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Credit Market Quality, Innovation and Trade

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# Credit Market Quality, Innovation and Trade\*

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## Abstract

Using a general equilibrium model with private R&D financing, this article investigates the impact of trade openness to trade on growth and on welfare for two countries equal in all aspects, except for the quality of credit markets. We show that opening to trade increases growth in the country with better credit markets (North) and decreases it in the other country (South). With respect to trade pattern, South imports high tech goods and exports traditional goods. In terms of welfare, opening to trade may lower the welfare of individuals in the short run, but in the long run all of them are better off under free trade than if they were under autarky.

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# 1 Introduction

The large differences in economic development across countries have been an important concern of economists. The endogenous growth models developed in the 1980's identify technological progress as a major source of growth (Romer, 1988, 1990; and Lucas, 1988). They posit that differences in growth rates stem from disparities in the amount of resources allocated to innovation, as in Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). More recently, Acemoglu, Johnson and Robinson (2001, 2002 and 2006), Engerman and Sokoloff (2007) and Hall and Jones (1999), among others, spot institutions as a fundamental cause of economic growth disparities, revitalizing an old idea in economics with compelling new empirical evidence and theoretical analysis. According to this view, institutions affect economic incentives and, ultimately, decisions related to growth enhancing activities such as investment in innovation.

In parallel, empirical studies have investigated the role of trade openness as a possible engine for growth. Frankel and Romer (1999) find a positive effect of trade on growth, but only moderately significant. They use geography as instrument for trade, which has been also identified as a good instrument for institutions. Going one step further, Dollar and Kraay (2003), Rodrik et al (2004) and Rigobon and Rodrik (2005) try to disentangle the relative roles of trade and institutions in explaining growth disparities. The results are puzzling. In most cases, the impact of trade on growth disappears when institutions are controlled for, and in others it turns out to have a negative impact.

These results suggest that there is an interrelation between trade and institutions on their impact on trade. This paper proposes a mechanism through which trade openness may have opposing impact on growth rates, depending of the country's institutional environment. More specifically, we develop an endogenous growth model in which the amount of resources allocated to innovation depend on the quality of credit markets. In this context, we investigate the impact of trade on growth and on welfare for countries differing in the quality of their credit markets. We show that opening to trade increases growth in countries with better credit markets and decreases it in countries with worse credit markets.

In this paper we focus on credit markets, which we believe is very much related to the quality of the institutional framework in each country, since it reacts to law, political and ethical systems. Several papers link institutional environment to financing, such as Townsend (1979), Stiglitz and Weiss (1981), Aghion and Bolton (1992) and Hart and Moore (1994,1998). Furthermore, La Porta et al (1997) and Djankov et al (2007) present empirical evidence that countries with weaker institutions have also less developed financial markets.

We propose a general equilibrium model to investigate the role of differences in the quality of credit markets as the driving force of differences in innovation rate across countries, and, consequently, in trade patterns and in welfare. We focus on moral hazard as the informational friction disturbing the investor-entrepreneur relationship in R&D. The quality of credit markets affects the intensity of this friction which, in turn, impacts the rate of return in innovation projects. More specifically, our model is inspired on Grossman and Helpman (1991) with respect to the way innovation creates dynamic comparative advantages for countries, and to the way that it becomes an endless

and self-sustained process. We extend the original Grossman-Helpman model by incorporating moral hazard in R&D activity, using the moral hazard model from Tirole (2006). Thereby, we investigate the interaction between credit market quality and R&D intensity. R&D determines innovation rates, which, in turn, affects trade patterns and welfare.

We model credit market imperfections as affecting R&D decisions but not production, since R&D activity is more likely to be sensitive to the quality of credit markets than production. In R&D projects, in general, investors are less informed about entrepreneur actions and failed project have lower liquidation value.

There are two types of final goods in our model economy: a ‘traditional’ final good which uses only labor as input, and a ‘high-tech’ good which requires intermediate goods for its production. Intermediate goods are of different varieties, and they are produced only after being invented through R&D. Credit market quality affects the amount of resources devoted to R&D activity. We analyze the impact of trade in final goods between two countries differing in the quality of their credit markets. We consider alternative assumptions with respect to the possibility of trade of intermediate goods, knowledge spillover and technology transfers across countries.

We find that both innovation rates and wages are higher in countries with better credit markets. International trade increases the innovation rate in countries with better credit markets, while countries with worse credit markets are not able to compete in R&D and lose their innovation sector when opening to trade. Additionally, countries with better credit markets export high-tech goods and import traditional ones. This result is in line with recent empirical evidence on financial development and trade patterns, as in Beck (2002), Hur et al (2006), Levchenko (2007), Manova (2005) and Svaleryd and Vlachos (2005).

With respect to welfare, we find that both countries are better off under free trade compared to autarky in the long run. World innovation rate is higher under free trade, which has a positive effect on the productivity of high-tech good production. In the short run, however, trade liberalization may lower welfare in the country with worse credit markets, since this country loses part of its wealth at opening. Opening to trade is more likely to be welfare enhancing in the short run when there is knowledge spillover across countries and when technology may be transferred internationally.

The paper is organized as follows. Section 2 describes the basic setup of the economy, describing the equilibrium in a closed economy, while section 3 derives the open economy equilibrium. Welfare analyzes is in section 4, and section 5 concludes.

## 2 Model Setup

In this model economy, individuals are at the same time consumers and owners of all firms. There are two types of firms: those that produce final goods, and those that invent and then produce different varieties of the intermediate good. Final consumption goods are either of the traditional type, which uses only labor in production, or high-tech,

using only intermediate goods as input. Both R&D and the production of intermediate goods use only labor in production. Final goods market is competitive, while each intermediate good producer has monopoly power over his variety. We use a representative consumer setup, where all variables are in per capita values.

## 2.1 Consumers

Preferences of the representative consumer are represented by the following utility function:

$$U_t = \int_t^{\infty} e^{-\rho(\tau-t)} \log C(\tau) d\tau,$$

where  $C(\tau) \equiv C_y(\tau)^\sigma C_z(\tau)^{1-\sigma}$ , and  $C_y(\tau)$  and  $C_z(\tau)$  represent the consumption of high-tech ( $y$ ) and traditional ( $z$ ) final goods at time  $\tau$ .  $\rho \in (0, 1)$  is the subjective discount rate.

Consumers can lend or borrow at the instantaneous interest rate  $r(t)$ , so that his intertemporal budget constraint is:

$$\int_t^{\infty} e^{-[R(\tau)-R(t)]} P(\tau) C(\tau) d\tau \leq \int_t^{\infty} e^{-[R(\tau)-R(t)]} Inc(\tau) d\tau + W(t),$$

where  $R(t) \equiv \int_0^t r(\tau) d\tau$ ,  $W(t)$  is the initial wealth,  $Inc(\tau)$  is the net income per capita, which is given by the sum of his income from labor and from the ownership of the firms, as described in subsection 2.4. Finally,  $P$  is the consumer price index:

$$P(\tau) = \left( \frac{P_y(\tau)}{\sigma} \right)^\sigma \left( \frac{P_z(\tau)}{1-\sigma} \right)^{1-\sigma}, \quad (1)$$

where  $P_y$  and  $P_z$  are the prices of goods  $y$  and  $z$ , respectively.

It is straightforward to show that the solution of the consumer intertemporal problem yields the optimal spending evolution:

$$\frac{\dot{E}(\tau)}{E(\tau)} = r(\tau) - \rho, \text{ for } \tau \geq t \quad (2)$$

and the final good consumption:

$$C_z(\tau) = \frac{(1-\sigma)E(\tau)}{P_z(\tau)}, \text{ and} \quad (3a)$$

$$C_y(\tau) = \frac{\sigma E(\tau)}{P_y(\tau)}, \text{ for all } \tau \geq t, \quad (3b)$$

where  $E(\tau) \equiv P(\tau)C(\tau)$  are total consumer expenditures.

## 2.2 Final Goods Production

The traditional good,  $Z$ , is produced using only labor, whereas only intermediate goods are used in the high-tech

good production,  $Y$ , according to:<sup>1</sup>

$$Z = L_z, \quad \text{and} \quad (4a)$$

$$Y = \left[ \int_0^{n(t)} x(j)^\alpha dj \right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1, \quad (4b)$$

where  $n(t)$  is the number of varieties of intermediate goods invented until period  $t$ , and  $x(j)$  and  $L_z$  are the quantity of the intermediate good  $j$  and labor used as inputs in the high-tech and traditional goods production, respectively. Note that the productivity in the high-tech goods sector increases with the number of varieties of intermediate goods. This is an interesting feature for our purposes, since the innovation activity will be the major source of comparative advantages between countries when they are open to trade.

The final goods market is perfectly competitive, hence prices equal average cost:

$$P_z = w, \quad \text{and} \quad (5a)$$

$$P_y = \left[ \int_0^{n(t)} p(j)^{-\frac{\alpha}{1-\alpha}} dj \right]^{-\left(\frac{1-\alpha}{\alpha}\right)}, \quad \forall t, \quad (5b)$$

where  $w$  is the wage rate and  $p(j)$  is the price of intermediate good  $j$ .

Finally, the demand for each variety of intermediate goods is given by:

$$x(j) = \frac{p(j)^{-\frac{1}{1-\alpha}}}{\int_0^{n(t)} p(j)^{-\frac{\alpha}{1-\alpha}} dj} \sigma E, \quad j \in [0, n(t)], \quad (6)$$

where, from equation (3b),  $\sigma E$  is the aggregate expenditure on high-tech good or, equivalently, the aggregate sales revenue of high-tech good.

## 2.3 Intermediate Goods

### 2.3.1 Production

We assume that each intermediate good is manufactured by a single producer, that has monopoly power over it. This assumption may be justified by a positive cost of imitation which, combined with the assumption that firms engage in ex-post price competition in a Bertrand fashion, yields no incentive to imitate. Once invented, an intermediate good is produced using one unit of labor per unit of production. Each producer of an intermediate good faces the demand function given by equation (6).

Due to the symmetry across firms, in the Bertrand-Nash equilibrium the prices of all intermediate goods are

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<sup>1</sup>The indication that the variable is a function of time is suppressed whenever it is not confusing to do so.

equal and given by:

$$p(j) = p_x \equiv \frac{w}{\alpha}, \quad j \in [0, n(t)]. \quad (7)$$

The demand for each intermediate good and profits thereby generated are, respectively:

$$x(j) = x \equiv \frac{\sigma E}{np_x} = \frac{\alpha \sigma E}{nw}, \quad \text{and} \quad (8a)$$

$$\pi(j) = \pi \equiv \frac{(1 - \alpha)\sigma E}{n}, \quad j \in [0, n(t)]. \quad (8b)$$

### 2.3.2 R&D

To be produced, an intermediate good has first to be invented, and invention is achieved through R&D. Following Romer (1990), we assume that past R&D generates public knowledge that renders the next generation of innovation more productive. We model this phenomenon as Grossman and Helpman (1991) do and assume the existence of a public pool of information which contains the stock of accumulated knowledge. The measure of this pool  $K$  is taken to be the same as that of the existing intermediate goods diversity, that is:

$$K(t) = n(t). \quad (9)$$

We are aware that the assumption in equation (9) has some important drawbacks. First, it does not consider nor the obsolescence of past contributions neither any complementarities between different kinds of knowledge. Second, spillovers are likely not to happen instantaneously, as suggested by the equation, but, rather, gradually. Finally, it does not consider heterogeneity between industries with respect to degree of informational content. Nevertheless, we follow previous literature and use this representation for simplicity.

The R&D activity uses only labor as input, but its outcome is uncertain. The research is successful with a probability  $q$ , and  $L_\gamma$  units of labor generate  $aK(t)L_\gamma(t)$  new varieties, where  $a$  is a parameter of labor productivity in R&D and  $K(t)$  is given by equation (9). With probability  $1 - q$ , no new brands are invented. Therefore, the expected outcome of R&D is  $qv(t)an(t)L_\gamma(t)$ , where  $v(t)$  is the value of a blueprint. More precisely,  $v(t)$  is the present value of the stream of future profits  $\pi$  generated by the intermediate good production, that is:

$$v(t) = \int_t^\infty e^{-[R(\tau) - R(t)]} \pi(\tau) d\tau. \quad (10)$$

Entrepreneurs borrow from investors in the credit market to engage in R&D to try and invent new brands. According to the debt contract, if the project is successful, the inventing firm pays an agreed upon amount for its debt. If unsuccessful, there is no payment to the creditor. Following Tirole (2006), we assume that the probability of success of an investment project depends on unobservable actions taken by the entrepreneurs. In particular, ‘good behavior’ yields a higher probability of success,  $q_H$ , and no private benefits to entrepreneurs. ‘Bad behavior’, on its

turn, yields a lower probability of success,  $q_L$ , but entrepreneurs are able to retain a share  $B$  of the investment made in this project. A higher  $B$  means that investors' rights are less protected by the legal or regulatory institutions.

To have an interesting case, we assume that the expected outcome of the project is greater than its costs only if entrepreneurs have good behavior. Clearly, investors will only lend to the inventing firm if the financing contract promotes good behavior from entrepreneurs. Since entrepreneurs' behavior cannot be observed, it cannot be written in a contract. The only way to induce good behavior is to have debt repayments that will make entrepreneurs themselves prefer good behavior. If  $R_b$  is the amount the entrepreneur retain after paying its debt in case of success, good behavior will be induced when the following incentive compatibility constraint is satisfied:<sup>2</sup>

$$q_H R_b \geq q_L R_b + B w L_\gamma. \quad (11)$$

Thus, the firms must retain a minimum of  $R_b^* \equiv \frac{B w L_\gamma}{q_H - q_L}$  of the outcome of a successful project to make the objectives of both parts aligned, as implicit in the incentive compatibility constraint (11).

