Pricing in Social Networks under Limited Information

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February 27, 2014

Abstract

This paper models the strategy of a monopolist that offers rewards to current clients in order to induce them to activate their social network and convince peers to buy from the company. In presence of heterogeneous search costs and reservation prices, this network-activation reward program may serve to expand the client base through a flow of information from informed to uninformed consumers. The offer of the monopolist affects individual incentives of aware people to share information, determining a minimal degree condition for investment. The optimal unitary reward balances the information spread effect (i.e. more receivers) and the crowding effect (i.e. less individual incentives) of an increase in the number of speakers. The monopolist always finds it profitable to use the bonus. Nevertheless, its introduction has ambiguous effects on the price and profits, depending on the process of spread of information and, in turn, on the network structure.

Keywords: social networks, monopoly pricing, network-based pricing, search costs.

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1 Introduction

Consumers are never perfectly aware about the different purchase options they have. As pointed out by influential papers in the 60s,[1] the access to information about the existence of a product and its characteristics is a crucial point when observing consumers decisions. This lack of information is an issue for consumers as well as for producers, that in turn need their product to be known to sell it. The traditional solution that producers opted for is advertisement in its informative view as in Nelson (1974) which, however, requires heavy sunk investments in exchange for uncertain outcome.

In the modern economy, consumers' and producers' access to information has changed considerably and towards different directions. On the one hand, new technologies have substantially improved the possibility for consumers to acquire information, but nevertheless they are required to have more skills to use the information they have and to understand more and more sophisticated products. Namely, consumers suffer an information overload Zandt (2004) and only some of them actually have the ability to face this complexity. On the other hand, the improved knowledge of producers of the social ties among consumers have increased the interest to exploit client's social network in order to generate business.[2] The use of consumers' network is an alternative solution to the informative problem, and it is more effective than advertising as consumers are usually embedded with a considerable component of trust.[3] Moreover, the decrease in costs of communicating to a large number of subject, brought upon by modern technologies (such as social networks, emails, messages), improves the effectiveness of this kind of programs, increasing their profitability.

In wanting to make use of the social network, consumers that are popular and able to acquire information often become the target of companies strategies. In particular, advantageous deals in the form of a reduced price or a gift are proposed to old buyers who support the firm to extend its clients base by convincing others to buy. Old buyers are clearly aware about the existence and characteristics of the product (as they experienced it in the past) and the more popular of them are more likely to be effective in helping firms to enlarge the

[1]Examples of this interest towards information and consumers behavior are Stigler (1961), which main focus is on the search cost that consumers bear to discover prices and Nelson (1970), interested in the difficulties that consumers have to actually evaluate the quality of a product

[2]This is confirmed by the recent development of companies (Anafore, ReferTo, NextBee) specialized in offering technical and consultancy services for the implementation of referral programs

[3]In an empirical paper of Schmitt et al. (2011) it is well documented that referred customers tend to be both more profitable and more loyal than customers acquired through other channels.

demand.

This use of network-based pricing is increasingly observable in several markets, taking different shapes. An important example are the online storage services such as iCloud and Dropbox, which offer free storage space to clients that convince their friends to subscribe their services. According to Huston (2010), founder and CEO of Dropbox, their referral program, run in 2009, extended their client basis of 60% and referral was responsible of 35% of daily new signups. Similar is the case of money transfers systems such as Paypal and UWC.[4] In these instances, the enterprise can even decide to give a monetary prizes for each new customer brought in the costumer base through external knowledge. In the same context, also more traditional banks recently began to offer more advantageous conditions (in the form of higher interest rates on the deposit or lowered service's fee) for each new customer that an old client manages to bring into the bank.

This paper is a first attempt at modeling theoretically the strategic decisions of a monopolist choosing to discriminate the price according to the ability of a consumer to induce others to buy. In our setup, the reduction in price takes the form of a monetary gift offered by the monopolist for each new customer brought into the clients base in a context where the population of potential buyers exhibits heterogeneous search costs. In our framework, the search cost is interpreted as the time and skills that an agent needs to dedicate for the acquirement of information.[5]

We directly investigate the effect of the introduction of the network-based reward on the flow of information when the monopolist has a very limited knowledge of the social network. The monopolist's offer creates some incentives for old consumers to communicate with uninformed peers about the existence of company's product thus reducing their informational problem. The objective of our analysis is twofold. On the one hand, we aim at characterizing the optimal unitary reward chosen by the monopolist and its dependence on the characteristics of the population and the social network structure. In aggregate terms, this reward will entail some implications for the spread of information about a product on a social network. On the other hand, we are interested in the effect that the introduction of this reward have on the general level of prices and profits of the monopolist. To reach this goal, we make

[4]http://www.uwcfs.com/en/faq/other-services/referral-program

[5] For example, young people tend to have more time to spend searching for information about technological products or services than old people. Moreover, they own stronger skills to obtain and interpret information about prices and characteristics of these products. In our interpretation, young people would have a lower search cost.

comparisons with the case of no reward (or uniform pricing) to analyze the level of prices and profits.

The remaining part of this paper is divided as follow. After a discussing the related literature in Section 2, we discuss the mathematical aspects of the model in Section 3. Then, in Section 4, we solve it and we discuss the results' implications. Finally we conclude in Section 5.

2 Related literature

It is now well known in economic theory that the solipsistic view of the consumer, which characterized the discipline in the past, can be relaxed considering the consumer as a member of a social group, that influences and is influenced by his behavior through local interactions. Economic theory introduced the concept of network while discussing economic interactions in a variety of fields. As pointed out in the comprehensive analysis of Jackson (2005) networks influences agents economic behaviour in fields such as decentralized financial markets, labour markets, criminal behaviour and spread of information and diseases.

In recent years the attention of industrial economists shifted from the network externalities approach, following the tradition of Katz and Shapiro (1985), to a new focus on the direct study of the effects of social interaction on the behavior of economic agents. In the traditional approach, consumers' own valuation of a good depends on the number of peers consuming the same good. The new tendency is to move the analysis a step further, linking the externality to a subset of neighbors rather than to the population overall. This new tradition is clearly exemplified by Sundararajan (2006) which proposes a model of network adoption where the externalities are local and consumers have incomplete information about adoption complementarities between all other agents. Following the same idea of locality, Banerji and Dutta (2009) find out the possible emergence of local monopolies even if homogenous firms compete only in prices. However, the focus of those papers remains on the study of consumption externalities in the new framework. Our approach is different both with respect assumptions and objectives of the research as we take the existence of a structure of social interactions as given and we study how the latter can be exploited by a monopolist to increase the consumers' base.

