Overinvestment in Marriage-Specific Capital

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Abstract

We consider the decisions of a married couple in a risky environment. The distribution of spouses’ bargaining power may change as a consequence of new outside opportunities that are offered to them, so that individual consumption may fluctuate over time. This is what we call "bargaining risk". To reduce this risk, the spouses may decide to over-invest in marriage-specific capital (which, by definition, is completely lost in the case of divorce) and thereby limit the attractiveness of spouses’ outside opportunities. This strategy is shown to be optimal. More surprisingly, over-investment in marriage-specific capital is still an optimal strategy when spouses are confronted to a (small) risk of divorce. This contrasts with the usual intuition.

Key-Words: Marriage, Investment, Durable Goods, Specific Capital, Risk

JEL-Code: D13, D91, J12

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1 Introduction

In married couples, some investments are specific to the relationship in the sense that they are much less valuable for spouses if marriage dissolves. Hence these investments increase the gains to both individuals of continuing the relationship and play an important role to explain the duration of marriage (Becker, 1974, 1991; Ben-Porath, 1980; Pollak, 1985). For instance, Becker, Landes, and Michael (1977) point out that children enhance the value of the relationship and may decrease the chance that the relationship ends.\(^1\) Conversely, the possibility of dissolution also discourages the accumulation of marriage-specific goods (Landes, 1978; Johnson and Skinner, 1986; Peters, 1986; Lommerud, 1988; Lundberg and Rose, 1999; Stevenson, 2007).\(^2\)

Our claim in the present paper is that, in a uncertainty context with risk-averse spouses, the decision process of the household may lead to over-investment in marriage-specific capital (defined as a larger level of investment than the optimal level observed when consumption profile is determined by binding contract at the beginning of the planning horizon). Investment may even be encouraged after a small, exogenous increase in the risk of divorce. To obtain this counterintuitive result, we construct a two-period model of household behavior, in which the decision process is described as a Rubinstein-Binmore bargaining game, and we suppose that spouses make decisions about the consumption of a private good and the consumption of a public good. The public good is durable (over two periods) and

\(^1\)This issue is addressed by several studies in sociology (Huber and Spitze, 1980; Waite and Lillard, 1991; Brüderl and Kalter, 2001, for instance).

\(^2\)The possibility of underinvestment in the context of the firm is well documented in the holdup literature. The intuition dates back to Williamson (1975) and Klein, Crawford and Alchian (1978); it was formalized by Grout (1984); for these models, see the surveys by Hart and Holmstrom (1988) and Malcomson (1997). Crawford (1988, 1990) and Tirole (1986) show that overinvestment may sometimes arise from the multiplicity of equilibriums. Rotemberg and Saloner (1987) and Mutthoo (1998) present some particular situations where overinvestment is the rule. The problem of the couple differs from the traditional holdup problem because (i) the decision of investment in marriage-specific capital is jointly made by both spouses, (ii) divorce is the result of exogenous shocks that affect the surplus of marriage. Underinvestment is, however, supposed to be the rule as underlined by the papers cited above.
specific to the relationship, that is, its value decreases in the case of divorce. We then suppose that, in general, a couple is not able to credibly commit to a fair division of future consumption since any current agreement can be renegotiated. The future financial situation of spouses, which will influence the intra-household balance of power, cannot be predicted at the moment of the marriage. If the state of nature turns to be markedly favorable to the husband (say), the latter can be inclined to take advantage of the situation and renege on the agreement made with his wife. The fluctuations in consumption which result from variations in spouses’ bargaining power is what we call "bargaining risk" hereafter. This form of risk may persist even if, at the end, divorce never takes place.

We consider two versions of the model. In the first version, the possibility of divorce is excluded. The spouses’ relationship continues because the surplus from marriage generated by the marriage-specific good is positive and constant during the two periods. Even in that case, however, the bargaining risk tends to decrease intertemporal utility of individuals living in multi-person households (so far as they are averse to risk, of course). An informal system of insurance then consists in investing more in marriage-specific capital, which will indeed reduce the attractiveness of spouses’ outside opportunities, so that the fluctuations in bargaining power will be reduced as well. We prove that this strategy is optimal. The corollary is that the variance of individual consumption is less important in high-income households than in low-income households. In the second version of the model, the possibility of divorce is included. In the course of the marriage, new informations are received by spouses that modify their subjective evaluation of the surplus from marriage. A large, negative shock on the marriage surplus may, ultimately, lead spouses to the dissolution of the couple (which entails the complete loss of the investment in marriage-specific capital). We then show, quite surprisingly, that overinvestment in marriage-specific capital may even be larger in this version of the model – at least when the risk of divorce remains moderate – than in the first version. This result is explained by the fact that investment in marriage-specific capital is more profitable in terms of reduction of the bargaining risk when spouses

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3The consequences of the non-existence of enforceable intertemporal contracts for couples are examined in many models (Konrad and Lommerud, 2000; Lundberg, 2002; Rainer, 2007; Wells and Maher, 1998). In these models, under-investment in marriage-specific capital is the rule.
are facing small, negative shocks that does not necessarily lead to divorce.\footnote{It is obvious that the results that follows can be used to describe any form of partnership, for instance, a small group of highly-specialized workers or a duopole, as far as (i) investments in relationship-specific capital is involved and (ii) opportunistic behavior cannot be prevented.}

The paper is structured as follows. The main assumptions on preferences, the form of uncertainty and the decision process are presented in section 2. Over-investment in marriage-specific capital in a non-divorce model is discussed in section 3. The results are generalized to a divorce context in section 4. The last section concludes.

\section{The Model of the Household}

\subsection{Goods, Preferences and Uncertainty}

In this section, we shall present the main assumptions of the model. We first describe the utility functions and then introduce uncertainty.

