

Thema

UMR 8184

Théorie Économique, Modélisation et Applications

Thema Working Paper n° 2008-28
University of Cergy Pontoise, France

**Italian Senate apportionment: is the 2007
proposal fair?**

Fabrice BARTHELEMY
Gabriele ESPOSITO
Mathieu MARTIN

June 2008



Italian Senate apportionment: is the 2007 proposal fair?

Fabrice BARTHÉLÉMY*, Gabriele ESPOSITO† and Mathieu MARTIN‡

June 27, 2008

Summary: Since the political collapse of the 90's, and in particular since the bicameral commission experience of 1997, Italian governments have always tried to face the need for wide constitutional reform. Reductions in the number of deputies and senators have been planned on several occasions. The purpose of this paper is to analyze whether or not the proposed reforms to the apportionment of seats in the Italian senate is fair. We use the theory of power indices to compare different scenarios. We show that the intended reform produces an outcome that is worse than both the ideal situation and the actual situation.

JEL classification: C7, D7.

Keywords: power index, Banzhaf, Italian Senate.

*THEMA, Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy-Pontoise Cedex.
E-mail: fabrice.barthelemy@u-cergy.fr

†EHESS-GREQAM, Centre de la Vieille Charité, 2 rue de la Charité, 13236 Marseille.
E-mail: esposito@ehess.fr

‡THEMA, Université de Cergy-Pontoise, 33 boulevard du Port, 95011 Cergy-Pontoise Cedex.
E-mail: mathieu.martin@u-cergy.fr

1 Introduction

Since the political collapse of the 90's, and in particular since the bicameral commission experience of the 1997, Italian governments have always tried to face the need for wide constitutional reform. A reduction in the number of deputies and senators was planned several times. Esposito (2008) tells the history of the creation of this chamber. These proposed modifications to the Senate never saw the light, as the equilibrium formed by the several political colors in the assembly forbade it. The final shape of the Senate was just a copy of the Chamber of Deputies, with a smaller number of members proportionate to the population plus a maximum of 5 honorific senators named by the President of the Republic for a life time as well as all former Presidents of the Republic (who are senators by right).

The functions of this second chamber are identical to the ones of the first: elaborate, discuss and approve new laws at the national level. The only difference is the election of the senators on a regional basis. The original version of article 56 of the Italian Constitution, ruling the composition of the Chamber of Deputies, says that these are elected at a rate of one every 80.000 inhabitants or a fraction larger than 40.000. Article 57, that explains the constitution of the Senate, gives one senator every 200.000 people or a fraction larger than 100.000 to every region. Some minima were guaranteed to every region, in particular one senator fixed for the Aosta Valley and there is a minimum of six senators for all other regions.

The introduction of Molise in 1963 gave two senators to this region (and raised the minimum of the other regions to seven). 1963 is also the year of a reform concerning the apportionment of the seats in both the Chamber of Deputies and the Senate. The size of the two assemblies were adjusted respectively to 630 and 315 members (plus honorific senators). The method chosen for the apportionment of the seats between the regions was the method of integers combined with the highest remainders: in other words, apportion to each region a number of seats equal to the whole number contained in each region's fair share with the remaining seats allocated to the regions with the highest remainders¹. The minimum number of seats increased from six to seven. In January 17th 2000 a new constitutional law introduced the Abroad constituency, modifying the articles 48, 56 and 57 of the Constitution, ensuring twelve seats at the Chamber of Deputies and six seats at

¹This method, called Hamilton method, was used for the U.S. house of representatives between 1850 and 1890.

the Senate to this new region.

In October 17th 2007 the commission for constitutional affairs of the Chamber of Deputies approved a new proposal implying a wide change of the Senate functions. Regional competencies are in some way resurrected, and the apportionment rule deeply changed. The legislator did not decide to build a Senate like the one of the United States, with an identical representation of the States or the Regions, but he/she did not opt for a proportional rule either. Schematically, article 3 of the text, replacing article 57 of the Constitution, gives a fixed number of seats to every region, this number increases whenever the population rises above a certain threshold. A total of six seats is allocated to regions with less than one million inhabitants, nine seats to the ones smaller than three million people, eleven to regions not bigger than five million citizens, twelve to the ones smaller than seven million and fourteen to the others². Two senators are allocated both to Aosta Valley and Molise, and six both to Trentino-Alto Adige and to Italians resident abroad. The size of the Senate would become a variable (according to the population of the different regions), if the proposal becomes law. With the data of the last census, the number of senators would be 186.

Then we face two situations for the Senate, the actual one, which nearly corresponds to the situation of 1963, except for the Abroad citizens, and the proposal, approved October 17th 2007, which would modify not only the number of seats but as well the relative apportionment between the regions (the comparison of these two apportionments is presented in table 1).

