Merger Control with Transfers from the Capital Gains Tax

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Abstract

This work proposes to rely on the capital gains tax legislation to introduce transfers in merger control. Transfers are never used in merger regulation, however they can represent a relevant device to extract information on synergies. The implicit transfer collected thanks to the capital gains tax, associated with divestitures, allows to screen among high and low synergy achievers. The analysis focuses on the fact that the transfer is fiscally constrained. It must be strictly positive but lower than a threshold; the capital gains tax paid by target shareholders depends on the ratio of cash used by the bidder as a medium of payment in the takeover bid and on the tax rate. The upper fiscal constraint combined with a non monotonic consumer surplus function allow the inefficient type to enjoy a rent, and affect the usual rent-efficiency trade-off. The lower fiscal constraint induces the efficient type’s divestitures to be distorted downward from the first best.

Key Words: Merger control, asymmetric information, capital gains tax, divestitures, principal agent.

JEL Classification: L51, D82, L41, D86

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1 Introduction

Economists interested in mergers all agree on one fact: mergers increase market prices through an intensification of the market power if they are not backed up with substantial efficiency gains. The lack of synergies has a negative impact on the consumer surplus, since higher prices decrease the purchasing power. Nevertheless, if anticompetitive effects are dominant, the competition authority can use divestitures, as a tool to restore effective competition in relevant markets.

Divestitures affect the allocation of propriety rights through partial or total sale of the combined business to another market player. For instance in the merger between GDF and Suez in November 2006, the EC initially found that the deal would have anticompetitive effects in the gas and electricity wholesale in Belgium and in France. However, the EC accepted the merger provided the divestiture of Distripal and SPE. This, in response to the anticompetitive concerns and synergies promised by concerned parties. In light of these divestitures, the EC concluded that the merger would not significantly impede competition in the European economic area or any substantial part of it and would not hurt consumers through an increase in prices.

This article proposes to rely on the fiscal system concerning the capital gains tax to offer to the antitrust agency another tool for its regulation task. This paper also aims at emphasizing other regulatory methods, in order to take a fair competition policy decision. The use of the capital gains tax as a transfer seems to be a relevant way to create incentives in the merger procedure and to get over two basic difficulties.

The first is the information problem. On the one hand, when firms propose a merger, they have a definite idea of the gains that they will achieve and the effect of their merger on the price. On the other hand, the commission does not dispose of all elements required to evaluate the exact level of synergy, and thus the knock-on effect on consumers.

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1 Among others Farrell and Shapiro (1990).
In this asymmetric information context, the antitrust agency should propose a menu of contracts to learn information about synergies that merged firms will achieve. Unfortunately it is confronted with a second problem: the lack of regulatory tools. In competition policy, available tools are not as extended as those employed by regulation authorities. The latter are indeed able to directly control prices or produced quantities, and to specify an explicit amount of transfer.\footnote{Merger commissions never use transfers in spite of the screening device that they represent.} Common wisdom in merger control argue that transfers cannot be used as a regulation tool. However a fiscal system exists in merger, since target shareholder are taxed for the sell of their stocks. Why the antitrust agency could not rely on this tax system to implement a transfer?

When a bidder firm launches a tender offer over a target firm, it proposes to buy each target shareholder’s stocks at a fixed cash payment, which is generally over the financial markets prices. Each stockholder achieving capital gains on the sell of their shares, is immediately taxed thanks to the capital gains tax. After the takeover bid procedure, a tax authority collects this quantifiable monetary amount which goes in the State’s reserves.

A transfer can be used but it is constrained by the takeover bid legislation, in France the tax rate is 27\%, 20\% in the US, 30\% in Sweden and Belgium shareholders are exempt. The special case where optimal transfers are not implementable can occur here. A fiscal constraint problem exists in our merger control model, it is twofold. The transfer must be positive (the lower fiscal constraint), but under the transfer defined by the costlier medium of payment (the upper fiscal constraint).

The medium of exchange in mergers can take several shapes: either all in cash, either all in stock, or with a mix of the two media of payment. We will focus on the mix bid. The all stock procedure is a pure stock exchange and need no immediate tax payment, it allows to avoid tax penalties. Consequently, the merger will generate a transfer proportionate to the ratio of cash in the mix bid, because of the capital gains tax the new entity must pay immediately.

To my knowledge, there are no published articles on taxation as a tool for merger

\footnote{See, for instance, Baron-Myerson (1982).}
control, it is the reason why this work constitutes a true contribution. It associates different merger literature fields. For the finance aspect we rely on the paper developed by Eckbo et al. (1990) and more precisely on the idea of Brown et al. (1991). They analyze the media of payment’s effects in mergers, and show that bidders with unfavorable private information about their value, choose offers containing some stock to avoid the capital gains tax consequences of cash offers. For the remedies aspect we rely on papers developed by Medvedev (2004), who proposes a Cournot analysis of mergers and remedies in complete information; and Cosnita and Tropeano (2005), who build a contract in asymmetric information with divestitures supposing that the commission can control the assets sale price. The reader could also refer to Vasconcelos (2005) for an endogenous merger process with remedies, to Vergé (2007), for an extension of Farrell and Shapiro (1990) in a divestitures context, or to Besanko and Spulber (1993) for an analysis of policy making in mergers and enforcement aspects of antitrust in an asymmetric information context.

When efficiency gains are the merged firms private information, we are able to deal with the design of optimal remedies and transfers. Those transfers will be implicitly collected thanks to the capital gains tax. We assume, in a competition game with homogenous good and constant marginal costs, a merger between two firms creating either high or low synergies and requiring divestitures. For each type of merger, the antitrust agency proposes a menu of contracts with divestitures and transfers, allowing to screen among types. The prime objective of the antitrust agency is to protect consumers from price increases. One of our objectives is to characterize the solutions of the principal agent model with multi fiscal constraints and to compare them with the solutions of the unconstrained problem.

We show that the use of a transfer in merger control leads the antitrust agency to be less demanding in term of divestitures. The AA objective is not only to maximize the consumer surplus, but also to check that insiders get a surplus in the merger with divestitures, which could be collected thanks to the transfer. Decreasing the required level of divestiture is a relevant way to increase the insiders surplus. The

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5 Poitevin (1998), deals with the transferability of tax losses in the corporate control by mergers.
6 This hypothesis of a consumer standard is subject to discussions, see Neven and Roller (2005).
non monotonic consumer surplus function combined with the upper fiscal constraint leads the agency to leave a rent to the inefficient type\footnote{in addition to the information rent left to efficient insiders. Those two properties will affect the usual rent-efficiency trade-off. When the problem of the antitrust agency is subject to the low fiscal constraint, then the efficient level of divestiture is distorted downward from the first-best. Finally, when both fiscal constraints are binding, the public intervention leads insiders to behave oppositely in term of the medium of payment’s choice.}

The paper proceeds as follows. After a description of the model in Section 2, we show in Section 3, that a possible solution for the antitrust agency to counteract information problem, is to build an incentive contract with an amount of divestitures and any transfer, for each type of synergy. Unfortunately, a direct transfer is not possible in the merger regulation procedure, therefore Section 4 deals with merger control and transfers from the capital gains tax. Finally, Section 5 concludes.

2 The model

Initially, in the no merger state, we consider three symmetric firms with the same constant marginal costs: $c$, producing an homogeneous product. Each firm maximizes its profits as a Cournot-Nash player. The solution of this program defines individual status quo value: $\Pi^*$, for firms and $CS^*$, for consumers ($\Pi$ represents profits, and $CS$ the Consumer Surplus).

Without loss of generality, firm 1 (the bidder), finds an opportunity to improve its costs in a merger with firm 2 (the target). The merged entity, also called insiders and indexed I, enjoys a positive multiplier of synergy gain $\theta$, that could not be obtained without a merger. Low values of $\theta$ depict high synergy gains. The merger creates costs asymmetries between firms, modifying the new entity’s marginal cost function: $Cm^I(\theta) < c$, with $Cm^I_\theta(\theta) > 0$ (subscripts indicate partial derivatives).

