Tax competition with two mobile and interdependent tax bases: which fiscal architecture?

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Abstract

Our paper analyses the issue of fiscal architecture from a tax competition perspective, in a two-tier setting with multiple countries and regions. Considering two mobile and interdependent tax bases allows us to compare four fiscal architectures: i) full decentralization, ii) full centralization, iii) partial decentralization with shared tax bases and iv) partial decentralization with exclusive tax bases. The interdependence between the two tax bases generates "indirect" tax externalities in addition to standard "direct" tax externalities. It results in partial decentralization with exclusive tax bases differing from other fiscal architectures in that tax competition can lead to inefficiently high tax rate at either tier. While there is always a level of expenditure decentralization such that partial decentralization with shared tax bases dominates full centralization, this is no longer the case with exclusive tax bases for a sufficiently high degree of substitutability between the tax bases.

Keywords: tax competition, multiple tax bases, fiscal decentralization, fiscal architecture, fiscal federalism

JEL Classification: H20, H40, H71, H77

1 Introduction

The decentralization phenomenon observed in most OECD countries is the evidence of an undeniable agreement on the legitimacy and the ability of local authorities to participate in the provision of public goods and services. However the widespread decentralization of expenditures has not led to a consensus about the assignment of revenues to different tiers

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of government. In particular, in a multitier governmental structure, the question arises of the fiscal architecture. In this paper, we analyze this issue from a tax competition perspective. With several mobile tax bases, the mobility of which being potentially interdependent, the question of whether a tax base should be shared by several tiers or be assigned for the exclusive use of one tier cannot be looked at separately for each tax bases. We then provide a comparison of different fiscal architectures to determine how the nature and the degree of interdependence between the tax bases affect tax decisions and which fiscal architecture is welfare-enhancing in the presence of tax competition.

Most recommendations in terms of tax assignment are to assign mobile tax bases to the highest level of government (Musgrave, 1983; Bird, 2009; Martinez-Vazquez, 2008). However, with the increasing mobility of economic agents, the rules of the game have changed as tax competition has become a concern not only for sub-national levels of government (local or regional) but also for central governments. Thereby, decentralization within a country cannot be considered anymore in isolation from the rest of the world. In practice, we observe different fiscal architectures. Some countries allow different tiers to share the same tax bases, while other countries have an exclusionary approach by assigning different tax bases to the different tiers. In the context of mobile tax bases, the main criticism of the former option is that it gives rise to vertical tax competition. However, with interdependent mobility of the tax bases, vertical tax competition cannot be completely avoided by the second option. Moreover, although it improves the legibility in terms of governments responsibility, the use of exclusive tax bases, by definition, restricts the number and size of tax bases available to each tier, thereby making governments more vulnerable to the instability of the tax bases. Choosing between different modes of decentralization is thus not an easy task and requires to weigh advantages and disadvantages of the different options.

In this paper, we consider a world economy consisting of $n \geq 1$ countries, each consisting of $m > 1$ regions. This two-tier setting provides a good framework to analyze different of fiscal architectures that are defined by the share of public goods provided by each tier and the tax bases available at each tier. We then distinguish four fiscal architectures: i) *full decentralization*, where regions are given full control over the provision of public goods and the taxation of all tax bases, ii) *full centralization*, where the provision of public goods is achieved by the national governments and only financed by national taxes, iii) *partial decentralization with shared tax bases*, where regional and central authorities share the provision of public goods and co-occupy all the tax bases, iv) *partial decentralization with exclusive tax bases,* where regional and central authorities share the provision of public goods and tax bases can only be assigned to either tier, no co-occupancy of tax bases is allowed. Since our analysis focuses on the tax competition aspect of the problem, we exclude the possibility of economies of scale. A simple way to model the interdependent mobility of tax bases is then to consider two mobile production factors which can be either gross substitute or gross complement. In order to make the results more tractable, we assume households to be immobile and in this world economy, the two mobile tax bases can then be thought of as different types of capital. Let us note that our framework could also be transposed onto a sub-national level. The different fiscal architectures would then characterize the type of fiscal arrangements between regional and local jurisdictions and the two production factors could then be labor and capital. Cross-border commuting of labor is a phenomenon observed in many European countries like Belgium or Germany while it is reduced to metropolitan area in the US or Canada due to a lower density of population (Kächelein, 2003).

With no interdependence between the tax bases, a change in a tax rate affects the allocation of the tax base on which the tax is levied: this induces direct tax competition. When the tax bases are interdependent, the allocation of the other tax base is also modified: indirect tax competition arises. When the tax bases are gross complement, the indirect tax competition effect reinforces the direct effect but point in opposite direction in case of gross substitutability.

We then find that the interdependence between the two mobile tax bases not only generates indirect tax externalities but also increases direct tax externalities, whatever the nature and the degree of this interdependence. In case of partial decentralization with exclusive tax bases, the existence of indirect tax competition with a high degree of substitutability between the two tax bases can lead to an inefficiently high tax rate at either tier. In this case the higher the degree of substitutability, the stronger the distortion at both tiers. On the opposite, a race to the bottom is always observed at both tiers in the three other fiscal architectures but the higher degree of substitutability, the lower the downward distortion: governments should then favor taxation on tax bases which are gross substitute.

While full centralization always dominates the other fiscal architectures when considering only one country, this is not necessarily the case in a world economy where several countries engage in tax competition. Depending on the degree of interdependence and the combination between the level of expenditure decentralization and the tax assignment, we show that partial decentralization can induce a higher welfare than full centralization. This result then goes against most recommendations of assigning mobile tax bases to the highest tier. However, while there is always a level of expenditure decentralization such that the use of shared tax base dominates full centralization, this is no longer the case with exclusive tax bases for a sufficiently high degree of substitutability between the tax bases.

With the interdependence between the tax bases, partial decentralization with exclusive tax bases does not prevent vertical tax competition and depending on the level of decentralization and the interdependence between the tax bases, the intensity of tax competition can even be stronger than in other fiscal architectures. However, even when tax competition is weaker, the exclusive use of tax bases reducing the tax bases available for taxation at each tier, the level of public good provision can be smaller leading to a lower level of welfare than in another fiscal architecture.

Tax competition, whether horizontal or vertical, has been extensively described in the literature. If horizontal tax competition has already been analyzed in the presence of two mobile tax bases (Burbidge and Myers, 1994; Braid, 2000; Duran-Vigneron, 2012), no particular attention has been given to the interdependence between the tax bases and how the nature and degree of this interdependence may affect tax competition. In a two-tier setting where both horizontal and vertical tax competition can occur, only the case of one mobile tax base has been considered. Keen and Kotsogiannis (2002) showed that in a framework with only one top-tier jurisdiction, vertical and horizontal tax externalities point in opposite direction and whether horizontal tax competition dominates vertical tax competition depends on the sensitivity of savings to interest rate, capital demand elasticity and the ability of governments to tax immobile factors. Breuillé and Zanaj (2013) extended the previous model by assuming more than one top-tier jurisdiction. As aforementioned, this setting appears to be more suitable in the context of increasing mobility of economic agents. In their model, the two tiers engage in horizontal and vertical tax competition over a shared tax base. To the best of our knowledge, only Wilson and Janeba (2005) formally analyzed different fiscal architectures in a tax competition perspective. In their model, they consider a two-stage game where: 1) central governments first choose their level of decentralization and 2) central and regional authorities decide on their tax rates. In this paper, we are only interested in the second stage of the game, the level of decentralization is taken as given by the authorities and is the same in every countries as we look at the symmetric equilibrium where countries are assumed perfectly identical. First, our model combines the different frameworks found in the literature on tax competition by assuming more than one top-tier jurisdictions and two tax bases, the mobility of which being potentially interdependent. Second, the existence of more than one mobile tax base allows us to study several fiscal architectures, and in particular, the case where two tiers cannot co-occupy the same tax bases. This form of decentralization is absent from the work of Wilson and Janeba (2005) although the issue of assignment of tax bases is a crucial question regularly in the debates on fiscal decentralization. With this framework, we can then challenge the main result of Wilson and Janeba (2005): decentralizing the provision of public goods always reduces the welfare for some degree of interdependence between the tax bases and the use of exclusive tax bases.

This paper is organized as follows. In Section 2, we present the model and the different fiscal architectures that are considered for the analysis. Section 3 provides a discussion about tax externalities at stake in the model and derives equilibrium tax rates. Section 4 analyses the effect of interdependence between the tax bases on the outcome of tax competition and section 5 compares tax rates, public goods provision and welfare derived from four different fiscal architectures. Section 6 concludes.

2 The model

Our world economy comprises $n \geq 1$ identical top-tier jurisdictions, e.g. countries, indexed by $i = 1, ..., n$, and each country consists of $m > 1$ identical bottom-tier jurisdictions, e.g. regions, indexed by $j = 1, ..., m$. Each region has a representative citizen who benefits from the provision of a continuum of public goods, that is financed by taxes on two mobile tax bases.

2.1 The representative citizen

The representative citizen of each region is endowed with two production factors x and y, respectively in quantity $\overline{x}$ and $\overline{y}$. As it is usually considered in the capital tax competition literature, the representative citizen owns the unique firm located in her region of residence but can supply the two factors to firms in any region. The firm is immobile and produces a composite good that can be used for private consumption c_{ij} by the citizen or be purchased by the public sector to be transformed into public goods. The representative citizen in ij thus receives the profit $\Pi_{ij}(x_{ij}, y_{ij})$ of her firm and the net returns, ρ_{ij}^x and ρ_{ij}^y, of her endowments in factors $\overline{x}$ and $\overline{y}$. The citizen's budget constraint is then given by:

$$c_{ij} = \Pi_{ij}(x_{ij}, y_{ij}) + \rho_{ij}^x \overline{x} + \rho_{ij}^y \overline{y}$$

The representative citizen derives utility from the consumption of the private good c_{ij} and from the consumption $g_{ij}(\delta)$ of the public goods δ with $\delta \in [0, 1]$. The public goods are publicly provided private goods[1] and the marginal rate of transformation between these goods and the private good is unity. As in Wilson and Janeba (2005), we assume the preferences of the representative citizen to be given by the following additively separable log-linear utility function:

$$u_{ij} = c_{ij} + \int_0^1 (\ln g_{ij}(\delta))\, d\delta$$

Let us note that all public goods δ enter the utility function in a symmetric way but are imperfect substitutes.

2.2 Fiscal architecture

We distinguish four fiscal architectures among our two tiers of jurisdictions, which differ according to both i) the tax assignment (which tier taxes which factor(s)) and ii) the

[1] There is no scale-economy arguments in favour of centralization, so that we can exclusively focus on the issue of fiscal architecture from a tax competition.

share of public goods provision between regions and countries.

Tax revenue can be raised through taxes on the two mobile production factors x and y. Let t_{ij}^{k} be the proportional tax rate levied by the regional authority ij on the production factor k_{ij} invested in the region and T_{i}^{k} be the proportional tax rate chosen by the central authority i on the production factor k_i invested in the country, with $k = x, y$. By construction, the central tax base k_i is the sum of the regional tax bases located in its territory, i.e. $k_i = \sum_{j=1}^{m} k_{ij}$, with $k = x, y$.

Tax revenue is the only source of financing of public goods provision; no deficit is allowed.[2] The cut-off between public goods provided by regions and those provided by countries is denoted by D with $D \in [0, 1]$. Therefore D captures the level of decentralization in terms of expenditures. A public good δ is provided by regions if $\delta < D$, while it is provided by countries if $\delta > D$. Due to the symmetry of the utility function with respect to the public goods and its concavity in $g_{ij}(\delta)$, each jurisdiction splits equally its tax revenues between all public goods provided. Let $DG_{ij}^{r} = \int_0^D g_{ij}(\delta)\, d\delta$ denote the aggregate consumption of public goods provided by region ij and $(1 - D)\, G_i^c = \int_D^1 g_{ij}(\delta)\, d\delta$ denote the aggregate consumption of public goods $g_{ij}(\delta)$ provided by country i.

The four fiscal architectures considered in this paper are:

i) *Full decentralization* (hereafter R), where regions provide all public goods δ over the interval [0,1] and finance them through the taxation of both production factors. This fiscal architectures corresponds to the case where $D = 1$. The regional budget constraint is given by:

$$G_{ij}^{r} = t_{ij}^{x} x_{ij} + t_{ij}^{y} y_{ij}$$

Countries play no role in R: they neither provide public goods nor raise tax revenue, i.e. $G_i^c = T_i^x = T_i^y = 0$.

ii) *Full centralization* (hereafter C), where countries provide all public goods δ over the interval [0,1] and finance them through the taxation of both production factors. This fiscal architectures corresponds to the case where $D = 0$. The central budget constraint is given by:

$$mG_i^c = T_i^x \sum_{j=1}^{m} x_{ij} + T_i^y \sum_{j=1}^{m} y_{ij}$$

Regions play no role in C: they neither provide public goods nor raise tax revenue, i.e. $G_{ij}^{r} = t_{ij}^{x} = t_{ij}^{y} = 0$.

iii) *Partial decentralization with shared tax bases* (hereafter PS), where both regional and central authorities provide public goods, i.e. $D \in]0, 1[$, and levy taxes on the same two tax bases. Each one of the two tax bases x and y is thus co-occupied by both tiers.

