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Strongly Rational Expectations Equilibria, Endogenous Acquisition of Information and the Grossman–Stiglitz Paradox

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ABSTRACT

This paper analyzes conditions for strong rationality of the equilibrium in a linear/Gaussian model of a competitive commodity market, where firms are differentially informed about costs of production and the precision of private information is endogenously acquired. A Rational Expectations Equilibrium is said to be Strongly Rational, or eductively Stable, (SREE) when it is the unique rationalizable outcome. A locally SREE exists when the informativeness of the price is below a threshold that is increasing in the informativeness of private information and the elasticity of marginal cost of information acquisition. In the spirit of the Grossman Stiglitz paradox, informativeness of the SREE price is bounded. In the case with constant marginal costs, we characterize the set of rationalizable information precisions. Furthermore, a SREE requires that marginal costs of information are neither too low nor too large. Exogenous public information always favors stability.

KEYWORDS: Rational Expectations, Eductive Stability, Asymmetric Information, Information Acquisition.

JEL-CLASSIFICATION: D 82, D 84.

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I INTRODUCTION

Although playing a central role in modern economic theory, the hypothesis of rational expectations is often viewed with skepticism. Indeed, the concept of rational expectations equilibrium (REE) is quite ambitious if one takes into account the underlying severe requirements on agent's information gathering and processing capabilities. Many attempts have been made to justify this concept and to state a clear set of assumptions that imply rational expectations. One such attempt is the concept of strongly rational expectations equilibrium (SREE) proposed by Guesnerie (1992, 2002). This concept relies on the two hypotheses of common knowledge (CK) of individual Bayesian rationality and model and asks, whether a REE is the only outcome implied by these two hypotheses. Whenever this is the case, a REE can be guessed (or 'educed') by rational agents computing the logical consequences of the CK assumptions through some kind of mental 'eductive' process.[1] A REE ist then said to be a SREE (or eductively stable, or stable, for short). Eductive stability is based on a suitably specified game form of the model. Agents use an iterative process to eliminate non best responses from their strategy sets and stability obtains whenever this process converges to the REE. One often restricts attention to local stability by adding a further CK assumption, namely CK that agents choose strategies in a given neighborhood of the REE. Guesnerie (2002) provides an overview of the conditions for existence of SREE that have been derived in various economic contexts (in general, a REE is not always stable, so that stability imposes restrictions on the parameters of the model). In particular, conditions for existence of a SREE have been derived in models with asymmetric information, both in models where agents are unable to use the information transmitted through current market prices (cf. Heinemann (2004)), and in models where this information is used (cf. Desgranges et al. (2003), Desgranges (1999), Heinemann (2002)). However, all these papers assume an exogenously given amount of private information. None of them analyzes, whether an endogenous acquisition of private information causes additional restrictions to REE eductive stability. The present paper tries to fill this gap.

The model: We consider a simple linear/Gaussian model of a competitive commodity market with endogenous information acquisition and we derive the conditions for existence of a SREE. The model follows the early contributions of Grossman (1976), Grossman and Stiglitz (1980),

[1] It may be hard to believe that economic agents are sophisticated enough to draw all the consequences of CK assumptions, namely to exploit the fact that everyone knows that everyone knows ... a certain property. Still, a few steps of reasoning may not be out of reach. Costa-Gomes and Crawford (2006) provide experimental evidences that economic agents sometimes use such reasoning based on high order beliefs.

and Verrecchia (1982), although we consider risk neutral (and not CARA) agents. Every agent chooses the precision of the private information he wants to buy, independently from what the others do (there is no market for information, with an endogenous price of the private signal, like in Veldkamp (2006a,b)). An additional stochastic factor (stochastic shifts in demand) makes the price a noisy signal of the agents' private information. The price is then an endogenous public signal, and agents use the information conveyed by the price. The model (unsurprisingly) admits a unique linear REE. In order to define stability, we describe the model as a game, so that the REE appears as a Nash equilibrium. We say that the REE is stable whenever it is the unique rationalizable outcome. We focus on local stability, that is we exogenously restrict the strategy sets to a neighbourhood of the equilibrium. We give therefore precise game-theoretical grounds to our stability concept.

Results: In the case with exogenously given information, Desgranges et al. (2003), Desgranges (1999) and Heinemann (2002) stress the role of the informativeness of the market price for existence of a SREE. In particular, Desgranges (1999) and Heinemann (2002) show that the REE is locally stable whenever prices reveal less information than a private signal. The question raised in this paper is then whether or not this stability condition is affected by the endogeneity of information acquisition (and if so, how).

The central result of this paper states that the REE is locally stable whenever two conditions are satisfied: the above condition derived in the case with exogenous private information, and a new condition specifying another upper bound for the informativeness of the equilibrium price. This new upper bound relies on the elasticity of the marginal cost of information acquisition (w.r.t. the precision of the acquired private signal). Thus, endogenous acquisition of information makes existence of a SREE more difficult when the second new condition is stronger than the first one. We show that this is the case whenever the marginal cost function is flat enough. In particular, in the limit case with constant marginal cost, a SREE exists if and only if the informativeness of the market price is less than one half the informativeness of the private signals. The result can be explained as follows. Endogenizing information acquisition adds one first order condition to the optimization problem of an agent (equating the marginal cost and the expected marginal benefit of private information). The additional stability condition comes from this additional first order condition. In the case of a steep marginal cost of information, a slight change in the acquired precision of information is enough to accommodate a change in the expected marginal benefit. It is therefore not necessary to know others' expectations precisely to guess what information precision they want to acquire: Endogeneity of information acquisition is innocuous for stability. In the case of a flat marginal cost, the acquired precision

of information is very sensitive to the expected marginal benefit. As others' expectations are a priori unknown, guessing the precision of the acquired information is not easy, and existence of a SREE is more difficult to obtain.

For the sake of completeness, recall that the intuition for the other condition (the stability condition in the case with exogenous information) is the following. Agents' private information is aggregated into the price because agents use their private signals to make their decisions. The exact informational content of the price (the correlation between price and private information) depends then on agents' decisions, and the information extracted from the price by an agent depends on his beliefs on others' decisions. In the case of a very informative REE price, agents have an incentive to learn excessively from the price, which makes their decisions very sensitive to their beliefs on the correlation between price and information. As agents' beliefs are a priori unknown, agents' decisions (and therefore the informational content of the price) are not easy to predict. This argument makes the REE unstable. In the opposite case of a not very informative REE price, the same argument leads to existence of a SREE: with the price being not very informative, agents' decisions does not depend much on their own interpretation of the informational content of the price. These decisions are therefore easy to predict, and the REE is a SREE.

In the case of constant marginal costs, we characterize existence of a global SREE in terms of exogenous parameters. We illustrate the eductive process in a simple fashion. We explicitly compute the set of rationalizable precisions of information (we are unfortunately unable to fully describe the set of rationalizable outcomes), and we show that this set shrinks to the equilibrium precision when the REE tends to be stable.

We then state a stability result that is somewhat reminiscent from the Grossman and Stiglitz paradox (cf. Grossman and Stiglitz (1980)) of impossibility of informationally efficient markets. The GS paradox says that prices cannot be fully revealing if acquisition of information is a costly activity. Indeed, in such a case, no agent would have an incentive to acquire information, prices will reveal anyway, while prices cannot be informative if no one acquires private information. As it is now well known, this paradox is a striking consequence of the fact that information acquisition is a strategic substitute (the more information others agent buy, the more revealing the price is, and the less information I want to buy). In our model, as in Verrecchia (1982), one formal result corresponding to the GS paradox is the existence of an upper bound on the informativeness of the REE price.[2] We show thast there also exists an upper bound on the informativeness of a SREE price. This upper bound is strictly smaller than the previous

[2]In Grossman and Stiglitz (1980), the informativeness of the REE does not depend of the (positive) variance of the noise, while no REE exists when there is no noisy supply.

upper bound of the informativeness of the REE price. Hence, taking account of the stability requirement reinforces the logic underlying the GS paradox.

In the case with constant marginal costs, the REE is stable either when the marginal cost is below a first threshold or above a second threshold (it is unstable between the two thresholds). As there is a one-to-one relation between the marginal cost and the precision of the acquired information (a high marginal cost corresponds to a low precision), this result is actually a consequence of the one obtained in Desgranges (1999) and Heinemann (2002) in the case of exogenous information (in this context, the REE is unstable for intermediate values of information precision, stable otherwise). It follows that decreasing the marginal costs of information acquisition sometimes destabilizes the equilibrium.

Lastly, in the case with constant elasticity of marginal costs, a SREE obtains if and only if the precision of the prior public information is large enough. This result shows that the influence of public information on REE stability is not the same when public information is exogenous or endogenous: it is only public information in form of the information contained in the market price, which causes expectational coordination difficulties, whereas public information in form of a priori knowledge always tends to attenuate such difficulties.

Literature: As already emphasized, this paper is essentially a piece added to previous works about eductive stability under asymmetric information. Still, the results in this paper can be linked to the following strands of literature as well.

In relation with the literature on acquisition of information, we follow Grossman and Stiglitz (1980): information acquisition is a strategic substitute, and our stability results reinforce the idea that prices cannot convey too much private information. Still, many papers (Barlevy and Veronesi (2000), Veldkamp (2006a,b), Chamley (2007) among others) exhibit motives that make information acquisition a strategic complement (a feature that sometimes leads to equilibrium multiplicity). A point that is beyond the scope of this paper, but that may be fruitful for further research, is that instability can be interpreted as creating uncertainty and restoring an incentive to buy more information. It follows that, even in a framework where information is a strategic substitute, focusing on equilibrium stability may sustain the idea that the information acquired by agents creates an incentive for acquiring further information.

In relation with the literature on public information, Morris and Shin (2002), and Angeletos and Pavan (2007) (among others) find a possible bad effect of public information (more public information leads to a decrease of welfare). The driving force of this effect is the commonality of the information available to each agent - that is the size of the noise common to the information of all the agents (private information can generate negative effects along the same lines).

On the other hand, Angeletos and Werning (2006), and Hellwig et al. (2006) show how endogenous public information (i.e. prices) can restore multiple equilibria in a model of a currency attack à la Morris and Shin (1998) where informational asymmetries prevent CK of actions. Hellwig (2002) shows that precise public information can favor multiple equilibria even when it is exogenous. Our point here is to distinguish between exogenous public information (which is always good for stability) and the endogenous public information (the price, which must not be too informative). Considering simultaneously two sources of public information shows that these two sources can play two distinct roles.

The remainder of the paper is organized as follows: Section II then presents the model and the unique REE. Section III defines the SREE and states conditions for existence of a SREE. Section IV discusses the informativeness of the price (the Grossman Stiglitz paradox), the role played by the marginal costs of information acquisition and public information. Section V concludes. The proofs are gathered together in the Appendix.

II A COMPETITIVE MARKET MODEL WITH LEARNING FROM CURRENT PRICES

2.1 The model

The model that builds the framework of our analysis is a simple model of a competitive market under asymmetric information. In this market, the price transmits information, that is: firms are able to use the information revealed by the current market price for their current decisions. This is the kind of models analyzed especially in the so-called REE literature (starting with Grossman (1976) and Grossman and Stiglitz (1980)). Precisely, the model is a static version of the one in Vives (1993).[3]

There is a continuum of risk neutral firms in $I = [0,1]$ supplying the same commodity. The inverse demand function for the commodity is known to the firms:

$$p = \beta - \frac{1}{\phi} X + \varepsilon. \tag{1}$$

Here, p is the market price, X is the aggregate demand, ε is a normally distributed demand shock with zero mean and precision τ_ε, $\beta > 0$ and $\phi > 0$ are known constants (while ε

[3]In fact, as demonstrated by Vives (1993), it is possible to restate the present model such that it can be interpreted as a financial market model where agents are buyers of an asset with unknown ex–post return. Our stability results of Section 3 hold in a CARA/Gaussian model (computations available from authors upon request).

is unknown to the firms). Every firm faces increasing marginal costs that are affected by the parameter θ: firm i's production costs are $\theta x(i) + \frac{1}{2\psi} x(i)^2$, where $\psi > 0$ and $x(i)$ is the output of firm i. The cost parameter θ is unknown to the firms (this may be a productivity shock, a long term pollution effect or any element unknown at the time where the production decision is made). The firms, however, know that this parameter is drawn from a normal distribution with zero mean and precision τ. Notice that the parameter θ is common to all the firms.

Private information on the side of the firms regarding the unknown parameter θ is introduced into the model by allowing for endogenous acquisition of information. It is assumed that each firm is able to perform an experiment (independent from experiments of other firms) that reveals additional but costly information regarding θ. Formally, it is assumed that each firm $i \in I$ can acquire a costly private signal $s(i) = \theta + u(i)$ where the noise $u(i)$ is normally distributed with mean zero and precision $\tau(i)_u$. The cost of acquiring a signal with precision $\tau(i)_u$ is $K(\tau(i)_u)$. We assume: $K' > 0$, $K'' \geq 0$ and $K(0) = 0$ ($\tau(i)_u = 0$ corresponds to no acquisition of information).

The objective of a firm is to maximize the expected profit where profit $\pi(i)$ of firm i is:

$$\pi(i) = [p - \theta] x(i) - \frac{1}{2} \frac{1}{\psi} [x(i)]^2 - K(\tau(i)_u), \tag{2}$$

We assume that a strong law of large numbers holds, and we write: $\int_0^1 u(i)\, di = 0$ almost surely. It follows that $\int_0^1 s(i)\, di = \theta$ almost surely, that is: the average of the firm's private signals reveals the value of the unknown parameter.