Firm 3, also called the outsider and indexed O, keeps its marginal cost constant: $Cm^O = c$. As a result: $\Pi^I_\theta(\theta) < 0$; and because of the duopoly competition:

\footnote{This rent is not due to information asymmetries.}
\[ \Pi^O_\theta(\theta) > 0. \] From the point of view of the Antitrust Agency (AA), the value of the merger is summarized in the consumer surplus \( CS(\theta) \). The more merged firms achieve synergies, the more they can pass on their cost savings through a fall in the offered price, and the more the consumer surplus can be improved. Formally: \( CS_\theta(\theta) < 0 \). A standard graphical analysis\(^8\) summarizes the above assumptions.

In Figure 1 (see Appendix A), curves depict gains of the merger in surplus terms depending on \( \theta \) for the three parties involved. From the insiders’ point of view, gains to merge are increasing with the level of synergies they can achieve (decreasing in \( \theta \)). Below \( \theta_3 \), synergies improve the insiders’ situation, but cannot be sufficient to be transformed in a price decrease (as in Farrell and Shapiro (1990)). It is the reason why the level of synergy which improves the consumer surplus: \( \theta_1 \), is on the left of \( \theta_3 \). Below \( \theta_1 \), costs savings are so large that insiders can pass on efficiency gains to consumers, by decreasing prices. But the fall in price exacerbates competition in the sector; below \( \theta_1 \), the outsider is negatively affected by the merger. Between \( \theta_2 \) and \( \theta_1 \), insiders gains are more and more important compared to those of the outsider.

If synergies are too low for the price to decrease; i.e. on the right of \( \theta_1 \), the AA can use divestitures \( \Delta \), to correct the inconveniences generated by the merger and to make prices the lower as possible. Divestitures improve the consumer surplus up to \( \Delta^* \): \( CS_\Delta(\Delta, \theta) > 0 \), moreover \( CS_{\Delta\Delta}(\Delta, \theta) < 0 \) (non monotonic assumption). There is no entry possibility, assets divested are sold to the outsider.

After the divestiture procedure, firms become more symmetric and prices are brought down by the increase in competition. Divestitures reduce costs asymmetries between firms, they are costly for insiders: \( Cm\Delta^I(\Delta, \theta) > 0 \) and favorable for the outsider: \( Cm\Delta^O(\Delta, \theta) < 0 \). Moreover, divestitures are more and more costly for insiders, and are costlier for less efficient insiders. This leads to:

\(^8\)See Duso et al. (2003).
Assumption 1:

- \( \Pi_I^{\Delta}(\Delta, \theta) < 0 \) \hspace{1cm} (1.1) ;
- \( \Pi_I^{O}(\Delta, \theta) > 0 \) \hspace{1cm} (1.2) ;
- \( \Pi_{I\Delta}(\Delta, \theta) < 0 \) \hspace{1cm} (1.3) ;
- \( \Pi_{I\theta}(\Delta, \theta) < 0 \) \hspace{1cm} (1.4) ;

When insiders accept to merge with divestitures, they get a profit: \( \Pi^I(\Delta, \theta) \) from production, and they receive a payment \( P \) from the outsider for divested assets. \( P \) is endogenous, and depends on divestitures and on synergies. Let \( IS(\Delta, \theta) \equiv \Pi^I(\Delta, \theta) + P(\Delta, \theta) - 2\Pi^* \), be the insiders surplus to merge. Furthermore, they will pay a costly positive transfer \( t \).

We assume that insiders have all the bargaining power in the determination of the assets sale price: they are able to propose a price \( P \) to the outsider, that binds its participation constraint. For the outsider, \( P \) is exactly the gains from receiving divestitures: \( P(\Delta, \theta) \equiv \Pi^O(\Delta, \theta) - \Pi^O(0, \theta) \). The assets’ price is all the larger that insiders are more inefficient. Indeed, from assumption (1.4) and because of Cournot competition: \( \Pi_{O\theta}(\Delta, \theta) > 0 \), implying \( P_\theta(\Delta, \theta) > 0 \).

We can rewrite insiders surplus to merge as a function of the sector’s total profit:

\[ IS(\Delta, \theta) = \Pi^I(\Delta, \theta) + \Pi^O(\Delta, \theta) - \Pi^O(0, \theta) - 2\Pi^* \]

We assume that the insiders surplus, \( IS(\Delta, \theta) \), has the same properties as \( \Pi^I(\Delta, \theta) \). This leads to:

Assumption 2:

- \( |\Pi_{I\Delta}(\Delta, \theta)| > |\Pi_{O\Delta}(\Delta, \theta)| \) \hspace{1cm} (2.1) ;
- \( |\Pi_{I\theta}(\Delta, \theta)| > |P_\theta(\Delta, \theta)| \) \hspace{1cm} (2.2) ;
- \( |\Pi_{I\Delta\theta}(\Delta, \theta)| > |\Pi_{O\Delta\theta}(\Delta, \theta)| \) \hspace{1cm} (2.3) ;

Assumption 2 is not unreasonable, even if it is restrictive. It guarantees the Spence-Mirrlees condition, the concavity of the insiders’ surplus functions and allows the maximization problem to have the good properties. Assumption 2 implies: \( IS(\Delta, \theta) < 0, IS_\theta(\Delta, \theta) < 0 \) and \( IS_{\Delta\theta}(\Delta, \theta) < 0 \). Furthermore the insiders surplus is concave in divestitures: \( IS_{\Delta\Delta}(\Delta, \theta) < 0 \), and finally: \( IS_{\Delta\Delta\theta}(\Delta, \theta) < 0 \).\(^9\)

\(^9\)This assumption is necessary to ensure the convexity of the information rent.
3 Merger Control with unrestricted Transfers

In this benchmark Section, we look at the nature of the incentive problem when transfers are not constrained. To screen between good and bad insiders, we assume that the AA is allowed to order any arbitrary and explicit transfer associated with an amount of divestiture. Initially, we deal with the optimal solutions in complete information to derive the first best, then in asymmetric information over synergies. We will index the first best by FB and the second best by SB. Later, we will consider the case of the shutdown of the less efficient type. This principal agent analysis is standard (Laffont and Martimort (2002)). We will only announce some necessary assumptions for the merger control case, and some results which follow.

3.1 First and second best contracts

During the merger in complete information, insiders achieve a level of synergy \( \theta \) which is publicly observable. The AA’s objective is to find a contract \((\Delta, t)\) which maximizes the sum of the consumer surplus and the transfer, providing that insiders accept the contract (subject to the participation constraint \(PC\)).

\[
\max_{(\Delta, t)} CS(\Delta, \theta) + t \\
\text{s.t. : (PC): } IS(\Delta, \theta) - t \geq 0
\]

In a traditional way in the principal agent theory, the AA binds the insiders’ participation constraint. The first best amount of divestitures are solution of:

\[
CS_\Delta(\Delta^{FB}, \theta) = -IS_\Delta(\Delta^{FB}, \theta)
\]

First best divestitures are given by the above first order conditions, which balance the first consumer surplus derivative in relation with divestitures and the negative of the first insiders surplus derivative in relation with divestitures. By assumption (2.1): \(IS_\Delta(\Delta, \theta) < 0\) implying that \(CS_\Delta(\Delta^{FB}, \theta) > 0\). We know that in \(\Delta^*\): \(CS_\Delta(\Delta^*, \theta) = 0\), so \(CS_\Delta(\Delta^*, \theta) < CS_\Delta(\Delta^{FB}, \theta)\). Since \(CS_{\Delta\Delta}(\Delta, \theta) < 0\), it is always true that: \(\Delta^{FB} < \Delta^*\), the solution of the first best, in divestitures term, is
on the left of the solution which maximizes the consumer surplus (see Figure 2 in Appendix A).

