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Do Stock Returns Rebound After Bear Markets? An Empirical Analysis From Five OECD Countries

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Abstract

This paper proposes an empirical study of the shape of recoveries in financial markets from a bounce-back augmented Markov Switching model. It relies on models first applied by Kim, Morley and Piger [2005] to the business cycle analysis. These models are estimated for monthly stock market returns data of five developed countries for the post-1970 period. Focusing on a potential bounce-back effect in financial markets, its presence and shape are formally tested. Our results show that *i)* the bounce-back effect is statistically significant and large in all countries, but Germany where evidence is less clear-cut and *ii)* the negative permanent impact of bear markets on the stock price index is notably reduced when the rebound is explicitly taken into account.

Keywords: Stock Market Returns, Markov Switching Models, Shape of Bounce-Back

JEL classification: C22, G10.

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1 Introduction

Even though first applied to the characterization of expansion and recession regimes in economic activity as measured by e.g. the real GDP growth rate, the Markov-Switching (MS hereafter) model — first popularized by Hamilton [1989] two decades ago — has quickly been used to model stock market returns dynamics. Indeed, this model is able to capture central statistical features of asset returns such as skewness, fat tails, volatility clustering or mean reversion[1]. This approach has the advantage that the different regimes of the financial market can be inferred from the market index data. Actually, this model and its extensions have proven quite successful in identifying the bull and bear market periods – see e.g. Turner, Startz and Nelson [1989], Schwert [1989], Hamilton and Susmel [1994], Schaller and Van Norden [1997], Hess [2003], Guidolin and Timmermann [2005], Guidolin and Timmermann [2007] or Chen [2009].

Basically, the two-regime models assume that following the end of a bear market, the average of stock returns is the same over the entire bull market period. This assumption in turn implies that the timing of investment in bull market is not so important. However, from 160 years of monthly U.S. data, Maheu and McCurdy [2000] find that "the best market gains come at the start of a bull market". Similarly, Gonzalez, Powell, Shi and Wilson [2005] investigate two centuries of bull and bear market cycles in the U.S. and find that "the first 6 months of bull markets exhibit significantly higher returns than do the remaining months of bull markets (e.g., 3.18% total return per month v.s. 1.91%)". As will be seen later, our monthly data for the U.S., Canada, France, Germany and the U.K. over the 1970M1-2012M12 period also point to the same feature. Hence, the two-regime assumption seems too restrictive and inconsistent with the empirical evidence for

[1]See the survey by Ang and Timmermann [2011] on this point.

stock market returns. For this reason, the MS modeling of these data has quickly moved to three- or four-regime assumptions. For instance, Guidolin and Timmermann [2005] identify three regimes in their MS setup: a bear, a normal and a bull regime. More recently, Guidolin and Timmermann [2007] or Maheu, McCurdy and Song [2009] use four-state MS models and typically identify a crash, a slow growth, a bull and a recovery regime. However, as stressed by the former, "models with more states have far more parameters" (p.3512 therein).

From an economic point of view, the bounce back effect in financial market captured by the so-called recovery regime is reasonable, since stock market movement is usually related to macroeconomic fundamentals, such as GDP growth which also seems to exhibit a bounce-back effect after the cycle trough (see e.g. Beaudry and Koop [1993] or more recently Kim et al. [2005], Morley and Piger [2012] and Bec, Bouabdallah and Ferrara [2013]). Cecchetti, Lam and Mark [1990] find that switching in economic growth dynamics influences the distribution of stock returns via the dividends. Hamilton and Lin [1996] show that the driving force of conditionally switching moments of stock returns are economic recessions. Perez-Quiros and Timmermann [2001] report the effect of output on higher order moments in a discrete state model and show its significant impact. Chen [2009] finds that stock market return could be predicted by some macro-variables. There is also a vast theoretical literature that relates the stock returns distribution to the business conditions (e.g. Ebell [2001]). Hence, it seems likely that a bounce-back effect found in stock market returns basically originates in real economic activity.

Our main contribution to the empirical literature devoted to stock market returns is to study the possible bounce-back effect using the newly developed bounce-back augmented MS model — first analyzed by Kim et al. [2005] and later generalized by Bec, Bouabdallah and Ferrara [2011] — across several developed countries's stock market returns. As will be seen below, this model allows for a transitional dynamics between the bear and bull markets during which the returns may be temporarily higher than in the bull market. It also allows for the magnitude of this bounce-back effect to be proportional to the depth of the previous bear market, an assumption that can be easily tested in this framework. Moreover, the bounce-back augmented MS model does not involve the estimation of a third regime and hence does not require any increase in the dimension of the transition probability matrix which governs the regime switching: the drawback put forward by Guidolin and Timmermann [2007] is avoided. An application to five OECD countries stock market returns reveals that the null hypothesis of no bounce-back is always strongly rejected. The presence of this bounce-back effect in turn implies that the negative permanent impact of a bear market episode on the level of stock market index is smaller than what it would be absent the bounce-back effect.

The paper is organized as follows. Section 2 presents and discusses the methodology. Section 3 describes the data while Section 4 reports the models estimates and tests for the presence and shape of the bounce-back effect. The implications of the estimated shape of the bounce-back effect is also discussed, particularly in terms of the permanent impact of a bear market episode on the stock market index. Section 5 concludes.