2.2 Linear rational expectations equilibrium

The timing of the model is as follows: each firm decides the precision of the private information it will acquire, observes its private signal and submits a supply schedule to an auctioneer. The auctioneer collects the individual supply schedules and sets the market clearing price. We assume the following restriction of firms' behavior:

Assumption 1 *Each firm's supply schedule is an affine function of its private signal $s(i)$ and the market price p.*

We write: $x(i) = \psi [(1 - \gamma(i)_2)\, p - \gamma(i)_0 - \gamma(i)_1 s(i)]$ where the weights $\gamma(i)_0, \gamma(i)_1$ and $\gamma(i)_2$ are real numbers for all $i \in I$. As a profit maximizing firm supplies a quantity $x(i) = \psi (p - E(\theta | p, s(i)))$, we have:

$$E(\theta | p, s(i)) = \gamma(i)_0 + \gamma(i)_1 s(i) + \gamma(i)_2 p. \tag{3}$$

The linearity of $x(i)$ is equivalent to the linearity of the conditional mean $E(\theta|p,s(i))$. Equation 3 holds true for example when the joint distribution $(\theta,p,s(i))$ is normal. We show below that Assumption 1 implies that the joint distribution $(\theta,p,s(i))$ is normal. Hence, Assumption 1 is equivalent to normality of the joint distribution $(\theta,p,s(i))$. This linearity assumption is usual and well known, and we will not motivate it further. It simplifies the analysis considerably, as the decision of firm i is characterized by four real parameters $(\gamma(i)_0,\gamma(i)_1,\gamma(i)_2,\tau(i)_u)$ only.

We now compute the market clearing price and define the equilibrium. Let $\gamma_0=\int_0^1\gamma(j)_0\,dj$, $\gamma_1=\int_0^1\gamma(j)_1\,dj$ and $\gamma_2=\int_0^1\gamma(j)_2\,dj$.[4] Aggregate supply is defined as:[5]

$$\int_0^1 x(j)dj=\psi[(1-\gamma_2)p-\gamma_0-\gamma_1\theta], \tag{4}$$

so that aggregate behavior is summarized by the coefficients γ_0, γ_1 and γ_2. Combining equations (1) and (4) shows that the market clearing price is uniquely defined as

$$p=\frac{\beta+\alpha\gamma_0+\alpha\gamma_1\theta+\varepsilon}{1+\alpha(1-\gamma_2)}, \tag{5}$$

where $\alpha=\psi/\phi$. As announced, the joint distribution $(\theta,p,s(i))$ is normal. The conditional mean $E(\theta|p,s(i))$ can be computed using Equation (5) and is linear in $(p,s(i))$. Assumption 1 is self-fulfilling: when every firm expects the conditional mean $E(\theta|p,s(i))$ (or the supply) to be linear, the actual conditional mean (or the actual supply) resulting from firms' behavior is indeed linear.

A *linear Rational Expectations Equilibrium (REE hereafter)* is then defined, quite as usual, as an outcome where the beliefs of every firm are self-fulfilling, that is:

- every firm submits a linear individual supply characterized by $(\gamma(i)_0,\gamma(i)_1,\gamma(i)_2)$ and all these parameters satisfy Equations (3) and (5) ($E(\theta|p,s(i))$ is computed using Equation (5)),
- for every firm, the optimal $\tau_u(i)$ is derived from the maximization of the profit.

A REE is a static equilibrium concept where the price p transmits information about θ (some information can be transmitted because firms are able to condition their supply decisions on p). A shorter definition of REE is given in the next section, after the best response mapping is defined.

[4] All the measurability assumptions required are made. In particular, we assume that $\int_0^1\gamma(j)_0\,dj$, $\int_0^1\gamma(j)_1\,dj$ and $\int_0^1\gamma(j)_2\,dj$ exist.

[5] This definition of aggregate supply is discussed in the Appendix.

Our first result establishes that there exists a unique linear REE.[6]

Proposition 1 *Let $\alpha = \psi/\phi > 0$. There exists a unique linear REE where every firm uses a linear supply function $x(i) = \psi[(1-\gamma_2^*)\,p - \gamma_0^* - \gamma_1^* s(i)]$ and acquires the same level of precision τ_u^* with the following properties:*

(i) If $K'(0) \geq \frac{\psi}{2\tau^2}$, then $\tau_u^* = 0$.

(ii) If $K'(0) < \frac{\psi}{2\tau^2}$, then $\tau_u^* > 0$ and τ_u^* is the unique solution of the equation:

$$\sqrt{\frac{2K'(\tau_u^*)}{\psi}}\left[\frac{2K'(\tau_u^*)}{\psi}\tau_u^{*2}\alpha^2\tau_\varepsilon + \tau + \tau_u^*\right] = 1, \tag{6}$$

(iii) The coefficients γ_0^*, γ_1^* and γ_2^* are given by:

$$\begin{aligned}
\gamma_0^* &= -\frac{\beta\alpha\gamma_1^*\tau_\varepsilon}{\tau + \tau_u^* + \alpha^2\gamma_1^{*2}\tau_\varepsilon + \alpha^2\gamma_1^*\tau_\varepsilon}, \\
\gamma_1^* &= \tau_u^*\sqrt{\frac{2K'(\tau_u^*)}{\psi}}, \\
\gamma_2^* &= \frac{\gamma_1^*\alpha(1+\alpha)\tau_\varepsilon}{\tau + \tau_u^* + \alpha^2\gamma_1^{*2}\tau_\varepsilon + \alpha^2\gamma_1^*\tau_\varepsilon}.
\end{aligned}$$

Proof. See Appendix. □

In this kind of model, existence of a unique linear REE is definitely unsurprising. A REE where the firms acquire a positive amount of private information exists, as conditions (i) and (ii) make clear, only if marginal costs of information acquisition at zero (i.e. $K'(0)$), fall short of respective marginal returns of information acquisition, which are at zero equal to $\psi/2\tau^2$. Since we are interested in equilibria where the current market price aggregates and reveals dispersed private information, we confine the following analysis to the case where the condition $K'(0) < \psi/2\tau^2$ is satisfied such that a REE with $\tau_u^* > 0$ exists.

[6] A usual question is whether there exist nonlinear equilibria besides this unique linear equilibrium. At least when supply schedules are restricted in an appropriate way such that they have bounded means and bounded variances, this is not the case. Vives (1993) provides a proof of this for a generic stage of this dynamic model that can easily be adapted to our model.

2.3 Informativeness of prices

The aggregation of information through the market price is illustrated in Equation (5) stating that p is a noisy observation of the unknown θ. Simple computations shows that the conditional precision $\tau_{\theta|p}$ is $\tau + \tau_p^*$, where τ_p^* is:[7]

$$\tau_p^* = \alpha^2 \gamma_1^{*2} \tau_\varepsilon. \tag{7}$$

Thanks to the normality assumption, τ_p^* does not depend on p. Thus, τ_p^* can be regarded as a measure of the precision of the information revealed by the market price. In what follows, we call τ_p^* the informativeness of the price.

According to (7), informativeness of the price increases in the endogenously determined weight γ_1^* which is given to private information in the firms' decisions. From Proposition 1 we get that γ_1^* is an increasing function of the — also endogenously determined — precision of the private signals τ_u^*. Hence, $\frac{d\tau_p^*}{d\tau_u^*} > 0$.

Two other (useful) properties of τ_p^* are that $\frac{d\tau_p^*}{d\tau} < 0$ and $\frac{d\tau_p^*}{d\tau_\varepsilon} > 0$. The first property $\frac{d\tau_p^*}{d\tau} < 0$ comes from the fact that an increase in the precision of public (*a priori*) information τ about θ decreases the private information precision τ_u^*.[8] Concerning the second property, the sign of $\frac{d\tau_p^*}{d\tau_\varepsilon}$ is *a priori* ambiguous because τ_u^* (and γ_1^{*2}) is decreasing in the precision τ_ε of the noise in market demand.[9] In our model (as in Verrecchia (1982)), the positive direct effect of τ_ε on τ_p^* offsets the indirect negative effect via τ_u^* and γ_1^* on τ_p^*. The fact that $\frac{d\tau_u^*}{d\tau_\varepsilon} < 0$ and $\frac{d\tau_p^*}{d\tau_\varepsilon} > 0$ forms the basis of the famous Grossman–Stiglitz–Paradox (cf. Grossman and Stiglitz (1980)) which we will discuss later in more detail: As τ_ε increases, prices become more informative *ceteris paribus* such that incentives for private accumulation of information are reduced.

III STRONGLY RATIONAL EXPECTATIONS EQUILIBRIA

3.1 Description of the concept

Since detailed descriptions of of the concept of a strongly REE (SREE hereafter) are already available (see Guesnerie (2002) for a synthetical assessment of this literature), it is adequate to limit the present analysis to a pragmatic treatment of this concept and the game–theoretical issues that are involved here. The fundamental question associated with the concept of a SREE

[7] $\tau_{\theta|p} = 1/Var(\theta \mid p)$. The computations are in the Appendix.

[8] $\frac{d\tau_u^*}{d\tau} < 0$ follows from differentiating Equation (6).

[9] $\frac{\partial \tau_u^*}{\partial \tau_\varepsilon}$ obtains from differentiating Equation (6) and $\frac{d\tau_p^*}{d\tau_\varepsilon}$ follows from differentiating Equation (7).

is, how agents in a model end up in a REE, assuming nothing more than common knowledge (CK hereafter) of the model's structure and individual rationality. Usually, these two hypotheses are not sufficient to predict a unique outcome. While the set of outcomes predicted by the two hypotheses of CK of individual rationality and model (i.e. a set of rationalizable solutions defined below) includes the REE, it typically includes other outcomes as well. Still, under some conditions, the REE is the unique outcome compatible with CK of individual rationality and model. In this case, following Guesnerie (2002), we call the REE *eductively stable*, or a *strongly rational expectations equilibrium*. The REE can then be justified as result of an eductive process or mental process of reasoning (that is, introspection) on the side of the agents in a model. In this sense, it is not subject to problems of expectational coordination.

We consider here local eductive stability, that is: we add to the CK of rationality and model the CK that firms' supply is in a neighborhood of the REE. In the end, local eductive stability, or existence of a locally SREE, means that CK that firms choose supply schedules in a neighborhood of the REE implies that firms exactly choose the REE supply schedules.

A formal description of the stability concept relies on the concept of rationalizable solutions, obtained through a process of iterated elimination of non best responses. The definition of rationalizable solutions requires to consider the model described in the previous section as a (normal form) game among the firms where the strategy of a firm consists of the parameters $(\gamma(i)_0, \gamma(i)_1, \gamma(i)_2, \tau(i)_u)$. It follows that a Nash equilibrium of this game is a linear REE, and the best response mapping is as summarized in the following Lemma:

Lemma 1 *Let $\gamma_0 = \int_0^1 \gamma(j)_0\, dj$, $\gamma_1 = \int_0^1 \gamma(j)_1\, dj$ and $\gamma_2 = \int_0^1 \gamma(j)_2\, dj$. Aggregate supply is then*

$$\int_0^1 x(j)dj = \psi[(1-\gamma_2)p - \gamma_0 - \gamma_1\theta],$$

so that aggregate behavior is summarized by the coefficients γ_0, γ_1 and γ_2. Then, the best response of a firm $i \in I$ to others' strategies, summarized by the aggregate supply $(\gamma_0, \gamma_1, \gamma_2)$, is characterized by the coefficients $(\gamma_0(i), \gamma_1(i), \gamma_2(i), \tau_u(i))$ defined by:

$$\gamma(i)_0 = -\frac{\alpha\gamma_1\tau_\varepsilon(\beta+\alpha\gamma_0)}{\tau+\tau_u(i)+\alpha^2\gamma_1^2\tau_\varepsilon}, \tag{8}$$

$$\gamma(i)_1 = \frac{\tau_u(i)}{\tau+\tau_u(i)+\alpha^2\gamma_1^2\tau_\varepsilon}, \tag{9}$$

$$\gamma(i)_2 = \frac{\gamma_1\alpha(1+\alpha(1-\gamma_2))\tau_\varepsilon}{\tau+\tau_u(i)+\alpha^2\gamma_1^2\tau_\varepsilon}, \tag{10}$$

where $\tau_u(i) = 0$ if $K'(0) > \frac{\psi}{2(\tau+\alpha^2\gamma_1^2\tau_\varepsilon)^2}$ and $\tau_u(i)$ is the unique solution of

$$\frac{\psi}{2}\frac{1}{[\tau+\tau(i)_u+\alpha^2\gamma_1^2\tau_\varepsilon]^2} = K'(\tau(i)_u), \tag{11}$$

otherwise.

Proof. See Appendix. □

Notice that the best response of i does not depend on others' information precision $\tau(j)_u$. This comes from the fact that i's best response depends on the aggregate supply only.

Let $z(i) = \mathcal{T}(z)$ denote the best response mapping. That is, the best response of a firm $i \in I$ to an aggregate behavior $z = (\gamma_0, \gamma_1, \gamma_2, \tau_u)$ is $z(i) = \mathcal{T}(z)$ where $z(i) \equiv (\gamma(i)_0, \gamma(i)_1, \gamma(i)_2, \tau(i)_u)$ denotes the strategy of a single firm i.[10] Clearly, the REE $z^* = (\gamma_0^*, \gamma_1^*, \gamma_2^*, \tau_u^*)$ is the fixed point of this best response mapping, i.e. $z^* = \mathcal{T}(z^*)$.

As we consider local stability only, we restrict attention to strategies in a neighborhood of the REE:

Assumption 2 *For all $i \in I$, firm i's strategy $z(i)$ is in a set $W_0 \subset \mathbb{R}^3 \times \mathbb{R}_+$. Furthermore, W_0 contains the REE, i.e. $z^* \in W_0$.*

Starting from this assumption, the eductive process proceeds as follows:

– *Step 1.* Since Assumption 2 is CK, every firm knows that the resulting aggregate supply is in the convex hull $\mathcal{W}_0 = \text{conv}(W_0)$ of W_0 and it plays accordingly a best response to an element in $\mathcal{W}_0$ (firm i's beliefs on aggregate supply are point beliefs, see the remark below). Define $W_1 = W_0 \cap \mathcal{T}(\mathcal{W}_0)$. The strategies in W_1 are the best responses in W_0 to aggregate behavior in $\mathcal{W}_0$.

– *Step 2.* Since Step 1 is known to the firms, every firm knows that the aggregate supply is in the set $\mathcal{W}_1 = \text{conv}(W_1)$. Define $W_2 = W_1 \cap \mathcal{T}(\mathcal{W}_1)$. The strategies in W_2 are the best responses in W_1 to aggregate behavior in $\mathcal{W}_1$.