When transfers are possible in merger control, the AA’s objective is not only to maximize the consumer surplus, but also to check that insiders get a surplus in the merger with divestitures, which could be collected thanks to \( t \). Without \( t \), the AA will level out marginal costs in order to obtain the perfect symmetry in the sector. To propose \( \Delta^* \) guarantees the lowest price and the maximum output\(^{[10]} \). But the AA can get away with decreasing \( \Delta \) so as to provide a bigger surplus to insiders.

In asymmetric information, insiders can be either efficient (with probability \( \nu \)) when they generate high synergies; i.e. \( \theta = \bar{\theta} \); or inefficient (with probability \( (1 - \nu) \)); i.e. \( \theta = \bar{\theta} \), with \( \bar{\theta} - \theta > 0 \). The sequence of events is standard. The AA must find a contract for each type of insiders which maximizes the expected value of the consumer surplus including the transfer that it can recover, subject to the participation and the incentive constraints \((PC)\) and \((IC)\) for efficient insiders; \((\bar{PC})\) and \((\bar{IC})\) for inefficient insiders:

\[
\max_{\{(\Delta, t): (\overline{\Delta}, \overline{t})\}} \nu \left[ CS(\Delta, \bar{\theta}) + \overline{t} \right] + (1 - \nu) \left[ CS(\overline{\Delta}, \bar{\theta}) + \overline{t} \right]
\]

subject to:

\[
\begin{cases}
(PC) : IS(\Delta, \theta) - t \geq 0 \\
(\bar{PC}) : IS(\overline{\Delta}, \bar{\theta}) - \overline{t} \geq 0 \\
(IC) : IS(\Delta, \theta) - t \geq IS(\overline{\Delta}, \bar{\theta}) - \overline{t} \\
(\bar{IC}) : IS(\overline{\Delta}, \bar{\theta}) - \overline{t} \geq IS(\Delta, \theta) - t
\end{cases}
\]

In the second best, the AA is ready to accept some distortions away from what it could do in complete information, in order to decrease the information rent (hereafter quoted \( R \)) intended to efficient insiders. Traditionally, the maximization of the AA’s program calls for no distortion away from the first best for \( \theta \)-type: \( \Delta^{FB} = \Delta^{SB} \).

The AA’s maximization program for the inefficient type yields to:

\[
(1 - \nu)[CS_{\overline{\Delta}}(\Delta^{SB}, \bar{\theta}) + IS_{\overline{\Delta}}(\Delta^{SB}, \bar{\theta})] = \nu R_{\overline{\Delta}}(\Delta^{SB})
\]

\(^{[10]}\)See Medvedev (2004).
The second best solution in divestiture terms for the inefficient type occurs when the expected marginal efficiency gains of the merger with divestitures and the expected marginal cost of the rent are equated. The above first order condition depicts the trade-off between the efficiency of divestitures intended to the inefficient type and the cost of the information rent left to the efficient type. At the second best equilibrium, the AA is neither willing to increase nor to decrease the inefficient amount of divestiture.

The AA must decrease $\Delta$ in the second best: $\Delta^{SB} < \Delta^{FB}$. The $\theta$-type has to divest even less than the $\theta$-type in the second best. We must not forget that the AA can recover insiders surplus since it disposes of a transfer tool. In addition, decreasing $\Delta$ is a relevant way to increase transfers devoted to inefficient insiders in order to relax incentive constraint for efficient insiders.

**Proposition 1** First best and second best optimal contracts in merger control with unrestricted transfers are such that:

\[
\begin{align*}
[FB, FB] &= [(\Delta^{FB}, l^{FB}), (\Delta^{FB}, l^{FB})], \text{ and } \\
[SB, SB] &= [(\Delta^{SB}, l^{SB}), (\Delta^{SB}, l^{SB})], \text{ with: } \\
\Delta^{FB} &= \Delta^{SB} > \Delta^{FB} > \Delta^{SB}; \quad l^{SB} > l^{FB} \quad \text{and} \quad l^{FB} > l^{SB}
\end{align*}
\]

From the insiders' point of view, divestitures and transfers are costly. The iso-surplus curves of both types correspond to increasing level of surplus when one moves in the southwest direction. Iso-surplus curves are decreasing and concave in $\Delta$ and $t$, they represent the set of $(\Delta, t)$, which maintain insiders in a situation at least as favorable as the no merger situation. From the AA’s point of view, divestitures and transfers are beneficial since they improve the consumer surplus. The increase in surplus goes in the northeast direction, iso-surplus functions are increasing and convex in $\Delta$ and $t$. It is costless for an efficient to divest an asset (assumption 1(4)), therefore iso-surplus curves for inefficient insiders are steeper. Those curves, for different types cross only once, guaranteeing the Spence-Mirrlees property.

The standard argument remains that both incentive constraints imply that the optimal level of divestiture is such that efficient insiders divest more: $\Delta > \Delta$. 

10
The AA will link divestitures and synergy. Two different levels of synergies must not divest the same amount \( \Delta \). As insiders which enjoy higher level of synergies increase even more the anticompetitive detrimental effect, it looks natural that, view to competition in a Cournot model, they must divest more, in order to restore the higher cost asymmetry they have generated. Thus we have: \( \Delta_{\theta}(\theta) < 0 \).

Our results are in accordance with the traditional principal agent results described in Laffont and Martimort (2002). We are in a very standard case of revelation of information. Efficient insiders have to divest more than inefficient insiders but receive an information rent, in the form of lower transfers.\(^{11}\)

### 3.2 Shutdown of the less efficient type

We extend our framework assuming that the AA is confronted to the possibility that inefficient mergers were suboptimal even with any divestitures. For those levels of synergies, divestitures worsen the inefficient insiders situation, deteriorate the competition even more and benefit the outsider. The merger associated with this level of synergy and any divestitures will always depreciate the consumer surplus. The AA must compute the value of:\[
\Delta CS(\Delta, \bar{\theta}) = CS(\Delta, \bar{\theta}) + \bar{t} - CS^*.\]
When this value is negative, a contract for the inefficient which could allow to improve the consumer situation does not exist. This trade-off defines a threshold \( \hat{\theta} \) above which an inefficient type must not be allowed to merge, even with divestitures. Proposition 2 summarizes the main features of the optimal contract with shutdown.

**Proposition 2** The optimal menu of contracts with shutdown entails:

- If \( \bar{\theta} > \hat{\theta} \), then the AA offers \( (\Delta_{FB}, \ell_{FB}) \), only \( \theta \)-type merges.

- If \( \bar{\theta} < \hat{\theta} \), and \( \nu \) high, then the AA offers \( (\Delta_{FB}, \ell_{FB}) \), only \( \theta \)-type merges.

- If \( \bar{\theta} < \hat{\theta} \), and \( \nu \) low, then the AA proposes the second best contract \([SB, \overline{SB}]\), every type merges and the AA leaves an information rent to \( \theta \)-type.

\(^{11}\)Note that the optimal contract could specify a negative transfer for insiders, which will lead the AA to pay for the merger to be allowed. We will develop this aspect further.
In the first case the level of synergy achieved by inefficient insiders is not enough from the AA’s point of view. The best way to exclude the less efficient type from the merger is to propose a unique contract to both type of insiders: \((\Delta^{FB}, t^{FB})\). This contract would never be chosen by an inefficient type, moreover it does not provide rent to efficient insiders. In the second case, the inefficient merger is consumer surplus enhancing in complete information, but the proportion of efficient is, a priori, important in comparison with the proportion of inefficient. The AA must often leave an information rent to the efficient type, whereas the probability to draw an inefficient type is low. For incentives reasons, the AA proposes: \((\Delta^{FB}, t^{FB})\) to both type of insiders to exclude the inefficient type. The presence of the first inefficient insider creates imitation incentives and the payment of a socially costly information rent. In the last case the proportion of inefficient is a priori important. The AA must leave an information rent to efficient insiders in very few cases and the elected contract should be the second best contract: \([(\Delta^{SB}, t^{SB}), (\Delta^{SB}, t^{SB})]\).