2 The bounce-back augmented Markov-Switching model

Let y_t denote the log of stock market index and Δy_t its first difference, i.e. the stock market returns. The model considered throughout this paper is the following:

$$\phi(L)(\Delta y_t - \mu_t) = e_t, \tag{1}$$

where $\phi(L)$ is a lag polynomial of order p with roots lying outside the unit circle and with e_t i.i.d. $\mathcal{N}(0, \sigma^2_{S_t})$. Note that μ_t is allowed to switch across regimes. The Markov Switching model proposed by Hamilton [1989] postulates the existence of an unobserved variable, denoted S_t, which takes on the value zero or one. S_t characterizes the "state" or "regime" in which the process is at time t. The standard version of Hamilton's model could be written as:

$$\mu_t = \gamma_0 + \gamma_1 S_t, \tag{2}$$

where the mean of Δy_t is γ_0 if $S_t = 0$ and $\gamma_0 + \gamma_1$ otherwise. Here, $S_t = 1$ is identified as the bear market by assuming $\gamma_0 > 0$ and $\gamma_0 + \gamma_1 < 0$. Hamilton [1989] assumes that the unobserved state variable S_t is the realization of a two states Markov chain with $P(S_t = j | S_{t-1} = i) = p_{ij}$. This Markov chain implies that S_t depends on the past realizations of Δy and S only through the most recent value S_{t-1}. The model given by Equations (1) and (2) allows for an asymmetric behavior across regimes. For instance, the bull market may be characterized by long and gradual upward movements if γ_0 is positive and small and p_{00} is close to one, while the bear market may correspond to sharp and short declines if γ_1 is negative and large in absolute value and p_{11} is small.

As shown in Bec et al. [2011], the above MS framework may be generalized by the bounce-back augmented Markov-Switching model given below and denoted BBF(p, m) hereafter:

$$\mu_t = \gamma_0 + \gamma_1 S_t + \lambda_1 S_t \sum_{j=1}^{m} \gamma_1 S_{t-j} + \lambda_2 (1 - S_t) \sum_{j=1}^{m} \gamma_1 S_{t-j} + \lambda_3 \sum_{j=1}^{m} \Delta y_{t-j} S_{t-j}, \tag{3}$$

where m is non-negative integer and corresponds to the duration of the bounce-back effect[2]. The λ_i's parameters govern the size of the bounce-back effect. The

[2]Bec et al. [2011]'s general model also allows for a delay of ℓ periods before the bounce-back is activated. Since this delay is not present in our data, we neglect it to make the presentation of the model simpler.

additional terms on the right-hand side of Equation (3) define a very flexible form for the rebound. Assuming $\lambda_2 = \lambda_3 = 0$, the first term of the bounce-back function is $\lambda_1 S_t \sum_{j=1}^{m} \gamma_1 S_{t-j}$. Consequently, any negative value of λ_1 will contribute to lead the growth rate of y_t above γ_1 as soon as one period after the dynamics of y_t enters the bear regime and stays therein for at least two consecutive periods. Hence a bounce-back effect requires that $\lambda_1 < 0$. Finally, its duration may vary according to the value of parameter m. Similarly, assuming $\lambda_1 = \lambda_3 = 0$, any negative value of λ_2 will lead μ_t above γ_0 as soon as the economy comes back in bull market after a bear market episode. The longer the bear market, the larger this bounce-back effect. This precise case corresponds to the so-called V-shaped recession model. A closely related model is obtained by setting $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_3 = 0$ in Equation (3): If $\lambda < 0$, then the bounce-back activates as soon as one month after the beginning of the bull market, and it lasts at least m periods whatever the duration and the depth of the bear market. Finally, the last term of Equation (3) also yields a bounce-back effect for the Δy_t variable from the month following the beginning of a bear market on, when λ_3 is negative: by construction of the S_t variable, the term $\Delta y_{t-j-1} S_{t-j}$ is indeed negative. In this particular case, namely the Depth-shaped recession model, the magnitude of the rebound is positively related to the depth of the recession and its duration is again an increasing function of the recession's duration.

Hence, the model given by equations (1) and (3) above nests the three models first proposed by Kim et al. [2005], namely the U-, V- and Depth-shaped bounce-back[3] as well as the no bounce-back — standard Hamilton — model with the following linear restrictions:

- H_0^N: $\lambda_i = 0 \ \forall i$ corresponds to the standard (no bounce-back) Hamilton model,

[3]See Bec et al. [2011] for a detailed description of these functions.

- H_0^U: $\lambda_1 = \lambda_2 = \lambda \neq 0$ and $\lambda_3 = 0$ gives the U-shaped model, denoted BBU,

- H_0^V: $\lambda_2 \neq 0$ and $\lambda_1 = \lambda_3 = 0$ gives the BBV model,

- H_0^D: $\lambda_3 \neq 0$ and $\lambda_1 = \lambda_2 = 0$ defines the BBD model.