More specifically related to our work is the recent strand of the literature dealing with the issue of pricing in social networks, which take directly into account the topology of social relationships. Sääskilahti (2007) studies uniform monopoly pricing introducing network topology in a model of network adoption. The paper demonstrates that taking into account local interactions reduces the traditional network size effect tin the monopolist's ability to extract surplus, thus concluding that rents and total surplus are exaggerated considering only the size of the network. In Ghiglino and Goyal (2010), consumers compare their consumption with that of their neighbors, suffering a negative consumption externality. They characterize prices and allocations and demonstrate that identical consumers located make different consumption decisions when they are located in different positions in the social network (e.g. have different centrality). Bloch and Quérou (2013) study the optimal monopoly pricing in a context in which the producer is able to perfectly identify the network centrality of consumers and chooses a target price for each of them on the base of this variable. Their main result is that, if consumers benefit from neighbor consumption (network externality) then pricing decisions are indifferent to consumer's centrality. However, when the consumers compare their price with those received by their social neighborhood then the producer has incentives to charge higher prices to central nodes. In this manuscript, on the one side we relax the informational requirements of the producer to use network based marketing strategies and, on the other side, we consider a sequential setting instead of simultaneous consumption decision for all consumers.

Similarly, in a setup that typically fits communication markets, Shi (2003) studies the pricing strategy of a monopolist that sells a network good. His main finding is that the strength of network ties can be used as to discriminate prices among consumers. A crucial assumption of this paper is that, two or more clients must consume the network good together in order to enjoy discounted prices proposed by a monopolist, an assumption that we relax completely (thus eliminating the coordination problem involved). Shi's main result is that producer's pricing choices depend on the composition of client's ego network. He proposes discount to clients on communications with strong ties (friends and family) in order to profit (imposing higher prices) from his weaker links. While the setup of Shi (2003) uses the locality of the network only to allow consumers to cooperate (consume together the good) in our setup the locality of network structure has both positive (a more dense network implies stronger incentives for informed consumer to spread information) and negative (more dense networks also intensify the competition for being the person referred by the consumer) effects.

In the setting we propose, the monopolist sets the prices and, after some consumers buy the product, the monopolist propose to them a "bring-a-friend" discount. The timing of our paper differs from Bloch and Quérou (2013) and is similar to the one proposed by similar papers as Hartline et al. (2008) and Arthur et al. (2009). In their papers, they study the monopoly pricing in an environment in which myopic consumers take their decisions according to the number of people that bought the good in the past. In our proposal, myopic consumers decide whether to buy the product or not without any direct externality from consumption.

The paper is also related to the marketing literature studying referral bonuses. Two papers are worth mentioning from a theoretical point of view: Biyalogorsky et al. (2001) and Kornish and Li (2010). The first one defines a customer as delighted when he is willing to recommend a product; in this setup the reward to optimally enlarge the client basis is positively correlated with the share of delighted consumers. Kornish and Li (2010) focus instead on the impact of referrals on customers evaluations (namely, reservation price) of the product in a setting of asymmetric information in which agents put a value on friend's utility. They find that the higher is the interest on friend's payoff, the higher the optimal referral bonus should be. Both these paper, however, disregard the effects on strategic interaction of social networks among consumer. The empirical literature on the subject is represented by Leskovec et al. (2007), which study the adoption of a referral market strategy by an online retailer and discuss the product categories for which this strategy works better.

3 The model

We consider a setup where a monopolist seeks to sell a product to a large, but finite, population $N = \{1, 2, \ldots, i, \ldots, n\}$ of agents. Consumers differ according to their willingness to pay and their search cost. The utility function for an agent i from buying the product at price p is defined as:

$$u_i = r_i - p - s_i \tag{1}$$

The reservation price r is distributed according to a c.d.f. G on the support $[0, 1]$, while the search cost s is a binary variable. A proportion $1 - \beta$ of consumers exhibits a low search cost normalized to 0, while the remaining β have a high search cost s_H which, by assumption, is larger than the maximal willingness to pay 1. These latter consumers would never get informed and thus never buy unless they passively receive the information from some external source. The heterogeneity in search costs captures the different consumers' skills to access and use the available informational tools.

Interactions and communication among consumers are restricted by an existing social network structure, which we consider as given. In particular, each agent i has a finite number of neighbors $K_i \subseteq N$ to interact with. The degree k_i (the number of neighbors) is just the cardinality of K_i. We further assume the consumer's social network to be undirected, in the sense that if node i is linked to node j, then j is in turn linked to i. The degree of the agents is distributed according to some p.d.f. $f(k)$, which has to be interpreted as the fraction of agents having k neighbors. In other terms, selecting a random agent from the social network, the probability that she has exactly k neighbors is $f(k)$. This general formulation allows us to provide results for any interaction structure. Moreover, it is possible to substitute $f(k)$ with specific networks and compare results across different topologies.

We consider a two period model where the supply side of the market is constituted by a monopolist which aims at maximizing the sum of inter-temporal profits.[6] Defining $D_1(p)$ as the demand in the first period and assuming a marginal cost normalized to 0, the expected profit obtained charging price p will be given by:

$$\pi_1 = p\mathbb{E}(D_1(p)) \tag{2}$$

Adding to the formulation in Equation 2, in the second period of our model, we allow the monopolist to offer rewards to old customers through a "bring a friend" program. Namely, the monopolist knows the distribution of the degrees in the social network and, accordingly, offers a gift to the old consumers who inform their friends about the existence of the product and convince them to buy. The rationale of this offer is to eliminate the high search costs that prevent some of the potential consumers from buying. This gift takes the form of a unitary amount b for each referral. Since each new consumer corresponds to one reward b given to some old customer the margin in the second period is given by $(p-b)$. Thus, defining $D_2(p, b)$ as the demand in the second period coming from new consumers, the expected profit π_2 turns out to be:

$$\pi_2 = (p - b)\mathbb{E}(D_2(p, b)) \tag{3}$$

The dependence of D_2 on p and b takes into account, on the one hand the willingness to buy of customers given price p (fixed in period one), and on the other hand the probability of getting informed about the product, which in turn depends on the incentives to speak

[6]Future gains are discounted by a factor δ, normalized to one without loss of generality.

given to customers in D_1 through b. To enjoy rewards, old buyers need to contact their social network which implies a costly investment of a fixed amount C.[7]

It is important to discuss the informational structure of the model as it constitutes a peculiar feature of our study. Specifically, the information available to agents about the idiosyncratic characteristics of all the others is summarized in the Assumption 1.