[2] We rule out vertical transfers between the two tiers of government and horizontal transfers between jurisdictions of the same tier.

The budget constraints are given by:

$$
\begin{aligned}
DG_{ij}^{r} &= t_{ij}^{x}x_{ij} + t_{ij}^{y}y_{ij} \\
m\,(1-D)\,G_{i}^{c} &= T_{i}^{x}\sum_{j=1}^{m}x_{ij} + T_{i}^{y}\sum_{j=1}^{m}y_{ij}
\end{aligned}
$$

Let us note that C and R are two polar cases of PS: PS amounts to C when no decentralization, i.e. $D = 0$, and to R when full decentralization, i.e. $D = 1$.

iv) *Partial decentralization with exclusive tax bases* (hereafter PE), where both regional and central authorities provide public goods, i.e. $D \in \,]0,1[$, and levy taxes on a separate tax base (no co-occupancy). The tax base x is entirely used to finance the regional public goods G_{ij}^{r} and the tax base y is entirely used to finance the central public goods G_{ij}^{c}. The budget constraints are given by:

$$
\begin{aligned}
DG_{ij}^{r} &= t_{ij}^{x}x_{ij} \\
m\,(1-D)\,G_{i}^{c} &= T_{i}^{y}\sum_{j=1}^{m}y_{ij}
\end{aligned}
$$

The four fiscal architectures can be summarized by the following table:

	R			**C**		
	Taxation		Expenditures	Taxation		Expenditures
Regional tier	t^{x}	t^{y}	G^{r}			
Central tier				T^{x}	T^{y}	G^{c}
	PE			**PS**		
	Taxation		Expenditures	Taxation		Expenditures
Regional tier	t^{x}		DG^{r}	t^{x}	t^{y}	DG^{r}
Central tier		T^{y}	$(1-D)G^{c}$	T^{x}	T^{y}	$(1-D)G^{c}$

Let us note that for R, C and PS, there is a symmetry between the two tax bases in terms of tax assignment, i.e. if, at a given tier, a tax is levied on factor x, a tax is also levied on factor y. In contrast, in PE, no tier raises simultaneously tax revenue from taxation on both x and y.

2.3 The factor markets

The market for each factor $k = x, y$ is modeled as in the literature on capital tax competition in a two-tier setting (Wrede, 1997; Breuillé and Zanaj, 2013). However, we depart from the previous papers by considering two factor markets rather than one. x_{ij} and y_{ij} are the quantities of the two factors located in region ij and they are jointly used by the firm located in region ij. All firms across the world use the same technology of production that is described by the function $F(x_{ij}, y_{ij})$. $F(.,.)$ is twice-differentiable and concave. We thus have $F_{xx}^{ij} < 0$, $F_{yy}^{ij} < 0$ and $F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij} > 0$.[3] The profit of the firm located in the region ij amounts to $\Pi_{ij} = F(x_{ij}, y_{ij}) - r_{ij}^x x_{ij} - r_{ij}^y y_{ij}$, where r_{ij}^x is the gross return for factor x_{ij} and r_{ij}^y is the gross return for factor y_{ij}. Firm profit maximizing behavior implies that both factors are remunerated at their marginal productivity, that is $F_x^{ij} = r_{ij}^x$ and $F_y^{ij} = r_{ij}^y$ for all i,j. The implicit demand functions are thus $x_{ij}(r_{ij}^x, r_{ij}^y)$ and $y_{ij}(r_{ij}^x, r_{ij}^y)$ with

$$\frac{\partial x_{ij}}{\partial r_{ij}^x} = \frac{F_{yy}^{ij}}{F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij}} < 0 \text{ and } \frac{\partial x_{ij}}{\partial r_{ij}^y} = \frac{-F_{xy}^{ij}}{F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij}}$$

$$\frac{\partial y_{ij}}{\partial r_{ij}^y} = \frac{F_{xx}^{ij}}{F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij}} < 0 \text{ and } \frac{\partial y_{ij}}{\partial r_{ij}^x} = \frac{-F_{yx}^{ij}}{F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij}}$$

When $F_{k,-k}^{ij} \neq 0$, the mobility of the two factors is interdependent. In the following, we say that the factors are gross complements when $F_{k,-k}^{ij} > 0$ and that the factors are gross substitutes when $F_{k,-k}^{ij} < 0$ for $k = x, y$. The gross complementarity between factors implies that a higher cost of factor $-k$ in the jurisdiction ij reduces both the demand for factor $-k$ and the demand for factor k, i.e. $\frac{\partial k_{ij}}{\partial r_{ij}^{-k}} < 0$. In contrast, the gross substitutability between factors implies that a higher cost of factor $-k$ in the jurisdiction ij reduces the demand for factor $-k$ while it increases the demand for factor k, i.e. $\frac{\partial k_{ij}}{\partial r_{ij}^{-k}} > 0$.

For $F_{xy}^{ij} = F_{yx}^{ij} = 0$, the two factor markets work independently:

$$\frac{\partial x_{ij}}{\partial r_{ij}^x} = \frac{1}{F_{xx}^{ij}} < 0 \text{ and } \frac{\partial x_{ij}}{\partial r_{ij}^y} = 0$$

$$\frac{\partial y_{ij}}{\partial r_{ij}^y} = \frac{1}{F_{yy}^{ij}} < 0 \text{ and } \frac{\partial y_{ij}}{\partial r_{ij}^x} = 0$$

The implicit demand functions are then $x_{ij}(r_{ij}^x)$ and $y_{ij}(r_{ij}^y)$.

The profit $\Pi^{ij}(x_{ij}, y_{ij})$ is a decreasing function of both r_{ij}^x and r_{ij}^y, i.e. $\Pi_{r^x}^{ij} = -x_{ij}$ and $\Pi_{r^y}^{ij} = -y_{ij}$.

The supply of each factor in the world is exogenous. The aggregate supply thus amounts to $nm\overline{x}$ for the factor x and $nm\overline{y}$ for the factor y. The net return of these

[3] Let F_k^{ij} and F_{kk}^{ij} be respectively the first and second derivatives of the production function in region ij w.r.t. input k_{ij}, with $k_{ij} = x_{ij}, y_{ij}$. Let F_{xy}^{ij} and F_{yx}^{ij} be the cross derivatives of the production function w.r.t. the two inputs.

factors is then $\rho^k_{ij} = r^k_{ij} - t^k_{ij} - T^k_i$ for $k = x, y$, which is the return after regional and central taxes.

The two factors are perfectly mobile in the world. They both move across all regions, and thus across countries, to locate in the region where the net return is the highest. Perfect mobility implies that at the equilibrium, the net return for each factor is the same across the world, i.e.,

$$\begin{array}{ll} \rho^x = \rho^x_{ij} & \forall i,j \\ \rho^y = \rho^y_{ij} & \forall i,j \end{array} \iff \begin{array}{ll} r^x_{ij} = \rho^x + t^x_{ij} + T^x_i & \forall i,j \\ r^y_{ij} = \rho^y + t^y_{ij} + T^y_i & \forall i,j \end{array}$$

We differentiate the system of market-clearing conditions:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}(r^x_{ij}, r^y_{ij}) = nm\overline{x} \\ \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}(r^x_{ij}, r^y_{ij}) = nm\overline{y} \end{cases}$$

We then solve this system to obtain the response of ρ^k, and thus also the response of r^k_{ij} to regional and central taxation (see Appendix A.1). At the symmetric equilibrium, we obtain:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho^k}{\partial t^k_{ij}} = \frac{\partial r^k_{ij}}{\partial t^k_{ij}} - 1 = \frac{\partial r^k_{ij}}{\partial t^k_{-(ij)}} = -\frac{1}{nm} < 0$$
$$\frac{\partial \rho}{\partial T} = \frac{\partial \rho^k}{\partial T^k_i} = \frac{\partial r^k_{ij}}{\partial T^k_i} - 1 = \frac{\partial r^k_{ij}}{\partial T^k_{-i}} = -\frac{1}{n} < 0$$
$$\frac{\partial \rho^k}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{-(ij)}} = \frac{\partial \rho^k}{\partial T^{-k}_i} = \frac{\partial r^k_{ij}}{\partial T^{-k}_i} = \frac{\partial r^k_{ij}}{\partial T^{-k}_{-i}} = 0$$

For $n = 1$, $\frac{\partial \rho^k}{\partial T^k_i} = -1$ and thus $\frac{\partial r^k_{ij}}{\partial T^k_i} = 0$. This is due to the fact that the supply of factor is inelastic and thus, with only one country, a change of central tax rate does not affect the allocation of factors.

Note that a tax rate levied on factor x (resp. y) has no impact on the net return of factor y (resp. x) at the symmetric equilibrium.[4] The equilibrium values of the net

[4] When $F^{ij}_{k,-k} = 0$, i.e. the mobility of the two factors is not interdependent, the outflow of one factor from a jurisdiction does not affect the allocation of the other factor, i.e., $\frac{\partial \rho^k}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{-(ij)}} = 0$. When $F^{ij}_{k,-k} \neq 0$, the outflow of factor x (resp. y) from a jurisdiction affects the gross return of factor y (resp. x) in the jurisdiction and in the other jurisdictions of the country in opposite direction. Everything else being equal, at the symmetric equilibrium, the allocation of factor y (resp. x) must then be such that it fully compensates the effect of mobility of factor x (resp. y) on the gross returns, i.e. such that $\frac{\partial \rho^k}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{ij}} = \frac{\partial r^k_{ij}}{\partial t^{-k}_{-(ij)}} = 0$.

returns are:

$$\rho^x\left(\mathbf{T^x}, \mathbf{t_1^x}, ..., \mathbf{t_i^x}, ..., \mathbf{t_n^x}\right) \quad \text{and} \quad \rho^y\left(\mathbf{T^y}, \mathbf{t_1^y}, ..., \mathbf{t_i^y}, ..., \mathbf{t_n^y}\right)$$

with $\mathbf{T^k} = (T_1^k, ...T_n^k)$ and $\mathbf{t_i^k} = (t_{i1}^k, ..., t_{ij}^k, ..., t_{im}^k)$ $\forall i$ and for $k = x, y$.

Let us note that, since horizontal tax competition involves less players at the central tier (n) than at the regional tier (nm), regional taxation is more distortive than central taxation, i.e. $\frac{\partial \rho}{\partial t} > \frac{\partial \rho}{\partial T} = m\frac{\partial \rho}{\partial t}$.

Before deriving equilibrium tax rates chosen by regional and central authorities for each fiscal architecture successively, we introduce two additional assumptions. First we assume the same supply in both factors, i.e. $\overline{x} = \overline{y} = \overline{e}$. Second, we assume that the production function is perfectly "symmetric" regarding the two factors, such that $F_{xx}^{ij} = F_{yy}^{ij}$ and $F_{xy}^{ij} = F_{yx}^{ij}$ when $x_{ij} = y_{ij}$. Let us then use the following notations at the symmetric equilibrium: $F_{xx} = F_{yy} = -b\left(\overline{e}\right) \equiv -b < 0$ and $F_{yx} = F_{xy} = p\left(\overline{e}\right) \equiv p$. These assumptions allow us to exclusively focus on the role of tax competition in the tax decisions in the context of two mobile tax bases.

3 The equilibrium

3.1 Description of the game

Regional and central governments are both benevolent. They play together a Nash game. Regional authorities simultaneously select their tax policy to maximize the welfare of their representative citizen, taking as given tax policies chosen by the other regions and the countries. Central authorities simultaneously select their tax policy to maximize the sum of the welfare of the representative citizens from the regions belonging to their territory, taking as given tax policies chosen by the other countries and the regions. When they choose their tax strategy, regional and central authorities take into account the mobility of both factors x and y. Public goods are determined as residuals after taxes are collected. Given these tax policies, migration of factors and then the production take place. Finally, profits are distributed, and citizens enjoy the consumption of both private and public goods. These two last stages are implicitly introduced in our analysis.

