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Local public education and childless voting: the arising of an "ends with the middle" coalition

Berardino Cesi

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Local public education and childless voting: the arising of an "ends with the middle" coalition[1]

Berardino Cesi[2]
THEMA, University of Cergy-Pontoise.

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We show that capitalization of local education into the housing price induces childless voters to support local education. In particular, low income childless households vote for a tax raise when capitalization is strong, whereas high income childless supports a higher tax when capitalization is weak. The median income voter is never pivotal because "ends with the middle" coalitions arise: high income households (with and without a child) makes coalition with middle income class with a child, whereas low income households (with and without a child) make coalition with childless middle income class. We find that the income of the childless median voter is higher than the median income, whereas median voter with a child has income lower than the median. Thus the equilibrium tax preferred by the median voter (childless or not), is higher than the tax preferred by the childless median income voter and lower than the tax preferred by the median income voter with a child. This result implies that it is not possible to exclude voting equilibria in which the tax of the childless median voter is higher than the tax of the median voter with a child.

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[2] Université de Cergy-Pontoise, THEMA. Site des Chênes I, 33 Bd du Port, F-95000, Cergy-Pontoise, France

1. INTRODUCTION

Empirical and theoretical studies show that the local provision of public education affects the households in two ways: it increases the human capital of school-age children, and it increases the value of their housing. The second effect leans on the *capitalization* of the public education into the value of the housing, that is, the higher the expenditure in local public education the higher the value of the housings.

In this paper we show that, when the local provision of public education is financed by a tax on residents, house price capitalization is a sufficient condition for childless households to support local taxation.

Over the next few decades, the share of childless households (mainly elderly) is rising. As a direct consequence, it is less and less likely that the pivotal voter at the local level has children at school age. To the extent that funding of local public education is determined by local voters (as in many US states), the question then becomes whether house price capitalization may provide a sufficient alternative mechanism to encourage childless households to support local public education. The answer we provide is: "yes, but only if the expected duration of childless households in the property is short". This is an important issue because it determines whether local public education is possibly under-provided from a welfare point of view. We show that, when households are likely not to sell the house, they are unwilling to vote in favor of funding for local public education. If the likelihood of relocation/house sale on the other hand is high, then house price capitalization provides a sufficient incentive to vote for public spending on education.

This result is in line with empirical evidence. The pioneer in capitalization study is Oates (1969). He analyzes a 1960 sample of northern New Jersey communities and finds that the value of housing increases in the public expenditure in the school system. Sonstelie and Portney (1980), Heinberg and Oates (1970), Orr (1968) and Hamilton (1979) confirm Oates's results of the capitalization in terms of school quality and per pupil expenditure. Benson and O'Halloran (1987) find that childless voters in California support school spending because of its positive effect on their property's value. Brunner and Baldosn (2004) find that in California elderly generally vote to decreases the state spending but are much more willing to support local spending. Hilber and Mayer (2006) show that the older the elderly are, the stronger is the positive link between the share elderly and local public spending on schools, that is, the more likely they vote in favor of funding for local public education. More recently, empirical evidence in Fletcher and Kenny (2008) confirms that an increasing share of elderly results in a very small drop in school spending.

As in Brueckner and Joo (1991) we allow the capitalization effect to run through the sale of the housing, within a context in which public school is locally provided (at community level). We consider a two period model analyzing a metropolitan area composed of two communities whose boundaries are exogenously fixed. The area is inhabited by a continuum of households both with and without a child. The public education is provided by the local government through a head tax set by a majority voting. In the first period households vote on the tax and send their child only to the school belonging to community where they live. Since we assume that voting takes place only once, the tax remains fixed over the two periods. In the second period with a certain probability households must leave and resell their housing. New households come into the area, buy housing from the leaving households and sort into the communities. In this model the capitalization effect means that the reselling price is higher the higher is the tax decided in the first period. Our analysis of the voting equilibrium shows that, when capitalization is strong, low income childless voters are more willing to bear a tax rise, whereas high income childless voters support a higher tax only if they can vote on a range of taxes sufficiently high. When capitalization is sufficiently weak, only high income childless voters prefer a higher tax.

The probability fo leaving could be considered as a weight given to the capitalization effect. Only when this probability is sufficiently high, the marginal benefit from the higher tax, capitalized into the housing price, allows childless households to vote for a positive tax. In other words, when the expected duration of childless households in the property is short then they vote for a higher tax to increase the value of the housing they are going to sell. Furthermore, we show that, when capitalization occurs, "ends with the middle" coalitions make the median income voter never pivotal: high income households (with and without a child) makes coalition with middle income class with a child, whereas low income households (with and without a child) make coalition with childless middle income class. We also find that, when the median voter is childless, its income is higher than the median one, whereas when it has a child, its income is lower than the median one. This implies that the equilibrium tax preferred by the median voter (childless or not), is higher than the tax preferred by the childless median income voter and lower than the tax preferred by the median income voter with a child. This result implicitly shows that it is not possible to exclude voting equilibria in which the tax of the childless median voter is higher than the tax of the median voter with a child. The difference in these taxes depends of income distribution.

When, instead, the capitalization effect disappears, only households with a child vote for a positive tax and cross incomes coalitions of voters arise to block public provision of local education.

A wide literature analyzes the capitalization effect within a framework where the level of provision is decided by a voting system. This literature shows that the capitalization effect may allow households with a child to support local public spending in education. Yinger (1981, 1982, 1988) shows that in multi-community model the capitalization of a local tax into the housing values is a sufficient condition for the median voter with child to support local public spending. When the government allows the residents to decide the tax by a majority voting, each voter internalizes the capitalization of public spending into the value of his housing, and then he votes for a positive tax. Fischel (2001) creates the term "homevoters" and extends the capitalization effect to the public provision of local education.

A wide branch of urban economics literature deals with the local provision of education by a majority voting. Tiebout (1956) is the first to theoretically model an economy composed of many independent communities where a generic public good is provided by the local government by a local tax and households are free to sort across communities. Many studies attempted to refine Tiebout (1956) by introducing the local provision of public education. Although these attempts strongly extend this literature, surprisingly the direct introduction of childless households in a theoretical model is still missing[3].

Differently from Brueckner and Joo (1991), we introduce the analysis of the

[3]Nevetheless Neychba (2003) stresses the necessity of introducing the childless households in a multi-community model, he also points out the complications arising from this refinement in terms of voting equilibrium.

voting equilibrium when a head tax is imposed. The introduction of the voting behavior of childless households allows this paper to integrate the results in Denzau and Grier (1984), Fischer (1988) and Epple and Romano (1996) according to which high income households are more willing to bear a tax rise for increased public education when the median income voter is not pivotal. The main idea in these papers is that, when a group of households vote for zero tax, whatever is the reason, cross incomes coalitions are formed to block the provision of education. In particular, in Epple and Romano (1996) households vote for zero tax because of the existence of private school, in our paper, instead, heterogeneity in the presence of children induces childless households to vote for zero tax when capitalization is absent. Fletcher and Kenny (2008) and Brunner and Ross (2009) confirm our result by finding that the median voter has income lower than the median one. In particular, Fletcher and Kenny (2008) find that a large presence of elderly voters makes the income of the median voter lower than the community's median income. Our paper makes capitalization an additional ingredient for these empirical works.

This paper aims at setting up a bridge between the literature on local majority voting with no capitalization and no childless voters and the literature strictly related to capitalization. Our work can be thought as a study attempting to stress the necessity of introducing the voting behavior of childless households in the urban economics theoretical models in which the local tax is decided by a majority voting.

The paper is organized as follows: in section 2 we present the model, section 3 and 4 describe respectively the households and the local government, in section 5 we focus on the equilibrium of the model, section 6 concludes.

2. THE MODEL

We consider a two periods ($t = 1, 2$) model analyzing a metropolitan area divided into 2 communities (a, b). In the metropolitan area the boundaries and land of the two communities are exogenously determined. The housing capacity in b is $H^b < 1$ while the housing capacity in community a is unconstrained. Houses are homogenous and each household consumes one unit of housing. We assume that everybody must own housing to live in a community and that the property of housing gives access to education. At time 1 the area is inhabited by a mass of household normalized to 1. This implies that capacity in community a is enough to allocate more than all households living in the area at time 1. Housing capacity in community b, instead, is restricted in the sense there is not more space to build new housings, whereas new housings can be built in community a. For simplicity we rule out the possibility to rent and assume that once households buy housing they become residents. We introduce the innocuous assumption that the price for buying a housing in community a is equal to the cost of building a new one. This assumption will be used to ease the housing market in community a.

We also assume that there exists another area far away which new households come from. The characteristics of this area and the migration will be described below.

