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Inheritance taxation in a model with intergenerational time

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Abstract

We consider a two-period overlapping generation model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. Starting from a steady state where public spendings are financed through taxation on capital income and labor income, we analyze a tax reform that consists in a shift of the tax burden from capital income tax towards inheritance tax. In the standard Barro model with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. In our setting, inheritance tax modifies parent's trade-off between time transfers and bequests. We identify situations where the tax reform increases welfare for all generations. Welfare improvement mainly depends on the magnitude of the effect of higher time transfers on the labor supply of the young.

Keywords:family transfers, altruism, time transfers, inheritance tax.
JEL Classifications: D64, H22, H24, J22.

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1 Introduction

Inheritance taxation is a controversial subject in the public policy debate and among economists. Currently, an increasing number of countries have no inheritance tax or have significantly reduced it (United States and United Kingdom). For the opponents, the inheritance tax discourages capital accumulation, and it is an immoral tax which increases the pain suffered by mourning families. They claim that tax on bequest involves "double taxation" of income that has already been taxed. A second line of argument suggests that if people have a long enough horizon (through altruistic behavior), inheritance taxation is inefficient. This point relies on the well known result of Chamley (1986) and Judd (1985) who show that capital income taxation should be zero in the long run. In dynastic interpretation of the infinite-lived agent model, this implies that inheritance taxation should also be zero (see Chamley (1986, p. 613)).

Over the past few years, an extensive literature has shown that we can overturn the Chamley-Judd result of zero capital income (inheritance) taxation by relaxing some of their hypotheses.[1] Moreover, the previous theoretical literature about inheritance taxation has essentially focused on financial bequests as the single source of intergenerational transfers within family. Nevertheless, a number of empirical studies suggest that time transfers from parents to their children are substantial and on average almost as important as monetary transfers (Cardia and Ng (2003) and Schoeni et al. (1997)).[2] Some studies based on the Survey of Health, Ageing and Retirement in Europe (SHARE)[3], such as Attias-Donfut et al. (2005) or also Albertini et al. (2007), show that parent's time transfers to children consists mainly in childcare. According to Wolff and Attias-Donfut (2007), two-fifth of grandparents keep their grandchildren every week. A common finding is that grandparents still support parents' home production with household tasks for instance.

Thanks to the intergenerational transfers of time in the form of grandparenting, parents free up more time for working and taking care of their children. Labor supply of the heirs as well as life cycle resources are affected differently by time transfers compared to inheritances as shown by Cardia and Michel (2004) or Belan et al. (2010). Taking into account time transfers allows to deal with the trade-off between both types of transfers. Time transfers have some macroeconomic implications through labor supply of the next generation, while bequests enhance its private wealth (Cardia and Ng, 2003).

Despite their importance and macroeconomic implications, the theoretical literature about fiscal incidence of inheritance tax has not devoted attention to the trade-off between giving time or giving money. Whether or not time transfers are introduced, inheritance taxation reduces the incentive

[1] A non-zero bequest tax result is potentially achieved by assuming other bequest motives (Cremer and Pestieau, 2011) or for example, focusing on a model with heterogeneous random tastes for bequest and for wealth per se (Piketty and Saez, 2013) or, lastly, considering imperfect competition on capital market (Farhi and Werning, 2010).

[2] For example Cardia and Ng (2003, Table 1) uses the Health and Retirement Study of 1992 and report that the mean time transfer for total sample (7547 households) of 325 hours has a value of $1950 (using a time cost of $6 per hour), which is similar of the sample mean of $1868 for monetary transfers.

[3] The SHARE Survey is conducted since 2004 in ten Western European countries.

to leave resources to the next generation. Taking account of time transfers adds a substitution effect since inheritance tax affects the trade-off between monetary and time transfers, making time transfers more attractive. From this point of view, taxing bequests may enhance the young's labor supply, giving room to an increase in resources disposable for market good consumption. Nevertheless, the positive effect on labor supply has to be balanced with the potential reduction in private wealth that may be detrimental for capital accumulation. At least, with inheritance tax, capital-labor ratio could be lower, moving the economy away from the Golden-rule of capital accumulation.

In this paper, considering time transfers in a second-best world, we analyze whether shifting from capital income tax towards inheritance tax may be a welfare-improving tax reform. The fall in capital income taxation may compensate the negative effects of inheritance tax on savings, capital accumulation and capital-labor ratio. But simultaneously, the reform may increase steady-state resources since the inheritance tax has a positive effect on labor supply. To analyze and disentangle the above effects, we consider a two-period overlapping generation model with rational altruism taking into account both types of family transfers (inheritance and time transfers) from grandparents to parents. Individuals work when young (*i.e.* parent) and then retire in their second period of life (*i.e.* when they are grandparents). In each period, every household consumes a composite good that aggregates market good and home production. Parent's labor supply decision depends on the trade-off between formal work and home production. Then, grandparents contribute to home production of the parents through both family transfers. Furthermore, the government finances public spending using taxation on inheritance, capital income and labor income.

As in the standard Barro model with rational altruism, inheritance tax decreases the accumulated capital stock and thus, reduces the capital-labor ratio at steady state. But the fall in capital income tax allows to neutralize this steady-state effect. Assuming that the tax reform is designed in order to leave the steady-state capital-labor ratio constant, we identify situations where life-cycle utility increases. First, steady-state utility is likely to increase when the substitution effect between consumption of market good and time devoted to home production is strong. Indeed, in this case, inheritance tax makes time transfers more attractive. Grandparents prefer to leave higher time transfers and lower bequests. The higher the substitution effect, the higher the increase in labor supply of the parents. Secondly, even if the substitution effect is not too strong, the tax reform may have a positive effect on steady-state utility through the size of the additional production of market goods generated by the increase in labor supply. We show that the strength of the latter effect depends crucially on the gaps between the marginal rates of transformation and the marginal rates of substitution between consumption in market goods and time devoted to home production. The reform is likely to increase utility if lower time devoted to home production allows the production sector to generate more market goods than necessary for leaving individual with the same level of utility. However, keeping the steady-state capital-labor ratio constant may shift the burden of the initial public debt towards the first generations and introduces some intergenerational redistributions. Using a numerical example, we illustrate that the effect of the tax reform on household's welfare of each generation can be positive along the transitional dynamics.

The paper is organized as follows. In Section 2, the model is presented. Section 3 analyzes the steady-state equilibrium with operative bequests and positive time transfers. Then, in Section 4, we present the tax reform and study its effects on households' utility in steady state. In Section 5, we conduct numerical illustrations in order to study the impact of the tax reform on the whole transitional dynamics. The final section concludes.

2 The model

We consider an overlapping generation model. Time is discrete. Population consists in one dynasty where the representative household of generation t lives two periods and has one child, born in $t+1$. We consider dynastic altruism *à la* Barro (1974) from parents to children.

2.1 Households

The representative household of generation t works during his first period of life (*i.e.* when young, or equivalently parent) and then retires (*i.e.* when old, or equivalently grandparent). Labor supply when young is elastic and depends on the allocation of a unit-time endowment between formal work and home production. In both periods, the household consumes a composite good that aggregates market good and home production. Life-cycle utility writes

$$u\left(f^y\left(c_t, T_t^y\right)\right) + v\left(f^o\left(d_{t+1}, T_{t+1}^o\right)\right)$$

where u and v are increasing and strictly concave. Function $f^y\left(c_t, T_t^y\right)$, respectively $f^o\left(d_{t+1}, T_{t+1}^o\right)$, is the quantity of composite good when young, resp. when old. The former is obtained with market good expenditures c_t and time devoted to home production T_t^y. In the latter, d_{t+1} represents market good expenditures when old, while T_{t+1}^o is time spent in home production. Home production functions f^y et f^o are assumed to be linear homogenous and concave. Marginal products are strictly positive and strictly decreasing.