3.2 The optimization problem

3.2.1 Full decentralization

We first analyze the case where the only tier is the regional one. Regions provide all the public goods δ and can tax both factors x and y. The optimization problem of the regional government ij is:

$$\max_{t_{ij}^x, t_{ij}^y} \ \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + \ln\left(t_{ij}^x x_{ij} + t_{ij}^y y_{ij}\right)$$

We obtain the following first-order conditions:

FOC $/t_{ij}^x$:

$$-\left(\frac{\partial \rho^x}{\partial t_{ij}^x} + 1\right) x_{ij} + \frac{\partial \rho^x}{\partial t_{ij}^x}\overline{x} + \frac{1}{G_{ij}^r}\left(x_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial t_{ij}^x} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial t_{ij}^x}\right) = 0 \qquad (1)$$

FOC $/t_{ij}^y$:

$$-\left(\frac{\partial \rho^y}{\partial t_{ij}^y} + 1\right) y_{ij} + \frac{\partial \rho^y}{\partial t_{ij}^y}\overline{y} + \frac{1}{G_{ij}^r}\left(y_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial t_{ij}^y} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial t_{ij}^y}\right) = 0 \qquad (2)$$

3.2.2 Full centralization

We now turn to the case where the only tier is the central one. Countries provide all the public goods δ and can tax both factors x and y. The optimization problem of the central government i is:

$$\max_{T_i^x, T_i^y} \ \sum_{j=1}^{m}\left(\Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + \ln\left(\frac{T_i^x}{m}\sum_{j=1}^{m} x_{ij} + \frac{T_i^y}{m}\sum_{j=1}^{m} y_{ij}\right)\right)$$

We obtain the following first-order conditions:

FOC $/T_i^x$:

$$\sum_{j=1}^{m}\left[\begin{array}{c} -\left(\frac{\partial \rho^x}{\partial T_i^x} + 1\right) x_{ij} + \frac{\partial \rho^x}{\partial T_i^x}\overline{x} \\ +\frac{1}{mG_i^c}\left(\sum_{j=1}^{m} x_{ij} + T_i^x \sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x} + T_i^y \sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x}\right)\end{array}\right] = 0 \qquad (3)$$

FOC $/T_i^y$:

$$\sum_{j=1}^{m}\left[\begin{array}{c} -\left(\frac{\partial \rho^y}{\partial T_i^y} + 1\right) y_{ij} + \frac{\partial \rho^y}{\partial T_i^y}\overline{y} \\ +\frac{1}{mG_i^c}\left(\sum_{j=1}^{m} y_{ij} + T_i^x \sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y} + T_i^y \sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}\right)\end{array}\right] = 0 \qquad (4)$$

3.2.3 Partial decentralization with shared tax bases

Regional and central tiers coexist and both tiers share the two tax bases x and y. Regions can levy tax rates t^x and t^y on each tax base, in order to finance the provision of public goods $\delta < D$. At the same time, countries select their additional tax rates T^x and T^y on each tax base, in order to finance the provision of public goods $\delta > D$.

Program of region ij

$$\max_{t_{ij}^x, t_{ij}^y} \ \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + D \ln \left(\frac{t_{ij}^x x_{ij} + t_{ij}^y y_{ij}}{D} \right) + (1-D) \ln \left(\frac{T_i^x}{m(1-D)} \sum_{s=1}^{m} x_{is} + \frac{T_i^y}{m(1-D)} \sum_{s=1}^{m} y_{is} \right)$$

We obtain the following first-order conditions:

FOC $/t_{ij}^x$:

$$-\left(\frac{\partial \rho^x}{\partial t_{ij}^x} + 1 \right) x_{ij} + \frac{\partial \rho^x}{\partial t_{ij}^x} \overline{x} + \frac{1}{G_{ij}^r} \left(x_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial t_{ij}^x} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial t_{ij}^x} \right) + \frac{1}{m G_i^c} \left(T_i^x \sum_{s=1}^{m} \frac{\partial x_{is}}{\partial r_{is}^x} \frac{\partial r_{is}^x}{\partial t_{ij}^x} + T_i^y \sum_{s=1}^{m} \frac{\partial y_{is}}{\partial r_{is}^x} \frac{\partial r_{is}^x}{\partial t_{ij}^x} \right) = 0 \tag{5}$$

FOC $/t_{ij}^y$:

$$-\left(\frac{\partial \rho^y}{\partial t_{ij}^y} + 1 \right) y_{ij} + \frac{\partial \rho^y}{\partial t_{ij}^y} \overline{y} + \frac{1}{G_{ij}^r} \left(y_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial t_{ij}^y} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial t_{ij}^y} \right) + \frac{1}{m G_i^c} \left(T_i^x \sum_{s=1}^{m} \frac{\partial x_{is}}{\partial r_{is}^y} \frac{\partial r_{is}^y}{\partial t_{ij}^y} + T_i^y \sum_{s=1}^{m} \frac{\partial y_{is}}{\partial r_{is}^y} \frac{\partial r_{is}^y}{\partial t_{ij}^y} \right) = 0 \tag{6}$$

Program of country i

$$\max_{T_i^x, T_i^y} \sum_{j=1}^{m} \left(\begin{array}{c} \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + D \ln \left(\frac{t_{ij}^x x_{ij} + t_{ij}^y y_{ij}}{D} \right) \\ + (1-D) \ln \left(\frac{T_i^x}{m(1-D)} \sum_{j=1}^{m} x_{ij} + \frac{T_i^y}{m(1-D)} \sum_{j=1}^{m} y_{ij} \right) \end{array} \right)$$

We obtain the following first-order conditions:

FOC $/T_i^x$:

$$\sum_{j=1}^{m} \left[\begin{array}{c} -\left(\frac{\partial \rho^x}{\partial T_i^x} + 1 \right) x_{ij} + \frac{\partial \rho^x}{\partial T_i^x} \overline{x} + \frac{1}{G_{ij}^r} \left(t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} \right) \\ + \frac{1}{m G_i^c} \left(\sum_{j=1}^{m} x_{ij} + T_i^x \sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} + T_i^y \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} \right) \end{array} \right] = 0 \tag{7}$$

FOC $/T_i^y$:

$$\sum_{j=1}^{m} \left[\begin{array}{c} -\left(\frac{\partial \rho^y}{\partial T_i^y} + 1 \right) y_{ij} + \frac{\partial \rho^y}{\partial T_i^y} \overline{y} + \frac{1}{G_{ij}^r} \left(t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} \right) \\ + \frac{1}{m G_i^c} \left(\sum_{j=1}^{m} y_{ij} + T_i^x \sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} + T_i^y \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} \right) \end{array} \right] = 0 \tag{8}$$

3.2.4 Partial decentralization with exclusive tax bases

Two tiers now coexist and each tier of jurisdiction taxes a different tax base. Regions levy a tax rate t^x on tax base x to finance the provision of public goods $\delta < D$, while countries levy a tax rate T^y on the tax base y to finance the provision of public goods $\delta > D$.

Program of region ij

$$\max_{t_{ij}^x} \ \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + D \ln \left(\frac{t_{ij}^x x_{ij}}{D} \right) + (1 - D) \ln \left(\frac{T_i^y}{m(1 - D)} \sum_{s=1}^{m} y_{is} \right)$$

We obtain the following first-order condition:

FOC $/t_{ij}^x$:

$$\begin{aligned} & -\left(\frac{\partial \rho^x}{\partial t_{ij}^x} + 1 \right) x_{ij} + \frac{\partial \rho^x}{\partial t_{ij}^x} \overline{x} + \frac{1}{G_{ij}^r} \left(x_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial t_{ij}^x} \right) \\ & + \frac{1}{m G_i^c} T_i^y \sum_{s=1}^{m} \frac{\partial y_{is}}{\partial r_{is}^x} \frac{\partial r_{is}^x}{\partial t_{ij}^x} = 0 \end{aligned} \tag{9}$$

Program of country i

$$\max_{T_i^y} \ \sum_{j=1}^{m} \left(\Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + D \ln \left(\frac{t_{ij}^x x_{ij}}{D} \right) + (1 - D) \ln \left(\frac{T_i^y}{m(1 - D)} \sum_{j=1}^{m} y_{ij} \right) \right)$$

We obtain the following first-order condition:

FOC $/T_i^y$:

$$\sum_{j=1}^{m} \left[\begin{array}{c} -\left(\frac{\partial \rho^y}{\partial T_i^y} + 1 \right) y_{ij} + \frac{\partial \rho^y}{\partial T_i^y} \overline{y} + \frac{1}{G_{ij}^r} \left(t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} \right) \\ + \frac{1}{m G_i^c} \left(\sum_{j=1}^{m} y_{ij} + T_i^y \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} \right) \end{array} \right] = 0 \tag{10}$$

3.3 Direct and indirect externalities

In our model, tax externalities can be defined along two dimensions: i) horizontal *versus* vertical externalities, i.e. externalities among authorities at the same tier *versus* externalities among authorities at two different tiers, ii) "direct" *versus* "indirect" externalities, i.e. externalities due to the migration of a tax base k arising from a modification of a tax rate on this base (t^k or T^k), *versus* externalities due to the migration of a tax base k arising from a modification of the tax rate on the other tax base (t^{-k} or T^{-k}). Let us note that indirect externalities only occur when $F_{xy}^{ij} = F_{yx}^{ij} \neq 0$, i.e. when the demand for a factor is affected by the taxation of the other factor.

To sum up, the nature of externalities depends on the interdependence between the tax bases and on which tier (regional/central) manipulates the tax rate. Four different types of externalities are thus at work in our model:

Direct horizontal tax externalities. An increase in the tax rate raised by a jurisdiction on a factor induces an outflow of this factor from the jurisdiction, which thus results in an inflow to all other jurisdictions at the same tier. We see from the FOCs that jurisdictions only take into account the externality of horizontal tax competition on their own tax base, i.e. respectively $\varepsilon_r^{k,DH}$ for regions and $\varepsilon_c^{k,DH}$ for countries:[5]

$$\varepsilon_r^{k,DH} = \frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial t_{ij}^k} < 0, \qquad \varepsilon_c^{k,DH} = \sum_{j=1}^{m}\frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} < 0 \quad \text{with } k = x, y$$

This type of externalities arises in every fiscal architecture.

Direct vertical tax externalities. These externalities arise when a tax base is shared by several tiers. Then an increase in the tax rate raised by a jurisdiction on a factor induces an outflow of this factor from the jurisdiction(s) sharing the same tax base at the other tier, which thus results in an inflow to all other jurisdictions of this other tier. Externalities induced by regional taxation are called *bottom-up* tax externalities and externalities induced by central taxation are called *top-down* tax externalities.

We see from the FOCs that within a country, regions internalize direct vertical bottom-up externalities imposed on the tax base of their country, denoted by $\varepsilon_r^{k,DV}$, and countries internalize direct vertical top-down externalities imposed on the tax base of their regions, denoted by $\varepsilon_c^{k,DV}$. Since regions only care about the welfare of their representative citizen, only a proportion $\frac{1}{m}$ of the vertical externalities is internalized within its country. On the opposite, countries care about the welfare of all the citizens of its regions and thus vertical externalities, are fully internalized within the country.[6]

$$\varepsilon_r^{k,DV} = \sum_{s=1}^{m}\frac{\partial k_{is}}{\partial r_{is}^k}\frac{\partial r_{is}^k}{\partial t_{ij}^k} < 0, \qquad \varepsilon_c^{k,DV} = \sum_{j=1}^{m}\frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} < 0 \quad \text{with } k = x, y$$

This type of externalities only arises in PS.

Indirect horizontal tax externalities. These externalities occur when, at a given tier, jurisdictions can levy taxes on two factors that are interdependently mobile, i.e. $F_{xy}^{ij} = F_{yx}^{ij} \neq 0$. In this case, an increase in the tax rate raised by a jurisdiction on a factor affects the amount of the other factor available to all other jurisdictions at the

[5] Let us note that due to the fixed supply of factors within the world, direct horizontal tax externalities on all the other regions (resp. countries) arising from a change of t_{ij}^k (resp. T_i^k) are positive, equal to $-\varepsilon_r^{k,DH}$ (resp. $-\varepsilon_c^{k,DH}$).

[6] The supply of factors being fixed within the world, the externalities imposed by a region on the other countries' tax revenue are of opposite sign, equal to $-\varepsilon_r^{k,DV}$. Similarly, the externalities imposed by a country on the other regions' tax revenue are of opposite sign, equal to $-\varepsilon_c^{k,DV}$.

same tier.

As for the direct horizontal externalities, we see from the FOCs that a jurisdiction only cares about the indirect externality on its tax base, respectively $\varepsilon_r^{k,IH}$ for regions and $\varepsilon_c^{k,IH}$ for countries, and neglects the externalities imposed on the tax base of the other jurisdictions at the same tier, respectively $-\varepsilon_r^{k,IH}$ and $-\varepsilon_c^{k,IH}$:

$$\varepsilon_r^{k,IH} = \frac{\partial \left(-k\right)_{ij}}{\partial r_{ij}^k} \frac{\partial r_{ij}^k}{\partial t_{ij}^k}, \qquad \varepsilon_c^{k,IH} = \sum_{j=1}^{m} \frac{\partial \left(-k\right)_{ij}}{\partial r_{ij}^k} \frac{\partial r_{ij}^k}{\partial T_i^k} \quad \text{with } k = x, y$$

The sign of the indirect horizontal tax externalities then depend on the nature of the interdependence between the two tax bases. The externality on the jurisdiction's own tax base is negative, pointing in the same direction as the direct horizontal one when factors are gross complement, i.e. $\varepsilon_r^{k,IH} < 0$ and $\varepsilon_c^{k,IH} < 0$, while it is positive, pointing in opposite direction to the direct horizontal one when factors are gross substitute, i.e. $\varepsilon_r^{k,IH} > 0$ and $\varepsilon_c^{k,IH} > 0$.

This type of externalities occurs in all fiscal architectures except PE.