– Every further step is analogous: we define iteratively a decreasing sequence of sets W_n:

$$W_n = W_{n-1} \cap \mathcal{T}(\mathcal{W}_{n-1}),$$

where $\mathcal{W}_{n-1} = \text{conv}(W_{n-1})$. At Step n, every firm knows that the aggregate supply is in $\mathcal{W}_{n-1}$ and it therefore plays a best response to an element in $\mathcal{W}_{n-1}$.

Because the sequence W_n is decreasing, it converges to a limit set $W_\infty = \cap_n W_n$. We say that the REE is *locally eductively stable* or a *locally strongly REE (LSREE)* whenever there exists a set W_0 such that W_∞ reduces to one element (this element is necessarily the REE). When the

[10] We write $\tau_u = \int_0^1 \tau(j)_u \, dj$. $\mathcal{T}$ is constant with respect to τ_u. Consideration of τ_u as an element of z serves only notational purposes.

REE is a LSREE, the CK assumptions imply that every firm expects aggregate behavior to be z^* and, therefore, reacts playing the equilibrium strategy z^*.

A remark on the set W_∞: The definition of W_∞ relies on point beliefs on aggregate supply and not stochastic beliefs (at step n, beliefs are in $\mathcal{W}_{n-1}$, not in $\Delta(\mathcal{W}_{n-1})$). A formal argument relying on a law of large numbers could certainly include mixed strategies along the following lines: A mixed strategy of a firm is an element of $\Delta(W_0)$, a profile of (non correlated) mixed strategies is then an element of $\Delta(W_0)^{[0,1]}$. Because the individual supplies are non correlated, the aggregate supply resulting from a profile of mixed strategies is deterministic: it is an element of $\mathcal{W}_0$. Therefore, W_∞ is exactly the set of rationalizable solutions[11] of the game where every strategy set is restricted to W_0.

3.2 Conditions for existence of a locally SREE

The next Proposition states the conditions under which the REE is a LSREE.

Proposition 2 *Let η denote the elasticity of marginal costs of information acquisition with respect to τ_u (i.e. $\eta(\tau_u) = K''(\tau_u)\tau_u/K'(\tau_u)$). The REE is a LSREE if and only if*

$$\tau_p^* < \tau_u^*, \tag{C.I}$$

$$\tau_p^* < \tau_u^* \left(\frac{\eta^* + 2}{\eta^* + 4}\right) + \tau \left(\frac{\eta^*}{\eta^* + 4}\right), \tag{C.II}$$

where τ_p^* is given by (7) *and η^* denotes elasticity of marginal costs at the REE.*

Proof. See Appendix. □

The proof of this Proposition consists of an analysis of the dynamics of $\mathcal{T}$ around z^*. Indeed, it is straightforward from the above discussion that the REE is locally strongly rational iff the map $\mathcal{T}$ is contracting at z^*.[12]

The stability conditions are stated in a form which makes explicit the importance of the informativeness of the equilibrium market price for stability: Both conditions imply that τ_p^* must be bounded from above in a certain way in order for a LSREE to exist.

Condition (C.I) says that the price must be less informative than any private signal. This condition is exactly the necessary and sufficient condition derived by Heinemann (2004) for existence of LSREE in the same model with exogenously given private information precision τ_u^*. In fact, Condition (C.I) is the condition for local stability of the best response dynamics

[11] These are non correlated rationalizable solutions, à la Bernheim (1984) and Pearce (1984).

[12] see Desgranges (1999) for a explicit proof of this technical characterization of local eductive stability.

associated with equations (8) – (10) only, that is when the precision of private information is exogenously fixed to τ_u^*. Moreover, condition (C.I) is identical to the condition is obtained in a CARA/Gaussian model in Desgranges (1999). It confirms also a result by Desgranges et al. (2003) obtained within the context of a model with private information but only a finite number of states and signals (they also conclude that the coordination of expectations becomes difficult, if the price becomes too informative).

Condition (C.II) is therefore the additional condition imposed by endogeneity of private information precision. Indeed, the upper bound on τ_p^* provided by Condition (C.II) relies on the elasticity of the costs of information acquisition. Existence of Condition (C.II) suggests that endogenous acquisition of information might lead to stronger conditions for existence of a LSREE. Before we answer this question, we explain why stability requires a low value of τ_p^*.

3.3 Why should τ_p^* be low?

In general, instability of the REE (that is: instability of the dynamics of the best response mapping $\mathcal{T}$ around z^*) means that the individual firm's reaction turns out to be too sensitive to others' decisions. In the case under consideration, a more precise intuition is as follows. If the informativeness of the price is high, then it is quite important for the firms to extract information regarding the unknown θ from the price. Hence, supply is very sensitive to the firms' beliefs about the information contained in the price. Thus, the actual correlation between the price and θ is very sensitive to the firms' beliefs as well. Given that we have not assumed CK of beliefs, this in turn makes it hard to assess the information contained in the price. Without further assumptions that go beyond that of CK, it can hardly be expected that firms are able to coordinate their expectations in any definite way. If, on the other hand, the price is not very informative, it is not quite important for the individual firm to extract information from the price and to anticipate correctly other firms' beliefs and decisions. Every firm acts nearly autonomous, with decisions based almost exclusively on private signals and hardly on beliefs. In this case, the REE is likely to be strongly rational. Summing up, the underlying problem is identical to the well known problem of 'forecasting the forecasts of others' that is described by Keynes (1936) in his famous 'beauty contest' example.

3.4 Stronger conditions with endogenous private information precision

Endogeneity of information precision makes existence of a LSREE more requiring whenever Condition (C.II) implies Condition (C.I). Some algebra shows that this is the case if and only if:

$$\eta^* < \frac{2\tau_u^*}{\tau} \tag{12}$$

According to this inequality, the cost function of information acquisition is relevant for stability. Precisely, Condition (C.II) is stronger than Condition (C.I), if the elasticity of marginal costs of information acquisition η^* at the REE falls short of a certain, also endogenously determined upper bound. As Condition (12) contains endogenous variables, its interpretation is delicate. Still, this inequality mainly confirms the above given intuitive reason for coordination problems. If prices are very informative, it is important for every firm to figure out what other firms believe and do in order to extract valuable information from prices. In case of endogenous information, a firm's reaction to a highly informative price is not only to learn excessively from the price but also to acquire less private information. When η^* is low, it is not very costly to adjust $\tau_u(i)$ for firm i. Therefore, $\tau_u(i)$ is very sensitive to firm i's beliefs about the informational content of the price. This makes $\tau_u(i)$ difficult to predict by every other firm and the REE can not be a LSREE.

The next Proposition summarizes our results regarding existence of LSREE and again highlights the role of the informativeness of the market price in this respect:

Proposition 3

(i) If $\eta^* \geq \frac{2\tau_u^*}{\tau}$, a LSREE exists if and only if condition (C.I) *is satisfied.*

(ii) If $\eta^* < \frac{2\tau_u^*}{\tau}$, a LSREE exists if and only if condition (C.II) *is satisfied. $\tau_p^* < \tau_u^*$ is still a necessary condition, while a sufficient condition for existence of a LSREE is $\tau_p^* < \frac{1}{2}\tau_u$.*

Proof. See Appendix. □

3.5 Constant marginal costs

One special case is the case of constant marginal costs of information acquisition K' (or, equivalently, $\eta(\tau_u) = 0$).[13] In such a case, the condition (12) always holds such that existence of a LSREE is equivalent to Condition (C.II). Some computations show that Condition (C.II) reduces to $\tau_p^* < \frac{1}{2}\tau_u^*$ (i.e. the precision of prices must be lower than half the precision of the private signals). Thus, in case of constant marginal costs, endogeneity of information acquisition definitely results in stronger conditions for existence of a LSREE.

[13] For instance, marginal costs are constant, if the private signals $s(i)$ are outcomes of individual sampling processes where firms make observations of the unknown θ plus some noise term with zero mean and constant variance. If costs per observation are constant, marginal costs of information acquisition will be constant too.

Furthermore, we are able to address in this special case the question of global stability of the eductive process and to describe the set of rationalizable information precisions. To define global stability, we restrict attention to the sets W_0 with $W_0 = W_0' \times \mathbf{R}_+$ where W_0' is a compact set in $\mathbf{R}^3$ (recall that W_∞ depends on W_0).[14] We say that the REE is *globally stable* or a *strongly rational expectations equilibrium (SREE)* if, for every initial set W_0, the set W_∞ reduces to one element. This element is necessarily the REE.

The following Lemma is a first step in the analysis of global stability. It shows that, when marginal costs are constant, the best response mapping (see Lemma 1) simplifies so that the best response dynamics of τ_u is independent from the other variables $(\gamma_0, \gamma_1, \gamma_2)$.

Lemma 2 *Consider the case with constant marginal costs and denote* $Q \equiv \sqrt{\frac{\psi}{2K'}}$. *Assume* $Q > \tau$ *(so that* $\tau_u^* > 0$*). Consider a profile of firms' strategies* $(\gamma_0(j), \gamma_1(j), \gamma_2(j), \tau_u(j))$ *in* $\mathcal{T}(W_0)$.[15] *Denote* $\tau_u = \int_0^1 \tau_u(j)dj$ *the average precision of information. Then, the information precision of the best response to this profile of strategies is:*

$$T(\tau_u) = \max\left\{0, Q - \tau - \frac{\alpha^2\tau_\varepsilon}{Q^2}\tau_u^2\right\}. \tag{13}$$

Proof. See Appendix. □

This Lemma suggests that the set S of rationalizable information precisions coincides with the limit set of the best response dynamics for the endogenously acquired amount of private information associated with (13) (i.e. the limit $T^\infty(\mathbb{R}_+)$ of the sequence of sets $T^n(\mathbb{R}_+)$). The following Proposition 4 states this result and describes S.

Proposition 4 *Consider the case with constant marginal costs of information acquisition. Assume* $Q > \tau$ *(so that* $\tau_u^* > 0$*). The sequence of sets* $T^n(\mathbb{R}_+)$ *is decreasing and converges to a limit denoted* $T^\infty(\mathbb{R}_+)$. *This limit set* $T^\infty(\mathbb{R}_+)$ *is the set S of rationalizable information precisions.*

(a) If $Q - \tau < \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon}$, *then* $S = \{\tau_u^*\}$, *i.e.* τ_u^* *is the unique and globally stable fixed point of the mapping* T.

(b) Otherwise, one of the following two cases applies:

[14] This restriction of compact W_0' is necessary for the definition of stability to make sense. If W_0' is not compact (for example, $W_0' = \mathbf{R}^3$), then the set of rationalizable outcomes is not compact either: it is unbounded and no REE can never be globally stable. Due to the properties of the model under consideration, this requirement does not apply to the $\tau_u(i)$-axis: $\tau_u(i)$ is not restricted to a compact set.

[15] That is: every firm's strategy is a best response to some point-beliefs in W_0. This assumption simply means that every firm is rational (as shown in the proof, this implies that $\gamma(i)_1 = \tau_u(i)/Q$).

(b.1) If $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} \leq Q-\tau < \frac{Q^2}{\alpha^2\tau_\varepsilon}$*, then* $S=[\underline{\tau}_u, \overline{\tau}_u]$*, where* $\underline{\tau}_u$*,* τ_u^* *and* $\overline{\tau}_u$ *are the 3 fixed points of* T^2 *(*$\underline{\tau}_u$ *and* $\overline{\tau}_u$ *satisfy* $0<\underline{\tau}_u<\tau_u^*<\overline{\tau}_u<Q-\tau$*).*

(b.2) If $Q-\tau \geq \frac{Q^2}{\alpha^2\tau_\varepsilon}$*, then* $S=T(\mathbb{R}_+)=[0,Q-\tau]$*.*

The proof of the Proposition consists of two parts: we first compute the set $T^\infty(\mathbb{R}_+)$, and we then show that every precision in this set is the precision of a rationalizable strategy.

The condition $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q-\tau$ is exactly the stability condition $\tau_p^* < \frac{1}{2}\tau_u^*$. Thus, Case (a) states that τ_u^* is the unique rationalizable precision when the REE is a LSREE. The remaining two cases $(b.1)$ and $(b.2)$ characterize the set of rationalizable precisions when the REE is not stable. Notice that, in case $(b.1)$, the CK assumptions are not sufficient to predict the REE as the unique rationalizable outcome, but these assumptions still lead to some restrictions on the set of rationalizable precisions of private information.

A natural extension of Proposition 4 would be to describe the set of rationalizable outcomes $(\gamma_0,\gamma_1,\gamma_2,\tau_u)$. We are unfortunately unable to provide a full description of this set. Still, we make the two following points:

- In the case $Q-\tau < \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon}$, the corollary below shows that there is a unique rationalizable outcome.
- In the case $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q-\tau$, the proof of Points $(b.1)$ and $(b.2)$ in Proposition 4 exhibit a set of rationalizable outcomes. This is the set of outcomes such that $\gamma_0 \in \left[\overline{\gamma}_0, \underline{\gamma}_0\right]$, $\gamma_1=\tau_u/Q$, $\gamma_2 \in \left[\overline{\gamma}_2, \underline{\gamma}_2\right]$ and $\tau_u \in [\underline{\tau}_u, \overline{\tau}_u]$, where $\left(\underline{\gamma}_0,\underline{\gamma}_1,\underline{\gamma}_2,\underline{\tau}_u\right)$ and $(\overline{\gamma}_0,\overline{\gamma}_1,\overline{\gamma}_2,\overline{\tau}_u)$ is the cycle of the best response map $\mathcal{T}$ (this cycle is shown to exist and to be unique). In simple words, the outcomes "within the cycle" are rationalizable.

The next corollary states that, whenever the REE is locally stable, it is globally stable:

Corollary 1 *Consider the case with constant marginal costs of information acquisition. A necessary and sufficient condition for existence of a SREE is* $\tau_p^* < \frac{1}{2}\tau_u^*$*. This condition rewrites in terms of exogenous variables:*

$$\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} > Q-\tau. \tag{14}$$

Proof. See Appendix. □

If $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} > Q-\tau$, the REE is the unique rationalizable outcome, whatever the set W_0 is, that is: the REE is globally stable. If $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} \leq Q-\tau$, then the REE is not stable (it is not even locally stable) and there are many rationalizable outcomes.