In each community the local government imposes a head tax to fund the provision of education. This tax is decided by the residents in a majority voting. The sequence of the events is illustrated in figure 1. Households vote on the tax and the child, if any, goes to school. This is in line with US primary school system where households can send their children only to the school into the community where they live. The voting takes place only at time 1, therefore, as in Brueckner and

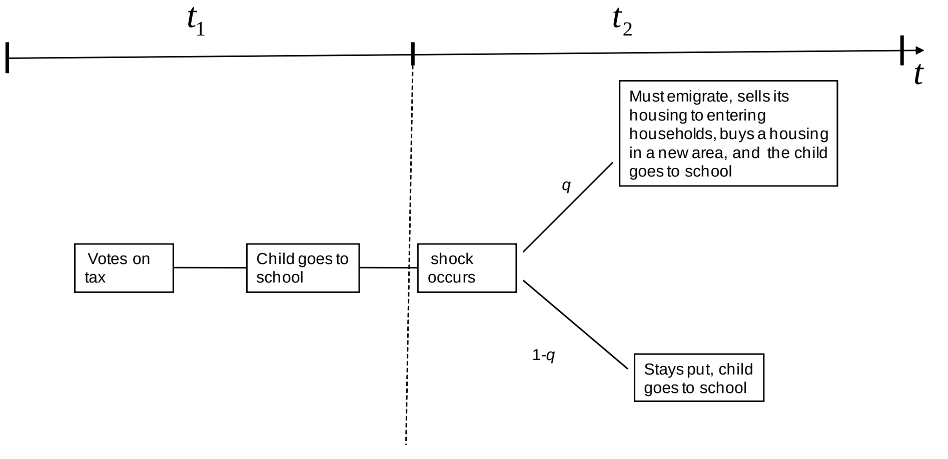

Fig. 1 The sequence of the events

Joo (1991), tax remains fixed over the two periods. In other words, the spending decision leads to construction of durable investments in a local public school, so that once investment is chosen, the level is fixed.

At time 2, a shock occurs and with probability q households must emigrate. Households sell their current housing and go to the other area. Once bought housing in the new area, these households send the child, if any, to school, consume and die at the end of the period. New households with a child and mass 1 move into the metropolitan area, buy housing from the leaving households, sort into communities and send their child to school. With probability $(1-q)$ households stay put and send their child to school.

Since the mass of entering households is 1, then housings supplied by leaving households in community b is not just enough to contain all entering households. Since at time 2 voting do not take place, the new entrants cannot modify the local provision of education. In the second period, the housings price in each community is determined in a competitive market.

3. HOUSEHOLDS

The mass of households living in the area and the mass of coming households are equal to 1 and both differ in income $y \in \left[\underline{y}, \overline{y}\right]$ according to the density $f(y)$. Initial residents also differ whether or not they have a child. In particular, we denote c and n respectively household with and without a child. At time 1, nature chooses the allocation of the households in both communities[4]. In particular, all childless household are allocated in community b that is completely allocated. Since the capacity of community a is enough to allocate all the households in both periods, then the housing price is normalized to zero. The private good z is considered as numeraire and its price is normalized to 1.

[4] We use this assumptions because the main purpose of this paper is studying the existence of a majority voting equilibrium in which childless voters support a higher investment in local public education through house price capitalization. Adding the possibility of sorting across communities at time 1 makes the model more complex and it does not change the quality of our results.

The inter-temporal utility function for households with a child at time 1 is defined as follows:

$$U_c\left(E,z;q\right)=v\left(z_1\right)+E_1+\left(1-q\right)\left(v\left(z_2\right)+E_2\right)+qv\left(\overline{z}_2\right) \tag{1}$$

Housing does not appear in the utility function because households consume just one unit of this and it is homogenous.

The first and the second term denote the utility at time 1: the household with a child consumes the numeraire z_1 and receives the educational expenditure per student E_1. The third and fourth term represent the expected utility at time 2. The discount factor is normalized to 1. In the second period, with probability $(1-q)$ households stay put. Since they do not leave the community, their consumption of numeraire is z_2 and the provision of public education per student is E_2. With probability q, households must leave. They sell their housing at price p_2 and emigrate in the other metropolitan area. Once in the new area, these households buy housing, pay the tax and send their child to school. Their numerarire is $\overline{z}_2$.

Utility function for childless households is:

$$U_n\left(z;q\right)=v\left(z_1\right)+\left(1-q\right)v\left(z_2\right)+qv\left(\overline{z}_2\right) \tag{2}$$

Households without a child do not receive public education; therefore utility function (2) does not directly depend on E. The budget constraints, at time 1 and 2, write:

$$z_1=y^i-p_1^j-T^j \tag{3}$$

$$z_2=y^i-T^j \tag{4}$$

$$\overline{z}_2=y^i+p_2 \tag{5}$$

Where $j=a,b$, denotes the community, the household's income is given by y^i, with $i=c,n$, and it is assumed to remain the same in both periods. Constraint (3) gives the consumption of numeraire in the first periods. At time 1, household living in community b pays housing price p_1^b, whereas we are assuming $p_1^a=0$[5]. The local government imposes a head tax T^j[6]. In this period, households do not save and the consumption of numeraire z_1 is given by the difference between income, housing price and tax. At time 2, the consumption of numeraire for no leaving households is given by the difference between income and the tax, as defined in constraint (4). The numeraire of the emigrating households is $\overline{z}_2$. They buy housing in the new area and send their children at school, for simplicity we normalize to zero the level

[5] Since there is no endogenous sorting at time 1, the assumption of $p_1^a=0$ does not affect our result. However, we could also normalize p_1^b to zero without affecting our results.

[6] The use of a head tax allows to rule out inefficiencies from income taxation. For instance in Italy there exists a tax on housing (ICI) whose value depends on the size of the housing. In our simple model of homogeneous housing our head tax could represent the ICI.

of education and the housing price in the new area. Hence, $\overline{z}_2$ is simply given by the sum of the reselling housing price and their income.

The probability q could be a proxy for the expected duration of households in the property or for the age of residents. The higher q the older are the residents then the higher is the probability of leaving. In this model we abstract from the role of inheritage, therefore if households do not sell their housing (leave) before they die, its value does not enter their utility function. New residents live in community j only at time 2, their utility writes:

$$U(E,z) = v(\widetilde{z}_2) + E_2 \tag{6}$$

Entering households consider the local tax as a constant because they do not vote. Each new resident pays T^j to finance the educational expenditure per student, E^j. Their budget constraint writes:

$$\widetilde{z}_2 = y - T^j - p_2^j \tag{7}$$

They do not save, and all income they have left once bought housing and paid the tax is used to consume $\widetilde{z}_2$.

In the rest of the model we assume $v(z_i)$ increasing in all its arguments, twice continuously differentiable and strictly concave in z_i with $i = 1, 2$, and $v'''(z_i) > 0$.

4. LOCAL GOVERNMENT AND EDUCATION PRODUCTION

In each community the local government imposes a head tax to collect resources and provide public education (E). In this model education is considered as a private good and it is produced from the numeraire according to constant returns to scale technology with respect to the number of students and the quantity provided. To make the analysis more tractable we exclude private school. The budget constraints for the local government in community $j = a, b$ in the two periods write:

$$n_1^j E_1^j = T^j N_1^j \tag{8}$$

$$\left((1-q)\, n_1^j + n_2^j\right) E_2^j = T^j N_2^j \tag{9}$$

Constraint (8) and (9) respectively represent the budget constraint for the local government at time 1 and at time 2. We assume that the government cannot transfer resources between periods and that in each community households receive the same educational expenditure per student.

N_1^j is the number of households living in each community j at time 1. n_1^j is the number of households with a child living in community j at time 1[7] and E_1^j is the educational expenditure per student. At time 1, each household pays T^j, therefore the tax revenue is $T^j N_1^j$. The total provision of education is $n_1^j E_1^j$.

[7] I recall that n_1^j (as N_i^j) is exogenously given.

At time 2, tax remains fixed and the per student education is E_2^j. Tax revenue in community j is $T^j N_2^j$. The total provision of education in community j, in the second period, is $\left((1-q)\,n_1^j + n_2^j\right) E_2^j$, where $(1-q)\,n_1^j$ is the number of households with a child remaining in community j after the shock and n_2^j is the number of households with a child entering the community j in the second period.

5. THE EQUILIBRIUM OF THE MODEL

In this section we characterize the equilibrium of the model by solving it by backward induction. We start by the housing market at time 2 and solve the maximization problem of the new residents. Then, we proceed by solving the voting equilibrium of the residents at time 1.

5.1. Housing market and sorting into communities at time 2

At time 2, once the shock has occurred and given the assumption of fully allocation, the housing supply in community b is vertical at the number of housings sold by leaving households and it is equal to qH^b. In this period, new residents choose community given T^j. We assume $T^b \leq T^a$ and the fraction of childless households at time 1 lower than the housing capacity in community b[8]. Since in each community the housings market is competitive, the equilibrium prices are given by the market clearing condition. However, given the unconstrained capacity, the sufficiently high housing supply in community a shrinks the housing price to zero. The housing demand is derived by solving the location problem of the new residents who take decision according to their indirect utility. The new residents' indirect utility functions in community j is[9]:

$$V^j\left(T^j, y\right) = v\left(y - p_2^j - T^j\right) + \frac{T^j N_2^j}{(1-q)\,n_1^j + n_2^j} \tag{10}$$

We recall that from the government's budget constraint we have $E_2^j = \frac{T^j N_2^j}{(1-q)n_1^j + n_2^j}$. Entering households choose to live in the community in which their indirect utility is maximized. The allocation decisions of entrants depend on both housing price and the educational expenditure per student E_j^i.