Indirect vertical tax externalities. These externalities occur when, in a two-tier setting, each tier can levy taxes on two factors that are interdependently mobile, i.e. $F_{xy}^{ij} = F_{yx}^{ij} \neq 0$. In this case, an increase in the tax raised by a jurisdiction on a factor affects the amount of the other factor available to the jurisdictions at the other tier. When $F_{xy}^{ij} = F_{yx}^{ij} > 0$, the externalities correspond to an outflow of factor from the jurisdiction(s) sharing the same tax base to all other jurisdictions. When $F_{xy}^{ij} = F_{yx}^{ij} < 0$, the externalities correspond to an inflow of factor to the jurisdiction(s) sharing the same tax base. These externalities are called indirect vertical bottom-up externalities when arising from regional taxation and indirect vertical top-down externalities when arising from central taxation.

We see from the FOCs that, as for the direct vertical externalities, regions internalize a proportion $\frac{1}{m}$ of the indirect vertical bottom-up externalities imposed to their country, denoted by $\varepsilon_r^{k,IV}$, and countries internalize all indirect vertical top-down externalities, denoted by $\varepsilon_c^{k,IV}$, imposed on their regions.

$$\varepsilon_r^{k,IV} = \sum_{s=1}^{m} \frac{\partial \left(-k\right)_{is}}{\partial r_{is}^k} \frac{\partial r_{is}^k}{\partial t_{ij}^k}, \qquad \varepsilon_c^{k,IV} = \sum_{j=1}^{m} \frac{\partial \left(-k\right)_{ij}}{\partial r_{ij}^k} \frac{\partial r_{ij}^k}{\partial T_i^k} \quad \text{with } k = x, y$$

The sign of the indirect vertical tax externalities depends on the nature of the interdependence between the two tax bases. The indirect vertical tax externalities internalized by a jurisdiction are negative, pointing in the same direction as the direct vertical ones when factors are gross complement, i.e. $\varepsilon_r^{k,IV} < 0$ and $\varepsilon_c^{k,IV} < 0$, while they are positive, pointing in opposite direction to the direct vertical ones when factors are gross substitute, i.e. $\varepsilon_r^{k,IV} > 0$ and $\varepsilon_c^{k,IV} > 0$.

This type of externalities arises in PS and PE.

At the symmetric equilibrium, we get:

$$\begin{cases} \varepsilon_r^{k,DH} &= -\frac{\partial r_{ij}^k}{\partial t_{ij}^k}\frac{b}{(b^2-p^2)} = -\frac{nm-1}{nm}\frac{b}{(b^2-p^2)} \equiv \varepsilon_r^{DH} < 0 \\ \varepsilon_c^{k,DH} &= -\left(\sum_{j=1}^{m}\frac{\partial r_{ij}^k}{\partial T_i^k}\right)\frac{b}{(b^2-p^2)} = -\frac{m(n-1)}{n}\frac{b}{(b^2-p^2)} \equiv \varepsilon_c^{DH} < 0 \end{cases}$$

$$\begin{cases} \varepsilon_r^{k,IH} &= -\frac{\partial r_{ij}^k}{\partial t_{ij}^k}\frac{p}{(b^2-p^2)} = -\frac{nm-1}{nm}\frac{p}{(b^2-p^2)} \equiv \varepsilon_r^{IH} \\ \varepsilon_c^{k,IH} &= -\left(\sum_{j=1}^{m}\frac{\partial r_{ij}^k}{\partial T_i^k}\right)\frac{p}{(b^2-p^2)} = -\frac{m(n-1)}{n}\frac{p}{(b^2-p^2)} \equiv \varepsilon_c^{IH} \end{cases}$$

$$\begin{cases} \varepsilon_r^{k,DV} &= -\left(\frac{\partial r_{ij}^k}{\partial t_{ij}^k} + (m-1)\frac{\partial \rho^k}{\partial t_{ij}^k}\right)\frac{b}{(b^2-p^2)} = -\frac{n-1}{n}\frac{b}{(b^2-p^2)} \equiv \varepsilon_r^{DV} > 0 \\ \varepsilon_c^{k,DV} &= -\left(\sum_{s=1}^{m}\frac{\partial r_{is}^k}{\partial T_i^k}\right)\frac{b}{(b^2-p^2)} = -\frac{m(n-1)}{n}\frac{b}{(b^2-p^2)} \equiv \varepsilon_c^{DV} > 0 \end{cases}$$

$$\begin{cases} \varepsilon_r^{k,IV} &= -\left(\frac{\partial r_{ij}^k}{\partial t_{ij}^k} + (m-1)\frac{\partial \rho^k}{\partial t_{ij}^k}\right)\frac{p}{(b^2-p^2)} = -\frac{n-1}{n}\frac{p}{(b^2-p^2)} \equiv \varepsilon_r^{IV} \\ \varepsilon_c^{k,IV} &= -\left(\sum_{s=1}^{m}\frac{\partial r_{is}^k}{\partial T_i^k}\right)\frac{p}{(b^2-p^2)} = -\frac{m(n-1)}{n}\frac{p}{(b^2-p^2)} \equiv \varepsilon_c^{IV} \end{cases}$$

With $k = x, y$.

We first note that in a two-tier setting with a unique top-tier jurisdiction, i.e. $n = 1$, horizontal externalities disappear at the central tier ($\varepsilon_r^{DV} = \varepsilon_r^{IV} = \varepsilon_c^{DV} = \varepsilon_c^{IV} = 0$) and due to the fixed supply of factors – which results in an inelastic tax base for the top-tier jurisdiction–, the vertical externalities, bottom-up and top-down, amount to zero ($\varepsilon_r^{DV} = \varepsilon_c^{DV} = 0$). This is similar to the mechanism described by Keen and Kotsogiannis (2002). With more than one top-tier jurisdiction, although the supply of factors is fixed for the world, it is flexible from the country's perspective and vertical tax competition as well as horizontal tax competition occur at each tier.

Second, we note that at the central tier, $\varepsilon_c^{DH} = \varepsilon_c^{DV}$ and $\varepsilon_c^{IH} = \varepsilon_c^{IV}$. As the central tax base is the sum of the tax bases of the m symmetric regions that belong to its territory, central taxation affects similarly each of those regions.

This is no longer the case for regional taxation, as we have $\varepsilon_r^{DV} = \left(\varepsilon_r^{DH} + \sum_{s\neq j}\frac{\partial k_{is}}{\partial r_{is}^k}\frac{\partial r_{is}^k}{\partial t_{ij}^k}\right)$ and $\varepsilon_r^{IV} = \left(\varepsilon_r^{IH} + \sum_{s\neq j}\frac{\partial (-k)_{is}}{\partial r_{is}^k}\frac{\partial r_{is}^k}{\partial t_{ij}^k}\right)$. A region ij not only internalizes the negative externality ε_r^{DH} on its own tax base but also the externalities on the tax base of each other region of the country $\sum_{s\neq j}\frac{\partial k_{is}}{\partial r_{is}^k}\frac{\partial r_{is}^k}{\partial t_{ij}^k}$

3.4 The equilibrium tax rates

At the symmetric equilibrium, we get the following tax rates[7]:

- for full decentralization (R):

$$t^R = t^{xR} = t^{yR} = \frac{1}{2\overline{e} - \frac{\varepsilon_r^{DH}+\varepsilon_r^{IH}}{\overline{e}}} = \frac{1}{2\overline{e} + \frac{nm-1}{nm(b-p)\overline{e}}} \tag{11}$$

- for full centralization (C):

$$T^C = T^{xC} = T^{yC} = \frac{1}{2\overline{e} - \frac{\varepsilon_c^{DH}+\varepsilon_c^{IH}}{m\overline{e}}} = \frac{1}{2\overline{e} + \frac{n-1}{n(b-p)\overline{e}}} \tag{12}$$

- for partial decentralization with shared tax bases (PS):

$$t^{PS} = t^{xPS} = t^{yPS} = \frac{D^{PS}}{2\overline{e} - \frac{D^{PS}\left(\varepsilon_r^{DH}+\varepsilon_r^{IH}\right)}{\overline{e}} - \frac{\left(1-D^{PS}\right)\left(\varepsilon_r^{DV}+\varepsilon_r^{IV}\right)}{m\overline{e}}} = \frac{D^{PS}}{2\overline{e} + \frac{D^{PS}(nm-1)}{nm(b-p)\overline{e}} + \frac{\left(1-D^{PS}\right)(n-1)}{nm(b-p)\overline{e}}} \tag{13}$$

$$T^{PS} = T^{xPS} = T^{yPS} = \frac{\left(1-D^{PS}\right)}{2\overline{e} - \frac{\left(1-D^{PS}\right)\left(\varepsilon_c^{DH}+\varepsilon_c^{IH}\right)}{m\overline{e}} - \frac{D^{PS}\left(\varepsilon_c^{DV}+\varepsilon_c^{IV}\right)}{m\overline{e}}} = \frac{\left(1-D^{PS}\right)}{2\overline{e} + \frac{(n-1)}{n(b-p)\overline{e}}} \tag{14}$$

- for partial decentralization with exclusive tax bases (PE):

$$t^{PE} = t^{xPE} = \frac{D^{PE}}{\overline{e} - \frac{D^{PE}\varepsilon_r^{DH}}{\overline{e}} - \frac{\left(1-D^{PE}\right)\varepsilon_r^{IV}}{m\overline{e}}} = \frac{D^{PE}}{\overline{e} + \frac{D^{PE}(mn-1)}{mn(b^2-p^2)\overline{e}}b + \frac{\left(1-D^{PE}\right)(n-1)}{mn(b^2-p^2)\overline{e}}p} \tag{15}$$

$$T^{PE} = T^{yPE} = \frac{\left(1-D^{PE}\right)}{\overline{e} - \frac{\left(1-D^{PE}\right)\varepsilon_c^{DH}}{m\overline{e}} - \frac{D^{PE}\varepsilon_c^{IV}}{m\overline{e}}} = \frac{\left(1-D^{PE}\right)}{\overline{e} + \frac{n-1}{n(b^2-p^2)\overline{e}}((1-D^{PE})b + D^{PE}p)} \tag{16}$$

The levels of equilibrium tax rates (11-16) are potentially influenced by:

- the share of public good provided by the tier where the tax is levied.
- the aggregate tax base per capita available at the tier where the tax is levied, i.e. $2\overline{e}$ in R, C and PS and $\overline{e}$ in PE.
- the weighted horizontal tax externalities internalized (e.g. $\frac{D^{PS}\left(\varepsilon_r^{DH}+\varepsilon_r^{IH}\right)}{\overline{e}}$ for t^{PS}). The wider the range of public goods provided by the tier, the stronger this effect, to which we refer hereafter as the horizontal tax competition effect.

[7] When there is a symmetry between the two tax bases in terms of tax assignment, the tax rates set on both tax bases by a given tier for a given fiscal architecture are identical, i.e. $t^{xR} = t^{yR} = t^R$, $t^{xPS} = t^{yPS} = t^{PS}$, $T^{xC} = T^{yC} = T^C$ and $T^{xPS} = T^{yPS} = T^{PS}$. This comes from the fact that the two factors enter the production function symmetrically and the endowment in factors is the same, i.e. $\overline{x} = \overline{y} = \overline{e}$.

- the weighted vertical tax externalities internalized (e.g. $\frac{(1-D^{PS})(\varepsilon_r^{DV}+\varepsilon_r^{IV})}{m\overline{e}}$ for t^{PS}). The smaller the range of public goods provided by the other tier, the smaller the effect, to which we refer hereafter as vertical tax competition effect.

With interdependent tax bases and the simultaneous taxation by a given tier of both tax bases, horizontal tax competition in R, C and PS and vertical tax competition in PS both arise from direct as well as indirect tax externalities, i.e. $\varepsilon_l^{DH}+\varepsilon_l^{IH}$ and $\varepsilon_l^{DV}+\varepsilon_l^{IV}$ with $l=r,c$. Under our assumption that $F_{xx}^{ij}F_{yy}^{ij}-F_{xy}^{ij}F_{yx}^{ij}=b^2-p^2>0$, direct tax externalities are always larger in absolute terms than their indirect counterparts, i.e. $\varepsilon_l^{DH}+\varepsilon_l^{IH}<0$, $\varepsilon_l^{DV}+\varepsilon_l^{IV}<0$ with $l=r,c$, which implies that all equilibrium tax rates in R, C and PS are positive.

In PE, horizontal tax competition only arises from direct externalities (due to the absence of simultaneous taxation by a given tier of both tax bases) and vertical tax competition only arises from indirect vertical externalities (due to the absence of tax base co-occupation). The positivity of the two tax rates t^{PE} and T^{PE} then depends on the relative magnitude of the weighted horizontal and vertical tax externalities and is only satisfied for values of the level of decentralization D^{PE} within the interval $[\frac{-\overline{e}+\frac{\varepsilon_r^{IV}}{r\overline{e}}}{-\left(\frac{\varepsilon_r^{DH}}{\overline{e}}-\frac{\varepsilon_r^{IV}}{m\overline{e}}\right)};\frac{\overline{e}-\frac{\varepsilon_c^{DH}}{m\overline{e}}}{-\left(\frac{\varepsilon_c^{DH}}{m\overline{e}}-\frac{\varepsilon_c^{IV}}{m\overline{e}}\right)}]$.[8] Hereafter, this condition is always assumed to be satisfied to ensure the existence of a Nash equilibrium.[9]

4 How does tax bases interdependence affect tax competition?

In the absence of direct and indirect externalities arising from the mobility of two interdependent tax bases, jurisdictions would choose the following optimal tax rates: $t^{*R}=T^{*C}=\frac{1}{2\overline{e}}$, $t^{*PS}=\frac{D^{PS}}{2\overline{e}}$, $T^{*PS}=\frac{(1-D^{PS})}{2\overline{e}}$, $t^{*PE}=\frac{D^{PE}}{\overline{e}}$ and $T^{*PE}=\frac{(1-D^{PE})}{\overline{e}}$.

