Intergenerational transfers, tax policies and public debt

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May 9, 2018

Abstract

This paper studies the impact of public debt on intergenerational family transfers and on human capital growth, in a successive generation model of a closed economy, in which parents augment their children's income through education and bequests. We limit ourselves to simple tax structures with labor and bequest taxes. When public debt is an available instrument for the government, we show that the fiscal policy used to achieve the long run optimal endogenous growth improves the individuals' consumption of the first generations. In this case, the government reduces the tax burden on labor, encourages human capital development and implements a redistributive policy. If the public debt is not available, the government cannot completely satisfy these objectives such that the two taxes do not fully implement the intergenerational redistributive policy and the long run human capital growth is higher. In all cases, the optimal bequest tax rate is higher than the optimal tax rate on labor income.

Keywords: family transfers, debt, altruism, growth, optimal taxation.
JEL classifications: D64, H21, H23, H63.

1 Introduction

Intergenerational family transfers in the form of bequests and education expenditures have an important influence on physical and human capital investments (Laitner and Juster, 1996). Both investments are engines of economic growth involving an increase in disposable resources for future generations. Endogenous growth models address the scenario in which human capital development induces positive externalities and spillover effects on economic growth. Further, public transfers in the form of public debt can be used to transfer these new resources from future generations to the present one, and distortive fiscal instruments can be implemented to internalize the external contribution of human capital investment to the production sector. Thus, public debt can be used by governments to implement an intergenerational redistributive policy. However, in many countries today, the pressure on public finance can make public debt unavailable. When public debt is not available, the intergenerational redistribution has to be conducted through other channels. When this occurs and the governments want to redistribute some resources from future generations to the first generation, they can only used tax instruments to correct externalities and arbitrate between generations. In this context, we address the issue of the best tax policy that allows simultaneously, on the one hand enhance physical capital and human capital investments and on the other hand redistribute.

The effectiveness of public debt in stimulating economic activity relies on the span of the forecasting horizons of households. Through intergenerational family transfers, families may be able to neutralize any attempt by the government to redistribute resources among generations as in the Barro (1974) model. This offsetting of public by private transfers involves the neutrality of public debt referred to as Ricardian equivalence. Intergenerational altruism extends the planning horizon of economic agents. In models of household's rational altruism *à la* Barro (1974), dynasties want to smooth their consumption over time through intergenerational family transfers, such as education and bequests. Dynastic households are the effective decision makers since they are as long lived as the government involving public debt neutrality. However, there exist a number of other bequest motives differently affecting the optimal fiscal policy (Cremer and Pestieau, 2011) in which the Ricardian equivalence may be rejected.

Therefore, the motives for intergenerational transfers are crucial to study the effect of public debt availability on economic activity. In Barro (1974), there is an altruistic feeling among generations. In our study, we consider family altruism, a less drastic approach than dynastic altruism *à la* Barro (1974), in which public debt is not neutral (see Becker (1991) or Mankiw (2000)). Lambrecht et al. (2006) implement this kind of bequest, wherein parents care only about their children's income and not about their utility levels. Halfway between a pure altruistic bequest *à la* Barro (1974) and a pure life cycle model *à la* Diamond (1965), the concept of family altruism involves the non-neutrality of the public debt. In addition, an empirical study from Arrondel et al. (1997) shows that family altruism is a significant fraction of bequest motives and that taking this perspective is a less drastic approach than rational altruism.

Various authors have highlighted the crucial importance of human capital investment in contributing to economic growth (see Romer (1986) and Lucas (1988)). In growth models, the latter is closely related to households' education levels. The human capital of individuals relies on their parents' knowledge and their parents' educational spending since these individuals are not able to self-finance their educations. Regardless of whether bequests take place, familial transfers of human capital are thus a significant altruistic behavior that impacts future generations and economic activity (Glomm and Ravikumar, 1992). Some authors, such as Lambrecht et al. (2005), consider transfers of education expenditures and bequests to analyze the effectiveness of fiscal policy in stimulating economic activity. Drazen (1978) shows that parents invest in education rather than bequest until the education return corresponds to the interest rate. The parents' trade-off between both transfers creates inequalities across agents and generations in accordance with their preferences and lifetime resources.

In this paper, we consider an endogenous growth model that includes both transfers, education and bequests. We investigate the impact of public debt availability on intergenerational family transfers and on optimal long-run economic growth, in a successive generation model of a closed economy, in which parents augment their children's income through education and bequest transfers. Human capital growth corresponds to the representative agent's improvement in education level, which depends on the parents' education expenditure and accumulated dynastic knowledge. Family altruism involves a positive externality of parents' education spending on all future generations. We assume that a fraction of production must be devoted to public spending necessary for the good development of the economy, such as justice or defence. The two available taxes are the ones affecting directly the agents' trade-offs between both transfers. Thus, the bequest tax, labor tax and public debt are the only available fiscal instruments to finance public spending, correct externalities and implement a redistributive policy. Public debt is required to achieve optimal human capital growth along the balanced growth path and to implement an intergenerational redistribution policy. Thanks to positive public debt, the tax burden on both labor and capital can be adjusted, and the government can promote human capital development and improve the household consumption of first generations. Otherwise, when public debt is not available, the social planner internalizes the positive human capital externality and pursues a redistributive policy by using lower bequest tax and labor tax but with a higher gap between both. This creates an over-investment in education and reduces the capital-labor ratio of each generation leading to increase the individual's consumption of first generation. However, this tax policy does not allow to achieve the first-best objective with respect to redistribution.

In Section 2, the framework and the dynamics of the model are developed. Section 3 analyzes the first-best optimum. Then, in Section 4, we present the second-best optimum. A numerical illustration is used to show that the transition dynamics jumps to the optimal solutions along the balanced growth path, in both cases. The final Section concludes.

2   The model

We consider one dynasty composed of successive generations of individuals in a closed economy. Each generation lives for one period and gives birth to a child. In addition, we concentrate on dynastic family altruism, meaning that parents care about their children's income.

2.1   Households

The representative household of generation t works, consumes and leaves intergenerational transfers to increase his offspring's disposable income. For this purpose, he invests in his child's education and leaves bequests to generation $t + 1$, which are represented by e_{t+1} and x_{t+1}, respectively. In our model, bequests are the only motive for saving.

The labor supply is inelastic, and an individual's labor income depends on his human capital level. We focus on a private education regime in which parent's investment in their child's education e_{t+1}, as well as accumulated knowledge from the dynasty H_t, characterizes the child's human capital:

$$H_{t+1} = G\left(e_{t+1}, H_t\right), \, t \geq 0 \tag{1}$$

The fact that parent's knowledge influences child's human capital is consistent with a number of empirical studies, such as Hertz et al. (2007). Parents have to invest first in child's education in order to provide a positive human capital to their child. Thus, both human capital factors are imperfect substitutes. A Cobb-Douglas human capital function is used to represent the human capital technology:

$$G\left(e_{t+1}, H_t\right) = B(e_{t+1})^{\delta}(H_t)^{1-\delta}$$

where B is a strictly positive technological parameter and $\delta \in (0, 1)$ represents the responsiveness of a child's human capital to a change in the parent's education spending.

The individual's resources come from two sources: work and bequest. The total after tax lifetime income is represented by Ω_t:

$$\Omega_t = (1 - \tau_t^B)R_t x_t + (1 - \tau_t^L)w_t H_t \tag{2}$$

where w_t is the real wage, R_t is the gross interest rate and x_t is the bequest received from his parent. τ_t^B and τ_t^L are the respective period-t tax rates on labor income and bequests. The individual born in t receives after tax labor income $(1-\tau_t^L)w_t H_t$ and after tax bequest $(1-\tau_t^B)R_t x_t$. These resources are allocated to consumption c_t, bequest x_{t+1} and the child's education expenditure e_{t+1}:

$$\Omega_t = c_t + e_{t+1} + x_{t+1} \tag{3}$$

In this framework, the parent derives utility from his offspring's resources. Unlike the joy-of-giving

formulation, this form of altruism allows a trade-off between bequest and education spending driven by relative returns. Individual's preferences are represented by a logarithmic utility function that depends on consumption c_t and child's disposable income Ω_{t+1}:

$$u_t = \ln c_t + \gamma \ln \Omega_{t+1} \tag{4}$$

where $\gamma > 0$ is the intergenerational degree of altruism. The parent makes a trade-off between his own consumption and both family transfers. Since individual's level of human capital depends on his parent's human capital, every additional unit of education spending increases child's human capital and that of generations to come. However, the parent's transfer decision does not consider the impact of human capital on subsequent generations given the feature of family altruism.

Plugging (2)-(3) into (4) gives individual's utility as a function of education expenditure e_{t+1} and bequest x_{t+1}. The representative individual maximizes his utility with respect to these two variables. This leads to the following first-order conditions:

- with respect to e_{t+1}, for $t \geq 0$,

$$-\frac{1}{c_t} + (1 - \tau_{t+1}^L) w_{t+1} G_e'(e_{t+1}, H_t) \frac{\gamma}{\Omega_{t+1}} = 0 \tag{5a}$$

- with respect to x_{t+1}, for $t \geq 0$,

$$-\frac{1}{c_t} + (1 - \tau_{t+1}^B) R_{t+1} \frac{\gamma}{\Omega_{t+1}} = 0 \text{ if } x_{t+1} > 0, \ \leq 0 \text{ otherwise} \tag{5b}$$

Since the after tax marginal return of education expenditure is decreasing and close to infinite when the education level is equal to zero, education spending constraint for an interior solution is necessarily satisfied at equilibrium (involving equation (5a)). The individual first invests in human capital until the marginal return of education expenditure corresponds to that of bequest. Both rates of return are equal with operative bequest.