The purpose of this paper is to analyze the fairness of this reform proposal. The idea of power which derives from cooperative game theory is used to compare the actual case with the proposed one. This theory shows that there exists an important difference between the number of seats of a player (here an Italian region) and his/her influence in a voting situation. The classical example is the majority election where an individual gets 50 seats, another one gets 49 seats and the third one gets 1 seat. A coalition is winning if it contains at least two individuals since the sum of seats is greater than 50 and losing otherwise. The power of an individual depends on his/her importance in a coalition. If a winning coalition becomes a losing coalition whenever he/she leaves it, then we say that he/she has power. In

²Some senators would be elected by the Regional Council and some by the Council of the local autonomies, elected by the regional populations.

such situation, the individual is said to be a swing player. Several power indices have been proposed in the literature (see for example Felsenthal and Machover 1998, or Laruelle 1998 for a clear presentation) and all of them take into account this notion of a swing player in various forms. In this paper, because of theoretical reasons, we only use the Banzhaf (1965) power index which we describe in the second section. Actually, the voting situation is a two-step vote like in the U.S. presidential election or in the European Council: every citizen of a region votes for senators and, in the second step, every senator votes on the laws. An important assumption is that all the senators belonging to a same region have the same preferences, that is to say we suppose that every region has only one senator who has several votes. This assumption seems to be strong but enables us to compute the power *a priori* and the result of the election is not considered here. Moreover, the new proposal intends to build a Senate with regional competencies: the assumption that senators belonging to different parties but to the same region can vote in the same way now makes sense. To be a swing player, an individual has to be a swing player in his/her region and the region has to be a swing region from the national point of view. Theoretical recommendations are presented in Barthélémy and Martin (2007, 2008) for using the Banzhaf index in a two-step vote when the seats are allocated following a method of proportionality, which is the case in the actual Italian situation. Our goal is not, like Balinski and Young (2001), to take into account the ideal situation of “one man-one vote” but to compare the actual apportionment and the proposal with a situation where every Italian citizen gets the same power, whatever region he/she happens to live in. This last situation is the ideal situation in terms of power. Note that in the two situations that we compare, the actual situation and the proposed amendment, the seats are given.

Two different majority rules are considered in computing the power indices: the simple majority of the members of the Senate, used for simple laws, and 2/3 majority, used to enact constitutional reforms. A similar approach is presented in Barthélémy and Martin (2008) for the U.S. presidential election. We will see that the hypothetical reform would be worse than the ideal situation and the actual situation.

Section 2 presents the concept of power that is widely used in the literature, section 3 provides our main results and section 4 concludes.

2 The concept of power

Cooperative game theory proposes an important concept of power. The idea is simple: if a player (an individual, a city, a country or, like in this paper, Italian regions...) belongs to a winning coalition, that is to say a group of players which can impose a decision, for example a majority, and if he/she leaves this coalition it becomes a losing one, then it means that this player has some power. We say that he/she is a swing player. If this player leaves the winning coalition and this coalition is still winning, the power of this player is null (in this coalition). To present more formally the concept of power, it is useful to recall the idea of a weighted game, which is a particular voting game.

Let a the number of seats and a_i the number of seats of the player i and $a = \sum_{i=1}^n a_i$, with n the number of players. A weighted game is $[q; a_1, \dots, a_n]$, where q , called the quota, is an integer. A winning coalition S (written $S \in W$, with W the set of winning coalitions) is a group of players such that

$$S \in W \iff \sum_{i \in S} a_i \geq q$$

The most famous voting game is the majority game, which perfectly corresponds to the method used in Italian Senate, where $q = \frac{a}{2} + 1$ if a is even and $q = \frac{a+1}{2}$ if a is odd. Actually, if the majority game is often used in the Senate, the 2/3 rule is used too ($q = 2/3a$). We only consider these two quotas in this paper.

Once the weighted game is defined, we can present the Banzhaf index. Firstly, one has to determine all $2^n - 1$ possible coalitions (non empty) and the number of times for which player i is a swing player. If this number is divided by 2^{n-1} (that is the number of coalitions containing player i), we obtain the non-normalized Banzhaf power index and if it is divided by the total numbers of swing players, we obtain the normalized Banzhaf power index. The formula of the normalized Banzhaf power index (always used in this paper) for player i is

$$\beta_i = \frac{\text{number of times } i \text{ is a swing player}}{\text{total number of swing players}}$$

or

$$\beta_i = \frac{\sum_{S \subseteq N} [v(S) - v(S \setminus \{i\})]}{\sum_{j \in N} \sum_{S \subseteq N} [v(S) - v(S \setminus \{j\})]}$$

where $v(S) = 1$ if $S \in W$ and $v(S) = 0$ otherwise. Note that $[v(S) - v(S \setminus \{i\})]$ is different from 0 only if the player i is a swing player in S .

Consider the following example: let a 3-player majority game $[6; 5, 4, 2]$. The different coalitions are $\{1\}, \{2\}, \{3\}, \{\underline{1}, \underline{2}\}, \{\underline{1}, \underline{3}\}, \{\underline{2}, \underline{3}\}$ and $\{1, 2, 3\}$, the swing players are underlined. For example, in coalition $\{1, 2\}$, if one player leaves it, then it becomes losing. Thus the two players are both swing players. We obtain $\beta_i = 1/3$, for $i = 1, 2, 3$. We can compute the Banzhaf index as the probability that player i is a swing player (for a presentation of the probabilistic interpretation, see Straffin 1977).

The literature proposes several power indices with associate normative properties (for a presentation of power indices, see for example Felsenthal and Machover 1998 or Laruelle 1998). However, our choice of the Banzhaf index is theoretic. It is shown, as described in the introduction, that this is the appropriate power index in a two-step game (the citizens vote for the senators and then the senators vote on legislation) when the seats are allocated proportionately. Some arguments are presented in Barthélemy and Martin (2008). Our purpose is to show that the reform proposal, that is to say the new apportionment in the Italian Senate, is not fair in terms of power. The power of an individual is the probability that he/she is a swing player in his/her region and the region is a swing player in the Senate. The Italian situation is different from the U.S. “winner-take-all” situation: indeed, a region gets several senators and their preferences may be different. However, in this paper, we compute the power *a priori* and thus we assume that all the senators in a given region vote the same way. The ideal situation is certainly the one where every citizen in Italy gets the same power, wherever he/she leaves. Our purpose is then different from the classical problem of apportionment, widely studied by Balinski and Young (2001), where the goal is meeting the ideal of “one man, one vote”.

