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Modified Sharpe Ratios in Real Estate Performance Measurement: Beyond the Standard Cornish Fisher Expansion

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Abstract An important component in the analysis of real estate performance and allocation is the efficient calibration of the distribution of returns. The classical method is to compute market or sub-market returns and volatilities, and to then calculate the standard performance measure, namely the Sharpe ratio. This measure is only based on the first two moments of the return distribution. Therefore, a significant weakness of this method is that it implicitly assumes that this distribution is Gaussian (if not, the approach may lead to a bad fit for the distribution). In fact, risk comes not only from volatility but from higher moments of the distribution, such as skewness and kurtosis. In order to resolve this issue, we focus on another risk-adjusted performance measure, one that takes the Value-at-Risk (VaR) as the risk measure, as was adopted by the Basel II regulation directive. This criterion is based on specific quantiles of the distribution of returns. When the VaR is computed from the Cornish Fisher expansion, the corresponding risk-adjusted performance measure is called the modified Sharpe ratio. Usually, its computation is based on the first four moments of the return's distribution. However, this methodology can exhibit several pitfalls, and thus, this paper shows how to make proper use of this tool. The usefulness of the proposed methodology is illustrated through an empirical application to optimal portfolio allocation in commercial real estate, using the IPD database. We find that markets that appear more desirable using simple Sharpe ratios bear, in reality, higher risk when

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the distribution of returns is taken into account in a more appropriate manner. Institutional investors may find that the technique proposed here is useful in that it allows them to consider non-normality in real estate performance analysis.

Keywords: Real estate portfolio; performance measures; Cornish Fisher expansion; modified Sharpe ratio

JEL classification: C61, G11, R39

1 Introduction

Most practitioners' analyses of real estate markets and funds focus on total return figures only. As such, they ignore risk, and thus, risk-adjusted performance, measures. The rationale behind such a treatment is a concern only with absolute returns. In particular, such an approach does not take into account the link between return and risk, contrary to Markowitz (1952). Nevertheless, comparison of absolute returns is widely used by marketers in the real estate field, to show that their management and allocations are superior to those of the competition.

Comparing investments is straightforward in very special cases: given equal risk, higher expected return is always better, and reciprocally, given equal expected return, lower risk is always preferable. Matters become complicated when two or more markets with different expected returns and risks are considered. Needless to say, when funds or indices exhibit different risk characteristics, naïve comparisons of this nature become extremely misleading. Investors who rely solely on returns to choose their allocation may not be prepared for the difficulties that lie ahead. Investing is, by nature, a two-dimensional process based not only on returns, but also on the risk taken to achieve those returns. In particular, given the fact that higher return is always desirable, but higher risk never is, the next question is how much additional return is a sufficient compensation for additional risk. This is precisely where risk-adjusted performance measures are helpful.

Measuring performance in real estate is difficult. Typically, funds or markets are compared without mention of risk. This is mainly because of two reasons: first, the various return measures that exist in real estate (single-period returns, multi-period returns, income returns, capital returns, total returns, real versus nominal returns, etc.), and second, the absence of relevant risk metrics in real estate. In addition, the origin of performance (either capital return or income return) is more difficult to ascertain in real estate than in other sectors, given the numerous players that participate in the management process, namely, fund managers, asset managers, brokers, property managers, facility managers, etc. For a long time, the lack of data and information about real estate markets made the estimation of risk difficult. However, in the era of big data, with the growth and improvement of large datasets, risk estimation is becoming easier.

Condensing return and risk into a single useful risk-adjusted number is one of the key tasks of performance measurement. Basically, good risk measurement must make it possible to compare the performance of markets with similar risk characteristics, as well as the performance of other funds having different risk characteristics. Even if the number of performance measures is large, the Sharpe ratio is, in practice, the most commonly used measure of risk-adjusted performance. Defined by the Nobel laureate Sharpe (1966), the Sharpe ratio measures the "excess return per unit of volatility." It is calculated by dividing the excess return of a market or a fund by its volatility. Algebraically, it is given by

$$\mathrm{SR} = \frac{r_P - r_f}{\sigma_P}, \tag{1}$$

where r_P is the average return of portfolio P, r_f is the risk-free rate, and σ_P is the standard deviation of returns of portfolio P.[1] By analyzing this risk-adjusted performance ratio, we can identify which markets or categories outperformed the others. An important weakness of this method, however, is that it presupposes normal distributions of returns, as it only considers the first two moments of the distribution of returns.

The Sharpe ratio is useful when assets are normally distributed, since the distribution of returns is then completely described by its mean and volatility (the inputs to equation 1).[2] When the distribution of returns cannot be considered as normal, it becomes necessary to rely on performance measures that take non-normality into account. To solve the non-normality issue, Favre & Galeano (2002) and Gregoriou & Gueyie (2003) introduced a modification of the traditional Sharpe ratio, the modified Sharpe ratio (mSR).[3] It is defined as the ratio between the excess return of a market, an asset, or a fund, and its Value-at-Risk or VaR (the definition of VaR appears in appendix A), where VaR is computed using the Cornish Fisher expansion, denoted as $mVaR_\alpha$ in the literature, with α being the probability level:

$$mSR_\alpha = \frac{r_P - r_f}{mVaR_\alpha}. \tag{2}$$

r_P is the average return of the portfolio, r_f the risk-free rate, and $mVaR$ is our way of computing VaR that neither relies on strong assumptions (such as the need for a normal distribution) nor requires excessive data (as do historical methods). It is based on use of the Cornish Fisher expansion. This approach makes it possible to approximate the true (unknown) distribution of returns. It takes the form of the Gaussian quantile estimation plus some correction terms taking account of the skewness and kurtosis of the return distribution (the Cornish Fisher expansion and procedure are presented in appendix C). $mVaR$ is popular among practitioners as well as academics because of its precision and the explicit form it takes (it is straightforward to compute and interpret). Thus, the Cornish Fisher expansion is a relatively easy and parsimonious way of dealing with non-normality in asset prices or returns. Using mSR allows us to recognize that risk comes not only from volatility but also from higher moments like skewness and kurtosis (an overview of skewness and kurtosis parameters is provided in appendix B). In particular, this article shows how this measure is preferable to the more traditional one.

Even though $mVaR$ is popular and has proven to be a useful technique, its use is restrained by its domain of definition (see Chernozhukov et al., 2010). One must also be careful that the parameters of the formula are not confounded with those of the underlying distribution (see Maillard, 2012). Typically, the literature (see

[1] As numbers are typically expressed on an annual basis, the Sharpe ratio itself is also expressed on an annual basis (especially because increases in standard deviation are not linear).

[2] However, recent evidence shows that the Sharpe ratio can result in almost identical fund rankings compared to alternative performance measures (see Eling & Schuhmacher, 2007).

