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"Payroll Taxation and the structure of quali_cations and wages in a segmented frictional labor market with intra-firm bargaining"

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Payroll Taxation and the structure of qualifications and wages in a segmented frictional labor market with intra-firm bargaining

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Abstract

The present paper investigates the incidence of payroll taxation - and more generally labor income taxation - in a search and matching model. The model considers a production function with different type of workers, allowing to understand the interactions between segmented labor markets. Furthermore, the equilibrium is reach through a double process of intra-firm wage bargaining ex post and labor demand ex ante. The model is derived analytically for linear tax function differentiated for worker type, and numerically for non-linear tax functions. The bargaining power parameter is interpreted as reflecting the intra-segment substitutability, in parallel to the inter-segment substitutability deriving from the production function and the segment size and productivity. Some standard results are found, such as the wages, unemployment and incidence increasing with respect to bargaining power; or the payroll tax burden falling mainly on workers. Moreover, it is shown that over-shifting of payroll taxes on net wages may happen. It is also shown that a stronger bargaining power induced weaker direct effect of taxes but larger crossed effects on other segments. In addition, marginal incidence decreases with respect to the payroll tax level and is therefore significantly lower than mean incidence, which may induce an underestimation of overall incidence by empirical analyses. This also induces a marginally decreasing effect on loabor costs of payroll tax cuts.

Keywords: Search and matching; segemented labor market; intra-firm bargaining; tax incidence

JEL: H22; J31; J38.

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1 Introduction

The consequences of taxation of labor markets is a central issue of applied public economics and more specifically of the understanding of public policies' impacts. The need for kowledge on that subject has been strenghted by the economics crisis. Little wage moderation occured in main developped countries - including the united states despite their quite liberal and competitive labor market - and the question of labor costs and its consequences on structure of jobs and unemployement is a main concern for governments, particularly european ones. France, for exemple set a new payroll tax rebate of 4% of the payroll bill for 2013 then 6% for years after 2014.

More broadly, a large number of governements use the fiscal tool not only to levy resources but also conversely to subsidy labor. It generates payroll taxation differentiated by industrial sector or level of qualification. This differentiation may modify the structure of employement and unemployement as well as the structure of wages. The present paper aims at analysing these effects of differentiated payroll taxation in a model of search and matching taking into account the productive interaction between different kind of inputs: employees of different qualifications and capital. This allows to understand the distortions generated on the labor markets as well as the distributive consequences.

From a general point of view, it is well known that the tax burden does not fall only onto the individuals officially taxed. The burden is shared among the agents interacting on markets. This also applies to payroll taxation; Gruber (1994, 1997); Anderson and Meyer (1997, 2000); Murphy (2007) demonstrated that workers pay the main part of payroll taxes through several natural experiments in the United States and in Chile, whatever their official designation, employees' or employers' social security contributions. Furthermore, the sharing of the tax burden varies with the bargaining power of employees: the larger the employee's bargaining power, the higher the share of taxes borne by employees and the higher the share of exemptions that will eventually be translated into net wage rises instead of labour cost reductions. By definition, workers paid at the minimum wage have no bargaining power, their bargained wage would have been lower. Hence, tax exemptions at the minimum wage levels are more fully converted into labor cost decrease than exemptions for higher wages. It is therefore of main importance to introduce bargaining power and minimum wage in the model. The results of the present model fit the empirical literature in the way that the share of taxation borne by employees through wage decreases is larger when the bargaining power is larger.

This issue is of main importance, not only for purely theoretical purpose but also for applied public economics. First of all, the motive of differentiated payroll taxes is often employement and incidence of payroll taxes is a key parameter of the success of such policies. Due to incidence differences, the impact on employment of payroll tax cuts should be greater for low wages than for high wages. There has been several empirical analysis of such policies in Europe, Crépon and Deplatz (2001), Kramarz and Philippon (2001) and Chéron et al. (2008) find significant impact for France when results of Bohm and Lind (1993); Bennmarker et al. (2009) for Suede and Korkeamäki and Uusitalo (2009) for finland are more mitigated. The cause of the difference may lie in incidence and the fact that French payroll tax reduction was setted very close to the minimum wage and therefore induce that incidence is full burden for employers. Actually, Crépon and Deplatz (2001) show that the effect in France occured through substitutions of low wage workers to high wage workers. Nevertheless, Huttunen et al. (2013) consider payroll tax cuts on low wages and found also very weak impact. They use difference-in-difference methodology (per age categories) to assess the impact of a Finnish payroll tax cut targeting older workers on low wages: they found no impact at the extensive margins and a small impact at the intensive margins.

The present article shows that the share of taxation borne by employees through wage decreases marginally decreases with respect to taxation. Consequently, the share of tax cut benefitting to employers through labor cost decreases marginally decreases with respect to tax cuts. Hence, the efficiency of tax cuts in order sustain employment is marginally decreasing. It is more efficient when the initial payroll taxation is heavy, which may also explain the difference between France (where the firsts tax cuts applied to very large social security contributions on low wages) and Scandinavia (where social security is mainly financed by the global budget).

Furthermore, it is essential to know incidence for understanding equity of taxation. Equity of taxation should be measured throught the actual distribution of the burden and not the official distribution. Different incidence of taxes - and particularly payroll taxes - may be of great influence on the way the redistributiveness of the whole fiscal system is measured. For exemple, Vicard et al. (2013) estimated the global redistribution of the whole French system of taxes and transferts and find opposite results (strongly redistributive or quite flat) depending on incidence hypotheses of payroll taxes. The marginal decrease of the share of taxation borne by employees proven in the present article implies that the mean incidence is larger than the marginal incidence. Hence, estimation of incidence through natural experiment (which gives the marginal incidence) underestimates the mean share of taxation borne by employees.

Another exemple of the main importance of payroll tax incidence may be found in the analysis by Farhi et al. (2014) of fiscal devaluation. They found that it is equivalent to monetary devaluation if incidence of VAT and payroll taxes are homogenous between sectors. This highlights the importance of understanding incidence not only globally but also at micro level where incidence depends on the characteristics of production and substitution between factors, what is one contribution of the present paper.

This also crosses the issue of optimal labor taxation as payroll taxation and labor income taxation probably have similar incidence even when labor taxation is not levied at source. However, optimal labor taxation literature has first focused on the labor supply side and the adverse selection problem. Mirrlees (1971) considered a descrete distribution of workers, Saez (2001) generalized the approach with continuous productivity of workers and Kleven et al. (2009) generalized to couples and labor supply in the extensive margins. However, this litterature does not consider any labor market as each unit of labor supplied finds an employer - there is no unemployement - and the wage is equal to the productivity of the worker.

The standart way of modeling labor markets has been developped by the search and matching literature (e.g. Pissarides (2000)). It provides a dynamic framework and reproduced the conditions of frictional unemployemment, the rent of employement being shared between firm and worker. Stole and Zwiebel (1996b,a) renew the process of wages setting by the hypothesis that contract incompleteness does not enables neither firms nor workers to commit to future wages and employement decision, which leads to intra-firm bargaining engaged individually by workers within employement. It results in lower wages and more employement than in standart model. All these models does not take into account the structure of production and possible substitution between factors of production.

Acemoglu (2001) built a matching model with two kinds of job (good job/bad job) and derives the impact of minimum wage on the structure of production. However, it does not fit either the problematic of the present paper as there is only one type of worker and the two kinds of jobs are modeled as separate sectors of intermediate goods. Belan et al. (2010) introduced a model with frictional and classical unemployment and two kinds of workers. However, there is also two kinds of goods and this model does not allow to understand the interactions between the type of workers within the production process.

The choice of the model necessitates therefore the hiring of different kinds of workers for the same production process, taking into account the interaction effects through a multifactor production function. Hence, the model developped above is based on Cahuc et al. (2008), including altogether matching, bargaining and multifactorial production function. The original paper was developped to understand the extend of overemployment in a normative point of view. However, overemployment in their model is directly linked to the wages being larger than the marginal productivity, which may be interpreted as an issue of value added sharing instead of overemployment. The question of overemployment is not considered here as the present paper focused on the positive understanding of the impact of taxation on the structure of wages and unemployement.

The setting of intra-firm bargaining is criticized because it assumed permanent and individual bargaining when in most countries the wages are bargained collectively and sequentially. The present paper answers this critic by both considering different type of workers (with wages bargained collectively or individually) and by interpretating further the process of individual intra-firm bargaining and the bargaining power parameter itself.

Actually, the model is modified mainly in two ways. First, three kinds of inputs are considered. The factors representing the different kind of capital are included in the production function and the decision of input demand by the firm; the allocation is not considered frictional, the remuneration is given internationally and exogenously. The constrained workers have no individual bargaining power. Their wages is determined collectively for each worker type, and this fixed wage is considered exogenous in the model. It may represent collective bargaining without modelling the collective bargaining process, it fits even more with workers whose qualification prevent them to access jobs paid over the minimum wage: in that case, the minimum wage is actually exogenous.

The last inputs represent workers whith an individual bargaining power. This does not come from their substitutability with other types of workers (inter-input substituability) but from their substitutability with other worker of the same type (intra-input subsitutability). The intra-input substitutability does not need that workers of one type are heterogenous but that the productivity of their kind of job is marginaly increasing with respect to the personal investment of the worker. In that case, as presented by Goldin (2014), the wage is convex with respect to the personal investment because it is costly for the employer to change one worker for another one even with the same qualification and ability. For those kind of jobs, the increase of productivity with personal investment lowers the substitutability with similar workers. This low intra-input substitutability allows those kind of workers to extract surplus from the employer by giving them individual bargaining power. This justify their ability to bargain intra-firm and the modelaization of their wage setting.

The second main modification is the introduction of taxation: capital income taxation, consumption taxation and taxation on wages which may represent either payroll taxes or labor income taxes. For the case of payroll taxation financing public social security systems, some countries separate employers' and employees' social security contributions. This differentiation is not considered here because it is formall but has no economic reality except at the level of minimum wage. Formally, the model considers only employers'social security contributions, as the base of taxation is the net wage. This choice of modelization has no impact on unconstrained workers. It matters only for constrained workers. Nevertheless, the model developped in this paper may be easyly adapted to considered taxes officially on employees: only the case of constrained workers should be modified by considering the collectively bargained wage as the gross wage.

Furthermore, this model does not differentiate between contributive and non-contributive social security contributions. The type of policies studied consists in payroll tax cuts to decrease the labor costs, with compensation if necessary to the institutions of social security in order not to decrease the benefits. In that way, social security contribution cuts actually have the same impact as tax cuts. It may be interesting to consider cuts both in social security contributions and benefits, but it is out of the purpose of the paper. It could be analized through the model presented here as it is equivalent for the constrained workers to a decrease of their exogenous wage (through the decrease of the in-kind part of this remuneration). For unconstrained workers, it would be only the decrease of a mandatory consumption of insurance.

The remainding of the paper is organized as follow. Section 2 presents the general model: first the global setup (2.1), then the demand equation of firms (2.2), the wage bargaining process (2.3), the general equilibrium (2.4), finally the resolution for the case of a piecewise linear tax function (2.5). Section 3 investigate the case of a unique type of worker, annalytically and numerically for linear taxation (3.1) then numerically for quadratic tax functions (3.2). Section 4 investigate numerically the case of the interactions between different type of workers. First, the case of two kinds of unconstrainted workers is considered (4.1). Then the case with three factors is considered: one type of unconstrained worker, one type of constrained worker and one type of capital (4.2). This reduced model allows analysing the impact on different parameter of taxes on low and high qualification workers, of taxes on capital income and of sales taxes. Section 5 concludes.

2 Theoretical framework

2.1 The general setup of the model

We consider an economy with a numeraire good produced thanks to $n \geq 1$ labor types ($i = 1, ..., n$) supplied by a continuum of infinitely lived workers of size $\vec{L} = (L_1, ..., L_n)$ (supplying each one unit of labor). The production function is $F(N_1, ..., N_n)$ where $N_i \geq 0$

is the level of employment of factor of type i ($\vec{N} = (N_1, ..., N_n)$). The inputs are of three kinds. The $m \leq n$ first facors ($(N_1, ..., N_m)$) are human input: different kinds of workers. The last $n - m$ factors ($\vec{K} = (N_{m+1}, ..., N_n)$) are capital. Their cost is constant at the internationaly fixed interest rate r and can be acquired each period without friction. Among the workers, some are unconstrained workers ($\vec{L}_u = (L_1, ..., L_l)$ among who $\vec{N}_u = (N_1, ..., N_l)$ are employed and $\vec{U}_u = (U_1, ..., U_l)$ are unemployed, with $U_i = L_i - N_i$). They are workers who can negociate individually their wages with their employers. They keep bargaining even when employed, which is the reason why the model of intra-firm bargaining has been chosen. Their remuneration $w_i(\vec{N})$ therefore depends on the quantity of each input. Last, the constrained workers ($\vec{L}_c = (L_{l+1}, ..., L_m)$ among who $\vec{N}_c = (N_{l+1}, ..., N_m)$ are employed and $\vec{U}_c = (U_{l+1}, ..., U_m)$ are unemployed, with $U_i = L_i - N_i$) are workers who cannot negociate their wage individually. They are employed at a wage $\underline{w}_i$, collectivelly bargained, applying to all worker of their type. Depending of the use of the model, it can be considered as the collective bargaining with unions for each type of job or as the legal minimum wage. The model does not endogeneize this collective bargaining and the wages $\underline{w}_i$ for $i = l+1, ..., m$ are considered exogenous. Those workers are subject to classical unemployement in addition to frictional unemployment.

To recruit workers, firms post vacancies for each type of worker (with a segment specific hiring cost γ_i per unit of time and per vacancy posted) matched with the pool of unemployed workers of the type. Matching functions $h_i(U_i, V_i)$ give for each segment of the labor market the mass of aggregate contacts depending on the mass of unemployed U_i and the mass of vacancies V_i for the type of workers. With $\theta_i = V_i/U_i$ the tightness of segment i of the labor market, the probability to fill a vacant job by unit of time is $q_i(\theta_i) = h_i(U_i, V_i)/V_i$ ($q_i'(\theta_i) < 0$ and $q_i(0) = +\infty$) and the probability to find a job by unit of time is $p_i = h_i(U_i, V_i)/U_i = \theta_i q(\theta_i)$ (with $d[\theta_i q(\theta_i)]/d\theta_i > 0$). The segment-specific exogenous probability of job destruction by unit of time is s_i. For unconstrained workers, the wage $w_i(\vec{N})$ is continuously negociated after hiring (individually bargained but common by symmetry to all workers of type $i \leq l$). For constrained workers, it is $\underline{w}_i$ ($l < i \leq m$).

The global hypothesis are the same for labor and capital. It is usual for labor, less for capital. However, there may also be some cost to obtain the right kind of capital at the right moment, and the destruction rate may be easily interpreted as a depreciation rate. However, the case of perfect market for capital is deducted in a straitforward way from the general calse by setting $\gamma_i = 0$ for all $i \in [m+1, n]$.

Furthermore, a tax function T_i is considered such that the gross wage is $T_i[w_i(\vec{N})]$ when the net wage is $w_i(\vec{N})$. This tax function may represent most of tax schedule around the globe, whatever social security contribution - mainly linear - or labor income tax schedule - mainly piecewise linear. A quadratic version is also analysed numericallly to understand regressive abattement to social security contribution that target low wages in some countries, amid whose France. For the capital factors, this tax gives the level of capital income tax. The setting considers "employer" income tax, that is the cost of labor is the net wage plus the tax. For modeling payroll tax, it fits the "employer" part. For other taxes, this specification has no impact on results for unconstrained workers as the contractual wage is negociated knowing both the net and gross wages resulting from the negociation. For capital, it is a straitforward specification given the hypothesis that capital remuneration is fixed at the international interest rate level: it has to be the net remuneration of capital. The specification matters only for constrained workers. The inverse specificationmay be easily set by assuming that the collectively bargained wage is the gross one.

Considering numerical application, one should keep in mindthis specifications of taxes - i.e. the tax rate applies to the net remuneration of input - as it matters for the level of plausible tax rates. The same tax obviously correspond to a much larger tax rate when applied to the net wage than the gross wage. For example, a tax rate of 25% on the gross wage is equivalent to a tax rate of 33.3% on the net wage and a tax rate of 50% on the gross wage is equivalent to a tax rate of 100% on the net wage. Hence, numercial analyses can consider tax rates on net remuneration as large as 100%.

Last, a specification of consumption tax of rate t may be easyly introduced by considering a net firm income $F(\vec{N}) = (1-t)G(\vec{N})$ where $G(\vec{N})$ is the actual production function. On contrary to other taxes, this consumption tax is specificated with a rate t applying to the gross sellings. To fit usual consumption taxes applying on net prices, one should just consider the net rate $u = t/(1-t)$.

The equilibrium on the market (2.4) is reach through the confrontation of a labor demand curve and a wage bargaining curve on each segment of the labor market - depending on the equilibria on the other labor markets. The demand for each level of labor is define ex ante by the quantity of vacancies posted on the labor market (2.2). It depends on the anticipation of the ex post wage bargaining, itself depending on the level of unemployement, the unemployement benefits and the marginal productivity of each type of input (2.3). The overall model is dynamics and time is continuous. The equilibrium is calcullated through the use of Bellman equation for the values of profit flows for firms, and employement and unemployement for workers.

2.2 Labor demand

The demand for each type of labor is determined by the maximization by the firm of the value of its profit flows. The Bellman equation of the value of the firm for time between t and $t+dt$ is given by equation 1, subject to equation 2 giving the evolution of the number of each type of input depending on the rate of destruction of jobs, the number of vacancies and the matchng function itself depending on the tightness of the segment of the labor market.

$$\Pi(\vec{N}) = \max_{\vec{V}} \frac{1}{1+rdt} \left\{ \left[F(\vec{N}) - \sum_{j=1}^{n} \Big(T[w_j(\vec{N})]N_j + \gamma_j V_j \Big) \right] dt + \Pi(\vec{N}^{t+dt}) \right\} \tag{1}$$

$$N_i^{t+dt} = N_i(1 - s_i dt) + V_i q_i(\theta_i) dt \tag{2}$$

At that stage, no distinctions between constrained and unconstrained factors should be made. The only difference between the two kinds of factors is that the remuneration $w_i(\vec{N})$ of constrained factors is constant (equal to r for capital and to $\underline{w}_i$ for low-skill workers).

To resolve the maximization problem of firms, and consequently obtain the labor deman, the marginal profits (with respect to each type of workers) are noted $J_i(\vec{N}) = \partial\Pi(\vec{N})/\partial N_i$.

There is two ways of calculating these marginal profits. The first one is the first order condition with respect to the number of vacancies V_i posted by firms: it gives equation 3 at steady state. The second one is derived from the envelop theorem: it gives equation 4.

$$J_i(\vec{N}) = \frac{\gamma_i}{q_i} \tag{3}$$

$$J_i(\vec{N}) = \frac{\frac{\partial F(\vec{N})}{\partial N_i} - T[w_i(\vec{N})] - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}}{r + s_i} \tag{4}$$

Indeed, first order condition with respect to V_i is $-\gamma_i dt + J_i(\vec{N}^{t+dt}) dN_i^{t+dt}/dV_i = 0$ where $dN_i^{t+dt}/dV_i = q_i dt$ from equation 2. At steady state, $\vec{N}^{t+dt} = \vec{N}$ which gives equation 3. In addition, the envelop theorem applied by differentiating equation 1 with respect to N_i gives:

$$\left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} - T[w_i(\vec{N})]\right] dt + \frac{\partial N_i^{t+dt}}{\partial N_i} J_i(\vec{N}^{t+dt}) = J_i(\vec{N})(1 + rdt)$$

With $\frac{\partial N_i^{t+dt}}{\partial N_i} = (1 - s_i dt)$ from equation 2, which gives equation 4 at steady state. Combining equation 3 and 4 gives the decomposition of the marginal productivity with respect to the workers of type i in equation 5

$$\frac{\partial F(\vec{N})}{\partial N_i} = T[w_i(\vec{N})] + \frac{\gamma_i(r + s_i)}{q_i(\theta_i)} + \sum_{j=1}^{l} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} \tag{5}$$

Where $\partial F(\vec{N})/\partial N_i$ is the marginal productivity of worker of type i; $T[w_i(\vec{N})]$ is its gross wage; $\gamma_i(r + s_i)/q_i(\theta_i)$ the hiring costs increasing with the vacancy posting cost γ_i and the rate of job destruction s_i and decreasing with the probability $q_i(\theta_i)$ that a vacancy meets an unemployed worker; $N_j \partial T[w_j(\vec{N})]/\partial N_i$ is the change in the wage bill for workers of type j due to the change in the level of employement of workers of type i through the intra-firm wage bargaining process. As only high-skill workers may negociate their wages, the sum of the wage bill effects are made only over factors $j \in [1, l]$.

This equation 5 gives a relation between wage bargaining function as anticipated by firms and the level of employement targeted by firm through their vacancies' posting. It correspond to the Labor demand curves. This demand is not such that overal marginal labor costs - gross wages plus the costs of hiring - equals the marginal productivity of workers. It depends also on the variations of the overall wage bill due to the change in the employement level because changing the level of employement (and therefore of unemployement) change the wages through changes in the outside options of workers and firms. As shown by Stole and Zwiebel (1996b,a) and confirmed by Cahuc et al. (2008), the labor demand may be such that the marginal productivity of a type of worker is lower than the overall margiinal cost of such type of labor.

