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Welfare and Trade Margins with Multinational Production

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Abstract

We examine how the presence of heterogenous multinational firms matters for the aggregate welfare implications of trade by comparing models with only export, intra-firm activity, and pure multination production. We show that extensive margins depend on both supply and demand parameters, and are not longer constant. We use a theoretical comparative static to isolate the additional component of welfare associated with intra-firms trade and to show that models of multinational production have new aggregate welfare implications. The model is then calibrated to analyze counterfactual scenarios. We find that truncation through multinational production is associated with the largest welfare gains from liberalization.

JEL classification: F12, F23

Keywords: MNFs, multinational production, intra-firm trade, welfare.

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1. Introduction

We examine the role of multinational production by introducing a finite upper bound to the Pareto distribution, and find that it alters the aggregate welfare implication of trade. We embed knowledge transfer in Helpman et al. (2004) to build a model of export and horizontal multinational production with intra-firm trade. In our general equilibrium framework with two symmetric countries, each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. This implies that geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively. For tractability reason, and similarly to Irarrazabal et al. (2013), intra-firm trade is left exogenous.

Upon drawing its own efficiency parameter, each firm decides whether to exit or to produce. In the latter case, firms must face additional fixed costs linked to the supply strategy chosen. When firms decide to serve the foreign market, they choose whether to export domestically produced goods or produce abroad via affiliate production. Free entry ensures that firms continue to enter until the expected sum of future profits equals the cost of entry. The presence of two alternative ways of reaching the foreign location introduces a double truncation in the productivity distribution of exporters.[1] The finite upper bound to the Pareto distribution, generated by the presence of multinational production, allows us to highlight three contributions.

Firstly, we derive gravity equations and margins' sensitivity for exports and affiliate sales to show that both trade elasticities are altered. As regards the margin of exports,

[1] Intra-firm trade *per se* does not affect our findings on welfare, which depend on the upper bound of the productivity distribution.

we find that, similarly to Chaney (2008), the intensive margin only depends on the elasticity of substitution. Differently from Chaney (2008), the extensive margin is no longer constant, but a function of both export and affiliate sales. A similar result is obtained for affiliate sales. More specifically, the intensive margin of affiliate sales is unambiguously related to the elasticity of substitution and the share of the imported intermediate good; whereas, the sensitivity of the extensive margin is no longer constant and depends on trade frictions. Therefore, our model delivers elasticity measures, both for exports and affiliates, that continue to depend on supply and demand parameters despite the Pareto assumption. This result is in line with recent papers investigating the consequences of replacing the Pareto with other heavy-tailed productivity distributions, as in Head et al. (2014) and Bas et al. (2017), among others.

Secondly, the standard results obtained for welfare in heterogeneous firm models are altered by the presence of export and multinational production. When the firm heterogeneity model includes multinational production, aggregate domestic trade shares and trade elasticity are no longer sufficient statistics to evaluate welfare gains. This result is in line with Feenstra (2014) and Melitz and Redding (2015) which remove the long tail distribution related to the untruncted Pareto assumption, and introduce an upper bound to the productivity distribution. Differently from them, the hypothesis of the existence of an upper bound to the distribution, is justified by the presence of multinational firms in the market. Comparing welfare gains with intra-firm trade to those in Helpman et al. (2004), we show that the latter has larger welfare gains from reduction in trade costs and smaller welfare losses from increase in trade costs than the model with intra-firm trade.

Thirdly, we quantify the country level gains from multinational production with intra-firm trade, and examine the quantitative implications for trade elasticities. We calibrate two versions of the model: export only *à la* Melitz and Redding (2013); and, export, multinational production, and intra-firm trade. Our findings stress the role of intra-firm trade for welfare gains: they range from 5 to 20 percent. We also compare the total gains from a model of multinational production and intra-firm with a model of pure multinational production. The latter yields the largest rise in welfare due to trade liberalization. Finally, we compute the sensitivity of export and affiliate sales: our numbers for trade elasticity are consistent with the estimation of the empirical literature (Mejean and Imbs, 2017 and Novy, 2013), and affiliate trade elasticity ranges from 0.2 to 1.2 in absolute values.

This paper relates to several strands of literature. As in Horstmann and Markusen (1992), Brainard (1997), Helpman et al. (2004), and Grossman et al. (2006) we capture the interaction between export, multinational production, and intra-firm trade. Keller and Yeaple (2013) measure the spatial barriers to transferring knowledge. They find that the knowledge intensity of production affects the level of affiliate sales around the world. Our theoretical setup is closely related to Irarrazabal et al. (2013), which structurally estimate a model of trade and multinational production with firm heterogeneity. They reject the proximity versus concentration hypothesis which did not consider intra-firm trade.[2] We add on to their findings and show that the welfare equation varies from the one obtained in models with no truncation. Moreover, we quantitatively compare welfare of alternative market access strategies. Our model is used to evaluate theoretical and quantitative welfare implications arising from the existence of different modes of supply.

This paper also contributes to the growing literature that theoretically analyzes the welfare gains from openness. Arkolakis et al. (2012) show that there exists a group of

[2] Our model is isomorphic to the set up in Bombarda (2007), which proposes a model of intra-firm trade with distant dependent fixed cost to highlight non-monotonic choices of modes of supply.

models in which a country's domestic trade share and the elasticity of trade are sufficient statistics to measure aggregate welfare gains from trade. This result relies on the assumption of an unbounded productivity distribution. Feenstra (2014) uses a bounded Pareto distribution and non CES preferences to restore the role for product variety and pro-competitive gains from trade in heterogeneous firm models. Feenstra (2016) and Melitz and Redding (2015) show that the additional adjustment margin in heterogeneous firm models plays an important role for welfare gains. Differently from Arkolakis et al. (2012) and similarly to Feenstra (2014), Feenstra (2016) and Melitz and Redding (2015), our welfare measure is altered by the double truncation in the productivity distribution of exporters. In our framework, the upper bound to the distribution is obtained via the presence of multinational firms in the market. This delivers a welfare measure that depends on the interaction between export and multinational activities trade barriers.

Another related strand of literature quantifies the gains from international activities. Edmond et al. (2015) study gains from international trade in a quantitative model with endogenously variable markups. Ramondo (2014) uses a multi-country general equilibrium model with a continuum of goods produced under constant return to scale at the industry level to calculate the gains that a country would experience from liberalizing access to foreign firms. Ramondo and Rodríguez-Clare (2013) consider trade and multinational production into an Eaton-Kortum framework to measure the overall gains from openness. Garetto (2013) quantifies the gains from multinational activity, using an Eaton-Kortum type model, where multinational firms engage in vertical FDI. Irarrazabal et al. (2013) find that impeding multinational activity has a small effect on welfare. Similarly to most of these studies, we propose a mechanism through which intra-firm trade affects multinational production, and we rely on aggregate evidence to quantify its importance.

Lastly, the paper relates to the literature measuring aggregate trade elasticity. Models of heterogeneous firms with selection into export market participation assuming a Pareto productivity distribution exhibit a constant trade elasticity. Nevertheless, Helpman et al. (2008) and Melitz and Redding (2015) show that for the bounded version of the Pareto, the trade elasticity recovers a bilateral-specific dimension. A recent strand of the literature investigates the limitation of the Pareto assumption, which represents a good approximation only for the right tail of the observed distribution of firm sizes. Head et al. (2014) and Bas et al. (2017) switch to log-normal distribution of productivity to obtain a better fit and a destination specific aggregate trade elasticity. In line with those findings, this paper shows that despite the Pareto assumption, coexistence of exporters and multinational firms allows trade elasticity to recover the bilateral-specific dimension.

The rest of the paper is organized as follows. Section 2 describes the theoretical framework. Section 3 reports general equilibrium results. Section 4 discusses gravity equations, and intensive and extensive margins of trade. In Sections 5 and 6, we derive the theoretical implications of the model on welfare and provide comparative statics to highlight the additional channel. Section 7 contains the calibration. Finally, Section 8 concludes.