[3] This performance metric is part of the $RhoVaR$ performance metric class

$$\rho VaR_\alpha = \frac{r_P - r_f}{VaR_\alpha}$$

,defined as the ratio between the excess return of a market, an asset, or a fund, and its Value-at-Risk.

for instance Lee & Higgins, 2009; Christoffersen, 2012; Hull, 2012) does not specify the domain of definition of the Cornish Fisher expansion, and thus, this is a possible pitfall that remains to be addressed. As a result, the use of the Cornish Fisher expansion is generally restricted to cases where the distribution is close to a normal one. Solutions overcoming the two obstacles just mentioned have been proposed by Chernozhukov et al. (2010) and Maillard (2012). Following their methodology, it is possible to properly compute $mVaRs$ irrespective of the distributions.

The remainder of the paper is organized as follows. Following a literature review in section 2, the modified Sharpe ratio and its subsequent Cornish Fisher expansion is presented in section 3, with an emphasis on the proper use of this tool. Section 4 implements the proposed methodology and discusses the empirical results.

2 Literature review

The research on performance measures has largely focused on the problem of knowing whether or not managers are able to beat the market (Markowitz, 1952; Sharpe, 1966; Jensen, 1968; Malkiel & Fama, 1970).

The extensive literature on efficient portfolios includes studies that incorporate real estate as an asset. Goetzmann & Ibbotson (1990) used estimations of real estate price appreciation and emphasized the commonly held perception that investing in property is less risky than investing in stock markets. Along the same lines, Brueggeman et al. (1984) showed how the inclusion of real estate in a portfolio can substantially reduce portfolio risk. They also observed the hedging properties of commercial real estate against expected inflation. The conclusions of the literature depend on the way real estate returns are estimated (often based on appraisal) and on the manner in which real estate is compared to other investments. In this domain, the studies by Clayton et al. (2008) or Geltner et al. (2013) are appealing and describe how the peculiarities of real estate, concerning liquidity, transaction costs, and investment size, make the estimation of returns and volatility of this asset class difficult.

Few studies have concentrated on performance measures in the context of real estate. Most of the research in this area has focused on fund managers trading public real estate securities and on the performance of managers in publicly listed real estate companies (Brounen et al., 2007; Chiang et al., 2008). The literature on the performance of institutional real estate firms investing directly in property assets is sparse, and up till now, data limitations have restricted the scope of the research questions and published empirical evidence. A few studies, nevertheless, have examined the question. The research conducted by Bond & Mitchell (2010) in this field is particularly interesting. Using a dataset from IPD database, they analyzed the performance of managers in direct real estate investments in the UK and found no evidence of excess returns. Chou & Hardin III (2014) questioned whether capital flows into private real estate funds predict subsequent returns. They studied this question at the individual real estate investment trust (REIT) and mutual fund levels. They showed how returns are negatively associated with increased fund flows and fund size. Downs et al. (2016) treated the same issue in the context of direct real estate investments and direct property investment vehicles only. They found that flows are not a good predictor of future returns, and that these flows chase past returns instead. In order to measure the active management of a fund, Higgins (2010) used tracking error to analyze Australian unlisted wholesale property funds and identified many investment styles driven principally by debt levels and sector specificities.

Studies on real estate have often used the Sharpe ratio as it is considered as one of the industry standards (even if the volatility estimation is sometimes questionable). For instance, Lee & Stevenson (2005) highlighted the usefulness of risk-adjusted performance measure analysis in selecting the best available real estate investment opportunities. In their paper, they investigated the effects of the selected time horizon in the context of portfolio optimization. One of their mean–variance efficient portfolios is computed on the basis of the Sharpe Ratio (*ex post*). Fugazza et al. (2009) studied portfolio allocation in the context of a multi-period setting. Using the Sharpe ratio (and the certainty equivalent), they showed that diversifying into REITs increases wealth for all investment horizons. Except for the article by Lee & Higgins (2009), none of the literature concentrates specifically on the modified Sharpe ratio. Our work is closely related to the former study. They used the modified Sharpe ratio in a real estate context for the Australian market. The authors argued that the Sharpe performance formula neglects two important characteristics of real estate returns: non-normality and autocorrelation. They calculated numerous Sharpe ratios in order to examine the joint effects of autocorrelation and non-normality on the risk-adjusted performance for a certain real estate asset class. They found that the direct property in Australia is characterized by exceptional performance even when the effects of non-normality and autocorrelation are taken into account.

The non-normality of real estate return distributions is another perplexing issue. This point was studied a considerable time ago by Myer & Webb (1994), Young & Graff (1995), and Byrne & Lee (1997). Recent studies such as those by Lizieri & Ward (2000), Young et al. (2006), and Young (2008) show that real estate returns usually exhibit non-normal returns. Real estate returns typically lean to the left (showing negative skewness) and exhibit fat tails (denoting leptokurtosis). These works focused mainly on Anglo–Saxon economies, but similarities in real estate return distributions are also found elsewhere. In the context of VaR computation, the return distribution has a strong impact. VaR is the estimation of an extreme quantile, and as such, it requires reasonable estimation not only near the center of the distribution but also in the tail, particularly if returns exhibit non-zero skewness and excess kurtosis.[4] The distribution used to estimate the VaR of a portfolio needs to be determined from such return distributions or corresponding sector indices. Nonetheless, an inappropriate normality assumption is regularly adopted in order to determine VaR, mostly because it allows quick and easy computation (for an example in real estate in the context of Solvency II, see Amédée-Manesme et

[4] Value-at-risk, however, does not assess the kurtosis of the loss distribution. With regard to VaR, a high kurtosis indicates fat tails of the loss distribution, where losses greater than the maximum expected loss may occur.

al., 2015).

Methods to compute VaR or to determine distribution quantiles have already been the subject of considerable research following the introduction of VaR into current banking practice (for a comprehensive review of methods, see Christoffersen, 2012). A considerable volume of research has concentrated on the best methods to compute VaR. Pichler & Selitsch (1999) compared five VaR methods in the context of portfolios and options, namely, the Johnson transformations, Variance–Covariance analysis, and the three Cornish Fisher expansions of the second, fourth, and sixth orders. They concluded that a sixth-order Cornish Fisher expansion is the best among the analyzed approaches. Jaschke (2001) concentrated on the properties of the Cornish Fisher expansion and its underlying assumptions in the context of VaR, with particular focus on non-monotonicity of the distribution function, in which case convergence is not guaranteed. Jaschke discussed how the conditions for its applicability make the Cornish Fisher approach difficult to use in practice (points we treat in this paper). However, he demonstrated that when a dataset obeys the required conditions, the accuracy of the Cornish Fisher expansion is generally more than sufficient for one's needs, in addition to being faster to implement than the other approaches.