3 Results

In this section we compare the actual seats apportioned and the proposed reform. This comparison is made on the basis of the relative power of each region. The power is computed using the Banzhaf power index developed in section 2. The ideal situation corresponds to the case where each citizen has the same amount of power. Whatever the region, the ideal power for each citizen is equal to $\frac{1}{59\,533\,387}$. In this ideal case, the same population in each region implies the same power for all the regions (as the power is expressed in a relative way, this would lead to a power of $1/21$ for each of the 21 regions). As shown in table 1,

the populations are not equal³ which implies mechanically a different power for each region. For instance, the ideal power for Lombardy is $\frac{9032554}{59533387} = 15.17\%$, while for the Aosta Valley it is $\frac{119548}{59533387} = 0.20\%$ (see table 1, columns ‘Population’ and ‘Ideal’). This ideal power is directly computed using the relative population of each region under the assumption that each citizen has the same power.

The seats apportioned (those we observe in practice) should lead to power as close as possible to the ideal one described above. The two apportionments compared in this article are:

- the actual one (‘Act.’ in table 1), with a total of 315 seats,
- the proposed one (‘Prop.’ in table 1), with a total of 186 seats.

First of all, the new proposition modifies the relative distribution of seats. For instance, with 47 seats over a total number of 315 Lombardy has 14.92% of the seats but only 7.53% under the new proposal (14 seats over 185). With less seats, 11 against 18, the proportion would be higher for the Tuscany, going from 5.71% to 5.91%. The effect is more important for the Abroad citizens region, where the number of seats is constant and even more for the Aosta Valley. More generally, the modification of the apportionment reinforces the relative number of seats for the smallest regions in terms of population (all the regions with less than 3 500 000 citizens).

Given these two apportionments, we can compute the power of each region. With the actual apportionment, the power of each region looks similar to the theoretical power except for the six smallest regions and for the Abroad citizens region. With the modified apportionment, the situation is clearly different. Indeed, the power is higher than the theoretical one for the smallest regions, and lower for the largest regions. These effects are underlined in figure 1 which represents a (continuous) scatter plot of the power for the actual, proposed and ideal cases as a function of the region population in proportion.

By construction, the ideal case curve (dotted) corresponds to the 45-degree line. Hence, we may compare the two other power curves (‘Actual’ and ‘Proposed’) to this reference line. When the curve is similar to the reference line, the power is not far from the ideal situation where each Italian citizen has the same power. On the contrary, the greater the distance from this line, the more unequal is citizen power. The ‘Actual’ curve is quite closed to the

³The 2001 population is used in this paper.

reference line except for two regions: Basilicata which is over the 45-degree line (an actual power of 2.15% instead an ideal power of 1.00%) and the fictive Abroad citizens region which is under the bissectrice (an actual power of 1.84% instead an ideal power of 4.26%). Let us remark that the pick due to the Abroad citizens region arises from the fact that this region has a number of seats which is not proportional to its population. Moreover, the ‘Actual’ power curve without this region nearly corresponds to the ideal curve as shown in figure 3.

The 2/3 majority case, leads to the same result, the main difference being the decrease of the power given to the largest region, Lombardy. This implies a value under the 45-degree line as shown in figure 2.

The power of a region is the sum of the powers of its citizens. Hence, the power of a citizen for a given region is the power of the region divided by its population. In the ideal case, the power of each Italian citizen is the same and is equal to $\frac{1}{59\,533\,387}$. In order to manage with this small value, the power for 100 000 citizens is computed. It corresponds to the percentage of the whole Italian power that have 100 000 citizens in a given region. In the ideal case this leads to $\frac{100\,000}{59\,533\,387} = 0.168\%$, for each region, as reported in table 2, which underlines the fact that one citizen has the same power whatever the region he/she lives in. In practice we do not observe the ideal power value, whatever the apportionment method applied. This table underlines the fact that the power of a citizen is dependent on the region and on the apportionment method. For instance, with the actual apportionment, 100 000 citizens in Lombardy have a power of 0.181% (of the total power of Italy), which is more than the ideal power mentioned above (0.168%). Moreover, the proposed apportionment would lead to a smaller power (0.085%).

In order to compare these three columns of power (Ideal, Actual and Proposed), the power of a citizen in a given region may be relativized by the ideal power value. Hence, for the previous example in the actual situation, the relative value for Lombardy is $\frac{0.181}{0.168} = 1.08$, which means that one citizen of Lombardy has 8% of power more than what he/she would have in the ideal situation. In the proposed situation, one Lombardian citizen has $\frac{0.085}{0.168} = 0.51$ of the ideal power, which means that one citizen of Lombardy has 49% of power less than what he/she would have in the ideal situation.

The largest values for the actual situation are 2.14 for Basilicata (more than twice the

ideal power) and 0.43 for the abroad citizens (less than half the ideal power). For the proposed situation we get respectively 5.39 for Aosta Valley (more than five times the ideal power) and 0.51 for Lombardy (nearly half the ideal power).

Except for the abroad citizens region, the distribution of relative power is more concentrated around one for the actual situation, as illustrated by figure 4, which is a way of understanding figure 1 in a relative sense.