2. Theoretical Framework

We use a model of export and horizontal multinational production with intra-firm trade to examine aggregate welfare implications.[3] The specification of entry is the same as in Helpman et al. (2004). We assume two-tier preferences with Cobb-Douglas in the upper tier and CES in the lower tier. A fraction β of income is spent on the differentiated

[3] Helpman et al. (2004) introduces multinational production in the Melitz model, and Irarrazabal et al. (2013) analyse intra-firm trade in a model without free entry.

good, and the rest $(1-\beta)$ is spent on the homogeneous good. Production is as in Bombarda (2007) and Irarrazabal et al. (2013). The output of every variety is described by a Cobb-Douglas function of the intermediate goods

$$x_i(v) = \frac{1}{a(v)} \left(\frac{y_1}{\eta}\right)^{\eta} \left(\frac{y_2}{1-\eta}\right)^{1-\eta} \qquad 0 < \eta < 1, \tag{2.1}$$

where $1/a(v)$ is the firm-specific productivity parameter and η is the Cobb-Douglas cost share of y_1, which is common across all countries. Using the intermediate results from the consumer and firm optimization problems, the operating profit from producing domestically is

$$\pi^*_{Di}(a, A, \eta) = A_i a^{1-\sigma} \frac{1}{\sigma} \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} - f_D, \quad A_i \equiv \frac{\beta E_i}{P_i^{1-\sigma}}$$

where A_i and η are industry- (and thus country-) specific. Using $B_i = \frac{A_i}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}$ we obtain

$$\pi^*_{Di}(a, A, \eta) = B_i a^{1-\sigma} - f_D. \tag{2.2}$$

If the firm chooses to reach a given foreign market, its equilibrium net operating profit on sales in that market is

$$\pi^*_X(a, A, \eta) = B_j (\tau a)^{1-\sigma} - f_X \tag{2.3}$$

and

$$\pi^*_M(a, A, \eta) = B_j a^{1-\sigma} \left[(\tau)^{\eta}\right]^{1-\sigma} - f_M, \tag{2.4}$$

for export or multinational production, respectively. We set parameters so as to obtain the same ranking as in Helpman et al. (2004) when there are only two countries. Namely, firms with sufficiently high productivity will supply the foreign market, with the most productive supplying it via FDI rather than exports. In this way our model is in line with the empirical findings in Helpman et al. (2004). The regularity condition is

$$f_D < (\tau)^{(\sigma-1)} f_X < (\tau)^{\eta(\sigma-1)} f_M.$$

Using equations (2.2) to (2.4), the equilibrium cutoff conditions write as

$$a_{Di} = \left(\frac{f_D}{B_i}\right)^{\frac{1}{1-\sigma}}, \tag{2.5}$$

$$a_X = \left(\frac{f_X}{B_j (\tau)^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}, \tag{2.6}$$

and

$$a_M = \left(\frac{f_M - f_X}{B_j \left[(\tau)^{\eta(1-\sigma)} - (\tau)^{1-\sigma}\right]}\right)^{\frac{1}{1-\sigma}}. \tag{2.7}$$

The model is then solved assuming perfect symmetry, i.e. $i = j$, and free entry.

3. General Equilibrium

This section presents general-equilibrium results, where we will omit the subscripts i and j. Substituting the price index, which has been solved assuming the Pareto distribution (equation A.4), into the domestic cutoff condition (2.5), yields the equilibrium number of varieties (and so the number of active firms) consumed in a typical country which depends on trade frictions related to different trade activities:

$$n^* = \frac{(b-1)\beta E}{\sigma b f_D\left[1 + T^{1-b}\phi^b + V^{1-b}\left(\phi^\eta - \phi\right)^b\right]}, \tag{3.1}$$

where $b = \frac{k}{\sigma - 1}$; $\phi = \tau^{1-\sigma}$; $T = f_X/f_D$ and $V = (f_M - f_X)/f_D$.[4]

Finally, using the price index and free-entry condition in equations (A.4) and (A.6) , we can solve for the equilibrium domestic cutoff

$$a_D^* = a_0\left[\frac{(b-1)f_I}{f_D(1+\Psi+\Omega)}\right]^{\frac{1}{k}}, \tag{3.2}$$

where $\Omega = T^{1-b}\phi^b$ and $\Psi = V^{1-b}\left[\phi^\eta - \phi\right]^b$. Replacing (3.2) into the ratio between (2.6) and (2.5), we find

$$a_X^* = a_0\left[\frac{(b-1)f_I}{f_X(1+\Psi+\Omega)}\phi^b T^{1-b}\right]^{\frac{1}{k}}. \tag{3.3}$$

Adopting a similar strategy, we obtain the equilibrium cutoff for the M-mode

$$a_M^* = a_0\left[\frac{(b-1)f_I}{(f_M - f_X)(1+\Psi+\Omega)}\left[\phi^\eta - \phi\right]^b V^{1-b}\right]^{\frac{1}{k}}. \tag{3.4}$$

4. INTENSIVE AND EXTENSIVE MARGINS

In this section we derive intensive and extensive margins of export and affiliate sales.

4.1. Export Sales

We differentiate the expression of total exports of a country $X^X = n\int_{\overline{a}^M}^{\overline{a}^X} x^X \mathrm{d}G(a)$ with respect to variable trade costs and derive the intensive and extensive margins of

[4] Appendix A provides further details on the price index and free entry solved under the Pareto distribution (equations (A.4) and (A.6), respectively).

export sales

$$\frac{\partial X^X}{\partial \tau} = \underbrace{n \int_{\bar{a}^M}^{\bar{a}^X} \frac{\partial x^X}{\partial \tau} \mathrm{d}G(a)}_{\text{Intensive Margin}} + \underbrace{n \left[x^X G'\left(\bar{a}^X\right) \frac{\partial \bar{a}^X}{\partial \tau} - x^M G'\left(\bar{a}^M\right) \frac{\partial \bar{a}^M}{\partial \tau} \right]}_{\text{Extensive Margin}}, \tag{4.1}$$

where we applied the Leibniz rule to separate the margins, and considered constant the number of active firms.[5]

PROPOSITION 1. *Elasticity of export sales is no longer constant. Let* $\Omega \equiv -\partial \log X^X / \partial \log \tau$*, a change in the variable costs* τ *makes the margins of export sales to react as follows*

$$\Omega = \underbrace{(\sigma - 1)}_{\textit{Intensive Margin Elasticity}} + \underbrace{(k - \sigma + 1) \left[1 - \frac{X^M}{X^X} (\Gamma - \omega) \right]}_{\textit{Extensive Margin Elasticity}}, \tag{4.2}$$

where

$$\Gamma = \frac{\eta \tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}{b\left(\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}\right)}, \tag{4.3}$$

$$\omega = \tau^{(1-\eta)(1-\sigma)}. \tag{4.4}$$

Proof. See Appendix D. □

Intensive Margin. Similarly to models with untruncated Pareto distribution, the volume of export sales depends on the constant elasticity of substitution. This implies that when goods are very substitutable (high σ), the export of each exporter is very sensitive to the trade barriers.

Extensive Margin. Differently from models with an untruncated Pareto, the extensive margin in our model depends on variable trade costs and it is not constant at $k - \sigma + 1$.

[5]The denominator in (3.1) does not change. This is similar to the assumption on θ made by Chaney (2008).

Equation (4.2) shows that the sensitivity of the extensive margin of exports to trade policy depends on the interaction between aggregate affiliate and export sales.[6] This is because the change in the number of varieties supplied via exports depends on the level of profits generated by export and multinational production strategies, which in turn affect overall affiliate sales.

Let us focus on the second part of (4.2). If $X^M > X^X$, a decrease in trade cost reduces the extensive margin elasticity of export. Notice that the sign of the overall elasticity depends on the size of the intensive margin, which can compensate the negative extensive margin. When $X^M < X^X$ the opposite is true, and a decrease in trade costs increases the extensive margin. To summarize, while the elasticity of the intensive margin is always positive (a decrease in trade costs increases the volume of trade), the behaviour of the extensive margin depends on how export and affiliate sales interact.

Differently from Chaney (2008), the elasticity of exports with respect to variable costs depends on the elasticity of substitution between goods, σ, and on trade costs τ. This bounded productivity framework reaffirms the importance of trade costs and elasticity of substitution in models of firm heterogeneity. This result is in line with recent findings using alternative right tail distributions, as in Head et al., 2014 and Bas et al., 2017.

4.2. Affiliate Sales

In this section we analyze how elasticity of substitution and the share of intermediate inputs affect the sensitivity of affiliate sales' margins. Differentiating total affiliate sales of a country $X^M = n\int_0^{\bar{a}^M} x^M \mathrm{d}G(a)$ with respect to variable trade cost, we derive the

[6] Note that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration. For further details on Γ and ω, see appendix D.

intensive and extensive margins of affiliate sales

$$\frac{\partial X^M}{\partial \tau} = \underbrace{n \int_0^{a^M} \frac{\partial x^M}{\partial \tau} \mathrm{d}G(a)}_{\text{Intensive Margin}} + \underbrace{n x^M G'\left(\overline{a}^M\right) \frac{\partial \overline{a}^M}{\partial \tau}}_{\text{Extensive Margin}}, \tag{4.5}$$

where we applied the Leibniz rule to separate the margins.