Let ℓ_t denotes labor supply of the young in the formal sector. Household's decision for labor supply results from the trade-off between participation in the formal sector or to home production. Time devoted to home production when young, T_t^y, aggregates time spent by the household for its home production $1-\ell_t$ and time transfer from its parent (denoted by λ_t):

$$T_t^y = 1 - \ell_t + \mu\lambda_t \tag{1}$$

where $\mu > 0$ represents the relative efficiency of time transfer of the parent. Since the parent is retired, time spent in home production when old is the fraction of the unit-time endowment that is not transferred to his offspring

$$T_t^o = 1 - \lambda_t \tag{2}$$

In the following, τ_t^w, τ_t^x and τ_t^R are the respective period-t tax rates on wages, bequests and capital income. R_t and w_t denote the gross interest rate and the wage rate. When young, a household born in t receives after-tax wage income $(1-\tau_t^w) w_t \ell_t$ and after-tax bequest $(1-\tau_t^x) x_t$. These resources are allocated between consumption spendings c_t and saving s_t:

$$c_t + s_t = (1-\tau_t^w) w_t \ell_t + (1-\tau_t^x) x_t \tag{3}$$

When old, the household allocates after-tax capital income $\left(1-\tau_{t+1}^R\right) R_{t+1} s_t$ between consumption spendings d_{t+1} and bequest x_{t+1}:

$$d_{t+1} + x_{t+1} = \left(1-\tau_{t+1}^R\right) R_{t+1} s_t \tag{4}$$

Following Barro (1974), rational altruism means that households enjoy utility of their children. Utility of the household born in t, U_t, depends on consumptions in composite goods in both periods and utility of its offspring U_{t+1}:

$$U_t = u\left(f^y\left(c_t, T_t^y\right)\right) + v\left(f^o\left(d_{t+1}, T_{t+1}^o\right)\right) + \beta U_{t+1} \tag{5}$$

where β denotes the degree of altruism, $0 < \beta < 1$.

Using equations (1)-(4), both consumptions in market goods rewrite

$$c_t = (1-\tau_t^w) w_t \left[1 - T_t^y + \mu\left(1 - T_t^o\right)\right] + (1-\tau_t^x) x_t - s_t \tag{6}$$

$$d_{t+1} = \left(1-\tau_{t+1}^R\right) R_{t+1} s_t - x_{t+1} \tag{7}$$

Plugging (6)-(7) into U_t gives household's utility as a function of s_t, x_{t+1}, T_t^y and T_{t+1}^o. The representative household maximizes U_t with respect to these four variables. For an interior solution, this leads to the following first-order conditions:

- with respect to s_t

$$-u_t' f_{c_t}^y + \left(1-\tau_{t+1}^R\right) R_{t+1} v_{t+1}' f_{d_{t+1}}^o = 0 \tag{8}$$

  where u_t', $f_{c_t}^y$, v_{t+1}' and $f_{d_{t+1}}^o$ respectively stand for the partial derivatives $\frac{\partial u_t}{\partial f_t^y}$, $\frac{\partial f_t^y}{\partial c_t}$, $\frac{\partial v_{t+1}}{\partial f_{t+1}^o}$ and $\frac{\partial f_{t+1}^o}{\partial d_{t+1}}$.

- with respect to T_t^y

$$-\left(1-\tau_t^w\right) w_t f_{c_t}^y + f_{T_t^y}^y = 0, \text{ if } 0 < T_t^y < 1 + \mu\left(1 - T_t^v\right) \tag{9}$$

where $f^y_{T^y_t}$ stands for $\frac{\partial f^y_t}{\partial T^y_t}$.

- with respect to x_{t+1}

$$-v'_{t+1} f^o_{d_{t+1}} + \beta \left(1 - \tau^x_{t+1}\right) u'_{t+1} f^y_{c_{t+1}} = 0, \text{ if } x_{t+1} > 0 \tag{10}$$

- with respect to T^o_{t+1}

$$v'_{t+1} f^o_{T^o_{t+1}} - \beta\mu \left(1 - \tau^w_{t+1}\right) w_{t+1} u'_{t+1} f^y_{c_{t+1}} = 0, \text{ if } 0 < T^o_{t+1} < 1 \tag{11}$$

where $f^o_{T^o_{t+1}}$ stands for $\frac{\partial f^o_{t+1}}{\partial T^o_{t+1}}$.

Constraints for an interior solution are not necessarily satisfied at equilibrium. The less critical one is the constraint $T^y_t < 1 + \mu\left(1 - T^o_t\right)$, which is equivalent to positive labor supply ($\ell_t > 0$). Assuming that it is satisfied remains to consider equilibria where the production sector uses labor. Two other constraints should be satisfied with small additional assumptions: $T^y_t \geq 0$ and $T^o_{t+1} \geq 0$. Time spent in home production remains positive if substitutability with market goods is not too strong.

Finally, non-negativity constraints on bequests and time transfers deserve some discussion. It depends on the utility gains that parents may expect with both kinds of transfers. As shown by Weil (1987), in the standard Barro framework without time transfers, positive bequests are obtained at steady state, if the steady-state capital-labor ratio in the corresponding Diamond economy is below the modified Golden-rule. With time transfers, assuming logarithmic utility and Cobb-Douglas technology, Cardia and Michel (2004) have given conditions for the existence of intertemporal equilibria where both transfers are positive. They also state conditions for the case with zero bequest and positive time transfers. In the following, since our concern is the effect of inheritance tax on bequests and time transfers, we focus on a steady state where both transfers are positive.

2.2 Equilibrium

The production sector consists in a representative firm that behaves competitively, and produces output with labor and capital. The production function $F(k, \ell)$ is linear homogenous and concave, and includes capital depreciation. Marginal products are strictly positive and strictly decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$w_t = F_L\left(k_t, \ell_t\right) \tag{12}$$

$$R_t = F_K\left(k_t, \ell_t\right) \tag{13}$$

where k_t is capital stock. F_L and F_K stand for the partial derivatives of F with respect to labor and capital.

At equilibrium, household savings s_t split into private capital that will be used in $t+1$ and public debt Δ_t

$$k_{t+1} + \Delta_t = s_t$$

In each period, government spendings amount to a fraction Γ of total production. Government resources come from taxation on labor income, capital income and bequests. Then, public debt accumulates according to the following law of motion:

$$\Delta_t = R_t \Delta_{t-1} + \Gamma F\left(k_t, \ell_t\right) - \tau_t^w \ell_t w_t - \tau_t^R R_t s_{t-1} - \tau_t^x x_t \tag{14}$$

where the initial public debt $\Delta_{-1} = \bar{\Delta}_{-1}$ is given.

The assumption that government spendings are proportional to production leads to an externality created by the level of production. Indeed, consider the resource constraint in period t

$$c_t + d_t + k_{t+1} = (1-\Gamma) F_K\left(k_t, \ell_t\right) \tag{15}$$

By increasing capital (resp. labor) used in production, the social marginal product for consumption and investment is $(1-\Gamma) F_K(k_t, \ell_t)$ (resp. $(1-\Gamma)\ F_L(k_t, \ell_t)$), while the private marginal product are higher, equal to $F_K(k_t, \ell_t)$ (resp. $F_L(k_t, \ell_t)$) as stated by the first-order condition of the representative firm (12) and (13). Such externalities are internalized by the private sector if the government sets the tax rates on capital and labor incomes to $\tau_t^R = \tau_t^w = \Gamma$.