2.3 Wage determination

This labor demand equation 5 gives a first relation between the number of employees of each type and their wages. The actual wages and employement levels for each type of worker need another relation to be fully determined, this second relation comes from the intra-firm bargaining determining function $w_j(\vec{N})$ for unconstrained workers, which is determined in the present subsection. Constrained factors have per definition their remuneration equal to r for capital and $\underline{w}_i$ for low-skill workers. Consequently, the present section concerns only high-skill workers, that is factors N_i for $i \in [1, l]$. The Bellman equation of the value of being in employement E_i for worker of type i is equation 6, from which is directly derived equation 7.

$$rE_i = w_i(\vec{N}) + s_i(U_i - E_i) \tag{6}$$

$$E_i - U_i = \frac{w_i(\vec{N}) - rU_i}{r + s_i} \tag{7}$$

Given the type specific bargaining power β_i of workers of type i, the usual Nash bargaining equation is equation 8, according to the fact that the rent of employement for workers is the difference of values $E_i - U_i$ between employement and unemployement and the rent of employement for the firm is the marginal productivity $J_i(\vec{N})$ of workers of type i.

$$\beta_i J_i(\vec{N}) = (1 - \beta_i)(E_i - U_i) \tag{8}$$

The bargaining power is a very influential parameter on the results of the present article. It is therefore of main importance to accurately understand what it stands for. This is often a quite neglected parameter in search and matching literature due to the difficulty to rightly interpret the economic reality behind what is called bargaining power in these kind of model. However, the focus is here particularly on tax incidence on wages, for which the bargaining power of workers of course matters. Hence, it is not possible to elude the issue and the first step to fully understand it is to clearly define what bargaining power is not. It does not come neither from rarity of the type of workers nor from its productivity. In search and matching models, rarity is taken into account through the intensity of use of the factor type, that is through the tightness of labor markets. Yet, tightness is not a parameter but a variable. Nevertheless, it comes from the actual rarity of factors compared to technical needs, that is compared to the production function. Production function is also the way to take workers' productivities into account independently from the bargaining power parameter.

The thesis of the present article is that bargaining power does not represent any form of substitutability of workers between worker types (which would be implicitly assumed by considering productivity or rarity of worker types) but substitutability within worker types. It may be fully understood considering the wage analysis of Goldin (2014). She focused on the gender gap explanations and draws very general results on wage variations within jobs and qualifications. She determines some activities with wages proportional to the implication of the workers (working time, acceptance of unusual period of work, any-time availability). They are jobs where tasks may be easily shared between different workers, where substituting one worker to another does not decrease the productivity. The example provided is pharmacists: each task is independent from the preceding one and all the needed information appears on the computer screen when loading the patient fill.

On the other hand, some kinds of jobs present a wage function strongly convex with respect to worker implication. Goldin (2014) and previously Goldin and Katz (2008) found that business and law jobs present such schemes. The reason comes from the need to fully follow contracts or clients and know all the details and specificities, which cannot be transferred without huge costs to a substitute worker. This creates a low substitutability within the type of worker allowing the employee to catch a larger share of the surplus than more substitutable types of workers with the same rarity and skills. This is exactly what is reflected by the bargaining power parameter. Furthermore, it also justifies the intra-firm bargaining process as it is actually the position of insider and the knowing of all the specificities of the very position that allows such non-substitutable worker to extract a share of the profit.

Considering this interpretation of the bargaining power parameters β_i, no straitforward monotonous relation should exist between qualification and bargaining power (one exemple being the high skills pharmacists whose bargaining power is weak). Nevertheless, from a broadly perspective, a positive correlation between qualification and bargaining power is highly plausible. Hence, we use a positive link between productivity and bargaining power in numerical simulations, even if the bargaining power does not come from productivity in itself.

According to equation 7 of the difference of value between employement and unemployement and equation 4 of the marginal productivity of workers of type i, equation 8 may be rewritten as the differential equation 9 of the wage as a function of the level of employemennt.

$$(1-\beta_i)w_i(\vec{N}) + \beta_i T[w_i(\vec{N})] = (1-\beta_i)rU_i + \beta_i \left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{l} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i} \right] \tag{9}$$

As the intra-firm bargaining take place individually for each worker allready employed by the firm, it did not anticipate the possible change in employement resulting for the new wage, wich means that rU_i is considered as constant in that differential equation. This differential equation will be solve in the section of the actual resolution of the model. To solve this differential equation, a condition at the limit is needed. The condition considered is that the overall gross wage bill $N_i T[w_i(\vec{N}_i)]$ for workers of type i tends towards zero when the employement N_i of such workers tends towards zero.

This differential equation gives the result of wage determination inside firms through wage bargaining. It is done inside each firm considering the state of the labor market as given, and therefore it is a solution in partial equilibrium: the reservation wages rU_i and the labor market tightnesses θ_i are considered as exogenous variables by bargaining workers.Consequently, the result of the wage bargaining process - the solution of differential equation 9 - gives the wage function depending on the employment structure $\vec{N}$ and the value of unemployment for workers of type N_i. In addition, this value - considered as given by the workers of type i during the bargaining proces - is defined at general equilibrium. This mechanism is presented in the following subsection.

2.4 Labor market equilibrium

To determine the general equilibrium of this model, two relations are needed. The first one is the demand of labor (equation 5) and the second the wage function given by the solution of differential equation 9. The way of resolution is to determine two sets of n equations linking directly N_i and θ_i. The first set of equations comes from the labor market allocation process.

Equation 2 gives $N_i s_i = V_i q_i(\theta_i)$ and consequently the first set of equations linking N_i to θ_i is equation 10.

$$\theta_i q_i(\theta_i) = \frac{s_i N_i}{L_i - N_i} \tag{10}$$

The second set of equations comes from the labor demand equation 5 knowing the remunaration of constrained factors and the wage functions of unconstrained factors (results of differential equations 9). However, these last functions depend on the value of unemployment rU_i for high-skill workers, which is determined at general equilibrium. The bellman equation for the value of unemployement is:

$$rU_i = b_i + \theta_i q_i(\theta_i)(E_i - U_i)$$

Where b_i is the income flow at unemployment. As equation 8 gives $E_i - U_i = \beta_i/(1 - \beta_i)J_i(\vec{N}) = \beta_i/(1 - \beta_i)\gamma_i/q_i(\theta_i)$ because of equation 3, the value of unemployement is given by equation 11.

$$rU_i = b_i + \gamma_i \frac{\beta_i}{1 - \beta_i}\theta_i \tag{11}$$

Hence, it is possible to find equation 12 giving the high-skill workers' wage at equilibrium by including equation 5 in equation 9.

$$w_i(\vec{N}) = b_i + \frac{\gamma_i \beta_i}{1 - \beta_i}\left(\theta_i + \frac{r_i + s_i}{q_i(\theta_i)}\right) \tag{12}$$

To calculate the structure of employement $\vec{N}$ and the structure wages $\vec{w}(\vec{N})$, the solution of differentital equation 9 should be incorporated in this system, which gives the second set of relations between the wage and employement and consequently the equilibrium wages and employements.

2.4.1 Full employer bargaining power

If the bargaining power is fully owned by the employer, that is if $\beta_i = 0$, differential equation 9 become equation 13 giving directly the bargained net wage.

$$w_i(\vec{N}) = rU_i = b_i \tag{13}$$

The employee without any bargaining power should accept its reservation wage and nothing more. In that case, the net wage is independant from the payroll tax which is fully beared by the employer. It is another way of thinking of the low-skill workers. They may be workers without bargaining powers, whose wages would be b_i if there where no minimum wage nor constraining collective bargaining. In the present framework, the condition for the low-skill workers actually being constrained factors is that the minimum wage or collectively bargained wage $\underline{w}_i$ is larger than the unemployment flow of low-skill workers b_i. This interpretation allow to be sure that low-skill workers are constrained as this constraint should be the results of the bargaining process.

2.4.2 Full employee bargaining power

At the opposite if the full bargaining power is owned by the employee, differential equation 9 become:

$$T[w_i(\vec{N})] = \frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}$$

Yet:

$$\frac{\partial \sum_{j=1}^{n} N_j T[w_j(\vec{N})]}{\partial N_i} = T[w_j(\vec{N})] + \sum_{j=1}^{n} N_j \frac{\partial T[w_j(\vec{N})]}{\partial N_i}$$

And therefore differential equation 9 is equivalent to equation 14

$$\frac{\partial \left(\sum_{j=1}^{n} N_j T[w_j(\vec{N})] - F(\vec{N}) \right)}{\partial N_i} = 0 \qquad (14)$$

And consequently $\sum_{j=1}^{n} N_j T[w_j(\vec{N})] - F(\vec{N})$ is constant with respect to $\vec{N}$. Yet it is zero when $\vec{N} = \vec{0}$. Hence, $\sum_{j=1}^{n} N_j T[w_j(\vec{N})] = F(\vec{N})$ and there is no equilibrium because the full output is paid in wage and nothing remains for the hiring costs. However, this hypothesis of full bargaining power of the employees is very unlikely and the following of the paper considers that high-skill workers have bargaining powers striclty between 0 and 1.

2.5 The wage functions when taxes are piecewise linear

The final stage to completely solve the model is to finds the solution of the wage differential equation 9. With the present knowledge of mathematics on differential equations, it is not possible to solve such a differential equation for a general tax function T. Basically, it is possible mainly in the linear case. However, the linear case is indeed the most probable as the tax schedules actually setted in most countries are flat or piecewise linear. Hence, the more general case is to consider a piecewise linear income tax schedule where the marginal tax rate at the level of wage of the workers of type i is τ_i: $T_i[w_i(\vec{N})] = (1+\tau_i)w_i(\vec{N})$. With that framework, τ_i for $i \in [m+1, n]$ are directly interpretated as marginal tax rate on capital income. Some continuously progressive tax schedule are also possible, enven if less likely. It is the case for payroll tax in France where a payroll tax rebate at the level of the minimum wage is continuously decreased giving birth to actually continuously progressive marginal tax rates on labor income. This very specific case is studied numerically in subsection 3.2. The system of differential equations become as presented by equation 15 for factors $i \in [1, l]$.

$$(1+\beta_i\tau_i)w_i(\vec{N}) = (1-\beta_i)rU_i + \beta_i \left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{l} (1+\tau_j) N_j \frac{\partial w_j(\vec{N})}{\partial N_i} \right] \qquad (15)$$

This system of differential equation can not be solved directly because each function $w_i(\vec{N})$ depends on the derivatives of the wage function of other type of workers. The first stage for solving this differential equation consists in desintengling partially this system. Appendix

A.1 shows how it is possible and demonstrates that the system is equivalent to those of equation 16.

$$(1+\beta_i\tau_i)w_i(\vec{N}) = (1-\beta_i)rU_i + \beta_i\left[\frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{l}(1+\tau_j)\chi_{ij}N_j\frac{\partial w_i(\vec{N})}{\partial N_j}\right] \tag{16}$$

Where the parameter $\chi_{ij} = \frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}$ give the comparison between the bargaining powers of workers of types i and j. There is no problem of dividing by zero because the only factors whose bargaining power is considered in the previous equation are those for $i \in [1,l]$ whose bargaining power is strictly positive (the other are indeed constrained factors because their unemployment benefit is lower than the minimum wage).

The second stage is the actual resolution of the differential equation. It consists in several changes of variables, the most important being the change in polar coordinates allowing to actually resolve the differential equation, and some intergration per part. It is quite technical and has no economic meaning by itself. This is the reason why it is entirely presented in the appendix section (appendix A.2). It allows to demonstrates lemma 1.

Lemma 1. *The solution of the system of wage bargaining differential equations 15 - with condition at limit being that the payroll bill of each segment tends towards zero when the employment on that segment tends toward zero - is given by equation 17* for all unconstrained workers (when $i \in [1,l]$).

$$w_i(\vec{N}) = \frac{1-\beta_i}{1+\beta_i\tau_i}rU_i + \int_0^1 u^{\frac{1-\beta_i}{\beta_i}+\tau_i}\frac{\partial F(\vec{N}_u A_i(u), \vec{N}_c, \vec{K})}{\partial N_i}du \tag{17}$$

Where matrix $A_i(u)$ is given by equation 18.

$$A_i(u) = \begin{pmatrix} u^{(1+\tau_1)\frac{\beta_1}{1-\beta_1}\frac{1-\beta_i}{\beta_i}} & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & u^{(1+\tau_j)\frac{\beta_j}{1-\beta_j}\frac{1-\beta_i}{\beta_i}} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & u^{(1+\tau_l)\frac{\beta_l}{1-\beta_l}\frac{1-\beta_i}{\beta_i}} \end{pmatrix} \tag{18}$$

Proof. See appendix A.2.

Equation 17 provides a decreasing relationship between the employement N_i and the net wage w_i as soon as factorial marginal productivity is decreasing. As equation 10 provides an increasing relationship between these two variables, it allows to define a general equilibrium as in the following subsection. Furthermore, an increase of taxes or bargaining powers for one kind of workers generates a net wage increase for type of workers who are complement (the marginal productivity of one type of workers increases with respect to the numbers of other type of workers) and a net wage decrease for type of workers who are substitutes (the marginal productivity of one type of workers decreases with respect to the numbers of other type of workers).

2.6 General equilibrium when taxes are piecewise linear

The two sets of n equations 17 and 10 provides n labor demand and n wage setting equations with $2n$ variables: the n input quantities N_i and the n tightness θ_i of labor market segments. Incorporating the wage functions from equation 17 into the general equilibrium wage equation 12 and replacing the value of unemployment thanks to equation 11 gives the general equilibirium system 19. The equations for constrained input comes from the demand equation 5 and the derivatives of the wage functions from equations 17.

$$\begin{cases} \theta_i q_i(\theta_i) = \frac{s_i N_i}{L_i - N_i} & (a) \\ \\ \left[\begin{array}{l} if\ i \in [1, l] \\ \int_0^1 u^{\frac{1-\beta_i}{\beta_i}+\tau_i} \frac{\partial F(\vec{N}A_i(u))}{\partial N_i} du = \frac{(1+\tau_i)\beta_i}{1+\beta_i\tau_i} b_i + \frac{\gamma_i\beta_i}{1-\beta_i}\left(\frac{1-\beta_i+2(1+\tau_i)\beta_i}{1+\beta_i\tau_i}\theta_i + \frac{r_i+s_i}{q_i(\theta_i)}\right) \\ \\ if\ i \in [l+1, n] \\ \frac{\partial F(\vec{N})}{\partial N_i} - \sum_{j=1}^{l}(1+\tau_j)N_j \int_0^1 u^{\frac{1-\beta_j}{\beta_j}+\tau_j} \frac{\partial^2 F(\vec{N}A_j(u))}{\partial N_j \partial N_i} du = (1+\tau_i)w_i + \frac{\gamma_i(r+s_i)}{q_i(\theta_i)} \end{array} \right. & \begin{array}{l} \\ (b_u) \\ \\ \\ (b_c) \end{array} \end{cases} \tag{19}$$

Furthermore, additionnal light assumption may be done. The main one is that different inputs are allways at least slightly complement. That means that no input has its marginal productivity strictly increasing when the quantity of another input decreases. With that assumption, it is possible to demonstrate the existence of an equilibrium, as it is stated by proposition 1.

> ***Proposition 1: The existence of a general equilibrium.*** *Under likely hypotheses on the production function - decreasing factorial productivity, imperfect substitution of factors and second order effects dominated by first order effects - there exists a general equilibrium on the segmented labor market, which is solution of the system of equations 19.*

Proof.: Equation 19a provides a strictly increasing relation between θ_i and N_i. Considering the implicit increasing function $\theta_i(N_i)$, the problem is the n equations 19b for the n unknown N_i. These equations are of the type $lht_i(\vec{N}) = rht_i(N_i)$. The right hand term functions rht_i striclty increase - from $(1+\tau_i)\beta_i b_i/(1+\beta_i)$ if $i \in [1, l]$ and from $(1+\tau_i)w_i$ if $i \in [l+1, n]$ - to infinity when N_i tends towards L_i. The hypotheses about the production function induce that the left hand term functions lht_i decrease with respect to N_i and increase with respect to N_j $j \neq i$.

In addition, let us assume that for any i, $lht_i(\vec{N}^{-i}, 0, \vec{N}^{+i})$ is larger than $rht_i(0)$ (where $\vec{N}^{-i} = (N_1, ..., N_{i-1})$ and $\vec{N}^{+i} = (N_{i+1}, ..., N_n)$). If it is not the case, N_i is zero at equilibrium and let us consider the labor market without this fictive segment (let call this the no-fictive segment assumption). It means that for any values of $\vec{N}^{-i}$ and $\vec{N}^{+i}$, the equation $lht_i(\vec{N}^{-i}, N_i, \vec{N}^{+i}) = rht_i(N_i)$ has a unique solution strictly between zero and L_i. This solution $N_i^*(\vec{N}^{-i}, \vec{N}^{+i})$ increases with respect to each N_j $j \neq i$, because $rht_i(N_i)$ does not depend on any N_j $j \neq i$ and $lht_i(\vec{N})$ increases with respect each N_j $j \neq i$. This partial equilibrium on segment i is shown by Figure 1.

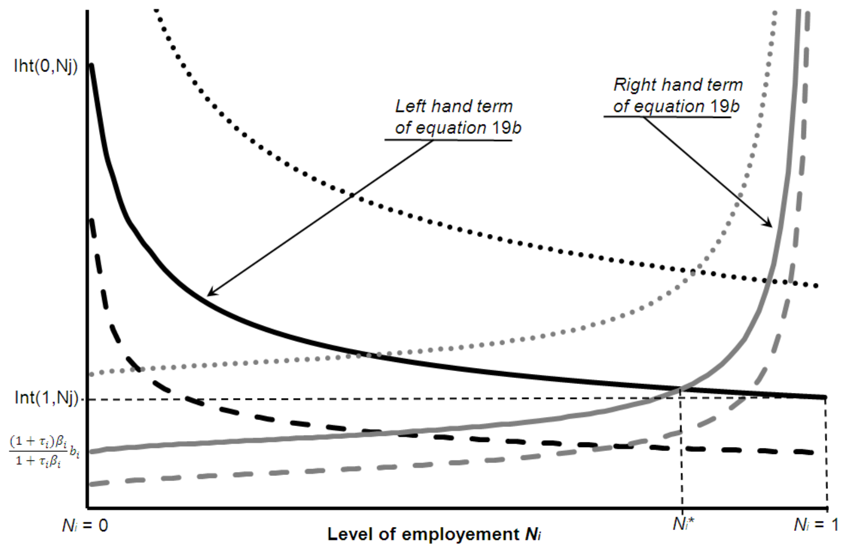


Figure 1: Equilibrium on a segemented labor market

Now let us build an infinite sequence of vectors $\vec{N}(\nu)$. Let assume that the first terms of the series are $(1/2^{\nu}, ...1/2^{\nu})$ until first rank μ where for each i $lht_i(\vec{N}(\nu)) > rht_i(N_i(\nu))$. The rank μ exists due to the no-fictive segment assumption. After this rank μ, let us define $N_i(\nu+1)$ as the partial equilibrium on segment i given $N_j = N_j(\nu+1)$ if $j < i$ and $N_j = N_j(\nu)$ if $j > i$. Each sequence $N_i(\nu)$ increases after rank μ, because $N_i(\mu)$ is under partial equilibirum on segment i, then $N_i(\nu+1)$ is the new equilibrium with increased N_j $j \neq i$. The sequence of vectors $\vec{N}(\nu)$ increases and is bounded (by $\vec{L}$), so it converges between zero and $\vec{L}$. The algebraic limit theorem induces that the limit $\vec{N}(\infty)$ of this sequence verifies the equation $lht_i(\vec{N}(\infty)) = rht_i(N_i(\infty))$ for each i and is therefore solution of the problem 19. ***Q.E.D.***

The unicity is not directly demonstrable. However, it is very likely as soon as there is no increasing returns to scale. If there is actually increasing returns of scale, the multiple equilibria result is usual.

Furthermore, the impact on the employment equilibrium of various parameters may be easily understood thanks to Figure 1. A parameter increasing lht_i (pushing the black solid line onto the black dotted line) leads to an increases of the level of employment. Reciprocally, a parameter decreasing lht_i (pushing the black solid line onto the black dashed line) leads to a decrease of the level of employment. A parameter increasing rht_i (pushing the grey solid line onto the grey dotted line) leads to a decreases of the level of employment. Reciprocally, a parameter decreasing rht_i (pushing the grey solid line onto the grey dashed line) leads to a decrease of the level of employment. In that way, all parameters but the bargaining power have unambiguous impact on the equilibrium, as presented in table

The case of crossed impact is more complex. Generally speaking, an increase of a tax rate τ_j on a given segment of the labor market decreases lht_i on other segments through the de-

Table 1: Impact of model parameters on the level of employement

Parameter	Variations in Eq. 19	Employement variation
Total factor produvitity	$lht \nearrow$	Increase
Matching function efficiency $q(.)$	$rht \searrow$	Increase
Segment size L	$rht \searrow$	Increase
Unemployement benefits b	$rht \nearrow$	Decrease
Vacancy posting cost γ	$rht \nearrow$	Decrease
Job destruction rate s	$rht \nearrow$	Decrease
Interest rate r	$rht \nearrow$	Decrease
Payroll tax rate τ	$lht \searrow$ and $rht \nearrow$	Decrease

crease of N_j, and therefore leads to a decrease of employment on other segments of the labor markets. For couple of unconstrained workers, this effect is increased by the effect of the matrix A_i (equation 18) on the left hand term: as u is lower than one, $u^{(1+\tau_j)[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]}N_j$ decreases with respect to τ_j, and so is lht_i.

