Thema
UMR 8184
THéorie Économique, Modélisation et Applications

Thema Working Paper n°2008-24
University of Cergy Pontoise, France

Conformism, Public News and Market Efficiency

G. Desgranges
C. Rochon

2008

UNIVERSITÉ de Cergy-Pontoise

CNRS CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Conformism, Public News and Market Efficiency

Gabriel Desgranges
THEMA, University of Cergy-Pontoise
33 bd du Port, 95011 Cergy Pontoise, France
phone: 33 1 34 25 61 35
gabriel.desgranges@u-cergy.fr

Céline Rochon*
Saïd Business School, University of Oxford
Park End Street, Oxford, OX1 1HT, UK
phone: 44 1865 288 719
celine.rochon@sbs.ox.ac.uk

*Corresponding author

Abstract

We study the implications of conformism among analysts in a CARA Gaussian model of the market for a risky asset, where a trader's information is a message sent by an analyst. Conformism increases the weight of the public information in the messages, decreasing their informativeness. More precise public information does not always result in more precise messages. A larger precision of the analysts information does not always make the market more liquid and the price more informative. Conformism creates an overreaction of the price to public information. Using the price as a public signal does not alter the results.

Keywords: public information, asymmetric information, conformism, revelation of information by prices, rational expectations

JEL classification: D82, D84, G14

1 Introduction

We develop a model of a market for a risky asset where traders make decisions based on information obtained from financial analysts. These analysts are subject to conformism. We study the consequences of the analysts' behavior on the market equilibrium and show how the liquidity and informational efficiency of the market respond to conformism and to the precision of the various kinds of information available to the analysts.

The model has two types of agents, traders and financial analysts, and it consists of two building blocks: (i) Analysts determining messages (to be sent to traders) based on their public and private information. These messages are Nash equilibrium outcomes of the game played by the analysts; they are indexed by the degree of conformism, which spans the entire spectrum of potential analysts' behavior (from the pure beauty contest to the case where analysts are only concerned by their prediction of the asset return). (ii) a standard competitive financial market with differentially informed traders (a CARA-Gaussian setting à la Grossman (1976), Grossman and Stiglitz (1980)). A trader's private information is endogenous; it is one of the messages sent by analysts.

The key ingredient in the model is the objective of the analysts. This objective embodies two features, namely to predict the asset return and to imitate the prediction of the other analysts. All our results derive from the interplay between these two features. We insist on the fact that these two features are assumed: we do not explain why analysts aim at imitating others (or at correctly predicting the asset return); we are only interested in deriving the consequences of these behavioral assumptions on equilibrium prices.[1]

The rationale for assuming that analysts imitate each other is the follow-

[1]The individual decision can depend endogenously on the aggregate decision, as in Angeletos, Lorenzoni and Pavan (2007). In that paper, conformism issues are absent.

ing: a financial analyst can either "follow the crowd" or distinguish himself from others. In the former case, he will never be considered as different from the others (all analysts will always perform equally). In the latter case, he will end up being either the unique winner, or the unique loser. Being a conformist corresponds to having a preference for the first solution. A possible structural explanation for conformism is risk aversion. This and other explanations are possible but will not be explored further in this paper. Note also that our model is static: agents do not sequentially make their decisions. We therefore do not explicitly model reputational issues, although such issues implicitly provide an informal interpretation of the analysts' objective.

An analyst's information set consists of two signals, one privately observed and another commonly observed by all the analysts (a public signal, for short). This results in a conflict between the two sides of the analyst's objective: the so-called conformism (i.e. imitation as part of the analyst's objective) leads to an increase in the weight of the public signal in the messages sent by analysts to traders and thus deteriorates the informativeness of the messages on the asset return. This comes from the public signal being not only an information about the asset return, but about others' behavior as well (as others use the public signal to make their decisions).

Concerning the second building block of the model, we have deliberately chosen the simplest market mechanism: market clearing is competitive, the trading process is not described, and the market model is linear so that closed-form solutions are available.

Results. Our results can be partitioned in two groups: the influence of conformism on the transmission of information from analysts to traders (conformism makes the transmission of information non monotonic), and the effects on the market outcome of the interplay between conformism and public news (some effects of the dissemination of public news can be usefully counteracted by some degree of conformism).

A preliminary result shows that conformism (unsurprisingly) leads analysts to partly hide their information from traders, and this effect monotonically increases in the degree of conformism. More interestingly, the striking effect of conformism is that the impact of the precisions of analysts' signals on traders' information is most of the time non monotonic. Precisely, the ratio of the precision of traders' information to the precision of every analyst's information is U-shaped in the precisions of both the public and the private signals. It is even the case that, when conformism is strong enough, increasing the precision of the public signal (i.e. the information commonly known among analysts) sometimes decreases the precision of traders' information. These results follow from the existence of different sources of information. Indeed, a change in the precision of a signal leads every analyst to reallocate the weights on the different signals when determining the message sent to traders (to increase the weight of the more precise signal and to decrease the weight of the other signal). A change in the precisions of analysts' signals has therefore an ambiguous impact on the precision of traders' information.

The market outcome is described as the unique Nash equilibrium of the game played by traders making their trading decisions based on the information transmitted by the analysts (a common result in a CARA/Gaussian setting).

In four corollaries, we analyze four properties of the equilibrium (precision of the information revealed by the price, liquidity, price volatility and reaction of the price to public news). We however do not introduce a welfare criterion. Indeed, a proper welfare analysis meets conceptual issues in this kind of models with three types of agents, namely analysts, traders and liquidity traders (the latter are modeled only through the liquidity shock, yet they cannot be ignored from a normative point of view). So we focus on a positive description of the equilibrium (even if as usual in the finance literature it is tempting to assimilate the equilibrium properties with the "ability" of the market to correctly price assets and to link them informally to welfare issues).

The corollaries produce three remarkable insights:

- It is not true that conformism systematically negatively affects the market: volatility is sometimes reduced by a higher degree of conformism. Although counter intuitive at first sight ("following the crowd" should result in a higher volatility), this result rests on a well known property of the CARA/Gaussian setting: price volatility is non monotonic in the precision of traders' information.

- Public information must be carefully introduced: if conformism is strong enough and the public signal is not very informative, then a more precise public information leads to a less informative equilibrium price and a less liquid market.

- The price overreacts to public news when there is a positive degree of conformism. Still, the overreaction of the price is not monotonically increasing in the degree of conformism: Increasing conformism sometimes decreases the sensitivity to public news.

In addition, as one would expect, a higher degree of conformism makes the market less liquid and the price less informative, ceteris paribus. Increasing the precision of the private information of the analysts always favors market price informativeness.

Finally, we use the price as the public signal. A key idea is that the "follow the crowd" behavior relies on the public information, but this public information itself is now endogenous. An increase in the precision of the private information available to every analyst has two effects on the price informativeness: a direct positive effect (every analyst transmits a more precise message to traders that leads to a more informative price) and an indirect negative effect (increasing the price informativeness creates an incentive for every analyst not to use his own private signal, which is bad for price informativeness). We show that the direct positive effect is always dominant: conformism creates no surprising price behavior and the informativeness of the price is monotonically increasing in the precision of the information of an analyst.

Literature. The paper closest in spirit to ours is that of Allen, Morris and Shin (2006). In our paper, as in theirs, the results are driven by an overweighing of the public signal as compared to the private signal. However the overweighing is here motivated by conformism of analysts rather than by the forward-looking structure of the asset pricing model, so that the beauty contest takes place among analysts and not among traders. Furthermore, there is a recent literature dealing with beauty contests, public news and social welfare (Morris and Shin (2002), Hellwig (2005), Angeletos and Pavan (2007) among others) . In these papers, there is sometimes a non-monotonic relationship between social welfare and the public information. In our paper, a non-monotonic relationship involving the public information is shown to exist, but it bears on positive properties of the equilibrium and not on social welfare.

To avoid misunderstandings, note that our model is not concerned by usual questions raised by the theoretical literature on financial analysts and traders (Chamley (2003) describes extensively various models). In particular, we do not investigate such issues as reputational effects and verifiability of the analysts' messages (Trueman (1994), Crawford and Sobel (1982), Prendergast (1993), Ottaviani and Sorensen (2001) among others). In our paper, traders rely completely on the messages of the analysts, as only the analysts have the skills to interpret any exogenous signal, private or public. Moreover, we do not study the market for information or tackle the problem of information acquisition by analysts (Admati and Pfleiderer (1986, 1988, 1990), Barlevy and Veronesi (2000), Verrecchia (1982) among others).

In comparison with the literature on herding, we offer a reduced form model where an exogenous real index is interpreted as the degree of conformism, rather than presenting a structural explanation of conformism. We provide a thorough analysis of the impact of conformism on the market outcome, and we show that the link between conformism and the market outcome is subtle. In particular, imitation by analysts does not always result in increased volatility. This contrasts with a pervasive idea in the literature (Brunnermeier (2001), Chamley (2003), Ottaviani and Sorensen (2000), to name but a few), according to which herding is inefficient and may result in increased volatility. Following the crowd instead of acting on the basis of one's own private information prevents the aggregation of all available information by the market price. In a recent paper, Abreu and Brunnermeier (2003) present a model in which the price goes up even though all agents understand that the price is excessively high. These inefficient equilibria rest on the existence of naive traders that always follow the crowd. Inefficient equilibria and mispricing are also shown to exist in models where traders care about their reputation for ability (Prat and Dasgupta (2006), Prat, Dasgupta and Verardo (2006)).

In comparison with the theoretical literature describing the influence of asymmetric information on the market outcome, we do not offer a microstructure model, but we show that considering two exogenous sources of information and adding one real parameter (the index of conformism) to the celebrated CARA Gaussian setting is enough to generate non monotonic effects of information. The microstructure literature on liquidity[2] offers related results. Chordia, Roll and Subrahmanyam (2001) show that liquidity

[2]The literature on liquidity is vast, and touches upon a number of topics: the investment decisions, the structure of finance (external finance, debt vs equity),.... We consider here

and trading activity increase prior to major macroeconomic announcements. Subrahmanyam (1991) shows that in a world with risk averse privately informed traders and market makers, market liquidity is non monotonic in the number of informed traders, their degree of risk aversion and the precision of their information. Increased liquidity trading leads to reduced price efficiency and under endogenous information acquisition, market liquidity may be non-monotonic in the variance of liquidity trades. The scope for conformism allows us to elaborate upon such results in our paper.