To fix ideas and simplify notation, we consider a two-person household in a two-period setting. During the first period, the spouses make decisions about the optimal levels of consumption of a private good and a public good. During the second period, the spouses spend their resources on the sole private good. The public good has two features: (i) its consumption can be made during two periods, i.e., the good is durable, (ii) its consumption is specific to the marriage, i.e., the good is totally lost in the case of divorce.\footnote{The public good can alternatively be seen as produced with a linear technology using money input. Lommerud (1989) and Lundberg (2002), for example, suppose that the public good is produced from a technology using spouses’ time inputs. The spouses’ time devoted to domestic chores in the first period enhances the productivity in the second period. In that case, the increase in household productivity is largely specific to the relationship because the public good is less valuable in case of divorce.} This good can typically be interpreted as a non-divisible marketable capital good (such as the spouses’ house at least if transaction costs are large) or a non-divisible non-marketable capital good (such as children or love). Let $x_{it}$ denote the individual consumption of the private good of spouse $i$ ($i = 1, 2$) at period $t$ ($t = 1, 2$), and $X$ the consumption of the public good (i.e., the marriage-specific capital).
The spouses are characterized by identical, intertemporally additive utility functions, and the discount factor of the second period is equal to one (i.e., there is no time impatience). Our argument would be more complicated but not significantly altered if spouses’ preferences were different.\textsuperscript{6} We thus suppose the following.

**Assumption A1.** The utility functions at each period are of the Von-Neuman-Morgenstern form with an additive structure, that is,

\[ U = u(x_{it}) + v(X), \]  

where \( v(\cdot) \) is a two times differentiable function that satisfies

\[ v'(X) > 0, \quad v''(X) \leq 0, \quad \text{and} \quad v(0) = 0, \]

and \( u(\cdot) \) is a three times differentiable function that satisfies

\[ u'(x_{it}) > 0, \quad u''(x_{it}) < 0, \quad u'''(x_{it})0, \quad \text{and} \quad u'(0) = \infty. \]

Because of the sign of the derivatives, the spouses can be said to be risk-averse and prudent.\textsuperscript{7} In the remainder of this paper, we shall consider additional restrictions on these utility functions.

The household as a whole receives an exogenous income, denoted by \( Y_t \), at each period \( t \). The amount of these incomes is non-stochastic and completely determined at the beginning of the first period. If the price of all the goods is set to one, the budget constraint of the first period is thus equal to \( x_{11} + x_{21} + X = Y_1 \), and the budget constraint of the second period is equal to \( x_{12} + x_{22} = Y_2 \). The household income of the second period can be broken down into individual incomes, that is, \( Y_2 = y_1 + y_2 \), where \( y_i \) is the exogenous personal income of spouse \( i \). The distribution of the individual incomes between spouses is stochastic and such that

\[ y_1 = \frac{Y_2}{2} - \Sigma \varepsilon, \quad y_2 = \frac{Y_2}{2} + \Sigma \varepsilon \]

\textsuperscript{6}Browning (1996, 2000) suggests that the discount rate of future may be larger for husbands than for wives.

\textsuperscript{7}Even if the utility functions have two arguments, this interpretation is well founded because, as it will be shown hereafter, only individual consumption at the second period is stochastic.
where $\varepsilon$ is a random term which follows a symmetric distribution with support $[-\frac{1}{2}, +\frac{1}{2}]$ and $0 < \Sigma \leq Y_2$ is a parameter of dispersion. At the end of the first period, each spouse is informed of what she or he will receive as individual income $y_i$ for the second period. The distribution of these individual incomes is thus the sole source of uncertainty for the moment (since the sum of individual incomes is deterministic). This form of risk is purely idiosyncratic and can be eliminated by an efficient system of insurance between spouses. The fact that there does not exist marital contracts legally enforceable is exactly at the core of this paper.

To guarantee that solutions are interior, we shall assume the following.

**Assumption A2.** The utility functions and the household income are such that, at the equilibrium, the optimal level of investment in marriage specific capital is positive.

### 2.2 The Sharing of Private Consumption

In this subsection, we shall examine how the spouses divide private consumption between them conditionally on the decisions they made about the investment in marriage-specific capital.

The private consumption is shared between spouses according to some rule that depends on the household environment. Since the environment that we consider is initially symmetrical (same utility functions and same anticipations for both spouses), it is natural to suppose that the first period household income is equally divided between spouses. The level of utility obtained by each spouse in the first period is then given by

$$U_1 = u \left( \frac{Y_1 - X}{2} \right) + v(X). \quad (3)$$

This assumption, if plausible, requires that, at the moment of the marriage, the partners have approximately the same outside opportunities.

The specification of the sharing of the second period household income is more complicated. The sharing rule will generally be a function of the respective individual incomes that spouses observe at the end of the first period. The main idea
of our approach is inspired by the bargaining models à la Rubinstein-Binmore\(^8\) where outside opportunities are given here by the level of utility obtained in the case of divorce.\(^9\) In other words, the distribution of consumption is subject to the constraint that the spouses obtain at least the level of utility of divorce. If they decide to divorce, the spouses give up the marriage surplus. The level of utility of each spouse is then equal to \(u(y_i)\). The dissolution of the couple is necessarily inefficient because of the loss of the marriage surplus. Therefore, the spouses can always bargain and redistribute the gains of marriage in such a way that divorce never occurs.\(^{10}\) The participation constraints involve that spouses receive in marriage at least what they would obtain in divorce. We consider two regimes.

(i) If both participation constraints are not binding, the second period household income will be divided in equal shares, and the level of utility of each spouse at the second period will simply be equal to

\[
U_2 = u\left(\frac{Y_2}{2}\right) + v(X). \tag{4}
\]

(ii) If the participation constraint of spouse \(i\) is binding, she will obtain the reservation level of utility obtained from divorce, that is,

\[
U_2 = u(y_i),
\]

and spouse \(j\) will obtain a level of utility inferior to that given by equation (4) which will be precisely defined hereafter.

\(^{8}\)See Binmore, Rubinstein and Wolinsky (1986) for a pedagogical introduction.

\(^{9}\)For most of them, the bargaining models of household behavior suppose that the decision process can be represented by the Nash solution with threat points defined by the utility of divorce (Manser and Brown, 1981; McElroy and Horney, 1981). This is unsatisfactory, however, because the Nash solution is not consistent with the idea we support here that a symmetric increase in the cost of divorce redistributes welfare between spouses and, generally, lead to a more egalitarian equilibrium. The application of the Rubinstein-Binmore bargaining model to the household context, discussed by Muthoo (1999), has been initially proposed by Bergstrom (1997). Our specification is also similar to the bargaining model of Adam, Hoddinott, and Ligon (2004).