Assumption 1. *The distribution of the variables r_i, k_i and s_i are common knowledge and independent from each other. Agents do not possess additional private information.*

Assumption 1 implies that consumers cannot condition their decisions on their local social neighborhood and the monopolist is not able to base his choice upon individual characteristics of consumers.

Our game is played in two periods and it is solved by backward induction. Each time period, in itself, is a sequential game in which the monopolist chooses first and consumers react. In period 1 the monopolist sets a price p (Period 1.A) and consumers, after having observed it, decide whether to purchase the good (Period 1.B). In the second period, the monopolist introduces the reward b (Period 2.A) and the first period buyers decide upon the possibility of contacting their friends (Period 2.B). Given the total investment of old consumers, information about the existence of the product may reach some potential new buyers. If reached, each customer purchases if his reservation price is sufficiently high (Period 2.C).

4 Results

We now proceed to solve our model by studying the decisions of the agents, from the last to the first, and assuming that what happened before is taken as given.

Period 2.C - Purchase decisions of uninformed consumers. In the last step, consumers with high search cost decide upon purchase. Some of them may receive the information through old buyers making their search cost drop to zero. We define ρ as the probability for an agent of receiving the information at least once. From the point of view of the single agent ρ is function of the number of social ties he has k and of the number of first period

[7]The choice of studying the case of fixed cost has been made in order to capture the idea that the emergence of the online social networks and the use of e-mails tends to make the difference in the number of people contacted negligible in terms of total cost.

consumers that invest in social network, which we define as D_1^{Inv}. Indeed, the more friends one person has, the more likely it is that at least one of them decides to invest and to speak with him about the product. Moreover, as the number of investors increases, the odds for each single neighbor to be an investor are higher. Since the agents who did not receive the information are stuck with a high search cost and cannot buy, the second period demand is composed by the fraction of newly informed agents exhibiting reservation price $r_i > p$. Given the degree distribution $f(k)$ and the probability of receiving the information $\rho(k, D_1^{Inv})$ we can derive the new expected demand in the second period as:

$$\mathbb{E}(D_2) = \beta(1 - G(p))\bar{\rho}n \tag{4}$$

where $\bar{\rho} = \sum_{k=1}^{n-1} \rho(k, D_1^{Inv}) f(k)$ represents the average probability of receiving the information about the existence of the product and then $\bar{\rho}n$ is the total number of receivers in the population.

2.B - Investment decisions of old buyers. After having observed the reward offered by the monopolist, old buyers take their decision about the investment in the social network considering the expected purchase behaviour of the agents they inform. The two alternatives are either to bear a cost and inform their friends (thus possibly getting rewards) or to give up the benefit enjoying no extra utility. Defining $B(b, k_i)$ as the total number of rewards received by an agent with degree k_i given the unitary reward b, the expect utility of informed agent i is:

$$\mathbb{E}(u_i) = \begin{cases} \mathbb{E}(B(b, k_i))b - C & \text{if } i \text{ invests} \\ 0 & \text{if } i \text{ does not invests.} \end{cases} \tag{5}$$

According to Equation 5, each agent invests if the amount he expect to receive $\mathbb{E}(B(b, k_i))b$ is bigger than the cost C. While the cost of activating the social network is assumed to be fixed, the expected benefit requires a more precise analysis. Indeed, this amount is composed by two elements: the total number of rewards the informed agent expects to get and the unitary bonus offered by the monopolist for each friend brought in the customers base. The first element, $\mathbb{E}(B(b, k_i))$, is agent specific, as it depends on the number of uninformed people that agent i is actually able to contact. This in turns is clearly an increasing function of his degree k_i. It follows that, for given b, the degree of an agent affects positively also the total amount that this agent expects to receive. Taking as given the degree of agent i, the unitary benefit b affects instead both elements of the total monetary reward. Clearly, as the unitary amount increases, so does the total amount that each agent expects to receive from speaking. Nevertheless, b also increases the incentives to invest for all agents, making the expectations about the total number of rewards $\mathbb{E}(B(b, k_i))$ change downwards due to a crowding effect.

So we assume that:

Assumption 2. *The number of expected rewards $\mathbb{E}(B(b, k_i))$ is decreasing in b for each degree level. Moreover, $\lim\limits_{b \to 0} \frac{\partial E(B)}{\partial b} = 0$ and $\lim\limits_{b \to 1} \frac{\partial E(B)}{\partial b} = -1$.*

The assumption about the limit values of the derivative $\frac{\partial E(B)}{\partial b}$ is made in order to avoid the degenerate cases in which the crowding effect created by b is so strong to exceed its positive effect, thus leading to an unrealistic decrease of the total expected benefit of an agent i as b increases.

Given the presence of a fixed cost C, the actual investors will be those for which $\mathbb{E}[B]\, b \geq C$. Since $\mathbb{E}[B]$ is monotonically increasing in k, this implies that there exists some $\underline{k}$ s.t. all agents i with $k_i \geq \underline{k}$ invest. Simply by equating benefits and cost, we find the critical degree:

$$\mathbb{E}\left[B(b, \underline{k})\right] b = C \tag{6}$$

Since the LHS of Equation 6 is increasing in b and $\underline{k}$ while the RHS is constant then $\underline{k}$ must be decreasing in b to maintain the equality. In economic terms this relationship indicates that offering an increased b creates stronger incentives for informed agents to invest given their degree. The fixed cost plays the opposite role.

Knowing the existence of $\underline{k}$, we can now compute the average probability of receiving the information in the population, which in turns requires the derivation of this probability for each k, namely $\rho(k, D_1^{inv})$. A degree k uninformed agent knows, on average, $(1-\beta)(1-G(p))k$ old - informed - buyers. Among them only the ones with $k \geq \underline{k}$ invest, i.e. a proportion $\sum\limits_{k \geq \underline{k}} f(k)$. In expected terms the probability of receiving the information from each single friend turns out to be equal to the share of investors in the total population $\frac{D_1^{inv}}{n}$. Thus, the probability of receiving the information from at least one among k friends is:

$$\rho(k, D_1^{inv}) = 1 - \left[1 - \frac{D_1^{inv}}{n}\right]^k \tag{7}$$

where $D_1^{inv} = n(1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k)$.