Various empirical studies related to liquidity issues have been conducted. Among others, Koski and Michaely (2000) show that price and liquidity effects are related to informational asymmetries measured by the information environment of the trade and trade size, a result that supports one of our observations: a more precise public information leads to a less informative price and a less liquid market. Barclay and Hendershott (2003) stress the importance of informational asymmetries on market behavior: when they are important, price changes are larger, reflect more private information and are less noisy.

Section 2 presents the model. Section 3 describes the messages sent by analysts to traders. Section 4 determines the market outcome. Section 5 considers a variant where analysts use the asset price to decide which message to send to traders. Section 6 concludes. The proofs are gathered in the Appendix.

---

the literature on price discovery and information revelation (Kyle (1985), Biais and Hillion (1994), Koski and Michaely (2000), Barclay and Hendershott (2003), O'Hara (2003), Van Bommel (2003)).

2 The model

Following the so-called "Rational Expectations" literature on efficient markets initiated by Grossman (1976) and Grossman and Stiglitz (1980), we consider a CARA-Gaussian specification and study the linear rational expectations equilibrium of the market for a risky asset.

The model consists of two building blocks: (i) a game played by financial analysts determining the information that will be transmitted to traders; (ii) the market where traders exchange the risky asset.

We begin with a description of the second part of the model, which is the most standard. There is a continuum $[0,1]$ of infinitesimal traders exchanging the risky asset whose price is p and future value θ is unknown. θ is a normally distributed random variable with mean 0 (for simplicity) and precision τ_θ.

Each trader $i \in [0,1]$ has a demand function $x_i(m_j, p)$ for the asset, which depends on the market price p and the agent's information m_j (to be described below). The traders have constant absolute risk aversion a. Their initial wealth is normalized to 0 (for simplicity). They maximize the expected utility of their final wealth $(\theta - p)x_i(m_j, p)$.

The price p is solution to the market clearing condition:[3]

$$\int x_i di = \varepsilon,$$

with ε an unobserved supply shock, usually interpreted as liquidity trading. It is a normally distributed random variable with mean 0 (for simplicity) and precision τ_ε. As every agent is infinitesimal, no manipulation of the information revealed by the price is possible. The price p reveals some information — it is a noisy signal about θ.

We now turn to the description of the first part of the model.

[3] $\int x_i di$ is the standard notation in a world with a continuum of traders. A rigorous writing will be provided in Section 4.

In contrast with the standard literature, traders' information on θ does not consist of exogenous signals but of endogenously determined messages sent by financial analysts.

There is a continuum $[0, J]$ of financial analysts holding each information in the form of two signals: a signal $s_j = \theta + \eta_j$, and a signal $y = \theta + \eta_y$.

- The signal s_j is privately observed by the analyst j. All signals have the same precision (η_j is a normally distributed variable with mean 0 and precision τ_s), but every analyst observes a different signal.

- The signal y is commonly observed by the analysts (the value of y is common knowledge among the analysts). η_y is a normally distributed variable with mean 0 and precision τ_y. We call y a public signal for short even if it is not observed by traders.

The use of these two signals by analysts and not by traders translate the fact that information takes time to be processed and requires some specific abilities or training. Analysts have this time and the expertise while traders do not. Along the same lines, the private signal of an analyst (with respect to the public signal) summarizes two kinds of information: the privileged information of the analyst (a genuine private information) and his estimation of the impact on the asset return of those information pieces that are commonly observed but whose interpretations differ across analysts. In this view, the so-called public signal y represents the information that is not only commonly observed but also identically interpreted by all analysts, the interpretation being commonly known to be unambiguous.

The random variables (θ, ε, η_y, and all the η_j) are not correlated. Moreover, we assume a strong law of large numbers for the variables η_j. We write this assumption: $\int \eta_j dj = 0$ with probability one. We will make heavy use of this formula throughout the paper.

Building upon his information, analyst j strategically offers a message

m_j on θ to a set of size $1/J$ of traders. Analysts and traders are exogenously matched (we do not model the market for information where traders would buy information from analysts). The information of a trader consists then in exactly one message m_j sent by an analyst.

We offer a very simple way of determining m_j. Every analyst j determines m_j by minimizing the following quadratic objective:

$$(1-\lambda)(m_j-\theta)^2+\lambda(m_j-\overline{m})^2,$$

where $\lambda \in [0,1]$ and $\overline{m} = \frac{1}{J}\int_0^J m_j dj$ is the average message. λ does not depend on j.

The objective of the analyst involves two features: to give a good prediction about θ, or else to conform to the other analysts' predictions and hence not to distinguish oneself too much from the group of analysts. The quadratic function is the most simple function that embodies the two features. We stress that the objective assumes conformism (this is described by the term $(m_j-\overline{m})^2$), and λ denotes the analyst' weight for conformism. The parameter λ varies from 0 to 1, and hence covers the full spectrum of analysts' behavior. A structural justification of the objective is beyond the scope of this paper. We instead focus on the influence of conformism λ on the information transmitted by analysts to the market, and its consequences on the market price.

Summing up, we have a two-stage model. In the first stage, analysts play a game in which the individual strategy is a message. In the second stage, every trader observes a message sent by an analyst, submits a demand function for the risky asset and the market price is determined through market clearing. In the sequel of the paper, we compute the linear Nash equilibrium of this two-stage game. Section 3 considers the first stage; Section 4 considers the second stage. For the sake of expositional simplicity and thanks to the simple nature of the game, we do not offer a formal definition of the equilibrium concept. In fact, sections 3 and 4 compute the equilibrium path of the game. A careful explanation of this is delayed until the end of Section 4.

Remark. We do not tackle the problem of information pricing (price of m_j). We argue that the definition of m_j can be seen, to some extent, as a reduced form model of the market for information (namely the market where the analysts sell information to the traders). As will be made clear in the next section, every m_j is an unbiased noisy signal of θ and all the m_j have the same precision. It follows that the signals m_j could result from the equilibrium of a full model of the market for information satisfying the following properties (we stress that these properties are not too demanding):

- the market for information is competitive, where "competitive" means that there is no bias in the information transmitted from the analysts to the traders (recall that analysts are infinitesimal)

- the equilibrium is symmetric, where "symmetric" means that all the analysts sell a message with the same precision at the same price to the same number of traders.

- it is profitable for every trader to buy some information. Indeed, given that traders are CARA, their trading behavior is NOT affected by the price of the information once traders have decided to buy information. As a consequence, it is not essential to determine the price of information to analyze the market outcome for the risky asset.

3 Analysts' behavior

In this section, we solve the first building block of the model, namely the game played by the analysts. We compute the equilibrium messages and analyze their characteristics as a function of the information of the analysts and conformism.

The optimal message m_j solves the quadratic objective of the analysts and is given by

$$m_j = (1-\lambda)E(\theta|s_j, y) + \lambda E(\bar{m}|s_j, y). \tag{1}$$

The usual formula for the first moment of Gaussian variables gives:[4]

$$E(\theta|s_j, y) = \frac{\tau_s s_j + \tau_y y}{\tau_y + \tau_s + \tau_\theta}.$$

The fact that m is linear in (s_j, y) is self-fulfilling: if an agent j expects every other analyst to send a message that is linear in y and his private signal, then the average message $\overline{m}$ is linear as well, and Equation (1) (together with θ and $\bar{m}$ being Gaussian) implies that the optimal message sent by j is linear in y and s_j. Hence, from now on, we focus on linear solutions only. Existence of nonlinear solutions is beyond the scope of the paper.

We now compute the optimal message sent by an analyst j expecting an average message:

$$\bar{m} = \bar{m}_0 + \bar{\mu} y + \bar{\nu}\theta,$$

where $\bar{m}_0$, $\bar{\mu}$ and $\bar{\nu}$ are real parameters. In this case,

$$E(\bar{m}|s_j, y) = \bar{m}_0 + \bar{\mu} y + \bar{\nu} E(\theta|s_j, y),$$

and Equation (1) implies that

$$\begin{aligned} m_j &= (1-\lambda)E(\theta|s_j, y) + \lambda E(\bar{m}|s_j, y), \\ &= (1-\lambda+\lambda\bar{\nu})\frac{\tau_s s_j + \tau_y y}{\tau_y + \tau_s + \tau_\theta} + \lambda\bar{\mu} y + \lambda \bar{m}_0 \\ &= \lambda \bar{m}_0 + \left(\frac{1-\lambda+\lambda\bar{\nu}}{\tau_y + \tau_s + \tau_\theta}\tau_y + \lambda\bar{\mu}\right) y + \frac{(1-\lambda+\lambda\bar{\nu})\tau_s}{\tau_y + \tau_s + \tau_\theta} s_j. \end{aligned}$$

Consequently we have the following temporary equilibrium map $\mathcal{T}$: if the other analysts "play" an average message function characterized by the three

[4] $E(\theta|V) = E(\theta) + cov(\theta, V) Var(V)^{-1}(V - E(V))$

parameters $(\bar{m}_0, \bar{\mu}, \bar{\nu})$, then the best reply of j is to play a linear message function

$$m_j = (\mathcal{T}\bar{m}_0) + (\mathcal{T}\bar{\mu})\, y + (\mathcal{T}\bar{\nu})\, s_j,$$

characterized by the three real parameters:

$$\begin{aligned}
\mathcal{T}\bar{m}_0 &= \lambda \bar{m}_0, \\
\mathcal{T}\bar{\mu} &= \frac{1-\lambda+\lambda\bar{\nu}}{\tau_y+\tau_s+\tau_\theta}\tau_y + \lambda\bar{\mu}, \\
\mathcal{T}\bar{\nu} &= \frac{(1-\lambda+\lambda\bar{\nu})\,\tau_s}{\tau_y+\tau_s+\tau_\theta}.
\end{aligned}$$

We are now in a position to compute the Nash equilibria of the game played by the analysts. It is straightforward from the definition of $\mathcal{T}$ that every equilibrium is symmetric (when all the agents expect the same average message function, then every agent plays the same message function).[5] Thus, the Nash equilibria coincide with the fixed points of $\mathcal{T}$.