2.2 Production

The production sector consists in a representative firm that behaves competitively, and produces a homogeneous good with physical capital K_t and efficient labor L_t. The production function $F(K_t, L_t)$ is linear homogeneous and concave. Profit maximization implies that factor prices w_t and R_t are equal to their marginal products:

$$w_t = F_L'(K_t, L_t) \tag{6a}$$

$$R_t = F_K'(K_t, L_t) \tag{6b}$$

assuming total depreciation of the physical capital stock in one period. F'_L and F'_K stand for the partial derivatives of F with respect to efficient labor and physical capital, respectively. We use a Cobb-Douglas production function with the following form:

$$F(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha} \tag{7}$$

where A is a strictly positive technological parameter and $0 < \alpha < 1$.

2.3 Government

In each period, the government incurs an amount of public spending that corresponds to a fraction Γ of total production. The available fiscal instruments are the bequest tax τ_t^B, the labor income tax τ_t^L and the public debt Δ_t with one period of maturity debt. The government's budget constraint at time t is:

$$\Gamma F(K_t, L_t) + R_t \Delta_{t-1} = \tau_t^B R_t x_t + \tau_t^L w_t L_t + \Delta_t \tag{8}$$

When public debt is not available, using the Cobb-Douglas production function (7), equation (8) can be rewritten as follows:

$$\Gamma = \tau_t^B \alpha + \tau_t^L (1 - \alpha) \tag{9}$$

2.4 Equilibrium

At equilibrium, household's bequest is divided into private capital and public debt:

$$\Delta_t + K_{t+1} = x_{t+1} \tag{10}$$

The assumption that government spending is proportional to production involves an externality created by the level of production. Considering the equilibrium on market for goods and taking account of the equilibrium on labor market which involves that $L_t = H_t$, we get in period t:

$$c_t + e_{t+1} + K_{t+1} = (1 - \Gamma) F(K_t, H_t) \tag{11}$$

The social marginal product of physical capital $(1-\Gamma) F'_K (K_t, H_t)$, respectively human capital $(1-\Gamma) F'_H (K_t, H_t)$, is lower than the private marginal product $F'_K (K_t, H_t)$ (resp. $F'_H (K_t, H_t)$) as stated by the first-order conditions of the representative firm (6a) and (6b). If the government sets the tax rates on bequest and labor income to $\tau_t^B = \tau_t^L = \Gamma$, these externalities are assimilated by the private sector.

2.5 Intertemporal equilibria with constant tax rates

We concentrate on intertemporal equilibria where bequests are positive. Tax rates on labor income τ_t^L and bequest τ_t^B are assumed to be constant for $t \geq 1$. At the initial period, τ_0^L and τ_0^B are lump-sum taxes since both taxes only affect the individual's resources. Both tax values implemented, are decisive for the initial public debt condition. Following this period, both taxes has an impact on the trade-off between both transfers. From the first-order conditions (5), the private rate of return on education is equal to the private rate of return on capital:

$$(1 - \tau^B)R_{t+1} = (1 - \tau^L)w_{t+1}G'_e(e_{t+1}, H_t) \tag{12}$$

Let $\eta_{t+1} \equiv \frac{e_{t+1}}{H_t}$ denotes the individual's education spending per unit of human capital. At equilibrium, from equation (12), we get:

$$\eta_{t+1} = (\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} \tag{13}$$

where $\rho_{t+1} \equiv \phi_{t+1}\frac{w_{t+1}}{R_{t+1}} = \phi_{t+1}\frac{1-\alpha}{\alpha}k_{t+1}$ is the after-tax ratio of factor prices, $k_{t+1} \equiv \frac{K_{t+1}}{H_{t+1}}$ is the capital-labor ratio and ϕ_{t+1} corresponds to the ratio $\frac{1-\tau_{t+1}^L}{1-\tau_{t+1}^B}$. As we focus on constant tax rates along the dynamics, the ratio $\phi_{t+1} = \phi$. A higher ρ_{t+1} involves that the return on human capital $(1-\tau^L)w_{t+1}$ increases relative to the one on physical capital $(1-\tau^B)R_{t+1}$. Thus, after an increase of ρ_{t+1}, the individual is incited to invest in education rather than bequest increasing the human capital growth rate $g_{t+1}^H \equiv \frac{H_{t+1}}{H_t}$. From equation (1), the human capital growth rate in period t, corresponds to:

$$g_{t+1}^H = B(\eta_{t+1})^\delta = B(\rho_{t+1}B\delta)^{\frac{\delta}{1-\delta}} \tag{14}$$

From equation (2), the child's disposable income per unit of human capital writes:

$$\frac{\Omega_{t+1}}{H_t} = (1 - \tau^B)R_{t+1}\left(\rho_{t+1}g_{t+1}^H + \frac{x_{t+1}}{H_t}\right) \tag{15}$$

And using equation (3), we get the consumption per unit of human capital:

$$\frac{c_t}{H_t} = (1 - \tau^B)R_t\left(\rho_t + \frac{x_t}{H_t}\right) - \eta_{t+1} - \frac{x_{t+1}}{H_t} \tag{16}$$

Then, equation (5b) gives simple expression for bequest per unit of human capital (using (15) and (16)):

$$\frac{x_{t+1}}{H_t} = \frac{1}{1+\gamma}\left[\gamma(1-\tau^B)R_t\left(\rho_t + \frac{x_t}{H_t}\right) - \left(\frac{1+\gamma\delta}{\delta}\right)\eta_{t+1}\right] \tag{17}$$

2.5.1 Dynamics

We combine the capital market equilibrium (10) and equation (17) (using the Cobb-Douglas production function (7)):

$$d_{t+1} + g_{t+1}^H k_{t+1} = \frac{1}{1+\gamma} \left[\gamma \left(1 - \tau^B\right) A\alpha k_t^{\alpha-1} \left(\rho_t + \frac{d_t}{g_t^H} + k_t\right) - \left(\frac{1+\gamma\delta}{\delta}\right) \eta_{t+1} \right] \tag{18}$$

where $d_{t+1} = \frac{\Delta_t}{H_t}$ is the public debt per unit of human capital. In addition, using the capital market equilibrium (10), we can rewrite the government budget constraint (8) as follows:

$$d_{t+1} = \left(1 - \tau^B\right) A\alpha k_t^{\alpha-1} \frac{d_t}{g_t^H} + A k_t^\alpha \left[\Gamma - \tau^B \alpha - \tau^L (1-\alpha)\right] \tag{19}$$

Since the after-tax ratio of factor prices ρ_t is linear with respect to capital labor ratio k_t, it is equivalent to analyze its dynamics. From equations (18) and (19) and using equations (13) and (14), we get the two-dimensional dynamics of $(\rho_{t+1}, d_{t+1})_{t\geq 0}$:

$$(\rho_{t+1} B\delta)^{\frac{1}{1-\delta}} = \frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^\alpha}{D} \rho_t^\alpha \left[\gamma\left(1-\Gamma\right) - E - \frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}}\right] \tag{20a}$$

$$d_{t+1} = A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\rho_t\right)^\alpha \left[\frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}} + E\right] \tag{20b}$$

where $D = \frac{1}{\delta}\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta\right) > 0$, $J = \delta\left(1-\tau^L\right)(1-\alpha) > 0$ and $E = \Gamma - \tau^B \alpha - \tau^L(1-\alpha)$. The initial human capital $\overline{H}_{-1}$ and $\overline{H}_0$, physical capital $\overline{K}_0$ and public debt $\overline{\Delta}_{-1}$ are given and consequently the initial bequest $\overline{x}_0 = \overline{K}_0 + \overline{\Delta}_{-1}$. Then, $\overline{\rho}_0$ and $\overline{d}_0$ are given since $\overline{\rho}_0 = \phi\frac{1-\alpha}{\alpha}\frac{\overline{K}_0}{\overline{H}_0}$ and $\overline{d}_0 = \frac{\overline{\Delta}_{-1}}{\overline{H}_{-1}}$. Notice that the right-hand side (RHS) of equation (20a) needs to be positive (i.e. $\gamma\left(1-\Gamma\right) - E - \frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}} > 0$) in order to get a positive level of education expenditures per unit of human capital η_{t+1} (see equation (13)) as well as positive human capital growth g_{t+1}^H (see equation (14)).