Proposition 2. *In the case of an increase of the tax rate on labor type $j \in [1, l]$, the decrease of employement on the segment of the labor market for labor type $i \in [1, l]$ is steaper when $[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]$ is larger, that is when the relative bargaining power of workers of type j is larger compared to the bargaining power of workers of type i.*

Proof. The left hand term of equation 19 decreases because input j in the production fonction under the intregral is $u^{(1+\tau_j)[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]}N_j$, which decreases both because N_j decreases (more strongly for workers j with larger bargaining power, e.g. subsection 3.1) and because $u^{(1+\tau_j)[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]}$ decreases because u is lower than 1; this last decrease is stronger when the parameter $[\beta_j/(1-\beta_j)]/[\beta_i/(1-\beta_i)]$ is larger. ***Q.E.D.***

Hence, increasing the taxes of high bargaining power workers affects more both their own employement and the employement of other unconstrained workers than increasing taxes of low bargaining power workers. At the opposite, decreasing taxes on the low bargaining powers have less positive impact both on themselves and on the other workers. If bargaining power is positively correlated with qualification and income, it constitutes an efficiency point against payroll tax abattement targeted on low wages, as is heavyly set in France.

This result may be easily understand by the ability of a segment to undermine the negative exogenous shocks on employment by decreasing the segment wage. The more bargaining power the workers have, the more share of surplus is capted by their segment, and hence the more the workers of the segment have to give back (in terms on surplus sharing, that is in terms of wages) to compensate the negative shock and limit the job destruction. This is reflected in the differentiation of the equilibrium wage from equation 12 with respect to the tightness of the segment of the labor market. This derivative is both positive and increasing in magnitude with respect to the bargaining power.

Following the same reasonning, one could anticipate that the marginal share of taxation borne by employees should decrease with the level of taxation. Indeed, the more the taxation increases, the more the employee has reduced its share of the surplus to limit unemployment, and the less it have surplus to leave to limit unemployment rise. This is confirmed by the numerical analyses presented in the following sections.

These empirical sections look at particular cases of the present model, and assess numerical analysis. It is important to understand that the model considers capital income and wage taxation defined on the net payment. Therefore, tax rates considered for the simulations appear to be quite high. However, a 100% tax rate on the net payment corresponds to a 50% tax rate on the gross payment, which is high but not unlikely. Hence, simulations computes results for tax rates from 0% to 100%. Furthermore, to simplify the numerical analysis, the parameter of the size of segments is not considered and each segment size is normalized to one.

3 Unique type of worker

In the simpler case with homogenous workers, differential equation 9 become equation 20.

$$(1-\beta)w(N) + \beta T[w(N)] = (1-\beta)rU + \beta\left[\frac{\partial F(N)}{\partial N} - N\frac{\partial T[w(N)]}{\partial N}\right] \tag{20}$$

Even if it can be shown that equation 20 has solutions, they can not be exibited formally. Numerical solving for continuously increasing tax functions T are presented in subsection 3.2. In a first subsection 3.1, differential equation 20 is solved formally in the special case of linear tax function. This case also covers the widely set tax schedule of picewise linear income/payroll taxes.

3.1 Numerical analysis of the linear case

With the assumption that the tax function is linear ($T(w) = (1+\tau)w$), the net wage is defined by differential equation 21.

$$(1-\beta\tau)w(N) = (1-\beta)rU + \beta\left[\frac{\partial F(N)}{\partial N} - (1+\tau)N\frac{\partial w(N)}{\partial N}\right] \tag{21}$$

With the condition at the limit being that the overall gross wage bill $NT[w(N)]$ tends towards zero when employement N tends towards zero, which is equivalent that that the net wage bill $Nw(N)$ tends towards zero when employement N tends towards zero because $T[w(N)] = (1+\tau)w(N)$.

Lemma 2. The solution of differential equation 21 subject to limit condition $\lim_{N\to 0} Nw(N) = 0$ is the wage function given by formula 22.

$$w(N) = \frac{1-\beta}{1+\beta\tau}rU + \int_0^1 v^{\frac{1-\beta}{\beta}+\tau} F'(v^{1+\tau}N)dv \tag{22}$$

Proof. This derives directly from the writting of equation 17 with a unique unconstrained factor N.

And therefore, the wage decreases with respect to level of employement. This gives a decreasing relationship between wage and employement as equation 12 gives an increasing relationship between these variables, allowing to define a general equilibrium.

Proposition 2. *As soon as the production function has not increasing marginal productivity, a unique equilibrium exists; it is solution of equation 23.*

$$\int_0^1 u^{\frac{1-\beta}{\beta}+\tau} F' = \frac{(1+\tau)\beta}{1+\beta\tau} b + \frac{\gamma\beta}{1-\beta}\left(\frac{1-\beta+2(1+\tau)\beta}{1+\beta\tau}\theta + \frac{r+s}{q(\theta)}\right) \tag{23}$$

Proof. This derives directly from the writting of equation 19 with a unique unconstrained factor N.

To go further, hypothesis should be made. We consider Cobb-Douglas production function $F(N) = AN^\alpha$. The matching function is assumed to be of the form $h(u, V) = au^{1-\eta}V^\eta$. Consequently, $q(\theta) = a\theta^{\eta-1}$ and $\theta q(\theta) = a\theta^\eta = sN/(1-N)$. Hence, equation 10 become 24

$$\theta = \left(\frac{s}{a}\right)^{\frac{1}{\eta}} \left(\frac{N}{1-N}\right)^{\frac{1}{\eta}} \tag{24}$$

Consequently, system of equations 24 and 23 imply equation 25 of the level of employment at equilibrium.

$$\frac{\alpha\beta AN^{\alpha-1}}{1-\beta+\alpha\beta(1+\tau)} = \frac{(1+\tau)\beta}{1+\beta\tau b} + \frac{\gamma\beta\left(\frac{s}{a}\right)^{\frac{1}{\eta}}}{1-\beta}\left[\frac{(1+\tau)\beta}{1+\beta}\left(\frac{N}{1-N}\right)^{\frac{1}{\eta}} + \frac{r+s}{s}\left(\frac{N}{1-N}\right)^{\frac{1}{\eta}-1}\right] \tag{25}$$

Left hand term decreases from infinity to a finite positive term when N goes from 0 to 1, while right hand term increases from 0 to infinity when N goes from 0 to 1. Hence, there exists a unique solution between 0 and 1. Now the question is: how this equilibrium N varies with respect to the parameters of the model (mainly β and τ) and how it impacts the equilibrium wage.

The program implemented to these numerical solving is presented in appendix E.1. The calibration is done with Cobb-Douglas production function with standart labor productivity parameter $\alpha = 2/3$, a marginal productivity setted to one when full employment, interest rate equal to three percent. The matching function parameters are calibrated according to Petrongolo and Pissarides (2000) survey of the empirical literature on the matching function and Borowczyk Martins et al. (2011) who corrects for a bias in the estimation due to endogenous search behavior from each side of the market.

For each parameter, variants are implemented to understand the effect of this parameters on the interest variables. Figures for each interest variable and each parameter are given in appendix B.1. It confirms the results of table 1 and are quite straitforward. Furthermore, the impact of to more parameters - the labor parameter α in the Cobb-Douglas production function and the bargaining power β - are presented. Concerning the level of wages, they are reported in figure 2.

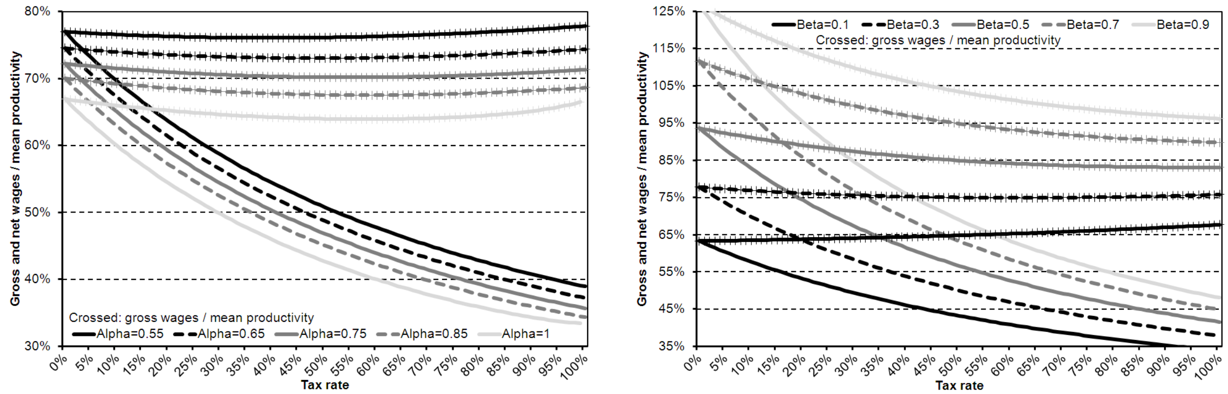


Figure 2: Impact of labor productivity and bargaining power on wages

The dependence on the bargaining power is straithforward: a larger bargaining power allows to get higher wages (as a share of the production). However, it is not the case for the parameter α of labor input in the Cobb-Douglas production function, an increase of those generate an equilibrium with lower wages. This come from a large impact of this parameter on the labor demand, which can be seen in figure 3 presenting the levels of unemployement for the different values of the parameters.

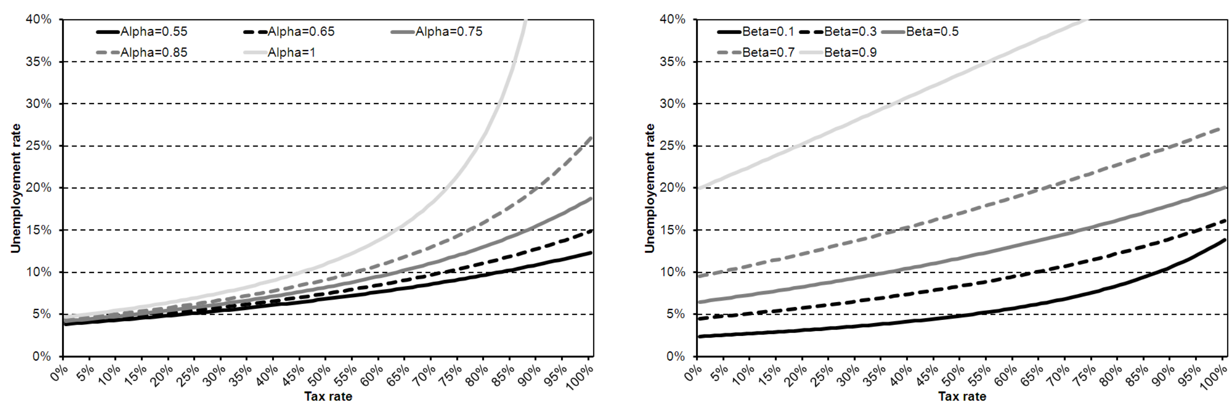


Figure 3: Impact of labor productivity and bargaining power on unemployement

The unemployement rate increases strongly with respect to that parameter α, and the negative impact of taxes on employement also increases more quickly when α is larger. Indeed, the impact of α is almost zero when taxation is low, it even has no impact at all without taxation. It highlights the phenomenon that some parameter other than taxation may be thought to have no impact on unemployment in models without taxation only because their effect is reveled by taxation. Furthermore, the bargaining power has also a negative impact on employement, due to its positive impact on wages. It is a standart result of search and matching literature. The crossed effect of bargaining power and taxes seems low even if each parameter reinforced the negtive impact on unemployment of the other. As a results of these dependencies, the incidence is given by figure 4.

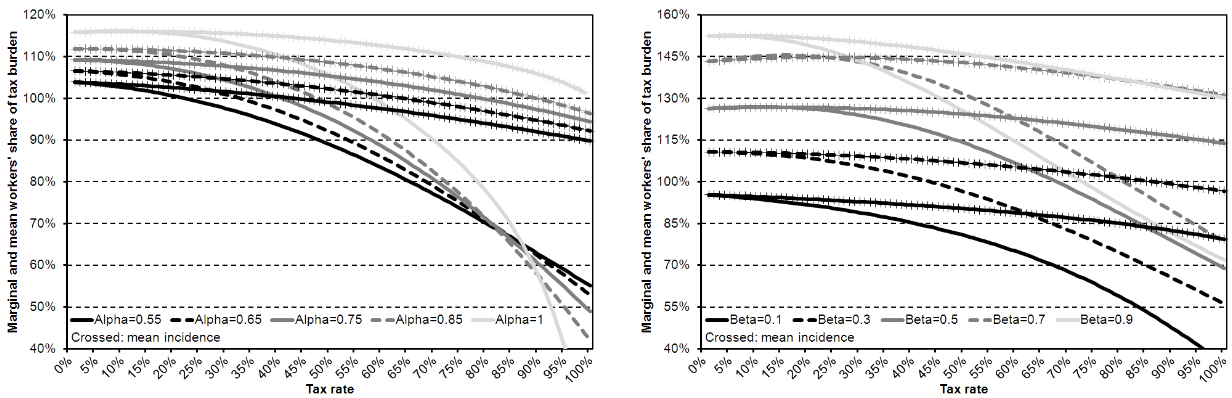


Figure 4: Impact of labor productivity and bargaining power on incidence

The share of the tax burden falling onto employees increases with respect to both α and β parameters. It is not surprising in the case of the bargaining power parameter β: the employee get a larger share of the total surplus, and therefore leave a small share of that surplus to employers, that have therefore few surplus to actually pay the tax. However, it is more surprising in the case of the α parameter as with larger α employees have lower wages and larger share of the tax burden, even if this last effect is relatively small.

Moreover, figure 4 is very representative of the whole figure in appendix B.1 presneting the influence of taxation. For all reasonable combination of parameters, the incidence is both high - even larger than 100% - and strongly marginally decreasing. this last result has at least to important implications. First of all, concerning the interpretation of empirical results, they often under-estimates the overall incidence of taxation as they mainly measure marginal incidence (due to empirical strategy of identification) which is significantly lower than mean incidence.

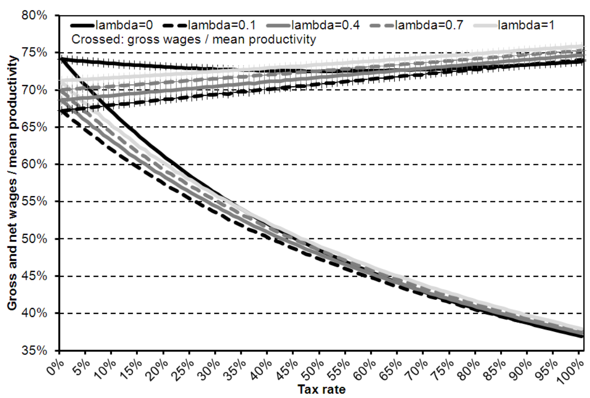
Moreover, from a public policy point of view, it appears that as incidence is marginally decreasing, the impact of taxes on labor costs are marginaly increasing and therefore the impact of taxes on unemployement is marginally increasing. Consequently, the efficiency of employement policies consisting in payroll tax abatemment is marginally decreasing with the level of abatement. This results seems to be confirmed by the meta-analysis of Zemmour (2013) on the French payroll tax abatement policy.

3.2 Numerical analysis of a non linear case

If differential equation 20 can be solved formally only in the case of a linear tax function $T[.]$, numerical analyses can be done for different cases. In the present subsection, the focus is made on quadratic tax function, that are lineary increasing tax rate. Most tax function in the world are linear or piecewise linear, but there exists some quadratic form.

For exemple, France has setted degressive abatement of payroll tax, which correspond to a quadratic payroll tax. The first abattement was created in 1993, and has been several times completed since. In 2012, it consisted in 26% reduction in the payroll tax from one to 2.1 minimum wage, the reduction beeing lineary declining to zero from 2.1 to 2.4 minimum wage. Consequently, between 2.1 to 2.4 minimum wage, if the normal payrrol tax rate is X%[2], the tax function between 2.1 and 2.4 minimum wage is $T[w] = (X\% - 208\%)w + (260/3)w^2$.

The program to resolve numerically differential equation 20 is the case of quadratic tax function, and then to solve find the general equilibrium for different level of tax and tax progressivity is presented in appendix E.2, the results are compiled in figures presented in appendix B.2. The main results may be observed in figure 5.



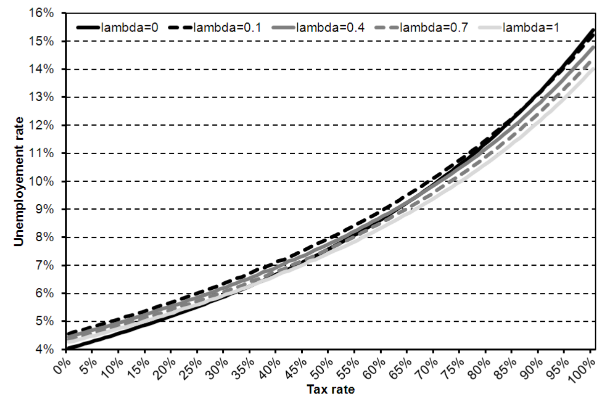


Figure 5: Impact of tax progressivity on wages and employement

It appears two surprising results, which are confirmed by looking at other variables or other scale of productivity. First, the effect of progressivity is very strong when progressivity is very small, then it decreases and the curves go closer to the linear case when progressivity increases.

Second, the impact of progressivity is negative both on wages and employement when the overal tax rate is small, then less and less negative while the overal tax rate increases for ending positive when the overal tax rate is high.

4 Interactions between multiple worker types

The aim of the numerical analysis is here to understand the effects of interactions between different workers subject to different taxes. The focus is made on two different kind of interactions. The first one is the interaction between two types of unconstrained workers (4.1). The second one is the interaction between a constrained and an unconstrained worker (4.2). For that case, a third factor (capital) is also added. This allows analysing the impact

[2]The actual normal payroll tax is not given here because calculating it requires differentiating what is actual taxation and what is mandatory insurrence in the French social contributions. However, this debate does not impact the present exemple concerns reduction of social contribution decollerated from any social benefit

on different paramaters of several taxes: not only the taxes on low and high qualification workers are considered, but also taxes on capital income and sales taxes.

4.1 The case of two unconstrained worker types

The first kind of numerical analysis restains to only two worker types. This relative simplicity allows keeping quite complex functional form for the production function to catch the impact of the level of substituability between worker types on the different economic outcomes. The idea is to simulate a production with two type of labor, one more qualified than the other. The difference of qualification is obtain by using different parameters of productivity in the production function. It is also assumed that the more qualified workers have more bargaining power than the less qualified one. The main idea is that qualification is rare and provides worker with some additional bargaining power due to the workers.

To solve this problem, the system of two equations 19 is considered for the two worker types 1 and 2 with the two unknown variables N_1 and N_2. It should be given of functional form to the production function. The decreasing marginal productivity form is kept for the global workforce N, with $F(N) = AN^{\alpha}$. Moreover, the aim is to understand the impact of the level of complementarity/substituability between workers on their interaction in relation with payroll taxation. Hence, a constant elasticity of substitution form is assumed for the global workforce N depending on the number N_1 and N_2 of each type of worker, with various calculations for various elasticities of substitution, including 1 modeled by a Cobb-Douglas production function. The production function is therefore $F(N_1, N_2) = A\left(\alpha_1 N_1^{-\delta} + \alpha_2 N_2^{-\delta}\right)^{-\frac{\alpha}{\delta}}$ with $\alpha_2 = 1 - \alpha_1$.

As said in the proof of proposition 2, the left hand term of that equation 19 is monotonously increasing when the right hand term monotonously decreasing. The unique solution may be numirically approach as close as wanted for each N_i at N_j given by incremental variations of N_i depending on the sign of the difference between the left hand term and the right hand term. Each solving is done sequentially with the other N_j given as the solution of the previous solving since the sum of the square of the changes $((N_{i,t} - N_{i,t-1})^2 + (N_{j,t} - N_{j,t-1})^2)$ is inferior to the precision level. The program is presented in appendix E.3. The level of precision is here of 10^{-8}.

The full resluts are presented in appendix C. They show some similarities with the one worker case, particularly concerning the decreasing marginal incidence and therefore the marginal incidence being lower than the mean incidence. The theoretical results are confirmed, particularly concerning the crossed impact on employement. The level of unemployement resulting from the taxation of high or low qualification workers are presented in figure 6.

Taxing high qualification workers increase unemployement of low qualification workers except for very large elasticity of substitution between type of workers. The impact of high qualification workers'taxation on low qualification workers employment is quite large, even if it smaller than the direct impact on igh qualification workers' unemployment. The same result is appears qualitatively for high qualification workers when taxing low qualification workers but the effect is very small.