Lemma 1 defines the equilibrium in the housings market at time 2.

LEMMA 1. *Given $T^b \leq T^a$, at time 2 there exist two types of housings market equilibria : i) $T^a \geq p_2^b + T^b$ with $E_2^a > E_2^b$, and ii) $T^a = p_2^b + T^b$ with $E_2^a = E_2^b$.*

Lemma 1 show that, when $T^b \leq T^a$, there could exist two possible equilibria. We can rule out the equilibria such that $p_2^a + T^a = p_2^b + T^b$ with $E_2^a = E_2^b$ because empirical evidence confirms that usually communities differ with respect provision of education and household's expenditure. Instead, we focus on equilibria in *i)*. When $T^a \geq T^b$, in equilibrium, the education per capita must be lower in community b, otherwise all entrant would prefer this community, that given the capacity constraint is not possible. Given $T^a \geq p_2^b + T^b$ and $E_2^a > E_2^b$, the following Lemma studies how entrants sort into community according to their income.

[8] This assumption allows the community with the positive selling housing price (community b) to be inhabited by households with and without a child.

[9] Since $U(E, z)$ is assumed to be continuously differentiable, then $V(.)$ has the same properties.

LEMMA 2. *Assuming the total expenditure denoted by* $\overline{p}_2^j = p_2^j + T^j$, *then a high income household with a child prefers the community with the highest* $\overline{p}_2^j$.

Lemma 2 says that high income households with a child prefer community a, where the tax is higher but the provision of education per capita is also higher. We can conclude that, at the second period, the housing market equilibrium is such that community b is inhabited by both childless households (regardless their income) and low income households with a child. Definition 3 allows finding the housing demand in community b.

DEFINITION 1. There exists an income $\widetilde{y}\left(T^a, T^b, p_2^b\right)$ such that, at time 2, the household with a child is indifferent between community a and b. It is defined as follows:

$$V^a\left(T^a, \widetilde{y}\left(T^a, T^b, p_2^b\right)\right) = V^b\left(T^b, \widetilde{y}\left(T^a, T^b, p_2^b\right)\right) \tag{11}$$

Hence, the sorting equilibrium is such that community b is inhabited by all childless households and by all households with a child and income at most equal to $\widetilde{y}(.)$[10]. Community b also provides an interesting case of no income segregation. Low income households with a child prefer to live in the community with a lower tax even thought the per capita education is lower and the housing price is high[11]. Recalling that the housing price in community a is zero, we check whether capitalization effect in community b arises, that is $\frac{dp_2^b}{dT^b} > 0$. Since the income distribution of all entrants (with mass 1) is also $f(y)$, then the housing demand of entrants in community b is $F(\widetilde{y}(.))$. In words, all entrants with income lower than $\widetilde{y}(.)$ prefer to allocate into community b together with the remaining residents (childless and not). The total housing demand in community b is then $D^b = F(\widetilde{y}(.))$ and the market clearing condition writes:

$$qH^b = F(\widetilde{y}(.)) \tag{12}$$

The following Lemma solves the housing market equilibrium and it gives the result in terms of capitalization.

LEMMA 3. *The capitalization effect in community* b *exists only when* $\frac{\partial E_2^b}{\partial T^b} > v'\left(\widetilde{y} - p_2^b - T^b\right)$.

Lemma 3 says that the capitalization effect holds only when the marginal effect of a higher education more than compensates the marginal loss from a reduction in private consumption. When this is the case, an increase in tax increases the

[10] We remind that by assumption, households sorting into community a are indifferent from buying a housing from the leaving households and building a new one.

[11] In this model with do not allow households to change community at time 2. In other words, we can think a scenario with sufficiently high transportation costs. However, allowing households which were exogenously allocated at time 1 to freely change community at time 2, does not change the quality of our result. In fact, childless households would choose community b as well since the expenditure there is lower. High income household with a child, exogenously allocated at time 1 in community b, would go to community a without affecting the housing market there because the housing capacity is sufficiently high to keep $p_2^a = 0$. Low income households with a child, instead, would remain in community b.

housing demand in community b (the indifference income goes up). This increase in housing demand pushes up the housing price. Instead, when the marginal loss from a reduction in private consumption overcomes the marginal benefit from a higher education, then an increase in tax reduces the housing demand (the indifference income decreases) therefore the housing price gets lower. The next Corollary shows that there always exists a tax such that capitalization exists.

Corollary 1. *At time* 2*, there exists a tax* $\widehat{T}^b$ *such that the capitalization effect exists for all* $0 < T^b < \widehat{T}^b$.

Lemma 3 and Corollary 1 introduce the capitalization effect and show that the level of tax may be capitalized into the housing prices. In our model there exists a range of taxes such that the value of the housing, given by its selling price, is increasing in the tax imposed by the local government to finance public education. In particular, the higher tax set at time 1 the higher is the selling price at time 2. For households with a child the marginal benefit of a higher tax runs through the educational expenditure and the reselling price, whereas childless households may only benefit from a higher tax through capitalization.

5.2. Voting at time 1

The aim of this section is studying the majority voting equilibrium. In particular, in the section 5.2.1 we show that the preferences of households with and without a child may be single peaked. This implies that the median voter theorem applies. We still restrict the analysis on the voting problem in the community b when the capitalization effect exists, that is $0 \leq T^b < \widehat{T}^b$.

5.2.1. Single peaked preferences

In this framework, voting takes place once the households are already owners, then the housing prices at time 1 are given. The voters know the local government's budget constraints and anticipate the housing market equilibrium at time 2; let us denote the equilibrium price and education as p_2^{b*} and E_2^{b*}. Voters' indirect utilities write:

$$V_c^b\left(T^b, y; q\right) = v\left(y^c - p_1^b - T^b\right) + \frac{T^b N_1^b}{n_1^b} + (1-q)\left(v\left(y^c - T^b\right) + E_2^{b*}\right) + qv\left(y^c + p_2^{b*}\right), \tag{13}$$

$$V_n^b\left(T^b, y; q\right) = v\left(y^n - p_1^b - T^b\right) + (1-q)\,v\left(y^n - T^b\right) + qv\left(y^n + p_2^{b*}\right). \tag{14}$$

Therefore their maximization problems are:

$$\max_{T^b} v\left(y^c - p_1^b - T^b\right) + \frac{T^b N_1^b}{n_1^b} + (1-q)\left(v\left(y^c - T^b\right) + E_2^{b*}\right) + qv\left(y^c + p_2^{b*}\right) \tag{15}$$

and

$$\max_{T^b} v\left(y^n - p_1^b - T^b\right) + (1-q)\, v\left(y^n - T^b\right) + qv\left(y^n + p_2^{b*}\right) \tag{16}$$

The first order conditions for households with and without a child are respectively:

$$-v'(.) + \frac{N_1^b}{n_1^b} - (1-q)\, v'(.) + (1-q)\frac{\partial E_2^{b*}}{\partial T^b} + \frac{\partial p_2^{b*}}{\partial T^b} qv'(.) = 0 \tag{17}$$

$$-v'(.) - (1-q)\, v'(.) + \frac{\partial p_2^{b*}}{\partial T^b} qv'(.) = 0^{12} \tag{18}$$

Since the indirect utilities (13) and (14) are strictly concave in the range of taxes we are interested in ($0 \leq T^b < \widehat{T}^b$), then these utilities reach the maximum at a unique value of the tax. Thus the preferences of both households are single peaked. Let T_c^{b*} and T_n^{b*} be the taxes satisfying the first order conditions (17)-(18). They are defined as follows:

$$T_c^{b*} = \arg\max_{T^b} V_c^b\left(T^b, y^c; q\right) \tag{19}$$

$$T_n^{b*} = \arg\max_{T^b} V_n^b\left(T^b, y^n; q\right) \tag{20}$$

Corollary 2. *The optimal taxes are such that $T_n^{b*} < T_c^{b*}$ for every $0 \leq T^b < \widehat{T}^b$.*

Corollary 2 says that households with a child prefer a higher tax. This result is due to the fact that households with a child benefit from both the education of their child and the value of the housing. The next Proposition enables us to show that the indirect utility of childless households reaches a single peak at a positive tax.