VaR has been the subject of numerous papers on real estate, even if they primarily focus on listed real estate and not direct real estate. VaR estimations for securitized real estate rely on the same methods as those used for ordinary stocks and bonds. Among others, the articles by Liow (2008), Cotter & Roll (2010), or Zhou & Anderson (2012) should be consulted. Literature focusing on VaR in the context of direct real estate investment (or funds) is sparse. Nonetheless, some studies do concentrate on risk management and assessment in real estate. Booth et al. (2002) examined risk measurement and management of real estate portfolios, suggesting that practical issues force real estate investors to treat real estate differently from other asset classes. The report focused on the difference between symmetric measures, such as standard deviation, and downside risk measures, such as VaR. Their work concentrated on all risk measures used in real estate, thus constituting a survey of then-current real estate risk measures. Gordon & Tse (2003) considered VaR as a tool to measure leveraged risk in the case of a real estate portfolio, in comparison to use of the Sharpe ratio. Debt in a real estate portfolio is a traditional issue much studied in real estate finance. Their paper demonstrated that VaR allows better assessment of such risk. In particular, traditional risk-adjusted measures (e.g., the Sharpe or Treynor ratio, as well as Jensen's alpha) suffer from a leverage paradox. Leverage adds risk along with potential for higher returns per unit of higher risk. Therefore, the risk/return ratio does not change noticeably, and thus, does not constitute an accurate tool by which to measure the risk inherent in debt. Contrarily, VaR is quite a good tool for studying leveraged risk. Brown & Young (2011) focused on spectral measures to assess real estate investment risk. VaR was not their selected measure; instead, an Expected Shortfall (in a way, expectation beyond VaR) was adopted. More recently, Amédée-Manesme et al. (2015) used the Cornish Fisher expansion and a so-called rearrangement procedure to calculate direct real estate VaR. They calculated a rolling VaR over time for real estate returns using the UK IPD database and showed how the Cornish Fisher expansion makes it possible to adequately account for non-normality of returns in real estate.

3 The proper calculation of the modified Sharpe ratio

In this paper, we propose the use of the modified Sharpe ratio based on $mVaR$ risk measurement as the appropriate risk-adjusted performance measure. $mVaR$ owes its popularity in practice to its precision and explicit form. This makes it straightforward to compute and interpret. However, use of $mVaR$ should be done with caution with regard to two points: (i) the domain of validity of the Cornish Fisher expansion and (ii) the confusion between skewness and kurtosis parameters as given by the formula and the corresponding parameters of the underlying distribution.

The $mVaR$ calculation is based on the Cornish Fisher expansion. In short, the Cornish Fisher expansion transforms naïve Gaussian quantiles according to the skewness and kurtosis coefficients selected to characterize the true distribution. This expansion is a simple polynomial function based on the Taylor series of the corresponding unit normal quantile (for more precision, see Stuart & Ord, 2009), where the coefficients of each resulting term are functions of the moments of the true distribution under consideration. For instance, denoting the Gaussian and the Cornish Fisher quantiles by z_α and $z_{CF,\alpha}$ respectively, we obtain the following expression for the normalized Cornish Fisher quantile at the probability level α:

$$z_{CF,\alpha} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)(K-3) - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2, \forall \alpha \in (0,1), \quad (3)$$

where S and K denote the skewness and kurtosis coefficients of the true distribution (see the definition of S and K in Appendix B). The corresponding modified Cornish Fisher quantile is then just:

$$q_{CF,\alpha} = \mu + z_{CF,\alpha}\sigma, \forall \alpha \in (0,1). \quad (4)$$

It is straightforward to note that in the presence of an underlying Gaussian distribution ($S = 0$ and $K = 3$), equation (3) reduces to the Gaussian quantile (and thus, the Cornish Fisher expansion can obviously be used when the distribution is normal).

Although the Cornish Fisher expansion has proven to be a useful technique, since it is usually truncated at the third order (see appendix C), its use presents two major pitfalls: (**i**) the resulting approximations of the distribution and quantile functions can be non-monotone, and (**ii**) the skewness and the kurtosis of the Cornish Fisher expansion are generally not those of the true distribution. Resolving these two issues requires us to combine the works of Chernozhukov et al. (2010) and Maillard (2012) and to use a so-called rearrangement procedure (**i**) with a correction of the parameters (**ii**). This leads to the correct use of the Cornish Fisher expansion.

(i) In fact, the resulting approximations of the distribution and quantile functions can be non-monotone. There are constraints on the permitted values of the true distributions' moments so that the Cornish Fisher expansion itself yields a well-defined distribution (for more details, see equation 10 in appendix C). This is due to the third-order truncation of the Cornish Fisher expansion and the fact that the polynomials involved in the expansion need not be monotone. The non-monotonic behavior can lead to incorrect results, as illustrated by Amédée-Manesme et al. (2015) in Figure 2. Indeed, in such a case, the quantile at a higher threshold can be smaller in absolute terms than the one at a smaller threshold ($| q_{\alpha_1} | < | q_{\alpha_2} | \; \forall \alpha_1 > \alpha_2$), which is obviously unpalatable for any cumulative distribution function and even less desirable when it is used for risk measurement. A solution to this issue has been proposed by Chernozhukov et al. (2010), who suggested using a rearrangement procedure restoring the monotonicity of the approximation. The rearrangement procedure is a sorting operation: the previously obtained values are simply sorted in increasing order. Furthermore, according to Chernozhukov et al. (2009), in addition to restoring monotonicity, the rearrangement improves the estimation properties of the approximation. The resulting improvement is due to the fact that the rearrangement necessarily brings the non-monotone approximations closer to the true monotone target function.

(ii) Another difficulty associated with the use of the Cornish Fisher expansion truncated at the third order is the confusion with regard to the skewness and kurtosis parameters of that formula (denoted by S_c and K_c, respectively, in the following) and those of the underlying true distribution (S and K, respectively). This can lead to considerable mis-estimation of quantiles. Though this point has already been raised by Maillard (2012), it does not seem to have received sufficient attention elsewhere in the literature. The author presents a solution for avoiding this problem by computing the correct moments of the distribution resulting from the Cornish Fisher expansion. This leads to the following true skewness (S, equation 5) and true kurtosis (K, equation 6) parameters (the technical details are available in the study by Maillard, 2012):

$$S = \frac{S_c - \frac{76}{216}S_c^3 + \frac{85}{1296}S_c^5 + \frac{1}{4}K_cS_c + \frac{13}{144}K_cS_c^3 + \frac{1}{32}K_c^2S_c}{\left(1 + \frac{1}{96}K_c^2 + \frac{25}{1296}S_c^4 - \frac{1}{36}K_cS_c^2\right)^{1.5}} S_c^3. \tag{5}$$

$$K = \frac{\begin{bmatrix} 3 + K_c + \frac{7}{16}K_c^2 + \frac{2}{32}K_c^3 + \frac{31}{3072}K_c^4 - \frac{7}{216}S_c^4 - \frac{25}{486}S_c^6 + \frac{21665}{559872}S_c^8 \\ -\frac{7}{12}K_cS_c^2 + \frac{113}{452}K_cS_c^4 - \frac{5155}{452}K_cS_c^4 - \frac{7}{24}K_c^2S_c^2 + \frac{2455}{20736}K_c^2S_c^4 - \frac{55}{1152}K_c^3S_c^2 \end{bmatrix}}{\left(1 + \frac{1}{96}K_c^2 + \frac{25}{1296}S_c^4 - \frac{1}{36}K_cS_c^2\right)^{1.5}} - 3. \tag{6}$$

As demonstrated by Maillard (2012), proper use of the Cornish Fisher expansion requires one to invert these relations. This way, the correct skewness and kurtosis can be entered into the expansion (the particular correction is required owing to the fact that the Cornish Fisher expansion is an approximation of order 3). This can easily be done numerically.

