MULTIPLE VOTES, MULTIPLE CANDIDACIES AND POLARIZATION

ARNAUD DELLIS*

Université Laval and CIRPEE
1025 Ave des Sciences Humaines, local 2174, Québec (QC) G1V 0A6, Canada
E-mail: arnaud.dellis@ecn.ulaval.ca

MANDAR OAK

School of Economics, University of Adelaide
Level 3,10 Pulteney Street, SA 5005
Australia
E-mail: mandar.oak@adelaide.edu.au

Abstract

We use the citizen-candidate model to study the differential incentives that alternative voting rules provide for candidate entry, and their effect on policy polarization. In particular, we show that allowing voters to cast multiple votes leads to equilibria which support multiple candidate clusters. These equilibria are more polarized than those obtained under the Plurality Rule. We also show that equilibria under the Alternative Vote Rule do not exhibit multiple candidate clusters and that these equilibria are less polarizing than those under the Plurality Rule. These results differ from those obtained in the existing literature, where the set of candidates is exogenous. Thus, our paper contributes to the scholarly literature as well as public debate on the merits of using different voting rules by highlighting the importance of endogenous candidacy.

JELCodes C72, D72, H11

Keywords Citizen-candidates, Policy Polarization, Approval Voting, Alternative Vote Rule

---

Date: October 16, 2013.
* Corresponding Author.

1. Introduction

Plurality Rule, under which each voter can vote for at most one among the set of eligible candidates, is a commonly used voting rule in several Western democracies (e.g., the US, the UK and Canada). Political activists as well as academic scholars have often questioned the use of this voting rule and have, time and again, argued for replacing it with alternatives which offer voters greater choice and flexibility. The two commonly cited examples are Approval Voting[1] and the Alternative Vote Rule[2], adoption of which ranks among the most publicly, and academically, debated ideas for electoral reforms for elections in single seat, single member districts.

What is the rationale behind this quest for an alternative to the Plurality Rule? A well known result in political science, namely the *Duverger's Law*, suggests that Plurality Rule typically leads to a political landscape dominated by only two credible political parties. This deprives the voters of a variety in their choices and, the optimism of the *Hotelling's Law* notwithstanding, may lead to extreme outcomes if political parties are beholden to their partisan voter bases. It is then argued that the option of casting more votes amounts to giving voters a greater say which, in turn, translates into greater policy moderation if the additional votes cast tend to favor more centrist candidates.

To understand the intuitive appeal behind this claim, let us consider a simple example. Suppose that three candidates—a leftist (L), a centrist (M) and a rightist (R)—are contesting an election. We denote their electoral platforms by x_L, x_M and x_R, respectively. Suppose that the electorate is also divided into three groups: leftists who prefer L to M to R, rightists who have exactly opposite preferences to the leftists, and centrists who prefer M to both L and R. If the election were held under the Plurality Rule, we would have the leftists voting for L and the rightists voting for R, leaving M with only the centrists' votes. Figure 1 depicts such a situation. Assuming there is a plurality of either the leftists or the rightists, the outcome of the election will be candidate L or R; M will receive too few votes to win.

[1]Under Approval Voting, every citizen can vote for as many candidates as she wishes, and the candidate with the most votes is elected. Approval Voting was popularized by Brams and Fishburn (1978). It is currently used by several professional and academic associations to elect their officers, and by the UN to elect its secretary general. For the more recent scholarship on this topic see Laslier and Sanver (2010) edited handbook.

[2]Under the Alternative Vote Rule, every voter ranks the candidates from first to last. A candidate is elected if he receives a majority of first place votes; otherwise, the candidate with the fewest first place votes is eliminated and his votes are transferred to the candidates ranked next on the individual ballot papers. The elimination process is repeated until one candidate receives a majority of first place votes.This rule (also known as the Instant Runoff Rule) is currently used for House elections in Australia and for presidential elections in Ireland; variants of it have recently been adopted for municipal elections in San Francisco and London.

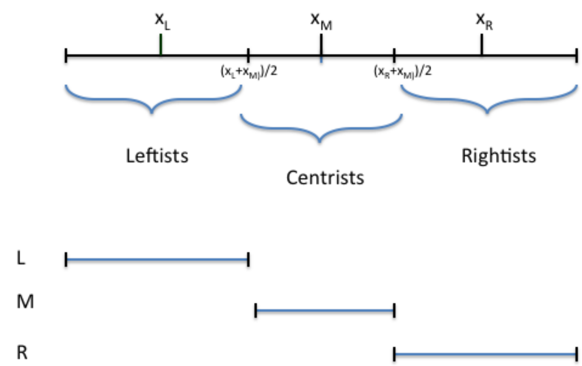

Figure 1. Votes under Plurality Rule

Suppose instead, that the voting rule were such that each voter got to cast *two* votes. Now, candidate M will receive the second votes of the leftists and the rightists, while the centrists will cast their second votes for either L or R, depending on their preferences. Thus, candidate M's electoral prospects will be significantly improved by the additional votes he attracts from both sides of the ideological spectrum. This is depicted in Figure 2; all voters cast one vote for candidate M making him the outright winner and thus making x_M the implemented policy. Assuming that in the examples depicted in Figures 1 and 2, x_M lies closer to the median voter's ideal policy, this voting rule would lead to greater policy moderation than under the Plurality Rule.

The above argument generalizes to Approval Voting and Alternative Vote Rules where the greater choice offered under these rules similarly helps improve the electoral prospects of the centrist candidate.

Improving electoral prospects of centrist candidates/policies, or equivalently, reducing policy polarization, is an attractive property for a voting rule to satisfy. Under standard assumptions regarding underlying voter preferences, more centrist policies improve overall (Utilitarian) Social Welfare. Hence, from a purely instrumentalist point of view, a voting rule which allows voters to cast more votes seems like a better alternative to Plurality Rule.

Note, however, that the example presented above contained two implicit assumptions: 1) the set of candidates was exogenously fixed, and 2) citizens' voting behavior was sincere. This paper is primarily concerned with studying the implications of relaxing the first of these assumptions on the relationship between policy polarization and voting rule. We will also draw on our earlier work on strategic voting and analyze the differences in outcomes under the two assumptions.

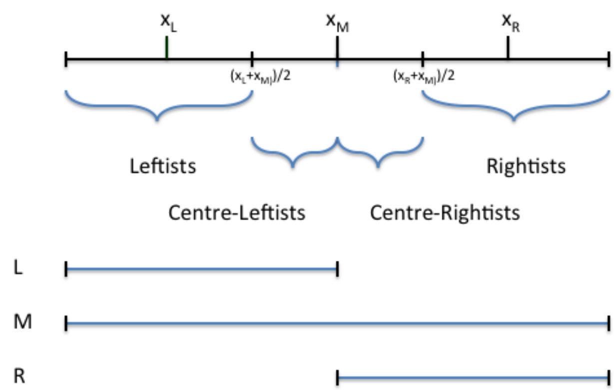

Figure 2. Votes when Each Voter Casts 2 Votes

To be specific, we develop our argument in a simple setting in which voting is sincere but candidate entry decisions are strategic. In the baseline case we consider candidates that are purely policy-motivated but later on show the robustness of our analysis to adding not-too-large rents from holding office. We study two voting rules: first, a broad class of rules we call (s, t)-rules and second, the Alternative Vote Rule. Under an (s, t)-rule, every voter must cast a 1-point vote to at least $s(\geq 1)$ and at most $t(\geq s)$ candidates; the candidate with the most votes is elected. The class of (s, t)-rules includes several well-known voting rules like the Plurality Rule, Approval Voting and Negative Voting (vote against one candidate). Among the (s, t)-rules we further differentiate between those which allow partial abstention (i.e., $s < t$) and those which don't (i.e., $s = t$).[3] The Alternative Vote Rule differs from the class of (s, t)-rules in two aspects: 1) voters rank-order the candidates; 2) to be the winner, a candidate must receive a majority of first-place votes.

Our analysis shows that when candidacy is endogenous, employing a voting rule that allows voters to cast more votes does not guarantee policy moderation. The crucial factor at play is the potential for multiple candidacies. In a nutshell, when there are more votes available to be cast, it also induces more candidates to stand for elections![4] This counteracts the effect outlined in the example above, where the additional votes were cast in favor

[3] In the voting literature the latter are often referred to as the k-approval rules where $k \equiv s = t$.

[4] The phenomenon we call multiple candidacies has been previously referred to as "duplicate candidacies" by Myerson (2002) and is related to Tideman (1987)'s concept of "clone candidates".

of the centrist candidate. With endogenous candidacy, the additional votes are *absorbed* by the additional candidates who need not be running on more centrist platforms. Hence, the direction of the net effect is dependent on the specifics of the voting rule.

Our analysis also brings new light on the relative standing between Approval Voting and the Alternative Vote Rule in facilitating policy moderation. We show that the (s,t)-rules, including those which allow partial abstention (of which Approval Voting is a special case), are susceptible to the multiple candidacy problem. Hence, many of these rules (including Approval Voting) admit equilibria that are more polarizing vis-à-vis the Plurality Rule. The Alternative Vote Rule does not suffer from the multiple candidacy problem and facilitates moderation vis-à-vis the Plurality Rule.

Thus, our paper makes an important contribution to the literature as well as policy debate on the comparative analysis of alternative voting rules. It also highlights the importance of accounting for endogenous candidacy decisions in understanding how voting rules facilitate/hinder policy moderation.

The remainder of the paper is organized as follows: Section 2 discusses the related literature. Section 3 presents the basic model under the (s,t)-rule. Sections 4 and 5 present our main results, first by using an illustrative example, and then more generally. Section 6 provides the analysis under the Alternative Vote Rule and highlights the differences vis-à-vis the (s,t)-rules of the previous section. Section 7 concludes by summarizing our main findings, addressing some of the shortcomings, and suggesting avenues for further research. All proofs are relegated to the Appendix.

2. Related Literature

The interest in various voting rules and their properties dates back at least as far as the late 18th Century French philosopher-mathematicians Condorcet and Borda, culminating in the classic works of Arrow, May and Gibbard-Satthertwaite in the mid-twentieth century. The modern approach to this topic embeds voting rules in a model of political competition, typically of a Downsian variety, i.e., one where a fixed, preexisting set of office-motivated candidates/parties competes by choosing policy positions. More recently scholars have used the citizen-candidate models of political competition to revisit some of the conclusions drawn under the Downsian framework. It is to this citizen-candidate tradition that our paper belongs.

The citizen-candidate models pioneered by Osborne and Slivinski (1996), and Besley and Coate (1997) provide an alternative modeling strategy to the standard Hotelling-Downsian paradigm of political competition. The key feature of the citizen-candidate framework is its explicit endogenization of the entry choices of policy-motivated candidates into the political arena. The ease of tractability as well as the generic existence of political equilibria under these models make them particularly amenable to the study of the alternative voting rules.

Osborne and Slivinski (1996) compare the number of candidates and their polarization in Plurality Rule and Plurality Runoff elections under the sincere voting assumption. Our paper extends theirs in that we consider a much larger array of voting rules. In particular, the family of voting rules we consider allows us to identify the key role played by the differential incentives for multiple candidacies. Morelli (2004) proposes a citizen-candidate model with endogenous party formation, and compares the number of parties/candidates in Plurality Rule and Proportional Representation elections under the sincere and strategic voting assumptions. In contrast, there is no party formation in our paper and we focus on single-member district elections. The focus on single-member district elections allows us to study electoral reform proposals for single seat elections such as presidential, gubernatorial or mayoral elections. Dellis and Oak (2006) and Dellis (2009) assume voting to be strategic, and study policy moderation in Plurality Rule and Approval Voting elections and in scoring rule elections, respectively. In the current paper we focus on sincere voting and consider other voting rules, particularly the Alternative Vote Rule which has recently gained popularity in the electoral reform debate and which allows us to further highlight the key role of multiple candidacies.

The issue of policy moderation under alternative voting rules has been previously studied under the Downsian framework by several scholars. For instance, Cox (1987, 1990) studies the families of scoring rules and Condorcet procedures. He assumes voting to be sincere and restricts attention to convergent equilibria (where all candidates choose the same electoral platform). He finds that increasing the number of votes every voter casts produces *centripetal* incentives for candidates and supports policy moderation. We show that this finding needs a qualification when candidacy is endogenous. He also finds that allowing partial abstention produces *centrifugal* incentives that lead to more polarization. We show that this finding is robust to endogenous candidacy à la citizen-candidate model. Myerson and Weber (1993) compare the Plurality Rule with Approval Voting under the strategic voting assumption. They find that Approval Voting supports less polarization than the Plurality Rule. Our analysis shows that this finding is reversed under the sincere voting and endogenous candidacy assumptions.

At a more general level, this paper is related to a number of papers comparing specific features of different electoral systems. Notable examples are Myerson (1993) who analyzes the incentives for candidates to create inequalities among voters, Myerson (2006) who studies the effectiveness in reducing government corruption, Lizzeri and Persico (2001) and Milesi-Ferretti et al. (2002) who study public good provision, and Pagano and Volpin (2005) who look at employment and shareholder protection. Myerson (2002) studies vote coordination in scoring rule elections, while Dellis (2013) studies its implications for the number of parties. Chamberlin and Cohen (1978) and

Merrill (1988) run simulations to study the propensity to elect the Condorcet winner, i.e., the candidate who defeats any other candidate in a pairwise contest.

Finally, this paper is related to the literature on clone candidates. Roughly speaking, a subset of candidates are said to be clones if they are ranked next to each other in every voter's ranking of the candidates. Tideman (1987) introduces the independence of clones criterion, which requires that an election winner must not change following the deletion of clones. Interestingly for our purpose, Tideman establishes that the Alternative Vote Rule is independent of clones, while (s,t)-rules are not. Laffond et al. (1996) and Laslier (2000) propose similar criteria, namely, composition consistency (which applies to tournaments) and cloning consistency. These contributions seek to characterize social choice correspondences and identify voting rules that are independent of clones. More recently, Elkind et al. (2011) studies the computational complexity of manipulating an election by cloning. Our paper complements these contributions by identifying a relationship between independence of clones and polarization.

3. Basic Model

This section sets out the basic ingredients of a canonical citizen-candidate model which will be used throughout the paper. The set up is based on Osborne and Slivinski (1996).

3.1. **Basic Set-up.** Take the policy space X to be unidimensional, say $X = [0, 1]$.[5] The electorate $\mathcal{N}$ consists of a continuum of citizens, its size normalized to one.[6] Each citizen $\ell \in \mathcal{N}$ has an ideal policy $x_\ell \in X$ and obtains utility

$$u^\ell (x) = u (|x - x_\ell|)$$

from policy $x \in X$. We assume u is a strictly decreasing and concave function and use a normalization such that $u(0) = 0$. Citizens' ideal policies are distributed according to some cumulative distribution function $F(\cdot)$ over the support X; we assume $F(\cdot)$ to be continuous and strictly increasing over X. We denote the median citizen's ideal policy by μ, and let $\mu = 1/2$.

The set of candidates running in an election, denoted $\mathcal{C}$, is endogenously derived from the simultaneous and independent entry decisions of *potential* candidates who belong to an exogenously given set denoted by $\mathcal{P}$. In line with the citizen-candidate approach we take the potential candidates to be citizens (i.e., $\mathcal{P} \subset \mathcal{N}$). As a result, each potential candidate $i \in \mathcal{P}$ has an ideal policy x_i and obtains utility $u^i(x)$ from $x \in X$. In line with the central

---

[5]Assuming a unidimensional policy space is made to facilitate comparison with related contributions (e.g., Cox 1987 and 1990, Myerson and Weber 1993), in which the policy space is assumed to be a closed interval on the real line.

[6]Assuming a continuum of citizens is made to be consistent with the sincere voting assumption. Indeed, a sincere voting profile (as any other voting profile) is then a Nash equilibrium given that no vote can ever be pivotal.

tenet of the citizen-candidate approach a candidate cannot credibly commit implementing any policy.[7]

To make our analysis as straightforward as possible, we assume that the ideal policy positions of the potential candidates belong to one of the following: a left position x_L, a middle or moderate position x_M and a right position x_R.[8] Moreover, we let $x_M = 1/2$ and $x_L = (1 - x_R) \in [0, 1/2)$, i.e., the moderate position corresponds to the median μ and the other two positions are symmetric around μ. There is a finite number $p \geq 1$ of potential candidates at each position.[9] In addition to the utility $u^i(x)$ he obtains from the chosen policy x, a candidate i obtains a benefit $\beta \geq 0$ from winning the election. In our basic formulation we assume purely policy-motivated candidates, i.e., $\beta = 0$. We will subsequently relax this condition, i.e., let $\beta > 0$, and check for robustness of our conclusions.

