Beyond the Arrow effect: income distribution and multi-quality firms in a Schumpeterian framework*

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Abstract

This paper introduces multi-quality firms within a Schumpeterian framework. Featuring non-homothetic preferences and income disparities in an otherwise standard quality-ladder model, I indeed show that the resulting differences in the willingness to pay for quality among consumers generate both positive investments in R&D by industry leaders *and* positive market shares for more than one quality, hence allowing for the emergence of multi-product firms within a vertical innovation framework. This positive investment in R&D by incumbents is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers: in our framework, the incentive for a leader to invest in R&D stems from the possibility for an incumbent having innovated twice in a row to efficiently discriminate between rich and poor consumers displaying differences in their willingness to pay for quality. I am then also able to analyze the impact of inequality both on long-term growth and on the allocation of R&D activities between challengers and incumbents. I find that an increase in the income *gap* shifts R&D activities from challengers to incumbents, and has an overall positive effect on an economy's growth rate. On the other hand, a greater income *concentration* is unequivocally detrimental for growth, diminishing both the incumbents' and the challengers' R&D activities.

Keywords: Growth, Innovation, Income inequality, Multi-Product firms.

JEL classification: O3, O4, F4.

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1 Introduction

The importance and specificities of multi-product firms (MPFs) have lately been exemplified by a growing body of literature.[1] In particular, because of unique supply and demand linkages, MPFs' product-market decisions such as intra-firm portfolio adjustments or investment in product innovation have been shown to obey to specific incentives (Eckel and Neary, 2010; Dhingra, 2013). Dynamic R&D-driven growth models studying the behavior and impact on aggregate innovation of MPFs have already been provided for the cases where firms are multi-industry (Klette and Kortum, 2004; Akcigit and Kerr, 2010) or multi-varieties (Minniti, 2006). However, the standard quality-ladder framework has so far not been able to account for the existence of "multi-quality" firms, i.e. firms selling more than one quality-differentiated version of the same good. Indeed, the "creative destruction" mechanism at the heart of Schumpeterian models traditionally not only deters leaders from investing in R&D, but also guarantees the systematic exit of any quality that has moved away from the frontier.[2]

Examples of firms offering more than one quality-differentiated version of the same product however abound. Apple recently jointly launched its latest flagship phone, the Iphone 5S, along with a lower-cost version (Iphone 5C), while still keeping the Iphone 4S at the lower end of its product offer. Similarly, Intel commercializes a whole array of microchips, selling its latest, highly efficient processors at high prices (Xeon, Core) while simultaneously offering cheaper models further from the industry frontier (Celeron, Atom). In the car industry, Renault-Nissan launched in the last decade several low-cost cars specifically marketed for developing countries (Logan, Go), re-using obsolete technologies previously featured in leading brands of the constructor (Renault Clio for the Logan, Nissan Micra for the Go). Those examples show how firms resort to vertical brand diversification so as to give a second life to technologies having moved down the quality ladder, and how this behavior enables them to better price-discriminate among consumers having different purchasing powers.

The present paper builds on this body of anecdotal evidence, and provides a model accounting for the existence of multi-quality leaders within a dynamic Schumpeterian framework. More precisely, along the salient features of the examples described above, I argue that as long as preferences are non-homothetic, income distribution impacts the strength and scope of the "creative destruction" process. Income differences then account for both the survival of more than one quality at the equilibrium *and* for positive investment in R&D

[1] Among others, Bernard et al. (2010) estimate that MPFs account for 41% of the total number of US firms as well as for 91% of total output; also, they estimate that the contribution to the US output growth of product mix decisions of MPFs (i.e. product adding and dropping) is greater than the one of firm entry and exit.

[2] Mussa and Rosen (1978) study pricing decisions of multi-quality firms, but in a static framework precluding any specific modeling of the R&D process leading to the initial design of the product line. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) feature MPFs in a quality-ladder world; however, multi-product firms are also multi-industry firms in their models, with only one quality being sold within each product line.

by incumbents. The result is the endogenous emergence in a dynamic framework of multi-quality leaders whose product portfolio composition and investment in R&D activities are both influenced by the extent of income disparities.

The intuition behind this result is straightforward. In a classic quality-ladder model, the actualized net profits of new entrants are pinned down to zero because of the free-entry condition in the R&D sector. Compared to the entrants, an incumbent firm investing in R&D realizes similar profits once innovating successfully, but bears an extra cost: indeed, its own investment lowers the expected profits derived from the sales of its current product. Therefore, while the new entrants are indifferent between investing or not, an incumbent strictly prefers *not* to invest in the standard framework. This is what has been traditionally described as the "Arrow effect" in the literature. However, I argue that *provided there exists differences in the willingness to pay for quality among consumers*, the expected value of a successful innovation actually differs between challengers and incumbents: indeed, if the latter innovates once more, he then retains exclusive monopoly rights over more than one quality, and can more efficiently price-discriminate between consumers having different tastes for quality (Mussa and Rosen, 1978). The Arrow effect operating under free entry then becomes compatible with positive investment in R&D by incumbents.

I integrate such a mechanism in a Schumpeterian model by featuring non-homothetic preferences in an otherwise traditional quality-ladder framework, hence allowing for more than one quality to be consumed at the equilibrium in the presence of differences in wealth endowment. This property is obtained by imposing unit consumption of quality goods in a two-class society, the rest of a consumer's income being spent on a composite, standardized good: within each industry, a given consumer then buys the quality that, *given its price*, offers him the highest utility (Mussa and Rosen, 1978; Shaked and Sutton, 1982). By contrast, in the standard quality-ladder models (Segerstrom et al., 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992) the quality goods are divisible, ensuring that only the highest quality is consumed at the equilibrium, even in the case of wealth endowment disparities: the poorest consumers only consume a lower amount of the top quality good.

In such a framework, a *challenger* winning the latest innovation race and being the producer of the highest quality needs to decide between two alternatives: capturing the whole market by charging a price sufficiently low to appeal to the poorest households, or selling its product at a higher price only to the wealthiest consumers, at the cost of abandoning the rest of the market to its direct competitor (i.e. the previous quality leader). On the other hand, an *incumbent* winning an innovation race retains exclusive monopoly rights over two successive qualities: he can then efficiently discriminate between rich and poor consumers by offering two distinct price/quality bundles, capturing the whole market *and* reaping the maximum surplus from the wealthy consumers at the same time. I then model R&D races in which both incumbents and challengers are participating, and show that without any advantage of any kind in the R&D field and under free entry, the incumbent still invests a strictly positive amount in R&D. Such a behavior directly stems from the existing increment between the profits realized when being a successful challenger and a successful incumbent.

I then move to studying the impact of income distribution on the innovation incentives of both challengers and incumbents, and by extension on long-term growth. I show that in a quality-ladder model, the impact on growth of an increase in the inequality level depends on the nature of the considered shock. More precisely, an increase in the income *gap* shifts R&D activities from challengers to incumbents, since it has opposite effects on the expected profits of both actors; it also has an overall positive effect on an economy's growth rate. On the other hand, a greater income *concentration* is unequivocally detrimental for growth, diminishing both the incumbents' and the challengers' R&D investments. Indeed, in that case the positive price effect stemming from a wealthier rich class is systematically more than offset by the negative market size effect resulting from the decrease in the number of rich consumers.

My main contribution is to provide a framework endogenously accounting for the emergence of multi-quality leaders in the presence of income disparities among consumers. Beyond its novelty, such a result bears several implications. First, while so far the incentives for innovation by quality leaders have essentially been modeled as stemming from the structure of the R&D process, this paper is the first to provide a *demand-driven* incentive for investment in R&D by incumbents. Second, such a framework makes it possible to investigate the impact of income distribution on the intensity of incumbents' innovation activities, a feature that dramatically modifies the predictions that had so far been obtained in the quality-ladder literature regarding the interactions of growth and inequality (Zweimuller and Brunner, 2005).

Relation to literature.

This paper contributes to the literature accounting for innovation by incumbents in quality-ladder models. Segerstrom and Zolnierek (1999) as well as Segerstrom (2007) have obtained positive investment in R&D by the incumbent by assuming that the expertise granted by quality leadership confers R&D cost advantages. Etro (2004, 2008) models sequential patent races with concave R&D costs where the incumbent, acting as a Stackelberg leader, is given the opportunity to make a strategic precommitment to a given level of R&D investment: the quality leader then has an incentive to invest in R&D in order to deter outsiders' entry. Denicolo and Zanchettin (2012) as well as Acemoglu and Cao (2010) provide models where incumbents and challengers participate to two different kinds of R&D races, differing in terms of costs and rewards: leaders invest in R&D to improve their products (incremental innovation), while challengers participate to R&D races in the hope of leapfrogging the existing incumbent (radical innovation). All those models have hence explored various possible incentives for innovation by incumbent stemming from the structure of the R&D process, i.e. from the *supply side*. While all those channels are indeed certainly relevant, this paper explores another venue and provides a *demand-based* rationale for leader R&D, stemming from the perspective of more efficient price discrimination in the case of successive successful innovations. All those papers also feature homothetic preferences, hence guaranteeing that even in the presence of consumer heterogeneity, only the highest quality will be produced and consumed within each industry: the emergence of multi-quality leaders cannot be a consequence of positive innovation by incumbent in those models.

A paper more closely related to this work is the one of Aghion et al. (2001), who analyze the influence of product market competition on innovation intensity, developing a framework in which goods of different quality are imperfect substitutes and can therefore coexist in the market. They show that the perspective to lessen the competition pressure (and broaden the market share) provides the incentive for the incumbent to resort to step-by-step innovation in order to improve its own product. They however preclude free entry by exogenously imposing that only two firms are active and invest in R&D, while our paper on the other hand provides a product market-driven incentive that is robust to the free entry condition.

This work also contributes to the small literature studying the R&D investment of multi-product firms in a dynamic, general equilibrium framework. Klette and Kortum (2004) as well as Akcigit and Kerr (2010) have already provided quality-ladder models in which industry leaders invest in exploration R&D so as to expand their activities in *other* sectors; those frameworks however cannot account for leaders widening their product portfolio within a *given* industry. Minniti (2006) embeds multi-product firms selling more than one horizontally-differentiated variety of a given good in an endogenous growth model; however, his model is an expanding-variety one, hence precluding the emergence of multi-*quality* firms.

This paper is finally also related to the literature examining the relationship between long-term growth and income distribution operating through the demand side. Foellmi and Zweimuller (2006) demonstrate that in an expanding-variety framework, higher inequality levels are systematically beneficial for long-term growth. Foellmi et al. (2009) provide a model combining both product innovations (introducing new luxury goods) and process innovations (transforming those goods into necessities through mass production technologies): in such a framework, the impact of higher inequality is ambiguous on growth, and depends on the scope of the productivity gains stemming from the process innovations. Both those contributions however investigate the impact of income distribution on growth in a *horizontal* differentiation framework, where firms retain *permanent* monopoly rights over their *single* product. Li (2003) and Zweimuller and Brunner (2005) on the other hand have studied the impact of disparities in purchasing power of households in a quality-ladder framework. Zweimuller and Brunner (2005) in particular show that a reduction in the level of inequality within the economy is beneficial for innovation intensity and hence for growth. They however only consider the R&D investment of challengers, and overlook the existing incentives for incumbent innovation in the presence of differences in the willingness to pay of consumers. I show that taking into account the R&D investment by incumbents actually strongly modifies the predictions regarding the overall growth rate of the economy, even reversing them in the case of an increased income gap.

The rest of the paper is organized as follows. Section 2 presents the structure of our general equilibrium model, while section 3 studies its steady state properties. Section 4 then analyzes the effects of the extent of inequality on the innovation intensity. Section 5 concludes.

2 The model

2.1 Consumers

The economy is populated by a fixed number L of consumers that live infinitely and supply one unit of labor each period, paid at a constant wage w. While all consumers are identical with respect to their preferences and their labor income, they are assumed to differ in terms of wealth, based on firms' assets ownership. More precisely, I assume a two-class society with rich (R) and poor (P) consumers being distinguished by their wealth $\omega_R(t)$ and $\omega_P(t)$.[3]

The share of "poor" consumers within the population is denoted by β. The extent of inequality within the economy is determined by this share, as well as by the repartition between rich and poor of the aggregate stock of assets within the economy $\Omega(t)$. $d \in (0,1)$ is defined as the ratio of the value of the stock of assets owned by a poor consumer *relative* to the average per-capita wealth: $d = \frac{\omega_P(t)}{\Omega(t)/L}$. The wealth position of the rich can be computed for a given d and β, and we finally have $\omega_P(t) = d\frac{\Omega(t)}{L}$ and $\omega_R(t) = \frac{1-\beta d}{1-\beta}\frac{\Omega(t)}{L}$.

Current income $y_i(t)$ of an individual belonging to the group i ($i = P, R$) is then of the form:

$$y_i(t) = w + r(t)\omega_i(t) \tag{1}$$

with $r(t)$ being the interest rate.