Summing over all ks the expression in Equation 7 we find explicitly $\bar{\rho}$. This can be plugged in Equation 4 obtaining the expected number of new consumers buying the product in period 2.

At equilibrium the number of new consumers at period 2 must be equal to the total number of benefits given away by the monopolist. The latter is simply the number of potential investors given by $n(1-G(p))(1-\beta)$ times the average number of benefits. Rearranging terms of this equilibrium condition:

$$\sum_{k \geq \underline{k}} f(k)\mathbb{E}(B(k,b)) = \frac{\beta}{1-\beta}\bar{\rho} \tag{8}$$

A naturally corollary of the equilibrium condition in Equation 8 is that the average expected benefit increases in b. In principle b entails two different effects on $\mathbb{E}(B)$. On the one hand it decreases the expected number of benefits obtainable for each degree level, making each term in the sum of average expected benefit lower. On the other hand, it decreases the investment threshold thus increasing the number of elements in the sum. At equilibrium, the second effect always dominates the first. This means that, from an individual point of view, b decreases the expectations about the number of rewards, but at the aggregate level, it increases the total expected number of rewards issued by the monopolist.

The derivations obtained so far, allows us to have a first set of results, regarding the behaviour of agent when monopolist's choices are taken as given, which are summarized in Proposition 3:

Proposition 3. *$\bar{\rho}$ is decreasing in the fixed cost of investment C and in price p while it is increasing in the unitary benefit b chosen by the monopolist.*

The proportion of uninformed agents (β) has an ambiguous effect on the investment threshold $\underline{k}$, depending on the balance between the inverse of the share of informed consumers ($\frac{1}{1-\beta}$) and the elasticity of the spread of information $\bar{\rho}$ on β, while its effect on $\bar{\rho}$ is unambiguously negative.

Proof. See Appendix 6.1. ■

Decreasing $\underline{k}$ implies increasing the share of investors. The natural consequence is that, as the number of investors grows, so does the probability of receiving the information for each individual and thus its average. Through this channel and considering how C, b and

p affects the incentives to speak the effect on this variables on $\bar{\rho}$ are easily derived. Indeed, since an increase in C or p (respectively a decrease in b) has a negative effect on the incentives to invest, $\underline{k}$ increases in them and, consequently, $\bar{\rho}$ decreases.

β has two opposite effects on the aggregate demand in the second period. Indeed, it increases the number of potential buyers but at the same time it decreases the probability that information reaches each of them. Which of this two effects is the strongest, depends on the reaction of information to a change in β measured by the elasticity. From the single agent point of view, an increase in β leads to change in the expectation on the total mass of benefits to be shared with other investors. The sign of this change depends on the strength of the reaction of $\bar{\rho}$ with respect to β. If the latter is strong enough, then the expectations will be updated downward and with them also the individual expectations of reward. This, in turns, reduces the incentives to invest for each degree level and thus the minimal $\underline{k}$ increases. The opposite is true when the response of information in β is not strong enough, compared with the inverse of the share of informed consumers.

2.A - Monopolist's choice of b. The monopolist, anticipating consumers decisions, faces a tradeoff. On the one hand, offering a bonus clearly reduces the margins that the monopolist can attain on the single new buyer. Indeed, the reward b works as a cost, since for each new buyer, the monopolist gives an amount b to one old buyer. On the other hand, the dimension of the unitary reward has a positive effect on the demand for the good (as it helps reducing the informational problem) as summarized in Lemma 4.

Lemma 4. *The demand faced by the monopolist in the second period is increasing in the unitary reward b.*

Proof. Let's compute the sign of $\frac{\partial \mathbb{E}(D_2)}{\partial b}$. Since $\frac{\partial \rho(k)}{\partial b} = \frac{\partial \rho(k)}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial b} > 0$ as proven in Proposition 3, by simple computation we get:

$$\frac{\partial \mathbb{E}(D_2)}{\partial b} = \beta \left(1 - G(p)\right) n \left(\sum_{k=1}^{n} \frac{\partial \rho(k)}{\partial b} f(k) \right) > 0 \tag{9}$$

■

Differently from the usual maximization problem, in this model the monopolist maximizes choosing b in a context where the margins are decreasing in this variable and the demand increasing in it:

$$\max_b (p-b)\mathbb{E}(D_2(b)) \tag{10}$$

Which yields the following modified Lerner rule:

$$\frac{p-b^*}{b^*} = \frac{1}{\eta_{\bar{\rho},b^*}} \tag{11}$$

where $\eta_{\bar{\rho},b^*}$ is the elasticity of the average probability of receiving the information to the unitary bonus computed at the optimal point. From this maximization problem yields the following proposition:

Proposition 5. *For any price charged in the first period, the monopolist always finds it profitable to run the program setting a unitary reward b^*.*

For the comparative statics on b^* two cases arise.

(i) If the optimal b^* lies in a interval with decreasing elasticity of $\bar{\rho}$ with respect to b, then b^* is increasing in the price p, in investment cost C and in the proportion of uninformed agents β.

(ii) If the optimal b^* lies in a interval with increasing elasticity of $\bar{\rho}$ with respect to b, then b^* is decreasing in the price p, in investment cost C and in the proportion of uninformed agents β. Unless:

(ii.1) either the elasticity is too small. In which case the results are the same as in (i) for all variables.

(ii.2) or the elasticity is big enough. And then then b^* increases in p.

Proof. See Appendix 6.2 ■

When setting b the monopolist faces a trade-off. Indeed, by increasing the unitary gift he obtains two effects. The demand increases and in this additional demand the monopolist makes a margin. The total additional profit represents the *marginal gain* of increasing b. At the same time, an increase in b decreases the margin of an equivalent amount for each new agent which is expected to buy. This reduction of the total profit represents the *marginal loss* of increasing b. The optimal b describes a situation in which these two opposite forces perfectly offset. In this setting the effects on the demand only take the form of effects in $\bar{\rho}$ so that the two terms can be use interchangeably.

