Games with perception*

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Abstract

We are interested in 2×2 game situations where players act depending on how they perceive their counterpart although this choice is payoff irrelevant. Perceptions concern a dichotomous characteristic. The model includes uncertainty as players know how they perceive their counterpart, but not how they are perceived. We study whether the mere possibility of playing differently depending on the counterpart's perception generates new equilibria. We analyze equilibria in which strategies are contingent on perception. We show that the existence of this discriminatory equilibrium depends on the characteristic in question and on the class of game. (JEL C72)

Keywords: 2×2 matrix games, incomplete information model, Bayesian games.

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1 Introduction

Recent research in Social Psychology provides support for rapid evaluative judgment. Ambady, Bernieri and Richeson (2000) conclude that a brief excerpt of expressive behavior is sufficient to infer information about internal states, personality, interaction motives and social relations. Strangers often need less than 10 seconds to make non-random inferences about emotions, personality, and physical traits (Weisbuch and Ambady, 2010). The ability to accurately extract personal information about target individuals from brief visual exposure to those target individuals is known as *thin-slice vision*.

Perceptions of others influence interactive decisions, as widely tested in the laboratory with positive results. Some of these results obtained using specific games which allow the impact of perception to be studied are mentioned below.

In an *ultimatum game* two players have to divide a sum of money that is given to them. One player proposes a division and the other can either accept or reject that proposal. If he/she rejects, neither player receives anything. If he/she accepts, the money is split according to the proposal. Solnick and Schweitzer (1999) conduct an experiment with this game and find that one's own attractiveness does not influence decision making, but it does influence the decision process of others: more is offered to attractive people (there is a beauty premium) and more is expected of them. Mussel, Göritz and Hewig (2013) study the effect of facial expression in playing this game. Results show that offers are accepted more often from proposers with a smiling facial expression than from those with a neutral facial expression. They also find lower acceptance rates for offers from proposers with an angry facial expression.

In a *trust game* two players are given an amount of money. One player is told to give some amount to his/her counterpart. This amount is multiplied and the resulting quantity is given to the other player who in turn has to send part of it back to the first player. Wilson and Eckel (2006) explore the impact of beauty when this game is played. Again there is a beauty premium and trustees have higher expectations of their more attractive counterparts. Eckel (2007) reports that an objective characteristics such as race matter in these games: Non-white subjects are trusted less than white subjects. Facial expressions also matter. Scharleman, Eckel, Kacelnika and Wilson (2001) examine the effect of a smile in such games by providing a smiling or a non smiling picture of the counterpart. They observe that smiling expressions may lead subjects to choose a more cooperative strategy.

In a *public good game* players are given an amount of money, then they secretly choose how much to put into a public pot. The amount obtained in this pot is multiplied and the resulting quantity is divided equally among players. Andreoni and Petrie (2008) examine the effect of beauty and gender with the following results: People tend to reward beauty although the beauty premium is not due to the actions of attractive people. It seems rather that people expect attractive people to be more cooperative, and thus behave more cooperatively toward them. People are more cooperative in groups of women than in groups of men. However on average, women are not more cooperative than men but benefit from being stereotyped as generous.

Experimental research has accumulated evidence in disagreement with the classical assumption that people only care about their own monetary payoffs. In all the experiments above players condition strategies to their perceptions of their counterpart. The interest in the observation of the partner is corroborated by an experiment run by Eckel and Petrie (2011) where subjects can pay to see their counterpart's picture before making their choices. These findings suggest the players' perception should be taken into account in modeling games. As put by Eckel (2007, p. 845): "Behavior not only deviates from the selfish prediction of the naïve rational actor model, but also varies by the characteristics of the counterpart. People are altruistic, and for most subjects, altruism is contingent, in the sense that it varies with what they observe."

Psychological game theory has broadened the model of human behavior by incorporating belief-dependent emotions such as reciprocity, guilt, aversion, regret and shame. Geanakoplos, Pearce and Stacchetti (1989) consider games in which the utility of a player depends not only on the outcome of the game but also on his/her beliefs about the other players' strategies, beliefs about other players' beliefs and so on. The best-known application of a psychological game is Rabin's (1993) work on reciprocity, according to which players wish to act kindly in response to kind strategies.

Still O'Neill (2001) argues that games with emotions would be more accurately analyzed if modeled as games of incomplete information. In his words "In social situations we feel anger or appreciation not when we learn the outcome of a random variable, but when we learn something about the other player, that their loyalty or thoughtfulness is lower or higher than we thought. This calls for incomplete information." And this is the approach that is followed in the current paper. In these games with perception emotions and psychological traits determine beliefs but, unlike psychological games, they are assumed to be payoff irrelevant.

To illustrate our approach we start from Aumann's (1990) discussion about the solution of the following coordination game:

Example 1 *Alice and Bob can either cooperate or defect. If the two players cooperate each player gets* 9*. If one defects and his/her counterpart cooperates, the defector gets* 8 *while the cooperator gets* 0*. If both individuals defect, each player gets* 7*. The payoffs are summarized in Figure 1.*

	cooperate	*defect*
cooperate	$(9, 9)$	$(0, 8)$
defect	$(8, 0)$	$(7, 7)$

Figure 1: Payoff matrix of Aumann's coordination game

This game has two equilibria in pure strategies in which the two players cooperate and the two players defect. But which equilibrium should be played? Each equilibrium has something going for it. The first leads to the largest payoff for both players (9) *but if the counterpart deviates and defects the player is left with nothing. By contrast the second equilibrium leads to a smaller payoff* (7) *but is much safer: if the counterpart deviates and cooperates the player gets more* (8)*. In Aumann's words (1990, p. 616): "The final choice of the player may depend on whether the player is "careful and prudent" and fears that the other does not trust him or her or impulsive and optimistic and believe that the other is too. So Alice's question is: will Bob trust me or not?"*

The question addressed in the paragraph above cannot be answered as long as players are modeled as blind players, but some light can be shed on the problem by allowing Alice and Bob see each other before making their choices (so that they can have a perception of how trustworthy their counterpart looks). To be precise each player may be of two different types: One type perceives the counterpart as trustworthy, the other type perceives the counterpart as untrustworthy. However information is incomplete: Alice and Bob are not sure how they are perceived by their counterpart and only have beliefs. In the setting that we propose a strategy is contingent on how the counterpart is perceived.

Four possible pure strategies emerge. Two of them are discriminatory: (i) cooperate if the counterpart looks trustworthy and defect if the counterpart looks untrustworthy, (ii) defect if the counterpart looks trustworthy and cooperate if the counterpart looks untrustworthy. The other two pure strategies are non discriminatory: (iii) always cooperate and (iv) always defect. Both players choosing strategy (i) seems a reasonable candidate for an equilibrium. The question is whether this pair of strategies is indeed an equilibrium. As shownbelow the answer is yes.

In this paper we introduce a second ingredient into the basic 2×2 game: beliefs based on the perception of the counterpart. We provide a taxonomy of beliefs depending on the type of characteristic observed in first impression; namely beliefs associated with an emotional state, with a personality trait and with objective characteristics. We study the equilibria in the resulting game, modeled as a Bayesian game. We show that the existence of the discriminatory equilibria (agents play different actions depending on their perceptions) is contingent on the class of games (coordination, anti-coordination, competitive or dominant solvable) and on the type of beliefs induced by the characteristic in question. We study the conditions for such discriminatory equilibria to exist and provide the equilibria. We also find that discrimination is not asymmetric in the sense that in equilibrium a player discriminates if and only if the other player does so.

The paper is organized as follows. Section 2 recalls the classification of 2×2 basic games (without perception) and their equilibria. Section 3 deals with beliefs based on perception and proposes a taxonomy of those beliefs. Section 4 contains the description of games with perception. These are modeled as Bayesian games. The equilibria of games with perception are given in Section 5. Section 6 concludes. The Appendix contains all the proofs.

2 Games without perception

In this section we recall the classification of 2×2 payoff matrices in coordination, anti-coordination, competitive and dominant solvable matrices. We give the equilibria of the corresponding games. Readers familiar with basic game theory can skip this section.