With mobile but independent tax bases, direct horizontal and vertical[10] externalities both lead to a downward distortion of optimal tax rates. Since jurisdictions neglect the positive direct horizontal externalities on all the other same-tier jurisdictions arising from an increase in their tax rate, the standard outcome of the competitive horizontal game is a race to the bottom, with inefficiently low equilibrium tax rates (Wilson, 1986, Zodrow

[8] Equilibrium tax rates in PE are positive whatever the level of decentralization $D^{PE}\in\,]0,1[$ in three circumstances: i) no indirect vertical tax externalities occur, i.e. the tax bases are independent or there is only one top-tier jurisdiction ($n=1$), ii) indirect vertical tax externalities point in the same direction as direct horizontal tax externalities, i.e. the tax bases are gross complement, iii) indirect vertical tax externalities point in opposite direction to direct horizontal tax externalities but are sufficiently small, i.e. the degree of gross substitutability between the tax bases is sufficiently small.

[9] Given the form of the utility function of the representative citizen, the existence of a Nash equilibrium requires the provision of all local public goods to be positive.

[10] Vertical externalities only occur in a two-tier setting with shared tax bases.

and Mieszkowski, 1986). This effect is reinforced in PS as jurisdictions also neglect the positive vertical externalities outside the country.

The interdependence between the two mobile tax bases, i.e. for $F_{yx} = F_{xy} = p \neq 0$, affects the tax competition game in two ways. First, it generates indirect (horizontal and/or vertical) tax externalities. Second, it amplifies the direct (horizontal and/or vertical) tax externalities. Indeed, when a jurisdiction increases its tax rate on a factor k, the modification of the allocation of the other factor reinforces the outflow of factor k via its effect on marginal productivity, whatever the sign of p. The higher the degree of interdependence $|p|$ of the two factors (whether gross complement or gross substitute), the higher the magnitude of both direct and indirect externalities, i.e. $\frac{\partial \left|\varepsilon_l^{DH}\right|}{\partial |p|} > 0$, $\frac{\partial \left|\varepsilon_l^{DV}\right|}{\partial |p|} > 0$, $\frac{\partial \left|\varepsilon_l^{IH}\right|}{\partial |p|} > 0$ and $\frac{\partial \left|\varepsilon_l^{IV}\right|}{\partial |p|} > 0$ with $l = r, c$. Therefore the higher $|p|$, the fiercer the direct and indirect tax competition.

Lemma 1 *Interdependence between the two mobile tax bases not only generates indirect tax externalities but also increases direct tax externalities, whatever the nature and the degree of this interdependence.*

When factors are gross complement, direct and indirect externalities reinforce each other, thereby worsening the race to the bottom of tax rates. This downward distortion increases with the degree of complementarity, i.e. $\frac{\partial t}{\partial p} < 0$ and $\frac{\partial T}{\partial p} < 0$, in all fiscal architectures.

In contrast, when factors are gross substitute, direct and indirect externalities push tax rates in opposite direction. In R, C and PS, the direct tax competition effect always dominates the indirect tax competition effect leading to inefficiently low tax rates. Moreover, the higher the degree of substitutability, the less downward distorted the equilibrium tax rates, due to $\frac{\partial \left|\varepsilon_l^{DH}\right|}{\partial |p|} < \frac{\partial \left|\varepsilon_l^{IH}\right|}{\partial |p|}$ and $\frac{\partial \left|\varepsilon_l^{DV}\right|}{\partial |p|} < \frac{\partial \left|\varepsilon_l^{IV}\right|}{\partial |p|}$ with $l = r, c$.

In PE, whether the direct horizontal tax competition effect dominates the indirect vertical tax competition effect depends on the level of decentralization D^{PE} as well as on the degree of substitutability $|p|$. PE differs from other fiscal architectures in that it may lead to inefficiently high tax rate at either tier. For $D^{PE} < \frac{\frac{\varepsilon_r^{IV}}{m\bar{e}}}{-\left(\frac{\varepsilon_r^{DH}}{\bar{e}} - \frac{\varepsilon_r^{IV}}{m\bar{e}}\right)}$ (resp. $D^{PE} > \frac{-\frac{\varepsilon_c^{DH}}{m\bar{e}}}{-\left(\frac{\varepsilon_c^{DH}}{m\bar{e}} - \frac{\varepsilon_c^{IV}}{m\bar{e}}\right)}$), the regional (resp. central) tax rate is upward distorted and the central (resp. regional) tax rate is downward distorted. With the magnitude of the direct and indirect externalities increasing with $|p|$ and the tax competition effects arising from these externalities depending on the level of decentralization D^{PE}, the overall impact of the degree of substitutability $|p|$ on tax rates also depends on the combination of D^{PE} with $|p|$ as depicted in figures 1 and 2 for given values of parameters n, m, b and $\bar{e}$. Figure

1 corresponds to the case of $D^{PE} \in [\frac{n-1}{(nm-1)+(n-1)}, \frac{1}{2}]$, i.e. tax rates are always downward distorted ($t^{PE} < t^{*PE}$ and $T^{PE} < T^{*PE}$, $\forall p$), while figure 2 corresponds to the case of $D^{PE} > \frac{1}{2}$, i.e. an upward distortion occurs at the central tier for a sufficiently high degree of substitutability.[11]

The above results can be summarized in the following proposition:

Proposition 1

- *Case of complementarity ($F_{yx} = F_{xy} = p > 0$). In all fiscal architectures (R, C, PS and PE), tax competition leads to inefficiently low tax rates. The higher the degree of complementarity between the two tax bases, the worse the downward distortion of tax rates.*

- *Case of substitutability ($F_{yx} = F_{xy} = p < 0$). In R, C and PS, tax competition leads to inefficiently low tax rates. The higher the degree of substitutability between the two tax bases, the smaller the downward distortion of tax rates.*

  In PE, the existence of indirect tax competition leads to inefficiently high: i) central tax rate when $D^{PE} > \frac{-\frac{\varepsilon_c^{DH}}{m\bar{e}}}{-\left(\frac{\varepsilon_c^{DH}}{m\bar{e}}-\frac{\varepsilon_c^{IV}}{m\bar{e}}\right)}$, ii) regional tax rate when $D^{PE} < \frac{\frac{\varepsilon_r^{IV}}{m\bar{e}}}{-\left(\frac{\varepsilon_r^{DH}}{\bar{e}}-\frac{\varepsilon_r^{IV}}{m\bar{e}}\right)}$. However, this upward distortion in PE can never occur at both tiers simultaneously. For sufficiently low values of $|p|$, a higher degree of substitutability reduces the downward distortions of both tax rates. For sufficiently high values of $|p|$, a higher degree of substitutability reinforces the (downward and/or upward) distortions arising at both tiers.

Proof. *See Appendix A.2* ■

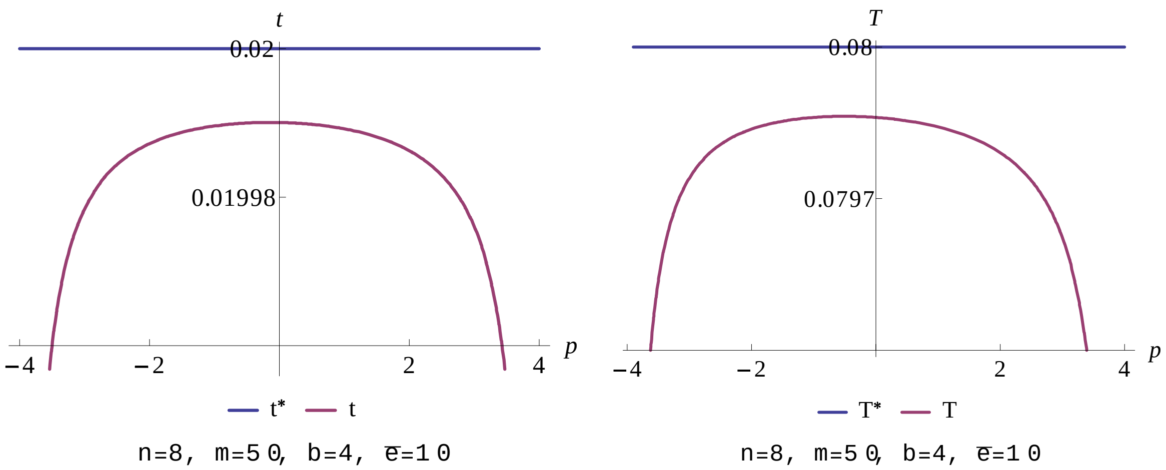

Figure 1: Tax rates in PE for $D = 0.2$

[11] When $D^{PE} > \frac{n-1}{(nm-1)+(n-1)}$, an upward distortion occurs at the regional tier for a sufficiently high degree of substitutability.

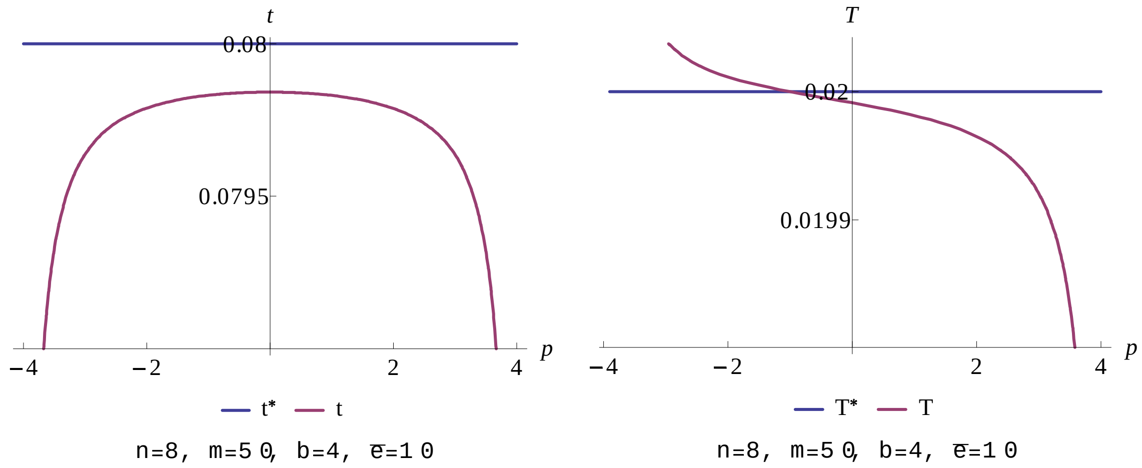

Figure 2: Tax rates in PE for $D = 0.8$

5 What is the best fiscal architecture?

In this section, we compare the four fiscal architectures regarding the tax rates selected by jurisdictions and the welfare derived by citizens at the symmetric equilibrium. Tables with all comparisons of tax rates are provided in Appendix A.3 and the levels of public good consumption in each fiscal architecture are given in Appendix A.4. The welfare of the representative citizen is given by the level of its utility function:

- in R
$$u^R = F(\bar{e}, \bar{e}) - G^{r,R} + \ln G^{r,R} \tag{17}$$
- in C
$$u^C = F(\bar{e}, \bar{e}) - G^{c,C} + \ln G^{c,C} \tag{18}$$
- in PS
$$u^{PS} = F(\bar{e}, \bar{e}) + D^{PS}\left(-G^{r,PS} + \ln G^{r,PS}\right) + \left(1 - D^{PS}\right)\left(-G^{c,PS} + \ln G^{c,PS}\right) \tag{19}$$
- in PE
$$u^{PE} = F(\bar{e}, \bar{e}) + D^{PE}\left(-G^{r,PE} + \ln G^{r,PE}\right) + \left(1 - D^{PE}\right)\left(-G^{c,PE} + \ln G^{c,PE}\right) \tag{20}$$

In the absence of direct and indirect externalities, the optimal level of consumption of public good δ is $g^*(\delta) = 1 \; \forall \delta$ whatever the fiscal architecture and provides the representative citizen a welfare equal to $u = F(\bar{e}, \bar{e}) - 1$.

We first compare the two polar cases that are full decentralization (R) and full centralization (C). In both R and C, the size of the aggregate tax base is $2\bar{e}$ and the whole range of public goods δ has to be provided by a unique tier. In addition, only direct and indirect horizontal tax externalities occur due to the absence of another tier, with direct externalities always dominating indirect ones. With a unique mobile tax base, Hoyt (1991) showed that the tax rate, and thus the public good provision and the welfare of residents, increase as the number of jurisdictions at a given tier decreases. The reduction in the number of competing jurisdictions reduces the distortive effect of tax competition and thus lessens the race to the bottom. In our paper, we generalize Hoyt (1991)'s result in a broader framework with two mobile tax bases rather than one. We show that his result holds whether the tax bases are interdependently mobile or not and whatever the nature of the interdependence, i.e. the two tax bases being gross complement or gross substitute. The number of jurisdictions involved in the tax game differing between R and C, we find that tax competition is fiercer between the nm regions in R than between the n countries in C. In other words, full decentralization worsens the race to the bottom arising from tax competition over both tax bases, in comparison to full centralization, whatever the nature and the degree of interdependence between the two tax bases. As a consequence, the provision of public goods is even more downward distorted and the welfare is lower in R than in C.