For m assumed known and fixed as described in section 3 below, the BBF(p, m) model is estimated by the maximum likelihood method using the filter presented in Hamilton [1989], as described in Kim et al. [2005]. Note that here, due to the terms involving the sum of lagged values of S_t, one has to keep track of 2^{p+m+1} states versus 2^{p+1} when constructing the likelihood function in each period. Standard errors are based on numerical second derivatives. Basically, the autoregressive lag parameter p is chosen as the smallest one which succeeds in eliminating residuals autocorrelation according to the LM test. Computing residuals for MS models is not so straightforward. The approach described in Kim, Shephard and Chib [1998] is retained here: the BBF model residuals are constructed from the one-step-ahead prediction distribution functions. Indeed, from equations (1) and (3), it can be seen that time-t residuals depend on $\xi_t \equiv (S_t, S_{t-1}, \ldots, S_{t--m-p})$. The one-step-ahead prediction distribution functions are defined by:

$$\epsilon_t = \sum_{i=1}^{2^{p+m+\ell+1}} P(\xi_{it}|I_{t-1}) \Phi\left(\frac{\hat{\varepsilon}_{it}}{\hat{\sigma}}\right), \tag{4}$$

where $\hat{\varepsilon}_{it}$ denotes the ML estimates of the BBF model residuals in regime i and Φ is the distribution function of the standard normal. Note that the $P(\xi_{it}|I_{t-1})$'s can be easily obtained as by-products of the filtering algorithm. If the nonlinear model is true and ignoring the effect of estimating parameters, the ϵ_t are approximately standard uniform and *iid*. Then, they can be mapped to the residuals for the BBF model by the standard normal inverse distribution function, $\varepsilon_t^{BBF} = \Phi^{-1}(\epsilon_t)$.

The linear null hypothesis amounts here to test the joint hypothesis $\lambda_1 = \lambda_2 = \lambda_3 = 0$ *and* $\gamma_0 = \gamma_1$, i.e. μ_t becomes a constant term. Due to the presence of nuisance parameters, this test statistics distribution is not standard. Garcia [1998]'s linearity test will be used as described below. By contrast, the four assumptions H_0^N, H_0^U, H_0^V and H_0^D LR test statistics are nuisance parameter free and have a standard Chi-squared distribution. Consequently, the test of the shape of the rebound is quite straightforward in this BBF-MS setup.

3 Data

Several developed countries' stock index are considered. For the U.S. financial market, y_t is the log of monthly S&P500 index over the period 1969M12 to 2012M12, i.e. 517 observations. For the sake of comparison, stock market data for Canada, France, Germany and the UK are also considered. They come from the Morgan Stanley Capital International (MSCI) database. Monthly stock indices are chosen because bull and bear markets are relatively low frequency movements so that it would not be worth it using higher frequency data such as daily data. In this paper, we follow the general practice which consists in identifying financial cycles from nominal stock returns. As emphasized by e.g. Schwert [1990], the variation in dividend returns is relatively small compared to the variation in the stock market index, so that capital returns convey the relevant information to capture the financial cycle.

The five stock market indices are presented in Fig 1 and 2. The series show an overall upward trend over time, but large market movements such as the crash of 1987 are also evident. Returns are computed as the first difference of the log stock price index. Summary statistics for stock index returns are reported in Table 1. The means of monthly stock returns range from 0.38% in Germany to 0.55% in

Table 1: Summary statistics

	US	Canada	France	Germany	UK
Mean	0.51	0.53	0.49	0.38	0.55
Median	0.86	0.86	1.11	0.78	1.01
Standard deviation	4.52	4.90	5.87	5.80	5.58
Kurtosis	2.42	3.01	1.19	2.80	8.56
Skewness	-0.66	-0.81	-0.38	-0.81	0.20
Minimum	-24.16	-24.57	-24.81	-28.67	-30.27
Maximum	15.96	14.95	20.40	17.95	42.89
Jarque-Bera	163.39	251.33	43.00	224.98	1578.19

Summary statistics for capital returns (%), from January 1970 to December 2012.

the UK over the whole sample, while their standard deviations range from 4.52% in the US to 5.87% in France. Expectedly, the returns' distribution is clearly not Gaussian according to Jarque and Bera test statistics: it displays leptokurtosis, a well-know feature of stock returns data.

A dating algorithm is applied to the stock price index data in order to identify bull and bear markets. A popular algorithm is the one proposed by Bry and Boschan [1971] to identify turning points of business cycles. Pagan and Sossounov [2003] adapt this algorithm to study the characteristics of bull/bear markets in monthly stock prices. Their method can be summarized as follows: firstly, they identify the peaks and troughs by using a window of 8 months; secondly, they enforce alternation of phases by deleting the lower of adjacent peaks and the higher of adjacent troughs; then, they eliminate phases lasting less than 4 months unless the changes exceed 20%; lastly, they eliminate the cycles lasting less than 16 months [4]. The dating algorithm sorts stock market index into a particular regime

[4]Window width and phase duration constraints will depend on the particular series and will obviously be different for smoothed business cycle data than for stock prices. Pagan and Sossounov [2003] provide a detailed discussion of their choices for these constraints.