In addition, investors will only be willing to invest in R&D if the expected rate of return in innovation projects is not smaller than the rate of return of the riskless asset, that is, his participation constraint is:

$$q_H v a n L_\gamma - w L_\gamma - q_H R_b \geq r w L_\gamma.$$

Substituting  $R_b^*$  in the equation above and rearranging terms, we write the participation constraint of investors as:

$$a q_H \frac{v n}{w} \geq \Psi, \text{ where } \Psi \equiv (1 + r) + \frac{q_H}{q_H - q_L} B. \quad (12)$$

Equation (12) states that the project is undertaken only if its returns is strictly higher than  $1 + r$ , since  $\Psi > 1 + r$  due to the credit market imperfection.

We will adopt, without loss of generality, the simplifying assumption that the effective measure of productivity of labor in the R&D activity,  $a q_H$ , is equal to 1. Thus, the investors participation constraint becomes:

$$\frac{v n}{w} \geq \Psi. \quad (13)$$

Note that the left-hand side of equation (13) is the return of the project. When this ratio is greater than  $1 + r$ , the project has positive expected net return.  $\Psi$  may be interpreted as a measure of the credit market imperfection. When  $\Psi = 1 + r$ , all projects with positive expected net return are financed, whereas, when  $\Psi > 1 + r$ , the projects with expected return in the range  $[1 + r, \Psi)$  are not financed, although they have positive expected net return. The

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<sup>2</sup>Note that the condition below implies risk neutrality from entrepreneurs. Although all individuals have concave utility functions, implying risk aversion, they behave as if risk neutral with respect to this investment outcome for two reasons. First, there is no aggregate uncertainty. By the law of large numbers, an exact share of  $q_H$  or  $q_L$  (depending on the entrepreneur's behavior) of the projects undertaken will be successful. Second, as each individual manager owns an equal share of all R&D projects, they can take advantage of the law of large numbers.



higher the value of  $\Psi$ , the larger is the range of projects with positive expected net return that are not financed due to informational asymmetry problems. The credit market imperfection is increasing in the private benefit accrued to managers with bad behavior,  $\frac{\partial \Psi}{\partial B} > 0$ , and decreasing in the degree of observability and/or accountability,  $\frac{\partial \Psi}{\partial (q_H - q_L)} < 0$ .

## 2.4 Individuals Income

All individuals are simultaneously workers, owners of final good firms, entrepreneurs in intermediate goods firms, and investors. Each individual has an equal share of all final goods firms. As those firms are in competitive markets, profits are zero, hence this ownership generates no income. To preserve the conflict of interests between entrepreneurs and investors, we assume that each individual owns an equal share of half of the intermediate goods firms, and invests in the other half. We detail these roles below.

**Entrepreneurs** For simplicity, we assume that there is a sufficiently large number of firms, so that the law of large numbers applies and there is no aggregate uncertainty. At every moment, exactly a fraction  $q_H$  of all investment projects are successful. Each entrepreneur receives every moment the  $\frac{1}{L}$  share of the total  $q_H R_b^*$  for his fraction of these firms.

**Investors** Each individual invests  $\frac{wL_\gamma}{L}$  in innovation projects and pays  $\frac{q_H R_b^*}{L}$  to entrepreneurs of successful projects. At each period the investor receive his share  $\frac{1}{L}$  of the monopoly profit  $\pi = \frac{(1-\alpha)\sigma E}{n}$  from production of each of the  $n$  successfully invented varieties.

**Workers** Individuals are endowed with one unit of labor and they supply it inelastically. Thus, their labor income is given the wage rate  $w$ . There are  $L$  individuals in the economy, hence that is also the total labor supply.

**Aggregate Income** Summing up the individual's revenue for each of his activities, we get:

$$Inc = \frac{w(L - L_\gamma) + (1 - \alpha)\sigma E}{L}. \quad (14)$$

On aggregate, the individual's payments as investors to entrepreneurs cancel out with what they receive as entrepreneurs. All individuals have the same behavior and they participate as workers and entrepreneurs in the same number of firms. Hence, their net income per capita is equal.

## 2.5 Equilibrium in a Closed Economy

We start with the description of the equilibrium in a closed economy. Following Grossman and Helpman (1991), we normalize  $P_y(t)$  and  $P_z(t)$  so that  $E(t) = 1, \forall t$ . Then, in equilibrium, equation (2) yields:

$$r(t) = \rho. \quad (15)$$

Final goods prices from equations (5) can be written as:

$$P_z = w, \quad P_y = \frac{w}{\alpha n^{\frac{1-\alpha}{\alpha}}}, \quad (16)$$

using the equilibrium price of intermediate goods in equation (7).

In a closed economy goods production must equal consumption. From equations (3) and using equation (16) above, we have that:

$$C_Z = Z = \frac{(1-\sigma)}{w}, \quad C_Y = Y = \frac{n^{\frac{1-\alpha}{\alpha}} \alpha \sigma}{w}. \quad (17)$$

Finally, production of intermediate goods and its profits from equations (8) can be written as:

$$x = \frac{\alpha \sigma}{nw}, \quad \pi = \frac{h}{n}, \quad (18)$$

where  $h \equiv \sigma(1-\alpha)$ .

There are two equilibrium conditions from the financing of R&D projects. First, the inequality  $\frac{vn}{w} > \Psi$  (see equation 13) cannot arise in equilibrium when firms maximize profits. If that were the case, entrepreneurs' profits would be higher the larger were investments, leading to unbounded R&D. Since labor supply is fixed, this would not be an equilibrium. Hence, the financing equilibrium condition (FEC) in the R&D activity is given by:

$$\frac{vn}{w} \leq \Psi, \text{ with equality when } \dot{n} > 0. \text{ (FEC)} \quad (19)$$

Second, assuming that agents have access to a riskless bond that pays  $r(t)$ , the non-arbitrage condition implies that the expected rate of return of the risky investment in R&D must be equal to the riskless rate,<sup>3</sup> that is:

$$\pi(t) + \dot{v}(t) = r(t)v(t),$$

which can be rewritten as:

$$\frac{\dot{v}(t)}{v(t)} = r(t) - \frac{\pi(t)}{v(t)}. \quad (20)$$

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<sup>3</sup>We refer to the argument in footnote 2 for the risk neutral behavior of the individual here.

Log-differentiating the FEC (expression 19) when  $\dot{n} > 0$ , that is, in equilibria with positive innovation, we get:

$$\frac{\dot{w}}{w} = \frac{\dot{v}}{v} + \gamma, \quad (21)$$

where  $\gamma \equiv \frac{\dot{n}}{n}$ . Substituting it in the no-arbitrage condition (20), using the profit equation (18) and the FEC, we can write the non-arbitrage condition (NAC) for the innovating country as:

$$\frac{\dot{w}}{w} = \gamma + \rho - \frac{h}{w\Psi}, \text{ when } \dot{n} > 0. \text{ (NAC)} \quad (22)$$

Equilibrium is characterized when the aggregate equity  $V \equiv vn$  (or the aggregate market value of firms) is constant. It means that in equilibrium, we must have that:

$$\frac{\dot{v}}{v} + \gamma = 0.$$

The condition above is equivalent to:

$$\frac{\dot{w}}{w} = 0,$$

given equation (21), which is derived the financing equilibrium condition in R&D when new varieties are produced.

Finally, there is the labor market equilibrium condition. The labor market clears when the sum of demands for labor in R&D ( $L_\gamma$ ), intermediate goods production ( $L_X$ ), and traditional good production ( $L_Z$ ) equals the labor supply  $L$ . We have already seen that one unit of labor produces either one unit of intermediate good or one unit of traditional good. Hence, from equations (17) and (18), we get that  $L_Z = \frac{(1-\sigma)}{w}$  and  $L_X = \frac{\alpha\sigma}{w}$ . As for the demand for labor in R&D, note that, from our assumptions in section 2.3.2, the innovation rate is  $\frac{dn}{dt} = aq_H n L_\gamma$ . Therefore,  $L_\gamma = \gamma$ , using the simplifying assumption that  $aq_H = 1$ . Thus, the labor market clearing condition (LMC) is:

$$\gamma + \frac{(1-h)}{w} = L. \text{ (LMC)} \quad (23)$$

The dynamics of the economy is represented in Figure 1. The LMC curve represent equation (23), while the NAC curve represents equation (22) for  $\frac{\dot{w}}{w} = 0$ . Wages increase at points above and decrease at points below the NAC curve. The economy is in equilibrium at the intersection point of the two curves, and it is represented by point E in the figure. The arrows indicate the equilibrium paths of the economy. Clearly, the steady-state is unstable. The economy must then be always at the equilibrium point E, where wage and innovation rates are constant.<sup>4</sup> The equilibrium values for  $w$  and  $\gamma$  are:

$$\check{\gamma} = \frac{hL - \rho(1-h)\Psi}{h + (1-h)\Psi}, \quad (24)$$

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<sup>4</sup>Note that the equilibria in Grossman and Helpman's (1991) model are also unstable.

and:

$$\tilde{w} = \frac{h + (1-h)\Psi}{(L + \rho)\Psi}. \quad (25)$$

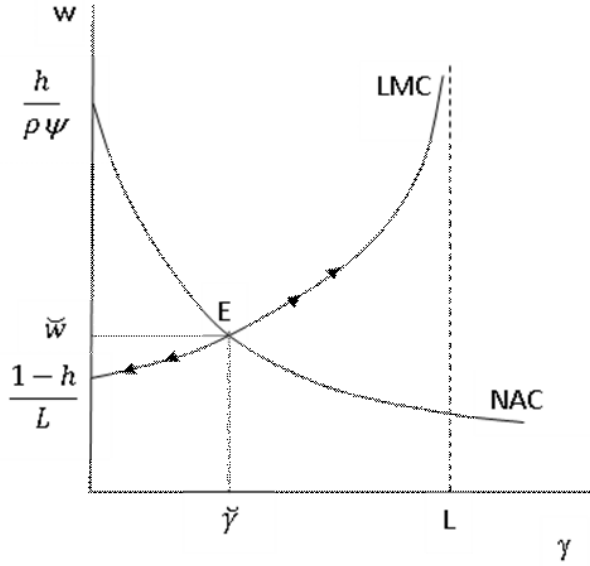


Figure 1: Closed Economy Dynamics

This equilibrium is feasible if  $\tilde{\gamma} \geq 0$ , i.e.:

$$\frac{L}{\rho} \geq \frac{(1-h)\Psi}{h}. \quad (26)$$

In terms of Figure 1, this condition ensures that the NAC curve crosses the vertical axis at a higher point than the point the LMC curve does, so that they cross at a positive value of the innovation rate  $\gamma$ .

The fact that  $\tilde{\gamma}$  and  $\tilde{w}$  are constant means that the allocation of labor remains constant across all activities (R&D, traditional good production, and intermediate goods production). Nevertheless, the ratio  $\frac{Y}{Z}$  increases at the instantaneous rate  $\frac{1-\alpha}{\alpha}\tilde{\gamma}$ , as the productivity of the intermediate goods in good  $Y$ 's production rises with the increase in the number of varieties. Consequently, its price also decreases as the number of intermediate goods varieties increases. The rate of output growth converges to  $\frac{1-\alpha}{\alpha}\tilde{\gamma}$  in the long run.

Figure 2 presents the equilibria in two closed economies differing only in the quality of their credit markets, that is, the value of  $\Psi$ . The LMC is the same for the two economies, but the difference in the credit market quality affects the NAC. The NAC for the country with the best credit market ( $NAC_{best}$ ) is higher than the one for the country with a worse credit market ( $NAC_{worst}$ ). The impact of credit market imperfection on equilibrium is summarized in Proposition 1.

**Proposition 1** *In economies where credit market frictions are less severe (lower  $\Psi$ ), investment in R&D activity and wages are higher and the number of varieties increases faster.*

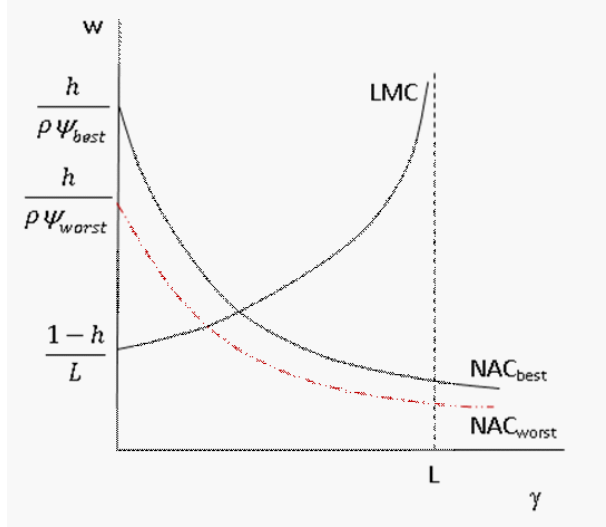


Figure 2: Comparing Economies with Different Credit Market Quality

**Proof.** It is straightforward to check that  $\frac{\partial \tilde{\gamma}}{\partial \Psi} = -\frac{(1-h)h}{[(1-h)\Psi+h]^2} < 0$  and  $\frac{\partial \tilde{w}}{\partial \Psi} = -\frac{h}{\Psi^2(L+\rho)} < 0$ . Investment in R&D is equal to  $wL_\gamma$ , by definition. Given the two previous inequalities, it is clear that investment is also a decreasing function of  $\Psi$ . ■

### 3 The Open Economy

We extend the previous model to a world economy with two countries engaging in international trade, with free flow of financial capital. Since we want to focus on the effects of the quality of credit market, we abstract from other possible differences across countries. Hence, countries are assumed to differ only with respect to the quality of their credit markets, which will be responsible for trade pattern and growth rates, through the differences in innovation rates and wages. Hereafter we denote the country with better credit market as ‘North’ and the other one as ‘South’. The superscript  $i$ ,  $i = N, S$ , is used to denote the two countries, hence  $\Psi^N < \Psi^S$ .

We also assume that both countries have been in autarky for the same length of time before they start to trade. From Proposition 1, we have then that the country with the best credit market will have a larger number of intermediate goods when they open to trade.<sup>5</sup> We study separately the equilibrium under alternative assumptions with respect to knowledge spillover across countries, to trade in intermediate goods, and to technology transfers.