Proposition 2 *Full centralization dominates full decentralization whatever the nature and degree of interdependence between the two tax bases. Formally,* $t^R < T^C$, $G^R < G^C$ *and* $u^R < u^C$ $\forall p$.

We then compare these two polar cases R and C with partial decentralization with shared tax bases (PS). Our results can be summarized as follows:

Proposition 3 *Whatever the nature and the degree of interdependence between the tax bases:*

- *Partial decentralization with shared tax bases (PS) dominates full decentralization (R) whatever the level of decentralization in PS, i.e.* $t^R < t^{PS} + T^{PS}$, $G^{r,R} < D^{PS}G^{r,PS} + \left(1 - D^{PS}\right) G^{c,PS}$ *and* $u^R < u^{PS}$ $\forall p$, $\forall D^{PS}$.
- *For a level of decentralization* $D^{PS} < 1 - \frac{1}{n}$, *full centralization (C) leads to smaller levels of taxation and public goods provision, and thus to a lower welfare than partial decentralization with shared tax bases (PS), i.e.* $T^C < t^{PS} + T^{PS}$, $G^{c,C} < D^{PS}G^{r,PS} + \left(1 - D^{PS}\right) G^{c,PS}$ *and* $u^C < u^{PS}$ $\forall p$. *For* $D^{PS} > 1 - \frac{1}{n}$, *C dominates PS and for* $D^{PS} = 1 - \frac{1}{n}$, *they are equivalent.*

The second result of proposition 3 not only generalizes the result of Wilson and Janeba (2005) to the case of more than two countries, but also extends it by assuming more than one mobile tax base and a potential interdependence in the mobility of the two tax bases.

In PS, direct and indirect vertical tax competition effects add to direct and indirect horizontal ones already at work at the regional tier in R and at the central tier in C. However, the range of public goods provided in R (resp. C) is by definition wider than the range of public goods provided by the regional (resp. central) tier in PS. This affects the tax rates in two ways. First, less resources are required to produce the optimal level of public goods at the regional (resp. central) tier in PS compared to R (resp. C), which pushes down the tax rates. Second, a weaker horizontal tax competition effect, i.e. horizontal tax externalities weighted by $D^{PS} < 1$, occurs at the regional (resp. central) tier in PS compared to the horizontal tax competition effect in R (resp. C), which pushes up the tax rates.

At the central tier, vertical externalities being equal to horizontal externalities ($\varepsilon_c^{DH} = \varepsilon_c^{DV}$ and $\varepsilon_c^{IH} = \varepsilon_c^{IV}$), the decrease in the horizontal tax competition effect due to partial decentralization is perfectly compensated by the emergence of the vertical tax competition effect, such that the overall tax competition effect is the same in PS as in C, thus weaker than the tax competition effect at the regional tier in R (from proposition 2). Therefore, the difference in central tax rates between PS and C is solely determined by the difference in the range of public goods provided at the central tier. Although it results that $T^{PS} = \left(1 - D^{PS}\right) T^C < T^C$, a public good δ is provided in the same quantity at the central tier in PS and in C, i.e. $G^{c,PS} = G^{c,C}$ $\forall p, \forall D^{PS}$, and therefore in a larger quantity at the central tier in PS than in R, i.e. $G^{c,PS} > G^{r,R}$ $\forall p, \forall D^{PS}$.

At the regional tier, vertical externalities being smaller than horizontal externalities ($\varepsilon_r^{DH} > \varepsilon_r^{DV}$ and $\varepsilon_r^{IH} > \varepsilon_r^{IV}$), the decrease in the horizontal tax competition effect due to partial decentralization is never fully offset by the emergence of the vertical tax competition effect, such that the overall tax competition effect is smaller in PS than in R. Therefore, although the regional tax rates in PS are smaller than in R due to a smaller range of public goods to provide, i.e. $t^{PS} < t^R$, a public good is provided in a larger quantity at the regional tier in PS than in R, i.e. $G^{r,PS} > G^{r,R}$.

It immediately follows from $t^{PS} < t^R$ that $t^{PS} < T^C$ for any level of decentralization D^{PS} (from proposition 2). However, vertical externalities arising from regional taxation in PS being smaller than the central horizontal externalities in C ($\varepsilon_r^{DV} < \varepsilon_c^{DH}$ and $\varepsilon_r^{IV} < \varepsilon_c^{IH}$), there exists a critical level of decentralization $D^{PS} = 1 - \frac{1}{n}$ such that the overall tax competition effect is the same at the regional tier in PS and in C. The smaller D^{PS}, the smaller the horizontal tax competition effect but the stronger the vertical tax competition effect. Therefore a low (resp. high) enough level of decentralization, i.e. $D^{PS} < 1 - \frac{1}{n}$ (resp. $D^{PS} > 1 - \frac{1}{n}$), leads to an overall smaller (resp. stronger) tax competition effect at the regional tier in PS than in C. It then translates into a public good provided in a larger (resp. lower) quantity at the regional tier in PS than in C, i.e. $G^{r,PS} > G^{r,R}$, $\forall D^{PS} < 1 - \frac{1}{n}$ (resp. $G^{r,PS} < G^{r,R}$, $\forall D^{PS} > 1 - \frac{1}{n}$).

Overall, households are subject to a higher tax burden ($t^R < t^{PS} + T^{PS}$) in PS than in R, but enjoy a higher level of public goods ($G^{r,R} < D^{PS}G^{r,PS} + \left(1 - D^{PS}\right) G^{c,PS}$) which leads to a higher level of welfare. The comparison with C depends on the level of decentralization D^{PS} as described in proposition 3. Whatever the nature and the degree of interdependence between the tax bases, there is always a level of decentralization $D^{PS} < 1 - \frac{1}{n}$ such that partial decentralization with shared tax bases provides a higher level of welfare than full centralization and full decentralization. However, the level of welfare is always socially sub-optimal, characterized by an underprovision of public goods at both tiers.

We now analyze the case of partial decentralization with exclusive tax bases (PE). Since the main drawback raised against the use of shared tax bases rather than exclusive tax bases comes from vertical tax competition, we first look at the difference in terms of tax competition between PE and PS. When the tax bases are independent, no vertical tax competition occurs in PE (since there is no indirect tax competition) and the overall tax competition effect is always smaller in PE. This results holds with tax bases being complement, since more tax externalities are at work in PS than in PE, all pointing in the same direction. On the opposite, when the tax bases are substitute, the overall tax competition effect can be either stronger or smaller at a given tier in PS than in PE, depending on the level of decentralization. However, each tier in PE being able to tax only one factor, the aggregate tax base is twice smaller in PE than in any other fiscal architecture and even in the case where the overall tax competition is weaker in PE, jurisdictions might not be able to provide a higher level of public goods in PE than in PS. A similar mechanism can be observed when comparing PE with R and C.

The comparison of the four fiscal architectures thus comes down to weighing differences in two effects: i) the differences in the overall tax competition effect and ii) the differences in the aggregate tax base. Although comparisons are proved to be complicated, three conclusions can be drawn:

Proposition 4

- *With interdependent tax bases, the use of exclusive tax bases does not prevent vertical tax competition, which occurs in its indirect form.*
- *The magnitude of the overall tax competition effect can be stronger in PE than in another fiscal architecture, depending on the levels of D^{PE} and p.*
- *Even when the magnitude of the overall tax competition effect is weaker in PE than in another form of fiscal decentralization, the use of exclusive tax bases reducing the tax bases available for taxation at each tier, the level of public good provision can be smaller in PE leading to a lower level of welfare.*

Proof. See Appendix A.5. ■

Although comparisons in tax rates between PE and PS provided in Appendix A.3 were made for an identical level of decentralization, we must allow for $D^{PE} \neq D^{PS}$ for the welfare comparisons between PE and PS in order to be able to draw some conclusions about the optimal fiscal architecture. However, the direct comparison of welfare functions appears to be too complicated to give general results and we cannot sign all comparisons in terms of welfare for any level of decentralization and degree of interdependence between the tax bases. We thus have recourse to simulations. The production $F(\bar{e}, \bar{e})$ being the same in every fiscal architecture at the symmetric equilibrium, comparing welfare amounts to comparing $V^Z = u^Z - F(\bar{e}, \bar{e})$, for $Z = R, C, PS, PE$, the social optimal level of which being -1. For PS and PE, we then determine the respective levels of decentralization $\widetilde{D^{PE}}$ and $\widetilde{D^{PS}}$ that maximize the welfare for each degree of interdependence between the tax bases, i.e. for each possible values of p. We then obtain the function of p: $\widetilde{V^Z} = V^Z(\widetilde{D^Z}, p)$, for $Z = PS, PE$ and in order to compare the four fiscal architectures, we plot V^R, V^E, $\widetilde{V^{PS}}$ and $\widetilde{V^{PE}}$ for different values of the parameters n,m,b and $\bar{e}$. All our simulations provided the same qualitative results as the ones in figure 3.[12]

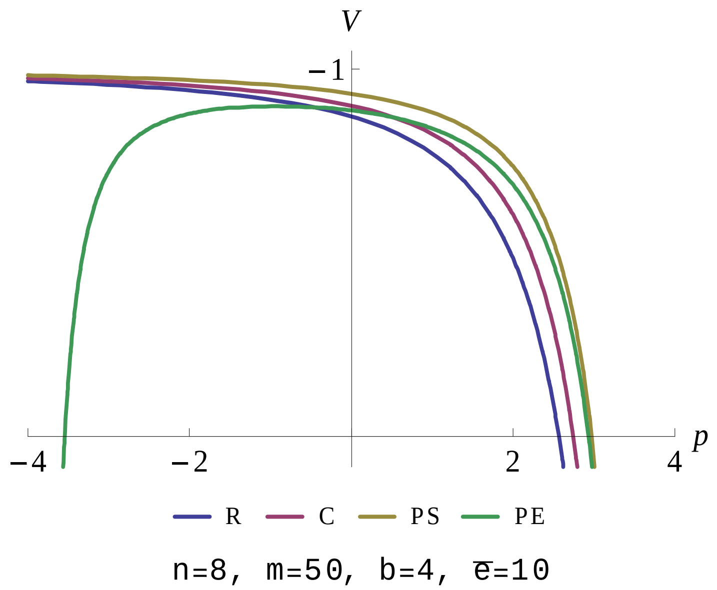

Figure 3: Welfare comparisons

In figure 3, for the optimal levels of decentralization $\widetilde{D^{PE}}$ and $\widetilde{D^{PS}}$, we see that PS dominates any other fiscal architecture, whatever the nature and the degree of interdependence between the two tax bases.

Moreover, figure 3 illustrates the proposition 1. A decrease in p reducing the downward distortion arising from tax competition in R, C and PS, it results an increase of welfare. Should the local authorities have the choice between different mobile tax bases, it is always better to levy taxes on tax bases that are gross substitute. However, a too high

[12]The results are provided for the same values of parameters as in figures 1 and 2.

degree of substitutability between the tax bases induces large distortions of tax rates in PE and thus a lower welfare.

In case the level of decentralization is exogenous, the question then remains whether PE can ever dominate PS in terms of welfare. We then used a 3D plot of the difference $u^{PS} - u^{PE}$ to compare welfare in PS and in PE for every combination of p and $D = D^{PS} = D^{PE}$ with given values of parameters n,m,b and $\overline{e}$. It turned out that we could not find any set of parameters such that $u^{PE} - u^{PS} > 0$. Therefore, even when there are some constraints on the choice of the level of decentralization, we can make the conjecture that PS always provides a higher level of welfare than PE, whatever the nature and degree of interdependence between the tax bases.

Proposition 5

- *Whatever the level of decentralization D^{PE}, partial decentralization with exclusive tax bases is always dominated by all other fiscal architectures for a sufficiently high degree of substitutability between the tax bases.*
- *There exist sets of parameters n,m, b and $\overline{e}$ for which the use of shared tax bases dominates partial decentralization with exclusive tax bases for all levels of decentralization $D = D^{PS} = D^{PE}$ and whatever the nature and degree of interdependence between the tax bases.*

Proof. See Appendix A.6. ■

In order to interpret our results in the light of the existing literature, we finally focus on the particular case of a unique top-tier jurisdiction ($n = 1$). In this case, there is no horizontal tax competition at the country tier and due to the fixed supply of factors, no vertical externalities occur. Central tax rates and thus public goods are always set at an efficient level while regional tax rates are inefficiently low leading to an underprovision of public goods. Any fiscal architecture characterized by taxation at the regional level leads to a sub-optimal level of welfare. As shown by Keen and Kotsogiannis (2002), full centralization then always provides the highest welfare, equal to the socially optimal level $F(\overline{e}, \overline{e}) - 1$ (case of $n = 1$ in proposition 3). On the opposite, with more than one top-tier jurisdiction, depending on the degree of interdependence and the combination between the level of decentralization and the tax assignment, we show that partial decentralization can induce a higher welfare than full centralization. However, while there is always a level of expenditure decentralization such that a fiscal architecture characterized by shared tax bases is better than R and C ($\forall p$), this is no longer the case with exclusive tax bases for a sufficiently high degree of substitutability between the tax bases (see figure 3), due to the very large distortions of tax rates.

