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abstract

This article presents a model of optimal decentralization of economic governance. It focuses on the provision of public input for private production. It considers that the decision power is given to a local government if it has the full right to decide new investments and new taxes to finance it. Three economic forces act on this optimal decentralization of the decision. First is the centripetal force which consists in the increasing accuracy and relevance of public investments when decided more locally. The second and third are the centrifugal forces of the administrative costs on the one hand and of the fiscal competition among decentralized jurisdictions on the other. Formal proofs of the existence and uniqueness of solutions are given under special hypotheses and in general. Numerical analysis is also done to understand the impact on the optimal decentralization level of the different model parameters.

keyword: Decentralization; Corporate taxes; Tax competition; Public input; Firm location. **JEL classification**: H25; H72; R12; R51; R53.

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1 Introduction

Decentralization of economic governance is a key question for states and both decentralization and centralization of competences occurred in developed countries in the past few decades. France has begun a decentralization second run in 2003, after a first run in 1982 and 1983, that transferred administration prerogatives from central states to *régions* and *départements*. At the same time, central government incited municipalities to unite in Pubic Inter-Municipal Cooperation Bodies. In the United States, the Indiana state legislature instituted in 1970 "Unigov" as the Indianapolis consolidated city-county government. In Canada, there has been various municipality mergers around Winnipeg in 1971, around Toronto in 1998, around Ottawa in 2001 and of Montreal island in 2002. These opposite changes highlight the trade-off between centralization and decentralization forces: merging local governments generates economies of scale in terms of management costs; on the other hand allowing public investment to be decided closer to the effective needs generates efficiency gains.

According to the Oates (1972) decentralization theorem, decentralized provision of public goods is always better than uniform provision of public good when there are no spillovers. Decentralized provision is also better than non-uniform centralized provision if asymmetrical information make the central state unable to know perfectly local needs. On that matter, Barankay and Lockwood (2007) tested the efficiency of decentralizing the provision of education through the Swiss cantons and actually found efficiency gains in decentralization.

The aim of the present paper is to characterize the optimal local government that should receive the competence of investing in public input for private production. I consider only the decentralization of competences among potentially already existing local governments. As presented by Arzaghi and Hendersen (2005) almost all countries have a large number of embedded administrative divisions with different competences and real power. Therefore, I do not consider vertical interactions between different decentralized levels of a federalist system. These vertical interactions are assumed dependent on the other competences and independent on the very competence I analyze. Consequently, three forces remain that incitate to centralize or decentralize the decision of investing in public input for public production. The centripetal force first: public investments are more accurate and more relevant when they are decided more locally. The reasons of this force are close to the arguments of the decentralization theorem. On the opposite side, there are two centrifugal forces. First, the administrative costs are increasing with respect to decentralization, since there are administrative economies of scale in economic governance. Second, fiscal competition among decentralized jurisdictions - and the fiscal cost of this competition - is increasing with respect to decentralization. Indeed, in order to keep the real decision about public investment, local governments should have the real power to finance it. As proposed by Schwab and Oates (1991), I consider a local corporate tax to finance public expenses for corporate activities. Local governments may therefore compete on public input and corporate tax rates to attract firms, and the bias towards low local corporate tax rates is stronger for small territorial jurisdictions than for bigger ones (Bucovetski, 1991; Wilson, 1991; Carbonnier, 2008).

The resolution of this model shows that there exists a unique optimal level for decentralizing the public input provision competence. This level is not modified by the actual productivity of public capital in the private production function, but is strongly impacted by the ratio of the GDP to the cost of administrating such competence. Therefore, the optimal level for decentralizing this competence is increasing with respect to productivity gains in administration but decreasing with respect to the complexity of public input needs.

Some papers compare benefits and costs of decentralization in terms of welfare linked to public good provisions. Such papers, as Alesina and Spolaore (1997) are based on the model presented by Tiebout (1956). People migrate in order to settle in territorial jurisdictions inhabited by people with the same tastes for public goods. This model assumes the perfect mobility of households and therefore their identical welfare at equilibrium. However, Glaeser and Gottlieb (2009) noted that the mobility of households is imperfect, strongly in Europe but also in the United States. Concerning mobility, Oates (1999) has stated that the decentralization theorem does not rely on the perfect mobility assumption. Welfare is improved by decentralization as soon as the sums of marginal utilities for public goods are different between localities. Instead, mobility may even worsen the equilibrium state because of competition among local governments. Panizza (1999) endogenized decisions of decentralization in a political model considering that central governments choose the level of decentralization in order to maximize their own objective function.

The papers just mentionned study decentralization of the provision of public good for the consumption of households. However, a similar question is coming to light for other competences such as provision of public input for private production. No optimal decentralization model exists concerning public input, but some papers showed the link between fiscal competition and underprovision of public input. Zodrow and Mieskowski (1986) theoretically demonstrated the bias towards low corporate tax rates. Bénassy-Quéré et al. (2005) empirically showed at the international level that more taxes and more public input may lead to a higher amount of foreign direct investment. Bell and Gabe (2004) presented the same results at the local level. In the reduce form, Thornton (2007) linked decentralization and growth.

These papers demonstrate that an underprovision of public input for private production depends on fiscal competition. The impact of fiscal competition on tax rates depends on the size of territorial jurisdictions, and therefore depends on decentralization. Hence, if decentralization improves the matching between public input and local needs, it also decreases the amount of public input provided. The aim of the present paper is to compare these two impacts of decentralization on public input provision.

