^X) \quad P_{isn}^X X_{isn} &= \sum_t P_{it}^C XS_{itsn} \\
 (L_{isn}^{dem}) \quad L_{isn}^{dem} &= \alpha_{isn}^L \left(\frac{P_{isn}^Q}{w_{isn}} \right)^{\sigma_{isn}^Q} Q_{isn} \\
 (F_{ifsn}^{dem}) \quad F_{ifsn}^{dem} &= \alpha_{ifsn}^F \left(\frac{P_{isn}^Q}{r_{if}} \right)^{\sigma_{isn}^Q} Q_{isn} \\
 (P_{isn}^Q) \quad P_{isn}^Q Q_{isn} &= w_{isn} L_{isn}^{dem} + \sum_f r_{if} F_{ifsn}^{dem} \\
 (p_{isn}) \quad p_{isn} &= \frac{\sigma_{isn}^D}{\sigma_{isn}^D - 1} P_{isn}^Z \\
 (S_{isn}^{dom}) \quad S_{isn}^{dom} &= \alpha_{isn}^{Sdom} \left(\frac{p_{isn}}{P_{isn}^{Sdom}} \right)^{\sigma_{isn}^{CET}} Z_{isn}, \quad n = H \\
 (S_{isn}^{exp}) \quad S_{isn}^{exp} &= \alpha_{isn}^{Sexp} \left(\frac{p_{isn}}{P_{isn}^{Sexp}} \right)^{\sigma_{isn}^{CET}} Z_{isn}, \quad n = H \\
 (S_{isn}^{RoW}) \quad S_{isn}^{RoW} &= \alpha_{isn}^{SRow} \left(\frac{p_{isn}}{P_{isn}^{SRow}} \right)^{\sigma_{isn}^{CET}} Z_{isn}, \quad n = H
 \end{aligned}$$

<Investor>

$$\begin{aligned}
 (Inv_i) \quad P_i^{Inv} Inv_i &= Sav_i^H + Sav_i^G + \sum_j Sav_{ij} + Sav_i^{RoW}, \quad i \neq j \\
 (I_{is}) \quad P_{is}^C I_{is} &= \alpha_{is}^I P_i^{Inv} Inv_i \\
 (P_i^{Inv}) \quad \ln P_i^{Inv} &= \sum_s \alpha_{is}^I \ln P_{is}^C
 \end{aligned}$$

<Government>

$$\begin{aligned}
(Inc_i^G) \quad Inc_i^G &= \tau_i^{Inc} Inc_i^H + \sum_{s,n} \tau_{isn}^v P_{isn} Z_{isn} N_{isn} \\
&\quad + \sum_s \tau_{is}^{imp} P^W D_{is}^{RoW} + \left[\sum_{j,s} \tau_{is}^{imp} (1 + \tau_{jsn}^v) p_{jsn} x_{jsn}^{exp} N_{jsn} \right]_{n=H, i \neq j} \\
(Sav_i^G) \quad Sav_i^G &= \mu_i^G Inc_i^G \\
(Gov_i) \quad P_i^{Gov} Gov_i &= (1 - \mu_i^G) Inc_i^G \\
(G_{is}) \quad P_{is}^C G_{is} &= \alpha_{is}^G P_i^{Gov} Gov_i \\
(P_i^{Gov}) \quad \ln P_i^{Gov} &= \sum_s \alpha_{is}^G \ln P_{is}^C
\end{aligned}$$

<Monopolistic competition demand>

$$\begin{aligned}
(D_{it}^A) \quad D_{it}^A &= \sum_{s,n} X S_{itsn} N_{isn} + C_{it} + G_{it} + I_{it} \\
(D_{itn}) \quad D_{itn} &= \alpha_{itn}^D \left(\frac{P_{it}^C}{P_{itn}^D} \right)^{\sigma_{it}^{DA}} D_{it}^A \\
(D_{it}^{RoW}) \quad D_{it}^{RoW} &= \alpha_{it}^{DRoW} \left(\frac{P_{it}^C}{(1 + \tau_{it}^{imp}) P^W} \right)^{\sigma_{it}^{DA}} D_{it}^A \\
(P_{it}^C) \quad P_{it}^C D_{it}^A &= \sum_n P_{itn}^D D_{itn} + (1 + \tau_{it}^{imp}) P^W D_{it}^{RoW} \\
(D_{itn}^{dom}) \quad D_{itn}^{dom} &= \alpha_{itn}^{Ddom} \left(\frac{P_{itn}^D}{P_{itn}^{Ddom}} \right)^{\sigma_{itn}^D} D_{itn}, \quad n = H \\
(D_{itn}^{imp}) \quad D_{itn}^{imp} &= \alpha_{itn}^{Dimp} \left(\frac{P_{itn}^D}{P_{itn}^{Dimp}} \right)^{\sigma_{itn}^D} D_{itn}, \quad n = H \\
(P_{itn}^D) \quad P_{itn}^D D_{itn} &= P_{itn}^{Ddom} D_{itn}^{dom} + P_{itn}^{Dimp} D_{itn}^{imp}, \quad n = H \\
(E_{jitn}) \quad E_{jitn} &= \alpha_{jitn}^E \left(\frac{P_{itn}^{Dimp}}{P_{jitn}^E} \right)^{\sigma_{itn}^{Dimp}} D_{itn}^{imp}, \quad n = H, \quad i \neq j \\
(P_{itn}^{Dimp}) \quad P_{itn}^{Dimp} D_{itn}^{imp} &= \sum_j P_{jitn}^E E_{jitn}, \quad n = H, \quad i \neq j \\
(x_{itn}^d) \quad x_{itn}^d &= \alpha_{itn}^{xd} \left(\frac{P_{itn}^D}{(1 + \tau_{itn}^v) p_{itn}} \right)^{\sigma_{itn}^D} D_{itn}, \quad n = L \\
(P_{itn}^D) \quad P_{itn}^D D_{itn} &= (1 + \tau_{itn}^v) p_{itn} x_{itn}^d N_{itn}, \quad n = L \\
(x_{itn}^d) \quad x_{itn}^d &= \alpha_{itn}^{xd} \left(\frac{P_{itn}^{Ddom}}{(1 + \tau_{itn}^v) p_{itn}} \right)^{\sigma_{itn}^{Ddom}} D_{itn}^{dom}, \quad n = H \\
(P_{itn}^{Ddom}) \quad P_{itn}^{Ddom} D_{itn}^{dom} &= (1 + \tau_{itn}^v) p_{itn} x_{itn}^d N_{itn}, \quad n = H \\
(x_{jitn}^e) \quad x_{jitn}^e &= \alpha_{jitn}^{xe} \left(\frac{P_{jitn}^E}{(1 + \tau_{jitn}^{imp})(1 + \tau_{jitn}^v) p_{jitn}} \right)^{\sigma_{itn}^{Dimp}} E_{jitn}, \quad n = H, \quad i \neq j \\
(P_{jitn}^E) \quad P_{jitn}^E E_{jitn} &= (1 + \tau_{jitn}^{imp}) (1 + \tau_{jitn}^v) p_{jitn} x_{jitn}^e N_{jitn}, \quad n = H, \quad i \neq j
\end{aligned}$$