Proposition 1. *When at time 2 the housing price in community b is increasing in the tax, then there exists a critical value $\widehat{q}$ such that childless household's most preferred tax is unique and positive for every $q > \widehat{q}$, whereas it is never positive for $q \leq \widehat{q}$.*

[12] The second order conditions with respect to the tax are:

$$v''(.) + (1-q)\, v''(.) + (1-q)\frac{\partial^2 E_2^{b*}}{\partial T^{b2}} + q\left(\frac{\partial^2 p_2^{b*}}{\partial T^{b2}} v'(.) + v''(.)\frac{\partial p_2^{b*}}{\partial T^b}\right) < 0$$

and

$$v''(.) + (1-q)\, v''(.) + q\left(\frac{\partial^2 p_2^{b*}}{\partial T^{b2}} v'(.) + v''(.)\frac{\partial p_2^{b*}}{\partial T^b}\right) < 0.$$

It is possible to show that when $E_2^{a*}\left(T^b\right)$ is concave then also $p_2^{a*}\left(T^b\right)$ is a concave function of T^b. The intuition is that an excessive increase in tax reduces education because reduces the number of tax payers in community b.

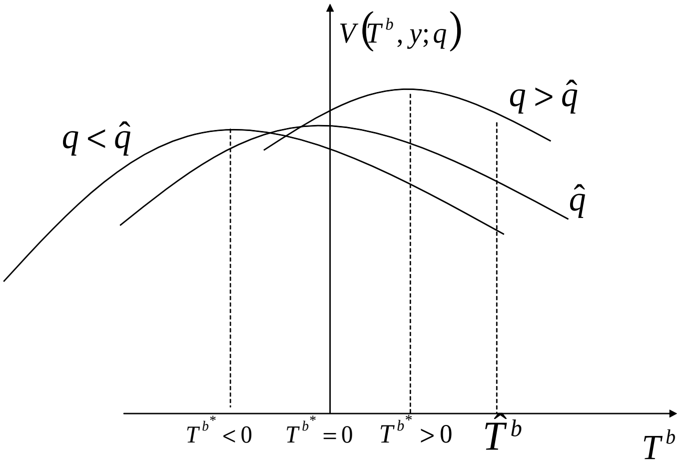

Fig. 2 Indirect utility of childless households

Proposition 1 says that when the probability of leaving community and reselling housing is sufficiently high, then also the childless household's most preferred tax is positive. The reason is that this probability can be considered as the weight given to the capitalization effect. When they do not leave community ($q = 0$) the benefit from capitalization disappears whereas when households must leave, then the benefit from the capitalization is totally gained. In other words, if the expected duration of childless households in the property is short, then they vote for a positive tax. Figure 2 illustrates the indirect utility functions of childless households[13]. Considering the probability of leaving community as childless' age we can conclude that aging induces childless to support local public expenditure in education via capitalization. This interpretation has an empirical confirmation in Baldosn and Brunner (2004) and Hilber and Mayer (2006).

The concavity of the indirect utility function of both households with and without a child makes the preferences of all the voters single peaked. This implies that there exists a majority voting equilibrium by the median voter theorem in which also childless households support local spending in education because the level of education is capitalized into the value of their housings.

5.2.2. The Voting Equilibrium

We firstly show that the median income voter may not be pivotal. We follow the standard literature and study how income drives the voting behavior of households by using the sign of the cross derivative of the indirect utility, $\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y \partial T^b}$, in particular when $\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y \partial T^b} > 0$ ($\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y \partial T^b} < 0$) the utility of higher income households increases more when the tax is higher (lower), then higher income households prefer a higher (lower) tax.

[13]For exponential convenience we restrict the graphical representation to the case in which $V_n^b\left(T^b, y; q\right)$ is concave even for $T^b < 0$. Actually, we found concavity only for a positive value of the tax, nevertheless we remark that we only need that when the probability of leaving community is sufficiently low, then there exists at least one peak at a non positive tax.

By differentiate the indirect utility $V_i\left(T^b, y; q\right)$ with respect to y^i and T^b we obtain:

$$\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y^i \partial T^b} = -v''\left(y^i - p_1^b - T^b\right) - (1-q)\, v''\left(y^i - T^b\right) + qv''\left(y^i + p_2^{b*}\right)\frac{\partial p_2^{b*}}{\partial T^b} \tag{21}$$

It is possible to see that for households both with and without a child the sign of (21) is ambiguous and depends on q and T^b.

The following Proposition enables us to study the effect of household's income on the voting behavior.

Lemma 4. *There exists a strictly increasing function defined over $0 \leq T^b < \widehat{T}^b$, denoted by $\widetilde{q}\left(T^b, y\right)$, called the indifference locus, such that high income households, with and without a child, prefer a higher tax for every pair q, T^b below $\widetilde{q}\left(y, T^b\right)$ whereas they prefer a lower tax for every pair above $\widetilde{q}\left(y, T^b\right)$.*

The result in Lemma 4 implies that the indirect utility functions of both households with and without a child, evaluated at different taxes, cross at most once in the plane (*U; y*). In particular, for sufficiently high probability, the utility with higher tax cross the other from above, whereas for sufficiently low probability the utility with higher tax cross the other from below.

To complete the analysis we focus only on the probabilities such that all households prefer a positive tax (that is $q > \widehat{q}$). Figure 3 illustrates this scenario according to the indifference locus[14]. In particular, when $\widehat{q} > \widetilde{q}\,(0)$, we have two scenarios: *i)* all pairs of q and T^b stay above $\widetilde{q}\left(y, T^b\right)$ (the dotted area), where high income households prefer a lower positive tax, and *ii)* all pairs of q and T^b stay below $\widetilde{q}\left(y, T^b\right)$ (the dashed area), where high income households prefer a higher positive tax. Consider the analysis in terms of childless voters. The presence of a strong capitalization effect (high q) makes low income childless household more willing to bear a tax rise for almost all taxes in the considered range. On the other hand, high income childless households are willing to bear a rise in tax only when the tax they vote for is sufficiently high. When capitalization almost disappears (q close to zero), only high income childless household are willing to support a tax for any level of possible tax lower than $\widehat{T}^b$[15]. The same arguments holds for the case $\widehat{q} < \widetilde{q}\,(0)$ and $q > \widehat{q}$[16]. This result is in line with Denzau and Grier (1984), Fischer (1988) and Epple and Romano (1996) according to which high income households are more willing to bear a tax rise for increased public education[17].

[14] In the figure 3 we draw the indifference locus as convex function in T^b. Actually, we do not show that the indifference locus is convex in T^b because it would imply a lot of complex computations. Moreover, even though the indifference locus was concave, the qualitative results providing by the figure would not change.

[15] The same arguments, of course, apply for households with a child.

[16] The assumption $\widehat{q} < \widetilde{q}\,(0)$ does not necessarily imply that the capitalization effect is low. In this case, in fact, we can only infer that the capitalization effect is lower than in the case of $\widehat{q} = \widetilde{q}\,(0)$. The fact that, in figure 5, the point $\widetilde{q}\,(0)$ is drawn near to $q = 0$ is purely a random graphical representation. Actually it is necessary a very complex computation to show the effective location of $\widetilde{q}\,(0)$ along the vertical axis; therefore we omit that and the comments for the case $\widehat{q} < \widetilde{q}\,(0)$ and $\widehat{q} > \widetilde{q}\,(0)$ are equivalent.

[17] Epple and Romano (1996) explain that the estimation about the effect of household income on the willingness to bear such a rise tax is still controversial and, for this reason, they consider the theoretical cases in which high income households prefer both a higher and a lower tax.

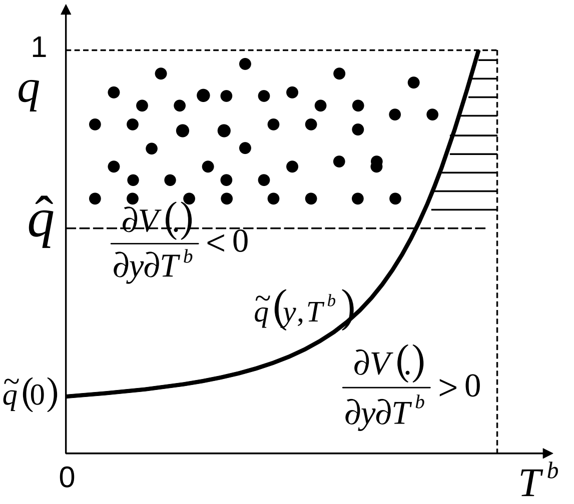

Fig. 3 The indifference locus

The behavior of households given in Lemma 4 enables us to characterize the voting equilibrium and check whether the median income is pivotal.

Definition 2. Let $\overline{y}$ be the median income in community b, such that $F(\overline{y}) = \frac{N_1^b}{2}$, with $\overline{T}_c$ and $\overline{T}_n$ be the preferred tax respectively by the voter with and without a child and income equal to $\overline{y}$.

We remind that Corollary 2 implies $\overline{T}_c > \overline{T}_n$. This inequality mainly drives the following results. We find that neither childless voter nor voter with a child can be pivotal when it has the median income, regardless whether childless are majority or not in community b.