When there is knowledge does not transpose national borders, which is the assumption made in section 3.1, the stock of knowledge available for each country is proportional to the number of intermediate goods that were invented in that country, as in:

$$K^i(t) = n^i(t), \quad \forall t. \quad (27)$$

<sup>5</sup>We argue this is a reasonable assumption, since countries with better institutions tend to have relatively more developed industries intensive in technology. Notice that our results still hold when both countries start trade with the same level of technological development.

Alternatively, in sections 3.2 and 3.3 we assume that there is international knowledge spillover. In this case the stock of knowledge in each country encompasses all varieties invented in both countries, that is:

$$K^i(t) = n(t) \equiv n^N(t) + n^S(t), \quad \forall t. \quad (28)$$

In sections 3.1 and 3.2 we assume away technology transfers, that is, each intermediate good must be produced in the country where it was invented, and we analyze the cases with and without international spillover of knowledge and intermediate goods trade. Section 3.3 studies the case when an intermediate good can be invented in one country and produced in the other through multinational firms, under international knowledge spillover.

As in the closed economy case, we normalize final good prices so that  $E^N + E^S = 1$  at all times.

With international trade of final goods, competition among suppliers of both countries implies that the equilibrium prices are set at the lowest manufacturing cost, hence we have that:

$$P_z = \min \{w^N, w^S\}, \text{ and} \quad (29a)$$

$$P_y = \begin{cases} \min \left\{ \frac{w^N}{\alpha(n^N)^{\frac{1-\alpha}{\alpha}}}, \frac{w^S}{\alpha(n^S)^{\frac{1-\alpha}{\alpha}}} \right\} \\ \text{when intermediate goods trade is not allowed;} \\ \alpha \left[ \frac{n^N}{(w^N)^{\frac{1-\alpha}{\alpha}}} + \frac{n^S}{(w^S)^{\frac{1-\alpha}{\alpha}}} \right]^{-\left(\frac{1-\alpha}{\alpha}\right)} \\ \text{when it is allowed,} \end{cases} \quad (29b)$$

where the price of the high-tech good is derived using equations (7) and (5b).

The demand for the traditional good produced in country  $i$  is the country's market-share,  $s_z^i$ , of global demand for that good. This market share is defined as:

$$s_z^i = \begin{cases} 0, & \text{if } w^i > w^k; \\ s_z^i \in [0, 1], & \text{if } w^i = w^k; \text{ and} \\ 1, & \text{if } w^i < w^k. \end{cases} \quad (30)$$

Hence, production of the traditional good in country  $i$ ,  $Z^i$ , equals demand when:

$$Z^i = s_z^i C_Z, \quad (31)$$

where  $C_Z \equiv C_Z^N + C_Z^S$ .

As for the high-tech final good, its production is:

$$Y^i = \left[ \int_0^{N^i(t)} x^i(j)^\alpha dj \right]^{\frac{1}{\alpha}}, \quad (32)$$

where:

$$N^i(t) = \begin{cases} n^i(t), & \text{when international trade of intermediate goods is not allowed;} \\ n(t) \equiv n^N(t) + n^S(t), & \text{when there is trade of intermediate goods.} \end{cases}$$

When trade in intermediate goods is not allowed, high-tech production costs may differ across countries, and the intermediate goods variety is produced only where its cost is lower. In this case, the demand for each variety of the intermediate good produced at country  $i$  becomes:

$$x^i(j) = s_y^i \frac{\alpha\sigma}{n^i w^i}, \quad (33)$$

where  $s_y^i$  is the high-tech good market share of firms from country  $i$ , and using equations (6) and (7). Given intermediate and final goods prices in equation (29b),  $s_y^i$  is:

$$s_y^i = \begin{cases} 0, & \text{if } \frac{w^i}{\alpha(n^i)^{\frac{1-\alpha}{\alpha}}} > \frac{w^k}{\alpha(n^k)^{\frac{1-\alpha}{\alpha}}} \\ s_y^i \in [0, 1], & \text{if } \frac{w^i}{\alpha(n^i)^{\frac{1-\alpha}{\alpha}}} = \frac{w^k}{\alpha(n^k)^{\frac{1-\alpha}{\alpha}}} \\ 1, & \text{if } \frac{w^i}{\alpha(n^i)^{\frac{1-\alpha}{\alpha}}} < \frac{w^k}{\alpha(n^k)^{\frac{1-\alpha}{\alpha}}} \end{cases}. \quad (34)$$

With trade in intermediate goods both countries will produce the high-tech good since they have equal access to all intermediate goods, which equates production costs. The demand for intermediate goods from equation (6) can be written as:

$$x(j) = \frac{p^i(j)^{-\frac{1}{1-\alpha}}}{\int_0^{n^i(t)} p^i(j)^{-\frac{1}{1-\alpha}} dj + \int_0^{n^k(t)} p^k(j)^{-\frac{1}{1-\alpha}} dj} \sigma,$$

which, given intermediate goods prices from equation (7), becomes:

$$x^i(j) = s_x^i \frac{\alpha\sigma}{n^i w^i},$$

where:

$$s_x^i = \frac{n^i(w^i)^{-\frac{\alpha}{1-\alpha}}}{n^N(w^N)^{-\frac{\alpha}{1-\alpha}} + n^S(w^S)^{-\frac{\alpha}{1-\alpha}}}. \quad (35)$$

In sum, the market for intermediate goods produced in country  $i$  is in equilibrium when its supply equals:

$$x^i(j) = s^i \frac{\alpha\sigma}{n^i w^i}, \quad j \in [0, n], \quad (36)$$

where  $s^i$  equals either  $s_x^i$  or  $s_y^i$ , depending, respectively, on whether trade of intermediate goods is allowed or not. Firm's profits will, then, be equal to:

$$\pi^i = \frac{s^i h}{n} \quad (37)$$

### 3.1 Without International Knowledge Spillover

Given the knowledge pool available for each country when there are no knowledge spillover given by equation (27), the FEC (19) becomes:

$$\frac{v^i n^i}{w^i} \leq \Psi^i, \text{ with equality when } \dot{n}^i > 0. \quad (38)$$

For the innovating country we have that  $v^i n^i = w^i \Psi^i$ , hence its NAC (22) becomes:

$$\frac{\dot{w}^i}{w^i} = \gamma^i + \rho - \frac{s^i h}{w^i \Psi^i}. \quad (39)$$

The new LMC (23) is:

$$\gamma^i + s^i \frac{\alpha \sigma}{w^i} + s_z^i \frac{(1 - \sigma)}{w^i} = L. \quad (40)$$

In current setup, the equilibrium conditions yield the same characterization of the steady state as in the closed economy case, that is:

$$\frac{\dot{w}^i}{w^i} = 0, \quad i = N, S.$$

#### 3.1.1 Equilibrium Without Intermediate Goods Trade

The high-tech good is manufactured only in the country in which its price is lower, as can be seen by the definition of the market share of high-tech goods  $s_y^i$  in equation (34). The country that loses the high-tech good market will also lose all its market for intermediate goods, when its trade is not allowed. All blueprints invented in that country become useless, and no further innovation takes place.

#### Only North innovates

**Proposition 2** *Without trade in intermediate goods and without international knowledge spillover, there is no equilibrium where both countries innovate under free trade of final goods. Only the country with the best credit market innovates, and it captures all the market of high-tech goods.*

**Proof.** Appendix 6.1 proves that there is no equilibrium with both countries innovating. We argue below that the only innovating country is North. ■

In the case of factor price equalization (FPE), North, which is the country that has the larger stock of blueprints when international trade starts, takes the whole high-tech good market (see equation 34). Only in North inter-



mediate goods have a positive value, since intermediate goods trade is not allowed. Therefore, only this country innovates and the situation is self-perpetuating.

In the case of non-FPE, wages would have to be lower in South to render its production of high-tech goods competitive, since, with a smaller stock of blueprints, South is relatively less productive in that sector. With lower wages, South would capture all the market for the traditional good. South would have a higher demand for labor both in the high-tech and in the traditional good production. The demand for labor for innovation would be same in both countries, as they must innovate at the same rate in order for both to remain equally competitive in the high-tech sector with constant wages. Hence, South would have a higher demand for labor than North, which is not possible in equilibrium because, by assumption, their labor supplies are the same. With no FPE, only North innovates in equilibrium.

The equilibrium is computed by the solution of the system composed by the NAC in equation (39) for North, the FEC in equation (38) and the LMC in equation (40) for both countries, where the market share of traditional goods  $s_z^i$  is defined in equation (30), and given that North is the only innovator ( $\gamma^S = 0$ ) and it captures all the market for high-tech good ( $s_y^N = 1$ ).

**FPE equilibrium** Under FPE, by definition, we have that  $\bar{w}^N = \bar{w}^S \equiv \bar{w}$ . Solving for the equilibrium value of  $s_z^N$  in the LMC of South (equation (40) for  $\gamma^S = 0$ ) and substituting it back in the LMC of North, we get that:

$$w = \frac{1-h}{2L-\gamma^N}. \quad (41)$$

In equilibrium, the NAC for North for  $\frac{\dot{w}}{w} = 0$ , on its turn, becomes:

$$w = \frac{h}{(\gamma^N + \rho)\Psi^N}. \quad (42)$$

The world economy will be in equilibrium when equations (41) and (42) are satisfied at the same time, as well as the LMC for South. They yield the following equilibrium values for the innovation rate in North, wages, and North market share of the traditional good:

$$\bar{\gamma}^N = \frac{2hL - \rho(1-h)\Psi^N}{h + (1-h)\Psi^N}, \quad (43a)$$

$$\bar{w} = \frac{h + (1-h)\Psi^N}{(2L + \rho)\Psi^N}, \quad (43b)$$

$$\bar{s}_z^N = \frac{(1-\sigma)(L + \rho)\Psi^N - L(h + \alpha\sigma\Psi^N)}{(1-\sigma)(2L + \rho)\Psi^N} \quad (43c)$$

remembering that  $h \equiv \sigma(1 - \alpha)$ .

Figure 3 compares the closed economy equilibrium to the one of the open economy with FPE. The NAC for North is the same in both cases, while the LMC changes as pictured in Figure 3. The picture shows that, for North,

wages are lower and innovation rate higher under free trade. We will see in the next section, though, that real wages increase with trade.<sup>6</sup> Moreover, the rate of innovation under free trade (equation (43a)) is more than two times higher than that of North under autarky (24).

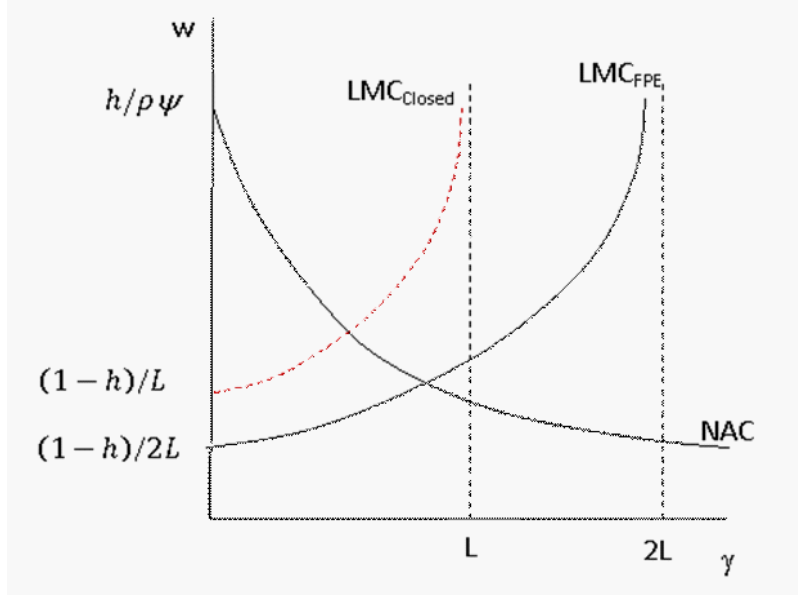


Figure 3: No knowledge spillover and no trade in intermediate goods: FPE equilibrium in North

We can also see from the picture that the condition for a non-negative innovation rate is easier to be met under free trade than in the closed economy. The condition for  $\bar{\gamma}^N > 0$  in the FPE for the open economy is given by:

$$\frac{L}{\rho} > \frac{(1-h)\Psi^N}{2h}, \quad (44)$$

which is less strict than the condition for equilibrium in the closed economy, given by inequality (26).

Finally, it must be the case that  $\bar{s}_z^N \in [0, 1]$ . It is straightforward to check that  $\bar{s}_z^N$  is non-negative for all parameters values, and it will be smaller than one whenever:

$$\frac{L}{\rho} \leq \frac{(1-\sigma)\Psi^N}{h + [\alpha\sigma - (1-\sigma)]\Psi^N}. \quad (45)$$

The requirement that  $\bar{s}_z^N \leq 1$  also guarantees that  $\bar{\gamma}^N \leq L$ , so that the LMC for North will be satisfied.

**Non-FPE equilibrium** In the non-FPE equilibrium, the only possible configuration for wages in equilibrium is  $\hat{w}^N > \hat{w}^S$ . It implies that  $\hat{s}_z^N = 0$ , and the LMC for North becomes:

$$w^N = \frac{\alpha\sigma}{L - \gamma^N}. \quad (46)$$

<sup>6</sup>This will also be true for the non-FPE equilibrium represented in Figure 4.

Its NAC for  $\frac{\dot{w}}{w} = 0$  remains unchanged, and can be represented by equation (42). These two curves are represented in Figure 4, and their interception yields the equilibrium wage in North and its innovation rate. Wages in South are determined by the LMC for that country. The equilibrium values for these variable are:

$$\hat{\gamma}^N = \frac{hL - \rho\alpha\sigma\Psi^N}{h + \alpha\sigma\Psi^N} \quad (47a)$$

$$\hat{w}^N = \frac{h + \alpha\sigma\Psi^N}{(L + \rho)\Psi^N} \quad (47b)$$

$$\hat{w}^S = \frac{1 - \sigma}{L}, \quad (47c)$$

$$s_z^N = 0 \quad (47d)$$

Here, again, the condition for a non-negative innovation rate is weaker than in the closed economy in inequality (26). It is given by:

$$\frac{L}{\rho} > \frac{\alpha\sigma\Psi^N}{h}. \quad (48)$$

Note that this inequality is equal to (44) when  $1 - \alpha = \alpha\sigma$ . In addition, this equilibrium is feasible only if  $\hat{w}^N > \hat{w}^S$ , i.e.,

$$\frac{L}{\rho} > \frac{(1 - \sigma)\Psi^N}{h + [\alpha\sigma - (1 - \sigma)]\Psi^N}. \quad (49)$$

This last feasibility condition is exactly the opposite of the condition for an FPE equilibrium to exist, in inequality (45).