The article is organized as follows. Section 2 presents the theoretical framework combining the three forces: are presented first a model of fiscal competition between an arbitrarily high number of territorial jurisdictions, then the dependence of the efficiency of public investment on the level of the decentralization level, and finally the dependence of administrative costs on the level of decentralization. Section 3 presents the formal solutions of the model as well as a numerical analysis and a discussion of the results under three different assumptions: first, a case where there is coordination and where no tax competition occurs; second, a case without administrative costs of decentralizing; and third, a general case. For each case, the impacts of model parameters on the optimal level of decentralization are analyzed. Section 4 presents the concluding comments.

2 Theoretical framework

In order to understand the balance between the different consequences of decentralization - and therefore defining the best decentralized government to which to attribute the competence of public investment three models are constituted to represent the three forces. First, the centralization forces are presented through a model of tax competition among decentralized governments and a function of the administrative costs depending of the level of decentralization. Then the decentralization force is presented through a model assuming that the efficiency of the public investments increases with respect to decentralization.

2.1 Fiscal competition

The model used to understand the aftermath of fiscal competition is based on Zodrow and Mieszkowski (1986). This kind of model is preferred to the new economic geography ones - and specially Baldwin and Krugman (2004) based on Krugman (1991) - because it is possible to extend the analysis to more than two territorial jurisdictions. The asymmetrical effects of tax competition in the new economic geography models are due to industrial differences between the two regions considered. The aim of the present paper is to consider the different effects of tax competition depending on the mean size of the administrative divisions, and therefore their number for a given country. These asymmetrical effects of tax competition - smaller territorial jurisdictions are weaker and undergo a larger bias toward low tax rates - has been shown theoretically by Bucovetski (1991) and Wilson (1991). Empirically, the existence of fiscal competition among local governments has been proved by Buettner (2001) and Buettner (2003), and the asymmetrical effects have been highlighted by Boadway and Hayashi (2001) and Leprince et al. (2007). Furthermore, agglomerations in the new economic geography models are driven by the increasing returns to scale of the industrial production function although the present study aims at comparing efficiency versus quantity of public input provision.

Hence, the model of tax competition is derived from Carbonnier (2008), which uses a parametrization that fits the goals of this paper. The bias on local corporate tax rates - and then on public spending - due to tax competition is computerized in function of different variables, among them the number nof administrative divisions, which can be arbitrarily high. This parameter n gives obviously the level of decentralization: for a given global economy, larger n implies smaller local governments. This model allows to calculate tax competition costs for any level n of decentralization, and the optimal n^* , final result of the global model of decentralization. This n^* should then be considered not as the exact number of local governments to be created, but as an indicator of the best existing level of decentralization: regions or states, counties, cities. The dependence of n^* on the other parameters allows us to understand which characteristics of the central country imply more or less need for decentralization. At each period t in each administrative division i (i = 1..n), there is a l_{it} fixed factor, k_{it} private capital and p_{it} public capital. These production factors allow private firms to produce y_{it} according to the production function $y_{it} = F(k_{it}, l_{it}, p_{it})$. The production function used for this model is a Cobb-Douglas production function $y_{it} = Ak_{it}^{\alpha} l_{it}^{\beta} p_{it}^{\gamma}$, with two kinds of capital, private and public. In order to focus on capital only, l_{it} is implemented as an exogenous parameter. There is a total amount of fixed factor L_t distributed irregularly among the administrative divisions. This fixed factor could be land or a specific advantage for production such as geological or geographical properties. In addition, it could be understood as the labor force or the number of inhabitants with the assumption that labor force is less mobile than private capital. The exogenous allocation of the fixed factor indicates the size of the economy of the administrative divisions and allows us to understand tax competition effects between a large number of local jurisdiction of different sizes.

The public capital is financed by local business taxation (Schwab and Oates, 1991). Local government of administrative division *i* taxes private capital k_{it} at rate τ_{it} and invests the revenue $\tau_{it}k_{it}$ as public capital for the following period t + 1. As public capital is depreciating at the rate of δ , the amount p_{it} of public capital at time *t* in administrative division *i* is $p_{it} = (1 - \delta)p_{it-1} + \tau_{it-1}k_{it-1}$. In each administrative division, entrepreneurs borrow private capital, pay local taxes and organize production. The Cobb-Douglas production function allows us to consider capital as a whole, even if it is invested in each division by a large number of entrepreneurs. Furthermore, each unit of private capital benefits from the total amount of public input invested in the jurisdiction.

The objective of local governments is to maximize the welfare of inhabitants. In that way they have to maximize their employment and income. Given the production function - and l_{it} being an exogenous parameter - employment and income of inhabitants are maximal if production is maximal. Hence, local government's objective should be to maximize the production Y_{it} . To improve productivity and production, they can set corporate taxes to finance investment in public input for private production. When rates of local corporate tax varies, two phenomena arise vis-à-vis private capital. First, the total amount of private capital K_t in the country varies, and second, this remaining private capital K_t is reallocated between administrative divisions. The global amount of private capital K is both the result of inter-temporal optimization of agent utility and the result of the international partial mobility of private capital. Consequently, the total amount K of private capital invested in the whole country depends on the private capital returns. At period t, the impact of tax τ_{it} on public capital p_{it} has not occurred yet. Hence, the elasticity of K with respect to τ does not depend on p. The total capital elasticity with respect to local corporate tax rate $\epsilon_K = -\frac{1-\tau_i}{k_i} \frac{\partial K}{\partial \tau_i}$ measures the national overall reaction of private capital to tax rate changes. Afterward, the total amount of national private capital K_t is allocated among administrative divisions in such manner that it equalizes the marginal returns of private capital in every administrative divisions. This corresponds to the assumption that private capital is perfectly mobile inside the national territory among the administrative divisions. The resulting private capital allocation is given by condition (1).