The parameters of the model change the equilibrium situation by affecting the incentives. In particular, while C and β only affect the demand faced by the monopolist, p also raises the margins.

As we have seen, an increase in C or β has a negative effect on the spread of information in the network and thus the average probability of getting informed for uninformed people. This means that, for any p and b set by the monopolist, the number of people that are going to buy is lower with higher values of these parameters. This effect is certain, while the effect that this has on the elasticity in the Lerner rule depends on the relationship between the latter and the reward.

To any couple of values of b and $\bar{\rho}$ corresponds one elasticity. When we refer to the case of increasing elasticity we are considering that, for high levels of $\bar{\rho}$ and b together (since $\bar{\rho}$ is increasing in b) the elasticity is higher. Thus, the second order effects of b on $\bar{\rho}$ (increasing or decreasing elasticity) are going to determine the effect of the parameters on the elasticity.

Following this reasoning, if the elasticity is decreasing, then an increase of C or β makes the elasticity larger. The opposite is true when the elasticity is increasing.

Assume that the optimal b is such that C or β increase the elasticity of information to b. This increase depends on the reduced incentive that decreases the information and thus the demand. In this context the marginal loss is reduced unambiguously more than the marginal gain. Consequently the monopolist is profitable to increase b, up to the point in which the two are again equal.

More interesting is the case in which C or β decreases the elasticity of information to b. This may be due either to a decrease of the response of the information to an increase in b or to an increase in the demand level. Since C reduces the incentives to speak and the spread of information, the second effect is excluded allowing us to conclude that the marginal effect of b on $\bar{\rho}$ becomes lower. Consequently the marginal gain shrinks as margins are fixed. Moreover, its reduction is stronger than the reduction in demand in order to have the desired effect on elasticity. Since the reduction in demand represent the marginal loss of increasing b, in general we can conclude that the latter becomes bigger than the marginal gain inducing the monopolist to decrease the reward.

The only limit case in which this is not true is the one in which even if the response of the demand to an increase in b is very low, the margins are so high, and the demand so low, that the monopolist finds it profitable to increase b.

A similar reasoning applies for p, with the exception that p does not only affect incentives and thus demand but also, directly, the margins. In particular p affects the margins the response of the demand and the level of demand. When the p makes the demand less elastic the negative effect on the level of the demand always overcome the combination of the other two. This pushed b to increase to balance.

When, instead the spread of information turns out to be more elastic as effect of an increase in p then the opposite is true for intermediates level of elasticity. The limit case that we have discussed for β and C, recourse itself here. However, in this case the conditions for his appearance are less tight as it appears both for very low and very high elasticity. The limit case of low elasticity, leading to an increase in b is similar to one studied for β and C with the addition that with an increase in p we also have higher margins, thus increasing more the the marginal gain compared to the previous case. A similar reasoning applies when we have a very high elasticity.

In the first period, the monopolist sets the price in order to maximize the sum of inter-temporal profits. In doing so, it knows the distribution of willingnesses to pay and search costs and internalizes customers decisions in the second period. Moreover, consumers will observe the price and decide whether to buy the product.

Period 1.B - Purchase decisions of consumers. After having observed the price p charged by the monopolist, agent i decides whether to buy the product. The utility that he enjoys from the purchase is $u_i = r_i - p - s_i$ and 0 otherwise. Since s_H is assumed to be larger than the greatest possible willingness to pay only agents with low search cost can buy. Thus only a proportion $1-\beta$ of the the population is eligible to buy.

Once an agent with no search cost gets informed, his decision depends on his preferences. Specifically, he buys only if the price set by the monopolist is lower than his reservation price (i.e. if $r_i > p$). As we already defined, the probability for the willingness to pay to be larger than p is indicated by $(1 - G(p))$ allowing us to conclude that the total number of buyers at price p, is:

$$\mathbb{E}(D_1(p)) = (1-\beta)(1-G(p))n \tag{12}$$

The remaining part of the population is composed by βn agents who are uninformed and $(1-\beta)G(P)n$ who are informed but not interested to buy at price p.

Period 1.A - Monopolist sets p. Anticipating what will occur in the second period and having expectations about the purchase decisions of the present period, the monopolist sets the price so to maximize its inter-temporal profits as defined in Equations 2 and 3:

$$\pi = \pi_1 + \pi_2(b^*) = n\,(1 - G(p))\,(1-\beta)p + (p - b^*(p))\mathbb{E}(D_2(b^*(p), p)) \tag{13}$$

After some rearrangement, the first order condition of the maximization problem yields the optimal price p^* which is the one such that:

$$\begin{gathered}(1-\beta)(G'(p^*))p^* + \beta\left[\bar{\rho}(G'(p^*)) - (1 - G(p^*))\,\tfrac{\partial \bar{\rho}}{\partial p^*}\right](p^* - b^*)\\ = \\ (1-\beta)\,(1 - G(p^*)) + \beta\bar{\rho}\,(1 - G(p^*)) - \beta\,(1 - G(p^*))\,\bar{\rho}\tfrac{\partial b^*}{\partial p^*}\end{gathered} \tag{14}$$

The LHS of Equation 14 summarizes all the marginal losses in profits given by an increase in p while the RHS represents the marginal gains.

The first term on the LHS can be interpreted as the marginal loss that an increase in p yields the number of informed consumers that buy in the first period.

The second term is instead referred to the second period and can be decomposed in two different components. The first one is a direct effect, due to the fact that less people are willing to buy at the increased price. The second one is indirect. On average less people receive the information in the second period both because there are both less potential investors (first period buyers) and less incentives to invest (because of the direct effect).

The first two term on the RHS are referred to the increase in total margin obtained respectively in the first (from informed people) and the second period (from initially uninformed people).

Finally, the last term represents the marginal increase in b (a loss for the monopolist) that should be provided in order to retain the same demand from uninformed consumers, which are less willing to buy because of the increased price. Indeed, when the price increases less people are willing to buy at the new price. This constitutes a disincentive for potential investors because there are less rewards to be gained. If the objective of the monopolist is to keep the demand at the same level, then the increase in b should more than compensate this effect so to improve the informational process. The level of demand remains the same but the composition changes: more people receive the information but, among them, a lower share will buy.