The policy-making process has three stages. In the first stage each potential candidate decides whether to stand for election. Candidacy decisions occur simultaneously and independently. A potential candidate who chooses to stand for election incurs a utility cost $\delta > 0$. If no candidate enters the race, then the game ends and a default policy x_0 is implemented. Following Osborne and Slivinski (1996), we assume without loss of generality that all citizens obtain a utility of $-\infty$ from the default policy. In the second stage an election is held. Voting is sincere, the precise meaning of which will be described in Definition 1. In the third and final stage the elected candidate chooses and implements a policy. We describe below the structure of each stage, working backwards.

Policy selection stage. Since there is no credible commitment possible, the elected candidate implements his ideal policy.

Election stage. Let $\mathcal{C} \subseteq \mathcal{P}$ denote a non-empty set of candidates who are running for office, and $c \equiv \#\mathcal{C}$ the number of candidates.

In the basic model, we consider a family of voting rules which are characterized by two parameters s and t such that $1 \leq s \leq t \leq 3p$. In an (s, t)-rule, each citizen must cast a vote to at least s candidates and can cast a vote

---

[7] Lee et al. (2004) provide empirical support for this assumption. Observe that our main conclusion—i.e., (s, t)-rules can support more, not less, polarization than the Plurality Rule—would become trivial if we were to relax this assumption. Indeed, if candidates can commit on implementing policies other than their ideal one, then only policies close to the median μ are supported by equilibria under the Plurality Rule; polarization is then minimal in Plurality Rule elections.

[8] It is important to emphasize that this assumption is made only to simplify the analysis. Indeed, the analysis with an arbitrary number of positions provides the same intuition and conlusions; it is available from the authors upon request.

[9] Assuming a finite number of potential candidates provides a justification for why potential candidates are strategic when making their candidacy decision, but sincere when making their voting decision.

for up to t candidates.[10] The candidate with the most votes is elected. Ties are broken randomly, with each tied candidate declared a winner with equal probability. Given that candidacy is endogenous, there need not be more than s or t candidates running for election. An (s,t)-rule must therefore, in essence, be considered as a family of voting rules parametrized by the number of candidates. To keep things simple, we shall require each citizen to vote for $(c-1)$ candidates if $c \leq s$, and allow them to vote for up to $(c-1)$ candidates if $c \leq t$. In Section 6 we will consider the Alternative Vote Rule.

The $(1,1)$-rule corresponds to the Plurality Rule, in which each citizen votes for one candidate. The $(1,3p)$-rule corresponds to Approval Voting, in which each citizen votes for as many candidates as she wishes. The $(3p,3p)$-rule corresponds to Negative Voting, in which each citizen votes against one candidate (i.e., votes for all but one candidate).

Let $\alpha^{\ell}(\mathcal{C}) = \left(\alpha_1^{\ell}, ..., \alpha_c^{\ell}\right)$ denote citizen ℓ's voting decision, where $\alpha_i^{\ell} = 1$ means citizen ℓ casts a vote for candidate i and $\alpha_i^{\ell} = 0$ means she does not. In an (s,t)-rule, it must be that

$$\min\{s, c-1\} \leq \sum_{i \in \mathcal{C}} \alpha_i^{\ell} \leq \min\{t, c-1\}$$

We denote the profile of voting decisions by $\alpha(\mathcal{C})$.

Each citizen votes sincerely, i.e., reports her preferences truthfully. We borrow the definition of sincere voting from Brams (1994). According to this definition, a citizen votes sincerely if whenever she votes for a candidate i she also votes for every candidate she prefers to i.[11] Formally,

Definition 1 (**Sincere Voting**). *A voting decision for citizen ℓ, $\alpha^{\ell}(\mathcal{C})$, is sincere if for each pair of candidates i and j with $u^{\ell}(x_j) > u^{\ell}(x_i)$, we have*

$$\alpha_i^{\ell} = 1 \Rightarrow \alpha_j^{\ell} = 1.$$

A voting profile $\alpha(\mathcal{C})$ is sincere if $\alpha^{\ell}(\mathcal{C})$ is sincere for each citizen $\ell \in \mathcal{N}$.

To keep algebra simple, a citizen who is indifferent between two candidates votes for each with the same probability. Also, when partial abstention is allowed (i.e., $s < t$), a citizen votes for as many of her most favorite candidates as possible and for as few of her least favorite candidates as possible. The latter restriction would be analogous to ruling out weakly dominated strategies were the electorate be very large but finite.

Finally, the set of candidates with the most votes is called the winning set and is denoted by $W(\mathcal{C}, \alpha)$. Given our random tie-breaking rule, the probability that candidate i is elected the policy maker is $\pi_i(\mathcal{C}, \alpha) = \frac{1}{\#W(\mathcal{C}, \alpha)}$ if $i \in W(\mathcal{C}, \alpha)$ and 0 otherwise.

[10] Observe that $s \geq 1$ rules out complete abstention. Such abstention can be ignored here since voting is costless and information is complete; if population were to be very large but finite, complete abstention would be weakly dominated by a sincere vote.

[11] In Section 5 of the paper we discuss an alternative to this definition of sincere voting.

Candidacy stage. Let $e_i \in \{0,1\}$ denote the candidacy decision of potential candidate i: $e_i = 1$ indicates his decision to stand for election. The candidacy profile is denoted by $e = (e_i)_{i \in \mathcal{P}}$ and the associated set of candidates by $\mathcal{C}(e) = \{i \in \mathcal{P} : e_i = 1\}$. We sometimes write $e = (e_i, e_{-i})$, where e_{-i} denotes the candidacy profile of potential candidates other than i.

Given a candidacy profile e and a voting profile α, the expected utility of potential candidate i is given by

$$U^i(e,\alpha) = \sum_{j \in \mathcal{P} \cup \{0\}} \pi_j(\mathcal{C}(e), \alpha(\mathcal{C}(e)))\, u^i(x_j) + \pi_i(\mathcal{C}(e), \alpha(\mathcal{C}(e)))\, \beta - e_i \delta$$

where $\pi_0(\mathcal{C}(e), \alpha(\mathcal{C}(e))) = 1$ if $\mathcal{C}(e) = \emptyset$ and 0 otherwise (i.e., it denotes the probability that the default policy is selected).

A candidacy profile e^* is a candidacy equilibrium given a voting profile $\alpha(.)$ if for every potential candidate $i \in \mathcal{P}$,

$$U^i\left(e_i^*, e_{-i}^*; \alpha\right) \geq U^i\left(e_i, e_{-i}^*; \alpha\right) \qquad \text{for all } e_i \in \{0,1\}.$$

To simplify algebra, we assume that a potential candidate who is indifferent as to whether to become a candidate, stands for election.

Definition 2 (**Political Equilibrium**). *A political equilibrium (hereafter equilibrium) is a pair $(e^*, \alpha^*(.))$ where 1) $\alpha^*(\mathcal{C})$ is a sincere voting profile for every non-empty set of candidates $\mathcal{C}$, and 2) e^* is a candidacy equilibrium given $\alpha^*(.)$.*

3.2. **Polarization.** It remains to define our concept of polarization. Intuitively, we would say that a voting rule supports more polarization than another voting rule if it supports the adoption of more extreme policies and does not support the adoption of more moderate policies. A policy is more extreme (resp. moderate) than another one if it lies further away from (resp. closer to) the median μ.

Given that the middle platform coincides with the median μ and that the other two platforms are symmetric around μ, we can associate the extent of polarization supported by a voting rule with the set of all possible left platforms the voting rule can support. We say that a platform/policy x can be supported if for a given configuration of platforms $\left(x, \frac{1}{2}, 1-x\right)$, an equilibrium exists in which a candidate at x is elected with positive probability. Let $Y(s,t) \subseteq [0, 1/2)$ denote the set of (left) platforms which can be supported under an (s,t)-rule.

We are now ready to formalize our concept of polarization.

Definition 3 (**Polarization**). *An (s,t)-rule supports more polarization than an (s',t')-rule if and only if $Y(s,t) \neq Y(s',t')$, $Y(s,t) \neq \emptyset$ and the following two conditions hold:*

1. *for each $x \in Y(s,t) \setminus Y(s',t')$, $|x-\mu| > |y-\mu|$ for all $y \in Y(s',t')$; and*
2. *for each $y \in Y(s',t') \setminus Y(s,t)$, $|y-\mu| < |x-\mu|$ for all $x \in Y(s,t)$.*

In words, it must be that 1) each policy supported by the (s,t)-rule but not by the (s',t')-rule is more extreme than any of the policies supported by the (s',t')-rule, and 2) each policy supported by the (s',t')-rule but not by the (s,t)-rule is more moderate than any of the policies supported by the (s,t)-rule.

Observe that for $F(.)$ symmetric around the median μ, a more extreme policy is associated with a lower (utilitarian or Rawlsian) social welfare. This provides a normative argument in support of less polarization.

4. An Illustrative Example

Before proceeding with the formal analysis we provide an example to illustrate the intuition underlying our main results. The specific functional forms and parameter values below have been chosen to make our calculations as straightforward as possible. The main claim, however, generalizes.

We consider a community that must elect a representative to choose a tax rate $x \in X = [0,1]$. Each citizen ℓ has an ideal tax rate $x_\ell \in X$ and preferences represented by $u^\ell(x) = -(x - x_\ell)^2$. Ideal tax rates are distributed over X according to a density function

$$f(x) = \begin{cases} 5/6 & \text{for } x \in [0, 2/5) \cup (3/5, 1] \\ 5/3 & \text{for } x \in [2/5, 3/5]. \end{cases}$$

There are six potential candidates, with two at each of the three positions $x_L \in [0, 1/2)$, $x_M = 1/2$ and $x_R = (1 - x_L) \in (1/2, 1]$. The utility cost of candidacy is $\delta = 1/50$.

Our main point of interest in this example concerns comparing the Plurality Rule ($s = t = 1$) with the $(2,2)$-rule. Our findings are summarized in Figure 3 which presents the set of equilibrium tax rates under the two rules. It is clear from the figure that relative to the Plurality Rule, the $(2,2)$-rule can support the adoption of more extreme tax rates. Specifically, while the lowest and the highest tax rates supported under the Plurality Rule are 30% and 70% respectively, the $(2,2)$-rule supports tax rates as low as 22% and as high as 78%. To understand this finding, we partition the equilibrium set into three subsets—the subsets of 1-, 2- and 3-position equilibria, with candidates running at one, two and three positions, respectively—and characterize each subset.

We start by characterizing the 1-position equilibria. These equilibria exhibit two key features. First, only one candidate stands for election; a second potential candidate situated at the same position would not want to enter the race since he would have to bear the candidacy cost, while the adopted tax rate would remain the same. Second, the position must be close enough to the median $\mu = 1/2$ so that a potential candidate at x_M or x_R, even though he is certain to win outright or tie for first place respectively, does not want to enter the race given the entry cost. The first feature implies the 1-position equilibria are the same under both rules. The second feature implies the 1-position equilibria are the least polarized equilibria. In our

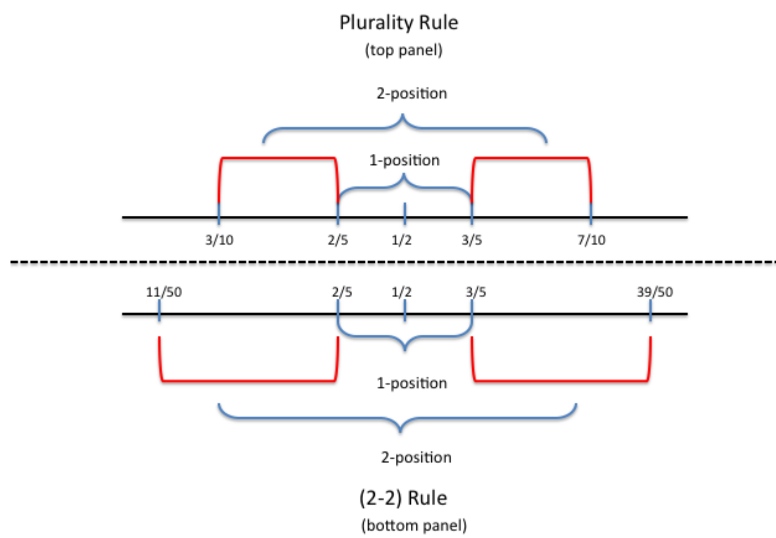

FIGURE 3. Comparing Outcomes under Endogenous Candidacy

example, the set of tax rates supported by 1-position equilibria corresponds to the interval $(2/5, 3/5)$.

There are no 3-position equilibria under either rule. This is because a candidate at x_L or x_R would be better off deviating and not running for election. Indeed, this would improve the electoral prospects of the other candidate at his position (if he runs for election) or of candidate(s) at x_M (otherwise). Thus, by not running for election he would not only save the candidacy cost but, given the concavity of the utility function, would also get a higher expected utility.

It remains to characterize the 2-position equilibria. It is clear from our characterization of the 1- and 3-position equilibria that all the differences between the two rules lie in the 2-position equilibria. These equilibria exhibit three key features. First, candidates must be standing at x_L and x_R so that they split the electorate evenly and tie for first place. Second, the two positions must be far enough apart so that each active candidate is willing to bear the candidacy cost and contest the election. Third, the two positions must not be so far apart that a candidate at x_M is both willing to enter the race and able to win it. The second feature puts a lower bound on polarization, which in our example requires $x_L \leq 2/5$ and $x_R \geq 3/5$. The third feature puts an upper bound on polarization. We now show that this upper bound is higher under the $(2, 2)$-rule than under the Plurality Rule.

To characterize the upper bound on polarization, we first need to determine the number of candidates at each of the two positions. Under the Plurality Rule, there is one candidate at each position. This is because two candidates at the same position would split their votes and help the election of the candidate(s) at the other position. By contrast, under the $(2, 2)$-rule there are two candidates at each position. This is because two candidates at a position no longer split their votes (given that citizens now vote for two candidates). To see this, suppose there was only one candidate at each position. Each candidate would then be elected with probability 1/2. Suppose the second potential candidate at x_L were to enter the race. Preferring x_L to x_R, every citizen ℓ with ideal tax rate $x_\ell < 1/2$ would cast her two votes for the two candidates at x_L. Preferring x_R to x_L, every citizen ℓ with ideal tax rate $x_\ell > 1/2$ would vote for the candidate at x_R and would have to cast her second vote for a candidate at x_L. The vote total of the candidate at x_R would be equal to 1/2, whereas the vote total of each candidate at x_L would be equal to $1/2 + (1/2)(1/2) = 3/4$. Each of the two candidates at x_L would then be elected with probability 1/2, and the second potential candidate at x_L would therefore want to enter the race since winning with probability 1/2 was sufficient to induce the other potential candidate at x_L to stand for election. The same argument applies for the second potential candidate at x_R.

To characterize the upper bound on polarization, it remains to determine when a candidate at x_M would attract enough votes to win the election. To do so, suppose a potential candidate at x_M enters the race. Notice citizens with ideal tax rate $x < \frac{x_L+1/2}{2}$ prefer x_L to x_M to x_R; citizens with ideal tax rate $x \in \left(\frac{x_L+1/2}{2}, \frac{x_R+1/2}{2}\right)$ prefer x_M to x_L and x_R; and citizens with ideal tax rate $x > \frac{x_R+1/2}{2}$ prefer x_R to x_M to x_L. We call the former leftists, the second centrists and the latter rightists.

Figure 4 presents the distribution of votes in a 2-position equilibrium under the Plurality Rule. Leftists vote for the candidate at x_L, centrists for the candidate at x_M, and rightists for the candidate at x_R. Given the distribution of ideal tax rates F, the candidate at x_M is elected outright and wants to enter the race iff $x_L < 3/10$; his expected utility gain exceeds $0 - \left[-(1/2 - 3/10)^2\right] = 1/25$, which is bigger than the candidacy cost $\delta = 1/50$.[12] Under the Plurality Rule, the set of tax rates supported by 2-position equilibria is then given by $[3/10, 2/5] \cup [3/5, 7/10]$.