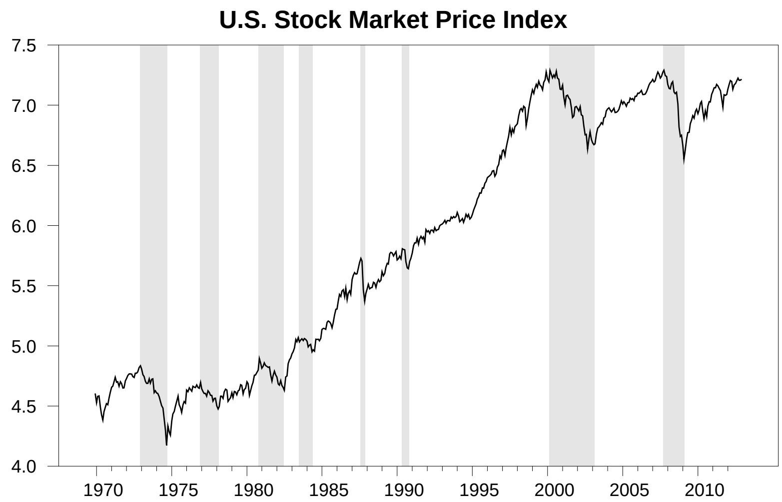
Table 2: Financial cycle turning points

U.S.		Canada		France		Germany		UK	
Peak(a)	Trough(b)	Peak(a)	Trough(b)	Peak(a)	Trough(b)	Peak(a)	Trough(b)	Peak(a)	Trough(b)
					06/70(-)		10/71(-)		05/70(-)
12/72(-)	09/74(21)	10/73(-)	09/74(10)	04/73(34)	09/74(17)	07/72(9)	09/74(26)	08/72(27)	11/74(27)
12/76(27)	02/78(14)	02/76(17)	10/77(20)	02/76(17)	04/77(14)	03/76(18)	10/76(7)	01/76(14)	11/76(10)
		11/80(37)	06/82(19)			09/78(23)	02/81(29)	04/79(29)	12/79(8)
11/80(33)	06/82(20)			10/80(42)	06/81(8)	07/81(5)	08/82(13)		
07/83(11)	05/84(11)	11/83(17)	07/84(8)			04/86(44)	01/88(21)	07/87(91)	11/87(4)
08/87(39)	11/87(3)	07/87(36)	11/87(4)	04/87(70)	01/88(9)			12/89(25)	09/90(9)
05/90(30)	10/90(5)	09/89(21)	10/90(14)			03/90(26)	09/90(6)	08/91(11)	08/92(12)
		01/92(15)	01/93(12)	01/94(72)	02/95(13)	12/93(39)	03/95(15)	01/94(17)	01/95(12)
		01/94(12)	06/94(5)					04/99(51)	02/00(10)
03/00(113)	09/03(35)	08/00(74)	02/02(18)	08/00(66)	03/03(31)	02/00(59)	03/03(37)	08/00(6)	01/03(29)
10/07(56)	02/09(16)	05/08(68)	02/09(9)	05/07(50)	02/09(21)	10/07(55)	02/09(16)	05/07(52)	02/09(21)
		02/11(24)	05/12(15)	04/11(26)	09/11(5)	04/11(26)	09/11(5)	04/11(26)	09/11(5)
Average durations in months, all cycles									
44	16	32	12	48	15	30	18	32	13

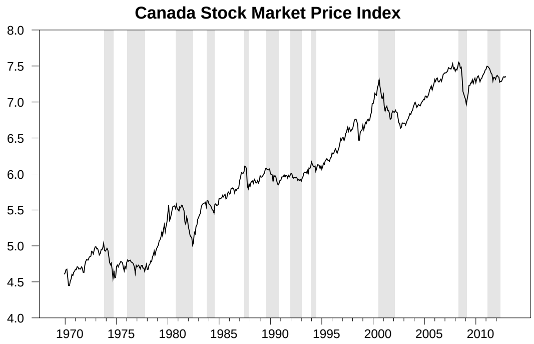
(a): bull market duration in months, from previous trough to this peak.
(b): bear market duration in months, from previous peak to trough.

with probability zero or one.The resulting peak and trough dates are reported in Table 2. It is then straightforward to compute such features of bull and bear markets as duration, average return or returns volatility. Table 2 clearly points to the fact that bull markets last longer than bear markets: depending on the country considered, the average length of bull markets ranges from 30 months to 48 months, while the average duration of bear markets lies between 12 months and 18 months. Obviously, this asymmetric duration is a feature of financial cycles. The classification into bull and bear markets obtained from this dating algorithm

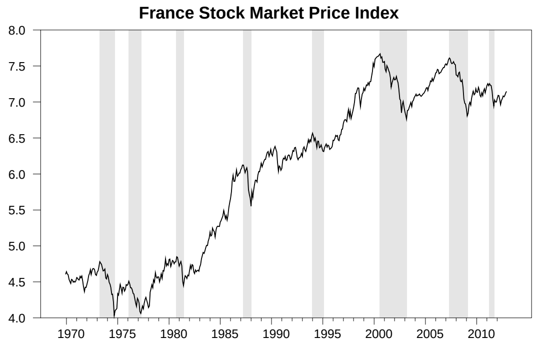
Figure 1: Bull and bear markets in the U.S.



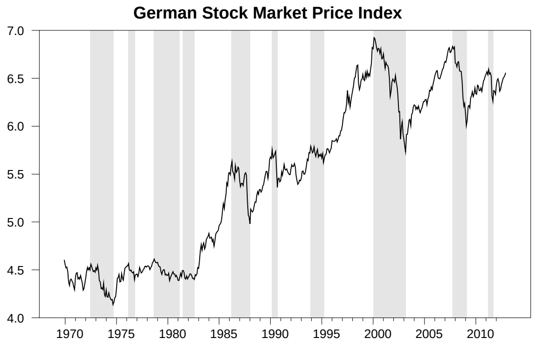
is reported in Figure 1 for the U.S. stock market, and in Figure 2 for the other countries: the shaded areas represent the bear markets episodes. The average returns of stock market observed from one to twelve months after the end of a bear market are reported in Table 3 below: these statistics confirm the presence of a bounce-back effect since the returns following the end of a bear market are larger than the average returns in bull markets in all cases. More precisely, the returns are two to five times larger in the month following the end of the bear