We compare the results obtained above with the benchmark case in which the monopolist does not run the reward program. In this case only the first period informed agents can be attracted and the maximization problem reduces to:

$$\max_p np(1-\beta)(1-G(p)) \tag{15}$$

Consequently the first order conditions yields the optimal price p^{**}, which is the one such that:

$$p^{**}(1-\beta)(G'(p^{**})) = (1-\beta)(1-G(p^{**})) \tag{16}$$

Analyzing the difference between 16 and 14 we have:

Proposition 6. *The price p^*, optimal when the program is run, is higher than the price p^{**}, optimal when the program is not run, if and only if:*

$$\eta_{p-b,p} > -(\eta_{(1-G),p} + \eta_{\bar{\rho},p}), \tag{17}$$

lower otherwise. The terms in this inequality are the elasticities with respect to p of the variables in the subscript.

Proof. See Appendix 6.3. ■

In our model, the effect of price on monopolist's profit is different from the usual price setting because of the introduction of the program in the second period. Increasing the prices not only has the classical effect on the first period, but also two effects on the second period. What is peculiar of our model in the second period is the double effect both on the margins and on the demand.

Indeed, the increase in margins can be higher or lower than usual because we have an effect on the unitary reward that will be chosen. Moreover, an increase in p decreases the number of people willing to buy and reduces also the number of receivers of information. The effects on margins is the marginal gain of choosing a price higher than in the benchmark case (described by the LHS of Equation 17), while the effects on demand is the marginal loss (represented by the RHS of Equation 17). Clearly when the first dominates the price set with the program is higher than in the benchmark case.

This is because in such condition the net marginal gain is positive in the second period and thus negative in the first (to maintain the balance on the first order conditions). The first period price is comparable (actually the same thing) than the one of the benchmark case. It means that if we were in the benchmark case p^* would be such that the marginal loss would be higher than the marginal gain (and thus it would have been necessary to reduce the price to balance the correspondent first order condition).

5 Conclusions

In this paper we considered a setup in which a monopolist tries to reduce the search cost affecting part of his potential client base using a referral program. His aim is to incentivize a mobilization of the current customer base, creating a flow of communication from informed to uninformed consumers on a social network, leading to an expansion of the total number of buyers. The reward program consists in offering to informed consumers a bonus for each new consumer convinced to buy. The incentives created by the offer are clearly stronger the more one person is connected and lead to the emergence of a minimal degree above which an agent invests in communicating with peers. This leads to the quite realistic result that the equilibrium investors are only those with relatively high degrees and thus only a limited fraction of agents gets the discounts.

We confirm that centrality matters when pricing is done on social networks as in Bloch and Quérou (2013). The main difference is that, in our setup, the monopolist has only limited information about the topology of the network (degree distribution) while they assume the producer to know perfectly all nodal characteristics of each single agent. Different informational assumptions lead our results in opposite directions. In Bloch and Quérou (2013), where the authors use the concept of reference prices, central agents turn out to be charged more while, in this manuscript, being central is advantageous as it allows to receive discounted prices (in terms of reward).

The offer of the monopolist produces two competing effects. On the one hand, it creates incentives for informed people to invest in their social network and transmit information about the existence of the product. On the other hand, it also reduces the total amount of rewards that each agent expects to receive because of a crowding effect emerging as more people invest.

The balance combination of these two effects leads to different responses of the optimal reward to exogenous changes of incentives. When the crowding effect is more important the marginal effect of the reward is lower and thus decreased incentives lead to decreased rewards and vice-versa when the information effect dominates.

Investing agents ignite a process of spread of information. The efficiency of this process strictly depends on the type of network we have (how, is subject of our current investigation) and on relative share of uninformed consumers. The determination of whom is better (or worse) off due to the existence of the program is strictly linked to the efficiency of this process. This is to say that what matters is the amount of information at the beginning and how well it circulates.

Indeed, the reward program has different effects on the different categories of agents. Uninformed agents receive transfers from investing consumers and are thus unambiguously better off. When the price is decreased with respect to the benchmark case all consumers are weakly better off while the situation of the monopolist is ambiguous. If we have a large share of uninformed consumers this makes the second period more important than the first. In this case we have that raising b would increase margins of a relatively small amount that may not be enough to compensate the loss in first period margins so to put the monopolist in a problem of time inconsistency. In this case the exploitation of the social network can be detrimental by the monopolist. When instead the price increases clearly the monopolist is better off while the informed agents who do not invest are surely worse off (they pay a higher price). In this case the ambiguity goes on the effect that running the program has on informed investors. Their position is indeed very ambiguous. In expected terms however, it is more probable that for more connected agents the increase in price is compensated by the received gifts.

While it is reasonable to assume the independence between the willingness to pay of one agent and his degree, one could challenge our assumption that search costs are independent with centrality and reservation prices. Indeed, one could consider the case in which a more central node may have lower search cost due to his popularity. The only channel through which this may happen is that they receive the information through their social network. But, the communication among agents is the core of this paper and initially uninformed people with many connections will be more likely to see their search cost drop to zero in the second period.

The study of a monopoly is a starting point to understand the effects on pricing of network's exploitation under limited information, but most markets where such programs are run are, up to some degree, oligopolistic. Consequently, our current research endeavors are focused on extending our setup to an imperfect competition environment, where firms compete on prices. We expect that, increasing the competitive pressure, would push producers to offer higher rewards (thus extending the share of consumers interested in activating their social network). In such models the informational problem described here could be accompanied by a problem of switching costs, that may induce producers to offer rewards to switchers as wells as to those who convince them to buy. An alternative is to think of competition in the context of an entry model. Here, the challenge would be to understand whether the referral program is a way to prevent entrance for the incumbent or a way to steal a part of his market for the entrant.