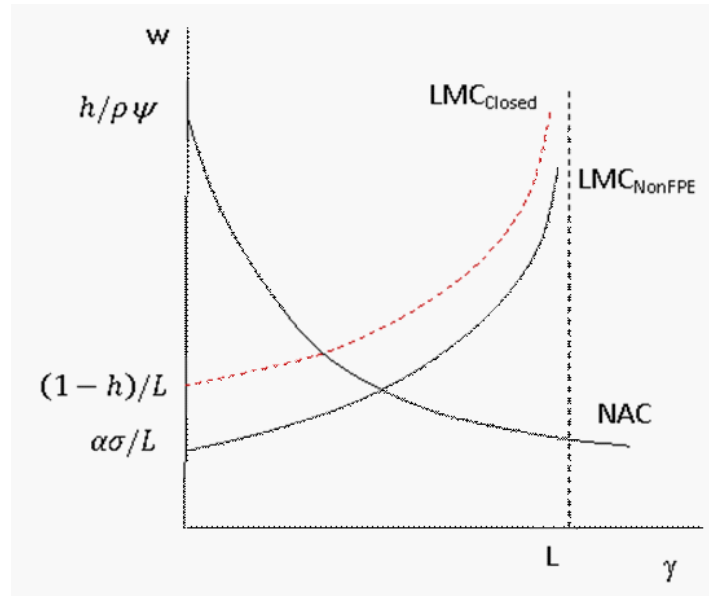


Figure 4: No knowledge spillover and no trade in intermediate goods: Non-FPE equilibrium in North

### 3.1.2 Equilibrium With Intermediate Goods Trade

When trade of intermediate goods is allowed, countries may produce intermediate goods even if they do not produce the high-tech good. In this case there is no waste of past invention when countries engage in international trade.

#### Only North innovates

**Proposition 3** *Without international knowledge spillover, there is no equilibrium where both countries innovate under free trade of final and intermediate goods. Only North innovates, and both economies tend to the equilibrium of the case without international trade of intermediate goods as time goes to infinity, which is defined either by the set of equations (43), in the case of FPE, or (47), in case of non-FPE.*

**Proof.** *Appendix 6.2 proves that only North innovates in all possible equilibria. Below we show that both economies tend to the equilibrium without trade in intermediate goods. ■*

**FPE equilibrium** We start with the equilibrium with FPE. The combination of the LMC for North and South, using equation (40), yields equation (41), as in the case with no trade in intermediate goods. As for the NAC for North for  $\frac{\dot{w}^i}{w^i} = 0$ , it is now given by:

$$w = \frac{s_x^N h}{(\gamma^N + \rho) \Psi^N} \quad (50)$$

From equation (35) we have that:

$$s_x^N = \frac{n^N}{n^N + n^S}. \quad (51)$$

Since only North innovates, clearly  $s_x^N \rightarrow 1$  as time goes to infinity. Hence, the NAC in equation (50) approaches equation (42) in the case of no trade in intermediate goods.

Figure 5 represents the FPE equilibrium. When the economies start to trade, wages and innovation rate in North are given by the intersection of the curves  $LMC_{FPE}$  and  $NAC_{TradeInt}$ . The latter converges to the curve  $NAC$  as  $s_x^N \rightarrow 1$ . Wages and innovation rate follow continuously the intersection of the  $LMC_{FPE}$  and  $NAC_{TradeInt}$  curves, approaching the equilibrium point  $E$  when time goes to infinity. The equilibrium values of the variables at point  $E$  are given by equation (43). The feasibility conditions for this equilibrium are the same ones for the FPE equilibrium without trade in intermediate goods, given by equations (44) and (45).

**Non-FPE equilibrium** In the non-FPE equilibrium, the dynamics of the NAC for  $\frac{\dot{w}}{w} = 0$  is the same as in the FPE equilibrium just described, while the LMC changes. Wages in North are larger than in South and the latter captures all the market for the traditional good. The LMC for North becomes:

$$w^N = \frac{s_x^N \alpha \sigma}{L - \gamma^N}. \quad (52)$$

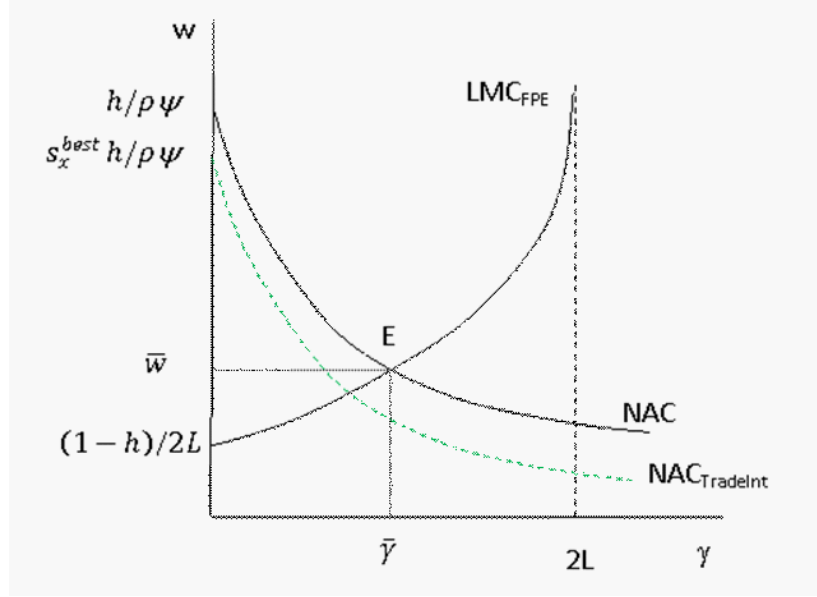


Figure 5: No knowledge spillover but with trade in intermediate goods: FPE equilibrium in North

The dynamics is represented in Figure 6. The dotted lines  $NAC_{TradeInt}$  and  $LMC_{TradeInt}$  represent equations (50) and (52) for  $s_x^N < 1$ , while the solid ones represent  $NAC$  and  $LMC$  when  $s_x^N = 1$ . It is possible to show that  $LMC_{TradeInt}$  and  $NAC_{TradeInt}$  cross at exactly the same innovation level as in the long run equilibrium where  $s_x^N = 1$ .<sup>7</sup> That is, in the path to the long run equilibrium the innovation rate is always constant and equal to  $\hat{\gamma}^N$  in equation (47a), while wages in North increase towards  $\hat{w}^N$  in equation (47b) as  $s_x^N$  approaches one.

The condition for  $w^N > w^S$  now becomes:

$$\frac{L}{\rho} > \frac{[(1 - s_x^N) \alpha \sigma + (1 - \sigma)] \Psi^N}{s_x^N h - (1 - s_x^N) \alpha \sigma + [s_x^N \alpha \sigma - (1 - \sigma)] \Psi^N}, \quad (53)$$

which becomes equal to the condition in the case with no trade in intermediate goods in inequality (49) when  $s_x^N = 1$ .

### 3.2 With International Knowledge Spillover

We now assume that domestic innovating firms can access a worldwide pool of knowledge under free trade, as represented in equation (28).

The FEC becomes:

$$\frac{v^i n^i}{w^i} \leq s_n^i \Psi^i, \text{ with equality when } \dot{n}^i > 0, \quad (54)$$

<sup>7</sup>We have that  $\frac{\partial w^{best}}{\partial s_x^{best}} = \frac{h}{(\gamma^{best} + \rho) \Psi^{best}}$  for the  $NAC$  in equation (50) and  $\frac{\partial w^{best}}{\partial s_x^{best}} = \frac{\alpha \sigma}{(L - \gamma^{best})}$  for the  $LMC$  in equation (52). It is straightforward to show that these two partial derivatives are equal when  $\gamma^{best} = \hat{\gamma}^{best}$  from equation (47a). Hence, the  $NAC$  and  $LMC$  curves have the same vertical change at that level of innovation rate.

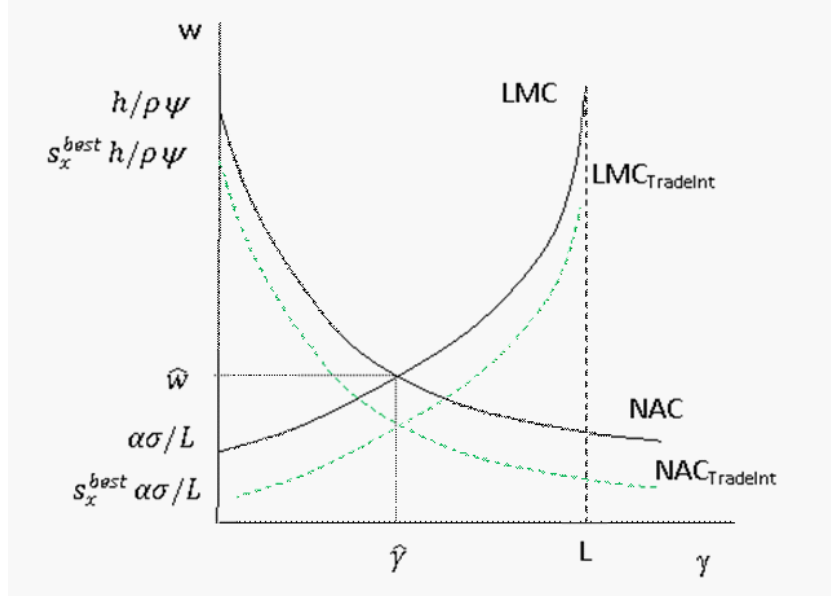


Figure 6: No knowledge spillover but with trade in intermediate goods: Non-FPE equilibrium in North

where  $s_n^i \equiv \frac{n^i}{n}$ . Differentiating this equation in the case of equality, we get:

$$\frac{\dot{v}^i}{v^i} + \gamma^i = \frac{\dot{s}_n^i}{s_n^i} + \frac{\dot{w}^i}{w^i}. \quad (55)$$

We also have, from the differentiation of the definition of  $s_n^i$ , that:

$$\frac{\dot{s}_n^i}{s_n^i} = s_n^j (\gamma^i - \gamma^j). \quad (56)$$

Substituting (56) into (55) yields:

$$\frac{\dot{w}^i}{w^i} = \frac{\dot{v}^i}{v^i} + \sum_{k=N,S} s_n^k \gamma^k. \quad (57)$$

Finally, we substitute the non arbitrage condition (20) into the previous expression to derive the new NAC for the innovating country:

$$\frac{\dot{w}^i}{w^i} = \sum_{k=N,S} s_n^k \gamma^k + \rho - \frac{s^i h}{s_n^i w^i \Psi^i}, \quad (58)$$

where  $s^i$  is either  $s_y^i$ , the share in the high-tech good production in equation (34), or  $s_x^i$ , the share in the intermediate good production in equation (35), depending on whether trade in intermediate good is allowed or not.

The LMC in each country is given by:<sup>8</sup>

$$s_n^i \gamma^i + s^i \frac{\alpha \sigma}{w^i} + s_z^i \frac{(1 - \sigma)}{w^i} = L. \quad (59)$$

<sup>8</sup>The labor resources used in R&D activity by country  $i$ ,  $L_{\gamma^i}$ , is calculated as follows. We have that the innovation rate is given by  $\frac{dn^i}{dt} = aq_H n L_{\gamma^i}$ . Using the simplifying assumption that  $aq_H = 1$ , we have that  $L_{\gamma^i} = \frac{1}{n} \frac{dn^i}{dt} = s_n^i \gamma^i$ .

As usual, equilibrium is characterized when the aggregate equity value is constant. Using the result in equation (55), we have that:

$$\frac{\dot{V}^i}{V^i} = \frac{\dot{v}^i}{v^i} + \gamma^i = \frac{\dot{w}^i}{w^i} + \frac{\dot{s}_n^i}{s_n^i} = 0, \quad i = N, S. \quad (60)$$

Note that equation (60) does not ensure that wages are constant. According to the equation, they may decrease while the blueprints share of the country increases, in such a way as to keep the value of the aggregate equity constant. This cannot be considered an equilibrium as labor allocation would be changing across productive activities. Thus, we also require that, in equilibrium:

$$\frac{\dot{w}^i}{w^i} = \frac{\dot{s}_n^i}{s_n^i} = 0. \quad (61)$$

### 3.2.1 Equilibrium Without Intermediate Goods Trade

When there is no trade in intermediate goods, these goods have no value in the country that loses the high-tech good market, so there is no further innovation in that country after trade opening. It is true that the inventions in the country that does not produce the high-tech good would increase the world stock of knowledge, thereby lowering the cost of new inventions also in the other country. This positive externality, however, has no market value. Innovation can only take place in both countries if the high-tech good price is equal across them.

**Both countries may innovate** There is no FPE equilibrium with both countries innovating for the same reason there is no such equilibrium in the case of no knowledge spillover, established in Proposition 2. There is, however, the possibility of non-FPE equilibrium, which is characterized by the FEC in equation (54), the NAC in equation (58) and the LMC in equation (59) for both countries, where the high-tech good market share  $s_y^i$  is defined in equation (34) and the market share of traditions goods  $s_z^i$  in equation (30). Nevertheless, this equilibrium is only possible if North detains a specific share of world blueprints at the time countries start to trade, as established in equation (98) in the appendix. Thus, this equilibrium is not likely to arise, and we let its exposition to the appendix.

**North always innovates** The existence of knowledge spillover does not change the result that if only one country innovates, it will be the one with the greater number of blueprints when international trade starts, which is the country with better credit markets.

The equilibria allocation in this setup are equal to those that arise when there are no knowledge spillover, presented in section 3.1.1.