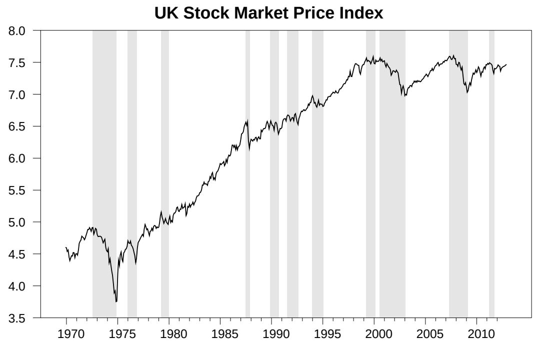
(a) CA



(b) FR



(c) GE



(d) UK

Figure 2: Bull and bear markets

market than they are on average during bull markets. For instance, in the US, the bull market average monthly return is 0.016 while it reaches 0.074 in the first month after the troughs and is still around 0.05 two months later or 0.027 during the seventh month after the troughs. When looking at the five countries under consideration, it is also worth noticing that if one wants to make sure that the bounce-back function captures all the rebound phenomenon, then setting m to seven months seems to be a safe choice in Equation (3). As can be seen from Table 3, from the eighth month following the trough on, the returns become close to zero or even slightly negative while they seem to become randomly positive or negative from the ninth month on.

Even though very useful for regime dependent descriptive statistics, dating algorithms cannot be used for forecasting or inference. Typically, they are not very helpful to tackle such questions of interest as "How likely is it that the market could turn into a bull market next month?" or "How strong is the bounce-back effect?". These questions can be answered by bounce-back augmented MS models, which involve the estimation of a transition probabilities matrix as well as of the parameters governing the magnitude of the bounce-back effect.

Table 3: Stock market index returns

	US	Canada	France	German	UK
Months after bear market					
1	0.074	0.057	0.083	0.074	0.059
2	0.038	0.030	0.018	0.027	0.067
3	0.049	0.010	0.013	0.005	0.028
4	0.020	0.032	0.039	0.045	0.008
5	0.037	0.012	0.037	0.044	0.034
6	0.022	0.011	0.012	0.007	0.025
7	0.027	0.025	0.013	0.029	0.010
8	0.005	-0.001	-0.002	-0.004	-0.016
9	0.031	0.029	-0.002	0.013	0.026
10	0.011	0.018	0.033	0.037	0.042
11	-0.001	0.010	0.043	0.000	0.017
12	-0.002	0.004	-0.011	-0.010	-0.000
Full bull market	0.016	0.019	0.018	0.022	0.023

Average returns are measured as monthly percentages. The sample period is 1970:M1 to 2012:M12.

4 Empirical results

This Section first presents the linearity tests results before turning to the bounce-back effect tests. From the latter, a preferred model is selected in each of the five countries considered, whose estimates are briefly commented. Finally, the implications of the estimated shape of the rebound are discussed.

4.1 Linearity tests

As a first step of the empirical analysis, Garcia [1998]'s linearity test is implemented and the corresponding results are reported in Table 4. Under the null, the process follows a standard linear autoregression whereas it is a two-regime MS model with switching mean and variance under the alternative. The models were estimated with no lag under both the null and the alternative since the estimated residuals displayed no serial correlation in this case. The likelihood-ratio (LR) statistics

Table 4: Linearity tests

	US	CA	FR	GE	UK
Log-Lik$_{Hamilton}$	-1480.94	-1509.75	-1625.49	-1594.68	-1557.63
Log-Lik$_{AR}$	-1510.15	-1551.85	-1644.57	-1638.44	-1618.97
LR	58.42	84.20	38.16	87.52	122.68
p-value	0.00	0.00	0.00	0.00	0.00

ranges from 38.16 in France to 122.68 in the UK. Since this statistics is not Chi-squared distributed due to the presence of nuisance parameters, Garcia [1998] has tabulated critical values for the simple two-mean, two-variance model: The 99%-critical value is 14.02, which is well below all the values obtained here: The linear null is hence always strongly rejected. Since the bounce-back augmented MS model encompasses the standard Hamilton MS model, Garcia [1998]'s test has probably some power against such larger alternative as the BBF model as well, which comforts the subsequent empirical analysis.

4.2 Tests for presence and shape of bounce-back effect

As discussed in the previous Section, since the BBF model nests the BBU, BBV, BBD as well as the no bounce-back MS model (Hamilton), it is possible to formally test whether there is a bounce-back effect and the shape of it. The advantage is that there is no nuisance parameter issue, so that the conventional LR test applies. The autoregressive lag parameter p of the generic BBF model is chosen as the smallest integer value such that there is no serial correlation in the estimated residuals, which leads to retain $p = 0$ in all cases. Then, as noticed from the descriptive statistics reported in Table 3, the choice of a quite liberal value of seven months for the bounce-back duration parameter m should guarantee that the whole rebound effect is well captured by the bounce-back function in Equation (3). Table 5 reports the log-likelihood of the BBF model and the LR test statistics corresponding to the restrictions H_0^H, H_0^U, H_0^V and H_0^D, which are described in Section 2.