The existence of such income disparities among consumers is however not sufficient to generate variations in the quality choice along income. Indeed, in the case of standard quality-ladder models traditionally featuring quality-augmented CES utility functions, the homotheticity of the preference specification guarantees that both poor and rich consumers end up purchasing the **same** quality, but in **different amounts**. So as to obtain such variations, I therefore introduce non-homothetic preferences in the form of a **unit consumption** requirement (i.e. the consumption of a given quality good yields a positive utility only for the first unit, and zero utility for any additional unit).[4]

[3] All the results presented in the paper pertaining to investment in R&D by incumbents are robust under the alternative specification of inequality being generated through differences in income, i.e. through different endowments in labor efficiency units.

[4] Unit consumption of the quality-differentiated goods ensures the non-homotheticity of the preference

More precisely, two types of final goods are available within the economy. One group of products, indexed by $s \in [0,1]$, is subject to quality innovation over time. At any date t, we assume that a sequence of qualities $q_j(s,t)$, $j = 0, -1, -2, ...$ exist and can be produced within each industry s, with $q_0(s,t)$ being the best quality, $q_{-1}(s,t)$ the second-best, etc. Two successive quality levels differ by a fixed factor $k > 1$: $q_j(s,t) = k.q_{j-1}(s,t)$. As stated above, consumers value by assumption only **one unit** of each differentiated good. For each industry s at each period t, an individual belonging to group i hence chooses to consume a *single unit* of the quality level $q_j(s,t)$ that offers him the highest utility, considering its price $p(s,t,q_j(s,t))$. I denote this quality $q_{ij}(s,t)$, and the index of consumed qualities over industries $Q_i(t) = \int_0^1 q_{ij}(s,t)ds$. Consumers then spend the rest of their income over the consumption of $c_i(t)$ units of a composite standardized commodity. This homogenous good is produced with a unit labor input of $1/w$; being competitively priced, it hence serves as the numeraire. The instantaneous utility function $\mathcal{U}_i(t)$ of a type i consumer is hence of the form:

$$\mathcal{U}_i(t) = \ln c_i(t) + \ln Q_i(t) = \ln(y_i(t) - P(t, Q_i(t))) + \ln Q_i(t) \tag{2}$$

with $P_i(t, Q(i,t)) = \int_0^1 p(s,t,q_j(s,t))ds$ being the price index associated to the quality good consumption index $Q_i(t)$. Please note that for the sake of notation simplicity, we will from now on refer to this price index as $P_i(t)$, and to the price being charged for quality j in sector s as $p_j(s,t)$.

At time τ, the intertemporal decision problem of a type i consumer is to maximize:

$$\begin{aligned} & \int_\tau^\infty (\ln c_i(t) + \ln Q_i(t))\, e^{-\rho(t-\tau)}dt \\ \text{s.t.} \quad & \omega_i(\tau) + \int_\tau^\infty we^{-r(t)(t-\tau)}dt \geq \int_\tau^\infty (c_i(t) + P_i(t))e^{-r(t)(t-\tau)}dt \end{aligned}$$

with ρ being the rate of time preference. Given an expected time path for both the interest rate $r(t)$ and the relation between quality and price $P_i(t)$, it is then possible to determine the optimal time path of $c_i(t)$ (i.e. the consumption devoted to the standardized commodity) and of $Q_i(t)$ (i.e. the chosen quality for each quality-differentiated good) for a consumer of type i.

Separability of utility (both over time and across goods) guarantees that for any given foreseen time path $P_i(t)$ of expenditures devoted to the continuum of quality goods that does not exhaust life-time resources, the optimal time path of consumption expenditures on homogenous commodities has to fulfill the standard first-order condition of such an intertemporal maximization problem:

$$\frac{\dot{c}_i(t)}{c_i(t)} = r(t) - \rho \tag{3}$$

The optimal time path of $Q_i(t)$, on the other hand, cannot be characterized by a differential

structure in this model. This particular way to model non-homotheticity is the most classic in qualitative choice models featuring strategic pricing of firms (Gabszewicz and Thisse, 1980; Shaked and Sutton, 1982). One could also have obtained differences in the willingness to pay by imposing exogenously different tastes for quality (Glass, 1997).

equation, since the quality choices are discrete. It is possible to notice however that within each industry s, the choice of the quality $q_{ij}(s,t)$ being consumed by a type i individual depends on the pricing decisions $p_j(s,t)$ made by profit-maximizing firms. I hence set aside the discrete quality choices on the part of consumers until having defined the market and price structure for each of the quality sectors.

The focus of this article is on the balanced growth path (BGP) properties of such a model, along which all variables remain constant or grow at a constant rate. Even though the equilibrium as well as the BGP will only be formally defined in section 3, from now on I will omit the functional dependance of the different variables on time, so as to simplify notations. Also, I will focus on "multi-quality firms" equilibria, i.e. balanced growth paths in which incumbents invest a positive amount in R&D. I take the existence of such equilibria for granted in the rest of this section, but will clearly discuss the parameter conditions guaranteeing their existence and uniqueness in section 3.

2.2 Market structure and pricing

The market for quality goods is non-competitive. Labor is the only input, with constant unit labor requirement $a < 1$.[5]

The quality goods being characterized by unit consumption and fixed quality increments, firms use prices as strategic variables. Firms know the shares of groups P and R in the population, the respective incomes y_R and y_P as well as the preferences of the consumers, but *cannot distinguish individuals by income*. In order to describe the strategic decisions operated by firms within a given industry, it proves convenient to define the "threshold" price $p^T_{\{j-m,j\}}(i,s)$ for which a consumer belonging to group i is indifferent between quality $j-m$ and quality j in industry s, given the price $p_{j-m}(s)$ charged for quality $j-m$. Determining such a threshold price amounts to solving the following equality:

$$\ln(q_j(s)) - \ln(q_{j-m}(s)) = \lambda_i[p^T_{\{j-m,j\}}(i,s) - p_{j-m}(s)] \tag{4}$$

The left-hand side of equality (4) is the utility gain when consuming quality $q_j(s)$ rather than quality $q_{j-m}(s)$; the right-hand side on the other hand captures the costs associated to choosing $q_j(s)$ over $q_{j-m}(s)$, expressed as the price difference $p^T_{\{j-m,j\}}(i,s) - p_{j-m}(s)$ times the marginal utility of income λ_i of a consumer belonging to group i. It is important to notice that while λ_i depends on the overall price index P_i, firms within a particular sector take it as given in their decision-making. Indeed, because of the existence of a *continuum* of quality good industries, firms within a given sector are "small in the big, but big in the small" (Neary, 2009): even though they resort to strategic pricing within their own industry, they do not take into account the impact of their pricing decisions on economy-wide variables.[6] Considering the fact that $q_j(s) = k^m q_{j-m}(s)$ and defining $\mu_i = \frac{1}{\lambda_i}$ as

[5]Given the model assumes unit consumption of the quality goods, a necessarily has to be inferior to 1.

[6]For a similar pricing decision problem in the case of successful innovators in a R&D-driven growth model, see Foellmi et al. (2009).

the willingness to pay of a consumer belonging to group i, solving for $p^T_{\{j-m,j\}}(i,s)$ in the above equality yields:

$$p^T_{\{j-m,j\}}(i,s) = \mu_i \ln k^m + p_{j-m}(s) \tag{5}$$

The price $p^T_{\{j-m,j\}}(i,s)$ is the maximum price that the firm selling the quality j in industry s can charge to a type i consumer in order to have a positive market share, when competing against the firm selling the quality $j-m$. As one can see, this threshold price positively depends on the willingness to pay of type i consumers $\mu_i = \frac{1}{\lambda_i}$ (with $\mu_R > \mu_P$), as well as on the price charged by the competitor $p_{j-m}(s)$.

Having defined this threshold price, it is possible to establish the following lemma:

Lemma 1: *Within each industry $s \in [0,1]$, if $p_j(s) \geq wa$ holds for the price of some quality $q_j(s)$, $j = -1, -2, ...$, then for the producer of any higher quality $q_{j+m}(s)$, $1 \leq m \leq -j$, there exists a price $p_{j+m}(s) > wa$ such that:*
(i) *any consumer prefers quality $q_{j+m}(s)$ to $q_j(s)$,*
(ii) *he makes strictly positive profits.*

Proof: Considering (5), it is straightforward that $p^T_{\{j,j+m\}}(i,s) > p_j(s)$. Hence, it is always possible for the producer of the quality $j+m$ to set a price $p_{j+m}(s) > p_j(s) \geq wa$ such that $p_{j+m}(s) \leq p^T_{\{j,j+m\}}(i,s)$, i.e. such that quality $q_{j+m}(s)$ is preferred to quality $q_j(s)$ by the consumers of group i. $\square$

Hence, within each industry s, if we take for granted that a producer never sells its quality at a price below the unit production cost wa, it is always possible for the producer of the highest quality to drive all of its competitors out of the market while still making strictly positive profits. Along this result, any firm entering the market with a new highest quality q_0 has to consider the following trade-off concerning the pricing of its product: **setting the highest possible price for any given group of costumers, vs. lowering its price in order to capture a further group of consumers**. It is then possible to show that in an economy characterized by two distinct groups of consumers (R and P), we have:

Lemma 2: *Within each sector $s \in [0,1]$, we have that at equilibrium,*
(1) *The highest quality is produced,*
(2) *At most the two highest qualities $q_0(s)$ and $q_{-1}(s)$ are actually produced,*

The detailed proof is available in Zweimuller and Brunner (2005) in the one-industry case. The intuitions are: **(1)** since we impose for every individual to buy one unit of quality good in every sector s, at least one quality is always consumed within each sector; **(2)** on the other hand, since there are only two distinct groups of consumers, at most two distinct qualities can be sold within each industry. By Lemma 1, higher qualities drive out lower ones, hence the two qualities being still possibly active are $q_0(s)$ and $q_{-1}(s)$.

As it can be seen from lemma 2, two different situations are possible for the equilibrium market structure and associated prices within each industry $s \in [0,1]$: either only the top quality good $q_0(s)$ is sold to both groups of consumers (groups P and R), *or* the top quality

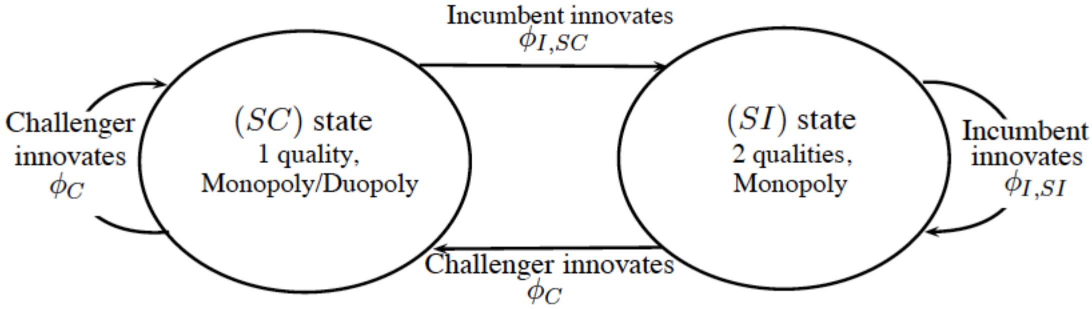

Figure 1: Two possible states

good is sold only to the rich consumers (group R) while the second-best quality good is sold to the poor consumers (group P). Lemma 1 shows that the decision regarding the market structure belongs to the producer of the highest quality $q_0(s)$, considering that he is always able to set a price that will drive its competitors out. The pricing structure resulting from this decision depends on two factors: (i) the **deterministic** extent of inequality within the economy, and (ii) the result of the latest **stochastic** innovation race, where the winner (who is also the producer of the highest quality good) is either a former incumbent or a challenger.

More precisely, each industry $s \in [0, 1]$ fluctuates between two states over time, with its position being determined by the identity of the winner of the last innovation race. The two possible states (SC) and (SI) can be characterized in the following way:

- **"Successful Challenger" (SC) state:** a challenger is the winner of the last R&D race, i.e. the new quality leader is *different* from the former quality leader. In that case, the new quality leader retains exclusive monopoly rights for the highest quality $q_0(s)$ only. As we will comment below, the market structure then depends on the income distribution within the economy.

- **"Successful Incumbent" (SI) state:** the former quality leader, still carrying out R&D, is the winner of the last R&D race, and hence retains exclusive monopoly rights for *both* the highest quality $q_0(s)$ and the second-best quality $q_{-1}(s)$.

Figure 1 illustrates the fluctuations between the two possible states over time. I will now discuss the market structure as well as the prices being charged in the two existing states.