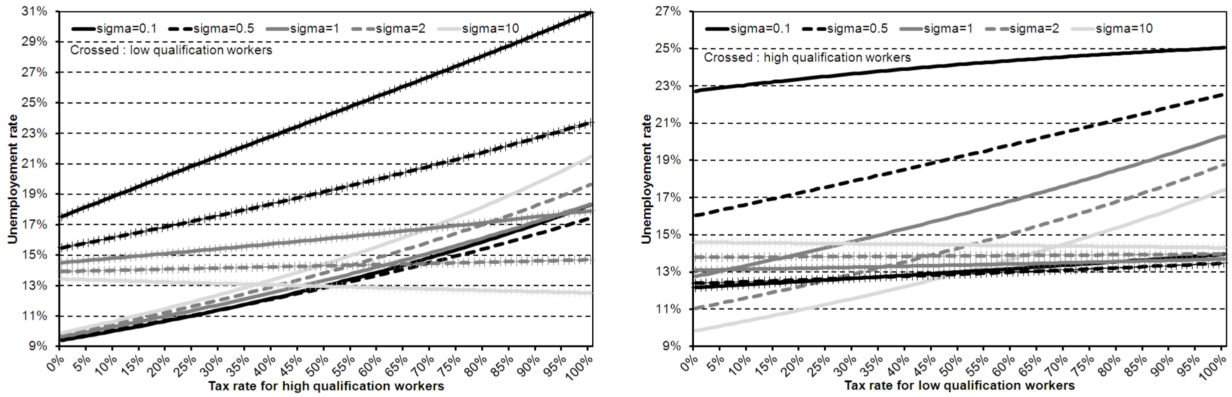


Figure 6: Impact of tax rate on high and low qualification workers' unemployment

The simulation is calculated for a high qualification worker with twice the productivity and the bargaining power as the low qualification worker ($\beta_h = 0.4$, $\beta_l = 0.2$, $\alpha_h = 2/3$, $\alpha_l = 1/3$). The simulations are done by changing the tax rate for one worker type from 0% to 100% keeping the other tax rate at 50%.

4.2 The case of three factors, low and high skill workers and capital

The simplication of the model to only three factors allows to understand the interactions between three main kind of factors: qualified labor, low-qualification labor and capital. Furthermore, to better understand the issue of substituability for the less skilled workers and the existence of classical unemployment for them, this reduced model considers that their bargained wage would be under minimum wage. Hence, the low-qualification workers are actually constrained workers and the model considered one of each kind of production factor: unconstrained worker (indexed by u), constrained workers (indexed by c) and capital (indexed by k).

To make the model usable for numerical analyses, functional form are given to matching function (the same as previously) and for the production function. This one is kept quite simple to keep the model tractable. A Cobb-Douglas production function $F(L_u, L_c, K) = (1-t)AN_u^{\alpha_u}N_c^{\alpha_c}K^{\alpha_k}$ is assumed.

The presence of the multiplicator $(1-t)$ allows to consider VAT, which is usefull in the simulation of social VAT (decrease of τ_c financed by an increase of VAT rate t). This model allow to understand various fiscal reforms, as t is the VAT rate, τ_u and τ_c earned income tax rate (their difference allow to understand the progressivity of the tax system) and τ_k is the tax rate capital income.

Given these hypotheses and calling $\underline{w}$ the minimum wage, the problem of general equilibrium 19 become the equation system 26.

$$
\begin{cases}
\frac{(1-t)A\alpha_u N_u^{\alpha_u-1}N_c^{\alpha_c}K^{\alpha_k}}{(1+\tau_u)\alpha_u+\frac{1-\beta}{\beta}} = \frac{(1+\tau_u)\beta}{1+\tau_u\beta}b_u + \gamma_u\beta\frac{1+2(1+\tau_u)\frac{\beta}{1-\beta}}{1+\tau_u\beta}\left(\frac{s_u}{a_u}\right)^{\frac{1}{\eta_u}}\left(\frac{N_u}{1-N_u}\right)^{\frac{1}{\eta_u}} & \\
\qquad\qquad + \frac{\beta}{1-\beta}\gamma_u\frac{r+s_u}{a_u}\left(\frac{s_u}{a_u}\right)^{\frac{1-\eta_u}{\eta_u}}\left(\frac{N_u}{1-N_u}\right)^{\frac{1-\eta_u}{\eta_u}} & (u) \\
\frac{(1-t)A\alpha_c N_u^{\alpha_u}N_c^{\alpha_c-1}K^{\alpha_k}}{(1+\tau_u)\alpha_u+\frac{1-\beta}{\beta}} = \frac{\beta}{1-\beta}(1+\tau_c)\underline{w} + \gamma_c\frac{r+s_c}{a_c}\left(\frac{s_c}{a_c}\right)^{\frac{1-\eta_c}{\eta_c}}\left(\frac{N_c}{1-N_c}\right)^{\frac{1-\eta_c}{\eta_c}} & (c) \\
\frac{(1-t)A\alpha_k N_u^{\alpha_u}N_c^{\alpha_c}K^{\alpha_k-1}}{(1+\tau_u)\alpha_u+\frac{1-\beta}{\beta}} = \frac{\beta}{1-\beta}(1+\tau_k)r + \gamma_k\frac{r+s_k}{a_k}\left(\frac{s_k}{a_k}\right)^{\frac{1-\eta_k}{\eta_k}}\left(\frac{K}{1-K}\right)^{\frac{1-\eta_k}{\eta_k}} & (k)
\end{cases}
\tag{26}
$$

Another hypothesis can be made which simplified substantially the resolution of the problem, whithout any consequences on the qualitative results of these modelisation: there is no friction on the market for capital. Actually, search and matching models have been developed for understanding labor market frictions. Hence, the hypothesis that there is no friction on the market for capital is quite straitforward. With this assumption, equation 26k allows to write K as a function of N_u and N_c. It appears directly that in that system that K increases both with respect to N_u and N_c. Given this function, equation 26u allows to write N_c as an increasing function of N_u. Given this two function, equation 26c is an equation with the unique unknown N_u, whose left hand term stricly decreases with respect to N_u from infinity towards a finite limit and whose right hand term striclty increases with respect to N_u from a finite limit tawards infinity. Hence, this equation has a unique solution.

This allows resolving the general equilibrium and find the value of N_u, N_c and K for each given value of the parameters A, α_u, α_c, α_k, $\underline{w}$, r, β, s_u, s_c, a_u, a_c, η_u, η_c, γ_u and γ_c. This can be done for various fiscal environments defined by t, τ_u, τ_c, τ_k. Given these results, high skill wage is calculated according to equation 12. For various level of bargaining power β of the unconstrained workers, the equilibrium are computed for all the ranges of taxes τ_u, τ_c, τ_k and t. The program is presented in appendix E.4. All the results are presented in appendix D. The results concerning the level of unemployment of both constrained and unconstrained workers are presented in Figure 7.

It appears that neither capital income nor wage taxation has a substantial impact on the level of unemployment of high skill workers. It remains relatvely low. Only the bargaining power have a discernible impact, however moderate. Consumption tax has an important impact but only since a high level of tax (over all consumption tax rates actually set). It is all the opposite for the unskilled workers, whose unemployment rate increase steeply with all taxes. The level of unskilled workers unemployment is ambiguously impact by the bargaining power of skilled workers. This last parameter has however a clear impact on the slope of the unemployment as a function of taxes: the higher the skilled workers bargaining power, the higher the unskilled workers unemployment rate with respect to taxes. Furthermore, it appears that tax rate on high wages has relatively low impact on the level of employment of constrained workers. In the opposite, the impact of consumption taxes seems very high.

Similar patterns are observables for taxation impact on other parameters. Bargaining power as well as capital and consumption taxation have a great impact on GDP and firm profits. The impact of wage taxation (whatever low or high wage) is moderate. High wage tax revenue increase strongly with high wage taxes as other tax revenues shows more concave

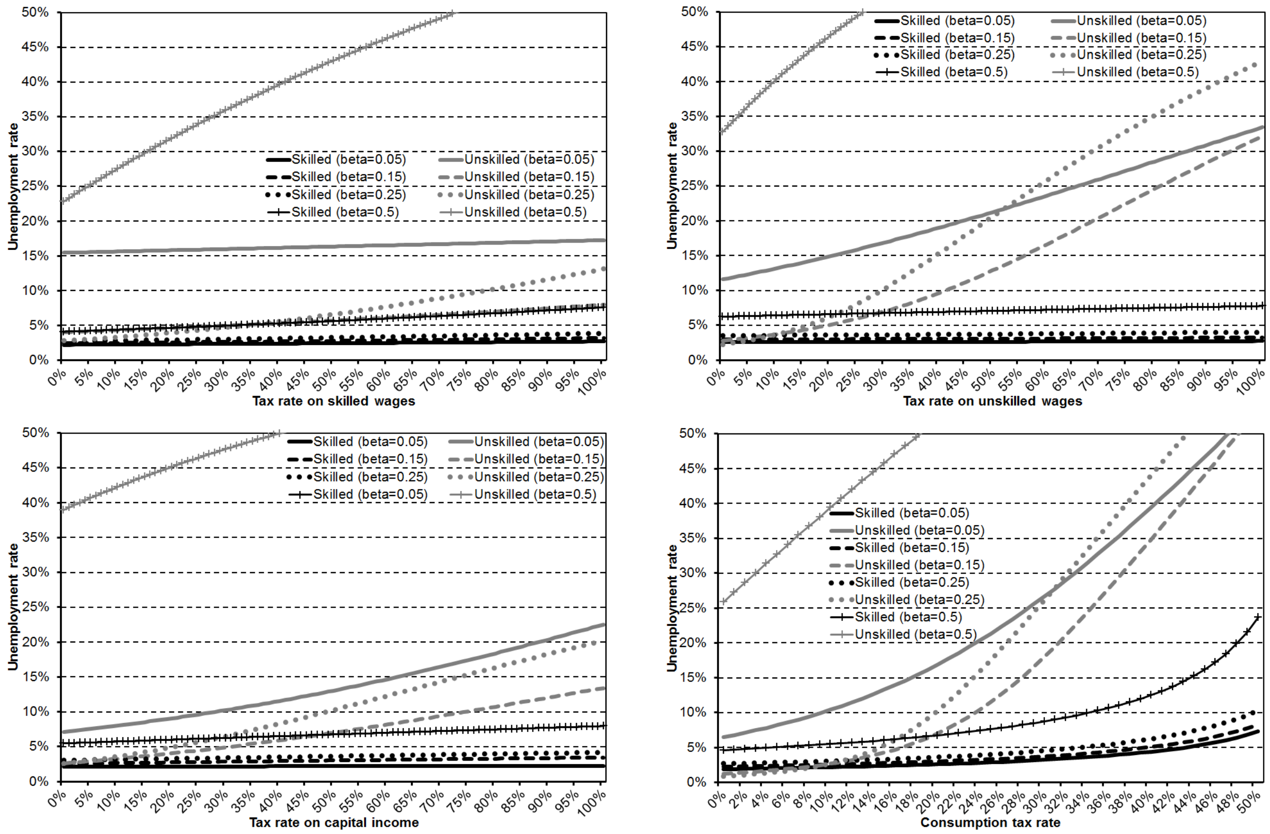


Figure 7: Impact of taxes on unemployment rates of high and low qualification workers

curves. Particularly, capital income tax revenue is very slowly increasing with respect to capital income tax rate as soon as the skilled workers bargaining power is not very low. Last, the impact of taxes on high qualification wage also differs from one tax to another. The impacts of low wage and capital income taxations are weak whereas the impacts of consumption and high wage taxations are strong.

5 Conclusion and comments

The present article models a labor market with heterogenous workers in a search and matching framework. A global production function is considered to take into account the interactions between different segments of the labor market. Furthermore, the hypothesis of intra-firm bargaining is assumed as the main purpose is to understand incidence of taxation as a result of wage bargaining. The model is solve formally and some numerical analyses are run.

The results confirm that payroll taxation fall mainly on workers. Furthermore, it appears that marginal incidence decreases with respect to the level of taxation and therefore that marginal incidence is significantly lower than mean incidence. This has at least two applications. The first one is linked to interpretation of incidence estimations. Such empirical studies, if using an identification strategy in natural experiment, would estimate the marginal incidence and therefore underestimate the overall incidence. Those estimates would be accurate for marginal policy reforms but not to understand the overal distribution of the burden of taxation.

The second one is linked to the efficiency of employment policies consisting in lowering payroll taxation in order to deal with unemployment. Such policies' efficiency is marginally decreasing with the level of tax rebate. Indeed, an additional reduction of payroll tax take place with lower starting level of taxation than the preceeding tax cut. Therefore it would induce more wage increase and less labor cost decrease than previous tax rebates.

In addition, wages, unemployement rates and incidence increase with bargaining power and productivity of workers. This also matters for policies of tax rebates. By those policies, governments try to lower labor costs but this works only for low qualification and low bargaining power workers. For an exemple, French enlargement of tax rebate (CICE, up to 2.4 times the minimum wage) promises a low efficiency for lowering labor costs in France.

However, if the direct effect of taxation on unemployment is lower for high qualification workers than for low qualification workers, the crossed effect of high qualification workers taxation on low qualification workers unemployement is larger than the reciproqual.

References

Acemoglu, D. (2001). Good jobs versus bad jobs. *Journal of Labor Economics*, 19:1–22.

Anderson, P. A. and Meyer, B. D. (1997). The effects of firm specific taxes and government mandates with an application to the u.s. unemployment insurance program. *Journal of Public Economics*.

Anderson, P. A. and Meyer, B. D. (2000). The effects of the unemployment insurance payroll tax on wages, employment, claims and denials. *Journal of Public Economics*, 78:81–106.

Belan, P., Carré, M., and Gregoir, S. (2010). Subsidizing low-skilled jobs in a dual labor market. *Labour Economics*, 17(5):776–788.

Bennmarker, H., Mellander, E., and Öckert, B. (2009). Do regional payroll tax reductions boost employment? *Labour Economics*, 16(5):480–489.

Bohm, P. and Lind, H. (1993). Policy evaluation quality - a quasi-experimental study of regional employment subsidies in sweden. *Regional Science and Urban Economics*, 23:51–65.

Borowczyk Martins, D., Jolivet, G., and Postel-Vinay, F. (2011). Accounting for endogenous search behavior in matching function estimation. IZA Discussion Papers 5807, Institute for the Study of Labor (IZA).

Cahuc, P., Marque, F., and Wasmer, E. (2008). A theory of wages and labor demand with intra-firm bargaining and matching frictions. *International Economic Review*, 49(3):943–972.

Chéron, A., Hairault, J.-O., and Langot, F. (2008). A quantitative evaluation of payroll tax subsidies for low-wage workers: An equilibrium search approach. *Journal of Public Economics*, 92(3-4):817–843.

Crépon, B. and Deplatz, R. (2001). Evolution des effets des dispositifs d'allègement de charges sur les bas salaires. *Economie et Statistique*.

Farhi, E., Gopinath, G., and Itskhoki, O. (2014). Fiscal devaluations. *Review of Economic Studies*.

Goldin, C. (2014). A grand gender convergence, its last chapter. *American Economic Review*, 104(4):1091–1119.

Goldin, C. and Katz, L. F. (2008). Transitions: Career and family life cycles of the educational elite. *American Economic Review*, 98(2):363–369.

Gruber, J. (1994). The incidence of mandated maternity benefits. *American Economic Review*, 84:622–641.

Gruber, J. (1997). The incidence of payroll taxation: Evidence from chile. *Journal of Labor Economics*, 15(3):S72–101.

Huttunen, K., Pirttilä, J., and Uusitalo, R. (2013). The employment effects of low-wage subsidies. *Journal of Public Economics*, 97(C):49–60.

Kleven, H. J., Kreiner, C. T., and Saez, E. (2009). The optimal income taxation of couples. *Econometrica*, 77(2):537–560.

Korkeamäki, O. and Uusitalo, R. (2009). Employment and wage effects of a payroll-tax cut - evidence from a regional experiment. *International Tax and Public Finance*, 16:753–772.

Kramarz, F. and Philippon, T. (2001). The impact of differential payroll tax subsidies on minimum wage employment. *Journal of Public Economics*, 82(1):115–146.

Mirrlees, J. (1971). An exploration in the theory of optimum income taxation. *Review of Economic Studies*, 38:175–208.

Murphy, K. J. (2007). The impact of unemployment insurance taxes on wages. *Labour Economics*, 14:457–484.

Petrongolo, B. and Pissarides, C. A. (2000). Looking into the black box: A survey of the matching function. CEPR Discussion Papers 2409, C.E.P.R. Discussion Papers.

Pissarides, C. A. (2000). *Unemployement Theory, 2nd Edition*. MIT Press, Cambridge, MA.

Saez, E. (2001). Using elasticities to derive optimal income tax rates. *Review of Economic Studies*, 68(1):205–29.

Stole, L. A. and Zwiebel, J. (1996a). Intra-firm bargaining under non-binding contracts. *Review of Economic Studies*, 63(3):375–410.

Stole, L. A. and Zwiebel, J. (1996b). Organizational design and technology choice under intrafirm bargaining. *American Economic Review*, 86(1):195–222.

Vicard, A., Langumier, F., and Eidelman, A. (2013). Prélèvements et transferts aux ménages : des canaux redistributifs différents en 1990 et 2010. *Économie et Statistique*, 459(1):5–26.

A Formal resolution of the differential equations

A.1 Disentengling the system of differential equations

The partial derivative of equation 15 with respect to $k \in [1, l] \backslash i$ is:

$$(1 + \beta_i \tau_i) \frac{\partial w_i(\vec{N})}{\partial N_k} = \beta_i \left[\frac{\partial^2 F(\vec{N})}{\partial N_i N_k} - \sum_{j=1}^{l} (1 + \tau_j) N_j \frac{\partial^2 w_j(\vec{N})}{\partial N_i N_k} - (1 + \tau_k) \frac{\partial w_l(\vec{N})}{\partial N_i} \right]$$

Yet, when $i, k \in [1, l]$ and $i \neq l$:

$$\frac{\partial^2}{\partial N_i \partial N_k} \sum_{j=1}^{l} (1 + \tau_j) N_j w_j(\vec{N}) = \sum_{j=1}^{l} (1 + \tau_j) N_j \frac{\partial^2 w_j(\vec{N})}{\partial N_k \partial N_i} + (1 + \tau_i) \frac{\partial w_i(\vec{N})}{\partial N_k} + (1 + \tau_k) \frac{\partial w_k(\vec{N})}{\partial N_i}$$

And therefore the derivative with respect to N_k of differential equation 15 for $i \in [1, l] \backslash i$ is:

$$(1 - \beta_i) \frac{\partial w_i(\vec{N})}{\partial N_k} = \beta_i \frac{\partial^2}{\partial N_i \partial N_k} \left[F(\vec{N}) - \sum_{j=1}^{l} (1 + \tau_j) N_j w_j(\vec{N}) \right]$$

Comparing derivative with respect to N_k of differential equation 15 for i and derivative with respect to N_i of differential equation 15 for k with $i, k \in [1, l]$ ans $i \neq k$ gives equation 27.

$$\frac{\partial w_k(\vec{N})}{\partial N_i} = \frac{1 - \beta_i}{\beta_i} \frac{\beta_k}{1 - \beta_k} \frac{\partial w_i(\vec{N})}{\partial N_k} = \chi_{ik} \frac{\partial w_i(\vec{N})}{\partial N_k} \tag{27}$$

Which implies that:

$$\sum_{j=1}^{l} (1 + \tau_j) N_j \frac{\partial w_j(\vec{N})}{\partial N_i} = \sum_{j=1}^{l} (1 + \tau_j) \chi_{ij} N_j \frac{\partial w_i(\vec{N})}{\partial N_j}$$

And differential equation 15 may be rewritten as differential equation 16.

A.2 Differential equations for multiple worker types

This differential equation must be solved in several stage: two successive change of coordinate, the actual resolution of the differential equation and the return to the original set of coordinates. For all the demonstration, $i \in [1, l]$ as there is no bargaining for other factors and w_i for $i \in [l + 1, n]$ is constant and equal to $\underline{w}$ for constrained workers and r for capital.