6 Appendixes

6.1 Proof of Proposition 3

Proof. To obtain the results of the Proposition we simply need to derive $\bar{\rho}$ with respect to the parameters of the model. From Equation 7 there is no direct effect of C and b on $\bar{\rho}$, the only effect is through $\underline{k}$. Thus $\frac{\partial \bar{\rho}}{\partial b} = \frac{\partial \bar{\rho}}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial b}$ and $\frac{\partial \bar{\rho}}{\partial C} = \frac{\partial \bar{\rho}}{\partial \underline{k}} \frac{\partial \underline{k}}{\partial C}$. The first terms can be computed from Equation 7 as:

$$\frac{\partial \rho(k)}{\partial \underline{k}} = k \left[1 - (1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k) \right]^{k-1} (1-\beta)(1-G(p))(-f(\underline{k})) < 0 \tag{18}$$

Notice that the derivative of $\frac{\partial \bar{\rho}}{\partial \underline{k}} = \sum_{k=1}^{n} \frac{\partial \rho(k)}{\partial \underline{k}} f(k)$. From the assumptions we know the signs of the second terms, and thus of the effects, in particular:

- $\frac{\partial \underline{k}}{\partial C} > 0 \longrightarrow \frac{\partial \bar{\rho}}{\partial C} < 0$
- $\frac{\partial \underline{k}}{\partial b} < 0 \longrightarrow \frac{\partial \bar{\rho}}{\partial b} > 0$

The result of Proposition 3 with respect to p is instead calculated as:

$$\begin{aligned} \frac{\partial \rho(k)}{\partial p} = \quad & k(1-\beta) \left[1 - (1-\beta)(1-G(p)) \sum_{k \geq \underline{k}} f(k) \right]^{k-1} \\ & \left(\underbrace{\sum_{k \geq \underline{k}} f(k)(-G'(p))}_{<0} \right) < 0 \end{aligned} \tag{19}$$

Finally, we can calculate the derivative for β:

$$\frac{\partial\rho(k)}{\partial\beta} = k\left(1-G(p)\right)\left[1-(1-\beta)\left(1-G(p)\right)\sum_{k\geq\underline{k}} f(k)\right]^{k-1} \left(\underbrace{(1-\beta)\frac{\partial\sum\limits_{k\geq\underline{k}} f(k)}{\partial\beta}}_{ambiguous} - \underbrace{\sum_{k\geq\underline{k}} f(k)}_{>0}\right) \tag{20}$$

The sign of Equation 20 is clearly ambiguous. However, multiplying by:

$$\frac{\beta}{(1-\beta)\sum\limits_{k\geq\underline{k}} f(k)} \frac{\left[1-(1-\beta)\left(1-G(p)\right)\sum\limits_{k\geq\underline{k}} f(k)\right]^{1-k}}{k\left(1-G(p)\right)} \tag{21}$$

and rearranging terms, we can obtain the following condition for positiveness of the derivative:

$$\eta_{\sum\limits_{k\geq\underline{k}} f(k),\beta} > \frac{\beta}{1-\beta} \tag{22}$$

According to the equilibrium condition stated in Equation 8, define:

$$\phi = \frac{1}{b}\sum_{k\geq\underline{k}} f(k)\mathbb{E}(B(k,b)) - \frac{\beta}{1-\beta}\bar{\rho} \tag{23}$$

In order to compute the derivative of $\frac{\partial\underline{k}}{\partial\beta}$ we use the total derivation of ϕ w.r.t. both variables and equate to zero, so that $\frac{d\underline{k}}{d\beta} = -\frac{\partial\phi/\partial\beta}{\partial\phi/\partial\underline{k}}$. Computing:

$$\frac{\partial\phi}{\partial\underline{k}} = \underbrace{\frac{1}{b}\frac{\partial\sum_{k\geq\underline{k}} f(k)\mathbb{E}(B(k,b))}{\partial\underline{k}}}_{>0} - \underbrace{\frac{\beta}{1-\beta}\frac{\partial\bar{\rho}}{\partial\underline{k}}}_{<0} > 0 \tag{24}$$

The first term is indeed positive because the average expected benefit can only increasing in the lower bound of the sum, while the other term's sign derives from Equation 18. On the other side:

$$\frac{\partial\phi}{\partial\beta} = -\frac{\frac{\partial\bar{\rho}}{\partial\beta}\beta(1-\beta)+\bar{\rho}}{(1-\beta)^2} \tag{25}$$

Interpreting the sign of this equation we find the condition such that $\frac{\partial\underline{k}}{\partial\beta} > 0$:

$$\frac{1}{1-\beta} > -\frac{\partial\bar{\rho}}{\partial\beta}\frac{\beta}{\bar{\rho}} \tag{26}$$

Now we are able to proof that $\frac{\partial\underline{k}}{\partial\beta}$ is ambiguous (depends on the satisfaction of Equation 26), while the $\frac{\partial\bar{\rho}}{\partial\beta}$ is unambiguously negative. We do this in 2 steps:

1. Let's assume that Condition in Equation 26 is satisfied. Then, since $\frac{\partial \underline{k}}{\partial \beta} > 0$, the share of investors $\sum_{k \geq \underline{k}} f(k)$ is decreasing in β. Substituting this result in Equation 20, the condition in Equation 22 can never be satisfied.

2. Let's assume that Condition in Equation 26 is reversed. Then, since $\frac{\partial \underline{k}}{\partial \beta} < 0$, the share of investors $\sum_{k \geq \underline{k}} f(k)$ is increasing in β.Substituting this result in Equation 20, two sub cases arise:

   - the condition in Equation 22 is satisfied. It implies that $\frac{\partial \bar{\rho}}{\partial \beta} > 0$. This leads to a contradiction. Indeed, the LHS of Equation 26 becomes negative implying that $\frac{\beta}{1-\beta} > -\frac{\partial \bar{\rho}}{\partial \beta} \frac{\beta^2}{\bar{\rho}}$.
   - The condition in Equation 22 is reversed. In this case both $\frac{\partial \bar{\rho}}{\partial \beta}$ and $\frac{\partial \underline{k}}{\partial \beta}$ are negative.

■

6.2 Proof of Proposition 5

Proof.

Interior solution. Proving that it is always profitable to run the program can be stated formally:

$$\forall p \in (0,1] \; \exists \; b \in (0,p) \text{ s.t. } \frac{p-b^*}{b^*} \geq \frac{1}{\eta_{\bar{\rho},b^*}} \tag{27}$$

For any p, take any b equal to $p-\epsilon$ with ϵ arbitrarily close to 0. We can always find an ϵ such that the equation above is true, since $\eta_{\bar{\rho},b^*} > 0$ as proven in Proposition 3. Moreover, b^* is always strictly lower than p otherwise the profit would be zero. This is equivalent to prove that there exists an interior solution to the maximization problem in Equation 10.