$$\begin{cases} k_i = \frac{f(i)}{\sum_{j=1}^n f(j)} K\\ f(i) = (1 - \tau_i)^{\frac{\alpha}{1-\alpha}} p_i^{\frac{\gamma}{1-\alpha}} l_i^{\frac{\beta}{1-\alpha}} \end{cases}$$
(1)

Knowing what would be the private capital reallocation, local governments settle local corporate tax rates. Two ways of resolving this model may be implemented. First, the optimization process may be done in order to maximize the overall production. It means that local governments cooperate and there is no tax competition. Second, tax competition may occur and each local government maximizes its own production, using its own corporate tax rate. This case is implemented by solving the model at Nash equilibrium. In the first case, the cooperation case, the optimal tax rate is settled identically for each city as shown in equation (2).

$$\tau_i^* = \frac{\gamma}{\alpha + \gamma} \frac{1}{1 + \epsilon_K} \tag{2}$$

The optimal rate formula is composed of two different terms. The first one $\frac{\gamma}{\alpha+\gamma}$ reflects the optimal ratio of private capital k_i to public capital p_i . In that matter τ^* is decreasing with respect to α because it indicates the productivity of private capital in the Cobb-Douglas production function: the more productive is private capital, the higher is the cost of taxing it. In addition, τ^* is increasing with respect to γ because it indicates the productivity of the public capital in the Cobb-Douglas production function: the more productive is public capital, the higher are the benefits of taxation. As this first term represents an optimal ratio of private capital to public capital, it does not depend on the local jurisdiction size. The second term $\frac{1}{1+\epsilon\kappa}$ is the usual fiscal arbitrage between tax rate and tax base.

If the tax base elasticity with respect to the tax rate is large, optimal tax rate is low, and vice versa. If fiscal competition is introduced, the optimization problem for each local government consists in maximizing its own production, with its own tax rate as the only control variable, given the other tax rates. Tax rate choices are made according to the local elasticity of private capital $\epsilon_{k_i} = -\frac{1-\tau_i}{k_i} \frac{\partial k_i}{\partial \tau_i}$. The actual corporate tax rate chosen by each local government in the case of non-cooperation is then given by equation (3).

$$\tau_i^o = \frac{\gamma}{\alpha + \gamma} \frac{1}{1 + \epsilon_{k_i}} = \frac{\gamma}{\alpha + \gamma} \frac{1}{1 + \frac{1}{1 - \alpha} \sum_{j \neq i} f(j) + \epsilon_K f(i)}}$$
(3)

As the first best optimal tax rate τ^* , the second best optimal tax rate τ_i^o is increasing with respect to α and ϵ_k . However, it may be either larger or smaller than the first best optimal tax rate, depending on ϵ_K being larger or smaller than $\epsilon_{k_i} = \frac{\frac{\alpha}{1-\alpha}\sum_{j\neq i}f(j)+\epsilon_K f(i)}{\sum_{j=1}^n f(j)}$. If the second best optimal tax rate is lower than the first best optimal tax rate - as well as if it is higher - the difference between both tax rates is larger if ϵ_k is further from ϵ_K . This means that the difference between both tax rates is larger if ϵ_k is lower, and therefore if the territorial jurisdictions in competition are smaller. Under the hypothesis that ϵ_K is lower than $\frac{\alpha}{1-\alpha}$, which is the most probable hypothesis, the second best optimal tax rate is lower tax rates. This bias towards low local corporate tax rates is that the first best optimal tax rate. Tax competition is then generating a bias toward low local corporate tax rates. This bias towards low local corporate tax rates is larger of the tax rates low local corporate tax rates.

marginal decreasing returns of private capital are less binding if there is a larger fixed factor of production l_{it} .

2.2 Administrative costs

The second centripetal force is the administrative cost of decentralization. The main idea is that there are economies of scale in the administration of jurisdictions. With decentralization, some administrative tasks are made by each jurisdiction independently while in a centralized government they may have been done only once. Furthermore, the global supply of human administrative competence is limited: the smaller is the administrative division, the more difficult is the implementation of administrative and technical tasks. Both arguments imply that the global administrative cost function c(n) is increasing with respect to the number n of administrative divisions.

In the limit case of constant cost function, no economies of scale exist and the administrative costs of governing an administrative division depend only on the economic size of the division: the costs of a centralized administration is the sum of the costs of the decentralized administrations. This may be due to the needs of hiring administrators at the central level devoted to each region, even in very centralized countries. At the opposite, Alesina and Spolaore (1997) consider a linear cost function: c(n) = c.n. In that case, all administrative tasks have to be duplicated when competence is decentralized. It assumes that the costs of every government, large or small, are the same.

A more subtle modeling may be done considering concave or convex cost functions. The convex hypothesis appears to be unlikely. It would mean that each small local government has higher costs than the large central one. If relative administrative costs decrease with respect to the size of the administrated economy - and therefore c(n) is increasing with respect to n - absolute administrative costs are surely increasing with the size of the administrated economy - and therefore c(n) is concave. Certainly, there may be costs due to vertical interactions because of the creation of new level of administrative divisions. However, creation of new local levels is not considered in the present paper because decentralization consists only in allocation of competences to existing local governments. Indeed, most countries in the world, whatever actually centralized or decentralized, are already constituted with a large number of administrative bodies, with more or less real autonomy and power. For example, the United States are constituted with the federal level, states, counties, townships and cities. In France there is the national level, regions, *départements*, Public Inter-municipal Cooperation Bodies and municipalities.