Combining the rearrangement procedure (i) and the correction of the two parameters (ii) leads to the proper use of the Cornish Fisher expansion.

4 Application

4.1 Data presentation and treatment

We study the Total Return Annual Indices published by the MSCI real estate (formerly IPD) Office for 10 countries (Australia, Canada, France, Germany, Republic of Ireland, Netherlands, New Zealand, Norway, UK, and USA) from 2000 to 2014 (14 returns). The selected data cover office investment only. These IPD indices constitute a valuation-based index and are, "like all indices," subject to criticism, mainly in regard to their smoothness and reliability. The proposed approach remains applicable, though, to any kind of index or to individual property returns, assuming the first four moments can be estimated. Our objective in this part is merely to apply our methodology to a commonly accepted and well-understood index. In this sense, the IPD All Property Total Return Indices present three advantages: reliability, acceptance by practitioners, and substantial representation of their components in institutional investors' portfolios.[5]

Table 1 presents some basic statistics for each market within the original (smoothed) dataset. On average, both the returns and the volatilities are about 8%. More specifically, Canada exhibits the highest return. Germany presents the lowest return but also the lowest volatility, while Ireland seems to be the most volatile market (nine times as volatile as Germany). Other than France, the Netherlands, and New Zealand, all the markets display negative skewness. The kurtosis results are less conclusive, with most of the markets having skewness close to 3, except the Netherlands, which is far below 3, and Norway, the UK, and the USA, which are all much above 3. The cases of the USA and the UK are particularly interesting: both markets are strongly negatively skewed but exhibit high kurtosis, which translates into the fact that the returns are negatively biased but relatively peaked.

[5] These specific indices have many other limitations that are not dealt with here. Such issues have been reported by Fisher et al. (1994).

	μ	σ	S	K
Australia	9.80	7.11	-0.18	3.96
Canada	11.21	6.47	-0.37	3.11
France	9.17	7.11	0.13	2.75
Germany	2.81	1.95	-0.33	2.35
Ireland	5.62	17.42	-0.61	3.44
Netherlands	5.51	5.64	0.10	1.94
New Zealand	10.43	7.49	0.10	2.83
Norway	8.44	6.56	-0.64	4.76
UK	7.83	11.37	-1.33	5.98
USA	8.12	10.86	-1.62	5.61
Average	7.89	8.20	-0.48	3.67

Table 1: Descriptive statistics of original (smoothed) data

Issues pertaining to liquidity, transaction costs, and investment size make real estate transactions rare (see Clayton et al. (2008) or Geltner et al. (2013)). Given these real estate specificities, appraisal-based data are mainly used to present direct real estate performance. This implies that reported returns are smoother than “true” returns. As stressed by Lee & Higgins (2009), among others, the smoothed nature of real estate indices tends to decrease the volatility of perceived returns and thus bias traditional risk-adjusted performance measures such as the Sharpe ratio. In addition, Geltner (1993) emphasized the relationship between smoothed returns, serial correlation, and illiquidity, and consequently, asserted that autocorrelation must be removed from return series before they can be used in any analysis (see also Lo, 2002, , who stated that investors should adjust return series before comparing Sharpe ratios). As noted by Lee et al. (2000), appraisal-based data usually exhibit autocorrelation, which may lead to underestimation of the true risk of direct property, and therefore, to overestimation of the associated Sharpe performance. It is accepted that the expected return of the unsmoothed series is equal to the expected value of the observed returns. However, the second-order moment of the distribution (i.e., the variance) is affected by the smoothness issue.[6] Several methods have been proposed to unsmooth smoothed returns. Following Lee & Higgins (2009), we adopt the method proposed by Fisher et al. (1994). This method is based on the assumption that the price of an asset is estimated using the price reported in the previous period (or equivalently that the appraiser updates his previous appraisal in each year or quarter based on new information). More precisely, the observed return at period t (r_t^*) is a weighted average of the “true” unobserved return at time t (r_t) and the observed return at time t-1 (r_{t-1}^*). Considering a first-order autoregressive process, we get

$$r_t^* = \alpha r_{t-1}^* + (1 - \alpha) r_t, \tag{7}$$

where α is the parameter of the first-order autoregressive obtained from the return series. The variable α is set to the slope coefficient from the regression of r_t^* to r_{t-1}^*.

[6] Smoothing (or unsmoothing) also has an impact on the third and fourth moments of the returns distribution even if, to the best of our knowledge, the literature has been mute on this point.

	Original data	Unsmoothed data
Australia	0.78	0.19
Canada	0.84	0.37
France	0.70	0.20
Germany	0.83	0.10
Ireland	0.46	0.35
Netherlands	0.74	0.16
New Zealand	0.91	0.42
Norway	0.66	-0.08
UK	0.52	0.25
USA	0.51	0.15
Average	**0.72**	**0.22**

Table 2: First-order autocorrelation data for original and unsmoothed data

Table 2 shows how the unsmoothing process substantially reduces the impact of autocorrelation. It should be noted that the autocorrelation has decreased, on average, by more than three times (from 0.72 to 0.22), with all markets now having an autocorrelation below 0.4 (except New Zealand at 0.42).

	μ	σ	S	K
Australia	9.25	33.56	-1.47	6.77
Canada	9.24	42.13	-0.20	1.98
France	5.89	22.67	-0.82	5.85
Germany	1.87	8.65	1.04	3.48
Ireland	2.79	26.66	-1.31	5.88
Netherlands	0.86	13.75	-0.57	4.07
New Zealand	16.14	69.28	-1.68	6.01
Norway	7.20	21.86	-2.42	10.05
UK	7.13	23.44	-1.07	3.80
USA	7.43	22.64	-0.60	3.88
Average	**6.78**	**28.46**	**-0.91**	**5.18**

Table 3: Descriptive statistics of unsmoothed data

Table 3 presents some descriptive statistics for the unsmoothed data. As expected, the returns are about the same as the ones of the original dataset: $\sim 7\%$ versus 8% in the original dataset. However, the effect on volatility is huge, with the average volatility increasing threefold ($\sim 28\%$ versus 8% in the original dataset), and with each country exhibiting an increase, the greatest being the ninefold increase for New Zealand to a value of 70%. In addition, on average, the resulting distributions are more negatively skewed and exhibit higher kurtosis. Interestingly, and contrary to the other countries, the unsmoothing process for the UK and the USA brings their coefficients closer to those of the normal distribution (both skewness and kurtosis become closer to 0 and 3, respectively).[7]

[7] The effect of unsmoothing on skewness and kurtosis is not very well (or not at all) documented in the literature (see also footnote 6).

Our approach remains relevant even in the presence of a normal distribution (see appendix D). Indeed, in this case, the skewness and kurtosis, respectively, are 0 and 3, and thus, equation 3 adds up to the Gaussian quantile (z_α). For this reason, determining the distribution of the returns is not key to the process, and the previous results, namely deciding whether a distribution is normal or not, are not necessary to proceed.

