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Offshoring and Sequential Production Chains: A General Equilibrium Analysis

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Abstract

We present a two-region general equilibrium model in which firms exploit international wage differences by offshoring parts of the production process. Firms have to take into account that production steps follow a strict sequence and that transporting intermediate goods across borders is costly. We analyze how a change in transport costs and various properties of the production process affect offshoring patterns as well as factor prices, accounting for the general equilibrium effects of firms' decisions. Interestingly, the influence of declining transport costs on the range of tasks offshored per firm may differ from the effect on the number of firms engaged in offshoring.

Keywords: Offshoring, sequential production, global production chain, task trade.
JEL Classification: D24, F10, F23, L23.

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1 Introduction

Over the past years a lot of attention has been devoted to the rapid advance of the "second unbundling" in international trade (Baldwin 2006) – i.e., the "offshoring" of production stages – and to its consequences for trade and national labor markets. Especially workers in industrialized economies are concerned about being exposed to competition from cheaper labor abroad if firms shift an increasing share of their production to other countries. To assess the impact of this development, general equilibrium models have been developed that capture the interdependence between firm decisions, trade flows and labor market outcomes.[1]

Many recent analyses of offshoring are based on a specific idea of the production process, according to which production can be interpreted as a set of "tasks" or "production steps". The decision to offshore a certain task depends on relative factor prices and productivity levels as well as on offshoring costs for that particular task (including additional monitoring and communication costs resulting from foreign production). Individual tasks can be ordered with respect to the cost advantage of performing them abroad, and there is a unique "cutoff task" that defines the extent of offshoring.

In contrast to this approach, production processes are, in many cases, *sequential* – i.e., individual steps follow a predetermined sequence that cannot be modified at will.[2] With sequential production, offshoring implies unfinished intermediate goods to be transported back and forth between countries – a phenomenon that is documented, e.g., by Haddad (2007), Ando and Kimura (2005), as well as Athukorala and Yamachita (2006): these studies describe the behavior of Japanese multinationals who ship high-technology core materials to their affiliates in developing East Asia, where they produce basic parts and components; the basic parts and components are then sent back to Japan (or to other high-skill abundant countries) for quality control and/or further processing.

[1] A non-exhaustive list of important contributions to this literature includes Jones and Kierzkowski (1990), Feenstra and Hanson (1996), Kohler (2004), and Grossman and Rossi-Hansberg (2008).

[2] See, e.g., Antras and Chor (2013), Baldwin and Venables (2013), Costinot et al. (2012, 2013), Harms et al. (2012), or Kim and Shin (2012). Earlier approaches are Dixit and Grossman (1982), Sanyal (1983), Yi (2003, 2010), and Barba Navaretti and Venables (2004). Fally (2012), Antras and Chor (2013), and Antras et al. (2012) provide empirical measures to characterize sequential production processes.

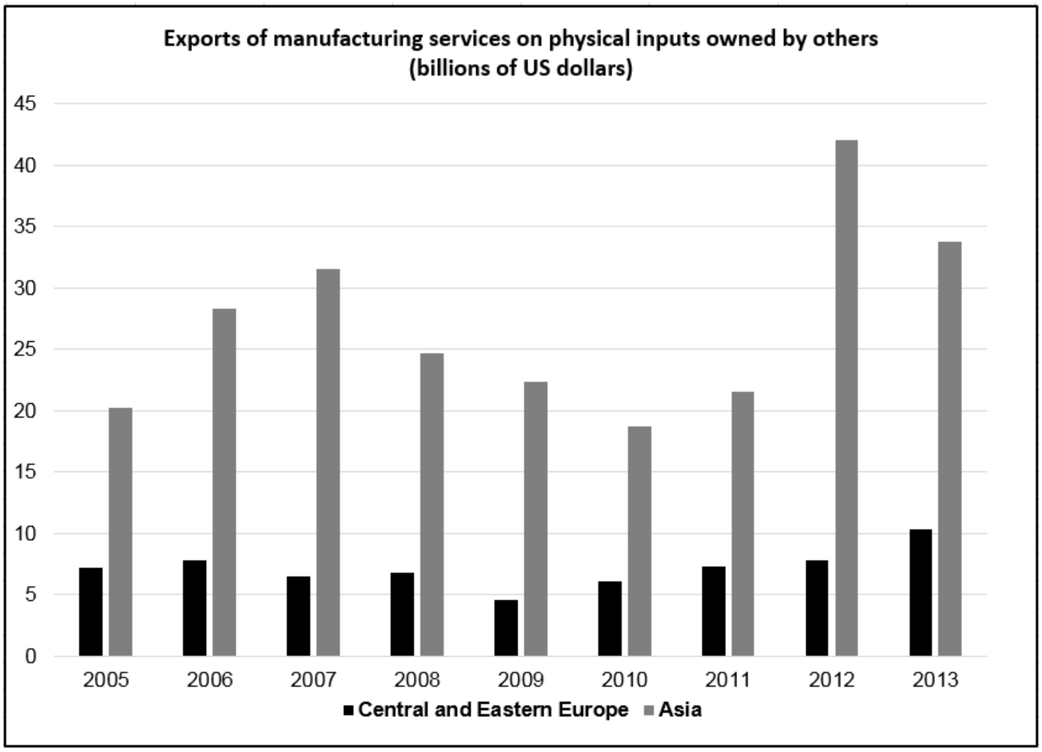

Figure 1: Exports of manufacturing services from Central and Eastern Europe and Asia
Source: IMF (Balance of Payments Statistics)

To assess whether this phenomenon is relevant in quantitative terms, one has to go beyond anecdotal evidence. Unfortunately, available data on offshoring derived, e.g., from the World Input-Output Database (WIOD), the Global Trade Analysis Project (GTAP) database, or from the WTO-OECD TiVA Database do not differentiate between the use of imported intermediate inputs and the type of offshoring that we have in mind, namely the delegation of production steps that is associated with unfinished goods crossing a border.[3] However, recent balance of payments data lend some support to the notion that the overall volume of "manufacturing services on physical inputs owned by others" has been substantial in recent years. Figure 1 shows the volume of manufacturing services exported by a number of Asian as well as Central and Eastern European countries.[4] It

[3] The data sets mentioned are presented and used, e.g., in Johnson (2014), Koopman et al. (2014), Timmer et al. (2014), De Backer and Miroudot (2013).

[4] This novel category within the services trade part of the current account was introduced in the context

illustrates the absolute value as well as the growth of manufacturing services exports.

The existence of *sequential production* would not necessarily change the view on offshoring if the relative costs of performing individual tasks abroad happened to monotonically increase or decrease along the production process. However, it is quite unlikely to meet this constellation in practice. More plausibly, potential offshoring destinations have a cost advantage for *some* particular segments of the production process whereas preceding and subsequent segments can be performed at lower costs in the domestic economy. If such a situation is combined with costs of shipping intermediate goods across borders, firms may be reluctant to offshore certain steps even if – considered in isolation – these could be performed at much lower costs abroad. The reason is that the domestic country may have a cost advantage with respect to adjacent steps, and the costs of shifting back and forth intermediate goods may more than eat up potential cost savings from fragmenting the production process. Such a constellation has important implications for observed offshoring patterns. For example, it may explain why – despite the large international discrepancies in factor prices – certain production processes are less fragmented internationally than what one might expect. At the same time, such a setup may generate substantial shifts in the total volume of offshoring as a reaction to rather moderate changes of the environment. And finally, it may give rise to a non-monotonic relationship between transport costs and the volume of offshoring.

Baldwin and Venables (2013) as well as Harms et al. (2012) have shown how such insights regarding the offshoring decision of individual firms can be obtained from partial equilibrium models in which factor prices are exogenous. However, to arrive at conclusions about the entire economy and to determine the implications of offshoring for labor markets at home and abroad, we need to consider the repercussions of induced factor price changes on firms' optimal behavior – i.e., we need to model offshoring in a general equilibrium framework. This is what the current paper does. More specifically, we develop a framework

of the sixth revision of the Balance of Payments Manual (BPM6). Since time series that comply with BPM6 only go back to 2005, we do not have data on this item for previous years. The Asian countries are Bangladesh, China, India, Indonesia, Korea, Malaysia. The Central and Eastern European countries are Bulgaria, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovak Republic, Slovenia, Turkey. Country-years for which no data were available entered the sum with a zero.

that allows to determine how changes in transport costs and other exogenous parameters influence both offshoring patterns and factor prices.

To accomplish this task, we present a two-region (North-South) model in which firms whose production is entirely *domestic* may co-exist with *multinational* firms who decide on the international distribution of production. By allowing for firms with different "production modes" to coexist, we avoid the drastic adjustment that characterizes the model of Harms et al. (2012): instead of modeling an economy in which a representative firm possibly jumps from purely domestic production to a large volume of offshoring, we describe the adjustment as taking place both at the *extensive* and at the *intensive* margin – i.e., as a reaction to changing parameters, both the number of (small) firms who engage in offshoring and the volume of offshoring chosen by individual firms may change. Production is based on a rigid sequence of individual steps, and the foreign cost advantage evolves in a non-monotonic fashion along the production chain: some steps are cheaper to perform in the South, some are cheaper to perform in the North, and so on. Finally, every step requires the presence of the unfinished intermediate good, and shifting intermediate goods across borders is associated with transport costs. Wages and prices in both economies are endogenous, and the increasing demand for labor that is generated by accelerating North-South offshoring may eventually result in wage increases that make offshoring less attractive. Using this framework, we explore how changes in transport costs affect the volume of offshoring at the intensive and extensive margin as well as factor prices in the North and the South. Numerical simulations corroborate our comparative-static results and provide further insights into the effects of changing factor endowments, relative productivities, and properties of the production process.

It is important to note in what respect our approach differs from other contributions that model offshoring under the assumption of a sequential production process. In Antras and Chor (2013), a firm has to decide whether to delegate sequential production steps to independent service suppliers abroad or to integrate these suppliers into its own organization. In a world of incomplete contracts, the firm wants to elicit relationship-specific investments from its suppliers, but is unable to commit to a given payment ex ante. Sequentiality matters in this set-up since investment decisions of upstream producers may influence decisions further downstream. While our approach also considers a sequence of production steps and assumes that the respective tasks are performed on unfinished goods, we abstract from the incentive problems associated with specific organizational arrangements and do not model the potential technological interdependence between subsequent steps. Instead, we assume that it is costly (in terms of labor input) to monitor activities abroad, and that these costs vary along with other costs in the production process.

Costinot et al. (2012, 2013) also assume that the sequence of production steps is exogenously given, such that there are "upstream" and "downstream" tasks. In their general equilibrium model, tasks can be delegated to other countries at the risk of a "mistake" that results in a failure of the entire production process. Mistakes made downstream therefore have more severe consequences compared to mistakes further upstream. Our approach differs from Costinot et al. (2012, 2013) as we (implicitly) assume that sufficient monitoring prevents foreign producers from committing mistakes and that the costs of monitoring do not increase systematically as we move from upstream to downstream tasks.[5]

The paper closest to ours is Baldwin and Venables (2013) who characterize *snake*-like production as "...processes whose sequencing is dictated by engineering" (Baldwin and Venables 2013:245). They highlight the possibility that the effective costs of performing tasks abroad may vary non-monotonically along the production chain, and they argue that this property, combined with the existence of *separation* – or transport – costs may result in a non-monotonic reaction of offshoring to further advances in globalization. While our paper imposes more structure on the functional form that characterizes relative costs, we go beyond Baldwin and Venables (2013) and Harms et al. (2012) by analyzing the "snake" in a general equilibrium setting. We thus combine a plausible description of production processes and their particular implications for the extent and evolution of offshoring with a general-equilibrium perspective that endogenizes factor prices and allows assessing the consequences of offshoring for countries' industrial structure and labor markets.

The rest of the paper is organized as follows: in section 2, we outline the structure of our model. Section 3 discusses the properties of the equilibrium and derives comparative-

[5] As we will show in Section 2.2, the difference between "upstream" and "downstream" tasks is much less pronounced in our framework than it is in Antras and Chor (2013) or Costinot et al. (2012, 2013). Nevertheless, firms are limited in their ability to re-arrange production steps, and this gives rise to offshoring patterns that would not occur if these constraints did not exist.

static results. In section 4, we perform a numerical analysis that illustrates how the volume of offshoring is affected by changes in factor endowments, decreasing transport costs, and various properties of the production process. Section 5 provides a summary and some conclusions.