Figure 5 presents the distribution of votes under the $(2, 2)$-rule. Leftists cast their two votes for the two candidates at x_L, and rightists for the two candidates at x_R. Centrists vote for the candidate at x_M and cast their second vote either for a candidate at x_L (if their ideal tax rate $x < 1/2$) or for a candidate at x_R (if their ideal tax rate $x > 1/2$). Given the distribution of ideal tax rates F, the candidate at x_M is elected iff $x_L < 11/50$, which is

[12]When $x_L = 3/10$, all three candidates tie for first place and each is elected with probability 1/3. In this case, the candidate at x_M does not want to enter the race since his expected utility gain is equal to $(1/3)(1/25) = 1/75$, which is smaller than the candidacy cost δ.

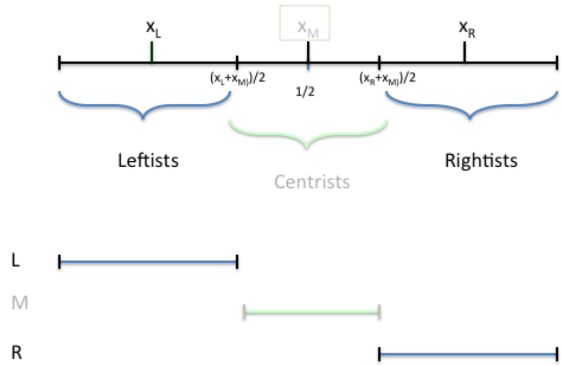

FIGURE 4. Endogenous Candidacy: Plurality Rule

lower than under the Plurality Rule. The key to understand this difference is to note that the vote total of the candidate at x_M is the same under both rules. This is because under both rules, the candidate(s) at x_L and x_R capture all the votes from the leftists and the rightists, leaving the candidate at x_M with votes from the centrists only. At the same time, the vote total of each candidate at x_L and x_R is bigger under the $(2,2)$-rule than under the Plurality Rule since they receive votes from the centrists under the $(2,2)$-rule but not under the Plurality Rule. As a result, the vote share of the candidate at x_M is smaller under the $(2,2)$-rule, implying that x_L and x_R must be more polarized for the candidate at x_M to win the election. Under the $(2,2)$-rule, the set of tax rates supported by 2-position equilibria is given by $[11/50, 2/5] \cup [3/5, 39/50]$.

To sum up, increasing the number of votes each voter casts weakens vote-splitting, inducing multiple candidacies. In turn, these multiple candidacies reduce the vote share of a moderate candidate, thereby allowing for more polarization.

The comparison of Figures 4 and 5, where candidacy is endogenous, with Figures 1 and 2 in the Introduction, where candidacy is exogenous, sheds light on the difference between our results and previous findings in the literature. When candidacy is exogenous, the Plurality Rule supports more polarization than the $(2,2)$-rule. The opposite is true when candidacy is endogenous. This difference follows from the multiple candidacies under the $(2,2)$-rule. Indeed, Figures 1 and 4 under the Plurality Rule are identical; this is because vote-splitting deters two candidates at the same position from standing for election. By contrast, Figures 2 and 5 under the $(2,2)$-rule differ; this is because under the $(2,2)$-rule, vote-splitting occurs only

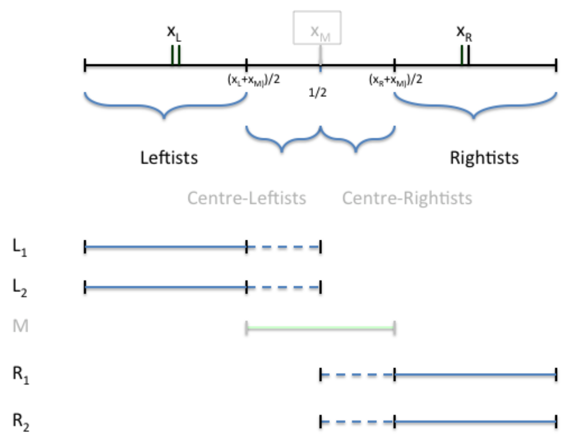

FIGURE 5. Endogenous Candidacy: (2,2)-Rule

when three or more candidates at the same position stand for election, inducing a second candidate at each of the left and right positions to enter the race. Differences in candidacy decisions across voting rules are not taken into account under the exogenous candidacy assumption.

Is our example robust to the introduction of rents from office, in particular, when $\beta > \delta$?[13] The answer is a 'yes' at least so long as $\beta < 2\delta$. Here we shall provide only a brief sketch of the argument, leaving the more general analysis to the next section. Note that when $\beta < 2\delta$, the 1-position equilibria under both voting rules are identical: there is exactly one candidate running at the median position. It can also be shown that, as before, there are no 3-position equilibria under either voting rule. Hence, we need to look only at the 2-position equilibria.

In the preceding analysis, the most polarizing positions under the 2-position equilibria were characterized by the constraint that a potential entrant, were he to enter, would *at most* achieve a tie for the top position with at least two other contenders. With $\beta < 2\delta$, it is still not worthwhile to enter the race when there are two or more candidates already in the race. Hence, the most polarizing positions under each voting rule are the same as when $\beta = 0$. The comparison, therefore, rests on the least polarizing outcomes. These are characterized by the constraint that the candidates in the race do not prefer to drop out. Note that there are two candidates at each position under the (2,2)-rule but only one candidate at each position under the Plurality Rule. It follows that each candidate under the former rule wins with probability 1/4 while under the latter rule he wins with probability 1/2. Hence, *holding the locations constant* in a 2-position equilibrium

[13]When $0 < \beta \leq \delta$ even a certain prospect of being elected does not encourage a potential candidate to enter the race, hence we focus here on the more interesting case where $\beta > \delta$.

the expected payoff of a candidate is greater under the Plurality Rule than under the (2,2)-rule which leads to the former supporting equilibrium positions which are closer to each other than the latter. This means that the most moderate outcome under the Plurality Rule is more moderate than that under the (2,2)-rule. Thus, in overall comparison, the Plurality Rule is less polarizing than the (2,2)-rule.

5. Analysis

In this section we start by characterizing the set of equilibria under each (s,t)-rule. We then discuss the main implications of our analysis with respect to polarization. Finally, we discuss the robustness of our conclusions to including rents from office.

5.1. **Characterization of equilibria.**

We proceed by partitioning the equilibrium set into three subsets: the 1-, 2- and 3-position equilibria, with candidates running at one, two and three positions, respectively.

We begin by providing a complete characterization of the 1-position equilibria. Our first lemma shows that in any 1-position equilibrium there is a single candidate whose ideal policy is not too extreme (given the candidacy cost δ).

Lemma 1. *In any 1-position equilibrium a single candidate runs unopposed. An equilibrium in which candidate $i \in \mathcal{P}$ runs unopposed exists if and only if*

(1) $x_i = x_M$, *or*
(2) $x_i \in \{x_L, x_R\}$ *and* $\delta > -\frac{u(|x_L - x_R|)}{2}$.

In a 1-position equilibrium all candidates must share the same ideal policy. If multiple candidates were running for election, all but one would be better off dropping out since their ideal policy would still be adopted with probability one, but they would save on the candidacy cost.

To guarantee that no other potential candidate wants to contest the election, the ideal policy of the single candidate, x_i, must not be too extreme. Either $x_i = x_M$ $(= \mu)$, in which case any other candidate at x_L or x_R would be defeated. Hence Condition (1). Or $x_i = x_L$ (resp. x_R), in which case another candidate at x_R (resp. x_L) would tie for first place and be elected with probability 1/2. Condition (2) guarantees that the candidacy cost exceeds his expected utility gain from contesting the election.[14]

Notice that neither of the conditions in Lemma 1 depends on the voting rule and, therefore, that the set of 1-position equilibria is equivalent under all (s,t)-rules. This happens because a single candidate runs for election. As a result, only sets of zero, one and two candidates are key for equilibrium

[14]Observe that in the latter case, a candidate at x_M would defeat candidate i. However, the concavity of the utility function $u(.)$ implies $-\frac{u(|x_L - x_R|)}{2} \geq -u(|x_L - x_M|)$; Condition (2) is therefore sufficient to deter him from contesting the election.

characterization. With at most two candidates, the election outcome is the same under every (s,t)-rule.

We now provide a complete characterization of the 2-position equilibria. Our next lemma establishes that in a 2-position equilibrium, candidates are found at x_L and x_R, not at x_M, and that the two positions are neither too polarized nor too close to each other. The number of candidates and the degree of polarization are shown to depend on the voting rule.

Before stating the result, we introduce some extra notation. Let $\underline{x} \equiv \frac{x_L+x_M}{2}$ be the ideal policy of citizens who are indifferent between x_L and x_M. Given the single-peakedness of preferences, every citizen ℓ with ideal policy $x_\ell < \underline{x}$ prefers x_L to x_M, and every citizen ℓ with ideal policy $x_\ell > \underline{x}$ prefers x_M to x_L. Likewise, let $\overline{x} \equiv \frac{x_M+x_R}{2}$ be the ideal policy of citizens who are indifferent between x_M and x_R. Observe that $\underline{x} < 1/2 < \overline{x}$.

Lemma 2. *Let the election be held under a (s,t)-rule. A 2-position equilibrium exists if and only if*

(1) $x_i \in \{x_L, x_R\}$ *for every candidate* $i \in \mathcal{C}(e)$.

(2) *The number of candidates at* x_L, c_L, *and the number of candidates at* x_R, c_R, *are such that*

$$\min\{s,p\} \leq c_L = c_R \leq \min\{t,p\}$$

and

$$\begin{cases} -\frac{u(|x_L-x_R|)}{2} \geq \delta & \text{if } c_L = c_R = \min\{s,p\} \\ -\frac{u(|x_L-x_R|)}{2(c_L+c_R-1)} \geq \delta & \text{if } c_L = c_R > s \\ -\frac{u(|x_L-x_R|)}{2(c_L+c_R+1)} < \delta & \text{if } c_L = c_R < \min\{t,p\}. \end{cases}$$

(3) *Candidates' positions* x_L *and* x_R *are such that*

(a) *when* $c_L = c_R = t$ *and* $F(\overline{x}) \geq 1 - F(\underline{x})$,

- *either* $F(\overline{x}) > \frac{1}{2}+\frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$ *and* $\delta > -u(|x_L - x_M|)$,
- *or* $F(\overline{x}) = \frac{1}{2}+\frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right] > 1 - F(\underline{x})$ *and* $\delta > -\frac{u(|x_L-x_M|)}{t+1}$,
- *or* $F(\overline{x}) = \frac{1}{2}+\frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right] = 1 - F(\underline{x})$ *and* $\delta > -\frac{u(|x_L-x_M|)}{2t+1}$,
- *or* $F(\overline{x}) < \frac{1}{2}+\frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$,

with similar conditions when $F(\overline{x}) < 1 - F(\underline{x})$.

(b) *when* $s \leq c_L = c_R < t$,

- *either* $F(\overline{x}) > \frac{1}{2} + F(\underline{x})$ *and* $\delta > -u(|x_L - x_M|)$,
- *or* $F(\overline{x}) = \frac{1}{2} + F(\underline{x})$ *and* $\delta > -\frac{u(|x_L-x_M|)}{c_L+c_R+1}$,
- *or* $F(\overline{x}) < \frac{1}{2} + F(\underline{x})$.

(c) *when* $c_L = c_R = p < s$, $\delta > -u(|x_L - x_M|)$.

Condition (1) requires that candidates are standing at x_L and x_R so they split the electorate equally. The key to understand this condition is to observe that all candidates must be tying for first place; sure losers would be better off not running. Assume by way of contradiction that there were candidates at x_M and, say, x_L. The candidates at x_M would be preferred by a majority of citizens (specifically, every citizen ℓ with ideal policy $x_\ell > \underline{x}$). For all candidates to tie, there must then be more candidates at x_L than at x_M so that each citizen preferring x_M casts more votes for the candidates at x_L than the number of votes each citizen preferring x_L casts for the candidates at x_M. That there are less candidates at x_M than at x_L implies that x_M is adopted with probability less than 1/2 and that some potential candidates at x_M do not stand for election. However, if one of those potential candidates were to enter the race, he would ensure that the candidate(s) at x_M capture enough votes to win the election. Such a potential candidate would therefore want to enter the race since his candidacy would increase the probability that x_M is adopted by more than 1/2 (from a probability less than 1/2 to probability 1), a contradiction.

Condition (2) imposes restrictions on the number of candidates at each position and requires the two positions to be sufficiently polarized. First, there is an equal number of candidates at each position since candidacy incentives are the same at both positions. Second, the maximum number of votes that each citizen can cast, t, puts an upper-bound on the number of candidates standing at a position. More than t candidates at a position would split votes (given that a citizen cannot vote for more than t candidates), helping the election of candidates at the other position. Candidates in excess of t would therefore be better off not running. Third, the minimum number of votes that each citizen can cast, s, puts a lower-bound on the number of candidates standing at a position. This lower-bound is equal to s if s is lower than p, the number of potential candidates at each position, and is equal to p, otherwise. With less than s (and p) candidates at a position, another potential candidate at the position would want to enter the race since he would capture votes that would otherwise go to the candidates at the other position, and would then ensure that a candidate at his position is elected. Fourth, the two positions must be sufficiently polarized so that the expected utility gain from standing for election — equal to $-\frac{u(|x_L - x_R|)}{2}$ — exceeds the candidacy cost δ. Finally, the last two inequalities in Condition (2) ensure that 1) no candidate in excess of s would be better off not running, and 2) no other potential candidate at x_L and x_R would want to enter the race.

Finally, Condition (3) requires the two positions to be close enough to each other so that no potential candidate at x_M wants to enter the race. Two types of situations must be considered. In the first type, there are s or more candidates at each position (cases (a) and (b) in the statement). In such situations, the odds a candidate at x_M is elected are minimized when citizens vote the following way: citizens with ideal policy $x < \underline{x}$ vote only for the candidates at x_L; citizens with ideal policy $x > \overline{x}$ vote only for the candidates at x_R; and citizens with ideal policy $x \in (\underline{x}, \overline{x})$ vote for the candidate at x_M and for as many of the candidates at x_L or x_R

(whichever they prefer) as they can. The different sets of conditions in the statement correspond to degrees of polarization for which a candidate at x_M would win outright, tie with the candidates at x_L and/or x_R, and be defeated. In the second type of situations, there are less than s candidates at each position (case (c) in the statement). In such situations a candidate at x_M would receive unanimous vote and win outright. For him to not enter the race it must be that the candidacy cost δ exceeds his utility gain $[0 - u(|x_L - x_M|)]$. Observe that the Plurality Rule belongs to case (a) (since $s = t = 1$), Approval Voting to case (b) (since $s = 1$ and $t = 3p$) and Negative Voting to case (c) (since $s = t = 3p$).

Remain the 3-position equilibria. Our next lemma establishes that 3-position equilibria do not exist under (s, t)-rules where $s = t$.

Lemma 3. *Let the election be held under a (s, t)-rule where $s = t$. Then, there are no 3-position equilibria.*

The intuition is as follows. If a 3-position equilibrium were to exist, there would be strictly more than t candidates on one side and less than t candidates on the other side, i.e., either $c_L + c_M > t \geq c_R + c_M$ or $c_R + c_M > t \geq c_L + c_M$. That there must be strictly more than t candidates on one side follows because otherwise the candidates at x_M would receive unanimous votes and x_M would be adopted with probability 1; a candidate at x_L or x_R would therefore be better off dropping out since the election outcome would be ex ante the same while he would save on the candidacy cost. That there must be less than t candidates on the other side follows because a candidate at x_L or x_R would otherwise be better off deviating and not running for election since the votes he would have received would go to the other candidates at his position (if any) and to the candidates at x_M, improving the electoral prospects of those candidates. He would then save on the candidacy cost and would increase the probability a more-preferred policy is adopted. All this implies that $c_L \neq c_R$. However, this cannot be supported in equilibrium; another potential candidate at the position with less candidates would want to enter the race.