A.2.1 First change of coordinates

Let consider a first change of coordinate such that $\vec{M}_i = (M_{i1}, ..., M_{in})$, $v_i(\vec{M}_i) = w_i(\vec{N})$ and:

$$\sum_{j=1}^{l} M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} = \sum_{j=1}^{l} (1 + \tau_j) \chi_{ij} N_j \frac{\partial w_i(\vec{N})}{\partial N_j}$$

It works in particular if for all $j \in [1, l]$:

$$M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} = (1 + \tau_j)\chi_{ij} N_j \frac{\partial w_i(\vec{N})}{\partial N_j} \tag{28}$$

And $M_{ij} = N_j$ for all $j \in [l+1, n]$. Yet by definition, for $j \in [1, l]$:

$$\frac{\partial w_i(\vec{N})}{\partial N_j} = \frac{dM_{ij}}{dN_j} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}}$$

And therefore equation 28 become:

$$M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} = (1 + \tau_j)\chi_{ij} N_j \frac{dM_{ij}}{dN_j} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}}$$

Allowing to define the differential equation for the functions $M_{ij}(N_j)$, which are:

$$M_{ij} = (1 + \tau_j)\chi_{ij} N_j dM_{ij}/dN_j$$

One solution is $M_{ij} = N_j^{\chi_{ji}/(1+\tau_j)}$ as $1/\chi_{ij} = \chi_{ji}$. Furthermore, we call $G(\vec{M}_i) = F(\vec{N})$. Hence, for $j \in [1, l]$, $\partial F(\vec{N})/\partial N_j = (\partial G(\vec{M}_i)/\partial M_{ij})(dM_{ij}/dN_j)$, that is $\partial F(\vec{N})/\partial N_j = [\chi_{ji}/(1+\tau_j)]N_j^{\chi_{ji}/(1+\tau_j)-1}(\partial G(\vec{M}_i)/\partial M_{ij})$. And in particular, as $i \in [1, l]$, $\partial F(\vec{N})/\partial N_i = [N_i^{-\tau_i/(1+\tau_i)}/(1+\tau_i)](\partial G(\vec{M}_i)/\partial M_{ii}) = [M_{ii}^{-\tau_i}/(1+\tau_i)](\partial G(\vec{M}_i)/\partial M_{ii})$.The differential equation is the new set of coordinates is consequently given by equation 29.

$$(1 + \beta_i \tau_i) v_i(\vec{M}_i) = (1 - \beta_i) r U_i + \beta_i \left[\frac{M_{ii}^{-\tau_i}}{1 + \tau_i} \frac{\partial G(\vec{M}_i)}{\partial M_{ii}} - \sum_{j=1}^{l} M_{ij} \frac{\partial v_i(\vec{M}_i)}{\partial M_{ij}} \right] \tag{29}$$

A.2.2 Second change: spherical coordinates

Another change of coordinate should now be made with spherical coordinates $(\rho_i, \phi_{i1}, ..., \phi_{i,l-1})$ where ρ_i is the canonical norm of the vector $\vec{M}_i^u = (M_{i1}, ..., M_{il})$ (eg: $\rho_i^2 = \sum_{j=1}^{l} M_{ij}^2$) and $\vec{\phi}_i = (\phi_{i1}, ..., \phi_{i,l-1})$ the angles. Let determine the angles as in equation 30.

$$\begin{cases} \phi_{i,1} & \textit{such that} \quad M_{i,l} = \rho_i \sin\phi_{i1} \\ \phi_{i,2} & \textit{such that} \quad M_{i,l-1} = \rho_i \cos\phi_{i1} \sin\phi_{i2} \\ & \qquad\qquad\quad ... \\ \phi_{i,j} & \textit{such that} \quad M_{i,l+1-j} = \rho_i \cos\phi_{i1} ... \cos\phi_{i,j-1} \sin\phi_{ij} \\ & \qquad\qquad\quad ... \\ \phi_{i,l-1} & \textit{such that} \quad M_{i,2} = \rho_i \cos\phi_{i1} ... \cos\phi_{i,l-2} \sin\phi_{i,l-1} \end{cases} \tag{30}$$

It follows that $M_{i,1}^2 = \rho^2 - \sum_{j=2}^{l} M_{i,j}^2$. Yet:

$$\begin{cases} M_{i,l}^2 + M_{i,l-1}^2 = \rho^2(1 - \cos^2\phi_{i1} + \cos^2\phi_{i,1}\sin^2\phi_{i,2}) & = \rho^2[1 - \cos^2\phi_{i1}(1 - \sin^2\phi_{i,2})] \\ & = \rho^2[1 - \cos^2\phi_{i1}\cos^2\phi_{i,2}] \\ ... + M_{i,l-2}^2 = \rho^2[1 - \cos^2\phi_{i1}\cos^2\phi_{i,2}(1 - \sin^2\phi_{i,3})] = \rho^2[1 - \cos^2\phi_{i1}\cos^2\phi_{i,2}\cos^2\phi_{i,3}] & \\ ... & \\ ... + M_{i,2}^2 = \rho^2[1 - \cos^2\phi_{i1} ... \cos^2\phi_{i,n-2}(1 - \sin^2\phi_{i,l-1})] = \rho^2[1 - \cos^2\phi_{i1} ... \cos^2\phi_{i,n-1}] & \end{cases}$$

And therefore $M_{i,1}$ is given by equation 31.

$$M_{i,1} = \rho \cos\phi_{i1} ... \cos\phi_{i,n-2} \cos\phi_{i,l-1} \tag{31}$$

Equations 30 and 31 imply that $\rho_i \partial M_{ij} / \partial \rho_i = M_{ij}$ and therefore:

$$\rho_i \partial v_i(\vec{M}_i)/\partial\rho_i = \rho_i \sum_{j=1}^{l} (\partial v_i(\vec{M}_i)/\partial M_{ij})(\partial M_{ij}/\partial\rho_i) = \sum_{j=1}^{l} M_{ij} \partial v_i(\vec{M}_i)/\partial M_{ij}$$

Hence, the differential equation in spherical coordinate is given by equation 32.

$$(1+\beta_i\tau_i) v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) = (1-\beta_i) r U_i + \beta_i \left[\frac{M_{ii}^{-\tau_i}}{1+\tau_i} \frac{\partial G(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K})}{\partial M_{ii}} - \rho_i \frac{\partial v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K})}{\partial \rho_i} \right] \tag{32}$$

A.2.3 Differential equations without crossed derivative

The homogenous equation is $(1-\beta_i\tau_i) v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) + \beta_i \rho_i \partial v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K})/\partial\rho_i = 0$ whose solution is $v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) = C \rho_i^{-\frac{1+\beta_i\tau_i}{\beta}}$. The method of variation of the constant gives:

$$C(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}} + \tfrac{\beta_i}{1+\beta_i\tau_i} \rho_i \left(\tfrac{\partial C(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K})}{\partial \rho_i} \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}} - \tfrac{1+\beta_i\tau_i}{\beta_i} C(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}-1} \right) = \tfrac{M_{ii}^{-\tau_i}}{1+\tau_i} \tfrac{\partial G(\vec{M}_i)}{\partial M_{ii}}$$

And consequently the derivative of the constant is:

$$\frac{\beta_i}{1+\beta_i\tau_i} \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}+1} \frac{\partial C(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K})}{\partial \rho_i} = \frac{M_{ii}^{-\tau_i}}{1+\tau_i} \frac{\partial G(\vec{M}_i)}{\partial M_{ii}}$$

Hence, the result of the differential equation with the spherical coordinates is:

$$v_i(\rho_i, \vec{\phi}_i, \vec{N}_c, \vec{K}) = \frac{1-\beta_i}{1+\beta_i\tau_i} r U_i + \rho_i^{-\frac{1+\beta_i\tau_i}{\beta_i}} \left(\kappa_i(\vec{\phi}_i, \vec{N}_c, \vec{K}) + \int_0^{\rho_i} \frac{M_{ii}^{-\tau_i}}{1+\tau_i} z^{\frac{1+\beta_i\tau_i}{\beta_i}-1} \frac{\partial G(\vec{M}_i)}{\partial M_{ii}} dz \right)$$

Where M_{ii} is indeed a function of z. In the present case, the condition at limit is that $\rho_i v_i$ tends towards zero when ρ_i tends towards zero, which means that $\kappa_i = 0$. Furthermore, it appears that $(u\rho_i, \vec{\phi}_i) = (uM_{i1}, ..., uM_{il})$ so doing the change of variable $z = u\rho_i$ (u from 0 to 1, $dz = \rho_i du$, $\vec{M}_i(z) = (u\vec{M}_i^u, \vec{N}_c, \vec{K})$). The integrale become:

$$\int_0^1 \frac{u^{-\tau_i} M_{ii}^{-\tau_i}}{1+\tau_i} u^{\frac{1+\beta_i\tau_i}{\beta_i}-1} \rho_i^{\frac{1+\beta_i\tau_i}{\beta_i}-1} \frac{\partial G(u\vec{M}_i^u, \vec{N}_c, \vec{K})}{\partial M_{ii}} \rho_i du = \frac{M_{ii}^{-\tau_i}}{1+\tau_i} \rho_i^{\frac{1+\beta_i\tau_i}{\beta_i}} \int_0^1 u^{\frac{1-\beta_i}{\beta_i}} \frac{\partial G(u\vec{M}_i^u, \vec{N}_c, \vec{K})}{\partial M_{ii}} du$$

In addition:

$$\frac{\partial G(u\vec{M}_i^u, \vec{N}_c, \vec{K})}{\partial M_{ii}} = \frac{1+\tau_i}{u^{-\tau_i} M_{ii}^{-\tau_i}} \frac{\partial F(u\vec{M}_i^u, \vec{N}_c, \vec{K})}{\partial N_i}$$

Let call $\mu_{ij} = uM_{ij}$ for $j \in [1, l]$, it is equal to $\nu_j^{\chi_{ji}/(1+\tau_j)}$ in the initial coordinates, yet $M_{ij} = N_j^{\chi_{ji}/(1+\tau_j)}$, so $\nu_j^{\chi_{ji}/(1+\tau_j)} = uN_j^{\chi_{ji}/(1+\tau_j)}$ and $\nu_j = u^{(1+\tau_j)\chi_{ij}} N_j$ and the net wage is given by equation 17.

B Numerical analyses in the case of unique worker type

B.1 One worker type and linear taxes

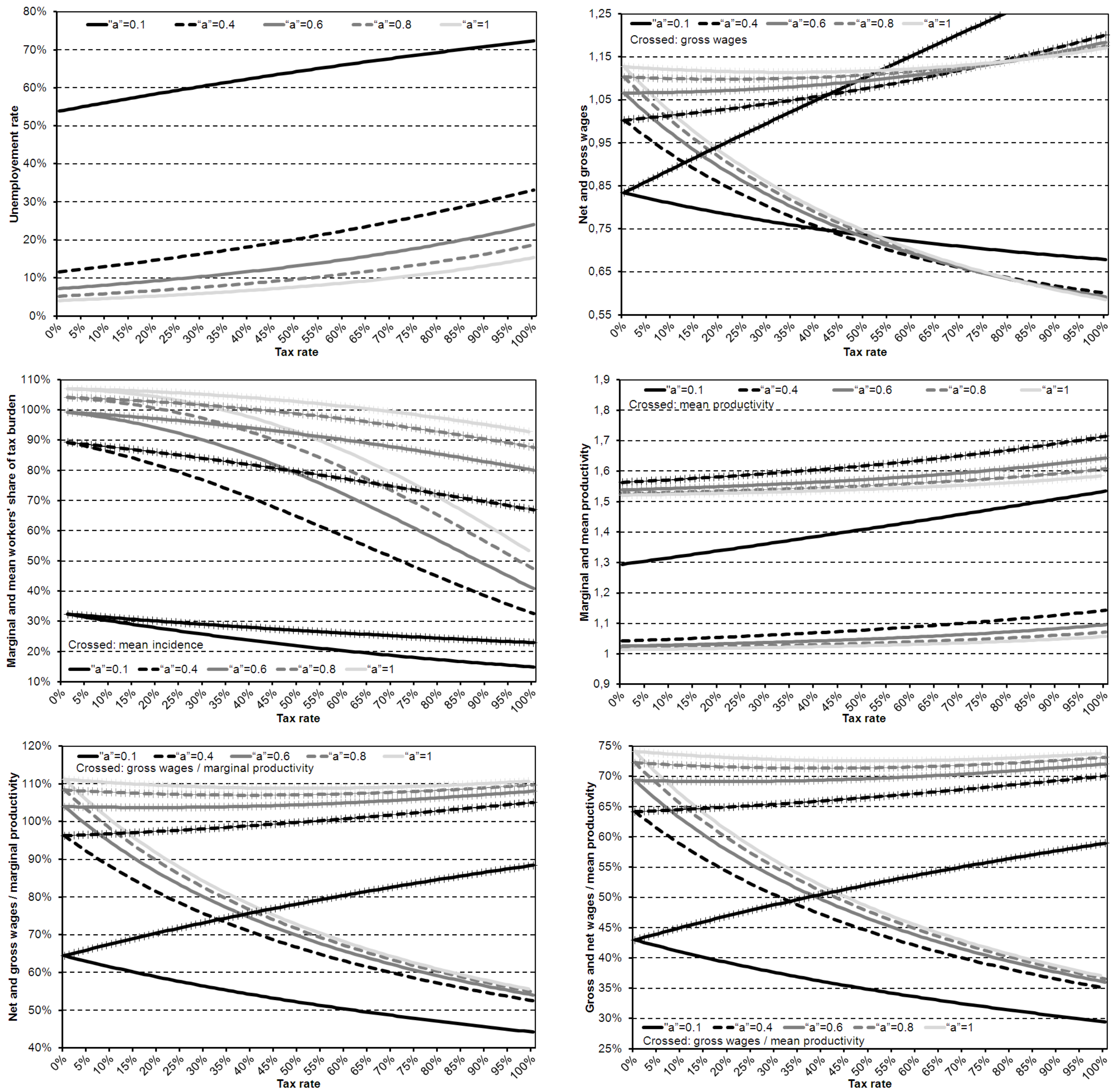


Figure 8: Impact of matching efficiency on unique labor market equilibrium

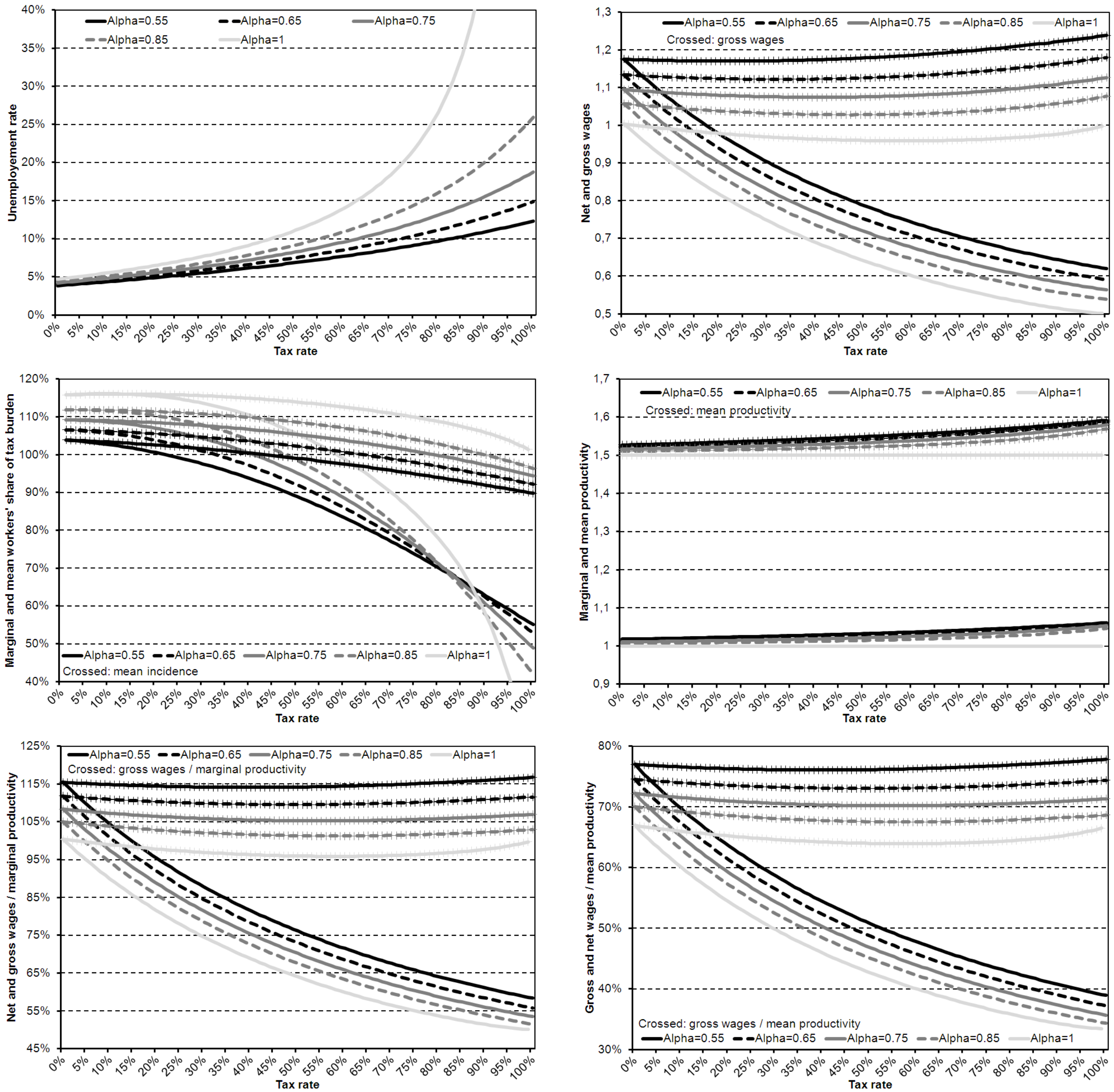


Figure 9: Impact of taxes and labor productivity on unique labor market equilibrium

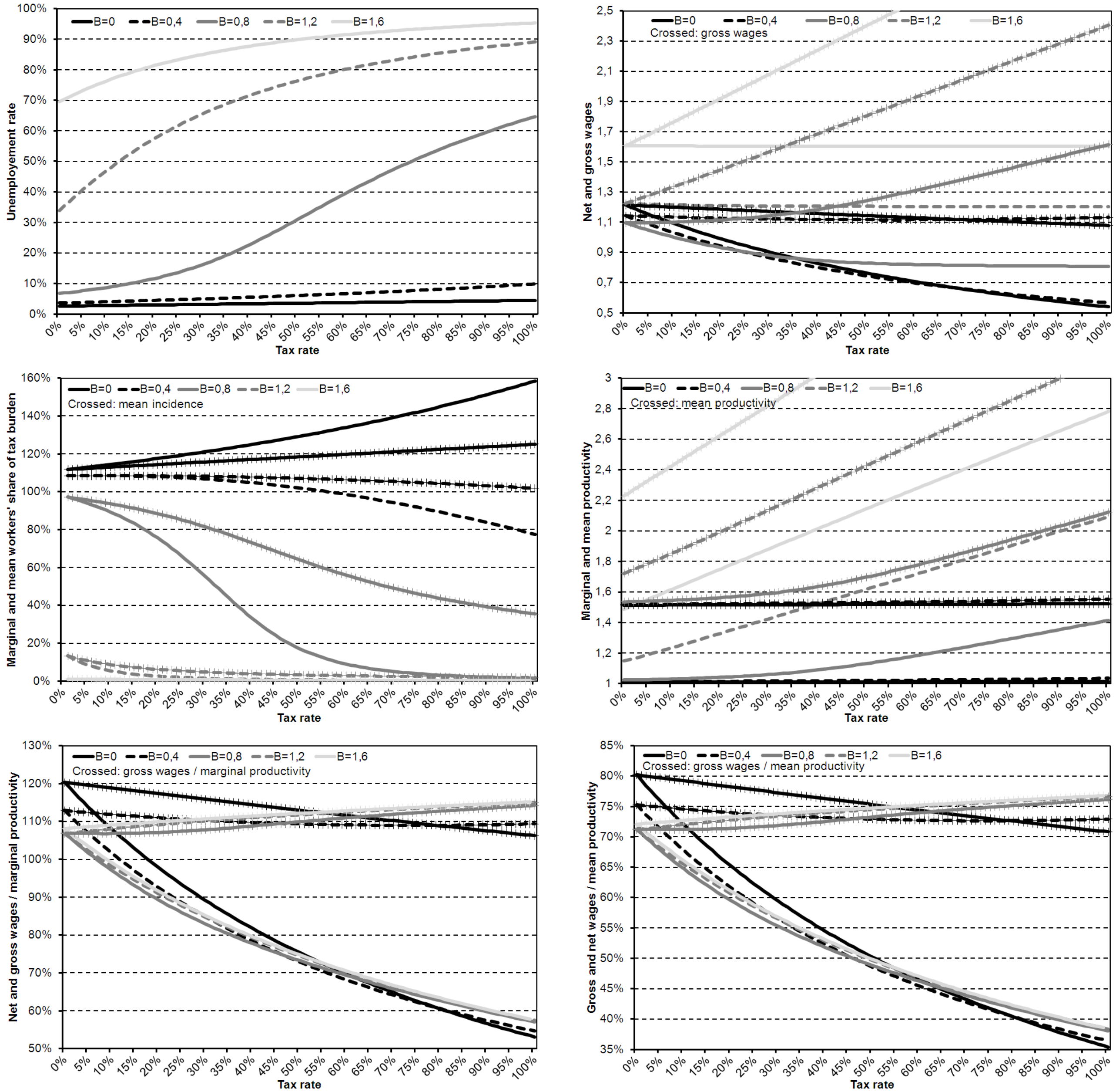


Figure 10: Impact of taxes and unemployement benefits on unique labor market equilibrium