\(^{10}\)This is a traditional application of the Coase Theorem. The possibility of divorce will be introduced in the next section.
The participation constraint of spouse $i$ will be binding if the realized value of the individual income $y_i$ is above a reservation value $y^*$ implicitly defined by

$$u\left(\frac{Y_2}{2}\right) + v(X) = u(y^*).$$  \hspace{1cm} (5)$$

The reservation value is thus the level of individual income for which spouse $i$ is indifferent between remaining married (with an equal sharing of the second-period income) and divorcing. This value is the same for both spouses because of the symmetry of the framework. Hence the solution to equation (5) can be denoted as: $y^* = y^*(Y_2, X)$. This function is always greater than or equal to $Y_2/2$ because $v(X) \geq 0$; it increases when $X$ increases.

The spouse $i$ will be in position of demanding a greater share of private consumption if the individual income she or he receives at the second period is greater than the reservation value $y^*$ defined above. The function $\eta_i$ represents thus the smallest transfer received by spouse $i$ such that she accepts to remain in the marriage. It is formally defined by

$$u(y_i) = u(\eta_i) + v(X),$$ \hspace{1cm} (6)$$

so that the level of utility is the same when she lives with her partner, benefiting from the marriage-specific capital, and when she lives as single. This equation has a unique solution for any $y_i y^*$ which is denoted by

$$\eta_i = \eta(y_i, X).$$

This function can be shown to be everywhere comprised between $Y_2/2$ and $y_i$ and satisfy:

$$0 < \frac{\partial \eta_i}{\partial y_i} < 1, \hspace{0.5cm} \frac{\partial \eta_i}{\partial X} < 0.$$  

That is, an increase in the level of marriage-specific capital and a decrease in individual income have a negative impact on what can be demanded by the spouse with a credible opportunity of leaving. It can also be formally proved using the implicit function theorem that $\partial^2 \eta_i/\partial y_i^2 < 0$ and $\partial^2 \eta_i/\partial X^2 > 0$. Moreover, the following identity must always be satisfied:

$$\eta_i(y^*, X) = \frac{Y_2}{2}. \hspace{1cm} (7)$$
Finally, if the participation constraint of spouse $i$ is binding, the partner of spouse $i$ will obtain what is left, $Y_2 - \eta(y_i, X)$, that is, the total resources from which the share of total consumption of spouse $i$ is substracted.

**Remark.** In the model, the equal sharing of total income is the rule as long as $y_i$ remains below its reservation value $y^*$. The property of income pooling – according to which only the sum of spouses’ individual incomes, and not its distribution among spouses, matters for explaining household consumption decisions (Phipps and Burton, 1998) – is thus satisfied if the level of marriage-specific capital is sufficiently large. This theoretical prediction is supported by the empirical observation made by Bonke and Uldall-Poulsen (2007) with Danish data that the property of income pooling is relatively more frequently satisfied by couples who have children and who are house-owner than by other couples. Therefore the present model, contrary to more traditional Nash bargaining models of household behavior, provides a theoretical justification of this important and intuitive observation.

### 2.3 The Full-Commitment Model

To make a comparison, we first consider the case where the spouses are able to commit to an allocation of resources for the future. In other words, idiosyncratic risks are completely eliminated by efficient risk sharing. In that case, the level of consumption assured to both spouses in the second period – whatever the initial distribution of individual incomes may be – will be equal to $Y_2/2$. The choice of the intertemporal allocation of resources is simply determined by the following optimization problem:

$$\max_{X \geq 0} \left[ u\left( \frac{Y_1 - X}{2} \right) + v(X) \right] + \left[ u\left( \frac{Y_2}{2} \right) + v(X) \right].$$

The first order condition for an interior solution can be written as:

$$\frac{1}{2} u'\left( \frac{Y_1 - X}{2} \right) = 2v'(X).$$

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Lommerud (1989) supports the idea that, in a couple where spouses are strongly emotionally attached to each other, a full-commitment outcome can be enforced by the simple fact that spouses take care of their reputation. This is what this author calls “voice enforcement”.

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9
The solution to this equation is denoted by $X_1$. The full-commitment model and its optimal solution $X_1$ will be used as a benchmark.\footnote{The full-commitment model is also valid if the dispersion of the individual incomes is so limited that the threat of divorce is never credible.}

3 Investment in Marriage-Specific Capital and Bargaining Risk

3.1 The Expected Utility

In this section, we shall suppose that the spouses are not able to make binding contracts and then examine the household equilibrium. First, the participation constraints are binding if and only if the individual income of one spouse is above the reservation value, that is,

$$
\frac{Y_2}{2} + \Sigma \epsilon > y^*(X), \quad \text{with} \quad \epsilon \in \left[ -\frac{1}{2}, +\frac{1}{2} \right],
$$

from equation (2). The density distribution function of $\epsilon$ is symmetric and denoted by $\phi(\epsilon)$, the support of which is $[-\frac{1}{2}, +\frac{1}{2}]$. The (conditional) expected utility function of each spouse at the second period is obtained by integrating $\epsilon$ over its domain. The expected utility function for spouse $i$ (say) can be expressed, using a convenient change of variable, as follows:

$$
E(U_2|X) = u \left( \frac{Y_2}{2} \right) \times \frac{2}{\Sigma} \int_{\frac{y^*}{2}}^{y^*} f(t) \cdot dt \\
+ \frac{1}{\Sigma} \int_{\frac{y^*}{2}}^{\frac{Y_2 + \Sigma}{2}} [u(\eta(t)) + u(Y_2 - \eta(t))] \cdot f(t) \cdot dt + v(X)
$$

if $y^* \leq \frac{Y_2 + \Sigma}{2}$

$$
= u \left( \frac{Y_2}{2} \right) + v(X)
$$

otherwise. \tag{10}

where, to keep notation as simple as possible, only the first argument of the sharing rule is made explicit and

$$
f(t) = \phi \left( \frac{2t - Y_2}{2\Sigma} \right).
$$
The first integral on the right-hand side represents the contribution to the expected utility when the two participation constraints are non-binding; the second integral represents the contribution when the participation constraint of one spouse is binding.