2 The Model

2.1 Preferences

There are two regions, North and South, with an asterisk denoting South-specific variables.[6] Consumers in both regions have Cobb-Douglas preferences over two consumption goods, X and Y. The X sector produces a continuum of differentiated varieties, whereas goods from the Y sector are homogeneous.[7] Household preferences are

$$U = X^{\beta} Y^{1-\beta}, \qquad 0 < \beta < 1 , \quad \text{and}$$
$$X = \left[\int_{i \in N} x(i)^{\rho_x} di \right]^{\frac{1}{\rho_x}} , \qquad 0 < \rho_x < 1 . \tag{1}$$

The index i denotes individual varieties, N is the measure of these varieties, and $\sigma_x = 1/(1-\rho_x)$ is the elasticity of substitution between them. Maximizing utility for a given income level (I) yields the following demand system:

$$x(i) = \left(\frac{P_X}{p(i)} \right)^{\sigma_x} X ,$$
$$P_X = \left[\int_{i \in N} p(i)^{1-\sigma_x} di \right]^{\frac{1}{1-\sigma_x}} , \quad P_X X = \beta I , \quad \text{and} \quad p_Y Y = (1-\beta) I . \tag{2}$$

Here, P_X denotes the ideal price index for the X-sector and p_Y denotes the price of the homogenous output in industry Y.

2.2 Technologies and Production Modes

Each region is endowed with given quantities of labor $\bar{L}$ (in efficiency units) and of a fixed composite factor $\bar{R}$. We assume that labor can be employed in both sectors, whereas the

[6] Each region should be understood as an aggregate composed of possibly numerous countries.

[7] As we will detail below, offshoring exclusively takes place in the X-sector. The two-sector framework is used in order to guarantee that foreign workers find an alternative employment if domestic firms do not engage in offshoring.

fixed composite factor, which may be land or a natural resource, is specific to industry Y. Good Y is produced in both regions by a competitive industry and can be freely traded. Production of Y combines the quantities R_Y and L_Y according to a CES-technology:

$$Q_Y = \left[\alpha_R R_Y^{\rho_Y} + \alpha_L L_Y^{\rho_Y}\right]^{\frac{1}{\rho_Y}}, \qquad 0 < \rho_Y < 1 . \tag{3}$$

Profit maximization yields the following demand for the two factors of production in the Y-sector:

$$R_Y = \alpha_R^{\sigma_Y} \left(\frac{p_Y}{r}\right)^{\sigma_Y} Q_Y \quad \text{and} \quad L_Y = \alpha_L^{\sigma_Y} \left(\frac{p_Y}{w}\right)^{\sigma_Y} Q_Y . \tag{4}$$

The term $\sigma_Y = 1/(1-\rho_Y)$ stands for the elasticity of substitution between the two factors R_Y and L_Y, and r and w denote factor prices, respectively. If good Y is produced, perfect competition implies that

$$p_Y = \left[\alpha_R^{\sigma_Y} r^{1-\sigma_Y} + \alpha_L^{\sigma_Y} w^{1-\sigma_Y}\right]^{\frac{1}{1-\sigma_Y}} . \tag{5}$$

We choose good Y as our numeraire throughout the paper, and from free trade in Y and product homogeneity we have $p_Y = p_Y^* = 1$.

Varieties of good X can only be produced by firms whose headquarters are located in the North. While we do not explicitly model research and development, this assumption can be rationalized by arguing that only Northern firms are able to develop and use the blueprints necessary for production. Firms in sector X act under monopolistic competition, and each firm has to incur fixed costs in addition to the variable production costs.[8]

We follow Grossman and Rossi-Hansberg (2008) in modeling the production process of any variety $x(i)$ as a continuum of tasks, indexed by t and ranging from 0 to 1. As in Harms et al. (2012) and Baldwin and Venables (2013), these tasks have to be performed following a strict sequence, i.e., they cannot be re-arranged at will. A firm producing a given variety of good X may choose between different *production modes*: domestic (D) and multinational (M). While *domestic firms* perform the entire production process in the North, *multinational firms* can offshore some of the tasks to the South. The goal of

[8] The monopolistic-competition framework with product differentiation allows for the co-existence of firms with different production modes in equilibrium.

our analysis is to determine the number of good-X producers that exploit the possibility to produce internationally, and to derive the amount of offshoring chosen by these firms.

Each task t involves a given quantity of labor.[9] The labor coefficients $a_D(t)$ or $a_M(t)$ denote the efficiency units of labor that are necessary for a domestic (D) or multinational (M) firm to perform task t in the North. For simplicity, we assume that $a_D(t) = a_D$ and $a_M(t) = a_M$ for all t, i.e., input coefficients in the North do not differ across tasks.[10] Being a multinational firm may come with a higher labor productivity in the North, i.e., $a_M \leq a_D$. Input coefficients $a^*_M(t)$ of performing a task in the South possibly differ from a_M. More importantly, these coefficients vary across tasks. For example, some tasks benefit strongly from a better educated workforce or a better production infrastructure in the North, implying a lower labor coefficient in the North than in the South. Other tasks may be less sophisticated such that the discrepancy between labor input coefficients is not that pronounced. In addition, offshoring firms possibly have to employ additional labor to monitor tasks performed in the South or to communicate with the headquarter in the North, and these monitoring and communication requirements may also vary across tasks. Summing up, there may be tasks which – given wages w and w^* in the North and the South respectively – are cheaper to perform domestically and tasks which are cheaper to perform abroad. This is represented by Figure 2, which juxtaposes the (constant) costs per task wa_D if these tasks are performed by a Northern domestic firm, the (constant) costs wa_M if they are performed domestically by a multinational firm, and the varying costs $w^*a^*_M(t)$ if these tasks are offshored to the South.

Multinational firms have to adjust to the fact that, at given wages, some tasks that are cheaper to perform in the South may be adjacent to tasks for which the North has a cost advantage (and vice versa). In Figure 2, the tasks $t \in [t_1, t_2]$ and $t \in [t_3, t_4]$ would be performed at lower cost if they were offshored to the South by Northern multinational firms. Conversely, all tasks $t \in [0, t_1]$, $t \in [t_2, t_3]$ and $t \in [t_4, 1]$ would be cheaper to perform

[9] Given this linear specification of production in sector X, the existence of sector Y and the specification of its technology add some convexity to the model. See, e.g., Markusen and Venables (1998) as well as Markusen (2002) for a similar approach .

[10] By contrast to this assumption of fixed labor coefficients, Jung and Mercenier (2014), who analyze skill/technology upgrading effects of globalization, employ a heterogeneous workers framework in which a worker's productivity is determined endogenously by his own talent and the technology he uses.

domestically. To simplify the analysis, we assume that the first and the last task are tied to being performed in the North.

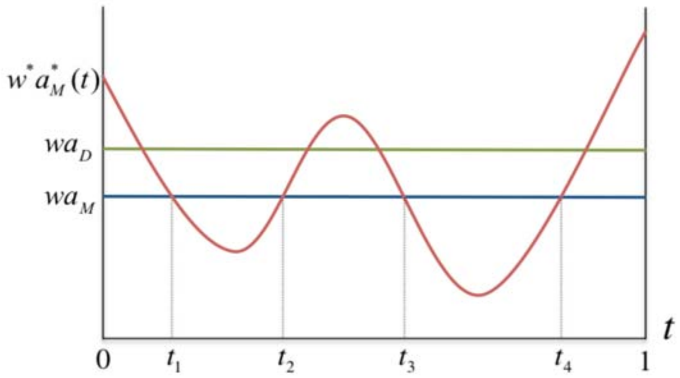

Figure 2: Costs along the Production Chain

As in Yi (2003, 2010), Barba Navaretti and Venables (2004), or Harms et al. (2012), we assume that performing a task requires the presence of the unfinished intermediate good, and that moving (intermediate) goods between regions is associated with constant *transportation costs* per unit. More specifically, any crossing of a border requires T units of labor in the sending region. It is for this reason that Northern multinational firms may find it profitable to perform a large part of production at home or to adopt a strategy of *agglomerating* (almost) the entire production process $t \in [t_1, t_4]$ abroad, rather than paying transport costs each time the unfinished good is crossing a border.

In the remainder of this paper, we restrict our attention to a symmetric specification of the $a^*_M(t)$ curve:

$$a^*_M(t) = A \cos(2n\pi t) + B \, . \tag{6}$$

Figure 3 illustrates this specification.[11] It compares the labor costs that a multinational firm has to incur if it performs individual tasks abroad to the costs associated with performing these tasks domestically (in units of foreign labor).

[11] This cosine specification implies that firms marginally adjust the amount of tasks they offshore as a reaction to marginal changes in relative wages. While other symmetric specifications – e.g., the one used in Harms et al. (2012) – are consistent with our non-monotonicity assumption, not all of them allow for such an *intensive-margin adjustment.*

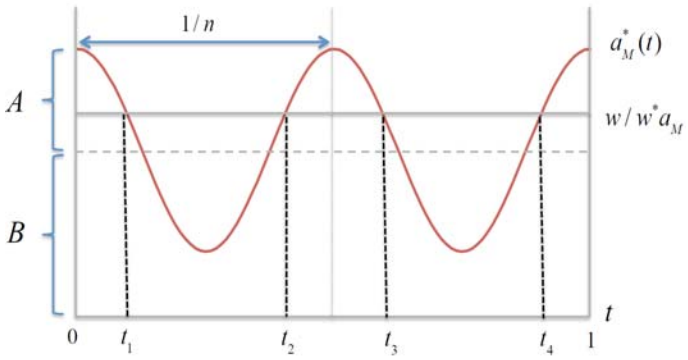

Figure 3: Costs along the Production Chain: Symmetric Case

The symmetry property of the cosine functional form offers a flexible way to capture the non-monotonic evolution of relative costs along the production process while substantially simplifying the analysis: first and foremost, instead of determining separate cutoff values $t_1, t_2, ...$, we can exploit the fact that $t_2 = \frac{1}{n} - t_1, t_3 = \frac{1}{n} + t_1$, etc. Moreover, the individual parameters characterizing the cosine-function have a straightforward economic interpretation: while the shift parameter B reflects the average labor coefficient in the South, the parameter A, which determines the function's amplitude, captures the *heterogeneity* of task-specific input requirements across regions. The variable n ($n \in \mathbb{N}^+$) measures the number of "cycles" that $a_M^*(t)$ completes between $t = 0$ and $t = 1$. We argue that production processes that are characterized by a higher number of cycles – i.e., a larger n – are more *sophisticated*, exhibiting more variability in terms of cost differences. To keep the analysis interesting, we assume that foreign production costs fluctuate around domestic costs more than once (i.e. $n \geq 2$).

Note that our symmetry assumption implies that there is no "upstreamness" or "downstreamness" of tasks in the sense of Antras and Chor (2013) or Costinot et al. (2012, 2013). More specifically, neither production costs nor offshoring costs are systematically higher or lower earlier or later in the production process. By contrast, foreign production costs along the segment between t_3 and t_4 in Figure 3 are identical to those along the segment between t_1 and t_2. Still, sequentiality plays a central role in determining offshoring patterns as it prohibits to perform these segments at home and the segment between t_2 and

t_3 abroad without incurring additional transportation costs for the unfinished good. This is what characterizes the snake-property of the production process and what distinguishes our model set-up from other contributions on offshoring without sequentiality.

The first cutoff t_1 is determined by the following condition:

$$\frac{w}{w^*} a_M = a_M^*(t_1) \; . \tag{7}$$

Offshoring with positive production volumes in both regions can only occur if each region has a cost advantage for *some* tasks. Technically, this requires that the two curves in Figure 3 intersect. Therefore, a necessary condition for offshoring to occur is

$$\frac{B-A}{a_M} < \frac{w}{w^*} < \frac{B+A}{a_M} \; . \tag{8}$$

Due to the symmetry of the $a_M^*(t)$ function, we can distinguish three *firm-types*: domestic firms (*domestic production, D*), multinational firms that fragment their production chain (*fragmentation, M^f*), and multinational firms that perform most tasks in the South (*production abroad, M^a*). Fragmented multinationals offshore all segments that are cheaper to perform in the South, i.e., all tasks between t_h and t_{h+1} ($h = 1, 3, \ldots, 2n-1$), and produce all other segments at home. Production-abroad firms offshore the entire segment between t_1 and t_{2n}, and produce only the first segment between 0 and t_1 and the last segment between t_{2n} and 1 at home.[12]

2.3 Costs and Prices

Marginal costs of firm type j are given by the following expression:

$$C_j = wL_j + w^* L_j^* \; , \tag{9}$$

for $j = D, M^f, M^a$. The variables L_j and L_j^* stand for the labor input at home and abroad per unit of output of a representative type-j firm. Labor input depends on the firm-type,

[12] This is where our assumption that the first and the last production steps have to be performed domestically becomes relevant. Without this assumption, the set of potential firm types would be larger, possibly including firms that perform *all* production steps abroad, and firms that perform only the first (or the last) part of the production process abroad. This would reduce tractability without offering much additional insight.

according to

$$L_D = a_D \ , \tag{10}$$

$$L_{M^f} = 2na_M t_1 + nT \ , \tag{11}$$

$$L_{M^a} = 2a_M t_1 + T \ , \tag{12}$$

$$L_D^* = 0 \ , \tag{13}$$

$$L_{M^f}^* = n \int_{t_1}^{\frac{1}{n}-t_1} a_M^*(t)dt + nT, \quad \text{and} \tag{14}$$

$$L_{M^a}^* = \int_0^1 a_M^*(t)dt - 2\int_0^{t_1} a_M^*(t)dt + T. \tag{15}$$

Note that, with n cycles, fragmented firms (M^f) have to incur n times the transport costs of production-abroad firms (M^a).