From the system of equations (20), we deduce the two-dimensional dynamics of $(X_{t+1}, \rho_{t+1})_{t\geq 0}$, where $X_{t+1} = d_{t+1}\left(\rho_{t+1} B\delta\right)^{\frac{-1}{1-\delta}}$. Thus, we get the following system of equations:

$$X_{t+1} = \psi\left(X_t\right) \equiv D\left(\frac{JX_t + E}{\gamma\left(1-\Gamma\right) - E - JX_t}\right) \tag{21a}$$

$$\rho_{t+1} = \frac{1}{B\delta}\left(\frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^\alpha}{D}\rho_t^\alpha\left[\gamma\left(1-\Gamma\right) - E - JX_t\right]\right)^{1-\delta} \tag{21b}$$

where $\overline{X}_0 = \overline{d}_0\left(\overline{\rho}_0 B\delta\right)^{\frac{-1}{1-\delta}}$ is given. As shown in Appendix, the function $\psi\left(X_t\right)$ is strictly increasing

and strictly convex if $JX + E < \gamma(1 - \Gamma)$, which is a necessary condition to get positive human capital growth g_{t+1}^H and defines an upper bound on X above which production become zero in finite time. The autonomous backward dynamics of X_t, for $t \geq 0$, is monotonous.

When public debt is not used to finance public spending (*i.e.* $E = 0$ or equivalently $\Gamma = \tau^B \alpha + \tau^L(1 - \alpha)$), we show in Appendix that two steady states are achievable: $\tilde{X} \equiv \frac{\gamma(1-\Gamma)}{J} - D$ and $X = 0$. If $X_0 < 0$ (*i.e.* with an initial public capital accumulation $d_0 < 0$), the dynamics of X_t converges towards a stable negative balanced growth path $\tilde{X}$. The other steady state $X = 0$ is unstable. Otherwise, if $X_0 > 0$ (*i.e.* with an initial public debt $d_0 > 0$), a zero production is reached in finite time, since the dynamic of X_t goes beyond its upper bond. Figure 1 illustrates this dynamics.

Then, as described in Figure 1, the curve of function $\psi(X_t)$ shifts to left with $E > 0$. According to the magnitude of E, there are two, one or zero steady state and all of which negative. When there are two steady states, X_t reaches a locally stable negative balanced growth path as soon as X_0 is below the unstable one. Otherwise, the dynamics leads to zero production as seen previously.

Figure 1: The dynamics of X depending on the sign of E
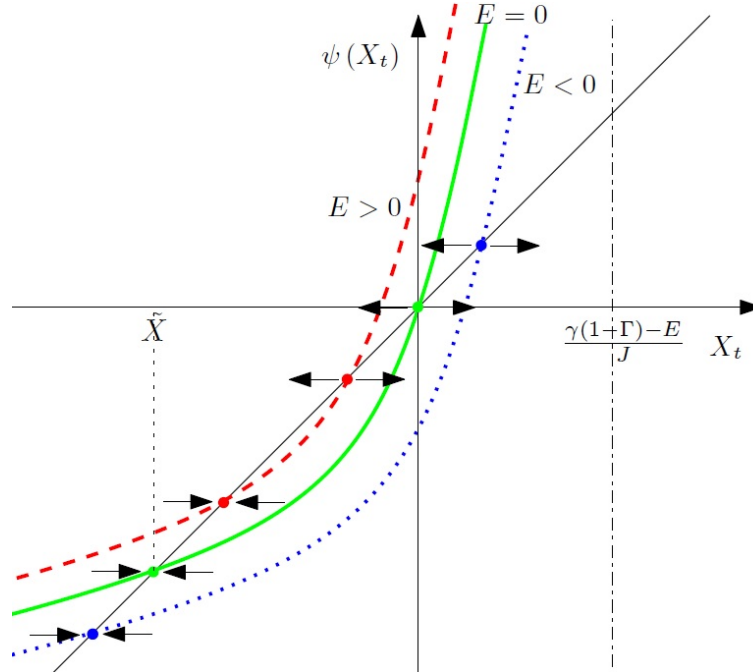
Lastly, the curve of function $\psi(X_t)$ shifts to right with $E < 0$ (see Figure 1). In this case, there are two steady states, one negative and one positive if it is below $\frac{\gamma(1-\Gamma)-E}{J}$. The dynamics of X_t converges towards a locally stable negative balanced growth path as soon as X_0 is below the unstable one. Above the unstable steady-state, a zero production is reached in finite time.

As a result, in each case, the dynamics of X_t may converge only towards a stable negative balanced growth path as soon as X_0 is below the unstable one. This implies a steady-state public capital

accumulation (*i.e.* $d < 0$). The only way to have a positive or zero public debt along the balanced growth path is that government implements, in the first period, the fiscal policy which jumps directly to unstable steady-state. There are two different situations of unstable steady-state based on whether governments use a positive public debt or not:

(i) At the unstable steady-state, we have $X = 0$ (*i.e.* $d_t = 0$ and $E = 0, \forall t \geq 1$). Given the government budget constraint (8), The initial tax instruments τ_0^B and τ_0^L are chosen such that $d_t = 0, \forall t \geq 1$, regardless of initial public debt d_0. Tax instruments satisfy $\Gamma = \tau^B \alpha + \tau^L (1-\alpha)$ for the following periods (*i.e.* $E = 0$).

(ii) At the unstable steady-state, we have $X > 0$ (*i.e.* $d_t > 0$ and $E < 0, \forall t \geq 1$). The initial tax instruments τ_0^B and τ_0^L is used by government to jump directly on the positive unstable balanced growth path (*i.e.* $X > 0$), where $d_t > 0$ and $E < 0, \forall t \geq 1$.

Whether or not, the dynamics of X_t jumps on unstable balanced growth path ($X > 0$ or $X = 0$) or converges towards a stable negative one, equation (21b) shows that ρ_t converges towards a strictly positive balanced growth path. Indeed, the RHS of equation (21b) is increasing and concave. The slope of the RHS of equation (21b) tends to infinity when ρ approaches zero and to zero when ρ approaches infinity. At steady state, we deduce from equation (21b), the following after-tax ratio of factor prices:

$$\rho = \left(\frac{1}{B\delta}\right)^{\frac{1}{1-\alpha(1-\delta)}} \left(\left(\frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^\alpha}{D}\right) [\gamma(1-\Gamma) - E - JX]\right)^{\frac{1-\delta}{1-\alpha(1-\delta)}}$$

As shown by the last equation, the after-tax ratio of factor prices is negatively affected by X. Thus, a negative X along the balanced growth path, leads to an higher ρ which implies a higher human capital growth g^H(see equation (14)). In contrast, a positive X involves a lower human capital growth g^H.

Since, in our paper, we analyze the effect of public debt constraint on steady state human capital growth and on the intergenerational family transfers, we focus on steady states with zero or positive public debt. Consider situation which jumps directly on positive X in period 0. In this case, $E < 0$ and the non-negativity constraint of bequests is satisfied since $d_t > 0, \forall t \geq 1$. Indeed, given the capital market equilibrium (10), a sufficient condition to satisfy the non-negativity constraint on bequests is to assumed a positive or zero public debt. The implementation of positive public debt has a negative effect on the after-tax ratio of factor prices ρ. Lower ρ decreases the capital-labor ratio k since $\rho = \phi\frac{1-\alpha}{\alpha}k$. As a result, implementing a positive public debt has a negative effect on education expenditure (*i.e.* $\eta = (\rho B\delta)^{\frac{1}{1-\delta}}$) and on human capital growth along the balanced growth path (*i.e.* $g^H = B\eta^\delta$). Thus, the human capital growth is higher without public debt (*i.e.* $X = 0$ and $E = 0$). We concentrates on this situation in the next Subsection.

2.5.2 Intertemporal equilibrium without public debt (*i.e.* $X = 0$ and $E = 0$)

In the following, we focus on intertemporal equilibria where bequests are positive, tax instruments are constant over time and public debt is not used to finance public spending (*i.e.* $E = 0$). The government implements τ_0^B and τ_0^L such that the dynamics of X_t jumps directly to the balanced growth path of $X = 0$ (*i.e.* $E = 0$ and $d_t = 0$ for $t \geq 1$). From equation (21b) with $X = 0$ and $E = 0$, the dynamics of the after-tax ratio of factor prices ρ reaches the following balanced growth path:

$$\rho = \left(\frac{1}{B\delta}\right)^{\frac{1}{1-\alpha(1-\delta)}} \left(\frac{\delta\gamma A \left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^\alpha (1-\Gamma)}{\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta}\right)^{\frac{1-\delta}{1-\alpha(1-\delta)}}$$

Plugging the last equation into expression (14), we get the human capital growth rate along a balanced growth path:

$$g^H = \left(B^{1-\alpha}\delta^{\delta(1-\alpha)} \left[\gamma A \left(\frac{\alpha}{1-\alpha}\right)^\alpha (1-\Gamma) f(\phi)\right]^\delta\right)^{\frac{1}{1-\alpha(1-\delta)}} \tag{22}$$

where $f(\phi) = \frac{\phi^{1-\alpha}}{(1+\gamma)\frac{\alpha}{1-\alpha} + \phi(1+\gamma\delta)}$. As stated by equation (22), the human capital growth is positively affected by the degree of altruism γ. Individuals leave more resources to their offsprings when they have a higher degree of altruism. Thus, they invest more in education which increases the human capital growth rate along a balanced growth path. However, individuals do not take into account the consequences of their transfer's decisions for all generations. By increasing the child's human capital through the education expenditures, the parent increases in the same way the human capital of generations to come by increasing the accumulated knowledge. Thus, the allocation of intergenerational family transfers plays an important role in the human capital growth level. The fiscal policy implemented affects this transfers' allocation (12) as well as the human capital growth g^H (see equation (22)). As a result, a fiscal policy should be set up in order to internalize the positive human capital externality on future generations.