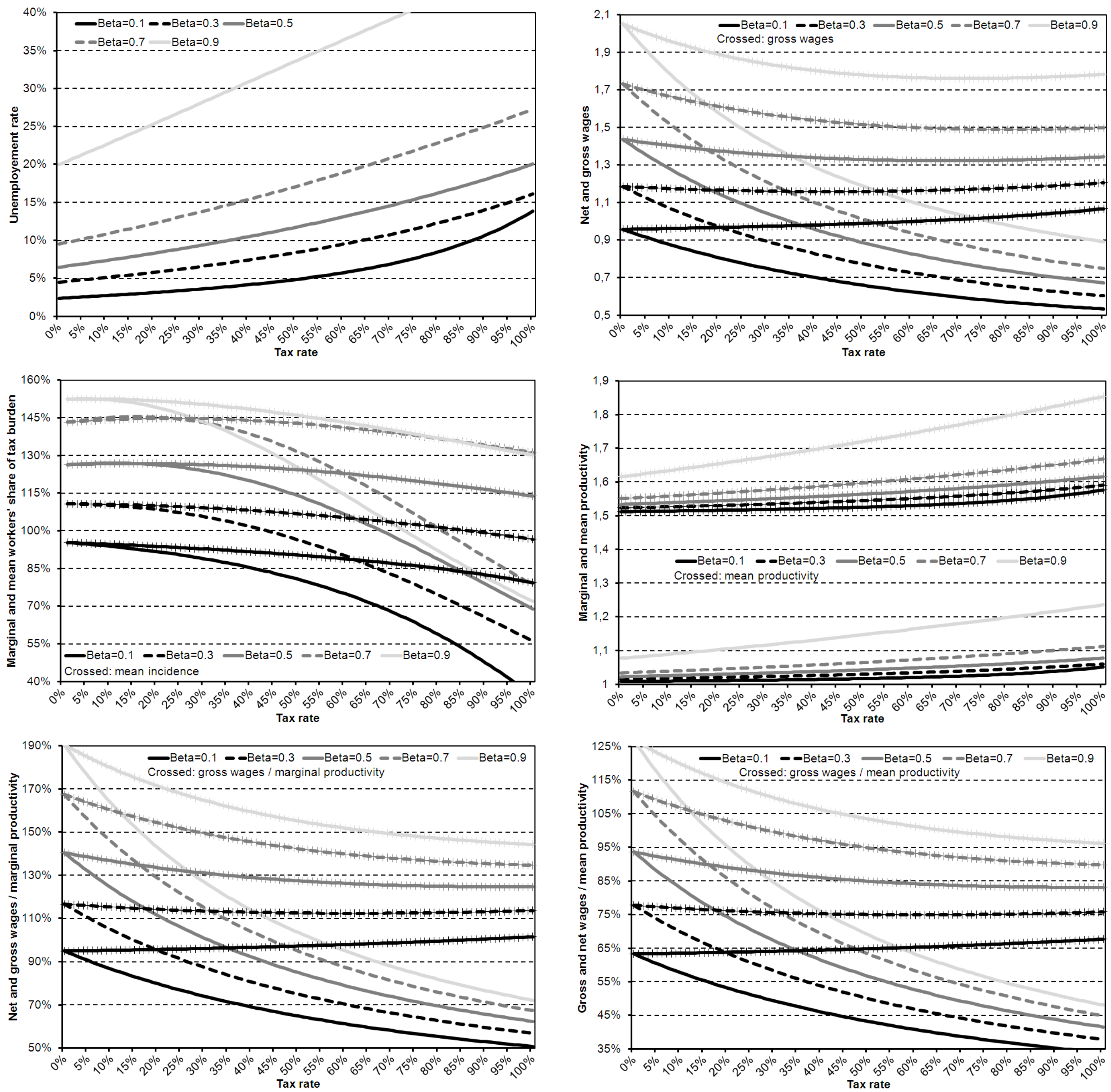


Figure 11: Impact of taxes and bargaining power on unique labor market equilibrium

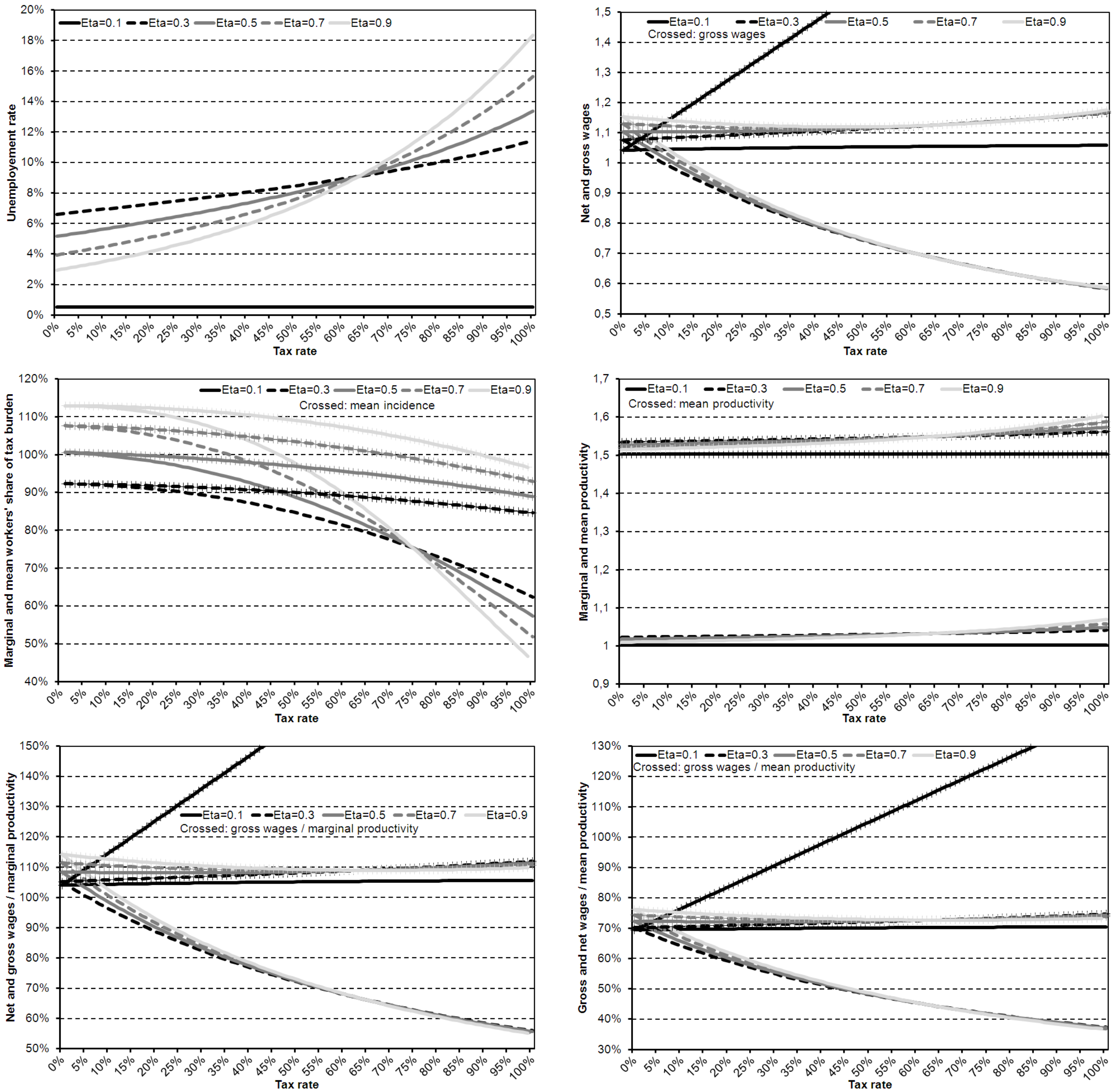


Figure 12: Impact of taxes and matching elasticity on unique labor market equilibrium

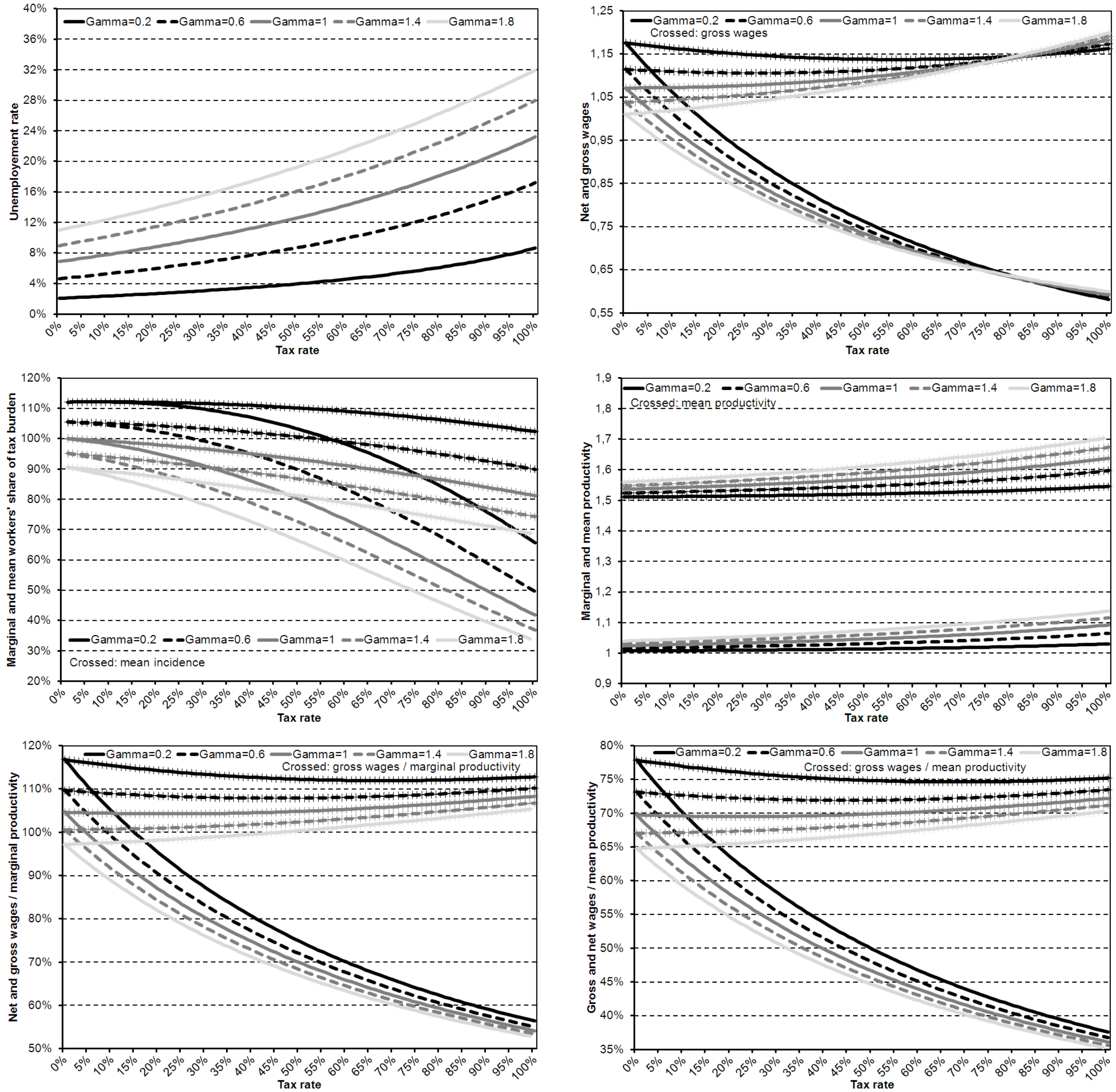


Figure 13: Impact of taxes and vacancy posting cost on unique labor market equilibrium

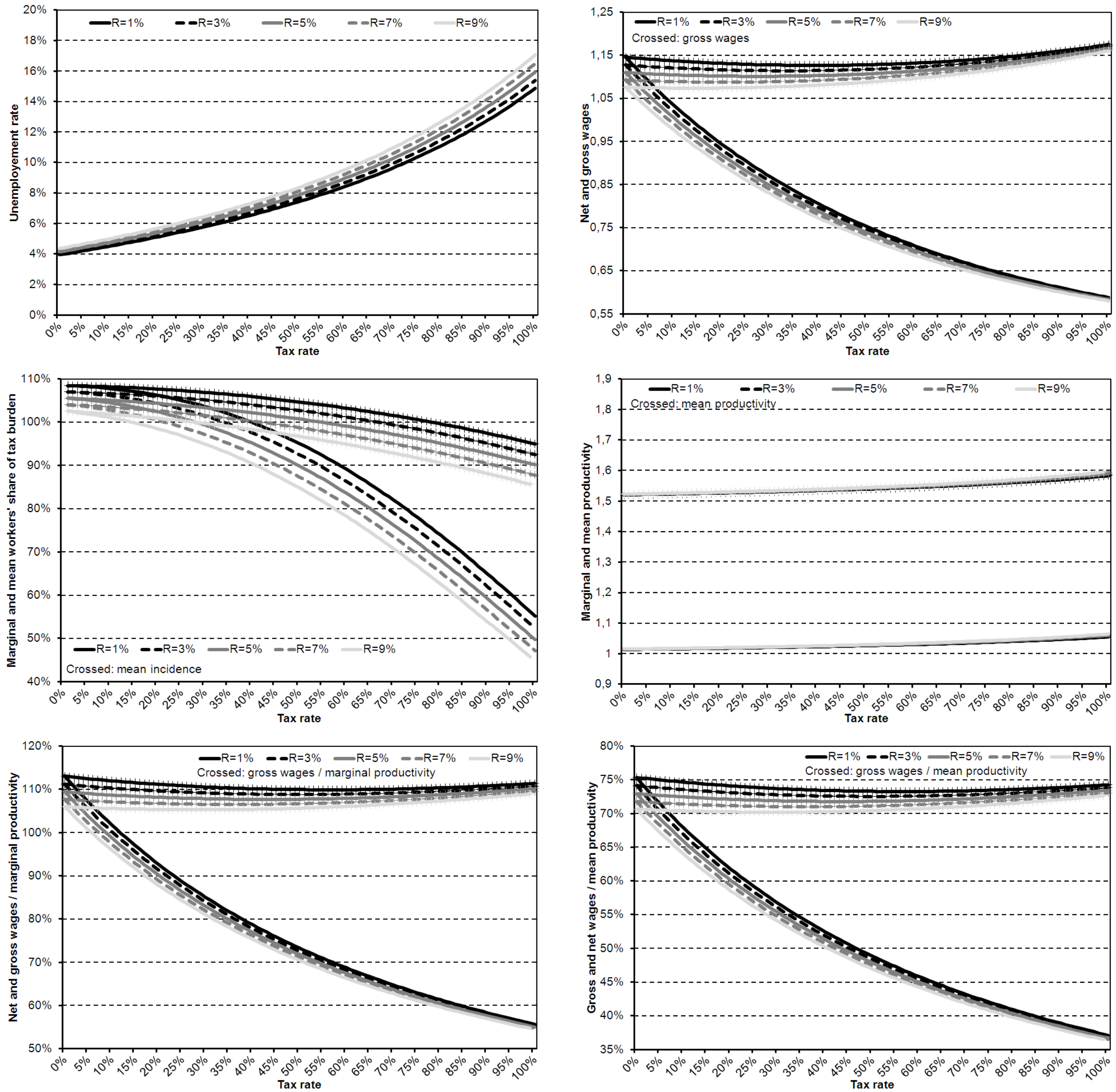


Figure 14: Impact of taxes and interest rate on unique labor market equilibrium

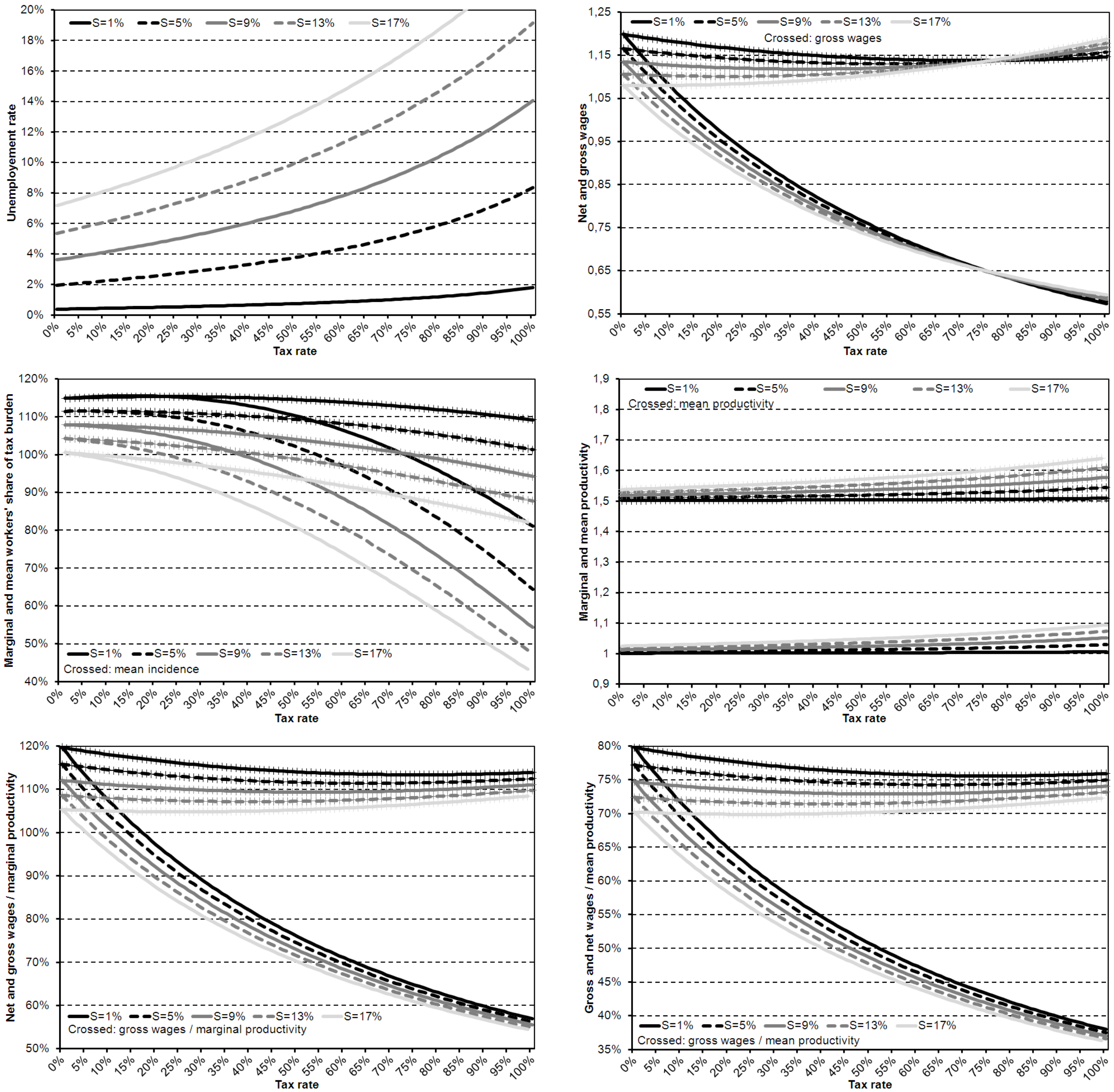


Figure 15: Impact of taxes and job destruction rate on unique labor market equilibrium