Proposition 2. *Elasticity of affiliate sales is no longer constant. Let* $\psi \equiv -\partial \log X^M / \partial \log \tau$, *a change in variable costs* τ *makes the margins of affiliate sales react as follows*

$$\psi = \underbrace{\eta\,(\sigma - 1)}_{\textit{Intensive Margin Elasticity}} + \underbrace{\frac{(k - \sigma + 1)}{k} \frac{(\sigma - 1)(\eta \tau^{(1-\sigma)\eta} - \tau^{1-\sigma})}{\tau^{(1-\sigma)\eta} - \tau^{(1-\sigma)}}}_{\textit{Extensive Margin Elasticity}}, \tag{4.6}$$

Proof. See Appendix E. □

Intensive Margin. The intensive margin of affiliate sales depends on the constant elasticity of substitution and on the level of imported intermediate η. Therefore, when goods are very substitutable (high σ), the sales of each individual affiliate is very sensitive to the trade barriers. Let us now focus on the role of the parameter η.

When η is equal to one, no firm will supply via multinational production. In this case, the foreign affiliate is importing both intermediate inputs from the home country. This strategy is extremely costly, since it implies bearing full trade costs as well as higher fixed costs. Therefore, when $\eta = 1$ export is the only market access strategy. Differently, when η is equal to zero, the foreign affiliate is producing using only foreign inputs (similarly to Helpman et al., 2004). When all intermediates are realized in the foreign location, the volume of sales of already existing affiliates are not affected by changes in trade costs. Therefore, in this case the intensive margin elasticity is equal to zero.

For intermediate levels of η, both the extensive and the intensive margins of affiliate sales are affected by the intensity of imported headquarter intermediates. The behaviour of the intensive margin is unambiguous: σ magnifies the sensitivity of the intensive margin. When σ is high, the change in X^M due to a change in τ is mostly captured by the intensive margin: if τ decreases, new affiliates enter the market, but a high σ leads to a high level of competition. In this environment, having a low productivity is an even bigger disadvantage as firms can only capture a small market share, and their impact on the overall affiliate sales is small.

Extensive Margin. The sensitivity of the extensive margin of affiliate sales to changes in trade costs is not constant and it is related to the elasticity of substitution σ. In general, we should expect that when the substitutability across varieties is low, an increase in σ makes entrance of new affiliates more sensitive to changes in τ. On the one hand, trade liberalization makes it easier to import the intermediate goods; on the other hand, the low degree of substitution keeps the level of competition down. This explains why more firms can survive as new affiliates after entry. Contrarily, a larger degree of substitutability among varieties makes entry of new affiliates less sensitive to changes in τ. In fact, when the level of competition is high, new entrants will capture only a small fraction of market share despite the reduction in trade costs.

To further stress this result, in the calibration section we proposes an exercise to understand the effects of trade policy on both export and affiliate sales' margins.

5. Welfare Gains with Exports and Intra-Firm Trade

Aggregate trade between two countries is inversely related to the domestic trade share, which can be written as the proportion of domestic sales in total sales

$$\lambda = \frac{\int_0^{a_D} r_D \mathrm{d}G(a/a_D)}{\int_0^{a_D} r_D \mathrm{d}G(a/a_D) + \int_{a_M}^{a_X} r_X \mathrm{d}G(a/a_D) + \int_0^{a_M} r_M \mathrm{d}G(a/a_D)}. \tag{5.1}$$

Setting $\Lambda = \frac{\int_{a_M}^{a_X} a^{1-\sigma} dG(a/a_D)}{\int_0^{a_D} a^{1-\sigma} dG(a/a_D)}$ and $Z = \frac{\int_0^{a_M} a^{1-\sigma} dG(a/a_D)}{\int_0^{a_D} a^{1-\sigma} dG(a/a_D)}$, the domestic trade share becomes

$$\lambda = \frac{1}{1 + \tau^{1-\sigma}\Lambda + \tau^{\eta(1-\sigma)} Z}. \tag{5.2}$$

Then, dividing the price index in equation (A.3) by $\int_0^{a_D} a^{1-\sigma} dG(a/a_D)$, we can express welfare as a function of the domestic trade share, λ,

$$\frac{w}{P} = \frac{\sigma - 1}{\sigma} n^{\frac{1}{\sigma-1}} \left(\frac{\delta(a_D)}{\lambda}\right)^{\frac{1}{\sigma-1}}, \tag{5.3}$$

where we set $\int_0^{a_D} a^{1-\sigma} \mathrm{d}G(a/a_D) = \delta(a_D)$.

Welfare gains can be derived by log-differentiating equation (5.3) and using the (observable) partial trade elasticity from equation (4.2):[7]

$$\begin{aligned} \mathrm{d}\ln\frac{w}{P} &= \frac{1}{\sigma - 1}[\mathrm{d}\ln n - \mathrm{d}\ln\lambda] \\ &= \frac{1}{\Omega - (k - \sigma + 1)\left[1 - \frac{X^M}{X^X}(\Gamma - \omega)\right]}[\mathrm{d}\ln n - \mathrm{d}\ln\lambda]. \end{aligned} \tag{5.4}$$

The above equation shows that welfare gains from trade liberalization are also affected by

[7] Welfare change using the full trade elasticity would be:
$\mathrm{d}\ln\frac{w}{P} = \frac{1}{\Omega-(k-\sigma+1)\left[1-\frac{X^M}{X^X}(\Gamma-\omega)\right]}[\mathrm{d}\ln n + \mathrm{d}\ln\delta(a_D) - \mathrm{d}\ln\lambda]$.

the interaction between exports and multinational activities (denominator), other than domestic trade share and trade elasticity. Therefore, changes in the micro structure related to the mode of supply will affect the welfare gains from trade. This result is in line with Melitz and Redding (2015). Notice that in the absence of the upper bound threshold generated by the multinational firms, equation (5.4) reduces to:

$$\mathrm{d}\ln\frac{w}{P} = \frac{1}{\Omega - (k - \sigma + 1)}\left[\mathrm{d}\ln n - \mathrm{d}\ln\lambda\right], \tag{5.5}$$

which replicates the formula in Arkolakis et al. (2012). To identify the different sources of gains from trade, we replicate equation (12) in Feenstra (2016) by taking the price index in equation (5.3) and dividing it by the corresponding price index in autarky, P_{Aut}, to get:

$$\frac{P_D}{P_{Aut}} = \left(\frac{n_D}{n_{Aut}}\right)^{\frac{1}{1-\sigma}}\left(\frac{\delta(a_D)}{\delta(a_{DAut})}\right)^{\frac{1}{1-\sigma}}\lambda^{\frac{1}{\sigma-1}}, \tag{5.6}$$

where the subscript Aut indicates autarky. Since firms earn zero expected profits, a reduction in the price index corresponds to a rise in the indirect utility. Therefore, equation (5.6) shows that welfare gains from reduction in trade costs depend on three elements: (i) the losses due to reduced domestic varieties (first term); (ii) the selection effect which implies a rise in the average productivity (second term); and, (iii) gains from import variety (third term). Gains from trade are therefore larger in models with multinational firms because all of the three effects are at work. Differently, as shown in Melitz and Redding (2015) and Feenstra (2016), restricting the distribution of productivity to be untruncated implies that only the selection effect is efective, since the others two effects completely offset.

6. THEORETICAL COMPARATIVE STATICS

In this section we compare the aggregate welfare properties of the model proposed in section 2 to a model with only export. Using the price index in equation (A.4), we can obtain a measure of welfare in a model with export and intra-firm trade:

$$\begin{aligned} W_{MP} &= \frac{w}{P} \\ &= \frac{\sigma}{\sigma-1}\left(\frac{1}{1-1/b}\right)^{\frac{1}{\sigma-1}}\left[\frac{(b-1)\beta E}{\sigma b f_D}\right]^{\frac{1}{\sigma-1}}\left[\frac{f_D}{(b-1)f_I}\right]^{\frac{1}{k}}\left(1+T^{1-b}\phi^b+V^{1-b}\left(\phi^\eta-\phi\right)^b\right)^{\frac{1}{k}}, \end{aligned} \tag{6.1}$$

where we used the equilibrium values for n^* and a_D from equations (3.1) and (3.2) respectively.