When both taxes are identical and equivalent to the share of production devoted to the public spending, $\tau^B = \tau^L = \Gamma$, none intergenerational transfer is promoted. From equation (22), the effect of both tax instruments on the after-tax ratio of factor prices ρ along the balanced growth path (and also on the human capital growth) are given by function $f(\phi)$. The sign of $f'(\phi)$ corresponds to the one of $1 + \gamma - (1 + \gamma\delta)\phi$. When $\tau^B = \tau^L = \Gamma$, we have $f'(1) = \gamma(1-\delta) > 0$ which implies that ρ is not maximized with this tax policy. Another fiscal policy must be implemented in order to get the highest human capital growth rate at equilibrium.

Since $f'(\phi) > 0$ when $\tau^B = \tau^L = \Gamma$, a marginal increase of the ratio ϕ has a positive impact on ρ until a threshold (where $f'(\phi) = 0$). The household is incited to invest more in education expenditures. Consequently, this new attractiveness for the education spending involves an increase of human capital and a decrease of physical capital used in production. This leads to fall down

the marginal return on human capital relative to the one on physical capital. After achieving this threshold, this negative effect on the marginal return on labor overcompensates the positive effect of reducing tax on labor income, which decreases the after-tax ratio of factor prices ρ. Thus, the fiscal policy which maximizes ρ corresponds to the situation where $f'(\phi) = 0$, that is:

$$\phi = \widehat{\phi} \equiv \frac{1 + \gamma}{1 + \gamma\delta} > 1$$

We deduce from $\widehat{\phi} > 1$ that the bequest tax is higher than Γ and the labor income tax is lower than Γ. Thanks to the fiscal policy which satisfied $\phi = \widehat{\phi}$, we get the highest human capital growth along the balanced growth path g^H of $X = 0$.

However, the highest human capital growth rate at equilibrium is not necessarily the optimum. Furthermore, in this Subsection, we consider an equilibrium without public debt whereas public debt can be used into the fiscal policy to achieve optimal human capital growth rate. In the following, we focus on the optimal fiscal policy which maximizes the households' welfare.

3   First-best optimum

The social objective adopted depends on whether or not individual's altruism is taken into account. There are at least two types of social criteria. In models based on rational altruism *à la* Barro (1974), the social welfare function is usually assumed to exclude altruistic preferences in order to avoid undesirable double counting and thus to avoid increasing social weight with time (Hammond, 1988). For Harsanyi (1995), the same approach can be adopted excluding "all external preferences" (*i.e.* preferences for assignments of goods to others indiviuals). In other words, the social objective should include only the individual's life cycle utility for any form of altruism. Michel and Pestieau (2004) follow this approach in the context of paternalistic altruism. Hence, we concentrate our analysis on a social welfare function where the government eliminates the altruistic part of the individual's utility function. The government's social objective thus is:

$$SWF = \sum_{t=0}^{+\infty} \beta^t \ln c_t \tag{23}$$

where β is the social discount rate.

3.1   The first-best optimal solutions

The social planner has to maximize the social welfare function (23) with respect to $(c_t, e_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$ subject to the human capital technology (1) and the resource constraint (11).

Proposition 1. *A sequence $\frac{c_t^*}{H_t^*}$, k_{t+1}^* and g_{t+1}^{H*}, for $t \geq 0$ satisfies the first-best optimality conditions iff:*

$$\frac{c_t^*}{H_t^*} + \eta_{t+1}^* + g_{t+1}^{H*} k_{t+1}^* = (1 - \Gamma) F (k_t^*, 1) \tag{24}$$

$$\frac{c_{t+1}^*}{H_{t+1}^*} / \frac{c_t^*}{H_t^*} g_{t+1}^{H*} = \beta (1 - \Gamma) F_K' (k_{t+1}^*, 1) \tag{25}$$

$$(1 - \Gamma) F_K' (k_{t+1}^*, 1) = G_e' (\eta_{t+1}^*, 1) \left[(1 - \Gamma) F_H' (k_{t+1}^*, 1) + \frac{G_H' (\eta_{t+2}^*, 1)}{G_e' (\eta_{t+2}^*, 1)} \right] \tag{26}$$

where g_{t+1}^{H*} and η_{t+1}^* are linked through the static relation: $g_{t+1}^{H*} = G (\eta_{t+1}^*, 1)$.

Proof. Let us denote by μ_{t+1} and λ_{t+1} the respective Lagrange multipliers of the human capital technology (1) and the resource constraint (11). Then, the optimality conditions are:

- with respect to c_t, for $t \geq 0$,

$$\frac{\beta^t}{c_t} - \lambda_{t+1} = 0 \tag{27}$$

- with respect to e_{t+1}, for $t \geq 0$,

$$\mu_{t+1} G_e' (\eta_{t+1}, 1) - \lambda_{t+1} = 0 \tag{28}$$

- with respect to H_{t+1}, for $t \geq 0$,

$$\lambda_{t+2} (1 - \Gamma) F_H' (k_{t+1}, 1) + \mu_{t+2} G_H' (\eta_{t+2}, 1) - \mu_{t+1} = 0 \tag{29}$$

- with respect to K_{t+1}, for $t \geq 0$,

$$\lambda_{t+2} (1 - \Gamma) F_K' (k_{t+1}, 1) - \lambda_{t+1} = 0 \tag{30}$$

From the optimality conditions (27) and (30), we get the optimal consumption growth (25). Using (27), (28) and (29), we deduce:

$$\frac{\beta^t}{c_t} = \frac{\beta^{t+1}}{c_{t+1}} G_e' (\eta_{t+1}, 1) \left[(1 - \Gamma) F_H' (k_{t+1}, 1) + \frac{G_H' (\eta_{t+2}, 1)}{G_e' (\eta_{t+2}, 1)} \right]$$

Using (25), we obtain the condition (26). Lastly, the resource constraint (11) can be rewritten as (24). $\square$

Equation (26) shows that the optimal level of education expenditure is obtained when the social marginal returns of both transfers are equal. The individual's decision about investment in a child's education involves a positive human capital externality for future generations. As a reminder, the

individual's transfer motive is to improve the lifetime resources of the recipient and not to increase his welfare, and given the human capital function (1), there is a positive human capital externality on all the next generations. Two different effects compose the social return of an extra unit of education: (i) the direct effect on a child's wage and (ii) the indirect effect on human capital level of future generations and its impacts on family transfer decisions.

As a result, the social planner's fiscal policy objectives are to finance public spending (24), to take positive human capital externality into account (26) and to pursue a redistributive policy that satisfies the optimal consumption growth (25). In the next Subsection, we analyze the fiscal policy used to decentralize the first-best optimal solutions such that individuals make the optimal choice regarding their transfer decisions.

3.2 Decentralization of the first-best optimal solutions

In this Section, we are seeking to achieve the fiscal policy that decentralizes the first best optimum. In addition to finance public spending Γ, the government has to implement incentives that allow to reach equilibrium paths for $\left(\frac{c_t}{H_t}, g_{t+1}^H, k_{t+1} \right)_{t \geq 0}$ that take positive externality into account (26) and ensure that optimal consumption growth is satisfied (25). This can be drawn by using the fiscal instruments $\left(\tau_{t+1}^B, \tau_{t+1}^L, d_{t+1} \right)_{t \geq 0}$.

Proposition 2. *The fiscal policy* $\left(\phi_{t+1}^*, d_{t+1}^* \right)_{t \geq 0}$ *that decentralizes the first-best optimal paths* $\left(\frac{c_t^*}{H_t^*}, g_{t+1}^{H*}, k_{t+1}^* \right)_{t \geq 0}$ *as an equilibrium, satisfies:*

$$\phi_{t+1}^* = 1 + \frac{1}{(1 - \Gamma) F_H' \left(k_{t+1}^*, 1 \right)} \left[\frac{G_H' \left(\eta_{t+2}^*, 1 \right)}{G_e' \left(\eta_{t+2}^*, 1 \right)} \right] \tag{31}$$

$$d_{t+1}^* = \gamma (1 - \Gamma) F \left(k_t^*, 1 \right) - (1 + \gamma) g_{t+1}^{H*} k_{t+1}^* - \left(\frac{1 + \gamma \delta}{\delta} \right) \eta_{t+1}^* \tag{32}$$

where $g_{t+1}^{H*} = G \left(\eta_{t+1}^*, 1 \right)$.

Proof. Notice that the resource constraint (11) and the human capital technology (1) are satisfied at equilibrium and at the first-best optimum. Thus, the fiscal policy is only used to decentralize the first-best conditions (25) and (26).

At equilibrium with positive bequest, equation (12) is satisfied and can be rewritten as follows:

$$\phi_{t+1} = \frac{(1 - \Gamma) F_K' \left(k_{t+1}, 1 \right)}{(1 - \Gamma) F_H' \left(k_{t+1}, 1 \right) G_e' \left(\eta_{t+1}, 1 \right)}$$

Then, the government uses the fiscal policy such that expression (12) coincides with the social condition (26). Plugging (26) into (12), we get the ratio ϕ_{t+1}^* (*i.e.* equation (31)) that internalizes the positive human capital externality at period t. In addition, using the capital market equilibrium

(10) and the linear homogeneity of the production function F, the government's budget constraint (8) can be rewritten as follows:

$$d_{t+1} + (1 - \Gamma) F(k_t, 1) = \left(1 - \tau_t^B\right) F_K'(k_t, 1) \left(\rho_t + \frac{d_t}{g_t^H} + k_t\right)$$

Then, plugging the last equation into (18), we deduce (32). This concludes the proof. □

Equation (31) shows that the optimal bequest tax rate is higher than the labor income tax rate. The gap between the two tax rates is used to internalize the positive human capital externality in the individual's intergenerational transfer decision. Having a bequest tax rate higher than the labor income tax rate, provides an incentive to transfer more to education and less to bequest.