Proposition 3.1. *Assume $\lambda < 1$. There is a unique linear Nash equilibrium of the game played by the analysts. This equilibrium is symmetric, and every analyst plays the following linear message function, i.e. the analyst j observing y and s_j sends the message:*

$$m_j = \mu y + \nu s_j,$$

where

$$\begin{aligned}
\mu &= \frac{\tau_y}{\tau_y + (1-\lambda)\,\tau_s + \tau_\theta}, \\
\nu &= \frac{(1-\lambda)\,\tau_s}{\tau_y + (1-\lambda)\,\tau_s + \tau_\theta}.
\end{aligned}$$

[5] At this point, the assumption that the analysts have the same λ is crucial. Heterogeneous λ_j will increase the computational complexity of the model without adding much insights.

The proof of the proposition is straightforward: the Nash equilibrium coincides with the fixed point of $\mathcal{T}$, defined above. Solving for the fixed points of $\mathcal{T}$ is immediate.

The case $\lambda = 1$ is uninteresting: in this case, the game is degenerate and every symmetric profile of strategies non dependent on the private signal is a Nash equilibrium (i.e. every linear and nonlinear functions of the public signal). However, the limit case of the equilibrium described in the proposition as λ tends to one makes good sense: this limit case is $\nu = 0$ and $\mu = \frac{\tau_y}{\tau_y + \tau_\theta}$. This amounts to saying that all agents perfectly coordinate on the same message and this message is the public signal (this is the pure beauty contest).

Other comparative statics give intuitive results as well: μ decreases with τ_s and increases with τ_y and λ; ν increases with τ_s and decreases with τ_y and λ.

We now investigate the properties of the information revealed by an analyst to a trader and, in particular, the influence of the degree of conformism λ. A first remark is that the law of θ conditional on a message m_j is normal, with mean $E(\theta|s_j, y)$ and precision (that is $1/Var(\theta|m_j)$):[6]

$$\tau_{\theta|m_j} = \tau_\theta + \frac{1}{\left(\frac{1}{1+\nu/\mu}\right)^2 \frac{1}{\tau_y} + \left(\frac{1}{1+\mu/\nu}\right)^2 \frac{1}{\tau_s}}. \tag{2}$$

Hence, an analyst sends an unbiased message to the trader (i.e. $E(\theta|m_j) = E(\theta|s_j, y)$), but as will be made clear below, he does not give all his information to the trader (i.e. $\tau_{\theta|m_j} < \tau_{\theta|s_j,y}$)[7] except in the extreme cases.

The next proposition studies the variations of the precision $\tau_{\theta|m_j}$ with the exogenous parameters λ, τ_y and τ_s. The precision τ_y (resp. τ_s) represents

[6]The formula for the second moment of a Gaussian variable is $Var(\theta|V) = Var(\theta) - cov(\theta, V)\, Var(V)^{-1}\, cov(\theta, V)$.

[7]$\tau_{\theta|s_j,y}$ is the precision of θ conditional on s_j and y. Standard computations show that $\tau_{\theta|s_j,y} = \tau_\theta + \tau_s + \tau_y$.

the precision of the public (resp. private) signal (the conditional precisions are $\tau_{\theta|y} = \tau_\theta + \tau_y$ and $\tau_{\theta|s_j} = \tau_\theta + \tau_s$).

Equation (2) and some simple computations show that $\tau_{\theta|m_j}$ is decreasing in $\frac{\mu}{\nu}$ when the condition $\frac{\mu}{\nu} > \frac{\tau_y}{\tau_s}$ is met and increasing otherwise. The proposition below shows that the effect on $\tau_{\theta|m_j}$ of an increase in the conformism λ, while *a priori* unclear, is always a decrease.

Using again Equation (2), and for given μ and ν, an increase in the precision τ_y or τ_s of the information available to the analysts increases $\tau_{\theta|m_j}$. But, any increase in τ_y or τ_s implies a change in the weights μ and ν. In particular, an increase in τ_y reinforces the "conformist" behavior of analysts: μ increases and ν decreases (the opposite holds for an increase in τ_s). Given that an increase in $\frac{\mu}{\nu}$ is followed either by an increase or a decrease in $\tau_{\theta|m_j}$, the impact on $\tau_{\theta|m_j}$ of an increase in τ_y is *a priori* ambiguous. The next proposition shows that the total resulting effect depends on the initial values of the parameters.

Proposition 3.2. *The precision $\tau_{\theta|m_j}$ increases with the precision τ_s of the private signal and decreases in the conformism λ.*

1. *If $\lambda < 1/2$, i.e. conformism is weak enough, then the precision $\tau_{\theta|m_j}$ increases with the precision τ_y of the public signal.*

2. *If $\lambda > 1/2$, i.e. conformism is strong enough, then the precision $\tau_{\theta|m_j}$ increases with the precision τ_y of the public signal if and only if the public signal is informative enough w.r.t. the private signal, namely:*

$$\frac{\tau_y}{\tau_s} > (2\lambda - 1)(1 - \lambda) > 0. \tag{3}$$

The above proposition shows that the effect of conformism λ on the precision $\tau_{\theta|m_j}$ is twofold: (i) *the conformism itself is detrimental to the information sent to the trader (the higher is λ, the smaller is $\tau_{\theta|m_j}$), and* *(ii) when the conformism is large enough, an increase in the precision of the information available to the analyst sometimes results in a decrease in the precision of the information available to the trader (Point 2 above).*

Implicit in the above proposition is the fact that conformism leads analysts to partially hide their information. Indeed, it is sometimes the case that improving the precision of analysts' information decreases the precision of the messages sent by the analysts. Along these lines, Corollary 3.3 below considers the "proportion" of the information possessed by the analyst that is sent to traders, as measured by the ratio $\tau_{\theta|m_j}/\tau_{\theta|s_j,y}$ of the precision $\tau_{\theta|m_j}$ of the information sent to traders and the precision $\tau_{\theta|s_j,y}$ of the information available to the analyst.

The corollary states that the ratio is smaller than one (Point 1.), and describes the impact of an increase in the precision of the information of the analyst on that ratio (Points 2 and 3). Such an increase always affects positively $\tau_{\theta|s_j,y}$, while it affects $\tau_{\theta|m_j}$ either positively or negatively, as shown in the previous proposition. In any case, the variations of the $\tau_{\theta|m_j}$ are so strong that the ratio is monotonic neither with the precision of the public signal τ_y (Point 2), nor with the precision of the private signal τ_s (Point 3): the ratio is U-shaped both in τ_y and τ_s.

Corollary 3.3.

1. *The ratio $\tau_{\theta|m_j}/\tau_{\theta|s_j,y}$ is smaller than 1, with equality holding if and only if $\lambda = 0$ (there is no conformism), or $\tau_y \in \{0, \infty\}$ or $\tau_s \in \{0, \infty\}$ (there is a unique source of information).*

2. *Let $\tau_y^* = (1-\lambda)\sqrt{(\tau_\theta + \tau_s)\,\tau_s}$. The ratio $\tau_{\theta|m_j}/\tau_{\theta|s_j,y}$ decreases in the precision of the public signal τ_y for τ_y in the interval $\left[0, \tau_y^*\right]$, reaches a minimum at τ_y^* and then increases for $\tau_y \geq \tau_y^*$.*

3. *Let $\tau_s^* = \sqrt{(\tau_y + \tau_\theta)\,\tau_y}/(1-\lambda)$. The ratio $\tau_{\theta|m_j}/\tau_{\theta|s_j,y}$ decreases in*

the precision of the private signal τ_s for τ_s in the interval $[0, \tau_s^*]$, reaches a minimum at τ_s^* and then increases for $\tau_s \geq \tau_s^*$.

At the core of the results, we find that analysts hide information on θ by overweighing y. Analysts overweigh y as the public signal y gives to every analyst a more precise information about the other analysts' information and actions than the private signal s_j does. To give a formal content to this insight, note that: (i) the precision of the analyst's information on θ is $\tau_{\theta|s_j,y}$ and the two signals y and s_j play an identical role for this precision (namely, $\tau_{\theta|s_j,y} = \tau_\theta + \tau_y + \tau_s$), while (ii) the precision of the analyst's information on the average message gives a more substantial role to y: an increase in τ_y is always preferred to an increase in τ_s (namely, one easily computes $\tau_{\overline{m}|s_j,y}$ and checks that $\frac{\partial \tau_{\overline{m}|s_j,y}}{\partial \tau_y} > \frac{\partial \tau_{\overline{m}|s_j,y}}{\partial \tau_s}$).

Conformism λ creates then an incentive to mimic others' messages, i.e. to use y rather than s_j to form one's own message m_j.

The corollary states that the overweighing is maximal for intermediate values of the ratio τ_y/τ_s (Points 2 and 3) and does not exist for extreme values (Point 1). Indeed, in this case, there is a single source of information and thus there is no conflict between the two terms of the objective of the analysts:

- Whenever $\tau_y = 0$, the public signal y is useless: it does not provide any information about the information of the other analysts. Hence there is no overweighing of y.

- Whenever $\tau_y = \infty$, or $\tau_s = 0$, the analysts neglect completely their private information. Even for a non conformist analyst, the weight of y is maximal (so there can be no overweighing of y).

The corollary does not study the variations of the ratio with the conformism λ because (given that $\tau_{\theta|s_j,y}$ does not depend on λ) these variations reflect only those of $\tau_{\theta|m_j}$ that have already been studied in the previous proposition.