First, it is worth noticing that these tests results provide support in favor of the presence of a bounce-back effect following a bear market. Actually, the LR tests of H_0^H, i.e. the standard Hamilton model without bounce-back effect, against the BBF alternative, do reject the null in the U.S., Canada, France and the UK at the 5%-level. By contrast, this model is not rejected in Germany at conventional levels. Then, based on the LR statistics of H_0^U, H_0^V and H_0^D, it turns out that none of these models is ever rejected at the 5%-level. Hence, the subsequent analysis will be carried out using the model which maximizes the log-likelihood, i.e. the BBU model for Canada and the United Kingdom, the BBV for the United States and

Table 5: Tests for presence and shape of BB effect

	US	CA	FR	GE	UK
			H_a : BBF		
Log-Lik	-1467.96	-1505.59	-1619.09	-1593.17	-1553.89
			H_0^H : Hamilton		
Log-Lik	-1480.94	-1509.75	-1625.49	-1594.68	-1557.63
$LR-stat$	25.96	8.32	12.80	3.02	7.48
$(p-value)$	(0.00)	(0.04)	(0.00)	(0.39)	(0.05)
			H_0^U: BBU		
Log-Lik	-1470.79	-1505.60	-1620.48	-1593.46	-1554.67
$LR-stat$	5.66	0.02	2.78	0.58	1.56
$(p-value)$	(0.06)	(0.99)	(0.25)	(0.75)	(0.46)
			H_0^V: BBV		
Log-Lik	-1468.96	-1506.15	-1620.33	-1593.92	-1555.72
$LR-stat$	2.00	1.12	2.48	1.50	3.66
$(p-value)$	(0.37)	(0.57)	(0.29)	(0.47)	(0.16)
			H_0^D: BBD		
Log-Lik	-1470.06	-1507.29	-1619.77	-1594.65	-1555.94
$LR-stat$	4.20	3.4	1.36	2.96	4.04
$(p-value)$	(0.12)	(0.18)	(0.51)	(0.23)	(0.13)

the BBD model for France. For Germany, where the presence of any bounce-back is questioned by these tests, we will consider both Hamilton and BBU models. Indeed, the null of no bounce-back effect is not rejected against the BBU in this German case, but at the 12% only.

4.3 Estimates of bounce-back augmented MS Models

Estimation results for the bounce-back augmented MS models selected in the previous paragraph are reported in Table 6 for the period 1970M1-2012M12.

Table 6: Estimates of nonlinear models

		γ_0	γ_1	σ_0	σ_1	p_{00}	p_{11}	λ	ARCH(12)	JB test	Q(12)
US	BBV	0.65 (0.23)	−3.07 (1.00)	3.24 (0.18)	6.18 (0.51)	0.95 (0.02)	0.82 (0.07)	−0.24 (0.08)	14.77 (0.25)	2.93 (0.23)	9.84 (0.63)
CA	BBU	0.79 (0.20)	−5.26 (1.61)	3.67 (0.15)	8.18 (0.73)	0.97 (0.01)	0.85 (0.05)	−0.10 (0.03)	8.96 (0.71)	0.91 (0.63)	12.16 (0.43)
FR	BBD	0.86 (0.29)	−7.76 (1.76)	4.78 (0.22)	7.78 (0.81)	0.96 (0.02)	0.78 (0.08)	−0.06 (0.02)	21.08 (0.05)	4.04 (0.13)	19.09 (0.09)
GE	H	0.92 (0.24)	−0.72 (0.69)	3.90 (0.21)	8.28 (0.59)	0.96 (0.01)	0.93 (0.03)		5.63 (0.93)	5.22 (0.07)	10.30 (0.59)
	BBU	0.73 (0.25)	−7.72 (2.65)	4.47 (0.20)	9.94 (1.06)	0.97 (0.01)	0.75 (0.10)	−0.08 (0.03)	18.15 (0.11)	2.25 (0.32)	17.88 (0.12)
UK	BBU	0.71 (0.21)	−5.38 (2.03)	4.17 (0.17)	11.40 (1.40)	0.98 (0.01)	0.83 (0.07)	−0.10 (0.04)	10.66 (0.56)	0.73 (0.70)	14.56 (0.27)

The entries into parenthesis are the standard errors. For ARCH and JB test, p-values are into parenthesis.

The U.S. As can be seen from the first line of Table 6, the bounce-back parameter λ is significantly negative at 5%-level, with a value of -0.24, which provides further support to the bounce-back augmented MS model for the US stock returns. It is also worth noting that the residuals variance is almost twice as large in the bear market (regime 1 with a large negative $\hat{\gamma}_1$ value) than in the bull regime: $\hat{\sigma}_1$=6.18 while $\hat{\sigma}_0$= 3.24. The smoothed probability of bear market is plotted in Figure 3