The cost function is then assumed to be increasing and concave with respect to the level of decentralization. This means that there are administrative economies of scale but that some local tasks have to be done separately for each location even in a centralized country. The costs of administrating public investments for private production are therefore lower if administrated centrally than if administrated locally for each part of the territory. However, the absolute administrative costs in each division are lower than the centralized costs for the whole territory. Hence, the administrative cost function is assumed to be as shown by equation (4).

$$c(n) = c \frac{\ln(1 + \frac{n}{d})}{\ln(1 + \frac{1}{d})}$$
(4)

This equation presents a cost function such as parameter c gives the administrative costs for the centralized administration of public input for private production. The cost function c(n) is increasing and concave with respect to the level n of decentralization. Its rate of increase is controlled with parameter d. The slope of cost function c(n) with respect to n is increasing with respect to parameter d. It is relevant to notice that even the relative increase rate of the cost function is increasing with respect to parameter d. When parameter d tends towards zero, the cost function tends towards the global fixed administrative costs c(n) = c. When parameter d tends towards infinity, the cost function tends towards the linear cost function c(n) = c.n.

2.3 Public investment efficency gains

The centrifugal force is the efficiency of public investment. It is similar to the explanation given by the decentralization theorem of Oates (1972) applied to provision of public input for firms instead of public goods for households. Local provision of public input for private production is better than centralized provision when firms variously located have different needs in term of public capital. It may derive from differences in the labor force, from geographical specificities or from an heterogeneous allocation of industries among the different locations. Asymmetrical information is the key notion because local needs are better known by local governments than by central ones. Some empirical studies try to estimate the outcome of decentralization. With Swiss data, Barankay and Lockwood (2007) find that decentralization improves the efficiency of education.

The efficiency gains the decentralization of public investments are modeled by the probability that the public capital invested fits the local needs and it is, therefore, productive. Consequently, a difference is made between actual public capital p_i and efficient public capital p_i^e . If public investment decisions are taken optimally, the marginal capital investment leads to an equal increase of efficient public capital: $dp_i^e = dp_i$. If not, the marginal public investment does not change the amount of efficient public capital: $dp_i^e = 0$. The increasing efficiency of public investment with respect to decentralization means that the probability π for a public investment to be efficient is increasing with respect to the level of decentralization. The central government does not know the actual local needs and has a lower probability to invest properly. Decentralized governments are aware of the infrastructures needed by local firms and their probability of investing efficiently is greater. Hence, the probability $\pi(n)$ is increasing with respect to the number nof local jurisdictions in which the global territory is divided. As all the marginal investments of a local government have the same probability $\pi(n)$ to be efficient, the global amount of efficient public capital is $p_i^e = \pi(n)p_i$.

A functional form is used in the optimal decentralization model to implement this probability for a marginal investment to be efficient. The limit of the probability $\pi(n)$ when the number of administrative

divisions tends to infinity should be 1 and the probability for the totally centralized state should be positive: $0 \le \pi(1) < 1$. Moreover, the efficiency gains due to decentralization should have decreasing returns as there exists a level of decision where less new information may be collected by increasing decentralization. Consequently, the probability function should be concave. Equation (5) presents this probability function $\pi(n)$.

$$\pi(n) = 1 - a \frac{\ln(1 + \frac{1}{b})}{\ln(1 + \frac{n}{b})}$$
(5)

This function tends to 1 when n tends to infinity. The probability of efficient investment for the central state depends on the a parameter: $\pi(1) = 1 - a$. Hence, parameter a gives the efficiency gap between the fully efficient administration of public investment for private production and the actual efficiency of the fully centralized administration. The rate of increase of the probability of efficiency is controlled by parameter b. A larger parameter b induces that the probability increase rate is larger - with the probability function very concave - and a lower parameter b means a lower increase rate of the probability function and less concavity.

3 The optimal decentralization problem

The decentralization problem consists in the choice by the central state of the best administrative level at which to decentralize the decisions of public investment. In the present model, it consists in determining the number n of administrative divisions that maximizes wealth in the whole country, knowing that every production decision - public or private - is made at local level. Therefore, the maximization problem is given by equation (6).

$$\max_{n} Y(n) = GDP(n) - c(n)$$

$$GDP(n) = \sum_{i=1}^{n} A(p_{i}^{e})^{\gamma} [(1 - \tau_{i})k_{i}]^{\alpha} l_{i}^{\beta}$$

$$\sum_{i=1}^{n} l_{i} = L$$

$$k_{i} = \frac{f(i)}{\sum_{j=1}^{n} f(j)} K$$

$$-\frac{1 - \tau_{i}}{k_{i}} \frac{\partial K}{\partial \tau_{i}} = \epsilon_{K}$$

$$u.c.: \qquad p_{i}^{e} = \pi(n)p_{i}$$

$$p_{i} = \frac{\tau_{i}}{\delta} k_{i}$$

$$\tau_{i} = \begin{cases} \frac{\gamma}{\alpha + \gamma} \frac{1}{1 + \epsilon_{K}} & (cooperation) \\ \frac{\gamma}{\alpha + \gamma} \frac{1}{1 + \frac{\sum_{i \neq j} f(j)}{\sum_{j=1}^{n} f(j)}} & (competition) \end{cases}$$

$$f(i) = (1 - \tau_{i})^{\frac{\alpha}{1 - \alpha}} (p_{i}^{e})^{\frac{\gamma}{1 - \alpha}} l_{i}^{\frac{\beta}{1 - \alpha}}$$