<Trade balance>

$$\begin{aligned}
(Sav_{ij}) \quad & \sum_s (1 + \tau_{isn}^v) p_{isn} x_{ijsn}^e N_{isn} = \sum_s (1 + \tau_{jsn}^v) p_{jsn} x_{jisn}^e N_{jsn} + Sav_{ij}, \quad n = H, \quad i \neq j \\
(E_{isn}^{RoW}) \quad & E_{isn}^{RoW} = \alpha_{isn}^{ERoW} \left(\frac{PW}{(1 + \tau_{is}^{imp})(1 + \tau_{isn}^v) p_{isn}} \right)^{\sigma_{isn}^{ERoW}} E0_{isn}^{RoW}, \quad n = H \\
(Sav_i^{RoW}) \quad & Sav_i^{RoW} = PW Sav0_i^{RoW} \\
(PW) \quad & \sum_{i,s} PW D_{is}^{RoW} = \sum_{i,s} (1 + \tau_{isn}^v) p_{isn} E_{isn}^{RoW} N_{isn} + \sum_i Sav_i^{RoW}, \quad n = H
\end{aligned}$$

<Equilibrium>

$$\begin{aligned}
(N_{isn}) \quad & \left(\frac{1}{\sigma_{itn}^D} \right) p_{isn} Z_{isn} = w_{isn} f_{isn} \\
(L_{isn}^{sup}) \quad & L_{isn}^{sup} = \phi_{is} \left(\int_{z_{\min}}^{z_{is}^*} \varphi_{isn}(z) g(z) dz \right) L_{is}^{sh}, \quad n = L \\
(L_{isn}^{sup}) \quad & L_{isn}^{sup} = \phi_{is} \left(\int_{z_{is}^*}^{z_{\max}} \varphi_{isn}(z) g(z) dz \right) L_{is}^{sh}, \quad n = H \\
(\varphi_{isn}) \quad & \varphi_{isn}(z) = c_{is} + a_{isn} z \\
(z_{is}^*) \quad & [w_{isn} \varphi_{isn}(z_{is}^*)]_{n=L} = [w_{isn} \varphi_{isn}(z_{is}^*)]_{n=H} \\
(L_{is}^{sh}) \quad & \frac{\sum_n w_{isn} L_{isn}^{sup}}{L_{is}^{sh}} = \lambda_{ist}^0 \frac{\sum_n w_{itn} L_{itn}^{sup}}{L_{it}^{sh}}, \quad \sum_s L_{is}^{sh} = 1, \quad s \neq t \\
(w_{isn}) \quad & (L_{isn}^{dem} + f_{isn}) N_{isn} = L_{isn}^{sup} \\
(r_{if}) \quad & \sum_{s,n} F_{ifsn}^{dem} = F_{if}^{sup} \\
(P_{isn}^{Sdom}) \quad & S_{isn}^{dom} = x_{isn}^d, \quad n = H \\
(P_{isn}^{Sexp}) \quad & S_{isn}^{exp} = \sum_j x_{ijsn}^e, \quad n = H, \quad i \neq j \\
(P_{isn}^{Srow}) \quad & S_{isn}^{RoW} = E_{isn}^{RoW}, \quad n = H \\
(P_{isn}^Z) \quad & Z_{isn} = x_{isn}^d, \quad n = L \\
(P_{isn}^Z) \quad & Z_{isn} = S_{isn}^{dom} + S_{isn}^{exp} + S_{isn}^{RoW}, \quad n = H
\end{aligned}$$