We now illustrate the three cases in Proposition 4 and the properties of the best response mapping (13) with three examples, bearing in mind that the average information precision τ_u is necessarily non–negative and that $Q-\tau$ represents the maximum precision ever acquired (i.e. $Q-\tau = \sup_{\tau_u \geq 0} T(\tau_u)$). Thus, we can restrict the analysis of T to the set $T(\mathbb{R}_+) = [0, Q-\tau]$ without loss of generality.

Example 1 (illustrating case (a)): In case (a), a LSREE exists both when the amount of private information is exogenously given and equal to τ_u^*, and when it is endogenous (the two stability conditions $\tau_p^* < \tau_u^*$ and $\tau_p^* < \frac{1}{2}\tau_u^*$ are both satisfied). Thus, in this case, the fact that information acquisition is endogenously determined does not destabilize the REE.

Consider a numerically specified version of the model where $\alpha = -0.85$, $\psi = 1$, $\tau = 0.1$, $\tau_\varepsilon = 1$ and $K' = 0.5$. From equation (9) and (11), equilibrium values can be computed as: γ_1^* =0.621, $\tau_u^* = 0.621$ and $\tau_p^* = 0.279$. Figure 1 shows how the function T looks like. The eductive process proceeds similarly to the well known cobweb–dynamics.[16] The first step of the process is to consider that $\tau_u \geq 0$ is common knowledge. Given that T is decreasing, this fact implies that the maximum amount of private information a firm will ever acquire is given by $T(0) = Q-\tau > 0$. Since T and rationality are common knowledge, it is therefore also common knowledge that $\tau_u \leq T(0)$. A further step of the process shows then that no firm will ever choose $\tau(i)_u < T(T(0)) = T(Q-\tau)$. Thus, this second step restricts the set of possible precision to $[T(T(0)), T(0)]$. As indicated in the figure, the dynamics that result if this kind of reasoning is iterated converges to the REE precision τ_u^* (because the condition stated in Proposition 3 is satisfied): each firm can educe that only the precision REE $\tau_u^* = 0.621$ constitutes a possible solution under the assumptions of common knowledge of individual rationality and model.

Example 2 (illustrating case (b.1)): The precision of the noise is now $\tau_\varepsilon = 1.3$, which is larger than in example 1. From equations (9) and (11), equilibrium values can be computed as $\gamma_1^* = 0.582$, $\tau_u^* = 0.582$ and $\tau_p^* = 0.318$. We have then $\frac{1}{2}\tau_u^* < \tau_p^* < \tau_u^*$: A LSREE exists if the amount τ_u^* of private information is exogenously given, but does not exist if information is endogenously acquired.

On figure 2, we have now also plotted the function T and the second iterate of this function $T^2(\tau_u) \equiv T(T(\tau_u))$. As can be seen, this function possesses two additional fixed points, denoted $\underline{\tau}_u$ and $\overline{\tau}_u$. Notice too that the associated 2–cycle is stable. If we repeat the argumentation used in the discussion of the first example, we therefore get a process which converges to this 2–cycle: the first step of the process shows that $\tau_u \leq T(0) = Q-\tau$, a second step shows that $\tau_u \geq T(Q-\tau) = T^2(0)$. Clearly, iterating this argument eliminates the precisions outside the

[16] This description of the process originates in Guesnerie (1992).

Figure 1: *Best response mapping $T(\tau_u)$ for example 1 (case (a))*

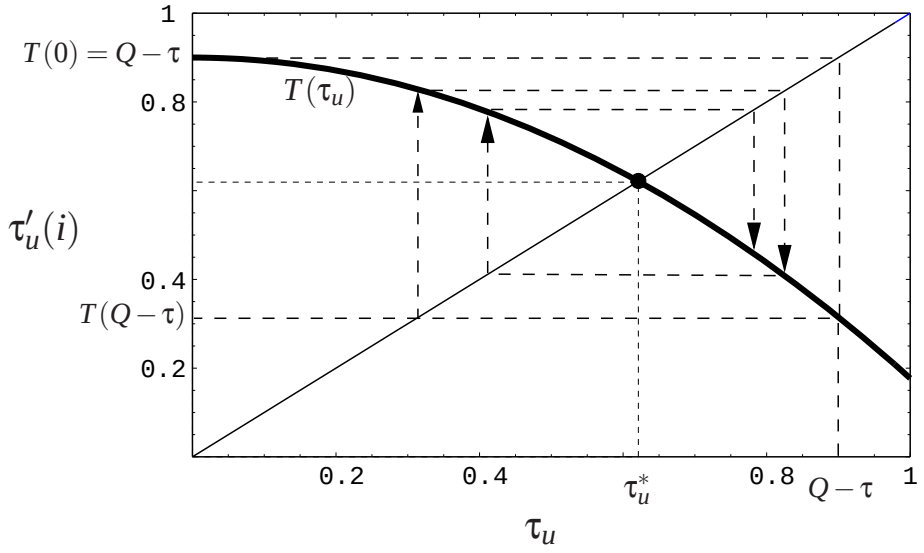
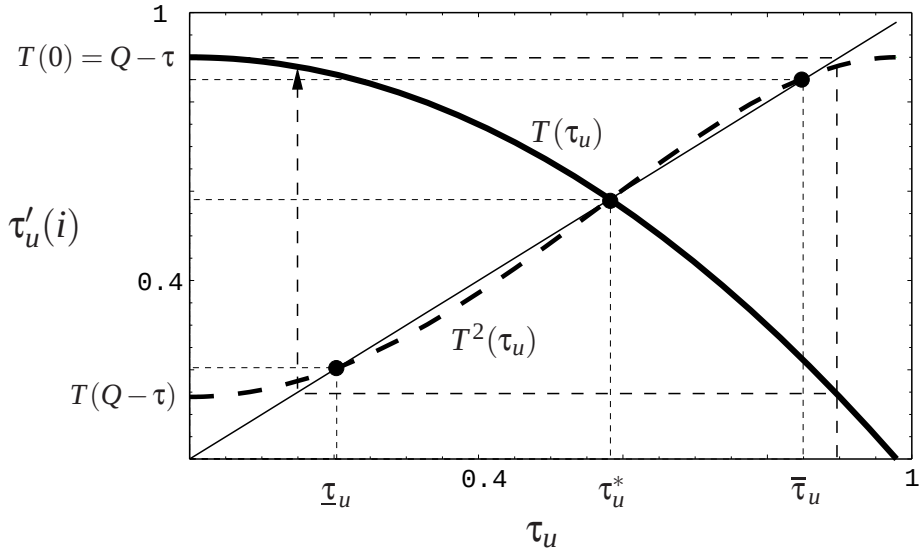


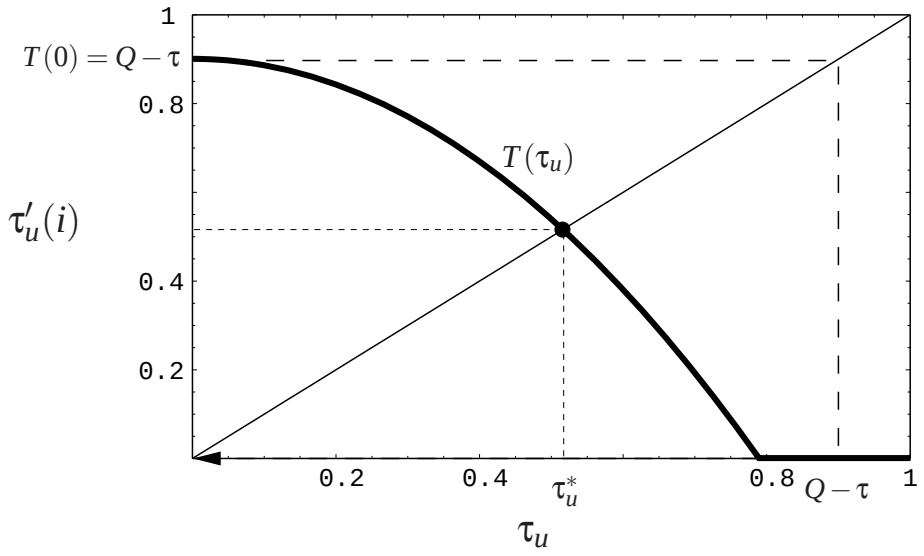
Figure 2: *Best response mapping $T(\tau_u)$ for example 2 (case (b.1))*



interval $[\underline{\tau}_u, \overline{\tau}_u]$, but not precisions in $[\underline{\tau}_u, \overline{\tau}_u]$. It follows that all precisions in the set $[\underline{\tau}_u, \overline{\tau}_u]$ constitute possible solutions under individual rationality and common knowledge.

Example 3 (illustrating case (b.2)): The precision of noise is $\tau_\varepsilon = 2.0$ and, hence, larger than in examples 1 and 2. At the REE, $\tau^*_u = 0.512$ and $\tau^*_p = 0.384$. The REE is still strongly rational, if information precision τ^*_u is assumed to be exogenously given, but not (since $\tau^*_u/2 = 0.258$),

Figure 3: *Best response mapping $T(\tau_u)$ for example 3 (case (b.2))*



when information acquisition is endogenous. The best response function T depicted in figure 3 reveals that in this example we have $T(Q-\tau) = 0$, i.e. the non-negativity constraint on $\tau'(i)_u$ becomes relevant.

Again, we repeat the argumentation used in the discussion of the above examples. However, the process here immediately converges to the whole interval $[0, Q-\tau]$. Indeed, the first step of the process still shows that $\tau_u \leq T(0) = Q-\tau$. If, however, each firm acquires this maximum amount $T(0)$ of private information such that $\tau_u = T(0)$, there is so much information in the market, that it is individually optimal to stop the acquisition of information, i.e. $T(Q-\tau) = 0$. In other words, the second step of the process shows that $\tau_u \geq T(Q-\tau) = 0$. Thus, no additional restriction is created by this second step. Clearly, iterating this argument does not eliminate any precision: all the precisions in $[0, Q-\tau]$ constitute possible solutions under individual rationality and common knowledge.

IV APPLICATIONS

In this final section, we provide three applications of the previous stability results. We first show that the requirement of existence of a LSREE creates an upper bound on the informativeness of the price, that is reminiscent of the mechanism of the celebrated Grossman-Stiglitz paradox. Then, we discuss how the precision of public (a priori) information τ and the level of marginal

costs of information acquisition K' affect the existence of a LSREE.

4.1 SREE and the Grossman–Stiglitz paradox

The well known Grossman–Stiglitz paradox on the impossibility of informationally efficient markets states that a REE with endogenous acquisition of information and a fully informative market price cannot exist simultaneously. In such a case, no firm would have an incentive to acquire costly the information the price reveals anyway, while the price cannot be informative if no firm acquires any information.

In our model, as in quite many models, the presence of exogenous noise ε prevents the price from being fully informative regarding the unknown θ. Still, the mechanism at work in the Grossman-Stiglitz paradox still holds, as previously stated in Section 2: when the precision τ_ε of the noise increases, informativeness of the REE price increases; this in turn destroys individual incentives to acquire private information (τ_u^* decreases). As a consequence, when τ_ε varies between 0 and $+\infty$, informativeness of prices in a REE is bounded from above. In the next Proposition, we compute this upper bound and we show that this upper bound is much smaller when we restrict attention to LSREE.

Proposition 5 *Assume $K'(0) < \frac{\psi}{2\tau^2}$ (so that $\tau_u^* > 0$ for every τ_ε).*[17] *Denote $\tau_p^{max} = \sup_{0<\tau_\varepsilon} \tau_p^*$ the upper bound of the informativeness of the market price and $\tau_p^{\max LSREE} = \sup_{0<\tau_\varepsilon / \text{the REE is a LSREE}} \tau_p^*$ the upper bound of τ_p^* when τ_ε is such that the REE is a LSREE. We have:*

$$\begin{aligned} \tau_p^{max} &= \sqrt{\frac{\psi}{2K'(0)}} - \tau, \\ \tau_p^{\max LSREE} &< \frac{1}{2}\tau_p^{max}. \end{aligned}$$

Furthermore, when marginal costs of information acquisition K' are constant, $\tau_p^{\max LSREE} = \frac{1}{3}\tau_p^{max}$.

Proof. See Appendix. □

The proof (that relies on algebraic computations) is in the Appendix. Notice that $\tau_p^{max} = +\infty$ iff $K'(0) = 0$.

In order for a LSREE to exist, informativeness of prices has to be lower than at least one half of the upper bound τ_p^{max} (and exactly one third in the case of constant marginal costs). All in all this means that while existence of a REE implies restrictions on the informativeness of the equilibrium price, the justification of such an equilibrium by means of an eductive learning process leads to even stronger restrictions.

[17] If $K'(0) > \psi/2\tau^2$, then $\tau_u^* = \tau_p^* = 0$ for every τ_ε and $\tau_p^{max} = 0$.