2.2.1 Prices and profits in the (SC) state

In industries being in the (SC) state, a challenger is the winner of the latest innovation race. The distance between this new leader and the "competitive fringe" (i.e. potential competitors with patent rights over lower qualities) is then of only one rung along the quality ladder. That is, even if we assume that being able to produce a quality q_j automatically grants the ability to produce any lower quality q_{j-m} ($m = -1, -2, ...$), the new leader will face Bertrand competition for any quality below the frontier:[7] he will hence be able to extract monopoly rents (i.e. positive profits) solely from the sale of the highest quality q_0. One or two qualities can then be sold on the market, *depending on the pricing strategy chosen by the new quality leader* (which will itself depend on the wealth distribution in the economy). More precisely, the market structure in this state is either a **monopoly** (only quality q_0 is sold), with the new quality leader charging a price that enables him to capture the whole market, or a **duopoly** (both qualities q_0 and q_{-1} are sold), with the new quality leader charging a higher price and serving only the upper part of the market, leaving the lower part to the producer of quality q_{-1}.

For the sake of exposition clarity, I will limit myself to discussing at length the resolution of the case where the equilibrium market structure in the (SC) state is a **monopoly**, i.e. where the income distribution makes it optimal for the new quality leader to sell the highest quality q_0 at a price being attractive for *both* the poor and the rich consumers. Indeed, as it will become clear in the following sections, not only can the monopoly case be fully analytically solved and analyzed in terms of comparative statics, but it is also the one being robust in most parametric cases (the duopoly case is actually only a possible equilibrium under some further conditions identified in Zweimuller and Brunner, 2005). The full discussion, exposition and resolution of the duopoly case can however be found in Appendices A and D.

It is straightforward to notice that within a given industry s, charging a price guaranteeing that the "poor" consumers buy the highest quality $q_0(s)$ automatically ensures that the rich consumers will consume the highest quality too, since $p^T_{\{0,-1\}}(i,s)$ is increasing along a consumer's willingness to pay $\mu_i = \frac{1}{\lambda_i}$. It then immediately follows that the optimal price chosen by a quality leader willing to capture the whole market is $p^T_{\{-1,0\}}(P,s)$. Assuming that the producer of quality $q_{-1}(s)$ engages in limit pricing (i.e. $p_{-1}(s) = wa$) and defining $\kappa = \ln k$, the price $p_P(s)$ being charged by the quality leader is then of the form:

$$p_P(s) = \kappa\mu_P + wa \tag{6}$$

2.2.2 Prices and profits in the (SI) state

In an industry being in the (SI) state, the former quality leader has won a second R&D race in a row, and retains exclusive monopoly rights for *both* the highest quality $q_0(s)$ and the second-best quality $q_{-1}(s)$. According to lemma 2, the market structure is then necessarily a *monopoly*; however, unlike the monopoly case in the (SC) state, the two highest qualities both have positive market shares. Indeed, the quality leader is two rungs above the competitive fringe along the quality ladder: facing two groups of consumers having different levels of income, he will hence be able to offer two distinct price-quality

[7] Indeed, since we impose unit consumption of every quality good, firms necessarily use prices as strategic variables; also, our utility specification guarantees that different qualities are perfect substitutes (for an alternative set-up where goods are imperfect substitutes and different producers can coexist on the market selling the *same* quality, see Aghion et al., 2001).

bundles so as to maximize its profit (Mussa and Rosen, 1978). The price charged by the monopolist for its second-best quality $q_{-1}(s)$ will be the maximal price enabling him to capture the poor group of consumers $p^T_{\{-2,-1\}}(P,s)$, given that the producer of quality $q_{-2}(s)$ engages in limit pricing. This corresponds to the price $p_P(s)$ as defined above in (6). The price charged for the highest quality $q_0(s)$ will then be $p^T_{\{-1,0\}}(R,s)$, given the price $p_P(s)$ charged for quality $q_{-1}(s)$. Denoting this price by $p_R(s)$, we have:

$$p_R(s) = \kappa\mu_R + \kappa\mu_P + wa \tag{7}$$

2.2.3 Equilibrium price indices

The prices for a given industry s as defined above by (6) and (7) are still a function of μ_P and μ_R, and we hence need to substitute for the latter so as to obtain a full characterization of the equilibrium prices in our economy.

The first-order condition governing the amount of divisible homogenous good c_i being consumed by a type i consumer yields the following expression for the marginal utility of income λ_i:

$$\frac{1}{c_i} = \frac{1}{y_i - P_i} = \lambda_i \tag{8}$$

The computation of the willingness to pay $\mu_i = \frac{1}{\lambda_i}$ hence depends on the the quality goods price index P_i each type of consumer is facing.

I denote by $p_i^r(s)$ the price paid by a consumer belonging to group i ($i = R, P$) for the quality good of an industry being in state r ($r = SC, SI$). In the case of the "poor" consumers, we have $p_P^{SC}(s) = p_P^{SI}(s) = p_P(s)$, i.e. a poor consumer will pay the same price for the consumed quality in every industry s, who then enter symmetrically in the price index: $P_P = p_P$.

In the case of the "rich" consumers on the other hand, we have $p_R^{SC}(s) = p_P(s)$, while $p_R^{SI}(s) = p_R(s)$. Indeed, rich consumers end up paying less for the highest quality than what would have been their maximum threshold price in industries being in state (SC), since the strategy of a quality leader having innovated for the first time is to capture the whole market by charging $p_P(s)$ for quality $q_0(s)$. In industries being in state (SI) on the other hand, leaders having innovated twice in a row offer two distinct quality/price bundles, being hence able to extract the maximum surplus from rich consumers, who will pay a higher price $p_R(s)$ for the highest available quality. We hence have $P_R = \int_{\theta_{SC}} p_P(s)ds + \int_{\theta_{SI}} p_R(s)ds$, with θ_{SC} and θ_{SI} denoting respectively the fraction of all industries being in the (SC) and the (SI) state.

Plugging (8), P_P and P_R in (6) and (7), it is possible to obtain the following equilibrium values for p_R and p_P:

$$p_P = \frac{\kappa y_P + wa}{\kappa + 1} \tag{9}$$

$$p_R = \frac{\kappa(\kappa+1)y_R + (1-\kappa\theta_{SC})(\kappa y_P + wa)}{(\kappa+1)(1+\theta_{SI}\kappa)} \tag{10}$$

As we can see considering (10), the price charged to the rich consumers in (SI)-state industries depends on the shares θ_{SC} and θ_{SI} describing the proportion of industries being in each possible state. More precisely, keeping in mind that those shares sum up to one, we have $\frac{\partial p_R}{\partial \theta_{SC}} = \frac{\kappa^2((\kappa+1)y_R-\kappa y_P-wa)}{(\kappa+1)(-1+(\theta_{SC}-1)\kappa)^2} > 0$: p_R is higher when an important proportion of industries are in the (SC) state. Indeed, the willingness to pay μ_R increases along θ_{SC}, since rich consumers pay less than their maximum threshold price in (SC)-state industries. Quality leaders in (SI)-state industries benefit from those higher levels of μ_R, since they retain exclusive patent rights over two successive qualities and are hence able to efficiently price-discriminate between rich and poor consumers. On the other hand, p_P does not depend on the shares θ_{SC} and θ_{SI}: indeed, as commented above, the poor consumer pays the same price in every industry.

Finally, it is also possible to define the profits π_M (resp. π_{SI}) accruing to a monopolist operating in an industry being in state (SC) (resp. (SI)):

$$\pi_M = L(p_P - wa) \tag{11}$$

$$\pi_{SI} = \beta L(p_P - wa) + (1-\beta)L(p_R - wa) \tag{12}$$

From then on, we drop the industry dependency s, since those equilibrium prices and profits do not depend on any industry-specific variable any more. We will do so as well in the next subsections.

2.3 R&D sector

Within each industry $s \in [0,1]$, firms carry out R&D in order to discover the next quality level. Two types of firms have the possibility to engage in R&D races: the current quality leader (incumbent), and followers (challengers). I assume free entry, with every firm having access to the same R&D technology. Innovations are random, and occur for a given firm f within sector s according to a Poisson process of hazard rate ϕ_f. Labor is the only input, and I assume constant returns to R&D at the firm level: in order to have an immediate probability of innovating of ϕ_f, a firm needs to hire $F\phi_f$ labor units, F being a positive constant inversely related to the efficiency of the R&D technology.I define v_C as the value of a challenger firm, v_{SC} as the expected present value of a quality leader having innovated once, and v_{SI} as the expected present value of a quality leader having innovated twice. Free entry and constant returns to scale imply that R&D challengers have no market value, whatever state the economy finds itself in: $v_C = 0$. Free entry of challengers in the successive R&D races also yields the traditional equality constraint between expected profits of innovating for the first time $\phi_C v_{SC}$ and engaged costs $\phi_C wF$: (free entry condition):

$$v_{SC} = wF \tag{13}$$

The incumbent on the other hand participates to the race while having already innovated at least once, and hence being the current producer of the leading quality for industries in the (SC) state/of the two highest qualities for industries in the (SI) state.

In industries being currently in the (SC) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$rv_{SC} = \max_{\phi_{I,SC}\geq 0}\{\pi_M - wF\phi_{I,SC} + \phi_{I,SC}(v_{SI} - v_{SC}) + \phi_C(v_C - v_{SC})\} \tag{14}$$

The incumbent in the (SC) state earns the profits π_M, and incurs the R&D costs $wF\phi_{I,SC}$. With instantaneous probability $\phi_{I,SC}$, the leader innovates once more, the industry jumps to the state (SI), and the value of the leader (now producing and selling two distinct qualities) climbs to v_{SI}.[8] However, with overall instantaneous probability ϕ_C, some R&D challenger innovates, and the quality leader falls back to being a follower: its value drops to $v_C = 0$. The industry then remains in the state (SC), and only one quality is produced.

In the (SI) state, the incumbent faces the following Hamilton-Jacobi-Bellman equation:

$$rv_{SI} = \max_{\phi_{I,SI}\geq 0}\{\pi_{SI} - wF\phi_{I,SI} + \phi_{I,SI}(v_{SI} - v_{SI}) + \phi_C(v_C - v_{SI})\} \tag{15}$$

The incumbent in the (SI) state earns the profits π_{SI} of a monopolist being able to discriminate between rich and poor consumers by offering two distinct price/quantity bundles. He incurs the R&D costs $wF\phi_{I,SI}$. With instantaneous probability $\phi_{I,SI}$, the incumbent innovates once more, in which case its value remains v_{SI}, since we have established with Lemma 2 that at most two successive quantities are sold at equilibrium. Hence, the incumbent will still be the producer of the two qualities being sold, *but* he will drive himself out of the market for the former quality q_{-1}, that has become quality q_{-2} with the latest quality jump. The industry then remains in state (SI). With instantaneous probability ϕ_C,[9] some R&D follower innovates, and the quality leader then falls back to being an R&D challenger: its value falls to $v_C = 0$. The industry then jumps to the state (SC), and only the new highest quality is sold by the latest successful innovator.

In both states, the incumbent firm chooses its R&D effort so as to maximize the right-hand side of its Bellman equation. (14) and (15) then yield the following first-order conditions:

$$(-wF + v_{SI} - v_{SC})\phi_{I,SC} = 0, \quad \phi_{I,SC} \geq 0 \tag{16}$$

$$-wF\phi_{I,SI} = 0, \quad \phi_{I,SI} \geq 0 \tag{17}$$

Condition (17) immediately yields $\phi_{I,SI} = 0$, i.e. incumbents do not invest in R&D in industries being in the (SI) state. This result follows from the fact that in this state, the incremental value of a further innovation for an incumbent is null, since the economy

[8] Accordingly to the crucial condition identified and discussed in the introduction as being necessary so as to to generate innovation by incumbent, this expected value of innovating for a second time v_{SI} is different from the expected value of innovating for the first time v_{SC}.

[9] The challengers invest the same amount in the R&D sector ϕ_C in both states (SC) and (SI), since they face the same expected reward v_{SC} in both cases: a successful innovation by a challenger indeed always brings the industry back to state (SC).

features only two distinct population groups.[10] On the other hand, for industries being in the (SC) state, (16) yields a relationship between the R&D costs $wF\phi_{I,SC}$ and the incremental value of a further innovation $(v_{SI} - v_{SC})\phi_{I,SC}$. From then on, I hence refer to the investment in R&D of the incumbent firm in a sector being in the (SC) state as simply ϕ_I. Plugging (13), (16) and (17) in (14) and (15), it is possible to obtain the 2 following expressions, equating incurred R&D costs and expected profits in both possible states:

$$wF = \frac{\pi_M}{r + \phi_C} \tag{18}$$

$$2wF = \frac{\pi_{SI}}{r + \phi_C} \tag{19}$$

3 Balanced growth path equilibrium

3.1 Labor market equilibrium

We first move to characterizing the equilibrium on the labor market. While challengers invest an equal amount in R&D in every industry, incumbents only invest in R&D in industries being in the (SC) state; the total labor demand in the R&D sector is hence equal to $F(\phi_C + \int_{\theta_{SC}} \phi_I(s)ds)$. Unit consumption of the differentiated goods and identical marginal costs of production regardless of the quality level yield a total amount of aL units of labor being devoted to the production of the quality goods. Finally, $(L/w)\left(\beta(y_P - p_P) + (1-\beta)(y_R - P_R)\right)$ are the units of labor being devoted to the production of the standardized good.