We assume that exporting final X-goods to the South is associated with iceberg trade costs $\tau > 1$ per unit. Using this information as well as (2), we can write the demand faced by a representative firm of type j as

$$x_j^d = \left(\frac{P_X}{p_j}\right)^{\sigma_x} X \quad \text{and} \quad x_j^{d*} = \tau\left(\frac{P_X^*}{p_j\tau}\right)^{\sigma_x} X^* \ . \tag{16}$$

The price indices for the X sector in the North and the South are given by

$$P_X = \left[\sum_j N_j p_j^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}} \quad \text{and} \quad P_X^* = \left[\sum_j N_j \left(p_j\tau\right)^{1-\sigma_x}\right]^{\frac{1}{1-\sigma_x}} , \tag{17}$$

where N_j stands for the number of firms of each firm-type. If type-j firms in sector X produce a strictly positive quantity ($x_j > 0$), they charge a constant mark-up rate over their marginal costs:

$$p_j = \frac{\sigma_x}{\sigma_x - 1} C_j \ . \tag{18}$$

We assume that all active firms in the X-sector have to incur fixed costs, which reflect the necessity to use F_j units of their own output to support production.[13] In units of the numeraire good, the fixed costs amount to a multiple of marginal costs $F_j C_j$, with $F_j > 0$,

[13] One interpretation of this specification is that, regardless of the eventual output, firms require a certain number of test models before they can start producing for the market.

and we assume $F_D < F_{M^f} < F_{M^a}$.[14] Free entry ensures zero profits:

$$\frac{1}{\sigma_x} p_j x_j \leq C_j F_j \ , \tag{19}$$

where $N_j > 0$ if the condition holds with equality, and $N_j = 0$ otherwise. For $N_j > 0$, we can combine (18) with (19) to derive production of a firm of type j in equilibrium:

$$x_j = (\sigma_x - 1) F_j \ . \tag{20}$$

Output of an individual firm of type j is thus proportional to the fixed cost term F_j. Our assumptions concerning fixed costs imply $x_D < x_{M^f} < x_{M^a}$, and, because of (16) and (18), $C_D > C_{M^f} > C_{M^a}$. That is, if domestic and multinational firms are both active in the market, multinational firms have to have lower marginal costs than domestic firms. The reason is that the higher fixed costs faced by multinational firms necessitate higher sales, which only materialize at lower prices. Since prices are proportional to marginal costs, this implies that marginal costs have to be lower for multinational firms.

3 Equilibrium

3.1 Definition

An equilibrium is defined by an optimal cutoff value t_1, a vector of wages as well as prices and quantities in the X and Y sectors, and an industrial structure (N_D, N_{M^f}, N_{M^a}) such that

(i) firms of a given type in the X-sector set profit-maximizing prices,
(ii) no firm has an incentive to change its production mode,
(iii) multinational firms of a given type (M^f, M^a) choose the optimal pattern of offshoring, i.e., the optimal cutoff value t_1,
(iv) free entry results in zero-profits of all active firms in equilibrium,
(v) in the Y-sector, factor prices reflect their marginal products,
(vi) goods and factor markets clear.

[14] These inequalities are based on the notion that the number of test runs that have to be accomplished before firms start producing for the market increases in the range of tasks performed abroad.

In what follows, we will explore how exogenous changes in transport costs, endowments, and technological parameters affect the volume of offshoring both at the *intensive* margin – i.e., the range of tasks performed abroad by a specific firm type as represented by t_1 – and at the *extensive* margin, i.e., the number of firms choosing a production mode that involves offshoring.

3.2 The Cutoff Task t_1 and Marginal Costs

The optimal cutoff task t_1 for firms who engage in offshoring is implicitly determined by equation (7). Using (6) and defining $\omega \equiv w/w^*$, we can obtain t_1 as

$$t_1 = min\left[\frac{1}{2\pi n}\arccos\left(\frac{a_M\omega - B}{A}\right), \frac{1}{2n}\right], \tag{21}$$

where the condition that $t_1 \leq 1/2n$ is due to the cosine function reaching its minimum at π.

The range of offshored tasks thus depends on the domestic labor coefficient a_M, the wage ω in the North relative to the South, as well as the parameters characterizing the $a_M^*(t_1)$ function. An increase in ω *ceteris paribus* lowers t_1, i.e.,

$$\frac{\partial t_1}{\partial \omega} = -\frac{a_M}{2\pi n A\sqrt{1-\left(\frac{a_M\omega - B}{A}\right)^2}} < 0 \, .$$

The economic intuition behind this result is straightforward: as the domestic wage relative to the foreign wage increases, firms have an incentive to perform a smaller share of the production process at home and a larger share abroad. In a similar fashion, we may determine the influence of the variables determining average costs, a_M and B, as well as the heterogeneity parameters A and n.

The cut-off value t_1 determines marginal costs of firms that are producing internationally – either in a fragmented production chain moving back and forth internationally or (almost) entirely abroad. Marginal costs with *fragmented production* can be written as

$$C_{M^f} = wa_M - n\int_{t_1}^{\frac{1}{n}-t_1}\left[wa_M - w^*a_M^*(t)\right]dt + nT\left[w + w^*\right] \, . \tag{22}$$

Using (7) and invoking the symmetry of the $a_M^*(t)$-curve, we can write C_{M^f} as

$$
\begin{aligned}
C_{M^f} &= wa_M\Omega^f(t_1) + wa_M\Phi^f(t_1)\ , \quad \text{with} \\
\Omega^f(t_1) &= 1 - 2n\int_{t_1}^{\frac{1}{2n}}\left(1 - \frac{a_M^*(t)}{a_M^*(t_1)}\right)dt \quad \text{and} \\
\Phi^f(t_1) &= nT\left[\frac{1}{a_M} + \frac{1}{a_M^*(t_1)}\right]\ .
\end{aligned} \tag{23}
$$

The term $\Omega^f(t_1) < 1$ reflects the fact that fragmented firms save on labor costs by delegating each task to the lowest-cost location. This is the mechanism that gives rise to the "productivity effect" in Grossman and Rossi-Hansberg (2008). However, in our setting, fragmentation comes at the cost of shifting unfinished goods across borders $2n$ times. This is reflected by the second term $\Phi^f(t_1)$. Both $\Omega^f(t_1)$ and $\Phi^f(t_1)$ are strictly increasing in t_1 for $t_1 < 1/(2n)$, such that

$$
\frac{d(C_{M^f}/w)}{dt_1} = -\frac{a_M}{\left[a_M^*(t_1)\right]^2}\frac{da_M^*(t_1)}{dt_1}\left[2n\int_{t_1}^{\frac{1}{2n}} a_M^*(t)dt + nT\right] \tag{24}
$$

is also positive. Recall from (14) that the term in squared brackets equals $L_{M^f}^*(t_1)$. The positive effect of t_1 on Ω^f has a straightforward intuition: as t_1 approaches $1/(2n)$, the volume of offshoring approaches zero, and the "productivity effect" vanishes, i.e., Ω^f approaches one. To understand the positive slope of Φ^f, we have to account for the fact that it is expressed relative to domestic labor costs, and remember the negative relationship between t_1 and ω: as t_1 increases, ω decreases. This raises the foreign component of the transport cost wage bill, and therefore results in an increase of Φ^f.

In a similar manner, we can write the marginal costs of a firm that chooses *production abroad* as

$$
\begin{aligned}
C_{M^a} &= wa_M\Omega^a(t_1) + wa_M\Phi^a(t_1)\ , \quad \text{with} \\
\Omega^a(t_1) &= \Omega^f(t_1) + 2(n-1)\int_0^{t_1}\left[\frac{a_M^*(t)}{a_M^*(t_1)} - 1\right]dt \quad \text{and} \\
\Phi^a(t_1) &= \frac{\Phi^f(t_1)}{n}\ .
\end{aligned} \tag{25}
$$

Note that $\Omega^f(t_1) < \Omega^a(t_1) < 1$, but $\Phi^f(t_1) > \Phi^a(t_1)$ for $n > 1$. This highlights the key tradeoff faced by firms: while fragmentation economizes on the costs for the labor force that is actually involved in the production process, it requires more labor devoted to the transportation of goods. For $t_1 < 1/(2n)$, Ω^a and Φ^a are strictly increasing in t_1, and we can show that

$$\frac{d(C_{M^a}/w)}{dt_1} = -\frac{a_M}{\left[a_M^*(t_1)\right]^2}\frac{da_M^*(t_1)}{dt_1}\left[\int_0^1 a_M^*(t)dt - 2\int_0^{t_1} a_M^*(t)dt + T\right] \tag{26}$$

is positive. Recall from (15) that the term in squared brackets equals $L_{M^a}^*(t_1)$.

3.3 Market Clearing

Labor market equilibrium requires

$$\bar{L} = L_Y + \sum_j N_j L_j\left(x_j + F_j\right) \quad \text{and} \quad \bar{L^*} = L_Y^* + \sum_j N_j L_j^*\left(x_j + F_j\right)\ , \tag{27}$$

with $j \in \{D, M^f, M^a\}$ and with $L_j(x_j+F_j)$ as the fixed and variable labor input required at home by a firm of type j to produce x_j units of output. Note that, given our assumption on fixed costs, each firm has to augment the x_j units it actually sells on the market by F_j additional output units. Using (20), we can rewrite (27) as

$$\bar{L} = L_Y + \sum_j N_j L_j \sigma_x F_j \quad \text{and} \quad \bar{L^*} = L_Y^* + \sum_j N_j L_j^* \sigma_x F_j\ . \tag{28}$$

For the market of the fixed composite factor to be in equilibrium we need

$$\bar{R} = R_Y \quad \text{and} \quad \bar{R^*} = R_Y^*\ . \tag{29}$$

For the final-goods markets, the equilibrium conditions are

$$x_j = x_j^d + x_j^{d*} \quad \text{as well as} \quad Q_Y + Q_Y^* = Y + Y^*\ . \tag{30}$$

Finally, incomes depend on factor prices and (fixed) factor endowments:

$$I = r\bar{R} + w\bar{L} \quad \text{and} \quad I^* = r^*\bar{R}^* + w^*\bar{L}^*\ . \tag{31}$$

3.4 Production Regimes

In what follows, we distinguish between equilibria in which all X-sector firms have the *same* production mode, and equilibria in which firms with *different* production modes co-exist. We will refer to these constellations as *production regimes*: a *pure domestic* production regime is characterized by $N_{M^a} = N_{M^f} = 0$ and $N_D > 0$, a *mixed domestic/fragmented* regime is characterized by $N_{M^a} = 0$ and $N_D > 0, N_{M^f} > 0$, etc.[15]

In a regime in which different production modes j and k co-exist, equation (20) implies $x_j/x_k = F_j/F_k$. Since it follows from (16) and (18) that $x_j/x_k = (p_j/p_k)^{-\sigma_x}$ and $p_j/p_k = C_j/C_k$, we obtain the condition

$$C_k = \left(\frac{F_k}{F_j}\right)^{-1/\sigma_x} C_j \; . \tag{32}$$

If the fixed costs of production mode k exceed fixed costs of mode j, then marginal costs have to be accordingly lower under free entry for both firm types to make zero profits in equilibrium. For further reference we set $F_D = 1$ and define

$$\varphi_{M^f} = (F_{M^f})^{-1/\sigma_x} \qquad \text{and} \qquad \varphi_{M^a} = (F_{M^a})^{-1/\sigma_x} \tag{33}$$

as the *marginal cost advantage* for offshoring firms that is necessary vis-a-vis domestic production to compensate for the higher fixed costs of fragmented production or production abroad, respectively. Since $F_D < F_{M^f} < F_{M^a}$, we have $\varphi_{M^a} < \varphi_{M^f} < 1$.