B.2 One worker type and quadratic taxes

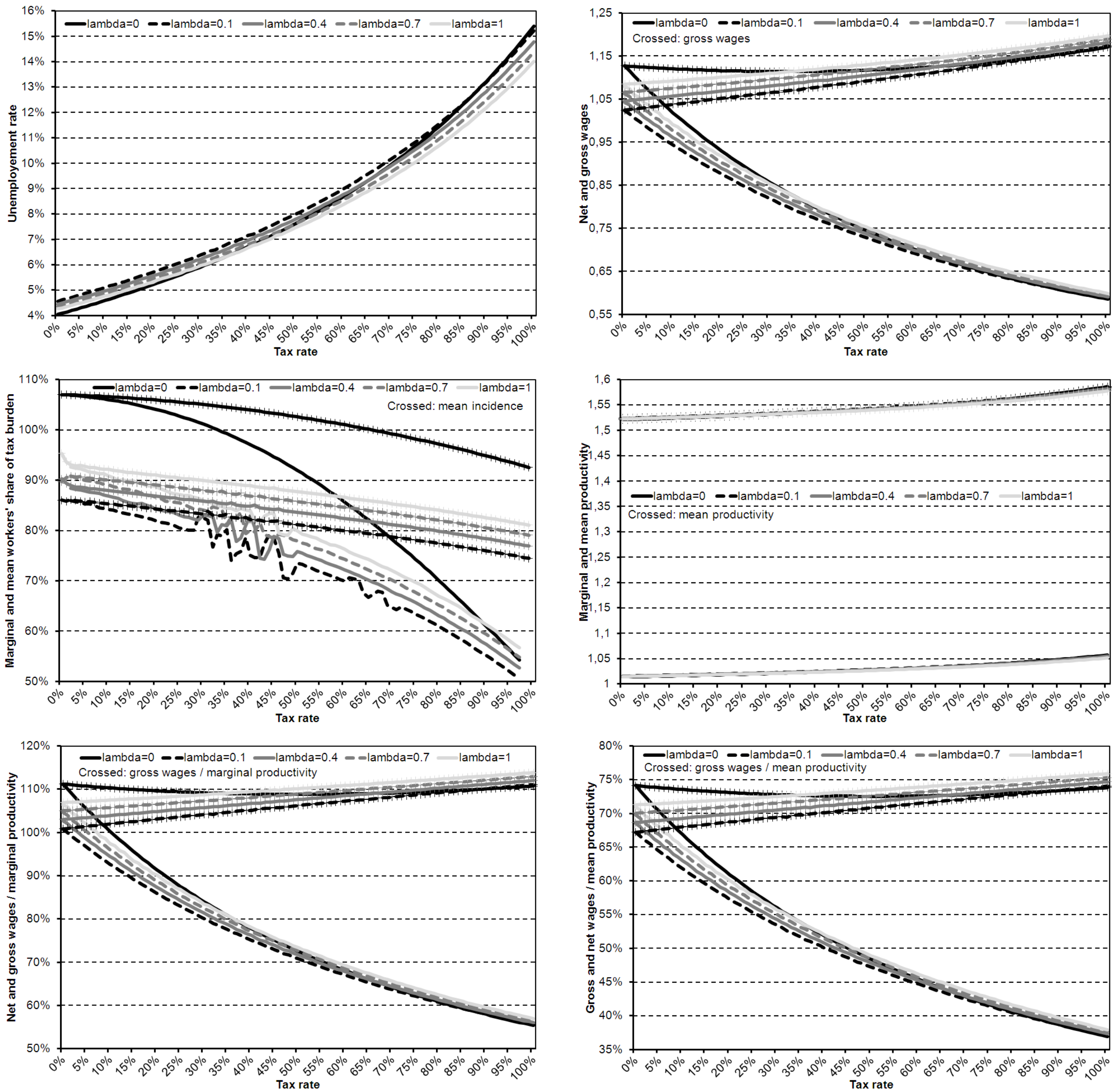


Figure 16: Impact of progressive taxes on unique labor market equilibrium

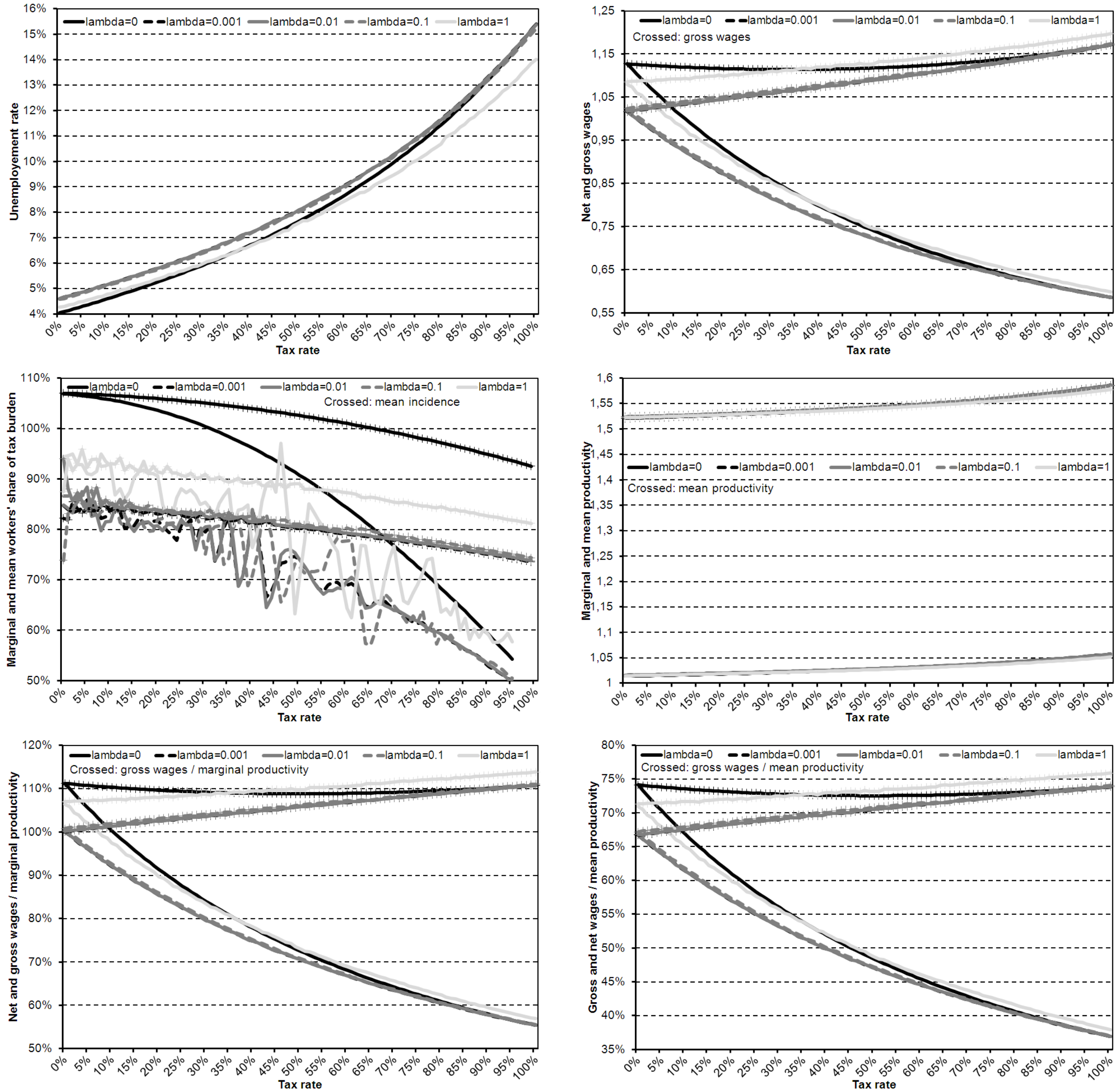


Figure 17: Impact of slightly progressive taxes on unique labor market equilibrium

C Numerical analyses in the case of two worker types

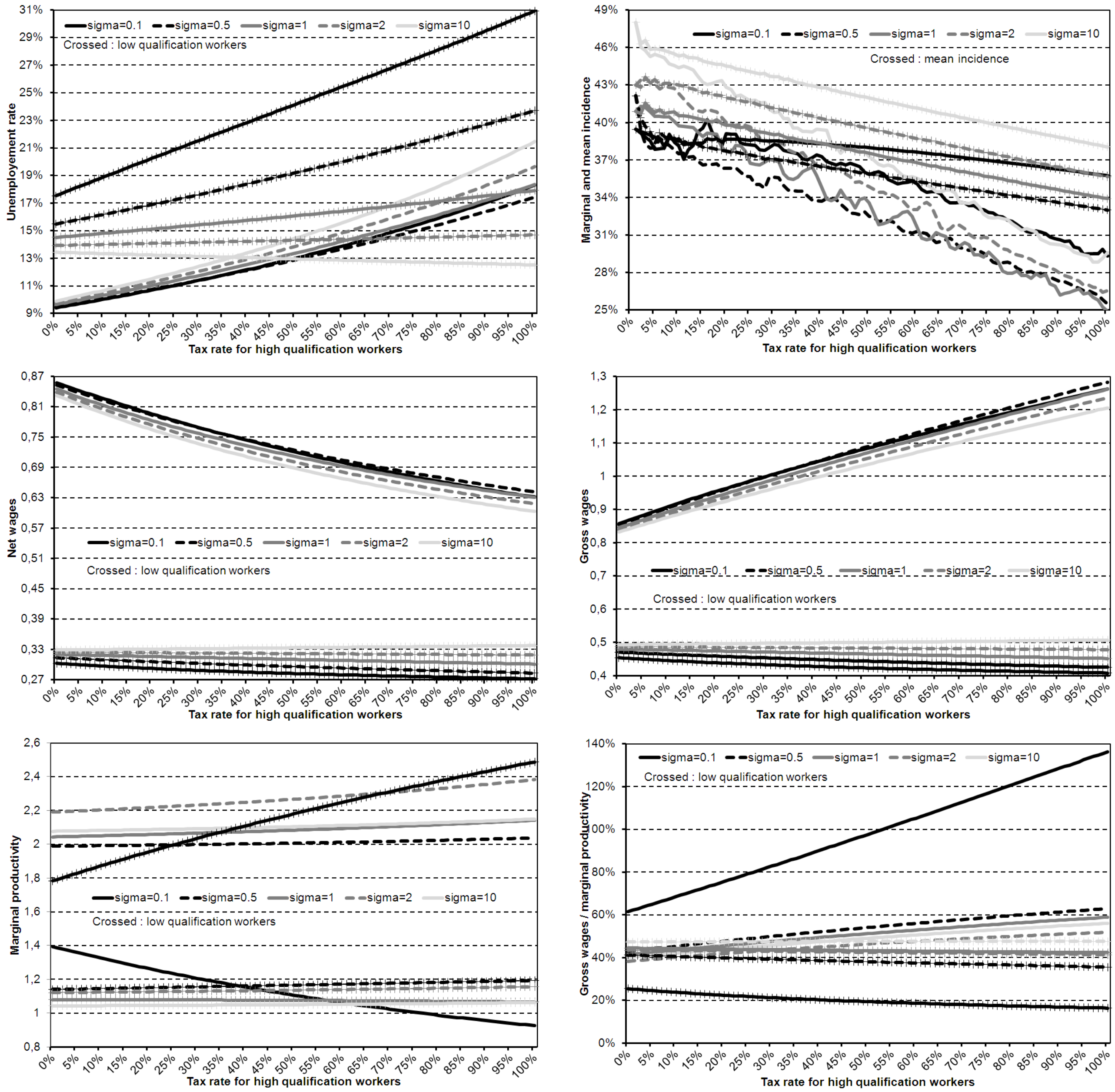


Figure 18: Impact of taxes on high qualification workers and substitution elasticity between workers

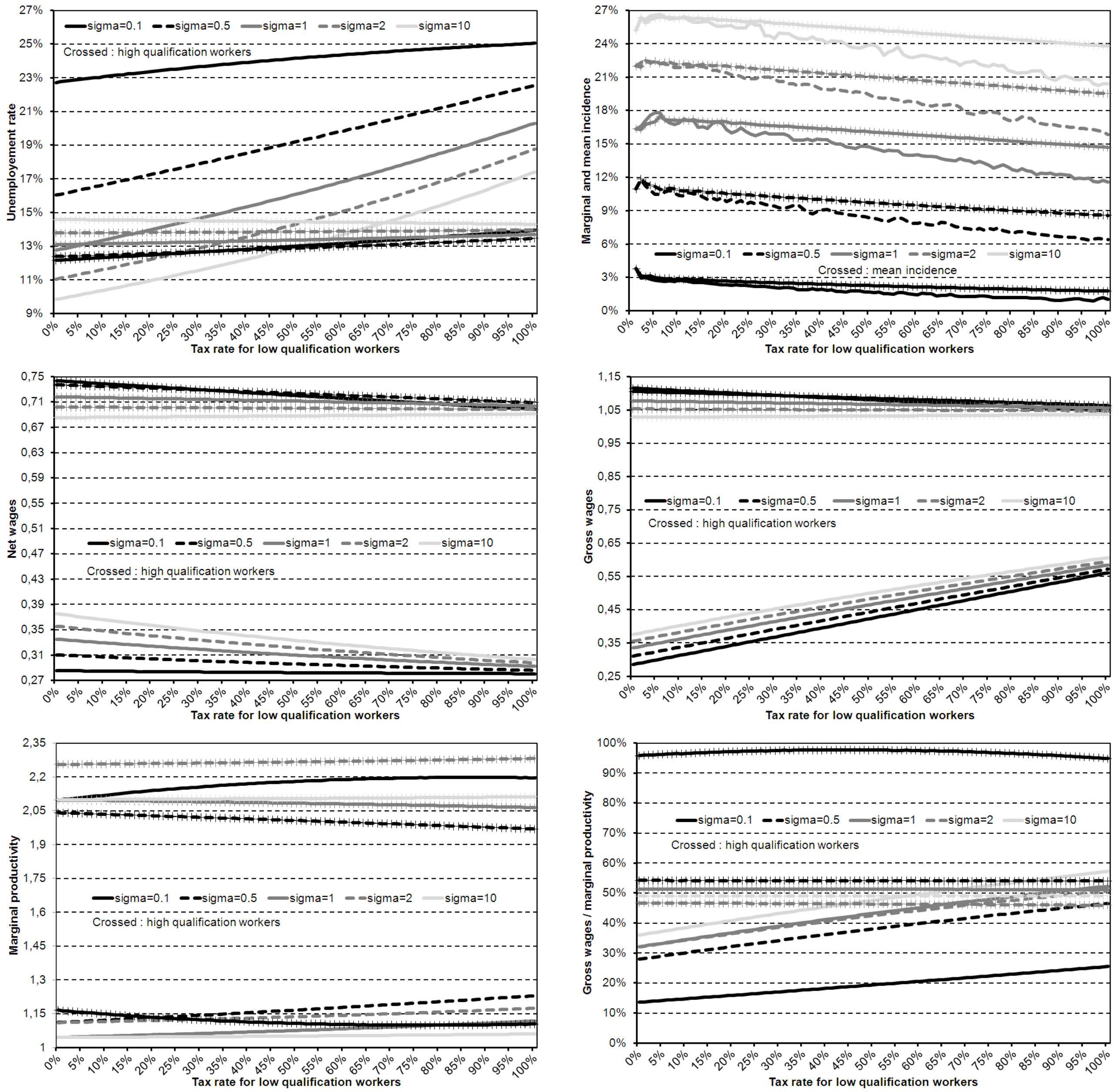


Figure 19: Impact of taxes on low qualification workers and substitution elasticity between workers

D Numerical analyses based on the three factor model

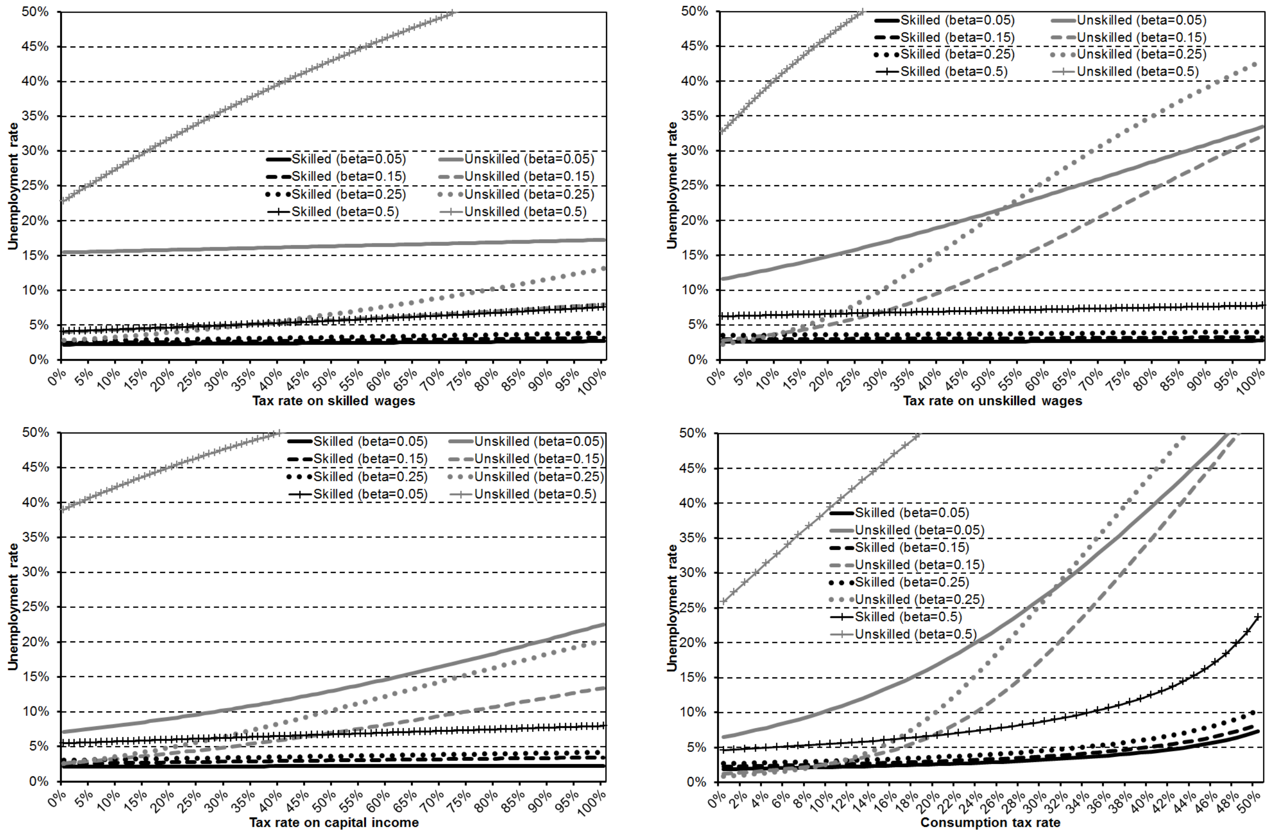


Figure 20: Impact of taxes on unemployment rates of high and low qualification workers