Differently, welfare from a model with only export will be:

$$W_{Exp} = \frac{w}{P} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-1/b}\right)^{\frac{1}{\sigma-1}}\left[\frac{(b-1)\beta E}{\sigma b f_D}\right]^{\frac{1}{\sigma-1}}\left[\frac{f_D}{(b-1)f_I}\right]^{\frac{1}{k}}\left(1+T^{1-b}\phi^b\right)^{\frac{1}{k}}, \tag{6.2}$$

where fixed costs of trade are not necessarily the same in the two models.

PROPOSITION 3. *Consider an heterogeneous firm model à la Melitz where also multinational firms exist. Given the same value for the parameters $\{f_D, f_I, f_X, k, L, \sigma\}$, welfare with multinational production is higher than welfare with only export.*

Proof. Comparison between equations (6.1) and (6.2) immediately shows that $W_{MP} > W_{Exp}$ since $1+T^{1-b}\phi^b+V^{1-b}\left(\phi^\eta-\phi\right)^b > 1+T^{1-b}\phi^b$.

Considering welfare with pure multinational production, W_{PMP}, which implies $\eta=0$, we can show that since $V^{1-b}(1-\phi)^b > V^{1-b}\left(\phi^\eta-\phi\right)^b$, then it follows that $W_{PMP} > W_{MP} > W_{Exp}$. □

Consider now the effect of a further reduction in trade costs in an open economy.

PROPOSITION 4. *Departing from the same level of welfare, the proportional welfare gains from reducing trade costs are strictly larger in a model where multinational production exists. The largest gains are obtained in the pure multinational production scenario.*

Proof. The proof follows from Proposition 3. In particular we need to show that welfare in a model with intra-firm trade is higher than in a model with only exports for any level of trade costs. This is straightforward from comparing equations (6.1) to (6.2). □

Therefore, in a model with exporters and intra-firm trade, decreasing trade costs makes welfare gains to increase more than in a model with only exports. This happens because trade costs affect both types of activities, export and intra-firm trade. For the same reason, welfare losses from an increase in trade costs are smaller in a world with intra-firm trade. The existence of different types of firms engaged in international trade, which have different degrees of exposure to trade barriers, increases the advantages of trade liberalization.[8]

7. QUANTITATIVE EXERCISE

We examine the quantitative relevance of our model. First, we show that there are relevant differences in welfare, mass of active firms, and domestic trade share between a benchmark model *à la* Melitz and Redding (2015), and our model with exporting and intra-firm activity in symmetric countries. Then, in section 7.1 we examine welfare gains from pure multinational production activity. Finally, in section 7.2, we exploit the upper bound to the support of the Pareto distribution to analyse the sensitivity of intensive and extensive margins of exports and affiliate sales to variable trade costs.

[8]This results is stronger when η is sufficiently small. Notice that when $\eta \longrightarrow 0$ we are in Helpman et al. (2004).

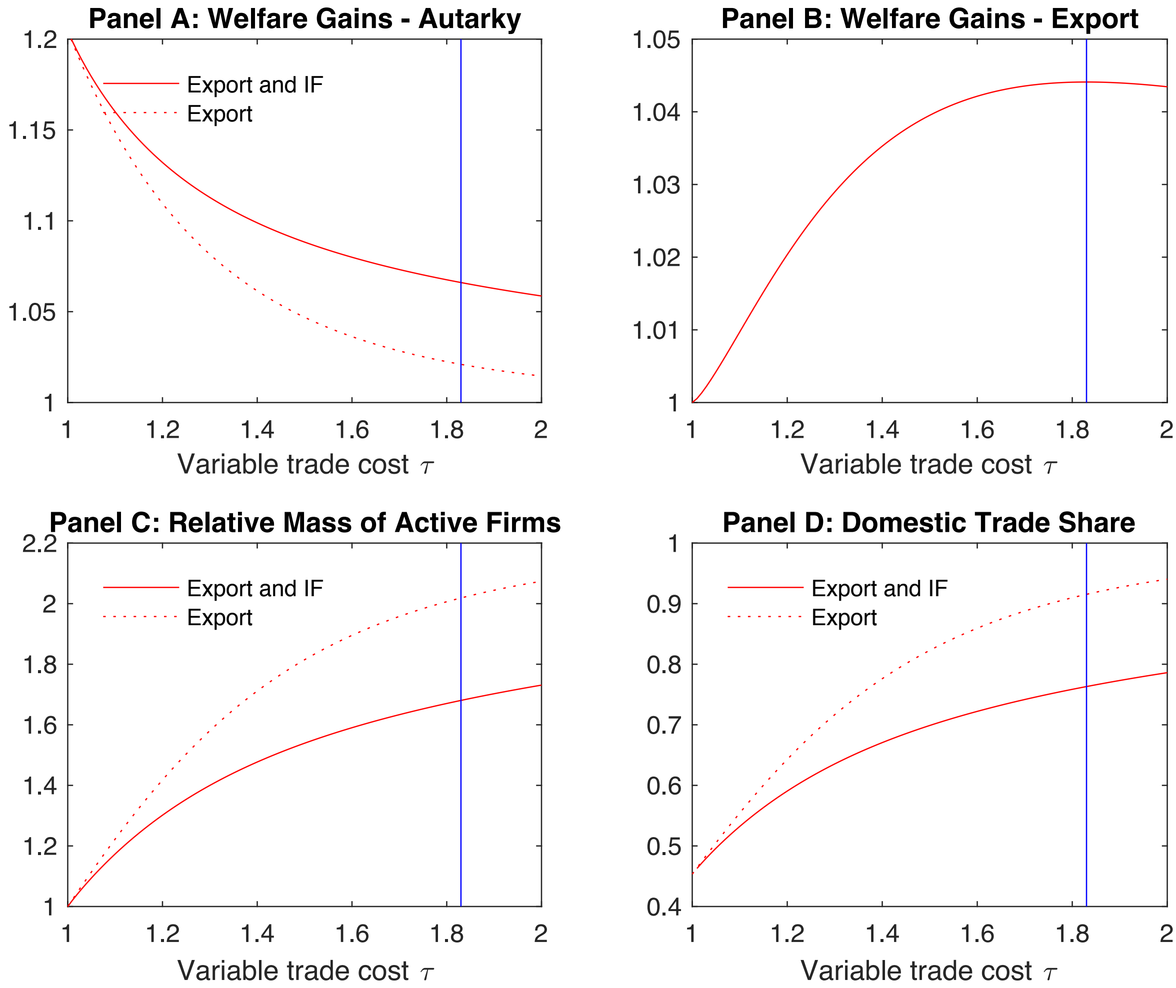

Figure 1: Variable trade costs

We set the elasticity of substitution $\sigma = 4.25$, as in Broda and Weinstein (2006): over the 1999-2001 period they find average and median elasticities for SITC 5-digit goods of 13.1 and 2.7, respectively (see their Table IV).[9] Consistently with the literature, we choose the shape parameter of the Pareto distribution to be $k = 4.25$. Geographical and trade barriers are set to $\tau = 1.83$, as in Melitz and Redding (2015) and Irarrazabal et al. (2013). We consider trade between two symmetric countries, and choose labor in one country as the numeraire ($w = 1$), which implies that the wage in both countries is equal to one. We set L equal to the U.S. labor force; we normalize f_D and f_I to one and set $f_X = 0.545$ as in Melitz and Redding (2015).

[9]The value $\sigma = 4.25$ implies a mark-up of 31 percent.

Given our choice for the parameters $\{\sigma, k, f_D, f_I, f_X, \tau\}$, we choose f_M to ensure that the model generates the average fraction of U.S. manufacturing firms that export of 18 percent (Bernard et al., 2007), and η to match the volume of intra-firm sales as a percentage of total export (20.1 percent, as in Table 11 of Mataloni and Yorgason, 2006). In our baseline specification we compute the open economy equilibrium using our calibrated values of $f_M = 2.85$, $\eta = 0.33$, and trade costs of $\tau = 1.83$.

In Figure 1, we show the effects of adjusting variable trade costs from their calibrated value of $\tau = 1.83$ (vertical line) to $\tau \in [1, 2]$. For the sake of comparison, we also calibrate an economy where we shut-down intra-firm trade and multinational production, the percentage of exporting firms is 18 percent, and find $f_X = 0.545$ as in Melitz and Redding (2015).