Figure 4: *The set of rationalizable strategies (*$\alpha = 0.98$, $K' = 0.5$, $\psi = 1.0$ *and* $\tau = 0.1$*).*

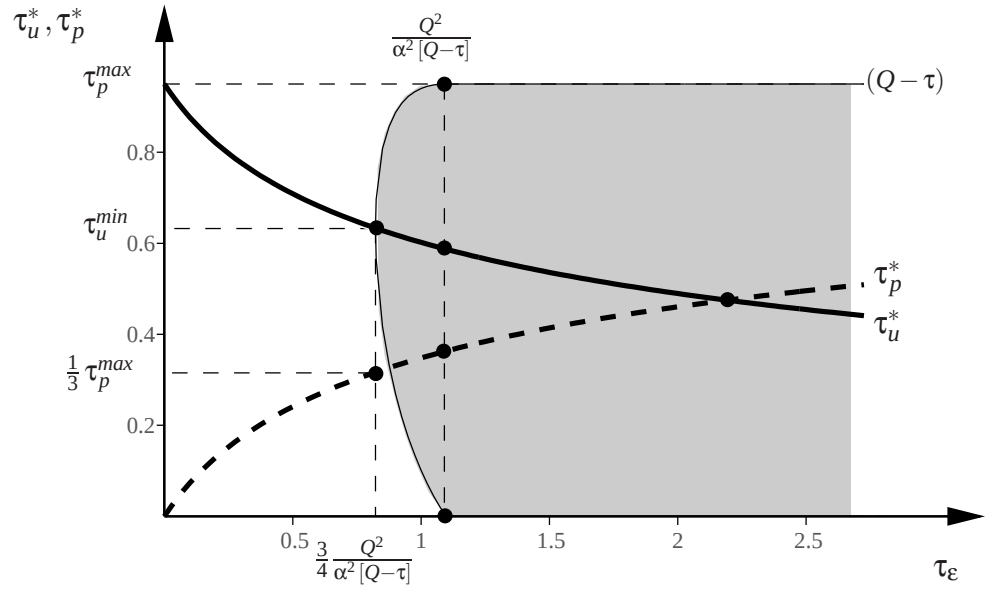
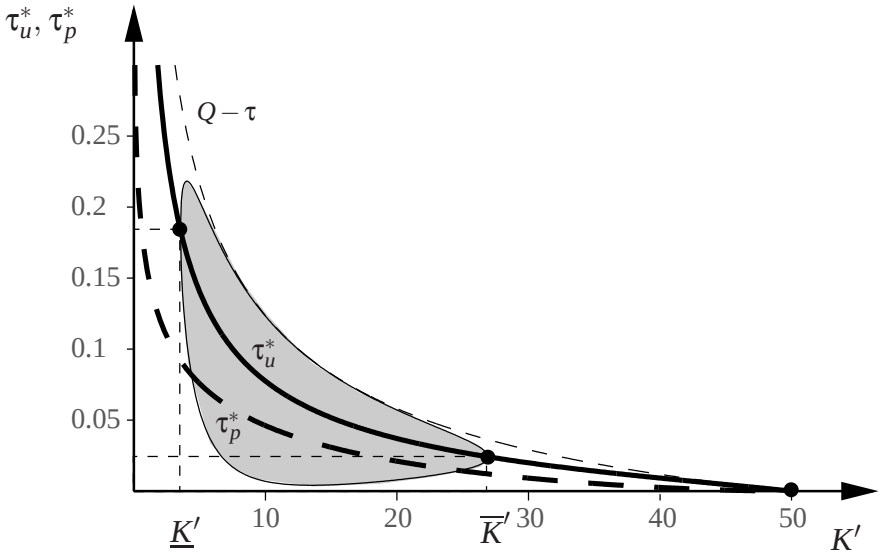


Figure 4 illustrates the above result and the implications of Proposition 4. Based on a numerical specification of the model, the figure shows the REE precision of private information τ_u^* (the solid line) and the REE informativeness of the market price τ_p^* (the dashed line) both dependent on the precision of the noise τ_ε. The precision of private information τ_u^* decreases from its maximal value $Q-\tau$ towards zero as τ_ε approaches infinity, while τ_p^* increases from zero towards $\tau_p^{max} = Q-\tau$. In addition, the figure shows the set S of rationalizable precisions of private information τ_u as stated in Proposition 4 dependent on the precision of the noise τ_ε. As long as $\tau_\varepsilon < \frac{3}{4}\frac{Q^2}{\alpha^2[Q-\tau]}$, a SREE exists and the REE precision is the unique rationalizable precision. If $\tau_\varepsilon \geq \frac{3}{4}\frac{Q^2}{\alpha^2[Q-\tau]}$, no SREE exists. The shaded area in the figure then represents all precisions that are rationalizable in this case. When the precision of prices is not too large, i.e. $\tau_\varepsilon < \frac{Q^2}{\alpha^2[Q-\tau]}$, case (b.1) of Proposition 4 arises, the common knowledge assumptions restrict the set of rationalizable precisions. When the price becomes too informative, i.e. $\tau_\varepsilon \geq \frac{Q^2}{\alpha^2[Q-\tau]}$, case (b.2) of Proposition 4 arises and all the precisions compatible with individual rationality (i.e. the set $[0, Q-\tau]$) are rationalizable. All in all, the figure illustrates that informativeness of REE prices must be below a well defined upper bound in order to get rid of coordination problems.

Figure 5: *Eductive stability and marginal costs of information acquisition* ($\alpha = 0.98$, $\tau_\varepsilon = 0.4$, $\psi = 1.0$ *and* $\tau = 0.1$*)*



4.2 Costs of information acquisition

In the case where marginal costs of information acquisition K' are constant, we compute the values of K' for which a SREE exists.

Proposition 6 *Consider the case with constant marginal costs K'. Assume that $K' < \frac{\psi}{2\tau^2}$ (so that $\tau_u^* > 0$).*

(i) If $\alpha^2\tau_\varepsilon < 3\tau$, a SREE exists for all levels of marginal costs K'.

(ii) Otherwise, $\alpha^2\tau_\varepsilon \geq 3\tau$ and there exist upper and lower bounds ($\overline{K}'$ and $\underline{K}'$, respectively) on marginal costs given by

$$\overline{K}' = \frac{9\psi}{8\left(\alpha^2\tau_\varepsilon + \sqrt{\alpha^2\tau_\varepsilon\left[\alpha^2\tau_\varepsilon - 3\tau\right]}\right)},$$

$$\underline{K}' = \frac{9\psi}{8\left(\alpha^2\tau_\varepsilon - \sqrt{\alpha^2\tau_\varepsilon\left[\alpha^2\tau_\varepsilon - 3\tau\right]}\right)},$$

such that a SREE exists whenever $K' \leq \underline{K}'$ or $K' \geq \overline{K}'$.

Proof. See Appendix. □

Point (ii) is the striking part of the result concerning the role played by K': the set of K' compatible with existence of a SREE is not convex, both small and large values of K' imply

existence of a SREE. This non-monotonic effect of K' on existence of a SREE is analogous to the non-monotonic effect of τ_u exhibited in Desgranges (1999) in a financial market model à la Grossman (1976) (where the private information precision is given). Indeed, in our model, τ_u^* is monotonic (and decreasing) in K' and the intuition for the result follows from two facts: τ_p^* is increasing but not linear in τ_u^* and stability obtains when $\tau_p^* < \tau_u^*/2$ (as stated in Corollary 1). The effect of a change in K' on the condition for stability is then ambiguous. For example, an increase in τ_u^* (due to a decrease in K') favors stability while the simultaneous increase in τ_p^* is detrimental to stability. Because of the non-linearity of τ_p^* in τ_u^*, the resulting effect of a change in K' can either favor stability or not.

The fact that a SREE exists for low values of τ_u^* is easily understood since a low τ_u^* means that there is not much to be learned from the price such that the above described coordination problem doesn't show up. On the other hand, when τ_u^* is high, the price is highly informative, but it is less informative than private signals such that it is not important for the firms to learn from the price.[18]

Figure 5 illustrates this result. Based on a numerical specification of the model, the figure shows the equilibrium precision of private information τ_u^* (the solid line) and the equilibrium informativeness of the market price τ_p^* (the dashed line) both dependent on the level of marginal costs of information acquisition K'. Note, that there is an upper bound $\psi/2\tau^2 = 50$ on this level of marginal costs because $\tau_u^* = 0$ above this upper bound. The shaded area in the figure again represents the set of rationalizable precisions of private information according to Proposition 4. As can be seen, existence of a SREE is favored by low or high marginal costs of information acquisition K'.

4.3 Public (a priori) information

Let us now turn to the comparative–statics with respect to the precision of public (a priori) information τ. The effect of this parameter on existence of a SREE turns out to be monotonic.

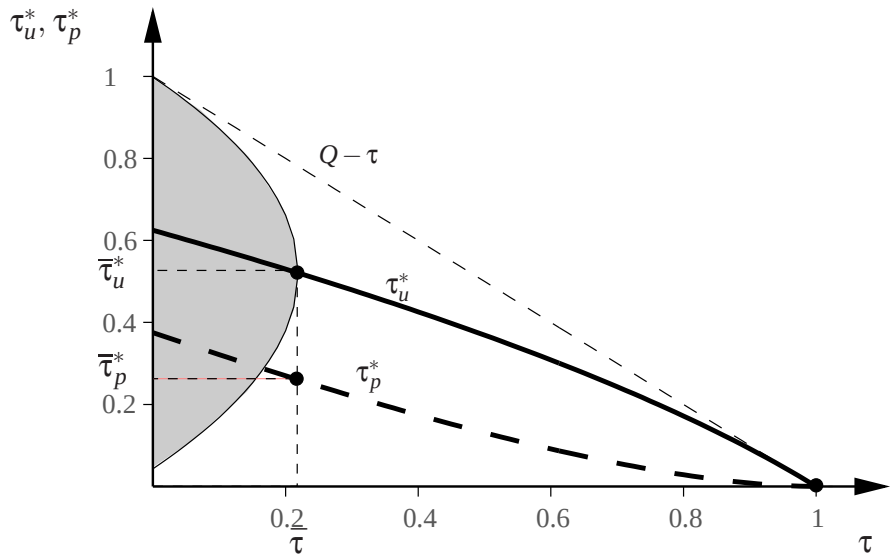
Proposition 7 *Assume that the elasticity of information acquisition costs $\eta = \tau_u K''(\tau_u)/K'(\tau_u)$ is constant. There exists a level $\bar{\tau} \geq 0$ of the precision of a priori information such that a SREE exists iff $\tau \geq \bar{\tau}$. In the case with constant marginal costs ($\eta = 0$), $\bar{\tau} = \max\left(0, Q - \frac{3}{4}\frac{Q^2}{\alpha^* \tau_\varepsilon}\right)$.*

Proof. See Appendix. □

Depending on the other parameters of the model, the critical level of the precision of public information $\bar{\tau}$ may well be 0 such that a SREE exists for all $\tau \geq 0$.

[18] In particular, when K' tends to 0, τ_u^* is not bounded while τ_p^* is bounded from above by $\alpha^2 \tau_\varepsilon$ (τ_p^* can be explicitly computed from Equations (6) and (7)).

Figure 6: *Eductive stability and precision of public information (*$\alpha = 0.98$, $\tau_\varepsilon = 1$, $\psi = 1$ *and* $K' = 0.5$*)*



Three comments are in order. First, this proposition does not straightforwardly follow from previous results. Indeed, when τ increases, τ_u^* decreases. As τ_p^* is increasing in τ_u^*, the effect of an increase of τ on the stability condition (C.I) $\tau_p^* < \tau_u^*$ is ambiguous: the decrease of τ_p^* favors stability while the decrease of τ_u^* is bad for stability. The same ambiguity holds for the other stability condition (C.II). Thus, the above proposition shows that the positive effect of an increase in τ is always the dominant one. Second, improving τ always favors stability. Thus, under the presumption that the aim of public policy is to prevent expectational coordination difficulties, acquisition and dissemination of information by the government is always helpful in this respect. Third, the present model contains two sources of public information: the prior probability distribution of θ (that is exogenous public information) and the price (that is an endogenous public signal). Our results stress a difference (from the viewpoint of coordination) between endogenous and exogenous public information. They unequivocally say that too much public information in form of the market price generates expectational coordination difficulties, while public information in form of a priori knowledge tends to attenuate such difficulties.

Figure 6, which is again based on a numerical specification of the model, illustrates this result for the case of a positive critical level $\overline{\tau}$. The figure shows the equilibrium precision of private information τ_u^* (the solid line) and the equilibrium informativeness of the market price τ_p^* (the dashed line) both dependent on the level of the precision of public (a priori) information τ. Note that τ is bounded from above by $Q = \psi/(2K') = 1$ because otherwise $\tau_u^* = 0$. The shaded

area in the figure again represents the set of rationalizable precisions of private information according to Proposition 4. As can be seen, existence of a SREE is favored by a high precision of public information τ.

V CONCLUSIONS

In the present paper, we have shown how known results for existence of SREE must be modified, if endogenously acquired private information is considered. Generally, endogeneity of acquisition of information leads to stronger conditions for existence of a SREE. In particular, it was shown that it is necessary, but not always sufficient, for REE prices to be less informative than private signals for a SREE to exist. In case of a relatively low elasticity of the marginal costs function associated with the acquisition of information with respect to the informativeness of that information, this is not sufficient for existence of a SREE. For example, in the limiting (and special) case of constant marginal costs and zero elasticity, informativeness of the price must be lower than one half the informativeness of the private signals.

Given that the conditions for existence of a SREE take the form of restrictions on informativeness of the market price, it is quite natural to look for a link between our results and the well known Grossman–Stiglitz paradox of the impossibility of informationally efficient markets. While the Grossman–Stiglitz paradox is concerned with the question of existence of a REE, our paper is concerned with the justification of an existing REE via eductive learning, that is based on the assumptions of individual rationality and common knowledge. If we regard the absence of possible expectational coordination difficulties as an important constraint to be respected, our results supplement the Grossman–Stiglitz paradox as follows: It is not only that mere existence of a REE necessitate a certain amount of informational inefficiency, but also the justification of such an REE based on individual rationality and common knowledge necessitates a specific amount of informational inefficiency. Furthermore, the amount of informational inefficiency required in order to avoid expectational coordination difficulties is generally greater than that required for existence of a REE.

Two comparative statics results are surprising. First, the set of values of the marginal costs of information acquisition that are compatible with a SREE is not convex: both low and high values sustain a SREE, while intermediate values do not always. Second, the influence of the public prior information on stability is positive, this is not very surprising in itself, but this is in contrast with the fact that the information that is publicly revealed by the price has a negative influence on stability.

Future work on this subject will analyze the case of increasing marginal costs of information acquisition in more detail in order to check the robustness of the results obtained for the case of constant marginal costs. Moreover, it should be analyzed whether the results carry over to financial market models with learning from current prices where risk aversion of traders is allowed for.

REFERENCES

Angeletos, G. and Pavan, A., (2007), Efficient use of information and social value of information, Econometrica 75(4), 1103–42.

Angeletos, G. and Werning, I., (2006), Crises and prices: Information aggregation, multiplicity, and volatility, American Economic Review 96(5), 1720–36.

Barlevy, G. and Veronesi, P., (2000), Information acquisition in financial markets, Review of Economic Studies 67(1), 79–90.

Bernheim, D., (1984), Rationalizable strategic behavior, Econometrica 52, 1007–1028.

Chamley, C., (2007), Complementarities in information acquisition with short-term trades, Theoretical Economics 2(4), 441–67.

Costa-Gomes, M. and Crawford, V., (2006), Cognition and behavior in two-person guessing games: An experimental study, American Economic Review 96(5), 1737–68.

Desgranges, G., (1999), Common knowledge and information revealed by prices: An example, mimeo.