Instead of comparing the observed power of a citizen to the ideal one, the reference could be the smallest observed power (for a given apportionment). Hence, for the actual situation, we set the abroad citizens power to one, which corresponds to dividing the different powers by the power of an abroad citizen (this is a normalization). Then, for each region, the power of a citizen is relativized to the reference citizen (as shown in figure 5). For instance, a citizen of Lombardy has $\frac{0.181}{0.073} = 2.50$ times the power of the reference one. The power of a citizen of Basilicata is nearly five times the reference one. These values are larger with the proposed apportionment where the reference becomes a citizen living in Lombardy. The power of a citizen of Aosta Valley is more than ten times the reference one ($\frac{0.905}{0.085} = 10.67$).

Let us be reminded that in the actual situation, the number of seats given to the Abroad citizens region is fixed independently of its population. This implies logically that the analysis between actual power and ideal power is biased (as presented in Figure 1 and Figure 3). This analysis is reinforced in Figure 6 where all the ratios for the actual situation are near one except for Basilicata. When compared to the proposed situation the gap is more than evident.

All the previous analyzes were made for the $\frac{1}{2}$ -majority case. The results for the $\frac{2}{3}$ -majority case are roughly the same. The power of a citizen relative to the smallest power is lower both for the actual and the proposed situations. The only two exceptions are Molise and the Aosta Valley.

4 Conclusions

We show in this paper that the intended reform concerning the apportionment of seats in the Italian senate is not fair, according to the fact that every citizen in this country should have the same power of vote, whatever the region he/she lives. This criteria is, in our opinion, as least as important than the classical one widely described by Balinski and

Young (20021, “one man, one vote”). Such an approach implies to use a power index and to accept some hypothesis. For example, we assume that every senator in a same state has the same preferences, which is a classical hypothesis in the power index theory. Indeed, we want to compute the power *a priori*, and not *a posteriori*.

Our main result is that the proposal implies some important inequalities concerning the individual power, inequalities which do not exist in the actual situation and the proposed change is not recommended.

Table 1: Power of the regions

Region ⁴	Population	Seats				Power				
		Number		In %		Ideal	$\frac{1}{2}$ -majority		$\frac{2}{3}$ -majority	
		Act.	Prop.	Act.	Prop.		Act.	Prop.	Act.	Prop.
Lombardy	9 032 554	47	14	14.92	7.53	15.17	16.39	7.67	13.89	7.35
Campania	5 701 931	30	12	9.52	6.45	9.58	9.53	6.48	9.59	6.40
Lazio	5 112 413	27	12	8.57	6.45	8.59	8.53	6.48	8.63	6.40
Sicily	4 968 991	26	11	8.25	5.91	8.35	8.18	5.94	8.32	5.88
Veneto	4 527 694	24	11	7.62	5.91	7.61	7.53	5.94	7.68	5.88
Piedmont	4 214 677	22	11	6.98	5.91	7.08	6.86	5.94	7.06	5.88
Apulia	4 020 707	21	11	6.67	5.91	6.75	6.53	5.94	6.75	5.88
Emilia-Rom.	3 983 346	21	11	6.67	5.91	6.69	6.53	5.94	6.75	5.88
Tuscany	3 497 806	18	11	5.71	5.91	5.88	5.58	5.94	5.77	5.88
Calabria	2 011 466	10	9	3.17	4.84	3.38	3.09	4.81	3.23	4.89
Sardinia	1 631 880	9	9	2.86	4.84	2.74	2.78	4.81	2.91	4.89
Liguria	1 571 783	8	9	2.54	4.84	2.64	2.46	4.81	2.59	4.89
Marche	1 470 581	8	9	2.54	4.84	2.47	2.46	4.81	2.59	4.89
Abruzzo	1 262 392	7	9	2.22	4.84	2.12	2.15	4.81	2.26	4.89
Friuli-Venezia	1 184 764	7	9	2.22	4.84	1.99	2.15	4.81	2.26	4.89
Trentino-Alto	940 016	7	6	2.22	3.23	1.58	2.15	3.18	2.26	3.31
Umbria	825 826	7	6	2.22	3.23	1.39	2.15	3.18	2.26	3.31
Basilicata	597 768	7	6	2.22	3.23	1.00	2.15	3.18	2.26	3.31
Molise	320 601	2	2	0.63	1.08	0.54	0.62	1.08	0.64	1.03
Aosta Valley	119 548	1	2	0.32	1.08	0.20	0.31	1.08	0.32	1.03
Abroad	2 536 643	6	6	1.90	3.23	4.26	1.84	3.18	1.94	3.31
Total	59 533 387	315	186	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Table 2: Power of the citizens in the $\frac{1}{2}$ -majority case