Two players, Alice and Bob (hereafter A and B or she and he), choose between two actions, s_1 and s_0. If players are not allowed to see their counterpart (that is, in a game without perception), they have only to choose an action or a mixed strategy however they may perceive their counterpart. Combinations of actions lead to four outcomes. Players' payoffs are summarized in Figure 2 where $u(s_i, s_j)$ is the utility obtained by A if she plays s_i $(i = 0, 1)$ and B plays s_j $(j = 0, 1)$, and $v(s_i, s_j)$ is the utility obtained by B if A plays s_i $(i = 0, 1)$ while he plays s_j $(j = 0, 1)$.

	s_1	s_0
s_1	$u(s_1, s_1)$, $v(s_1, s_1)$	$u(s_1, s_0)$, $v(s_1, s_0)$
s_0	$u(s_0, s_1)$, $v(s_0, s_1)$	$u(s_0, s_0)$, $v(s_0, s_0)$

Figure 2: Payoff matrix of a 2×2 game

Players can also choose a mixed strategy, that is, play action s_1 with a certain probability and action s_0 with the complementary probability. If A plays s_1 with probability α and B plays s_1 with probability β the probability of outcome (s_1, s_1) is $\alpha\beta$, the probability of (s_1, s_0) is $\alpha(1-\beta)$, the probability of (s_0, s_1) is $(1-\alpha)\beta$, and the probability of (s_0, s_0) is $(1-\alpha)(1-\beta)$. Thus, players' expected utilities are the sums of the probabilities of the four outcomes multiplied by their respective utilities. If $u(\alpha, \beta)$ denotes A's expected utility, $v(\alpha, \beta)$ denotes B's expected utility, we have

$$\begin{aligned}
u(\alpha, \beta) &= \alpha\beta u(s_1, s_1) + \alpha(1-\beta)u(s_1, s_0) \\
&\quad + (1-\alpha)\beta u(s_0, s_1) + (1-\alpha)(1-\beta)u(s_0, s_0), \\
v(\alpha, \beta) &= \alpha\beta v(s_1, s_1) + \alpha(1-\beta)v(s_1, s_0) \\
&\quad + (1-\alpha)\beta v(s_0, s_1) + (1-\alpha)(1-\beta)v(s_0, s_0).
\end{aligned}$$

A player's objective is to maximize her/his expected utility. The best response gives the most favorable strategies taking the counterpart's strategy as given. Player A can choose her strategy α given player B's choice (that is, given that B plays s_1 with probability β). Similarly player B chooses β given that player A plays α. We can rewrite as

$$\begin{aligned}
u(\alpha, \beta) &= (1-\beta)u(s_0, s_0) + \beta u(s_0, s_1) + f(\beta)\alpha \\
v(\alpha, \beta) &= (1-\alpha)v(s_0, s_0) + \alpha v(s_1, s_0) + g(\alpha)\beta
\end{aligned}$$

where

$$
\begin{aligned}
f(\beta) &= [u(s_1,s_1)-u(s_0,s_1)+u(s_0,s_0)-u(s_1,s_0)]\,\beta - u(s_0,s_0)+u(s_1,s_0) \\
g(\alpha) &= [v(s_1,s_1)-v(s_1,s_0)+v(s_0,s_0)-v(s_0,s_1)]\,\alpha - v(s_0,s_0)+v(s_0,s_1).
\end{aligned}
$$

Thus A chooses $\alpha = 1$ (that is, s_1) if $f(\beta) > 0$, chooses $\alpha = 0$ (that is, s_0) if $f(\beta) < 0$ and chooses any α if $f(\beta) = 0$. Similarly B chooses $\beta = 1$ (that is, s_1) if $g(\alpha) > 0$, chooses $\beta = 0$ (that is, s_0) if $g(\alpha) < 0$ and chooses any β if $g(\alpha) = 0$. Player A's best response, denoted by $\mathcal{R}(\beta)$, and player B's best response, denoted $\mathcal{S}(\alpha)$ are given by

$$
\mathcal{R}(\beta) = \begin{cases} \{1\} & \text{if } f(\beta) > 0, \\ [0,1] & \text{if } f(\beta) = 0, \\ \{0\} & \text{if } f(\beta) < 0, \end{cases} \quad \text{and } \mathcal{S}(\alpha) = \begin{cases} \{1\} & \text{if } g(\alpha) > 0, \\ [0,1] & \text{if } g(\alpha) = 0, \\ \{0\} & \text{if } g(\alpha) < 0. \end{cases} \tag{1}
$$

A pair of strategies (α^*, β^*) is a *Nash equilibrium* (or simply an equilibrium) if each strategy is a best response to the other. That is, A's strategy α^* is a best response to B's playing β^* and B's strategy β^* is a best response to A's playing α^*.

The best responses (and consequently the equilibria) only depend on the signs of parameters u_0, u_1, v_0 and v_1 defined as follows:

$$
\begin{array}{l} u_1 = u(s_1,s_1) - u(s_0,s_1), \\ u_0 = u(s_0,s_0) - u(s_1,s_0), \end{array} \quad \text{and} \quad \begin{array}{l} v_1 = v(s_1,s_1) - v(s_1,s_0), \\ v_0 = v(s_0,s_0) - v(s_0,s_1). \end{array}
$$

Then we have

$$
\begin{aligned}
f(\beta) &= (u_0+u_1)\beta - u_0 \\
g(\alpha) &= (v_0+v_1)\alpha - v_0.
\end{aligned} \tag{2}
$$

Thus we can work with the following normalized 2×2 payoff matrix[1] $\mathcal{M}$ that preserves the best responses:

$$
\mathcal{M} = \begin{pmatrix} (u_1,v_1) & (0,0) \\ (0,0) & (u_0,v_0) \end{pmatrix}.
$$

Players' expected utility functions are given by

$$
\begin{aligned}
u(\alpha,\beta) &= \alpha\beta u_1 + (1-\alpha)(1-\beta)u_0 = (1-\beta)u_0 + f(\beta)\alpha \\
v(\alpha,\beta) &= \alpha\beta v_1 + (1-\alpha)(1-\beta)v_0 = (1-\alpha)v_0 + g(\alpha)\beta
\end{aligned} \tag{3}
$$

[1] Details of this normalization are given in Calvo-Armengol (2006) and Eichberger, Haller and Milne (1993).

and best responses are given by (1). The game associated is denoted by $\Gamma(\mathcal{M})$. It is referred to as the *game without perception* because players are not allowed to see their counterpart (and consequently cannot make their choice contingent on how they perceive their counterpart). The number of equilibria depends on the signs of the parameters u_0, u_1, v_0, and v_1. In the following we also use parameters $\bar{u}$ and $\bar{v}$ defined as

$$\bar{u} = \frac{u_0}{u_0 + u_1} \text{ and } \bar{v} = \frac{v_0}{v_0 + v_1} \text{ whenever } u_0 + u_1 \neq 0 \text{ and } v_0 + v_1 \neq 0.$$

Four classes of matrices are to be distinguished.

Definition 1 *$\mathcal{M}$ is (i) a* coordination *matrix if $u_0, u_1, v_0, v_1 > 0$, (ii) an* anti-coordination *matrix if $u_0, u_1, v_0, v_1 < 0$, (iii) a* competitive *matrix if either $u_0, u_1 > 0$ and $v_0, v_1 < 0$, or $u_0, u_1 < 0$ and $v_0, v_1 > 0$, (iv) a* dominance solvable *matrix if $u_0 u_1 < 0$ or $v_0 v_1 < 0$.*

The equilibria of the game without perception depend on the class of matrix as follows:

1. If $\mathcal{M}$ is a coordination matrix, game $\Gamma(\mathcal{M})$ has three equilibria: $(1,1)$, *i.e.* both players choosing s_1, $(0,0)$, *i.e.* both players choosing s_0 and $(\bar{v}, \bar{u})$, *i.e.* A chooses s_1 with probability $\bar{v}$ while B chooses s_1 with probability $\bar{u}$. A classical example of these games is the Battle of Sexes.

2. If $\mathcal{M}$ is a anti-coordination matrix, game $\Gamma(\mathcal{M})$ has three equilibria: $(1,0)$, *i.e.* Player A chooses s_1 and B chooses s_0, $(0,1)$, *i.e.* A chooses s_0 and B chooses s_1 and $(\bar{v}, \bar{u})$, *i.e.* A chooses s_1 with probability $\bar{v}$ and B plays s_1 with probability $\bar{u}$. A classical example of these games is the Chicken Game.