4 Merger control with transfers from the capital gains tax

The bidder firm proposes to target shareholders a mixed bid composed of a ratio of cash \(p\), and a ratio of stock \((1 - p)\). The ratio \(p\) implicitly defines an amount of capital gains that shareholders are going to achieve and hence a monetary transfer \(t\) that insiders must pay as taxes. The mixed bid is all the more fiscally costly that \(p\) is important. Indeed, when \(p\) is high, the bidder firm proposes a more important part of cash for target shares. This increases the amount of capital gains and leads insiders to pay more in capital gains tax term: thus \(t\) is high.

It is worth noticing that the use of this transfer tool is constrained. The AA cannot modify neither the tax rate, nor the premium by share proposed by the bidder. Moreover, for a given tax rate, the minimum transfer that the AA is able to capture is defined by an offer without cash, since it allows to avoid tax penalties: \(t^{\text{min}} = 0\) when \(p = 0\). The maximum transfer, is defined by the costlier procedure
all in cash, that generates the higher tax revenue: $t^{\max}$, such that $p = 1$. Transfers must be between 0 and $t^{\max}$. There is no more important tax than the one defined by the all cash bid, except for higher premium by share. In spite of those limits, the AA is able to include this constrained fiscal parameter into its objective function, since it can consider that the monetary amount can be reallocated to consumers. Be that as it may, it is a tax that consumers will not have to pay.

The use of transfer from the capital gains tax for merger control will modify the ex ante financial markets anticipations about the shares value of the merged entity. Nevertheless, we neglect all the impact of this regulation policy on financial markets, keeping in mind that this transfer tool will indisputably have a financial market effect.

The timing of events is the following. First insiders learn if they are $\theta$-type or $\overline{\theta}$-type. Then, the AA proposes a menu of contracts to insiders. Each contract specifies a transfer $t$ that insiders must pay and a level of divestiture $\Delta$, as structural remedies. Insiders choose a unique contract among those proposed, in announcing the ratio $p$ in the mixed takeover bid. This will imply a capital gains tax of an amount $t$, computed by the AA. Lastly insiders receive $P(\Delta, \theta)$ for the sell of divestitures to the outsider. Insiders can retain their status quo value by giving up the merger.

The addition of transfers from the capital gains tax modifies substantially the results from the previous unconstrained problem, but are qualitatively similar to the results of limited liability with ex-ante contracting.\footnote{Note that we could have a weight on $t$ to illustrate the opportunity cost of public funds in order to extend the model. This additional parameter will allow to have a discussion about the} Of course in the present model, we have two bounds on $t$ rather than one. The fiscal constrained solutions are indexed $FC$.

### 4.1 The complete information context

The AA’s problem in complete information is to find a contract $(\Delta, t)$ that maximizes the sum of the consumer surplus and the transfer provided that insiders accept the contract, and that the transfer is within both bounds ($(FC_{sup})$ and $(FC_{inf})$)\footnote{see Laffont-Martimort(2002).}.
\[ \max_{(\Delta, t)} C S(\Delta, \theta) + t \]
\[ \text{s.t. : } \]
\[ \begin{cases} 
(\text{PC}) : IS(\Delta, \theta) - t \geq 0 \\
(\text{FC}_{\sup}) : t \leq t_{\max} \\
(\text{FC}_{\inf}) : t \geq 0 
\end{cases} \]

**Proposition 3** The optimal menu of contracts in complete information entails:

- If \( 0 < IS(\Delta^{FB}, \theta) < t_{\max} \), then the AA proposes \((\Delta^{FB}, t^{FB})\);
- If \( IS(\Delta^{FB}, \theta) \leq 0 \), then the AA proposes \((\Delta^{FC}_{\inf}, 0)\).
  With \( \Delta^{FC}_{\inf} \) such that \( IS(\Delta^{FC}_{\inf}, \theta) = 0 \);
- If \( IS(\Delta^{FB}, \theta) \geq t_{\max} \) and \( IS(\Delta^{*}, \theta) < t_{\max} \), then the AA proposes \((\Delta^{FC}_{\sup}, t_{\max})\).
  With \( \Delta^{FC}_{\sup} \) such that \( IS(\Delta^{FC}_{\sup}, \theta) = t_{\max} \);
- If \( IS(\Delta^{FB}, \theta) > t_{\max} \) and \( IS(\Delta^{*}, \theta) \geq t_{\max} \), then the AA proposes \((\Delta^{*}, t_{\max})\).

**Proof:** (See Appendix B).

<Insert Figure 3>

In complete information when the transfer is not fiscally constrained, like in Section 3, the insiders’ participation constraint \((\text{PC})\) is always binding. If it wasn’t, the AA could increase \( t \) for the same \( \Delta \), since its objective function is increasing in \( t \). Whereas here, the AA cannot increase \( t \) as it would want, because of the upper fiscal constraint \((\text{FC}_{\sup})\). Thus the insiders’ participation constraint is not always binding. Figure 3 (see Appendix A) depicts the solution of the fiscally constrained problem in complete information, for different levels of \( t_{\max} \).

For a threshold \( t_{1}^{\max} > IS(\Delta^{FB}_{1}, \theta) \), the first best solution is implementable. The \((\text{FC}_{\sup})\) does not play any role, the first best transfer is \( t^{FB} = IS(\Delta^{FB}_{1}; \theta) < t_{\max} \).

The optimal contract is \( FB_{1} \) in Figure 3.

redistributive efficiency of this particular transfer to consumers.
If the first best contract specifies a negative amount of transfer: \( IS(\Delta_2^{FB}; \theta) < 0 \), the AA must distort the level of divestiture downward, from \( \Delta_2^{FB} \) to \( \Delta_2^{FC} \), in order to increase the transfer to zero. Insiders propose a medium of payment all in stock and avoid tax penalties, whereas it would be optimal that the AA pays for the merger with divestiture, for it to be allowed. To compensate, the AA decreases the required amount of divestitures to \( \Delta_2^{FC} \). The optimal contract is \( FC_1 \) in Figure 3.

For a threshold \( t_{max}^2 \in \left[ IS(\Delta^*; \theta); IS(\Delta_1^{FB}; \theta) \right] \), the optimal level of divestiture calls for an upward distortion away from the first best \( \Delta_1^{FB} \), since the first best transfer is over \( t_{max} \). Insiders propose a medium of payment all in cash for the merger and pay a capital gains tax corresponding to \( t_{max}^2 \). Note that if the AA had disposed of any unconstrained transfer, it would have been optimal to make insiders pay more than \( t_{max}^2 \). To compensate, the AA increases the required amount of divestiture to \( \Delta_2^{FC} \). The optimal contract is \( FC_2 \) in Figure 3.

For a threshold \( t_{max}^3 \leq IS(\Delta^*; \theta) \), it is in the AA’s interest to propose \( \Delta^* \). Keeping in mind that \( \Delta^* \) is the level of divestiture which maximizes the consumer surplus, thus proposing a higher level of divestiture for a given transfer \( t_{max} \) will never be optimal. Moreover, for any \( t_{max} \leq IS(\Delta^*; \theta) \), the optimal divestiture solution is always \( \Delta^* \). The optimal contract is \( FC_3 \) in Figure 3.