Region	Power for 100 000 citizens (in %)			Power relative to the			
	Ideal	Actual	Prop.	ideal power*		smallest power**	
				Actual	Prop.	Actual	Prop.
Lombardy	0.168	0.181	0.085	1.08	0.51	2.50	1.00
Campania	0.168	0.167	0.114	1.00	0.68	2.30	1.34
Lazio	0.168	0.167	0.127	0.99	0.75	2.30	1.49
Sicily	0.168	0.165	0.120	0.98	0.71	2.26	1.41
Veneto	0.168	0.166	0.131	0.99	0.78	2.29	1.55
Piedmont	0.168	0.163	0.141	0.97	0.84	2.24	1.66
Apulia	0.168	0.163	0.148	0.97	0.88	2.23	1.74
Emilia-Romagna	0.168	0.164	0.149	0.98	0.89	2.26	1.76
Tuscany	0.168	0.159	0.170	0.95	1.01	2.19	2.00
Calabria	0.168	0.154	0.239	0.92	1.42	2.11	2.81
Sardinia	0.168	0.170	0.294	1.01	1.75	2.34	3.47
Liguria	0.168	0.157	0.306	0.93	1.82	2.16	3.60
Marche	0.168	0.168	0.327	1.00	1.95	2.30	3.85
Abruzzo	0.168	0.171	0.381	1.02	2.27	2.35	4.48
Friuli-Venezia Giulia	0.168	0.182	0.406	1.08	2.41	2.50	4.78
Trentino-Alto Adige/Sudtirol	0.168	0.229	0.338	1.36	2.01	3.15	3.98
Umbria	0.168	0.261	0.385	1.55	2.29	3.59	4.53
Basilicata	0.168	0.360	0.532	2.14	3.17	4.95	6.26
Molise	0.168	0.192	0.338	1.14	2.01	2.64	3.98
Aosta Valley	0.168	0.257	0.905	1.53	5.39	3.54	10.67
Abroad	0.168	0.073	0.125	0.43	0.75	1.00	1.48