6 Conclusion

The issue of tax assignment in a multi-tier setting, i.e. which tier should tax which tax base(s), cannot be dealt with in isolation from the issue of expenditure decentralization. Our paper demonstrates that the interdependence between two mobile tax bases and the level of decentralization (measured by the share of public goods provision assigned to lower-tier jurisdictions) are crucial parameters that affect the tax competition game and thus the welfare of citizens. On the one hand, the interdependence between tax bases complicates the tax competition game arising in a two-tier setting by: i) introducing "indirect" horizontal and vertical tax externalities and ii) reinforcing the standard "direct" horizontal and vertical tax externalities. On the other hand, the level of decentralization affects the weights of these tax externalities and thus the intensity of tax competition.

We show that, depending on the interdependence between the tax bases *and* the share of public goods provision assigned to lower-tier jurisdictions, partial decentralization with exclusive tax bases may lead to inefficiently high tax rates (although not simultaneously at both tiers), while tax rates are always inefficiently low in all other fiscal architectures, i.e. full centralization, full decentralization and partial decentralization with shared tax bases. A higher degree of complementarity between the two tax bases pushes down tax rates and thus deteriorates the welfare in all fiscal architectures. Conversely, a higher degree of substitutability reduces the downward distortion of tax rates and thus improves the welfare for all fiscal architectures, but only for low degree of interdependence in partial decentralization with exclusive tax bases. In the latter case, with a high degree of interdependence between the tax bases, substitutability reinforces the (downward and/or upward) distortions of tax rates and thus reduces the welfare. It follows that authorities should always favour taxation on tax bases which are substitute, although not with a high degree of interdependence in case of partial decentralization with exclusive tax bases.

Both the level of decentralization and the interdependence of tax bases affecting tax decisions, they also influence the comparison between the different fiscal architectures. More specifically, with the interdependence between the tax bases, partial decentralization with exclusive tax bases does not prevent vertical tax competition and the intensity of tax competition can even be stronger than in other fiscal architectures, leading to lower welfare. Moreover, a weaker intensity of tax competition does not guarantee a higher welfare as the exclusive use of tax bases reduces the sources of tax revenues at each tier, thereby pushing up the tax rates.

With a unique top-tier jurisdiction and a fixed supply of factors, no vertical tax competition occurs while horizontal tax competition only takes place at the bottom tier. It is then optimal to follow the recommendations of assigning mobile tax bases to the highest tier and thus opt for full centralization. Considering more than one top-tier jurisdiction competing for mobile tax bases modifies this conclusion. While full centralization still dominates full decentralization, partial decentralization may induce a higher welfare than full centralization. However, this result crucially depends on the degree of interdependence and the combination between the level of decentralization and the tax assignment. Compared to full centralization, partial decentralization with exclusive tax bases always deteriorates the welfare for sufficiently high degree of substitutability of tax bases while there is always a level of decentralization such that the use of shared tax bases is welfare enhancing, whatever the nature and degree of interdependence between the tax bases.

These results then suggest that partial decentralization combined with appropriate tax assignment is always preferable to full decentralization and full centralization when the share of public good provision between the two tiers can be freely adjusted. Transposed onto a sub-national level where tax competition can occur at both regional and local tiers, our analysis also implies that the federal structure of a country should consist of two sub-national tiers sharing the provision of public goods.

In practice, there might exist some constraints to the level of decentralization. For instance, heterogeneous preferences for public goods as well as scale economies can influence the level of decentralization. While some public goods should be provided at a local level to better match preferences following Tiebout (1956)'s argument, scale economies or lower congestion costs may be achieved by a provision of public goods at a higher level. Absent from our framework as we focused exclusively on the issue of tax competition, these elements could be introduced in the model but at the expense of some complexity.

Finally, it ought to be remarked that assuming a world economy with perfectly identical countries amounts to assume homogeneity in fiscal architecture across countries. However, the issue of the coexistence of different fiscal architectures may arise when introducing some form of asymmetry between the countries. Moreover, although recommendations can be made about the optimal fiscal architecture(s), our analysis assumes the fiscal architecture to be exogenous and is thus silent about which one would actually be adopted if the fiscal architecture could be used as a strategic device by countries engaged in tax competition. With two mobile tax bases, countries would then have to decide on both the tax bases and the share of public good provision assigned to each tier.[13]

Acknowledgements

We thank Sam Bucovetsky, Aurélie Cassette and all participants at seminars and conferences where this paper was presented, for their valuable comments and suggestions.

[13] Although Wilson and Janeba (2005) assume the level of decentralization to be endogeneous, they only consider two countries and one mobile tax base which excludes the case of partial decentralization with exclusive tax bases.

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A Appendix

A.1 Response of net returns to taxation

The system of market-clearing conditions is:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}(r_{ij}^{x}, r_{ij}^{y}) = nm\overline{x} \\ \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}(r_{ij}^{x}, r_{ij}^{y}) = nm\overline{y} \end{cases}$$

With $r^x_{ij} = \rho^x + t^x_{ij} + T^x_i$ and $r^y_{ij} = \rho^y + t^y_{ij} + T^y_i$, $\forall i, j$

From the differentiation of market-clearing we derive the response of ρ^x to regional and central taxation:

$$\frac{\partial \rho^x}{\partial t^x_{ij}} = \frac{-\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} + \frac{\partial y_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial t^y_{ij}} = \frac{-\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} + \frac{\partial y_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial T^x_{i}} = \frac{-\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} + \sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial T^y_{i}} = \frac{-\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} + \sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

A.2 Proof of proposition 1

For R, C and PS, all tax rates depend on the sum of direct and indirect externalities $\varepsilon_l^{DH} + \varepsilon_l^{IH}$ and $\varepsilon_l^{DV} + \varepsilon_l^{IV}$ with $l = r, c$. Therefore, direct externalities always dominating indirect ones in absolute terms, we obtain $\varepsilon_l^{DH} + \varepsilon_l^{IH} < 0$ and $\varepsilon_l^{DV} + \varepsilon_l^{IV} < 0$ $\forall p$ with $l = r, c$. All tax rates are thus inefficiently low, i.e. $t^{*R} > t^R$, $T^{*C} > T^C$, $t^{*PS} > t^{PS}$ and $T^{*PS} > T^{PS}$ $\forall p$.

For R, C and PS, differentiating the expressions of equilibrium tax rates (11-14) with respect to p, directly provides the results, i.e. $\frac{\partial t}{\partial p} < 0$ and $\frac{\partial T}{\partial p} < 0$.

For PE, whether the regional (resp. central) tax rate is inefficiently low, i.e. $t^{*PE} > t^{PE}$ (resp. $T^{*PE} > T^{PE}$), or inefficiently high, i.e. $t^{*PE} < t^{PE}$ (resp. $T^{*PE} < T^{PE}$), depends on the sign of $\frac{D^{PE}\varepsilon_r^{DH}}{\bar{e}} + \frac{\left(1-D^{PE}\right)\varepsilon_r^{IV}}{m\bar{e}}$ (resp. $\frac{D^{PE}\varepsilon_c^{DH}}{m\bar{e}} + \frac{\left(1-D^{PE}\right)\varepsilon_c^{IV}}{m\bar{e}}$) . It follows that when tax bases are independent or complement, both tax rates are always inefficiently low due to $\varepsilon_l^{DH} < 0$ and $\varepsilon_l^{IV} \leq 0$ $\forall p \geq 0$ with $l = r, c$. When tax bases are substitute, direct and indirect externalities have opposite signs and we observe an upward distortion of: i) the regional tax rate t^{PE} for $D^{PE} < \frac{\frac{\varepsilon_r^{IV}}{m\bar{e}}}{-\left(\frac{\varepsilon_r^{DH}}{\bar{e}} - \frac{\varepsilon_r^{IV}}{m\bar{e}}\right)}$, ii) the central tax rate T^{PE} for

$$D^{PE} > \frac{-\frac{\varepsilon_c^{DH}}{m\bar{e}}}{-\left(\frac{\varepsilon_c^{DH}}{m\bar{e}} - \frac{\varepsilon_c^{IV}}{m\bar{e}}\right)}.$$

Moreover, since $\frac{\frac{\varepsilon_r^{IV}}{m\bar{e}}}{-\left(\frac{\varepsilon_r^{DH}}{\bar{e}}-\frac{\varepsilon_r^{IV}}{m\bar{e}}\right)} < \frac{-\frac{\varepsilon_c^{DH}}{m\bar{e}}}{-\left(\frac{\varepsilon_c^{DH}}{m\bar{e}}-\frac{\varepsilon_c^{IV}}{m\bar{e}}\right)}$, an upward distortion of tax rates can never occur simultaneously at both tiers.

In case of complementarity of factors, i.e. $p > 0$, the magnitude of both direct and indirect externalities increasing with p, a higher degree of complementarity increases the downward distortion, i.e. $\frac{\partial t^{PE}}{\partial p} < 0$ and $\frac{\partial T^{PE}}{\partial p} < 0$, $\forall p > 0$.

In case of substitutability, the relative effect of indirect vertical tax competition to direct horizontal tax competition depending on the level of decentralization D^{PE} as well as on the degree of substitutability $|p|$. We then reason for a given value of D^{PE}. It follows that we observe an upward distortion of: i) the regional tax rate t^{PE} for $-\frac{D^{PE}(mn-1)b}{(1-D^{PE})(n-1)} > p$, ii) the central tax rate T^{PE} for $-\frac{\left(1-D^{PE}\right)}{D^{PE}}b > p$.

By assumption $|p| < b$, an upward distortion at the central tier can only occur for a level of decentralization $D^{PE} > \frac{1}{2}$. Similarly, an upward distortion at the regional tier can only occur for a level of decentralization $D^{PE} < \frac{(n-1)}{(mn-1)+(n-1)} < \frac{1}{2}$.

$$t^{PE} = t^{xPE} = \frac{D^{PE}}{\bar{e}-\frac{D^{PE}\varepsilon_r^{DH}}{\bar{e}}-\frac{(1-D^{PE})\varepsilon_r^{IV}}{m\bar{e}}} = \frac{D^{PE}}{\bar{e}+\frac{D^{PE}(mn-1)}{mn(b^2-p^2)\bar{e}}b+\frac{\left(1-D^{PE}\right)(n-1)}{mn(b^2-p^2)\bar{e}}p} \tag{21}$$

$$T^{PE} = T^{yPE} = \frac{\left(1-D^{PE}\right)}{\bar{e}-\frac{(1-D^{PE})\varepsilon_c^{DH}}{m\bar{e}}-\frac{D^{PE}\varepsilon_c^{IV}}{m\bar{e}}} = \frac{\left(1-D^{PE}\right)}{\bar{e}+\frac{n-1}{n(b^2-p^2)\bar{e}}((1-D^{PE})b+D^{PE}p)} \tag{22}$$

From the differentiation of tax rates in PE, we then get:

- With $D^{PE} \geq \frac{1}{2}$, $\frac{\partial T^{PE}}{\partial p} < 0 \;\; \forall p < 0$ and $\lim_{p \to -b} T^{PE} = +\infty$

- With $D^{PE} < \frac{1}{2}$, $\frac{\partial^2 T^{PE}}{\partial p^2} < 0$, $\begin{cases} \frac{\partial T^{PE}}{\partial p} \leq 0 \;\text{ if }\; 0 > p \geq \frac{-\left(1-D^{PE}\right)+\sqrt{(1-2D^{PE})}}{D^{PE}}b \\ \frac{\partial T^{PE}}{\partial p} > 0 \;\text{ if }\; -b < p < \frac{-\left(1-D^{PE}\right)+\sqrt{(1-2D^{PE})}}{D^{PE}}b \end{cases}$ and $\lim_{p \to -b} T^{PE} = 0$

- With $D^{PE} \leq \frac{(n-1)}{(mn-1)+(n-1)}$, $\frac{\partial t^{PE}}{\partial p} < 0 \;\; \forall p < 0$ and $\lim_{p \to -b} t^{PE} = +\infty$

- With $D^{PE} > \frac{(n-1)}{(mn-1)+(n-1)}$, $\frac{\partial^2 t^{PE}}{\partial p^2} < 0$, $\begin{cases} \frac{\partial t^{PE}}{\partial p} \leq 0 \;\text{ if }\; 0 > p \geq \frac{-D^{PE}(mn-1)+\sqrt{(D^{PE})^2(mn-1)^2-(1-D^{PE})^2}}{(1-D^{PE})(n-1)} \\ \frac{\partial t^{PE}}{\partial p} > 0 \;\text{ if }\; -b < p < \frac{-D^{PE}(mn-1)+\sqrt{(D^{PE})^2(mn-1)^2-(1-D^{P}}}{(1-D^{PE})(n-1)} \end{cases}$

and $\lim_{p \to -b} t^{PE} = 0$

A.3 Comparisons of tax rates

A.3.1 Symmetry between tax bases in terms of tax assignment

Comparison between R and C gives $t^R < T^C$, $\forall p$.