In order to solve this maximization problem, hypotheses have to be assumed. First of all, whatever the number of administrative divisions chosen, the fixed factor L should be distributed equally among them because of the decreasing marginal returns of production factors. Hence, all decentralized administrative divisions are identical at the optimum, and therefore have the same amount of taxes, private capital and public capital. Thus, function f(i) may be simplified as $f(i)/\sum_j f(j) = \frac{1}{n}$. If all cities change their taxes

by $d\tau$, the global private capital investment changes by $dK = -\epsilon_K \frac{K}{n} \frac{nd\tau}{1-\tau}$ and $K = K_0(1-\tau)^{\epsilon_K}$, where K_0 is the total amount of private capital invested with no taxes. Thus, private capital investment in each jurisdiction is $k = \frac{K_0}{n}(1-\tau)^{\epsilon_K}$. The function to maximize is then given by equation (7).

$$Y(n) = GDP(n) - c(n)$$

= $An^{1-\alpha-\beta-\gamma}L^{\beta}K_{0}^{\alpha+\gamma}\delta^{-\gamma}\pi(n)^{\gamma}\tau^{\gamma}(1-\tau)^{\alpha+\epsilon_{K}(\alpha+\gamma)} - c(n)$ (7)

The model aims at comparing effects of tax competition and administrative costs on the one hand and efficiency gains in the other. To focus only on these effects, the production returns to scale are assumed constant, that is $\alpha + \beta + \gamma = 1$. The optimal tax rate is noted τ^* ($\tau^* = \frac{\alpha}{\alpha + \beta}$) and the actual tax rate $\tau^* B(n)$: $B(n) = \frac{1+\epsilon_K}{1+\frac{\epsilon_K}{n}+\frac{n-1}{n}\frac{\alpha}{1-\alpha}}$. The maximum potential global production X is the production with the optimal tax rate and perfectly efficient public capital: it is $X = AL^{\beta}K_0^{\alpha+\gamma}\delta^{-\gamma}(\tau^*)^{\gamma}(1-\tau^*)^{\alpha+\epsilon_K(\alpha+\gamma)}$. Using these notations, the objective function is given by equation (8) and the first order condition of that maximization is given by equation (9). The second order condition implies that the left hand term of equation (9) should be first superior and then inferior to the right hand term of this first order condition.

$$Y(n) = GDP(n) - c(n) = X\pi(n)^{\gamma}B(n)^{\gamma} \left(\frac{1 - \tau^*B(n)}{1 - \tau^*}\right)^{\alpha + \epsilon_K(\alpha + \gamma)} - c(n)$$

$$\tag{8}$$

$$DER(n) = GDP(n) \left\{ \gamma \frac{\pi'(n)}{\pi(n)} + \gamma \frac{B'(n)}{B(n)} - \left[\alpha + \epsilon_K(\alpha + \gamma) \right] \frac{\tau^* B'(n)}{\left[1 - \tau^* B(n) \right]^{\alpha + \epsilon_k(\alpha + \gamma)}} \right\} - c'(n) = 0$$

$$\tag{9}$$

Formally solving that maximization problem proves very difficult. However, it is possible to prove formally the existence and uniqueness of a solution to that problem. The following subsections present the proofs of the existence and uniqueness of a solution for three different cases. First, the coordination case is considered, without tax competition nor a bias towards low local corporate tax rates. Second, it is assumed that the global administrative costs do not depend on the decentralization level. Third, we consider the general case with both tax competition and administrative cost increase with respect decentralization. For each case, numerical analyses are performed in order to understand the optimal n dependance on the different parameters of the model.

3.1 Tax coordination between decentralized local governments

Tax coordination may be understood as local governments being only partially autonomous. They choose in which public infrastructure to invest public funds but they have no control on the resources - the local corporate tax rate - nor the amount invested. The resources may either be national subsidies or local corporate taxes, the rates of which are decided centrally. In France, the 2010 reform of local direct taxation provides this kind of political framework. The *taxe professionnelle* - a corporate tax collected by the different local governments - has been removed and replaced by national subsidies and a new corporate tax collected nationally and distributed to local administrations. The consequence is that B(n) = 1 and B(n)' = 0, regardless of the amount of n. According to the general first order condition (9), the probability (5) of efficiency and the cost function (4), the first order condition with tax coordination is given by equation (10), with the second order condition being that the left hand term should be first inferior then superior to the right hand term of this first order condition.

$$\ln\left(1+\frac{n}{b}\right)^{1+\gamma} \left[\ln\left(1+\frac{n}{b}\right) - a\ln\left(1+\frac{1}{b}\right)\right]^{1-\gamma} \frac{n+b}{n+d} = \gamma \frac{X}{c} a\ln\left(1+\frac{1}{b}\right) \ln\left(1+\frac{1}{d}\right)$$
(10)

The right hand term of condition (10) - noted rht_1 - is positive and superior to the value of the left hand term - noted $lht_1(n)$ - when n = 1 as soon as the central administrative cost c is small compared to the potential GDP X. Furthermore, the derivative of the left hand term $lht'_1(n)$ - given by equation (11) is positive² and the left hand term tends to infinity when n tends to infinity. Hence, there exists a unique solution.