Desgranges, G., Geoffard, P. and Guesnerie, R., (2003), Do prices transmit rationally expected information?, Journal of the European Economic Association 1, 124–153.

Grossman, S., (1976), On the efficiency of competitive stock markets where traders have diverse information, Journal of Finance 31, 573–585.

Grossman, S. and Stiglitz, J., (1980), On the impossibility of informationally efficient markets, American Economic Review 70, 393–408.

Guesnerie, R., (1992), An exploration of the eductive justifications of the rational–expectations hypothesis, American Economic Review 82, 1254–1278.

Guesnerie, R., (2002), Anchoring economic predictions in common knowledge, Econometrica 70, 439–480.

Heinemann, M., (2002), Existence of strongly rational expectations equilibria on asset markets with asymmetric information, Schumpeter Working Paper 2002-2, Humboldt–University Berlin, Berlin.

Heinemann, M., (2004), Are rational expectations equilibria with private information eductively stable?, Journal of Economics 82, 169–194.

Hellwig, C., (2002), Public information, private information, and the multiplicity of equilibria in coordination games, Journal of Economic Theory 107(2), 191–222.

Hellwig, C., Mukherji, A. and Tsyvinski, A., (2006), Self-fulfilling currency crises: The role of interest rates, American Economic Review 96(5), 1769–87.

Keynes, J., (1936), The General Theory of Employment, Interest and Money (Macmillan, London).

Morris, S. and Shin, H., (1998), Unique equilibrium in a model of self-fulfilling currency attacks, American Economic Review 88(3), 587–97.

Morris, S. and Shin, H., (2002), Social value of public information, American Economic Review 92(5), 1521–34.

Pearce, D., (1984), Rationalizable strategic behavior and the problem of perfection, Econometrica 52, 1029–1050.

Veldkamp, L., (2006a), Information markets and the comovement of asset prices, Review of Economic Studies 73(3), 823–45.

Veldkamp, L., (2006b), Media frenzies in markets for financial information, American Economic Review 96(3), 577–601.

Verrecchia, R., (1982), Information acquisition in a noisy rational expectations economy, Econometrica 50, 1415–1430.

Vives, X., (1993), How fast do rational agents learn?, Review of Economic Studies 60, 329–347.

APPENDIX

Definition of aggregate supply. Given that every $x(i)$ is a function of $(s(i),p)$, aggregate supply is *a priori* a function of p and all the private signals $s(i)$ (and not only a function of θ and p): aggregate supply may vary from one state $(s(i))_{i\in[0,1]}$ to another corresponding to the same θ (because of heterogeneous $x(i)$). Aggregate supply may depend on the individual precisions $(\tau_u(i))_{i\in[0,1]}$ as well. A convenient way to avoid this difficulty is to assume that the heterogeneity of the $x(i)$ is not correlated with the $s(i)$. Formally, aggregate supply is defined as:[19]

$$X(\theta,p) \overset{def}{=} \int_{[0,1]} \int_{\mathbb{R}} x(i)(\theta+u(i),p)\, dP_i(u(i))\, di, \tag{A.1}$$

where dP_i is the normal centered distribution with precision $\tau_u(i)$. Aggregate supply is then always defined as a function of (θ,p) even in case of behavioral heterogeneity.

In the case where every firm j submits a linear supply $x(j)$:

$$x(j) = \psi[(1-\gamma(j)_2)p - \gamma(j)_0 - \gamma(j)_1(\theta+u(j))],$$

with $\tau_u(j)$ the precision of $u(j)$, the above definition (A.1) reduces to Equation (4). □

Proof of equation (7)**.** This is purely routine. Rewrite Equation (5) as:

$$p = \frac{\beta+\alpha\gamma_0^*}{1+\alpha(1-\gamma_2^*)} + \frac{\alpha\gamma_1^*}{1+\alpha(1-\gamma_2^*)}\omega,$$

where the random variable ω is:

$$\omega \equiv \theta + \frac{1}{\alpha\gamma_1^*}\varepsilon.$$

The observation of p is equivalent to the observation of ω (i.e. the conditional distribution $\theta|p$ is the same as $\theta|\omega$). It follows that $\tau_{\theta|p} = \tau_{\theta|\omega}$. Standard computations give $\tau_{\theta|\omega} = \tau + \alpha^2\gamma_1^{*2}\tau_\varepsilon$ (where $\alpha^2\gamma_1^{*2}\tau_\varepsilon$ is the precision of the noise term in ω), and (7) follows. □

Proof of Proposition 1. We prove this Proposition using the best response mapping specified in Lemma 1 and the fact that a REE $(\gamma_0^*,\gamma_1^*,\gamma_2^*,\tau_u^*)$ is a fixed point of this best response mapping. In the case $K'(0) \geq \psi/2\tau^2$, it is straightforward to check that $\gamma_0^* = \gamma_1^* = \gamma_2^* = \tau_u^* = 0$ is the unique equilibrium. In the case $K'(0) \leq \psi/2\tau^2$, combining equations (9) and (11) shows that τ_u^* and γ_1^* are the solutions to:

$$\gamma_1^* = \sqrt{\frac{2K'(\tau_u^*)}{\psi}}\tau_u^* \tag{A.2}$$

and substituting this expression into equation (11) gives:

$$\sqrt{\frac{2K'(\tau_u^*)}{\psi}}\left[\frac{2K'(\tau_u^*)}{\psi}\tau_u^{*2}\alpha^2\tau_\varepsilon + \tau + \tau_u^*\right] = 1, \tag{A.3}$$

The LHS of the latter equation (A.3) is an increasing function of τ_u^* (increasing from $\sqrt{\frac{2K'(0)}{\psi}}\tau$ to $+\infty$ when τ_u increases from 0 to $+\infty$). This implies that there is a unique τ_u^* solving this equation. Given τ_u^*, there exists a unique positive solution γ_1^* to equation (A.2) and therefore unique solutions γ_0^* and γ_2^* of the two equations (8) and (10). □

[19] All the required measurability assumptions are made.

Proof of Lemma 1. Deriving the best response of a firm i to a given profile of strategies of the other firms is purely routine (given that the aggregate behavior of others' firms is characterized by the 3 parameters $(\gamma_0, \gamma_1, \gamma_2)$). Profit $\pi(i)$ of firm i is:

$$\pi(i) = [p - \theta] x(i) - \frac{1}{2}\frac{1}{\psi}[x(i)]^2 - K(\tau(i)_u).$$

Clearly, the profit maximizing output is $x(i) = \psi(p - E[\theta|p, s(i)])$. Given Equation (5), we have

$$E[\theta|p, s(i)] = \frac{\alpha^2\gamma_1^2\tau_\varepsilon\omega + \tau(i)_u s(i)}{\tau + \tau(i)_u + \alpha^2\gamma_1^2\tau_\varepsilon},$$

where $\omega = \theta + \frac{\varepsilon}{\alpha\gamma_1}$. We have then:

$$(1 - \gamma(i)_2)p - \gamma(i)_0 - \gamma(i)_1 s(i) = p - \frac{\alpha^2\gamma_1^2\tau_\varepsilon\frac{(1+\alpha(1-\gamma_2))p - (\beta+\alpha\gamma_0)}{\alpha\gamma_1} + \tau(i)_u s(i)}{\tau + \tau(i)_u + \alpha^2\gamma_1^2\tau_\varepsilon}.$$

Identifying the coefficients $\gamma(i)_0$, $\gamma(i)_1$ and $\gamma(i)_2$ to their counterparts in the RHS of the above expression gives equations (8) to (10).

To compute the optimal precision, consider the expected profit:

$$E[\pi(i)] = E\left([p - \theta] x(i) - \frac{1}{2}\frac{1}{\psi}[x(i)]^2\right) - K(\tau(i)_u).$$

The partial derivative with respect to $\tau_u(i)$ is then:

$$\frac{\partial E[\pi(i)]}{\partial \tau(i)_u} = \frac{\partial}{\partial \tau(i)_u} E\left([p - \theta] x(i) - \frac{1}{2}\frac{1}{\psi}[x(i)]^2\right) - K'(\tau(i)_u),$$

where $x(i) = (1 - \gamma(i)_2)p - \gamma(i)_0 - \gamma(i)_1 s(i)$. Straightforwardly, $E((p - \theta)x(i))$ does not depend on $\tau(i)_u$. Thus, some computations show that

$$\frac{\partial E[\pi(i)]}{\partial \tau(i)_u} = -\frac{\psi}{2}\gamma(i)_1^2\frac{\partial E\left(s(i)^2\right)}{\partial \tau(i)_u} - K'(\tau(i)_u) = \frac{\psi}{2}\left(\frac{\gamma(i)_1}{\tau(i)_u}\right)^2 - K'(\tau(i)_u).$$

The first order condition is then:

$$\frac{\partial E[\pi(i)]}{\partial \tau(i)_u} \leq 0 \text{ and } \tau(i)_u\frac{\partial E[\pi(i)]}{\partial \tau(i)_u} = 0 \text{ (given the constraint } \tau(i)_u \geq 0).$$

$\frac{\partial E[\pi(i)]}{\partial \tau(i)_u} \leq 0$ rewrites as equation (11):

$$\frac{\psi}{2}\frac{1}{[\tau + \tau(i)_u + \alpha^2\gamma_1^2\tau_\varepsilon]^2} \leq K'(\tau(i)_u).$$

The LHS is decreasing and the RHS is increasing. This implies that $\tau(i)_u = 0$ if $\frac{\psi}{2}\frac{1}{[\tau + \alpha^2\gamma_1^2\tau_\varepsilon]^2} < K'(0)$ and $\tau(i)_u$ is the unique solution of

$$\frac{\psi}{2}\frac{1}{[\tau + \tau(i)_u + \alpha^2\gamma_1^2\tau_\varepsilon]^2} = K'(\tau(i)_u), \tag{A.4}$$

otherwise. □

Proof of Proposition 2. At a REE with $\tau_u^* > 0$, the total differential of best response mapping (defined in equations (8) to (11)) is:

$$\begin{bmatrix} 1 & 0 & 0 & \frac{\gamma_0^*\gamma_1^*}{\tau_u^*} \\ 0 & 1 & 0 & -\frac{(1-\gamma_1^*)\gamma_1^*}{\tau_u^*} \\ 0 & 0 & 1 & \frac{\gamma_2^*\gamma_1^*}{\tau_u^*} \\ 0 & 0 & 0 & -\left(K'' + \frac{\psi {\gamma_1^*}^3}{{\tau_u^*}^3}\right) \end{bmatrix} \begin{pmatrix} d\gamma'(i)_0 \\ d\gamma'(i)_1 \\ d\gamma'(i)_2 \\ d\tau'(i)_u \end{pmatrix} =$$

$$\begin{bmatrix} -\frac{\alpha^2\tau_\varepsilon{\gamma_1^*}^2}{\tau_u^*} & \frac{\gamma_0^*}{\gamma_1^*} - \frac{2\alpha^2\tau_\varepsilon\gamma_0^*{\gamma_1^*}^2}{\tau_u^*} & 0 & 0 \\ 0 & -\frac{2\alpha^2\tau_\varepsilon{\gamma_1^*}^3}{\tau_u^*} & 0 & 0 \\ 0 & \frac{\gamma_2^*}{\gamma_1^*} - \frac{2\alpha^2\tau_\varepsilon\gamma_2^*{\gamma_1^*}^2}{\tau_u^*} & -\frac{\alpha^2\tau_\varepsilon{\gamma_1^*}^2}{\tau_u^*} & 0 \\ 0 & \frac{2\psi\alpha^2\tau_\varepsilon{\gamma_1^*}^4}{{\tau_u^*}^3} & 0 & 0 \end{bmatrix} \begin{pmatrix} d\gamma_0 \\ d\gamma_1 \\ d\gamma_2 \\ d\tau_u \end{pmatrix}$$

We write this system as $Ax' = Bx$. The Jacobian of the best response dynamics at the REE is then a matrix $P = A^{-1}B$. Since it turns out that P is a triangular matrix (after tedious computations), its eigenvalues are equal to the elements on its main diagonal. The respective eigenvalues $\lambda_1 \dots \lambda_4$ are:

$$\lambda_1 = 0, \quad \lambda_2 = \lambda_3 = -\frac{\alpha^2{\gamma_1^*}^2\tau_\varepsilon}{\tau_u^*}, \quad \lambda_4 = -\frac{2\alpha^2{\gamma_1^*}^2\tau_\varepsilon}{\tau + \tau_u^* + \alpha^2\tau_\varepsilon{\gamma_1^*}^2 - (1-\gamma_1^*)\frac{\psi\frac{\gamma_1^*}{{\tau_u^*}^2}}{K''(\tau_u^*) + \psi\frac{{\gamma_1^*}^2}{{\tau_u^*}^3}}}.$$

The condition for stability of this dynamical system is that all eigenvalues are less than one in absolute value. The stability conditions therefore are: $|\lambda_2| < 1$ and $|\lambda_4| < 1$. Using the definition (7) for τ_p^*, the condition $|\lambda_2| < 1$ rewrites as Condition(C.I), and the condition $|\lambda_4| < 1$ rewrites:

$$\tau_p^* < \tau_u^* + \tau - (1-\gamma_1^*)\frac{\psi\frac{\gamma_1^*}{{\tau_u^*}^2}}{K''(\tau_u^*) + \psi\frac{{\gamma_1^*}^2}{{\tau_u^*}^3}}. \tag{A.5}$$

Using the definition $\eta^* = \frac{K''(\tau_u^*)\tau_u^*}{K'(\tau_u^*)}$, some algebra shows that the right hand side of (A.5) is equal to $\tau_u^* - \tau_p^*\left(\frac{2}{\eta^*+2}\right) + \tau\left(\frac{\eta^*}{\eta^*+2}\right)$. Condition (A.5) then becomes (C.II). □

Proof of Proposition 3. The proof follows from quite simple computations. On the one hand, Condition (C.II) implies Condition (C.I) iff $\eta^* < 2\tau_u^*/\tau$. On the other hand, the right hand side of Condition (C.II) is greater than $\tau_u^*/2$ as soon as $\eta^* < 2\tau_u^*/\tau$. □

Proof of Lemma 2.