When the election is held under a (s, t)-rule where $s < t$, 3-position equilibria are possible.[15] The key difference between (s, t)-rules where $s < t$ and those where $s = t$ lies in the way the votes of a candidate who drops out are transferred. When $s = t$, those votes go to other candidates since voters are forced to cast all t votes. By contrast, when $s < t$, some of these votes may not go to other candidates since voters are forced to cast only s votes; they may choose to not cast the vote they would otherwise have cast to the deviating candidate.

5.2. **Polarization.** We can now discuss the implications of our analysis. We first consider (s, t)-rules where $s = t > 1$, i.e., voters are given multiple

[15]Example 1 in the supplementary material contains a 3-position equilibrium under Approval Voting.

votes and are asked to cast them all. We second consider (s,t)-rules where $s < t$, i.e., voters are given multiple votes but have the option to not cast them all.

Our first proposition establishes that (s,t)-rules where $s = t > 1$ can support more or less polarization than the Plurality Rule depending on whether the number of votes t is larger or smaller than p, the number of potential candidates at each position.

Proposition 1. *Let the election be held under a (s,t)-rule where $s = t > 1$.*

(1) *If $s = t \leq p$, the (s,t)-rule supports more polarization than the Plurality Rule.*
(2) *If $s = t > p$, the (s,t)-rule supports less polarization than the Plurality Rule.*

To understand this result, recall from Lemma 3 that all equilibria are 1- and 2-position equilibria. Also, recall from Lemma 1 that 1-position equilibria are equivalent under all voting rules. Finally, comparing Condition (2) in Lemma 1 with Condition (2) in Lemma 2 indicates that 1-position equilibria are strictly less polarized than 2-position equilibria. It follows that the extent of polarization a voting rule can support is determined by its 2-position equilibria. From now on, we shall therefore focus on 2-position equilibria.

The key to understand the result is to note that the number of candidates standing at each of the two positions varies with the (s,t)-rule. On the one hand, the number of votes t puts an upper-bound on the number of candidates standing at a position. This is because more than t candidates at a position would split votes, thereby helping the election of the candidates at the other position; candidates in excess of t would be better off dropping out. On the other hand, the number of votes s puts a lower-bound on the number of candidates willing to stand for election at a position. This is because with less than s candidates at a position, say x_L, voters who prefer x_L to x_R would be forced to cast some of their votes to candidates at x_R. Other potential candidates at x_L (if any) would therefore be better off entering the race since they would capture those votes, and so worsen the electoral prospects of the candidates at x_R. To sum up, the number of candidates at each position is equal to either t (if $p \geq s = t$) or p (if $p < s = t$).

We are now ready to discuss the first part of Proposition 1. Under the Plurality Rule ($s = t = 1$) there is one candidate standing at x_L and another one at x_R. By contrast, under a (s,t)-rule where $1 < s = t \leq p$, there are t candidates at each of the two positions. This difference in the number of candidates standing for election is what differentiates our approach with models where the set of candidates is exogenous and kept fixed across voting rules.

The key is to note that the vote total of the candidate at x_M is the same under both rules, whereas each of the candidates at x_L and x_R receives more votes under the (s,t)-rule than under the Plurality Rule. The vote total of the candidate at x_M is the same under both rules because in each case, the number of candidates at x_L and at x_R is equal to the number of votes a voter casts. As a result, the candidates at x_L and at x_R capture all the votes from the citizens on the left ($x_\ell < \underline{x}$) and on the right ($x_\ell > \overline{x}$). Only the citizens in the middle ($x_\ell \in (\underline{x}, \overline{x})$) cast a vote for the candidate at x_M. That each of the candidates at x_L and x_R receives more votes under the (s,t)-rule follows because they still receive a vote from every citizen on the left or on the right, but they also receive votes from citizens in the middle.

It follows that the vote share of the candidate at x_M is smaller under the (s,t)-rule than under the Plurality Rule. The opposite is true for each of the candidates at x_L and x_R. As a result, x_L and x_R can be more polarized and still deter a potential candidate at x_M from entering the race when the election is held under the (s,t)-rule compared to when the election is held under the Plurality Rule. Thus, a (s,t)-rule where $1 < s = t \leq p$ can support more polarization than the Plurality Rule.

It remains to discuss the second part of Proposition 1. Under a (s,t)-rule where $s = t > p$, a candidate at x_M would receive unanimous votes and would be elected outright. This is because there cannot be more than p candidates at x_L and at x_R. Given that $s > p$, every citizen ℓ on the left ($x_\ell < \underline{x}$) and on the right ($x_\ell > \overline{x}$) would have to cast a vote for the candidate at x_M. A potential candidate at x_M is therefore deterred from entering the race only if x_L and x_R are not too extreme so that the utility gain for a candidate at x_M is smaller than the candidacy cost. Thus, a (s,t)-rule where $s = t > p$ supports less polarization than the Plurality Rule.

Observe that for (s,t)-rules where $s = t > p$, the argument is similar to the one in models where candidacy is exogenous. This is because the assumption of exogenous candidacy is equivalent to setting $p = 1$ in our model.

Our second proposition establishes that allowing partial abstention ($s < t$) — i.e., giving every citizen the option to not cast all her votes — helps support more polarization.

Proposition 2. *The (s,t)-rule where $s < t$ supports more polarization than the (s,s)- and (t,t)-rules.*

To understand the intuition behind Proposition 2, recall that Lemma 1 and Lemma 2 imply that 1-position equilibria are equivalent under all voting rules and are strictly less polarized than any 2-position equilibrium.

Let us look at the 2-position equilibria. The (s,t)-rule where $s < t$ can support more polarized 2-position equilibria than the (t,t)-rule. To understand why, consider a situation where $p \geq t$.[16] Recall from Lemma 2 and our discussion of Proposition 1 that under the (t,t)-rule, there would be t

[16]Situations where $p < t$ are easier to understand. Since $p < t$, a candidate at x_M entering the race would receive unanimous votes and win the election outright. Only the

candidates standing at each position. By contrast, when $s < t$, the number of candidates standing at a position lies somewhere between s and t, i.e., $c_L = c_R \in \{s, ..., t\}$.

The difference in the number of candidates standing at each position implies the electoral prospects of a candidate at x_M entering the race can be no better and can be even worse under the (s,t)-rule than under the (t,t)-rule. This is trivial if $c_L = c_R = t$ since there are then as many candidates under the (s,t)-rule as under the (t,t)-rule; the vote profile is then the same. If instead $c_L = c_R < t$, the vote profile under the (s,t)-rule can take the following form: every citizen ℓ with ideal policy $x_\ell < \underline{x}$ votes only for the c_L candidates at x_L (which they can do since $c_L \geq s$); likewise, every citizen ℓ with $x_\ell > \overline{x}$ votes only for the c_R candidates at x_R; finally, every citizen ℓ with $x_\ell \in (\underline{x}, \overline{x})$ votes for the candidate at x_M and either for all c_L candidates at x_L if $x_\ell < 1/2$ (which they can do since $c_L < t$) or for all c_R candidates at x_R if $x_\ell > 1/2$.[17] This vote profile could for instance correspond to a situation where x_L and x_R are focal, and everybody anticipates the race to be between the candidates at x_L and x_R; the citizens on the left and on the right would then have no incentive to cast a vote for the candidate at x_M, while the centre-leftists would want to vote for as many of the candidates at x_L as they can and the centre-rightists for as many of the candidates at x_R as they can. Also, Fenster (1983) provides empirical support for such a vote profile in the context of elections held under Approval Voting. His explanation is that the more extreme voters are more ideological and, therefore, more reluctant to cast a vote for candidates in the middle.

Comparing the two vote profiles, we can make two observations. First, the vote total of the candidate at x_M is the same under both the (s,t)-rule and the (t,t)-rule; in both cases, the candidate at x_M receives votes only from citizens in the middle. Second, every candidate at x_L and x_R receives more votes under the (s,t)-rule than under the (t,t)-rule. This is because when $s < t$, $c_L = c_R < t$ implies the citizens in the middle can vote for **all the** candidates at x_L or x_R. By contrast, under the (t,t)-rule, $c_L = c_R = t$ implies the citizens in the middle can vote for **all but one** of the candidates at x_L and x_R.

These two observations imply that the vote share of the candidate at x_M is smaller when $s < t$, whereas the vote share of every candidate at x_L and x_R is bigger. As a result, x_L and x_R can be more polarized under the (s,t)-rule and still deter a potential candidate at x_M from entering the race. Hence, more polarized 2-position equilibria can be supported under

less polarized 2-position equilibria can therefore be supported under the (t,t)-rule. It follows trivially that the (s,t)-rule supports more polarized 2-position equilibria.

[17]Observe that the vote profile under the (t,t)-rule (as described in the discussion of Proposition 1) differs only in that the citizens in the middle vote for $(t-1)$ of the t candidates at x_L or x_R; they cannot vote for all t candidates.

the (s,t)-rule than under the (t,t)-rule. A similar argument applies to the comparison with the (s,s)-rule.[18]

Now remains the case of the 3-position equilibria. Recall from Lemma 3 that there are no 3-position equilibria under the (s,s)- and (t,t)-rules. By contrast, such equilibria can exist when $s < t$ (e.g., see Example 1 in the supplementary material). Given that the set of policies supported by 1- and 2-position equilibria is an interval centered around the median $\mu = 1/2$, every policy which is supported solely by a 3-position equilibrium is necessarily more polarized than any policy supported by a 1- or 2-position equilibrium. Hence, the existence of 3-position equilibria adds to polarization.

To sum up, a (s,t)-rule where $s < t$ supports more polarization than the (s,s)- and (t,t)-rules given that all three subsets of equilibria — 1-, 2- and 3-position equilibria — are as much or more polarized.[19]

An example illustrating Proposition 2 (Example 1) is provided in supplementary material available online.

We conclude the analysis of polarization in our baseline model by summarizing our results for the three polar (s,t)-rules, namely, the Plurality Rule ($s = t = 1$), Approval Voting ($s = 1$ and $t = 3p$) and Negative Voting ($s = t = 3p$).

---

[18] Observe that the key difference between (s,t)-rules that allow partial abstention and (s,t)-rules that don't, and a driving force behind Proposition 2, is the greater multiplicity of vote profiles under the former rules. Interestingly, the desirability of the greater multiplicity of voting profiles under Approval Voting as compared to the Plurality Rule has been the object of a heated debate. On the one hand, Donald Saari argues that the greater multiplicity of voting profiles is a vice since it makes the election outcome under Approval Voting highly indeterminate (e.g., Saari and van Newenhizen 1988, Saari 2001). On the other hand, Steven Brams argues that the greater multiplicity of voting profiles is a virtue since it makes Approval Voting responsive to voters' preferences (e.g., Brams et al. 1988, Brams and Sanver 2006).

[19] One may object that the result in Proposition 2 follows because the notion of sincere voting does not put enough restrictions on exactly how many candidates a citizen votes for. One may then want to consider an alternative definition of sincere voting, namely, pure sincerity (to use a terminology proposed in Merill and Nagel 1987). A voting decision for a citizen is purely sincere if 1) she votes for as many as possible of the candidates from whom she gets at least as much utility as the average utility over the whole set of candidates, and 2) she votes for as few as possible of the candidates for whom she gets less utility than the average utility over the whole set of candidates. (Fishburn and Brams (1981) argues that this is the way citizens should be voting under Approval Voting.) Under pure sincerity, we find that 1) (s,t)-rules where $p < t$, whether $s < t$ or $s = t$, support less polarization than the Plurality Rule, whereas 2) (s,t)-rules where $p \geq t > s = 1$ support more polarization than the Plurality Rule. (A complete characterization of equilibria under pure sincerity is available from the authors.) This is because in the second group the same logic as under the assumption of sincere voting applies, whereas in the first group a candidate at x_M running against candidates at x_L and at x_R would necessarily receive unanimous vote and win outright. Observe that Approval Voting belongs to the first group ($t = 3p > p \geq s = 1$). It follows that whether Approval Voting would support more or less polarization than the Plurality Rule depends on the way citizens would vote. Whether citizens would vote sincerely, purely sincerely or otherwise is an empirical question.

Corollary 1. *When candidacy is endogenous and voting is sincere, Approval Voting supports more polarization than the Plurality Rule, which itself supports more polarization than Negative Voting.*

5.3. **Allowing for Rents from Office.** So far we have assumed candidates to be purely policy-motivated, i.e., the ego-rent $\beta = 0$. This assumption allowed for a sharp and clear-cut analysis that captures the key differences in candidacy incentives across voting rules and their implications for polarization. In this subsection, we add office-motivation, i.e., we set $\beta > 0$. We show that our results are robust to including rents from office so long as they are not too large.

The key is to observe that office-motivation strengthens candidacy incentives. Whether this effect dominates the differences in candidacy incentives across voting rules depends on the ego-rent β relative to the candidacy cost δ.

The implications of our analysis with respect to polarization are robust to $\beta < 2\delta$. To see this, consider first the 1-position equilibria. As when $\beta = 0$, a single candidate runs unopposed. Indeed, a second candidate at the same position would be elected with probability 1/2 and would implement the same policy. He would therefore be better off not running since his expected utility gain — equal to $\beta/2$— is smaller than the candidacy cost δ. With a single candidate running for election, 1-position equilibria are equivalent under every (s, t)-rule and are strictly less polarized than any 2-position equilibrium, as when $\beta = 0$.

Consider second the 2-position equilibria. Given that office-motivation strengthens candidacy incentives, the number of votes s still puts a lower-bound on the number of potential candidates willing to stand for election at each of the two positions. Moreover, the office-motivation is weak enough that the upper-bound on the number of candidates at a position is still equal to t. As a result, for (s, t)-rules where $s = t$, there are t (or p) candidates at each position, as when $\beta = 0$. For (s, t)-rules where $s < t$, there might be more candidates than when $\beta = 0$, but there are still no more than t candidates at each position. It follows that as when $\beta = 0$: 1) a (s, t)-rule where $s = t \leq p$ (resp. $s = t > p$) supports more (resp. less) polarized 2-position equilibria than the Plurality Rule; and 2) a (s, t)-rule where $s < t$ supports more polarized 2-position equilibria than the (s, s)- and (t, t)-rules.

Finally, consider the 3-position equilibria. As when $\beta = 0$, there are no 3-position equilibria under the (s, t)-rules where $s = t$. This is because β is sufficiently small that a candidate at x_L or x_R would still be better off not running for election so as to improve the electoral prospects of the other candidates at his position or at x_M. By contrast, 3-position equilibria may exist under (s, t)-rules where $s < t$, as when $\beta = 0$.

To sum up, the main findings of the earlier part of this section still hold when candidates enjoy not-too-large rents from holding office. An example illustrating the above discussion (Example 2) is provided in supplementary material available online.[20]

5.4. **Strategic Voting.** Our analysis thus far assumed sincere voting. This assumption has been justified in a set up with a large number of voters or in situations where voters have little access to information about the voting behavior of others (Weber 1995). Likewise, this assumption has also been justified for complex voting rules such as the Alternative Vote Rule (Bartholdi III and Orlin 1991, Van der Straeten et al. 2010). However, there are voting situations, such as board and committee meetings, and less complex voting rules, especially the Plurality Rule, for which sincere voting is not an adequate behavioral assumption, as empirical and experimental evidence suggest (e.g., Van der Straeten et al. 2010, Kawai and Watanabe 2013). How do the various voting rules compare under strategic voting behavior? In our previous work we have analyzed the citizen-candidate model with strategic voting behavior under Approval Voting (Dellis and Oak 2006) and (s,t)–rules (Dellis 2009). We shall briefly summarize here the main differences in the outcome under sincere and strategic voting and refer the interested reader to our earlier work for further details.

Let us first consider the (s,t)–rules where $s = t (> 1)$. When voting is strategic the extent of polarization under such rules is same as under the Plurality Rule. There are two forces driving this outcome: First, strategic voting eliminates the upper bound on polarization under the Plurality Rule. This happens due to the so-called *wasted vote effect*—if citizens anticipate that a potential moderate entrant will not get enough votes, they will fear wasting their vote on him and will not vote for him, thus leading to a self-fulfilling outcome with two extreme candidates running in the election. Second, as under sincere voting, multiple votes encourage multiple candidacy and therefore every 2-position outcome under the Plurality Rule can also be supported as an outcome under the (s,t)–rule with $s = t$ candidates running at each position.