$$lht_{1}'(n) = \frac{lht_{1}(n)}{n+b} \left(\frac{1+\gamma}{\ln\left(1+\frac{n}{b}\right)} + \frac{1-\gamma}{\ln\left(1+\frac{n}{b}\right) - a\ln\left(1+\frac{1}{b}\right)} + \frac{d-b}{n+d} \right)$$
(11)

As the left hand term is increasing and concave, the optimal level of decentralization is more than proportionally increasing with respect to the ratio X/c. To derive more properties about this unique solution, numerical solving is implemented. The optimal decentralization levels are calculated with different values of the parameters. Changes in the values of the parameters lead to very large changes in the optimal decentralization level. Figures 1 present therefore the optimal decentralization level in function of the different parameters. Figures 1a and 1b show for different public capital productivity γ the optimal decentralization level as a function of the ratio of the potential GDP X to the cost c of a totally centralized administration of public investment for private production. In figure 1c, the optimal number of administrative divisions depends on the probability function of the efficiency of public investments - ie: parameters a and b. The last figure, figure 1d, shows the optimal decentralization level depending on the parameter d controlling the rate of increase of the administrative costs.

The optimal decentralization level is exponentially increasing with respect to the ratio X/c which represents the ratio of the economic size of the global country to the costs of the national government to administrate centrally the investments in public capital for private production (eg: figures 1a and 1b). If X/c decreases with respect to the economic size of the country - which means that the administrative costs of a fully centralized country increase more than proportionally to the country economic size - the optimal size of administrative divisions increases with respect to the country size. At the opposite, if X/cincreases with respect to the country's economic size - which means that there exists economies of scale for the administrative costs of fully centralized countries - the optimal size of administrative divisions may also decrease with respect to the country economic size.

In addition, the optimal decentralization level increases with respect to the efficiency gap a, that is the difference between public investment efficiency of centralized and fully decentralized administrations (eg: figure 1c). It increases more rapidly for smaller b parameter, which means that the slower efficiency increases, the larger is the optimal number of administrative divisions. For fast increasing probability,

²It may be negative for a short range of small n, but soon becomes positive. In that case, and because $lht_1(1) < rht_1$, there is also a unique solution.



Figure 1: Optimal decentralization in case of fiscal coordination between local jurisdiction

a high efficiency of public investments is reached with only few - and large - administrative divisions in charge of public input; therefore, full decentralization is not necessary.

The speed of increase d of administrative costs has a negative influence on the optimal decentralization level (eg: figure 1d). A larger public investment productivity γ delays the negative impact of this dparameter increase.

3.2 No administrative cost variations

The second simplified case considers tax competition as a centripetal force, but the function of administrative costs is assumed constant with respect to the decentralization level. It can be explained by the cost of civil servants applying central decisions locally even if there is no decentralization at all. There are no economies of scale in administrating public input for private production. Hence, the right hand term of condition (9) is zero, and the first order condition of the maximization problem is given by equation (12), the second order condition being that the left hand term of this condition should be first superior then inferior to the right hand term.

$$\gamma \frac{\pi(n)'}{\pi(n)} = -\gamma \frac{B(n)'}{B(n)} + \left[\alpha + \epsilon_k(\alpha + \gamma)\right] \frac{\tau^* B(n)'}{\left[1 - \tau^* B(n)\right]^{\alpha + \epsilon_K(\alpha + \gamma)}} \tag{12}$$

The right hand term $rht_2(n)$ of equation (12) is equal to formula (13) with $\Delta = \frac{\alpha}{1-\alpha} - \epsilon_K$ and $p = \alpha + \epsilon_K(\alpha + \gamma)$. Under reasonable assumptions, p is inferior to 1 and Δ is strictly positive. In order to have p superior to unity, ϵ_K needs to be superior to unity itself, which is very unlikely. The other assumption is invalidated ($\Delta < 0$) if the bias B(n) of the local corporate tax rate is greater than one, which is the case - theoretically possible, but also very unlikely - when tax competition leads to higher tax rates.

$$rht_2(n) = \frac{\Delta\{(\alpha+\gamma)^{1-p}(1-\frac{\Delta}{n})^{1-p}[\alpha^2+\gamma-(\alpha+\gamma)\frac{\Delta}{n}]^p - p\alpha(1-\alpha)\}}{n^2(\alpha+\gamma)^{1-p}(1-\frac{\Delta}{n})^{2-p}[\alpha^2+\gamma-(\alpha+\gamma)\frac{\Delta}{n}]^p}$$
(13)

The denominator of rht_2 is positive, increases with respect to n and tends toward infinity when n tends toward infinity. The numerator decreases with respect to n and the limit $Num_{+\infty}$ when n tends toward infinity is $\Delta \left[(\alpha + \gamma)^{1-p} (\alpha^2 + \gamma)^p - p\alpha(1-\alpha) \right]$. If taking the assumptions (RTS) that returns to scale are constant - i.e.: $\alpha + \beta + \gamma = 1$ - and (VAS) that the shares of the value added are one third for capital and two third for labor - i.e.: $\alpha = \beta/2 - Num_{+\infty}$ is positive whatever the α value if $\epsilon_K < 1$. Therefore, $rht_2(n)$ becomes positive and then tends toward zero.

Equation 12 is equivalent to $der(n) \equiv \frac{a \ln(1+\frac{1}{b})}{(n+b)\ln(1+\frac{n}{b})\delta(n)} - rht_2(n) = 0$ (der(n) being first positive then negative) where $\delta(n) = \ln(1+\frac{n}{b}) - a \ln(1+\frac{1}{b})$. Therefore, $\frac{a \ln(1+\frac{1}{b})}{(n+b)\ln(1+\frac{n}{b})\delta(n)}$ is positive and tends towards zero as $\frac{1}{n \ln(n)^2}$ and $rht_2(n)$ tends toward zero as $\frac{1}{n^2}$. Therefore, der(n) tends towards zero being positive. The question is then: is der always positive or not? And if not, is the optimal n^* finite?