We first prove the following Claim:

Claim Every element $(\gamma_0(i), \gamma_1(i), \gamma_2(i), \tau_u(i))$ of $\mathcal{T}(W_0)$ satisfies:

$$\gamma(i)_1 = \frac{\tau_u(i)}{Q}. \tag{A.6}$$

Proof of the Claim The Claim follows from Lemma 1 in the case of a constant K'. Denote $(\gamma_0, \gamma_1, \gamma_2, \tau_u)$ the element of W_0 such that

$$(\gamma_0(i), \gamma_1(i), \gamma_2(i), \tau_u(i)) = \mathcal{T}(\gamma_0, \gamma_1, \gamma_2, \tau_u).$$

Notice that $K' > \psi/2(\tau+\alpha^2\gamma_1^2\tau_\varepsilon)^2$ rewrites $\tau+\alpha^2\gamma_1^2\tau_\varepsilon > Q$. If $\tau+\alpha^2\gamma_1^2\tau_\varepsilon > Q$, then Lemma 1 implies that $\tau_u(i)=0$ and $\gamma(i)_1=0$ (so that $\gamma(i)_1=\tau_u(i)/Q$). Otherwise, if $\tau+\alpha^2\gamma_1^2\tau_\varepsilon < Q$, then substitution of (9) into (11) yields:

$$\left(\frac{\gamma(i)_1}{\tau(i)_u}\right)^2 = \frac{1}{Q^2}.$$

Given that $\gamma(i)_1 \geq 0$ (and $\tau(i)_u \geq 0$), we get $\gamma(i)_1=\tau(i)_u/Q$. □

Given Claim 1 above, the map T is exactly the best response mapping defined in Lemma 1 when K' is constant. Indeed, taking account of $\gamma_1=\tau_u/Q$, Lemma 1 implies:

(i) $\tau(i)_u=0$ in the case $\tau+\alpha^2\gamma_1^2\tau_\varepsilon > Q$,

(ii) $\tau(i)_u = Q-\tau-\frac{\alpha^2\tau_\varepsilon}{Q^2}\tau_u^2 > 0$ in the case $\tau+\alpha^2\gamma_1^2\tau_\varepsilon < Q$.

Considering these two cases together gives Equation (13). □

Proof of Proposition 4.

We first prove some technical Lemmas (A.1 – A.5).

Lemma A.1 The sequence of sets $T^n(\mathbb{R}_+)$ is decreasing, and therefore converges to a limit denoted $T^\infty(\mathbb{R}_+)$.

Proof of Lemma A.1. Given that T is a continuous map on $\mathbb{R}$ to $\mathbb{R}$, $T(X)$ is an interval whenever X is an interval. Hence, $T(\mathbb{R}_+)$ is an interval, and one sees (step by step) that every $T^n(\mathbb{R}_+)$ is an interval as well. We write $T^n(\mathbb{R}_+)=[\underline{\tau}_u^n, \overline{\tau}_u^n]$ for every n. Given that T is decreasing, we have:

$$\begin{aligned}\underline{\tau}_u^{n+1} &= T(\overline{\tau}_u^n),\\ \overline{\tau}_u^{n+1} &= T(\underline{\tau}_u^n),\end{aligned}$$

with $\underline{\tau}_u^1=0$ and $\overline{\tau}_u^1=Q-\tau$. Given that $\underline{\tau}_u^2$ and $\overline{\tau}_u^2$ are in $T(\mathbb{R}_+)=[0,Q-\tau]$, we have:

$$\begin{aligned}\underline{\tau}_u^1 &\leq \underline{\tau}_u^2,\\ \overline{\tau}_u^2 &\leq \overline{\tau}_u^1.\end{aligned}$$

Using the fact that T is decreasing, we have first

$$\begin{aligned}\overline{\tau}_u^2 &= T\left(\underline{\tau}_u^1\right) \leq T\left(\underline{\tau}_u^2\right)=\overline{\tau}_u^3,\\ \underline{\tau}_u^3 &= T\left(\overline{\tau}_u^2\right) \geq T\left(\overline{\tau}_u^1\right)=\underline{\tau}_u^2,\end{aligned}$$

and iterating the argument, we have that $\underline{\tau}_u^n$ is increasing and $\overline{\tau}_u^n$ is decreasing. Given that the sequences $\underline{\tau}_u^n$ and $\overline{\tau}_u^n$ are bounded, they converge. □

Lemma A.2 We have:

(a) $T^\infty(\mathbb{R}_+)=\{\tau_u^*\}$ if $Q-\tau < \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon}$,

(b.1) $T^\infty(\mathbb{R}_+)=[\underline{\tau}_u,\overline{\tau}_u]$ if $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} \leq Q-\tau < \frac{Q^2}{\alpha^2\tau_\varepsilon}$, where $\underline{\tau}_u$, τ_u^* and $\overline{\tau}_u$ are the 3 fixed points of T^2 ($\underline{\tau}_u$ and $\overline{\tau}_u$ satisfy $0<\underline{\tau}_u<\tau_u^*<\overline{\tau}_u<Q-\tau$),

(b.2) $T^\infty(\mathbb{R}_+)=T(\mathbb{R}_+)=[0,Q-\tau]$ if $Q-\tau \geq \frac{Q^2}{\alpha^2\tau_\varepsilon}$.

Proof of Lemma A.2. Let $f(\tau_u) = Q - \tau - \frac{\alpha^2\tau_\varepsilon}{Q^2}{\tau_u}^2$, and denote z the positive root of f ($z = \sqrt{(Q-\tau)\frac{Q^2}{\alpha^2\tau_\varepsilon}}$). For $\tau_u \in [0, Q-\tau]$, $T(\tau_u) > 0$ iff $\tau_u < z$. We distinguish between two cases:

In the case $Q - \tau \geq z$ (Case $(b.2)$ of the Lemma), then direct computations show (step by step) that, for every n, $\underline{\tau}_u^n = 0$ and $\overline{\tau}_u^n = Q - \tau$. This proves Case $(b.2)$.

In the case $Q - \tau \leq z$ (Cases (a) and $(b.1)$ of the Lemma), then $T = f$ and (given that $T([0, Q-\tau]) \subset [0, Q-\tau]$) $T^2 = f^2$. It follows that T^2 is increasing (as T is decreasing), $T^2(0) > 0$, $T^2(Q-\tau) < Q - \tau$. The next Claim characterizes the fixed points of T^2 in $[0, Q-\tau]$:

Claim If $Q - \tau < \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon}$, then T^2 has one fixed point in $[0, Q-\tau]$. If $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau < \frac{Q^2}{\alpha^2\tau_\varepsilon}$, then T^2 has three fixed points $\underline{\tau}_u$, τ_u^* and $\overline{\tau}_u$ in $[0, Q-\tau]$ (with $\underline{\tau}_u < \tau_u^* < \overline{\tau}_u$).

Proof of the Claim. Recall that T^2 has at least one fixed point in $[0, Q-\tau]$ (that is τ_u^*). The derivative $\left(T^2\right)'(\tau_u)$ of T^2 at τ_u is:

$$\left(T^2\right)'(\tau_u) = {f^2}'(\tau_u) = f'(\tau_u)\, f'(f(\tau_u)),$$

and the second derivative is:

$$\left(T^2\right)''(\tau_u) = \left(f^2\right)''(\tau_u) = 4\left(\frac{\alpha^2\tau_\varepsilon}{Q^2}\right)^2\left(Q - \tau - 3\frac{\alpha^2\tau_\varepsilon}{Q^2}{\tau_u}^2\right).$$

It follows that ${f^2}''$ has a unique positive real root. Hence, f^2 (that is T^2) has 0 or 1 inflection point in $[0, Q-\tau]$. If T^2 has no inflection point, then it has exactly one fixed point (T^2 is convex). If T^2 has one inflection point, then it has 1 or 3 fixed points.

Summing up, T^2 has 1 or 3 fixed points. To determine the exact number of fixed points, we compute:

$$\begin{aligned} T'(\tau_u) &= -2\frac{\alpha^2\tau_\varepsilon}{Q^2}\tau_u, \\ \tau_u^* &= \frac{Q^2}{2\alpha^2\tau_\varepsilon}\left(\sqrt{4\frac{\alpha^2\tau_\varepsilon}{Q^2}(Q-\tau)+1} - 1\right). \end{aligned} \tag{A.7}$$

Some more computations show that:

$$\left(T^2\right)'(\tau_u^*) > 1 \Leftrightarrow \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau. \tag{A.8}$$

Hence, T^2 has exactly one fixed point in $[0, Q-\tau]$ in the case $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau$ and 3 fixed points in the case $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} \geq Q - \tau$. $\square$

From the proof of Lemma A.1 above, we know that $\underline{\tau}_u^{n+2} = T^2(\underline{\tau}_u^n)$ and $\overline{\tau}_u^{n+2} = T^2(\overline{\tau}_u^n)$, and the limits of $\underline{\tau}_u^n$ and $\overline{\tau}_u^n$ are fixed points of T^2. Then, the Claim allows us to prove Cases (a) and $(b.1)$:

(i) in Case (a), the two sequences $\underline{\tau}_u^n$ and $\overline{\tau}_u^n$ converge necessarily to τ_u^*. This proves Case (a).

(ii) in Case $(b.1)$, one checks that $\underline{\tau}_u^n \leq \underline{\tau}_u < \overline{\tau}_u \leq \overline{\tau}_u^n$.[20] It follows that $\underline{\tau}_u^n$ and $\overline{\tau}_u^n$ converge to $\underline{\tau}_u$ and $\overline{\tau}_u$ respectively. This proves Case $(b.1)$.

[20] $\underline{\tau}_u^1 \leq \underline{\tau}_u < \overline{\tau}_u \leq \overline{\tau}_u^1$ implies that $T\left(\underline{\tau}_u^1\right) \geq T(\underline{\tau}_u) > T(\overline{\tau}_u) \geq T\left(\overline{\tau}_u^1\right)$, that is:$\underline{\tau}_u^2 \leq \underline{\tau}_u < \overline{\tau}_u \leq \overline{\tau}_u^2$. Iterating the argument shows the inequality for every n.

□

Lemma A.3 $S \subset T^{\infty}(\mathbb{R}_+)$.

Proof of Lemma A.3. Denote $\text{proj}(X)$ the projection of a set $X \subset \mathbb{R}^4$ on the τ_u-axis ($\text{proj}(X) \subset \mathbb{R}$). By definition, $S = \text{proj}(W_\infty)$. Notice the 3 following properties (they can be checked easily):

(i) $\text{proj}(\mathcal{T}(E)) = T(\text{proj}(E))$ for every set E of strategies,

(ii) $\text{proj}(E \cap F) \subset \text{proj}(E) \cap \text{proj}(F)$ for every sets E and F,

(iii) If $\text{proj}(E)$ is included in an interval I of $\mathbb{R}$, then $\text{proj}(\text{conv}(E)) \subset I$.

An optimal information precision lies in $T(\mathbb{R}_+) = [0, Q-\tau]$. This statement writes: $\text{proj}(W_1) \subset T(\mathbb{R}_+)$. Hence, $\text{proj}(\text{conv}(W_1)) \subset T(\mathbb{R}_+)$. It follows that $\text{proj}(\mathcal{T}(\text{conv}(W_1))) = T(\text{proj}(\text{conv}(W_1))) \subset T^2(\mathbb{R}_+)$.

Given $W_2 = \mathcal{T}(\text{conv}(W_1)) \cap W_1$ and the second property above, we have: $\text{proj}(W_2) \subset T^2(\mathbb{R}_+)$. Iterating the argument shows that, for every n, $\text{proj}(W_n) \subset T^n(\mathbb{R}_+)$. Given that the sequence W_n is decreasing, $\text{proj}(W_n)$ is decreasing and converges to $\text{proj}(W_\infty)$. Hence, $\text{proj}(W_\infty) \subset T^n(\mathbb{R}_+)$ for every n. Hence, $\text{proj}(W_\infty) \subset T^{\infty}(\mathbb{R}_+)$. □

In the case $Q - \tau < \frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon}$, $T^{\infty}(\mathbb{R}_+) = \{\tau_u^*\}$. Lemma A.3 above implies then $S = \{\tau_u^*\}$. This proves Point (a) in the Proposition. We now turn attention to Point $(b.1)$ in the Proposition.

Lemma A.4 Assume $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} < Q - \tau < \frac{Q^2}{\alpha^2\tau_\varepsilon}$. The strategies $\left(\underline{\gamma}_0, \underline{\gamma}_1, \underline{\gamma}_2, \underline{\tau}_u\right)$ and $(\overline{\gamma}_0, \overline{\gamma}_1, \overline{\gamma}_2, \overline{\tau}_u)$ are uniquely defined as the solution of the system:

$$\left(\underline{\gamma}_0, \underline{\gamma}_1, \underline{\gamma}_2, \underline{\tau}_u\right) = \mathcal{T}(\overline{\gamma}_0, \overline{\gamma}_1, \overline{\gamma}_2, \overline{\tau}_u), \tag{A.9}$$

$$(\overline{\gamma}_0, \overline{\gamma}_1, \overline{\gamma}_2, \overline{\tau}_u) = \mathcal{T}\left(\underline{\gamma}_0, \underline{\gamma}_1, \underline{\gamma}_2, \underline{\tau}_u\right), \tag{A.10}$$

where $\underline{\tau}_u$ and $\overline{\tau}_u$ are the 2 fixed points of T^2 distinct from τ_u^*. Furthermore, $\overline{\gamma}_0 < \underline{\gamma}_0 < 0$ and $0 < \overline{\gamma}_2 < \underline{\gamma}_2$.