### 3.2.2 Equilibrium With Intermediate Goods Trade

**Both countries may innovate** An analysis analogous to the case with no trade in intermediate goods applies here. A non-FPE equilibrium with both countries innovating is possible, but only if, at the moment the economies

open to trade, the share of blueprints in North equals exactly the solution of equation (104) in the appendix. At any other blueprints distribution across countries when trade starts, North is the only one to innovate in equilibrium. This is not an interesting equilibrium, since it will occur at a very specific state of the economies. The description of this equilibrium is in the appendix.

### North always innovates

**Proposition 4** *When there are international knowledge spillover, North innovates in any equilibrium with innovation, under free trade of final and intermediate goods. If North is the only one to innovate, both economies tend to the equilibrium of the case without international knowledge spillover when there is no trade of intermediate goods.*

**Proof.** In the appendix. ■

**FPE equilibrium** Under FPE we have that  $s_x^N = s_n^N$ . Using equation (58), the NAC with  $\frac{\dot{w}^i}{w^i} = 0$  for North becomes:

$$w = \frac{h}{(s_n^N \gamma^N + \rho) \Psi^N}, \quad (62)$$

while the combination of the LMC for both countries, from equation (59), yields:

$$w = \frac{1 - h}{2L - s_n^N \gamma^N}. \quad (63)$$

Since only North innovates,  $s_n^N \rightarrow 1$ , and equations (62) and (63) tend to equations (42) and (41), respectively. The dynamics is represented in Figure 7. The curves NAC' and LMC'<sub>FPE</sub> represent equations (62) and (63), respectively, for  $s_n^N < 1$ . They cross at the same level of wages at which curves NAC and LMC<sub>FPE</sub> do, which represent equations (42) and (41), respectively.<sup>9</sup> Hence, wages are constant and given by equation (43b) at all times, while the rate of innovation equals:

$$\tilde{\gamma}_{FPE}^N = \frac{2hL - \rho(1 - h) \Psi^N}{s_n^N [h + (1 - h) \Psi^N]}, \quad (64)$$

tending to  $\bar{\gamma}$  from equation (43a) as  $s_n^N \rightarrow 1$ .

Comparing equations (64) and (43a), we see that the rate of innovation is larger in this case, under knowledge spillover and trade in intermediate goods, than in the case with no trade in intermediate goods. Nevertheless, the amount of labor in R&D,  $L_\gamma = s_n^N \tilde{\gamma}_{FPE}^N$ , is the same as in the two cases. This result is due to the fact that the inventions from the non-innovating country are still produced in the current case, which increases the stock of knowledge and renders R&D more productive.

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<sup>9</sup>We have that  $\frac{\partial \gamma^{best}}{\partial s_n^{best}} = -\frac{\gamma^{best}}{s_n^{best}}$  for both the NAC in equation (62) and the LMC in equation (63). Hence, the NAC and LMC curves have the same horizontal displacement when departing from a crossing point of the two of them.



North's share of traditional goods equals:

$$\tilde{s}_z^N = \frac{[1 - \sigma + (1 - s_n^N) \alpha \sigma] (2L + \rho) \Psi^N - L [h + (1 - h) \Psi^N]}{(1 - \sigma) (2L + \rho) \Psi^N}$$

This is an equilibrium if  $0 \leq s_z^N \leq 1$ . The first inequality is ensured when:

$$\frac{L}{\rho} \geq \frac{(1 - s_n^N) \rho \alpha \sigma \Psi^N}{h + (1 - h) \Psi^N - 2(1 - s_n^N) \alpha \sigma},$$

while the second requires that:

$$\frac{L}{\rho} \leq \frac{[(1 - \sigma) + (1 - s_n^N) \alpha \sigma] \Psi^N}{h + [(2s_n^N - 1) \alpha \sigma - (1 - \sigma)] \Psi^N}. \quad (65)$$

Note that the first inequality is always satisfied for a  $s_n^N$  sufficiently close to one, while the second approaches the corresponding condition (inequality (45)) of the case without knowledge spillover nor trade in intermediate goods when  $s_n^N \rightarrow 1$ . Finally, the condition for a positive innovation rate is the same as in the case without knowledge spillover, given by inequality (44).

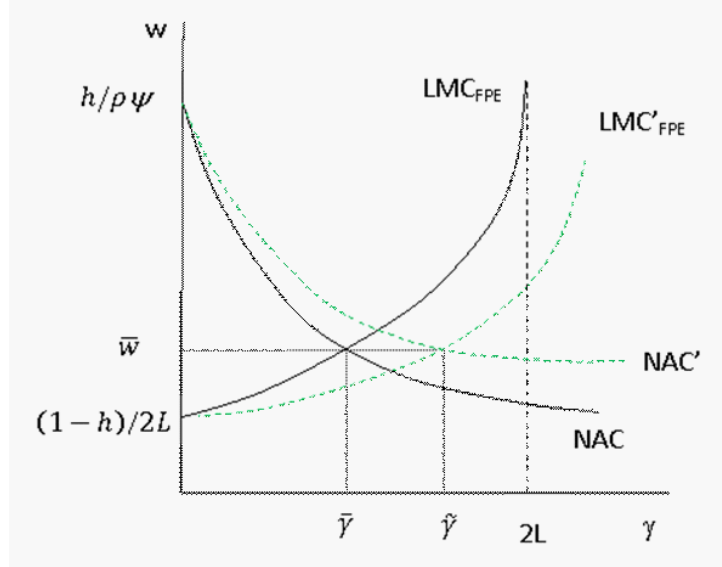


Figure 7: With knowledge spillover and with trade in intermediate goods: FPE equilibrium in North

**Non-FPE equilibrium** In the non-FPE equilibrium, the NAC for North for  $\frac{\dot{w}^N}{w^N} = 0$  and the combination of the LMC for both countries become, respectively:

$$w^N = \frac{s_x^N h}{(s_n^N \gamma^N + \rho) s_n^N \Psi^N}, \quad (66)$$

$$w^N = \frac{s_x^N \alpha \sigma}{L - s_n^N \gamma^N}. \quad (67)$$

Figure 8 represents the non-FPE equilibrium. NAC' and LMC' curves represent equations (66) and (67), while NAC and LMC stand for equations (42) and (46), in which  $s_x^N = s_n^N = 1$ . In order to understand the relative placement of the curves, let us investigate the effects of changes in  $s_n^N$  and in  $s_x^N$  in turn.  $NAC^0$  and  $LMC^0$  represent the curves for  $s_x^N = 1$  and  $s_n^N < 1$ . It is straightforward to check that the displacement of the NAC curve to the right is larger than that of LMC, so that they cross at a higher wage rate than the original NAC and LMC. For NAC' and LMC' we have that  $s_x^N < 1$  as well. They are placed further down compared to  $NAC^0$  and  $LMC^0$ . Since  $\frac{\partial w^N}{\partial s_x^N} = \frac{\gamma^N}{s_x^N}$  for both curves, they will cross at the same innovation rate. Furthermore, given that  $s_x^N > s_n^N$ , the final placement of the NAC' curve will be to the right of NAC and to the left of the point where  $LMC^0$  crosses the wage  $\hat{w}$ .<sup>10</sup> Hence, NAC' and LMC' cross at a lower wage rate than NAC and LMC do, and at the innovation rate equal to:

$$\tilde{\gamma}_{NFPE}^N = \frac{hL - s_n^N \rho \alpha \sigma \Psi^N}{s_n^N (h + s_n^N \alpha \sigma \Psi^N)}.$$

NAC' and LMC' approach NAC and LMC as  $s_x^N$  and  $s_n^N$  tend to one. In the path to the steady state equilibrium wages North increases while the innovation rate decreases, tending to wages and innovation rates given by equations (47b) and (47a), respectively.

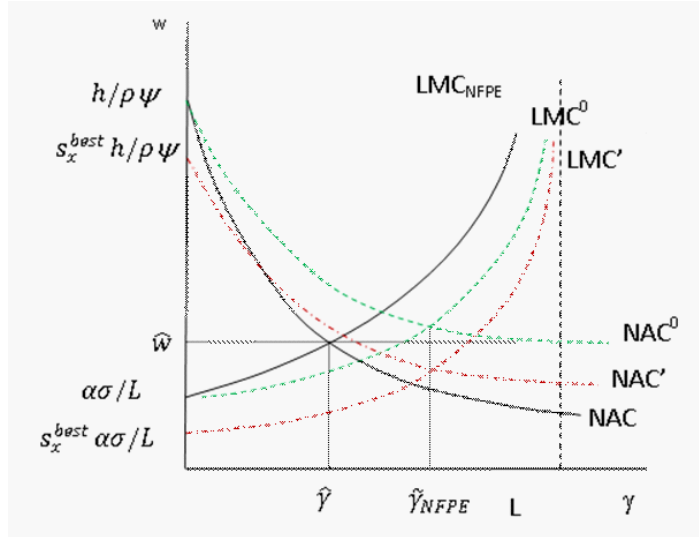


Figure 8: With knowledge spillover and with trade in intermediate goods: Non-FPE equilibrium in North

In this equilibrium it is necessary that  $w^N > w^S$ . While it is not possible to write an explicit expression for this inequality, we know that:

$$\frac{L}{\rho} > \frac{s_n^N [(1 - \sigma) + (1 - s_n^N) \alpha \sigma] \Psi^N}{s_x^N h + s_n^N [(2s_x^N - 1) \alpha \sigma - (1 - \sigma)] \Psi^N}, \quad (68)$$

which would be exactly the opposite of condition (65) for the FPE equilibrium if  $s_n^N = s_x^N$ . Although  $s_x^N$  is itself a function of  $\frac{w^N}{w^S}$ , we know that  $s_x^N \rightarrow 1$  as  $s_n^N \rightarrow 1$ . Hence, for  $s_n^N$  sufficiently close to one, condition (68) approaches

<sup>10</sup>Note that NAC' would cross  $LMC^0$  at  $w = \hat{w}$  if  $s_x^{best} = s_n^{best}$ .

condition (49). Finally, the condition for a positive innovation rate is:

$$\frac{L}{\rho} > \frac{s_n^N \alpha \sigma \Psi^N}{h},$$

which is equal to condition (48) when  $s_n^N = 1$ .

### 3.3 Multinational Corporations

Here, we relax the assumption that invention and manufacturing of an intermediate good variety must be located in the same country. Firms may now explore the comparative advantages across countries by producing the intermediate goods in a country different from the one where it was invented. We denote these firms multinational corporations (MNC).

The variables  $p^i$ ,  $x^i$ ,  $\pi^i$  denote no longer price, demand and profits of intermediate good varieties *produced* in country  $i$ , but, rather, *invented* in that country. In previous sections one could use the both interpretations interchangeably, since production and invention of a variety were located in the same place. Now the whole production of intermediate goods is located wherever the wage is lower, no matter where these goods were invented. These variables are given by:

$$p^i = \frac{1}{\alpha} \min \{w^N, w^S\}, \quad (69)$$

$$x^i = \frac{s_n^i \alpha \sigma}{\min \{w^N, w^S\}}, \text{ and} \quad (70)$$

$$\pi^i = s_n^i h. \quad (71)$$

The FEC is the same as in the case without multinational corporations, given by expression (54). With the same procedure used to derive the NAC in equation (58), we get the NAC for the case with multinationals:

$$\frac{\dot{w}^i}{w^i} = \sum_{k=N,S} s_n^k \gamma^k + \rho - \frac{h}{w^i \Psi^i}. \quad (72)$$

As now the production of intermediate and traditional goods is located wherever the wage is lower, we have that  $s^i = s_z^i$ , and  $s_z^i$  is still defined by equation (30). The LMC can, thus, be written as:

$$s_n^i \gamma^i + \frac{(1-h) s_z^i}{\min \{w^N, w^S\}} = L. \quad (73)$$

Finally, the steady-state equilibrium characterization remains the same as in equations (60) and (61) in the previous section.

**Both countries may innovate** The equilibrium is characterized by the FEC (54), the NAC (72), and the LMC (73) for both countries. From equation (56), it is clear that  $\gamma^N = \gamma^S$  in the steady state. The characterization of this equilibrium is in the appendix. Like the other cases where both countries may innovate, the equilibrium is only possible for specific share of varieties in North when they start to trade. In this case, this share is given by equation (105d) in the appendix. Thus, this equilibrium is no likely to arise.

**North always innovates** Here there is no equilibrium with only South innovating for the same reason that there was no such equilibrium in the case without multinationals, in section 3.2. The NAC (72) is satisfied for North, which is the innovator, while LMC (73) are satisfied for both countries. The FEC (54) is satisfied with equality for North, and strict inequality for the South.

It is easy to check that with FPE the NAC and LMC turn out to be the same as in the previous case, with knowledge spillover and trade in intermediate goods.

On its turn, the non-FPE equilibrium is given by:

$$\tilde{\gamma}^N = \frac{L}{s_n^N}, \quad (74a)$$

$$\tilde{w}^N = \frac{h}{(L + \rho) \Psi^N} \quad (74b)$$

$$\tilde{w}^S = \frac{1 - h}{L} \quad (74c)$$

$$\tilde{s}_z^N = 0 \quad (74d)$$

and is feasible if  $\tilde{w}^N > \tilde{w}^S$ , i.e.:

$$\frac{L}{\rho} > \frac{(1 - h) \Psi^N}{h - (1 - h) \Psi^N}.$$

Under this non-FPE equilibrium North specializes in R&D, while South produces all final and intermediate goods.

### 3.4 Trade and Growth: Summary of results

We have seen that, in autarky, the economy with better credit markets grows faster, since its innovation rate is larger. We have then analyzed the open economy under different hypothesis with respect to trade of intermediate goods, knowledge spillover across countries and technology transfer. In all cases studied North always innovates, and its innovation rate is larger compared to the one in autarky. With respect to the South, in general it does not invest in R&D any longer after opening. There is a possibility it also innovates when there is knowledge spillover across countries, but this equilibrium is not likely to arise. Hence, opening to trade increases growth in the country with better institutions and decreases it in the other country.