* Illustrated in figure 4

** Illustrated in figure 5

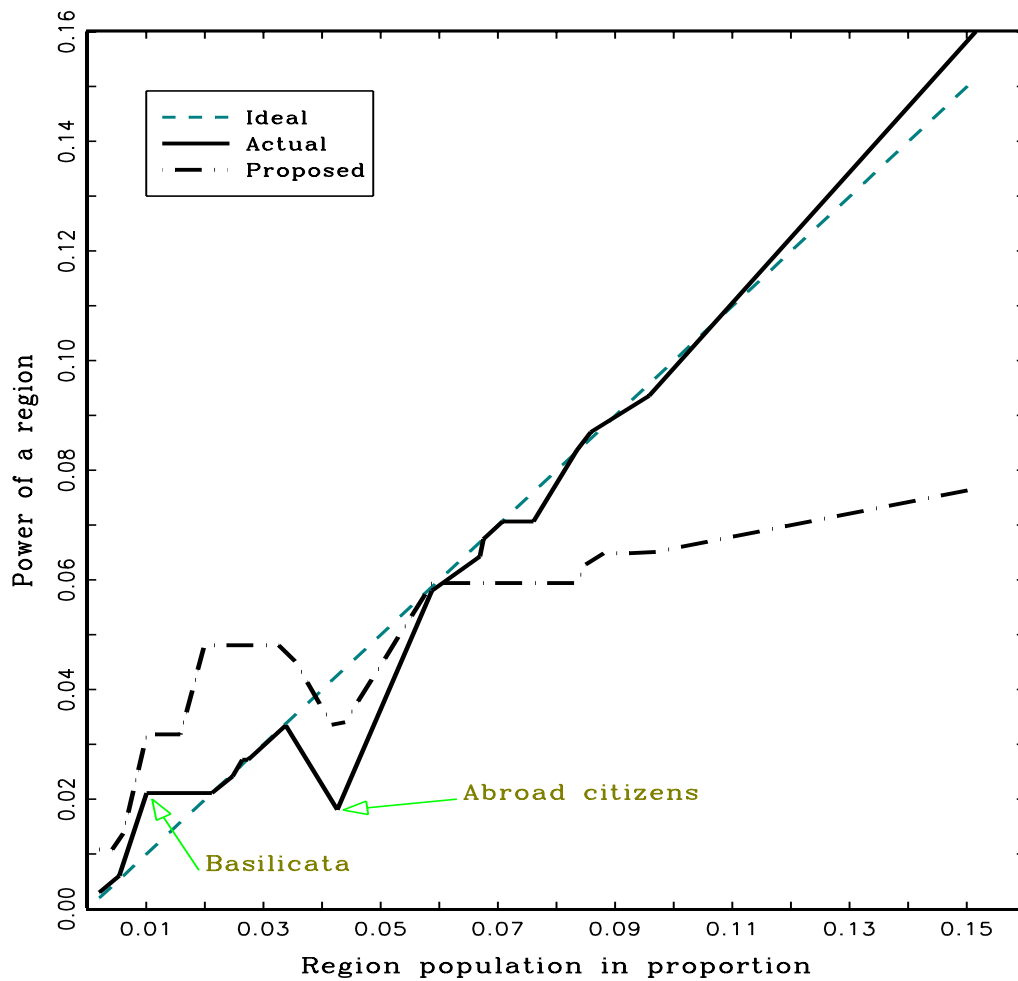


Figure 1. Power of the regions in the $\frac{1}{2}$ -majority case

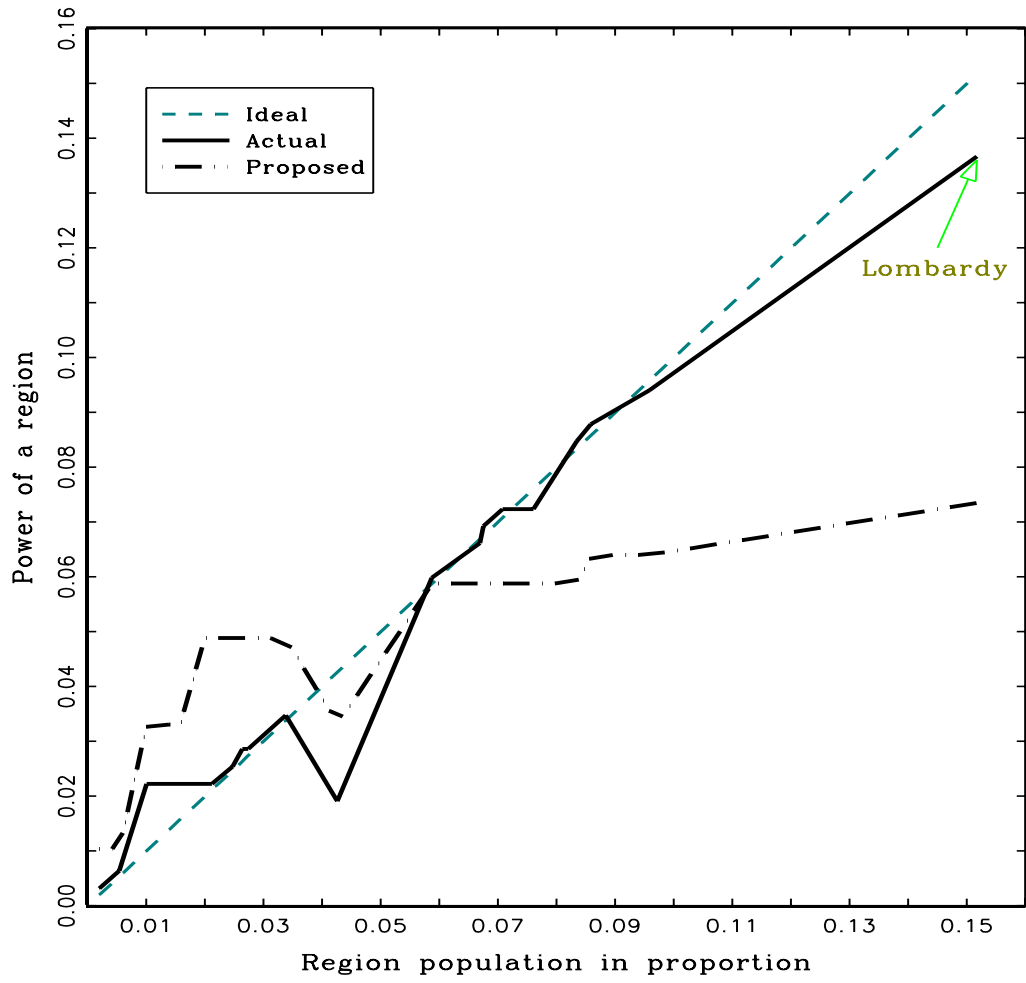


Figure 2. Power of the regions in the $\frac{2}{3}$ -majority case

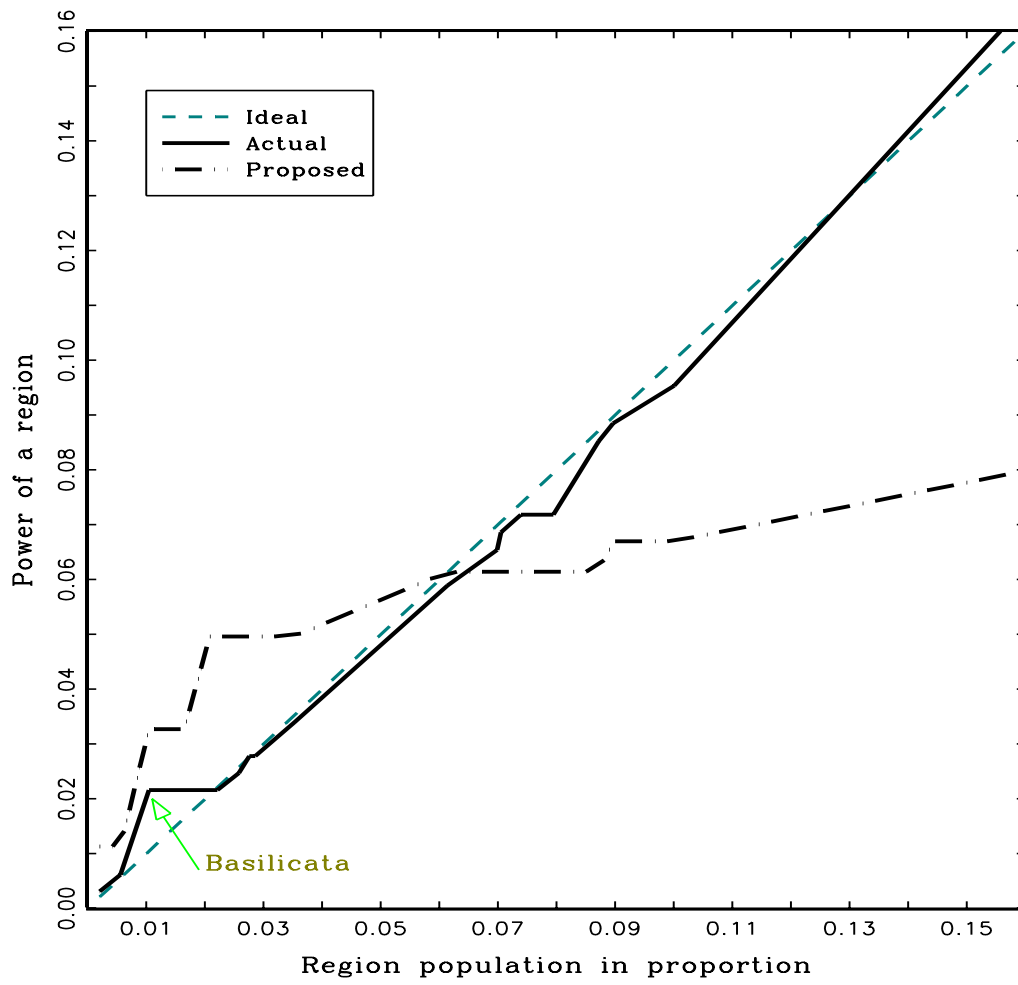


Figure 3. Power of the regions in the $\frac{1}{2}$ -majority case without the Abroad citizens region