3. If $\mathcal{M}$ is a competitive matrix, game $\Gamma(\mathcal{M})$ has a single equilibrium: $(\bar{v}, \bar{u})$, *i.e.* A plays s_1 with probability $\bar{v}$ and B plays s_1 with probability $\bar{u}$. A classical example of these games is the Matching Pennies.

4. If $\mathcal{M}$ is a dominance solvable matrix, game $\Gamma(\mathcal{M})$ has a single equilibrium in pure strategies. It is $(1,1)$ whenever u_1, $v_1 > 0$, it is $(0,0)$ when u_0, $v_0 > 0$, it is $(1,0)$ when u_0, $v_1 < 0$ and it is $(0,1)$ when u_1, $v_0 < 0$. A classical example of these games is the Prisoner's Dilemma.

3 Beliefs based on perception

Psychologists find that first impressions of others can be remarkably accurate and affect our beliefs about them. For instance, when two people meet the initial observation is sufficient to perceive a state of emotion, a personality trait or an objective feature. Depending on the environment, one characteristic usually prevails over the rest. For instance, being smart is the dominant characteristic in a chess competition, being attractive is important on a date and being tall is essential to be a player on a basketball team. Thus, a brief exposure is sufficient for one agent to assess whether his/her counterpart has the predictable salient characteristic or not, but he/she finds it more difficult to evaluate how he/she is perceived. Still that agent will surely have an idea of how he/she is perceived which in turn will depend on his/her own perception of his/her counterpart.

We assume that the two players perceive the same characteristic. Depending on whether or not he/she perceives the characteristic in the counterpart, each agent may be of two types, I or II. Thus A's types are denoted by A_{I}, A_{II}, where A_{I} means that A perceives the characteristic in B, while A_{II} means that she does not. Similarly B's types are denoted by B_{I}, B_{II}, where B_{I} means he perceives the characteristic on A, while B_{II} means that he does not. Thus, the following four encounters between types can be set up: $(A_{\mathrm{I}}, B_{\mathrm{I}})$, $(A_{\mathrm{II}}, B_{\mathrm{II}})$, $(A_{\mathrm{I}}, B_{\mathrm{II}})$ and $(A_{\mathrm{II}}, B_{\mathrm{I}})$. Note that in each encounter both agents know their own type (what they perceive) but they do not know their counterpart's type (how they are perceived).

We further, wassume that agents assign probabilities to their counterpart's type depending on their own. Let $p_{\mathrm{I}} = p(B_{\mathrm{I}} \,|A_{\mathrm{I}}\,)$ be the conditional probability that A assigns B to type I given that she is of type I. For instance, if the characteristic in question is attractiveness, p_{I} is the probability that A assigns that she is perceived as attractive by B (*i.e.* B being of type I) given that she finds him attractive (*i.e.* A is of type I). The probability that she assigns to being perceived as non attractive if B looks attractive in her eyes (*i.e.* A is of type I) is $1 - p_{\mathrm{I}}$. Probabilities $p_{\mathrm{II}} = p(B_{\mathrm{I}} \,|A_{\mathrm{II}}\,)$, $q_{\mathrm{I}} = q(A_{\mathrm{I}} \,|B_{\mathrm{I}}\,)$ and $q_{\mathrm{II}} = p(A_{\mathrm{I}} \,|B_{\mathrm{II}}\,)$ and their complementary probabilities are read analogously. Based on these probabilities and on the kind of characteristic being observed we propose the following taxonomy of beliefs:

1. $p_{\mathrm{I}} = p_{\mathrm{II}}$ and $q_{\mathrm{I}} = q_{\mathrm{II}}$ are associated with the perception of *objective fea-*

tures, i.e. the equality between probabilities holds for objective features such as gender, being blue-eyed, red-haired,etc. In these cases how the counterpart is perceived does not affect the probabilities assigned. For instance, the probability that an agent assigns to being perceived as blue-eyed does not depend on whether the counterpart is blue-eyed or not. Beliefs of this kind of are called called *free* by d'Aspremont, Crémer and Gérard-Varet (2003).

2. $p_{\mathrm{I}} > p_{\mathrm{II}}$ and $q_{\mathrm{I}} > q_{\mathrm{II}}$ are associated with *emotional states.* "Emotion"[2] is understood here as a transitory state of a person that occurs as a reaction to a subjective experience which is transferred to others. People are usually not immune to others' emotions and unintentionally catch joy, sadness, anxiety, fear, etc. from simple exposure to them. It is well-known that a smile tends to be contagious (Provine, 1992). A number of studies confirm that exposure to emotional facial expressions (Dimberg, 1997) or listening to a happy or sad voice (Neuman and Stract, 2000) evokes a congruent effect in receivers. This is what Hartfield, Cacioppo and Rapson (1994) call to "emotional contagion." In these situations, the probability that an agent assigns to being perceived as, say happy if the counterpart looks happy is greater than the probability assigned to being perceived as happy if the counterpart looks unhappy. Therefore it can be expected that $p_{\mathrm{I}} > p_{\mathrm{II}}$ and $q_{\mathrm{I}} > q_{\mathrm{II}}$ which are referred to hereafter as *contagious* beliefs.

3. $p_{\mathrm{I}} < p_{\mathrm{II}}$ and $q_{\mathrm{I}} < q_{\mathrm{II}}$ are associated with *personality traits.* This is understood to mean an personality trait an individual characteristic non transferable to others but which nevertheless generates a reaction in them. For instance the counterpart's attractiveness may generate pessimistic feelings (or optimistic feelings in the face of the counterpart's unattractiveness) because personal insecurity may be determined by what one believes others think about one's attractiveness. The experiments of Walster et *al.* (1966) and Murstein (1972) show that attractive subjects have more rigorous requirements for an acceptable date than less attractive ones. Taylor, Fiore, Mendelsohn and Chesire (2011) show that "low self worth individuals will voluntarily select undesirable partners" (p. 942).[3] Accordingly if A finds

[2]See Reisenzein (2007) for a discussion of the definition of emotion.
[3]This is what is known in Psychology as the "matching hypothesis" which means that

B attractive then the probability that A assigns to being perceived as attractive by B is smaller than if she finds B is unattractive. Thus, in contrast to contagious beliefs, it can be expected that $p_{\mathrm{I}} < p_{\mathrm{II}}$ and $q_{\mathrm{I}} < q_{\mathrm{II}}$ which are referred to hereafter as *demanding* beliefs.

In short, depending on the characteristic being observed (an objective feature, an emotion, or a personality trait) three relationships between conditional probabilities of the two agents are possible, as embraced in the following definition:

Definition 2 *Beliefs* $\mathcal{B} = ((p_{\mathrm{I}}, p_{\mathrm{II}}), (q_{\mathrm{I}}, q_{\mathrm{II}}))$ *are* (i) free *if* $p_{\mathrm{I}} = p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} = q_{\mathrm{II}}$, (ii) contagious *if* $p_{\mathrm{I}} > p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} > q_{\mathrm{II}}$ (iii) demanding *if* $p_{\mathrm{I}} < p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} < q_{\mathrm{II}}$.

Note that in all three cases considered, beliefs satisfy the following condition: $(p_{\mathrm{I}} - p_{\mathrm{II}})(q_{\mathrm{I}} - q_{\mathrm{II}}) \geq 0$. The remaining cases (that is, when $(p_{\mathrm{I}} - p_{\mathrm{II}})(q_{\mathrm{I}} - q_{\mathrm{II}}) < 0$) are excluded because each agent is assumed to experience the same kind of impact by observing the characteristic at stake. Cases in which, for instance, an agent focuses on an emotion in the counterpart while the latter focuses on a personal trait are not considered in the current paper.

4 Games with perception

In this section we first model the strategic behavior of two players in a game with two actions when players are endowed with beliefs based on perception. Then we introduce the definition of a Nash Equilibrium for these games. Formally a game with perception is a Bayesian game.[4]

A *game with perception,* denoted by $\Gamma(\mathcal{M}, \mathcal{B})$, describes the interaction of players when beliefs are taken into account. It comprises by $\mathcal{M}$, the payoff matrix and $\mathcal{B}$, player's beliefs as defined in Section 2 and 3, respectively.