To answer this question, numerical analysis is made, using the (RTS) and (VAS) assumptions. It appears that der(n) has a unique maximum - for n = 1 if der(1) is positive. If der(1) is positive, then der(n) is positive for all n and the optimal level of decentralization is the infinite decentralization. If der(1) is negative, objective function Y(n) first decreases with respect to n then increases towards its finite limit $Y_{+\infty}$ when n tends towards infinity; but in that case, Y(1) is always superior to $Y_{+\infty}$ and the optimal level of decentralization is no decentralization at all. Figure 2 presents the value of this der(n)function for different values of the efficiency gap a and gives the difference between the efficiency of public investment decided at a fully decentralized and a fully centralized level.

It appears out of these numerical analyses that the optimal level of decentralization without administrative cost of decentralization is the full decentralization when the efficiency gap a is not too small. In figure 2, the parameter a, which gives the efficiency gap, should be inferior to 2% for the completely centralized administration to be a better decision taker than the completely decentralized administration.



Figure 2: Objective function derivative in case of no decentralization administrative cost

When doing the same analysis with various values for the other parameters, the trigger value of the efficiency gap a for the centralization dominance over decentralization is close to zero, between 1% and 5%. So small efficiency gap parameter a would therefore mean that decentralization generates almost no efficiency gains in terms of public investment. In all cases with very small efficiency gap - and even with quite small a parameter - more decentralization always improves the global net output.

3.3 The general case

The previous subsection allows not only to understand the optimal level of decentralization in the case of constant administration costs with respect to decentralization but allows also to prove that there exists a finite solution for the general case. The first order condition of the general case is DER(n) =GDP(n)der(n) - c'(n) = 0 with DER(n) being first positive then negative. In the case of a being not too small (higher then 0.05 in the worst parametrization), previous section shows that der(n) is positive and decreases towards 0 as $\frac{1}{n \ln(n)^2}$ when n tends towards infinity. As GDP(n) is also positive and increases towards a finite limit when n tends towards infinity, GDP(n)der(n) decreases towards 0 as $\frac{1}{n \ln(n)^2}$ when n tends towards infinity. On the other hand, the derivative c'(n) function of the administrative costs is positive and decreases towards 0 as $\frac{1}{n}$, that is at a slower decrease than those of GDP(n)der(n). Therefore, DER(n) becomes negative at a certain n and stays negative until infinity. Hence, there exists a finite solution to that problem.

In order to derive properties of this finite optimal level of decentralization, numerical analysis is done. As for previous subsections, the optimal levels of decentralization are calculated for different values of the parameters. Figures 3a and 3b present the variations depending on the productivity γ of the public capital and the ratio of the potential GDP X to the administrative cost c of the fully centralized decision of public investment. Figure 3c presents variations depending on parameters a and b of the probability function of the public investment efficiency. Figures 3d and 3e present variations depending on parameter d, which gives the increase rate of the administrative costs with respect to decentralization.

Figures 3a and 3b show that the optimal level of decentralization increases exponentially with respect to the ratio of the potential GDP X to the administrative cost c if public input decisions are fully centralized. Figure 3c shows that this optimal level of decentralization increases with respect to the efficiency gap a- the difference of the probability for an investment to be efficient if it is decided at a fully decentralized level or at a fully centralized level - but it decreases with respect to the increase rate b of this probability. Figures 3d and 3e show that the optimal level of decentralization decreases strongly with respect to the rate of increase of the administrative costs with respect to decentralization. Furthermore, figures 3a, 3b, 3d and 3e show that the optimal level of decentralization depends very weakly on the γ parameter of the productivity of the efficient public capital in the Cobb-Douglas production function. This dependency is not only weak but also non monotonous.

4 Concluding comments

Depending on the calibration of the model, resulting optimal levels of decentralization may be very different: it goes from $n^* = 1$ - administration of the investments of public input for private production fully centralized - to $n = +\infty$ - administration decentralized as much as possible. However, restricting parameters to ranges of likely values decreases the range of resulting optimal level of decentralization. However, the orders of magnitude may be from 10 to 10,000 with different likely values of the parameters. This range of results is credible. The optimal level of decentralization does not really give any information on the precise number of administrative divisions, but on the local level of decentralized administrations, it is most appropriate to acquire competences of public investment for private production, and on the actual autonomy of these local governments. If the order of magnitude of the optimal level of decentralization is 10, the best level to which to decentralize prerogatives of public investment should be states or regions. If the order of magnitude is more than 10,000, it should be municipalities or districts. Between those values, the best level may be urban sprawls or counties.

Hence, one important contribution of this paper is to show that there are no general results concerning the decentralization of decision of investment in public input for private production. The optimal level may be very different from one country to another, depending on its properties. Therefore, a decentralization theorem cannot be drawn from the provision of public input for private production as it exists for provision of public goods for households' consumption. Especially, there is a trigger value for parameter a, which gives the efficiency gap between a centralized decision and a perfectly efficient decision: should parameter abe inferior to this trigger value, full centralization dominates every level of decentralization. However, this trigger value is very low - it depends on the other parameters but is always inferior to 5%. Consequently, there may be a weak decentralization theorem: should the efficiency gap a be not too small - actually the



a. Optimal decentralization depending on administrative costs (large scale)

b. Optimal decentralization depending on administrative costs (narrow scale)



c. Optimal decentralization depending on the public investment efficency



on costs increase speed (large scale)

e. Optimal decentralization depending on costs increase speed (narrow scale)

Figure 3: Optimal decentralization in the general case

efficiency rate of decision taken by the central state should be inferior to 95% of the full efficiency - at least some decentralization of the decision of investment of public input for private decision is better than full centralization.