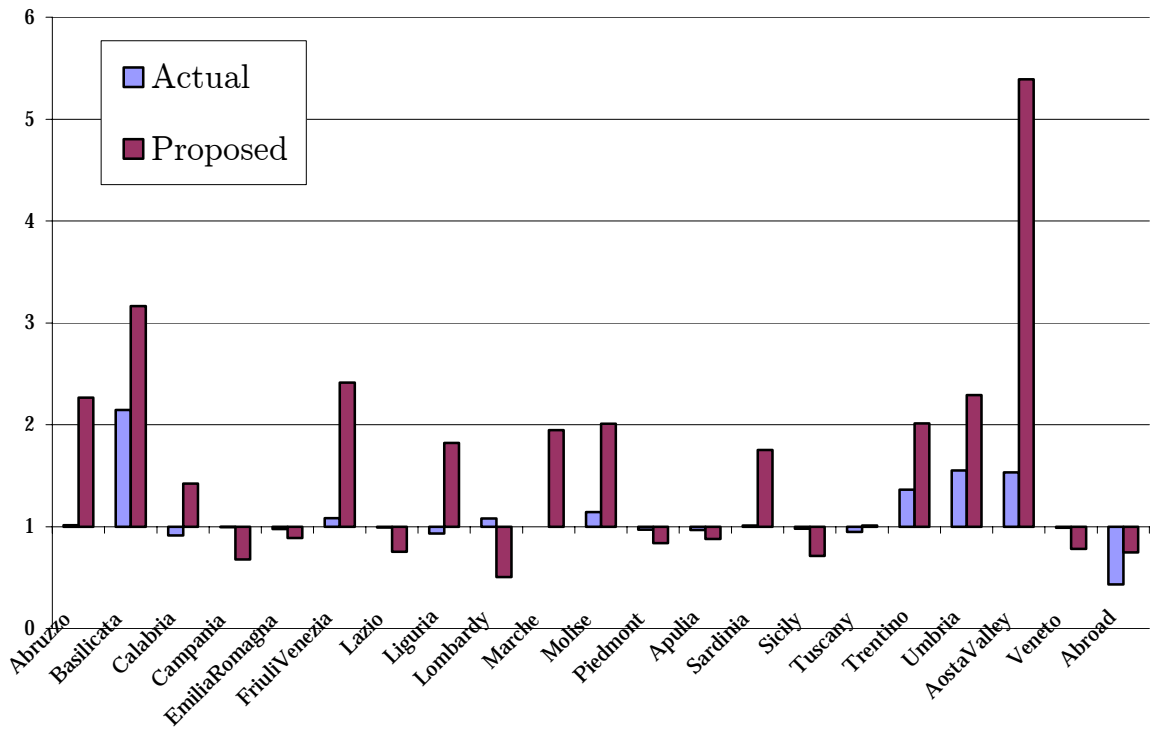


Figure 4: actual or proposed power relatively to the ideal power

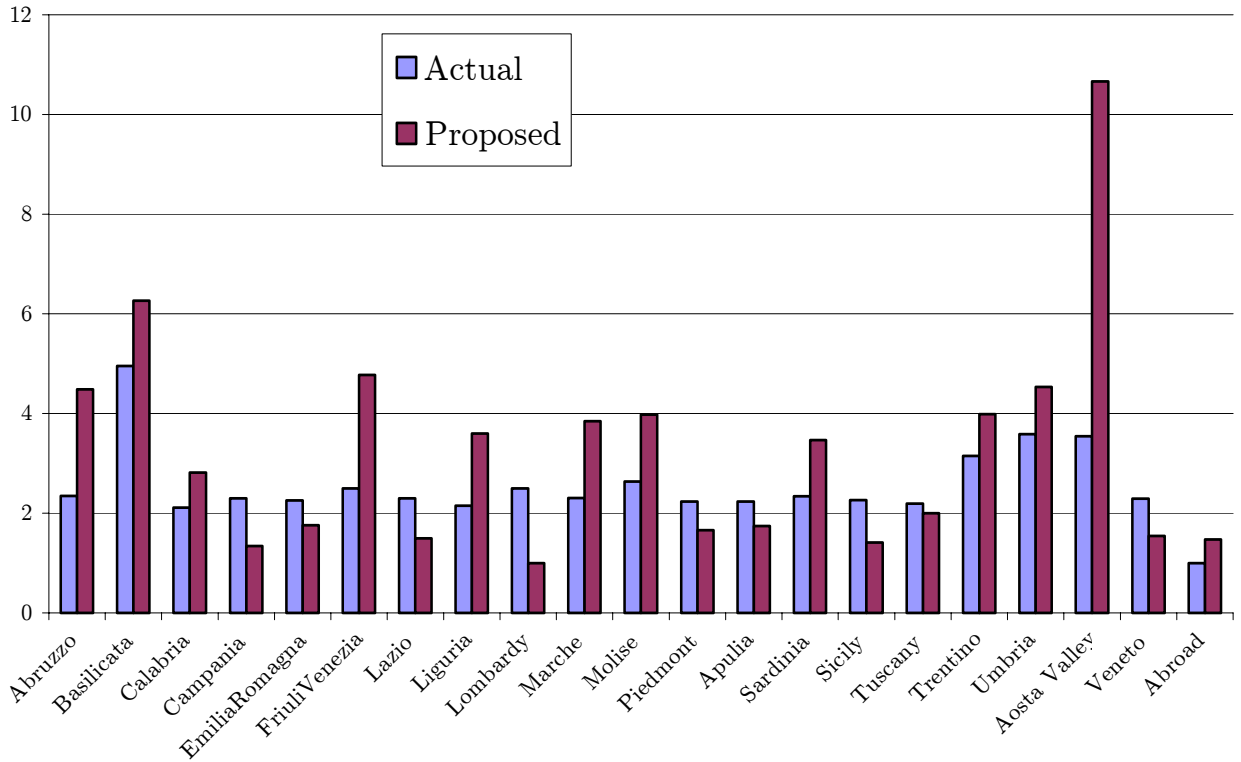


Figure 5: actual and proposed power computed relatively to their smallest value

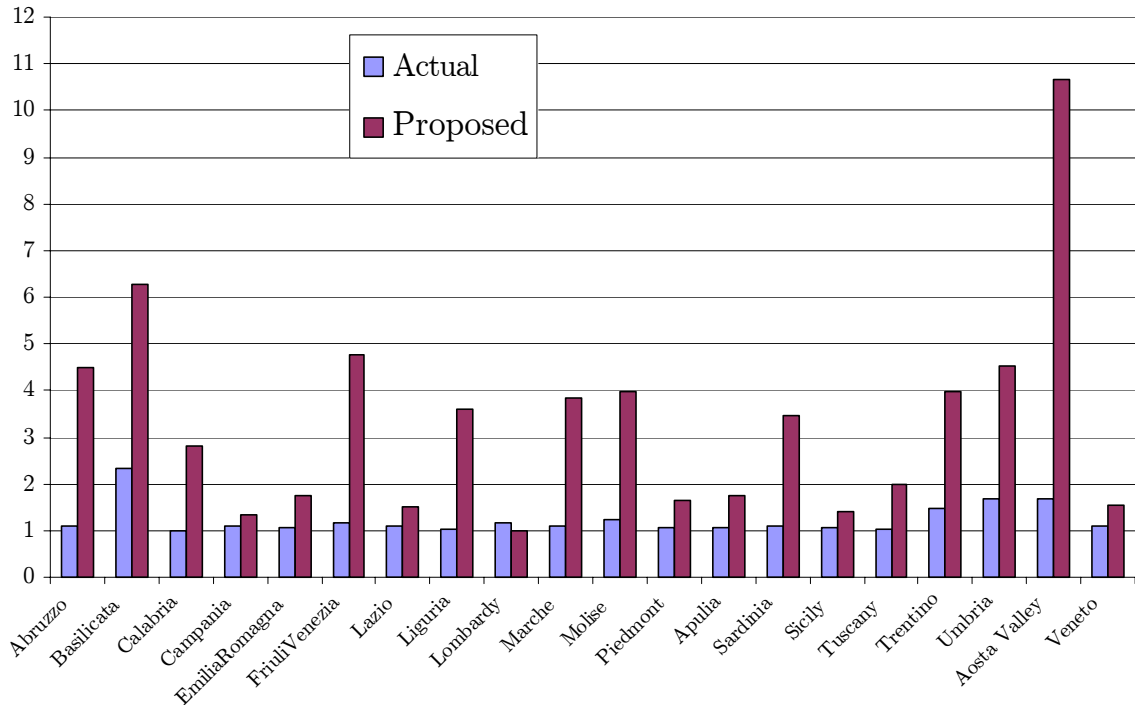


Figure 6: actual and proposed power computed relatively to their smallest value without the Abroad citizens region

REFERENCES

- BALINSKI M., YOUNG P. [2001], *Fair Representation: Meeting the Ideal of One Man, One Vote*, Brookings Institution Press, Washington D.C, second edition.
- BANZHAF J.F. [1965], “Weighted voting doesn’t work A mathematical analysis”, *Rutgers Law Review*, 19, p. 317 – 343.
- BARTHÉLÉMY F., MARTIN M. [2007], “Critères pour une meilleure répartition des sièges du Val d’Oise au sein des structures intercommunales: une application au cas du Val d’Oise”, *Revue Économique*, 58, p. 399 – 426.
- BARTHÉLÉMY F., MARTIN M. [2008], “A comparison between the methods of apportionment using power indices: the case of the U.S presidential election”, *Thema Working Paper* 2007-26.
- ESPOSITO G. [2008], “La ripartizione territoriale dei seggi al Senato: metodi, storia e possibili novità”, forthcoming in *Quaderni dell’osservatorio elettorale*.
- FELSENTHAL D.S., MACHOVER M. [1998], *The Measurement of Voting : Theory and practice, problems and paradoxes*, Edward Elgar Publishers, London.
- LARUELLE A. [1998], *Game theoretical analysis of decision-making processes with applications to the European Union*, Thèse de doctorat, Université catholique de Louvain.
- SHAPLEY L.S., SHUBIK M. [1954], “A method for evaluating the distribution of power in a committee system”, *American Political Science Review*, 48, p. 787 – 792.
- STRAFFIN P.D. [1977], “Homogeneity, independence power indices”, *Public Choice*, 30, p. 107 – 108.