In game $\Gamma(\mathcal{M}, \mathcal{B})$ players are allowed to make their choice contingent on how they perceive their counterpart, that is, on their own type. A *strategy* for A specifies her choice when she is of type A_{I} (*i.e.* when she perceives the characteristic in question in B) and when she is of type A_{II} (*i.e.* when she does not perceive the characteristic in B). As in Section 2, we represent a strategy

individuals look for a partner who is socially as desirable as they are (see Real, 1991, Buss and Schackelford, 2008 and Reisenzein and Weber, 2009). The hypothesis holds for traits such as intelligence, empathy, extraversion, etc.

[4]For a formal definition of Bayesian games see Osborne (2004) Chapter 9.

by the probability that the player assigns to action s_1 (the complementary probability represents the choice of s_0). The only difference is that now we specify this probability for each type, that is for A_{I} and A_{II}. A strategy for A is denoted by pair $(\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}})$, where α_{I} gives the probability of A playing s_1 when her type is A_{I} and α_{II} gives the probability of A playing s_1 when her type is A_{II}. A strategy for B is defined similarly.

A player may decide to choose the same strategy when he/she perceives the characteristic in the counterpart and when he/she does not, i.e. may assign the same probability to s_1 for both types. In this case the strategy is referred to as *non discriminatory.* Formally strategy $(\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}})$ is a *non discriminatory* strategy if $\alpha_{\mathrm{I}} = \alpha_{\mathrm{II}}$. If $\alpha_{\mathrm{I}} \neq \alpha_{\mathrm{II}}$ strategy $(\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}})$ is a *discriminatory* strategy. That is, the player behaves differently when he/she perceives the characteristic than when he/she does not.

Players know their own type, so A's expected utility when she is of type I and II can be defined respectively. Let U_{I} and U_{II} denote the expected utility of A_{I} and A_{II} respectively. Similarly let V_{I} and V_{II} denote the expected utility of B_{I} and B_{II} respectively. These expected utilities depend the strategy for each type and on the counterpart's strategy. For instance, U_{I} is obtained as follows: p_{I} is the probability assigned by A_{I} assigns to being perceived as having the characteristic in question by B while $1 - p_{\mathrm{I}}$ is the probability of being perceived by B as not having that characteristic. If A_{I} plays α_{I} and B plays $(\beta_{\mathrm{I}}, \beta_{\mathrm{II}})$ she obtains $u(\alpha_{\mathrm{I}}, \beta_{\mathrm{I}})$ when B perceives her as having characteristic and $u(\alpha_{\mathrm{I}}, \beta_{\mathrm{II}})$ when B perceives her as not having the characteristic. Therefore:

$$
\begin{aligned}
U_{\mathrm{I}}(\alpha_{\mathrm{I}}, (\beta_{\mathrm{I}}, \beta_{\mathrm{II}})) &= p_{\mathrm{I}} u(\alpha_{\mathrm{I}}, \beta_{\mathrm{I}}) + (1 - p_{\mathrm{I}}) u(\alpha_{\mathrm{I}}, \beta_{\mathrm{II}}), \\
U_{\mathrm{II}}(\alpha_{\mathrm{II}}, (\beta_{\mathrm{I}}, \beta_{\mathrm{II}})) &= p_{\mathrm{II}} u(\alpha_{\mathrm{II}}, \beta_{\mathrm{I}}) + (1 - p_{\mathrm{II}}) u(\alpha_{\mathrm{II}}, \beta_{\mathrm{II}}), \\
V_{\mathrm{I}}((\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}}), \beta_{\mathrm{I}}) &= q_{\mathrm{I}} v(\alpha_{\mathrm{I}}, \beta_{\mathrm{I}}) + (1 - q_{\mathrm{I}}) v(\alpha_{\mathrm{II}}, \beta_{\mathrm{I}}), \\
V_{\mathrm{II}}((\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}}), \beta_{\mathrm{II}}) &= q_{\mathrm{II}} v(\alpha_{\mathrm{I}}, \beta_{\mathrm{II}}) + (1 - q_{\mathrm{II}}) v(\alpha_{\mathrm{II}}, \beta_{\mathrm{II}}).
\end{aligned}
$$

Substituting in these equations the expected payoffs as defined in (3), and using the following definitions

$$
\begin{aligned}
f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}}) &= (u_1+u_0)(p_{\mathrm{I}}\beta_{\mathrm{I}}+(1-p_{\mathrm{I}})\beta_{\mathrm{II}})-u_0,\\
f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}}) &= (u_1+u_0)(p_{\mathrm{II}}\beta_{\mathrm{I}}+(1-p_{\mathrm{II}})\beta_{\mathrm{II}})-u_0,\\
g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}}) &= (v_1+v_0)(q_{\mathrm{I}}\alpha_{\mathrm{I}}+(1-q_{\mathrm{I}})\alpha_{\mathrm{II}})-v_0,\\
g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}}) &= (v_1+v_0)(q_{\mathrm{II}}\alpha_{\mathrm{I}}+(1-q_{\mathrm{II}})\alpha_{\mathrm{II}})-v_0,
\end{aligned}
\tag{4}
$$

we obtain

$$
\begin{aligned}
U_{\mathrm{I}}(\alpha_{\mathrm{I}},(\beta_{\mathrm{I}},\beta_{\mathrm{II}})) &= [1-p_{\mathrm{I}}\beta_{\mathrm{I}}-(1-p_{\mathrm{I}})\beta_{\mathrm{II}}]\,u_0+f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})\alpha_{\mathrm{I}},\\
U_{\mathrm{II}}(\alpha_{\mathrm{II}},(\beta_{\mathrm{I}},\beta_{\mathrm{II}})) &= [1-p_{\mathrm{II}}\beta_{\mathrm{I}}-(1-p_{\mathrm{II}})\beta_{\mathrm{II}}]\,u_0+f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})\alpha_{\mathrm{II}},\\
V_{\mathrm{I}}((\alpha_{\mathrm{I}},\alpha_{\mathrm{II}}),\beta_{\mathrm{I}}) &= [1-q_{\mathrm{I}}\alpha_{\mathrm{I}}-(1-q_{\mathrm{I}})\alpha_{\mathrm{II}}]\,v_0+g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})\,\beta_{\mathrm{I}},\\
V_{\mathrm{II}}((\alpha_{\mathrm{I}},\alpha_{\mathrm{II}}),\beta_{\mathrm{II}}) &= [1-q_{\mathrm{II}}\alpha_{\mathrm{I}}-(1-q_{\mathrm{II}})\alpha_{\mathrm{II}}]\,v_0+g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})\beta_{\mathrm{II}}.
\end{aligned}
$$

Given player B's strategy $(\beta_{\mathrm{I}},\beta_{\mathrm{II}})$ player A can choose α_{I} in order to maximize $U_{\mathrm{I}}(\alpha_{\mathrm{I}},(\beta_{\mathrm{I}},\beta_{\mathrm{II}}))$ and choose α_{II} in order to maximize $U_{\mathrm{II}}(\alpha_{\mathrm{II}},(\beta_{\mathrm{I}},\beta_{\mathrm{II}}))$. We denote player A's best response by $\mathcal{R}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})$ defined as follows:

$$
\begin{cases}
\{(0,0)\} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0,\\
\{(1,1)\} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0,\\
\{(1,0)\} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0,\\
\{(0,1)\} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0,\\
\{(0,\alpha)\,|\alpha\in[0,1]\ \} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})=0,\\
\{(\alpha,0)\,|\alpha\in[0,1]\ \} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})=0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})<0,\\
\{(1,\alpha)\,|\alpha\in[0,1]\ \} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})=0,\\
\{(\alpha,1)\,|\alpha\in[0,1]\ \} & \text{if} \quad f_{\mathrm{I}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})=0 \quad \text{and} \quad f_{\mathrm{II}}(\beta_{\mathrm{I}},\beta_{\mathrm{II}})>0,\\
[0,1]\times[0,1] & \text{if} \quad (\beta_{\mathrm{I}},\beta_{\mathrm{II}})=(\bar{u},\bar{u}).
\end{cases}
$$