The following equation then describes the equilibrium on the labor market:

$$L = F\phi_C + \theta_{SC}F\phi_I + aL + (L/w)\left(\beta(y_P - p_P) + (1-\beta)(y_R - \theta_{SC}p_P - \theta_{SI}p_R)\right) \tag{20}$$

It is then possible to transform (20) so as to obtain a relationship between profit flows and the overall wealth within the economy Ω. Multiplying both sides by w, replacing y_P and y_R by their respective values, splitting waL into $\beta waL + (1-\beta)waL$ and rearranging terms, we get:

$$wF(\phi_C + \theta_{SC}\phi_I) - \theta_{SC}\pi_M - \theta_{SI}\pi_{SI} + r\Omega = 0 \tag{21}$$

Using (18) so as to substitute for wF and noticing that (18) and (19) imply that $\pi_M = 2\pi_{SI}$, we obtain the expected identity between overall wealth of the consumers Ω and the present discounted value of firms' profits within the economy as a whole:

$$\Omega = \theta_{SC}\frac{\pi_M}{r + \phi_C} + \theta_{SI}\frac{\pi_{SI}}{r + \phi_C} \tag{22}$$

[10] I believe it would be possible to generalize this model to more than two groups of population, or a continuum of quality valuations as in Mussa and Rosen (1978). Intuitively, the incumbent would then keep investing in R&D beyond the second innovation in a row.

3.2 Balanced growth path analysis

Definition 1 *In the case we have a monopoly market structure in the (SC) state, an equilibrium is defined by a time path for consumption of the homogenous good for both types of consumers* $\{c_i(t)\}_{i=(R,P),t=0}^{\infty}$ *that satisfies* (8)*, time paths for R&D expenditures by incumbents and challengers* $\{\phi_C(s,t), \phi_I(s,t)\}_{s\in(0,1),t=0}^{\infty}$ *that satisfy* (13), (14) *and* (15)*, time paths of price indices and present discounted value of firms' profits* $\{P_R(t), P_P(t), \Omega(t)\}_{t=0}^{\infty}$ *given by* (9), (10), (11), (12), *and* (22), *and a time path of the interest rate* $\{r(t)\}_{t=0}^{\infty}$ *which satisfies* (3).

In addition, we define a balanced growth path (BGP) as an equilibrium path along which every variable grows at a constant rate, either null or positive. In such a *product*-innovation model (i.e. precluding any productivity improvement) with fixed wage and population levels w and L, the BGP is characterized by constant levels of innovation ϕ_C and ϕ_I, overall wealth Ω and consumption c_i $(i = R, P)$.[11] Consumers however still become better-off over time due to the quality improvements of the differentiated goods and the resulting growth of individual utility. As already stated in the previous section, we focus in this paper on such a BGP, and we now proceed to describing its properties.

Along such a BGP, it is possible to express θ_{SC} and θ_{SI} as functions of the innovation rates ϕ_C and ϕ_I. Indeed, so as to ensure that c_R is indeed constant along the BGP, the share of industries being in each state must remain constant (cf. expression of the price index P_R). Hence, the flows in must equal the flows out of each state: we then have the condition $\phi_C\theta_{SI} = \phi_I\theta_{SC}$ that has to be respected along the BGP.[12] Combining it with the fact that the two shares sum up to 1 (i.e. $\theta_{SC} + \theta_{SI} = 1$), we obtain:

$$\theta_{SC} = \frac{\phi_C}{\phi_I + \phi_C}, \qquad \theta_{SI} = \frac{\phi_I}{\phi_C + \phi_I} \tag{23}$$

Proposition 1 (Existence and uniqueness of a steady state equilibrium): *For* κ*,* F *and* β *sufficiently high and for not too low values of* d*, there exists a unique BGP along which (i) we necessarily have a monopoly in the (SC) state, (ii) both incumbents and challengers invest strictly positive amounts in R&D* ϕ_I *and* ϕ_C*, and (iii) the consumers' utility grows at the constant rate* $\gamma = \kappa\phi_C(1 + \frac{\phi_I}{\phi_I+\phi_C})$.

Proof: cf Appendix B. □

Note that Proposition 1 implies not only that there exists a unique positive solution for the system of variables as defined in Definition 1, but also that the equilibrium with a monopoly market regime is robust (existence) while its duopoly counterpart is not (uniqueness). While a detailed demonstration is available in Appendix B, I will here provide intuitions regarding the conditions on the exogenous parameters needed so as to obtain this

[11] The consumption of the continuum of quality-differentiated goods is anyway always constant, since we impose unit consumption in this model.

[12] Indeed, for each industry being in the (SC) state, the probability to exit this state is equal to the probability $\phi_I(s)$ of an incumbent innovating; for each industry being in the (SI) state, the probability to enter the (SC) state corresponds to the probability $\phi_C(s)$ of a challenger innovating.

result. A first group of conditions guarantees a strictly positive amount invested in R&D by incumbents along the BGP: those are κ and F sufficiently high. Indeed, κ represents the utility increment of consuming quality q_0 over quality q_{-1}: the higher κ, the higher the gap between p_P and p_R. In other words, high values of κ ensure that the gains from price-discriminating are high enough to represent viable incentives for the incumbent to invest in R&D. The condition on the "production" costs in the R&D sector can be rationalized considering we have $v_{SI} - v_{SC} = wF$: again, the need for high values of F can be linked to the necessity of sufficiently significant incremental profits in the case of a second successful innovation. On the other hand, the two last conditions (β sufficiently high and d not too low) are needed so as to guarantee the existence and uniqueness of the obtained BGP. Regarding the existence, we indeed need to check that for the obtained equilibrium values of ϕ_C, ϕ_I and Ω, the monopoly market structure in the (SC) state is robust, i.e. the new leader does not prefer the alternative regime when comparing expected profits. Regarding the uniqueness, we also have to make sure that the equilibrium values obtained when solving for a BGP with a duopoly market structure in the (SC) state (as defined in Appendix A) do not define a robust equilibrium. Intuitively enough, high values of β and not too low values of d ensure that the monopoly is the **only** viable price regime: indeed, a leader facing an important group of poor people (both in terms of size and in terms of purchasing power) is not going to be willing to abandon that part of the market to its direct competitor.

A "multi-quality firms BGP" with a monopoly market regime in the (SC) state hence emerges **only** for parameter values respecting the conditions stated in Proposition 1 above. Appendices A and D similarly define conditions under which there exists a robust "multi-quality firms BGP" with a duopoly market regime in the (SC) state.[13] Outside those parameter constellations though, condition (16) yields $\phi_I = 0$, and the model collapses to the Zweimuller and Brunner (2005) framework.

In an economy where sufficiently strong disparities in purchasing power exist, incumbents hence have an incentive to keep investing in R&D beyond their first successful innovation. In my framework, the immediate consequence of this result is the **endogenous emergence of multi-quality leaders in a dynamic quality-ladder model**, since income disparities generate both (1) the survival of more than one quality at the equilibrium, and (2) positive investment in R&D activities by incumbents. The existence and behavior of those multi-quality firms had not been exemplified in the growth literature so far, and it is a significant contribution of the model presented here.

A few further comments can be made. First, a salient implication of this result is the existence of *demand-related* determinants of innovation by incumbents. Here, positive investment in R&D by quality leaders is obtained with complete equal treatment in the R&D field between the incumbent patentholder and the challengers, as well as with-

[13] However, due to the impossibility to obtain closed-form solutions, we have to resort to simulations so as to determine the robustness conditions of the BGP in the case of a duopoly in the (SC) state.

out any concavity in the R&D cost function. This model is therefore the first to hint at the existence of so far overlooked incentives for innovation by incumbent stemming from the demand structure rather than from the supply side (i.e. R&D sector characteristics and R&D capabilities of challenger and incumbent firms), and opens the field for further investigations.[14] Second, this result emphasizes the macroeconomic consequences of the negative "heterogenous taste for quality" externality identified by Mussa and Rosen (1978) in a micro framework. This externality can be formulated in the following way: in the absence of the possibility of first-degree discrimination, the existence of "poor" consumers prevents the monopolist from capturing the maximum costumer surplus from those who have a stronger taste for quality. In a static framework, a multi-quality monopolist internalizes this negative externality by inducing less enthusiastic consumers to buy lower quality items charged at a lower price, opening the possibility of charging higher prices to more adamant buyers of high quality units. As a consequence, a wider range of qualities than what would be optimal is finally offered. In our dynamic model with endogenous innovation, the monopolist only retains exclusive patent rights for as many qualities as R&D races he has won: the negative externalities stemming from having to serve two distinct groups of consumers having different quality valuations is then internalized by expanding the line of product towards *higher* (and not lower) qualities, i.e. through R&D investment.

4 Distribution of income and long-term growth

I now investigate the implications of such a model regarding the existing interactions between income distribution and long-run growth operating through the demand market.

In their contribution, Zweimuller and Brunner (2005) had shown that in a vertical differentiation framework, a rising level of inequality systematically decreases the R&D investment rate of challengers. This leads them to conclude to an unambiguous detrimental impact of inequality (whether it stems from higher income gaps or greater wealth concentration) on long-term growth. However, as already stated before, their model pins down the R&D investment rate with a simple free entry condition in the R&D sector, overlooking the possibility of investment by incumbents in such a framework: they hence only capture *part* of the influence of income distribution on innovation incentives. As I will show below, taking into account the fact that incumbents also invest in R&D in the presence of income heterogeneity significantly modifies the predictions regarding the overall growth rate, and sheds light on possible determinants of varying intensities in the R&D activities of different actors (i.e. incumbents and challengers).

In the following analysis, I consider two types of variations in the extent of wealth disparities: (a) larger income *gap* (i.e. a decrease in d for a fixed level of β), and (b) a

[14] As we already pointed out in the introduction, Aghion et al. (2001) had already provided a quality-ladder framework in which it was possible to investigate the influence of product market competition on innovation intensity. However, they did not allow for free entry of firms, only allowing for the existence of two active firms in the R&D sector.

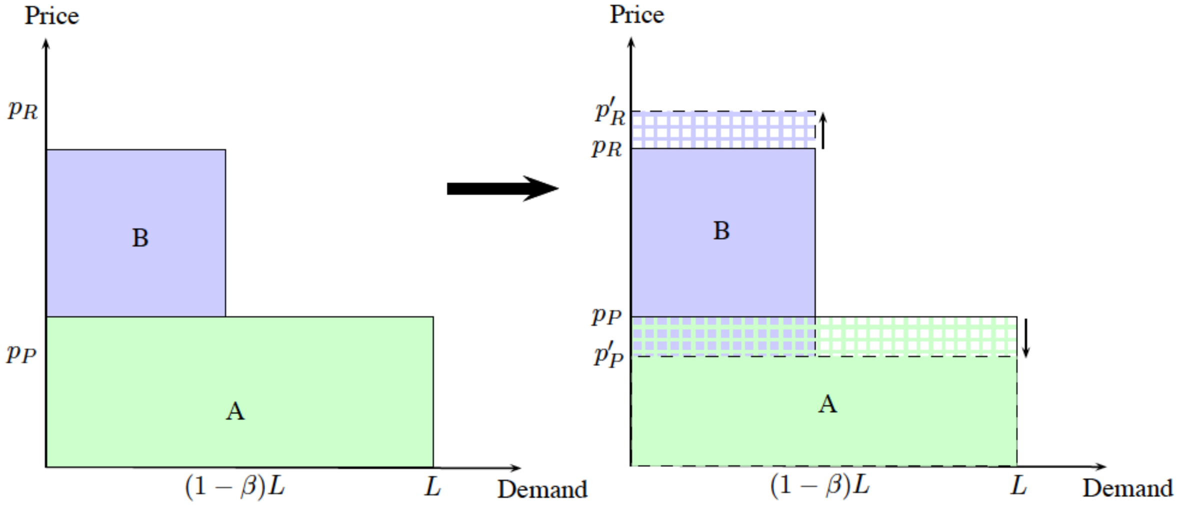

Figure 2: Effects of a shock on d (for Ω constant)

greater wealth *concentration* (i.e. an increase in β for a given d). The results of those comparative statics can be summarized in the following proposition:

Proposition 2 (Wealth distribution and long-term growth):
Under the parametric conditions guaranteeing the existence of a unique BGP with a monopoly market structure in the (SC) state, we have the following comparative statics for varying values of β and d:

- *(a) Effect of a larger income gap (corresponding to a decrease in d): the challengers' innovation rate ϕ_C decreases following an increase in the income gap, while the incumbent's innovation rate ϕ_I, the overall wealth Ω and the long-run growth rate γ increase. An increase in the income gap also leads to a greater share of R&D activities to be carried out by incumbents.*
- *(b) Effect of a greater income concentration (corresponding to an increase in β): the challengers' innovation rate ϕ_C, the incumbent's innovation rate ϕ_I, the overall wealth Ω and the long-run growth rate γ decrease following an increase in income concentration. An increase in income concentration also leads to a greater share of R&D activities to be carried out by challengers.*

Proof: cf Appendix C. □

(a) Let us first comment the effects of a larger income gap, i.e. of a decrease in d.