The high tech good is produced only in North, so that South imports high tech goods and exports traditional

goods. When trade in intermediate goods is not allowed, South suffers a capital loss. It loses its stock of blueprints which become useless since the country no longer produces the high tech good. Moreover, growth in North is even higher when there are international knowledge spillover and trade in intermediate goods. In this case South suffers no capital loss and continues to produce the previously invented blueprints, which increases productivity not in both high tech good production and R&D activity in North.

Finally, we have considered the case where intermediate goods can be produced in a country different from the one where it was invented. A full specialization equilibrium may arise in this case, in which the North specializes in R&D while the South produces all final and intermediate goods. With all its resources dedicated to R&D, innovation in North achieves its highest value.

## 4 Welfare Analysis

We compare the welfare in autarky and in free trade for North and South, which differ only with respect to the quality of their credit markets. The possibility of knowledge accumulation as new blueprints are invented allows for different impacts of trade opening in the short and in the long run. Since individuals have the same preferences in both countries, comparing welfare is equivalent to comparing their purchasing power. Therefore, we restrict our analysis to investigating the purchasing power of the residents of each country in each of the cases studied.

### 4.1 Closed economy

The purchasing power of country  $i$ 's residents in autarky,  $G_{aut}^i$ , is given by their total net income, given by equation (14), divided by the price index in autarky  $P_{aut}^i$ :

$$G_a^i = \frac{w_{aut}^i(L - L_\gamma^i) + h}{P_{aut}^i} \quad (75a)$$

Using the definition of the price index in equation (1) and the equilibrium prices of final and intermediate goods in equations (5a), (5b) and (7), we have that the price index in autarky is:

$$P_{aut}^i = \frac{w_{aut}^i}{z (n^i)^{\frac{h}{\alpha}}}, \quad (76)$$

where  $z \equiv (\alpha\sigma)^\sigma (1 - \sigma)^{1-\sigma}$ . It depends positively on wages and negatively on the number of varieties of the intermediate goods.

Substituting equation (76) into (75a), we have the welfare in autarky:

$$G_{aut}^i = \left[ (L - L_\gamma^i) + \frac{h}{w_{aut}^i} \right] z (n^i)^{\frac{h}{\alpha}} = \quad (77)$$

$$= \left[ \frac{(L + \rho) \Psi^i}{h + (1 - h) \Psi^i} \right] z (n^i)^{\frac{h}{\alpha}}, \quad (78)$$

where the steady state values of  $L_\gamma^i$  and  $w_{aut}^i$  are in equations (24) and (25), respectively. The purchasing power is a decreasing function of investment in R&D and of wages. Since, from proposition 1, these two variables are higher in North, its residents' welfare is lower than that of South residents when the stock of knowledge is the same across countries. The reason for this result is that residents of North invest a higher share of their income in R&D projects, so that, given the same stock of knowledge, they consume less than residents in South. Nevertheless, the number of blueprints increases faster in North, leading to a faster decrease of its price index. If initially both countries have zero blueprints, there will be a moment  $t^*$  where the lower price index compensates the higher level of investment in North. Thereafter, the residents of North are better off, and the difference in welfare across countries increases continuously.

Period  $t^*$  is implicitly defined as the moment when  $G_{aut}^N = G_{aut}^S$  as in:

$$\frac{n(t^*)^N}{n(t^*)^S} = \left[ \frac{\Psi^N (1 - \alpha\sigma + h\Psi^S)}{\Psi^S (1 - \alpha\sigma + h\Psi^N)} \right]^{\frac{\alpha}{h}} \quad (79)$$

Note that  $\frac{\partial t^*}{\partial \left(\frac{\Psi^S}{\Psi^N}\right)} < 0$ , therefore the higher is the gap of quality in the credit markets, the shorter is the period over which the welfare of the residents of North is smaller than the welfare of residents abroad.

## 4.2 Free Trade

We assume that there is free flow of financial capital when there is international trade, therefore all individuals can invest in any innovation project, no matter whether he is resident of the country where the investment project takes place or not.<sup>11</sup> Since all individuals have the same logarithmic preferences, all of them devote the same share of income to investment in R&D.

Let us denote  $k^i(t)$  the share of the world capital (total number of blueprints) that belongs to residents of country  $i$ , and let  $T$  be the moment the countries open to trade. We have that:

$$k^i(t) = \frac{n^i(T) + \frac{w^i}{w^N + w^S} [n(t) - n(T)]}{n(t)}, \text{ for } t \geq T \quad (80)$$

where  $n = n^N + n^S$ . Notice that in autarky  $k^i(t) = s_n^i(t)$ , where  $k^i$  represents the gross national product in the intermediate goods sector, while  $s_n^i$  represents the gross domestic product in that sector. When countries are

<sup>11</sup>Under the alternative assumption that each resident can only invest in its own country's R&D we would get the obvious result that Northern residents are richer in the long run because North grows faster.

autarkies in goods and asset markets,  $k^i$  and  $s_n^i$  must be equal.

When there is no trade in intermediate goods, all blueprints of South become useless, therefore  $k^N(T) = 1$  and  $k^S(T) = 0$ . With trade in intermediate goods we have that  $0 < k^S(T) < k^N(T) < 1$ . In all cases, nevertheless, those shares converge to:

$$\lim_{t \rightarrow \infty} k^i(t) = \frac{w^i}{w^N + w^S}. \quad (81)$$

It means that, under FPE, residents of both countries share equally the profits of intermediate goods firms in the long run, whereas when non-FPE equilibria arises North residents have a higher share of profits in the long-run.

The purchasing power of country  $i$ ' residents under free trade equals:

$$G_{free}^i(t) = \frac{w^i L - w^N L_\gamma + h k^i(t)}{P_{free}}, \quad (82)$$

when only North innovates, and it equals:

$$G_{free}^i(t) = \frac{w^i L - \left( \sum_{i=N,S} s_n^i w^i \right) L_\gamma + h k^i(t)}{P_{free}} \quad (83)$$

when both countries innovate. As we have seen, an equilibrium where both countries innovate is possible when there are spillover of knowledge across countries, but it is not likely to arise.

In all cases studied, South always supplies the traditional good. Hence, the price index is:

$$P_{free} = \frac{(w^S)^{1-\sigma} (P_y)^\sigma}{z}, \quad (84)$$

where:

$$P_y = \frac{(n)^{-\frac{1-\alpha}{\alpha}}}{\alpha} \left[ \sum_{i=N,S} s_n^i (w^i)^{-\frac{\alpha}{1-\alpha}} \right]^{-\frac{1-\alpha}{\alpha}}. \quad (85)$$

**Without trade in intermediate goods** In this case,  $s_n^N = 1$ , and equation (82), combined with (84) and (85), become:

$$G_{free}^i(t) = \left[ \frac{w^i L - w^N L_\gamma + h k^i(t)}{(w^S)^{1-\sigma} (w^N)^\sigma} \right] z (n^N)^{\frac{h}{\alpha}}. \quad (86)$$

At the moment  $T$  when countries open to trade, the stock of blueprints owned by the residents of South is lost, and we have that  $k^N(T) = 1$  and  $k^S(T) = 0$ .

**Non-FPE equilibria** In non-FPE equilibria, we have that  $\hat{w}^S < \hat{w}^N < w_{aut}^S < w_{aut}^N$ , where  $\hat{w}^S$  and  $\hat{w}^N$  are defined in equations (47c) and (47b), respectively. The first inequality is the condition for the non-FPE equilibrium

to exists, while the last one is derived from proposition 1. The middle inequality is shown through:

$$\hat{w}^N = \frac{h + \alpha\sigma\Psi^N}{(L + \rho)\Psi^N} < \frac{h + \alpha\sigma\Psi^S}{(L + \rho)\Psi^S} < \frac{h + (1 - h)\Psi^S}{(L + \rho)\Psi^S} = w_{aut}^S.$$

Clearly, North residents are better off than those who live in South. The latter not only lose their accumulated capital (stock of blueprints), but also face lower wages than those in North.

At the moment the countries open to trade, the ratio of North residents welfare under free trade and under autarky equals:

$$\frac{G_{free}^N(T)}{G_{aut}^N(T)} = \sigma \left( \frac{w_{aut}^N}{\hat{w}^N} \right) \left( \frac{\hat{w}^N}{\hat{w}^S} \right)^{1-\sigma}. \quad (87)$$

Then last two terms are larger than one and they represent the country's gain in purchasing power from opening to trade. The first term ( $\sigma$ ) indicates a loss arising from the fact that investment in R&D is higher under free trade (see Figures 3 and 4), which decreases disposable income at the moment trade starts. For a share of high-tech good in consumption ( $\sigma$ ) sufficiently large, the country has a net gain when trade starts. More specifically, it is possible to show that  $\sigma \geq \frac{1}{2(1-\alpha)}$  is a sufficient condition for  $\frac{G_{free}^N(T)}{G_{aut}^N(T)} > 1$ .

As for the residents in South, they have more to lose when trade starts. The ratio of their welfare under free trade and under autarky is:

$$\frac{G_{free}^S(T)}{G_{aut}^S(T)} = \left[ (1 - \sigma) - \left( \frac{hL - \rho\alpha\sigma\Psi^N}{(L + \rho)\Psi^N} \right) \right] \times \left( \frac{w_{aut}^S}{w^S} \right) \left( \frac{w^S}{w^N} \right)^\sigma \left( \frac{n^N}{n^S} \right)^{\frac{h}{\alpha}}. \quad (88)$$

First, they lose part of their wealth when their stock of blueprints loses its value. Second, investment in R&D is higher under free trade compared to autarky, which decreases its disposable income for consumption in the short run. These two effects are captured in the first term between brackets, which is lower than one. Finally, the effect of opening to trade on purchasing power in terms of the high tech good is uncertain. On the one hand, those goods are now produced by North, which has higher wages (second to last term in the equation). On the other hand, that country has a larger number of varieties of the intermediate goods, which renders production more efficient and less costly (last term). The net effect depends on the difference in the number of varieties both countries had just before starting to trade.

**FPE equilibria** Under the FPE equilibria the only difference in the purchasing power between the two countries stems from their difference in wealth. North residents have an accumulated capital, which makes them



richer. Equations (87) and (88) become:

$$\frac{G_{free}^N(T)}{G_{aut}^N(T)} = 1 + \frac{hL(\Psi^N - 1)}{(L + \rho)\Psi^N} > 1, \text{ and} \quad (89)$$

$$\begin{aligned} \frac{G_{free}^S(T)}{G_{aut}^S(T)} &= \left(1 - h - \frac{hL}{(L + \rho)\Psi^S}\right) \times \\ &\quad \left(\frac{h + (1 - h)\Psi^S}{h + (1 - h)\Psi^N}\right) \left(\frac{n^N}{n^S}\right)^{\frac{h}{\alpha}}. \end{aligned} \quad (90)$$

Similarly to the non-FPE case, the residents of North are clearly better off when trade starts, as shown in equation (89). The decrease in the price index more than compensates the decrease in disposable income due to more investment in R&D after opening, so that their purchasing power increases.

Also like the non-FPE case, South residents may be better or worse off, depending on the distance between their stock of blueprints and that of North at moment  $T$ . The first term in equation (90) is smaller than one and it captures both the lower disposable income due to higher investment in R&D and the losing of their previous stock of blueprints. The other two terms depict the gain from a lower price index after opening to trade. Just as in the non-FPE, the last term reflects the higher efficiency in high-tech production with the larger number of intermediate goods varieties in the good country. It may be the case that North has so much more blueprints, that the reduction in the price of the high-tech goods more than compensates the capital loss that occurs the moment trade starts.

**Long run** All the analysis so far refers to the comparisons of purchasing power and, therefore, welfare in the short run. Let us now look at the long run impact of opening to trade. In the non-FPE equilibrium, we substitute equation (81) into equation (86) to get:

$$\lim_{t \rightarrow \infty} G_f^S(t) = \left[ \frac{\hat{w}^S L - \hat{w}^N L_\gamma + \left(\frac{\hat{w}^S}{\hat{w}^N + \hat{w}^S}\right) h}{(\hat{w}^S)^{1-\sigma} (\hat{w}^N)^\sigma} \right] z (n^N)^{\frac{h}{\alpha}}, \quad (91)$$

$$\lim_{t \rightarrow \infty} G_f^N(t) = \left[ \frac{\hat{w}^N (L - L_\gamma) + \left(\frac{\hat{w}^S}{\hat{w}^N + \hat{w}^S}\right) h}{(\hat{w}^S)^{1-\sigma} (\hat{w}^N)^\sigma} \right] z (n^N)^{\frac{h}{\alpha}}. \quad (92)$$

Here, again, North residents are clearly better off than those of South, since  $\hat{w}^N > \hat{w}^S$ .

Comparing the purchasing power in the long run equilibrium in equations (91) and (92) to that in autarky in equation (77), we see that, differently from the short run case, in the long run both countries' residents are better off under free trade compared to autarky. The terms between brackets are constant in all three equations, given that the rate of innovation and wages are constant in the steady state. Hence, the difference between welfare under free trade and under autarky in the long run is proportional to the difference in the number of blueprints. Since the number of varieties increases faster under free trade than in closed economies, the distance in welfare tends to infinity in the long run.

**With trade in intermediate goods** We have seen in section 3.1 that, without international knowledge spillover, only North innovates in all possible equilibria. Furthermore, in the long run, the equilibria tend to those of the previous case, that is, without trade in intermediate goods. Therefore, the welfare analysis in the long run is equal to the one above.

In the short run, South retains a share of the production of intermediate goods. The purchasing power of each country is obtained by substituting equations (80), (84) and (85) into equation (83). With trade in intermediate goods, South continues producing the varieties it had invented up to the moment of opening to trade. The total number of varieties of intermediate goods used in the high-tech good production is larger than when there is no trade in intermediate goods. This increases productivity, lowering the high-tech good price. Welfare is, then, higher when there is trade in intermediate goods than when there is not such trade. The impact is even larger for the residents of South, because now they do not incur in a capital loss when trade starts.