As was already mentioned, the skewness (S_c) and kurtosis (K_c) coefficients of the Cornish Fisher expansion must be estimated by solving equations 5 and 6, respectively. Table 4 presents the initial and corrected coefficients.

	Empirical values		Corrected values	
	$\boldsymbol{S}$	$\boldsymbol{K}$	$\boldsymbol{S_c}$	$\boldsymbol{K_c}$
Australia	-1.47	6.77	-0.964	6.34
Canada	-0.204	1.98	-0.154	4.26
France	-0.824	5.85	-0.516	5.65
Germany	1.04	3.48	0.778	5.15
Ireland	-1.31	5.88	-0.871	5.99
Netherlands	-0.565	4.07	-0.378	5.08
New Zealand	-1.68	6.01	-1.22	6.56
Norway	-2.42	10.0	-1.72	8.84
UK	-1.07	3.80	-0.787	5.27
USA	-0.602	3.88	-0.408	5.04
Average	**-0.91**	**5.18**	**-0.62**	**5.82**

Table 4: Skewness and kurtosis coefficients: empirical and corrected values (Maillard, 2012)

4.2 Performance measures

The use of the Sharpe ratio and the modified Sharpe ratio requires that one determines a risk-free rate. The choice of the risk-free rate is an interesting question for academicians as well as practitioners: it is often presumed that the risk-free rate is given and/or easy to obtain. From a theoretical perspective, the risk-free rate is the rate of return of an investment with zero risk over a specified period of time. However, in reality, a risk-free rate does not exist, since all investments carry some amount of risk.

In practice, both academics and practitioners usually use government security rates as risk-free rates. In the context of this study, we consider various countries over 14 years. We thus implicitly consider the case of a foreign investor who can invest internationally and who has the choice of a risk-free asset (national or local investors may be more limited regarding their choice of a risk-free asset). We thus choose a risk-free rate of 3% over the period, as an international investor was indeed able to locate assets with very limited risk for an annual return of about 3%.

Considering the use of the modified Sharpe ratio raises another question, namely, which confidence level to use (the choice is a bit arbitrary). In particular, the choice of the confidence level may have a strong impact on the determination of VaR, especially if the distribution exhibits high kurtosis. Models of risk based on VaR start with the presumption that the confidence level is chosen. However, in practice, it is very difficult to justify a particular confidence level. Most textbooks illustrate VaR using a 5% confidence level because a comparison with two standard deviations becomes easier. In this study, we use a confidence level of 1%, the value requested by the Basel Committee's quantitative criteria. Discussing the choice of the confidence level is not the primary subject of this paper, but it can, nevertheless, be noted that the VaR computation under the normal assumption gives the same ranking irrespective of the threshold level. This is not the case when the normal assumption is removed.

Our objective is to compare the use of the following three performance measures: the return, the Sharpe ratio, and the modified Sharpe ratio. More precisely, we concentrate on the corrections (rearrangement and true moments) that have to be undertaken to properly calculate $mVaR$. For comparison purposes, we also add the modified Sharpe ratio computed under a normality assumption (hence, only with μ and σ), denoted as NmSR. To do so, we compute six performance measures: the returns (Returns), Sharpe ratio (SR), normal modified Sharpe ratio ($\mathrm{NmSR}_{1\%}$), standard modified Sharpe ratio ($\mathrm{mSR}_{1\%}$), rearranged modified Sharpe ratio ($\mathrm{mSR}_{1\%}^{\mathrm{rearranged}}$), and corrected rearranged modified Sharpe ratio ($\mathrm{CmSR}_{1\%}^{\mathrm{rearranged}}$). Table 5 presents the results for the six performance measures.

First, it must be noted that three of the Sharpe ratios are negative regardless of the risk metric used. Theoretically, the Sharpe ratio can take on any value; in particular, negative Sharpe ratios are possible. A Sharpe ratio is negative when excess return is negative. Excess return is the return on the asset, portfolio, or market, less the risk-free rate, and therefore, excess return is negative when the return on the market is lower than the risk-free rate. Negative Sharpe ratios do not provide useful information because the risk-free asset then outperforms the investment on a risk-adjusted basis.

Second, the changes among the three mSR computations are small. Only Germany shows a considerable difference while going from $\mathrm{mSR}_{1\%}$ to $\mathrm{mSR}_{1\%}^{\mathrm{rearranged}}$. The case of Germany is confirmed by Figure 1. As seen in Figure 1, differentiating between the Cornish Fisher (magenta) curve and the rearranged Cornish Fisher (red) curve is not always easy.When the two curves cannot be differentiated, the rearrangement procedure is useless, as the ordering of quantiles is not affected by the Cornish Fisher procedure (in such cases, the order of quantiles is already increasing). However, differentiating the two curves (in particular, in the lower tails) shows the importance of rearranging (or sorting) the results of the Cornish Fisher procedure. For Germany, the effect of rearrangement is obvious, since without rearrangement we get $VaR_{5\%} = -10.2250$ and $VaR_{1\%} = -10.2113$ ($\mid VaR_{5\%} \mid > \mid VaR_{1\%} \mid$!) versus $VaR_{5\%}^{rearranged} = -8.2977$ and $VaR_{1\%} = -9.4729$ with rearrangement. Even if only this one country is obviously affected by the Cornish Fisher ordering issue for the quantiles chosen under the current scenario, the effect of the rearrangement will be consequential at other alpha levels. For instance, it should be noted that the rearrangement applies to the highest quantile for the Republic of Ireland, Norway, and the UK (even though these quantiles are not important for VaR computation, which concentrates on losses). All this demonstrates the importance of the rearrangement procedure when need for it arises.

Third, the effect of the corrected parameters on the modified Sharpe ratio is not regular. Mostly, it decreases the ratio (in absolute terms), except for Australia and France– both showing evidence of non-normality (see appendix D)–where the ratios increase. Because the parameter correction affects the denominator only, the VaR values of Australia and France decrease because of the parameters' adjustments. This is not surprising when looking at equations 5 and 6, where S and K are complex mixtures of S_c and K_c, respectively. In addition, the effect on the ratio is not proportional, the change being about 30% for Germany (from -0.1164 to -0.0723) and near 0% for Australia.

	Returns	SR	$\mathbf{NmSR}_{1\%}$	$\mathbf{mSR}_{1\%}$	$\mathbf{mSR}^{\mathbf{rearranged}}_{1\%}$	$\mathbf{CmSR}^{\mathbf{rearranged}}_{1\%}$
Australia	9.2516	0.1863	0.0908	0.0582	0.0582	0.0584
Canada	9.2443	0.1482	0.0703	0.0740	0.0740	0.0591
France	5.8941	0.1277	0.0618	0.0414	0.0414	0.0430
Germany	1.8657	-0.1311	-0.0621	-0.1247	-0.1164	-0.0723
Ireland	2.7910	-0.0078	-0.0035	-0.0024	-0.0024	-0.0024
Netherlands	0.8610	-0.1556	-0.0687	-0.0554	-0.0554	-0.0523
New Zealand	16.1395	0.1897	0.0906	0.0638	0.0638	0.0581
Norway	7.2042	0.1923	0.0963	0.0597	0.0597	0.0547
UK	7.1283	0.1761	0.0871	0.0686	0.0686	0.0608
USA	7.4258	0.1955	0.0978	0.0779	0.0779	0.0721

Table 5: Results of the performance measures

The first observation with regard to the rankings in Table 6 is that the three methods give somewhat different results. This result contradicts the finding of Lee & Higgins (2009), who noted similar rankings in most instances for valuation-based property data in Australia, using a similar approach. The difference in rankings may be questioned, as most returns did not seem to exhibit non-normal behavior (see Table 7 in appendix D, which shows that the results are generally not significant). Nonetheless, the levels of skewness and kurtosis are far from those of the normal law, which justifies considering the higher moments.