This particular case seems to be interesting both for merger control concerns and for contract theory. It results from the confrontation of two properties in the model. Firstly, the upper fiscal constraint \( t_{max} \) does not allow the AA to implement the level of transfer that it would want. Secondly, the consumer surplus is a non-monotonic function, since it exists a \( \Delta^* \) which maximizes \( CS \). Together, those two properties lead the insiders participation constraint to be not always binding, especially when \( t_{max} \leq IS(\Delta^*; \theta) \). From the contract theory point of view it is an original result, because insiders enjoy a rent even in complete information. As well from the competition point of view, because the AA proposes the level \( \Delta^* \) which restores the perfect symmetry between insiders and the outsider’s marginal costs on top of being consumer surplus maximizing.

In those particular cases, the optimal level of divestiture is no longer represented by the tangency between the participation constraint and the AA surplus curve as
in the first best. In complete information with fiscal constraints we observe distortions in comparison with the solution of complete information with any unrestricted transfers. The fiscal constraints modify the result of the previous Section, it is even more the case as far as asymmetric information is concerned.

4.2 The asymmetric information context

Here the set of incentive-feasible contracts is constrained by some exogenous limits on the feasible transfers between the AA and insiders. Those fiscal constraints will affect the usual rent-efficiency trade-off.

When the AA faces a lack of information over the type of insiders in the constrained problem, it tries to find an appropriate contract for each type. This contract must maximize the expected value of the consumer surplus plus the transfer that it can recover, provided that insiders are at least in the same situation as before the merger (participation constraints \((PC)\) and \((PC')\)), and that they choose the contract intended for them (incentive constraints \((IC)\) and \((IC')\)). Furthermore the AA must choose transfers between 0 and \(t_{\text{max}}\) for each type (fiscal constraints: \((FC_{\text{sup}})\), \((FC_{\text{inf}})\), \((FC_{\text{sup}}')\) and \((FC_{\text{inf}}')\)). All is summarized in the following maximization program:

\[
\max_{\left\{ (\Delta, \theta) : (\bar{\Delta}, \bar{\theta}) \right\}} \nu \left[ CS(\Delta, \theta) + t \right] + (1 - \nu) \left[ CS(\bar{\Delta}, \bar{\theta}) + t \right] \\
\text{s.t. :} \\
\begin{align*}
(PC) : & IS(\Delta, \theta) - t \geq 0 \\
(PC') : & IS(\bar{\Delta}, \bar{\theta}) - \bar{t} \geq 0 \\
(IC) : & IS(\Delta, \theta) - t \geq IS(\bar{\Delta}, \bar{\theta}) - \bar{t} \\
(IC') : & IS(\bar{\Delta}, \bar{\theta}) - \bar{t} \geq IS(\Delta, \theta) - t \\
(FC_{\text{sup}}) : & t \leq t_{\text{max}} \\
(FC_{\text{sup}}') : & \bar{t} \leq t_{\text{max}} \\
(FC_{\text{inf}}) : & t \geq 0 \\
(FC_{\text{inf}}') : & \bar{t} \geq 0
\end{align*}
\]
Let us consider contracts with neither shutdown nor bunching; i.e., every type merges and divests. Moreover, contracts are different from the efficient to the inefficient type. The resolution of this program allows the AA to extract information about hidden efficiency gains, in proposing an appropriate contract.

As usual in incentive and contract theory, the main difficulty to solve this kind of maximization program, is to work out which of those constraints are relevant and those which are not. We must find which constraints are binding at the optimum of the AA’s problem.

First, \((PC)\) is never binding (except for the shutdown case). Traditionally, the ability of the efficient type to mimic the inefficient type implies that \((PC)\) is always strictly satisfied. Indeed, both \((PC)\) and \((IC)\) imply \((PC)\). Second, as we showed in the benchmark case \(\Delta \theta < 0\), implying that \(t < \bar{t}\) is always true in asymmetric information, due to incentive constraints \((IC)\) and \((IC)\). This assertion means that \(t\) is never equal to \(t^{max}\), and that \(\bar{t}\) is never equal to zero. In term of the constraints, \((FC_{sup})\) and \((FC_{inf})\) are always strictly satisfied. Third, as \((FC_{sup})\) is never binding, then \((IC)\) is binding, since the AA can always increase \(t\) so as to bind \((IC)\). Last, \((IC)\) seems irrelevant because the incentive problem comes from an efficient willing to pass itself for an inefficient rather than the opposite. We will neglect \((IC)\) and we will verify ex post that it is true. Thus we can neglect, \((PC)\), \((FC_{sup})\), \((FC_{inf})\) and \((IC)\). Further, \((IC)\) is always satisfied with strict equality. The simplification in the number of relevant constraints leaves us with another maximization program:

\[
\begin{align*}
\max_{(\Delta, l) : (\bar{\Delta}, \bar{t})} & \quad \nu[CS(\Delta, \theta) + l] + (1 - \nu)[CS(\bar{\Delta}, \emptyset) + \bar{t}] \\
\text{s.t.:} & \\
(\text{PC}) & : IS(\Delta, \theta) - l \geq 0 \\
(\text{IC}) & : IS(\Delta, \emptyset) - l = IS(\bar{\Delta}, \emptyset) - \bar{t} \\
(\text{FC}_{sup}) & : \bar{t} \leq t^{max} \\
(\text{FC}_{inf}) & : t \geq 0
\end{align*}
\]
Given this maximization program, Proposition 4 and 5 summarize the main features of the optimal contract when respectively \((FC_{inf})\) is not binding and when it is binding.

**Proposition 4** The optimal menu of contracts in imperfect information with fiscal constraints when \((FC_{inf})\) is not binding entails:

- If \(IS(\Delta^{SB}, \theta) < t^{max}\), then the AA proposes: \([\Delta^{FB}, t^{SB}]; (\Delta^{SB}, t^{SB})]\).

- If \(IS(\Delta^{SB}, \theta) \geq t^{max}\) and \(IS(\Delta^{*}, \theta) < t^{max}\), then the AA proposes:
  \([\Delta^{FB}, t^{max}]; (\Delta^{SB}, t^{SB})]\).

  With, \(\Delta^{FC}_{sup}\) such that:
  \(IS(\Delta^{FC}_{sup}, \theta) = t^{max}\);
  \(t^{FC}_{sup} = IS(\Delta^{FB}, \theta) - IS(\Delta^{FC}_{sup}, \theta) + IS(\Delta^{FC}_{sup}, \theta)\). So that, \(\Delta^{FC}_{sup} \geq \Delta^{SB}\), and \(t^{FC}_{sup} \leq t^{SB}\);

- If \(IS(\Delta^{SB}, \theta) > t^{max}\) and \(IS(\Delta^{*}, \theta) \geq t^{max}\), then the AA proposes:
  \([\Delta^{FB}, t^{FC}]; (\Delta^{*}, t^{max})]\).

  With, \(\Delta^{FC}_{1} \in [\Delta^{*}, \Delta^{FC}_{sup}]; t^{FC} \in [\tilde{t}; t^{FC}_{sup}]\);

  With, \(\tilde{t} = IS(\Delta^{FB}, \theta) - IS(\Delta^{*}, \theta) + t^{max}\).

**Proof:** (See Appendix B).

\(<\text{Insert Figure 4 and Figure 5}>\)

The results of Proposition 4 are very similar to the results of limited liability on transfers with ex-ante contracting described in Laffont and Martimort (2002)\[14]\.

There is no distortion at the top: efficient insiders divest efficiently; and an upward distortion at the bottom: the inefficient type’s divestiture is distorted upward from the first best. Figure 4 and 5 (see Appendix A) illustrate this result.

The results are similar but different in a way, since the inefficient type enjoy a rent when \(IS(\Delta^{*}, \theta) > t^{max}\). To deal with a rent for the inefficient type is original per se. In fact, following the classical contract theory, a rent is usually paid by the principal to the good type for incentive reasons, but the bad type is never rewarded,

\[14\]Of course, we are not in an ex-ante contracting framework, since insiders choose their contract when they learn their type.
because he is not subject to imitation behaviors. The rent paid to the inefficient type comes from several properties of the model.