Proposition 2. *A majority voting equilibrium in the community b exists and the income of median voter is different from the median income for every $q \in [\widehat{q}, 1]$.*

Proposition 2 says that, at time 1 in community b, a majority voting equilibrium exists and the median voter's income is different from the media income. The existence of a "middle class" with or without a child never allows the median income voter to be pivotal. Consider the case in which high income voters prefer a higher tax. Since childless voters prefer a lower tax than voters with a child, then, low income households (with and without a child) make coalition with a childless middle income class to beat the median income voter, whereas high income households (with and without a child) make coalition with a middle income class with a child. In other word, a middle class with a child makes coalition with high income households whereas middle class without a child makes coalition with low income households. This Proposition shows that the heterogeneity in the presence of children induces income classes at the ends of the distribution to get coalition with the middle income class[18].

[18]In the proof we show that the median income is not pivotal even when high income voters prefer a lower tax.

We now separately focus on the extreme case of no capitalization, that is $q = \widehat{q}$. In words, childless household prefer a tax equal to zero, whereas households with a child prefer a positive tax only via education of their child. The following Proposition shows that, absent any capitalization effect, the median income is still not pivotal.

Corollary 3. *When $q = \widehat{q}$, median income voter is still not pivotal regardless whether the majority of voters is childless*

Corollary 3 shows that the introduction of heterogeneity like absence of child, when capitalization does not occur, induces childless voters not to support local public education and vote for zero tax. In this case, when high income households prefer a higher tax, the low income households with a child make coalition with childless voters to block public investment. When, instead, high income households prefer a lower tax then high income households with a child make coalition with childless voters. This result is still in line with Epple and Romano (1996) in the sense that when a group of households is willing to vote for zero tax, whatever the reason, cross incomes coalitions may arise to block local provision of education. In Epple and Romano (1996) households vote for zero tax because of the existence of private school, in our paper, instead, the heterogeneity in the presence of child, absent capitalization, induces childless households to vote for zero tax.

In conclusion, we analyze where is collocated the income of the median voter. This is an important issue because we provide a proxy for the level of education in the community. To do that we check whether the income of median voter is higher or lower than the median income.

Corollary 4. *When the median voter is childless, its income is higher than the median income, whereas when the median voter has a child its income is lower than the median one.*

The intuition follows the approach in Proposition 2. Consider the case in which high income voters prefer a higher tax. Since $T_c^* > T_n^*$, the median voter with a child and income higher than the median is beaten by a coalition preferring a lower tax. This coalition is composed of voters with and without a child and income at most equal to this particular level and childless voters with an income higher than this level (childless middle class). The childless median voter with income lower than the median is not pivotal as well. It will be beaten by a coalition preferring a higher tax. This coalition is composed of voters with and without a child and income at least equal to this particular level and voters with a child and income lower than this level (middle class with a child). Corollary 4 is confirmed by the recent empirical work of Baldosn and Brunner (2009). They find that in California an income percentile below the median is decisive for majority voting rules. This is exactly what who show when the median voter has a child.

We can conclude that, regardless whether higher income households prefer a higher or a lower tax, the equilibrium tax preferred by the median voter (childless or not) is higher than the tax preferred by the childless median income voter and lower than the tax preferred by the median income voter with a child. The difference between the tax of the childless and non childless median voter depends on income distribution. Different income distributions could lead to different equilibria in which the tax preferred by the childless median voter is higher than the tax preferred by the median voter with a child. This should suggest that when the median voter is childless, the level of provision of education may be higher than the level obtained under a different income distribution allowing the median voter to have a child. Although the rising share of childless households is making more and more likely that the pivotal voter at the local level is childless, however the presence of capitalization should not reduce the support for local education.

6. CONCLUSIONS

Empirical evidence and theoretical studies show that when a local government finances the local provision of public education by a tax set by a majority voting, then households with school aged children support local spending for two main reasons: children benefit from a higher provision of education, and a higher local public spending is capitalized into the value of the housing.

This model shows that the house price capitalization may provide a sufficient alternative mechanism to encourage childless households to support local public education. We find that the median income voter is never pivotal. In particular, when median voter is childless, his income is higher than the median one whereas when it has a child, his income is lower than the median one. The difference between the tax of the median voter with and without a child depends on the income distribution. There could exist, in fact, equilibria such that the childless median voter prefers a higher tax than the median voter with a child.

The presence of the childless households differentiates our model from the previous theoretical literature. Although our results predict that capitalization may induce decisive childless voters to support local education more than decisive voters with a child; however, a formal welfare analysis could be a good complement for this work. Further research could also model capitalization by allowing some externalities, such as rich people with no children preferring to live near rich people who, if they have children vote for high tax. We are aware that other alternative mechanisms could motivate childless households to vote for local public spending. Cutler et al. (1993), Hoxby (1998), Goldin and Katz (1997, 1999), Alesina et al. (1999), Bergstrom et al. (1982), Harris et al. (2001) show that pure altruism, grandparents who care for their grandchildren and reduced juvenile crime also matter. Nevertheless, this paper focuses on the effect of housing value because housing is an essential and primary investment. Altruism, care for their children, interest in a lower crime or environmental quality certainly exist but they do not characterize all households, like instead housing does; the presence of housing, indeed is a necessary condition for other interests to exist.

7. APPENDIX A

Proof of Lemma 1

We immediately exclude the case $T^a < p_2^b + T^b$ because $T^b \leq T^a$, then we proceed by contradiction. **1)** Consider the scenario $T^a \geq p_2^b + T^b$, $E_2^a < E_2^b$ and the housings market is in equilibrium. When $T^a \geq p_2^b + T^b$ the total expenditure is weakly lower in community b, since $E_2^a < E_2^b$ at least one household with a child has incentive to go to the community b. Since $H^b < 1$ and given that at time 1 all childless households are allocated in community b the capacity of community b is not enough to allocate all households, then this scenario does not characterize a housing market equilibrium. **2)** Consider now $T^a > p_2^b + T^b$ and $E_2^a = E_2^b$, and the housing market is in equilibrium. All residents would prefer community b. However, since for the housing capacity constraint this is not possible, then $T^a > p_2^b + T^b$ and $E_2^a = E_2^b$ cannot be an equilibrium. ■

Proof of Lemma 2

The proof comes from the separability of the utility function and it is simply given by: $\frac{\partial V^j(T^j,y)}{\partial y \partial \overline{p}_2^j} = -v''\left(y - \overline{p}_2^j\right) > 0.$■

Proof of Lemma 3

To find how p_2^b changes according to T^b we apply the total differential to the condition (12). Let us to define a function $R(\widetilde{y}, q) = F(\widetilde{y}(.)) - qH^b = 0$. By totally differentiating $R(\widetilde{y}, q)$ we have:

$$\frac{\partial R(.)}{\partial T^b} dT^b + \frac{\partial R(.)}{\partial p_2^b} dp_2^b = 0 \tag{22}$$

This gives: $\frac{dp_2^b}{dT^b} = -\frac{\frac{\partial R(.)}{\partial T^b}}{\frac{\partial R(.)}{\partial p_2^b}}$. Given $\frac{\partial R(.)}{\partial T^b} = \frac{\partial F(.)}{\partial \widetilde{y}(.)}\frac{\partial \widetilde{y}}{\partial T^b}$ and $\frac{\partial R(.)}{\partial p_2^b} = \frac{\partial F(.)}{\partial \widetilde{y}(.)}\frac{\partial \widetilde{y}}{\partial p_2^b}$, then $\frac{dp_2^b}{dT^b} = -\frac{\frac{\partial \widetilde{y}}{\partial T^b}}{\frac{\partial \widetilde{y}}{\partial p_2^b}}$. To find $\frac{\partial \widetilde{y}}{\partial T^b}$ and $\frac{\partial \widetilde{y}}{\partial p_2^b}$ we apply the total differential to (11). Let us define a function $W(\widetilde{y})$ as follows:

$$W(\widetilde{y}) = v(\widetilde{y} - 0 - T^a) + E_2^a(T^a, \widetilde{y}) - v\left(\widetilde{y} - p_2^b - T^b\right) - E_2^b\left(T^b, \widetilde{y}\right) = 0, \tag{23}$$

by totally differentiating $W(\widetilde{y})$ we have:

$$\frac{\partial W(.)}{\partial \widetilde{y}} d\widetilde{y} + \frac{\partial W(.)}{\partial T^b} dT^b + \frac{\partial W(.)}{\partial p_2^b} dp_2^b = 0 \tag{24}$$