Furthermore, the degree of altruism γ has an impact on the fiscal policy implemented to decentralize the optimum solutions. Indeed, the optimal amount of public debt depends on this parameter. The government uses the public debt to support the tax instruments that internalize the positive human capital externality and pursue a redistributive policy. Therefore, the availability of public debt plays an important role in reaching the first-best optimal solution. In the following, we analyze first-best optimal fiscal policy at a steady state as to better grasp public debt impact.

3.3 The first-best optimal solutions and decentralization at steady state

We consider equilibria with operative bequests along the balanced growth path. Recall that at steady state equilibrium with positive public debt, the human capital growth is lower than the one without public debt (22). This human capital growth is, in general, different to the first-best human capital growth that internalizes the positive human capital externality.

Proposition 3. *At steady state, the first-best consumption growth corresponds to the first-best human capital growth.*

Proof. At steady state, the capital-labor ratio is strictly positive and constant: $k_{t+1} = k$. Then, the resource constraint (11) is equivalent to:

$$k = \frac{(1 - \Gamma) F(k, 1)}{g^H} - \frac{1}{g^H} \left(\frac{c}{H}\right) - \frac{1}{g^H} \eta$$

Since the human capital growth and the education expenditure per unit of human capital are constant along the balanced growth path, $\frac{c_t}{H_t}$ is also constant. Thus, the resource constraint involves that consumption growth and human capital growth are the same for every generation and are equivalent to each other. This concludes the proof. □

In order to achieve the first-best human capital growth as an equilibrium, the social planner implements the following fiscal policy, which decentralizes the optimal solutions $\left(\frac{c^*}{H^*}, g^{H*}, k^*\right)$ along

the balanced growth path.

Proposition 4. *At steady state, a fiscal policy $\left(\tau^{L*}, \tau^{B*}, d^{*}\right)$ allows to decentralize the first-best optimal solutions. This optimal fiscal policy is fully described as follows:*

$$\tau^{L*} = 1 - \frac{(1-\Gamma)\beta}{\gamma(1-\beta)} \left[\frac{1}{1-\alpha\beta(1-\delta)} + \left(\frac{\gamma}{1+\gamma} - \beta\right) \frac{1+\gamma}{1-\beta(1-\delta)} \right] \tag{33}$$

$$\tau^{B*} = 1 - (1-\beta(1-\delta)) \frac{(1-\Gamma)\beta}{\gamma(1-\beta)} \left[\frac{1}{1-\alpha\beta(1-\delta)} + \left(\frac{\gamma}{1+\gamma} - \beta\right) \frac{1+\gamma}{1-\beta(1-\delta)} \right] \tag{34}$$

$$d^{*} = (1+\gamma)\left(\frac{\gamma}{1+\gamma} - \beta\right)\left(\frac{1-\alpha\beta(1-\delta)}{1-\beta(1-\delta)}\right)(1-\Gamma)F(k^{*}, 1) \tag{35}$$

There are three possible fiscal policy cases:
(*) If $\frac{\gamma}{1+\gamma} = \beta$, only the two taxes are implemented in order to decentralize the first-best human capital growth. (**) If $\frac{\gamma}{1+\gamma} > \beta$ (resp. $\frac{\gamma}{1+\gamma} < \beta$), the social planner uses also a positive (resp. negative) public debt into the first-best fiscal policy.

Proof. As both consumption and human capital growths are equal along the balanced growth path, we deduce from the optimal consumption growth (25):

$$g^{H*} = \beta(1-\Gamma)F'_{K}(k^{*}, 1) \tag{36}$$

Then, using equation (12) and given that $g^{H*} = B(\eta^{*})^{\delta}$, we get:

$$\frac{\eta^{*}}{\beta\delta\phi^{*}} = (1-\Gamma)F'_{H}(k^{*}, 1) \tag{37}$$

At steady state, equation (31) corresponds to:

$$\phi^{*} = 1 + \frac{1}{(1-\Gamma)F'_{H}(k^{*}, 1)} \left[\frac{1-\delta}{\delta}\eta^{*} \right]$$

Plugging (37) into the last equation gives:

$$\phi^{*} = \frac{1}{1-\beta(1-\delta)} \tag{38}$$

Then, using equations (36)-(38) and the Cobb-Douglas production function (7), the optimal level of public debt per unit of human capital (32) can be rewritten as equation (35). Equation (35) shows that the sign of public debt depends on the gap between β and $\frac{\gamma}{1+\gamma}$. Indeed, $d^{*} > 0 \Leftrightarrow \frac{\gamma}{1+\gamma} > \beta$. From the optimal ratio ϕ^{*} (equation (38)), we get: $\tau^{B*} = \tau^{L*} + \left(1 - \tau^{L*}\right)\beta(1-\delta)$. Then, we deduce the first-best optimal tax instruments $\left(\tau^{L*}, \tau^{B*}\right)$ from the government's budget constraint (19), taking into account equation (36) and the optimal level of public debt (35) along the balanced growth path. This concludes the proof. □

To interpret results in Proposition 4, recall that the first-best ratio ϕ^* internalizes the positive human capital externality. Equation (38) illustrates this point. A higher discount factor β increases the gap between taxes which increases the individual's incentive to invest in education relative to bequest. In addition, the gap between β and $\frac{\gamma}{1+\gamma}$ affects both taxes in the same way.

As a result, the social planner decentralizes the first-best human capital growth as an equilibrium such that the equilibrium human capital growth rate (14) corresponds to it is first-best optimal value (36), using the optimal fiscal policy describes in Proposition 4. Since the after-tax ratio of factor prices ρ is linear in relation to the capital-labor ratio k, we deduce from equation (36), the first-best capital-labor ratio along the balanced growth path (using expression (14), the first-best ratio ϕ^* (equation (38)) and the Cobb-Douglas production function (7)):

$$k^* = \left[\frac{(1 - \beta(1 - \delta))^\delta (\beta(1 - \Gamma)\alpha A)^{1-\delta}}{B\left(\frac{1-\alpha}{\alpha}\delta\right)^\delta} \right]^{\frac{1}{1-\alpha+\alpha\delta}} \tag{39}$$

Then, plugging the last equation into equation (36), we get the first-best human capital growth rate along the balanced growth path:

$$g^{H*} = \left[B^{1-\alpha} \left(\frac{(1-\alpha)\delta}{\alpha(1-\beta(1-\delta))} \right)^{(1-\alpha)\delta} (\beta(1-\Gamma)\alpha A)^\delta \right]^{\frac{1}{1-\alpha+\alpha\delta}} \tag{40}$$

As shown in Proposition 3, the first-best human capital grows at the same rate than consumption along the balanced growth path and thus, both are positively affected by the discount factor β (see equation (40)). Indeed, the optimal growth is higher when the social planner gives more weight to future generations.

Notice that the first-best human capital growth rate is positively affected by the discount factor β whereas the human capital growth rate at equilibrium without public debt g^H (see equation (22)) positively depends on the degree of altruism γ. At steady state, the first-best human capital growth rate g^{H*} (see equation (40)) is lower than the human capital growth rate at equilibrium g^H (see equation (22)) iff:

$$\frac{\gamma}{1+\gamma} \left(\frac{\phi}{\phi^*} \right)^{1-\alpha} > \beta \left[\alpha + \phi(1-\alpha) \left(\frac{1+\gamma\delta}{1+\gamma} \right) \right] \tag{41}$$

When we consider the equilibrium situation without public debt in which both taxes implement the highest human capital growth (i.e. $\phi = \widehat{\phi} \equiv \frac{1+\gamma}{1+\gamma\delta}$), the first-best ratio ϕ^* (equation (38)) is lower than $\widehat{\phi} \Leftrightarrow \frac{\gamma}{1+\gamma} > \beta$. Then, we deduce from the last inequality that $g^{H*} < \widehat{g}^H$ iff $\frac{\gamma}{1+\gamma} > \beta$. (i.e. when the optimal public debt is positive). Thus, the key element is the weight given to future generations by the social planner and the representative individual. Therefore, only three cases of fiscal policy can arise.

(i) When $\frac{\gamma}{1+\gamma} > \beta$, for the social planner, the representative individual over-invests in the child's resources such that he penalizes his own consumption. As shown by inequality (41), the social

planner promotes a lower human capital growth rate g^{H*} (see equation (40)) than the highest human capital growth at equilibrium without public debt $\widehat{g}^{H}$ (see equation (22)) in order to increase the individual's consumption in the current generation (using positive public debt d^* and both tax instruments $\left(\tau^{L*}, \tau^{B*}\right)$). The first-best labor income tax rate τ^{L*} is still less than that on bequest τ^{B*} to take into account the positive externality, and both taxes are higher to reduce the individual's incentive to transfer resources to his child. Thus, this fiscal policy encourages individuals to consume rather than invest in future generations by reducing the incentive to pass on resources to the next generation using higher tax rates and positive public debt. Furthermore, when the gap between $\frac{\gamma}{1+\gamma}$ and β is reduced, the gap between both tax rates is higher and the public debt tends towards zero.