Player B's best response is denoted by $\mathcal{S}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})$ defined as follows:

$$
\begin{cases}
\{(0,0)\} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0,\\
\{(1,1)\} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0,\\
\{(1,0)\} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0,\\
\{(0,1)\} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0,\\
\{(0,\beta)\,|\beta\in[0,1]\ \} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})=0,\\
\{(\beta,0)\,|\beta\in[0,1]\ \} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})=0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})<0,\\
\{(1,\beta)\,|\beta\in[0,1]\ \} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})=0,\\
\{(\beta,1)\,|\beta\in[0,1]\ \} & \text{if} \quad g_{\mathrm{I}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})=0 \quad \text{and} \quad g_{\mathrm{II}}(\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})>0,\\
[0,1]\times[0,1] & \text{if} \quad (\alpha_{\mathrm{I}},\alpha_{\mathrm{II}})=(\bar{v},\bar{v}).
\end{cases}
$$

A pair of strategies $((\alpha_{\mathrm{I}}^*,\alpha_{\mathrm{II}}^*),(\beta_{\mathrm{I}}^*,\beta_{\mathrm{II}}^*))$ is a *Nash equilibrium (or simply an equilibrium) of a game with perception* if $(\alpha_{\mathrm{I}}^*,\alpha_{\mathrm{II}}^*)$ is a best response for A when B plays $(\beta_{\mathrm{I}}^*,\beta_{\mathrm{II}}^*)$ and vice versa.

The question addressed in this paper is whether the mere possibility of playing differently depending on types generates discriminatory behavior at equilibrium. This is why we classify equilibria in three categories.

Definition 3 *Let* $((\alpha^*_{\mathrm{I}}, \alpha^*_{\mathrm{II}}), (\beta^*_{\mathrm{I}}, \beta^*_{\mathrm{II}}))$ *be an equilibrium of a game with perception. This equilibrium is* (i) non discriminatory *if* $\alpha^*_{\mathrm{I}} = \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} = \beta^*_{\mathrm{II}}$*,* (ii) discriminatory *if* $\alpha^*_{\mathrm{I}} \neq \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} \neq \beta^*_{\mathrm{II}}$*, and* (iii) partially discriminatory *if* $(\alpha^*_{\mathrm{I}} = \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} \neq \beta^*_{\mathrm{II}})$ *or* $(\alpha^*_{\mathrm{I}} \neq \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} = \beta^*_{\mathrm{II}})$*.*

If all equilibria are non discriminatory the answer to the question is no, since players will not behave differently depending on types. However, as shown in the next section, there are discriminatory equilibria. By contrast we find no partially discriminatory equilibria, which means that players never behave asymmetrically at equilibrium: either both players take into account the perception of the other or neither of them does.

5 Equilibria in games with perception

In this section we characterize the set of equilibria of a game with perception. All proofs are relegated to the Appendix. First, we prove that there are no partially discriminatory equilibria.

Proposition 1 *If* $((\alpha^*_{\mathrm{I}}, \alpha^*_{\mathrm{II}}), (\beta^*_{\mathrm{I}}, \beta^*_{\mathrm{II}}))$ *is an equilibrium of a game with perception, then either* $(\alpha^*_{\mathrm{I}} = \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} = \beta^*_{\mathrm{II}})$ *or* $(\alpha^*_{\mathrm{I}} \neq \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} \neq \beta^*_{\mathrm{II}})$*.*

This result says that there is no equilibrium in which A behaves differently depending on how she perceives B while B behaves identically no matter how he perceives A. That is, either both players use discriminatory strategies or neither of them does. Interesting enough although discrimination is usually understood as an asymmetric phenomenon players' behavior is never asymmetric: either both players discriminate or neither does.

We therefore focus the analysis hereafter on non discriminatory and discriminatory equilibria. The following proposition characterizes non discriminatory equilibria:

Proposition 2 *Let* $\Gamma(\mathcal{M}, \mathcal{B})$ *be a game with perception. The pair of strategies* $((\alpha^*, \alpha^*), (\beta^*, \beta^*))$ *is a non discriminatory equilibrium of game* $\Gamma(\mathcal{M}, \mathcal{B})$ *if and only if* (α^*, β^*) *is an equilibrium of the game without perception* $\Gamma(\mathcal{M})$*.*

That is, in game with perception $\Gamma(\mathcal{M}, \mathcal{B})$ we have the following non discriminatory equilibria depending on the payoff matrix.

1. If $\mathcal{M}$ is a dominance solvable matrix then there is a single non discriminatory equilibrium. The equilibrium strategies are one of the following: (a) both players always choose s_1; (b) both players always choose s_0; (c) player A always chooses s_1 while B always chooses s_0; (d) player A always chooses s_0 while B always chooses s_1.

2. If $\mathcal{M}$ is a coordination matrix then there are three non discriminatory equilibria: (i) both players always choose s_1; (ii) both players always choose s_0; and (iii) player A always chooses s_1 with probability $\bar{v}$ while B always chooses s_1 with probability $\bar{u}$.

3. If $\mathcal{M}$ is an anti-coordination matrix then there are three non discriminatory equilibria: (i) player A always chooses s_1 while B always chooses s_0; (ii) player A always chooses s_0 while B always chooses s_1; and (iii) player A always chooses s_1 with probability $\bar{v}$ while B always chooses s_1 with probability $\bar{u}$.

4. If $\mathcal{M}$ is a competitive matrix then there is one non discriminatory equilibrium: player A always chooses s_1 with probability $\bar{v}$ while B always chooses s_1 with probability $\bar{u}$.

We turn now to the analysis of discriminatory equilibria. We first show games with perception, in which such equilibria never arise:

Proposition 3 *Let* $\Gamma(\mathcal{M}, \mathcal{B})$ *be a game with perception.*

1. *There is no discriminatory equilibrium if* $\mathcal{M}$ *is a dominance solvable or competitive matrix.*

2. *There is no discriminatory equilibrium if* $\mathcal{M}$ *is a coordination or anti-coordination matrix and* $\mathcal{B}$ *comprises free beliefs.*

The proposition gives us the cases where perception plays no role. If players play prisoners' dilemma or matching pennies, the counterpart's perception does not affect the choice of the strategy at equilibrium. Furthermore, if the perception observed concerns an objective characteristic, its corresponding beliefs do not play any role either.

Thus, the only cases that remain to be checked are games with perception $\Gamma(\mathcal{M}, \mathcal{B})$ in which $\mathcal{M}$ is a coordination or anti-coordination matrix, and $\mathcal{B}$ comprises either contagious or demanding beliefs.[5] There are discriminatory equilibria in these games, as the next two results show.

The first result focuses on equilibria in which both players are more likely to choose the same action when they perceive the characteristic. That is, if $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ is an equilibrium either ($\alpha_{\mathrm{I}}^* > \alpha_{\mathrm{II}}^*$ and $\beta_{\mathrm{I}}^* > \beta_{\mathrm{II}}^*$) or ($\alpha_{\mathrm{I}}^* < \alpha_{\mathrm{II}}^*$ and $\beta_{\mathrm{I}}^* < \beta_{\mathrm{II}}^*$).