A last comparative statics exercise involves the study of the impact on $\tau_{\theta|m_j}$ of the composition of the information of the analyst. We show that, for a given value of $\tau_{\theta|s_j,y}$, the precision $\tau_{\theta|m_j}$ of the information sent to the trader depends on τ_s and τ_y in a way that sustains the intuition drawn from the previous corollary . An analyst sends all his information to the trader when his information is either not correlated (conditionnally to θ) or perfectly correlated with the information of others.

Corollary 3.4. *Let $\tau = \tau_s + \tau_y$. The precision $\tau_{\theta|s_j,y}$ of the information of the analysts is $\tau_\theta + \tau$. Assume that τ is constant (and so is $\tau_{\theta|s_j,y}$). Assume that τ_y increases from 0 to τ (so that $\tau_s = \tau - \tau_y$ decreases from τ to 0). Then $\tau_{\theta|m_j}$ decreases as long as $\tau_y < \left(\frac{1-\lambda}{2-\lambda}\right)\tau$ and then increases. $\tau_{\theta|m_j} = \tau_{\theta|s_j,y}$ if and only if $\tau_y = 0$ or $\tau_y = \tau$.*

4 The market outcome

We now compute the price resulting from the trading strategies played by the traders, based on the information transmitted by the analysts. This is purely routine as this is the rational expectations equilibrium of a variant of the widely used CARA-Gaussian model of the market for a risky asset.

As already stressed in Section 2, every trader has a piece of information consisting of the message sent by an analyst. Traders do not observe the signal y, which is available to the analysts only. Notice that, if traders were to observe y, the role played by analysts would be reduced to nothing: m_j being conditional on s_j and y only, the traders would be able to exactly recover s_j from m_j.

More importantly, neither the analysts nor the traders use the price p to infer information on θ. It may be tempting to include p in the information set of the analysts and/or traders, as it is usual in this trend of the literature. However, doing so would not add much to the understanding of the model and would complicate the computations. In the next section, we identify y to p to study the specific effects of the price as a public signal.

Given that all the stochastic variables are Gaussian, the agent's demand for the risky asset is:

$$x\left(m_j,p\right)=\frac{E\left(\theta|m_j\right)-p}{aVar\left(\theta|m_j\right)}. \tag{4}$$

The demand is linear in (m_j,p). It is determined by the expected joint distribution (θ,m_j).

We first compute individual demand, using the formulas in footnotes 4 and 6.

Lemma 4.1. *When the analysts send the equilibrium message computed in Section 3, individual demand is*

$$x\left(m_j,p\right)=\frac{\tau_\gamma n_j-\left(\tau_\theta+\tau_\gamma\right)p}{a}, \tag{5}$$

where:

$$\begin{aligned}
n_j &= \theta+\gamma_j,\\
\gamma_j &= \frac{\tau_y\eta_y+\left(1-\lambda\right)\tau_s\eta_j}{\tau_y+\left(1-\lambda\right)\tau_s},\\
\tau_\gamma &= \frac{\left(\tau_y+\left(1-\lambda\right)\tau_s\right)^2}{\tau_y+\left(1-\lambda\right)^2\tau_s}.
\end{aligned}$$

The signal n_j is proportional to the message m_j:

$$m_j=\frac{\tau_y+(1-\lambda)\tau_s}{\tau_\theta+\tau_y+(1-\lambda)\tau_s}n_j.$$

We introduce it for computational simplicity only. (Indeed we have $\tau_{\theta|m_j}=\tau_{\theta|n_j}=\tau_\theta+\tau_\gamma$.)

We now compute the aggregate demand. For this purpose, recall that there is a set of measure $1/J$ of traders i observing the message m_j sent by analyst j and having the same demand $x(m_j, p)$. The aggregate demand of traders observing *the same signal* m_j (sent by analyst j) is then:

$$\int_0^{1/J} x(m_j, p)\, di = \frac{1}{J} x(m_j, p).$$

Given that the set of analysts is $[0, J]$, it follows that the aggregate demand is:

$$\int_0^J dj \int_0^{1/J} x(m_j, p)\, di \quad = \quad \int_0^J \frac{1}{J} x(m_j, p)\, dj$$

Given that the individual demand $x(m_j, p)$ is linear in m_j, the aggregate demand reduces to:

$$x\left(\frac{1}{J}\int_0^J m_j dj, p\right) = x(\bar{m}, p).$$

The price is then determined through market clearing:

$$x(\bar{m}, p) = \varepsilon. \tag{6}$$

It is a linear function of $\bar{m}$ and ε. Using Lemma 4.1, the market clearing condition writes:

$$\frac{\tau_\gamma \bar{n} - (\tau_\theta + \tau_\gamma) p}{a} = \varepsilon,$$

where $\bar{n}$ is the average message:

$$\bar{n} = \int n_j dj = \theta + \int \gamma_j dj.$$

Some simple computations show the following proposition:

Proposition 4.2. *There is a unique equilibrium, and the equilibrium price is:*

$$p = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma}\left(\bar{n} - \frac{a}{\tau_\gamma}\varepsilon\right), \tag{7}$$

where

$$\bar{n} \quad = \quad \theta + \frac{\tau_y \eta_y}{\tau_y + (1-\lambda)\,\tau_s}.$$

This equilibrium is the usual linear rational expectations equilibrium extended to our framework with analysts. The result of existence and uniqueness of the equilibrium is therefore unsurprising (given the existence of a unique equilibrium message sent by an analyst). The case with no conformism ($\lambda = 0$) is identical to a model with no analysts and traders directly observing y and s_j. So is the case with no public signal ($\tau_y = 0$), where the analysts simply send their private information to traders.

The four following corollaries make precise the impact of conformism on the market equilibrium. They illustrate the general idea that conformism is most of the time detrimental to the market. They also produce two remarkable insights: First, it is not true that conformism systematically negatively affects the market (volatility is sometimes reduced by conformism); secondly, as information aggregation is imperfect under conformism, public information must be introduced with caution (when the degree of conformism is high enough).

Each corollary investigates a different property of the market equilibrium, namely the informativeness of the price, liquidity (measured as the slope of the price with respect to the supply shock ε), price volatility, and the reaction of the price to public news.

To understand these properties, it is useful to consider again the model as consisting of two building blocks (as described in Section 2): the first building block (the game played by the analysts) produces $\tau_{\theta|m_j}$ (the precision of the traders information received from the analysts), as a function of the three exogenous parameters τ_y, τ_θ and λ. The second building block (the market for the risky asset) produces the equilibrium price as a function of $\tau_{\theta|m_j}$.

The first building block was described in Section 3. The second building block is standard: liquidity and the informativeness of the price increase in $\tau_{\theta|m_j}$; the variance of the equilibrium price is not monotonic in $\tau_{\theta|m_j}$.

The point made in the first three corollaries is that the main features of $\tau_{\theta|m_j}$ (i.e. decreasing in λ and non-monotonic in τ_y, see Proposition 3.2) are reflected in the behavior of the market equilibrium: liquidity and the informativeness of the price are decreasing in λ and non-monotonic in τ_y, and the variance of the market price is sometimes non monotonic in λ.

Corollary 4.3. *In equilibrium, the precision of the information revealed by prices is:*

$$\tau_{\theta|p} = \tau_\theta + \frac{1}{\frac{\tau_y}{(\tau_y+(1-\lambda)\tau_s)^2} + \frac{\left(\tau_y+(1-\lambda)^2\tau_s\right)^2}{(\tau_y+(1-\lambda)\tau_s)^4}\frac{a^2}{\tau_\varepsilon}}. \tag{8}$$

$\tau_{\theta|p}$ *increases in* τ_s*, decreases in* λ*, and the variations of* $\tau_{\theta|p}$ *with* τ_y *satisfy the following properties:*

- *If* $2\frac{a^2}{\tau_\varepsilon}(2\lambda-1) < \tau_s$*, then* $\tau_{\theta|p}$ *increases in* τ_y*.*
- *If* $2\frac{a^2}{\tau_\varepsilon}(2\lambda-1) > \tau_s$*, then* $\tau_{\theta|p}$ *first decreases then increases in* τ_y*. The minimum of* $\tau_{\theta|p}$ *is reached at some threshold* $\hat{\tau}_y$ *satisfying*

$$(2\lambda-1)(1-\lambda)\tau_s < \hat{\tau}_y < (1-\lambda)\tau_s$$

Notice that $2\frac{a^2}{\tau_\varepsilon}(2\lambda-1) > \tau_s$ requires $\lambda > 1/2$. This corollary has the same flavor as Proposition 3.2, stating that $\tau_{\theta|m_j}$ is sometimes not monotonic in τ_y. However, the critical values are not the same in the two results: It is possible to have simultaneously $\tau_{\theta|m_j}$ increasing and $\tau_{\theta|p}$ decreasing. This result is not paradoxical as (i) p depends on the average message and (ii) the messages m_j are correlated. A situation where $\tau_{\theta|m_j}$ is increasing and $\tau_{\theta|p}$

is decreasing is a situation where the correlation among the m_j increases so much that the precision of the average message (and thus $\tau_{\theta|p}$) is decreasing.

The next corollary studies the liquidity of the market. We measure the liquidity of a market by the absolute value of the slope of the price with respect to the supply shock ε. The lower this index, the more liquid is the market. Proposition 4.2 shows that this slope c_ε is $-\frac{a}{\tau_\theta+\tau_\gamma}$.[8]

Corollary 4.4. *$|c_\varepsilon|$ increases in λ.*

1. *If $\lambda < 1/2$, then $|c_\varepsilon|$ decreases with τ_y.*
2. *If $\lambda > 1/2$, then $|c_\varepsilon|$ increases with τ_y for $0 \leq \tau_y \leq \tau_y'$ and decreases for $\tau_y \geq \tau_y'$, where*

$$\tau_y' = (2\lambda - 1)(1 - \lambda)\tau_s. \tag{9}$$

The next corollary studies the variance of the equilibrium price. In a CARA-Gaussian setting, the variance of the price can be non monotonic in the precision of the information of the traders because this variance is the sum of three adverse effects: (i) the decisions of the traders are more sensitive to a more precise information (the precision of traders' information affects positively the variance of the price); (ii) a liquidity effect: traders absorb more the supply shock ε when they have a more precise information so that the price is less sensitive to ε (the precision of traders' information affects negatively the variance of the price); (iii) the precision of the aggregate information of traders (that is: the average message) affects negatively the variance of the price (the price is a linear combination of the average message of traders and the supply shock ε).