3 Steady state with positive transfers

We consider steady states with operative bequests and positive time transfers. Tax rates and public debt are assumed to be constant over time. From the marginal conditions (8) and (10), the gross interest rate satisfies the modified Golden-rule, and is equal to R_M defined as

$$\beta\left(1-\tau^x\right)\left(1-\tau^R\right) R_M = 1 \tag{16}$$

which characterizes the capital-labor ratio $k/\ell = z_M$ and the wage rate $w_M = F_L(z_M, 1)$.

From equations (8),(9) and (11), the other marginal conditions of the household problem can be rewritten as equalities between marginal rates of substitution and relative prices:

$$\frac{v' f_d^o}{u' f_c^y} = \beta\left(1-\tau^x\right) \equiv P^R \tag{17}$$

$$\frac{f^y_{T^y}}{f^y_c} = (1-\tau^w)\, w_M \equiv P^y \tag{18}$$

$$\frac{f^o_{T^o}}{f^o_d} = \mu \frac{(1-\tau^w)\, w_M}{1-\tau^x} \equiv P^o \tag{19}$$

where P^R is the relative price between market good consumption when old d and market good consumption when young c, and P^y (resp. P^o) is the relative price between time devoted to home production and market good consumption when young (resp. when old).

Time constraint when young (1) gives the household's labor supply

$$\ell = 1 - T^y + \mu (1 - T^o). \tag{20}$$

Then, the resource constraint (15) becomes

$$c + d = C_M \left[1 - T^y + \mu (1 - T^o)\right] \tag{21}$$

where C_M denotes aggregate consumption per labor unit

$$C_M \equiv (1-\Gamma)\, F(z_M, 1) - z_M$$

Consequently, for given tax rates $\left(\tau^w, \tau^R, \tau^x\right)$, marginal conditions (17)-(19) and the resource constraint (21) characterize household's choice at steady-state equilibrium for consumptions in market good, c and d, and times devoted to home production, T^y and T^o.

The household's intertemporal budget constraint (obtained by eliminating s_t from equations (6) and (7)) allows to compute steady-state bequest. Indeed, using the time constraint (20) and the relation between relative prices $P^R P^o = \beta\mu P^y$ (deduced from their definitions in (17)-(19)), the intertemporal budget constraint rewrites

$$c + P^y T^y + P^R \left(d + \beta^{-1} P^o T^o\right) = P^y (1+\mu) + (1-\tau^x)(1-\beta)\, x$$

Bequests are positive if the present value of market good spendings $c + P^R d$ is higher than net wage income $(1-\tau^w)\, w_M \ell$.

Finally, public debt is deduced from the budget constraint of the government

$$\Delta = \left(\left(1-\tau^R\right) R_M - 1\right)^{-1} \left(\tau^x x + \left[\tau^w w_M + \tau^R R_M z_M - \Gamma F(z_M, 1)\right] \ell\right) \tag{22}$$

For instance, if all tax rates are zero, steady-state public debt is negative, equal to $-(R_M - 1)^{-1}\, \Gamma F(z_M, 1)\, \ell$. This means that, at each period, the government uses interests on public capital to finance public spendings $\Gamma F(z_M, 1)\, \ell$. Of course, this is possible either if there is initial public capital that finances the whole sequence of public spendings, or if the government has taxed households in order to accumulate some public capital to this end.

As stated before, the case where capital and labor incomes are taxed at the same rate $\tau^R = \tau^w = \Gamma$ allows to eliminate the externality created by public spendings. Since F is linear homogenous, these taxes would then finance the whole current public spendings. Additionally, if the initial debt $\bar{\Delta}_{-1}$ is zero, a first-best optimum is obtained with a zero inheritance tax.

4 Fiscal reform

In the following, we assume that the government cannot set the tax rates, τ^w and τ^R, at Γ in order to eliminate the public spending externality. This can result, for instance, from the burden of a positive public debt that has to be distributed among generations through additional taxation. In this regard, before the tax reform, the government finances the debt burden using capital income tax at rate $\bar{\tau}^R > \Gamma$ constant over time, with $\bar{\tau}^w = \Gamma$ and $\bar{\tau}^x = 0$. Higher capital income tax rate distorts in household's saving decision, leading to a lower capital-labor ratio than the one obtained at a first-best optimum.

The issue we address is whether a tax shift from capital income tax towards inheritance tax would be welfare enhancing. The tax reform consists to set up a positive inheritance tax rate $\tau^x > 0$ and reduces the capital income tax τ^R in order to make it closer to Γ.

In overlapping-generation models with rational altruism, capital income is divided between second-period consumption and inheritance (see the second-period budget constraint (4) of the representative household). This implies that inheritance is lower than capital income. Therefore, if the government tries to keep the primary surplus (fiscal receipts minus public spendings) constant, it needs to increase inheritance tax rate by a larger amount than the fall in the capital income tax rate. This means that the product $(1-\tau^x)\left(1-\tau^R\right)$ decreases and becomes lower than $\left(1-\bar{\tau}^R\right)$, leading to a fall in the capital-labor ratio (see equation (16)), moving the economy away from the Golden-rule.

In the following, we conduct the analysis by first assuming that the fiscal reform is designed in order to keep the capital-labor ratio constant. Therefore, the shift from capital income to inheritance tax is such that

$$(1-\tau^x)\left(1-\tau^R\right) = 1-\bar{\tau}^R$$

This implies a proportional change in both tax rates. Consequently, the fiscal reform decreases the steady-state primary surplus and reduces the steady-state public debt (see equation (22)). The tax reform then shifts the burden of the initial debt toward the first generations. Otherwise stated, it introduces an intergenerational redistribution of resources from the first generations towards the ones far in the future.

At this stage, we will focus on the effect of the reform on steady-state life-cycle utility:

$$V = u\left(f^y\left(c, T^y\right)\right) + v\left(f^o\left(d, T^o\right)\right) \tag{23}$$

and postpone the issue of intergenerational redistribution to the next section, through numerical illustrations.

The rest of the section decomposes the marginal effect of the tax reform on steady-state household's life cycle utility in different settings. We start from the Barro model, assuming inelastic labor supply and no time transfer. We then extend the discussion to elastic labor supply, still without time transfer. Finally, we consider the complete framework with elastic labor supply, assuming that both private intergenerational transfers are positive.

4.1 Tax reform without time transfer at steady state

To decompose the different effects of a tax reform, we first analyze a shift from capital income taxation toward inheritance taxation (leaving constant the capital-labor ratio) in an economy where time transfers are inoperative. We thus leave aside the fact that inheritance taxation modifies the trade-off between both parental transfers. At a steady-state equilibrium with positive bequests and zero time transfer, *i.e.* $x > 0$ and $T^o = 1$, the capital-labor ratio, the gross interest rate and the wage rate are at their modified Golden-rule levels. Market good consumptions (c and d) and time spent to home production when young T^y are characterized by marginal conditions (17), (18) and the resource constraint (21) and may change with the tax reform implemented. Thus, the marginal changes in τ^x reduces the relative price P^R and then modifies the household's intertemporal allocation of resources between consumptions in market good when young and old. The magnitude of the effect crucially depends on the elasticity of substitution between the composite goods f^y and f^o. Let us denote by σ^u, the absolute value of this intertemporal elasticity of substitution. Then

$$\frac{df^y}{f^y} - \frac{df^o}{f^o} = \sigma^u \frac{d\left(\frac{f_c^y P^R}{f_d^o}\right)}{\frac{f_c^y P^R}{f_d^o}} = \sigma^u \left(\frac{df_c^y}{f_c^y} - \frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R}\right) \tag{24}$$

4.1.1 Tax reform in the standard Barro model

We first show that the tax reform in the standard Barro (1974) model with inelastic labor supply ($T^y = 1$) has a negative effect on household's welfare.