Theorem 1 *Let $\Gamma(\mathcal{M}, \mathcal{B})$ be a game with perception where either M is a coordination matrix and B comprises contagious beliefs or M is an anti-coordination matrix and B comprises demanding beliefs. The discriminatory equilibria of game $\Gamma(\mathcal{M}, \mathcal{B})$ are:*

$((1,0),(1,0))$	*if* $p_{\mathrm{II}} < \overline{u} < p_{\mathrm{I}}$ *and* $q_{\mathrm{II}} < \overline{v} < q_{\mathrm{I}}$
$((0,1),(0,1))$	*if* $p_{\mathrm{II}} < 1-\overline{u} < p_{\mathrm{I}}$ *and* $q_{\mathrm{II}} < 1-\overline{v} < q_{\mathrm{I}}$

Table 1a: Equilibria in pure strategies

$((\frac{\overline{v}}{q_{\mathrm{II}}},0),(1,\frac{\overline{u}-p_{\mathrm{I}}}{1-p_{\mathrm{I}}}))$	*if* $\overline{u} > p_{\mathrm{I}}$ *and* $\overline{v} < q_{\mathrm{II}}$
$((\frac{\overline{v}}{q_{\mathrm{I}}},0),(\frac{\overline{u}}{p_{\mathrm{I}}},0))$	*if* $\overline{u} < p_{\mathrm{I}}$ *and* $\overline{v} < q_{\mathrm{I}}$
$((1,\frac{\overline{v}-q_{\mathrm{II}}}{1-q_{\mathrm{II}}}),(1,\frac{\overline{u}-p_{\mathrm{II}}}{1-p_{\mathrm{II}}}))$	*if* $\overline{u} > p_{\mathrm{II}}$ *and* $\overline{v} > q_{\mathrm{II}}$
$((1,\frac{\overline{v}-q_{\mathrm{I}}}{1-q_{\mathrm{I}}}),(\frac{\overline{u}}{p_{\mathrm{II}}},0))$	*if* $\overline{u} < p_{\mathrm{II}}$ *and* $\overline{v} > q_{\mathrm{I}}$

Table 1b: Equilibria in mixed strategies with $\alpha_{\mathrm{I}}^* > \alpha_{\mathrm{II}}^*$ and $\beta_{\mathrm{I}}^* > \beta_{\mathrm{II}}^*$

$((0,\frac{\overline{v}}{1-q_{\mathrm{I}}}),(\frac{\overline{u}-(1-p_{\mathrm{II}})}{p_{\mathrm{II}}},1))$	*if* $1-\overline{u} < p_{\mathrm{II}}$ *and* $1-\overline{v} > q_{\mathrm{I}}$
$((0,\frac{\overline{v}}{1-q_{\mathrm{II}}}),(0,\frac{\overline{u}}{1-p_{\mathrm{II}}}))$	*if* $1-\overline{u} > p_{\mathrm{II}}$ *and* $1-\overline{v} > q_{\mathrm{II}}$
$((\frac{\overline{v}-(1-q_{\mathrm{I}})}{q_{\mathrm{I}}},1),(\frac{\overline{u}-(1-p_{\mathrm{I}})}{p_{\mathrm{I}}},1))$	*if* $1-\overline{u} < p_{\mathrm{I}}$ *and* $1-\overline{v} < q_{\mathrm{I}}$
$((\frac{\overline{v}-(1-q_{\mathrm{II}})}{q_{\mathrm{II}}},1),(0,\frac{\overline{u}}{1-p_{\mathrm{I}}}))$	*if* $1-\overline{u} > p_{\mathrm{I}}$ *and* $1-\overline{v} < q_{\mathrm{II}}$

Table 1c: Equilibria in mixed strategies with $\alpha_{\mathrm{I}}^* < \alpha_{\mathrm{II}}^*$ and $\beta_{\mathrm{I}}^* < \beta_{\mathrm{II}}^*$

To illustrate of these results let us go back to the coordination game in the Introduction.

[5] To avoid technicalities concerning the existence of an infinite number of equilibria, we assume that $\bar{u}$ and $\bar{v}$ do not coincide with the probabiliy assigned by players to their opponents. That is, $\bar{u} \notin \{p_{\mathrm{I}}, p_{\mathrm{II}}, 1-p_{\mathrm{I}}, 1-p_{\mathrm{II}}\}$ and $\bar{v} \notin \{q_{\mathrm{I}}, q_{\mathrm{II}}, 1-q_{\mathrm{I}}, 1-q_{\mathrm{II}}\}$, which holds generically. Results are analogous if these conditions are not assumed. The only difference is that instead of obtaining single point Nash equilibria we obtain intervals with infinite Nash equilibria.

Example 1 (cont.) *Consider the game with perception* $\Gamma(\mathcal{M},\mathcal{B})$ *where matrix* $\mathcal{M}$ *is given in Figure 1 and beliefs are* $\mathcal{B} = ((\frac{9}{10},\frac{3}{5}),(\frac{19}{20},\frac{4}{5}))$*. We assume that the characteristic associated is trust (as it is an emotion* $p_{\mathrm{I}} > p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} > q_{\mathrm{II}}$*). It can be checked that game* $\Gamma(\mathcal{M},\mathcal{B})$ *has three non discriminatory equilibria (given by Proposition 2) and four discriminatory equilibria.*[6] *One of them is Alice and Bob playing "cooperate if the counterpart looks trustworthy and defect if the counterpart looks untrustworthy".*

The second main result characterizes those equilibria in which one player is more likely to choose with larger probability one action and the counterpart is more likely to choose the other action when they perceive the characteristic in the other. That is, if $((\alpha^*_{\mathrm{I}},\alpha^*_{\mathrm{II}}),(\beta^*_{\mathrm{I}},\beta^*_{\mathrm{II}}))$ equilibrium either ($\alpha^*_{\mathrm{I}} > \alpha^*_{\mathrm{II}}$ and $\beta^*_{\mathrm{I}} < \beta^*_{\mathrm{II}}$) or ($\alpha^*_{\mathrm{I}} < \alpha^*_{\mathrm{II}}$ and $\beta^*_{\mathrm{I}} > \beta^*_{\mathrm{II}}$).

Theorem 2 *Let* $\Gamma(\mathcal{M},\mathcal{B})$ *be a game with perceptions where either* M *is a coordination matrix and* B *comprises demanding beliefs or* M *is an anti-coordination matrix and* B *comprises contagious beliefs. The discriminatory Nash equilibria of game* $\Gamma(\mathcal{M},\mathcal{B})$ *are:*

$((1,0),(0,1))$	*if* $p_{\mathrm{I}} < 1-\overline{u} < p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} < \overline{v} < q_{\mathrm{II}}$
$((0,1),(1,0))$	*if* $p_{\mathrm{I}} < \overline{u} < p_{\mathrm{II}}$ *and* $q_{\mathrm{I}} < 1-\overline{v} < q_{\mathrm{II}}$

Table 2a: Equilibria in pure strategies

$((1,\frac{\overline{v}-q_{\mathrm{II}}}{1-q_{\mathrm{II}}}),(\frac{\overline{u}}{1-p_{\mathrm{II}}},1))$	*if* $1-\overline{u} > p_{\mathrm{II}}$ *and* $\overline{v} > q_{\mathrm{II}}$
$((\frac{\overline{v}}{q_{\mathrm{I}}},0),(\frac{\overline{u}-(1-p_{\mathrm{I}})}{p_{\mathrm{I}}},1))$	*if* $1-\overline{u} < p_{\mathrm{I}}$ *and* $\overline{v} < q_{\mathrm{I}}$
$((\frac{\overline{v}}{q_{\mathrm{II}}},0),(0,\frac{\overline{u}}{1-p_{\mathrm{I}}}))$	*if* $1-\overline{u} > p_{\mathrm{I}}$ *and* $\overline{v} < q_{\mathrm{II}}$
$((1,\frac{\overline{v}-q_{\mathrm{I}}}{1-q_{\mathrm{I}}}),(\frac{\overline{u}-(1-p_{\mathrm{II}})}{p_{\mathrm{II}}},1))$	*if* $1-\overline{u} < p_{\mathrm{II}}$ *and* $\overline{v} > q_{\mathrm{I}}$

Table 2b: Equilibria in mixed strategies with $\alpha^*_{\mathrm{I}} > \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} < \beta^*_{\mathrm{II}}$

$((0,\frac{\overline{v}}{1-q_{\mathrm{II}}}),(1,\frac{\overline{u}-p_{\mathrm{I}}}{1-p_{\mathrm{I}}}))$	*if* $\overline{u} > p_{\mathrm{II}}$ *and* $1-\overline{v} > q_{\mathrm{II}}$
$((\frac{\overline{v}-(1-q_{\mathrm{II}})}{q_{\mathrm{II}}},1),(1,\frac{\overline{u}-p_{\mathrm{I}}}{1-p_{\mathrm{I}}}))$	*if* $\overline{u} > p_{\mathrm{I}}$ *and* $1-\overline{v} < q_{\mathrm{II}}$
$((\frac{\overline{v}-(1-q_{\mathrm{I}})}{q_{\mathrm{I}}},1),(\frac{\overline{u}}{p_{\mathrm{I}}},0))$	*if* $\overline{u} < p_{\mathrm{I}}$ *and* $1-\overline{v} > q_{\mathrm{I}}$
$((0,\frac{\overline{v}}{1-q_{\mathrm{I}}}),(\frac{\overline{u}}{p_{\mathrm{II}}},0))$	*if* $\overline{u} < p_{\mathrm{II}}$ *and* $1-\overline{v} > q_{\mathrm{I}}$

Table 2c: Equilibria in mixed strategies with $\alpha^*_{\mathrm{I}} < \alpha^*_{\mathrm{II}}$ *and* $\beta^*_{\mathrm{I}} > \beta^*_{\mathrm{II}}$

[6] These are (i) $((1,0),(1,0))$, *(ii)* $((\frac{35}{38},0),(\frac{35}{36},0))$, (iii) $((1,\frac{5}{8}),(1,\frac{11}{16}))$ and (iv) $((\frac{87}{95},1),(\frac{31}{36},1))$.