Panel A displays the welfare gains from opening the closed economy to multinational production and intra-firm trade. As shown in Proposition 3, welfare gains from opening an economy to export are strictly higher when there is multinational production. In Panel B we plot welfare gains with multinational production relative to welfare gains with only exports. Welfare gains in both economies are decreasing in τ (Panel A), but they decrease faster in an economy with only exports. Panel C shows the relative (to $\tau = 1$) mass of active firms. An increase in variable trade costs raises the number of actives in both economies relative to the mass of actives for $\tau = 1$. But this number is strictly lower in a model with multinational production due to the higher domestic cutoff productivity. The number of active firms increases faster with τ when only exporting firms are considered. Panel D displays the domestic trade share λ, which is higher for an economy with only exporting firms. It increases faster with τ in a model with only exports because, when also multinational firms exist, the denominator of equation (5.1) is larger.

In Figure 2, we show the effects of adjusting the share of intra-firm trade from their calibrated value of 0.33 to $\eta \in [0, 1]$.[10]

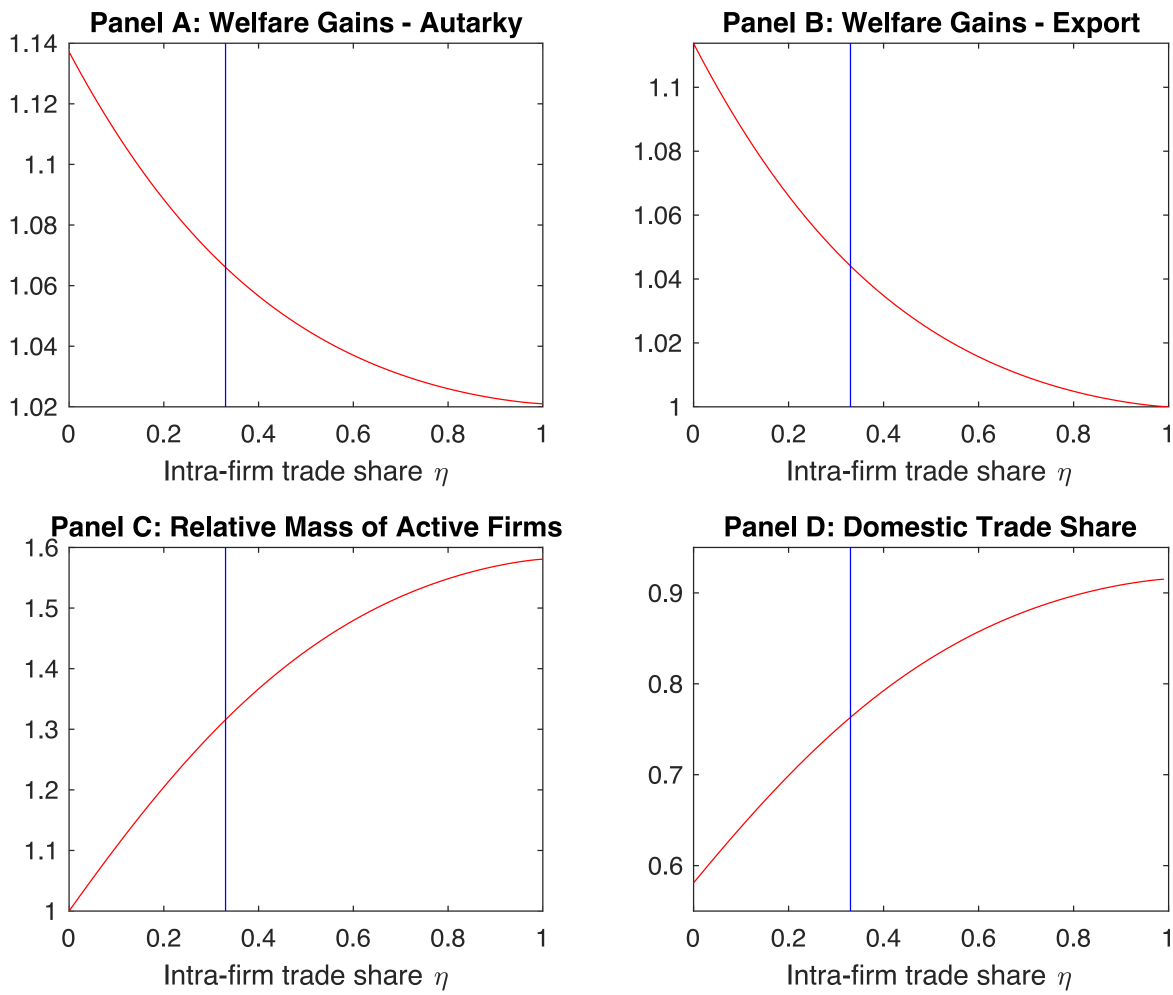

Figure 2: Increasing share of intra-firm trade

An increase in η acts similarly to an increase in variable trade costs which exclusively affects multinational firms, i.e. the most productive firms. Similarly to an increase in τ, the increase in η reduces average productivity by diminishing the level of competition in each market. This implies a larger amount of active firms in equilibrium (Panel C). Panel A shows that an increase in η reduces welfare gains because of the raise in the proportion of less productive firms. Panel B shows that welfare gains from allowing exporting firms

[10]Note that as η approaches 0, the economy converges to a situation where there is pure multinational production and no intra-firm trade; as η is close to 1, the economy converges to Helpman et al. (2004).

to produce in foreign countries are still decreasing in η. The insights are similar to these obtained from Panel A, as the denominator (welfare gains from export) does not depend on η. Panel D plots λ as an increasing function of η. As η goes towards 1, it becomes more and more difficult to do intra-firm trade, and therefore to undertake multinational production with the result of increasing domestic trade share.

7.1. Pure Multinational Production

We now consider an economy where $\eta = 0$, and all production is carried out in the host country.