Simple intuitions for the variations of ϕ_C and ϕ_I can be obtained by considering the expected gains associated to successfully innovating for the first and the second time. Figure 2 represents for a given level of overall wealth Ω the impact of a decrease in d on the profits of a monopolist in a (SC) industry (area A) and on the incremental profits stemming from innovating for a second time (area B). One can first notice that since we keep both β and the quantities produced fixed (the quality-differentiated industries face unit consumption), there can be no variation in the *market size* following an increase in the income gap: profit variations will derive from *price* adjustments. For a monopolist in the (SC) state, the critical income is the one of poor households, which decreases following the considered shock: hence, at a given level of wealth Ω, a decrease in d has a *negative price effect* on the profits of a successful challenger, since he now has to charge a lower price so as to be able to keep capturing the whole market (area A decreases in Figure 2). On the other hand, at fixed Ω, a larger gap between the wealth of the rich and the poor leads to a *positive price effect* on the incremental profits stemming from innovating for a second time: considering Figure 2, area B increases. The incentives to invest in R&D for an incumbent have hence become greater, while they are now smaller for a challenger: ϕ_I increases, and ϕ_C decreases.

The variation in Ω following a decrease in d can be rationalized considering equation (22), which establishes that wealth within the economy stems from ownership in the monopolistic active firms: $\Omega = \theta_{SC} v_{SC} + \theta_{SI} v_{SI}$. Along the BGP, the two R&D conditions of equality between incurred costs and expected profits (18) and (19) pin down the firms' discounted present values in both states: $v_{SC} = wF$ and $v_{SI} = 2wF$. Hence, Ω is a weighted sum of wF and $2wF$, and variations in its value can only stem from variations in those weights, i.e. in the shares θ_{SC} and θ_{SI} of industries being in each state. Here, the increase in the share of sectors being in the (SI) state automatically yields an increase in Ω.

The variation in γ can be commented in light of the readjustments operating on the labor market following a negative shock on d. Indeed, γ differs by only one multiplicative constant from the amount of labor devoted to R&D activities: $F\phi_C(1 + \frac{\phi_I}{\phi_C + \phi_I}) = \frac{\kappa}{F}\gamma$. Because of the imposed unit consumption of quality goods, the amount of labor devoted to their production is pinned down at an always constant value aL. Adjustments on the labor market following variations in the income distribution hence only operate through reallocations between the R&D and the homogenous good sector. As we have commented above, a decrease in d leads to an increase in the share of industries being in the (SI) state: there is now a larger chunk of industries in which rich consumers are being charged their maximum reservation price by price-discriminating monopolists. The residual demand for the composite homogenous good by rich consumers c_R hence decreases, and labor is shifted from the production of the standardized good to the R&D sector: as a consequence, γ increases.

(b) I now move to commenting the effects of an increase in β when we have a monopoly price regime in the (SC) state. I first note that a rise in the share of the population being poor β while keeping d constant corresponds to a higher concentration of wealth among a smaller group of rich people. Indeed, it implies an increase in the *relative* income of a rich consumer ($\frac{\partial d_R}{\partial \beta} = \frac{1-d}{(1-\beta)^2} > 0$): there are more poor with the same income, and fewer rich with more income. As we can see in Proposition 2, this type of variation in the inequality level is unequivocally detrimental for economic growth. We now comment the intuitions pertaining to the variations of the different variables.

Figure 3 represents for a given level of income gap d the impact of an increase in β on the

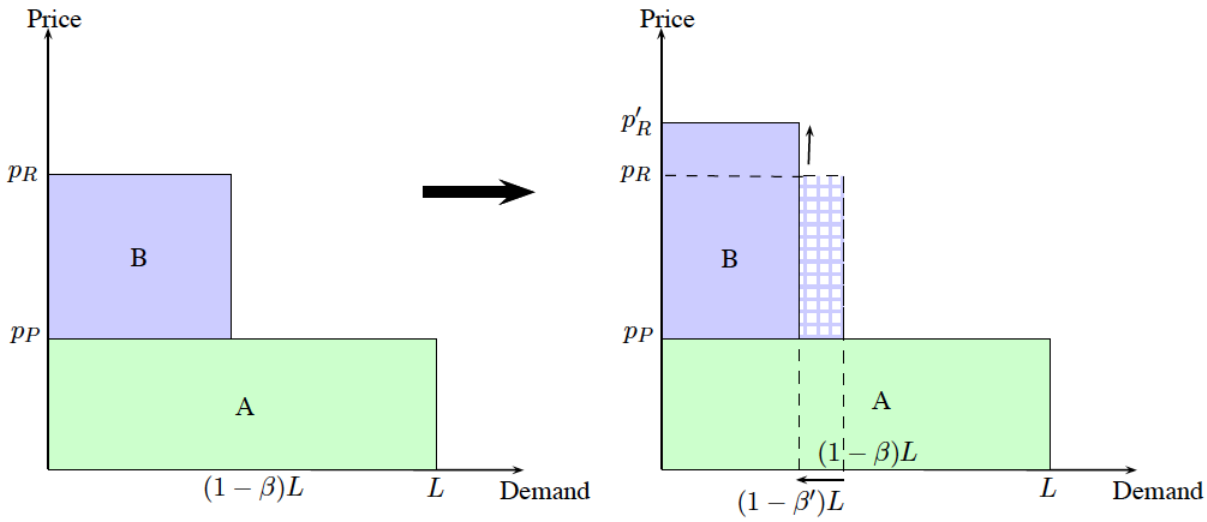

Figure 3: Effects of a shock on β (for Ω constant)

profits of a monopolist in a (SC) industry (area A) and on the incremental profits stemming from innovating for a second time (area B). As we can see, only area B is impacted by a shock on β; this time, unlike a shock on d generating only variations in prices, we have both a market size *and* a price effect. Price-discriminating monopolists operating in (SI)-state industries can now charge a higher price, but to a smaller part of the population. Contrarily to what happens in the horizontal differentiation case (Foellmi and Zweimuller, 2006), the negative market size effect systematically dominates here, and the incumbents' investment in R&D ϕ_I decreases. The negative variation in ϕ_C can be rationalized considering the fact that part of the expected profits considered by the challengers when entering their first innovation race pertains to the possibility to price-discriminate if they innovate for a second time. The decrease of those hypothetical profits leads the free-entry condition to pin down the challengers' innovation rate ϕ_C at a lower level than before the shock on β.

The decrease in Ω means that the share of industries being in the (SC) state has increased (Ω being a weighted sum of wF and $2wF$): a greater bulk of the overall R&D investment is now carried out by challengers. Indeed, the identified negative market size effect only impacts the incremental profits realized when innovating for a second time: incumbents are hence hit harder than challengers, and react more strongly to a greater wealth concentration. Finally, since both types of actors have diminished their investments in R&D, the overall growth rate γ unambiguously decreases.

Several conclusions can be derived from the results presented in this section.

First, when asking "How does inequality affect investment in R&D and growth in a quality-ladder set-up?", the answer depends crucially on whether higher levels of inequality result from a larger income gap or from a higher income concentration. In the case of a larger income gap, only price effects are at play, and while they impact negatively investment in R&D by challengers, they lead incumbents to increase their R&D expenses to an even larger extent, hence generating an increase in the economy's growth rate. In the case of an increased wealth concentration on the other hand, the positive price effect is more than counterbalanced by a negative market size effect, leading to a decrease in the R&D investments of both types of actors. The two different shocks also lead to different predictions in terms of reallocation of the overall R&D bulk from one type of actor to another: while a greater income gap leads to a greater share of the overall R&D being carried out by incumbents, the reverse is true in the case of a higher wealth concentration.

Second, when comparing those predictions to the ones obtained in the case of expanding-variety growth models (Foellmi and Zweimuller, 2006; Foellmi et al., 2009), we see that the nature of the differentiation considered (i.e. horizontal vs vertical) is crucial in order to predict the impact of varying inequality on R&D investment and growth. Indeed, Foellmi and Zweimuller (2006) have shown that in an horizontal differentiation framework, higher levels of inequality are systematically positive for an economy's rate of growth. The intuition pertains to a product's life-cycle: lower levels of inequality induce a positive market size effect (the market for a new good develops faster into a mass market), but a negative price effect (the willingness to pay for a new product decreases with a less wealthy rich class). The latter always dominates the former, since profit flows *early* in the product's life cycle matter more, and are lowered by a decrease in inequality. Foellmi et al. (2009) show that even when the monopolist can engage in process innovation so as to transform its luxury good into a product of mass consumption (hence engaging in a form of price discrimination), higher inequality levels still have a positive impact on growth *provided* the technological spillovers stemming from the introduction of mass production are not too important.

The mechanisms present in those two models however rely on the crucial assumption that a firm keeps *permanent monopoly rights* over a given good, without running the risk of being leapfrogged. In the case of a vertical-differentiation model where the introduction of new products pushes the older ones further from the frontier, the predictions are altered. As it was possible to demonstrate in this section, higher levels of wealth concentration are detrimental for growth in a quality-ladder framework: the positive price effect is dominated by the negative market size effect. Also in the case of a higher income gap, the mechanisms leading to a higher growth rate are fundamentally different in the two frameworks: in a quality-ladder model, the positive impact on economic growth is obtained despite a negative price effect on a new entrant's profits, and through a reallocation of R&D activities from challengers to incumbents.

Finally, those results show how decisive it is to take into account the behavior of incumbents when analyzing the interactions of aggregate demand and long-run growth in a quality-ladder model. Indeed, including incumbents in the analysis leads me to totally different conclusions from the ones obtained in Zweimuller and Brunner (2005), even though I confirm their predictions regarding the challengers' behavior when facing a larger income gap. More precisely, in the case we have a monopoly in the (SC) state, the model presented here yields opposite predictions regarding the impact on the overall growth rate of a decrease in d, and predicts a negative impact of an increase in β while their model finds none.[15] This framework also makes it possible to further characterize the evolution of the *allocation* of overall R&D expenditures between challengers and incumbents.

5 Conclusion

This paper contributes to the analysis of the interactions between income distribution and long-term growth operating through the demand side. It first demonstrates that disparities in purchasing power justify investment in R&D by both leaders and challengers, providing a demand-driven rationale for innovation by incumbents. Indeed, the strictly positive innovation rate of the incumbent is here obtained with constant returns to R&D efforts and without any advantage of the incumbent in the R&D field (supply side), by allowing for income inequality to generate different quality valuation of poor and rich consumers (demand side). The paper then also provides a significant contribution to the literature investigating the impact of income inequality on growth, showing that while an increase in the income gap can be beneficial for growth, a greater wealth concentration is systematically detrimental for the economy.

Some lines of further work can be quickly sketched. An obvious extension to this model would be to treat the more general case of more than two types of consumers, in order for the incumbent to keep investing in R&D after the second successful race.A model such as this one can also be applied to a two-country framework, in order to contribute to the developing literature studying the determinants and impact of vertical, intra-industrial trade (Fajgelbaum et al., 2011). Indeed, while the impact on growth of inter-industrial quality trade has already been extensively studied (product life-cycle), I believe the framework presented in this paper would be a good starting point for the elaboration of a dynamic model of intra-industrial quality trade (quality life-cycle).

[15] Indeed, in the case of a monopoly price regime, the size of the rich population does not matter at all, since successful challengers can never correctly price-discriminate them.

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Appendix A - Exposition of the duopoly case

In the main text, we have limited ourselves to detailing the exposition of the economy in the case we have a monopoly in the (SC) state, i.e. in the case a challenger who innovates finds it optimal to charge a price which will ensure that the highest quality is attractive for both consumer groups. I will now present the main equilibrium equations in the case we have a duopoly in the (SC) state.

Prices and profits in the (SC) state

First, note that a further assumption needs to be made so as to ensure that a duopoly can indeed be a possible equilibrium price regime. Indeed, as argued by Zweimuller and Brunner (2005), the pricing problem faced by firms in a given sector being in the (SC) state can be considered as an infinitely repeated game between the quality leader and the producer of the second-best quality. The monopoly pricing strategy as we have described it in the main text is a Nash equilibrium of the stage game. On the other hand, if we assume that both the leader and the follower have positive market shares (i.e. if we want to define a pricing strategy compatible with a duopoly), no pair of prices $(p_0(s), p_{-1}(s))$ represents a Nash equilibrium of the stage game: *given* the price charged by the other, at least one of the two firms always has an incentive to deviate. It is however possible to guarantee the existence of a duopoly equilibrium, under the further condition on the punishment strategies that *no firm is punished if it changes its price without affecting the other firm's profit* (Proof: cf Zweimuller and Brunner (2005), p. 242).