Proposition 1. *At a steady-state equilibrium with no time transfer and inelastic labor supply, consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first-period consumption in the market good c decreases, while the second-period consumption d increases. Moreover, steady-state life-cycle utility (23) decreases.*

Proof. Differentiating steady-state life-cycle utility $V = u(f^y(c,1)) + v(f^o(d,1))$, and using marginal condition (17), dV has the same sign as

$$dc + P^R dd$$

Moreover, differentiating the resource constraint (21), one gets

$$c\frac{dc}{c} + d\frac{dd}{d} = 0 \tag{25}$$

Thus dV has the same sign as

$$\left(P^R - 1\right)\frac{dd}{d}$$

We now need to state the sign of dd. Let us define the shares of market good cost in the total cost of production of the composite good for the young $\alpha^y \equiv f_c^y c/f^y$ and the old $\alpha^o \equiv f_d^o d/f^o$. Equation (24) then rewrites as

$$\alpha^y \frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(\frac{f_{cc}^y(c,1)\,c}{f_c^y(c,1)}\frac{dc}{c} - \frac{f_{dd}^o(d,1)\,d}{f_d^o(d,1)}\frac{dd}{d} + \frac{dP^R}{P^R} \right)$$

using the following relations:

$$\frac{df_c^y}{f_c^y} = \frac{f_{cc}^y(c,1)\,c}{f_c^y(c,1)}\frac{dc}{c} \text{ and } \frac{df_d^o}{f_d^o} = \frac{f_{dd}^o(d,1)\,d}{f_d^o(d,1)}\frac{dd}{d}$$

$$\frac{df^y}{f^y} = \alpha^y\frac{dc}{c} \text{ and } \frac{df^o}{f^o} = \alpha^o\frac{dd}{d}$$

Then, from equation (25), one easily checks that dd has an opposite sign to dP^R. Since the tax reform considered implies a fall in $P^R = \beta(1-\tau^x)$, one gets $dV < 0$, which concludes the proof. □

The fall in the relative price between both intertemporal market good consumptions P^R increases the market good consumption when old d and pushes down the market good consumption when young c. Both effects are stronger when the substitutability between composite goods is important (*i.e.* high σ^u). In addition, from equation (25), the marginal rate of transformation between d and c ($MRT_{d/c}$) is equal to one. As the marginal rate of substitution between d and c ($MRS_{d/c} = P^R$) is lower than the $MRT_{d/c}$ and declines with the tax reform, household's welfare is negatively affected by the reform.

4.1.2 Tax reform with elastic labor supply

Extending the model to elastic labor supply when young ($T^y \leq 1$) modifies the effect of the tax reform, introducing labor supply effects. From equation (18), since home production functions are linear homogeneous, one deduces that the ratio c/T^y can be written as a function of P^y: $c/T^y = \phi^y(P^y)$. Since the tax reform does not modify the relative price P^y, market good consumption c varies in the same proportion as time devoted to home production T^y. Then any reallocation of resources from c to d is associated with a reduction in T^y by the same percentage as the reduction in c. One gets the following result.

Proposition 2. *At a steady-state equilibrium with no time transfer, let us consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first period consumption in the market good (c) and time spent in home production (T^y) decrease, while the second period consumption (d) increases. Moreover, steady-state utility increases iff*

$$C_M - \frac{P^y}{P^R} > \phi^y(P^y)\left(\frac{1}{P^R} - 1\right). \tag{26}$$

Proof. Since the home production function when young f^y is linear homogenous and $dP^y = 0$, we deduce from (18) that

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

Then, equation (24) rewrites as

$$\frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(-\frac{f_{dd}^o(d,1)\,d}{f_d^o(d,1)}\frac{dd}{d} + \frac{dP^R}{P^R}\right)$$

Differentiating the resource constraint (21), one gets

$$(c + C_M T^y)\frac{dc}{c} = -d\frac{dd}{d} \tag{27}$$

Thus, straightforward computations lead to

$$\left[\frac{-f_{dd}^o(d,1)\,d}{f_d^o(d,1)}\sigma^u + \frac{d}{c + C_M T^y} + \alpha^o\right]\frac{dd}{d} = -\sigma^u\frac{dP^R}{P^R}$$

which shows that the sign of dd is opposite to dP^R, while dc and dT^y have the same sign as dP^R. Moreover, the sign of dV is the same as

$$dc + P^y dT^y + P^R dd = (c + P^y T^y)\frac{dc}{c} + P^R d\frac{dd}{d}$$

Using equation (27), $dV > 0$ is equivalent to condition (26), since the tax reform considered implies a fall in $P^R = \beta(1 - \tau^x)$. □

To interpret results in Proposition 2, recall that the tax reform consists in a fall in second-period consumption price P^R that increases d and reduces c and T^y. The fall in T^y improves total resources for market good consumption $C_M(1 - T^y)$ through the increase in labor supply. The positive effect of the tax reform on labor supply attenuates or reverses the Barro-model effect on utility highlighted in Proposition 1. Notice that the increase in labor supply should be stronger when the substitutability between both periods is important (*i.e.* high σ^u).

Since the capital-labor ratio is kept constant, the increase in labor supply is associated with an increase in the capital stock, and thus in savings. The young work more, consume less and then save more for their second period of life.

The consumption per additional labor unit C_M corresponds to the marginal rate of transformation between T^y and d, while P^y/P^R corresponds to the marginal rate of substitution between both variables. Then, $C_M > P^y/P^R$ means that the fall in T^y allows to produce more market goods for second-period consumption than the amount necessary to preserve the same welfare.

The condition $C_M > P^y/P^R$ is sufficient to guarantee welfare improvement if $P^R > 1$. But, with the initial values of the instruments that we consider ($\tau^w = \Gamma$, $\tau^x = 0$ and $\bar{\tau}^R > \Gamma$), the relative price P^R is equal to β, and is lower than 1. In this case, the condition $C_M > P^y/P^R$ is no longer sufficient: welfare increases if the ratio ϕ^y is small enough. Indeed, a low ϕ^y corresponds to a situation where the first-period market good consumption c is relatively small to T^y. Thus, the proportional reduction of c and T^y leads to a small reduction in c (small negative effect on welfare) and a sharp increase in labor supply.

In a country where people consume a large (resp. small) amount of market goods, the ratio ϕ^y would be high (resp. low) and then the tax reform would be detrimental for welfare (resp. welfare enhancing). The situation where consumption relies essentially on market goods can be associated with a developped country. By contrast, in a developing country, time devoted to home production becomes more important and consumption in market goods lower, leading to a small ratio ϕ^y. Following this interpretation, under the condition $C_M > P^y/P^R$, the tax reform is likely to be welfare enhancing in developing rather than developped countries.

4.2 Tax reform when both transfers are positive

Let us now introduce time transfers by considering the tax reform at steady state where both private transfers are positive: $x > 0$ and $T^o < 1$. Compared with the preceding Section without time transfers, the marginal shift from capital income tax towards inheritance tax also modifies the parent's trade-off between bequests and time transfers. As we shall see, this adds new positive or negative effects on the young's labor supply.

The steady state is characterized by marginal conditions (17)-(19) and the resource constraint (21). In these equations, the tax reform not only decreases the relative price P^R between both market good consumptions, but also increases P^o, the relative price between market good and time used in home production when old. In the following, consequences of the fall in P^R will be named *interperiod* effects, while those resulting from higher P^o will be named *intraperiod* effects.