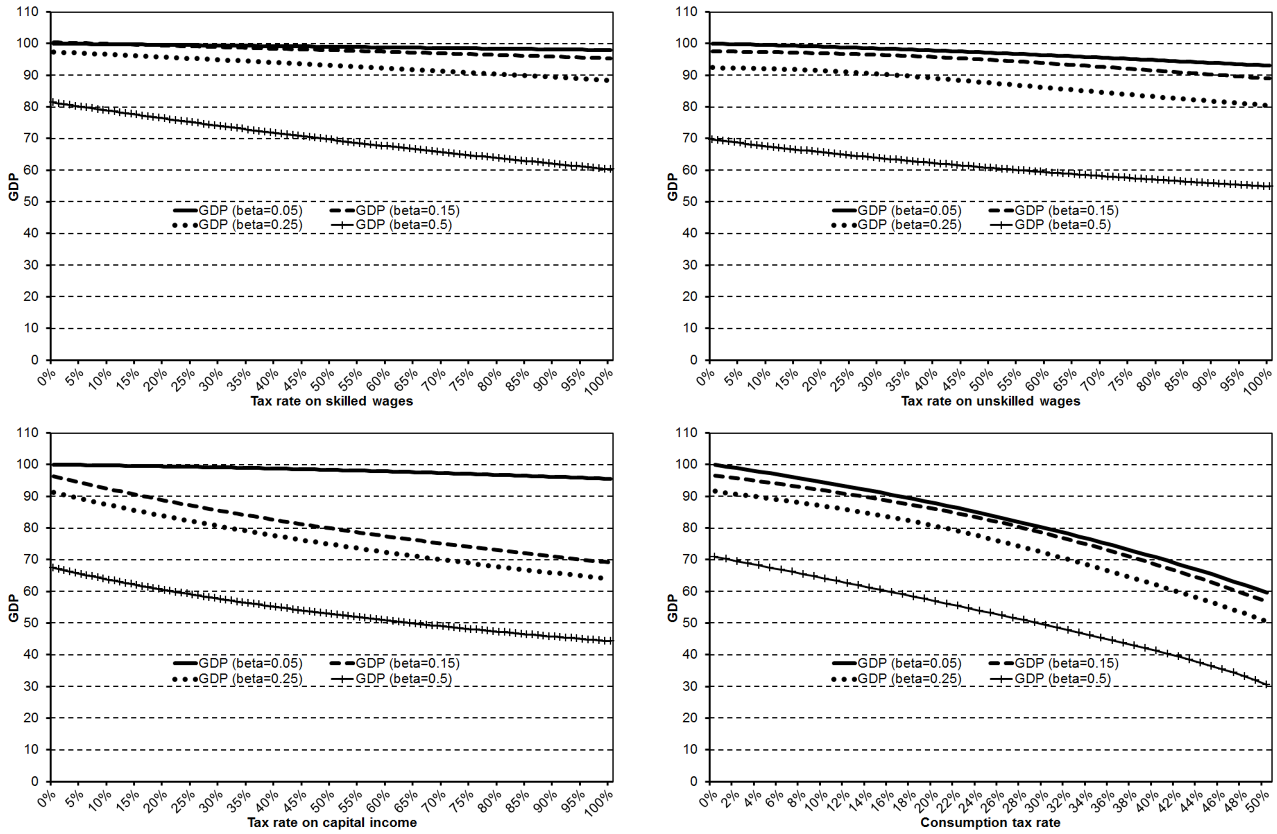


Figure 21: Impact of taxes on total production

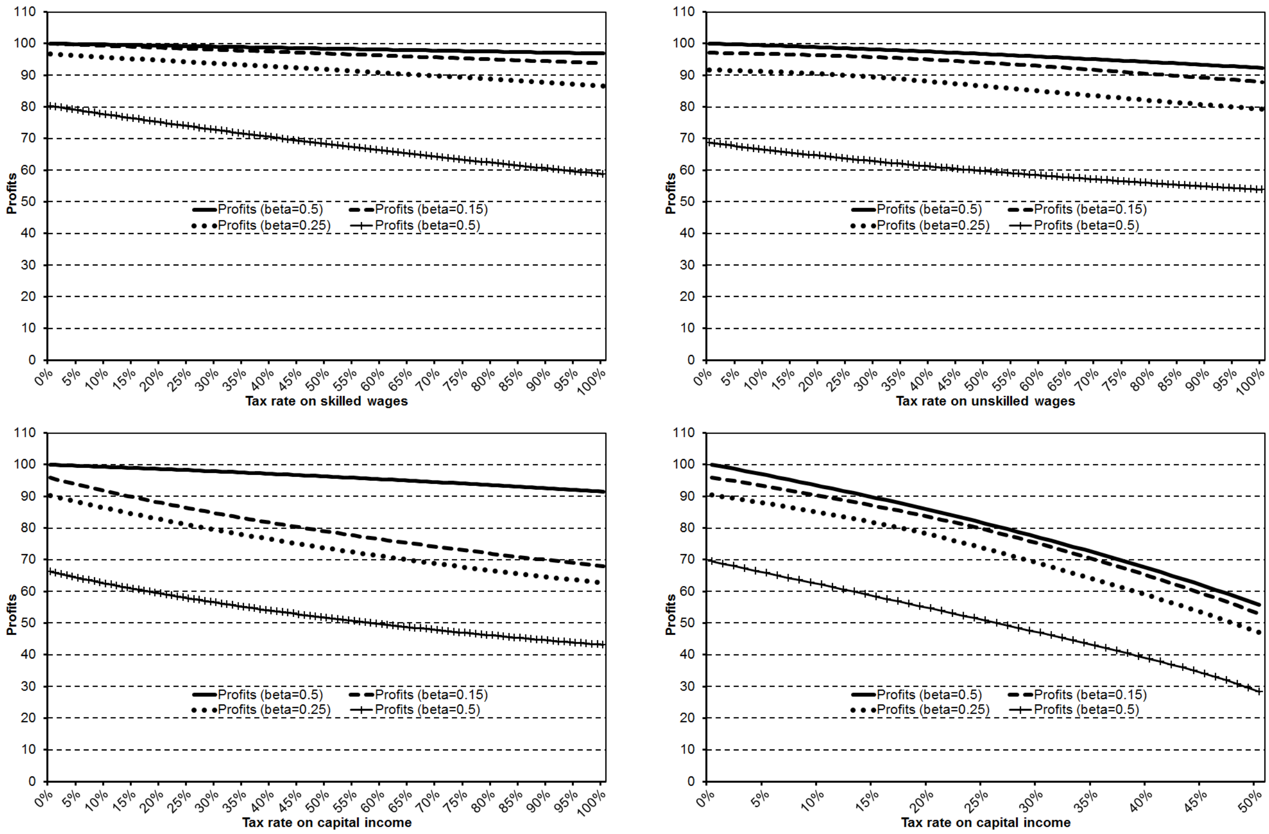


Figure 22: Impact of taxes on Firms profits

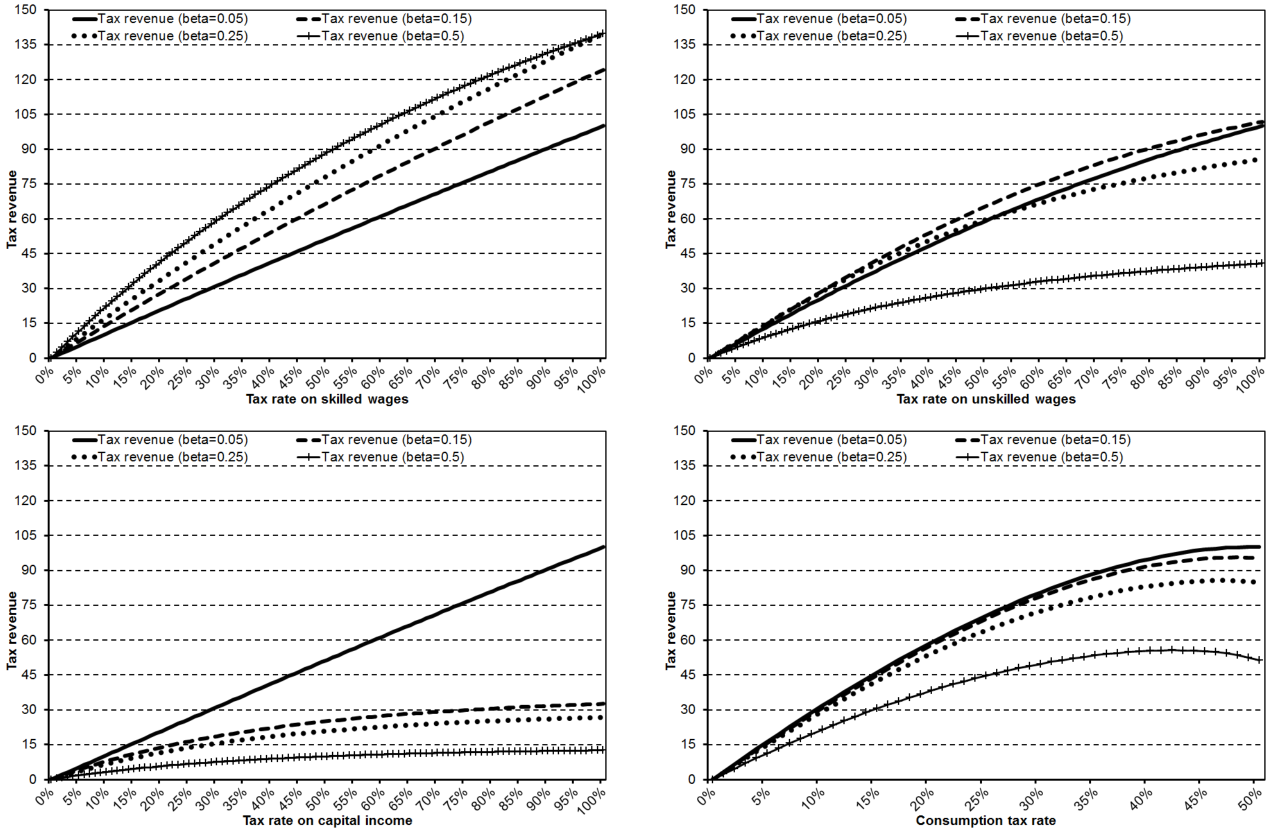


Figure 23: Impact of taxes on tax revenues

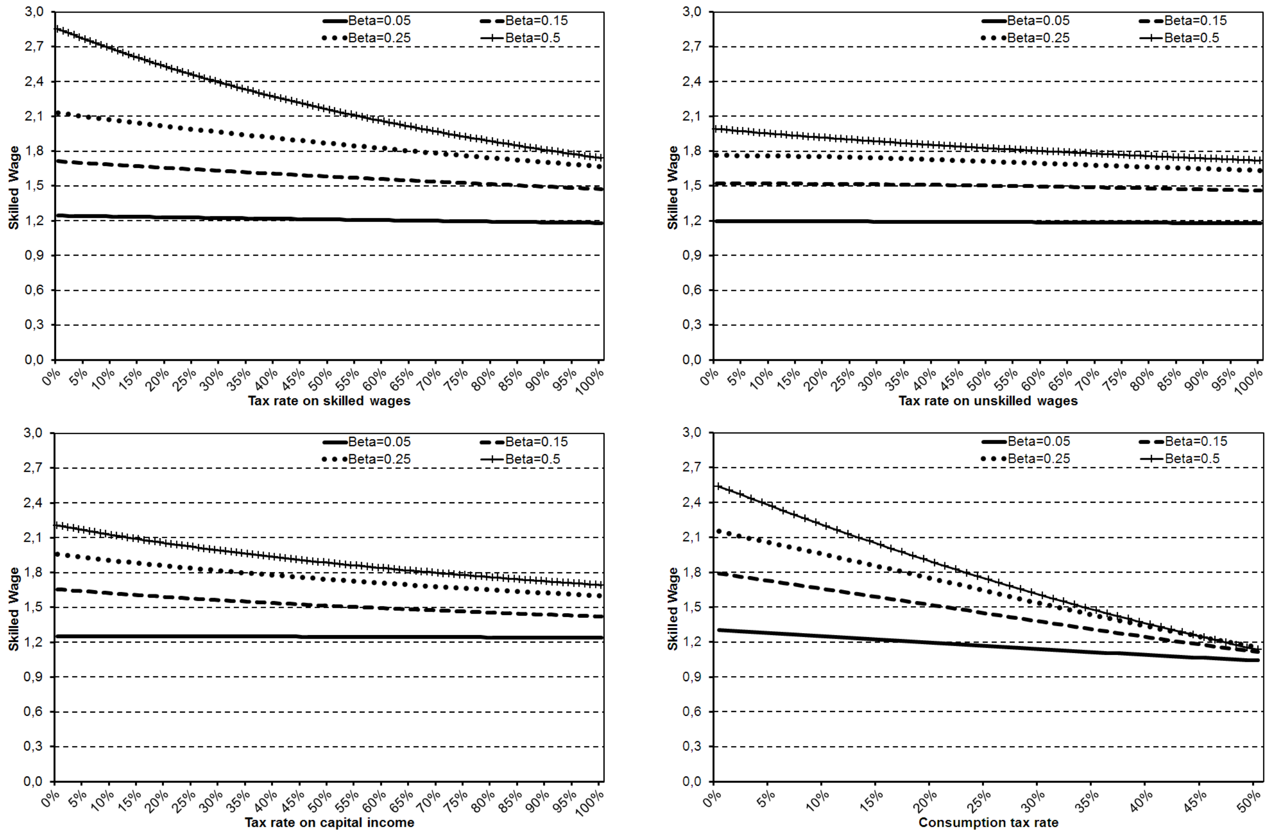


Figure 24: Impact of taxes on high qualification workers' wage

E Programs for numerical analyse

The numerical analyses has been implemented with *SciLab* software.

E.1 Programs for one worker type under linear tax

For the case of the analysis of the β, the program is as follows. It is the same with very little adaptation for the other parameter analyses.

```
N=zeros(10,101); Chom=zeros(10,101); Wnet=zeros(10,101); Wbrut=zeros(10,101);
Prodm=zeros(10,101);

bet=1/4; b=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');

deff('y=w(x)','y=b+gam*bet/(1-bet)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for be=1:1:10
     for ta=1:1:101
          alp=0.5+be/20;
          A=1/alp;
          tau=(ta-1)/100;
          deff('y=f(x)','y=((alp*bet*A*x^(alp-1))/(1-bet+alp*bet*(1+tau)))-
               (((1+tau)*bet*b)/(1+bet*tau))-((gam*bet)/(1-
               bet))*((1+tau)*bet/(1+bet)*(s/a)^(1/eta)*(x/(1-
               x))^(1/eta)+(r+s)/s*(s/a)^(1/eta)*(x/(1-x))^(1/eta-1))');
          deff('y=fj(x)','y=((alp-1)*alp*bet*A*x^(alp-2))/(1-bet+alp*bet*(1+tau))-
               (gam*bet)/(1-bet)*((1/eta)*(1+tau)*bet/(1+bet)*(s/a)^(1/eta)*(1/(1-
               x)^2)*(x/(1-x))^(1/eta-1)+(1/eta-1)*(r+s)/s*(s/a)^(1/eta)*(1/(1-
               x)^2)*(x/(1-x))^(1/eta-2))');
          if (be>9 & ta>99) then
               N(be,ta)=fsolve(0.5,f,fj);
               else
               N(be,ta)=fsolve(0.99999999,f,fj);
          end
          Chom(be,ta)=1-N(be,ta);
          Wnet(be,ta)=w(N(be,ta));
          Wbrut(be,ta)=(1+tau)*Wnet(be,ta);
          Prodm(be,ta)=alp*A*N(be,ta)^(alp-1);
     end
end

[res]=file('open','resultalp.txt','new');
dispfiles()
write(1,"Fonction de alpha");
write(1," ");
write(1,"Chomage");
write(1,Chom);
write(1," ");
write(1,"Salaire net");
write(1,Wnet);
write(1," ");
write(1,"Salaire brut");
write(1,Wbrut);
write(1," ");
write(1,"Productivity");
write(1,Prodm);
file("close",1);
```

E.2 Programs for one worker type under quadratic tax

```
N=zeros(4,101); Chom=zeros(4,101); Wnet=zeros(4,101); Wbrut=zeros(4,101);

alp=2/3; bet=0.25; A=1/alp; b=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');

deff('y=w(x)','y=b+gam*bet/(1-bet)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for lambda=1:1:4
      if lambda==1 then lam=0.1; end
      if lambda==2 then lam=0.4; end
      if lambda==3 then lam=0.7; end
      if lambda==4 then lam=1; end

      for tax=1:1:101
            Wtest=0.75;
            test=1;
            tau=(tax-1)/100;

            while test>0.001,
                  kap=tau-Wtest*lam;
                  minecar=100000;
                  for enne=0.84:0.001:0.96
                        theta=the(enne);
                        deff('z=dif(x,y)','z=(1-bet)*b/bet+gam*theta+x^(alp-1)-(1-
                              bet)*(sqrt((1+kap)^2+4*lam*y/x)-(1+kap))/(2*bet*lam)');
                        nen=0.001:0.001:0.999;
                        sol=ode(0.001,0.001,nen,dif);
                        wage=(sqrt((1+kap)^2+4*lam*sol(1000*enne)/enne)-(1+kap))/(2*lam);
                        ecar=wage-b-gam*bet*(theta+(r+s)/(a*theta^(eta-1)))/(1-bet);
                        if (abs(ecar)<minecar) then
                              minecar=abs(ecar);
                              N(lambda,tax)=enne;
                              Chom(lambda,tax)=1-enne;
                              Wnet(lambda,tax)=wage;
                        end
                  end
                  Wtest=Wnet(lambda,tax);
                  test=abs(tau-kap-Wtest*lam)
            end
            kap=tau-Wtest*lam;
            minecar=100000;
            Nmin=N(lambda,tax)-0.0005;
            Nmax=N(lambda,tax)+0.0005;
            for enne=0.84:0.00001:0.96
                  if enne>=Nmin then
                        if enne<=Nmax then
                              theta=the(enne);
                              deff('z=dif(x,y)','z=(1-bet)*b/bet+gam*theta+x^(alp-1)-(1-
                                    bet)*(sqrt((1+kap)^2+4*lam*y/x)-(1+kap))/(2*bet*lam)');
                              nen=0.00001:0.00001:0.99999;
                              sol=ode(0.00001,0.00001,nen,dif);
                              wage=(sqrt((1+kap)^2+4*lam*sol(100000*enne)/enne)-
                                    (1+kap))/(2*lam);
                              ecar=wage-b-gam*bet*(theta+(r+s)/(a*theta^(eta-1)))/(1-bet);
                              if (abs(ecar)<minecar) then
                                    minecar=abs(ecar);
                                    N(lambda,tax)=enne;
                                    Chom(lambda,tax)=1-enne;
                                    Wnet(lambda,tax)=wage;
                              end
                        end
                  end
            end
            Wbrut(lambda,tax)=(1+kap+lam*Wnet(lambda,tax))*Wnet(lambda,tax);
      end
end
```

E.3 Programs for one worker type under linear tax

```
Chom1=zeros(5,101); Chom2=zeros(5,101); Wnet1=zeros(5,101); Wnet2=zeros(5,101);
alp=2/3; alp1=1/3; alp2=2/3; A=9/2; bet1=0.2; bet2=0.4; b1=1/4; b2=1/2; gam=1/2; s=1/10; r=3/100; eta=0.68; a=1; tau2=0.5;

deff('y=the(x)','y=(s/a)^(1/eta)*(x/(1-x))^(1/eta)');
deff('y=weq1(x)','y=b1+gam*bet1/(1-bet1)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');
deff('y=weq2(x)','y=b2+gam*bet2/(1-bet2)*(the(x)+(r+s)/(a*the(x)^(eta-1)))');

for sigma=1:1:5
   if sigma==1 then sig=0.1; end
   if sigma==2 then sig=0.5; end
   if sigma==3 then sig=1; end
   if sigma==4 then sig=2; end
   if sigma==5 then sig=10; end
   del=(1-sig)/sig;

   for ta=1:1:101
        sigma, ta,
        tau1=(ta-1)/100;
        if sigma==3 then
           deff('f=F1(x,y,z)','f=alp*alp1*A*x^(alp*alp1-1)*y^(alp*alp2)*z^(alp*(alp1*(1+tau1)
                +alp2*(1+tau2)*bet2*(1-bet1)/(bet1*(1-bet2))))');
           deff('f=F2(x,y,z)','f=alp*alp2*A*x^(alp*alp1)*y^(alp*alp2-1)*z^(alp*(alp1*(1+tau1)*bet1*(1-bet2)/(bet2*(1-bet1))
                +alp2*(1+tau2)))');
        else
           deff('f=F1(x,y,z)','f=alp*alp1*A*x^(-del-1)*z^(-del*(1+tau1))*(alp1*x^(-del)*z^(-del*(1+tau1))
                +alp2*y^(-del)*z^(-del*(1+tau2)*bet2*(1-bet1)/(bet1*(1-bet2))))^(-1-alp/del)');
           deff('f=F2(x,y,z)','f=alp*alp2*A*y^(-del-1)*z^(-del*(1+tau2))*(alp1*x^(-del)
                *z^(-del*(1+tau1)*bet1*(1-bet2)/(bet2*(1-bet1))) +alp2*y^(-del)*z^(-del*(1+tau2)))^(-1-alp/del)');
        end

        N1=0.85; N2=0.85; n1=0; n2=0; test1=(N1-n1)^2+(N2-n2)^2;
        if ta>1 then N1=1-Chom1(sigma,ta-1); N2=1-Chom2(sigma,ta-1); end

        while test1>0.00000002
           n1=N1; n2=N2;
           deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
           test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));

           if test2<0 then
                while (test2<0 & N1>0.0001);
                   N1=N1-0.00005;
                   deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
                   test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                        +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));
                end
           else
                while (test2>0 & N1<0.9999);
                   N1=N1+0.00005;
                   deff('u=sous1(t)','u=t^(tau1+(1-bet1)/bet1)*F1(N1,N2,t)');
                   test2=intg(0,1,sous1)-(1+tau1)*bet1*b1/(1+bet1*tau1)-gam*bet1/(1-bet1)*(((1-bet1)
                        +2*(1+tau1)*bet1)*the(N1)/(1+bet1*tau1)+(r+s)/(a*the(N1)^(eta-1)));
                end
           end

           deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
           test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));

           if test2<0 then
                while (test2<0 & N2>0.0001);
                   N2=N2-0.00005;
                   deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
                   test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                        +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));
                end
           else
                while (test2>0 & N2<0.9999);
                   N2=N2+0.00005;
                   deff('u=sous2(t)','u=t^(tau2+(1-bet2)/bet2)*F2(N1,N2,t)');
                   test2=intg(0,1,sous2)-(1+tau2)*bet2*b2/(1+bet2*tau2)-gam*bet2/(1-bet2)*(((1-bet2)
                        +2*(1+tau2)*bet2)*the(N2)/(1+bet2*tau2)+(r+s)/(a*the(N2)^(eta-1)));
                end
           end

           test1=(N1-n1)^2+(N2-n2)^2;
        end

        Chom1(sigma,ta)=1-N1;
        Chom2(sigma,ta)=1-N2;
        Wnet1(sigma,ta)=weq1(N1);
        Wnet2(sigma,ta)=weq2(N2);
   end
end
```

E.4 Programs for one worker type under linear tax

```
ChomU=zeros(101); ChomC=zeros(101); WnetU=zeros(101); Cap=zeros(101);

alpU=0.525; alpC=0.175; alpK=0.3; A=4; b=1; gamU=0.5; gamC=0.5; sU=0.1; sC=0.1; r=0.03; etaU=0.68; etaC=0.68; aU=1; aC=1;
smic=1; beta=0.5;

t=0.2; tauC=0.3; tauK=0.5;

target=0.9592+0.000282;

for ta=1:1:101

    tauU=(ta-1)/100;

    target=target-0.000282;

    deff('z=K(x,y)','z=((((1+tauU)*alpU+(1-beta)/beta)*beta*(1+tauK)*r)/((1-t)*A*alpK*(1-beta)))^(-1/(1-alpK)) *x^(alpU/(1
        alpK))*y^(alpC/(1-alpK))');

    deff('y=Lc(x)','y=x/(alpU^((1-alpK)/alpC)*alpK^(alpK/alpC))*(beta*(1+tauK)*r/(1-beta))^(alpK/alpC)
        *((1-t)*A/((1+tauU)*alpU+(1-beta)/beta))^(-1/alpC)*((1+tauU)*beta*b/(1+tauU*beta)
        +gamU*beta*(1+2*(1+tauU)*beta/(1-beta))/(1+tauU*beta)*(sU/aU)^(1/etaU)*(x/(1-x))^(1/etaU)
        +beta/(1-beta)*gamU*(r+sU)/aU*(sU/aU)^((1-etaU)/etaU)*(x/(1-x))^((1-etaU)/etaU))^((1-alpK)/alpC)');

    deff('y=Eq(x)','y=(1-t)*A*alpC*x^(alpU)*(Lc(x))^(alpC-1)*(K(x,Lc(x)))^(alpK)/((1+tauU)*alpU+(1-beta)/beta)
        -beta/(1-beta)*(1+tauC)*smic-gamC*(r+sC)/aC*(sC/aC)^((1-etaC)/etaC)*((Lc(x))/(1-(Lc(x))))^((1-etaC)/etaC)');

    Lu=fsolve(target,Eq);

    ChomU(ta)=1-Lu;

    ChomC(ta)=1-Lc(Lu);

    Cap(ta)=K(Lu,Lc(Lu));

end
```