Figure 4 depicts marginal costs for the different production modes (already incorporating φ_j), relative to marginal costs incurred by a domestic firm (wa_D). It follows from (24) and (26) that both $C_{M^f}/(\varphi_{M^f} wa_D)$ and $C_{M^a}/(\varphi_{M^a} wa_D)$ are strictly increasing in t_1. It also follows that the line $C_{M^a}/(\varphi_{M^a} wa_D)$ is steeper than $C_{M^f}/(\varphi_{M^f} wa_D)$ if $L^*_{M^a} \geq L^*_{M^f}$.[16] Assuming that this condition is satisfied rules out implausible cases in which an increase in the relative domestic wage ω has a larger effect on marginal costs of firms producing abroad compared to marginal costs of fragmented firms. Moreover, $C_{M^f}/(\varphi_{M^f} wa_D) > C_{M^a}/(\varphi_{M^a} wa_D)$ for $t_1 = 0$ as long as n is sufficiently large.[17] Given these assumptions, all two line-pairs in Figure 4 intersect at most once, implying a unique production regime for each t_1.

[15] The potential coexistence of different production modes and thus different marginal costs is, of course, due to the assumption that goods markets are characterized by imperfect competition.

[16] By comparing (14) and (15) it is easy to show that $L^*_{M^a} \geq L^*_{M^f}$ if $2\int_0^{t_1} a^*_M(t)dt \geq T$, i.e., if transport costs T are lower than the costs of performing "expensive" tasks abroad. Of course, t_1 is an endogenous variable. Note, however, that our assumption (8) implies that t_1 is strictly positive such that there always exists a sufficiently small T which satisfies the above inequality.

[17] It follows from (23) and (25) as well as the definition of the a^*_M function that the exact requirement on n is $n > \frac{\left(\frac{\varphi_{M^f}}{\varphi_{M^a}}-1\right)a_M B}{T(a_M+A+B)} + \frac{\varphi_{M^f}}{\varphi_{M^a}}$. For φ_{M^a} close to φ_{M^f}, this holds for $n > 1$.

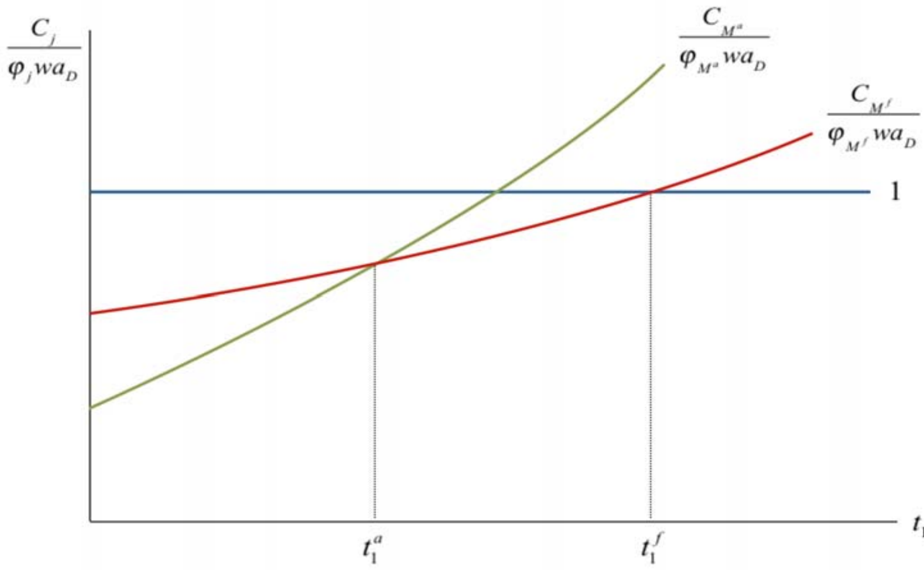

Figure 4: Equilibrium Regimes and Cutoff

We can characterize the different production regimes by a set of "complementary slackness" conditions:

$$\left(\frac{C_j}{\varphi_j} - \frac{C_k}{\varphi_k}\right) N_j \leq 0 \ , \quad N_j \geq 0 \ , \quad j, k = D, M^f, M^a \ , \quad k \neq j \ . \tag{34}$$

In Figure 4, a regime with only production-abroad firms ($N_{M^a} > 0, N_D = N_{M^f} = 0$) exists if $t_1 < t_1^a$. In this region the relative wage in the domestic economy is rather high such that fragmented firms or domestic firms make negative profits if the zero profit condition for production abroad firms is satisfied, and we obtain a pure abroad regime. At the intersection point t_1^a, a *mixed fragmented/abroad* regime exists, i.e., ($N_{M^f} > 0, N_{M^a} > 0, N_D = 0$), etc.[18] For the *three* production modes to co-exist, the three curves in Figure 4 would have to intersect in one point. We consider this to be an extremely unlikely outcome, and in what follows we focus on the analysis of equilibria with one or two production modes.

[18]Recall that Figure 4 depicts an example. Depending on parameter values, the existence and ordering of points of intersection may differ from the one depicted here.

3.5 Model Solution

Let us take stock: for a given relative wage ω, equation (21) defines the optimal cutoff value t_1 for firms that engage in international production. This cutoff value can be fed into the labor demand equations (10)–(15) and the marginal cost curves (23) and (25). A set of complementary slackness conditions (34) determines which production modes co-exist in equilibrium.

To determine the remaining endogenous variables, we start by equating the "supply function" (20) to the demand functions (16). This yields

$$(\sigma_x - 1)F_j = \left(\frac{P_X}{p_j}\right)^{\sigma_X} X + \tau \left(\frac{P_X^*}{\tau p_j}\right)^{\sigma_X} X^* \, . \tag{35}$$

Equation (17) implies $P_X = \left(N_j p_j^{1-\sigma_x} + N_k p_k^{1-\sigma_x}\right)^{\frac{1}{1-\sigma_x}}$ and $P_X^* = \tau P_X$. By combining this insight and equation (35) with the aggregate demand for X goods as described by (2) and the income definitions (31), we obtain

$$(\sigma_x - 1)F_j = \frac{\beta\left(w\bar{L} + r\bar{R} + w^*\bar{L}^* + r^*\bar{R}^*\right)}{\left(N_j p_j^{1-\sigma_x} + N_k p_k^{1-\sigma_x}\right) p_j^{\sigma_x}} \, . \tag{36}$$

Solving (36) for N_j, invoking (18) and (32), yields

$$N_j = \frac{\beta\omega\left(\bar{L} + \bar{R}\frac{r}{w}\right) + \beta\left(\bar{L}^* + \bar{R}^*\frac{r^*}{w^*}\right)}{\sigma_x F_j\left(\omega L_j + L_j^*\right)} - N_k\left(\frac{\varphi_k}{\varphi_j}\right)^{1-\sigma_x} \, . \tag{37}$$

Note that in an equilibrium in which *all* firms choose production mode j, we have $N_k = 0$. It follows from (37) that N_j depends on the relative wage ω as well as factor price ratios (r/w) and (r^*/w^*).[19] The latter can be derived by combining the labor market equilibrium conditions (28) with (4) , i.e., the conditions characterizing optimal factor demand in the Y-sector. This yields

$$\bar{L} - \sigma_x\left(N_k L_k F_k + N_j L_j F_j\right) = \left(\frac{\alpha_L}{\alpha_R}\right)^{\sigma_Y}\left(\frac{r}{w}\right)^{\sigma_Y}\bar{R} \quad \text{and} \tag{38}$$

$$\bar{L}^* - \sigma_x\left(N_k L_k^* F_k + N_j L_j^* F_j\right) = \left(\frac{\alpha_L}{\alpha_R}\right)^{\sigma_Y}\left(\frac{r^*}{w^*}\right)^{\sigma_Y}\bar{R}^* \, . \tag{39}$$

[19] Using the definition of fixed and marginal costs, we can show that (37) is equivalent to $N_j F_j C_j + N_k F_k C_k = \frac{\beta}{\sigma_x}\left(w\bar{L} + r\bar{R} + w^*\bar{L}^* + r^*\bar{R}^*\right)$. This expression has a straightforward interpretation: in equilibrium, a share $1/\sigma_x$ of global expenditure on X goods is spent on fixed costs in that sector.

Finally, the relative wage ω can be determined by combining (5) with the numeraire assumption $p_Y = p_Y^* = 1$:

$$\omega = \frac{\left[\alpha_R^{\sigma_Y}\left(\frac{r^*}{w^*}\right)^{1-\sigma_Y} + \alpha_L^{\sigma_Y}\right]^{\frac{1}{1-\sigma_Y}}}{\left[\alpha_R^{\sigma_Y}\left(\frac{r}{w}\right)^{1-\sigma_Y} + \alpha_L^{\sigma_Y}\right]^{\frac{1}{1-\sigma_Y}}}. \tag{40}$$

In equilibrium, the relative wage implied by (40) has to coincide with the value of ω on which the threshold task t_1 was based.

3.6 Existence of an Equilibrium with Offshoring

Due to our assumption on preferences, the allocation that emerges in equilibrium necessarily involves the production of both the (composite) X good and the Y good. Under which conditions is this allocation characterized by offshoring? First, the relative wage that is defined by (40) must satisfy the condition in (8). Moreover, $C_j/(\varphi_{M^j} w a_D)$, as depicted in Figure 4 has to be less than one in the limit $t_1 = 0$. Using (23) and (25) as well as the definition of the $a_M^*(t)$ curve, we can show that this requires

$$\frac{a_M(B+nT)}{B+A} + nT < \varphi_{M^f} a_D \tag{41}$$

for firms choosing fragmentation and

$$\frac{a_M(B+T)}{B+A} + T < \varphi_{M^a} a_D \tag{42}$$

for firms choosing production abroad. Note that, since $\varphi_{M^f} > \varphi_{M^a}$, neither of these inequalities implies the other one.

The two conditions in (41) and (41) are necessary but not sufficient for an equilibrium with offshoring. However, they are informative about the parameters that make the existence of such an equilibrium more or less likely: not surprisingly, increasing values of T raise the left-hand side and make it less likely that one of the conditions is satisfied. Conversely, if $T = 0$, the conditions simplify to $\frac{a_M B}{a_D(B+A)} < \varphi_j$, with $j \in \{M^f, M^a\}$, i.e., the higher productivity of multinational firms has to dominate the higher fixed cost.[20]

[20] Note that, with $T = 0$, no firm will choose production abroad, since shipping unfinished goods across borders is costless, and bundling tasks in one location does not yield any benefits.

A higher value of A, which reflects the *heterogeneity* of production processes reduces the left-hand side, widening the range of parameters at which an equilibrium with offshoring possibly exists. The intuition is straightforward, since A also reflects the gains from shifting tasks abroad. (41) and (42) reveal that a higher value of n, reflecting the *sophistication* of production processes, does not affect the feasibility of an equilibrium with *production abroad*, but reduces the range of parameters at which an equilibrium with *fragmentation* exists. Finally, the effect of the parameter B, which reflects the average costs of performing tasks abroad, on the left-hand sides of (41) and (42) is ambiguous: while a higher value of B raises the costs of foreign labor that is actually employed in the production process, it is also associated with a lower value of t_1, a higher value of ω, and thus a lower foreign component of transportation costs.

3.7 Comparative Static Analysis: The Effects of Lower Transport Costs

In a *mixed* regime with two production modes, the "free entry" condition $C_j/\varphi_j = C_k/\varphi_k$, as represented by an intersection of two curves in Figure 4, pins down t_1. This pattern is represented by the horizontal FE line in Figure 5. The "optimal cutoff" condition (21), which establishes a negative relationship between ω and t_1, is reflected by the downward-sloping OC curve in Figure 5. An equilibrium with a mixed production regime is characterized by the intersection of the FE line and the OC curve. Hence, in a mixed equilibrium, the relative wage ($\tilde{\omega}$), the optimal cutoff value ($\tilde{t}_1$), as well as employment for a given firm type in the X-sector can be determined regardless of factor endowments and conditions in the Y-sector.

In the following, we focus on the effects of an exogenous decrease in border-crossing costs (T) on the relative wage and the intensive/extensive margin of offshoring.