### 3.2 The Variance of Individual Consumption

Before computing the spouses’ marginal expected utility for the second period, we shall compute the variance of individual consumptions which will be used hereafter. If we now use the fact that the expected individual consumption for each spouse is equal to \( E(x_i) = \frac{Y_2}{2} \) because of the symmetry of the optimization problem, and compute the variance of the individual consumption for each spouse, we obtain (with a convenient change of variable):

\[
\text{var}(x_i|X) = \frac{2}{\Sigma} \int_{y_i}^{\frac{Y_2 + \Sigma}{2}} \left( \frac{Y_2}{2} - \frac{Y_2}{2} \right)^2 \cdot f(t) \cdot dt \quad \text{if } y^* \leq \frac{Y_2 + \Sigma}{2},
\]

Thus the dispersion of the private consumption of the second period depends on the deviation between the sharing rule and the equal sharing of total income. This expression represents what we call "bargaining risk". Since the sharing rule \( \eta \) is concave with respect to \( y_i \), we also have that, if \( X > 0 \), \( \text{var}(x_i|X) < \text{var}(y_i) \), that is, the bargaining risk is smaller than the income risk from which it results. This illustrates the natural idea (Ben-Porath (1980), Pollak (1985), Chiappori and Reny (2006), Halla and Scharler (2012) for instance) that the formation of a household can be used as a system of insurance against adverse economic events. If we now differentiate this expression with respect to \( X \), use the Leibniz Rule and equation (7), we obtain:

\[
\frac{\partial \text{var}(x_i|X)}{\partial X} = \frac{4}{\Sigma} \int_{y_i}^{\frac{Y_2 + \Sigma}{2}} \left( \frac{Y_2}{2} - \frac{Y_2}{2} \right) \cdot \frac{\partial \eta(t)}{\partial X} \cdot f(t) \cdot dt \quad \text{if } y^* \leq \frac{Y_2 + \Sigma}{2}. \tag{11}
\]

The derivative of the variance of individual consumption with respect to the level of marriage-specific capital is negative since \( \frac{\partial \eta}{\partial X} < 0 \). The intuition is that investment makes spouses’ outside opportunities less attractive and reduces the variance of individual private consumption.
3.3 The Marginal Expected Utility

In this subsection, we shall compute the marginal utility of investment in marriage-specific capital.

As shown in the appendix A, if we differentiate expression (10) with respect to $X$, we obtain a decomposition of the marginal utility of marriage-specific capital as follows,

$$\frac{\partial E(U_2|X)}{\partial X} = C(X) + I(X). \quad (12)$$

The first term on the right-hand-side simply represents the consumption motive of marriage-specific capital. This term is equal to

$$C(X) = v'(X) > 0. \quad (13)$$

The second term represents the insurance motive of marriage-specific capital. We can write it as:

$$I(X) = \frac{1}{2} u'' \left( \frac{Y_2}{2} \right) \frac{\partial \text{var}(x_i|X)}{\partial X} + \text{remainder} \geq 0, \quad (14)$$

where the remainder is a fourth order term. The insurance motive is strictly positive if $y^* < (Y_2 + \Sigma)/2$ at the equilibrium. Everything else being equal, if the level of marriage-specific capital increases, the share of consumption that the spouse with a credible threat of divorce can demand will be reduced. This effect is proportionate to a measure of the concavity of the utility function and to the effect of marriage-specific capital on the dispersion of consumption.

3.4 The Partial Commitment Model

Given that marital contracts are generally not enforceable through the legal system, the partial-commitment set-up we examine now is certainly more credible than the full-commitment case discussed above. The spouses choose the level of marriage-specific capital in order to maximize their expected utility. The optimization problem of each spouse is then as follows:

$$\max_{X \geq 0} \left[ u \left( \frac{Y_1 - X}{2} \right) + v(X) \right] + E(U_2|X). \quad (15)$$
Using expressions (13) and (14), and applying simple transformations, the first order condition for an interior solution is given by:

\[
\frac{1}{2}v' \left( \frac{Y_1 - X}{2} \right) = 2v'(X) + I(X)
\]  

(16)

The solutions to this equation is denoted by \(X_2\). The first order condition here can now be compared with the first order condition obtained in the full commitment case, which directly gives the following result.

**Proposition 1.** Assume A1-A2 and assume that \((Y_2 + \Sigma)/2 > y^*(Y_2, X_2)\). Then the level of investment in marriage-specific capital will be greater in the bargaining model than in the full commitment model, that is, \(X_2 > X_1\).

**Proof.** This result is immediate from the comparison of first order conditions (9) and (16) because \(I > 0\) provided that \((Y_2 + \Sigma)/2 > y^*(Y_2, X_2)\), and the sub-utility functions \(u(\cdot)\) and \(v(\cdot)\) are concave.

The intuition is elementary. Confronted to bargaining risk, the risk-averse spouses will overinvest in marriage-specific capital to reduce the fluctuations in future individual consumption. The condition used in the proposition implies that the participation conditions will be binding for some values of individual incomes.

### 3.5 Income and the Variance of Individual Consumption

In this subsection, we shall examine how the decisions of investment in marriage-specific capital are affected by variations in the first period income of the household. The result is summarized in the next proposition.

**Proposition 2.** Assume A1-A2 and assume that \((Y_2 + \Sigma)/2 > y^*(Y_2, X_2)\). Then the level of investment in marriage-specific capital increases when the first period income increases.

**Proof.** If we differentiate the first order condition (16) with respect to \(Y_1\), we obtain:

\[
\left[ 2v''(X) + \frac{1}{4}u'' \left( \frac{Y_1 - X}{2} \right) + \frac{\partial I(X)}{\partial X} \right] \cdot dX = \frac{1}{4}u'' \left( \frac{Y_1 - X}{2} \right) \cdot dY_1
\]
It is shown in the appendix B that $\partial I / \partial X < 0$. Hence, the term in squared brackets is negative and $\partial X / \partial Y_1 > 0$.