In the case of such an equilibrium, the new quality leader chooses to charge the highest possible price enabling him to capture the group of rich consumers $p^T_{\{-1,0\}}(R, s)$, given the expected strategy of the producer of the second-best quality. The former quality leader charges the highest possible price enabling him to capture the poor group of consumers $p^T_{\{-2,-1\}}(P, s)$, given that the producer of quality q_{-2} engages in marginal cost pricing (i.e. $p_{-2}(s) = wa$). Those two prices actually correspond to $p_P(s)$ and $p_R(s)$ as defined by (6) and (7). Hence, when the market structure is a duopoly in the (SC) state, a consumer belonging to group i ($i = R, P$) pays systematically the same price for the consumed quality, whether the industry is in state (SC) or (SI). The two price indices P_P and P_R hence simply boil down to $P_P = p_P$, and $P_R = p_R$. Plugging those two price indices as well as (8) in (6) and (7), it is possible to obtain the following equilibrium values for p^D_R and p_P when we have a duopoly in the (SC) state:

$$p_P = \frac{\kappa y_P + wa}{\kappa + 1} \tag{24}$$

$$p^D_R = \frac{\kappa}{\kappa + 1} y_R + \frac{\kappa y_P + wa}{(\kappa + 1)^2} \tag{25}$$

It is also finally possible to define the profits π_L and π_F accruing to the producers of the first-best and the second-best qualities in the (SC) state, as well as the profits π^D_{SI} of the discriminating monopolist in the (SI) state:

$$
\begin{aligned}
\pi_L &= (1-\beta)L(p_R^D - wa) && (26)\\
\pi_F &= \beta L(p_P - wa) && (27)\\
\pi_{SI}^D &= \beta L(p_P - wa) + (1-\beta)L(p_R^D - wa) && (28)
\end{aligned}
$$

R&D sector

In the case we have a duopoly in the (SC) state, the main modification is that the value of a "leapfrogged" quality leader v_F does not fall to zero: since the new leader abandons the lower part of the market so as to be able to charge a higher price to the rich consumers, the former leader still makes positive profits π_F. We hence now have three Hamilton-Jacobi-Bellman equations. In industries being in the (SC) state, both the incumbent and the former leader face the two following HJB equation:

$$
\begin{aligned}
rv_{SC} &= \max_{\phi_{I,SC}\geq 0}\{\pi_L - wF\phi_{I,SC} + \phi_{I,SC}(v_{SI} - v_{SC}) + \phi_C(v_F - v_{SC})\} && (29)\\
rv_F &= \max_{\phi_F \geq 0}\{\pi_F - wF\phi_F + \phi_F(v_{SC} - v_F) + (\phi_C + \phi_I)(v_C - v_F)\} && (30)
\end{aligned}
$$

In industries being in the (SI) state, the incumbent faces the following HJB equation:

$$
rv_{SI} = \max_{\phi_{I,SI}\geq 0}\{\pi_{SI}^D - wF\phi_{I,SI} + \phi_{I,SI}(v_{SI} - v_{SI}) + \phi_C(v_F - v_{SI})\} \qquad (31)
$$

The three corresponding first-order conditions are of the following form:

$$
\begin{aligned}
(-wF + v_{SI} - v_{SC})\phi_{I,SC} &= 0, \quad \phi_{I,SC} \geq 0 && (32)\\
(-wF + v_{SC} - v_F)\phi_F &= 0, \quad \phi_F \geq 0 && (33)\\
-wF\phi_{I,SI} &= 0, \quad \phi_{I,SI} \geq 0 && (34)
\end{aligned}
$$

As in the monopoly case, (34) immediately implies $\phi_{I,SI} = 0$. Combined with (13), (33) entails either $\phi_F = 0$ or $v_F = 0$. The second possibility cannot be true, since the follower's profits π_F are strictly positive: we hence necessarily have that $\phi_F = 0$. Plugging this value back into (30), we obtain that $v_F = \frac{\pi_F}{\rho + \phi_C + \phi_I}$. Finally, plugging the free-entry condition (13) and the first-order condition (33) in the HJB equations (29) and (31), it is possible to obtain the 2 following expressions, equating incurred R&D costs and expected profits in both possible states:

$$
\begin{aligned}
wF &= \frac{\pi_L + \phi_C \frac{\pi_F}{r + \phi_C + \phi_I}}{r + \phi_C} && (35)\\
2wF &= \frac{\pi_{SI}^D + \phi_C \frac{\pi_F}{r + \phi_C + \phi_I}}{r + \phi_C} && (36)
\end{aligned}
$$

Labor market equilibrium

The equilibrium on the labor market is slightly modified with respect to the monopoly case. While the R&D behavior as well as the units of labor being devoted to the production of the quality goods remain unchanged, the units of labor being devoted to the production of the standardized good are now equal to $L/w(\beta(y_P - p_P) + (1-\beta)(y_R - p_R))$. The following equation then describes the equilibrium on the labor market:

$$L = F\phi_C + \theta_{SC}F\phi_I + aL + L/w(\beta(y_P - p_P) + (1-\beta)(y_R - p_R)) \tag{37}$$

Proceeding to the same transformations than in the monopoly case and noticing that $\pi_F + \pi_L = \pi_{SI}^D$, we get:

$$wF(\phi_C + \theta_{SC}\phi_I) - \pi_{SI}^D + r\Omega = 0 \tag{38}$$

Definition of the equilibrium and the BGP

Definition 2 *An equilibrium when the market structure in the (SC) state is a duopoly is given by a time path for consumption of the homogenous good for both types of consumers* $\{c_i(t)\}_{i=(R,P),t=0}^{\infty}$ *that satisfies* (8)*, time paths for R&D expenditures by incumbents and challengers* $\{\phi_C(s,t), \phi_I(s,t)\}_{s\in(0,1),t=0}^{\infty}$ *that satisfy* (13)*,* (29)*,* (30) *and* (31)*, time paths of price indices and present discounted value of firms' profits* $\{P_R(t), P_P(t), \Omega(t)\}_{t=0}^{\infty}$ *given by* (24)*,* (25)*,* (26)*,* (27)*,* (28) *and* (38)*, and a time path of the interest rate* $\{r(t)\}_{t=0}^{\infty}$ *which satisfies* (3)*.*

Once again, I define a BGP as an equilibrium along which every variable grows at a constant rate, either null or positive. The computation of the values of the shares θ_{SC} and θ_{SI} is exactly similar to the one carried out in the monopoly case, and once again we obtain that all the variables remain constant along the BGP, except for the utility level of consumers which grows at the constant rate $\gamma = \kappa\phi_C(1 + \frac{\phi_I}{\phi_I + \phi_C})$.

Proposition 3 (Existence and uniqueness of a steady state equilibrium in the duopoly case): *Under the parametric conditions (i)-(iii) (cf. Appendix D) and for low enough values of d and F, there exists a unique BGP along which we necessarily have a duopoly in the (SC) state and in which both incumbents and challengers invest strictly positive amounts in R&D ϕ_I and ϕ_C.*

Proof: cf. Appendix D. Note that while the existence of a unique positive solution for the BGP as defined in Definition 2 can be proved analytically, the absence of closed-form solutions makes it necessary to resort to numerical simulations so as to define parametric intervals in which the defined equilibrium is robust and unique. □

Appendix B - Demonstration of the existence and uniqueness of the BGP in the monopoly case

Along the BGP, (3) implies that we have $r = \rho$. As it can be seen from (1), (8), (11), (12), (9), (10) and (23), all the other endogenous variables of the model (i.e. individual consumption of the homogenous good c_i ($i = R, P$), as well as prices and profits in both states p_P, p_R, π_M and π_{SI}) can be expressed as functions of the innovation rates ϕ_C and ϕ_I as well as the overall wealth level Ω. Solving for the BGP hence amounts to solving the following system of 3 equations:

$$wF = \frac{\pi_M}{\rho + \phi_C} \tag{39}$$

$$2wF = \frac{\pi_{SI}}{\rho + \phi_C} \tag{40}$$

$$wF\phi_C + \frac{\phi_I \phi_C wF}{\phi_I + \phi_C} - \frac{\phi_C \pi_M}{\phi_I + \phi_C} - \frac{\phi_I \pi_{SI}}{\phi_I + \phi_C} + \rho\Omega = 0 \tag{41}$$

Using (1), (9) and (11), (39) makes it possible to express Ω as a function of ϕ_C:

$$\Omega = \frac{w(F(\kappa+1)(\rho+\phi_C) - \kappa L(1-a))}{\kappa\rho d} \tag{42}$$

Using (39) so as to substitute for wF in (40), we obtain the condition $\pi_{SI} = 2\pi_M$. We use it so as to simplify (41), which becomes:

$$wF\phi_C(\phi_I + \phi_C) + \phi_I\phi_C wF - \phi_C\pi_M - 2\phi_I\pi_M + \rho\Omega(\phi_I + \phi_C) = 0 \tag{43}$$

Using (11), (12), (9), (10), (43) makes it possible to express ϕ_I as a function of ϕ_C:

$$\phi_I = \frac{\phi_C(F(\kappa+1)(\rho+\phi_C) - dF\kappa\rho - (1-a)\kappa L)}{2dF\kappa\rho + (1-a)\kappa L - F(\kappa+1)(\rho+\phi_C)} \tag{44}$$

Considering (42) and (44), we see that the following condition is sufficient so as to ensure a positive investment in R&D by incumbents ϕ_I and a positive overall wealth Ω:

$$dF\kappa\rho < F(\kappa+1)(\rho+\phi_C) - (1-a)\kappa L < 2dF\kappa\rho \quad (*)$$

Substituting for Ω and ϕ_I using (42) and (44), it is possible to transform (40) into a second degree polynomial in ϕ_C:

$$d\rho((1-a)L\kappa - F(1+\beta)\rho) + \phi_C((1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho) - F(\kappa+1)\phi_C^2 = 0 \tag{45}$$

We then have $\phi_C = \frac{(1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho \pm \sqrt{\Delta}}{2F(\kappa+1)}$, with Δ being of the following form:

$$\Delta = ((1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho)^2 + 4F(\kappa+1)d\rho((1-a)L\kappa - F(1+\beta)\rho)$$

A sufficient (but not necessary) condition so as to ensure that $\Delta > 0$ is:

$$F(1+\beta)\rho - (1-a)\kappa L < 0 \quad (**)$$

Under $(**)$, we then also necessarily have $\sqrt{\Delta} > (1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho$, which entails $\phi_{C1} = \frac{(1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho - \sqrt{\Delta}}{2F(\kappa+1)} < 0$ and, on the other hand:

$$\phi_{C2} = \frac{(1-a)L\kappa - F(1+(1+\beta)d+\kappa)\rho + \sqrt{\Delta}}{2F(\kappa+1)} > 0 \tag{46}$$

The parametric conditions $(*) - (**)$ are hence sufficient to guarantee that **there is systematically necessarily one (and no more than one) positive solution for** ϕ_C

given by (46)**, entailing positive solutions for** Ω **and** ϕ_I **given by** (42) and (44). $(*)$ is met for sufficiently low values of L/F, while $(**)$ is met for sufficiently high values of κ. The intuitions behind those parametric conditions are commented in the main text of the paper.

Robustness of the monopoly price regime in the (SC) state

Proving that the system of three equations (39)-(41) admits a unique and positive solution in (ϕ_I, ϕ_C, Ω) is however not sufficient so as to demonstrate the existence and the uniqueness of the defined BGP. Indeed, we have assumed from the beginning that the equilibrium market structure chosen by the new leader in the (SC) state is a monopoly. We now need to check that for the obtained values for ϕ_C, ϕ_I and Ω, this specific price regime indeed represents a *robust* equilibrium, i.e. the new leader does not prefer the alternative regime when comparing expected profits. More formally, the condition for a monopoly to occur is of the form:

$$\pi_M + \phi_I^M \pi_{SI} \geq \pi_L + \phi_I^M \pi_{SI}^D + \phi_C^M (1 + \phi_I^M) \frac{\pi_F}{\rho + \phi_C^M + \phi_I^M} \tag{47}$$

with the supercript M referring to the fact we compute the different profits for the values of Ω^M, ϕ_C^M and ϕ_I^M obtained when solving for an equilibrium with a **monopoly** market structure. Expressions for π_M, π_{SI}, π_L, π_F and π_{SI}^D are given by equations (11), (12), (26), (27) and (28). Note that profits in the (SI) state take different values in both cases, even tough we use the same overall wealth Ω^M: indeed, the functional forms of the prices charged to the rich consumers p_R and p_R^D as defined by equations (10) and (25) are different.