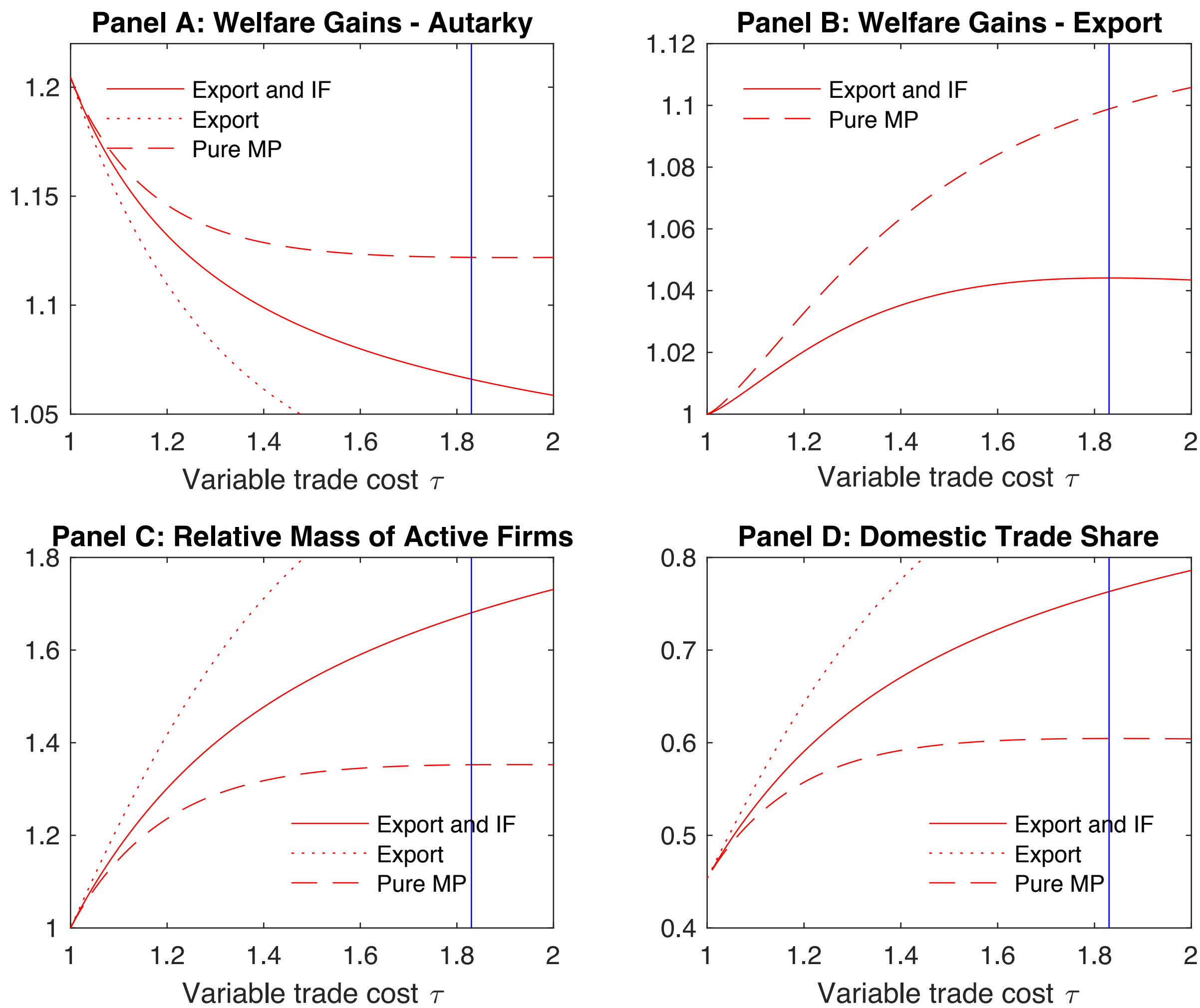

Figure 3: Variable trade costs

The percentage of exporting firms is calibrated to 18 percent, with $f_X = 0.545$ and

$f_M = 4.485$. Figure 3 compares the welfare gains of the economies considered in Figure 1 and a third economy where multinational firms shut down intra-firm imports from headquarters (i.e. $\eta = 0$ as in Helpman et al., 2004.)

Panel A shows that welfare gains in a pure multinational production economy are strictly higher than welfare gains in the other models. In all economies, welfare gains decrease as τ increases but remain higher for pure multinational production firms. This is related to Proposition 3, and it is related to the higher average domestic productivity required to enter the market and operate. Panel B plots the relative (to the export case) welfare gains. The figures underlines that, in comparison with an exporting economy, welfare gains with pure multinational production decrease at a lower rate than they do with intra-firm trade. This is related to the fact that here only exporters face trade frictions. A higher average productivity in a model with pure multinational production explains the lower relative mass of active firms (Panel C), and the lower domestic trade share (Panel D).

Table 1: Comparative Statics - Welfare Gains

	Baseline Calibration $\sigma = 4.25, k = 4.25$	$\sigma = 3.5$	$\sigma = 4.5$	$k = 3.75$	$k = 4.75$
	(1)	(2)	(3)	(4)	(5)
W_{MP}/W_A	1.066	1.049	1.072	1.095	1.047
W_{MP}/W_{Exp}	1.044	1.022	1.051	1.064	1.031
W_{PMP}/W_A	1.122	1.077	1.137	1.169	1.089
W_{PMP}/W_{Exp}	1.099	1.049	1.115	1.135	1.073

In Table 1 we show the results of another exercise in which welfare gains are computed varying σ and k. Welfare gains of a multinational production and a pure multinational production economies are computed with respect to a closed economy (W_A), and to an exporting economy (W_{Exp}). Welfare gains decrease when market power increases (lower σ), and the drop is higher when outsourcing is total (pure multinational production, W_{PMP}). A decrease in k, the shape parameter of the Pareto distribution, increases the productivity cutoff of domestic firms and raises welfare gains.

7.2. Trade Elasticities

Using equations (4.2) and (4.6), we now examine the quantitative implications for trade elasticities.

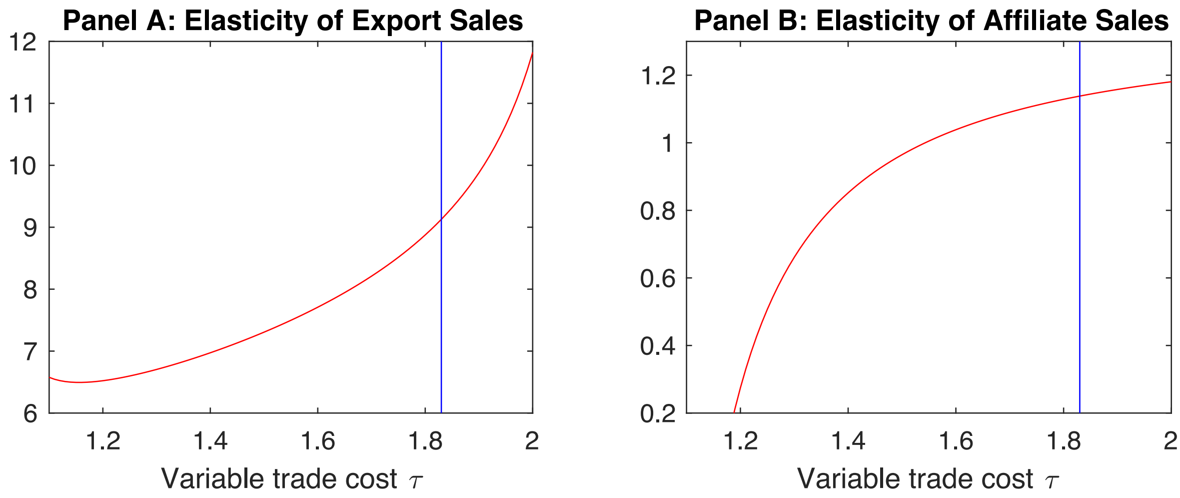

Figure 4: Variable trade costs

In Figure 4 we show the partial elasticity of export sales (Panel A), and the partial trade elasticity of affiliate sales (Panel B).[11] As we vary the variable trade costs from one to two, trade elasticity in Panel A ranges from less than seven to more than twelve. These numbers are in line with the findings of the empirical literature: Mejean and Imbs (2017) structural estimates' range from less than four to more than nine; and, Novy (2013) estimates' vary from less than five to more than twenty. As variable trade costs increase, the export productivity cutoff rises at a higher rate than intra-firm trade productivity

[11]Figure 4 reports absolute values of trade elasticities.

cutoff, which in turn increases the trade elasticity.[12] Panel B shows that affiliate trade elasticity rises at a decreasing rate, in line with the intra-firm productivity cutoff.

8. Conclusion

Our goal in this paper has been to evaluate welfare gains and trade elasticities from intra-firm trade in a general equilibrium model with export and multinational production. We have assumed that each foreign affiliate imports an intermediate input from the home country due to technological appropriability issues. This set up captures the interaction between alternative market access strategies, by allowing the knowledge-intensive input used in multinational production to move over geographical space. Therefore, geographical costs apply to both exports and multinational production because they involve transportation of a finished good and of an intermediate good, respectively. We have investigated the effects of an increase in trade barriers on multinational production. First, it increases the productivity cutoff: the need to import intermediate goods from the headquarter makes it more difficult to enter as a foreign affiliate when trade costs increase. Second, sales of the existing foreign affiliates decrease, which implies the existence of a new margin of adjustment for multinational firms.

An important theoretical result of the paper is that alternative market access strategies alter the standard results obtained for welfare in heterogeneous firm models, through a double truncated productivity distribution. Our model shows that with export and multinational production, the welfare gains from trade are affected by trade costs and trade elasticities are not constant anymore. This result is obtained by imposing an upper bound to the Pareto distribution.

[12] Equation (4.1) shows that the variable part of trade elasticity is a function of the difference between productivity cutoffs.

To quantitatively assess the country level gains from multinational production with intra-firm trade, we calibrate the model to match aggregate U.S. data. Our findings stress the role of intra-firm trade for additional welfare gains. Moreover, we exploit the delivered gravity equations to compare margins' sensitivity for exports and affiliate sales with respect to alternative models such as Chaney (2008) and Helpman et al. (2004). Our framework reaffirms the importance of trade costs and elasticity of substitution in models with firm heterogeneity, and shows that demand and supply parameters may affect trade elasticities even working with a standard productivity distribution.

Acknowledgments

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A. Appendix

A. Free Entry and the Price Index in the Symmetric Case

A.1 Free Entry

We describe the equilibrium which characterizes this economy. In order to do so, we need to specify some other equilibrium equations, namely the free-entry condition and the price index.