To obtain $\frac{\partial \widetilde{y}}{\partial T^b}$ we put $dp_2^b = 0$ and we get $\frac{d\widetilde{y}}{dT^b} = -\frac{\frac{\partial W(.)}{\partial T^b}}{\frac{\partial W(.)}{\partial \widetilde{y}}}$, and putting $dT^b = 0$ we get $\frac{d\widetilde{y}}{dp_2^b} = -\frac{\frac{\partial W(.)}{\partial p_2^b}}{\frac{\partial W(.)}{\partial \widetilde{y}}}$. Where:

$$\frac{\partial W(.)}{\partial \widetilde{y}} = v'(\widetilde{y} - T^a) + \frac{\partial E_2^a(T^a, \widetilde{y})}{\partial \widetilde{y}} - v'\left(\widetilde{y} - p_2^b - T^b\right) - \frac{\partial E_2^b\left(T^b, \widetilde{y}\right)}{\partial \widetilde{y}} > 0 \tag{25}$$

since $\frac{\partial E_2^a(T^a,\widetilde{y})}{\partial \widetilde{y}} > 0$ and $\frac{\partial E_2^b\left(T^b,\widetilde{y}\right)}{\partial \widetilde{y}} < 0$ (a higher $\widetilde{y}$ means a higher number of household with a child entering community b, then the education per capita in community a increases). We also have:

$$\frac{\partial W(.)}{\partial p_2^b} = v'\left(\widetilde{y} - p_2^b - T^b\right) > 0 \tag{26}$$

and,

$$\frac{\partial W(.)}{\partial T^b} = v'\left(\widetilde{y} - p_2^b - T^b\right) - \frac{\partial E_2^b}{\partial T^b} \tag{27}$$

with $\frac{\partial W(.)}{\partial T^b} \gtreqless 0$ if $\frac{\partial E_2^b}{\partial T^b} \lesseqgtr v'\left(\widetilde{y} - p_2^b - T^b\right)$. Therefore we get:

$$\frac{\partial \widetilde{y}}{\partial p_2^b} = -\left(\frac{v'\left(\widetilde{y} - p_2^b - T^b\right)}{v'\left(\widetilde{y} - T^a\right) + \frac{\partial E_2^a(T^a,\widetilde{y})}{\partial \widetilde{y}} - v'\left(\widetilde{y} - p_2^b - T^b\right) - \frac{\partial E_2^b(T^b,\widetilde{y})}{\partial \widetilde{y}}}\right) < 0 \tag{28}$$

and

$$\frac{\partial \widetilde{y}}{\partial T^b} = -\left(\frac{v'\left(\widetilde{y} - p_2^b - T^b\right) - \frac{\partial E_2^b}{\partial T^b}}{v'\left(\widetilde{y} - T^b\right) + \frac{\partial E_2^a(T^a,\widetilde{y})}{\partial \widetilde{y}} - v'\left(\widetilde{y} - p_2^b - T^b\right) - \frac{\partial E_2^b(T^b,\widetilde{y})}{\partial \widetilde{y}}}\right) \tag{29}$$

with $\frac{\partial \widetilde{y}}{\partial T^b} \gtreqless 0$ if $\frac{\partial E_2^b}{\partial T^b} \gtreqless v'\left(\widetilde{y} - p_2^b - T^b\right)$. Hence, after some simplifications we find:

$$\frac{dp_2^b}{dT^b} = \frac{-v'\left(\widetilde{y} - p_2^b - T^b\right) + \frac{\partial E_2^b}{\partial T^b}}{v'\left(\widetilde{y} - p_2^b - T^b\right)} \tag{30}$$

with $\frac{dp_2^b}{dT^b} \gtreqless 0$ when $\frac{\partial E_2^b}{\partial T^b} \gtreqless v'\left(\widetilde{y} - p_2^b - T^b\right)$.$\blacksquare$

Proof of Lemma 4

A high income household is indifferent between preferring a higher or a lower tax if:

$$\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y \partial T^b} = 0 \tag{31}$$

Lemma 5. $\frac{\partial^2 V_i\left(T^b, y; q\right)}{\partial y \partial T^b}$ *is decreasing in* q *for every* $0 \leq T^b < \widehat{T}^b$. $\|$

Lemma 5 shows that there exists a unique value of $q \in [0,1]$ which solves the (31) for any $0 \leq T^b < \widehat{T}^b$. This called indifference locus is a function of y and T^b, and is denoted by $\widetilde{q}\left(y, T^b\right)$. It is defined as follows:

$$-v''\left(y^i - p_1^b - T^b\right) - \left(1 - \widetilde{q}\left(y, T^b\right)\right) v''\left(y^i - T^b\right) + \widetilde{q}\left(y, T^b\right) v''\left(y^c + p_2^{b*}\right) \frac{\partial p_2^{b*}}{\partial T^b} = 0 \tag{32}$$

By totally differentiating (32), given a level of income, the slope of the indifference locus in the plane q, T^b is:

$$\frac{d\widetilde{q}}{dT^b} = -\frac{v'''\left(y^i - p_1^b - T^b\right) + (1-\widetilde{q})\, v'''\left(y^i - T^b\right) + \widetilde{q}\left[v'''\left(y^c + p_2^{b*}\right)\left(\frac{\partial p_2^{b*}}{\partial T^b}\right)^2 + \frac{\partial^2 p_2^{b*}}{\partial T^{b2}} v''\left(y^c + p_2^{b*}\right)\right]}{v''\left(y^i - T^b\right) + v''\left(y^c + p_2^{b*}\right)\frac{\partial p_2^{b*}}{\partial T^b} + \widetilde{q}\left[v'''\left(y^c + p_2^{b*}\right)\frac{\partial p_2^{b*}}{\partial q}\frac{\partial p_2^{b*}}{\partial T^b} + \frac{\partial p_2^{b*}}{\partial T^b \partial q} v''\left(y^c + p_2^{b*}\right)\right]} \tag{33}$$

That, by result in the proof of Lemma 6 and Lemma 3, is positive. Hence, we have that: *i)* $\frac{\partial^2 V_i\left(T^b,y;q\right)}{\partial y \partial T^b} > 0$ for every pair q, T^b below $\widetilde{q}\left(y, T^b\right)$, and *ii)* $\frac{\partial^2 V_i\left(T^b,y;q\right)}{\partial y \partial T^b} < 0$ for every pair above $\widetilde{q}\left(y, T^b\right)$■

Proof of Lemma 5

The cross derivative of the indirect utility function of households both with and without a child with respect to the income and the tax is given by:

$$\frac{\partial^2 V_i\left(T^b, y\right)}{\partial y \partial T^b} = -v''\left(y^i - p_1^b - T^b\right) - (1-q)\, v''\left(y^i - T^b\right) + q v''\left(y^c + p_2^{b*}\right) \frac{\partial p_2^{b*}}{\partial T^b} \tag{34}$$

Now by differentiating with respect to q, we obtain:

$$\frac{\partial^3 V_i\left(T^b, y\right)}{\partial y \partial T^b \partial q} = v''\left(y^i - T^b\right) + v''\left(y^c + p_2^{b*}\right) \frac{\partial p_2^{b*}}{\partial T^b} + q\left[v'''\left(y^c + p_2^{b*}\right) \frac{\partial p_2^{b*}}{\partial q} \frac{\partial p_2^{b*}}{\partial T^b} + \frac{\partial p_2^{b*}}{\partial T^b \partial q} v''\left(y^c + p_2^{b*}\right)\right] \tag{35}$$

Since we have shown that $\frac{\partial p_2^{b*}}{\partial q} < 0$, $\frac{\partial p_2^{b*}}{\partial T^b} > 0$ and $\frac{\partial^2 p_2^{b*}}{\partial T^b \partial q} = 0$, then given $v'''\left(.\right) > 0$, $\frac{\partial^3 V_i\left(T^b,y\right)}{\partial y \partial T^b \partial q}$ is negative.

Furthermore, $\frac{\partial^2 V_i\left(T^b,y\right)}{\partial y \partial T^b}$ is always increasing in T^b, in fact we have:

$$\frac{\partial^3 V_i\left(.\right)}{\partial y \partial T^{b2}} = v'''\left(y^i - p_1^b - T^b\right) + (1-q)\, v'''\left(y^i - T^b\right) + q\left[v'''\left(y^c + p_2^{b*}\right)\left(\frac{\partial p_2^{b*}}{\partial T^b}\right)^2 + \frac{\partial^2 p_2^{b*}}{\partial T^{b2}} v''\left(y^c + p_2^{b*}\right)\right] \tag{36}$$

That, given $\frac{\partial^2 p_2^{b*}}{\partial T^{b2}} < 0$, is always positive.■

Proof of Proposition 1

We only need to show the result for the childless household, and then the result for households with a child is a direct consequence of Corollary 2. The proof proceeds along two steps. Firstly, we show that the slope of $V_n^b\left(T^b, y; q\right)$ valued at a tax T^b sufficiently close to $\widehat{T}^b$ is negative. In the second step we show that the slope of $V_n^b\left(T^b, y; q\right)$ valued at $T_n^b = 0$ is increasing in q, positive for every probability higher than $\widehat{q}$, and negative for every probability lower than $\widehat{q}$. Hence, concavity of $V_n^b\left(T^b, y; q\right)$ over $0 \leq T^b < \widehat{T}^b$ gives the following results: i) when the probability q is sufficiently high $V_n^b\left(T^b, y; q\right)$ reaches a unique peak at a positive tax, ii) when q is sufficiently low $V_n^b\left(T^b, y; q\right)$ reaches its peak at a non positive tax .