More precisely, we have $p_R - p_R^D = \frac{\kappa^2 \phi_C^M (wL(1-\beta)(1-a) + (\kappa+1-d(\kappa+\beta))\rho\Omega^M)}{L(1-\beta)(\kappa+1)^2(\phi_C^M + (\kappa+1)\phi_I^M)} > 0$: in other words, for given values of Ω^M, ϕ_C^M and ϕ_I^M, the price charged by a firm in a (SI)-state industry in the monopoly case is unambiguously superior to its duopoly counterpart. The intuition behind this result is straightforward: in the monopoly case, rich consumers pay less than their reservation price in (SC)-state industries. This leads to a higher willingness to pay μ_R, which firms being in a (SI)-state industry are able to exploit since they have innovated twice and are hence able to fully price-discriminate between rich and poor consumers. Actually, as commented in subsection 2.2.3, p_R is increasing along θ_{SC}: in the monopoly case, the higher the share of industries being in the (SC) state, the higher the "residual" income left to exploit for industries being in the (SI) state. This mechanism is not present in the duopoly case, where rich consumers pay their maximum threshold price in every industry.

Considering we compute all the profits in (47) using the same value of overall wealth Ω^M, we have $\pi_F = \beta \pi_M$ and $\pi_{SI} - \pi_{SI}^D = (1-\beta)L(p_R - p_R^D)$. Dropping the M superscripts for the sake of exposition brevity, (47) then becomes:

$$\underbrace{\left(1 - \frac{\beta \phi_C (1 + \phi_I)}{\rho + \phi_C + \phi_I}\right) \pi_M - \pi_L}_{\text{(a)}} + \underbrace{\phi_I (1-\beta) L (p_R - p_R^D)}_{\text{(b)}} > 0 \tag{48}$$

First focusing on (a), is it possible to proceed to the following developments:

$$\begin{aligned}
(a) &= \frac{(\rho+(1-\beta)\phi_C+\phi_I(1-\beta\phi_C))\pi_M-(\rho+\phi_C+\phi_I)\pi_L}{\rho+\phi_C+\phi_I} \\
&= \frac{(\rho+(1-\beta)\phi_C+\phi_I(1-\beta\phi_C))L(p_P-wa)+(\rho+\phi_C+\phi_I)(1-\beta)L(p_R^D-wa)}{\rho+\phi_C+\phi_I} \\
&= \frac{L}{\rho+\phi_C+\phi_I}\Bigg[\underbrace{\rho(p_P-(1-\beta)p_R^D-\beta wa)}_{(c)}+\underbrace{\phi_I\Big((1-\beta\phi_C)(p_P-wa)-(1-\beta)(p_R^D-wa)\Big)}_{(d)} \\
&\quad+\underbrace{\phi_C(1-\beta)(p_P-p_R^D)}_{(e)}\Bigg]
\end{aligned}$$

We develop (c), (d) and (e) so as to get the following expressions:

$$\begin{aligned}
(c) &= \frac{\rho\kappa}{L(\kappa+1)^2}\left[(1-a)wL(\beta(2+\kappa)-1)+(d(\kappa+\beta(2+\kappa))-\kappa-1)\rho\Omega\right] \\
(d) &= \frac{\phi_I\kappa}{L(\kappa+1)^2}\left[(1-a)wL(\beta(2+\kappa(1-\phi_C)-\phi_C)-1)+(d(\kappa+\beta(2+\kappa(1-\phi_C)-\phi_C)-\kappa-1)\rho\Omega\right] \\
(e) &= -\frac{\phi_C(1-\beta)\kappa}{L(\kappa+1)^2}\left[wL(1-a)(1-\beta)+(\kappa+1-d(\kappa+\beta))\right]
\end{aligned}$$

A sufficient condition so as to guarantee that (c) and (d) are positive is:

$$\beta d(2+\kappa-\phi_C(\kappa+1))-1-(1-d)\kappa>0 \quad (***)$$

Indeed, note that (***) entails $\beta(2+\kappa)-1>0$ and $d(\kappa+\beta(2+\kappa))-\kappa-1>0$ (hence guaranteeing $(c)>0$), as well as $\beta(2+\kappa(1-\phi_C)-\phi_C)-1>0$ (hence guaranteeing $(d)>0$). (***) is met for sufficiently high values of β (provided d is not too low), as well as for values of F sufficiently high so as to guarantee that $\phi_C << 1$ (note that the condition on F was already needed so as to ensure the existence of a positive equilibrium).

On the other hand, (e) is necessarily negative. We however now turn to the other member of (48), i.e. we consider the following expression obtained for (b):

$$(b)=\frac{\phi_I\phi_C\kappa^2\left[wL(1-a)(1-\beta)+(\kappa+1-d(\kappa+\beta))\right]}{(\kappa+1)^2(\phi_C+(\kappa+1)\phi_C)}$$

Adding (b) and $\frac{L}{\rho+\phi_C+\phi_I}(e)$, we then get:

$$(b)+\frac{L}{\rho+\phi_C+\phi_I}(e) = \underbrace{\frac{\phi_C\kappa\left(wL(1-a)(1-\beta)+(\kappa+1-d(\kappa+\beta))\right)}{(\kappa+1)^2}}_{>0}\underbrace{\left[\frac{\kappa\phi_C}{\phi_I+(\kappa+1)\phi_C}-\frac{1-\beta}{\rho+\phi_C+\phi_I}\right]}_{(f)}$$

Developing (f), we finally get:

$$(f) = \frac{\phi_I(\kappa(\rho+\phi_C+\phi_I)-(1-\beta)(\kappa+1))-(1-\beta)\phi_C}{(\phi_I(\kappa+1)+\phi_C)(\rho+\phi_I+\phi_C)}$$

For low values of β, (f) starts out as unambiguously negative; however, along increasing values of β, (f) becomes more and more negligibly negative, before finally becoming positive. Hence again, for high values of β, it is possible to guarantee that $(f)>0$. To sum it up, we have been able to decompose (48) the following way:

$$\underbrace{\frac{L}{\rho+\phi_C+\phi_I}[(c)+(d)]}_{>0}+\underbrace{(b)+\frac{L}{\rho+\phi_C+\phi_I}(e)}_{>0}>0 \tag{49}$$

For sufficiently high values of β and d, (49) is necessarily positive. Note that this also entails that under those conditions, a duopoly cannot be a robust equilibrium (solving for the duopoly, we would need to prove that (49) is necessarily negative, which is an excludable case whatever the values of ϕ_C^D, ϕ_I^D and Ω^D provided we have high values of β and d).

Hence, under the parametric conditions (*)-(***), the positive equilibrium described by (42)**,** (44) **and** (46) **defines a unique BGP where the market structure in the (SC) case is necessarily a monopoly.** This ends the proof. □

Appendix C - Demonstration of the comparative statics

Comparative statics in the case of a variation in d

We first consider the variation of ϕ_C following a shock on d:

$$\frac{\partial\phi_C}{\partial d}=\frac{\rho}{2(\kappa+1)\sqrt{\Delta}}\left(\underbrace{(1-a)L(2\kappa+1-\beta)\kappa-(1+\beta)F(\kappa+1-d(1+\beta))\rho}_{(g)}-(1+\beta)\sqrt{\Delta}\right) \tag{50}$$

Since we have $(g)>0$ under $(**)$ and $(1+\beta)\sqrt{\Delta}>0$, the sign of the expression in brackets will be the same than the sign of $(g)^2-(1+\beta)^2\Delta$, which simplifies to the following expression:

$$(g)^2-(1+\beta)^2\Delta=4(1-a)\kappa(\kappa+1)(\kappa-\beta)((1-a)L\kappa-F(1+\beta)\rho)>0 \quad \text{under (**)}$$

We hence have $\frac{\partial\phi_C}{\partial d}>0$.

We then move to determining the variations of ϕ_I and Ω following a shock on d. The expressions of Ω and ϕ_I as obtained in (42) and (44) still depend on ϕ_C. We have the following partial derivatives of Ω w.r.t. ϕ_C and d:

$$\frac{\partial\Omega}{\partial\phi_C} = \frac{Fw(\kappa+1)}{d\kappa\rho}>0 \tag{51}$$

$$\frac{\partial\Omega}{\partial d} = -\frac{w(F(\kappa+1)(\rho+\phi_C)-(1-a)L\kappa)}{d^2\kappa\rho}<0 \tag{52}$$

We notice from (42) and (52) that we have $\frac{\partial\Omega}{\partial d}=-(1/d)\Omega$. Applying the total differentiation formula, we then have:

$$\begin{aligned}\frac{\mathrm{d}\Omega}{\mathrm{d}d} &= \frac{\partial\Omega}{\partial d}+\frac{\partial\Omega}{\partial\phi_C}\frac{\partial\phi_C}{\partial d}\\ &= -(1/d)\Omega+\frac{F}{2d\kappa\sqrt{\Delta}}\left((g)-(1+\beta)\sqrt{\Delta}\right)\end{aligned}$$

We furthermore know that the overall wealth Ω is equal to the present discounted value of firms' profits within the economy, i.e. $\Omega = \theta_{SC} v_{SC} + \theta_{SI} v_{SI}$. Using (18), (19) and (23), this yields the following expression along the BGP: $\Omega = \frac{\phi_C}{\phi_I+\phi_C} wF + \frac{\phi_I}{\phi_I+\phi_C} 2wF$. In other words, Ω is a weighted sum between the firms' discounted profits in the (SC) state (equal to wF through the free entry condition in the R&D sector) and the firms' discounted profits in the (SI) state (equal to $2wF$): we hence have $wF < \Omega < 2wF$. Using that inequality, we get:

$$\frac{\mathrm{d}\Omega}{\mathrm{d}d} = \frac{F}{2d\kappa\sqrt{\Delta}}\left((g) - (1+\beta+2\kappa\Omega)\sqrt{\Delta}\right) < \frac{F}{2d\kappa\sqrt{\Delta}}\left((g) - (1+\beta+2\kappa wF)\sqrt{\Delta}\right)$$

We hence have $\frac{\mathrm{d}\Omega}{\mathrm{d}d} < 0$ provided $(g) - (1+\beta)\sqrt{\Delta} < 2\kappa wF\sqrt{\Delta}$, which is unambiguously the case under (*)-(***).

Before moving to studying the variation of ϕ_I following an increase in d, we first consider the variation of the overall growth $\gamma = \kappa\phi_C(1+\frac{\phi_I}{\phi_I+\phi_C})$. Using the labor market equilibrium condition (21), we have that $wF\phi_C(1+\frac{\phi_I}{\phi_I+\phi_C}) = \theta_{SC}\pi_M + \theta_{SI}\pi_{SI} - \rho\Omega$. Using (18) and (19), we then obtain $\gamma = \frac{\kappa}{wF}\Omega\phi_C$. Totally derivating γ with respect to d, we have:

$$\begin{aligned}\frac{\mathrm{d}\gamma}{\mathrm{d}d} &= \left(\frac{\partial\Omega}{\partial d} + \frac{\partial\Omega}{\partial\phi_C}\frac{\partial\phi_C}{\partial d}\right)\phi_C + \frac{\partial\phi_C}{\partial d}\Omega \\ &= \frac{\partial\phi_C}{\partial d}\left(\frac{Fw(\kappa+1)\phi_C}{\kappa d\rho} + \Omega\right) - (1/d)\Omega\phi_C\end{aligned}$$

Using (42), we notice that $\frac{Fw(\kappa+1)\phi_C}{\kappa d\rho} = \Omega - \frac{Fw(\kappa+1)\rho-\kappa L(1-a)}{\kappa d\rho}$, and we finally get:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}d} = \Omega\underbrace{(2\frac{\partial\phi_C}{\partial d} - (1/d)\phi_C)}_{(h)} - \frac{\partial\phi_C}{\partial d}\left(\frac{Fw(\kappa+1)\rho - \kappa L(1-a)}{\kappa d\rho}\right) \tag{53}$$

Developing (h), we obtain:

$$\begin{aligned}(h) &= \frac{1}{2Fd(\kappa+1)\sqrt{\Delta}}\Big[2dF\rho\left((1-a)L\kappa(2\kappa+1-\beta) - (1+\beta)F(\kappa+1-d(1+\beta))\rho\right) \\ &\quad - \left((1+\beta)2dF\rho + (1-a)L\kappa - F(\kappa+1+d(1+\beta))\rho\right)\sqrt{\Delta} - \Delta\Big] \\ &= \frac{1}{2Fd(\kappa+1)\sqrt{\Delta}}\Big[2dF\rho\left((1-a)L\kappa(2\kappa-1-\beta) - (1+\beta)F(\kappa-1-d(1+\beta))\right) \\ &\quad -\sqrt{\Delta}\left((1-a)L\kappa - F\rho(\kappa+1-d(1+\beta))\right) - \left(F(\kappa+1+(1+\beta)d)\rho - (1-a)L\kappa\right)^2\Big]\end{aligned}$$

We have $(h) < 0$ under (*)-(***), and we finally obtain that the overall growth rate γ decreases following a positive shock on d: $\frac{\mathrm{d}\gamma}{\mathrm{d}d} < 0$. We can now finally consider the variation of ϕ_I. We have the following partial derivative:

$$\frac{\partial\phi_I}{\partial d} = -\frac{F\kappa\rho\phi_C(F(\kappa+1)(\rho+\phi_C) - (1-a)L\kappa)}{(F(\rho(1-(2d-1)\kappa) + \phi_C(1+\kappa) - (1-a)L\kappa))^2} < 0 \tag{54}$$