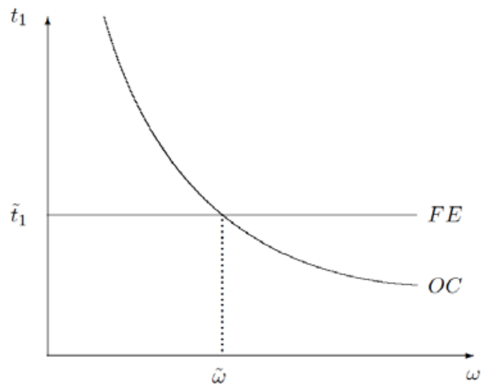

Figure 5: Cutoff Task and Equilibrium Wage in a Mixed Equilibrium

To show how a reduction of border-crossing costs affects the point of intersection of the two curves in Figure 4, we start by observing that lowering T shifts the curves $C_{M^f}/(\varphi_{M^f} w a_D)$ and $C_{M^a}/(\varphi_{M^a} w a_D)$ in Figure 4 downward, while it leaves (21) unaffected. In regimes in which *fragmented production* or *production abroad* co-exist with *domestic production*, the downward shift of the $C_{M^f}/(\varphi_{M^f} w a_D)$ and $C_{M^a}/(\varphi_{M^a} w a_D)$ curves results in *higher* values of t_1. The effect on the equilibrium relative wage can be inferred from Figure 5: the FE line shifts upward, causing a decline in the equilibrium value of ω . The economic intuition behind this effect is straightforward: lowering transport costs reduces the costs of firms engaged in offshoring. To sustain the mixed production regime, foreign production costs relative to domestic costs have to increase. This is brought about by a decline of ω. Hence, as long as domestic and multinational firms co-exist, decreasing transport costs expand the range of tasks performed *domestically* by multinational firms.

The effect of a decline in T for a regime in which domestic and multinational firms coexist may also be determined analytically. Condition (32) implies $a_D \omega \varphi_k = L_k \omega + L_k^*$ or

$$\omega = \frac{L_k^*}{a_D \varphi_k - L_k} \,. \tag{43}$$

with $k \in \{M^f, M^a\}$. Inserting from (10)–(15) and taking the derivative yields $\partial\tilde{\omega}/\partial T > 0$.

If M^f and M^a firms co-exist, the effect of a decline in T on $\tilde{\omega}$ and $\tilde{t}_1$ is not obvious ex ante (both curves shift downward in Figure 4). To understand how lowering T affects the cutoff value in such a regime, we start by observing that $C_{M^f}/\varphi_{M^f} = C_{M^a}/\varphi_{M^a}$ requires

$\varphi_{M^a}\left[L_{M^f}w + L^*_{M^f}w^*\right] = \varphi_{M^f}\left[L_{M^a}w + L^*_{M^a}w^*\right]$, or

$$\omega = \frac{L^*_{M^a} - \frac{\varphi_{M^a}}{\varphi_{M^f}}L^*_{M^f}}{\frac{\varphi_{M^a}}{\varphi_{M^f}}L_{M^f} - L_{M^a}}\,. \tag{44}$$

Inserting the labor demand equations (11) – (15) and taking the derivative with respect to T yields

$$\frac{\partial\tilde{\omega}}{\partial T} = -\frac{\left(n\frac{\varphi_{M^a}}{\varphi_{M^f}} - 1\right)(1+\tilde{\omega})}{\frac{\varphi_{M^a}}{\varphi_{M^f}}L_{M^f} - L_{M^a}} = -\frac{1+\tilde{\omega}}{2a_M t_1 + T}\,,$$

which is negative. Hence, reducing T shifts the FE-curve in Figure 5 downward, resulting in a *rise* in $\tilde{\omega}$ and a *fall* in $\tilde{t}_1$. This stands in sharp contrast to the previous cases in which domestic firms co-existed with offshoring firms.

In order to further explore the influence of transport costs T on firms' offshoring decisions at the extensive and the intensive margin, and to also analyze the comparative-static properties of regimes with only one firm-type, we consider a Cobb-Douglas production technology in sector Y, i.e., $\sigma_Y \to 1$ and $\alpha_R = \alpha_L = \alpha$.

Assumption 1 $Q_Y = R_Y^{\alpha} L_Y^{1-\alpha}$.

The following three lemmas describe the effect of a decline in T on relative wages and on offshoring at the intensive margin. Moreover, they show how the *total volume of offshoring*, which we define as domestic firms' demand for foreign labor ($\sum_j N_j L_j^*$), reacts to a reduction of border-crossing costs.[21]

Lemma 1 *In a production regime in which domestic (D) and multinational ($j = M^a$ or $j = M^f$) firms co-exist, a decline in transport costs T*

(i) lowers the wage in the North relative to the wage in the South, $\omega \equiv w/w^*$

(ii) reduces offshoring at the intensive margin (i.e., raises the optimal cutoff t_1)

(iii) raises the total volume of offshoring $N_j L_j^*$ if Assumption 1 is satisfied

(iv) raises the number of multinational firms N_j if Assumption 1 is satisfied.

[21] For proofs, see Appendix A.

The discrepancy between the adjustment at the extensive margin and at the intensive margin has a straightforward interpretation: a lower value of T raises the number of Northern firms that take advantage of the possibility to relocate parts of the production process to the South. In the aggregate, these decisions lower the relative wage in the North, and this reduces the volume of offshoring at the firm level. If Assumption 1 is satisfied, the expansion of offshoring at the *extensive* margin apparently dominates the reduction at the *intensive* margin, such that the *total* volume of offshoring increases as transport costs decrease.

Lemma 2 *In a production regime in which the two different types of multinational firms co-exist ($j = M^a$ and $k = M^f$), a decline in transport costs T*

(i) raises the wage in the North relative to the wage in the South $\omega \equiv w/w^*$

(ii) raises offshoring at the intensive margin (i.e., reduces the optimal cutoff t_1)

(iii) lowers the total volume of offshoring $N_{M^a}L^*_{M^a} + N_{M^f}L^*_{M^f}$ if Assumption 1 is satisfied.

A reduction of T has a stronger effect on the marginal costs of fragmented firms than on firms that choose production abroad. This provides an incentive to move from production abroad to fragmented production and to repatriate tasks that have been offshored before. As a result, the overall volume of offshoring decreases, demand for domestic labor increases, and this raises the relative wage.

Lemma 3 *Suppose that Assumption 1 is satisfied. In a production regime that involves only one multinational firm type ($j = M^a$ or $j = M^f$), a decline in transport costs T*

(i) raises the relative wage ω, raises offshoring at the intensive margin (lowers the cutoff t_1), and lowers the total volume of offshoring $N_jL^*_j$ if $\lambda_j \equiv L_j/L^*_j > 1$

(ii) lowers the relative wage ω, lowers offshoring at the intensive margin, and raises the total volume of offshoring $N_jL^*_j$ if $\lambda_j < 1$

(iii) raises the number of offshoring firms N_j.

In a pure production regime, the number of offshoring firms increases as the costs of transportation decrease. This is reminiscent of the result we presented in Lemma 1. However, the effect on wages, firms' offshoring decision at the extensive margin, and the overall volume of offshoring depends on relative employment in the X-industry, i.e., $\lambda_j \equiv L_j/L_j^*$. The reason is that the relative wage ω depends on λ_j, and that a change in T directly influences both L_j and L_j^* – see (11) and (12) as well as (14) and (15). If, initially, $\lambda_j > 1$, the effect of decreasing transport costs on foreign employment in the X-sector dominates, and both λ_j and ω increase if T decreases. The higher relative wage raises offshoring at the intensive margin and lures labor out of the Y-sector in the North. In the South, the higher value of ω results in an expansion of the Y-sector. Since this leaves less labor for the X-sector, the total volume of offshoring necessarily decreases. The increasing number of offshoring firms and the expansion of tasks delegated to the South are compatible with a *lower* total volume of offshoring since – with a decreasing T – the South uses less labor to ship intermediate goods across the border. Conversely, if $\lambda_j < 1$ initially, lowering T reduces domestic employment in the X-sector by more than foreign employment, such that λ_j and ω decrease. As a result, offshoring at the intensive margin decreases, but an increasing share of the Southern labor force is employed in the X-sector, implying a greater total volume of offshoring. The latter finding is compatible with a higher value of t_1 since the number of firms engaged in offshoring increases. [22]

Note that our analysis has characterized the comparative static properties of different production regimes, but has not spelled out the process that leads from one regime to another one. Nevertheless, there are some important qualitative findings to take away: first, decreasing transport costs do not necessarily raise the total volume of offshoring. They do so if multinational firms co-exist with domestic firms (see Lemma 1), but they don't if some firms choose fragmented production and others choose production abroad (Lemma 2). Whether they do so in a pure production regime depends on initial employment in

[22] In the last part of Appendix A, we discuss the parameters that determine whether λ_j is smaller or greater than one. Not surprisingly, it is more likely that λ_j is greater (smaller) than one if the North's share in the world's endowment with effective labor ($\frac{\bar{L}}{L+L^*}$) is high (low) relative to its share in the global endowment with the fixed factor ($\frac{\bar{R}}{R+R^*}$): a higher (relative) supply of effective labor in the North lowers the wage and raises employment in the X-sector.

the X-sector. Moving from a partial-equilibrium perspective (as in Harms et al. 2012 or Baldwin and Venables 2013) to general equilibrium thus adds two important aspects to the analysis of offshoring: First, within a particular production regime, there is a smooth adjustment to changing transport costs at the intensive and the extensive margin. Second, the volume of offshoring as well as the relative number of different firm types in equilibrium affect relative wages, and this has a repercussion on offshoring at the firm level. In fact, the share of tasks shifted abroad may actually decrease while the total offshoring volume increases.

Note, finally, that our comparative static analysis has focused on the effects of lowering transport costs T. While we think that this change – which can be interpreted as a symptom of *intensifying globalization* – is of particular interest, it is of course desirable to explore the effects of other parameter changes as well. As we show in the last part of Appendix A, a reduction of the North's endowment with effective labor unambiguously raises the volume of offshoring if Assumption 1 is satisfied. In mixed regimes with different firm types, this adjustment entirely materializes at the extensive margin, i.e. through a higher number of offshoring firms, while the relative wage (ω) and the intensive margin of offshoring are unaffected. In pure production regimes with just one firm type, the lower domestic labor supply raises both the number of offshoring firms, the relative wage, and offshoring at the intensive margin. For other parameters that characterize the two regions as well as the technologies used by firms, it is hardly possible to derive unambiguous comparative-static results. However, a numerical analysis offers important insights on how changes in these parameters affect the volume of offshoring.

4 A Numerical Appraisal

4.1 Motivation

In this section, we run numerical simulations to further explore the comparative static properties of our model. In particular, we drop Assumption 1 – i.e., we return to a more general specification of the production function in the Y-sector – and we extend the analysis by also considering the effects of other model parameters. As outlined above, our model allows for two dimensions along which the extent of offshoring changes as exogenous parameters vary: first, the share of the production process that is performed abroad for a given firm-type may increase or decrease (the *intensive margin*). Second, the number of firms of a certain type may vary (the *extensive margin*). In what follows, we will analyze how offshoring reacts at the extensive and at the intensive margin to a change in factor endowments, a decline in transport costs, changes in relative productivities and other properties of the production process.

4.2 Calibration

The two regions are scaled so that initially about half of the domestic consumption of good Y is imported from the South, while X goods are produced only by Northern firms and exported to the South. With preference parameter $\beta = 0.5$, the North is endowed with one third of the world's R and two thirds of the world's L, while the South is endowed with the rest.[23] In industry Y, we set $\sigma_Y = 3$ for the CES production function and in industry X, we set $\sigma_x = 4$ and $n = 2$ as benchmark values for the substitution elasticity and the number of cycles of $a_M^*(t)$, respectively. We choose somewhat arbitrarily – but within the ranges consistent with our theoretical constraints – the fixed costs and the cost of transporting intermediate goods across borders: $F_D = 1.0$; $F_{M^f} = 1.3$; $F_{M^a} = 1.5$; $T = 0.2$. With this functional form and parameter values, we calibrate the key technological parameters – A, B, a_D and a_M –, so that initially about half of the total X output is produced by D-type firms and the other half by M^f-type firms. Appendix B reports the benchmark parameter and equilibrium variable values.

4.3 Factor Endowments and Production Regimes in Equilibrium

Given our benchmark parameter values, Figure 6 presents the equilibrium production regimes for different allocations of production factors between the two regions. The vertical axis is the total world endowment of effective labor, and the horizontal axis is the total world endowment of the composite factor, with the North measured from the southwest

[23] Note that our framework adopts elements from both the Ricardian and the Heckscher-Ohlin framework, i.e., trade is driven by differences in factor endowments and by technological differences. While our assumption that the North is relatively abundant in labor may seem unjustified at first glance, L should be interpreted as *effective* labor supply, which is determined by both demographic and technological factors.

(SW) and the South from the northeast (NE).