Free entry ensures equality between the expected operating profits of a potential entrant and the entry cost, $E(\pi) - f_I$. This condition holds for all types of firms. The cumulative density function is $G(a)$, with support: $[0, ..., a_0]$. The free-entry condition can be defined as

$$f_I = \int_0^{a_D} \pi_D dG(a) + 2\Big(\int_{a_M}^{a_X} \pi_X dG(a) + \int_0^{a_M} \pi_M dG(a)\Big) \tag{A.1}$$

Since we are in the symmetric case, we omit the subscripts i and j. Using the profit conditions (2.2)-(2.4), we obtain

$$\begin{aligned} f_I \;=\; & \int_0^{a_D} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_D\Big]\mathrm{d}G(a) + 2\int_{a_M}^{a_X} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\phi\beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_X\Big]\mathrm{d}G(a) \\ & +2\int_0^{a_M} \Big[\Big(\frac{\sigma}{\sigma-1}\Big)^{(1-\sigma)} \frac{\phi^{\eta}\beta E a^{1-\sigma}}{P^{1-\sigma}\sigma} - f_M\Big]\mathrm{d}G(a), \end{aligned} \tag{A.2}$$

where $\phi = \tau^{1-\sigma}$ is freeness of trade, and $P^{1-\sigma}$ is a weighted average of the marginal costs corrected for markups of all firms active in the market.

We now analyze in detail the term $P^{1-\sigma}$. In every country this weighted average, $P^{1-\sigma}$, is characterized by all of the brands offered in that particular country: the brands offered by domestic firms, for which the consumer price is $a\sigma/(\sigma-1)$; the brands offered by foreign exporters, for which the consumer price is $a\sigma\tau/\left(\sigma-1\right)$; and, finally, the brands supplied by foreign subsidiaries, with consumer price $a\sigma\left(\tau\right)^{\eta}/\left(\sigma-1\right)$. Therefore

$$\begin{aligned} P^{1-\sigma} &= \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} n\int_{0}^{a_D} a^{1-\sigma}\mathrm{d}G(a/a_D) + \\ & \left(\frac{\sigma}{\sigma-1}\right)^{\left(1-\sigma\right)} n\Big[\int_{0}^{a_M}\phi^{\eta}a^{1-\sigma}\mathrm{d}G(a/a_D) + \int_{a_M}^{a_X}\phi a^{1-\sigma}\mathrm{d}G(a/a_D)\Big] \end{aligned} \quad (A.3)$$

where n is the measure of varieties available in the country.

A.2 Parameterization: Pareto Distribution

The fact that the free-entry condition and the price index depend on the probability distribution implies that, in order to have explicit solutions for this model, we need to assume a particular functional form for $G(a)$. Following the empirical literature on firm-size distributions (see Axtell 2001 and Helpman et al. (2004)), we use the Pareto distribution as an approximation. The cumulative distribution function of a Pareto random variable a is $G(a)=(a/a_0)^k$, where k and a_0 are the shape and scale parameters, respectively. Note that $k=1$ implies a uniform distribution on $[0,a_0]$. The shape parameter k represents the dispersion of cost draws. An increase in k would imply less dispersion in the firm productivity-draws: the higher is k the less heterogeneity there is. We can now use this Pareto distribution to derive the price index and the free-entry condition.

As noted above, firms offer a price only if they have productivity of at least $1/a_D$. The cumulative distribution is hence defined over a support $[0,...,a_D]$. We can now solve the symmetric price index to obtain

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} \frac{n}{1-\frac{1}{b}} a_D^{1-\sigma}\left[1+T^{1-b}\left(\phi\right)^{b}+V^{1-b}\left[\left(\phi\right)^{\eta}-\phi\right]^{b}\right], \quad (A.4)$$

where $b = \frac{k}{\sigma-1}$; $T = f_X/f_D$ and $V = (f_M - f_X)/f_D$. In order for the integral to converge we assume that $b > 1$. Rewriting now the free entry condition in (A.1) using the Pareto distribution we obtain

$$\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} \frac{\beta E}{\sigma P^{1-\sigma}} \Big[\int_0^{a_D} a^{1-\sigma} \mathrm{d}G(a) + \int_0^{a_M} (\phi)^\eta a^{1-\sigma} \mathrm{d}G(a) + \int_{a_M}^{a_X} \phi a^{1-\sigma} \mathrm{d}G(a) \Big] \quad \text{(A.5)}$$
$$= f_D G(a_D) + \big(f_X G(a_X) - f_X G(a_M) + f_M G(a_M)\big) + f_I.$$

B. Aggregate Sales with Free Entry

Let's define the aggregate affiliate sales as

$$\begin{aligned} AS &= n^* \int_0^{a_M} a^{1-\sigma} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (\phi)^\eta \, dG(a/a_D) \\ &= n^* \phi^\eta \left(\frac{a_M}{a_D}\right)^k a_M^{1-\sigma} \frac{k}{k-\sigma+1} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}, \end{aligned} \quad \text{(A.6)}$$

where $A \equiv \frac{\beta E}{P^{1-\sigma}}$. Similarly, aggregate export sales are

$$\begin{aligned} X &= n^* \int_{a_X}^{a_M} \phi a^{1-\sigma} A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} dG(a/a_D) \\ &= n^* \phi A \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{k}{a_D^k} \frac{\left(a_X^{k-\sigma+1} - a_M^{k-\sigma+1}\right)}{k-\sigma+1}. \end{aligned} \quad \text{(A.7)}$$

Since in the above expressions we are conditioning on a_D, to find aggregate sales we simply multiplied by number of active n^*.

Intra-firm exports can be derived from the following Hicksian factor demand:

$$y_1^* = x^M a \eta \left(\frac{1}{\tau}\right)^{1-\eta}$$

Thus aggregate intra-firms exports are

$$IF = n^* B_j \frac{k}{k-\sigma+2} \frac{a_M^{k-\sigma+2}}{a_D^k} \tau^{\eta(1-\sigma)} \eta \left(\frac{1}{\tau}\right)^{1-\eta}, \tag{A.8}$$

where $B_j = \frac{A_j}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}$.

C. Number of MNF and Exporting Firms

The number of affiliates producing in each foreign country is given by

$$n_M = n^* \left(\frac{a_M}{a_D}\right)^k, \tag{A.9}$$

where n^* is the number of active firms and comes from equation (3.1). The number of exporting firms in each country is obtained from

$$n_X = n^* \left(\frac{a_X}{a_D}\right)^k. \tag{A.10}$$

D. Intensive and Extensive Margins of Export Sales

1) Rearranging the definition of intensive and extensive margins of exports we get

$$-\frac{\partial X^X}{\partial \tau}\frac{\tau}{X^X} = \underbrace{-\frac{\tau}{X^X}\left(n\int_{\overline{a}^M}^{\overline{a}} \frac{\partial x^X}{\partial \tau} dG(a)\right)}_{\text{Intensive Margin Elasticity}} \underbrace{-\frac{\tau}{X^X} n \left[x^X G'\left(\overline{a}\right)\frac{\partial \overline{a}}{\partial \tau} - x^M G'\left(\overline{a}^M\right)\frac{\partial \overline{a}^M}{\partial \tau}\right]}_{\text{Extensive Margin Elasticity}}. \tag{A.11}$$

Using the definition of equilibrium individual export sales, which is

$$x^X = p^X q^X = \frac{\sigma}{\sigma-1} a\tau\beta E \frac{(a\tau\frac{\sigma}{\sigma-1})^{-\sigma}}{P^{1-\sigma}} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} A(a\tau)^{1-\sigma}, \tag{A.12}$$

and assuming that country i is small enough so that $\partial P_j/\partial\tau \approx 0$, we get:

$$\begin{aligned}\frac{\partial x^X}{\partial\tau} &= (1-\sigma)\,\tau^{-\sigma}a^{1-\sigma}A\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \\ &= (1-\sigma)\,\frac{x^X}{\tau}.\end{aligned} \tag{A.13}$$

Therefore, the elasticity of the intensive margin of export with respect to the variable costs is:

$$\begin{aligned}\varepsilon^X_{I,\tau} &= -\frac{\tau}{X^X}\left(w_iL_i\int_{\bar{a}^M}^{\bar{a}}\frac{\partial x^X}{\partial\tau}dG(a)\right) \\ &= -(1-\sigma)\,\frac{\tau}{X^M}\frac{w_iL_i\int_{\bar{a}^M}^{\bar{a}}x^XdG(a)}{\tau} \\ &= (\sigma-1)\,,\end{aligned} \tag{A.14}$$

which is identical to the elasticity in Chaney (2008).