**With international knowledge spillover** Wages and investment in R&D are exactly the same as in the case with no international knowledge spillover. Hence, the short run welfare impact of trade will be the same as in that case. There is one important difference though. With knowledge spillover and trade in intermediate goods, the stock of knowledge is larger compared to the case without knowledge spillover. Therefore, investment in R&D will be more productive and the stock of blueprints will grow faster, yielding a faster increase of welfare.

**With multinational corporations** The welfare impact of trade when there are MNC is the same as in the case with knowledge spillover and trade in intermediate goods, if the equilibrium arising under free trade is the one with FPE. In the case of a non-FPE equilibrium, wages in North will be smaller and in South they will be higher compared to the case with no MNC. Hence, there is less world wage inequality with MNC than without. Moreover, the rate of innovation is the highest one in this case. The impact of welfare in the short run is uncertain, since investment in R&D is higher for residents of both countries, compared to autarky. However, the faster increase in the number of blueprint will provide a faster increase in purchasing power. Comparing all cases, this one may yield the highest present value of welfare, if citizens' valuation of the future is sufficiently high.

## 5 Concluding Remarks

New ideas may pop up at every moment and every place, but it is hard to know ex-ante their chances of success. The risks involved in a project increase considerably when their assets are intangible, since in case of failure the liquidation value is negligible. R&D projects are then a good example of investments that ask for diversification as a form of risk sharing among economic agents, which is achievable through a well functioning financial system. Thus, when countries engage in technological competition, R&D financing becomes the main instrument for creating comparative advantage over time. A better functioning financial system generates more R&D research, which

increases the innovation rate and renders the country's high-tech sector more productive than abroad. This is the basic idea of the model developed in this paper.

Recent empirical studies fail to find a positive relation between trade and growth. Furthermore, when controlling for institutions the relation between trade and growth may even turn out to be negative in certain cases. We offer a possible explanation for these results. We suggest that the impact of trade on growth may depend on institutions. More specifically, we show that opening to trade increases growth in the country with better credit market, and decreases growth in the country with worse credit market.

In terms of welfare, we show that the innovating North is better off than the South. In the short run South's welfare may be lower under free trade than in autarky. In the long run, however, all residents are better off under free trade than if they were under autarky due to the higher rate innovation under free trade. Hence, whether it is worth opening to trade depends on the present value of wealth under both situations. Calculations show that opening to trade is more likely to be worth for all individuals when multinational corporations exist and economies are not too small.

Wealth inequality across country is strictly increasing in the wage gap. Hence, there is no inequality when FPE equilibria arise under free trade. Among all non-FPE equilibria, the largest inequality happens when there are no knowledge spillovers between countries and when there are no multinational corporations, whereas the least inequality arises when both of these features exist. All in all, these results point out that if South opens to trade, it should try to promote knowledge spillover and multinational corporations.

## References

- [1] Acemoglu D., S. Johnson, and J.A. Robinson (2001), "The Colonial Origins of Comparative Development: An Empirical Investigation," *American Economic Review* 91(5): 1369-1401.
- [2] Acemoglu D., S. Johnson, and J.A. Robinson (2002), "Reversal of Fortune: Geography and Development in the Making of the Modern Income Distribution," *Quarterly Journal of Economics* 117(4): 1231-1294.
- [3] Acemoglu D., S. Johnson, and J.A. Robinson (2006), "Institutions as a Fundamental Cause of Long-Run Growth" in Philippe Aghion and Steven Durlauf (eds.) *Handbook of Economic Growth*, Amsterdam, North-Holland.
- [4] Aghion, Philippe and Peter Howitt (1992). "A Model of Growth through Creative Destruction," *Econometrica* 60(2): 323-351.
- [5] Beck, Thorsten (2002). "Financial Development and International Trade. Is there a Link?," *Journal of International Economics* 57:107-131.

- [6] Bolton, Patrick, and Philippe Aghion (1992). "An Incomplete Contracts Approach to Financial Contracting," *The Review of Economic Studies* 59(3): 473-494.
- [7] Djankov, Simeon, Caralee McLiesh and Andrei Shleifer (2007). "Private credit in 129 countries," *Journal of Financial Economics* 84(2): 299-329.
- [8] Dollar D. and A. Kraay (2003), "Institutions, Trade and Growth," *Journal of Monetary Economics* 50(1): 133-162.
- [9] Engerman S.L. and K.L. Sokoloff (2002). "Factor Endowments, Inequality and Paths of Development among New World Economies," *Economia* 3(Fall): 41-109.
- [10] Frankel J.A. and D. Romer (1999), "Does Trade Cause Growth?," *American Economic Review* 89(3): 379-399.
- [11] Grossman, Gene M., and Helpman, E. (1991). *Innovation and Growth in the Global Economy*, The MIT Press.
- [12] Hall, Robert E. and Charles I. Jones (1999). "Why Do Some Countries Produce So Much More Output Per Worker Than Others?," *The Quarterly Journal of Economics* 114(1): 83-116.
- [13] Hart, Oliver and John Moore (1994). "A Theory of Debt Based on the Inalienability of Human Capital," *The Quarterly Journal of Economics* 109(4): 841-79.
- [14] Hart, Oliver and John Moore (1998). "Default And Renegotiation: A Dynamic Model Of Debt," *The Quarterly Journal of Economics* 113(1): 1-41,
- [15] Hur, Jung, Manoj Raj and Yohanes E.Riyanto (2006). "Finance and trade: A cross-country empirical analysis on the impact of financial development and asset tangibility on international trade," *World Development* 34(10): 1728-1741.
- [16] La Porta, Rafael, Florencio Lopez-de-Silanes, Andrei Shleifer and Robert W. Vishny (1997). "Legal Determinants of External Finance," *Journal of Finance* 52(3): 1131-50.
- [17] Levchenko, Andrei (2007). "Institutional Quality and International Trade," *Review of Economic Studies* 74(3): 791-819.
- [18] Levine, R.; Loayza, N.; and Beck, T. (2000). "Financial Intermediation and Growth: Causality and Causes." *Journal of Monetary Economics* 46.
- [19] Lucas, Robert (1988). "On the Mechanisms of Economic Development" *Journal of Monetary Economics* 22: 3-42.
- [20] Manova, K., (2006). "Credit Constraints, Heterogeneous Firms, and International Trade", NBER wp# 14531.

- [21] Manova, K., (2005). "Credit Constraints in Trade: Financial Development and Export Composition", mimeo Harvard University.
- [22] Rigobon R. and D. Rodrik (2005), "Rule of Law, Democracy, Openness, and Income: Estimating the Interrelationships," *The Economics of Transition* 113(3): 533-564.
- [23] Rodrik D., A. Subramanian, and F. Trebbi (2004), "Institutions Rule: The Primacy of Institutions Over Geography and Integration in Economic Development," *Journal of Economic Growth* 9(2): 131-165.
- [24] Romer, Paul M. (1988). "Increasing Returns and Long-Run Growth," *Journal of Political Economy* 94(5): 1002-1037.
- [25] Romer, Paul M. (1990). "Endogenous Technological Change." *Journal of Political Economy* 98(5): S71-S102.
- [26] Stiglitz, Joseph and Andrew Weiss (1981). "Credit Rationing in Markets with Imperfect Information," *American Economic Review* 71(3): 393-410.
- [27] Svaleryd, Helena and Jonas Vlachos (2005). "Financial Markets, the Pattern of Industrial Specialization and Comparative Advantage: Evidence from OECD countries," *European Economic Review* 49.
- [28] Tirole, J. (2006). *The Theory of Corporate Finance*. Princeton University Press.
- [29] Townsend, Robert M. (1979). "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory* 21(2): 265-293,

## 6 Appendix

### 6.1 Proof of Proposition 2

If both countries innovated, the FEC (38) would imply:

$$\Psi^S = \frac{v^S n^S}{\bar{w}^S} > \frac{v^N n^N}{\bar{w}^N} = \Psi^N. \quad (93)$$

Under FPE,  $v^N = v^S$ . When countries start to trade  $n^N \geq n^S$ , hence  $\frac{vn^N}{w} \geq \frac{vn^S}{w}$ , and equation (93) is not satisfied. The FEC (38) can only be satisfied simultaneously for both countries if North is the only country that innovates.

Let us now investigate the possibility of an equilibrium with both countries innovating when  $w^N \neq w^S$ . As there is no trade in intermediate goods, both countries would have to produce the high-tech good, and its price would have to be equal in both countries, which, given equation (29b), implies:

$$P_y^N = \frac{w^N}{(n^N)^{\left(\frac{1-\alpha}{\alpha}\right)}} = \frac{w^S}{(n^S)^{\left(\frac{1-\alpha}{\alpha}\right)}} = P_y^S. \quad (94)$$

For the price of the high-tech goods to be equal across countries, it would be necessary that  $w^N > w^S$ , given that  $n^N \geq n^S$  when the countries start to trade and given equation (94). Furthermore, since  $\dot{w}^N = \dot{w}^S = 0$  in equilibrium, it must also be the case that  $\frac{n^N}{n^S}$  remains constant so that equation (94) is always satisfied. Hence,  $\gamma^N = \gamma^S > 0$ . Using this result, we calculate the high-tech good market shares equalizing the no-arbitrage condition (expression 39), and we get that:

$$s_y^N = \frac{w^N \Psi^N}{w^N \Psi^N + w^S \Psi^S}. \quad (95)$$

We substitute it in the labor market clearing conditions of both countries (equation 40) and equalize them. We get:

$$\frac{\Psi^N}{\bar{w}^N \Psi^N + \bar{w}^S \Psi^S} \alpha \sigma = \frac{\Psi^S}{\bar{w}^N \Psi^N + \bar{w}^S \Psi^S} \alpha \sigma + \frac{(1 - \sigma)}{w^S}.$$

The above expression holds only if  $\Psi^N > \Psi^S$ , which is a contradiction. Thus, there is no non-FPE equilibrium with both countries innovating.

## 6.2 Proof of proposition 3

If both countries innovate simultaneously, equation (93) must be satisfied. With an argument analogous to the one used in the proof of Proposition 2, under FPE we have that  $v^N = v^S$ . When countries start to trade  $n^N \geq n^S$ , hence  $\frac{vn^N}{w} \geq \frac{vn^S}{w}$ , and equation (93) is not satisfied. The FEC (38) can only be satisfied simultaneously for both countries if North is the one that innovates.

We turn to non-FPE equilibria. With trade in intermediate goods, we have that  $\frac{\partial v^i}{\partial w^i} < 0$ , given the definition of  $v^i$  in equation (10), the profit function (37) and the definition of the market share  $s^i$  in (35). Therefore, in the non-FPE equilibrium, it is also true that  $\frac{v^N n^N}{w^N} > \frac{v^S n^S}{w^S}$  when  $w^N < w^S$ , and there is no equilibrium with both countries innovating as equation (93) is not satisfied.

There is also no equilibrium with  $w^N > w^S$  and positive innovation in both countries. In this case  $s_z^N = 1$ , so that the LMCs from equation (40) become:

$$\gamma^N = L - \frac{\alpha \sigma s_x^N}{w^N}$$

for North and:

$$\gamma^S = L - \frac{\alpha \sigma s_x^S}{w^S} - \frac{(1 - \sigma)}{w^S},$$

for South.

From the definition of market share  $s_x^N$  in equation (35) and the fact that  $n^N > n^S$ , we have that  $\frac{s_x^N}{w^N} < \frac{s_x^S}{w^S}$ . Therefore, from the two LMCs above, we have that  $\gamma^N > \gamma^S$ . Log-differentiating the equation for  $s_x^N$  we get:

$$\dot{s}_x^N = s_x^N (1 - s_x^N) \left[ (\gamma^N - \gamma^S) - \frac{\alpha}{1 - \alpha} \left( \frac{\dot{w}^N}{w^N} - \frac{\dot{w}^S}{w^S} \right) \right]. \quad (96)$$

Since  $\frac{\dot{w}^N}{w^N} = \frac{\dot{w}^S}{w^S} = 0$  in the steady-state,  $\dot{s}_x^N > 0$  when  $\gamma^N > \gamma^S$ . As  $\dot{s}_x^N \rightarrow 1$  we have that  $\gamma^S \rightarrow -\rho$  (using the NAC in equation (39) for the South), which is not possible.

Finally North is the country that innovates in the non-FPE equilibrium for the following reason. The non-innovating country must have the lower wage to capture the market for the traditional good. Otherwise, its demand for labor would tend to zero as its share of the intermediate good market tends to zero. Let us assume that South is the innovating one. We would have that  $w^S > w^N$ , hence  $\pi^S < \pi^N$ , yielding  $v^S < v^N$ . Since  $n^S < n^N$  when the economies open to trade, we would then have that:

$$\frac{v^N n^N}{w^N} > \frac{v^S n^S}{w^S}$$

The FEC (38) for the innovating (South) country would imply:

$$\frac{v^S n^S}{w^S} = \Psi^S,$$

which, combined with the previous inequality, would yield:

$$\frac{v^N n^N}{w^N} > \Psi^S > \Psi^N, \quad (97)$$

where the last inequality is an assumption of the model. According to inequality (97), the FEC would not be satisfied for North. Hence, there is no equilibrium where South is the one that innovates.

## 6.3 Both countries may innovate when there is knowledge spillover

### 6.3.1 Without trade in intermediate goods

For both countries to innovate, it is necessary that both produce the high-tech good, hence  $p_y^N = p_y^S$ . According to equation (29b), this implies:

$$s_n^N \equiv \frac{n^N}{n^N + n^S} = \frac{(w^N)^{\frac{\alpha}{1-\alpha}}}{W}, \quad (98)$$

where  $W \equiv (w^N)^{\frac{\alpha}{1-\alpha}} + (w^S)^{\frac{\alpha}{1-\alpha}}$ . It also implies that  $w^N > w^S$ , as  $n^N > n^S$  when countries start to trade.<sup>12</sup> Furthermore, given that wages are constant in the steady state, the rate of growth of varieties in each country must be equal, that is,  $\gamma^N = \gamma^S$ .