We first detail the interperiod effects. The fall in P^R has similar consequences on labor supply than those stressed in the preceding Subsection 4.1.2, but also introduces an additional effect through changes in the time transfer. Indeed, lower P^R involves a negative effect on c and T^y and a positive effect on d and T^o. The elasticity of substitution σ^u between both composite goods may amplify these effects. The resulting impact on the young's labor supply is ambiguous: the negative effect on T^y affects positively the labor supply whereas the positive effect on T^o leads to a negative impact on time transfers, hence on the young's labor supply.

We now turn to the intraperiod effects, that come from the increase in P^o. The equality between marginal rate of substitution and relative price, $MRS_{T^o/d} = P^o$, implies that the marginal rate of substitution between d and T^o increases with the tax reform. This has a positive impact on d and a negative effect on T^o. The negative effect on T^o affects positively the labor supply. The magnitude of the intraperiod effect on T^o depends crucially on the elasticity of substitution between T^o and d. Let us denote by σ^o, the absolute value of this elasticity of substitution associated with home production technology f^o. By definition:

$$\frac{dd}{d} - \frac{dT^o}{T^o} = \sigma^o \frac{dP^o}{P^o} = -\sigma^o \frac{dP^R}{P^R} \tag{28}$$

The following Lemma signs the marginal effect on the second-period consumption d.

Lemma 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, marginal effect on second-period consumption d is positive and given by*

$$\frac{dd}{d} = -\left[\left(1 - \frac{d}{(1+\mu)\,C_M}\right)\sigma^o + \frac{c + C_M T^y}{(1+\mu)\,C_M}\alpha^o\left(\sigma^u - \sigma^o\right)\right]\frac{dP^R}{P^R} > 0 \tag{29}$$

where $\alpha^o \equiv f^o_d d / f^o$.

Proof. As the home production function when old f^o is linear homogenous,

$$\frac{df^o}{f^o} = \alpha^o \frac{dd}{d} + (1-\alpha^o)\frac{dT^o}{T^o} = \frac{dd}{d} + (1-\alpha^o)\,\sigma^o \frac{dP^R}{P^R}$$

where the second equality is obtained with equation (28). Since $dP^y = 0$ and the home production function when young f^y is linear homogenous, one deduces

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df^y_c = 0$$

Then, equation (24) rewrites as

$$\frac{dc}{c} - \frac{dd}{d} - (1-\alpha^o)\,\sigma^o \frac{dP^R}{P^R} = \sigma^u\left(-\frac{df^o_d}{f^o_d} + \frac{dP^R}{P^R}\right)$$

Linear homogeneity of f^o implies $T^o f^o_{dT^o}(d, T^o) = -d f^o_{dd}(d, T^o)$ and $\frac{-f^o_{dd} d}{f^o_d}\sigma^o = 1 - \alpha^o$. Then, one

gets
$$\frac{df^o_d}{f^o_d}=\frac{f^o_{dd}dd+f^o_{dT^o}dT^o}{f^o_d}=\frac{-f^o_{dd}d}{f^o_d}\sigma^o\frac{dP^R}{P^R}=(1-\alpha^o)\frac{dP^R}{P^R}$$
Consequently, the preceding relation between $\frac{dc}{c}$ and $\frac{dd}{d}$ becomes
$$\frac{dc}{c}-\frac{dd}{d}=\left[\sigma^u\alpha^o+(1-\alpha^o)\,\sigma^o\right]\frac{dP^R}{P^R} \tag{30}$$
Differentiation of the resource constraint (21) yields
$$c\frac{dc}{c}+d\frac{dd}{d}+C_M\left(T^y\frac{dT^y}{T^y}+\mu T^o\frac{dT^o}{T^o}\right)=0$$
and, combining with equation (30), allows to compute dd/d:
$$\frac{dd}{d}=-\frac{(c+C_MT^y)\left[\sigma^u\alpha^o+(1-\alpha^o)\,\sigma^o\right]+\mu C_MT^o\sigma^o}{c+d+C_M\,(T^y+\mu T^o)}\frac{dP^R}{P^R}>0$$
which is equivalent to equation (29). □

Lemma 1 shows that tax reform results in an increase in d whatever the initial values of the instruments, as soon as they allow for positive bequests and positive time transfers. We now turn to the variations of c, T^y and T^o that depend crucially on both elasticities of substitution σ^u and σ^o, that respectively drive up the size of the interperiod and intraperiod effects.

Lemma 2. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. Then, one gets the following sufficient conditions:*

(i) If $\sigma^o \geq \sigma^u$, the marginal effect on time devoted to home production when old T^o is negative.

(ii) If $\sigma^u \geq \sigma^o$, the marginal effects on first-period consumption in market good c and time devoted to home production T^y are negative.

(iii) If σ^o/σ^u is close to zero, c and T^y decrease, while T^o increases.

(iv) If σ^o/σ^u is close to unity, then c, T^y and T^o decrease.

(v) If σ^o/σ^u tends to infinity, c and T^y increase, while T^o decreases.

Proof. Marginal effects on c, T^y and T^o can be computed from expressions (28), (29) and (30):
$$\frac{dc}{c}=\frac{dT^y}{T^y}=\sigma^o\frac{d+\mu C_MT^o}{(1+\mu)\,C_M}\left[\alpha^o\left(\frac{\sigma^u}{\sigma^o}-1\right)+\frac{d}{d+\mu C_MT^o}\right]\frac{dP^R}{P^R}$$

$$\frac{dT^o}{T^o} = -\,\sigma^o \frac{c + C_M T^y}{(1+\mu)\, C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1 \right) - \frac{d}{c + C_M T^y} \right] \frac{dP^R}{P^R}$$

This proves results (i)-(iv). Let us show result (v). Assuming that σ^o/σ^u tends to infinity, one gets that dc and dT^y are positive iff

$$\alpha^o > \frac{d}{d + \mu C_M T^o}$$

which is equivalent to $\mu C_M > P^o$, since $\alpha^o = d / (d + P^o T^o)$. The proof is complete. □

Notice that the assumption $\mu C_M > P^o$ is satisfied at the initial steady state, that is, with $\tau^x = 0$, $\tau^w = \Gamma$ and $\bar{\tau}^R > \Gamma$. Indeed, since $P^o = \beta \mu P^y / P^R$, straightforward calculations using linear homogeneity of the technology F show that the inequality $\mu C_M > P^o$ is always true.[4] At equilibrium, the relative price P^o is equal to the marginal rate of substitution between T^o and d ($MRS_{T^o/d}$). Moreover, from the resource contraint, the marginal rate of transformation between T^o and d is: $MRT_{T^o/d} = \mu C_M$. Thus, the assumption $\mu C_M > P^o$ means that the MRT between T^o and d is higher than the MRS, that is, for given (c, T^y), any fall in T^o increases labor supply, and then leaves enough additional resources for second-period consumption, to increase utility.

From the proof of the preceding Lemma, one may notice that increases in all variables c, d, T^y and T^o cannot arise simultaneously, since $dc > 0$ requires $\sigma^u < \sigma^o$, which implies $dT^o < 0$. Therefore, only three cases can arise:

- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o > 0$. This case arises when σ^o/σ^u is close to zero. Intergenerational time transfers have been reduced by the increase in the inheritance tax.
- $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u is close to one, as with logarithmic utility.[5] It induces a rise in intergenerational time transfers.
- $dc > 0$, $dT^y > 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u tends to infinity. Intergenerational time transfers increase with the inheritance tax.