These effects are not due to conformism but result from the CARA-Gaussian specification of the second building block of our model (the market

[8] This measure corresponds to that of Kyle (1985): $1/|c_\varepsilon|$ is the well-known market depth.

for the asset). Still, these effects imply that the correlation between the degree of conformism and the variance of the price is more sophisticated than the basic view about "herding"(according to which every agent takes the same action, which makes the price more volatile). The variance of the price as a function of λ is here either increasing, U-shaped, or non-single peaked.

Corollary 4.5. *Let $\hat{\tau} = \frac{2a^2}{\tau_\varepsilon}$. We have:*

1. *If $\hat{\tau} < \tau_\theta + \tau_y$, then the variance of the price is increasing in λ;*
2. *If $\hat{\tau} > \tau_\theta + \tau_y$ and $\hat{\tau} > \tau_s$, then the variance of the price is first decreasing, then increasing in λ;*
3. *If $\hat{\tau} > \tau_\theta + \tau_y$ and $\hat{\tau} < \tau_s$, then two cases are possible. There is a quantity Δ such that*
    - *for $\Delta > 0$, the variance of the price is increasing in λ.*
    - *for $\Delta < 0$, the variance of the price is non-monotonic in λ (it first increases, then decreases and finally increases again).*

The expression of Δ is given in the Appendix by equation (18). It is the discriminant of a polynomial of degree three.

In the next corollary, we study the sensitivity of the market price to public news. Formally, we look at the slope of p with respect to the noise in the public signal η_y. Proposition 4.2 above shows that this slope is:

$$c_y = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma} \frac{\tau_y}{\tau_y + (1 - \lambda)\, \tau_s}.$$

This clearly follows from a rewriting of (7) as:

$$p = c_y y + c_s \int s_j dj - \frac{a}{\tau_\theta + \tau_\gamma} \varepsilon,$$

where $\int s_j dj$ is the average private signal (equal to θ with probability one) and

$$c_s = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma} \frac{(1-\lambda)\,\tau_s}{\tau_y + (1-\lambda)\,\tau_s}.$$

Corollary 4.6.

1. *c_y reaches a minimum at $\lambda = 0$.*

2. *If $\tau_\theta < \tau_y$, then c_y increases in λ.*

3. *If $\tau_y < \tau_\theta$, then c_y first increases, then decreases in λ with a maximum reached at $\hat{\lambda}$ defined by:*

$$\hat{\lambda} = 1 - \frac{1}{\tau_s}\left(\sqrt{\frac{\tau_\theta \tau_y (\tau_s + \tau_y)}{\tau_s + \tau_\theta}} - \tau_y\right).$$

The corollary shows: (i) conformism generates an overreaction to public news (Point 1) (ii) Still, the maximal overreaction does not always correspond to a maximal level of conformism (Point 3). The intuition for these results finds its roots again in the original two building blocks of the model. The sensitivity of the price to the aggregate information of the traders (the average message) is increasing in the precision of this information. This precision is itself decreasing in λ, which implies a negative effect of conformism on the sensitivity of the price to information (public and private). But the weight of the public signal in the traders' information is increasing in the degree of conformism and this generates a positive effect of conformism on the sensitivity of the price to information (public). The corollary follows from the combination of these two effects.

A game theoretical remark

We now describe the game theoretical nature of the solution of the model we have proposed in sections 3 and 4. These sections provide the equilibrium path of a subgame perfect Nash equilibrium in linear strategies and they show that it is the unique equilibrium path of any subgame perfect Nash equilibrium (in linear strategies). We have not fully characterized any equilibrium as we have not defined the actions out of the equilibrium path in the second stage of the model (i.e the traders' demand when analysts send out-of-equilibrium messages).

We could compute the equilibrium path without computing the actions out of the equilibrium path because the determination of the messages of the analysts does not require the knowledge of what traders would do if they received out-of-equilibrium messages. Indeed, computing the equilibrium actions in every subgame of the second stage is generally needed to compute the equilibrium actions in the first stage. But in our simple model, the objectives of the players in the first stage (the analysts) do not depend on the actions in the second stage.

Computing the actions out of the equilibrium path is pure routine (it amounts to computing the rational expectations equilibrium of the market when traders receive information of various precisions). Computing these actions would allow us to verify that there exists a unique subgame perfect Nash equilibrium in linear strategies. It is unnecessary for our purpose (which is to describe what we observe in equilibrium).

5 The informational role of price

We investigate the robustness of the results of the previous sections to a change in the information set of the analysts. We now assume that the analysts use the information revealed by the price to choose the messages m_j they send to the traders. More precisely, we identify the public signal y to the price p.[9]

[9]We are aware that there is here a conceptual difficulty: analysts use a price that is not yet determined. This variant of the model is just a preliminary step towards an intertemporal model where analysts at date t use the price at date $t-1$. It however helps

Formally, Equation (7) states that, whenever the public signal is y, the price is $p = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma}\left(\bar{n} - \frac{a}{\tau_\gamma}\varepsilon\right)$. Hence, p is observationally equivalent[10] to $\left(\bar{n} - \frac{a}{\tau_\gamma}\varepsilon\right)$, that is:

$$\theta + \frac{\tau_y \eta_y}{\tau_y + (1-\lambda)\tau_s} - \frac{a}{\tau_\gamma}\varepsilon.$$

As long as y is exogenous, p is a noisy signal of θ, and there are two sources of noise: the noise η_y on the public signal and ε the stochastic supply of liquidity traders. We now assume that the public signal y is the price: the public signal (and the information revealed by the public signal) is thus now endogenous and the only exogenous source of noise is ε. This amounts to assuming that:

$$y = \theta + \frac{\tau_y \eta_y}{\tau_y + (1-\lambda)\tau_s} - \frac{a}{\tau_\gamma}\varepsilon.$$

Given that $y = \theta + \eta_y$, the above equation rewrites:

$$\eta_y = -\frac{\tau_y + (1-\lambda)\tau_s}{(1-\lambda)\tau_s}\frac{a}{\tau_\gamma}\varepsilon. \tag{10}$$

This equation defines η_y as a function of ε. Substituting Equation (10) into Equation (7) shows that the equilibrium price is

$$p = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma}\left(\theta - \frac{\tau_y + (1-\lambda)\tau_s}{(1-\lambda)\tau_s}\frac{a}{\tau_\gamma}\varepsilon\right). \tag{11}$$

The unique source of noise is now ε. The price is a public signal whose precision is endogenous, namely $\tau_{\theta|p} = \tau_\theta + \tau_y$, where τ_y is now endogenous. From Equation (10), we have:

$$\tau_y = \left(\frac{(1-\lambda)\tau_s}{a}\frac{\tau_y + (1-\lambda)\tau_s}{\tau_y + (1-\lambda)^2\tau_s}\right)^2 \tau_\varepsilon \tag{12}$$

given that $\tau_\gamma = \frac{(\tau_y + (1-\lambda)\tau_s)^2}{\tau_y + (1-\lambda)^2\tau_s}$. This implicitly defines τ_y as a function of the exogenous parameters of the model $(\tau_s, \lambda, a, \tau_\varepsilon)$.

producing some insights.

[10]The distribution of θ conditional on p is the same as the distribution of θ conditional on $\left(\bar{n} - \frac{a}{\tau_\gamma}\varepsilon\right)$.

The next proposition shows that in this case, the model behaves well, and no additional effect due to conformism appears.

Proposition 5.1. *There is a unique equilibrium. In equilibrium,*

1. $\tau_{\theta|p}$ *increases with* τ_s *and decreases with* a.
2. $\tau_{\theta|p}$ *decreases with* λ *(from* $\frac{\tau_\varepsilon \tau_s^2}{a^2}$ *to 0 when* λ *increases from 0 to 1).*
3. *The liquidity deteriorates with* λ *(our liquidity index* $|\frac{\partial p}{\partial \varepsilon}|$ *computed in (11) increases in* λ*).*

6 Conclusion

We have studied the implications of conformism on the price in a CARA Gaussian model of the market for a risky asset where traders are differentially informed. Conformism leads to an increase in the weight of the public signal and thus deteriorates the informativeness of the message about the asset return. Our main results are: (i) a more precise public information does not always imply that traders receive a more precise information from analysts; (ii) improving the precision of the exogenous information does not always imply that the market is more liquid and the price is more informative; (iii) conformism creates an overreaction of the price to public information.

We have been silent about the derivation of the analysts objective and also about the market for information. Future research will bear on these issues.