When partial abstention is allowed, i.e., $s < t$, strategic voting is capable of supporting outcomes more moderate than those when partial abstention is not allowed. This feature is best understood for Approval Voting. Under Approval Voting, weak undominance requires that every citizen votes for all the candidates she prefers most and does not vote for any of the candidates she likes least. This implies that candidates at x_L and x_R receive votes

[20]When $\beta \geq 2\delta$, the differences in candidacy incentives across voting rules get (partly) dominated by the effect of office-motivation, muddying the analysis. With $\beta \geq 2\delta$, the set of 1-position equilibria need no longer be equivalent under every voting rule. This is because two or more candidates want to stand for election. As a result, the characterization of 1-position equilibria involves sets of candidates of cardinality three or more (at least two candidates, plus one entrant). With three or more candidates, different (s,t)-rules may elect different candidates. Hence, 1-position equilibria need no longer be equivalent under all voting rules. Moreover, with $\beta \geq 3\delta$, 2-position equilibria may involve candidates at x_M, and 3-position equilibria may exist even under (s,t)-rules where $s = t$.

from *at most* half the electorate; specifically, from citizens on the left of the median μ for the candidates at x_L, and from citizens on the right of the median μ for the candidates at x_R. At the same time, an entrant at x_M would receive votes from *at least* the citizens in the middle, i.e., with ideal policy $x \in [\underline{x}, \overline{x}]$. For x_L and x_R sufficiently polarized, the citizens in the middle represent a majority of the electorate, and an entrant at x_M would win the election; a potential candidate at x_M would then want to enter the race if the candidacy cost δ is not too high relative to the policy gain $-u\left(|x_L - x_M|\right)$. This puts an upper bound on polarization, offering a contrast with (s,t)-rules where $s = t$ (including the Plurality Rule), for which there is no upper bound.[21]

We conclude this section by summarizing our results for the three polar (s,t)-rules, offering a contrast with Corollary 1 where voting is assumed to be sincere.

Corollary 2. *When voting is strategic, Negative Voting supports as much polarization as the Plurality Rule which itself supports more polarization than Approval Voting.*

Thus, for those (s,t)-rules where $s = t > p$ or $s < t$, whether they would support less or more polarization than the Plurality Rule depends on whether the voting behavior would be sincere or strategic. The difference between sincere and strategic voting follows because for (s,t)-rules where $s = t$, sincere voting puts an upper-bound on polarization whereas strategic voting does not. Whether citizens would vote sincerely or strategically under the different (s,t)-rules is an empirical question.

The comparison of Corollaries 1 and 2 brings an interesting insight on an important question raised by Cox (1990): Does the impact of allowing partial abstention depend on the assumption on the voting behavior? Our analysis shows it does. Specifically, allowing partial abstention $(s < t)$ helps support more polarization when voting is sincere, but does not when voting is strategic. With respect to polarization, allowing partial abstention is therefore desirable if voting is strategic, but not if voting is sincere.

6. The Alternative Vote Rule

In the preceding analysis, we considered the large class of (s,t)-rules, which has allowed us to highlight the differential incentives for multiple candidacies and their effect on polarization. We now consider another voting rule advocated by electoral reformers, namely, the Alternative Vote Rule. We show that the Alternative Vote Rule does not induce multiple candidacies and does not support as much polarization as the Plurality Rule. This result occurs due to the fact that multiple candidacies would only lengthen

[21] An example illustrating the above discussion (Example 3) is provided in supplementary material available online.

the elimination sequence of candidates without affecting the expected policy outcome.

Under the Alternative Vote Rule, every voter rank-orders the candidates. A candidate is elected if he is ranked first on a majority of ballots. If neither candidate receives a majority of first-place votes, then the candidate with the fewest first-place votes is eliminated and his votes are transferred to the candidates who are ranked next on the ballots. This process is repeated until a candidate receives a majority of first-place votes.

We start by establishing that in every equilibrium under the Alternative Vote Rule, there is at most one candidate running at each position.

Lemma 4. *Let the election be held under the Alternative Vote Rule. In any equilibrium, $c_h \leq 1$ for $h = L, M, R$, i.e., there is no equilibrium with two or more candidates at the same position.*

The logic of the proof is simple. For 1-position equilibria, we already know that a single candidate runs for office. For 2- and 3-position equilibria, the result follows because multiple candidacies lengthen the elimination process without affecting the probability with which each policy is adopted. Multiple candidates would therefore be better off dropping out.

Lemma 4 allows us to establish:

Proposition 3. *The Alternative Vote Rule supports (as much, or) less polarization than the Plurality Rule.*

The key to understand the intuition is to note that the extent of polarization each of the two rules can support is determined by its 2-position equilibria. This is because 1-position equilibria are equivalent under both rules (since there is a single candidate) and are strictly less polarized than 2-position equilibria. Moreover, there are no 3-position equilibria under either rule. This is because the absence of multiple candidacies and the concavity of the utility function $u(.)$ imply the candidate at x_L or the one at x_R would be better off not running for election and letting the candidate at x_M win outright.[22]

As in case of the (s,t)-rules, in any 2-position equilibrium, the candidates are standing at x_L and x_R. Were it the case that the candidates are running at x_M and, another position, then a candidate at x_M would receive a majority of (first-place) votes and be elected outright, in which case the candidate at the other position would be better off not running. Moreover, x_L and x_R must not be too extreme so that an entrant at x_M would be defeated. The (first-place) vote total of an entrant at x_M would be $[F(\overline{x}) - F(\underline{x})]$, and the vote totals of the candidates at x_L and x_R

[22]Observe that under both rules, there are no more than two effective candidates. This finding is consistent with empirical evidence from U.S. elections (under the Plurality Rule) and Australian elections (under the Alternative Vote Rule) where the effective number of candidates is around two. For empirical evidence, see for instance Farrell and McAllister (2006).

would be $F(\underline{x})$ and $[1 - F(\overline{x})]$, respectively. Under the Plurality Rule, an entrant at x_M is defeated if he does not receive the plurality of votes, i.e., $[F(\overline{x}) - F(\underline{x})] < \max\{F(\underline{x}), 1 - F(\overline{x})\}$. Under the Alternative Vote Rule, an entrant at x_M is defeated if he is the first candidate to be eliminated, i.e., $[F(\overline{x}) - F(\underline{x})] < \min\{F(\underline{x}), 1 - F(\overline{x})\}$. It follows that x_L and x_R can be more polarized under the Plurality Rule than the Alternative Vote Rule, and still deter a potential candidate at x_M from entering the race. Hence, the Alternative Vote Rule supports less polarization than the Plurality Rule. The difference between the two rules need not be substantial though. For example, if ideal policies are distributed symmetrically around the median μ, then $F(\underline{x}) = 1 - F(\overline{x})$ and, therefore, $\min\{F(\underline{x}), 1 - F(\overline{x})\} = \max\{F(\underline{x}), 1 - F(\overline{x})\}$; generically, the two rules support as much polarization.

For reasons similar to those outlined in the previous section, this result is robust to allowing not-too-large rents from holding office.

7. Conclusion

In this paper we examined different voting rules and their impact on the extent of polarization in electoral outcomes. While several other papers have investigated this question, they have done so in settings where candidacy is exogenous and the set of candidates is kept fixed across voting rules. However, we know that different voting rules provide different incentives for candidates to run for election (e.g., Lijphart 1994, Dutta et al. 2001). The main contribution of this paper is to take these differential candidacy incentives into account by endogenizing the set of candidates.

We find that (s, t)-rules, i.e., voting rules where every citizen is either allowed or required to cast multiple votes, leads to multiple candidacies, an effect that exogenous candidate models are not capable of capturing. Moreover, this effect counteracts the moderating effect of multiple votes, which can lead to outcomes that are more polarized than those under the Plurality Rule. Hence, we showed that for a broad class of voting rules, which includes Approval Voting, one ought to be skeptical of their potential, relative to the Plurality Rule, in promoting policy moderation. The Alternative Vote Rule, on the other hand, deters multiple candidacies and supports less polarization than the Plurality Rule.

Our analysis also complements similar analyses conducted using the Downsian model of political competition, for instance, Cox (1990). Those papers, like ours, show that allowing partial abstention can lead to more polarizing outcomes. However, there is an important qualitative difference. That literature focuses on convergent equilibria—what we call the 1-position equilibria. Our findings, on the other hand, crucially depend on the divergent equilibria, i.e., equilibria with multiple positions. In fact, in our set up the set of convergent equilibria are identical across the different voting rules.

Hence, while the two approaches make qualitatively similar predictions regarding the relationship between partial abstention and polarization, they offer observably different predictions regarding the number and locations of candidates. In future research, one could test these predictions using empirical or experimental methods. Casual evidence, however, suggests that convergent equilibria rarely occur.

Our analysis also provides an interesting contrast between outcomes under sincere and strategic voting. In particular, we find that when voting is strategic it is not just the ability to cast more votes, but to partially abstain that yields policy moderation. In contrast, with sincere voting it is the requirement that more votes be cast that drives policy moderation. It follows that the relative ranking between Plurality Rule, Negative Voting and Approval Voting is sensitive to the underlying assumption about voting behavior being sincere or strategic.

Other avenues for future research include relaxing some of the assumptions implicit in the paper such as uni-dimensionality of the policy space, complete information and one-shot nature of the political game. These were made to make our analysis comparable to the related literature so that we could isolate the impact of endogenous candidacy. However, relaxing these assumptions will provide additional insights into the comparative properties of the various rules.

References

[1] **Bartholdi III, J. and J. Orlin.** 1991. "Single transferable vote resists strategic voting." *Social Choice and Welfare*, 8: 341-354.

[2] **Besley, T. and S. Coate.** 1997. "An economic model of representative democracy." *Quarterly Journal of Economics*, 112: 85-114.

[3] **Brams, S.** 1994. "Voting procedures." In: *Handbook of Game Theory, Volume 2*, Aumann, R. and S. Hart (Eds), Elsevier Science: 1055-1089.

[4] **Brams, S. and P. Fishburn.** 1978. "Approval Voting." *American Political Science Review*, 72: 831-847.

[5] **Brams, S., P. Fishburn and S. Merrill.** 1988. "The responsiveness of Approval Voting: Comments on Saari and Van Newenhizen." *Public Choice*, 59: 121-131.

[6] **Brams, S. and R. Sanver.** 2006. "Critical strategies under Approval Voting: Who gets ruled in and ruled out." *Electoral Studies*, 25: 287-305.

[7] **Chamberlin, J. and M. Cohen.** 1978. "Toward applicable social choice theory: A comparison of social choice functions under spatial model assumptions." *American Political Science Review*, 72: 1341-1356.

[8] **Cox, G.** 1987. "Electoral equilibrium under alternative voting institutions." *American Journal of Political Science*, 31: 82-108.

[9] **Cox, G.** 1990. "Centripetal and centrifugal incentives in electoral systems." *American Journal of Political Science*, 34: 903-935.

[10] **Dellis, A.** 2009. "Would letting people vote for multiple candidates yield policy moderation?" *Journal of Economic Theory*, 144: 772-801.

[11] **Dellis, A.** 2013. "The two-party system under alternative voting procedures." *Social Choice and Welfare*, 40: 263-284.

[12] **Dellis, A. and M. Oak.** 2006. "Approval Voting with endogenous candidates." *Games and Economic Behavior*, 54: 47-76.

[13] **Dutta, B., M. Jackson and M. Le Breton.** 2001. "Strategic candidacy and voting procedures." *Econometrica*, 69: 1013-1037.

[14] **Elkind, E., P. Faliszewski and A. Slinko.** 2011. "Cloning in elections." *Journal of Artificial Intelligence Research*, 42: 529-573.

[15] **Farrell, D. and I. McAllister.** 2006. "The Australian Electoral System. Origins, Variations and Consequences." *University of New South Wales Press*: Sidney, Australia.

[16] **Fenster, M.** 1983. "Approval Voting: Do moderates gain?" *Political Methodology*, 9: 355-376.

[17] **Fishburn, P. and S. Brams.** 1981. "Expected utility and Approval Voting." *Behavioral Science*, 26: 136-142.

[18] **Kawai, K. and Y. Watanabe.** 2013. "Inferring strategic voting." *American Economic Review*, 103: 624-662.

[19] **Laffond, G., J. Laine and J-F. Laslier.** 1996. "Composition-consistent tournament solutions and social choice functions." *Social Choice and Welfare,* 13: 75-93.

[20] **Laslier, J-F.** 2000. "Aggregation of preferences with a variable set of alternatives." *Social Choice and Welfare,* 17: 269-282.

[21] **Laslier, J-F. and R. Sanver (Eds).** 2010. *Handbook on Approval Voting.* Springer: Heidelberg.

[22] **Lee, D., E. Moretti and M. Butler.** 2004. "Do voters affect of elect policies? Evidence from the US House." *Quarterly Journal of Economics*, 119: 807-859.

[23] **Lijphart, A.** 1994. "Electoral Systems and Party Systems." *Oxford University Press*: Oxford.

[24] **Lizzeri, A. and N. Persico.** 2001. "The provision of public goods under alternative electoral incentives." *American Economic Review*, 91: 225-239.

[25] **Merrill, S.** 1988. "Making Multicandidate Elections More Democratic." *Princeton University Press*: Princeton, NJ.

[26] **Merrill, S. and J. Nagel.** 1987. "The effect of approval balloting on strategic voting under alternative decision rules." *American Political Science Review*, 81: 509-524.

[27] **Milesi-Ferretti, GM., R. Perotti and M. Rostagno.** 2002. "Electoral systems and public spending." *Quarterly Journal of Economics*, 117: 609-657.

[28] **Morelli, M.** 2004. "Party formation and policy outcomes under different electoral systems." *Review of Economic Studies*, 71: 829-853.

[29] **Myerson, R.** 1993. "Incentives to cultivate favored minorities under alternative electoral systems." *American Political Science Review*, 87: 856-869.

[30] **Myerson, R.** 2002. "Comparison of scoring rules in Poisson voting games." *Journal of Economic Theory*, 103: 219-251.

[31] **Myerson, R.** 2006. "Bipolar multicandidate elections with corruption." *Scandinavian Journal of Economics*, 108: 727-742.

[32] **Myerson, R. and R. Weber.** 1993. "A theory of voting equilibria." *American Political Science Review*, 87: 102-114.

[33] **Osborne, M. and A. Slivinski.** 1996. "A model of political competition with citizen-candidates." *Quarterly Journal of Economics*, 111: 65-96.

[34] **Pagano, M. and P. Volpin.** 2005. "The political economy of corporate governance." *American Economic Review*, 95: 1005-1030.

[35] **Saari, D.** 2001. "Decisions and Elections. Explaining the Unexpected." *Cambridge University Press*: Cambridge.

[36] **Saari, D. and J. Van Newenhizen.** 1988. "The problem of indeterminacy in approval, multiple, and truncated voting systems." *Public Choice*, 59: 101-120.

[37] **Tideman, N.** 1987. "Independence of clones as a criterion for voting rules." *Social Choice and Welfare,* 4: 185-206.

[38] **Van Der Straeten, K., J-F. Laslier, N. Sauger and A. Blais.** 2010. "Strategic, sincere, and heuristic voting under four election rules: An experimental study." *Social Choice and Welfare,* 35: 435-472.

[39] **Weber, R.** 1995. "Approval Voting." *Journal of Economic Perspectives*, 9: 39-49.

Appendix

Proof. Lemma 1.
Let (e, α) be a 1-position equilibrium. If two or more candidates with the same ideal policy were standing for election, one of them would be strictly better off deviating from his candidacy strategy and not stand for election since his ideal policy would still be adopted with probability one and he would save on the candidacy cost δ. Hence, a single candidate stands for election.

(Necessity) Let e be a candidacy profile such that $e_i = 1$ for candidate $i \in \mathcal{P}$ and $e_j = 0$ for every potential candidate $j \in \mathcal{P}$, $j \neq i$. Suppose $x_i = x_L$. If a potential candidate j with $x_j = x_R$ were to enter the race, candidates i and j would split the votes equally (since $u(|x_L - \mu|) = u(|x_R - \mu|)$) and each would be elected with probability 1/2. As a result, if condition (2) were to fail, candidate j would want to run against candidate i and would be better off deviating from his candidacy strategy. The same argument applies if $x_i = x_R$.