The effect of the second period income on the level of investment in marriage-specific capital is generally ambiguous. The above proposition has an important corollary that characterizes the intra-household distribution of consumption at the second period.

**Corollary 3.** Assume A1-A2 and assume that $(Y_2 + \Sigma)/2 > y^*(Y_2, X_2)$. Then the variance $\text{var}(x_i)$ of individual consumption decreases when the first period income increases.

**Proof.** From proposition 2, an increase in the first period income has a positive impact on the level of marriage-specific capital. From equation (11), then, the variance of individual consumptions decreases.

This result implies, all other things being the same, that the distribution of individual consumption, primarily for older couples (once first period investments are made), will tend to be more egalitarian, or at least more stable, in high-income households than in low-income households. What is relevant here is the income of the beginning of spouses’ life-cycle because it determines the initial level of investments. An increase in income that is unanticipated by spouses should not influence the variance of consumption at the end of the life-cycle. One interpretation is that *intra-household equality is a normal good, the consumption of which increases with the household income.* With a completely different model, Haddad and Kanbur (1992) and Kanbur and Haddad (1994) obtain a similar result and conclude that economic development should ultimately lead to a decrease in intrahousehold inequality. The same conclusion can be drawn here.

## 4 Investment in Marriage-Specific Capital and Divorce Risk

The conclusions derived above can radically change, in principle, if the spouses run the risk of divorce. In this section, we shall incorporate this risk and examine how the household equilibrium is modified.
4.1 The Risk of Divorce

The traditional view says that the possibility of family dissolution has a disincentive effect on the accumulation of marriage-specific capital because such capital is less valuable after dissolution. To go into this issue, we shall investigate what happens to the main results we derived in the preceding section when the spouses may decide to separate at the end of the first period.

To begin with, let us note that the sole hypotheses placed on the distribution of individual incomes are not sufficient to generate a positive probability of dissolution. To introduce the possibility of divorce in the present model, we suppose that characteristics that influence the gains from marriage change over time in an unpredictable manner (Becker, Landes and Michael, 1977; Weiss and Willis, 1997). The idea is that participants in marriage have limited information about the utility they can expect with potential mates at the time of the formation of the couple. The surprises revealed in the course of the marriage can cause both partners to reconsider their original decision to marry. The probability of divorce is then a function of two factors: the expected gain from marriage represented by investment in marriage-specific capital and the distribution of a variable describing unexpected outcomes. To represent this scheme, we suppose that the level of utility of each spouse at the second period, in the case where the family remains intact, is perturbed by a random term which represents new information, that is,

$$V_2(X, \nu) = \begin{cases} U_2(X) + \Omega \nu & \text{if the couple remains married} \\ U_2(X) & \text{if the couple divorces} \end{cases}$$

where $U_2(X)$ has the same definition as in the preceding section, $V_2(X, \nu)$ is the level of utility after the new information is revealed to spouses, $\nu$ is a random term and $\Omega > 0$ is a constant that can be interpreted as an exogenous tendency to divorce. The density distribution function of $\nu$ is symmetric and denoted by $\varphi(\nu)$, the support of which is $[-\frac{1}{2}, +\frac{1}{2}]$, with the continuity property $\varphi(-1/2) = \varphi(+1/2) = 0$. The new information arriving at the household has thus exactly the same effect on the welfare of both spouses. To simplify, we suppose the following.

**Assumption A3.** The utility functions $v(X)$ are linear, that is, $v(X) = X$.

This assumption is not harmless since it implies that the shock on marriage and
the marriage-capital are perfect substitute. Indeed, the marriage dissolution is the optimal solution \( D = 1 \) if the random term is such that the marriage surplus of the second period is completely swallowed up, that is,

\[
X + \Omega \nu \leq 0.
\]

Otherwise, the resulting decrease in utility due to \( \Omega \nu \) is completely equivalent to an unanticipated variation in the level of marriage-specific good at the second period. In particular, the loss in utility due to the negative shock vanishes if spouses divorce. Thus, the probability of divorce is a function of \( X \) given by

\[
\Pr (D = 1|X) = \begin{cases} 
0 & \text{if } \Omega \leq 2X \\
\int_{-1/2}^{-X/\Omega} \varphi(\nu) \cdot d\nu & \text{if } \Omega \geq 2X 
\end{cases}.
\]

Using the law of iterated expectations, the (conditional) expected utility of each spouse is given by

\[
E(V_2|X) = E(V_2|X, D = 1) \times \Pr (D = 1|X) + E(V_2|X, D = 0) \times \Pr (D = 0|X),
\]

where

\[
E(V_2|X, D = 0) = \frac{\int_{-X/\Omega}^{+1/2} E(U_2|X + \Omega \nu) \cdot \varphi(\nu) \cdot d\nu}{\int_{-1/2}^{+1/2} \varphi(\nu) \cdot d\nu}
\]

is the conditional expected utility given that the couple does not divorce, and

\[
E(V_2|X, D = 1) = \int_{-1/2}^{+1/2} u \left( \frac{Y_2}{2} + \Sigma \varepsilon \right) \cdot \phi(\varepsilon) \cdot d\varepsilon.
\]

is the conditional expected utility given that the couple divorces. Note that, in this latter case,

\[
E(V_2|X, D = 1) = E(U_2|X = 0),
\]

that is, the utility of spouses coincides with the utility they would obtain in an intact family with the level of the marriage-specific good equal to zero (since, in that case, \( \eta_i = y_i \)).
4.2 The Full-Commitment Model with Divorce