(ii) When $\frac{\gamma}{1+\gamma} = \beta$, the individual and government care about the next generation in the same way. Households transfer the optimum amount of resources but still under-invest in education. Individual make non-optimal trade-offs between intergenerational transfers. Thus, the government uses only both tax instruments $\left(\tau^{L*}, \tau^{B*}\right)$ to internalize the positive human capital externality. In this situation, the first-best fiscal policy ϕ^* that ensures the first-best human capital growth rate g^{H*} along the balanced growth path is that which promotes the highest human capital growth rate at equilibrium without public debt $\widehat{g}^{H}$. As $\phi^* = \widehat{\phi}$, we deduce from inequality (41) that g^{H*} corresponds to the highest human capital growth rate at equilibrium $\widehat{g}^{H}$. Given the government budget constraint (9) and the optimal ratio ϕ^* (equation (38)), we get:

$$
\begin{aligned}
\tau^{B*} &= \Gamma + \frac{(1-\alpha)\beta(1-\delta)(1-\Gamma)}{1-\alpha\beta(1-\delta)} \\
\tau^{L*} &= \Gamma - \frac{\alpha\beta(1-\delta)(1-\Gamma)}{1-\alpha\beta(1-\delta)}
\end{aligned}
$$

where the bequest tax is positive whereas the labor income tax can be either positive or negative depending on the values of the parameters.

(iii) When $\frac{\gamma}{1+\gamma} < \beta$, for the social planner, individuals are selfish in that they do not enough consider their child. Inequality (41) illustrates that both consumption and human capital growths at equilibrium without public debt, are low compared to their first-best optimal levels. Hence, the government wants to increase consumption of future generations. By improving the gap between both taxes (such that $\phi^* > \widehat{\phi}$), the government increases the incentives to invest in education expenditures, which negatively affects the capital-labor ratio. Thus, a negative public debt is necessary to achieving the first-best human capital growth rate g^{H*}. A negative public debt implies a public capital accumulation that increases the capital-labor ratio k and removes the negative impacts on the capital-lalor ratio to achieve the first-best one k^*.

Therefore, the first-best fiscal policy implemented depends on the discount factor β and the degree of altruism γ. When both are not equal, public debt is required to decentralize the first-best optimum. The first-best human capital growth decentralized with positive public debt (*i.e.* when $\frac{\gamma}{1+\gamma} > \beta$), is lower than the first-best one in which public debt is zero. Thus, in the following, we

analyze the impact of a positive public debt constraint on the human capital growth and on the tax policy used. Then, we compare results with the ones at equilibrium.

4   Second-best optimum

The government adopts as a social criteria the discounted sum of generational consumption's utility (23) as in the previous Section. As stated before, we focus on situations where the first-best fiscal policy uses a positive public debt in order to decentralize the first best optimal solutions (*i.e.* when $\frac{\gamma}{1+\gamma} > \beta$). However, in this Section, public debt is not available. Thus, the social planner only uses the two tax instruments to maximize the households' welfare. The household's bequest is used only as private capital.

From equation (5b), we get: $c_t = \left[\gamma \left(1 - \tau_{t+1}^B\right) R_{t+1}\right]^{-1} \Omega_{t+1}$. Then, using equation (15) and the capital market equilibrium (10), the individual's consumption corresponds to:

$$c_t = \frac{1}{\gamma} \left(1 + \frac{1-\alpha}{\alpha} \phi_{t+1}\right) K_{t+1} \tag{42}$$

Expression (12) gives the education expenditure:

$$e_{t+1} = \delta \frac{1-\alpha}{\alpha} \phi_{t+1} K_{t+1} \tag{43}$$

Using the consumption level (42) and education expenditure (43), the resource constraint (11) corresponds to:

$$K_{t+1} \left[1 + \frac{1}{\gamma} \left(1 + \frac{1-\alpha}{\alpha} \phi_{t+1}\right) + \delta \frac{1-\alpha}{\alpha} \phi_{t+1}\right] = (1 - \Gamma) F(K_t, H_t) \tag{44}$$

As a result, the social planner maximizes the social welfare function (23) with respect to $(\phi_{t+1}, H_{t+1}, K_{t+1})_{t \geq 0}$ subject to the resource constraint (11) and the human capital technology (1) as well as taking into account the agent's consumption level (42). Let us denote by λ_{t+1} and μ_{t+1}, the respective Lagrange multipliers of both constraints. Taking into account the Cobb-Douglas production function (7) and derivatives, we get the following optimality conditions:

- with respect to H_{t+1}, for $t \geq 0$,

$$-\frac{\mu_{t+1}}{\mu_{t+2}} + \beta \frac{\lambda_{t+2}}{\mu_{t+2}} (1-\alpha)(1-\Gamma) F(k_{t+1}, 1) + \beta (1-\delta) g_{t+2}^H = 0 \tag{45}$$

- with respect to K_{t+1}, for $t \geq 0$, (using the optimality condition (47)),

$$-1 + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} \alpha (1-\Gamma) \frac{F(k_{t+1}, 1)}{k_{t+1}} + \left[\frac{1}{c_t \lambda_{t+1}} - 1\right] \frac{1}{\gamma} = 0 \tag{46}$$

- with respect to ϕ_{t+1}, for $t \geq 0$,

$$\left[\frac{1}{c_t \lambda_{t+1}} - 1 \right] \frac{1}{\gamma} + \left[\frac{\mu_{t+1}}{\lambda_{t+1}} \delta \frac{H_{t+1}}{e_{t+1}} - 1 \right] \delta = 0 \tag{47}$$

4.1 Second-best optimum along the balanced growth path

In this Subsection, we analyze the second-best fiscal policy along the balanced growth path where bequests are positive. We concentrate on situations where public debt is positively used when it is not constrained (i.e when $\frac{\gamma}{1+\gamma} > \beta$).

Proposition 5. *Let us assume $\frac{\gamma}{1+\gamma} > \beta$. Along the balanced growth path of the second-best optimum, we get:*

(i) $\phi^{**} > \widehat{\phi} > \phi^*$;

(ii) $k^{**} < k^*$ and $k^{**} < \widehat{k}$;

(ii) $g^{H*} < g^{H**} < \widehat{g}^H$.

where ϕ^{**} is the second-best ratio ϕ, k^{**} is the second best capital-labor ratio and g^{H**} is the second-best human capital growth rate.

Proof. At steady state, we have $g^H = \frac{\lambda_{t+1}}{\lambda_{t+2}} = \frac{\mu_{t+1}}{\mu_{t+2}}$ and we define $\Lambda = \frac{\lambda_{t+1}}{\mu_{t+1}}$ and $M = \lambda_{t+1} K_{t+1}$. Thus, under the optimality conditions (46) and (47), we get:

$$\Lambda = \frac{\delta^2 \frac{(g^{H**})}{\eta^{**}}}{\delta - 1 + \frac{\beta}{g^{H**}} \alpha (1 - \Gamma) \frac{F(k^{**},1)}{k^{**}}} \tag{48}$$

Plugging equation (48) into the optimality condition (45), we obtain the following relationship between the second-best values of g^{H**}, ϕ^{**} and k^{**}:

$$\frac{\delta}{\phi^{**} [1 - \beta (1 - \delta)]} + (1 - \delta) \frac{g^{H**}}{\beta \alpha (1 - \Gamma) \frac{F(k^{**},1)}{k^{**}}} = 1 \tag{49}$$

At a steady state, the resource constraint (44) can be rewritten as follows:

$$g^H k \left[1 + \gamma + (1 + \gamma \delta) \frac{1 - \alpha}{\alpha} \phi \right] = \gamma (1 - \Gamma) F(k, 1)$$

Plugging the last expression into (49), we get:

$$\Psi (\phi^{**}) = 1 \tag{50}$$

where

$$\Psi\left(\phi\right) = \frac{\delta\phi^*}{\phi} + (1-\delta)\frac{\frac{\gamma}{1+\gamma}}{\beta\left[\alpha + (1-\alpha)\frac{\phi}{\widehat{\phi}}\right]} \tag{51}$$

which is decreasing and convex relative to the ratio ϕ. As a reminder, $\widehat{\phi} = \frac{1+\gamma}{1+\gamma\delta}$, $\phi^* = \frac{1}{1-\beta(1-\delta)}$ and $\widehat{\phi} > \phi^* \Leftrightarrow \frac{\gamma}{1+\gamma} > \beta$. Then, from (51):

$$\Psi\left(\widehat{\phi}\right) > 1 \quad \Leftrightarrow \quad \delta\frac{\phi^*}{\widehat{\phi}} + (1-\delta)\frac{\gamma}{\beta\left(1+\gamma\right)} > 1$$

$$\Leftrightarrow \quad (1-\delta)\left[\frac{\gamma}{\beta\left(1+\gamma\right)} - 1\right] > \delta\left(1 - \frac{\phi^*}{\widehat{\phi}}\right)$$

where $\frac{\phi^*}{\widehat{\phi}} = \frac{1 - \frac{\gamma}{1+\gamma}(1-\delta)}{1-\beta(1-\delta)}$. Thus, we get:

$$\Psi\left(\widehat{\phi}\right) > 1 \Leftrightarrow \left[\frac{\gamma}{\beta\left(1+\gamma\right)} - 1\right]\left[1 - \frac{\delta\beta}{\delta\beta + 1 - \beta}\right] > 0$$

which is verified iff $\frac{\gamma}{1+\gamma} > \beta$. Since the second-best ratio ϕ^{**} satisfies equation (50) and given the function $\Psi\left(\phi\right)$ is decreasing and convex, a necessary and sufficient condition for $\phi^{**} > \widehat{\phi} > \phi^*$ is $\frac{\gamma}{1+\gamma} > \beta$.