7 Appendix

Proof of Proposition 3.2

Given the expressions of μ and ν found in Proposition 3.1, Equation (2) rewrites:

$$\tau_{\theta|m_j} = \tau_\theta + \frac{1}{\left(\frac{\tau_y}{\tau_y+(1-\lambda)\tau_s}\right)^2 \frac{1}{\tau_y} + \left(\frac{(1-\lambda)\tau_s}{\tau_y+(1-\lambda)\tau_s}\right)^2 \frac{1}{\tau_s}},$$

implying:

$$\tau_{\theta|m_j} = \tau_\theta + \frac{\left(\tau_y + (1-\lambda)\,\tau_s\right)^2}{\tau_y + (1-\lambda)^2\,\tau_s}. \tag{13}$$

It is easy to check that:

$$\frac{d\tau_{\theta|m_j}}{d\tau_y} = \frac{\left(\tau_y + (1-\lambda)\,\tau_s\right)\left(\tau_y - (1-\lambda)\,\tau_s + 2\tau_s\,(1-\lambda)^2\right)}{\left(\tau_y + (1-\lambda)^2\,\tau_s\right)^2}.$$

Then, $\frac{d\tau_{\theta|m_j}}{d\tau_y} > 0$ if and only if Condition (3) holds. Furthermore,

$$\begin{aligned}
\frac{d\tau_{\theta|m_j}}{d\tau_s} &= \left(\tau_y + (1-\lambda)\,\tau_s\right)(1-\lambda)\,\frac{\tau_y + (1-\lambda)^2\,\tau_s + \tau_y\lambda}{\left(\tau_y + (1-\lambda)^2\,\tau_s\right)^2} > 0,\\
\frac{d\tau_{\theta|m_j}}{d\,(1-\lambda)} &= 2\frac{\lambda\left(\tau_y + (1-\lambda)\,\tau_s\right)\tau_s\tau_y}{\left(\tau_y + (1-\lambda)^2\,\tau_s\right)^2} > 0.
\end{aligned}$$

The proposition follows immediately. This ends the proof. □

Proof of Corollary 3.3

Given Equation (13), one has:

$$\frac{\tau_{\theta|m_j}}{\tau_{\theta|s_j,y}} = \frac{\tau_\theta + \frac{(\tau_y+(1-\lambda)\tau_s)^2}{\tau_y+(1-\lambda)^2\tau_s}}{\tau_\theta + \tau_s + \tau_y}. \tag{14}$$

Simple computations show Point 1. The derivatives of the ratio can be computed. They are:

$$\begin{aligned}
\frac{d}{d\tau_y}\left(\frac{\tau_{\theta|m_j}}{\tau_{\theta|s_j,y}}\right) &= \tau_s\lambda^2\frac{\tau_y^2 - (\tau_\theta+\tau_s)\,\tau_s\,(1-\lambda)^2}{\left(\tau_y + (1-\lambda)^2\,\tau_s\right)^2(\tau_\theta+\tau_s+\tau_y)^2},\\
\frac{d}{d\tau_s}\left(\frac{\tau_{\theta|m_j}}{\tau_{\theta|s_j,y}}\right) &= \tau_y\lambda^2\frac{\tau_s^2\,(1-\lambda)^2 - (\tau_y+\tau_\theta)\,\tau_y}{\left(\tau_y + (1-\lambda)^2\,\tau_s\right)^2(\tau_\theta+\tau_s+\tau_y)^2}.
\end{aligned}$$

Points 2 and 3 follow immediately. This ends the proof. $\square$

Proof of Corollary 3.4

Rewrite Equation (13) as:

$$\tau_{\theta|m_j} = \tau_\theta + \frac{\left(\tau_y + (1-\lambda)(\tau - \tau_y)\right)^2}{\tau_y + (1-\lambda)^2 (\tau - \tau_y)}. \tag{15}$$

The variations of $\tau_{\theta|m_j}$ with τ_y (given $\tau_s = \tau - \tau_y$) are given by the partial derivative of the above expression (15) w.r.t. τ_y:

$$\left.\frac{d\tau_{\theta|m_j}}{d\tau_y}\right|_{\tau_s=\tau-\tau_y} = \left((1-\lambda)\tau + \tau_y\lambda\right)\lambda^2 \frac{(2-\lambda)\tau_y - (1-\lambda)\tau}{\left(\tau_y + (1-\lambda)^2(\tau - \tau_y)\right)^2}.$$

This expression is positive if and only if:

$$\tau_y > \left(\frac{1-\lambda}{2-\lambda}\right)\tau.$$

Notice that $\left(\frac{1-\lambda}{2-\lambda}\right) < 1$ so that this condition is compatible with τ_y belonging to $[0, \tau]$. This ends the proof. $\square$

Proof of Corollary 4.3

Equation (7) shows that p is observationnally equivalent to $\theta + \varsigma$, where

$$\varsigma = \frac{\tau_y \eta_y}{\tau_y + (1-\lambda)\tau_s} - \frac{a}{\tau_\gamma}\varepsilon. \tag{16}$$

Hence, $\tau_{\theta|p} = \tau_\theta + \tau_\zeta$. Given that $cov(\eta_y, \varepsilon) = 0$, we have

$$\tau_\zeta = \frac{1}{\frac{\tau_y}{(\tau_y + (1-\lambda)\tau_s)^2} + \frac{a^2}{\tau_\gamma^2 \tau_\varepsilon}}.$$

This proves Equation (8).

Some computations show that:

$$\begin{aligned}
\frac{\partial \tau_\gamma}{\partial \tau_s} &= (1-\lambda)\left((1+\lambda)\tau_y + (1-\lambda)^2\tau_s\right)\frac{(\tau_y + (1-\lambda)\tau_s)}{\left(\tau_y + (1-\lambda)^2\tau_s\right)^2} > 0, \\
\frac{\partial \tau_\gamma}{\partial (1-\lambda)} &= -2\lambda\tau_s\tau_y \frac{\tau_y + (1-\lambda)\tau_s}{\left(\tau_y + (1-\lambda)^2\tau_s\right)^2} < 0.
\end{aligned}$$

Given that $\frac{\tau_y}{(\tau_y+(1-\lambda)\tau_s)^2}$ is decreasing in τ_s and increasing in λ, this shows that τ_ζ (and $\tau_{\theta|p}$) is increasing in τ_s and decreasing in λ.

We next study the sign of $\frac{\partial \tau_{\theta|p}}{\partial \tau_y}$ (that is $\frac{\partial \tau_\zeta}{\partial \tau_y}$). To this end, we focus on:

$$Q(\tau_y) = \frac{\tau_y}{(\tau_y + (1-\lambda)\tau_s)^2} + u\left(\frac{\tau_y + (1-\lambda)^2 \tau_s}{(\tau_y + (1-\lambda)\tau_s)^2}\right)^2,$$

where $u = \frac{a^2}{\tau_\varepsilon}$. $\frac{\partial \tau_{\theta|p}}{\partial \tau_y} > 0$ if and only if $Q' < 0$. We have:

$$\begin{aligned}
\frac{\partial}{\partial \tau_y}\left(\frac{\tau_y}{(\tau_y + (1-\lambda)\tau_s)^2}\right) &= -\frac{\tau_y - (1-\lambda)\tau_s}{(\tau_y + (1-\lambda)\tau_s)^3}, \\
\frac{\partial}{\partial \tau_y}\left(\frac{\tau_y + (1-\lambda)^2\tau_s}{(\tau_y + (1-\lambda)\tau_s)^2}\right) &= -\frac{\tau_y + (1-2\lambda)(1-\lambda)\tau_s}{(\tau_y + (1-\lambda)\tau_s)^3}.
\end{aligned}$$

Hence, $Q'(\tau_y) < 0$ writes $H(\tau_y) > 0$, where:

$$H(\tau_y) = \tau_y - (1-\lambda)\tau_s + 2u(\tau_y + (1-2\lambda)(1-\lambda)\tau_s)\frac{\tau_y + (1-\lambda)^2\tau_s}{(\tau_y + (1-\lambda)\tau_s)^2}.$$

We first consider the derivative of H:

$$H'(\tau_y) = 1 + 2u\left(\frac{\tau_y + (1-\lambda)^2\tau_s}{(\tau_y + (1-\lambda)\tau_s)^2} - \frac{(\tau_y + (1-2\lambda)(1-\lambda)\tau_s)^2}{(\tau_y + (1-\lambda)\tau_s)^3}\right),$$

that rewrites:

$$H'(\tau_y) = 1 + 2u\frac{(3-4\lambda)(1-\lambda)\tau_s + 3\tau_y}{(\tau_y + (1-\lambda)\tau_s)^3}(1-\lambda)\lambda\tau_s, \tag{17}$$

Equation (17) implies that $(3-4\lambda)(1-\lambda)\tau_s + 3\tau_y > 0$ is a sufficient condition for $H'(\tau_y) > 0$. It follows that $H'(\tau_y) > 0$ when $\lambda < 1/2$ or $\tau_y > (2\lambda - 1)(1-\lambda)\tau_s$.

To study the sign of $H(\tau_y)$, we notice that $\lim_{\tau_y \to +\infty} H(\tau_y) = +\infty$, and we distinguish between two cases:

- If $\lambda < 1/2$, then $H'(\tau_y) > 0$ for every $\tau_y > 0$. Assume that:

  $$H(0) = (1-\lambda)(2u(1-2\lambda) - \tau_s).$$

  If $2u(1-2\lambda) > \tau_s$, then $H(0) > 0$ and $H(\tau_y) > 0$ for every $\tau_y > 0$. If $2u(1-2\lambda) < \tau_s$, then $H(0) < 0$ and there is exactly one solution $\hat{\tau}_y$ of $H(\tau_y) = 0$. Moreover, $(2\lambda - 1)(1-\lambda)\tau_s < 0 < \hat{\tau}_y$.

- If $\lambda > 1/2$, then $H'(\tau_y) > 0$ for every $\tau_y > (2\lambda - 1)(1-\lambda)\tau_s$ and (given that $(2\lambda - 1)(1-\lambda)\tau_s > 0$):

$$H((2\lambda-1)(1-\lambda)\tau_s) = -2(1-\lambda)^2\tau_s < 0.$$

  It follows that there is exactly one solution $\hat{\tau}_y$ of $H(\tau_y) = 0$ satisfying $\hat{\tau}_y > (2\lambda - 1)(1-\lambda)\tau_s$. To show that there is no other solution of $H(\tau_y) = 0$, we compute:

$$H''(\tau_y) = 12u(1-\lambda)\lambda\tau_s \frac{(2\lambda-1)(1-\lambda)\tau_s - \tau_y}{(\tau_y + (1-\lambda)\tau_s)^4},$$

  which implies that $H''(\tau_y) > 0$ when $\tau_y < (2\lambda - 1)(1-\lambda)\tau_s$. The convexity of H on $[0, (2\lambda-1)(1-\lambda)\tau_s]$ (together with $H(0) < 0$ and $H((2\lambda-1)(1-\lambda)\tau_s) < 0$) implies that $H(\tau_y) < 0$ on $[0, (2\lambda-1)(1-\lambda)\tau_s]$.