Comparisons with PS give:

	t^{PS}	T^{PS}	$t^{PS} + T^{PS}$
t^R	$t^R > t^{PS}$	$\begin{cases} t^R \geq T^{PS} \text{ if } D^{PS} \geq \widetilde{D^{PS}} \\ t^R < T^{PS} \text{ if } D^{PS} < \widetilde{D^{PS}} \end{cases}$	$t^R < t^{PS} + T^{PS}$
T^C	$T^C > t^{PS}$	$T^C > T^{PS}$	$\begin{cases} T^C \geq t^{PS} + T^{PS} \text{ if } D^{PS} \geq 1 - \frac{1}{n} \\ T^C < t^{PS} + T^{PS} \text{ if } D^{PS} < 1 - \frac{1}{n} \end{cases}$

Where $\widetilde{D^{PS}} = \frac{-\frac{\varepsilon_r^{DH} + \varepsilon_r^{IH}}{\overline{e}} + \frac{\varepsilon_c^{DH} + \varepsilon_c^{IH}}{m\overline{e}}}{\left(2\overline{e} - \frac{\varepsilon_r^{DH} + \varepsilon_r^{IH}}{\overline{e}}\right)} = \frac{(m-1)}{\left(2mn(b-p)\overline{e}^2 + mn - 1\right)}$

A.3.2 PE versus all other fiscal architectures

Comparisons between PE and PS are made for an identical level of decentralization, i.e. $D^{PE} = D^{PS} = D$.

- At the regional tier:
  - For $n = 1$:

	$b > p \geq p^*$	$p^* > p > -b$
$/t^R$	$\begin{cases} t^R \leq t^{PE} \text{ if } D \geq D^* \\ t^R > t^{PE} \text{ if } D < D^* \end{cases}$	$t^R > t^{PE}$
$/t^{PS}$	$t^{PS} < t^{PE}$	$\begin{cases} t^{PS} > t^{PE} \text{ if } D > D^{**} \\ t^{PS} \leq t^{PE} \text{ if } D \leq D^{**} \end{cases}$

  - For $n > 1$:

	$b > p > p^*$	$p^* \geq p \geq p^{**}$	$p^{**} > p > -b$
$/t^R$	$\begin{cases} t^R \leq t^{PE} \text{ if } D \geq D^* \\ t^R > t^{PE} \text{ if } D < D^* \end{cases}$	$t^R > t^{PE}$	$\begin{cases} t^R \geq t^{PE} \text{ if } D \geq D^* \\ t^R < t^{PE} \text{ if } D < D^* \end{cases}$
$/t^{PS}$	$t^{PS} < t^{PE}$	$t^{PS} < t^{PE}$	$\begin{cases} t^{PS} > t^{PE} \text{ if } D > D^{**} \\ t^{PS} \leq t^{PE} \text{ if } D \leq D^{**} \end{cases}$

Where $p^* < 0$ satisfies $\overline{e} - \frac{\varepsilon_r^{IH}}{\overline{e}} = 0$; $p^{**} < 0$ satisfies $\overline{e} - \frac{\varepsilon_r^{IV}}{m\overline{e}} = 0$

And $D^* = \frac{\overline{e} - \frac{\varepsilon_r^{IV}}{m\overline{e}}}{\left(2\overline{e} - \frac{\varepsilon_r^{IH}}{\overline{e}} - \frac{\varepsilon_r^{IV}}{m\overline{e}}\right)}$, $D^{**} = \frac{\overline{e} - \frac{\varepsilon_r^{DV}}{m\overline{e}}}{-\left(\frac{\varepsilon_r^{DV}}{m\overline{e}} - \frac{\varepsilon_r^{IH}}{\overline{e}}\right)}$

- At the central tier:
  - For $n = 1$: $\begin{cases} T^C \geq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C < T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$, $\forall p$ and $T^{PS} < T^{PE}$, $\forall p, D$
  - For $n > 1$:

	$b > p > p^{***}$	$p = p^{***}$	$p^{***} > p > -b$
$/T^C$	$\begin{cases} T^C \geq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C < T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$	$T^C > T^{PE}$	$\begin{cases} T^C \leq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C > T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$
$/T^{PS}$	$T^{PS} < T^{PE}$	$T^{PS} < T^{PE}$	$\begin{cases} T^{PS} \leq T^{PE} \text{ if } D \geq D^{***} \\ T^{PS} > T^{PE} \text{ if } D < D^{***} \end{cases}$

Where $p^{***} < 0$ satisfies $\bar{e} - \frac{\varepsilon_c^{IH}}{m\bar{e}} = 0$

And $D^{***} = -\frac{\bar{e} - \frac{\varepsilon_c^{IH}}{m\bar{e}}}{\left(-\frac{\varepsilon_c^{DV}}{m\bar{e}} + \frac{\varepsilon_c^{IH}}{m\bar{e}}\right)}$

A.4 Levels of public good consumption in each fiscal architecture

$$\text{In } R \quad : \qquad G^{r,R} = 2t^R\bar{e} = \frac{1}{1 - \frac{\varepsilon_r^{DH} + \varepsilon_r^{IH}}{2\bar{e}^2}} = \frac{1}{1 + \frac{nm-1}{2nm(b-p)\bar{e}^2}}$$

$$\text{In } C \quad : \qquad G^{c,C} = 2T^C\bar{e} = \frac{1}{1 - \frac{\varepsilon_c^{DH} + \varepsilon_c^{IH}}{2m\bar{e}^2}} = \frac{1}{1 + \frac{n-1}{2n(b-p)\bar{e}^2}}$$

$$\text{In } PS \quad : \quad \begin{cases} G^{r,PS} = \frac{2t^{PS}\bar{e}}{D^{PS}} = \frac{1}{1 - D^{PS}\frac{\varepsilon_r^{DH} + \varepsilon_r^{IH}}{2\bar{e}^2} - (1-D^{PS})\frac{\varepsilon_r^{DV} + \varepsilon_r^{IV}}{2m\bar{e}^2}} = \frac{1}{1 + \frac{D^{PS}(nm-1) + \left(1-D^{PS}\right)(n-1)}{2nm(b-p)\bar{e}^2}} \\ G^{c,PS} = \frac{2T^{PS}\bar{e}}{(1-D^{PS})} = \frac{1}{1 - (1-D^{PS})\frac{\varepsilon_c^{DH} + \varepsilon_c^{IH}}{2m\bar{e}^2} - D^{PS}\frac{\varepsilon_c^{DV} + \varepsilon_c^{IV}}{2m\bar{e}^2}} = \frac{1}{1 + \frac{(n-1)}{2n(b-p)\bar{e}^2}} \end{cases}$$

$$\text{In } PE \quad : \quad \begin{cases} G^{r,PE} = \frac{t^{PE}\bar{e}}{D^{PE}} = \frac{1}{1 - D^{PE}\frac{\varepsilon_r^{DH}}{\bar{e}^2} - (1-D^{PE})\frac{\varepsilon_r^{IV}}{m\bar{e}^2}} = \frac{1}{1 + \frac{D^{PE}(mn-1)b + \left(1-D^{PE}\right)(n-1)p}{mn(b^2-p^2)\bar{e}^2}} \\ G^{c,PE} = \frac{T^{PE}\bar{e}}{(1-D^{PE})} = \frac{1}{1 - (1-D^{PE})\frac{\varepsilon_c^{DH}}{m\bar{e}^2} - D^{PE}\frac{\varepsilon_c^{IV}}{m\bar{e}^2}} = \frac{1}{1 + \frac{n-1}{n(b^2-p^2)\bar{e}^2}((1-D^{PE})b + D^{PE}p)} \end{cases}$$

A.5 Proof of proposition 4

The first part of the proposition immediately follows from the definition of partial decentralization with exclusive tax bases.

The magnitude of the overall tax competition effect in PE is given by:

$$\left|-\frac{D^{PE}\varepsilon_r^{DH}}{\bar{e}}-\frac{\left(1-D^{PE}\right)\varepsilon_r^{IV}}{m\bar{e}}\right| = \left|\frac{D^{PE}(mn-1)}{mn(b^2-p^2)\bar{e}}b+\frac{\left(1-D^{PE}\right)(n-1)}{mn(b^2-p^2)\bar{e}}p\right| \quad \text{at the regional tier}$$

$$\left|-\frac{\left(1-D^{PE}\right)\varepsilon_c^{DH}}{m\bar{e}}-\frac{D^{PE}\varepsilon_c^{IV}}{m\bar{e}}\right| = \left|\frac{n-1}{n(b^2-p^2)\bar{e}}\left(\left(1-D^{PE}\right)b+D^{PE}p\right)\right| \quad \text{at the central tier}$$

Assuming that the level of decentralization D^{PE} is such that a high degree of substitutability between the tax bases leads to a downward distorstion of both tax rates, i.e. $D^{PE}\in[\frac{n-1}{(nm-1)+(n-1)},\frac{1}{2}]$, the limits of the magnitude of the overall tax competition effect at both tier is infinity when p approaches to $-b$. On the opposite the magnitude of the overall tax competition effect in all other fiscal architectures belongs to the finite interval $\left[\frac{n-1}{2nmb\bar{e}},\frac{nm-1}{2nmb\bar{e}}\right]$ when p approaches to $-b$ and is thus lower than in PE, proving the second part of the proposition.

$\frac{D^{PS}(nm-1)+\left(1-D^{PS}\right)(n-1)}{2nmb\bar{e}}$

To prove the third part of the proposition, let us for instance compare tax rates in PS and in PE for an identical level of decentralization D and when tax bases are complement. In this case, with shared tax bases, an indirect horizontal tax competition effect as well as a direct vertical tax competition effect come in addition to the tax competition effects already occurring in PE. The magnitude of the overall tax competition effect is then weaker in PE than in PS, which results in higher tax rates in PE than in PS (table in Appendix A.3.2).

Comparing now the levels of public good consumption given in Appendix A.4, we obtain for $D^{PS}=D^{PE}=D$ and $p>0$:

$$\begin{aligned} G_i^{mTBS} &\geq G_i^{mTS} \text{ if } D\geq\frac{(n-1)}{(n-1)+(nm-1)} \\ G_i^{rTBS} &\geq G_i^{rTS} \text{ if } D\geq\frac{1}{2} \end{aligned}$$

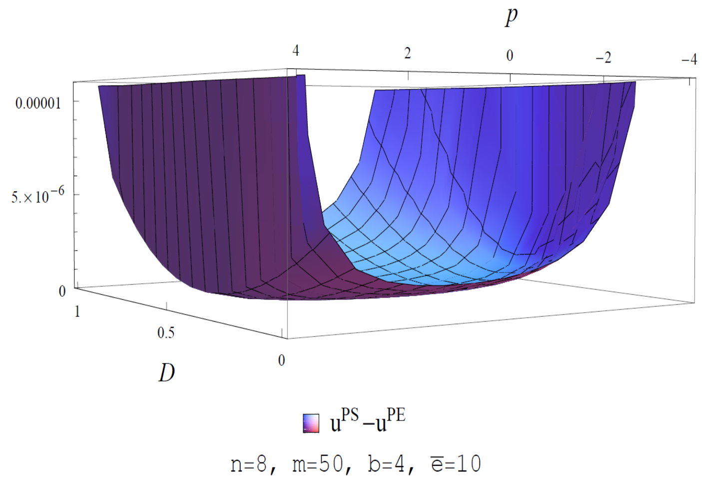
It follows that $u^{PS}>u^{PE}$ if $\frac{1}{2}\geq D\geq\frac{(n-1)}{(n-1)+(nm-1)}$. The weaker tax competition effect in PE than in PS cannot compensate for the smaller aggregate tax base, proving the last part of the proposition.

A.6 Proof of proposition 5

When p approaches to $-b$, tax rates in PE are highly distorted (proposition 1), such that the limit of the consumption for a public good δ is infinity or zero depending on the level of decentralization D^{PE}. It follows that the limit of welfare is minus infinity when p approaches to $-b$.

In all other fiscal architectures, the consumption for a public good δ belongs to the finite interval $\left[\frac{1}{1+\frac{nm-1}{4nmb\bar{e}^2}}, 1\right]$ when p approaches to $-b$. It follows that the welfare belongs to the finite interval $\left[F(\bar{e},\bar{e}) - \frac{1}{1+\frac{nm-1}{4nmb\bar{e}^2}} + \ln\left(\frac{1}{1+\frac{nm-1}{4nmb\bar{e}^2}}\right), F(\bar{e},\bar{e}) - 1\right]$ and is thus always higher than in PE.

For the set of parameters $n = 8$, $m = 50$, $b = 4$ and $\bar{e} = 0$ (same parameters as in the other figures), we plot of the difference $u^{PS} - u^{PE}$ to compare welfare in PS and in PE for every combination of p, $D = D^{PS} = D^{PE}$. As shown in the figure below, PS always provides a higher level of welfare than PE, whatever the nature and degree of interdependence between the tax bases and the level of decentralization D.



We did find many other sets of parameters such that $u^{PS} - u^{PE} > 0, \forall p, \forall D$. However, we could not find any set of parameters such that PE provides a higher welfare than PS for some combinations of p and D.