When $IS(\bar{\Delta}, \bar{\theta}) > t^{max}$, the level of divestiture $\Delta_{sup}^{FC}$ which binds both $(\bar{P}C)$ and $(\bar{FC}_{sup})$ is over $\Delta^*$. For a given transfer $t^{max}$, the divestiture $\Delta_{sup}^{FC}$ cannot be optimal, the AA will always prefer the divestiture $\Delta^*$ rather than any higher divestiture, when she takes the consumer surplus into account. It is the first point, $(\bar{P}C)$ is not binding. However, $\Delta^*$ cannot be optimal because the AA must leave a rent to the inefficient type which could be decreased by an increase of $\Delta$ toward $\Delta_{sup}^{FC}$. Furthermore, an increase in $\Delta$ relax the efficient incentive constraint, when the inefficient participation constraint is not binding. The optimal solution $\Delta_{1}^{FC}$, in Figure 5, is the result of the trade-off between a divestiture solution close to $\Delta^*$, for consumer surplus reasons, and a divestiture solution close to $\Delta_{sup}^{FC}$ for rent extraction reasons. Finally, the AA pays an information rent to the efficient type, and a rent to the inefficient type. This rent is not due to information asymmetries but rather to the constraint on transfers. Interestingly this divestiture will generate competition issues. The outsider will benefit from the situation since the required divestiture is over the divestiture of perfect symmetry. Then, with probability $(1 - \nu)$, the outsider will enjoy a lower marginal cost in comparison with insiders.

Moreover, as far as the maximum transfer decreases exogenously, we observe that the level of transfer intended for the efficient type decreases. Efficient insiders incorporate less cash and more stocks when $t^{max}$ decreases. In a way, the regulation power of the AA decreases, since the range of transfers available is shortened and because contracts must remain incentive compatible: the upper fiscal constraint implies higher power incentives for insiders, the AA must pay higher rents as $t^{max}$ decreases.

In conclusion, when transfers are positive, the bigger the decrease in the set of implementable transfers, the stronger the pressure on inefficient insiders to divest, the higher the information rent enjoyed by efficient insiders, and hence the lower the need to incorporate cash in their mix bid. In another way, when the low fiscal constraint does not matter, the AA must increase divestitures intended to the inefficient type to keep efficient divestitures at the first best level.
Proposition 5 The optimal menu of contracts in imperfect information with fiscal constraints when \((FC_{inf})\) is binding entails:

- If \(IS(\Delta^{SB}, \theta) < t^{\text{max}}\) and \(t^{FC} < t^{\text{max}}\), then the AA proposes: \([(\Delta^{FC}_2, 0); (\Delta^{FC}_2, t^{FC})]\). With, \(\Delta^{FC}_2 < \Delta^{FB}_2\), \(\Delta^{FC}_2 < \Delta^{SB}\) and \(t^{FC} > t^{SB}\).

- If \(IS(\Delta^{SB}, \theta) \geq t^{\text{max}}\) and \(IS(\Delta^*, \theta) < t^{\text{max}}\); or if \(IS(\Delta^{SB}, \theta) < t^{\text{max}}\) and \(t^{FC} \geq t^{\text{max}}\), then the AA proposes: \([(\Delta^{FC}_{\text{sup}}, 0); (\Delta^{FC}_{\text{sup}}, t^{\text{max}})]\). With, \(\Delta^{FC}_{\text{sup}}\) is such that: 
  \(t^{\text{max}} = IS(\Delta^{FC}_{\text{sup}}, \theta)\); and \(\Delta^{FC}_{\text{sup}}\) is such that: 
  \(t^{\text{max}} = IS(\Delta^{FC}_{\text{sup}}, \theta) - IS(\Delta^{FC}_{\text{sup}}, \theta)\). 
  So that, \(\Delta^{FC}_{\text{sup}} \geq \Delta^{SB}\), and \(\Delta^{FC}_{\text{sup}} < \Delta^{FB}\).

- If \(IS(\Delta^{SB}, \theta) > t^{\text{max}}\) and \(IS(\Delta^*, \theta) \geq t^{\text{max}}\), then the AA proposes:
  \([(\Delta^{FC}_3, 0); (\Delta^{FC}_3, t^{\text{max}})]\). With, \(\Delta^{FC}_3 < \Delta^{FB}\), and \(\Delta^{FC}_3 > \Delta^*\).

Proof: (See Appendix B).

The results of Proposition 5 are original and innovative both for merger control and for contract theory. To the best of my knowledge, there are no works dealing with principal agent models with two simultaneous bounds on transfers. Merger control with transfers from the capital gains tax seems to be a good framework to introduce this interesting approach into the contract theory.

Proposition 5 describes the optimal solutions when the second best transfer is negative for the efficient type; i.e, when \((FC_{inf})\) is binding. Figure 6, 7 and 8 (see Appendix A) illustrate the results of Proposition 5. \((FC_{inf})\) binding amounts to saying that the AA will be agree, for different reasons, to pay for the merger. This merger could create a so much important decrease in price that it would justify a monetary transfer from the AA toward those so efficient insiders. In the context of transfer from the capital gains tax, such a negative transfer is not possible. The smallest transfer is represented, in the takeover bid, by a medium of payment without any cash. The all stock bid is the costless bid since it allows to avoid tax penalties, and defines a transfer equal to zero. The nature of contracts is affected by this lower fiscal constraint and brings new insights to the principal agent model.
First, there is downward distortion at the top. Whatever the level of $t^{max}$, the level of divestiture intended to efficient insiders is distorted downward from the first best. It is yet an interesting result for contract theory, since we always deal with no distortion at the top, even in Proposition 4. This is the main result of Proposition 5, for any given $t^{max}$, efficient insiders use only stocks as medium of payment, pay no transfer to the AA, and divest less than their first best. This is due to the efficient insiders capacity to achieve high synergy gains so as to pull down price under the level expected by the AA. Efficient insiders are rewarded in the form of no transfer and lower divestitures. However, efficient insiders are not always the winners.

In particular if the upper fiscal constraint is not binding. Then the AA disposes to a higher room for maneuver to increase the transfer intended to the inefficient type, and can decrease inefficient divestitures to relax the efficient incentive constraint in comparison to the second best situation. Thus, the AA pays a lower information rent to the efficient type. Importantly, in that context, we obtain the opposite of the classical limited liability results; i.e there are downward distortion at the bottom and at the top. The lower constraint, alone, pull optimal divestitures down for both type of insiders.

As in the previous Proposition, the decrease of $t^{max}$ down to $IS(\Delta^{SB}; \theta)$ increases the information rent paid to efficient insiders. Further, when $t^{max}$ falls under $IS(\Delta^{*}; \theta)$ the AA must pay a rent to inefficient insiders. Figure 7 and 8 summarize those points. Here, efficient insiders must propose only stock in their takeover bid, whereas inefficient insiders propose only cash. In that case the level of divestiture for the inefficient type is distorted upward from the second best, while the level of divestiture for the efficient type is distorted downward from the first best (or from the second best since $\Delta^{FB} = \Delta^{FB}$). In this context it is interesting to note that, the public intervention leads insiders to behave oppositely to what they would do without the public intervention. Indeed, according to economists interested in media of payment in mergers, inefficient insiders choose offers containing some stock to avoid the capital gains tax consequences of cash offers, and efficient insiders signal

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15 Figure 6 illustrates this result.
their efficiency by choosing high cash levels.

Note that the corner solutions case can arise, when \((FC\sup{sup})\) is not binding. In this particular kind of principal agent model applied to merger control, the corner solution amounts to saying that the inefficient type is allowed to merge without divesting nothing. On the other hand, he must pay the higher transfer compatible with its participation constraint. The concept of corner solutions in this model is totally different to the corner solutions of Laffont and Martimort (2002). In fact, it corresponds to shutdown, since the inefficient type does not produce. Here the inefficient type is not excluded from the deal, and can merge without compensation except the one of paying capital gains tax. The AA can also propose the shutdown of the less efficient type contract as in the unconstrained problem.