Proof of Lemma A.4. The proof of Lemma A.2 above implies $T = f$ and (together with Lemma 2):

$$\tau + \tau_u(i) + \alpha^2\gamma_1^2\tau_\varepsilon = Q$$

Using this equation, the best response map defined in Lemma 1 writes:

$$\gamma(i)_0 = -\frac{\alpha\tau_\varepsilon}{Q^2}(\beta + \alpha\gamma_0)\tau_u, \tag{A.11}$$

$$\gamma(i)_1 = \frac{\tau_u(i)}{Q}, \tag{A.12}$$

$$\gamma(i)_2 = \frac{\alpha\tau_\varepsilon}{Q^2}(1 + \alpha(1-\gamma_2))\tau_u, \tag{A.13}$$

$$\tau(i)_u = T(\tau_u) = Q - \tau - \frac{\alpha^2\tau_\varepsilon}{Q^2}\tau_u^2. \tag{A.14}$$

It follows from Equations (A.9) and (A.10) that $\underline{\gamma}_0$ and $\overline{\gamma}_0$ are characterized by:

$$\overline{\gamma}_0 = -\frac{\alpha\tau_\varepsilon}{Q^2}(\beta + \alpha\underline{\gamma}_0)\underline{\tau}_u,$$

$$\underline{\gamma}_0 = -\frac{\alpha\tau_\varepsilon}{Q^2}(\beta + \alpha\overline{\gamma}_0)\overline{\tau}_u.$$

This linear system uniquely defines $\underline{\gamma}_0$ and $\overline{\gamma}_0$ (as functions of $\underline{\tau}_u$ and $\overline{\tau}_u$ that are already known). Analogously, a characterization of $\underline{\gamma}_1$, $\overline{\gamma}_1$, $\underline{\gamma}_2$ and $\overline{\gamma}_2$ in terms of $\underline{\tau}_u$ and $\overline{\tau}_u$ follows from Equations (A.9) and

(A.10). Straightforward computations relying on the inequality $\tau_u \leq Q - \tau < \frac{Q^2}{\alpha^2 \tau_\varepsilon}$ show that $\overline{\gamma}_0 < \underline{\gamma}_0 < 0$ and $0 < \overline{\gamma}_2 < \underline{\gamma}_2$. $\square$

We write that a set E of strategies has the best-response property iff, for every z in E, there is z' in E such that $\mathcal{T}(z') = z$. Straightforwardly, any set E with the best-response property is a subset of W_∞.

Lemma A.5 Assume $\frac{3}{4}\frac{Q^2}{\alpha^2 \tau_\varepsilon} < Q - \tau < \frac{Q^2}{\alpha^2 \tau_\varepsilon}$. Consider the set E of strategies $(\gamma_0, \gamma_1, \gamma_2, \tau_u) \in W_0$ satisfying:

$$\begin{aligned} \gamma_0 &\in \left[\overline{\gamma}_0, \underline{\gamma}_0\right], \\ \gamma_1 &= \tau_u / Q, \\ \gamma_2 &\in \left[\overline{\gamma}_2, \underline{\gamma}_2\right], \\ \tau_u &\in [\underline{\tau}_u, \overline{\tau}_u]. \end{aligned}$$

E has the best-response property.

Proof of Lemma A.5. Consider an element $(\gamma_0, \gamma_1, \gamma_2, \tau_u) \in E$. Using Equations (A.11) to (A.14), it is straightforward to show that there is a unique $(\gamma'_0, \gamma'_1, \gamma'_2, \tau'_u)$ such that $(\gamma_0, \gamma_1, \gamma_2, \tau_u) = \mathcal{T}(\gamma'_0, \gamma'_1, \gamma'_2, \tau'_u)$. Furthermore, Equation (A.14) implies that $\tau'_u \in [\underline{\tau}_u, \overline{\tau}_u]$, and Equation (A.11) writes: $\gamma'_0 = h(\gamma_0, \tau'_u)$, where:

$$h(\gamma_0, \tau'_u) = -\frac{Q^2}{\alpha^2 \tau_\varepsilon \tau'_u} \gamma_0 - \frac{\beta}{\alpha}.$$

Thus, $\gamma_0 \in \left[\overline{\gamma}_0, \underline{\gamma}_0\right]$ implies $h\left(\underline{\gamma}_0, \tau'_u\right) < h(\gamma_0, \tau'_u) < h(\overline{\gamma}_0, \tau'_u)$. This implies (recall $\overline{\gamma}_0 < \underline{\gamma}_0 < 0$): $h\left(\underline{\gamma}_0, \overline{\tau}_u\right) < h(\gamma_0, \tau'_u) < h(\overline{\gamma}_0, \underline{\tau}_u)$, that is: $\gamma'_0 \in \left[\overline{\gamma}_0, \underline{\gamma}_0\right]$. Analogous arguments imply: $\gamma'_1 = \tau'_u / Q$ and $\gamma'_2 \in \left[\underline{\gamma}_2, \overline{\gamma}_2\right]$. $\square$

Summing up, we have shown that: if $\frac{3}{4}\frac{Q^2}{\alpha^2 \tau_\varepsilon} < Q - \tau < \frac{Q^2}{\alpha^2 \tau_\varepsilon}$, then

(i) $[\underline{\tau}_u, \overline{\tau}_u] \subset S$ according to Lemma A.5.

(ii) $S \subset T^\infty(\mathbb{R}_+) = [\underline{\tau}_u, \overline{\tau}_u]$ according to Lemmas A.1 and A.3.

Hence, $S = [\underline{\tau}_u, \overline{\tau}_u]$. This proves Point $(b.1)$ in the Proposition.

We now turn attention to the case $\frac{Q^2}{\alpha^2 \tau_\varepsilon} < Q - \tau$. On the one hand, Lemmas A.1 and A.3 show that $S \subset T^\infty(\mathbb{R}_+) = [0, Q - \tau]$. On the other hand, Lemma A.5 can be rewritten in the case $\frac{Q^2}{\alpha^2 \tau_\varepsilon} < Q - \tau$ using $[0, Q - \tau]$ instead of $[\underline{\tau}_u, \overline{\tau}_u]$ (at the cost of some more computations). Hence, $[0, Q - \tau] \subset S$. This proves Point $(b.2)$ in the Proposition. $\square$

Proof of Corollary 1. From Equations (7) and (A.2), we have:

$$\tau_p^* = \frac{\alpha^2 \tau_\varepsilon}{Q^2} \tau_u^{*2}.$$

and the condition $\tau_p^* < \tau_u^*/2$ rewrites:

$$2\frac{\alpha^2 \tau_\varepsilon}{Q^2} \tau_u^* < 1.$$

Equation (A.7) shows then that $\tau_p^* < \tau_u^*/2$ rewrites:

$$\frac{3}{4}\frac{Q^2}{\alpha^2 \tau_\varepsilon} > Q - \tau$$

It follows that if $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} > Q-\tau$ is a necessary and sufficient condition for local stability, it is also a necessary condition for global stability.

To prove the reciprocal implication, recall from the proof of Proposition 4 (see proof of Lemma A.4 in this proof) that, under the assumption $\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} > Q-\tau$, the best response mapping is described by Equations (A.11) to (A.14). Clearly, the mapping described by Equations (A.11) to (A.14) is globally contracting on $\mathcal{T}(W_0)$ (and even on the convex envelope of $\mathcal{T}(W_0)$). It follows then that the REE is a SREE. □

Proof of Proposition 5. As we have already written in Subsection 2.3, τ_p^* increases in τ_ε. It follows that $\tau_p^{\max} = \lim_{\tau_\varepsilon\to+\infty}\tau_p^*$. From Equation (6), we know that $\lim_{\tau_\varepsilon\to+\infty}\tau_u^* = 0$. Equation (7) and $\gamma_1^* = \tau_u^*\sqrt{\frac{2K'(\tau_u^*)}{\psi}}$ (see Proposition 1) implies:

$$\tau_p^* = \alpha^2\tau_\varepsilon\frac{2K'(\tau_u^*)}{\psi}\tau_u^{*2}. \tag{A.15}$$

We can then rewrite Equation (6) as

$$\tau_p^* + \tau + \tau_u^* = \sqrt{\frac{\psi}{2K'(\tau_u^*)}}, \tag{A.16}$$

and conclude that:

$$\tau_p^{\max} = \lim_{\tau_\varepsilon\to+\infty}\tau_p^* = \sqrt{\frac{\psi}{2K'(0)}} - \tau > 0.$$

Equation (A.16) gives:

$$\tau_p^* + \tau_u^* = \sqrt{\frac{\psi}{2K'(\tau_u^*)}} - \tau \leq \tau_p^{\max}, \tag{A.17}$$

Hence, $\tau_p^* > \tau_p^{\max}/2$ implies that $\tau_u^* < \tau_p^{\max}/2 < \tau_p^*$. This violates Condition (C.I) that is a necessary condition for stability. Hence, we have that $\tau_p^* < \tau_p^{\max}/2$ whenever the REE is a LSREE. This implies $\tau_p^{\max LSREE} < \tau_p^{\max}/2$.

In the case of constant marginal costs of information acquisition, a necessary and sufficient condition for existence of a SREE is $\tau_p^* < \frac{1}{2}\tau_u^*$ (see Corollary 1). As Equation (A.16) writes:

$$\tau_p^* + \tau + \tau_u^* = Q,$$

the condition $\tau_p^* < \frac{1}{2}\tau_u^*$ becomes:

$$\tau_p^* < \frac{1}{3}[Q-\tau] = \frac{1}{3}\tau_p^{\max}.$$

□

Proof of Proposition 6. A SREE exists if and only if the inequality (14) holds (see Corollary 1). The polynomial $-\frac{3}{4}\frac{Q^2}{\alpha^2\tau_\varepsilon} + Q - \tau$ of degree 2 in Q has two positive roots $\underline{Q}$ and $\overline{Q}$:

$$\begin{aligned} Q &= \frac{2}{3}\alpha^2\tau_\varepsilon\left(1-\sqrt{1-\frac{3\tau_\varepsilon}{\alpha^2\tau}}\right), \\ Q &= \frac{2}{3}\alpha^2\tau_\varepsilon\left(1+\sqrt{1-\frac{3\tau_\varepsilon}{\alpha^2\tau}}\right). \end{aligned}$$

Hence, a SREE exists iff $\underline{Q} < Q < \overline{Q}$. The two bounds $\underline{K}'$ and $\overline{K}'$ referred to in the Proposition then arise from the definition of $Q = \sqrt{\frac{\psi}{2K'}}$. □

Proof of Proposition 7. Recall from Proposition 2 that a LSREE exists iff Conditions (C.I) and (C.II) holds. Consider first that, given Equation (A.15), the condition (C.I) (that is $\tau_p^* < \tau_u^*$) writes:

$$\alpha^2 \tau_\varepsilon \frac{2K'(\tau_u^*)}{\psi} \tau_u^* < 1. \tag{A.18}$$

The derivative (w.r.t. τ_u^*) of the LHS of the above inequality is:

$$\alpha^2 \tau_\varepsilon \frac{4K''(\tau_u^*)}{\psi} \tau_u^* + \alpha^2 \tau_\varepsilon \frac{4K'(\tau_u^*)}{\psi} > 0. \tag{A.19}$$

It follows that Condition (C.I) holds iff τ_u^* is smaller than a certain threshold. We have already written in Subsection Informativeness of prices that τ_u^* decreases in τ. Given that $\tau_u^* = 0$ for τ large enough (see Proposition 1), the above inequality (A.18) holds true for τ large enough, and there exists $\overline{\tau}_1 \geq 0$ such that Condition (C.I) holds iff $\tau > \overline{\tau}_1$.

We now turn attention to Condition (C.II). Given that $\tau_u^* = 0$ for τ large enough (see Proposition 1), it follows that, for τ large enough, $\tau_p^* = 0$ and Condition (C.II) holds true. With constant elasticity η, Condition (C.II) writes $C > 0$, where:

$$C \equiv \frac{2+\eta}{4+\eta} \tau_u^* + \frac{\eta}{4+\eta} \tau - \tau_p^*.$$

Consider that:

$$\frac{dC}{d\tau} = \frac{2+\eta}{4+\eta} \frac{d\tau_u^*}{d\tau} + \frac{\eta}{4+\eta} - \frac{d\tau_p^*}{d\tau_u^*} \frac{d\tau_u^*}{d\tau}.$$

Differentiating Equation (A.15) and substituting in the above expression gives:

$$\frac{dC}{d\tau} = (\eta+2) \left(\frac{1}{4+\eta} - \frac{\tau_p^*}{\tau_u^*} \right) \frac{d\tau_u^*}{d\tau} + \frac{\eta}{4+\eta}.$$

Differentiating Equation (6) to compute $\frac{d\tau_u^*}{d\tau}$ and substituting again in the above expression gives:

$$\begin{aligned} \frac{dC}{d\tau} &= \frac{-(\eta+2)\frac{1}{4+\eta}\tau_u^* + (\eta+2)\tau_p^*}{\frac{1}{2}\eta\tau + \left(\frac{3}{2}\eta+2\right)\tau_p^* + \left(\frac{1}{2}\eta+1\right)\tau_u^*} + \frac{\eta}{4+\eta}, \\ &= \frac{1}{4+\eta} \frac{\left(\frac{1}{2}\eta^2 - 2\right)\tau_u^* + \left(8\eta + \frac{5}{2}\eta^2 + 8\right)\tau_p^* + \frac{1}{2}\eta^2\tau}{\frac{1}{2}\eta\tau + \left(\frac{3}{2}\eta+2\right)\tau_p^* + \left(\frac{1}{2}\eta+1\right)\tau_u^*}. \end{aligned}$$

Whenever $C = 0$, some computations show that:

$$\frac{dC}{d\tau} > \frac{1}{(4+\eta)^2} \frac{\left(22\eta + 15\eta^2 + 3\eta^3 + 8\right)\tau_u^* + \left(8\eta + \frac{5}{2}\eta^2 + 8\right)\eta\tau + \frac{4+\eta}{2}\eta^2\tau}{\frac{1}{2}\eta\tau + \left(\frac{3}{2}\eta+2\right)\tau_p^* + \left(\frac{1}{2}\eta+1\right)\tau_u^*} > 0.$$

This means that C (as a function of τ) is always increasing around a value τ such that $C(\tau) = 0$. It follows that there is at most one value of τ such that $C = 0$. Given that $C > 0$ for τ large enough, there is at least one such value of τ. Denote $\overline{\tau}_2$ this value. $C > 0$ iff $\tau > \overline{\tau}_2$.

Let $\overline{\tau} = \max(\overline{\tau}_1, \overline{\tau}_2)$. This proves the result.

In the case $K' = 0$, a necessary and sufficient condition for existence of a SREE is Condition (14). This condition is exactly:

$$\tau > Q - \frac{3}{4} \frac{Q^2}{\alpha^2 \tau_\varepsilon},$$

where the RHS does not depend on τ. □