2) In order to derive the extensive margin of trade we need to use the equilibrium productivity thresholds from (3.3) and (3.4). Deriving these thresholds with respect to τ we find:

$$\frac{\partial\bar{a}}{\partial\tau} = -\frac{\bar{a}}{\tau}, \tag{A.15}$$

and

$$\frac{\partial\bar{a}^M}{\partial\tau} = -\frac{a_M}{k}\frac{(\sigma-1)(\eta\tau^{(1-\sigma)\eta-1}-\tau^{(-\sigma)})}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}. \tag{A.16}$$

Rewriting the equation for firm level exports in (A.12), we obtain

$$\begin{aligned}x^X &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}A(a\tau)^{1-\sigma} \\ &= \lambda^Xa^{1-\sigma},\end{aligned} \tag{A.17}$$

and similarly for firm level affiliate sales

$$
\begin{aligned}
x^X &= \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} A(a^{1-\sigma}(\tau)^{\eta(1-\sigma)} \\
&= \underbrace{\lambda^X \left(\tau^{(1-\eta)(1-\sigma)}\right)^{-1}}_{=\lambda^M} a^{1-\sigma}. \qquad \text{(A.18)}
\end{aligned}
$$

Then since the Pareto distribution assumption implies that $G'(\overline{a}) = k(\overline{a})^{k-1}$, we can rewrite the aggregate export sales in the following way:

$$
\begin{aligned}
X^X &= n\int_{\overline{a}^M}^{\overline{a}} x^X dG(a) \\
&= n\int_{\overline{a}^M}^{\overline{a}} \lambda^X a^{1-\sigma} k a^{k-1} da \\
&= n\,(k/(k-\sigma+1)\left[\lambda^X \overline{a}^{k-\sigma+1} - \lambda^X (\overline{a}^M)^{k-\sigma+1} (\overline{a}^M)^k\right] \\
&= \frac{n}{k-\sigma+1} x^X G'(\overline{a}) a - \frac{n}{k-\sigma+1} x^M \tau^{(1-\eta)(1-\sigma)} G'(\overline{a}^M) a^M \qquad \text{(A.19)}
\end{aligned}
$$

where we used the relationship between λ^X and λ^M highlighted in equation (A.18). Then using equation (A.19), we can derive the elasticity of the extensive margin of exports:

$$
\begin{aligned}
\varepsilon^X_{E,\tau} &= -\frac{\tau}{X^X} n\left[x^X G'\left(\overline{a}\right)\frac{\partial \overline{a}}{\partial \tau} - x^M G'\left(\overline{a}^M\right)\frac{\partial \overline{a}^M}{\partial \tau}\right] \\
&= -\frac{\tau}{X^X} w_i L_i \left[x^X G'\left(\overline{a}\right)\left(-\frac{\overline{a}}{\tau}\right) - x^M G'\left(\overline{a}^M\right)\frac{\Gamma}{\tau}\right]. \qquad \text{(A.20)}
\end{aligned}
$$

then since

$$
n x^M G'\left(\overline{a}^M\right)\overline{a}^M = \left(k-\sigma+1\right) X^M, \qquad \text{(A.21)}
$$

The above expression allows us to rewrite equation (A.19) to obtain:

$$
X^X + \frac{n}{k-\sigma+1} x^M \tau^{(1-\eta)(1-\sigma)} G'(\overline{a}^M) a^M = \frac{n}{k-\sigma+1} x^X G'(\overline{a}) a \qquad \text{(A.22)}
$$

and using equation (A.35) we find:

$$nx^X G'(\overline{a})\,\overline{a} = (k-\sigma+1)\left[X^X + \tau^{(1-\eta)(1-\sigma)}X^M\right]. \tag{A.23}$$

The expressions in (A.23) can now be plugged in equation (A.20), to find a more compact expression for $\varepsilon^X_{E,\tau}$. This yields:

$$\begin{aligned}
\varepsilon^X_{E,\tau} &= -\frac{\tau}{X^X}\Bigg[(k-\sigma+1)\left[X^X + \tau^{(1-\eta)(1-\sigma)}X^M\right]\left(-\frac{1}{\tau}\right) \\
&\quad -\frac{k-\sigma+1}{k}X^M\frac{\tau}{\tau}\frac{(1-\sigma)(\eta\tau^{(1-\sigma)\eta-1}-\tau^{-\sigma}}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}\Bigg] \\
&= -(k-\sigma+1)\left[\frac{X^M}{X^X}(\Gamma-\omega)-1\right],
\end{aligned} \tag{A.24}$$

where we used

$$\Gamma = \frac{\eta\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}{b(\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)})}, \tag{A.25}$$

$$\omega = \tau^{(1-\eta)(1-\sigma)}. \tag{A.26}$$

Notice that $\Gamma > \omega$ is true for certain parameter restrictions consistent with our calibration exercise.

We can conclude that

$$\text{if } X^M > X^X \longrightarrow \varepsilon^X_{E,\tau} < 0, \tag{A.27}$$

$$\text{if } X^M < X^X \longrightarrow \varepsilon^X_{E,\tau} > 0. \tag{A.28}$$

Finally, combining (A.14) with (A.24) gives equation (4.2).

E. Intensive and Extensive Margins of Affiliate Sales

1) Rearranging the definition of intensive and extensive margins of affiliate sales in equation (4.5), we get

$$-\frac{\partial X^M}{\partial \tau}\frac{\tau}{X^M} = \underbrace{-\frac{\tau}{X^M}\left(n\int_0^{\overline{a}^M}\frac{\partial x^M}{\partial \tau}dG(a)\right)}_{\text{Intensive Margin Elasticity}}\underbrace{-\frac{\tau}{X^M}\left(nx^M G'\left(\overline{a}^M\right)\frac{\partial \overline{a}^M}{\partial \tau}\right)}_{\text{Extensive Margin Elasticity}}. \tag{A.29}$$

Using the definition of equilibrium individual affiliate sales, which is:

$$x^M = p^M q^M = \frac{\sigma}{\sigma-1}a\tau^\eta \beta E\frac{(a\tau^\eta\frac{\sigma}{\sigma-1})^{-\sigma}}{P^{1-\sigma}} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}A(a^{1-\sigma}(\tau)^{\eta(1-\sigma)}, \tag{A.30}$$

and assuming that the country is small enough so that $\partial P/\partial\tau \approx 0$, we get:

$$\frac{\partial x^M}{\partial \tau} = \eta\left(1-\sigma\right)\frac{x^M}{\tau}. \tag{A.31}$$

Therefore, the elasticity of the intensive margin of affiliate sales with respect to the variable costs is:

$$\begin{aligned}\varepsilon^M_{I,\tau} &= -\frac{\tau}{X^M}\left(n\int_0^{\overline{a}^M}\frac{\partial x^M}{\partial \tau}dG(a)\right)\\ &= \eta\left(\sigma-1\right).\end{aligned} \tag{A.32}$$

2) Using the definition of the equilibrium productivity threshold from (3.4), we find:

$$\frac{\partial \overline{a}^M}{\partial \tau} = \frac{a_M}{k}\frac{(1-\sigma)(\eta\tau^{(1-\sigma)\eta-1}-\tau^{(-\sigma)})}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}} \tag{A.33}$$

We now rewrite the equation for firm level affiliate sales in (A.30), as

$$x^M = \lambda^M a^{1-\sigma}. \tag{A.34}$$

Then, since the Pareto distribution assumption implies that $G'(a) = ka^{k-1}$, the aggregate affiliate sales equation becomes:

$$\begin{aligned} X^M &= n\int_0^{\overline{a}^M} x^M dG(a) \\ &= nx^M G'\left(\overline{a}^M\right)\frac{\overline{a}^M}{k-\sigma+1}, \end{aligned} \tag{A.35}$$

where we used the fact that $\overline{a}^M G'\left(\overline{a}^M\right) = k(\overline{a}^M)^k$. Using equation (A.35), we can find a solution for the elasticity of the extensive margin:

$$\begin{aligned} \varepsilon^M_{E,\tau} &= -\frac{\tau}{X^M}\left(nx^M G'\left(\overline{a}^M\right)\frac{\partial \overline{a}^M}{\partial \tau}\right) \\ &= \frac{(k-\sigma+1)}{k}\frac{(\sigma-1)(\eta\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)})}{\tau^{(1-\sigma)\eta}-\tau^{(1-\sigma)}}. \end{aligned} \tag{A.36}$$

Therefore, putting together (A.32) and (A.36) gives (4.6).

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