5 Conclusion:

Merger regulation is a crucial issue in international politics, and is prey to information problems. The model presented here proposes to rely on the capital gains tax legislation and on structural remedies to build incentive contracts in a merger control framework. That is new in the merger control literature. This method seems to be a relevant way to create incentive in merger control and to screen among "good" and "bad" mergers. Thanks to a multi-constrained principal agent model, insiders are induce to reveal their efficiency gains in the merger, and the AA is able to learn this hidden parameter. We manage to find a fairly realistic and implementable transfer tool. The drawback, for merger control, is that it is constrained on the top and on the bottom, on the other hand, it generates interesting and innovative results for the classical contract theory. Generally, the model speaks in favor of a less massive utilization of divestitures in comparison with the practice, because of the transfer tool.

The analysis shows that, the use of the capital gains tax as transfer allows to implement the first best level of divestiture for efficient insiders in most asymmetric information cases. In particular when transfers are positive in the constrained problem and provided that the AA pays an increasing information rent as far as the
range of transfers decreases. Moreover, the AA must pay a rent to the inefficient type when the range of transfer falls under a critical threshold. The analysis also shows that the public intervention leads inefficient insiders to pay more than efficient insiders in term of transfers. This implies that inefficient insiders must incorporate more cash in their takeover bid. Finally, when the AA should "pay" efficient insiders to merge; i.e, when their transfers are negative in the unconstrained problem, the AA must decrease their requested level of divestiture in order to be in accordance with the capital gains tax legislation.
References


Appendix A

Figure 1. Gains in the merger for protagonists:

\[ \Delta \Pi^I(\theta) \quad \Delta \Pi^O(\theta) \]
\[ \Delta CS(\theta) \]

\[ \theta_1 \quad \theta_2 \quad \theta_3 \]

Figure 2. First best and second best contracts:

\[ IS(\Delta, \theta) - t = 0 \]
\[ IC \]
\[ PC : IS(\Delta, \theta) - t = 0 \]
Figure 3. Fiscally constrained contracts in complete information:

\[ t^{\max}_1, t^{\max}_2, t^{\max}_3 \]

Figure 4. The \((PC)\) and \((FC_{sup})\) are binding:

\[ t^{\max} \]
Figure 5. The (PC) is not binding and the (FC\textsubscript{sup}) is binding:

\[ t \Delta = CS(\Delta, \bar{\beta}) + t = CS^* \]

Figure 6. The (PC) and (FC\textsubscript{inf}) are binding but the (FC\textsubscript{sup}) is not binding:

\[ t \Delta = IS(\Delta, \bar{\beta}) \]

\[ t \Delta = IS(\Delta, \bar{\beta}) \]
Figure 7. The $\langle PC \rangle$, $\langle FC_{sup} \rangle$ and $\langle FC_{inf} \rangle$ are binding:

Figure 8. The $\langle PC \rangle$ is not binding, $\langle FC_{sup} \rangle$ and $\langle FC_{inf} \rangle$ are binding:
Appendix B

Proof: Proposition 3:

Consider first that \((FC_{sup})\) is not binding, then \((PC)\) is binding. The objective function of the AA is increasing in \(t\), if both \((FC_{sup})\) and \((PC)\) were not binding, the AA could increase \(t\) for any \(\Delta\). Thus \((PC)\) is binding, when \((FC_{sup})\) is not binding. In that case, \((FC_{inf})\) can be either binding or not.

- If \((FC_{inf})\) is not binding, then the AA can implement the first best solution. Fiscal constraints do not play any role. When \(0 < IS(\Delta^{FB}, \theta) < t_{max}\), the optimal contract is \((\Delta^{FB}, t^{FB})\).

- If \((FC_{inf})\) is binding, then the program of the AA becomes a program in \(\Delta\) subject to: \(\max_{\Delta} CS(\Delta, \theta)\), subject to: \(IS(\Delta, \theta) \geq 0\). Since \((PC)\) is binding, \(\Delta^{FC}_{inf}\) is such that \(IS(\Delta^{FC}_{inf}, \theta) = 0\). When \(IS(\Delta^{FB}, \theta) \leq 0\), the optimal contract is \((\Delta^{FC}_{inf}, 0)\).

Consider then that \((FC_{sup})\) is binding, then \((FC_{inf})\) is not binding, since \(t_{max}\) is strictly positive. The program of the AA becomes a program in \(\Delta\) such that: \(\max_{\Delta} CS(\Delta, \theta) + t_{max}\), subject to \((PC)\): \(IS(\Delta, \theta) - t_{max} \geq 0\). The \((PC)\) can be either binding or not.

- If \((PC)\) is binding, then \(\Delta^{FC}_{sup}\) is such that \(IS(\Delta^{FC}_{sup}, \theta) = t_{max}\). When \(IS(\Delta^{FB}, \theta) \geq t_{max}\) and \(IS(\Delta^{*}, \theta) < t_{max}\), the optimal contract is \((\Delta^{FC}_{sup}, t_{max})\).

- If \((PC)\) is not binding, the program becomes a consumer surplus maximization program with a divestiture solution \(\Delta^{*}\) such that \(IS(\Delta^{*}, \theta) \geq t_{max}\). When \(IS(\Delta^{FB}, \theta) > t_{max}\) and \(IS(\Delta^{*}, \theta) \geq t_{max}\), the optimal contract is \((\Delta^{*}, t_{max})\).

\(\blacksquare\)
Proof: Proposition 4:

Consider that \((FC_{inf})\) is not binding.

- If \( IS(\Delta^{SB}, \overline{\theta}) < t^{max}\), then \((FC_{sup})\) is not binding, and \((PC)\) is binding. As, none of the fiscal constraints are binding, the second best solution prevails, \( t^{SB} \) is under \( t^{max} \) and it is always implementable.

- If \( IS(\Delta^{SB}, \overline{\theta}) \geq t^{max} \) and \( IS(\Delta^{*}, \overline{\theta}) < t^{max}\), then both \((FC_{sup})\) and \((PC)\) are binding. The maximization program becomes:

\[
\max_{\{(\Delta, \bar{t}) : (\Delta, \overline{\theta})\}} \nu \left[ CS(\Delta, \overline{\theta}) + \bar{t} \right] + (1 - \nu) \left[ CS(\Delta, \overline{\theta}) + \bar{t} \right]
\]

subject to:

\[
\begin{align*}
\bar{t} &= IS(\Delta, \overline{\theta}) = t^{max} \\
\bar{t} &= IS(\Delta, \overline{\theta}) - IS(\Delta^{*}, \overline{\theta}) + t^{max}
\end{align*}
\]

As \((FC_{sup})\) and \((PC)\) are binding, \( \Delta^{FC} \) is such that: \( IS(\Delta^{FC}, \overline{\theta}) = t^{max}\). The first order conditions for the efficient type are those of the first best. Thus \( \Delta^{FB} \) is solution, which leads the transfer for the efficient type to be solution of: \( t^{FC}_{sup} = IS(\Delta^{FB}, \overline{\theta}) - IS(\Delta^{FC}_{sup}, \overline{\theta}) + IS(\Delta^{FC}_{sup}, \overline{\theta}) \). As \( IS(\Delta^{SB}, \overline{\theta}) \geq t^{max}\), \( IS(\Delta^{FC}_{sup}, \overline{\theta}) = t^{max}\), and by assumption 2(1): \( IS_{\Delta}(\Delta, \theta) < 0\), then \( \Delta^{FC}_{sup} = \overline{\Delta}^{SB}\). Furthermore, as \( t^{FC}_{sup} = IS(\Delta^{FB}, \overline{\theta}) - R(\Delta^{FC}_{sup})\), \( t^{SB} = IS(\Delta^{FB}, \overline{\theta}) - R(\Delta^{SB})\) and because \( R(\Delta) > 0\), then \( t^{FC}_{sup} \leq t^{SB}\).