(Sufficiency) If Conditions (1) or (2) are satisfied, then e is an equilibrium candidacy profile. If $x_i = x_M$ (i.e. Condition (1) is satisfied), then no potential candidate at x_L or x_R wants to enter the race since he would be defeated. If $x_i = x_L$ (resp. x_R), then Condition (2) implies no candidate $j \in \mathcal{P}$ with $x_j = x_R$ (resp. x_L) wants to enter the race. Moreover, the concavity of $u(.)$ implies $u(|x_L - x_M|) \geq \frac{u(|x_L - x_R|)}{2}$ which, together with Condition (2), implies no candidate $j \in \mathcal{P}$ with $x_j = x_M$ wants to stand against candidate i. □

Proof. Lemma 2.
Let (e, α) be a 2-position equilibrium. Observe that all candidates must be tying for first place; sure losers would be better off deviating from their candidacy strategy.

We first establish the necessity of Condition (1). Assume by way of contradiction that $x_i \in \{x_L, x_M\}$ for every candidate $i \in \mathcal{C}(e)$. Denote by c_L (resp. c_M) the number of candidates at x_L (resp. x_M). We proceed in four steps to establish the contradiction.

Step 1: $c_h \leq t$ for every $h \in \{L, M\}$. Assume by way of contradiction that $c_M > t$. Denote by V_h the vote total of a candidate at x_h. Since all candidates tie for first place, it must be that $V_L = V_M$. Suppose a candidate i at x_M were to deviate and not run for election. Given $c_M > t$, the vote total of each candidate at x_L would remain unchanged, i.e., $\widetilde{V}_L = V_L$. At the same time, the vote total of each candidate at x_M would strictly increase, i.e., $\widetilde{V}_M > V_M$. It follows that $\widetilde{V}_M > \widetilde{V}_L$, and x_M is now adopted with probability one. Candidate i is therefore strictly better off deviating since his ideal policy is adopted with higher probability and he saves on the candidacy cost, a contradiction. Hence $c_M \leq t$. A similar argument implies $c_L \leq t$.

Step 2: $c_h < s$ for every $h \in \{L, M\}$. Given $c_M \leq t$, $V_M \geq 1 - F(\underline{x})$. If $c_M \geq s$, then neither citizen with ideal policy $x > \underline{x}$ casts a vote for a candidate at x_L. This, together with $c_L \leq t$, implies $V_L = F(\underline{x})$. Since $F(\underline{x}) < 1/2$, $V_M > V_L$, a contradiction. Hence $c_M < s$.

If $c_L \geq s$, then vote totals are equal to

$$\begin{cases} V_L = F(\underline{x}) + \left(\frac{s-c_M}{c_L}\right)[1 - F(\underline{x})] \\ V_M = 1 - F(\underline{x}). \end{cases}$$

Moreover, $V_L = V_M$ implies x_M is adopted with probability $\pi_M = \frac{c_M}{c_L+c_M} < 1/2$. Suppose another potential candidate at x_M were to enter the race. (We know he exists since $c_M < s \leq c_L \leq p$.) Vote totals would be equal to

$$\begin{cases} \widetilde{V}_L = F(\underline{x}) + \left(\frac{s-c_M-1}{c_L}\right)[1 - F(\underline{x})] < V_L \\ \widetilde{V}_M = 1 - F(\underline{x}) = V_M. \end{cases}$$

It follows that $\widetilde{V}_M > \widetilde{V}_L$, and x_M is adopted with probability $\widetilde{\pi}_M = 1$. Given $\pi_M < 1/2$, a potential candidate at x_M is strictly better off entering the race, a contradiction. Hence $c_L < s$.

Step 3: $c_M < c_L$. It is easy to check that $V_M = V_L$ only if $c_M < c_L$. Observe that $c_M < c_L$ implies x_M is adopted with probability $\pi_M < 1/2$.

Step 4. Suppose another potential candidate at x_M were to enter the race. (We know he exists since $c_M < c_L \leq p$.)

Either $(c_L + c_M) \leq s$, in which case every citizen casts $(c_L + c_M)$ votes instead of $(c_L + c_M - 1)$. Vote totals are such that

$$\begin{cases} \widetilde{V}_L = V_L = F(\underline{x}) + \left(\frac{c_L-1}{c_L}\right)[1 - F(\underline{x})] \\ \widetilde{V}_M = \frac{c_M}{c_M+1}F(\underline{x}) + [1 - F(\underline{x})] > \frac{c_M-1}{c_M}F(\underline{x}) + [1 - F(\underline{x})] = V_M. \end{cases}$$

Thus, $\widetilde{V}_M > \widetilde{V}_L$ and $\widetilde{\pi}_M = 1$.

Or $(c_L + c_M) > s$, in which case every citizen will choose to cast exactly s votes before and after the entry of another candidate at x_M. Vote totals are such that

$$\begin{cases} \widetilde{V}_L = F(\underline{x}) + \left(\frac{s-c_M-1}{c_L}\right)[1 - F(\underline{x})] < F(\underline{x}) + \left(\frac{s-c_M}{c_L}\right)[1 - F(\underline{x})] = V_L \\ \widetilde{V}_M = \frac{s-c_L}{c_M+1}F(\underline{x}) + [1 - F(\underline{x})] < \frac{s-c_L}{c_M}F(\underline{x}) + [1 - F(\underline{x})] = V_M. \end{cases}$$

Simple algebra establishes that $V_L = V_M$ implies $\widetilde{V}_M > \widetilde{V}_L$ and, therefore, $\widetilde{\pi}_M = 1$.

In both cases, $\widetilde{\pi}_M = 1$ implies another potential candidate at x_M wants to enter the race, a contradiction. Hence, it cannot be that $x_i \in \{x_L, x_M\}$ for every $i \in \mathcal{C}(e)$. A similar argument applies to $x_i \in \{x_M, x_R\}$ for every $i \in \mathcal{C}(e)$.

We second establish the necessity of Condition (2). That $c_h \leq p$ for every $h \in \{L, R\}$ is obvious. That $c_h \leq t$ is shown as in Step 1 above. We now establish $c_h \geq \min\{s, p\}$. W.l.o.g. suppose $c_R \geq c_L$, which implies that x_L is adopted with probability $\pi_L = \frac{c_L}{c_L+c_R} \leq \frac{1}{2}$. Assume by way of contradiction that $c_L < \min\{s,p\}$. Proceeding as in Step 4 above (replacing M by R and making a minor adjustment if $c_R \geq s$), we can establish that another potential candidate at x_L (whom we know to exist since $c_L < p$) would be better off entering the race, a contradiction.

Given the above, $c_L = c_R$ is obvious when $s = t$. Consider $s < t$, and assume by way of contradiction that $c_L < c_R$. Given the above, it must be that $s \leq c_L < c_R \leq t$. Vote totals are given by $V_L = V_R = 1/2$. If a candidate at x_R were to deviate and not stand for election, vote totals would be left unchanged, and the probability that x_R is adopted would drop from $\pi_R = \frac{c_R}{c_L+c_R}$ to $\widetilde{\pi}_R = \frac{c_R-1}{c_L+c_R-1}$. Thus, a candidate at x_R does not want to deviate only if

$$\text{(I)} \qquad \delta \leq -\frac{c_L}{c(c-1)} u(|x_L - x_R|),$$

where $c = c_L + c_R$. If another potential candidate at x_L were to enter the race, vote totals would be left unchanged and the probability that x_L is adopted would increase from $\pi_L = \frac{c_L}{c_L+c_R}$ to $\widetilde{\pi}_L = \frac{c_L+1}{c_L+c_R+1}$. Thus, another potential candidate at x_L does not want to deviate only if

$$\text{(II)} \qquad \delta > -\frac{c_R}{c(c+1)} u(|x_L - x_R|).$$

Simple algebra shows that (I) and (II) cannot hold simultaneously, a contradiction. Hence $c_L = c_R$.

Suppose $c_L = c_R = \min\{s,p\}$. If a candidate, say at x_L, were to deviate and not run for election, then every citizen preferring x_L would have to cast more votes for the candidates at x_R than the number of votes every citizen preferring x_R would have to cast for the candidates at x_L. As a result, vote totals would be such that $\widetilde{V}_R > \widetilde{V}_L$, and the probability x_L is adopted would drop from $\pi_L = 1/2$ to $\widetilde{\pi}_L = 0$. Thus, a candidate at x_L does not want to deviate only if $\delta \leq -\frac{u(|x_L-x_R|)}{2}$.

Suppose $c_L = c_R > s$. If a candidate, say at x_L, were to deviate and not run for election, then vote totals would be left unchanged at $V_L = V_R = 1/2$ and the probability with which x_L is adopted would drop from $\pi_L = 1/2$ to $\widetilde{\pi}_L = \frac{c_L-1}{c_L+c_R-1}$. Thus, a candidate at x_L does not want to deviate only if $\delta \leq -\frac{u(|x_L-x_R|)}{2(c-1)}$.

Suppose $c_L = c_R < \min\{t,p\}$. If another potential candidate, say at x_L, were to enter the race (we know he exists since $c_L < p$), vote totals would be left unchanged at $V_L = V_R = 1/2$ and the probability with which x_L is adopted would increase from $\pi_L = 1/2$ to $\widetilde{\pi}_L = \frac{c_L+1}{c_L+c_R+1}$. Thus, a candidate at x_L does not want to deviate only if $\delta > -\frac{u(|x_L-x_R|)}{2(c+1)}$.

We third establish the necessity of Condition (3). For (e,α) to be an equilibrium, it must be that no potential candidate at x_M wants to stand for election. Suppose a potential candidate i at x_M were to enter the race, i.e., $\widetilde{e}_i = 1$ for some $i \in \mathcal{P}$ with $x_i = x_M$ and $\widetilde{e}_j = e_j$ for all $j \in \mathcal{P}$, $j \neq i$. Given Condition (2) above, there are three cases to consider.

Case 1: $c_L = c_R = t$. Let the voting profile $\alpha(\widetilde{e})$ be as followed: for every citizen ℓ with ideal policy $x_\ell < \underline{x}$, $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_L$; for every citizen ℓ with ideal policy $x_\ell \in (\underline{x}, 1/2)$, $\alpha_i^\ell(\mathcal{C}(\widetilde{e})) = 1$ and $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for $(t-1)$ of the candidates with $x_j = x_L$; for every citizen ℓ with ideal policy $x_\ell \in (1/2, \overline{x})$, $\alpha_i^\ell(\mathcal{C}(\widetilde{e})) = 1$ and $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for $(t-1)$ of the candidates with $x_j = x_R$; and, finally, for every citizen with ideal policy $x_\ell > \overline{x}$, $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_R$. Observe that this vote profile maximizes the vote totals of candidates at x_L and x_R, and minimizes the vote total of the candidate at x_M. Candidates' vote totals are given by

$$\begin{cases} \widetilde{V}_L = F(\underline{x}) + \frac{t-1}{t}\left[\frac{1}{2} - F(\underline{x})\right] \\ \widetilde{V}_M = F(\overline{x}) - F(\underline{x}) \\ \widetilde{V}_R = \frac{t-1}{t}\left[F(\overline{x}) - \frac{1}{2}\right] + \left[1 - F(\overline{x})\right]. \end{cases}$$

W.l.o.g. suppose $F(\overline{x}) \geq 1 - F(\underline{x})$, which implies $\widetilde{V}_L \geq \widetilde{V}_R$.

Either $F(\overline{x}) > \frac{1}{2} + \frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$, in which case $\widetilde{V}_M > \widetilde{V}_L$, and candidate i at x_M is elected outright. For him to not be willing to enter the race, it must be that the candidacy cost δ exceeds his expected utility gain $[0 - u(|x_L - x_M|)]$.

Or $F(\overline{x}) = \frac{1}{2} + \frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$ and $F(\overline{x}) > 1 - F(\underline{x})$, in which case $\widetilde{V}_M = \widetilde{V}_L > \widetilde{V}_R$, and candidate i at x_M ties with the t candidates at x_L. For him to not be willing to enter the race, it must be that the candidacy cost exceeds his expected utility gain $\left[\frac{t}{t+1}u(|x_L - x_M|) - u(|x_L - x_M|)\right]$.

Or $F(\overline{x}) = \frac{1}{2} + \frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$ and $F(\overline{x}) = 1 - F(\underline{x})$, in which case $\widetilde{V}_M = \widetilde{V}_L = \widetilde{V}_R$, and candidate i at x_M ties with the $2t$ candidates at x_L and x_R. For him to not be willing to enter the race, it must be that the candidacy cost exceeds his expected utility gain $\left[\frac{2t}{2t+1}u(|x_L - x_M|) - u(|x_L - x_M|)\right]$.

Or $F(\overline{x}) < \frac{1}{2} + \frac{1}{t}\left[(t+1)F(\underline{x}) - \frac{1}{2}\right]$, in which case $\widetilde{V}_M < \widetilde{V}_L$, and candidate i at x_M is not elected and does not want to enter the race.

These four cases exhaust all possibilities.

Case 2: $s \leq c_L = c_R < t$. Let the voting profile $\alpha(\widetilde{e})$ be as follows: for every citizen ℓ with ideal policy $x_\ell < \underline{x}$, $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_L$; for every citizen ℓ with ideal policy $x_\ell \in (\underline{x}, 1/2)$, $\alpha_i^\ell(\mathcal{C}(\widetilde{e})) = 1$ and $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_L$; for every citizen ℓ with ideal policy $x_\ell \in (1/2, \overline{x})$, $\alpha_i^\ell(\mathcal{C}(\widetilde{e})) = 1$ and $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_R$; and, finally, for every citizen ℓ with ideal policy $x_\ell > \overline{x}$, $\alpha_j^\ell(\mathcal{C}(\widetilde{e})) = 1$ for every candidate j with $x_j = x_R$. Otherwise, $\alpha_k^\ell(\mathcal{C}(\widetilde{e})) = 0$ for every other candidate k. Observe that this vote profile maximizes the vote totals of candidates at x_L and x_R, and minimizes the vote total of the candidate at x_M. Candidates' vote totals are given by

$$\begin{cases} \widetilde{V}_L = \widetilde{V}_R = 1/2 \\ \widetilde{V}_M = F(\overline{x}) - F(\underline{x}). \end{cases}$$

Proceeding as in Case 1 above, we obtain the three conditions in the statement.

<u>Case 3: $c_L = c_R = p < s$.</u> In any voting profile, every citizen casts a vote for the candidate at x_M. At the same time, every citizen ℓ with ideal policy $x_\ell < 1/2$ (resp. $x_\ell > 1/2$) does not vote for at least one of the candidates at x_R (resp. x_L). As a result, vote totals are such that $\widetilde{V}_M = 1 > \max\left\{\widetilde{V}_L, \widetilde{V}_R\right\}$, and candidate i at x_M is elected outright. For him to not be willing to enter the race, it must be that the candidacy cost δ exceeds his expected utility gain $[0 - u(|x_L - x_M|)]$.

Finally, it is easy to show that together the conditions in Lemma 2 are sufficient for a 2-position equilibrium to exist. □

Proof. Lemma 3.
Consider an election held under a (s,t)-rule where $s = t$. Let (e, α) be a 3-position equilibrium. We denote by c_h the number of candidates at x_h (for $h = L, M, R$). We start by establishing that $\max\{c_L + c_M, c_M + c_R\} > s = t$. Suppose the contrary. Given that $c_L + c_M \leq s$, every citizen ℓ with ideal policy $x_\ell < 1/2$ casts a vote for each of the candidates at x_M and does not vote for at least one of the candidates at x_R. Likewise, $c_M + c_R \leq s$ implies every citizen ℓ with ideal policy $x_\ell > 1/2$ casts a vote for each of the candidates at x_M and does not vote for at least one of the candidates at x_L. Vote totals are therefore such that $V_M = 1 > \max\{V_L, V_R\}$, and x_M is adopted with probability $\pi_M = 1$. A candidate at x_L would be better off deviating and not running for election since it would still be the case that $\widetilde{V}_M > \max\left\{\widetilde{V}_L, \widetilde{V}_R\right\}$ and $\widetilde{\pi}_M = 1$, but he would save on the candidacy cost. This contradicts (e, α) is an equilibrium.