As a point of comparison, let us consider again the case where spouses are able to commit to the allocation of future resources. Note that, in spite of the existence of binding agreements, divorce is still the optimal strategy for spouses if $\nu < -2X/\Omega$. Even in the case of the couple’s dissolution, however, the level of private consumption assured to both spouses in the second period must remain constant and equal to $Y_2/2$. The idea is that spouses make a contract with spousal support in the case of divorce such that the level of consumption is unaffected. The choice of the intertemporal allocation of resources is simply determined by the following optimization problem:

$$\max_{X \geq 0} \left[ u \left( \frac{Y_1 - X}{2} \right) + X \right] + u \left( \frac{Y_2}{2} \right) \times \Pr (D = 1|X)$$
$$+ \left( u \left( \frac{Y_2}{2} \right) + X + \Omega E(\nu|D = 0) \right) \times \Pr (D = 0|X) \quad (19)$$

where

$$E(\nu|D = 0) = \frac{\int_{-X/\Omega}^{+1/2} \nu \varphi(\nu) \cdot d\nu}{\int_{-X/\Omega}^{+1/2} \varphi(\nu) \cdot d\nu}$$

is the averaged shock conditional on the continuation of the marriage. The first order condition for an interior solution can be written as:

$$\frac{1}{2} u' \left( \frac{Y_1 - X}{2} \right) = 1 + \int_{-X/\Omega}^{+1/2} \varphi(\nu) \cdot d\nu \leq 2. \quad (20)$$

Noting that $\nu'(X) = 1$ here, the right-hand side of this expression is inferior to that of expression (9). Hence the optimal level of marriage-specific capital in the present model will be necessarily below the full commitment level in the no divorce model. This is exactly the explanation for underinvestment in marriage-specific capital given by Gary Becker and others for a model without uncertainty. The optimal level of marriage-specific capital is denoted by $X_3$. Quite interestingly, the strategy that consists for individuals in accumulating marriage-specific capital to reduce the risk of divorce is not appropriate here. Indeed, for individuals who are just about to divorce and whose the decision might be affected by the level of marriage-specific capital, the marginal utility obtained from continuing the relationship is exactly the same as that of divorcing. They have thus no incitation to
accumulate marriage-specific capital. The mechanism that support overinvestment described below is thus different.

4.3 The Marginal Expected Utility

In this subsection, we compute the marginal utility of investment in marriage-specific capital. Using identity (18), and applying the Leibniz rule, we can show that the derivative of equation (17) with respect to $X$ can be written as:

$$\frac{\partial E(V_2|X)}{\partial X} = C_M + I_M$$

where

$$C_M = \int_{-X/\Omega}^{+1/2} C(X + \Omega \nu) \cdot \varphi(\nu) \cdot d\nu = \int_{-X/\Omega}^{+1/2} \varphi(\nu) \cdot d\nu$$

because of assumption 3, and

$$I_M = \int_{-X/\Omega}^{+1/2} I(X + \Omega \nu) \cdot \varphi(\nu) \cdot d\nu$$

and the terms $C(\cdot)$ and $I(\cdot)$ are defined in the preceding section. The marginal expected utility is thus equal to the consumption and insurance motives of investment in marriage-specific capital averaged over all the state of the nature, accounting for the fact that these motives are equal to zero if divorce is involved.

4.4 The Optimal Level of Marriage-Specific Capital

The optimal level of marriage-specific capital in the bargaining model with divorce is denoted by $X_4$. The optimization problem of each spouse can be written as:

$$\max_{X \geq 0} \left[ u \left( \frac{Y_1 - X}{2} \right) + X \right] + E(V_2|X).$$

(21)

Using the expression previously derived, and applying some simple transformations, the first order condition for an interior solution is given by:

$$\frac{1}{2} u' \left( \frac{Y_1 - X}{2} \right) = 1 + C_M + I_M.$$

(22)

To obtain the next result we need to suppose the following.
Assumption A4. The Pratt measure of absolute risk aversion of spouses is decreasing, that is,

\[ A(n_i) = \frac{-u''(n_i)}{u'(n_i)} \text{ decreases when } n_i \text{ increases.} \]

This condition is generally supposed to be plausible and it is satisfied, for instance, by the CRRA utility function. Furthermore, this condition is sufficient, but not necessary, to obtain the result that follows. This result states that, for some values of \( \Omega \) that are relatively small, the optimal level of marriage-specific capital will be larger in the divorce case than in the non-divorce case.

Proposition 4. Assume A1-A4 and assume that \((Y_2 + \Sigma)/2 > y^*(Y_2, X_2)\). Then the investment in marriage-specific capital is a non-monotonic function of \( \Omega \). In particular, there exists a pair of values \( \overline{\Omega} \) and \( \underline{\Omega} \) such that, at the optimal solution, the probability of divorce is positive and

(a) For any \( \Omega < \overline{\Omega} \), an increase in \( \Omega \) has a positive impact on the level of marriage-specific capital, and

\[ X_4 > X_2 > X_1 \geq X_3; \]

(b) For any \( \Omega > \underline{\Omega} \), an increase in \( \Omega \) has a negative impact on the level of marriage-specific capital, and

\[ X_3 < X_4 < X_1 < X_2, \text{ with } \lim_{\Omega \to \infty} (X_4 - X_3) = 0, \]

where \( X_1, X_2, X_3, X_4 \) are the optimal levels of investment in marriage-specific capital in the full-commitment case, in the partial-commitment case, in the full-commitment case with divorce, and in the partial-commitment case with divorce.

Proof. If we differentiate the first order condition (22) with respect to \( \Omega \), we obtain:

\[ -\frac{1}{4} u'' \left( \frac{Y_1 - X}{2} \right) \cdot dX = \left( \frac{\partial C_M}{\partial \Omega} + \frac{\partial I_M}{\partial \Omega} \right) \cdot d\Omega. \tag{23} \]

The sign of the derivative of the optimal level of marriage-specific capital with respect to \( \Omega \) will be the same as the sign of the term in brackets on the right-hand side of this expression.
Part (a). The optimal level of marriage-specific capital is strictly positive from assumption 2. Then, by definition of the limit, there exists a positive value $\Omega_1$ such that, for any $\Omega < \Omega_1$ and any $\nu$, divorce is not optimal. Then, for any $\Omega < \Omega_1$,

$$C_M = 1 > 0,$$

$$I_M = \int_{-1/2}^{+1/2} I(X + \Omega \nu) \cdot \varphi(\nu) \cdot d\nu > 0.$$