We now analyze the marginal effect of the tax reform on the household life-cycle utility in each of these three cases. In the following Proposition, we establish the condition for the tax reform to be welfare improving.

[4] With $\tau^x = 0$, inequality $\mu C_M > P^o$ is equivalent to $C_M > P^y$. Using the linear homogeneity of F, one gets

$$C_M = (1-\Gamma)\, F_L + [(1-\Gamma)\, F_K - 1]\, z_M > P^y$$

where the last inequality is obtained using $\tau^w = \Gamma$ and $(1-\Gamma)\, F_K > \left(1 - \bar{\tau}^R\right) F_K = \frac{1}{\beta} > 1$.

[5] This is the case, for instance, if the life-cycle utility function is:

$$\alpha^y \ln c + (1-\alpha^y) \ln (1 - T^y) + \gamma \left[\alpha^o \ln d + (1-\alpha^o) \ln (1 - T^o) \right]$$

where α^y, α^o and γ are positive parameters, $\alpha^y < 1$ and $\alpha^o < 1$.

Proposition 3. *At a steady state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant, the marginal effect on utility dV has the same sign as*

$$\left[P^R - \Theta\right] d - \alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) \left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] \tag{31}$$

where

$$\Theta \equiv \frac{c + P^y T^y + P^R d + \beta\mu P^y T^o}{(1+\mu)\, C_M} \tag{32}$$

Proof. Using the marginal conditions of the household problem (17)-(19), dV has the same sign as

$$dc + P^y dT^y + P^R dd + \beta\mu P^y dT^o$$

Since $dP^y = 0$, relative changes dc/c and dT^y/T^y are equal. Consequently, replacing equations (28) and (30) in the latter equation and using expression (29) in Lemma 1, one obtains that dV has the same sign as

$$-\sigma^o \left[P^R - \Theta\right] d \frac{dP^R}{P^R} + \alpha^o \left(\sigma^u - \sigma^o\right) \left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] \frac{dP^R}{P^R}$$

which concludes the proof. □

To interpret condition (31), we distinguish the above three cases according to the value of the elasticity ratio σ^o/σ^u.

4.2.1 Tax reform with $\sigma^u = \sigma^o$

In this situation, that encompasses the case of a logarithmic utility function, the second-period consumption d increases thanks to lower c, T^y and T^o. From expression (31), welfare increases if and only if $P^R > \Theta$, which can be rewritten as:

$$dV > 0 \Leftrightarrow C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y + \left(\mu C_M - P^o\right)\frac{T^o}{T^y} > 0$$

In the latter inequality, we observe the same term as in (26): $C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y$. The tax reform increases welfare in the model with elastic labor supply and no time transfer iff this term is positive. This leads to the same kind of interpretation: the fall in the second-period consumption price P^R reduces c and T^y and increases d. Then, the reduction in T^y increases the young's labor supply involving a positive effect on resources in market good.

Moreover, the positive effect on labor supply is reinforced by the increase in time transfers since T^o decreases with the reform. This positive effect on welfare appears in the second-term of the latter inequality. As stated before, the substitution from T^o to d is welfare enhancing since the initial equilibrium satisfies $\mu C_M > P^o$, that is, $MRT_{T^o/d} > MRS_{T^o/d}$.

Therefore, taking the Barro model with elastic labor supply as a benchmark, the introduction of intergenerational time transfers creates an additional positive effect on steady-state welfare. As soon as condition (26) is satisfied, the tax reform improves steady-state welfare. The falls in T^y and T^o involve a rise in labor supply. Simultaneously, reducing c and increasing d imply higher savings, and lead to higher capital stock. All these additional inputs allow to produce more market goods, that will be consumed in second-period of life.

4.2.2 Tax reform with $\sigma^u >> \sigma^o$

In this case, interperiod effects (from the decrease in P^R) dominate intraperiod effects (from the increase in P^o). This arises with a high elasticity of substitution between both composite goods σ^u, or with a low elasticity of substitution σ^o.

A high σ^u involves a significant shift in resources from the first to the second period of life. Thus, the market good consumption d and the time devoted to home production when old T^o strongly increase thanks to lower c and T^y.

For a low elasticity of substitution σ^o, the tax reform has a negative effect on time transfers. Indeed, the increase in d associated with strong complementarity between d and T^o results in an increase in T^o as $\phi^o = d/T^o$ remains constant.

In both cases, the effect on labor supply is ambiguous as the labor supply is positively affected by the reduction in T^y and negatively by the increase in T^o.

The marginal effect on household life-cycle utility may be worse off than with a logarithmic utility as the effect on labor supply is attenuated or reversed. From expression (31), the welfare is improved iff:

$$dV > 0 \Leftrightarrow -\left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] > 0$$

Using expression (32), one gets

$$C_M - \frac{P^y}{P^R} > \left(\frac{\phi^o + \mu C_M}{\phi^o + P^o}\frac{1}{P^R} - 1\right)\phi^y \tag{33}$$

With the initial values of the instruments: $\mu C_M > P^o$ and $P^R < 1$. Therefore, the difference $C_M - \frac{P^y}{P^R}$ has to be positive for the tax reform to improve welfare. Comparing inequalities (26) and (33), the right-hand side in inequality (33) is higher. Consequently, situations where the tax reform has a positive effect on welfare are less likely to happen with operative time transfers than in the Barro model with elastic labor supply. Increase in T^o reduces time transfer to the young and then affects negatively their labor supply.

The ratio ϕ^y has still to be low in order to get a positive effect of the tax reform. As in the preceding Sections, low ϕ^y means a sharp decrease in T^y, and thus an important increase in labor supply. With time transfers, the ratio ϕ^o has also an impact. Indeed, since the tax reform increases P^o, the ratio ϕ^o also increases. Thus, if ϕ^o is initially high, the rise in T^o will be small and has also a small negative effect on labor supply.

4.2.3 Tax reform with $\sigma^u << \sigma^o$

Here, intraperiod effects (through higher P^o) dominate interperiod effects (through lower P^R). This case arises if σ^o is high, or if σ^u is small.

On the one hand, for a high elasticity of substitution σ^o, increasing relative price P^o involves higher second-period consumption of market good d, lower time devoted to home production T^o, and so, higher time transfer to the young. The young enjoy more resources, and then consume more composite good, increasing both c and T^y.

On the other hand, a low elasticity of substitution between both periods σ^u means that both composite goods are complements. This involves a small effect of P^R and a small shift of resources from the first to the second period. But, higher P^o increases the ratio d/T^o, leading to a fall in T^o. The latter involves a positive effect on labor supply of the young.

Corollary 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. If the ratio σ^o/σ^u tends to infinity, the marginal effect of the tax reform on utility is positive.*

Proof. Putting σ^u/σ^o at zero in condition (31), one gets that dV is positive iff

$$\left[P^R - \Theta\right] d + \alpha^o \left[(c + P^y T^y) - \Theta\,(c + C_M T^y)\right] > 0$$

Then, plugging Θ, from expression (32) into the preceding inequality yields

$$(1+\mu)\, C_M > c + C_M T^y + d + P^o T^o$$

which is true if $\mu C_M > P^o$. □

In the case with σ^o/σ^u closed to unity, both T^y and T^o were reduced by the tax reform. With higher σ^o/σ^u, the negative effect of the tax reform on time devoted to home production by the old, T^o, is strengthened. This increases first-period resources, and allows a rise in time devoted to home production by the young T^y. This shows that the effect on welfare is likely to be positive if the increase in labor supply only comes from a rise in time transfers from the grandparents.