We second establish that $\min\{c_L + c_M, c_M + c_R\} \leq s = t$. Suppose the contrary. We start by observing that all candidates must then be tying for first place. If $\pi_M = 0$, a candidate at x_M would get his least-preferred policy (x_L or x_R) adopted. He would therefore be better off not running since he would save on the candidacy cost and could not get a worse policy adopted. Likewise, if $\pi_L = 0$, a candidate at x_L would be better off not running for election. This is because $c_L + c_M > t$ implies that neither of the votes he would have received would go to the candidates at x_R. Vote totals would be such that $\widetilde{V}_L \geq V_L$, $\widetilde{V}_M > V_M$ and $\widetilde{V}_R = V_R$, the strict inequality because $c_L \leq t$ (more than t candidates at a position would split votes, and candidates in excess of t would be better off not running). This, together with $\pi_M > 0$, implies $\widetilde{\pi}_R = 0$, and either x_L or x_M would be adopted. As a result, a candidate at x_L would want to deviate and not run for election since he would get a weakly preferred policy adopted and would save on the candidacy cost. Hence, it must be that $\pi_L > 0$. Likewise, it must be that $\pi_R > 0$.

W.l.o.g. suppose $c_L \leq c_R$. Since all candidates tie for first place, x_h (for $h = L, M, R$) is adopted with probability $\pi_h = \frac{c_h}{c_L + c_M + c_R}$. $c_L \leq c_R$ implies $\pi_L \leq \pi_R$. If a candidate at x_L were to deviate and not run for election, $c_L + c_M > t$ and $c_L \leq t$ imply again that vote totals would be such that $\widetilde{V}_L \geq V_L$, $\widetilde{V}_M > V_M$ and $\widetilde{V}_R = V_R$. This, together with $\pi_M > 0$, implies $\widetilde{\pi}_R = 0$. Given the concavity of $u(.)$, we have

$$\sum_{h \in \{L,M,R\}} \widetilde{\pi}_h u^L(x_h) \geq \sum_{h \in \{L,M,R\}} \pi_h u^L(x_h).$$

A candidate at x_L would therefore be better off deviating and not running since it would increase his expected utility and he would save on the candidacy cost. Hence it must be that $\min\{c_L + c_M, c_M + c_R\} \leq s = t$.

We are now ready to establish that (e, α) cannot be an equilibrium. This is obvious for (s,t)-rules where $t \geq 2p$ since $\max\{c_L + c_M, c_M + c_R\} > t$ cannot hold. From now on, consider (s,t)-rules where $t < 2p$. W.l.o.g. suppose $c_L + c_M > t \geq c_M + c_R$. Vote totals are equal to

$$\begin{cases} V_L = F(\underline{x}) + \left(\frac{t - c_M}{c_L}\right)\left[\frac{1}{2} - F(\underline{x})\right] + \left(\frac{t - c_M - c_R}{c_L}\right)\frac{1}{2} \\ V_M = \left(\frac{t - c_L}{c_M}\right) F(\underline{x}) + \left[1 - F(\underline{x})\right] \\ V_R = \frac{1}{2}. \end{cases}$$

$F(\underline{x}) < 1/2$ implies $V_M > 1/2 = V_R$, and x_R is adopted with probability $\pi_R = 0$. By the same argument as above, $\pi_M > 0$ and $\pi_L > 0$ (the latter since $c_L + c_M > t$), which requires $V_L = V_M$. Simple algebra shows that $V_L = V_M$ only if $c_L > c_M$ and $t > (c_M + c_R)$.

If another potential candidate at x_R (we know he exists since $c_R < c_L \leq p$) were to enter the race, vote totals would be such that $\widetilde{V}_L < V_L$, $\widetilde{V}_M = V_M$ and $\widetilde{V}_R = V_R$. Since $V_L = V_M$, x_M would be implemented with probability $\widetilde{\pi}_M = 1$. Thus, no other potential candidate at x_R wants to enter the race only if

$$\text{(I)} \qquad \delta > \frac{c_L}{c_L + c_M}\left[u(|x_R - x_M|) - u(|x_R - x_L|)\right].$$

If a candidate at x_R were to deviate and not run for election, vote totals would be such that $\widetilde{V}_L > V_L$, $\widetilde{V}_M = V_M$ and $\widetilde{V}_R = V_R$. Since $V_L = V_M$, x_L would be implemented with probability $\widetilde{\pi}_L = 1$. Thus, a candidate at x_R does not want to deviate only if

$$\text{(II)} \qquad \delta \leq \frac{c_M}{c_L + c_M}\left[u(|x_R - x_M|) - u(|x_R - x_L|)\right].$$

Taken together, (I) and (II) imply $c_M > c_L$, which contradicts our earlier finding that $c_L > c_M$. $\square$

Proof. Proposition 1. We establish the result by comparing the equilibrium set under a (s,t)-rule where $s = t > 1$ with the equilibrium set under the Plurality Rule.

We know from Lemma 3 that there are no 3-position equilibria under either of the two rules. Hence, all equilibria are 1- and 2-position equilibria.

We know from Lemma 1 that 1-position equilibria are equivalent under both rules. We now establish that every 1-position equilibrium is strictly less polarized than any 2-position equilibrium. Fix the voting rule, and pick a 1-position equilibrium (e,α). We know from Lemma 1 that a single candidate i runs unopposed. Either $x_i = x_M$, in which case the result follows directly from $x_j \in \{x_L, x_R\}$ for every candidate j in a 2-position equilibrium (Condition (1) of Lemma 2). Or $x_i \in \{x_L, x_R\}$, in which case Condition (2) of Lemma 1 implies $\delta > -\frac{u(|x_L - x_R|)}{2}$. Pick a 2-position equilibrium $(\widehat{e},\widehat{\alpha})$. We know from Condition (1) of Lemma 2 that $x_j \in \{\widehat{x}_L, \widehat{x}_R\}$ for every candidate $j \in \mathcal{C}(\widehat{e})$. Moreover, Condition (2) of Lemma 2 and $\widehat{c}_L = \widehat{c}_R = \min\{s,p\}$ imply $-\frac{u(|\widehat{x}_L - \widehat{x}_R|)}{2} \geq \delta$. We then have $u(|x_L - x_R|) > u(|\widehat{x}_L - \widehat{x}_R|)$. Since $u(.)$ is a strictly decreasing function, the latter inequality implies $|\widehat{x}_L - \mu| > |x_L - \mu|$. Hence, the 1-position equilibrium (e,α) is strictly less polarized than the 2-position equilibrium $(\widehat{e},\widehat{\alpha})$.

We now establish that the set of equilibrium policies is an interval under either of the two rules. Given the symmetry of positions, we can restrict attention to policies $x \leq \mu = 1/2$. Suppose the election is held under a (s,t)-rule where $s = t$. Let x and $\widehat{x}$, with $x < \widehat{x} \leq 1/2$, be two equilibrium policies. Pick $y \in (x, \widehat{x})$. We must show that y can be supported by an equilibrium. There are three cases to consider.

<u>Case 1: x is supported by a 1-position equilibrium.</u> Since $x < \mu$, Condition (2) of Lemma 1 implies $\delta > -\frac{u(|x-(1-x)|)}{2}$. Since $|y - \mu| < |x - \mu|$ and $u(.)$ is a strictly decreasing function, we have $u(|y - (1-y)|) > u(|x - (1-x)|)$. Hence $\delta > -\frac{u(|y-(1-y)|)}{2}$, and y can be supported by a 1-position equilibrium for the platform configuration $\{y, 1/2, 1-y\}$.

<u>Case 2: $\widehat{x}$ is supported by a 2-position equilibrium.</u> We know from Condition (2) of Lemma 2 that $\widehat{c}_L = \widehat{c}_R = \min\{s,p\} = \min\{t,p\}$ and $-\frac{u(|\widehat{x}-(1-\widehat{x})|)}{2} \geq \delta$. Since $|\widehat{x} - \mu| < |y - \mu|$, we have $u(|\widehat{x} - (1-\widehat{x})|) > u(|\widehat{y} - (1-\widehat{y})|)$. Hence, Condition (2) of Lemma 2 can be satisfied for y.

Since 1-position equilibria are less polarized than 2-position equilibria and $x < \widehat{x}$, x must be supported by a 2-position equilibrium as well. Hence, Condition (3) of Lemma 2 must be satisfied for x. Either subcondition (c) applies, in which case it applies for y as well and Condition (3) can be satisfied for y. Otherwise, subcondition (a) must apply for x. Define $\underline{y} \equiv \frac{y+x_M}{2}$ and $\overline{y} \equiv \frac{(1-y)+x_M}{2}$. Observe that $x < y$ implies $\underline{x} < \underline{y}$ and $\overline{y} < \overline{x}$.

It follows that $F(\underline{x}) \leq F(\underline{y})$ and $F(\overline{y}) \leq F(\overline{x})$. It is easy but tedious to check that subcondition (a) satisfied for x implies it can be satisfied for y.

Hence, all conditions of Lemma 2 can be satisfied for y, and y can be supported by a 2-position equilibrium.

Case 3: $\widehat{x}$ is supported by a 1-position equilibrium and x by a 2-position equilibrium. Either $\delta > -\frac{u(|y-(1-y)|)}{2}$, in which case y can be supported by a 1-position equilibrium. Or $-\frac{u(|y-(1-y)|)}{2} \geq \delta$, in which case Condition (2) of Lemma 2 can be satisfied for y. Proceeding in the same fashion as in Case 2 above, we can establish that Condition (3) of Lemma 2 satisfied for x implies it can be satisfied for y as well, and that y can therefore be supported by a 2-position equilibrium.

Consider a (s,t)-rule where $1 < s = t \leq p$. Given the above, the result is established by showing that the set of 2-position equilibria under the (s,t)-rule is a superset of the set of 2-position equilibria under the Plurality Rule. This is trivial if there are no 2-position equilibria under the Plurality Rule. So suppose (e, α) is a 2-position equilibrium under the Plurality Rule. Denote the two positions by x_L and x_R. Condition (2) of Lemma 2 implies $-\frac{u(|x_L-x_R|)}{2} \geq \delta$ and $c_L = c_R = 1$. Moreover, subcondition (a) of Condition (3) must hold with $t = 1$. To prove the result, it is sufficient to construct an equivalent equilibrium under the (s,t)-rule. Let the number of candidates at each position be $\widehat{c}_L = \widehat{c}_R = s (= t)$ and $\widehat{c}_M = 0$. Thus, Condition (1) of Lemma 2 is satisfied. Given $-\frac{u(|x_L-x_R|)}{2} \geq \delta$, Condition (2) of Lemma 2 is satisfied as well. Finally, given $\widehat{c}_L = \widehat{c}_R = t$, subcondition (a) of Condition (3) applies. Since $t > 1$, the fact that this subcondition is satisfied for the Plurality Rule implies it is satisfied for the (s,t)-rule as well. Finally, $c_L = c_R$ and $\widehat{c}_L = \widehat{c}_R$ imply x_L and x_R are each adopted with probability 1/2 under either of the two rules. Hence, we have constructed an equivalent 2-position equilibrium.

Consider a (s,t)-rule where $s = t > p$. We show that the set of 2-position equilibria under the (s,t)-rule is a subset of the set of 2-position equilibria under the Plurality Rule. This is trivial if there are no 2-position equilibria under the (s,t)-rule. So suppose (e, α) is a 2-position equilibrium under the (s,t)-rule. Denote the positions by x_L and x_R. Condition (2) of Lemma 2 implies $c_L = c_R = p$ and $-\frac{u(|x_L-x_R|)}{2} \geq \delta$. Moreover, subcondition (c) of Condition (3) must be satisfied, implying $\delta > -u(|x_L - x_M|)$. We now construct an equivalent equilibrium under the Plurality Rule. Let the number of candidates at each position be $\widehat{c}_L = \widehat{c}_R = 1$ and $\widehat{c}_M = 0$. Thus, Condition (1) of Lemma 2 is satisfied. Since $-\frac{u(|x_L-x_R|)}{2} \geq \delta$, Condition (2) is satisfied as well. Finally, subcondition (a) of Condition (3) applies. Since $\delta > -u(|x_L - x_M|)$, this condition is satisfied as well. Hence, we have constructed an equivalent 2-position equilibrium. $\square$

Proof. Proposition 2.
(1) Consider the (s,t)- and (s',t)-rules where $s < s' = t$. We start with several observations. First, we know from Lemma 1 that 1-position equilibria are equivalent under both rules. Second, from the comparison of Condition (2) in Lemma 1 and in Lemma 2, we can infer that every 1-position equilibrium is strictly less polarized than any 2-position equilibrium. Third, proceeding in the same fashion as in the proof of Proposition 1, we can establish that under both rules, the set of policies that are supported by 1- and 2-position equilibria is an interval.

We now establish that the set of 2-position equilibria is more polarized under the (s,t)-rule than under the (s',t)-rule. Given our observations above, it is sufficient to show that the set of 2-position equilibria under the (s,t)-rule is a superset of the set of 2-position equilibria under the (s',t)-rule. Let (e',α') be a 2-position equilibrium under the (s',t)-rule. Denote by x_L and x_R the two positions. We now construct an equivalent equilibrium under the (s,t)-rule. There are three cases to consider.
Case 1: $s < s' \leq p$. Condition (2) of Lemma 2 implies $c'_L = c'_R = s' = t$ and $-\frac{u(|x_L - x_R|)}{2} \geq \delta$. Moreover, Condition (3a) must be satisfied.

Suppose now that the election is held under the (s,t)-rule. Let there be $c_L = c_R \in \{s, s+1, ..., t\}$ candidates at x_L and at x_R such that Condition (2) of Lemma 2 is satisfied. (This is possible since $-\frac{u(|x_L - x_R|)}{2} \geq \delta$.) Also, let $c_M = 0$, which implies Condition (1) of Lemma 2 is satisfied as well. It remains to show that Condition (3) is satisfied. This is obvious if $c_L = c_R = t$ since the condition is already satisfied under the (s',t)-rule. In case, $s \leq c_L = c_R < t$, subcondition (b) applies. Observe that $F(\underline{x}) > \frac{1}{t}\left[(t+1)F(\underline{x}) - 1/2\right]$. This, together with the fact that subcondition (a) is satisfied for $c'_L = c'_R = t$, implies subcondition (b) is satisfied for $s \leq c_L = c_R < t$. Hence, all three conditions of Lemma 2 are satisfied, and we have constructed an equilibrium under the (s,t)-rule. This equilibrium is equivalent to (e',α') since $c'_L = c'_R$ and $c_L = c_R$ imply x_L and x_R are each adopted with probability $1/2$ in both equilibria.
Case 2: $s \leq p < s'$. Condition (2) of Lemma 2 implies $c'_L = c'_R = p$ and $-\frac{u(|x_L - x_R|)}{2} \geq \delta$. Also, condition (3c) must be satisfied, which implies $\delta > -u(|x_L - x_M|)$.

Suppose now that the election is held under the (s,t)-rule. Proceeding in the same fashion as in Case 1, let $c_M = 0$ and $c_L = c_R =\in \{s, ..., p\}$ such that Conditions (1) and (2) of Lemma 2 are satisfied. (Again, this is possible since $-\frac{u(|x_L - x_R|)}{2} \geq \delta$.) Since $s \leq c_L = c_R \leq p < t$, Condition (3b) applies. Observe that this condition is necessarily satisfied since $\delta > -u(|x_L - x_M|)$. Hence, we have again constructed a 2-position equilibrium equivalent to (e',α').
Case 3: $p < s < s'$. It is straightforward to see that in this case, (e',α') is an equilibrium under the (s,t)-rule.

Hence, we have shown that any 2-position equilibrium under the (s', t)-rule has an equivalent equilibrium under the (s, t)-rule. It is easy to see that the reverse is not true. Hence, the set of 2-position equilibria is more polarized under the (s, t)-rule than under the (s', t)-rule.

Finally, we know from Lemma 3 that there are no 3-position equilibrium under the (s', t)-rule. 3-position equilibria may however exist under the (s, t)-rule (e.g. see Example 1 in the supplementary material). Since the set of policies that are supported by 1- and 2-position equilibria is an interval centered around μ, the only policies that can be supported only by 3-position equilibria are more polarized than policies supported by 1- and 2-position equilibria.

We have thus established that the (s, t)-rule where $s < t$ can support more polarization than the (s', t)-rule where $s' = t$.