At the steady state, the first-best optimal capital-labor ratio (39) can be rewritten as follows:

$$\frac{B^{\frac{1}{1-\delta}}\left(k^*\right)^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta\alpha\left(1-\Gamma\right)A} = \left(\frac{\alpha}{\delta\left(1-\alpha\right)\phi^*}\right)^{\frac{\delta}{1-\delta}}$$

For the second-best capital-labor ratio k^{**}, plugging human capital growth at equilibrium (14) into the second-best optimal condition (49), we get:

$$\frac{B^{\frac{1}{1-\delta}}\left(k^{**}\right)^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta\alpha\left(1-\Gamma\right)A} = \frac{1-\delta\frac{\phi^*}{\phi^{**}}}{1-\delta}\left(\frac{\alpha}{\delta\left(1-\alpha\right)\phi^{**}}\right)^{\frac{\delta}{1-\delta}}$$

From the last two equations, we obtain:

$$k^{**} \leq k^* \Leftrightarrow \frac{1-\delta\frac{\phi^*}{\phi^{**}}}{1-\delta}\left(\frac{\phi^*}{\phi^{**}}\right)^{\frac{\delta}{1-\delta}} \leq 1$$

which is always satisfied since $\phi^{**} > \phi^*$.

Given condition (49), the previous result, regarding $\phi^{**} > \phi^*$ with $\frac{\gamma}{1+\gamma} > \beta$, leads to the following inequality:

$$g^{H**} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{**},1\right)}{k^{**}}$$

Thus, assuming $\frac{\gamma}{1+\gamma} > \beta$ and given that $k^{**} < k^*$, we deduce that the second-best economic growth

rate g^{H**} (given condition (49)) is higher than the first-best one (36):

$$g^{H**} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{**},1\right)}{k^{**}} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{*},1\right)}{k^{*}} = g^{H*}$$

As stated before in Section 2.5.2, without public debt, the fiscal policy which satisfied $\phi = \widehat{\phi}$, leads to the highest human capital growth rate along the balanced growth path. Thus, $g^{H*} < g^{H**} < \widehat{g}^{H}$.

Lastly, given that $g^{H**} < \widehat{g}^{H}$ and $\phi^{**} > \widehat{\phi}$, we deduce from equation (14) that $k^{**} < \widehat{k}$ (since the human capital growth rate satisfies in both cases: $g^{H} = B\left(\phi\frac{1-\alpha}{\alpha}kB\delta\right)^{\frac{\delta}{1-\delta}}$). This concludes the proof. $\qquad\square$

Figure 2: Human capital growth (22) along the balanced growth path of $X = 0$
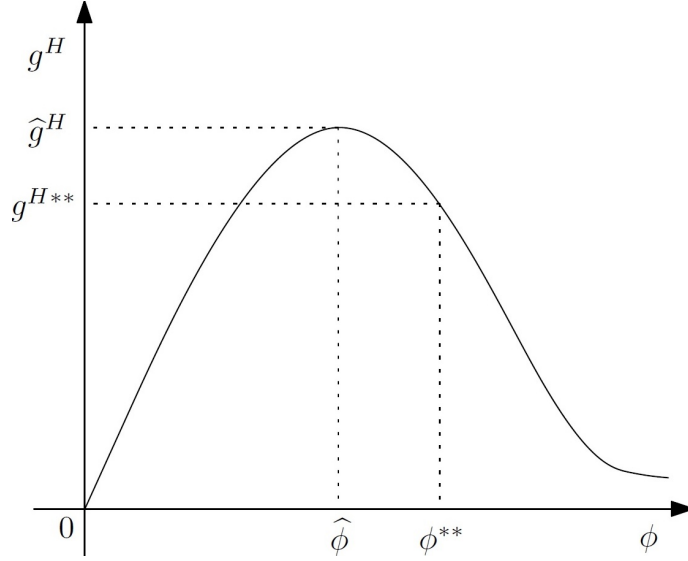
Therefore, the social planner only uses both taxes to internalize the positive human capital externality, to finance public spending and to pursue a redistributive policy between generations. For these purposes, when $\frac{\gamma}{1+\gamma} > \beta$, the government adopts a second-best ratio ϕ^{**} which is higher than the one used to obtain the highest human capital growth rate without public debt $\widehat{\phi}$, as shown in Figure 2. Since, $\phi^{**} > \widehat{\phi}$, the agent's incentive to invest in education expenditure rather than bequest increases. This implies an over-investment in education and results in reducing the capital-labor ratio (*i.e.* $k^{**} < \widehat{k}$). Notice that the fall in capital-labor ratio has a positive effect on the first generation consumption. Since $k^{**} < k^{*}$, the government encourages agents to consume rather than invest in future generations by decreasing the capital-labor ratio using both tax instruments. However, the government cannot fully support its redistributive policy towards current generation (since $g^{H*} \neq g^{H**}$). Thus, the availability of public debt plays an essential role in optimizing the intergenerational family transfers between generations.

Therefore, the government implements a tax policy (*i.e.* $\phi = \phi^{**}$) which leads to a higher human

capital growth g^{H**} than the first-best human capital growth g^{H*} but lower compared to the highest possible one $\widehat{g}^H$ along the balanced growth path.

4.2 Numerical illustration

We use a numerical example to show that optimal transition dynamics jumps to the balanced growth path and to analyze the optimal solutions given the availability of public debt. We focus on situation where achieving the first-best optimal solutions requires a positive public debt. As a result, we calibrate the optimal transition dynamics with $\frac{\gamma}{1+\gamma} > \beta$. For this purpose, the discount factor chosen should be quite low such that government cares more about the wellbeing of the present generations rather than that of the next one. This dynamics will result in a low β compared to the usual estimate given that γ cannot be too high. We assume $\gamma = 0.7$ such that households pay attention to their offspring. Table 1 presents value of all parameters.
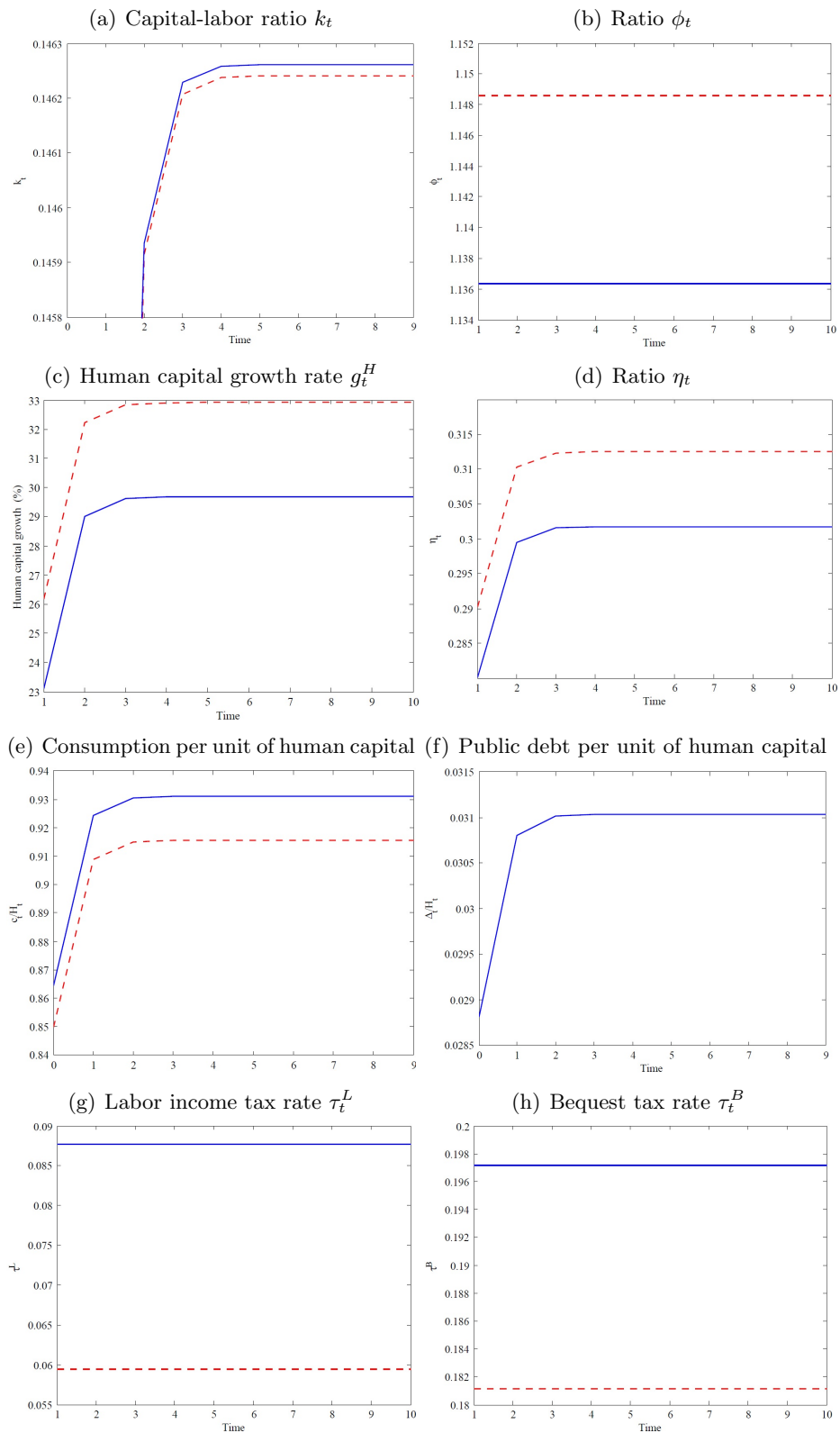
Table 1: Base-case parameter value