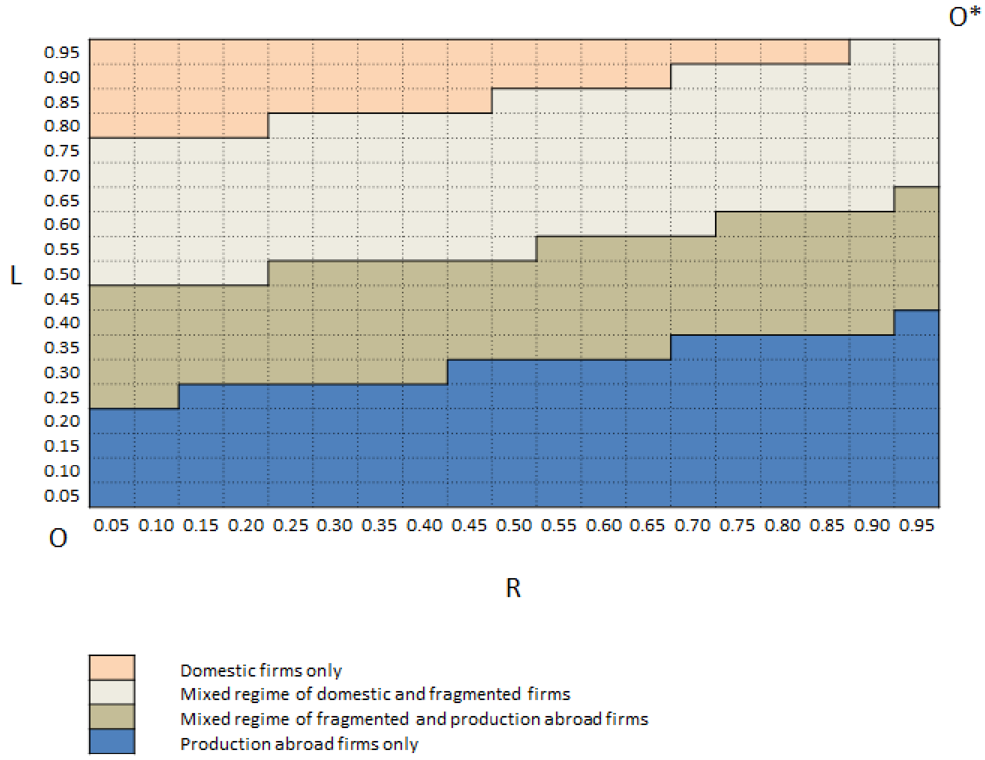

Figure 6: Equilibrium Regimes

We see that the equilibrium regimes are associated with differences in relative factor endowments.[24] Intuitively, if the North is highly abundant in effective labor – such that this factor is relatively cheap – no offshoring occurs. Conversely, if the South is highly abundant in labor, we may expect that most firms produce abroad. For intermediate allocations of factors of production, fragmented firms should dominate.[25]

Figure 7 displays the equilibrium number of each firm-type along the NW-SE diagonal – the two regions differ in relative factor endowments – and along the SW-NE diagonal –

[24] Note that our assumptions on parameter values exclude certain production regimes that could also occur in principle, as for example an equilibrium in which domestic firms and firms that choose production abroad coexist.

[25] Indeed, altering the distribution of the world endowment in much finer steps, we also have regimes in which only fragmented firms exist between the two regimes of $\{D, M^f\}$ and $\{M^f, M^a\}$.

the two regions have identical relative factor endowments but differ in size.

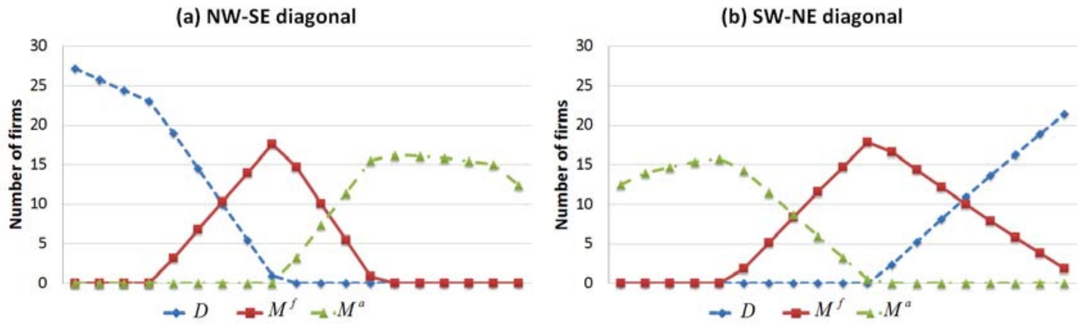

Figure 7: Endogenous Market Structure along the NW-SE Diagonal (North becoming less labor-abundant) and SW-NE Diagonal (North becoming bigger)

As we move from left to right in panel (a) of Figure 7, the relative share of the North in global labor supply decreases, and its share in the composite factor increases. This raises the Northern wage rate, making offshoring more attractive and thus reducing the number of domestic firms. First, we observe the emergence of fragmented firms that allocate their production process to different regions. Eventually, as the bulk of global labor is located in the South, firms decide to offshore the biggest possible part of the value-added chain, leaving only M^a firms in business.

The effect of increasing the size of the North – leaving relative factor endowments constant – is depicted in panel (b) of Figure 7. As the North is growing bigger, it hosts an increasing share of the global labor supply. Since the labor endowment is decisive for the attractiveness of offshoring, the picture is first dominated by multinationals producing in the South, and eventually by domestic firms. At intermediate stages – i.e., for constellations at which both regions are of roughly equal size – most of global production is performed by fragmenting multinationals that intensively exploit international cost differences.

4.4 The Effects of Globalization

We now investigate the effects of globalization, which we interpret as a decline in transport costs T. Figure 8 reports the effects of a decline in T (horizontal axis) on the prevalence of each firm-type. Note that the horizontal axis is inverted, moving from higher to lower values – i.e., the extent of globalization increases (with transport costs decreasing) from left to right.

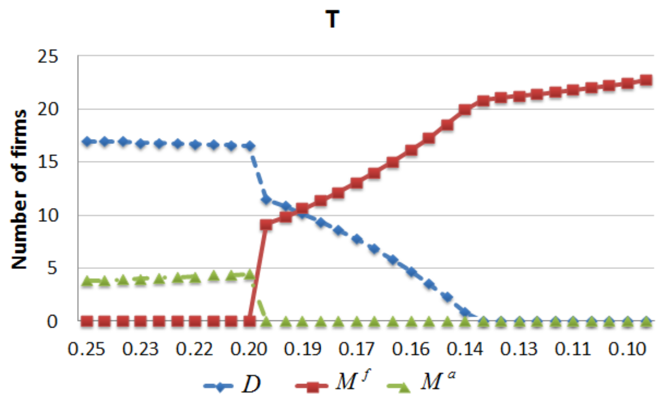

Figure 8: Decreasing transport costs (T) and the importance of different firm types

Figure 8 supports the theoretical results derived above: if T decreases, the number of multinational firms increases. As long as transportation costs are high, production abroad dominates the picture. With lower values of T, fragmented production replaces production abroad and, eventually, this becomes the dominant mode of production.

Figure 9 presents the induced variations in the equilibrium cutoff task t_1 and the relative wage ω (with initial values normalized to one at $T = 0.25$).

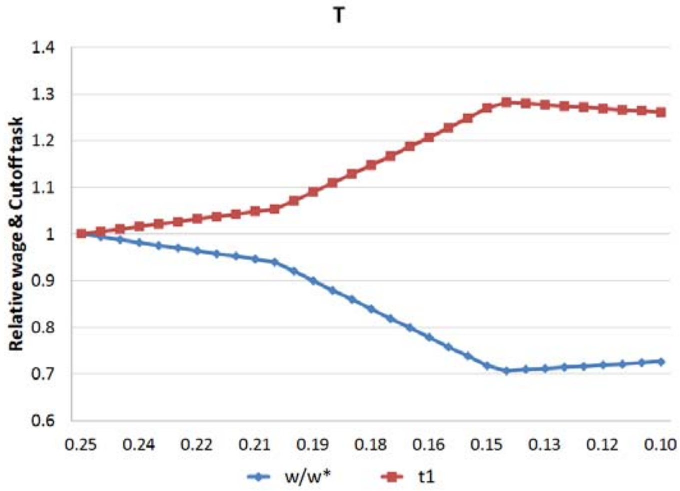

Figure 9: Decreasing transport costs (T), the cutoff task, and the relative wage

As already shown in the comparative static analysis, a decline in transport costs first *raises* the cutoff value t_1 and thus *lowers* the range of tasks offshored. This is due to the fact that a higher demand for labor in the South, resulting from an increasing number of offshoring firms, lowers the wage ratio (w/w^*), as also shown by Figure 9. According to equation (21), this makes it less attractive to offshore a large range of tasks. However, for very low levels of T, as only fragmented firms exist, the effect of a decline in T on the cutoff values is reversed. This reflects case (i) spelled out in Lemma 3: apparently, lowering transport costs and thus releasing labor in both regions, has a bigger impact on the South . As shown by Figure 9, this raises w/w^* so that it is profitable to offshore a higher range of tasks to the South.

4.5 Technological Change

In this subsection, we explore how technological changes affect the relative importance of alternative production modes. Again, the horizontal axis is inverted, moving from higher to lower values – i.e., productivity increases are reflected by decreasing input coefficients from left to right.

Figure 10 shows the effects of domestic technological change of a_D and a_M. Intuitively, technological progress in domestic firms – a fall in a_D – makes these firms more competitive compared to multinationals. More entry by domestic firms raises the domestic wage, which is more detrimental to fragmented firms given that they perform more tasks in the North than firms producing almost everything abroad. In contrast, productivity improvements of multinational firms – a fall in a_M – reduces the number of domestic firms. Between the two types of multinational firms, fragmented firms benefit more by the same logic as above.

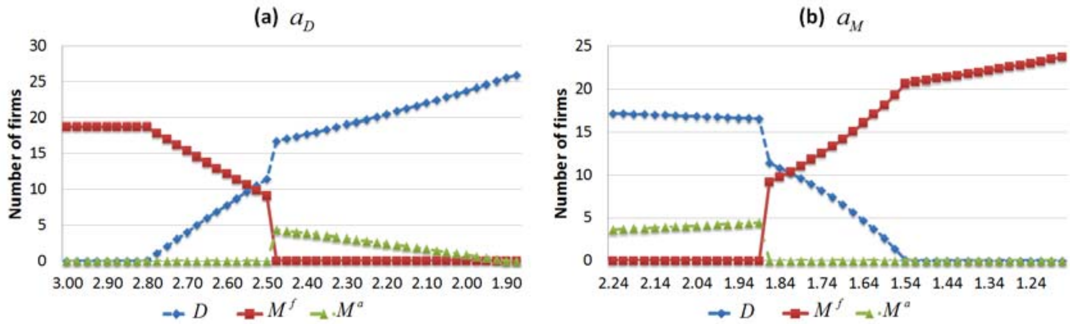

Figure 10: The effects of decreasing input coefficients a_D and a_M

Figure 11 focuses on the South and presents the effects of varying the parameters A and B. Recall that raising A increases the amplitude of the $a_M^*(t)$ function, representing greater cost differences between different tasks in the production chain. Conversely, reducing A implies a decline in the cost advantage of producing in the South for tasks $t \in \left[t_1, \frac{1}{n} - t_1\right]$. This makes fragmentation less attractive, and eventually only domestic firms and production abroad firms prevail. A similar pattern emerges if B – i.e., the average costs associated with producing abroad – increases: in this case, the number of firms choosing any type of offshoring declines.

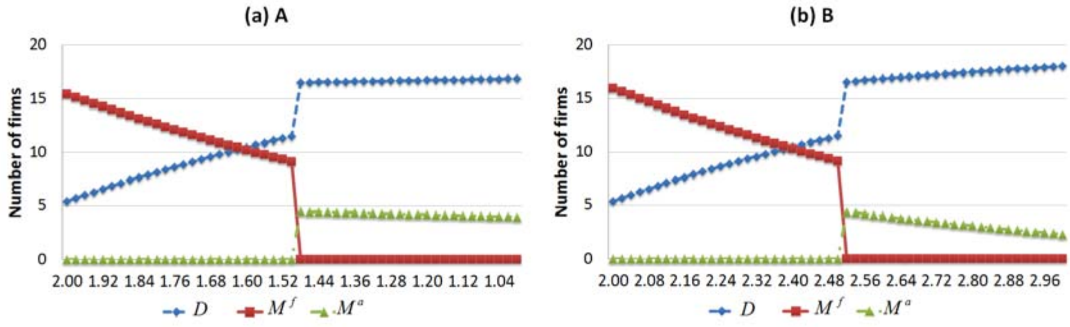

Figure 11: Technological Change of A and B

Finally, Figure 12 displays the effect of an increase in the number of cycles (n) on the relative importance of alternative production modes. Recall that we interpret production processes that are characterized by a higher value of n as being more "sophisticated", reflecting a more complex structure of cost differences along the value-added chain.