Looking at the dependency of the optimal level n^* of decentralization on the values of the parameters of this model, three results were expected. First, n^* increases with respect to a. This link is obvious as parameter a gives the efficiency gain with decentralization. Second, n^* decreases with respect to b. If parameter b is larger, the probability for public investment to be efficient increases faster for small n and a high efficiency of decisions is quickly attained when decentralizing: a deep decentralization is thus not needed. Third, n^* decreases strongly with respect to d: d gives the absolute and relative increase rate of administrative costs with respect to decentralization: the larger is d, the more costly is decentralization and the less decentralized is the decision of public investment at optimum.

However, two of the main results may appear paradoxical. First, parameter γ - giving the productivity of the efficient public capital in the Cobb-Douglas production function - has little impact on the optimal level of decentralization. Indeed, in numerical analyses, the curves of the optimal levels n^* of decentralization for the different values of parameter γ are indistinguishable the ones from the others. It is quite paradoxical since parameter γ accounts for the productivity of the efficient public capital: decentralization is needed in order to increase the efficiency of public capital because the efficient public capital is productive. Thus, productivity of public input should matter. However, this result may be intuitively understood. The marginal production losses at centralizing or decentralizing - and therefore the optimal level of decentralization - are due to an amount of efficient public capital smaller than the optimal amount. The marginal losses of decentralization are due to tax competition and the marginal losses of centralization are due to inefficient investments. If γ increases, the marginal productivity of efficient public capital increases for every amount of efficient public capital. But the optimal amount of efficient public capital also increases, so its productivity decreases because of the decreasing factorial returns of the Cobb-Douglas production function. Hence, the actual marginal productivities of the optimal efficient public capital are close to each other for different values of γ , and therefore for different amounts of optimal efficient public capital. The optimal levels of decentralization, equaling the marginal impacts on production of centralization and decentralization forces, are therefore close to each other for different values of parameter γ .

Second, the optimal level of decentralization increases more than proportionally with respect to the ratio of the potential GDP X of the country to the costs c for the fully centralized country to administrate investments of public input for private production. This means that the sizes of the optimal administrative divisions may either increase or decrease with respect to the economic size of the country. It depends on whether the administrative costs c of a fully centralized country are larger or smaller than proportional to its economic size as measured by its potential GDP X. If there are economies of scale in a central administration of investments of public input for private production, the administrative costs c of a fully centralized country are less than proportional to its economic size X. Thus, the optimal level of

decentralization increases more than proportionally with respect to the economic size X of the country. Indeed, the central government of a larger country is further from the local needs in matter of public investments and therefore suffers more from centralization. In addition, as the function of administrative costs increases less and less with respect to decentralization, marginal decentralization is less costly for larger countries as there are more administrative divisions.

However, the dependency between economic growth and decentralization may depend on the reasons of the potential GDP X increase. Mainly, it may increase because the fixed factor L increases, because the potential private capital K_0 increases or because the total factor productivity A increases. If economic growth is driven by population growth or settlement in new territories (an increase of L), administrative costs c of public input may also increase widely and ratio X/c may increase little or even decrease. In that case, the increase of the potential GDP X may result in no additional decentralization. If economic growth is driven by private capital accumulation (an increase of K_0), there is no reason why c should increase substantially. In that case, the increase of potential GDP X should result in an important decentralization. It appears then that Solovian growth should be accompanied with a significant decentralization of the administration of public input for private production. Last, if economic growth is driven by technological progress (an increase of the total factor productivity A), there should not be either substantial c changes and it should lead to more decentralization. However, if this increase of A implies an higher need for private and public capital adaptation - and therefore more expertise in administrating investment of public input for private production - the increase of the administrative costs c may be sufficient to compensate the increase of the potential GDP X. In that case, economic growth driven by technological progress may result in no additional decentralization or even in partial centralization of the administration of public input for private production.

In France, as in other countries, both decentralization and centralization occurred during the last decades. Local governments acquired more autonomy in the administration of public input for private production. In parallel, mergers of small administrative divisions have been encouraged, with both transfers of competency concerning public input and setting of local corporate tax from the municipalities to Public Inter-municipal administration Bodies. It results in a partial centralization of this competency.

In addition, one may think that the solution may be to keep only the benefits of decentralization and forgive the costs. It seems possible if investment decision of public input for private production are taken by decentralized governments but financed by the central authority. It is approximately what has been tried in France with the 2009 reform of the *taxe professionnelle*. This local corporate tax was mainly used by French local governments to finance investments in public input for private production, but it was distorsive (e.g.: Carbonnier, 2008). The local corporate tax has been replaced by national subsidies and a corporate tax based on value added was collected at the central level and then distributed to local governments. In 2010, due to constraints in the public finances of the central government, it has been announced that all these transfers from the national government to the local government facing important

new needs the forthcoming years in terms of public input to actually invest in them. Hence, it appears that not having authority on their own resources limits the actual autonomy of local governments. Indeed, actual decentralization should be both decentralization of competency and collection of the resources, which limits the optimal level of decentralization.

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