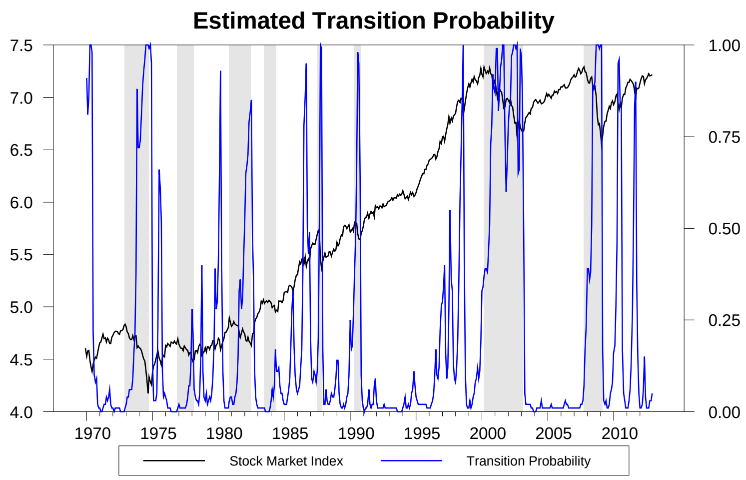


Figure 3: U.S. smoothed probability of S_t and bear market of BB dating algorithm

together with the bear market dates in shaded areas. Even though there are some wrong signals, Figure 3 reveals a strong correspondence between this smoothed probability and the bear market dates. For six out of eight bear markets in the sample, the smoothed probability decreases dramatically from around 1 to 0 after the trough date. The 1976M12 and 1983M6 bear markets are the exceptions: they are hardly detected by the model as the smoothed probability drops close to zero, but from 0.25 or less. Then, for seven out of eight bear markets, the smoothed probability increases sharply after the peak date. The transition probabilities also confirm that bull market are more persistent than bear market. More specifically, the estimated average length of a bear market, given by $1/(1 - \hat{p}_{11})$, is 5 months, while the estimated average length of a bull market is 20 months.

International Evidence From the results in Table 6, it can be seen that on the whole, the parameters estimates of the bounce-back augmented MS models have the same order of magnitude in the remaining countries. For instance, γ_0 typically lies between 0.71 and 0.92 while γ_1 ranks from -7.76 to -5.26. The residuals standard deviation in the bear market regime is two to three times as large as in the bull market regime. The estimated duration of the bear market regime is also very homogeneous across countries since it goes from four months in the German case (BBU model) to seven months for Canada. Finally, the bounce back parameter estimates lies between -0.06 for the French BBD model and -0.10 for the Canadian and UK BBU models.Since it is significantly negative in all these cases, it provides further support to the presence of a bounce-back effect in stock market returns.

4.4 Estimated shape of the bounce-back effects

The typical shape of the bounce-back effect obtained from the BBV model's estimates for the US data is plotted in Figure 4 below. The solid line represents the shape of stock market return taking the bounce-back effect into account, while the dotted line corresponds to the case without bounce back effect, i.e. Hamilton's model. In the latter, the function is simply given by γ_0 when $S_t = 0$ and by $\gamma_0 + \gamma_1$ when $S_t = 1$. Similarly, the Canadian, French, German and UK analogues are plotted in Figure 5. The main difference between the BBV model and the

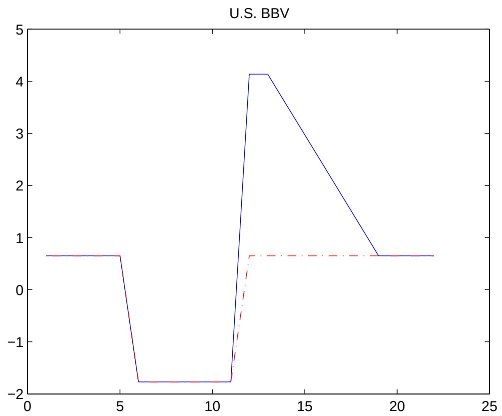
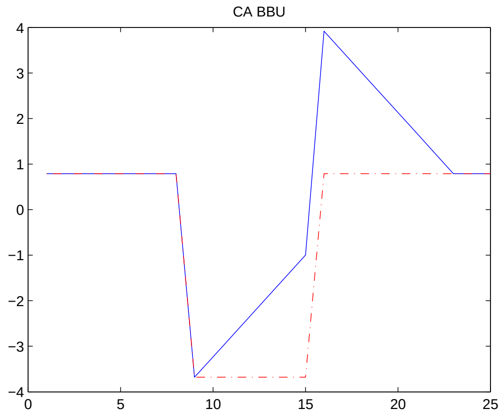


Figure 4: Estimated bounce-back function: The US BBV model

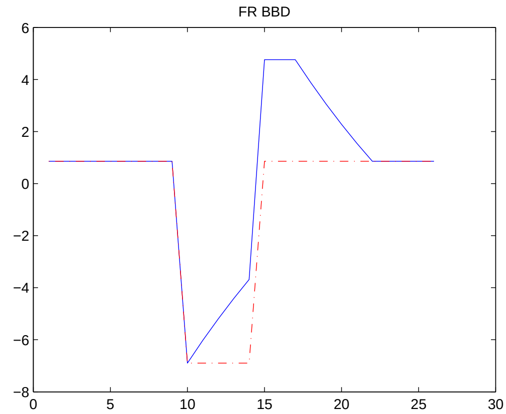
other bounce-back functions, such as the BBU or BBD ones, lies in the timing of the activation of the rebound: the bounce-back effect is triggered as soon as one month after the process has entered the bear market in the BBU and BBD model, whereas in the BBV model, it becomes active only when the returns are back in the bull market. Probably due to the "depth" nature of the recoveries in the model retained for France, this is the country which exhibits the largest monthly returns at the beginning of bull markets.