Combining the NAC in equation (58) with the results above, we get:

$$s^N = \frac{\Psi^N (w^N)^{\frac{1}{1-\alpha}}}{V}, \quad (99)$$

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<sup>12</sup>One could wonder whether there could be an equilibrium path in which the worse country innovates faster when trade begins, so that eventually  $n^{worst} > n^{best}$ , and a steady state could be reached with such configuration. This cannot happen, though, because there would be a moment when  $n^{worst} = n^{best}$ . At that moment  $p_y^{worst} = p_y^{best}$  would imply  $w^{worst} = w^{best}$ , and the FEC could not be satisfied simultaneously for both countries, as shown in the proof for proposition 2.

where  $V \equiv \Psi^N (w^N)^{\frac{1}{1-\alpha}} + \Psi^S (w^S)^{\frac{1}{1-\alpha}}$ .

Substituting equations (98) and (99) back into the NAC, we get that:

$$\gamma = \frac{hW}{V} - \rho. \quad (100)$$

We now substitute equations (98), (99) and (100) into the LMC for each country in equation (59), using the fact that  $s_z^N = 0$ , as  $w^N > w^S$ . We get two functions that implicitly define  $w^N$  and  $w^S$  in equilibrium:

$$(w^N)^{\frac{\alpha}{1-\alpha}} \left( \frac{h + \alpha\sigma\Psi^N}{V} - \frac{\rho}{W} \right) = L, \text{ and} \quad (101a)$$

$$(w^S)^{\frac{\alpha}{1-\alpha}} \left( \frac{h + \alpha\sigma\Psi^S}{V} - \frac{\rho}{W} \right) + \frac{1-\sigma}{w^S} = L. \quad (101b)$$

Note that this result is different from the case with no international knowledge spillovers, and the reason for it is the following. Remember that in the case of no knowledge spillovers South could not innovate basically because it was less productivity in the innovation activity, which would generate an excess demand for labor. In the case of international knowledge spillovers, the productivity in R&D is the same across countries. Hence, when both countries increase their varieties stock at an equal rate, and the one with a lower stock of blueprints will dedicate relatively less labor to this activity, compared to the case without knowledge spillovers.

Nevertheless, this equilibrium is not likely to arise for the following reason. The equilibrium wages determined in equations (101) are independent of the share of varieties in North  $s_n^N$ . This equilibrium would only be possible if  $s_n^N$  at the time the economies open to trade were at a specific level, established in equation (98), given wages established in equations (101).

### 6.3.2 With trade in intermediate goods

Log-differentiating the definition of the market share  $s^i$  in equation (35), we get:

$$\dot{s}_x^N = s_x^N (1 - s_x^b) \left[ (\gamma^N - \gamma^S) - \frac{\alpha}{1-\alpha} \left( \frac{\dot{w}^N}{w^N} - \frac{\dot{w}^S}{w^S} \right) \right].$$

It is then clear that  $\gamma^N = \gamma^S$  in the steady-state with  $s_x^N \in (0, 1)$ . Substituting this result in the NAC for each country (equation (58)), and using the definition of the market share of intermediate goods in equation (35), we get that:

$$\frac{w^S}{w^N} = \left( \frac{\Psi^N}{\Psi^S} \right)^{1-\alpha}, \text{ and} \quad (102)$$

$$s_x^N = \frac{s_n^N (\Psi^N)^\alpha}{s_n^N (\Psi^N)^\alpha + (1 - s_n^N) (\Psi^S)^\alpha}. \quad (103)$$



With equations (102) and (103), and the LMCs for both countries, in equation (59), we obtain the equilibrium values for the model variables. This equilibrium, however, is only possible for the blueprint shares implicitly defined by:

$$A\rho(\bar{s}_n^N)^2 + B\bar{s}_n^N + LC = 0, \quad (104)$$

where:

$$\begin{aligned} A &= (1 - \sigma)(\Psi^S)^{1-\alpha} \left[ (\Psi^N)^\alpha - (\Psi^S)^\alpha \right] + \alpha\sigma(\Psi^N - \Psi^S), \\ B &= L \{ A - 2\alpha\sigma(\Psi^N + \Psi^S) - 2h \} + \rho C + h + \alpha\sigma\Psi^N, \text{ and} \\ C &= [h + (1 - h)\Psi^S]. \end{aligned}$$

### 6.3.3 With multinationals

In this setup, there is no FPE equilibrium, as equation (72) cannot hold for both countries when  $\dot{w} = 0$  and  $w^N = w^S$ . In the non-FPE equilibrium, equation (72) can be satisfied simultaneously for both countries only if:

$$w^N\Psi^N = w^S\Psi^S.$$

Therefore,  $w^N > w^S$ , and South will capture all the market of final and intermediate goods. Recall that when intermediate goods have to be produced where invented, the country with lower wages captures only the traditional good market. Since the amount of labor available for innovation is smaller compared to the case without MNCs and the labor supply is the same in the two cases, the share of blueprints invented in South is smaller when there are MNCs.

The equilibrium allocation is given by:

$$\bar{\gamma}^N = \bar{\gamma}^S = \frac{2hL - \rho(1-h)\Psi^S}{h + (1-h)\Psi^S} \quad (105a)$$

$$\bar{w}^N = \frac{h + (1-h)\Psi^S}{(2L + \rho)\Psi^N} \quad (105b)$$

$$\bar{w}^S = \frac{h + (1-h)\Psi^S}{(2L + \rho)\Psi^S} \quad (105c)$$

$$s_n^N = \frac{[h + (1-h)\Psi^S]L}{2hL + \rho(1-h)\Psi^S} \quad (105d)$$

and it is feasible if  $s_n^N \in (0, 1)$  and  $\bar{\gamma} > 0$ , i.e.,

$$\begin{aligned} \frac{L}{\rho} &< \frac{(1-h)\Psi^S}{(1-h)\Psi^S - h} \text{ and} \\ \frac{L}{\rho} &> \frac{(1-h)\Psi^S}{2h}. \end{aligned}$$

## 6.4 Proof of Proposition 4

**South cannot be the only innovator** First, we show that South cannot be the only one to innovate in equilibrium. Let us assume it is in order to obtain a contradiction.

Substituting the definition of  $s_x^i$  (equation 35) in the profit function (equation 37), we get:

$$\pi^i = \frac{h(w^i)^{-\frac{\alpha}{1-\alpha}}}{n^N (w^N)^{-\frac{\alpha}{1-\alpha}} + n^S (w^S)^{-\frac{\alpha}{1-\alpha}}}, \quad i = N, S. \quad (106)$$

From equation (106) we have that  $\frac{\partial v^i}{\partial w^i} < 0$ . Hence, if  $w^S \geq w^N$ , the FEC in equation (54) would imply:

$$\Psi^S = \frac{v^S n}{w^S} \leq \frac{v^N n}{w^N},$$

given that the FEC is satisfied with equality for South. As  $\Psi^S > \Psi^N$ , we would have that  $\frac{v^N n}{w^N} > \Psi^N$ , and the FEC would not be satisfied for North.

If  $w^S < w^N$ , then  $s_z^N = 0$  and the LMC (equation (59)) for North and South would be:

$$L = \frac{s_x^N}{w^N} \alpha \sigma, \quad (107a)$$

$$L = (1 - s_n^N) \gamma^S + \frac{(1 - s_x^N)}{w^S} \alpha \sigma + \frac{(1 - \sigma)}{w^S}, \quad (107b)$$

respectively. As only South innovates,  $s_x^N$  and  $s_n^N$  tend to zero. Also, given that  $w^S < w^N$ , there would be a moment where:

$$\frac{s_x^N}{w^N} < \frac{(1 - s_x^N)}{w^S},$$

and, consequently, equations (107a) and (107b) could not hold simultaneously. Therefore, there is no equilibrium with only South innovating.

**Equilibria with only North innovating** Now we investigate the equilibria where North innovates. We have already seen that there is an equilibrium with both countries innovating. Here we study the equilibria with only North innovating.

We start with the non-FPE equilibrium. Using an analogous argument to the one used above to show that there is no equilibrium with  $w^S < w^N$  and with only South innovating, it is straightforward to show that there is also no

equilibrium with  $w^N < w^S$  and where only North innovates.

There is an equilibrium with  $w^N > w^S$  and with only North innovating. The FEC in equation (54) for North becomes:

$$\Psi^N = \frac{v^N n}{w^N} < \frac{v^S n}{w^S},$$

where the inequality is due to the fact that  $\frac{\partial v^i}{\partial w^i} < 0$ , and it is compatible with the FEC for South, as  $\Psi^S > \Psi^N$  by assumption.

The NAC (equation (58)) for North becomes:

$$\frac{\dot{w}^N}{w^N} = s_n^N \gamma^N + \rho - \frac{s_x^N h}{s_n^N w^N \Psi^N}, \quad (108)$$

while the LMC for each country is:

$$L = s_n^N \gamma^N + \frac{s_x^N}{w^N} \alpha \sigma, \text{ and} \quad (109)$$

$$L = \frac{(1 - s_x^N)}{w^S} \alpha \sigma + \frac{(1 - \sigma)}{w^S}. \quad (110)$$

From equation (110), we have that:

$$w^S = \frac{1 - \sigma + (1 - s_x^N) \alpha \sigma}{L}. \quad (111)$$

Combining equations (108) and (109), we get:

$$w^N = \frac{h + s_n^N \alpha \sigma \Psi^N}{s_n^N \left( L + \rho - \frac{\dot{w}^N}{w^N} \right) \Psi^N}, \quad (112)$$

which, substituted back into equation (108), yields:

$$\begin{aligned} s_n^N \gamma^N &= \frac{s_x^N h L + (s_x^N - 1) \rho - s_n^N \alpha \sigma \rho \Psi^N}{h + s_n^N \alpha \sigma \Psi^N} + \\ &+ \frac{\dot{w}^N}{w^N} \left( \frac{h + s_n^N \alpha \sigma \Psi^N - s_x^N}{h + s_n^N \alpha \sigma \Psi^N} \right) \end{aligned} \quad (113)$$

Let us now derive the system which represents the dynamics of the world economy. Combining equations (108), (112), and (113), we get:

$$\dot{w}^N = (L + \rho) w^N - \frac{(w^N)^{-\frac{\alpha}{1-\alpha}} [1 + \alpha (s_n^N \Psi^N - 1)] \sigma}{\left[ s_n^N (w^N)^{-\frac{\alpha}{1-\alpha}} + (1 - s_n^N) (w^S)^{-\frac{\alpha}{1-\alpha}} \right] \Psi^N} \quad (114)$$

The differentiation of  $s_n^N$  combined to equation (109) yields:

$$\dot{s}_n^N = (1 - s_n^N) \left[ L - \frac{s_n^N (w^N)^{-\frac{1}{1-\alpha}}}{s_n^N (w^N)^{-\frac{\alpha}{1-\alpha}} + (1 - s_n^N) (w^S)^{-\frac{\alpha}{1-\alpha}}} \alpha \sigma \right] \quad (115)$$

Equations (114), (115), (109) and (110) determine the equilibrium dynamics of the world economy. Solving that system, equilibrium arises when  $s_n^N = s_x^N = 1$ . Equations (111), (112) and (113) tend to equilibrium values in equations (47), which is the allocation of the non-FPE equilibrium with no international knowledge spillovers and no trade of intermediate goods. The feasibility conditions for this equilibrium is the same as the ones in equations (48) and (49).

We now consider the FPE equilibrium. Given the profit equation (106), we have that profits are equal across countries when  $w^N = w^S \equiv w$ , hence  $v^N = v^S$ . Combining this information with the FEC for North and the fact that  $\Psi^N < \Psi^S$ , we get that:

$$\frac{v^S n}{w} = \frac{v^N n}{w} = \Psi^N < \Psi^S.$$

The FEC is then also satisfied for South, with  $\gamma^S = 0$ . Under FPE  $s_x^i = s_n^i$ , for  $i = N, S$ , then the LMC (equation (59)) for North and South are, respectively:

$$L = s_n^N \gamma^N + \frac{s_n^N}{w} \alpha \sigma + s_z^N \frac{(1 - \sigma)}{w}, \text{ and} \quad (116a)$$

$$L = \frac{(1 - s_n^N)}{w} \alpha \sigma + (1 - s_z^N) \frac{(1 - \sigma)}{w}. \quad (116b)$$

Adding up equations (116a) and (116b), we get:

$$s_n^N \gamma^N = 2L - \frac{(1 - h)}{w}. \quad (117)$$

The wages is obtained by substituting the equation above in the NAC (58):

$$w = \frac{h + (1 - h) \Psi^N}{\Psi^N (2L + \rho - \frac{\dot{w}}{w})}, \quad (118)$$

which, substituted back into equation (117), yields the innovation rate:

$$s_n^N \gamma^N = \frac{2Lh - (1 - h) (\rho - \frac{\dot{w}}{w}) \Psi^N}{h + (1 - h) \Psi^N}. \quad (119)$$

Finally, substituting equation (118) into (116b), we get North share of the traditional good market:

$$s_z^N = \frac{\Psi^N (1 - \sigma) \left( L + \rho - \frac{\dot{w}}{w} \right) - L (\alpha \sigma \Psi^N + h)}{\Psi^N \left( 2L + \rho - \frac{\dot{w}}{w} \right) (1 - \sigma)} + \frac{\alpha \sigma (1 - s_n^N)}{(1 - \sigma)}. \quad (120)$$

The dynamics of the economy is characterized by equation (118), which can be written as:

$$\dot{w} = \left[ (2L + \rho) - \frac{h + (1 - h) \Psi^N}{\Psi^N} \right] w, \quad (121)$$

and by:

$$\dot{s}_n^N = (1 - s_n^N) \left[ 2L - \frac{(1 - h)}{w} \right], \quad (122)$$

which is obtained by substituting the equation (117) in the log-differentiation of  $s_n^N$ .

Equations (121) and (122) above describe the dynamics of wages and blueprints. Wages and blueprint shares are constant when  $s_n^N = 1$ . Equations (118), (119) and (120) tend to equilibrium values in equations (43), which is the allocation of the non-FPE equilibrium with no international knowledge spillovers and no trade of intermediate goods. The feasibility condition for this equilibrium is the same as those in equations (44) and (45).