Comparative statics on the FoC. Define function ϕ as follows:

$$\phi = \frac{p-b^*}{b^*} - \frac{1}{\eta_{\bar{\rho},b^*}} \tag{28}$$

Take the derivatives of ϕ w.r.t. b, p, C, β.

$$\frac{\partial \phi}{\partial b} = -\frac{p}{b^2} + \frac{\partial \eta_{\bar{\rho},b}}{\partial b} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{29}$$

$$\frac{\partial \phi}{\partial p} = \frac{1}{b} + \frac{\partial \eta_{\bar{\rho},b}}{\partial p} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{30}$$

$$\frac{\partial \phi}{\partial C} = \frac{\partial \eta_{\bar{\rho},b}}{\partial C} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{31}$$

$$\frac{\partial \phi}{\partial \beta} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} \frac{1}{(\eta_{\bar{\rho},b})^2} \tag{32}$$

The signs of the partial derivatives above, depend crucially on the sign of the derivative of $\eta_{\bar{\rho},b^*}$ with respect to all variables.

Partial Derivatives of Elasticity $\eta_{\bar{\rho},b^*}$. Two cases need to be studied:

1. Assume that the optimal point b^* lies in an interval in which the elasticity is decreasing, i.e. $\frac{\partial \eta_{\bar{\rho},b}}{\partial b} < 0$. Then , the partial derivatives of the elasticity $\eta_{\bar{\rho},b}$ w.r.t. b, p, C and β can be rewritten in such a way to easily study their signs:

$$\underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial b}}_{<0} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} \underbrace{\frac{\partial \bar{\rho}}{\partial b}}_{>0} \implies \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} < 0$$

Thus:

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial p} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial p}}_{<0} > 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial C} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial C}}_{<0} > 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{<0} \underbrace{\frac{\partial \bar{\rho}}{\partial \beta}}_{<0} > 0$$

Plugging these results into the Equations 29, 30, 31 and 32 we find that $\frac{\partial \phi}{\partial p} > 0$, $\frac{\partial \phi}{\partial b} < 0$, $\frac{\partial \phi}{\partial C} > 0$ and $\frac{\partial \phi}{\partial \beta} > 0$. It follows that:

$$\frac{db}{dp} = -\frac{\frac{\partial \phi}{\partial p}}{\frac{\partial \phi}{\partial b}} > 0 \tag{33}$$

$$\frac{db}{d\beta} = -\frac{\frac{\partial \phi}{\partial \beta}}{\frac{\partial \phi}{\partial b}} > 0 \tag{34}$$

$$\frac{db}{dC} = -\frac{\frac{\partial \phi}{\partial C}}{\frac{\partial \phi}{\partial b}} > 0 \tag{35}$$

2. Assume that the optimal point b^* lies in an interval in which the elasticity is increasing, i.e. $\frac{\partial \eta_{\bar{\rho},b}}{\partial b} > 0$. As in Case 1 above, the partial derivatives of the elasticity $\eta_{\bar{\rho},b}$ w.r.t. b, p, C and β can be rewritten as:

$$\underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial b}}_{>0} = \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} \underbrace{\frac{\partial \bar{\rho}}{\partial b}}_{>0} \implies \frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}} > 0$$

Thus:

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial p} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial p}}_{<0} < 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial C} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial C}}_{<0} < 0$$

$$\frac{\partial \eta_{\bar{\rho},b}}{\partial \beta} = \underbrace{\frac{\partial \eta_{\bar{\rho},b}}{\partial \bar{\rho}}}_{>0} \underbrace{\frac{\partial \bar{\rho}}{\partial \beta}}_{<0} < 0$$

Plugging these results into the Equations 29, 30, 31 and 32 we find:

2.A. If $\eta_{\bar{\rho},b^*} < \sqrt{\frac{\partial \eta_{\bar{\rho},b^*}}{\partial b} \frac{b^2}{p}} \equiv \bar{\eta}$ then $\frac{\partial \phi}{\partial b} > 0$, $\frac{\partial \phi}{\partial C} < 0$ and $\frac{\partial \phi}{\partial \beta} < 0$. Thus we can conclude that:

$$\frac{db}{d\beta} = -\frac{\frac{\partial \phi}{\partial \beta}}{\frac{\partial \phi}{\partial b}} > 0 \tag{36}$$

$$\frac{db}{dC} = -\frac{\frac{\partial \phi}{\partial C}}{\frac{\partial \phi}{\partial b}} > 0 \tag{37}$$

The signs are reversed if $\eta_{\bar{\rho},b^*} < \bar{\eta}$.

2.B. If $\eta_{\bar{\rho},b^*} < \sqrt{-\frac{\partial \eta_{\bar{\rho},b^*}}{\partial p} b} \equiv \hat{\eta}$, then $\frac{\partial \phi}{\partial p} < 0$ and thus we can conclude that if either $(\eta_{\bar{\rho},b^*}) < \min\{\bar{\eta}, \hat{\eta}\}$ or $\eta_{\bar{\rho},b^*} > \max\{\bar{\eta}, \hat{\eta}\}$:

$$\frac{db}{dp} = -\frac{\frac{\partial \phi}{\partial p}}{\frac{\partial \phi}{\partial b}} > 0 \tag{38}$$

The signs are reversed if $\min\{\bar{\eta}, \hat{\eta}\} < \eta_{\bar{\rho},b^*} < \max\{\bar{\eta}, \hat{\eta}\}$.

■

6.3 Proof of Proposition 6

Proof. The LHS (respectively RHS) of the FOC in Equation 16 is equal to the first term of the LHS (respectively RHS) of Equation 14. Indeed, these are the effects of the price on profits without considering the population of new entrants in the second period. Thus what makes p^* different from p^{**} are the terms referred to the second period demand (which depends on the spread of information in the network). Focusing only on these terms, if the negative effect of an increase in p in the LHS of Equation 14 is higher than the positive effect on the RHS, the marginal loss from the first period in p^* should be lower than the corresponding marginal benefits, thus the price p^* should be lower than the benchmark case p^{**}. Formally, taking the difference of these remaining effects and multiplying by $\frac{1}{(1-G(p^*))\bar{\rho}b^*}$ after some rearrangement we get that, if:

$$\eta_{p-b,p} < -(\eta_{(1-G),p} + \eta_{\bar{\rho},p}) \tag{39}$$

then the price running the program is lower than the one in the benchmark case (i.e.: $p^* < p^{**}$), higher otherwise. ■

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