The second observation relates to the effect of non-normality on the rankings. The ranking computed with the normal VaR assumption is very close to the Sharpe ratio of 1. On the contrary, the effect of higher moments clearly changes the ranking. Once more, this demonstrates the importance of considering non-normality when using performance measures.

The third observation is that, in most instances, the rankings for the three mSR ratios are close, but they differ somewhat for the corrected mSR. This is partly due to the rearrangement procedure, which is mostly superfluous for the threshold considered in the context of this study (except for Germany). However, the rankings are a bit different when the parameters are corrected. This is the case for Australia, Canada, New Zealand, Norway, and the UK. Thus, this result underscores the importance of the correction when normality cannot be guaranteed.

	Returns	**SR**	$\mathbf{NmSR}_{1\%}$	$\mathbf{mSR}_{1\%}$	$\mathbf{mSR}_{1\%}^{\mathbf{rearranged}}$	$\mathbf{CmSR}_{1\%}^{\mathbf{rearranged}}$
Australia	2	4	3	6	6	4
Canada	3	6	6	2	2	3
France	7	7	7	7	7	7
Germany	9	9	9	10	10	10
Ireland	8	8	8	8	8	8
Netherlands	10	10	10	9	9	9
New Zealand	1	3	4	4	4	5
Norway	5	2	2	5	5	6
UK	6	5	5	3	3	2
USA	4	1	1	1	1	1

Table 6: Ranks of the markets according to performance measures

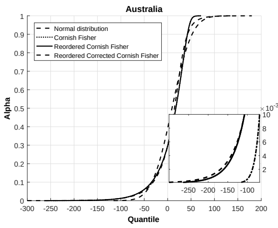
Figure 2 presents the rankings for all the countries and for all the methodologies according confidence level α. Predictably, the return and Sharpe ratio are not impacted by the confidence level as they do not depend on it. The results for the four other metrics are mixed. For instance, Australia's ranking for the modified Sharpe ratio is equal to 6 at 1% and 3 at 3% whereas France's ranking does not change with the confidence level. Nevertheless, in most cases, the changes in the rankings are relatively small. This underlines the robustness of our approach.

5 Conclusion

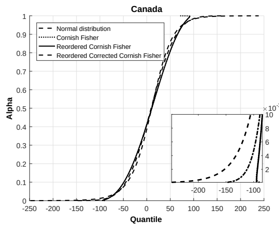
The traditional Sharpe ratio approach presents some limitations that make it tricky to use despite its popularity among practitioners. Possible non-normality of returns is ignored in the traditional Sharpe ratio, and this can cause investors to invest inappropriately in risky assets. The modified Sharpe ratio makes it possible to overcome these limitations. In particular, it relies on $mVaR$, a risk metric that considers the entire distribution of the returns since its computation is based on third and fourth moments of the distribution.

The modified Sharpe ratio is based on the Cornish Fisher expansion. This expansion is a useful technique but must be used with caution: the resulting approximations of the distribution and quantile functions can be non-monotone, and the skewness and kurtosis parameters of the formula must not be confused with those of the distribution. These two pitfalls are fully accounted for in this paper, and thus, the modified Sharpe ratio is properly computed.

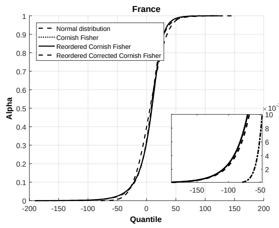
The proposed methodology should be appealing to both practitioners and academics. Indeed, it is relatively easy to compute and facilitates quantitative risk management in real estate transactions. The modified Sharpe ratio is a powerful tool when managing and dealing with property portfolios or in the case of large