Considering (44), it is also straightforward to establish that $\frac{\partial\phi_I}{\partial\phi_C} > 0$; however, the closed-form analytical expression of this partial derivative is relatively complex. We hence face an ambiguity regarding the sign of the total derivative $\frac{\mathrm{d}\phi_I}{\mathrm{d}d} = \frac{\partial\phi_I}{\partial d} + \frac{\partial\phi_I}{\partial\phi_C}\frac{\partial\phi_C}{\partial d}$. We can however deduce the direction of variation of ϕ_I when simultaneously considering the variations of γ, Ω and ϕ_C. Indeed, we have that γ differs by only one multiplicative constant from the amount of labor devoted to R&D activities: $F\phi_C(1+\frac{\phi_I}{\phi_C+\phi_I})=\frac{\kappa}{F}\gamma$. We know from the decrease in Ω that the share θ_{SC} of industries being in the (SC) state has necessarily increased. In those industries, both incumbents and challengers carry out R&D. Since challengers have increased their investment $F\phi_C$, the decrease in the total amount of labor devoted to R&D is necessarily due to a decrease in the amount invested by the incumbents $F\phi_I$: we hence necessarily have $\frac{\mathrm{d}\phi_I}{\mathrm{d}d}<0$. This ends the proof. □

Comparative statics in the case of a variation in β

We have the following expression for $\frac{\partial\phi_C}{\partial\beta}$:

$$\frac{\partial\phi_C}{\partial\beta}=-\frac{d\rho}{2(\kappa+1)}\left(1+\frac{L(1-a)\kappa+F(\kappa+1-d(1+\beta))\rho}{\sqrt{\Delta}}\right)<0$$

On the other hand, we have $\frac{\partial\Omega}{\partial\beta}=0$ and $\frac{\partial\phi_I}{\partial\beta}=0$: the variations of Ω and ϕ_I following an increase in wealth concentration β are only due to the impact of β on ϕ_C. More precisely, we have:

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\beta}=\underbrace{\frac{\partial\Omega}{\partial\phi_C}}_{>0}\underbrace{\frac{\partial\phi_C}{\partial\beta}}_{<0}<0;\qquad \frac{\mathrm{d}\phi_I}{\mathrm{d}\beta}=\underbrace{\frac{\partial\phi_I}{\partial\phi_C}}_{>0}\underbrace{\frac{\partial\phi_C}{\partial\beta}}_{<0}<0$$

Finally, considering that we have $\gamma=\frac{\kappa}{wF}\Omega\phi_C$, we have that the consumers' utility growth rate along the BGP necessarily decreases following an increase in wealth concentration β. This ends the proof. □

Appendix D - Demonstration of the existence of the duopoly case

We first notice using (26), (27) and (28) that π_L, π_F and π_{SI}^D can be re-expressed as $\pi_L=A_l+B_l\Omega$, $\pi_F=A_f+B_f\Omega$ and $\pi_{SI}=A_f+A_l+(B_l+B_f)\Omega$, with:

$$\begin{aligned}A_l &= \frac{\kappa}{(\kappa+1)^2}(1-a)wL\kappa(1-\beta), \quad B_l=\frac{\kappa}{(\kappa+1)^2}\rho(\kappa+1-d(\beta(\kappa+2)-1))\\ A_f &= \beta\left(\frac{\kappa}{\kappa+1}\right)(1-a)wL, \quad B_f=\beta\left(\frac{\kappa}{\kappa+1}\right)d\rho\end{aligned}$$

We also note that $\rho>B_l$ and $\rho>B_f$.

The 3 sufficient parametric conditions under which there exists a unique positive equilibrium are the following:

$$\begin{aligned}2wF\rho>A_f-A_l &> 0 \text{ (i)}\\ B_f-B_l &> 0 \text{ (ii)}\\ A_l-wF\rho &> 0 \text{ (iii)}\end{aligned}$$

We now proceed to demonstrating the existence and uniqueness of the equilibrium under those conditions. Replacing wF with its value as expressed in (36) into equation (35), it is possible to obtain the following expression for $\phi_C v_F$:

$$\phi_C \left(\frac{\pi_F}{\rho + \phi_C + \phi_I} \right) = \pi_{SI} - 2\pi_L \qquad (55)$$

The existence of a positive steady state equilibrium implies that all the elements of the LHS of (55) are positive. The RHS then also has to be positive, which is ensured under the conditions (i) and (ii).

Substituting for $\phi_C v_F$ into (36), it is then possible to express ϕ_C as a function of Ω: $\phi_C = \frac{\pi_{SI} - \pi_L}{wF} - \rho > 0$ under condition (i) and (iii). Substituting for the obtained value of ϕ_C into equations (38) and (55), we obtain two implicit functions $\phi_I = \psi_R(\Omega)$ and $\phi_I = \psi_L(\Omega)$. ψ_R and ψ_L are implicitly defined by writing (38) and (55) respectively as $R(\phi_I, \Omega) = 0$ and $L(\phi_I, \Omega) = 0$ with:

$$\begin{aligned}
R(.) &= A_f \left(\frac{A_l}{wF} - \rho \right) + \left(B_f (\frac{A_l}{wF} - \rho) + \frac{B_l A_f}{wF} \right) \Omega + (B_l - B_f)\Omega\phi_I + (A_l - A_f)\phi_I + \left(\frac{B_f B_l}{wF} \right) \Omega^2 \\
L(.) &= - \left(\frac{A_f}{wF} - \rho \right) (A_l + \rho wF) + \left(\frac{B_f}{wF}(A_f - A_l - 2wF\rho) + \left(\frac{A_f}{wF} - \rho \right) (\rho - B_d) \right) \Omega \\
&\quad + (B_f + \rho - B_l)\Omega\phi_I + (A_f - A_l - 2wF\rho)\phi_I + \frac{B_f}{wF}(\rho - B_l)\Omega^2
\end{aligned}$$

We first consider the intercept of the two curves RR and LL (respectively representing the two functions ψ_R and ψ_L in the (ϕ_I, Ω) plane) with the vertical axis. We have $\psi_R(0) = \frac{A_f(\frac{A_l}{wF} - \rho)}{A_f - A_l} > 0$ under conditions (i) and (iii). On the other hand, we have $\psi_L(0) = \frac{-\left(\frac{A_f}{wF} - \rho\right)(A_l + \rho wF)}{B_f + \rho - B_l} < 0$ under condition (iii). We then move to considering the slopes of RR and LL. Using implicit differentiation, we have $\frac{\partial \psi_L}{\partial \Omega} = -\frac{\partial L / \partial \Omega}{\partial L / \partial \phi_I}$ and $\frac{\partial \psi_R}{\partial \Omega} = -\frac{\partial R / \partial \Omega}{\partial R / \partial \phi_I}$. More precisely, we have:

$$\begin{aligned}
\frac{\partial R}{\partial \Omega} &= (2\frac{B_f B_l}{wF})\Omega + (B_l - B_f)\phi_I + B_f(\frac{A_l}{wF} - \rho) + \frac{B_l A_f}{wF} > 0 \text{ under condition (iii)} \\
\frac{\partial R}{\partial \phi_I} &= (B_l - B_f)\Omega + A_l - A_f < 0 \text{ under conditions (i) and (ii)} \\
\frac{\partial L}{\partial \Omega} &= 2\frac{B_f}{wF}(\rho - B_l)\Omega + \underbrace{(B_f + \rho - B_l)\phi_I}_{(*)} + \frac{B_f}{wF}\underbrace{(A_f - A_l - 2wF\rho)}_{(**)} + (\frac{A_f}{wF} - \rho)(\rho - B_d) \\
\frac{\partial L}{\partial \phi_I} &= (B_f + \rho - B_l)\Omega + \underbrace{A_f - A_l - 2wF\rho}_{(**)}
\end{aligned}$$

Under conditions (i)-(iii), we unambiguously have that $\frac{\partial \psi_R}{\partial \Omega} > 0$; the curve RR is hence monotonously increasing (cf Figure 4). The shape of curve LL can be analyzed considering the explicit value of ϕ_I obtained when solving for $L(\phi_I, \Omega) = 0$:

$$\phi_I = \psi_L(\Omega) = \frac{- \left(\frac{A_f}{wF} - \rho \right) (A_l + \rho wF) + \left(\frac{B_f}{wF}(A_f - A_l - 2wF\rho) + \left(\frac{A_f}{wF} - \rho \right) (\rho - B_d) \right) \Omega + \frac{B_f}{wF}(\rho - B_l)\Omega^2}{(B_l - \rho - B_f) + 2wF\rho + A_l - A_f}$$

For small values of Ω, ϕ_I is negative (remember that $\psi_L(0) < 0$). We hence have that the term (*) in $\frac{\partial L}{\partial \Omega}$ is negative. Since the term (**) is also negative under condition (i), it

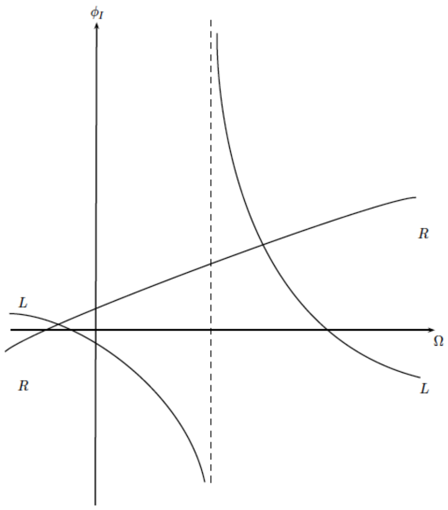

Figure 4: Case 2 steady state equilibrium

guarantees that both $\frac{\partial L}{\partial \Omega} < 0$ and $\frac{\partial L}{\partial \phi_I} < 0$. We hence have $\frac{\partial \psi_L}{\partial \Omega} < 0$ for small values of Ω. As Ω increases, ϕ_I actually becomes more and more negative, with $\phi_I \to -\infty$ as $\Omega \to \Omega_A$ with $\Omega_A = \frac{2wF+A_l-A_f}{\rho+B_f-B_l}$; we hence have $\frac{\partial \psi_L}{\partial \Omega} < 0$ for any $\Omega < \Omega_A$. For $\Omega > \Omega_A$, high values of ϕ_I as well as greater values of Ω ensure both $\frac{\partial L}{\partial \Omega} > 0$ and $\frac{\partial L}{\partial \Omega} > 0$, and we hence still have $\frac{\partial \psi_L}{\partial \Omega} < 0$. The curve LL is hence monotonously decreasing, with an asymptote at $\Omega = \Omega_A$ (cf Figure 4). RR and LL hence necessarily intersect only once, yielding a unique positive equilibrium with (Ω, ϕ_I) strictly positive.

Robustness of the duopoly price regime in the (SC) state

We have hence proved that under conditions (i)-(iii), there exists a unique and positive solution to the system of equations defining a BGP with a duopoly price regime in the (SC) state. As already extensively commented in Appendix B, we however still need to make sure that the obtained values for ϕ_I^D, ϕ_C^D and Ω^D indeed make it optimal for the successful challenger to charge a price p_R^D capturing only the rich. In other terms, we need the following condition to be respected:

$$\left(1 - \frac{\beta\phi_C^D(1+\phi_I^D)}{\rho+\phi_C^D+\phi_I^D}\right)\pi_M - \pi_L + \phi_I^D(1-\beta)L(p_R - p_R^D) < 0 \tag{56}$$

Condition (56) is the exact contrary of Condition (48), which necessarily holds for high enough values of β and not too low values of d (cf. Appendix B). Fixing low enough values of β and d would hence automatically imply that Condition (56) is respected, hence ensuring the existence and the uniqueness of the BGP with a duopoly price regime. However, Conditions (i) and (ii), needed so as to guarantee positive values of $\phi_C = \frac{\pi_{SI}-\pi_L}{wF} - \rho$, only hold for high values of β! It is hence a priori not obvious that there exists parameter values for which the duopoly case is robust, and it will depend on the obtained values for ϕ_C^D and ϕ_I^D. The absence of closed-form solutions however makes it necessary to resort to simulations so as to determine whether (56) holds for some parameter values.

Carrying out some simulations for a wide array of parametric values, the following numerical finding emerges:

Numerical finding: *Under the parametric conditions (i)-(iii) and for low enough values of F and d, there exists a unique and robust equilibrium BGP in which we have a duopoly in the (SC) state, and the incumbents invest a positive amount in R&D ϕ_I.*

It however appears that the parametric constellations under which the duopoly case is robust and unique are much narrower than their counterpart ensuring a unique and robust BGP with a monopoly in the (SC) state. This also justifies the fact that we focus on the monopoly case in the main text of the paper.