Finally,

$$H((1-\lambda)\tau_s) = 4u(1-\lambda)^2\tau_s \frac{\tau_y + (1-\lambda)^2\tau_s}{(\tau_y + (1-\lambda)\tau_s)^2} > 0,$$

which implies that $\hat{\tau}_y < (1-\lambda)\tau_s$. This ends the proof. □

Proof of Corollary 4.4

As already written, $\tau_\gamma = \tau_{\theta|m_j} - \tau_\theta$. It follows that the liquidity index $|c_\varepsilon|$ is decreasing in $\tau_{\theta|m_j}$. The corollary is then a straightforward consequence of Proposition 3.2. This ends the proof. □

Proof of Corollary 4.5

It follows from Proposition 4.2 that

$$p = \frac{\tau_\gamma}{\tau_\theta + \tau_\gamma}\left(\theta + \frac{\tau_y \eta_y}{\tau_y + (1-\lambda)\tau_s} - \frac{a}{\tau_\gamma}\varepsilon\right).$$

The variance of p writes:

$$\begin{aligned} Varp &= \left(\frac{\tau_\gamma}{\tau_\theta + \tau_\gamma}\right)^2 \left(\frac{1}{\tau_\theta} + \frac{\tau_y}{(\tau_y + (1-\lambda)\tau_s)^2} + \frac{a^2}{(\tau_\gamma)^2 \tau_\varepsilon}\right), \\ &= A + \tau_s k B, \end{aligned}$$

with $k=\frac{a^2}{\tau_s\tau_\varepsilon}$ and:

$$\begin{aligned}
A &= \left(\frac{\tau_\gamma}{\tau_\theta+\tau_\gamma}\right)^2\left(\frac{1}{\tau_\theta}+\frac{\tau_y}{(\tau_y+(1-\lambda)\tau_s)^2}\right),\\
B &= \frac{1}{(\tau_\theta+\tau_\gamma)^2}.
\end{aligned}$$

Let $l=(1-\lambda)$, $u=\tau_y/\tau_s$ and $t=\tau_\theta/\tau_s$. We have:

$$\begin{aligned}
A &= \frac{(u+l)^2}{\left(t\left(u+l^2\right)+(u+l)^2\right)^2}\left(\frac{(u+l)^2+tu}{t\tau_s}\right),\\
B &= \frac{\left(u+l^2\right)^2}{\tau_s^2\left(t\left(u+l^2\right)+(u+l)^2\right)^2}.
\end{aligned}$$

Some computations show that the derivative of the variance w.r.t. l is:

$$\frac{dVarp}{dl}=\frac{2u\,(u+l)}{\tau_s\left(t\left(u+l^2\right)+(u+l)^2\right)^3}H(l),$$

with

$$H(l)=2(k-1)\,l^3+(1-t-4u-2k)\,l^2+2u\,(1-t-u+k)\,l+u\,(u+t-2k).$$

The derivative $\frac{dVarp}{dl}$ signs as $H(l)$, and we have: $\frac{dVarp}{d\lambda}>0$ if and only if $H(l)<0$. We now study the sign of $H(l)$ when l varies between 0 and 1. For this purpose, notice that H is a polynomial of degree 3 whose coefficient of highest degree is $a_3=2(k-1)$. We have:

$$\begin{aligned}
H(0) &= u\,(u+t-2k),\\
H(1) &= -t-2u-tu-u^2-1<0,
\end{aligned}$$

and

$$\begin{aligned}
H'(0) &= 2u\,(1-u-t+k),\\
H'(1) &= 2\,(u+1)\,(k-2-u-t).
\end{aligned}$$

We distinguish between 6 different cases.

Case 1. $k > 1$ and $t + u > 2k$:

In this case, $a_3 > 0$, $H(0) > 0, H'(0) < 0$, $H'(1) < 0$. It follows that there is a unique root r between 0 and 1, and we have: $H(l) > 0$ if and only if $l < r$.

Case 2. $k > 1$ and $2k > t + u > k + 1$:

In this case, $a_3 > 0$, $H(0) < 0, H'(0) < 0$, $H'(1) < 0$. It follows that $H(l) < 0$.

Case 3. $k > 1$ and $k + 1 > t + u$:

In this case, $a_3 > 0$, $H(0) < 0$. We introduce the polynomial

$$V(l) = \frac{1}{2} l H'(l) - H(l).$$

We have

$$\begin{aligned} V'(l) &= 3(k-1)l^2 + u(t + u - k - 1), \\ V''(l) &= 6(k-1)l. \end{aligned}$$

The inflection point of V (the zero of V'') is 0, $V(0) > 0$ and $V'(0) < 0$. It follows that for $l > 0$, V is a convex function of l and it is above the tangent at 0. The equation of this tangent is $y = V'(0)l + V(0)$ and its value at $l = 1$ is $V(0) + V'(0) = u(k-1) > 0$. Hence, this tangent takes positive values for $0 < l < 1$. This implies that $V(l) > 0$ for $0 < l < 1$, and $H'(r) = 2V(r)/r > 0$ for any root r of H between 0 and 1. Given that $H(0) < 0$ and $H(1) < 0$, this implies in turn that there can be no root of H between 0 and 1, and $H(l) < 0$ for $0 < l < 1$.

Case 4. $k < 1$ and $t + u > k + 1$:

In this case, $a_3 < 0$, $H(0) > 0$.

The argument is similar to that of Case 3. $V(0) < 0$ and $V'(0) > 0$. It follows that for $l > 0$, V is a concave function of l and it is below the tangent at 0. This tangent takes negative values for $0 < l < 1$ (as

$V(0) + V'(0) = u(k-1) < 0$). This implies that $V(l) < 0$ for $0 < l < 1$, and $H'(r) = 2V(r)/r < 0$ for any root r of H between 0 and 1. Given that $H(0) > 0$ and $H(1) < 0$, this implies in turn that there is a unique root r of H between 0 and 1, and we have: $H(l) > 0$ if and only if $l < r$.

Case 5. $k < 1$ and $k + 1 > t + u > 2k$:

In this case, $a_3 < 0$, $H(0) > 0, H'(0) > 0$, $H'(1) < 0$.

It follows that there is a unique root r between 0 and 1, and we have: $H(l) > 0$ if and only if $l < r$.

Case 6. $k < 1$ and $2k > t + u$:

In this case, $a_3 < 0$, $H(0) < 0, H'(0) > 0$, $H'(1) < 0$.

H has either zero or two roots between 0 and 1. To determine the number of roots, one studies the discriminant Δ of H.

- If Δ is positive, then H has only one real root. In the present case, the root is negative and $H(l) < 0$ for $0 < l < 1$.

- If Δ is negative, then H has three real roots. In the present case, one root is negative and two roots, r_1, r_2, are positive and between 0 and 1. $H(l) > 0$ for $r_1 < l < r_2$ and $H(l) < 0$ otherwise.

The discriminant of a polynomial of degree three has the following general form:

$$\Delta = 4a_1^3 a_3 - a_1^2 a_2^2 + 4a_0 a_2^3 - 18a_0 a_1 a_2 a_3 + 27a_0^2 a_3^2$$

where a_n is the coefficient of degree n. Some further computations show that $\frac{\Delta}{4u(u+1)}$ is a polynomial of degree four in k given by

$$t\left(6u-3t+15tu+3t^2-t^3+12u^2+8u^3+15tu^2+6t^2u+1\right) \tag{18}$$
$$-2(7u+33tu-3t^2+2t^3+18u^2+20u^3+8u^4+51tu^2+33t^2u+24tu^3+8t^3u$$
$$+24t^2u^2+1)k$$
$$+3\left(29u-4t+44tu+41u^2+16u^3+32tu^2+16t^2u+4\right)k^2$$
$$-8\left(9u-2t+6tu+6u^2+3\right)k^3+16\left(1+u\right)^2k^4$$

This ends the proof. □

Proof of Corollary 4.6

Given the definition of τ_γ, we have:

$$\frac{\tau_\gamma}{\tau_\theta+\tau_\gamma}=\frac{\left(\tau_y+\left(1-\lambda\right)\tau_s\right)^2}{\tau_\theta\left(\tau_y+\left(1-\lambda\right)^2\tau_s\right)+\left(\tau_y+\left(1-\lambda\right)\tau_s\right)^2}.$$

Hence, the definition of c_y implies:

$$\frac{1}{\tau_y c_y}=\tau_\theta\frac{\tau_y+\left(1-\lambda\right)^2\tau_s}{\tau_y+\left(1-\lambda\right)\tau_s}+\tau_y+\left(1-\lambda\right)\tau_s.$$

The derivative w.r.t. $l=(1-\lambda)$ of the RHS of the above equation is:

$$\frac{d}{dl}\left(\tau_\theta\frac{\tau_y+l^2\tau_s}{\tau_y+l\tau_s}+\tau_y+l\tau_s\right)=\tau_s\tau_\theta\frac{2l\tau_y+l^2\tau_s-\tau_y}{\left(\tau_y+l\tau_s\right)^2}+\tau_s.$$

This derivative is positive if and only if:

$$\tau_\theta\left(2l\tau_y+l^2\tau_s-\tau_y\right)+\left(\tau_y+l\tau_s\right)^2>0.$$

This is a polynomial of degree 2 in l. One easily checks that its roots are real, one is negative and the other is:

$$\hat{l}=\frac{1}{\tau_s}\left(\sqrt{\frac{\tau_\theta\tau_y\left(\tau_s+\tau_y\right)}{\tau_s+\tau_\theta}}-\tau_y\right)<1.$$

Furthermore, one has: $\hat{l}>0$ if and only if $\tau_\theta>\tau_y$. One defines: $\hat{\lambda}=1-\hat{l}$. Usual considerations on polynomials shows Points 2 and 3.