First step. Let the slope of $V_n^b\left(T^b, y\right)$ be defined as follows:

$$\frac{\partial V_n^b\left(T^b, y; q\right)}{\partial T^b} = -v'\left(y^n - p_1^b - T^b\right) - (1-q)\, v'\left(y^n - T^b\right) + \frac{\partial p_2^{b*}}{\partial T^b} q v'\left(y^n + p_2^{b*}\right) \tag{37}$$

By corollary 1 we know that $\frac{\partial p_2^{b*}}{\partial T^b} = 0$ for $T^b = \widehat{T}^b$; therefore there exists an ε sufficiently small and a level of tax $T_\varepsilon^b = \widehat{T}^b - \varepsilon$ such that $\frac{\partial p_2^{b*}}{\partial T^b} \simeq 0$. Hence, given

$v'(.) > 0$, we have that the slope of the indirect utility function is negative when $T^b = T^b_\varepsilon$.

Second step. We rewrite the slope of the indirect utility at zero tax as follows:

$$\frac{\partial V_n^b\left(T^b,y;q\right)}{\partial T^b}\mid_{T^b=0}= -v'\left(y^n-p_1^b\right)-(1-q)\,v'\left(y^n\right)+q\left(\frac{\partial p_2^{b*}}{\partial T^b}\mid_{T^b=0}\right)v'\left(y^n+p_2^{b*}\mid_{T^b=0}\right) \tag{38}$$

The following Lemma enables us to study the effect of the probability q on $\frac{\partial V_n^b\left(T^b,y\right)}{\partial T^b}\mid_{T^b=0}$.

LEMMA 6. $\frac{\partial^2 V_n^b\left(T^b,y;q\right)}{\partial T^b \partial q}\mid_{T^b=0}> 0$. *When* $q=0$, *we have* $\frac{\partial V_n^b\left(T^b,y;q\right)}{\partial T^b}\mid_{T^b=0}< 0$ ||

Lemma 6 shows that the slope of $V_n^b\left(T^b,y;q\right)$ increases in q and it is negative when the probability of reselling housing is zero. Thus there exists a critical positive value $\widehat{q}$ such that the slope of the indirect utility is zero when $T^b=0$; we denote this critical value as $\widehat{q}$ and it is defined as follows:

$$-v'\left(y^n-p_1^b\right)-(1-\widehat{q})\,v'\left(y^n\right)+\widehat{q}\left(\frac{\partial p_2^{b*}}{\partial T^b}\mid_{T^b=0,q=\widehat{q}}\right)v'\left(y^n+p_2^{b*}\mid_{T^b=0,q=\widehat{q}}\right)=0 \tag{39}$$

Given $\widehat{q}$, we have the following results: *i)* $\frac{\partial V_n^b\left(T^b,y;q\right)}{\partial T^b}\mid_{T^b=0}\leq 0$, for every $q\leq \widehat{q}$, and *ii)* $\frac{\partial V_n^b\left(T^b,y;q\right)}{\partial T^b}\mid_{T^b=0}> 0$ for every $q>\widehat{q}$. Hence, by concavity of $V_n^b\left(T^b,y;q\right)$ over $0\leq T^b<\widehat{T}^b$, we conclude the proof by showing that: *i)* the indirect utility of childless households is maximized at negative tax for every $q<\widehat{q}$, *ii)* it reaches a peak at zero tax when $q=\widehat{q}$, and *iii)* it presents a unique peak at a positive tax for every $q>\widehat{q}$.■

Proof of Lemma 6

i) The cross derivative of the function $V_n^b\left(T^b,y;q\right)$ with respect to T^b and q is as follows:

$$\frac{\partial^2 V_n^b\left(T^b,y;q\right)}{\partial T^b\partial q}=v'\left(y^n-T^b\right)+\frac{\partial^2 p_2^{b*}}{\partial T^b\partial q}qv'\left(y^n+p_2^{b*}\right)+\frac{\partial p_2^{b*}}{\partial T^b}v'\left(y^n+p_2^{b*}\right)+q\frac{\partial p_2^{b*}}{\partial T^b}\frac{\partial p_2^{b*}}{\partial q}v''\left(y^n+p_2^{b*}\right)q \tag{40}$$

To find $\frac{\partial p_2^b}{\partial q}<0$ and $\frac{\partial p_2^b}{\partial T^b\partial q}$ we differentiate $R\left(\widetilde{y},q\right)=F\left(\widetilde{y}\left(.\right)\right)-qH^b=0$ and get:

$$\frac{\partial R\left(.\right)}{\partial q}dq+\frac{\partial R\left(.\right)}{\partial p_2^b}dp_2^b=0 \tag{41}$$

$$\frac{dp_2^b}{dq}=-\frac{\frac{\partial R(.)}{\partial q}}{\frac{\partial R(.)}{\partial p_2^b}}=-\frac{\frac{\partial F(.)}{\partial \widetilde{y}(.)}\frac{\partial \widetilde{y}}{\partial q}-H^b}{\frac{\partial F(.)}{\partial \widetilde{y}(.)}\frac{\partial \widetilde{y}}{\partial p_2^b}}>0 \tag{42}$$

By using $\frac{\partial W(.)}{\partial \widetilde{y}}d\widetilde{y}+\frac{\partial W(.)}{\partial q}dq=0$, we then get $\frac{\partial \widetilde{y}}{\partial q}=-\frac{\frac{\partial W(.)}{\partial q}}{\frac{\partial W(.)}{\partial \widetilde{y}}}=0$. Given $\frac{\partial \widetilde{y}}{\partial p_2^b}<0$, as showed in the proof of Lemma 3, we have $\frac{\partial p_2^b}{\partial q}<0$. The higher the housing supply the lower is the price equilibrium. Also, given $\frac{\partial \widetilde{y}}{\partial q}=0$ and the value of $\frac{\partial p_2^b}{\partial T^b}$, it is possible to compute that $\frac{\partial p_2^b}{\partial T^b\partial q}=0$. Therefore we obtain $\frac{\partial^2 V_n^b\left(T^b,y;q\right)}{\partial T^b\partial q}>0$.

Since the slope of the indirect utility function of childless households with respect to T^b, valued at $T^b = 0$, is:

$$\frac{\partial V_n^b\left(T^b, y\right)}{\partial T^b} \mid_{T^b=0} = -v'\left(y^n - p_1^b\right) - (1-q)\, v'\left(y^n\right) + \left(\frac{\partial p_2^{b*}}{\partial T^b} \mid_{T^b=0}\right) q v'\left(y^n + p_2^{b*} \mid_{T^b=0}\right) \tag{43}$$

Then for $q = 0$, we have:

$$\frac{\partial V_n^b\left(T^b, y\right)}{\partial T^b} \mid_{T^b=0} = -v'\left(y^n - p_1^b\right) - v'\left(y^n\right) < 0 \tag{44}$$

■

Proof of Proposition 2

The proof is composed of two steps. Since the indirect utilities of all voters reach a unique peak over the tax $0 \leq T^b < \widehat{T}^b$, then the preferences of all voters are single peaked. Consequently, by the median voter theorem (Black 1958), we are sure that a unique majority voting equilibrium exists and the median voter is pivotal[19].

i) Consider the scenario $q > \widehat{q}$ with all q and T^b below $\widetilde{q}\left(T^b, y\right)$. Lemma 4 shows that both households vote for a positive tax, and a higher income household prefers a higher tax. We proceed by showing that the median income voter neither with nor without a child is pivotal. In other words, it is equivalent to say that $\overline{T}_c$ and $\overline{T}_n$ cannot by pivotal because there exists a coalition composed of at least half the voters preferring a different tax. Since Corollary 2 implies $\overline{T}_c > \overline{T}_n$ then we firstly have the following results: a) if $\overline{T}_n$ is the preferred tax of the childless median income voter, then there exists an income $\widetilde{y}^c$ lower than $\overline{y}$ such that voters with a child and income $\widetilde{y}^c$ also prefer $\overline{T}_n$; b) if $\overline{T}_c$ is the preferred tax of the median income voter with a child, then there exists an income $\widetilde{y}^n$ higher than $\overline{y}$ such that also childless voters with income $\widetilde{y}_n$ prefer $\overline{T}_c$. Given point a) and b) let us now proceed by contradiction and firstly assume that the pivotal tax is $\overline{T}_c$. In this case voters with a child and income at most equal to $\overline{y}$ and childless voters and income at most equal to $\widetilde{y}^n$ prefer a tax T^b lower than $\overline{T}_c$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^b < \overline{T}_c$. Thus, childless voter with the median income cannot be pivotal. Assuming now that the pivotal tax is $\overline{T}_n$, in this case childless voters with income at least equal to $\overline{y}$ and voters with a child with income at least equal to $\widetilde{y}^c$ prefer a tax $T^b > \overline{T}_n$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^b > \overline{T}_n$. Thus, neither $\overline{T}_n$ can be pivotal. Figure 4 illustrates a specific simple example in which households with a child are majority.

ii) Consider the scenario $q > \widehat{q}$ with all q and T^babove $\widetilde{q}\left(T^b, y\right)$. In this case both households vote for a positive tax but higher income households prefer a lower tax. It is possible to see that the reverse arguments of part i) apply, then even in this case the median voter's income is different from the median one. In particular now we have $\widetilde{y}^c > \overline{y}$ and $\widetilde{y}^n < \overline{y}$. Assume that the pivotal tax is $\overline{T}_n$. In this case, childless voters with income at most equal to $\overline{y}$ and voters with a child and income at most equal to $\widetilde{y}^c$ prefer a tax $T^b > \overline{T}_n$. Since $\overline{y}$ is the median-income, then at

[19] We remark that by definition the voter with the median income must be one (or childless or with a child) because half distribution must have income lower than the median one and half higher than this.