(2) We can proceed in the same fashion as above to establish the second part of the result. The only difference lies in the way to prove that the set of 2-position equilibria under the (s, t)-rule is a superset of the set of 2-position equilibria under the (s, t')-rule. Let (e', α') be a 2-position equilibrium under the (s, t')-rule. Denote by x_L and x_R the two positions. We now construct an equivalent equilibrium under the (s, t)-rule. There are two cases to consider.

Case 1: $s \leq p$. Condition (2) of Lemma 2 implies $c'_L = c'_R = s = t'$ and $-\frac{u(|x_L - x_R|)}{2} \geq \delta$. Moreover, Condition (3a) must be satisfied.

Suppose the election is held under the (s, t)-rule. Let there be $c_L = c_R \in \{s, ..., \min\{t, p\}\}$ candidates at x_L and at x_R such that Condition (2) of Lemma 2 is satisfied. Also, let $c_M = 0$, which implies Condition (1) of Lemma 2 is satisfied. It remains to show that Condition (3) is satisfied. Either $c_L = c_R = t$, in which case subcondition (a) applies. Since $\frac{1}{t}\left[(t+1) F(\underline{x}) - 1/2\right]$ is strictly increasing in t and $t > t'$, the fact that subcondition (a) is satisfied for $c'_L = c'_R = t'$ implies it is satisfied for $c_L = c_R = t$ as well. Or $s \leq c_L = c_R < t$, in which case subcondition (b) applies. Since $F(\underline{x}) > \frac{1}{t'}\left[(t'+1) F(\underline{x}) - 1/2\right]$, the fact that subcondition (a) is satisfied for $c'_L = c'_R = t'$ implies subcondition (b) is satisfied for $s \leq c_L = c_R < t$. Hence, all three conditions of Lemma 2 are satisfied, and we have constructed an equilibrium under the (s, t)-rule. This equilibrium is equivalent to (e', α') since $c'_L = c'_R$ and $c_L = c_R$ imply x_L and x_R are each adopted with probability $1/2$ in both equilibria.

Case 2: $p < s$. It is easy to see that (e', α') is an equilibrium under the (s, t)-rule as well. □

Proof. Lemma 4.

To prove the result, we shall again partition the equilibrium set into three subsets: the 1-, 2- and 3-position equilibria.

We already know that in 1-position equilibria, a single candidate stands for election.

Let (e, α) be a 2-position equilibrium. Observe first that $c_M = 0$; otherwise, the first candidate to receive a majority of votes and be elected would be a candidate at x_M. Thus, x_M would be adopted with probability $\pi_M = 1$. Candidate(s) at the other position would therefore be better off deviating from their candidacy strategy and not running for election.

It remains to establish that $c_L = c_R = 1$. The (first-place) vote total of each candidate is given by $V_L = \frac{1/2}{c_L}$ and $V_R = \frac{1/2}{c_R}$ for candidate(s) at x_L and x_R, respectively. If $c_L > c_R$, then $V_L < V_R \leq 1/2$. Neither candidate receives a majority of first-place votes and candidates at x_L are eliminated until it remains only c_R candidates at x_L. Hence $c_L = c_R$ since candidates at x_L (in excess of c_R) would be better off not running for election. By the same argument, we get $c_L = c_R = 1$.

Finally, let (e, α) be a 3-position equilibrium. Observe first that x_M must be adopted with probability $\pi_M > 0$; otherwise, candidate(s) at x_M would be better off not running for election since candidacy is costly and they are indifferent between x_L and x_R. Likewise, it must be that $\pi_L > 0$ and $\pi_R > 0$; it is easy to see that otherwise $\pi_M = 1$ whether or not a candidate at x_L or x_R deviates and does not run for election.

The (first-place) vote total of each candidate is given by $V_L = \frac{F(\underline{x})}{c_L}$, $V_M = \frac{F(\overline{x}) - F(\underline{x})}{c_M}$ and $V_R = \frac{1 - F(\overline{x})}{c_R}$ for candidate(s) at x_L, x_M and x_R, respectively. $\pi_h > 0$ for $h = L, M, R$ implies $V_h \leq 1/2$ for $h = L, M, R$. Since $\pi_L > 0$ and $F(\underline{x}) < 1/2$, there must be an elimination sequence where all candidates at x_M are eliminated before all candidates at x_L. This requires $F(\overline{x}) - F(\underline{x}) \leq F(\underline{x})$. Likewise, $\pi_R > 0$ and $[1 - F(\overline{x})] < 1/2$ require $F(\overline{x}) - F(\underline{x}) \leq 1 - F(\overline{x})$. Finally, $\pi_M > 0$ and $F(\overline{x}) - F(\underline{x}) < 1/2$ (the inequality since $F(\overline{x}) - F(\underline{x}) \leq F(\underline{x}) < 1/2$) imply that there must exist an elimination sequence where all candidates at x_L or x_R are eliminated before all candidates at x_M. This requires $F(\overline{x}) - F(\underline{x}) \geq \min\{F(\underline{x}), 1 - F(\overline{x})\}$. Hence

$$F(\overline{x}) - F(\underline{x}) = \min\{F(\underline{x}), 1 - F(\overline{x})\}.$$

By the same argument as for 2-position equilibria, it is not difficult to see that we must have $c_L = c_M = c_R = 1$. □

Proof. Proposition 3.
Let the election be held under the Alternative Vote Rule. We start by characterizing each of the three subsets of equilibria.

For 1-position equilibria, the characterization is the same as in Lemma 1 since there is a single candidate.

Proceeding in the same fashion as for Lemma 2, we can show that a 2-position equilibrium exists if and only if

(1) $x_i \in \{x_L, x_R\}$ for every candidate $i \in \mathcal{C}(e)$.
(2) $c_L = c_R = 1$ and $-\frac{u(|x_L - x_R|)}{2} \geq \delta$.
(3) x_L and x_R are such that
- either $F(\overline{x}) - F(\underline{x}) > \min\{F(\underline{x}), 1 - F(\overline{x})\}$ and $\delta > -u(|x_L - x_M|)$,

- or $F(\overline{x})-F(\underline{x}) = \min\{F(\underline{x}), 1-F(\overline{x})\} < \max\{F(\underline{x}), 1-F(\overline{x})\}$ and $\delta > -\frac{u(|x_L - x_M|)}{2}$,
- or $F(\overline{x})-F(\underline{x}) = \min\{F(\underline{x}), 1-F(\overline{x})\} = \max\{F(\underline{x}), 1-F(\overline{x})\}$ and $\delta > -\frac{2}{3}u(|x_L - x_M|)$,
- or $F(\overline{x}) - F(\underline{x}) < \min\{F(\underline{x}), 1-F(\overline{x})\}$.

Finally, there are no 3-position equilibria. To see this, recall from the proof of Lemma 4 that $c_L = c_M = c_R = 1$ and $\pi_h > 0$ for $h = L, M, R$. W.l.o.g. suppose $\pi_L \geq \pi_R$. Pick the candidate at x_R. Suppose he were to deviate and not run for election. Then there would be only two candidates left, one at x_L and another one at x_M, and the candidate at x_M would be elected outright. The concavity of $u(.)$ implies the candidate at x_R would be strictly better off not running, a contradiction.

Using Lemmas 1-3 and proceeding in the same fashion as in the proof of Proposition 1, we can establish that the Alternative Vote Rule supports less polarization than the Plurality Rule. □

Online Appendix

Example 1. *We consider a community that must elect a representative to choose a tax rate $x \in X = [0,1]$. Each citizen ℓ has preferences over X that can be represented by a utility function $u^{\ell}(x) = -(x - x_{\ell})^2$. For a sharp contrast between voting rules, we assume citizens' ideal tax rates are uniformly distributed over X. There are two potential candidates at each of the three positions, i.e. $p = 2$. Finally, the candidacy cost is given by $\delta = 1/50$.*

We contrast Approval Voting ($s = 1$ and $t = 3p$) with the Plurality Rule ($s = t' = 1$) and Negative Voting ($s' = t = 3p$).

Applying Lemmata 1-3, we find that under the Plurality Rule, the set of equilibrium tax rates is given by $(1/6, 5/6)$, where all tax rates in $(2/5, 3/5)$ are supported by 1-position equilibria, and all tax rates in $(1/6, 2/5] \cup [3/5, 5/6)$ by 2-position equilibria. By contrast, under Negative Voting, the set of equilibrium tax rates is given by $\left(\frac{1}{2} - \frac{1}{5\sqrt{2}}, \frac{1}{2} + \frac{1}{5\sqrt{2}}\right)$, where all tax rates in $(2/5, 3/5)$ are supported by 1-position equilibria, and all tax rates in $\left(\frac{1}{2} - \frac{1}{5\sqrt{2}}, \frac{2}{5}\right] \cup \left[\frac{3}{5}, \frac{1}{2} + \frac{1}{5\sqrt{2}}\right)$ by 2-position equilibria. Observe that Negative Voting supports strictly less polarization than the Plurality Rule, in accordance with part (2) of Proposition 1.

Given the uniform distribution of ideal tax rates, under Approval Voting the set of equilibrium tax rates coincides with the full policy space $X = [0,1]$. All tax rates in $(2/5, 3/5)$ are supported by 1-position equilibria, as under the other two rules. All tax rates in $(0, 2/5] \cup [3/5, 1)$ are supported by 2-position equilibria, with one candidate at each position for $\left(\frac{1}{2} - \frac{\sqrt{3}}{10}, \frac{2}{5}\right] \cup \left[\frac{3}{5}, \frac{1}{2} + \frac{\sqrt{3}}{10}\right)$ and two candidates at each position for $\left(0, \frac{1}{2} - \frac{\sqrt{3}}{10}\right] \cup \left[\frac{1}{2} + \frac{\sqrt{3}}{10}, 1\right)$. Finally, 0 and 1 are supported by a 3-position equilibrium $\{0, 1/2, 1\}$. In this equilibrium, all potential candidates stand for election and the vote profile is as follows: every citizen ℓ with ideal policy $x_{\ell} < 1/4$ votes for the two candidates at 0; every citizen ℓ with $x_{\ell} \in (1/4, 1/2)$ votes for the two candidates at 0 and the two candidates at $1/2$; every citizen ℓ with $x_{\ell} \in (1/2, 3/4)$ votes for the two candidates at $1/2$ and the two candidates at 1; and, finally, every citizen ℓ with $x_{\ell} > 3/4$ votes for the two candidates at 1. Thus, every candidate receives a vote from half the electorate, and ties for first place. Neither candidate would be better off dropping out if citizens were then voting for the other candidates in the same way (in which case the other five candidates would still be tying for first place).

Example 2. *Consider the community described in Example 1, where $\beta = 0$ is replaced with $\beta = 1/30$. We contrast four rules: the Plurality Rule, the $(2,2)$-rule, Negative Voting and Approval Voting. The $(2,2)$-rule is an example of (s,t)-rule where $s = t \leq p$. Negative Voting is an example of (s,t)-rule where $s = t > p$. Finally, Approval Voting is an example of (s,t)-rule where $s < t$.*

1/2 is the only tax rate that can be supported by 1-position equilibria. This is because $\beta > \delta$ implies a potential candidate at x_M would necessarily want to enter the race (and would win outright) against a candidate at x_L or x_R.

Consider now the 2-position equilibria. Given the symmetry of x_L and x_R around the median $\mu = 1/2$, we shall focus here on tax rates in $[0, 1/2]$. Under the Plurality Rule, all tax rates in $\left(\frac{1}{6}, \frac{1}{2} - \frac{1}{10\sqrt{6}}\right]$ can be supported by a 2-position equilibrium. Under the $(2,2)$-rule, the set of tax rates that can be supported by 2-position equilibria is given by $\left(\frac{1}{10}, \frac{1}{2} - \frac{1}{20}\sqrt{\frac{7}{3}}\right]$. (For comparison, this set is given by $\left(\frac{1}{10}, \frac{2}{5}\right]$ when $\beta = 0$.) Under Negative Voting, there are no 2-position equilibria given that $\beta > \delta$ implies a potential candidate at x_M would want to enter the race since he would receive unanimous vote and win outright. Finally, under Approval Voting all tax rates in $\left(\frac{1}{2} - \frac{1}{5\sqrt{3}}, \frac{1}{2} - \frac{1}{10\sqrt{6}}\right]$ can be supported by 2-position equilibria with one candidate at each position, and all tax rates in $\left(0, \frac{1}{2} - \frac{\sqrt{7}}{20}\right]$ by 2-position equilibria with two candidates at each position.

Two observations are worth making with respect to 2-position equilibria. First, increasing β from zero to 1/30 triggers an expansion toward the median of the set of tax rates that can be supported by 2-position equilibria. (There is an exception for Negative Voting where the set empties.) This is because $\beta > 0$ strengthens candidacy incentives. As a result, some of the tax rates that were supported by 1-position equilibria are now supported by 2-position equilibria.

A second observation with respect to 2-position equilibria is that the degree of polarization varies across the four rules in a similar fashion as when $\beta = 0$. The least polarized 2-position equilibrium under the Plurality Rule is equally polarized than the one under Approval Voting, and less polarized than the one under the $(2,2)$-rule. Moreover, the most polarized 2-position equilibrium under the Plurality Rule is strictly less polarized than the one under Approval Voting or the one under the $(2,2)$-rule.

Finally, as when $\beta = 0$, no tax rate can be supported by a 3-position equilibrium under the Plurality Rule, the $(2,2)$-rule or Negative Voting. Moreover, $\{0, 1/2, 1\}$ can still be supported by a 3-position equilibrium under Approval Voting.

Example 3. *Again, we consider a community that must elect a representative to choose a tax rate $x \in X = [0,1]$. For strategic voting to make sense, we assume there is a finite number of citizens so that a vote can be pivotal. Specifically, there are 22 citizens whose ideal tax rates are distributed uniformly over the set $\left\{0, \frac{1}{10}, \frac{2}{10}, ..., 1\right\}$, i.e., two citizens prefer zero tax, two others prefer a 10% tax, and so on. The median tax rate is thus $\mu = 1/2$. Each citizen ℓ has preferences that can be represented by a utility function $u^\ell(x) = -(x - x_\ell)^2$. There are six potential candidates: two at $x_M = 1/2$, two at $x_L \in \left\{0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right\}$ and two at $x_R = 1 - x_L$. The candidacy* *cost* $\delta = 1/50$ *and the ego-rent* $\beta = 0$. *We contrast the same four voting rules as in Example 2, namely, the Plurality Rule, the* $(2,2)$*-rule, Negative Voting and Approval Voting. Given the symmetry, we shall determine for each rule which of the tax rates at and on the left of the median* μ *(i.e.,* $\left\{0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*) can be supported by an equilibrium.*

For comparison purposes, we start by characterizing the sets of equilibrium tax rates when voting is sincere. Under the Plurality Rule, the set of equilibrium tax rates is given by $\left\{\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*, where* $\{1/2\}$ *is supported by 1-position equilibria, and all tax rates in* $\left\{\frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right\}$ *by 2-position equilibria. Under the* $(2,2)$*-rule, it is given by* $\left\{\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*. Under Negative Voting, the set of equilibrium tax rates is only* $\left\{\frac{4}{10}, \frac{1}{2}\right\}$*. Finally, under Approval Voting, it is the whole set* $\left\{0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*.*

We now characterize the set of equilibrium tax rates when voting is strategic. Under the Plurality Rule, the $(2,2)$*-rule and Negative Voting, the set of equilibrium tax rates is given by the whole set* $\left\{0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*:* $\{1/2\}$ *is supported by 1-position equilibria; all tax rates in* $\left\{0, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right\}$ *are supported by 2-position equilibria, with one candidate at each position under the Plurality Rule, and two under the* $(2,2)$*-rule and Negative Voting. By contrast, under Approval Voting, the set of equilibrium tax rates is given by* $\left\{\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{1}{2}\right\}$*, where* $\{1/2\}$ *is supported by 1-position equilibria,* $\{4/10\}$ *by 2-position equilibria with one candidate at each position, and all tax rates in* $\left\{\frac{2}{10}, \frac{3}{10}\right\}$ *by 2-position equilibria with two candidates at each position.*

To sum up, when voting is sincere, Approval Voting supports the most polarization, followed by the $(2,2)$*-rule, the Plurality Rule and, finally, Negative Voting, which supports the least polarization. When voting is strategic, the ranking of voting rules is instead the* $(2,2)$*-rule, the Plurality Rule and Negative Voting, followed by Approval Voting, which now supports the least polarization.*