From these expressions, we can compute the derivative of the right-hand side of expression (23). First, we have:

$$\frac{\partial C_M}{\partial \Omega} + \frac{\partial I_M}{\partial \Omega} = \int_{-1/2}^{+1/2} \frac{\partial I(X + \Omega \nu)}{\partial (X + \Omega \nu)} \cdot \nu \cdot \varphi(\nu) \cdot d\nu.$$

Because of the symmetry of $\varphi(\nu)$, the derivative of $I_M$ can be written (using a convenient change of variable) as:

$$\frac{\partial C_M}{\partial \Omega} + \frac{\partial I_M}{\partial \Omega} = \int_{0}^{+1/2} \left( \frac{\partial I(X + \Omega \nu)}{\partial (X + \Omega \nu)} - \frac{\partial I(X - \Omega \nu)}{\partial (X + \Omega \nu)} \right) \cdot \nu \cdot \varphi(\nu) \cdot d\nu.$$

From the appendix B, we know that the insurance motive is decreasing and, if assumption 4 is satisfied, convex in the level of marriage-specific capital. The term in brackets is thus positive. If we incorporate it in (23), we prove that, as long as $\Omega < \Omega_1$, the level of investment in marriage-specific capital increases with $\Omega$.

Because of the budget constraint, the optimal value of the investment in marriage-specific capital is bounded. Hence there exists a value $\Omega_2$ such that $X_4 = \Omega_2/2$. At this very point, the probability of divorcing becomes positive and

$$\frac{\partial C_M}{\partial \Omega} + \frac{\partial I_M}{\partial \Omega} = \int_{-1/2}^{+1/2} \frac{\partial I(\Omega_2 (1/2 + \nu))}{\partial (X + \Omega \nu)} \cdot \nu \cdot \varphi(\nu) \cdot d\nu > 0.$$

Since $\varphi(-1/2) = 0$, the left-hand side of this expression is continuous at $X_4 = \Omega_2/2$. Consequently, there exists a value $\Omega > \Omega_2$ such that the probability of divorce is positive and such that, for any $\Omega < \Omega$, an increase in $\Omega$ has a positive impact on the level of marriage-specific capital. Then, using proposition 1, we prove that $X_4 > X_2 > X_1 \geq X_3$. 

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Part (b). In the general case, the term in brackets on the right-hand side of expression (23) is equal to:

\[
\frac{\partial C_M}{\partial \Omega} + \frac{\partial I_M}{\partial \Omega} = \int_{-X/\alpha}^{+1/2} \frac{\partial I(X + \Omega \nu)}{\partial (X + \Omega \nu)} \cdot \nu \cdot \varphi(\nu) \cdot d\nu - \frac{(1 + I(0)) \cdot X}{\Omega^2} \cdot \varphi\left(-\frac{X}{\Omega}\right),
\]

where the second term on the right-hand side is always negative and the first term is negative if \( \Omega \) is sufficiently large. Hence, there exists some critical value \( \Omega \) such that, for any \( \Omega > \tilde{\Omega} \), an increase in \( \Omega \) has a negative impact on the level of marriage-specific capital. When \( \Omega \) tends to infinity, then \( X_4 \) converges to \( X_3 \).

The second statement in this proposition is conformed to the intuition of Gary Becker and his followers that the risk of divorce discourages investment in marriage-specific capital. One can see, though, that the complete story is more complicated: an exogenous reduction in the tendency to divorce (represented by \( \Omega \) and due to a change in divorce laws, for instance) may also have a positive effect on investment in marriage-specific capital. To make this result more explicit, the profile of the optimal level of investment in marriage-specific capital be broken down into two segments: for any value \( \Omega \) below \( \tilde{\Omega} \), the optimal level of investment increases and, for any \( \Omega \) above \( \tilde{\Omega} \), the optimal level of investment decreases. For intermediate values, the form of the profile is not determined. The intuition of the increasing segment is that the insurance effect of investment in marriage-specific capital is reinforced when the marriage surplus is affected by a small shock that may lead to divorce. The occurrence of negative shocks on the marriage-surplus makes investment in marriage-specific capital much more profitable. The negative shocks are not completely counterweighted by positive shocks that have the opposite effect. Note, however, that the existence of the first increasing segment depends on our assumptions regarding the distribution of the random term \( \nu \) and, in particular, the symmetry of the density distribution function \( \varphi(\nu) \). However, the increasing segment would be even more marked if shocks exhibited negative skewness – which is not necessarily counter-intuitive.

5 Conclusion

In this paper, we propose a new theoretical framework that can be used to investigate the impact of bargaining risk on investment decisions. Our results show
that a large level of marriage-specific capital may reduce fluctuations in consumption, which may in turn incite risk-averse spouses to overinvest. Overinvestment is even reinforced when the surplus of marriage is stochastic (provided that the probability of divorce is small). This observation has several interesting consequences. (i) The intra-household distribution of consumption is more stable in high-income households than in low-income households. To translate this idea in the terminology of collective models (Donni, 2008; Chiappori and Donni, 2011), the effect of distribution factors – i.e., the variables that affect the intrahousehold distribution of consumption without influencing individual preferences – should be less pronounced in high-income households than in low-income households. The level of the household income at the beginning of the life-cycle is the most important element in the mechanism. (ii) The propensity to divorce in the society will not necessarily depress the demand for marriage-specific capital (such as children). This contrasts with the traditional view of Gary Becker. If individuals acquire more information on their partner before marriage, for instance, so that the proportion of divorce is negatively affected, then investment in marriage-specific capital will turn out to be less necessary to reduce fluctuations in consumption. Investment in marriage-specific capital may well diminish as a consequence.