To illustrate of these results let us go back to the coordination game in the Introduction.

Example 1 (cont.) *With demanding beliefs (for instance attractiveness) the pair of strategies "cooperate if the counterpart looks attractive and defect if the counterpart looks unattractive" never constitutes an equilibrium in a game of perception* $\Gamma(\mathcal{M}, \mathcal{B})$ *where matrix* $\mathcal{M}$ *is given in Figure 1. However Alice playing "cooperate if Bob is attractive and defect if Bob is not" while Bob plays "defect if Alice is attractive and cooperate if Alice is not" may be an equilibrium, for some demanding beliefs. For instance it can be checked that it is an equilibrium if* $\mathcal{B} = ((\frac{1}{10}, \frac{1}{2}), (\frac{7}{10}, \frac{9}{10}))$.

The difference in players' behavior regarding discriminatory strategies in the coordination game in Example 1 can be attributed to the different kinds of beliefs. Emotions generate emotional contagion which is the tendency for two individuals to emotionally converge. Therefore it seems natural that in coordination games the two agents should have identical behavior if their beliefs are contagious. By contrast the lack of congruency generated by a non-transferable personality trait such as attractiveness may explain opposite behavior by players when beliefs are demanding.

We end up this section with some additional results. First note that in all discriminatory Nash equilibria each player always chooses one pure strategy for at least one of his/her types.

Next, we represent the conditions and equilibria of Theorem 1 (though the same could be done for Theorem 2). Tables 1a, 1b, and 1c, and are represented in two figures to help us understand how the number of discriminatory equilibria can be computed. Conditions implying $\bar{u}$ and $\bar{v}$ are represented on the left-side of *Figure 3*, while conditions implying $(1-\bar{u})$ and $(1-\bar{v})$ are represented on the right-side. In both figures the $[0,1]^2$ space is divided into two different areas: a non-shadowed area, and a black one. It can be checked that if $(\bar{u}, \bar{v})$ lies in the black area of the left-side figure, there are three discriminatory equilibria, and otherwise only one. Similarly if $(1-\bar{u}, 1-\bar{v})$ lies in the black area of the right-side figure, there are three discriminatory equilibria, otherwise only one. Therefore the number of equilibria for games with perception considered in Theorem 1 may be two four or six. The number of equilibria for games with perception in Theorem 2 can be derived similarly.

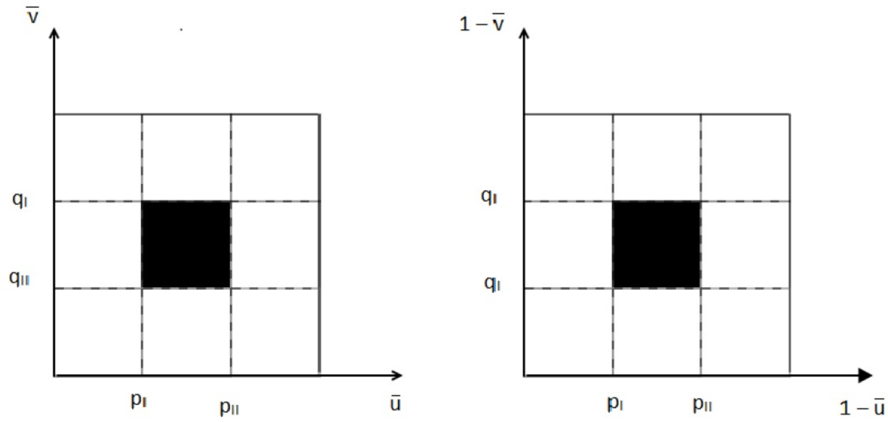

Figure 3

The even number of these equilibria is due to the dichotomy of the characteristic under analysis (either an agent is perceived as having characteristic or he/she is not). Whenever there is an equilibrium strategy in which $\alpha^*_{\mathrm{I}} > \alpha^*_{\mathrm{II}}$ we have another in which $\alpha^*_{\mathrm{II}} > \alpha^*_{\mathrm{I}}$. The reason of this apparent contradiction lies in that the formal modelling of the games with perception, the characteristic at stake (an emotion or a personality trait) does not incorporate the contextual connotation regarding the action to be played in a coordination (or anti-coordination) game. However in reality one of these equilibria is more plausible than the other and this will depend on the setting under analysis. For instance, "cooperate if the other looks untrustworthy and defect if the other looks trustworthy" is less reasonable than "cooperate if the other looks trustworthy and defect if the other looks untrustworthy".

6 Concluding remarks

Situations suitable for modeling as games rarely arise in real life between agents who do not see each other. The experimental results described in the introduction make clear that perception modifies choices at equilibrium. Deviations from "rational" outcomes may be attributed to the characteristic under study: beauty, trust or gender. These experimental studies led us to add perception to the traditional 2×2 games. The inclusion of perception in normal form games adds realism to the study of the strategic interaction between players. We have restricted ourselves to 2×2 games which are an archetype for strategic interaction, and to the simple case of dichotomous characteristics. Still, the mere possibility of discriminating generates discriminatory equilibria in coordination and anti-coordination games even when payoffs are type-irrelevant. A subjective parameter which does not affect the payoff matrix manages to generate new equilibria. In this sense our approach follows to a certain extent that of Cass and Shell (1983), introducing an external parameter into a model that in principle does not appear as relevant becomes essential and provides non standard solutions.

Our equilibrium outcomes are consistent with the findings of some of the experiments considered in the introduction. Indeed in a coordination game, the pair of strategies where both players "cooperate if the counterpart looks trustwothy and defect otherwise" is sustained at equilibrium. Similarly in an anti-coordination game (a public good game can be modeled as a simultaneous 2×2 game) the pair of strategies where both players choose "cooperate if the counterpart is attractive and defect otherwise" is also sustained at equilibrium. By contrast in our model characteristics such as gender or race do not affect the results, but experiments show that perception of race or gender matters. One possible explanation is that people perceive the stereotype associated with the characteristic rather than the characteristic per se; for instance, "women are more generous." Since many experimental works involve sequential games, it would be worth modeling games in extensive form with perception. This is left for further research.

Finally, note that in this paper we have focused on an *interim stage* of decision making, i.e. a scenario where players are already endowed with some belief, provoked in these cases by perception following a brief observation of their partner. We could alternatively study an *ex ante* stage that precedes perception in which players hold prior beliefs on how they will perceive their opponent, and interim post-perception beliefs are obtained by updating this original prior belief. In particular, when such prior beliefs are *common* (*i.e.* players hold a *common prior* belief), the results obtained are not sharper than those obtained without this assumption.