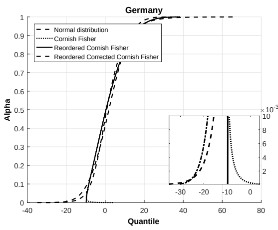
(a) Australia



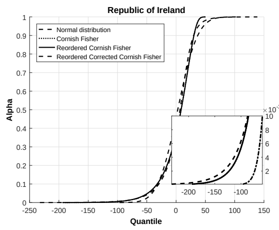
(b) Canada



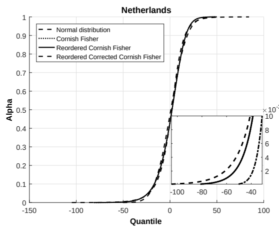
(c) France



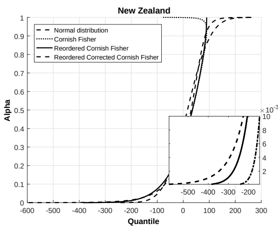
(d) Germany



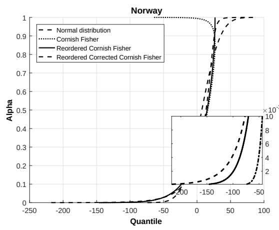
(e) Republic of Ireland



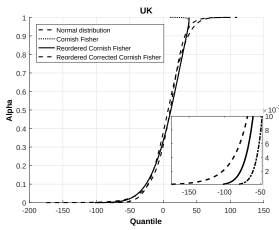
(f) Netherlands



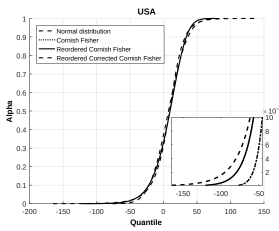
(g) New Zealand



(h) Norway



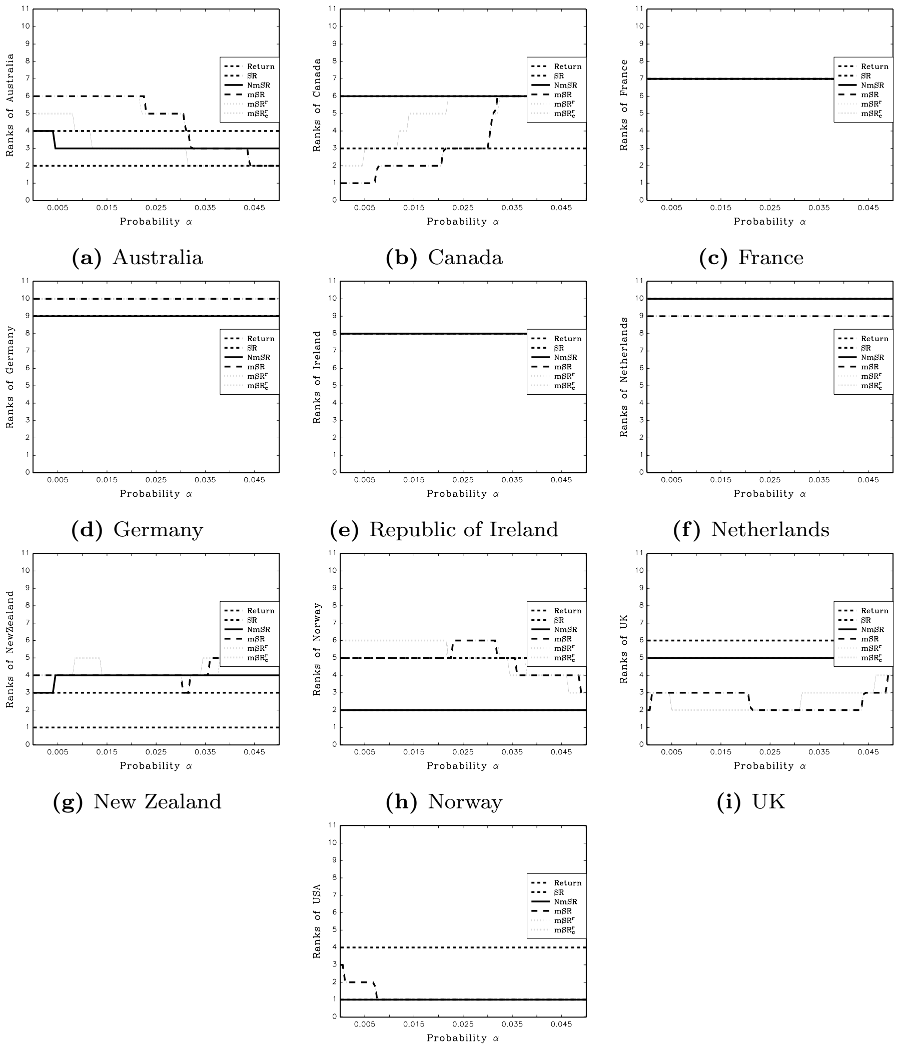
(i) UK



(j) USA

Fig. 1: Quantile computations with four methods for all the countries

international investors. The ratio is useful because real estate investment is volatile by nature and may not always be driven by a normal distribution. However, our approach remains relevant even with a normal distribution. This work opens the door to many other risk-adjusted performance measures, such as the Sortino ratio, the Modigliani ratio, and the Omega ratio.



(a) Australia **(b)** Canada **(c)** France

(d) Germany **(e)** Republic of Ireland **(f)** Netherlands

(g) New Zealand **(h)** Norway **(i)** UK

(j) USA

Fig. 2: Rankings according to the threshold for all the countries

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A Appendix: Value-at-Risk

A.1 Definition of Value-at-Risk

Informally, Value-at-Risk is the largest percentage loss with a given probability (confidence level), likely to be suffered on a portfolio position over a given holding period. In other words, for a given portfolio and time horizon, and having selected the confidence level $\alpha \in (0,1)$, VaR is defined to be that threshold value, assuming no further trade, such that the probability that the mark-to-market loss in the portfolio exceeds this VaR level is exactly the preset probability of loss α.[8] Thus, VaR is the quantile of the projected distribution of losses over the target horizon, in that if α is taken to be the confidence level, VaR then corresponds to the α quantile. By convention, this worst loss is always expressed as a positive percentage in the manner indicated. Thus, in formal terms, if we take L to be the loss, measured as a positive number, and α to be the confidence level, then VaR can be defined as the smallest loss–in absolute value–such that

$$P(L > VaR) \leq \alpha. \tag{8}$$

A more detailed definition of VaR can be found in the study by Jorion (2007).[9]

Over the past few years, the popularity of downside risk measures (including VaR) has been rising. Today, these metrics are replacing standard deviation to evaluate the risk of investments. The reason behind the growing interest in downside risk measures is the choice of many regulators (Basel and Solvency) to rely almost solely on downside risk metrics such as VaR or its derivative, the Expected Shortfall, for the calculus of the required capital.

A.2 Value-at-Risk and real estate

VaR is mainly estimated using one of the following three methods: historical, parametric (variance–covariance), and the Monte Carlo simulations. All these methods present advantages and drawbacks in the context of real estate. Primarily, both the lack of data from the commercial real estate sector and issues arising from non-normality of returns are known to cause issues. Limited data for this sector is one of the primary obstacles to reliable VaR computation. One of the two cases may arise: the investment is either in listed real estate and is quoted daily with sufficient available data to compute VaR for the portfolio, or the investment is in direct real estate and it has small datasets. This is particularly true in commercial real estate, in which investments are generally done by large institutions. The real estate market

[8] Note that VaR does not give any information about the likely severity of the loss by which its level will be exceeded.

[9] In terms of gains rather than losses, the VaR at confidence level α for a market rate of return X whose distribution function is denoted as $F_X(x) \equiv P[X \leq x]$ and whose quantile at level α is denoted as $q_\alpha(X)$ is

$$-VaR_\alpha(X) = \sup\{x : F_X(x) \leq \alpha\} \equiv q_\alpha(X).$$

is thus comparable to the private equity market, where indices are created from small numbers of transactions. Any real estate property index attempts to aggregate real estate market information in order to provide a representation of underlying real estate performance. However, observations are generally conducted monthly in the best of cases; else, they are made quarterly, semi-annually, or sometimes, even annually. Therefore, determining VaR of a real estate portfolio at a threshold of 1% (as requested by the Basel II framework) using the historic approach would require a minimum of 100 values (even with 100 values, the minimum of the series might be an outlier, which represents 8 years even for a monthly index and 25 years for a quarterly index). Given the need for such a large number of observations, VaR considerations are frequently irrelevant, since this requirement typically exceeds the recorded history of the index. The parametric and Monte Carlo methods also show inadequacies: they generally rely on strong assumptions (such as normality of returns). Thus, the problems include difficulty in identifying the distribution of a series when dealing with modest datasets.

B Appendix: Skewness and kurtosis

Given a probability distribution $f(x)$ of the random variable X and a real-valued function $g(x)$, one defines the expectation $E[g(X)] = \int g(x)f(x)dx$, in which case the first moment is $\mu = E[X]$, whereas the higher central moments are then defined as $\mu_n = E[(X-\mu)^n]$. The first task in almost all statistical analyses is to characterize the location and variability of a dataset. This is captured by the moments of orders one and two, usually called the mean μ and the variance $\sigma^2 = \mu_2$, respectively. A further characterization of the data often includes the standardized moments of orders three and four, called the skewness $\gamma_1 = \mu_3/\sigma^3$ and kurtosis $\beta_2 = \mu_4/\sigma^4$, respectively. These last two measures further describe the shape of a probability distribution. We briefly state the significance of these two last parameters.