- If \( IS(\Delta^{SB}, \overline{\theta}) > t^{max} \) and \( IS(\Delta^{*}, \overline{\theta}) \geq t^{max}\), then \((FC_{sup})\) is binding and \((PC)\) is not. Both \((FC_{sup})\) and \((IC)\) can be substituted into the maximization program. We obtain a reduced program with \( \overline{\Delta} \) as the only choice variables:

\[
\max_{\{(\overline{\Delta})\}} \nu \left[ CS(\Delta, \overline{\theta}) + IS(\Delta, \overline{\theta}) - IS(\overline{\Delta}, \overline{\theta}) \right] + (1 - \nu) \left[ CS(\overline{\Delta}, \overline{\theta}) \right] + t^{max}
\]

The first order conditions for the efficient type remains the first best one. On the contrary, the FOCs for the inefficient type become:

\[
(1 - \nu)CS_{\overline{\Delta}^{FC}_{i}}(\overline{\Delta}_{i}, \overline{\theta}) = \nu IS_{\overline{\Delta}^{FC}_{i}}(\overline{\Delta}^{FC}_{i}, \overline{\theta})
\]
As \( \nu \in [0, 1] \) and \( IS_\Delta(\Delta, \theta) < 0 \), then \( CS_\Delta(\Delta^e_1, \bar{\theta}) < 0 \). Thus \( \Delta^e_1 > \Delta^* \). Furthermore, \((FC)\) is not binding thus: \( IS(\Delta^e_1, \bar{\theta}) > \bar{t} = t^{max} = IS(\Delta^e_{sup}, \bar{\theta}). \)

Which leads by assumption (2.1) to: \( \Delta^{FC}_1 < \Delta^{FC}_{sup} \). Finally, \( \Delta^{FC}_1 \in [\Delta^*, \Delta^{FC}_{sup}] \). As \( t \) depends on the level of divestiture \( \Delta \), the previous results implies that \( t^{FC} \in [\bar{t}; t^{FC}_{sup}] \),

with \( \bar{t} = IS(\Delta^{FB}, \theta) - IS(\Delta^*, \theta) + t^{max} \) and \( t^{FC}_{sup} = IS(\Delta^{FB}, \theta) - IS(\Delta^{FC}_{sup}, \theta) + t^{max} \).

**Proof: Proposition 5:**

Consider that \((FC_{inf})\) is binding.

- If \( IS(\Delta^{SB}, \bar{\theta}) < t^{max} \), then \((FC_{sup})\) is not binding, and \((FC)\) is binding. We must apply the Lagrangian techniques to the AA’s problem. Let the Lagrangian takes the following form:

\[
L(\Delta, \Delta, \lambda) = \nu CS(\Delta, \bar{\theta}) + (1 - \nu)(CS(\Delta, \bar{\theta}) + IS(\Delta, \bar{\theta})) - \lambda [R(\Delta) - IS(\Delta, \bar{\theta})].
\]

The first order conditions are:

\[
\begin{cases}
\nu CS(\Delta^{FC}_1, \bar{\theta}) = -\lambda IS(\Delta^{FC}_1, \bar{\theta}) \\
CS(\Delta^{FC}_2, \bar{\theta}) + IS(\Delta^{FC}_2, \bar{\theta}) = \frac{\lambda}{(1-\nu)} R(\Delta^{FC}_2) \\
R(\Delta^{FC}_2) = IS(\Delta^{FC}_2, \bar{\theta})
\end{cases}
\]

Now, on that particular equations, when \( \lambda = \nu \), the second best results of the unconstrained program are solutions. This particular case is not possible since the second best contract is not compatible with the Lagrangian constraint. So either \( \lambda < \nu \) or \( \lambda > \nu \). The first is not possible, since it violates the efficient incentive constraint: \( \Delta^{FC} > \Delta^{FB} \) is not compatible with \( \Delta^{FC} > \Delta^{SB} \). So the unique solution of the Lagrangian is simultaneously: \( \lambda > \nu, \Delta_2^{FC} < \Delta^{FB} \) and \( \Delta_2^{FC} < \Delta^{SB} \). Because \((FC)\) is binding, and under assumption (2.1), \( \bar{t}^{FC} > \bar{t}^{SB} \). However, \( t^{FC} < t^{max} \) must hold, if not \((FC_{sup})\) is binding and the solution is the one of the second point of Proposition 5.

- If \( IS(\Delta^{SB}, \bar{\theta}) \geq t^{max} \) and \( IS(\Delta^*, \bar{\theta}) < t^{max} \), then \((FC_{sup}), (FC_{inf})\) and \((FC)\) are binding. As \((FC_{sup})\) and \((FC)\) are binding, then \( \Delta^{FC}_{sup} \) is such that:
\( t^{\max} = IS(\Delta_{sup}^{FC}, \theta) \). As \((FC_{inf})\) and \((IC)\) are binding, then \(\Delta_{sup}^{FC}\) is such that: 
\( t^{\max} = IS(\Delta_{sup}^{FC}, \theta) - IS(\Delta_{sup}^{FC}, \theta) \). For the same reason that the second point of Proposition 4, \(\Delta_{sup}^{FC} > \Delta^{SB} \). As a result: \(\Delta_{sup}^{FC} < \Delta^{FB} \).

- If \( IS(\Delta^{SB}, \theta) > t^{\max} \) and \( IS(\Delta^*, \theta) \geq t^{\max} \), then both \((FC_{sup})\) and \((FC_{inf})\) are binding and \((PC)\) is not binding. The maximization program of the AA is:

\[
\max_{(\Delta, \theta) \in (\Delta, \theta)} \nu \left[ CS(\Delta, \theta) + t \right] + (1 - \nu) \left[ CS(\Delta, \bar{\theta}) + \bar{t} \right]
\]

\[
\text{s.t. :}
\begin{align*}
t^{\max} &= IS(\Delta, \theta) - IS(\Delta, \theta) \\
\bar{t} &= t^{\max} \\
t &= 0
\end{align*}
\]

Then, \(t\) and \(\bar{t}\) can be substituted into the program of the AA by their respective value. We obtain a reduced program with divestitures as the only choice variables subject to \( t^{\max} = IS(\Delta, \theta) - IS(\Delta, \theta) \). Let solve this maximization program thanks to the Lagrangian method: \( L(\Delta, \Delta, \lambda) = \nu CS(\Delta, \theta) + (1 - \nu)[CS(\Delta, \bar{\theta}) + t^{\max}] - \lambda[IS(\Delta, \theta) - IS(\Delta, \theta) - t^{\max}] \). The FOCs are:

\[
\begin{align*}
\nu CS_{\Delta}(\Delta_{3}^{FC}, \theta) &= -\lambda IS_{\Delta}(\Delta_{3}^{FC}, \theta) \\
CS_{\Delta}(\Delta_{3}^{FC}, \bar{\theta}) &= \left(\frac{\lambda}{1 - \nu}\right) IS_{\Delta}(\Delta_{3}^{FC}, \theta) \\
t^{\max} &= IS(\Delta, \theta) - IS(\Delta, \theta)
\end{align*}
\]

Since \( IS_{\Delta}(\Delta, \theta) < 0 \), then \( CS_{\Delta}(\Delta_{3}^{FC}, \bar{\theta}) < 0 \), for all value of \(\lambda\). This result implies that: \(\Delta_{3}^{FC} > \Delta^* \). Considering this divestiture distortion we have: \(\lambda > \nu\) and \(\Delta_{3}^{FC} < \Delta^{FB} \).