5 Numerical illustrations when $\sigma^u << \sigma^o$

As stated before, our aim is to identify situations where a tax shift from capital income tax towards inheritance tax would be Pareto-improving. We use numerical examples to analyze the impact of the tax reform on welfare along the transitional dynamics. Welfare of any generation t corresponds to the infinite sum

$$W_t = \sum_{i=t}^{+\infty} \beta^{i-t} V_i$$

where V_i is life-cycle utility of generation $i \geq t$. Then a Pareto-improvement is achieved if the tax reform does not reduce W_t, for any generation $t \geq -1$, and increases W_t for at least one generation.

We start from the same values of the instruments as those considered in the steady-state analysis, for any $t \geq 0$: $\tau_t^R = \bar{\tau}^R > \Gamma$, $\tau_t^w = \Gamma$, $\tau_t^x = 0$. We consider an initial public debt $\bar{\Delta}_{-1} = 0.1$ whose debt burdens is equally shared between generations through an higher capital income tax such that we get $\tau_t^R = \bar{\tau}^R > \Gamma$.

In a first illustration, we focus on the same kind of fiscal reform as in Section 4, that is a fiscal reform that keeps the capital-labor ratio constant in the long-run. However, this tax reform reduces fiscal receipts at steady state, thus shifting the burden of the initial public debt to the first generations. It involves some intergenerational redistribution towards generations living in the long run.

Then, we turn to a second illustration where the tax reform can reduce the capital-labor ratio in the long-run. This attenuates the intergenerational redistribution due to the reallocation of public debt burden.

Furthermore, we concentrate on situations where the tax reform increases c, T^y and the labor supply through the positive effect on time transfer (*i.e.* $\sigma^u << \sigma^o$). In this case, the fiscal reform implemented in Section 4, involves an increase of the steady state households' life-cycle utility. This is likely the most favorable situation to improve the household welfare. For this purpose, we assume that d and T^o are substitutes and that the elasticity of substitution between both periods σ^u is low. Table 1 presents values of all parameters.

5.1 Tax reform with constant steady-state capital-labor ratio

A shift from capital income tax towards inheritance tax is implemented by introducing $\tau_t^x = 0.03$ for any period $t \geq 0$ and leaving the steady-state capital-labor ratio constant. The new capital income tax rate is $\tilde{\tau}_t^R = \frac{\bar{\tau}^R - \tau^x}{1-\tau^x}$ for any generation $t \geq 1$, with adjustment of τ_0^R in order to satisfy

Table 1: Base-case parameter value

Parameter		Value
Government		
Initial public debt	$\bar{\Delta}_{-1}$	0.1
Fraction of production devoted to public sector	Γ	0.1
Production function[a]		
Technological parameter	A	20
Share parameter of physical capital	a	0.4
Share parameter of labor supply	b	1
Elasticity of substitution between production factors	σ^F	0.5
Representative household		
Home production function when young[b]		
Share parameter of market good c	a^y	0.1
Elasticity of substitution between c and T^y	σ^y	0.5
Home production function when old[c]		
Share parameter of market good d	a^o	0.1
Elasticity of substitution between d and T^o	σ^o	10
Taste		
Degree of altruism	β	0.7
Efficiency of time transfer	μ	0.7
Elasticity of substitution[d] between f^y and f^o	σ^u	0.2
Time preference	γ	0.5

Note: We consider CES production and utility functions:

[a] $F(K,L) = A\left(aK^{\rho^F} + bL^{\rho^F}\right)^{\frac{1}{\rho^F}}$, with $\rho^F = 1 - \frac{1}{\sigma^F}$.

[b] $f^y(c, T^y) = \left(a^y c^{\rho^y} + (T^y)^{\rho^y}\right)^{\frac{1}{\rho^y}}$, with $\rho^y = 1 - \frac{1}{\sigma^y}$.

[c] $f^o(d, T^o) = \left(a^o d^{\rho^o} + (T^o)^{\rho^o}\right)^{\frac{1}{\rho^o}}$, with $\rho^o = 1 - \frac{1}{\sigma^o}$.

[d] $u(x) = \left(1 - \frac{1}{\sigma^u}\right)^{-1} x^{1-\frac{1}{\sigma^u}}$ and $v(x) = \gamma u(x)$.

the intertemporal budget constraint of the government.[6]

Figure 1 describes the effect of the tax reform on households' welfare. The steady-state household's life-cycle utility increases. The tax reform involves an increase of the first generation's capital income tax τ_0^R since the burden of the public debt is shifted towards them. This results in a negative impact on the life-cycle utility of the first generation (decreasing from -1.7103 to -1.7109) while the effect on life-cycle utilities along the transitional dynamics is positive. Taking account of the altruistic component of utility, the tax reform increases welfare of each generation (see Figure 1(b))

[6]From equation (14) and given tax instruments values considered in Subsection 5.1, τ_0^R is solution of the following intertemporal budget constraint:

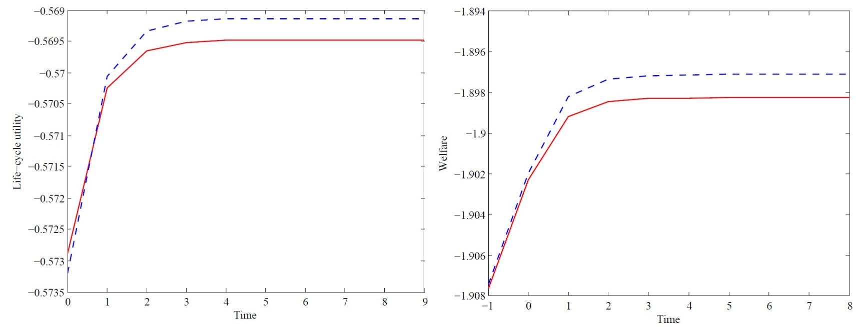
$$\left(1 - \tau_0^R\right) R_0 \bar{\Delta}_{-1} + \Gamma F\left(\bar{k}_0, \ell_0\right) - \Gamma \ell_0 w_0 - \tau_0^R R_0 \bar{k}_0 - \tau^x x_0 + \sum_{t=1}^{+\infty} P_t \left[\Gamma F\left(k_t, \ell_t\right) - \Gamma \ell_t w_t - \tilde{\tau}^R R_t k_t - \tau^x x_t\right] = 0$$

with $P_0 = 1$ and $P_t\left(1 - \tilde{\tau}^R\right) R_t = P_{t-1}$, for any period $t \geq 1$.

and then involves efficiency gains.

Figure 1: Tax reform with constant steady-state capital-labor ratio

(a) Life-cycle utility V_t (b) Welfare W_t



Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. The introducion of a positive inheritance tax $\tau_t^x = 0.03$ for any $t \geq 0$, implies that $\tau_0^R \simeq 0.2895$ and $\tilde{\tau}_t^R \simeq 0.2603$ for any $t \geq 1$ after the reform.

However, the adjustment of the first generation's capital income tax τ_0^R allows to shift part of the burden of the initial debt towards a lump-sum tax. One may wonder whether the result comes from the fact that the tax reform shifts the public debt burden on a lump-sum tax. For this reason, we thereafter focus on another fiscal reform that keeps the lump-sum tax τ_0^R constant.

5.2 Tax reform with constant tax rate on capital income from period 1

The tax reform now consists to set up a positive inheritance tax rate $\tau_t^x = 0.03$ for any period $t \geq 0$, keeping the initial tax rate on capital income constant $\tau_0^R = \bar{\tau}^R$. This implies a decrease of the capital income tax $\tau_t^R = \widehat{\tau}^R$ for any period $t \geq 1$ such as $\widehat{\tau}^R$ balances the intertemporal budget constraint of the government.[7]

Since the tax reform is not conducted to leave the capital-labor ratio constant in steady-state, a shift from capital income tax towards inheritance reduces the capital-labor ratio, and thus the resource available for market good consumption. In order to attenuate this negative effect on the whole dynamics and in the long run, we consider that production factors are complements.