The model has been kept as simple as possible. Other factors that may influence spouses’ investment decisions and modify our main conclusions have been voluntarily omitted. To take an example, let us suppose that spouses have the possibility to save in a riskless asset during the first period and transfer some consumption from the first period to the second one. In the case of divorce, total savings are divided between ex-spouses into two equal shares. The attractiveness of outside opportunities is then reduced by a large level of savings. Indeed, because of the concavity of individual utility functions, the second period income received by the spouse with the best opportunity is less desirable if the household savings are important. Therefore, oversaving can be used as an alternative instrument to reduce bargaining risk. The decision process may finally lead to underinvestment in marriage-specific capital. However, this conclusion does not fundamentally change our claim. Savings in this story have a marriage-specific component that make them attractive to reduce bargaining risk. Future empirical studies should investigate these issues and examine the implications of these results for the literature on collective models.
Appendix A. The Marginal Expected Utility: The Consumption and Insurance Motives

Differentiating the expected utility function with respect to $X$ gives the marginal expected utility of marriage-specific capital. If the Leibniz rule and equation (7) are used, the marginal expected utility can be expressed as the sum of two terms:

$$\frac{\partial E(U_2|X)}{\partial X} = C + I. \quad (A-1)$$

The interpretation and the sign of both terms in equation (A-1) can be precisely determined.

The Consumption Motive.

The first term represents the consumption motive of the investment in the marriage-specific capital, that is,

$$C = v'(X).$$

The Insurance Motive.

The second term of equation (A-1) represents the insurance motive of the investment in the marriage-specific capital. Defining $h(t) = \eta(t) - Y_2/2$ and using a convenient change of variable, the insurance motive can be written as:

$$I = \frac{1}{\Sigma} \int_{y^*}^{y^* + \Sigma} \left( u' \left( \frac{Y_2}{2} + h(t) \right) - u' \left( \frac{Y_2}{2} - h(t) \right) \right) \cdot \frac{\partial \eta(t)}{\partial X} \cdot f(t) \cdot dt.$$ 

This expression is clearly positive since the utility function is concave and $\partial \eta/\partial X$ is negative. If we calculate the second order Taylor approximation of $u'$ around point $Y_2/2$, and introduce this approximation in $I$, we obtain:

$$I = u'' \left( \frac{Y_2}{2} \right) \times \frac{2}{\Sigma} \int_{y^*}^{y^* + \Sigma} h(t) \cdot \frac{\partial \eta(t)}{\partial X} \cdot f(t) \cdot dt + \text{remainder},$$

or, alternatively,

$$I = \frac{1}{2} u'' \left( \frac{Y_2}{2} \right) \cdot \frac{\partial \text{var}(x_i)}{\partial X} + \text{remainder}.$$
Appendix B. The First and Second Order Derivatives of the Insurance Motive

The insurance motive is defined as:

\[ I = \frac{1}{\sum} \int_{y^*}^{y^{*+\Sigma}} \left( u' \left( \frac{Y_2}{2} + h(t) \right) - u' \left( \frac{Y_2}{2} - h(t) \right) \right) \frac{\partial \eta(t)}{\partial X} \cdot f(t) \cdot dt, \]

where \( h(t) = \eta(t) - Y_2/2 \).

The first derivative of this expression is

\[ \frac{\partial I}{\partial X} = \frac{1}{\sum} \int_{y^*}^{y^{*+\Sigma}} \left[ A(t) \frac{\partial^2 \eta(t)}{\partial X^2} + B(t) \left( \frac{\partial \eta(t)}{\partial X} \right)^2 \right] \cdot f(t) \cdot dt \]

and

\[
\begin{align*}
A(t) &= u' \left( \frac{Y_2}{2} + h(t) \right) - u' \left( \frac{Y_2}{2} - h(t) \right) < 0, \\
B(t) &= u'' \left( \frac{Y_2}{2} + h(t) \right) + u'' \left( \frac{Y_2}{2} - h(t) \right) < 0,
\end{align*}
\]

because of the concavity of \( u(x) \). Then, using the implicit function theorem implies that

\[
\frac{\partial \eta}{\partial X} = -\frac{v'(X)}{u'(\eta)} < 0, \quad \frac{\partial^2 \eta}{\partial X^2} = -\frac{v''(X)u'(\eta) - v'(X)u''(\eta) \frac{\partial \eta}{\partial X}}{|u'(\eta)|^2} > 0;
\]

hence,

\[ \frac{\partial I}{\partial X} < 0. \]

The second derivative is equal to

\[
\frac{\partial^2 I}{\partial X^2} = -\frac{2}{\sum} u'' \left( \frac{Y_2}{2} \right) \cdot \left( \left. \frac{\partial \eta}{\partial X} \right|_{y^*} \right)^2 \cdot \frac{\partial y^*}{\partial X} \cdot f(y^*) \tag{A-2}
\]

\[ + \int_{y^*}^{y^{*+\Sigma}} \left[ A(t) \frac{\partial^3 \eta(t)}{\partial X^3} + 3B(t) \frac{\partial \eta(t)}{\partial X} \frac{\partial^2 \eta(t)}{\partial X^2} + C(t) \left( \frac{\partial \eta(t)}{\partial X} \right)^3 \right] \cdot f(t) \cdot dt, \]

where \( A(t) \) and \( B(t) \) are defined as above and

\[ C(t) = u''' \left( \frac{Y_2}{2} + h(t) \right) - u''' \left( \frac{Y_2}{2} - h(t) \right) < 0, \]
because of the convexity of \( u'(x) \). The first term on the right-hand side of (A-2) is positive since \( \partial y^*/\partial X > 0 \). The last two terms between the squared brackets are positive. The first term is positive if \( \partial^2 \eta/\partial X^3 < 0 \). To examine this, we adopt assumption 3. Then,

\[
\frac{\partial \eta}{\partial X} = -\frac{1}{u'(\eta)} < 0, \quad \frac{\partial^2 \eta}{\partial X^2} = A(\eta) \times \left( \frac{\partial \eta}{\partial X} \right)^2 > 0,
\]

where \( A(\eta) \) is the Pratt measure absolute risk aversion measure and

\[
\frac{\partial^3 \eta}{\partial X^3} = \frac{\partial A}{\partial \eta} \left( \frac{\partial \eta}{\partial X} \right)^2 + 2A(\eta) \frac{\partial \eta}{\partial X} \frac{\partial^2 \eta}{\partial X^2}.
\]

This expression is negative if the measure of risk aversion is decreasing (as required by assumption 4). In that case,

\[
\frac{\partial^2 I}{\partial X^2} > 0.
\]

References


[29] Lundberg, Shelly and Elaina Rose (1999). "The Determinants of Specialization within Marriage", manuscript, University of Washington