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or dataset, is symmetric if it looks the same to the right and left of its center (which is the mean μ). The skewness of any symmetric distribution, such as a Gaussian one, is necessarily zero. Negative values for the skewness coefficient indicate that the data are skewed to the left, whereas positive values indicate that the data are right-skewed. The left skewness means that the left tail of the distribution is long relative to the right one.

Kurtosis refers to whether the data are peaked or flat relative to a normal distribution. That is, datasets with high kurtosis tend to have a distinct peak near the mean, then decline rather rapidly, but still have heavy tails. Datasets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. The kurtosis formula measures the degree of this peakedness; for instance, the kurtosis of a Gaussian distribution turns out to be 3.

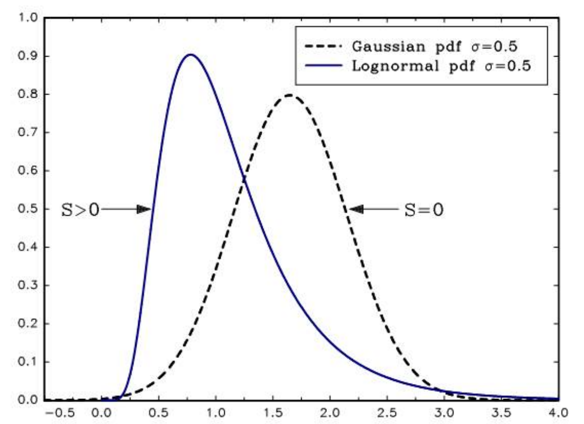

Fig. 3: Right-skewed distribution ($S = 1.75$)

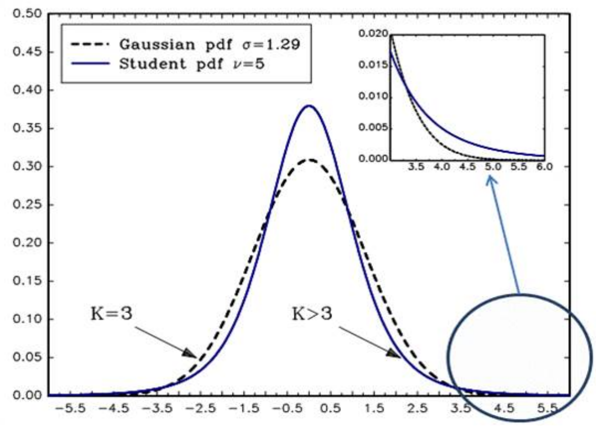

Fig. 4: Fat-tailed distribution ($K = 9$)

C Appendix: The Cornish Fisher procedure

The Cornish Fisher expansion is a useful tool for quantile estimation. For any $\alpha \in (0, 1)$, the upper αth-quantile of F_n is defined by $q_n(\alpha) = inf\{x : F_n(x) \geq \alpha\}$, where F_n denotes the cumulative distribution function of $\xi_n = (\sqrt{n}/\sigma)(\bar{X} - \mu)$, and $\bar{X}$ is the sample mean of independent and identically distributed observations $X_1, \ldots, X_n$. If z_α denotes the upper αth-quantile of $N(0, 1)$, then, the fourth-order Cornish Fisher expansion can be expressed as

$$q_n(\alpha) = z_\alpha + \frac{1}{6\sqrt{n}}(z_\alpha^2 - 1)S + \frac{1}{24n}(z_\alpha^3 - 3z_\alpha)(K - 3) - \frac{1}{36n}(2z_\alpha^3 - 5z_\alpha)S^2 + o(n^{3/2}), \quad (9)$$

where S and K are the skewness and kurtosis of the observations X_i, respectively.

The Cornish Fisher expansion is useful because it allows one to obtain more accurate results compared to those acquired using the central limit theorem (CLT) approximation, which is the same as z_α defined in the main text. A demonstration and example of the greater accuracy provided by the Cornish Fisher expansion compared to the CLT approximation is reported by Chernozhukov et al. (2010).
Relation (9), in general, grants a non-monotonic character to $q_n(\alpha)$, which means that the true distribution's ordering of quantiles is not preserved. The Cornish Fisher expansion formula is thus valid only if the skewness and kurtosis coefficients of the distribution meet a particular constraint. This domain of validity has been studied by Maillard (2012), among others. Monotonicity requires the derivative of $z_{CF,\alpha}$ relative to z_α to be non-negative. This leads to the following constraint, which implicitly defines the domain of validity (D) of the Cornish Fisher expansion:

$$\frac{S^2}{9} - 4\left(\frac{K-3}{8} - \frac{S^2}{6}\right)\left(1 - \frac{K-3}{8} - \frac{5S^2}{36}\right) \leq 0. \quad (10)$$

In practice, this constraint is rarely taken into account as S and K are generally considered to be small in finance.

D Appendix: Normality tests

Because our objective in this study is to consider the possible non-normality of the assets' returns, it may be interesting to check if the asset returns are normally distributed.[10] Table

[10] As a reminder though, our methodology is still applicable when assets are normally distributed.

7 presents five normality tests that seek to determine if the markets could be considered as being normally distributed. All the tests are performed using Matlab functions. Given the size of the dataset, p-values have been interpolated using Matlab tables derived from Monte Carlo simulations. The results are not very conclusive and are difficult to interpret. Most of the p-values are very large (the results at 5% significance level are in bold). This can be attributed to a widespread issue encountered in the real estate sector: the low number of data points (14). Some conclusions can, nonetheless, be inferred. The returns for Norway and Australia are certainly non-normally distributed. New Zealand also shows signs of a non-normal distribution, but the results are not as conclusive. The returns for France can be considered as being normally distributed. Finally, and interestingly, contrary to the findings of much of the literature (see Young et al., 2006), the returns for the UK seem to be normally distributed.[11]

	Anderson–Darling	**Jarque–Bera**	**Kolmogorov–Smirnov**	**Lilliefors**	**Shapiro—Wilk**
Australia	(0.0920)	**(0.0090)**	(0.5601)	(0.1199)	(0.0362)
Canada	(0.7139)	(0.5163)	(0.9782)	(0.8639)	(0.5469)
France	**(0.0230)**	(0.0800)	(0.4004)	**(0.0380)**	**(0.0247)**
Germany	(0.1287)	(0.1066)	(0.5185)	(0.1072)	(0.1200)
Ireland	(0.2249)	**(0.0300)**	(0.7145)	(0.2822)	(0.0656)
Netherlands	(0.4055)	(0.6352)	(0.5202)	(0.1020)	(0.3587)
New Zealand	**(0.0210)**	**(0.0110)**	(0.1845)	**(0.0020)**	**(0.0130)**
Norway	**(0.0020)**	**(0.0000)**	(0.3503)	**(0.0180)**	**(0.0011)**
UK	**(0.0270)**	(0.0810)	(0.4514)	(0.0510)	**(0.0348)**
USA	(0.1050)	(0.5979)	(0.6400)	(0.2002)	(0.1353)

Table 7: Normality tests (in bold if significant at the 5% level)

[11] Given the conflicts among the various tests and the relatively few significant p-values, all these comments need to be treated with care.