The results are reported in Figure 2. We get similar effects on utility and welfare to the preceding illustration. The tax reform implemented illustrates the trade-off for the government between

[7] From equation (14) and given tax instruments values considered in Subsection 5.2, $\widehat{\tau}^R$ is solution of the following intertemporal budget constraint:

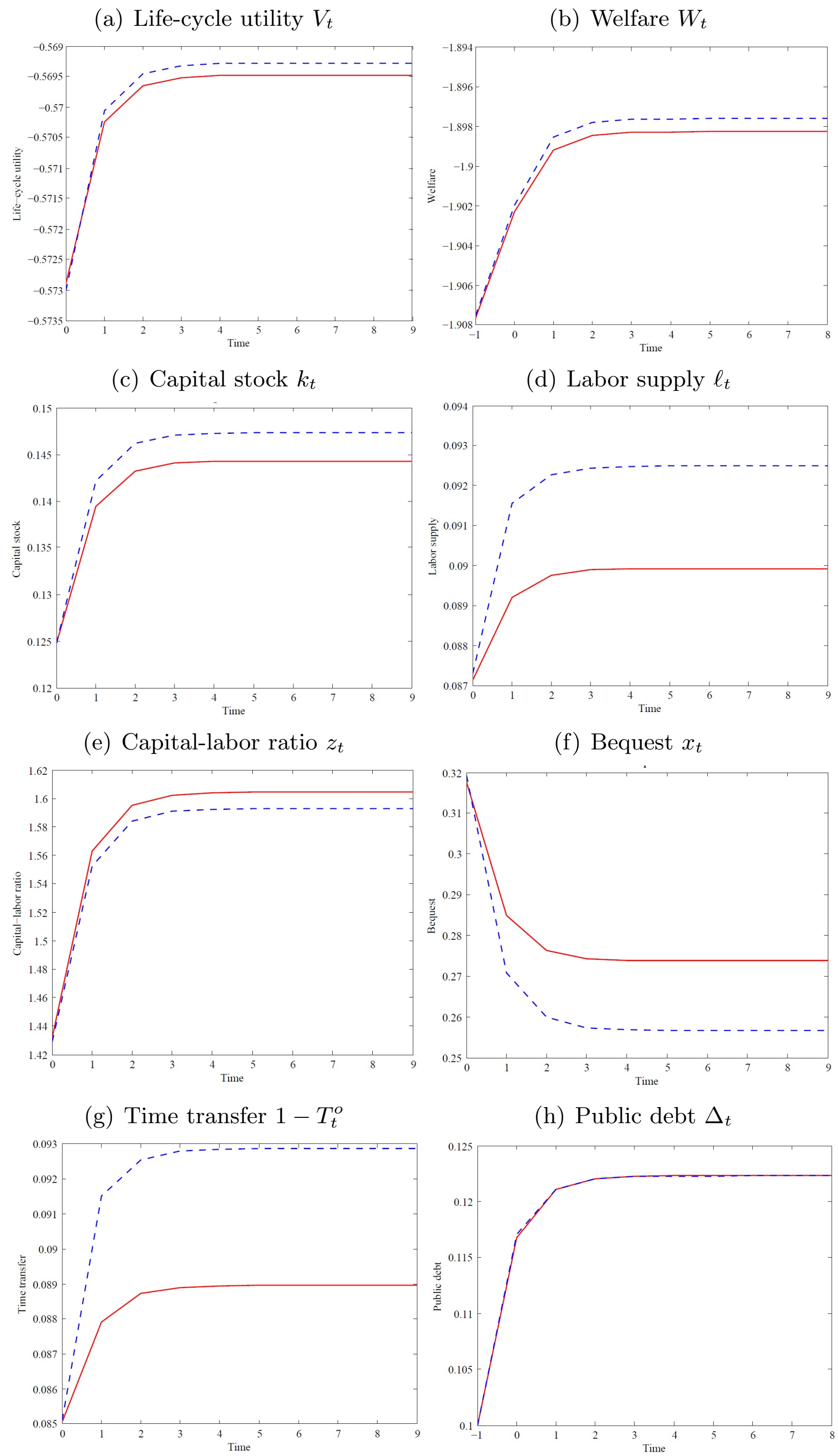
$$\left(1-\bar{\tau}^R\right) R_0 \bar{\Delta}_{-1} + \Gamma F\left(\bar{k}_0, \ell_0\right) - \Gamma \ell_0 w_0 - \bar{\tau}^R R_0 \bar{k}_0 - \tau^x x_0 + \sum_{t=1}^{+\infty} P_t \left[\Gamma F\left(k_t, \ell_t\right) - \Gamma \ell_t w_t - \widehat{\tau}^R R_t k_t - \tau^x x_t\right] = 0$$

with $P_0 = 1$ and $P_t\left(1-\widehat{\tau}^R\right) R_t = P_{t-1}$, for any period $t \geq 1$.

keeping the steady-state primary surplus constant (constant steady-state public debt) and leaving the capital-labor ratio constant (lower steady-state public debt). The tax reform considered involves a decrease of the steady-state capital-labor ratio: the increase of the capital stock (thanks to the fall in the relative price between both intertemporal market goods consumptions P^R) is lower than the rise in labor supply. This positive effect on labor supply relies on the rise of time transfers (through the increase of P^o). In addition, implementing positive inheritance tax rate reduces bequests (since households have more incentive to transfer time).

As in the previous Subsection, life cycle utility is improved for each generation except for the first old (their utility decreases from -1.7103 to -1.7113) and welfare of all generations increases with the tax reform.

Figure 2: The tax reform effect with constant tax rate on capital income



Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. The introdution of a positive inheritance tax $\tau_t^x = 0.03$ for any $t \geq 0$, implies that $\tau_0^R = \bar{\tau}^R \simeq 0.2603$ and $\widehat{\tau}^R \simeq 0.2689$ for any $t \geq 1$, after the reform.

6 Conclusion

To summarize our results, we consider a tax reform starting from an intertemporal equilibrium where the capital income tax is above its efficient level in order to finance the burden of an initial public debt. We have then addressed the following issue: should the government increase inheritance tax in order to reduce the capital income tax?

In the Barro model, the tax reform reduces steady-state welfare. The driving force is the change in the marginal rate of substitution between young and old consumptions, leading to a fall in the first-period consumption and a rise in the second-period one.

With elastic labor supply, the tax reform may be Pareto-improving. The most favorable cases are those where the fall in first-period consumption is associated with a fall in time devoted to domestic production (*i.e.* leisure in the usual terminology), allowing for an increase in the young labor supply.

Introducing time transfers enhances the positive effect on the young labor supply. Indeed, in this framework, inheritance tax also modifies the trade-off between time transfers and bequests. Grandparents are incited to transfer more time and less money to the next generation, that will benefited from higher time resources and will be able to work more.

With familial time transfers, we have shown that a shift from capital income tax towards inheritance tax can be Pareto-improving. The Pareto improvement strongly depends on the strength of the positive effect of time transfers on the young's labor supply and on the strength of the effect of higher labor supply on the production of market goods.

For further researches, a closer look to the intragenerational heterogeneity would allow to address redistribution issues. Heterogeneity could be introduced at least in the two following dimensions. First, empirical studies show differences in the distributions of time transfers and distributions of bequests. They suggest that bequests are more concentrated than time transfers. Secondly, capital income tax may affect a larger part of the population than inheritance tax.

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