4.5 Permanent impact of a bear market

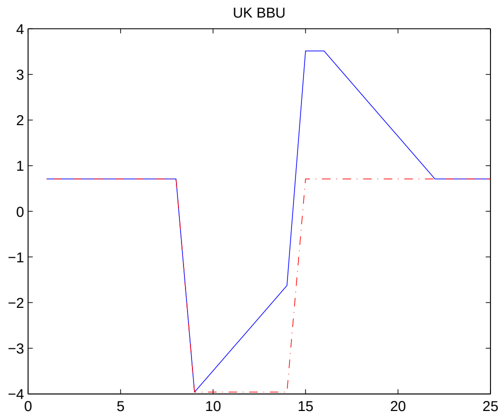
In order to evaluate the average permanent impact of a bear market on the stock index price, we follow Hamilton [1989]'s suggestion and measure it as the expected



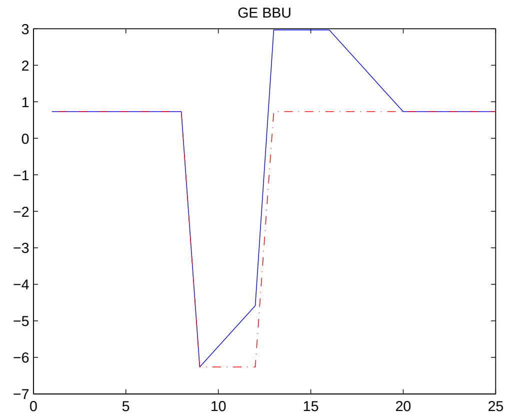
(a) Canada



(b) France



(c) UK



(d) Germany

Figure 5: Estimated bounce-back functions

difference in the long run stock index price given that the stock market is currently in the bear regime versus in the bull regime, which may be written as:

$$\lim_{j\to\infty} E[y_{t+j}|S_t = 1, \Omega_{t-1}] - E[y_{t+j}|S_t = 0, \Omega_{t-1}], \qquad (5)$$

where $\Omega_{t-1} = S_{t-1} = 0, S_{t-2} = 0, ...; y_{t-1}, y_{t-2}, ...$. Unfortunately, when the bounce-back functions depend on past returns as in the French case for instance, this limit has no trivial closed form solution. Nevertheless, as stressed by Bec et al. [2011], the area defined in Figure 5 by the difference between the bounce-back function and the horizontal line corresponding to γ_0, i.e. the average returns in bull market, provides a natural measure of the accumulated or permanent impact of a bear market on the stock market index. The main difference between this measure and the one suggested by Hamilton [1989] is that the latter does not constrain the S_t paths to be the same in $E[y_{t+j}|S_t = 1, \Omega_{t-1}]$ and $E[y_{t+j}|S_t = 0, \Omega_{t-1}]$ after the bear market of the first expectation term is over. Hence, starting from the bull market return and after time-t bear market is over, the expected stock return will not tend towards the bull market return γ_0. Instead, it will reach a weighted average of the bull and bear market returns. In Figure 5, it would lie somewhere between the first value and the minimum value of the bounce-back curves. As a result, the magnitude of Hamilton's measure of permanent loss should be slightly lesser than ours: put in other words, it would tend to over-estimate the bounce-back magnitude once S_t goes back to zero. Here, we retain the Bec et al. [2011] measure since we also think that an important intrinsic property of this two-regime MS class of models is to allow for the return to go back to its bull market average after the end of the bear market. To this end, we compute numerically the limit given in Equation (5) conditionally to the assumption that once the bear market initiated at time t in the first term is over, i.e. at time t plus the estimated average bear market duration, both expectation terms involved in this limit are governed by the same path for the state variable. The resulting values are reported in the first line of Table 7. By contrast with what seems to happen in the other

Table 7: Permanent impact evaluation

λ	US	France	UK	Canada	Germany
$\lambda = \hat{\lambda}$	5.37%	-7.20%	-5.42%	-5.51%	-10.06%
$\lambda = 0$	-14.52%	-34.50%	-28.02%	-31.29%	-27.96%

countries, a bear market in the US will typically result in a slight permanent gain of 5.37% in the stock market index. In Figure 4, this means that the area between the bounce-back function and γ_0 is larger for $S_t = 0$ than for $S_t = 1$. So, in this case, the rebound of the markets returns in the months following the end of a bear market is strong enough to more than offset the initial drop. Regarding real economic activity, some authors as e.g. Caballero and Hammour [1994] or Aghion and Saint Paul [1998] conclude that the cleansing effect of recessions has a permanent positive impact on output. This could in turn translate into a higher stock market price. However, this result is specific to the US case since all the other countries considered here exhibit slight permanent losses instead. These losses rank from 5.42% in the UK to 10.06% in Germany if we consider the BBU model for this latter case. It is worth noticing that without any bounce back effect, i.e. with $\lambda = 0$ as in the second line of Table 7, all the countries would experience a large permanent loss: around 30% for France, the UK, Canada and Germany and 15% in the US.

5 Conclusion

This paper applies the bounce-back augmented class of Markov-Switching model to identify bull and bear markets and measure any potential bounce-back effect from monthly stock returns data for several developed countries since the early seventies. This model provides a realistic identification of bull and bear markets which matches correctly the outcome of the dating algorithm. Focusing on a potential bounce-back effect in financial markets, its presence and shape are formally tested. Our results show that *i)* the bounce-back effect is statistically significant and large in all countries, but Germany where evidence is less clear-cut and *ii)* the negative permanent impact of bear markets on the stock price index is notably reduced (or even more than offset in the US) when the rebound is explicitly taken into account.

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