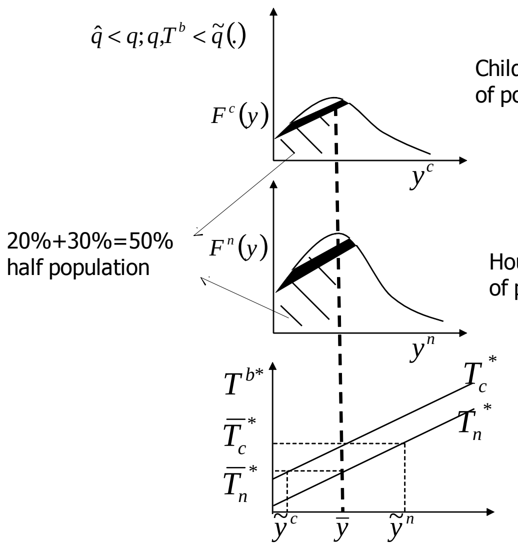

Fig. 4 Median income voter

least half the voters prefers a tax $T^b > \overline{T}_n$. Thus, childless voter with the median income cannot be pivotal. Assuming now that the pivotal tax is $\overline{T}_c$, in this case voters with a child and income at least equal to $\overline{y}$ and childless voters with income at least equal to $\widetilde{y}^n$ prefer a tax $T^b < \overline{T}_c$. Since $\overline{y}$ is the median-income, then at least half the voters prefers a tax $T^b < \overline{T}_c$. Thus, voter with a child and median income cannot be pivotal.■

8. APPENDIX B

Proof of Corollary 1

Consider only the case of a positive tax. Concavity of $v(.)$ implies that there exists a positive level of T^b, denoted by $\widehat{T}^b$, such that $v'\left(\widetilde{y} - p_2^b - T^b\right) = \frac{\partial E_2^b}{\partial T^b}$. Since $v'(.)$ is concave and $v(.)$ reaches its highest value at $T^b = 0$ (where $v'(.) = 0$), then for all taxes higher than zero $v'(.)$ is increasing in T^b (because $v'(.)$ decreases in z_2, by concavity assumption, and z_2 decreases in T^b). Therefore when tax is to high, then $v'(.)$ increases so much to become higher than $\frac{\partial E_2^b}{\partial T^b}$, then we get $\frac{dp_2^b}{dT^b} < 0$. The uniqueness of the $\widehat{T}^b$depends on the shape of $E_2^b\left(\widehat{T}^b\right)$, however since $\frac{\partial E_2^b}{\partial T^b}$ should be positive at $T^b = 0$, then, whatever the shape is, we are sure that there exists at least a value $\widehat{T}^b$ such that function $\frac{\partial E_2^b}{\partial T^b}$ and $v'\left(\widetilde{y} - p_2^b - T^b\right)$ cross.■

Proof of Corollary 2

Given the first order condition in (17)-(18) with $\frac{N_1^b}{n_1^b} + (1-q)\frac{\partial E_2^{b*}}{\partial T^b} > 0$, then $v''(z) < 0$ is a sufficient condition for T_n^{b*} to be lower than T_c^{b*}.■

Proof of Corollary 3

We still assume that $\overline{y}$ is the median income and follow the proof of the Proposition 2. **i)** Consider the scenario in which $q = \widehat{q}$ with q and T^b below $\widetilde{q}\left(T^b, y\right)$.

We firstly analyze the case in which the majority of voters is childless. Assume that $\overline{T}_c$ is the pivotal voter. In this case households with a child and income at most equal to $\overline{y}$ and all childless households prefer a tax lower than $\overline{T}_c$, than this cannot be pivotal. Since $\overline{T}_n = 0$ is the most preferred tax of the majority of voters no matters their income, then the median income childless voters is not pivotal either. Consider now that the majority of voters has a child. All households with a child and income at most equal to $\overline{y}$ and all childless voters prefer a tax lower than $\overline{T}_c$, since $\overline{y}$ is the median income and there exist at least a childless voter with an income higher than $\overline{y}$, then $\overline{T}_c$ cannot be pivotal. The most preferred tax of childless voters, $\overline{T}_n = 0$, can never be pivotal because there exist always a majority of voters with a child preferring a positive tax. **ii)** Consider the scenario in which $q = \widehat{q}$ with q and T^b above $\widetilde{q}\left(T^b, y\right)$. When childless voters are the majority, then $\overline{T}_c > 0$ will be beaten by a coalition at least including half voters. However, for the same reason in i) neither $\overline{T}_n = 0$ can be pivotal. When the majority of voters has a child the reverse arguments of part i) holds and it is possible to show that median income voter neither childless nor with a child is pivotal.■

Proof of Corollary 4

Proposition 2 shows that the median voter does not have income equal to the median income. In this proof we use again the definitions of $\widetilde{y}^n$ and $\widetilde{y}^c$ given in the proof of Proposition 2. We separately analyze the case in which a higher income voter prefer a higher and a lower tax. To show Corollary 4 we show that median voter has income between $\widetilde{y}^n$ and $\widetilde{y}^c$. This implies that the equilibrium tax preferred by the median voter should be between $\overline{T}_n$ and $\overline{T}_c$.

a) Consider the case in which higher income households prefer a higher tax. We remind that in this case we have $\widetilde{y}^n > \overline{y}$ and $\widetilde{y}^c < \overline{y}$. We show that when the median voter is childless, its income is higher than $\overline{y}$ but lower than $\widetilde{y}^n$, whereas when he has a child, its income is lower than $\overline{y}$ but higher than $\widetilde{y}^c$. We proceed by contradiction over two steps: **1)** we show that when the median voter is childless its income cannot be lower than the median one; **2)** we show that when the median voter has a child its income cannot be higher than the median one. The proof simply follows the argument in the proof of Proposition 2. **1)** Assume a level of income y^n_ε sufficiently close to $\overline{y}$ with $\overline{y} - \varepsilon = y^n_\varepsilon$ and the associated most preferred tax given by $T_{n,\varepsilon}$. Given $\widetilde{y}^n$ and $\widetilde{y}^c$, we know that there exists an income $\widetilde{y}^c_\varepsilon < \overline{y}$ such that households with a child and income at least equal to $\widetilde{y}^c_\varepsilon$ and childless households with an income at least equal to y^n_ε prefer a tax $T^b > T_{n,\varepsilon}$. Since y^n_ε is an income slightly lower than the median one, we have that the tax preferred by all childless voters with incomes lower than the median are beaten by a coalition of at least half voters. **2)** Assume a level of income y^c_ε sufficiently close to $\overline{y}$ with $\overline{y} + \varepsilon = y^c_\varepsilon$ and the associated most preferred tax given by $T_{c,\varepsilon}$. Again, there exists an income $\widetilde{y}^n_\varepsilon > \overline{y}$ such that childless households with income at most equal to $\widetilde{y}^n_\varepsilon$ and households with a child and income at most equal to y^c_ε prefer a tax $T^b < \overline{T}_{c,\varepsilon}$. Since y^c_ε is an income slightly higher than the median income, we have that the tax preferred by all voters with a child and income higher than the median are beaten by a coalition of at least half voters.

b) It is ease to show that the same procedure applies when higher income households prefer a lower tax. In this case we find the opposite results of point a). We remind that according to the definitions of $\widetilde{y}^n$ and $\widetilde{y}^c$, in this case we have $\widetilde{y}^n < \overline{y}$ and $\widetilde{y}^c > \overline{y}$. Hence, we get that the income of the median voter is lower than $\overline{y}$ but higher than $\widetilde{y}^n$ when he is childless, whereas it is higher than $\overline{y}$ but lower than $\widetilde{y}^c$ when he has a child.$\blacksquare$

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