Parameter		Value
Government		
Discount factor	β	0.4
Share of production devoted to public sector	Γ	0.1
Production function		
Technological parameter	A	3
Share parameter of physical capital	α	0.33
Representative individual		
Agent's human capital production function		
Technological parameter	B	3
Share parameter of education spending	δ	0.7
Taste		
Degree of altruism	γ	0.7

The results are reported in Figure 3. All the variables jump to the optimal balanced growth path in two periods. Because of optimal tax instruments, regardless of the availability of public debt, individuals have incentives to modify their family transfers toward the optimal choices.

Since the optimal capital-labor ratio is quite low in both cases (Figure 3(a)), this numerical illustration describes a knowledge economy that encourages agents to invest in child's education expenditure. In both situations, we get a human capital growth rate of approximately 30% per period along the balanced growth path (see Figure 3(c)). Assuming that a period lasts for approximately 30 years, the economic growth achieved corresponds to an annual growth rate of 1%. Then, Figures 3(g) and 3(h) illustrate the optimal tax instruments values. The labor income tax rate is really low in both cases whereas the bequest tax rate is approximately 20%. Concerning the

Figure 3: The optimal transition dynamics beginning in the first period

(a) Capital-labor ratio k_t

(b) Ratio ϕ_t

(c) Human capital growth rate g_t^H

(d) Ratio η_t

(e) Consumption per unit of human capital

(f) Public debt per unit of human capital

(g) Labor income tax rate τ_t^L

(h) Bequest tax rate τ_t^B

Note: The first-best optimal solutions: bold line. The second best optimal solutions: dashed line.

effect of non-availability of public debt , this simulation illustrates the results obtained previously. When public debt is available, it is used to pursue the intergenerational redistributive policy and to internalize the positive externality (through supports on both taxes). Otherwise, the results given in Proposition 5 are satisfied. The absence of public debt involves higher growth levels of human capital (Figure 3(c)) and of consumption (Figure 3(e)). Indeed, the negative effect of increasing the ratio ϕ compared to the first-best one ϕ^* (Figure 3(b)) on the capital-labor ratio (Figure 3(a)) does not lead to the first best human capital growth (or equivalently the first-best consumption growth). As stated in Section 4.1, the second-best fiscal policy results in a over-investment in education expenditure (Figure 3(d)). Finally, the second-best tax rates are lower than the first-best ones since public debt can not be used to finance public spending.

Therefore, this numerical example describes the public debt availability issue correctly. It shows that the transitional dynamics jump to the balanced growth path and that both optimal tax rates are lower when public debt is not available.

5   Conclusion

The long-run optimal human capital growth that we are able to identify in this paper depends crucially on the availability of public debt. Public debt allows optimal distributions of intergenerational family transfers, through which the first-best optimal human capital growth is achieved. Because of a positive public debt, the government improves the consumption of the current generations without affecting the optimal human capital growth and uses taxes to internalize the positive human capital externality. When a positive public debt is not available, social planner cannot completely satisfy these objectives such that the two taxes do not fully implement the intergenerational redistribution policy. For this reason, the economic growth is higher than with public debt. Furthermore, the model reveals the necessity of public intervention to ensure that agents' decisions concerning their family transfers correspond to the optimal choices along the balanced growth path.

We focus on intergenerational redistribution policies. We do not analyze the effect of intragenerational inequality on the optimal human capital growth, which depends on the availability of public debt. These disparities across agents should modify the optimal fiscal policy used and the human capital growth along the balanced growth path. Additionally, we can analyze the optimal fiscal policy by relaxing the inelastic labor supply. Thus, studying the influence of the labor income tax rate on the household's labor supply and on transfer allocations is an interesting question to be explored in future investigations.

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A   Appendix

A.1   Analysis of function ψ

As a reminder, $D = \frac{1}{\delta}\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta\right) > 0$, $J = \delta\left(1 - \tau^L\right)(1 - \alpha) > 0$ and $E = \Gamma - \tau^B\alpha - \tau^L(1 - \alpha)$. From equation (21a), we get the following first order derivative of $\psi(X)$:

$$\frac{DJ\gamma(1-\Gamma)}{(\gamma(1-\Gamma) - E - JX)^2} > 0 \tag{52}$$

and the following second order derivative:

$$\frac{D(3J)\gamma(1-\Gamma)}{(\gamma(1-\Gamma) - E - JX)^3} > 0 \text{ iff } JX + E < \gamma(1-\Gamma)$$

The condition $JX + E < \gamma(1 - \Gamma)$ is always satisfied as soon as we focus on positive human capital growth g^H(see equation (14)). Under this condition, the function $\psi(X)$ is strictly increasing and strictly convex.

A.2   Steady states of X with $E = 0$

Using equation (21a), the steady states of X depend on the roots of the following polynomial of degree two:

$$P(X) = JX^2 - [E + JD - \gamma(1-\Gamma)]X + DE = 0$$

A sufficient condition to get two opposite sign roots for the polynomial P is that $E = 0$, (*i.e.* $\Gamma = \tau^B\alpha + \tau^L(1 - \alpha)$). In this case, the polynomial P has two real roots, which are 0 and $\frac{\gamma(1-\Gamma)}{J} - D$. The sign of the second root is negative iff:

$$\frac{\gamma(1-\Gamma)}{J} - D < 0 \quad \Leftrightarrow \quad \frac{\left(1 - \tau^L\right)(1 - \alpha)\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta\right)}{\gamma(1-\Gamma)} > 1$$

$$\Leftrightarrow \quad \alpha\frac{1 - \tau^B}{1 - \Gamma} + \frac{1 + \gamma\delta}{1 + \gamma}(1 - \alpha)\frac{1 - \tau^L}{1 - \Gamma} > \frac{\gamma}{1 + \gamma}$$

From the government budget constraint (9), we get:

$$E = 0 \quad \Leftrightarrow \quad 1 - \Gamma = \alpha \left(1 - \tau^B\right) + (1 - \alpha)\left(1 - \tau^L\right)$$
$$\Leftrightarrow \quad \frac{1 - \tau^B}{1 - \Gamma} = \frac{1}{\alpha}\left[1 - (1 - \alpha)\left(\frac{1 - \tau^L}{1 - \Gamma}\right)\right]$$
$$\Leftrightarrow \quad \phi = \frac{\alpha}{\frac{1 - \Gamma}{1 - \tau^L} - (1 - \alpha)}$$
$$\Leftrightarrow \quad \frac{1 - \Gamma}{1 - \tau^L} = \frac{\alpha}{\phi} + 1 - \alpha = \frac{1}{\phi}\left(\frac{1 - \Gamma}{1 - \tau^B}\right)$$

Then, using the last equation, we deduce the sign of the second root:

$$\frac{\gamma(1 - \Gamma)}{J} - D < 0 \quad \Leftrightarrow \quad \alpha \frac{\frac{1}{\phi}}{\frac{\alpha}{\phi} + 1 - \alpha} + \left(\frac{1 + \gamma\delta}{1 + \gamma}\right)\frac{1 - \alpha}{\frac{\alpha}{\phi} + 1 - \alpha} > \frac{\gamma}{1 + \gamma}$$
$$\Leftrightarrow \quad \frac{\alpha}{\phi} + (1 - \alpha)\frac{1 + \gamma\delta}{1 + \gamma} > \frac{\gamma}{1 + \gamma}\left[\frac{\alpha}{\phi} + 1 - \alpha\right]$$
$$\Leftrightarrow \quad \frac{\alpha}{\phi} > (1 - \alpha)\left[\gamma(1 - \delta) - 1\right]$$

which is always satisfied. Thus, the sign of $\tilde{X} \equiv \frac{\gamma(1-\Gamma)}{J} - D$ is negative. Since $\psi(X)$ is strictly convex and one real root is strictly negative with $E = 0$, $\tilde{X}$ is a locally stable negative balanced growth path of the dynamics of X_t. The other steady state $X = 0$ is unstable. Then, the stable set of $\tilde{X}$ is $\mathbb{R}_-^*$. If $X_0 > 0$, X_t goes beyond the upper bond for positive human capital growth in finite time which leads the following period to zero production.