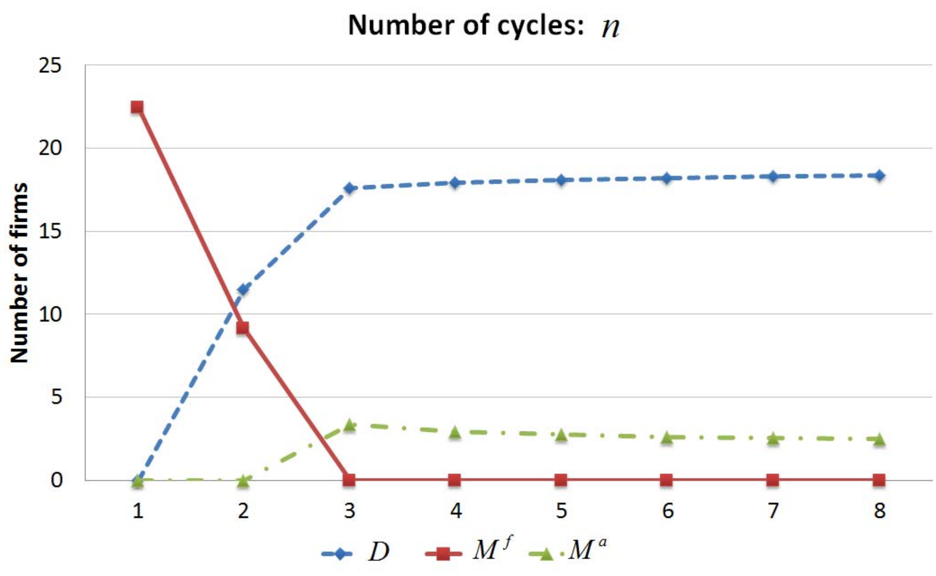

Figure 12: Increase in Sophistication of the Sequential Production Chain

An increase in n has similar effects as an increase in T. Given that fragmented offshoring requires transportation costs of n-times T in each region, raising n reduces N_{M_f} while the number of domestic firms increases.

4.6 Robustness

How robust are the numerical results presented in the preceding subsections with respect to variations in some crucial parameters? In Appendix C, we report the results of using alternative values for the two essential elasticities of the model, σ_Y and σ_X, as well as different factor endowments along the two diagonals of Figure 6. While lowering or raising the elasticity of substitution affects the number of different firm types – with a lower (higher) value of σ_X raising (lowering) markups and thus the number of firms in the X-sector, and a lower (higher) value of σ_Y reinforcing (dampening) the effect of offshoring on the Southern wage – the qualitative results presented in Figures 8 and 9 are largely unchanged. As for factor endowments, reducing the domestic (effective) labor supply in favor of the South reduces the range of T-values for which an equilibrium with domestic production emerges and raises the importance of *production abroad.* At the same time, lowering $\bar{L}/\bar{L}^*$ raises the relative wage ω and raises the range of T values for which fragmented production coexists with production abroad.

5 Summary and Conclusions

In this paper, we have analyzed the extent of offshoring in a two-region general equilibrium model that is based on three crucial assumptions. First, a firm's production process follows a rigid structure that defines the sequence of production steps. Second, the costs of offshoring vary in a non-monotonic fashion along the production chain. Third, each task requires the presence of an unfinished intermediate good whose transportation across borders is costly. We believe that these assumptions are quite plausible for a wide range of industries. As a consequence, some firms may be reluctant to offshore individual production steps, even if performing them abroad would be associated with cost advantages: the reason is that adjacent tasks may be cheaper to perform in the domestic economy and that high transport costs do not justify shifting the unfinished good abroad and back home.

Using this basic structure and setting up a general equilibrium model along these lines, we have analyzed the influence of globalization – interpreted as a variation in transportation costs – and of technological changes on the volume of offshoring at the extensive and the intensive margin. As relative production costs vary, firms adjust the share of tasks they perform abroad (the *intensive* margin). At the same time, the number of firms that fragment their production process or produce almost entirely abroad varies (the *extensive* margin). Both adjustments may affect relative wages at home and abroad, which can reinforce or dampen the initial impulse. We have shown that globalization in the form of declining transport costs may have different effects on offshoring at the extensive and intensive margin. For example, if domestic and multinational firms coexist, lower transport costs result in a *decrease* of offshoring at the intensive margin – i.e., firms offshore a smaller part of the entire production process – but an *increase* in the number of firms that perform at least some tasks abroad.

We believe that the simplicity of our model – in particular, the symmetry of the $a_M^*(t)$ function – has allowed us to derive some novel results, which are likely to carry over into a more general environment. The challenge ahead is to expand the framework to accommodate additional features of reality, e.g., by introducing a non-symmetric shape of the $a_M^*(t)$ function, or by allowing for transport costs that vary along the production chain. In fact, dropping the symmetry property and letting costs vary systematically between earlier and later production segments may create a richer picture of possible offshoring patterns. For example, transport costs could be higher for later tasks, as the intermediate product becomes more valuable along the production process. In this case, additional firm types may emerge that offshore only the initial parts of the production chain, performing the later tasks at home.

The second challenge is to assess the *welfare effects* of globalization: while reducing T lowers the resources that are not used productively *ceteris paribus*, this induces an increase of offshoring at the *extensive margin* and may therefore raise total transport costs. At the same time, the number of firms rises, which raises utility due to the "love of variety" built into the model. Combining these insights with the results concerning the evolution of factor prices and incomes and evaluating the resulting net effects on agents' welfare in North and South is an interesting challenge ahead. Finally, it is important to gauge the relative importance of sequential production processes for the economy as a whole. Our contribution rested on the assumption that *all* firms have to cope with a rigid sequence of production steps. This may be as unrealistic as the notion that production processes can be re-arranged freely by every firm. We believe that characterizing real-world production processes in terms of "sequentiality" holds ample promise for future research.

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Appendix A: Comparative Statics

Proof of Lemma 1: Parts (i) and (ii) follow from equations (21) and (43). For parts (iii) and (iv), first note that for a Cobb-Douglas technology, factor demands of equation (3) can be written as $(1-\alpha)\,Q_Y = wL_Y$ and $\alpha Q_Y = rR_Y$ for North and South. The zero profit condition (4) with $p_Y = p_Y^* = 1$ implies

$$w^{1-\alpha}r^{\alpha} = (w^*)^{1-\alpha}(r^*)^{\alpha} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \ .$$

Equilibrium on the market for good Y requires

$$Q_Y + Q_Y^* = (1-\beta)\left[w\bar{L} + r\bar{R} + w^*\bar{L}^* + r^*\bar{R}^*\right] \ .$$

Inserting for factor demands and factor prices from the above conditions and accounting for the fact that $R_Y = \bar{R}$ and $R_Y^* = \bar{R}^*$ yields after rearranging

$$\frac{r}{w} = \frac{(1-\beta)\,\alpha\left[\bar{L} + \bar{L}^*/\omega\right]}{(1-(1-\beta)\,\alpha)\left[\bar{R} + \bar{R}^*\omega^{(1-\alpha)/\alpha}\right]} \quad \text{and} \tag{A.1}$$

$$\frac{r^*}{w^*} = \frac{(1-\beta)\,\alpha\left[\bar{L}\omega + \bar{L}^*\right]}{(1-(1-\beta)\,\alpha)\left[\bar{R}\omega^{(\alpha-1)/\alpha} + \bar{R}^*\right]} \ . \tag{A.2}$$

For labor input in sector Y we obtain

$$L_Y = \frac{(1-\alpha)}{\alpha}\left(\frac{r}{w}\right)\bar{R} \qquad \text{and} \qquad L_Y^* = \frac{(1-\alpha)}{\alpha}\left(\frac{r^*}{w^*}\right)\bar{R}^* \ . \tag{A.3}$$

A decline in ω, induced by a reduction of T, lowers r^*/w^* and raises r/w. L_Y increases and L_Y^* declines. Since $\sigma_x N_j L_j^* F_j = \bar{L}^* - L_Y^*$, a decline in L_Y^* raises $N_j L_j^*$ and (because L_j^* decreases due to declining ω) also N_j.

Proof of Lemma 2: Parts (i) and (ii) follow from equations (21) and (44). For part (iii), we can apply (A.3) and the fact that the volume of offshoring equals $\bar{L}^* - L_Y^*$.

Proof of Lemma 3: Inserting for L_Y and L_Y^* from (A.1)–(A.3) into $\sigma_x N_j L_j F_j = \bar{L} - L_Y$ and $\sigma_x N_j L_j^* F_j = \bar{L}^* - L_Y^*$ determines the "relative labor supply" in the X-sector λ^s as

$$\lambda^s(\omega) = \frac{\bar{L} - L_Y(\omega)}{\bar{L}^* - L_Y^*(\omega)} \ . \tag{A.4}$$

It follows from (A.4) and the arguments brought forward in the proof of Lemma 1 that $\lambda^s(\omega)$ increases in ω, i.e., $\partial\lambda^s/\partial\omega > 0$. The second equation that determines the equilibrium can be derived by dividing the "labor demand" equations (11) and (14) for $j = M^f$ or (12) and (15) for $j = M^a$ and inserting $t_1 = t_1(\omega)$ from (21). The resulting ratio can be written as $\lambda_j^d(\omega, T)$, with $\partial\lambda_j^d/\partial\omega < 0$. For the partial derivative $\partial\lambda_j^d/\partial T > 0$, we obtain

$$\frac{\partial\lambda_{M^f}^d}{\partial T} = \frac{nL_{M^f}^* - nL_{M^f}}{\left(L_{M^f}^*\right)^2} < 0 \quad \text{if} \quad \lambda_{M^f} > 1 \quad \text{and}$$

$$\frac{\partial\lambda_{M^a}^d}{\partial T} = \frac{L_{M^a}^* - L_{M^a}}{\left(L_{M^a}^*\right)^2} < 0 \quad \text{if} \quad \lambda_{M^a} > 1 \; .$$

Setting $\lambda^s(\omega) = \lambda_j^d(\omega, T)$ yields the equilibrium value for ω, and from taking the derivative, we can determine the following condition:

$$\frac{d\omega}{dT} = \frac{\partial\lambda_j^d/\partial T}{\partial\lambda^s/\partial\omega - \partial\lambda_j^d/\partial\omega} \; . \tag{A.5}$$

Equation (A.5) implies $d\omega/dT < 0$ if $\lambda_j > 1$. In this case, a decline in T raises ω and lowers t_1. This completes the proof for part (i). Part (ii) can be proven analogously. For part (iii) in the case of $\lambda_j > 1$, we first note that L_j decreases if both T and t_1 decline – according to (11) or (12). Second, L_Y declines according to (A.3), which implies that $N_j L_j$ increases. Since L_j declines, this is only possible if N_j increases. In the case of $\lambda_j < 1$, a decline in T results in an increase in t_1 such that L_j^* declines – according to (14) or (15). As L_Y^* declines according to (A.3), $L_j^* N_j$ has to increase, which, again, implies that N_j increases.

To get some idea of the forces that determine whether λ_j is smaller or greater than one, we consider $\lambda^s(\omega)$ for $\omega = 1$. Using (A.1) – (A.4), we can show that $\lambda^s(1)$ is greater (smaller) than one if $\delta_L - \frac{1}{2} > \theta\left(\delta_R - \frac{1}{2}\right)$, with $\delta_L \equiv \frac{\bar{L}}{\bar{L}+\bar{L}^*}$ and $\delta_R \equiv \frac{\bar{R}}{\bar{R}+\bar{R}^*}$ and $\theta \equiv \frac{1-\beta-(1-\beta)\alpha}{1-(1-\beta)\alpha} < 1$ – i.e. if domestic economy's share in the global labor endowment is bigger than the domestic economy's share in the global endowment with the fixed factor. For $j = M^a$, we can show that $\lambda_{M^a}^d(1) < 1$ if $2a_M t_1 < \int_0^1 a_M^*(t)dt - \int_0^{t_1} a_M^*(t)dt$. It is easy to see that this condition is satisfied. Hence, if $\delta_L - \frac{1}{2} < \theta\left(\delta_R - \frac{1}{2}\right)$, $\lambda_{M^a} < 1$. What happens if $\delta_L - \frac{1}{2} > \theta\left(\delta_R - \frac{1}{2}\right)$ depends on all parameters of the model. In the same way, we can show that $\lambda_{M^f}^d(1) < 1$ if $2a_M t_1 < \int_{t_1}^{\frac{1}{n}-t_1} a_M^*(t)dt$. Whether this condition is satisfied depends on all model parameters. If it is, $\lambda_{M^f} < 1$ if $\delta_L - \frac{1}{2} < \theta\left(\delta_R - \frac{1}{2}\right)$. If it is not, $\lambda_{M^f} > 1$ if $\delta_L - \frac{1}{2} > \theta\left(\delta_R - \frac{1}{2}\right)$.