Finally:

$$\begin{aligned}
c_y &= \frac{1}{\tau_y\left(\tau_\theta+\tau_y+\tau_s\right)} \text{ if } \lambda=0, \\
c_y &= \frac{1}{\tau_y\left(\tau_\theta+\tau_y\right)} \text{ if } \lambda=1.
\end{aligned}$$

Using Points 2 and 3 shows Point 1. This ends the proof. □

Proof of Proposition 5.1

Equation (12) rewrites $G\left(\tau_y\right)=0$ where G is a polynomial of degree 3 in τ_y:

$$G\left(\tau_y\right)=\frac{a^2}{\tau_\varepsilon}\tau_y\left(\tau_y+(1-\lambda)^2\tau_s\right)^2-\left((1-\lambda)\tau_s\left(\tau_y+(1-\lambda)\tau_s\right)\right)^2.$$

We show that G has a unique positive real root. For simplicity, we write $u=\frac{a^2}{\tau_\varepsilon}$ and $l=(1-\lambda)$. We have:

$$G\left(\tau_y\right)=u\tau_y^3+\left(2u-\tau_s\right)l^2\tau_s\tau_y^2+\left(ul-2\tau_s\right)l^3\tau_s^2\tau_y-l^4\tau_s^4,$$

and

$$G'\left(\tau_y\right)=3u\tau_y^2+2\left(2u-\tau_s\right)l^2\tau_s\tau_y+\left(ul-2\tau_s\right)l^3\tau_s^2.$$

The discriminant of G' is

$$l^2\tau_s^2\left(l^2\tau_s^2+2ul^2\tau_s+u^2l^2+6lu\tau_s\left(1-l\right)\right)>0,$$

implying that G' has 2 real roots. If $u>\frac{2\tau_s}{l}$, then these 2 roots are negative (the product of the roots is positive, their sum is negative). Otherwise $u<\frac{2\tau_s}{l}$ and the 2 roots are of opposite signs (the product of the roots is negative).

These properties of the roots of G', together with the fact that $G(0)<0$ and the coefficient of degree 3 of G is positive, imply that G has a unique positive real root τ_y^*. This shows that there is a unique equilibrium. Furthermore, $G'\left(\tau_y^*\right)>0$.

We now show that $\frac{d\tau_y^*}{d\tau_s} > 0$. The implicit function theorem gives:

$$\frac{d\tau_y^*}{d\tau_s} = -\frac{G'_{\tau_s}\left(\tau_y^*\right)}{G'_{\tau_y}\left(\tau_y^*\right)}.$$

Hence $\frac{d\tau_y^*}{d\tau_s}$ signs as $-G'_{\tau_s}\left(\tau_y^*\right)$. Simple computations show that:

$$G'_{\tau_s}\left(\tau_y\right) = 2l^2\left(\left(u-\tau_s\right)\tau_y^2 + \left(ul - 3\tau_s\right)l\tau_s\tau_y - 2l^2\tau_s^3\right).$$

Given that:

$$G\left(\tau_y\right) = u\tau_y^3 + ul^2\tau_s\tau_y^2 + \left(u-\tau_s\right)l^2\tau_s\tau_y^2 + \left(ul-3\tau_s\right)l^3\tau_s^2\tau_y + \tau_s l^3\tau_s^2\tau_y - l^4\tau_s^4,$$

we have:

$$-l^2\tau_s\left(\left(u-\tau_s\right)\left(\tau_y^*\right)^2 + \left(ul-3\tau_s\right)l\tau_s\tau_y^* - 2l^2\tau_s^3\right) = u\left(\tau_y^*\right)^3 + ul^2\tau_s\left(\tau_y^*\right)^2 + \tau_s l^3\tau_s^2\tau_y^* + l^4\tau_s^4.$$

This implies

$$-\frac{\tau_s}{2}G'_{\tau_s}\left(\tau_y^*\right) = u\left(\tau_y^*\right)^3 + ul^2\tau_s\left(\tau_y^*\right)^2 + \tau_s l^3\tau_s^2\tau_y^* + l^4\tau_s^4.$$

and $G'_{\tau_s}\left(\tau_y^*\right) < 0$. Finally, $\frac{d\tau_y^*}{d\tau_s} > 0$.

We now show that $\frac{d\tau_y^*}{d\lambda} < 0$. The implicit function theorem gives:

$$\frac{d\tau_y^*}{d\lambda} = -\frac{d\tau_y^*}{dl} = \frac{G'_l\left(\tau_y^*\right)}{G'_{\tau_y}\left(\tau_y^*\right)}.$$

Hence $\frac{d\tau_y^*}{d\lambda}$ signs as $G'_l\left(\tau_y^*\right)$. Simple computations give:

$$\frac{G'_l\left(\tau_y\right)}{2l\tau_s} = \left(2u-\tau_s\right)\tau_y^2 + \left(2ul-3\tau_s\right)l\tau_s\tau_y - 2l^2\tau_s^3. \tag{19}$$

Furthermore, $G\left(\tau_y^*\right) = 0$ implies:

$$u = \frac{1}{\tau_y^*}\frac{2l^3\tau_s^3\tau_y^* + l^4\tau_s^4 + l^2\tau_s^2\left(\tau_y^*\right)^2}{\left(\tau_y^*\right)^2 + 2l^2\tau_s\left(\tau_y^*\right) + l^4\tau_s^2}.$$

Substituting u by this expression in Equation (19) gives, after some computations:

$$\frac{G'_l\left(\tau_y^*\right)}{2l\tau_s} = \tau_s\left(2\frac{\tau_s l^2\left(\tau_s l + \tau_y^*\right)}{l^2\tau_s + \tau_y^*} - \left(2\tau_s l + \tau_y^*\right)\right)\left(l\tau_s + \tau_y^*\right).$$

Hence, we have:

$$G_l'\left(\tau_y^*\right) = -2l\tau_s^2\tau_y^*\frac{\tau_s l\left(2-l\right)+\tau_y^*}{l^2\tau_s+\tau_y^*}\left(l\tau_s+\tau_y^*\right) < 0.$$

This shows that $\frac{d\tau_y^*}{d\lambda} < 0$.

Finally, $G_u'\left(\tau_y\right) > 0$ implies (by the implicit function theorem) that $\frac{d\tau_y^*}{du} < 0$. This in turn implies $\frac{d\tau_y^*}{da} < 0$ and $\frac{d\tau_y^*}{d\tau_\varepsilon} > 0$. When $l = 0$, $G\left(\tau_y\right) = u\tau_y^3$ and $\tau_y^* = 0$. When $l = 1$,

$$\begin{aligned} G\left(\tau_y\right) &= u\tau_y^3 + \left(2u-\tau_s\right)\tau_s\tau_y^2 + \left(u-2\tau_s\right)\tau_s^2\tau_y - \tau_s^4, \\ &= \left(\tau_y+\tau_s\right)^2\left(u\tau_y-\tau_s^2\right), \end{aligned}$$

and the positive root is $\frac{\tau_s^2}{u}$. This ends the proof. □

References

[1] Abreu D, M. Brunnermeier (2003), Bubbles and crashes. Econometrica 71: 173-204.

[2] Admati A, P. Pfleiderer (1986), A monopolistic market for information. Journal of Economic Theory 39: 400-438.

[3] Admati A, P. Pfleiderer (1988), Selling and trading on information in financial markets. American Economic Review 78: 96-103.

[4] Admati A, P. Pfleiderer (1990), Direct and indirect sale of information. Econometrica 58: 901-928.

[5] Allen F, S. Morris, H. Shin (2006), Beauty contests and iterated expectations in asset markets. The Review of Financial Studies 19: 719-752.

[6] Angeletos, G-M, G. Lorenzoni, A. Pavan (2007), Wall Street and Silicon Valley: a delicate interaction. mimeo.

[7] Angeletos, A. Pavan (2007), Efficient use of information and social value of information. Econometrica 75: 1103-1142.

[8] Barclay M, T. Hendershott (2003), Price discovery and trading after hours. The Review of Financial Studies 16: 1041-1073.

[9] Barlevy B, P. Veronesi (2000), Information acquisition in financial markets. The Review of Economic Studies 67, 79-90.

[10] Biais F, P. Hillion (1994), Insider and liquidity trading in stock and options markets. The Review of Financial Studies 7: 743-780.

[11] Brunnermeier M (2001), Asset pricing under assymmetric information: bubbles, crashes, technical analysis, and herding. Oxford University Press.

[12] Chamley C (2003), Rational herds: economic models of social learning. Cambridge University Press.

[13] Chordia T, R. Roll, A. Subrahmanyam (2001), Market liquidity and trading activity. The Journal of Finance 56: 501-530.

[14] Crawford V, J. Sobel (1982), Strategic information transmission. Econometrica 50: 1431-1451.

[15] Dasgupta A, A. Prat (2005), Reputation and asset prices: a theory of information cascades and systematic mispricing, mimeo.

[16] Dasgupta A, A. Prat, M. Verardo (2006), The price of conformism, mimeo.

[17] Grossman S (1976), On the efficiency of competitive stock markets where trades have diverse information. The Journal of Finance 31: 573-585.

[18] Grossman S, J. Stiglitz (1980), On the impossibility of informationally efficient markets. American Economic Review 70: 393-408.

[19] Hellwig C, (2005), Heterogeneous information and the welfare effects of public information disclosures. mimeo.

[20] Kosky J, F. Michaely (2000), Prices, liquidity and the information content of trades. The Review of Financial Studies 13: 659-696.

[21] Kyle P (1985), Continuous auctions and insider trading. Econometrica 53: 1315-1335.

[22] Morris, S, H. Shin (2002), Social value of public information. The American Economic Review 92: 1521-1534.

[23] O'Hara M (2003), Presidential address: liquidity and price discovery. The Journal of Finance 58: 1335-1354.

[24] Ottaviani M, P. Sorensen (2000), Herd behavior and investment: comment. American Economic Review 90: 695-704.

[25] Ottaviani M, P. Sorensen (2001), Information aggregation in debate: who should speak first. Journal of Public Economics 81: 393-421.

[26] Prendergast C (1993), A theory of yes men. American Economic Review 83: 757-770.

[27] Subrahmanyam A (1991), Risk aversion, market liquidity and price efficiency. The Review of Financial Studies 4: 417-441.

[28] Trueman B (1994), Analyst forecasts and herding behavior. The Review of Financial Studies 7: 97-124.

[29] Van Bommel J (2003), Rumors. The Journal of Finance 58: 1499-1519.

[30] Verrecchia F (1982), Information acquisition in a noisy rational expectations economy. Econometrica 50: 1415-1430.