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7 Appendix

Proof of Proposition 1. In a game with perception assume that there is an equilibrium $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ satisfying $\alpha_{\mathrm{I}}^* = \alpha_{\mathrm{II}}^*$ and $\beta_{\mathrm{I}}^* \neq \beta_{\mathrm{II}}^*$. By plugging $\alpha_{\mathrm{I}}^* = \alpha_{\mathrm{II}}^*$ into (4) we obtain $g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*))$, and thus we have either $\beta_{\mathrm{I}}^* = \beta_{\mathrm{II}}^*$, or $\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^* \in (0,1)$. The former is false by hypothesis, and the latter only holds if $g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = 0$. But note that $g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = 0$ implies that $\alpha_{\mathrm{I}}^* \neq \alpha_{\mathrm{II}}^*$. Hence, we reach a contradiction, and the assertion of the proposition follows. ■

Proof of Proposition 2. Let $\Gamma(\mathcal{M}, \mathcal{B})$ be a game with perception. From (2) and (4) it can be seen that for any pair of strategies $((\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}}), (\beta_{\mathrm{I}}, \beta_{\mathrm{II}}))$ where $\alpha_{\mathrm{I}} = \alpha_{\mathrm{II}} = \alpha$ and $\beta_{\mathrm{I}} = \beta_{\mathrm{II}} = \beta$ we have both that $f_{\mathrm{I}}((\beta_{\mathrm{I}}, \beta_{\mathrm{II}})) = f_{\mathrm{II}}((\beta_{\mathrm{I}}, \beta_{\mathrm{II}})) = f(\beta)$ and $g_{\mathrm{I}}((\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}})) = g_{\mathrm{II}}((\alpha_{\mathrm{I}}, \alpha_{\mathrm{II}})) = g(\alpha)$. Thus, it follows immediately that $\mathcal{R}(\beta, \beta) = \mathcal{R}(\beta) \times \mathcal{R}(\beta)$, and $\mathcal{S}(\alpha, \alpha) = \mathcal{S}(\alpha) \times \mathcal{S}(\alpha)$. As a result we have both that $(\alpha, \alpha) \in \mathcal{R}(\beta, \beta)$ if and only if $\alpha \in \mathcal{R}(\beta)$, and $(\beta, \beta) \in \mathcal{S}(\alpha, \alpha)$ if and only if $\beta \in \mathcal{S}(\alpha)$. Thus, $((\alpha, \alpha), (\beta, \beta))$ is an equilibrium of $\Gamma(\mathcal{M}, \mathcal{B})$ if and only if (α, β) is an equilibrium of game $\Gamma(\mathcal{M})$. ■

Proof of Proposition 3. Let $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ be a discriminatory equilibrium of game with perception $\Gamma(\mathcal{M}, \mathcal{B})$. Then we have that:

$$\begin{aligned} f_{\mathrm{I}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) - f_{\mathrm{II}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) &= (u_1 + u_0)(p_{\mathrm{I}} - p_{\mathrm{II}})(\beta_{\mathrm{I}}^* - \beta_{\mathrm{II}}^*), \\ g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) - g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) &= (v_1 + v_0)(q_{\mathrm{I}} - q_{\mathrm{II}})(\alpha_{\mathrm{I}}^* - \alpha_{\mathrm{II}}^*). \end{aligned} \tag{5}$$

Thus, if $\mathcal{B}$ comprises free beliefs, in equilibrium it must hold that $f_{\mathrm{I}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) = f_{\mathrm{II}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ and $g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) = g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*))$. Note that this is only possible if $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) = ((\bar{v}, \bar{v}), (\bar{u}, \bar{u}))$. Thus, free beliefs never provide equilibria in discriminatory strategies. In particular, this proves part 2 of Proposition 3.

Regarding part 1 assume first that $\mathcal{M}$ is a dominance solvable matrix and, with no loss of generality, that action s_1 is dominant for A. Then, since $\mathcal{R}(\beta) = \{1\}$ for any strategy $\beta \in [0,1]$, we have that $\mathcal{R}((\beta_{\mathrm{I}}, \beta_{\mathrm{II}})) = \{(1,1)\}$, for any strategy $(\beta_{\mathrm{I}}, \beta_{\mathrm{II}})$. Since $g_{\mathrm{I}}(1,1) = g_{\mathrm{II}}(1,1) \neq 0$, we conclude that $\mathcal{S}((1,1)) = \{(y,y)\}$ for some $y \in \{1,0\}$, and therefore, that there is no discriminatory equilibrium.

Assume now that $\mathcal{M}$ is a strictly competitive matrix and, with no loss of generality, that $u_1, u_0 > 0$, and $v_1, v_0 < 0$. As seen above, it can be assumes that $\mathcal{B}$ does not comprise free beliefs. Let $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ be an equilibrium. From (5) we have that if $\mathcal{B}$ comprises contagious beliefs, then:

$$\alpha_{\mathrm{I}}^* > \alpha_{\mathrm{II}}^* \iff f_{\mathrm{I}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) \geq f_{\mathrm{II}}((\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*)) \iff \beta_{\mathrm{I}}^* \geq \beta_{\mathrm{II}}^*,$$

and,

$$\beta_{\mathrm{I}}^* > \beta_{\mathrm{II}}^* \iff g_{\mathrm{I}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) \geq g_{\mathrm{II}}((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*)) \iff \alpha_{\mathrm{I}}^* \leq \alpha_{\mathrm{II}}^*.$$

Thus, from these two expressions and Proposition 1, we have that $\alpha_{\mathrm{I}}^* > \alpha_{\mathrm{II}}^*$ implies $\beta_{\mathrm{I}}^* > \beta_{\mathrm{II}}^*$ and that $\beta_{\mathrm{I}}^* > \beta_{\mathrm{II}}^*$ implies $\alpha_{\mathrm{I}}^* < \alpha_{\mathrm{II}}^*$. Hence, contagious beliefs cannot induce discriminatory equilibria. The proof for demanding beliefs is completed similarly. ■

Proof of Theorems 1 and 2. Let $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ be a discriminatory equilibrium in game with perception $\Gamma(\mathcal{M}, \mathcal{B})$. First note that (5) impose that if $\mathcal{M}$ is a coordination matrix and $\mathcal{B}$ comprises contagious beliefs, or $\mathcal{M}$ is an anti-coordination matrix and $\mathcal{B}$ comprises demanding beliefs, then $(\alpha_{\mathrm{I}}^* - \alpha_{\mathrm{II}}^*)(\beta_{\mathrm{I}}^* - \beta_{\mathrm{II}}^*) > 0$ hold. Second if $\mathcal{M}$ is a coordination matrix and $\mathcal{B}$ comprises demanding beliefs, or with $\mathcal{M}$ is an anti-coordination matrix and $\mathcal{B}$ comprises contagious beliefs, then $(\alpha_{\mathrm{I}}^* - \alpha_{\mathrm{II}}^*)(\beta_{\mathrm{I}}^* - \beta_{\mathrm{II}}^*) < 0$ holds. Then, from all the partial results above: (a) for games with a coordination matrix and contagious beliefs or with an anti-coordination matrix and demanding beliefs, the only possible non discriminatory equilibria are discriminatory equilibria $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ such that at least one of two types of players plays a pure strategy, and $(\alpha_{\mathrm{I}}^* - \alpha_{\mathrm{II}}^*)(\beta_{\mathrm{I}}^* - \beta_{\mathrm{II}}^*) > 0$, and ($b$) for games with a coordination matrix and demanding beliefs and with an anti-coordination matrix and contagious beliefs, the only possible non discriminatory equilibria are non discriminatory equilibria $((\alpha_{\mathrm{I}}^*, \alpha_{\mathrm{II}}^*), (\beta_{\mathrm{I}}^*, \beta_{\mathrm{II}}^*))$ such that at least one of the two types of players plays a pure strategy, and $(\alpha_{\mathrm{I}}^* - \alpha_{\mathrm{II}}^*)(\beta_{\mathrm{I}}^* - \beta_{\mathrm{II}}^*) < 0$. Note that in the tables in the statement of the theorems: (i) all the possible pairs of strategies that could be an equilibrium according to (a) and (b) above are represented (that is, no pair of strategies not represented in the table is an equilibrium in discriminatory strategies); (ii) it is immediately apparent that the conditions mean that the pair of strategies form an equilibrium; and (iii) the conditions in Tables 1a, 1b, 1c and 2a, 2b, 2c in each theorem are exhaustive given that we assume that $\bar{u} \notin \{p_{\mathrm{I}}, p_{\mathrm{II}}, 1 - p_{\mathrm{I}}, 1 - p_{\mathrm{II}}\}$ and $\bar{v} \notin \{q_{\mathrm{I}}, q_{\mathrm{II}}, 1 - q_{\mathrm{I}}, 1 - q_{\mathrm{II}}\}$ (which holds generically), i.e. it is impossible for none of them to be satisfied. ■