The Effects of Raising Domestic Labor Supply: We start by considering the consequences of an exogenous increase of $\bar{L}$ in a mixed production regime, which is characterized by the coexistence of domestic and multinational firms. As we have shown in the main text, a change in factor endowments has no effect on the relative wage ω and thereby on the intensive margin of offshoring. The influence on the extensive margin, i.e., on N_{M^f} and N_{M^a}, can be derived from equations (A.1), (A.2) and (A.3). It follows from these expressions that an increase of $\bar{L}$ raises L_Y and L_Y^*. As a consequence, employment in the X-sector decreases both in the North and in the South, i.e. the total volume of offshoring $N_j L_j^*$ decreases. Since the endowment change has no impact on the relative wage and thus on the labor demand of firms engaged in offshoring, all the adjustment takes place at the extensive margin, i.e. N_j decreases for $j \in \{M^a, M^f\}$. A similar logic applies for a mixed production regime with the two types of multinational firms, i.e., the total volume of offshoring decreases as a consequence of an exogenous increase of $\bar{L}$. However, without imposing further structure, we cannot determine how this affects the number of firms that choose fragmentation or production abroad, respectively.

Production regimes in which only one type of multinational firms exists can be analyzed along the lines described in the proof of Lemma 3: after inserting (A.1) – (A.3) into (A.4), we can show that λ^s is upward-sloping in ω. Moreover, an increase of $\bar{L}$ shifts the λ^s-curve upward. To see this, not that $\lambda^s(\omega)$ can be written as

$$\lambda^s(\omega) = \frac{\bar{L} - \frac{(1-\beta)(1-\alpha)\left[\bar{L}+\bar{L}^*/\omega\right]}{(1-(1-\beta)\alpha)\left[1+\frac{\bar{R}^*}{\bar{R}}\omega^{(1-\alpha)/\alpha}\right]}}{\bar{L}^* - \frac{(1-\beta)(1-\alpha)\left[\bar{L}\omega+\bar{L}^*\right]}{(1-(1-\beta)\alpha)\left[\frac{\bar{R}}{\bar{R}^*}\omega^{(\alpha-1)/\alpha}+1\right]}} \tag{A.6}$$

The numerator of this expression is unambiguously increasing in $\bar{L}$, while the denominator is unambiguously decreasing in $\bar{L}$. Since the λ_j^d-curve is unaffected by the endowment change, this implies a decreasing relative wage ω in equilibrium. Plugging this insight into (A.1) and (A.3) shows that r/w and thus L_Y are unambiguously increasing. In a *pure* production regime with only one firm type, this implies that $N_j L_j$ is decreasing. Combining this insight with the fact that the lower ω raises L_j indicates that the number of firms N_j is decreasing. Since L_j^* positively depends on ω, the overall volume of offshoring $N_j L_j^*$ decreases as a result of an increasing $\bar{L}$.

Appendix B: Benchmark Parameter and Variable Values

$\bar{R}$	$\bar{L}$	$\bar{R}^*$	$\bar{L}^*$	β	σ_x	a_D	a_M	n	A
100.00	200.00	200.00	100.00	0.50	4.00	2.50	1.86	2.00	1.50
B	t_{min}	t_{max}	τ	T	F_D	F_{M^f}	F_{M^a}	σ_Y	α_R
2.50	0.00	1.00	1.30	0.20	1.00	1.30	1.50	3.00	0.50
α_L	Inc	U	P_U	X	Y	x_D^d	$x_{M^f}^d$	$x_{M^a}^d$	P_X
0.50	190.48	101.14	1.88	107.41	95.24	1.68	2.19	2.52	0.89
Inc^*	U^*	P_U^*	X^*	Y^*	x_D^{d*}	$x_{M^f}^{d*}$	$x_{M^a}^{d*}$	P_X^*	x_D
149.49	69.62	2.15	64.84	74.75	1.32	1.71	1.98	1.15	3.00
x_{M^f}	x_{M^a}	C_D	C_{M^f}	C_{M^a}	p_D	p_{M^f}	p_{M^a}	N_D	N_{M^f}
3.90	4.50	1.88	1.76	1.70	2.51	2.35	2.27	11.48	9.13
N_{M^a}	t_1	t_2	t_3	t_4	Q_Y	Q_Y^*	p_Y	p_Y^*	L_Y
0.00	0.14	0.36	0.64	0.86	51.42	118.57	1.00	1.00	15.11
R_Y	L_Y^*	R_Y^*	w	r	w^*	r^*			
100.00	52.78	200.00	0.75	0.40	0.65	0.42			

Appendix C: Robustness Checks – Varying σ_Y, σ_X, and Factor Endowments

T	$\sigma_Y = 2$					(Benchmark) $\sigma_Y = 3$					$\sigma_Y = 4$				
	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$
0.250	16.831	0.000	3.901	0.137	1.221	16.965	0.000	3.772	0.137	1.221	17.091	0.000	3.660	0.137	1.221
0.225	16.663	0.000	4.200	0.141	1.184	16.732	0.000	4.135	0.141	1.184	16.799	0.000	4.076	0.141	1.184
0.200	11.481	9.133	0.000	0.144	1.149	11.481	9.133	0.000	0.144	1.149	11.481	9.133	0.000	0.144	1.149
0.175	8.338	12.505	0.000	0.157	1.025	7.715	13.065	0.000	0.157	1.025	7.089	13.602	0.000	0.157	1.025
0.150	3.800	17.208	0.000	0.171	0.901	2.253	18.548	0.000	0.171	0.901	0.678	19.849	0.000	0.171	0.901
0.125	0.000	21.426	0.000	0.179	0.835	0.000	21.414	0.000	0.174	0.872	0.000	21.368	0.000	0.171	0.902
0.100	0.000	22.456	0.000	0.177	0.853	0.000	22.448	0.000	0.173	0.887	0.000	22.405	0.000	0.169	0.914
0.075	0.000	23.585	0.000	0.175	0.872	0.000	23.582	0.000	0.171	0.903	0.000	23.542	0.000	0.168	0.928
0.050	0.000	24.828	0.000	0.172	0.891	0.000	24.831	0.000	0.169	0.919	0.000	24.794	0.000	0.166	0.942
0.025	0.000	26.204	0.000	0.170	0.912	0.000	26.212	0.000	0.167	0.937	0.000	26.179	0.000	0.165	0.957
0.000	0.000	27.735	0.000	0.167	0.933	0.000	27.749	0.000	0.165	0.955	0.000	27.721	0.000	0.163	0.973

T	$\sigma_X = 3$					(Benchmark) $\sigma_X = 4$					$\sigma_X = 5$				
	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$
0.250	23.196	0.000	4.331	0.131	1.285	16.965	0.000	3.772	0.137	1.221	13.357	0.000	3.263	0.141	1.184
0.225	22.894	0.000	4.821	0.134	1.246	16.732	0.000	4.135	0.141	1.184	13.168	0.000	3.550	0.144	1.149
0.200	22.586	0.000	5.322	0.138	1.208	11.481	9.133	0.000	0.144	1.149	8.355	8.088	0.000	0.148	1.108
0.175	13.023	14.881	0.000	0.150	1.090	7.715	13.065	0.000	0.157	1.025	5.095	11.415	0.000	0.161	0.987
0.150	6.703	21.465	0.000	0.164	0.963	2.253	18.548	0.000	0.171	0.901	0.338	16.089	0.000	0.175	0.865
0.125	0.000	28.552	0.000	0.174	0.872	0.000	21.414	0.000	0.174	0.872	0.000	17.131	0.000	0.174	0.872
0.100	0.000	29.931	0.000	0.173	0.887	0.000	22.448	0.000	0.173	0.887	0.000	17.959	0.000	0.173	0.887
0.075	0.000	31.443	0.000	0.171	0.903	0.000	23.582	0.000	0.171	0.903	0.000	18.866	0.000	0.171	0.903
0.050	0.000	33.108	0.000	0.169	0.919	0.000	24.831	0.000	0.169	0.919	0.000	19.865	0.000	0.169	0.919
0.025	0.000	34.949	0.000	0.167	0.937	0.000	26.212	0.000	0.167	0.937	0.000	20.970	0.000	0.167	0.937
0.000	0.000	36.998	0.000	0.165	0.955	0.000	27.749	0.000	0.165	0.955	0.000	22.199	0.000	0.165	0.955

< NW-SE Diagonal >

	(Benchmark) $\bar{L}=200$, $\bar{R}=100$, $\bar{L}^*=100$, $\bar{R}^*=200$					$\bar{L}=150$, $\bar{R}=150$, $\bar{L}^*=150$, $\bar{R}^*=150$					$\bar{L}=100$, $\bar{R}=200$, $\bar{L}^*=200$, $\bar{R}^*=100$				
T	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$
0.250	16.965	0.000	3.772	0.137	1.221	8.623	0.000	9.569	0.137	1.221	0.221	0.000	15.415	0.137	1.221
0.225	16.732	0.000	4.135	0.141	1.184	8.347	0.000	10.015	0.141	1.184	0.000	0.000	15.875	0.140	1.192
0.200	11.481	9.133	0.000	0.144	1.149	0.000	14.692	3.224	0.144	1.149	0.000	0.000	16.197	0.141	1.180
0.175	7.715	13.065	0.000	0.157	1.025	0.000	17.641	1.120	0.136	1.225	0.000	1.380	15.344	0.136	1.225
0.150	2.253	18.548	0.000	0.171	0.901	0.000	19.630	0.000	0.132	1.266	0.000	3.970	13.467	0.128	1.312
0.125	0.000	21.414	0.000	0.174	0.872	0.000	20.505	0.000	0.132	1.270	0.000	7.283	10.986	0.118	1.411
0.100	0.000	22.448	0.000	0.173	0.887	0.000	21.462	0.000	0.132	1.275	0.000	11.854	7.438	0.106	1.527
0.075	0.000	23.582	0.000	0.171	0.903	0.000	22.512	0.000	0.131	1.280	0.000	19.070	1.611	0.092	1.670
0.050	0.000	24.831	0.000	0.169	0.919	0.000	23.671	0.000	0.130	1.285	0.000	21.922	0.000	0.089	1.693
0.025	0.000	26.212	0.000	0.167	0.937	0.000	24.957	0.000	0.130	1.291	0.000	23.145	0.000	0.090	1.683
0.000	0.000	27.749	0.000	0.165	0.955	0.000	26.390	0.000	0.129	1.296	0.000	24.510	0.000	0.091	1.673

< SW-NE Diagonal >

	$\bar{L}=100$, $\bar{R}=100$, $\bar{L}^*=200$, $\bar{R}^*=200$					$\bar{L}=150$, $\bar{R}=150$, $\bar{L}^*=150$, $\bar{R}^*=150$					$\bar{L}=200$, $\bar{R}=200$, $\bar{L}^*=100$, $\bar{R}^*=100$				
T	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$	N_D	N_{M^f}	N_{M^a}	t_1	$\frac{w}{w^*}$
0.250	2.674	0.000	13.443	0.137	1.221	8.623	0.000	9.569	0.137	1.221	14.243	0.000	5.951	0.137	1.221
0.225	2.446	0.000	13.922	0.141	1.184	8.347	0.000	10.015	0.141	1.184	13.957	0.000	6.332	0.141	1.184
0.200	0.000	4.025	12.432	0.144	1.149	0.000	14.692	3.224	0.144	1.149	6.201	13.610	0.000	0.144	1.149
0.175	0.000	6.094	10.934	0.136	1.225	0.000	17.641	1.120	0.136	1.225	1.877	17.886	0.000	0.157	1.025
0.150	0.000	8.603	9.075	0.128	1.312	0.000	19.630	0.000	0.132	1.266	0.000	20.278	0.000	0.161	0.988
0.125	0.000	11.825	6.622	0.118	1.411	0.000	20.505	0.000	0.132	1.270	0.000	21.218	0.000	0.160	1.001
0.100	0.000	16.317	3.094	0.106	1.527	0.000	21.462	0.000	0.132	1.275	0.000	22.245	0.000	0.158	1.014
0.075	0.000	20.554	0.000	0.098	1.606	0.000	22.512	0.000	0.131	1.280	0.000	23.372	0.000	0.157	1.027
0.050	0.000	21.638	0.000	0.099	1.600	0.000	23.671	0.000	0.130	1.285	0.000	24.615	0.000	0.155	1.041
0.025	0.000	22.840	0.000	0.100	1.593	0.000	24.957	0.000	0.130	1.291	0.000	25.993	0.000	0.154	1.055
0.000	0.000	24.182	0.000	0.100	1.586	0.000	26.390	0.000	0.129	1.296	0.000	27.528	0.000	0.152	1.070